Subjective Well-Being, Peer Comparisons and Optimal Income Taxation

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Subjective Well-being, Peer Comparisons and Optimal Income Taxation.[*]

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Abstract

Empirical evidence suggests that an important determinant of subjective well-being is how an individual's consumption compares with that of their immediate peers. We introduce peer comparisons into the standard optimal tax framework and demonstrate that the optimal linear tax expression is adjusted in three key ways, the latter two of which are novel to this paper and act to lower the tax rate. First, the dependence of well-being on peer income introduces an externality that distorts labour supply above that which individuals would choose were they to recognise the interplay between their own choices and the Nash equilibrium level of peer consumption. The optimal tax rate is adjusted upwards to (partially) correct this distortion. Second, if individual labour supply is a function of peer consumption, there are 'Keeping up with the Joneses' multiplier effects that raise the Nash compensated labour supply elasticity above the individual labour supply elasticity. This implies a lower tax rate on efficiency grounds. Third, Nash indirect well-being is decreasing in the wage rate for workers with wages close to the reservation wage. To the extent that this lowers the covariance between gross earnings and the net social marginal value of income, this will act to lower the optimal tax rate.

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1 Introduction

It is increasingly recognised that public economic theory should incorporate important insights from the extensive literature on subjective well-being; in particular the finding that relative income considerations are a key driver of subjective well-being (Layard, 2006; O'Donnell et al., 2014). Doing so will provide richer frameworks in which to analyse important policy questions. This paper aims to respond to these recommendations and is concerned with a policy question at the heart of public economics: how do the conventional results from the optimal linear income tax framework change when individuals care about their own consumption relative to that of their peers? In particular, how does the optimal tax rate and transfer depend on the extent to which relative income concerns affect individual well-being and in turn social social welfare? We aim to answer this question in a very general framework that nests the traditional approach as a special case.

Crucially, one can treat relative income considerations solely as (i) a pure negative externality that lowers individual well-being but has no direct behavioural effect; or (ii) as a negative externality *and* an argument of labour supply, such that the level of peer consumption has a direct behavioural effect. Empirical evidence suggests the behavioural effect is important (Pérez-Asenjo, 2011). Intuitively, the separability assumptions one employs will determine which of these effects relative income considerations have. The second, more general form, has not been adequately explored in the literature.

Boskin and Sheshinski (1978) treat relative income considerations as a pure negative externality: preferences over consumption are separable from peer consumption, whilst the disutility of effort is measured in consumption units. Accordingly, individual choices are *independent* of the level of peer consumption. The pure negative externality is corrected for by a having a higher tax rate than would otherwise have been the case. Their model thus ignores an important facet of having individuals care about relative income - namely that it induces *Keeping up with the Joneses* (henceforth *KUJ*) behaviour that causes individuals to overwork. More recently, Layard (2006) employs more general preferences where relative income considerations do cause individuals to overwork, and where this can be corrected for by a (Pigovian) tax on income. However, the author assumes that all individuals are identical and so abstracts from redistributive considerations. The sole purpose for taxation is to correct the labour distortion.

In this paper we develop a highly general framework for examining the policy implications of relative income concerns, in which (i) peer consumption can have both a direct negative effect on welfare *and* induce a *KUJ* labour supply effect. We show that there are then a

number of additional effects at work that have not been fully recognised in discussions of optimal tax policy, and that these point towards lower tax rates than suggested thus far in the literature on subjective well-being.

In introducing relative income concerns into a model where individuals differ in their ability/productivity, the first issue to address is to who individuals compare themselves with. There is now considerable empirical evidence that individuals compare themselves with those to whom they are very similar, e.g. work colleagues (see Clark and Senik, 2010; Knight et al., 2009; Senik, 2009). Accordingly, we assume throughout that individuals compare themselves to others with the same level of productivity (gross wage rate). We also make the standard assumption that, in making their labour supply decisions, individuals take *as given* the average income of their peers. Taken together, these two assumptions mean that *for every* wage rate we have to determine the Nash equilibrium levels of labour supply and consumption. We in turn establish four key properties of these equilibrium levels that are very different from the conventional properties of labour supply.

(i) *Inefficiency of equilibrium labour supply.* For every wage rate the Nash equilibrium level of labour supply is inefficient because individuals treat the average income of their peers as fixed and so fail to recognise the externality that their behaviour collectively imposes on their peers. This just generalises the conclusion of Layard (2006) to the case where individuals differ in their productivity.

(ii) *Well-being decreases with the wage rate for those of low productivity.* For wages above but close to the reservation wage, well-being is a decreasing function of the wage rate. This arises because an increase in the wage rate has two effects: (a) by Roy's identity it increases income and hence well-being at a rate proportional to hours of work; but (b) it also increases the average consumption of the peer group and so exacerbates the negative distortionary impact of peer consumption on well-being. This first effect is close to zero when the wage rate is close to the reservation wage because labour supply is close to zero. This implies that the individuals in society who are worst-off are no longer those with the lowest productivity, nor are they the unemployed. This generalises the conclusions in Ulph (2014) to a very general utility function.

(iii) *Marginal indirect utility of income and the wage rate.* The marginal utility of income may not decrease with the wage rate: so both the absolute level of well-being and the marginal impact of income on well-being may move in different directions from the standard theory.

(iv) *Multiplier Effects arising from the KUJ phenomenon.* When preferences over individual

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consumption and leisure are non-separable from peer income - such that KUJ effects are at work on labour supply - there are important multiplier effects that drive the compensated Nash labour supply elasticity above the individual compensated labour supply elasticity[1].

We show that the traditional formula for the optimal income tax rate - which captures the equity-efficiency tradeoff inherent in income taxation[2] - gets modified in three key ways:

(a) *Distortion-Correcting Role.* In relation to (i) above there is a new positive term capturing the aggregate value of the distortion to labour supply relative to the net social marginal value of income (smvi). This indicates the extent to which the optimal tax should be increased to correct the externalities induced by relative income considerations.

(b) *Muted equity considerations.* Due to (ii) above the net smvi may fall less rapidly with rapidly with productivity. To the extent that this lowers the covariance between gross earnings and the net smvi, this may lessen equity considerations and act to lower the optimal tax rate. This will depend on the distributional weights employed in the social welfare function: in particular whether or not weight is given to inequality in utility *levels*. In addition, (iii) above may affect how the net smvi changes with productivity, in turn adjusting equity considerations.

(c) *Heightened Efficiency considerations.* Due to the multiplier effects in (iv) above the distortionary costs of income taxation are increased, thus acting to lower the tax rate.[3]

Whilst the first effect implies that the optimal tax rate should be higher than the traditional policy considerations would suggest (see Boskin and Sheshinski, 1978; Layard, 2006), the second and third effects suggests it should be lower.

To assess the balance of these considerations we adopt a more explicit functional form whereby:

I. The utility function is separable in (i) a tradition subutility function of consumption

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[1]Anything that causes an individual with a given wage rate to work harder causes all such individuals to work harder. This increases peer consumption, which in turn increases labour supply and so on.

[2]Where equity considerations are captured through the negative of the covariance between gross earnings and the net social marginal value of income; whilst efficiency considerations are captured through a weighted average of the compensated labour supply elasticity.

[3]A somewhat related paper to ours is the unpublished paper Beath and FitzRoy (2011). They too examine the optimal income tax rate when individuals compare their income to that of those with the same productivity, and consequently consider Nash equilibria. However, their paper differs from ours in a number of crucial respects: (i) they use a particular functional form for the utility function - e.g. quasi-linear in leisure; (ii) despite the fact that, in their model, all unemployment is voluntary they assume that the way the unemployed compare their income to that of their peers is different from the way the unemployed do so; (iii) they have no universal benefit.

and leisure which we term the *well-offness* function; and (ii) a function of relative consumption which we term subjective well-being. By varying the relative weight θ that is given to subjective well-being we can nest the traditional policy framework - which is concerned solely with well-offness - as a special case ($\theta = 0$) and see how the optimal tax rate varies with θ.

II. Moreover, through a separate parameter, χ, we treat parametrically the extent to which peer consumption - which enters solely through the subjective well-being function - exercises a pure negative effect on utility ($\chi = 0$) or also exercises a *KUJ* behavioural effect on labour supply ($\chi > 0$).

A notable property of this functional form is that because an individual's own consumption enters *both* the well-offness and subjective well-being functions, the overall weight given to consumption in utility can potentially vary as we change the relative weight, θ, given to subjective well-being. To account for this, we also numerically simulate results for an alternative subjective well-being function that is decreasing in peer consumption and independent of own consumption.

Overall, we find that (i) the optimal tax rate increases in θ, where we note that $\theta = 0$ yields the well-documented results from Stern (1976); but (ii) decreases with χ for any $\theta \in (0, 1]$. The intuition for the first observation is that an increase in θ increases the weight individuals place on subjective well-being and, in turn the size of the negative externality and labour distortion. As indicated by our optimal tax expression, the larger the distortion the larger the corrective role for taxation. The intuition for the second observation also follows directly from our optimal tax expression. An increase in χ increases the extent to which labour supply depends on peer consumption, in turn increasing the size of the *KUJ* multiplier effects at the optimum. These multiplier effects thus increase the responsiveness of individual gross earnings to taxation, in turn reducing the role of the income tax due to heightened efficiency considerations. In addition, we find that the *absolute* covariance between gross earnings and the net smvi is decreasing in the weight individuals place on relative consumption whenever the social welfare function exhibits concern for inequality in utility *levels*. This suggests that equity considerations are indeed muted relative to the standard framework.

The remainder of this paper is structured as follows: Section 2 sets out the general framework, determines the key properties of the Nash equilibrium and derives the optimal linear income tax; Section 3 numerically simulates the optimal income tax for a specific functional form that embeds the traditional model as a special case; and finally Section 4 concludes the paper.

2 General Framework

2.1 The Model and Individual Behaviour

Suppose that we have an economy in which:

(i) There is a linear income tax system in place under which all income is taxed at the rate $t \in (0,1)$ and all individuals receive a tax-free universal benefit, $\sigma > 0$.

(ii) Individuals have identical preferences represented by $u(c,l,\bar{c})$, which depend on consumption, $c \geq 0$; leisure, $l \in [0,1]$; and the *average* consumption of the individual's peers, $\bar{c} \geq 0$. We assume that u is strictly decreasing in $\bar{c}$, but increasing and concave in c and l for all values of $\bar{c}$. Formally, where subscripts denote partial derivatives these assumptions correspond to:[4]

$$u_{\bar{c}} < 0 \; ; \text{whilst} \; \forall \, \bar{c} \; : u_c > 0 \; , \; u_l > 0 \; , \; u_{cc} < 0 \; , \; u_{ll} < 0 \; , \; u_{cc} u_{ll} - u_{cl}^2 > 0$$

Further, both c and l are normal goods, i.e. $u_c u_{ll} - u_l u_{cl} < 0$ and $u_l u_{cc} - u_c u_{cl} < 0$.

(iii) All individuals are able to work but differ in their productive abilities, as reflected in their gross wage rate, $n \geq 0$; and hence in their net wage rate, $\omega = n(1-t)$.

In this subsection and the next we treat the parameters (t,σ) as fixed and so conduct the analysis in terms of the net wage rate, ω.

Individual Labour Supply. In making their labour supply decisions, individuals operate in the standard Nash fashion: they choose their labour supply taking *as given* the average consumption level of their peers, $\bar{c}$. Individual labour supply therefore satisfies

$$h^*(\omega, \sigma, \bar{c}) = \arg \max_{h \in [0,1]} u(\sigma + \omega h, 1 - h, \bar{c}) \tag{1}$$

and is characterised by:

$$\omega \leq \frac{u_l \left(\sigma + \omega h^*, 1 - h^*, \bar{c} \right)}{u_c \left(\sigma + \omega h^*, 1 - h^*, \bar{c} \right)} \; , \; h^* \geq 0 \tag{2}$$

where the pair of inequalities hold with complementary slackness.

