Meritocracy, Egalitarianism and the Stability of Majoritarian Organizations*

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Abstract

Egalitarianism and meritocracy are competing principles to distribute the joint benefits of cooperation. We examine the consequences of letting members of society vote between those two principles, in a context where individuals must joint with others into coalitions of a certain size to become productive. Our setup induces a hedonic game of coalition formation. We study the existence of core stable partitions (organizational structures) of this game. We show that the inability of voters to commit to one distributional rule or another is a potential source of instability. But we also prove that, when stable organizational structures exist, they may be rich in form, and different than those predicted by alternative models of coalition formation. Non-segregated coalitions may arise within core stable structures. Stability is also compatible with the coexistence of meritocratic and egalitarian coalitions. These phenomena are robust, and persist under alternative variants of our initial model.

Key words: Egalitarianism, Meritocracy, Coalition Formation, Hedonic Games, Core Stability, Assortative Mating.

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1. Introduction

Egalitarianism and meritocracy are two competing principles to distribute the joint benefits from cooperation. One could debate their relative merits and side for one or the other. Rather, we analyze the consequences of not taking sides between these two principles, and letting different organizations choose by vote between the two, in a context where this choice is part of coalition formation decisions. The lack of ability to commit "a priori" on one specific distributional criterion may lead to organizational structures and consequences that would not arise in other frameworks.

Our model is very simple.[1] We consider societies composed of n individuals who must form coalitions in order to perform certain tasks. Each individual is endowed with a productivity level. Coalitions need a minimal size, v, to be productive. Beyond that size, a coalition produces the sum of its members productivities. Agents prefer to get a higher pay than a lower one. If they must choose among organizations that will pay them the same, they prefer those whose average productivity is higher. If a coalition is formed, its members decide by majority vote whether to distribute their production according to meritocracy or to egalitarianism. Hence, the median voter in each coalition ends up determining the distributional rule: it will be meritocratic if the median's productivity is above the coalition's mean, egalitarian otherwise.

Agents know the productivities of all others and can therefore anticipate what rewards they will get from joining any given coalition. They will thus play a hedonic game (Drèze and Greenberg (1980)), defined by the preferences of agents over the coalitions they may belong to. The outcomes of such games are partitions of agents into coalitions. We concentrate on those partitions (coalitional structures) that enjoy a natural property of stability, being in the core of the hedonic game induced by our problem. Our main results refer to the characteristics of stable coalitional structures, which we interpret as the expected result of voluntary cooperation under the basic assumption that the majority chooses between the two possible distributional rules.

We examine the consequences of this form of coalition governance on the size, stability and composition of organizations and on their endogenous choice of rewards. We can then apply our understanding of coalition formation as a tool to analyze different issues within our stylized model, like the ability of coalitions to compete for talent, or their ability to keep a competitive edge under changes in their definitional parameters.

We find that, contrary to the predictions of other related models, where agents of similar

[1] We thoroughly discuss our assumptions in Section 2, after having presented the framework more formally.

characteristics tend to associate into segregated coalitions, it is possible in our case that core stability may require the cooperation among diverse individuals. More precisely, we identify general and natural conditions under which stability is not only compatible with, but even requires the formation of non-segregated coalitions. That departure from homophily, segregation or assortative matching arises in our model for societies where the abilities of the predominant agents are not extreme. We also notice that meritocracy and egalitarianism may coexist within stable societies, and that this can happen irrespectively of the segregated or non-segregated character of stable partitions. Hence, many combined characteristics can arise for general societies, but our results will make the analysis of what drives these different social characteristics, and in particular the rise of non-segregated coalitions, much more precise.

The analysis of hedonic games is never a trivial task. It always depends on the type of preferences over coalitions of agents that are admissible in the worlds under consideration. In our case, the family of preferences that agents may have over coalitions is dictated by the structure of the model and by the role that voting plays over the distribution of the benefits from cooperation. The games we confront are thus more specific than those one could postulate without reference to any particular interpretation (see Banerjee et al (2001), Bogomolnaia and Jackson (2002)), and as a result we are able to obtain clear-cut existence and characterization results for core stable coalitional structures. At the same time, our model differs from others that also give rise to specific hedonic games but restrict the preferences of agents in alternative manners (see for example Farrell and Scotchmer (1988), Alcalde and Romero-Medina (2006), Iehlé (2007), Bogomolnaia et al (2008), Papai (2011), Pycia (2012)). All of them apply to domains of preferences different than those implied by our model. For example, Pycia's includes matching problems as a special case, but then does not apply to our world because we implicitly assume an equal treatment of equals property that is not present in the matching literature. Since we cannot rely on preceding work that derives from different models, we offer a complete treatment of existence and characterization issues as an integral part of our study.

Let us now be more specific about our formal results regarding the existence of core stable organizational structures and their characteristics.

We first analyze the case where potential coalitions to be formed are so large relative to the population that only one at most can be formed. That case is interesting on its own. But we also emphasize it because the existence of stable organizational structures is shown to depend on the satisfaction of a condition, the weak top coalition property, that is sufficient for stability under any general hedonic game, and in that case also turns out to be necessary. The weak top coalition property, first introduced in Banerjee at al (2001), is used at that point but also along the rest of the paper. Next, we turn to the analysis of three type societies, where individuals are restricted to have three possible productivity levels: high, medium and low. Modeling a society through such a three-way partition is certainly limitative, but also a reference case, that is resorted to in other contexts[2]. A major contribution of our paper is the