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[4]Throughout this paper subscripts denote partial derivatives.

One can readily establish from (2) that the reservation wage is given by

$$\tilde{\omega}(\sigma, \bar{c}) = \frac{u_l(\sigma, 1, \bar{c})}{u_c(\sigma, 1, \bar{c})} \tag{3}$$

where $h^* = 0 \ \forall \ \omega \leq \tilde{\omega}$, but $h^* > 0$ otherwise.

The consumption and indirect utility functions associated with the labour supply function characterised in (2) are, respectively:

$$c^*(\omega, \sigma, \bar{c}) = \sigma + \omega h^*(\omega, \sigma, \bar{c}) \geq \sigma \tag{4}$$

$$v(\omega, \sigma, \bar{c}) = u\left[\sigma + \omega h^*(\omega, \sigma, \bar{c}), 1 - h^*(\omega, \sigma, \bar{c}), \bar{c}\right] > 0 \tag{5}$$

By the Envelope Theorem, the indirect utility function will have the following properties:

$$v_\sigma(\omega, \sigma, \bar{c}) = u_c\left[\sigma + \omega h^*(\omega, \sigma, \bar{c}), 1 - h^*(\omega, \sigma, \bar{c}), \bar{c}\right] > 0$$

$$v_\omega(\omega, \sigma, \bar{c}) = v_\sigma(\omega, \sigma, \bar{c}) \cdot h^*(\omega, \sigma, \bar{c}) \geq 0 \tag{6}$$

$$v_{\bar{c}}(\omega, \sigma, \bar{c}) = u_{\bar{c}}\left[\sigma + \omega h^*(\omega, \sigma, \bar{c}), 1 - h^*(\omega, \sigma, \bar{c}), \bar{c}\right] < 0$$

where the second expression is Roy's identity.

By the definition of the reservation wage it follows from the standard theory that:

(i) $\forall \ \omega \leq \tilde{\omega}(\sigma, \bar{c})$: $\quad h^*(\omega, \sigma, \bar{c}) = 0$; $\quad c^*(\omega, \sigma, \bar{c}) \equiv \sigma$; $\quad v(\omega, \sigma, \bar{c}) \equiv u(\sigma, 1, \bar{c})$

(ii) $\forall \ \omega > \tilde{\omega}(\sigma, \bar{c})$: $\quad h^*(\omega, \sigma, \bar{c}) > 0 \Rightarrow v_\omega(\omega, \sigma, \bar{c}) > 0$; $\quad h^*_\sigma(\omega, \sigma, \bar{c}) < 0$;

$$c^*_\omega(\omega, \sigma, \bar{c}) > 0 \ ; \ c^*_\sigma(\omega, \sigma, \bar{c}) > 0$$

Given our assumption that u is strictly concave in c and l for all values of $\bar{c}$, one can readily show that $v_{\sigma\sigma} < 0$. Further, given that leisure is a normal good, it follows immediately from Roy's identity in (6) that:

$$v_{\omega\sigma} = v_{\sigma\sigma}h^* + v_\sigma h^*_\sigma \leq 0 \tag{7}$$

The inequality is strict for wage rates above the reservation wage (i.e. $\omega > \tilde{\omega}$), such that the marginal utility of income is decreasing in the net wage rate for workers.

Finally, in this subsection we specify how behaviour is affected by peer consumption, $\bar{c}$, and examine two alternative approaches.

1. **Pure Negative Externality.** If, similarly to Boskin and Sheshinski (1978), individual preferences are separable in (c, l) and $\bar{c}$, the functions $h^*(\omega, \sigma, \bar{c})$, $c^*(\omega, \sigma, \bar{c})$ and $\tilde{\omega}(\sigma, \bar{c})$ will no longer depend on $\bar{c}$ and can instead be simply written as $h^*(\omega, \sigma)$, $c^*(\omega, \sigma)$ and $\tilde{\omega}(\sigma)$, respectively.[5] It does, of course, continue to hold that $v_{\bar{c}}(\omega, \sigma, \bar{c}) < 0$. In this case we say that peer consumption exerts a *pure negative externality.*

2. **Keeping Up with the Joneses.** Consistent with the idea of *Keeping Up with the Joneses (KUJ)*, it is perhaps more natural to assume that peer consumption also exerts a *behavioural externality* that induces individuals to work harder than they otherwise would. More specifically, when what we call the *KUJ* effect is in operation, an increase in peer consumption will (i) cause the reservation wage to fall, thus inducing some individuals who would have not otherwise worked to start working; whilst (ii) cause those who already do work to work more intensively. Formally:

$$\tilde{\omega}_{\bar{c}}(\sigma, \bar{c}) < 0 \ \text{ and } \forall \, \omega > \tilde{\omega}(\sigma, \bar{c}) \ : \ h_{\bar{c}}(\omega, \sigma, \bar{c}) > 0 \Rightarrow c_{\bar{c}}(\omega, \sigma, \bar{c}) > 0. \qquad (8)$$

However, by the Envelope Theorem the effect of a marginal increase in peer consumption on welfare is still as specified in the last line of (6).

Note that in both cases, individual labour supply will differ from what the individual would choose were they to recognise the interplay between their own choices and the level of peer consumption. We return to this point in Section 2.2.1.

2.2 Nash Equilibrium

In this paper we assume that the relevant peers to whom individuals compare themselves are those with the same productivity - and hence the same wage rate. Since individuals are assumed to have identical preferences, the level of consumption that is commonly chosen in response to a given peer level of consumption will be, in equilibrium, the peer level of consumption. Therefore, for each net wage, ω, can can define the Nash equilibrium level of consumption, $c^N(\omega, \sigma)$, by the condition that:

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[5]It follows from (2) that the pure negative externality case will arise whenever u_l/u_c is independent of $\bar{c}$. This will be the case if (i) $u_{c\bar{c}} = 0$ and $u_{l\bar{c}} = 0$; or (ii) $u_l = g^l(c, l) \cdot k(\bar{c})$ and $u_c = g^c(c, l) \cdot k(\bar{c})$, where g^i and k are functions.

Figure 1: Nash Equilibrium Consumption.
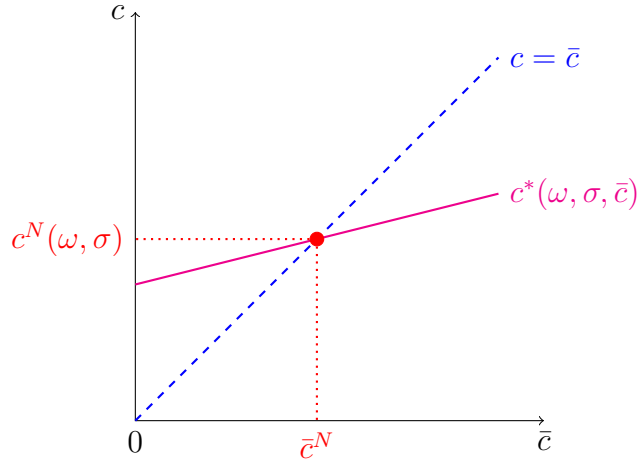
Notes. This figure graphically illustrates the purpose of assuming $c_{\bar{c}}^*[\omega, \sigma, c^N(\omega, \sigma)] < 1$.

$$c^N(\omega, \sigma) \equiv c^* \left[\omega, \sigma, c^N(\omega, \sigma)\right] \tag{9}$$

In all that follows we assume that for all $(\omega, \sigma) \geq 0$ there is a unique solution to (9). It is easy to see that this implies:

$$c_{\bar{c}}^* \left[\omega, \sigma, c^N(\omega, \sigma)\right] < 1 \tag{10}$$

Figure 1 graphically illustrates this assumption. When peer consumption exerts a *pure negative externality* then $c_{\bar{c}}^* \equiv 0$, so (10) is automatically satisfied and (9) just reduces to the condition $c^N(\omega, \sigma) \equiv c^*(\omega, \sigma)$. However, when the *KUJ* effect is in operation then $c_{\bar{c}}^* > 0$ and so the requirement in (10) has more force. Moreover, if we differentiate (9) totally w.r.t. ω and σ we obtain:

$$c_x^N(\omega, \sigma) = \frac{c_x^* \left(\omega, \sigma, c^N(\omega, \sigma)\right)}{(1 - c_{\bar{c}}^*)} > 0 \ , \ x = \omega, \sigma \tag{11}$$

So when the *KUJ* effect is in operation, there are *multiplier* effects at work on consumption and consequently anything that causes a change in individual consumption will have a magnified impact on Nash consumption.[6]

The Nash consumption level is of course determined by the Nash labour supply function,

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[6]Notice that $c_\omega^* = \omega + \omega h_\omega^* > 0$ is guaranteed by our assumption that $u_l u_{cl} - u_c u_{ll} > 0$.

which we define by:

$$h^N(\omega, \sigma) = h^* \left[\omega, \sigma, c^N(\omega, \sigma)\right] \tag{12}$$

From (2) and (3), the Nash labour supply function is characterised by the first-order-condition:

$$\omega \leq \frac{u_l\left(\sigma + \omega h^N, 1 - h^N, \sigma + \omega h^N\right)}{u_c\left(\sigma + \omega h^N, 1 - h^N, \sigma + \omega h^N\right)} \ , \ h^N \geq 0 \tag{13}$$

where the pair of inequalities hold with complementary slackness. The associated Nash reservation wage is:

$$\tilde{\omega}^N(\sigma) \equiv \frac{u_l(\sigma, 1, \sigma)}{u_c(\sigma, 1, \sigma)}$$

Once more, if peer consumption exerts a *pure negative externality* then $h^N(\omega, \sigma) \equiv h^*(\omega, \sigma)$ and $\tilde{\omega}^N(\sigma) \equiv \tilde{\omega}(\sigma)$.

Finally, the level of individual well-being in the Nash equilibrium is given by the Nash indirect utility function:

$$v^N(\omega, \sigma) = v\left[\omega, \sigma, c^N(\omega, \sigma)\right] \tag{14}$$

Before proceeding, notice that if $\omega \leq \tilde{\omega}(\sigma)$ then $v^N(\omega, \sigma) \equiv u(\sigma, 1, \sigma) \Rightarrow v^N_\sigma = u_c + u_{\bar{c}}$; but if $\omega > \tilde{\omega}^N$ then - by differentiating (14) totally w.r.t. σ - we have $v^N_\sigma = u_c + u_{\bar{c}}c^N_\sigma$. At this level of generality, in neither case is it obvious that the marginal (Nash) utility of income is positive. While there may be interesting policy implications that arise through pursuing further the possibility of a non-positive marginal utility of income, given the other issues we wish to pursue we simply assume in all that follows that:

$$v^N_\sigma(\omega, \sigma) > 0 \ \ \forall \ \omega, \sigma \geq 0 \tag{15}$$

We now establish four key properties of the Nash-equilibrium which, as we will subsequently show in subsection 1.3, have important implications for the optimal income tax rate.

2.2.1 Inefficiency of Nash Equilibrium

Given the symmetric Nash equilibrium in which individuals of the same productivity have the same consumption, it is immediately clear that the individual endeavour to raise their consumption relative to that of their peers is self-defeating. A social planner would recognise this externality and, taking as given the linear tax system in place, labour supply would be chosen to maximise $u(\sigma + \omega h, 1 - h, \sigma + \omega h)$. Formally, let

$$h^S(\omega, \sigma) \equiv \arg \max_{h \in [0,1]} u(\omega h, 1 - h, \sigma + \omega h)$$

be the level of labour supply that would result were individuals to recognise that peer consumption will coincide with own consumption in equilibrium. Formally, this is characterised by:

$$\omega \left[1 + \frac{u_{\bar{c}}\left(\sigma + \omega h^S, 1 - h^S, \sigma + \omega h^S\right)}{u_c\left(\sigma + \omega h^S, 1 - h^S, \sigma + \omega h^S\right)} \right] \leq \frac{u_l\left(\sigma + \omega h^S, 1 - h^S, \sigma + \omega h^S\right)}{u_c\left(\sigma + \omega h^S, 1 - h^S, \sigma + \omega h^S\right)} \quad h^S \geq 0 \quad (16)$$

A comparison of (13) and (16) confirms that the Nash equilibrium labour supply is suboptimal - i.e. $h^N(\omega, \sigma) \neq h^S(\omega, \sigma)$. Intuitively, because individuals treat peer consumption as fixed, their labour supply decision is distorted away from that which they would choose upon recognising that in equilibrium $c = \bar{c}$. It is also straightforward to show that, as with all externalities, the distortion can in principle be corrected by imposing a Pigovian tax on *net* income at the rate

$$\hat{\tau}^S = -\frac{u_{\bar{c}}\left(\sigma + \omega h^S, 1 - h^S, \sigma + \omega h^S\right)}{u_c\left(\sigma + \omega h^S, 1 - h^S, \sigma + \omega h^S\right)} > 0 \quad (17)$$

and using the resulting tax revenue raised (on individuals with wage rate ω) to increase the lump-sum benefit they receive to $\sigma^S = \sigma + \hat{\tau}^S \omega h^S$.[7] So we have:

Result 1. *For every net wage rate, ω, uncorrected Nash equilibrium labour supply is suboptimal, i.e. $h^N(\omega, \sigma) \equiv h^S(\omega, \sigma)$. The distortion causing this can be corrected by a Pigovian tax on net income at the rate $\tau^S > 0$ as specified in (17), with a corresponding adjustment*

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[7]This adjustment to the lump-sum benefit is necessary to ensure that consumption in the social optimum is the same as in the Nash Equilibrium once the Pigovian tax has been imposed. This is not discussed in Layard (2006).

to the lump-sum benefit. Formally, $h^S[\omega, \sigma] = h^N\left[\omega(1 - \tau^S), \sigma^S\right]$

This result generalises the argument of Layard (2006) to an economy where individuals differ in their productivity, and where there may be a linear income tax schedule in force for revenue-raising/redistributional purposes. Notice that:

(i) The corrective Pigovian tax is applied to net income and so is imposed on top of any other taxes on income that are in place for other reasons.

(ii) Both the Pigovian tax rate and the adjustment to unearned income can in principle vary with ω.

Notice also that the distortion responsible for the sub-optimality arises because of the negative effect of peer consumption on utility and *does not* rely on any *KUJ* effects of peers consumption on behaviour. Indeed, it will arise whenever $u_{\bar{c}} < 0$.

In what follows let

$$\delta^N(\omega, \sigma) = -\frac{u_{\bar{c}}\left(\sigma + \omega h^N, 1 - h^N, \sigma + \omega h^N\right)}{u_c\left(\sigma + \omega h^N, 1 - h^N, \sigma + \omega h^N\right)} > 0 \tag{18}$$

be the uncorrected labour supply distortion in the Nash equilibrium.

### 2.2.2 Well-Being Decreases in the Net Wage for Low Wage Workers

Notice that if $\omega \leq \tilde{\omega}^N(\sigma)$ then $h^N \equiv 0 \Rightarrow c^N \equiv \sigma \Rightarrow v^N \equiv u(\sigma, 1, \sigma)$. So, unsurprisingly, well-being is independent of the net wage rate for non-workers. Now consider how well-being changes with the net wage for those with $\omega > \tilde{\omega}^N$: differentiating (14) w.r.t. ω and making use of (6),(11) and (17) yields

$$v_\omega^N = v_\sigma + v_{\bar{c}} \cdot c_\omega^N = v_\sigma\left[h^N - \delta^N(\omega, \sigma) \cdot c_\omega^N\right] \tag{19}$$

This shows that an increase in the net wage rate has two effects on the Nash level of individual well-being:

(i) It makes individuals better off by increasing the income available for spending on consumption or leisure, with the rate of increase in income being proportional to hours worked (Roy's identity);

Figure 2: Nash Indirect Wellbeing over the net wage continuum.
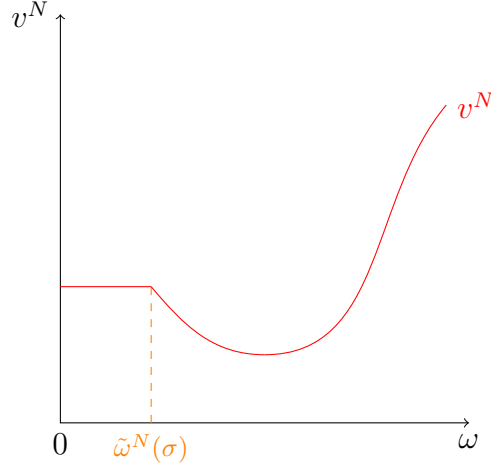
Notes. This figure graphically illustrates Result 2. Nash Indirect well-being is falling in ability for workers with $\omega \approx \tilde{\omega}^N(\sigma)$ and thus $h^N \approx 0$.

(ii) However, it makes individuals worse off by increasing peer income an so intensifying the distortionary effect on labour supply to which concerns over peer income give rise.

But now we get the following:

Result 2. *If $\omega > \tilde{\omega}^N$ and $\omega \approx \tilde{\omega}^N(\sigma)$ then $\dfrac{\partial v^N}{\partial \sigma} < 0$.*

Proof: $\omega \approx \tilde{\omega}^N(\sigma) \Rightarrow h^N(\omega, \sigma) \approx 0 \Rightarrow v_\omega^N \approx -v_\sigma \cdot \delta^N(\omega, \sigma) \cdot c_\omega^N < 0$, where the final inequality follows from (6),(11) and (17). ∎

This generalises the result in Ulph (2014), which was proved for a specific functional form similar to that we will employ below in Section 3. The immediate implication of Result 2 is that, contrary to the traditional theory, the worst off in society are no longer those of lowest productivity or the unemployed.

To facilitate our derivation of the optimal tax rate in Section 3, we totally differentiate (14) w.r.t. σ to obtain:

$$v_\sigma^N = v_\sigma \left(1 - \delta^N \cdot c_\sigma^N\right) \tag{20}$$

We can then combine (19) and (20) to obtain what we term the *distortion-adjusted Roy's identity*:

$$v_\omega^N = v_\sigma^N \left[h^N - \left(\frac{\delta^N \cdot c_\omega^{cN}}{1 - \delta^N \cdot c_\sigma^N}\right)\right] \tag{21}$$

where c_ω^{cN} is the compensated effect of an increase in the net wage on Nash consumption.[8] Notice that the second term within square brackets is a monotonically increasing function of the distortion to labour supply decisions, δ^N.

2.2.3 Multiplier Effects on Compensated Labour Supply

As noted, when peer consumption exerts a *pure negative externality* then $h^N(\omega, \sigma) \equiv h^*(\omega, \sigma)$, so Nash labour supply behaves in exactly the same way as individual labour supply. So consider what happens when the peer consumption exerts a behavioural *KUJ* effect. In this case we re-write (12) as:

$$h^N(\omega, \sigma) = h^*[\omega, \sigma, \sigma + \omega h^N(\omega, \sigma)] \tag{22}$$

Differentiating (22) totally w.r.t. ω and σ then gives:

$$h_\omega^N = h_\omega + h_{\bar{c}} \left[h^N + \omega h_\omega^N \right] \tag{23}$$

$$h_\sigma^N = h_\sigma + h_{\bar{c}} \left[1 + \omega h_\sigma^N \right] \tag{24}$$

Multiplying (24) by h^N and subtracting the result from (23) then yields:

$$h_\omega^{cN} = \left(h_\omega^N - h^N \cdot h_\sigma^N \right) = \frac{\left(h_\omega^* - h^N \cdot h_\sigma^N \right)}{(1 - c_{\bar{c}}^*)} = \frac{h_\omega^c}{\underbrace{(1 - c_{\bar{c}}^*)}_{\text{Multiplier Effect}}} \tag{25}$$

where we have made use of the fact that $c_{\bar{c}}^* = \omega h_{\bar{c}}^*$. When the *KUJ* effect is in operation we know that at the Nash consumption level $0 < c_{\bar{c}}^* < 1$. We have thus proved:

Result 3. *When peer consumption exerts a behavioural KUJ effect on individual behaviour, the effect of an increase in the net wage on compensated Nash labour supply is a multiple > 1 of its effect on compensated individual labour supply.*

For later purposes it will be helpful to express this relationship in elasticity form, so let:

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[8]One can readily establish from (19) and (20) that:

$$\frac{v_\omega^N}{v_\sigma^N} = \frac{h^N - \delta^N c_\omega^N}{1 - \delta^N c_\sigma^N} = h^N - \left[h^N - \frac{h^N - \delta^N c_\omega^N}{1 - \delta^N c_\sigma^N} \right] = h^N - \left(\frac{\delta^N (c_\omega^N - h^N c_\sigma^N)}{1 - \delta^N c_\sigma^N} \right)$$

$$\eta^{cN} = \frac{\omega h_\omega^{cN}}{h^N} \;\; ; \;\; \eta^c = \frac{\omega h_\omega^c}{h^N}$$

be, respectively, the Nash compensated individual labour supply elasticity and the compensated individual labour supply elasticity, both calculated at the Nash level of labour supply. Then it readily follows that (25) becomes:

$$\eta_\omega^{cN} = \frac{\eta_\omega^c}{(1 - c_{\bar{c}}^*)} \tag{26}$$

The Nash compensated labour supply elasticity is thus a multiple (greater than 1) of the compensated labour supply elasticity. One can anticipate how this multiplier effect will heighten the efficiency considerations in the optimal tax expression presented below in section 2.3.

2.2.4 Marginal utility of income not necessarily decreasing in the net wage.

As established, both well-being and the marginal indirect utility of income are independent of the wage rate for those with $\omega \leq \tilde{\omega}^N(\sigma)$. So let us consider how the marginal utility of income changes with the wage rate for those with $\omega > \tilde{\omega}^N(\sigma)$, and thus those who work. Since $v_{\omega\sigma}^N = v_{\sigma\omega}^N$, we differentiate (19) w.r.t. σ to obtain:

$$v_{\omega\sigma}^N = \underbrace{v_{\omega\sigma}}_{<0} + v_{\omega\bar{c}} c_\omega^N + v_{\bar{c}} c_{\omega\sigma}^N \tag{27}$$

As shown in (7), the first term on the right side of (27) is negative - i.e. the conventional result. However, the signs of the second and third terms are ambiguous, thus rendering the overall sign of (27) unclear.

We now state the following result:

Result 4.

(i) *If $v_{\omega\bar{c}} c_\omega^N + v_{\bar{c}} c_{\omega\sigma}^N > -v_{\omega\sigma}$ then $v_{\omega\sigma}^N > 0$;*

(ii) *A sufficient condition for $v_{\omega\sigma}^N > 0$ is:*

$$\frac{\bar{c} v_{\sigma\bar{c}}}{v_\sigma} > -\frac{\bar{c} h_c^*}{h^*} \quad \text{and} \quad \frac{\omega h_{\omega\sigma}^N}{h_\sigma^N} > -1$$

Having understood the properties of the Nash equilibria at every wage rate we can now formulate the optimal tax problem and characterise its solution.

2.3 Optimal Income Tax

In this section the tax parameters (t, σ) are choice variables and so we write everything in terms of individual productivity, $n \geq 0$. We assume that the distribution of productivity in the population is given by $F(n)$; where $F(0) = 0$, $F'(n) > 0 \ \forall \ n \geq 0$ and $\int_0^\infty dF(n) = 1$.

The optimal income tax problem is:

$$\max_{t, \sigma} \int_0^\infty \Psi \left\{ v^N \left[n(1 - t), \sigma \right] \right\} dF(n)$$

$$\text{s.t.} \quad \sigma \leq t \int_0^\infty n h^N \left[n(1 - t), \sigma \right] dF(n) \ , \quad t \in [0, 1] \ , \ \sigma \geq 0 \tag{28}$$

where $\Psi(\cdot)$ is a concave transformation of individual well-being that captures society's views about inequality of well-being, i.e. $\Psi' > 0$ and $\Psi'' < 0$. For simplicity we assume taxation to be purely redistributive and as such there is no revenue requirement.

We denote the solutions to (28) by $\hat{t}$ and $\hat{\sigma}$, and also let $\hat{\lambda}$ be the associated Lagrange multiplier attached to the government budget constraint. This latter term will have the standard interpretation as the social marginal value of public funds.

Assuming an interior solution (i.e. $\hat{t} > 0, \hat{\sigma} > 0$), the first order conditions w.r.t. t and σ are, respectively:

$$(t): \ \ \hat{\lambda}^N \int_0^\infty n h^N dF(n) = \int_0^\infty \left\{ \Psi' \left(v^N \right) n v_\omega^N + \hat{\lambda} t n^2 h_\omega^N \right\} dF(n) \tag{29}$$

$$(\sigma): \ \ \int_0^\infty \left[\Psi' \left(v^N \right) v_\sigma^N + \hat{\lambda} \hat{t} n h_\sigma^N \right] dF(n) = \hat{\lambda} \tag{30}$$

Analogous to other authors - Atkinson and Stiglitz (1980); Slack (2015); Viard (2001) - let

$$\phi(n) = \Psi' \left\{ v^N \left[n(1 - \hat{t}), \hat{\sigma} \right] \right\} \cdot v_\sigma^N \left[n(1 - \hat{t}), \hat{\sigma} \right] + \hat{\lambda} \hat{t} n h_\sigma^N \left[n(1 - \hat{t}), \hat{\sigma} \right] \tag{31}$$

denote the net social marginal value of income (smvi) of a productivity n individual, here evaluated at the *optimum* choices $\hat{t}$ and $\hat{\sigma}$. It immediately follows that (30) reduces to:

$$\hat{\lambda} = \int_0^\infty \phi(n)dF(n) \tag{32}$$

This simply states that the social marginal value of public funds equates with the average net smvi at the optimum choices (see, for example, Atkinson and Stiglitz, 1980).

To save on notation in what follows, we let

$$z(n) = nh^N[n(1-\hat{t}), \hat{\sigma}] \ , \ \ \bar{z} = \int_0^\infty z(n)dF(n) \ , \ \ r_z(n) = \frac{z(n)}{\bar{z}} \ , \ \ r_\phi(n) = \frac{\phi(n)}{\hat{\lambda}} \tag{33}$$

denote the gross earned income (evaluated at the optimal tax and benefit); average gross earned income; relative gross earned income and relative net smvi of a productivity n individual.

Further, we also let

$$\Delta = \frac{\int_0^\infty \left[\Psi'(v^N)v_\sigma^N\right] n \left(\dfrac{\delta^N \cdot c_\omega^{cN}}{1 - \delta^N \cdot c_\sigma^N}\right)}{\hat{\lambda}\bar{z}} \tag{34}$$

measure the average smvi of the uncorrected distortions to individual labour supply, evaluated relative to aggregate gross earnings valued at the shadow price of public funds. From (21), the term $\delta^N c_\omega^{cN} \left(1 - \delta^N c_\sigma^N\right)^{-1}$ is the distortion-adjustment that is applied to the conventional Roy's identity when determining how an increase in the net wage impacts the well-being of a productivity n individual.[9] Multiplying this term by n therefore gives the foregone income cost of the distortion; and the marginal welfare cost of each unit of income lost is $\Psi'(v^N)v_\sigma^N$.

Putting this all together - i.e. substituting (21),(25),(32),(33) and (34) into (29) - yields the below result.

Result 5. *The optimal tax rate is implicitly characterised by:*

$$\frac{\hat{t}}{1-\hat{t}} = \frac{\Delta - Cov\left[r_z(n), r_\phi(n)\right]}{\int_0^\infty r_z(n) \cdot \dfrac{\eta_\omega^c}{[1 - c_{\bar{c}}^*]}dF(n)} \tag{35}$$

Proof: See derivation in Appendix.

Expression (35) makes clear that, in contrast to the standard optimal tax expression, there

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[9]Like ϕ, this is evaluated at the optimal choices $\hat{t}$ and $\hat{\sigma}$.

are now *three* considerations in setting the optimal tax rate: (i) correcting labour distortions, (ii) the deadweight loss of taxation (efficiency considerations) and (iii) the negative covariance between relative gross earnings and the relative net smvi (equity considerations). The distortion correcting role is not found in the traditional optimal tax expression (see Atkinson and Stiglitz, 1980), whilst the latter two considerations will differ from the standard analysis. We discuss each in turn:

(i) *Correcting Distortions.* The first term in the numerator of (35) is positive and measures the average social value of the distortion to labour supply induced by the presence of peer concerns, as a fraction of the average earned income evaluated at the shadow price of public funds. This tells us that, in absence of the corrective Pigovian taxes at every wage rate, the income tax rate has to be higher to correct the distortions on average. This is analogous to the standard argument for higher tax rates proposed in the literature (see Boskin and Sheshinski, 1978; Layard, 2006).

(ii) *Greater Deadweight Loss.* The denominator of (35) captures the fact that - as derived in Result 3 - the compensated elasticity of Nash labour supply is a multiple (greater than 1) of the compensated elasticity of individual labour supply. Consequently, a given rate of income tax is more distortionary than in the standard framework without peer comparisons, thus suggesting on efficiency grounds that the optimal tax rate should be lower.

(iii) *Muted Inequality Concerns.* As is conventionally the case, the equity concerns are captured in the numerator by negative covariance between (relative) gross earnings and the (relative) net smvi. Note, however, that:

$$\phi'(n) = (1 - \hat{t})\left\{\Psi'' v_\omega^N v_\sigma^N + \Psi' v_{\omega\sigma}^N + \hat{\lambda}\hat{t}nh_{\omega\sigma}^N\right\} + \hat{\lambda}\hat{t}h_\sigma^N$$

We know from Result 2 that $v_\omega^N < 0$ for wages close to the reservation wage and so the first term in within curly brackets will be positive for low productivities, thus departing from the conventional framework.[10] Further, Result 4 indicates that $v_{\omega\sigma}^N$ is no longer unambiguously negative. Both of these factors suggest that the net smvi may decline less rapidly with productivity than is conventionally the case. This in turn suggests that the covariance term in the numerator of (35) may be smaller in absolute magnitude than in the conventional analysis of optimal taxation, leading *ceteris paribus* to a lower tax rate.

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[10]In the strict utilitarian case we have $\Psi'' = 0$ and so the first term will be zero because there is no concern for inequality in utility *levels*.

In summary, the distortion correcting role implies a higher tax rate than in the conventional analysis; the multiplier effects attached to KUJ effects imply a lower marginal tax rate; and finally the observation that the net smvi *may* decline less rapidly with productivity may also act to lower the tax rate.

To assess the net effect of these considerations and understand more fully how the optimal tax rate responds to an increase in the weight individuals place on relative income considerations, we proceed in the next section to adopt a more explicit functional form and undertake numerical simulations.

3 Specific Functional Forms

In this section we adopt a specific functional form which embeds the traditional framework as a special case where no weight is placed on relative consumption. Formally, we make the separability assumption:[11]

$$u(c, l, \bar{c}) = [y(c, l)]^{1-\theta} \left[s \left(\frac{c}{\bar{c}} \right) \right]^{\theta} \quad , \ 0 \leq \theta \leq 1 \tag{36}$$

where:

- The function $y(c, l)$ is a conventional utility function that depends on individual consumption and leisure. It captures how 'well-off' individuals are and so we call it the *well-offness* function.

- The function $s(c/\bar{c})$ captures subjective well-being: as indicated, it is a function of relative consumption.

This specification captures directly the concern that traditional policy-making has tended to ignore subjective well-being and instead focus on what economists have been very successful at monitoring and understanding: how well-off individuals are. The traditional policy-making framework is thus nested as a special case where $\theta = 0$. Moreover, for reasons that will become clear, we henceforth assume that $s(c/\bar{c})$ takes the form:

$$s \left(\frac{c}{\bar{c}} \right) = \frac{\frac{c}{\bar{c}}}{1 + \chi \cdot \frac{c}{\bar{c}}} = \frac{c}{\bar{c} + \chi c} \quad , \ \chi \geq 0 \tag{37}$$

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[11]For a similar functional form see Ulph (2014).

This captures the idea that, as conventionally measured, subjective well-being can be thought of as varying on scale between 0 and 1. Further, this information also allows us to readily link our analysis to the alternative form of separability considered by Boskin and Sheshinski (1978): i.e. separability between (i) consumption and leisure; and (ii) peer consumption. To see this, note that we can always set $\chi = 0$ and write (36) as:

$$u\left(c, l, \bar{c}\right) = \left[y(c,l) \cdot c^{\frac{\theta}{1-\theta}}\right]^{1-\theta} \bar{c}^{-\theta}$$

This also brings out clearly the fact that, under our chosen specification, varying the relative weight to subjective well-being and well-offness also varies the relative weight given to individual consumption vis-a-vis individual leisure in utility.

Under these preferences individual labour supply is characterised by:

$$n(1-t)\left[1 + \left(\frac{\theta}{1-\theta}\right)\left(\frac{\bar{c}}{\chi c^* + \bar{c}}\right)\left(\frac{y/c^*}{y_c}\right)\right] \leq \frac{y_c}{y_l} \ \ ; \ h^* \geq 0 \tag{38}$$

where the pair of inequalities hold with complementary slackness. As alluded to above, (38) makes clear that individual labour supply will only be a function of average peer consumption if $\chi > 0$. Put differently, there will only be KUJ effects when $\chi > 0$.

In turn, the symmetric Nash equilibrium level of labour supply can be characterised by:

$$\omega\left[1 + \left(\frac{\theta}{1-\theta}\right)\left(\frac{1}{1+\chi}\right)\left(\frac{y/c^N}{y_c}\right)\right] \leq \frac{y_l}{y_c} \ \ ; \ h^N \geq 0 \tag{39}$$

where the pair of inequalities hold with complementary slackness. At an interior solution one can therefore readily verify that:

$$\frac{\partial h^N}{\partial \theta} > 0 \ \ , \ \ \frac{\partial h^N}{\partial \chi} < 0 \tag{40}$$

The size of the labour distortion is thus increasing in the weight that individuals give to relative consumption, θ. Indeed, were individuals to recognise that in equilibrium their own consumption would coincide with that of their peers, such that $s = 1$, they would simply choose to maximise the conventional well-offness function (which corresponds to the case where $\theta = 0$). Contrastingly, equilibrium labour supply is decreasing in χ.

With the properties established in (40) serving by way of background, we proceed to the numerical analysis.

3.1 Numerical Simulations

In line with much of the optimal income tax literature, we let individual well-offness take the constant elasticity of substitution (CES) form (see Stern, 1976; Viard, 2001; Immonen et al., 1998):

$$y(c,l) = \left[\alpha c^{\frac{\rho-1}{\rho}} + (1-\alpha)l^{\frac{\rho-1}{\rho}}\right] \; ; \; \rho \neq 1 \tag{41}$$

where ρ denotes the elasticity of substitution between leisure and consumption.

With respect to the social welfare function in (28), we assume that preferences for equality of individual well-being are captured by the function:

$$\Psi(v) = \begin{cases} \dfrac{v^{1-\epsilon}}{1-\epsilon} & \epsilon \geq 0, \epsilon \neq 1 \\ \ln(v) & \epsilon = 1 \end{cases} \tag{42}$$

where ϵ is the Atkinson inequality aversion parameter. Notice that $\epsilon = 0$ corresponds to the *strict utilitarian* case where equity concerns are restricted to inequality in the social marginal utility of income.

The parameter choices are as follows: $n \sim \mathcal{N}(mean = -1, s.d. = 0.39)$; $\alpha = 0.614$, $\rho \in \{0.5, 0.99\}$; $\chi \in \{0, 0.5, 1\}^{12}$; $\epsilon \in \{0, 2, 3\}$ and $\theta \in [0, 0.3]$. These parameters are intentionally chosen to allow comparability between our results and the major existing studies in the optimal income tax literature, in particular Stern (1976). The assumption that productivity is lognormally distributed with mean and standard deviations of $\ln n$ set at -1 and 0.39, respectively, has been the benchmark since Mirrlees (1971). The range of values of ρ fall within those from major studies, with $\rho = 0.5$ being the most empirically plausible (Viard, 2001). Analogous to Stern (1976), α is chosen such that the average productivity individual ($n = 0.3969$) works two-thirds of their time endowment when $\theta = 0, \rho = 0.5$ and $t = \sigma = 0$. We view the choices of θ as corresponding to moderate concerns for relative consumption.

As indicated by our choices of (θ, χ), we decompose the effects of the model parameters by simulating the following optimal tax rates, $\hat{t}(\theta, \chi)$:

(i) *Baseline Case.* The optimal tax rate $\hat{t}(0,0)$ is that which arises when all that matters

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[12]Note that there is no upper bound on the parameter χ.

is well-offness and so:[13]

$$u(c, l, \bar{c}) = y(c, l)$$

(ii) *Negative Externality and KUJ Effects.* The optimal tax rates $\hat{t}(\theta, \chi)$ with $\chi \in \{0.5, 1\}$ for our preferred specification where peer comparisons induce both a negative externality and *KUJ* effects:

$$u(c, l, \bar{c}) = [y(c, l)]^{1-\theta} \left(\frac{c}{\chi c + \bar{c}} \right)^{\theta} \; ; \; \chi \in \{0.5, 1\}$$

(iii) *Pure Negative Externality.* The optimal tax rate function $\hat{t}(\theta, 0)$ that arises when $\chi = 0$ and so peer consumption has a pure negative externality effect, and so:

$$u(c, l, \bar{c}) = [y(c, l)]^{1-\theta} \left(\frac{c}{\bar{c}} \right)$$

(iv) *Pure Negative Externality with no additional weight on consumption.* The optimal tax rate function $\hat{t}_{alt}(\theta)$ that arises when, as in (iii) above, peer consumption has a pure negative externality *but* the weight given to consumption vis-à-vis leisure is unaffected by the weight given to peer income, so:

$$u(c, l, \bar{c}) = [y(c, l)]^{1-\theta} \bar{c}^{-\theta}$$

So we end up with the following decomposition of the change in the optimal tax rate brought about by recognising the importance of relative income to individuals:

$$\hat{t}(\theta, \chi) - \hat{t}(0, 0) = \left[\hat{t}_{alt}(\theta) - \hat{t}(0, 0) \right] + \left[\hat{t}(\theta, 0) - \hat{t}_{alt}(\theta) \right] + \left[\hat{t}(\theta, \chi) - \hat{t}(\theta, 0) \right] \tag{43}$$

The first term on the right side captures the impact on the optimal tax rate of the pure negative welfare externality that arises when individuals have concerns over relative income. This should be positive because we need a higher income tax to correct the externality, thus capturing the Boskin and Sheshinski (1978) argument for higher taxes. The second term captures the fact in the pure negative externality version of our specification - as opposed to that generating $\hat{t}_{alt}$ - an increase in the weight placed on relative consumption also increases the weight given to individual consumption vis-à-vis leisure. Finally, the third term captures the effect of having peer consumption generate *KUJ* behavioural effects on labour supply

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[13]Note that $\hat{t}(0, 0) = \hat{t}(0, \chi) \; \forall \; \chi$ because the optimal tax function is independent of χ when $\theta = 0$.

Figure 3: Variation of $\hat{t}(\theta, \delta)$ with θ and χ.
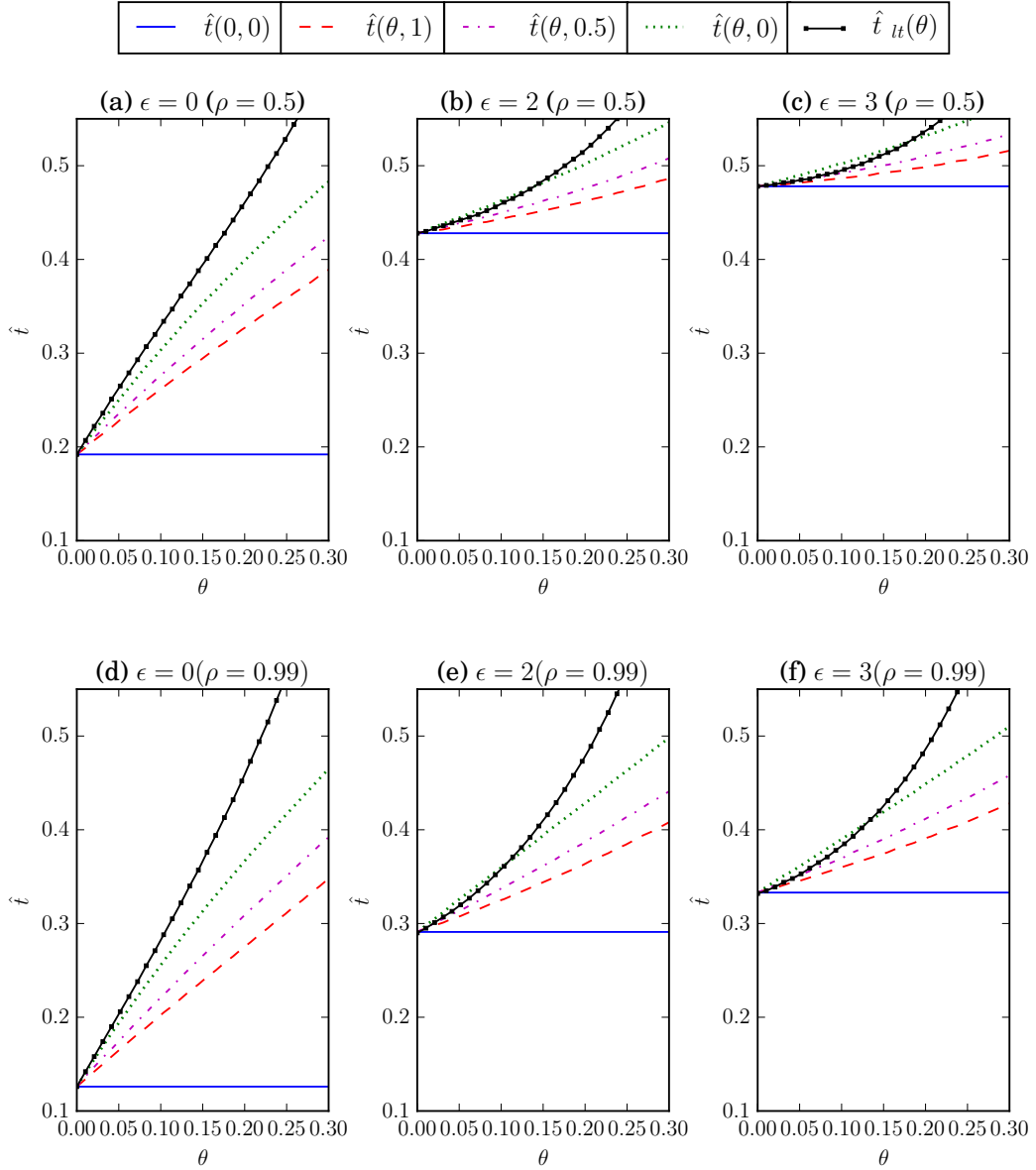
Notes. Subplots (a), (b) and (c) are generated for the CES parameter $\rho = 0.5$; whilst subplots (d), (e) and (f) are generated for $\rho = 0.99$. When $\theta = 0$ the optimal tax rates correspond to those in Stern (1976): I.e. in subplot (a) $\hat{t}(0,0) = 0.192$; (b) $\hat{t}(0,0) = 0.428$; (c) $\hat{t}(0,0) = 0.478$; (d) $\hat{t}(0,0) = 0.126$; (e) $\hat{t}(0,0) = 0.291$; and (f) $\hat{t}(0,0) = 0.333$.

and, as indicated, should be negative.

The numerical results are presented graphically in Figure 3. Subplots (a), (b) and (c) are generated for $\rho = 0.5$; whilst subplots (d),(e) and (f) are generated for $\rho = 0.99$ (i.e. approximately Cobb-Douglas). On the horizontal axis in each subplot the parameter θ is varied in

discrete increments of 0.01 from 0 to 0.3; where for each value of θ the optimal tax functions $\hat{t}(\theta, \chi)$ and $\hat{t}_{alt}$ are simulated. The different curves within each subplot correspond to the different tax specifications specified above: $\hat{t}(0,0)$, $\hat{t}(\theta, 1)$, $\hat{t}(\theta, 0.5)$, $\hat{t}(\theta, 0)$ and $\hat{t}_{alt}$.

The key observations are as follows. First, the functions $\hat{t}(\theta, \chi)$ and $\hat{t}_{alt}(\theta)$ are *increasing* in θ; and thus in the weight that individuals place on relative consumption/subjective well-being considerations. The intuition here is provided by the Δ term in the numerator of the optimal tax expression in (35): (i) the distortion to labour supply induced by taking peer consumption *as given* leads to a corrective role for taxation; and further (ii) to the extent that an increase in θ increases labour supply and thus the size of the distortion, it also increases the corrective role for the tax rate. Indeed, we know from (40) that Nash labour supply is increasing in θ.

Second, for any given $\theta > 0$ we observe:

$$\hat{t}(0,0) < \hat{t}(\theta, \chi = 1) < \hat{t}(\theta, \chi = 0.5) < \hat{t}(\theta, \chi = 0)$$

The optimal tax rate is thus *decreasing* in χ. The intuition is twofold: (i) when $\chi > 0$ there are KUJ effects which, as illustrated in the denominator of (35), act to heighten the efficiency consequences of taxation; and (ii) we know from (40) that Nash labour supply is decreasing in χ, thus reducing the size of the labour distortion.

Given our postulate that equity considerations will be muted due to Result 2 - which states that the *level* of indirect utility is decreasing in the wage for workers with wages close to the reservation wage - it is of interest to see how the covariance between gross earnings and the net smvi changes with the weight, θ, that individuals place on subjective well-being. Figure 4 illustrates this and is composed of three subplots which differ in their distributional weights. In subplot (a) we have the strict utilitarian case ($\epsilon = 0$) and so the net smvi is *independent* of utility levels. We here observe that the covariance becomes more negative with θ. Contrastingly, in subplots (b) and (c) we have $\epsilon = 2$ and $\epsilon = 3$ respectively, and so the net smvi is a function of the utility *level*. Result 2 now has force: we observe that the covariance between gross earnings and the net smvi becomes less negative with θ and so equity considerations are indeed muted relative to the standard framework.

Figure 4: Equity considerations: covariance between gross earnings and the net smvi.
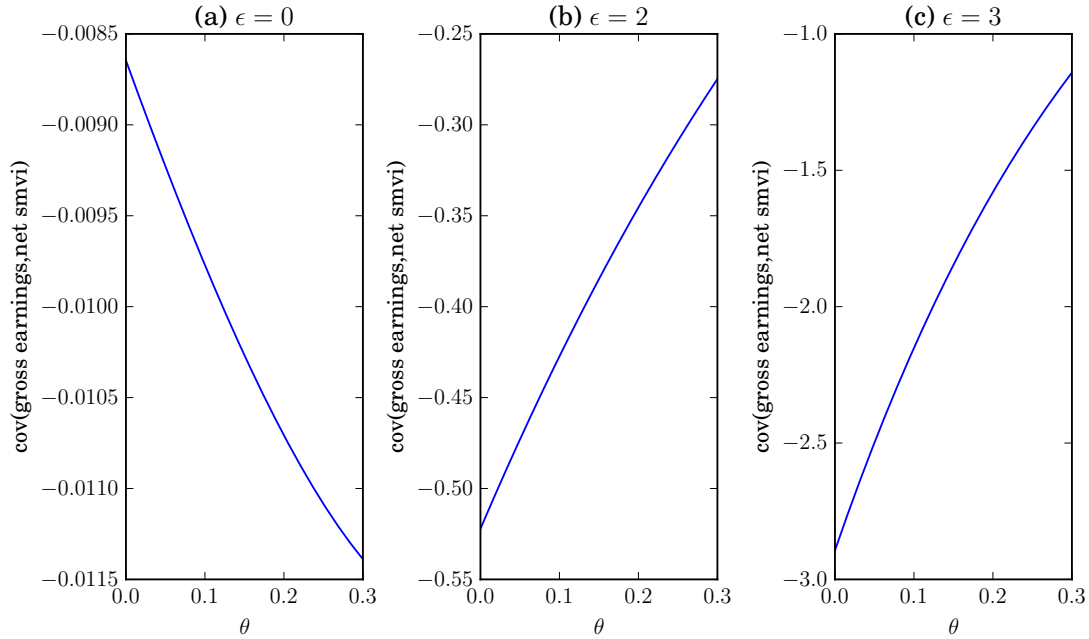
Notes. To generate this figure we set $t = 0.2$ and calculate the σ that exhausts the budget constraint. The net smvi is $(v^N)^{-\epsilon}v_\sigma^N + \lambda tnh_\sigma^N$; where λ is the root to the expression $\int_0^\infty \{(v^N)^{-\epsilon}v_\sigma^N + \lambda tnh_\sigma^N\}dF = \lambda$.

4 Concluding Remarks

This paper has developed a framework in which to analyse optimal linear income taxation when individual preferences give weight to subjective well-being: here taken to be how one's own consumption compares with that of their immediate (same ability) peers. We have shown that when individuals compare themselves to their peers there are three factors that suggest the optimal tax rate will differ from that proposed under a conventional framework.

(i) *Distortion Correcting Role.* Through treating peer consumption *as given*, individual labour supply decisions are distorted away from what they would choose were they to recognise that in equilibrium their own consumption will coincide with that of their peers. The optimal income tax rate is increasing in the size of this distortion.

(ii) *Heightened Efficiency considerations.* When peer concerns exercise a *Keeping up with the Joneses* effect on labour supply, individual labour supply is a function of peer consumption. In equilibrium compensated labour elasticities exhibit a multiplier effect: person A working harder raises peer consumption which in turn induces person B to work harder and so forth. This means that the distortionary effect of taxation is higher

than in the traditional framework, thus pointing to a lower optimal tax rate.

(iii) *Muted Inequality concerns.* Inequality concerns - as captured through the negative covariance between (relative) gross earnings and the (relative) net smvi - may be muted relative to the standard policy framework. In our framework individual well-being is decreasing in the wage rate for workers with wages close to the reservation wage. To the extent that this reduces the rate at which the net smvi falls with productivity, it may lower the absolute value of the covariance. We have shown that an important factor here will be whether or not the social welfare function gives weight to inequality in utility *levels.* Whilst the result that well-being may fall with wage was first demonstrated in Ulph (2014) for a specific functional form, we have shown here that holds in general. The worst-off in society are thus no longer those with the lowest wage, nor are they the unemployed. These muted inequality concerns suggest a lower tax rate relative to the traditional framework.

To the best of our knowledge, this paper is the first to (i) establish the importance of these second two effects in determining the optimal tax rate; and (ii) capture all three effects in a general optimal tax framework. Our numerical analysis embeds the tradtional framework (see Stern, 1976) as a special case and demonstrates that when all three effects are present the optimal tax rate is increasing in the weight individuals place on relative consumption. However, the heightened efficiency considerations embodied in the second effect (and potentially the muted inequality considerations in the third) draw the optimal tax rate closer to that emerging from a traditional policy framework, as well as reducing the rate at which it increases with the weight given to relative consumption.

There are a number of extensions that we intend to pursue. First, we have drawn on recent empirical evidence that individuals make relative income comparisons in the narrow sense: i.e. with colleagues/relatives/friends who are very similar to themselves. We have interpreted this in a very extreme way of meaning others who are identical in terms of ability: this has the attractive feature of providing a clear counterfactual of how individuals would behave were they to recognise that the equilibrium will be symmetric. However, an alternative analysis to undertake would determine the optimal tax rates when individuals make the broadest comparison and care about their income relative to that in the population at large.[14] This would strip out much of the Nash-equilibrium considerations arising in Section 2.2 and so much of the source of the second and third factors identified above. Accordingly, such an analysis would serve as a useful counterfactual against which to compare optimal tax rates

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[14]The paper by Beath and FitzRoy (2011) referred to in footnote 3 also contains a comparison with the case where individuals make the broadest of comparisons.

from both the traditional framework and that identified here.

A second direction for research is to consider other formulations that lie intermediate between the very strict interpretation of peer comparisons considered here and the broadest comparison discuss above. We postulate that many of the features and conclusions of the existing framework will carry over to this more general setting.

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Appendices

A   Derivation of the optimal tax expression.

To derive the optimal tax expression in (35) we substitute into the first-order-condition for t, (29), both (i) the *distortion-adjusted Roy's identity* from (21) and (ii) the equation for compensated Nash labour supply from (25). This yields:

$$
\begin{aligned}
\hat{\lambda} \int_0^\infty n h^N dF(n) &= \int_0^\infty \left\{ \Psi'(v^N) n v_\sigma^N \left[h^N - \left(\frac{\delta^N c_\omega^{cN}}{1 - \delta^N c_\omega^N} \right) \right] + \hat{\lambda}^N t n^2 \left(h_\omega^{cN} + h^N h_\sigma^N \right) \right\} dF(n) \\
&= \int_0^\infty \left\{ n h^N \left[\Psi'(v^N) v_\sigma + \hat{\lambda} \hat{t} n h_\sigma^N \right] - \Psi'(v^N) n v_\sigma^N \left(\frac{\delta^N c_\omega^{cN}}{1 - \delta^N c_\omega^N} \right) \right\} dF(n) \\
&\quad + \hat{\lambda} \hat{t} \int_0^\infty n^2 h_\omega^{cN} dF(n)
\end{aligned}
$$
$$(\text{A.1})$$

Substituting into (A.1) the definition of $\phi(n)$ from (31) then yields

$$
\int_0^\infty \left\{ n h^N \left[1 - \frac{\phi(n)}{\hat{\lambda}} \right] + \frac{\Psi'(v^N) n v_\sigma^N}{\hat{\lambda}} \left(\frac{\delta^N c_\omega^{cN}}{1 - \delta^N c_\omega^N} \right) \right\} dF(n) = \hat{t} \int_0^\infty n^2 h_\omega^{cN} dF(n) \quad (\text{A.2})
$$

To save on notation we let $z = nh^N$ and $\bar{z} = \int_0^\infty nh^N dF$. Dividing both sides of (A.2) by $\bar{z}$ and rearranging then yields:

$$\int_0^\infty \left\{ \frac{z}{\bar{z}} \left[1 - \frac{\phi(n)}{\hat{\lambda}} \right] + \frac{\Psi'(v^N)nv_\sigma^N}{\hat{\lambda} \cdot \bar{z}} \left(\frac{\delta^N c_\omega^{cN}}{1 - \delta^N c_\omega^N} \right) \right\} dF(n) = \frac{\hat{t}}{1 - \hat{t}} \int_0^\infty \frac{z}{\bar{z}} \left(\frac{\omega}{h^N} h_\omega^{cN} \right) dF(n) \tag{A.3}$$

Substituting into (A.3) the definitions r_z, r_ϕ and $\hat{\lambda}$ from (33) and (32) then finally gives:

$$\frac{\hat{t}}{1 - \hat{t}} = \frac{\Delta - \mathrm{Cov}\left[r_z(n), r_\phi(n) \right]}{\int_0^\infty r_z(n) \cdot \dfrac{\eta_\omega^c}{[1 - c_{\bar{c}}]} dF(n)}$$

∎

Supplementary Appendices for 'Optimal Taxation and Subjective Well-being'[NOT FOR PUBLICATION]

Sean Slack & David Ulph

A Properties of the specific functional form

In Section 3 of the main article we make the separability assumption:

$$u(c, l, \bar{c}) = [y(c, l)]^{1-\theta} \left[s\left(\frac{c}{\bar{c}} \right) \right]^{\theta} \quad ; \; \theta \in [0, 1] \tag{A.1}$$

where y is the standard sub-utility function we refer to as 'well-offness', whilst s refers to subjective well-being and is a function of relative consumption. The parameter θ captures the weight individuals place on subjective well-being relative to well-offness.

We let:

$$y(c, l) = \left[\alpha c^{\frac{\rho-1}{\rho}} + (1-\alpha) l^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \tag{A.2}$$

$$s\left(\frac{c}{\bar{c}} \right) = \frac{c}{\chi c + \bar{c}} \quad ; \; \chi \geq 0 \tag{A.3}$$

Well-offness thus takes the constant elasticity of substitution (CES) form, where $\rho \in (0, 1)$ is elasticity of substitution between leisure and consumption and α is the weight individuals place on consumption vis-à-vis leisure in the CES function.

For computational purposes, the partial derivatives of the CES function are:

$$y_c(c, l) = \alpha \left[\alpha + (1-\alpha) \left(\frac{c}{l} \right)^{\frac{1-\rho}{\rho}} \right]^{\frac{1}{\rho-1}} \tag{A.4}$$

$$y_l(c, l) = (1-\alpha) \left[\alpha \left(\frac{l}{c} \right)^{\frac{1-\rho}{\rho}} + (1-\alpha) \right]^{\frac{1}{\rho-1}} \tag{A.5}$$

$$y_{cc}(c, l) = -\left(\frac{1}{\rho} \right) \alpha(1-\alpha) \left[\alpha + (1-\alpha) \left(\frac{c}{l} \right)^{\frac{1-\rho}{\rho}} \right]^{\frac{2-\rho}{\rho-1}} \left(\frac{c}{l} \right)^{\frac{1-2\rho}{\rho}} \left(\frac{1}{l} \right) \tag{A.6}$$

$$y_{ll}(c, l) = -\left(\frac{1}{\rho} \right) \alpha(1-\alpha) \left[\alpha \left(\frac{l}{c} \right)^{\frac{1-\rho}{\rho}} + (1-\alpha) \right]^{\frac{2-\rho}{\rho-1}} \left(\frac{l}{c} \right)^{\frac{1-2\rho}{\rho}} \left(\frac{1}{c} \right) \tag{A.7}$$

$$y_{cl}(c,l) = \left(\frac{1}{\rho}\right)\alpha(1-\alpha)\left[\alpha+(1-\alpha)\left(\frac{c}{l}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{2-\rho}{\rho-1}}\left(\frac{c}{l}\right)^{\frac{1-\rho}{\rho}}\left(\frac{1}{l}\right) \qquad (A.8)$$

The partial derivatives of the subjective well-being function are:

$$s_c = \frac{\bar{c}}{(\chi c+\bar{c})^2} \;\;;\;\; s_{\bar{c}} = -\frac{c}{(\chi c+\bar{c})^2} \;\;;\;\; s_{cc} = -\frac{2\chi\bar{c}}{(\chi c+\bar{c})^3} \;\;;\;\; s_{\bar{c}\bar{c}} = -\frac{2c}{(\chi c+\bar{c})^3} \;;\; s_{c\bar{c}} = \frac{\chi c-\bar{c}}{(\chi c+\bar{c})^3}$$
$$\qquad (A.9)$$

Putting this all together, the partial derivatives of the overall well-being function are:

$$u_c(c,l,\bar{c}) = (1-\theta)\left(\frac{s}{y}\right)^{\theta}\left[y_c+\left(\frac{\theta}{1-\theta}\right)\left(\frac{y}{s}\right)s_c\right] \qquad (A.10)$$

$$u_{cc}(c,l,\bar{c}) = (1-\theta)\left(\frac{s}{y}\right)^{\theta}\left\{\begin{array}{l}\theta\left(\dfrac{ys_c-sy_c}{y\cdot s}\right)\left[y_c+\left(\dfrac{\theta}{1-\theta}\right)\left(\dfrac{y}{s}\right)s_c\right]\\[2mm]+y_{cc}+\left(\dfrac{\theta}{1-\theta}\right)\left[\left(\dfrac{sy_c-ys_c}{s^2}\right)s_c+\left(\dfrac{y}{s}\right)s_{cc}\right]\end{array}\right\} \qquad (A.11)$$

$$u_l(c,l,\bar{c}) = (1-\theta)\left(\frac{s}{y}\right)^{\theta}y_l \qquad (A.12)$$

$$u_{ll}(c,l,\bar{c}) = (1-\theta)\left(\frac{s}{y}\right)^{\theta}\left[y_{ll}-\theta\left(\frac{y_l^2}{y}\right)\right] \qquad (A.13)$$

$$u_{\bar{c}}(c,l,\bar{c}) = \theta\left(\frac{y}{s}\right)^{1-\theta}s_{\bar{c}} \qquad (A.14)$$

$$u_{l\bar{c}}(c,l,\bar{c}) = \theta(1-\theta)\left(\frac{s}{y}\right)^{\theta-1}\left(\frac{y_l}{y}\right)s_{\bar{c}} \qquad (A.15)$$

$$u_{c\bar{c}}(c,l,\bar{c}) = (1-\theta)\left(\frac{s}{y}\right)^{\theta}\left\{\begin{array}{l}\theta\left(\dfrac{s_{\bar{c}}}{s}\right)\left[y_c+\left(\dfrac{\theta}{1-\theta}\right)\left(\dfrac{y}{s}\right)s_c\right]\\[2mm]+\left(\dfrac{\theta}{1-\theta}\right)\left[\left(\dfrac{y}{s}\right)s_{c\bar{c}}-\left(\dfrac{y}{s^2}\right)s_c s_{\bar{c}}\right]\end{array}\right\} \qquad (A.16)$$

$$u_{lc}(c,l,\bar{c}) = (1-\theta)\left(\frac{s}{y}\right)^{\theta}\left[\theta\left(\frac{y_l}{s}\right)\left(\frac{ys_c-sy_c}{y}\right)+y_{lc}\right] \qquad (A.17)$$

Income effect derivative.   In order to compute the net smvi we need to know the derivative h_{σ}^N. Differentiating the (interior) f.o.c characterising h^N

$$\omega u_c\left(\sigma+\omega h^N, 1-h^N, \sigma+\omega h^N\right) - u_l(\sigma+\omega h^N, 1-h^N, \sigma+\omega h^N) = 0$$

w.r.t. σ yields:

$$\omega\left\{(u_{cc}+u_{c\bar{c}})\left(1+\omega h_{\sigma}^N\right)-u_{cl}h_{\sigma}^N\right\}-(u_{cl}+u_{l\bar{c}})\left(1+\omega h_{\sigma}^N\right)+u_{ll}h_{\sigma}^N = 0$$

and thus:

$$h_\sigma^N = \frac{\omega(u_{cc} + u_{c\bar{c}}) - (u_{cl} + u_{l\bar{c}})}{\omega(2u_{cl} + u_{l\bar{c}}) - \omega^2(u_{cc} + u_{c\bar{c}}) - u_{ll}} \tag{A.18}$$

$$= \frac{\left[\dfrac{u_l(u_{cc} + u_{c\bar{c}}) - u_c(u_{cl} + u_{l\bar{c}})}{u_c}\right]}{\left(\dfrac{u_l}{u_c}\right)\left[(2u_{cl} + u_{l\bar{c}}) - \left(\dfrac{u_l}{u_c}\right)(u_{cc} + u_{c\bar{c}}) - \left(\dfrac{u_c}{u_l}\right)u_{ll}\right]} \tag{A.19}$$

Notice immediately that if $u_{c\bar{c}} = u_{l\bar{c}} = 0$ then h_σ^N reduces to the traditionally found in the literature.

Numerical Code

```
1
2   from __future__ import division
3   import numpy as np
4   from scipy.optimize import minimize
5   from scipy.stats import lognorm
6   from scipy.integrate import quad
7   from scipy.optimize import brenth
8   import matplotlib.pyplot as plt
9
10  #NUMERICAL CODE: SUBJECTIVE WELL-BEING,
        RELATIVE INCOME AND
11  #OPTIMAL TAXATION.
12
13  #Individual well-being is a function of
        'well-offness',y, and subjective
        well-being,h. These two functions are
        defined by:
14
15  def y(c,l,a,p):
16      if c<=0:
17          return 0
18      else:
19          return (a*(c**((p-1)/p))+
20          (1-a)*(l**((p-1)/p)))**(p/(p-1))
21
22  def s(c,cbar,chi):
23      if c<=0:
24          return 0
25      else:
26          return (c/(chi*c+cbar))
27
28  #where 'c' denotes consumption; 'l' is
        leisure, 'a' is the weight placed on
        consumption vis-a-vis leisure in the
        well-offness function; 'p' is
        elasticity of substitution between
        leisure and consumption the
        well-offness function; 'cbar' is
        average peer consumption; and 'chi'
        is a non-negative parameter.
29
30  #The overall well-being function is
31  def u(c,l,cbar,theta,chi,a,p):
32      return ((s(c,cbar,chi)**theta)*
33      (y(c,l,a,p)**(1-theta)))
34
35  #where 'theta' is the weight placed on
        subjective well-being.
36
37  #The partial derivatives of the
        well-offness function w.r.t. 'c' and
        'l' are:
38
39  def yc(c,l,a,p):
40      return a*((a+(1-a)*((c/l)**((1-p)/p)))
41      **(1/(p-1)))
42
43  def yl(c,l,a,p):
44      return (1-a)*((a*((l/c)**((1-p)/p))+
45      (1-a))**(1/(p-1)))
46
47  def ycc(c,l,a,p):
48      return
            (-(1/p)*a*(1-a)*((a+(1-a)*((c/l)
49      **((1-p)/p)))**(
50      (2-p)/(p-1)))*((c/l)**((1-2*p)/p))*(1/l))
51
52  def yll(c,l,a,p):
53      return
            (-(1/p)*a*(1-a)*((a*((l/c)**((1-p)/p))
54      +(1-a))**(
55      (2-p)/(p-1)))*((l/c)**((1-2*p)/p))*(1/c))
```

```python
56
57  def ycl(c,l,a,p):
58      return ((1/p)*a*(1-a)*((a+(1-a)*((c/l)
59      **((1-p)/p)))**(
60      (2-p)/(p-1)))*((c/l)**((1-p)/p))*(1/l))
61
62  #The partial derivatives of the
        subjective well-being function w.r.t.
        'c' and 'cbar' are:
63
64  def sc(c,cbar,chi):
65      return cbar/((chi*c+cbar)**2)
66
67  def scbar(c,cbar,chi):
68      return -c/((chi*c+cbar)**2)
69
70  def scc(c,cbar,chi):
71      return -2*chi*cbar/((chi*c+cbar)**3)
72
73  def scbarcbar(c,cbar,chi):
74      return -2*c/((chi*c+cbar)**3)
75
76  def sccbar(c,cbar,chi):
77      return (chi*c-cbar)/((chi*c+cbar)**3)
78
79  #The partial derivatives of the overall
        well-being function are
80  #thus:
81
82  def uc(c,l,cbar,theta,chi,a,p):
83      return (1-theta)*(((s(c,cbar,chi)/
84      y(c,l,a,p))**(theta))*
85      (yc(c,l,a,p)+(theta/(1-theta))*
86      (y(c,l,a,p)/s(c,cbar,chi))*
87      sc(c,cbar,chi)))
88
89  def ucc(c,l,cbar,theta,chi,a,p):
90      return ((1-theta)*((s(c,cbar,chi)/
91      y(c,l,a,p))**theta)*
92      (theta*((y(c,l,a,p)*sc(c,cbar,chi)
93      -s(c,cbar,chi)*yc(c,l,a,p))/(y(c,l,a,p)*
94      s(c,cbar,chi)))*
95      (yc(c,l,a,p)+(theta/(1-theta))*
96      (y(c,l,a,p)/s(c,cbar,chi))*
97      sc(c,cbar,chi))+ycc(c,l,a,p)+
98      (theta/(1-theta))*
99      ((((s(c,cbar,chi)*yc(c,l,a,p)
100     -y(c,l,a,p)*sc(c,cbar,chi))/
101     (s(c,cbar,chi)**2)) *sc(c,cbar,chi)
102     +(y(c,l,a,p)/s(c,cbar,chi))*
103     scc(c,cbar,chi))))
104
105
106 def ul(c,l,cbar,theta,chi,a,p):
107     return
108         (1-theta)*((s(c,cbar,chi)/y(c,l,a,p))
        **theta)*yl(c,l,a,p)
109
110 def ull(c,l,cbar,theta,chi,a,p):
111     return
112         (1-theta)*(((s(c,cbar,chi)/y(c,l,a,p))
        **theta)*(yll(c,l,a,p)-
113     theta*((yl(c,l,a,p)**2)/y(c,l,a,p))))
114
115 def ucbar(c,l,cbar,theta,chi,a,p):
116     return
117         theta*((y(c,l,a,p)/s(c,cbar,chi))
        **(1-theta))*scbar(c,cbar,chi)
118
119 def ulcbar(c,l,cbar,theta,chi,a,p):
120     return
121         theta*(1-theta)*(((s(c,cbar,chi)
        /y(c,l,a,p))**(theta-1))
122     *(yl(c,l,a,p)/y(c,l,a,p))*scbar(c,cbar,chi))
123
124 def uccbar(c,l,cbar,theta,chi,a,p):
125     return
126         (1-theta)*((s(c,cbar,chi)/y(c,l,a,p))**theta)
127     *(theta*(sc(c,cbar,chi)/
128     s(c,cbar,chi))*(yc(c,l,a,p)+
129     (theta/(1-theta))*(y(c,l,a,p)/
130     s(c,cbar,chi))*sc(c,cbar,chi))
131     +(theta/(1-theta))*((y(c,l,a,p)/
132     s(c,cbar,chi))*sccbar(c,cbar,chi)
133     -(y(c,l,a,p)/(s(c,cbar,chi)**2))
134     *sc(c,cbar,chi)*scbar(c,cbar,chi)))
135
136 def ulc(c,l,cbar,theta,chi,a,p):
137     return (1-theta)*((s(c,cbar,chi)/
138     y(c,l,a,p))**theta)*(
139     theta*(yl(c,l,a,p)/s(c,cbar,chi))
140     *((y(c,l,a,p)*sc(c,cbar,chi)
141     -s(c,cbar,chi)*yc(c,l,a,p))/y(c,l,a,p))
142     +ycl(c,l,a,p))
143
144 #Individual labour supply.
145 #The foc characterising individual labour
        supply is:
146
147 def foch(h,n,t,sigma,cbar,theta,chi,a,p):
148     return n*(1-t)-(ul(n*(1-t)*h+sigma,
149     1-h,cbar,theta,chi,a,p)
150     /uc(n*(1-t)*h+sigma,1-h,cbar,
151     theta,chi,a,p))
152
153 #The reservation productivity is thus:
154 def reservationn(t,sigma,cbar,
        theta,chi,a,p):
```

```python
155        if sigma<=0:
156            return 0
157        else:
158            return(ul(sigma,1,cbar,theta,chi,a,p)/
159        uc(sigma,1,cbar,theta,chi,a,p))/(1-t)
160
161    #For all n exceeding the reservation
           productivity, optimal labour supply
           is given by the root of 'foch'. So
           the optimal labour supply function is:
162    def opth(n,t,sigma,cbar,theta,chi,a,p):
163        if n<=reservationn(t,sigma,cbar,theta,
164        chi,a,p):
165            return 0
166        else:
167            return brenth(foch,0,0.9999,
168            args=(n,t,sigma,cbar,theta,chi,a,p))
169
170    #Indirect well-being is thus given by:
171    def v(n,t,sigma,cbar,theta,chi,a,p):
172        return u(n*(1-t)*opth(n,t,sigma,cbar,
173        theta,chi,a,p)+sigma,
174        1-opth(n,t,sigma,cbar,theta
175        ,chi,a,p),cbar,theta,chi,a,p)
176
177    #~Peer Comparison: Nash Equilibrium~
178    #----------------------------------------
179
180    #Let cbar be the average consumption of
           one's peers: i.e. those individuals
           of the same ability, n. The Nash
           Equilibrium level of labour supply -
           which we denote by 'hnash' - is
           characterised by the root to the
           condition:
181
182    def nashhimplicit(hnash,n,t,sigma,theta,
183    chi,a,p):
184        return hnash-opth(n,t,sigma,n*hnash*
185        (1-t)+sigma,theta,chi,a,p)
186
187    #The Nash Rpervation gross wage is:
188    def
           reservationnnash(t,sigma,theta,chi,a,p):
189        if sigma<=0:
190            return 0
191        else:
192            return (ul(sigma,1,sigma,theta,
193            chi,a,p)/
194        uc(sigma,1,sigma,theta,
195        chi,a,p))/(1-t)
196
197    #For n>rpervationnnash the Nash level of
           labour supply is given
```

```python
198    #by the root to nashimplicit:
199
200    def opthnash(n,t,sigma,theta,chi,a,p):
201        if
               n<=reservationnnash(t,sigma,theta,chi,a,p):
202            return 0
203        else:
204            return
                   brenth(nashhimplicit,0,0.9999,
205            args=(n,t,sigma,theta,chi,a,p))
206
207    #The income effect derivative (not
           evaluated at the optimal labour
208    #supply) is:
209    def incomeeffect(c,l,cbar,theta,chi,a,p):
210        return (((ul(c,l,cbar,theta,chi,a,p)*
211        (ucc(c,l,cbar,theta,chi,a,p)
212        +uccbar(c,l,cbar,theta,chi,a,p))
213        -uc(c,l,cbar,theta,chi,a,p)*(
214        ulc(c,l,cbar,theta,chi,a,p)
215        +ulcbar(c,l,cbar,theta,chi,a,p)))/
216        uc(c,l,cbar,theta,chi,a,p))/
217        ((ul(c,l,cbar,theta,chi,a,p)/
218        uc(c,l,cbar,theta,chi,a,p))*
219        ((2*ulc(c,l,cbar,theta,chi,a,p)
220        +ulcbar(c,l,cbar,theta,chi,a,p))-
221        (ul(c,l,cbar,theta,chi,a,p)/
222        uc(c,l,cbar,theta,chi,a,p))*
223        (ucc(c,l,cbar,theta,chi,a,p)
224        +uccbar(c,l,cbar,theta,chi,a,p))
225        -(uc(c,l,cbar,theta,chi,a,p)/
226        ul(c,l,cbar,theta,chi,a,p))*
227        ull(c,l,cbar,theta,chi,a,p))))
228
229    #We then substitute Nash labour
           supply/consumption into
           'incomeeffect' to obtain the partial
           derivative of 'opthnash' w.r.t.
           'sigma':
230    def optnashsigma(n,t,sigma,theta,chi,a,p):
231        if
               n<=reservationnnash(t,sigma,theta,chi,a,p):
232            return 0
233        else:
234            return incomeeffect(n*(1-t)*
235            opthnash(n,t,sigma,theta,chi,a,p)
236            +sigma,1-
237            opthnash(n,t,sigma,theta,chi,a,p),
238            n*(1-t)*opthnash(n,t,sigma,theta,chi,a,p)
239            +sigma,theta,chi,a,p)
240
241    #Nash indirect well-being is:
242    def
           nashindirectu(n,t,sigma,theta,chi,a,p):
```

5

```python
243     return u(n*(1-t)*
244     opthnash(n,t,sigma,theta,chi,a,p)+sigma,
245     1-opthnash(n,t,sigma,theta,chi,a,p),
246     n*(1-t)*opthnash(n,t,sigma,theta,chi,a,p)
247     +sigma,theta,chi,a,p)
248
249 def
        nashindirectusigma(n,t,sigma,theta,chi,a,p):
250     return ((uc(sigma+n*(1-t)*
251     opthnash(n,t,sigma,theta,chi,a,p),
252     1-opthnash(n,t,sigma,theta,chi,a,p),
253     sigma+n*(1-t)*
254     opthnash(n,t,sigma,theta,chi,a,p)
255     ,theta,chi,a,p)
256     +ucbar(sigma+n*(1-t)*
257     opthnash(n,t,sigma,theta,chi,a,p),
258     1-opthnash(n,t,sigma,theta,chi,a,p),
259     sigma+n*(1-t)*
260     opthnash(n,t,sigma,theta,chi,a,p),
261     theta,chi,a,p))*
262     (1+n*(1-t)*
263     optnashsigma(n,t,sigma,theta,chi,a,p))-
264     ul(sigma+n*(1-t)*
265     opthnash(n,t,sigma,theta,chi,a,p),
266     1-opthnash(n,t,sigma,theta,chi,a,p),
267     sigma+n*(1-t)*
268     opthnash(n,t,sigma,theta,chi,a,p),theta,chi,a,p)
269     *optnashsigma(n,t,sigma,theta,chi,a,p))
270
271 #Social Welfare and the Optimisation
        Problem
272 #=========================================
273 #Letting 'ep' denote the distributional
        welfare weight, we construct the
        social welfare function as follows.
        The below function is the nash
        indirect utility function raised to
        the welfare weight and multiplied the
        productivity pdf.
274 def nashindirectupdf(n,t,sigma,theta,chi,a
275 ,p,mean,var,ep):
276     return
            ((1/(1-ep))*(nashindirectu(n,t,sigma,
277     theta,chi,a,p)**(1-ep))*
278     lognorm.pdf(n,var,scale=np.exp(mean)))
279 #Integrating the above function over n
        thus gives social welfare:
280 def socialwelfare(t,sigma,theta,chi,a
281 ,p,mean,var,ep):
282     return quad(nashindirectupdf,0,10,
283     args=(t,sigma,theta,chi,a,p,
284     mean,var,ep))[0]
285
286

287 #Individual earnings times by the
        distribution pdf are:
288
289 def
        earninsgpdf(n,t,sigma,theta,chi,a,p,mean,var):
290     return
            (n*opthnash(n,t,sigma,theta,chi,a,p)*
291     lognorm.pdf(n,var,scale=np.exp(mean)))
292 #Integrating the above function over n
        gives aggregate earnings:
293 def
        aggregateearnings(t,sigma,theta,chi,a,p,mean,var):
294     return quad(earninsgpdf,0,10,
295     args=(t,sigma,theta,chi,a,p,mean,
296     var))[0]
297
298
299 #For any given 't', the resulting value
        of 'sigma' is the root to
300 def sigmaforgiventcondition
301 (sigma,t,theta,chi,a,p,mean,var):
302     return
            sigma-t*aggregateearnings(t,sigma,
303     theta,chi,a,p,mean,var)
304 #and thus given by
305 def
        sigmaforgivent(t,theta,chi,a,p,mean,var):
306     return brenth(sigmaforgiventcondition,
307     0.01,10,
308     args=(t,theta,chi,a,p,mean,var))
309
310
311 #Let the vector x=(x[0],x[1])=(t,sigma)
        denote the variables we are
312 #optimising with respect to.
313
314 #Written in terms of the choice variables
        the government budget
315 #constraint is:
316 def budget(x,theta,chi,a,p,mean,var,r):
317     return x[0]*aggregateearnings
318     (x[0],x[1],theta,chi,a,p,mean,var)-r
319
320 #The optimisation problem is described by:
321 def
        results(theta,chi,a,p,ep,mean,var,r,s0):
322     cons=({'type':'eq',
323 'fun':lambda x:np.array([x[1]-
324 budget(x,theta,chi,a,p,mean,var,r)])},
325 {'type':'ineq',
326 'fun':lambda x: np.array([0.95-x[0]])},
327 {'type':'ineq',
328 'fun':lambda x: np.array([x[0]])},
329 {'type':'ineq',
```

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```python
330  'fun':lambda x: np.array([x[1]])})
331      def obj(x,theta,chi,a,p,mean,var,ep):
332          return -socialwelfare(x[0],x[1],
333          theta,chi,a,p,mean,var,ep)
334      res=minimize(obj,
335      [s0,sigmaforgivent(s0,theta,chi,a,p,mean,var)],
336      args=(theta,chi,a,p,mean,var,ep),
337      constraints=cons,method='SLSQP',
338      options={'ftol':1e-10,'disp':False})
339      return res.x[0],res.x[1]
340
341  #where 's0' denotes a starting value in
         the search for the optimal
342  #t. Notice also that, via the function
         'sigmaforagivent', the
343  #value of 's0' also pins down a value for
         'sigma'.
344
345  #Running the below commands exactly
         replicates Stern 1976 (see Table 3):
346  """
347  print
         results(0,0,0.613537,0.5,0,-1,0.39,0,
         0.3)
348  print
         results(0,0,0.613537,0.6,0,-1,0.39,0,0.3)
349  print
         results(0,0,0.613537,0.7,0,-1,0.39,0,0.3)
350  print
         results(0,0,0.613537,0.8,0,-1,0.39,0,0.3)
351  print
         results(0,0,0.613537,0.9,0,-1,0.39,0,0.3)
352  print
         results(0,0,0.613537,0.99,0,-1,0.39,0,0.3)
353  (0.19161418428268773,
         0.048261860362912537)
354  (0.16954033549211536,
         0.042013877702567475)
355  (0.15363822035148467,
         0.037437739266487489)
356  (0.14179682534545596, 0.03395299261497884)
357  (0.13277474460714653,
         0.0312189909512964285)
358  (0.12641651761276695,
         0.029222167646414807)
359  """
360  #The following function loops over
         'theta', for each 'theta'
361  #printing out 'results':
362  def resultsloop(chi,a,p,ep,mean,var,r):
363      theta=0
364      while theta<=0.3:
365          print
             results(theta,chi,a,p,ep,mean,var,r)[0],
366          theta=theta+0.01
```

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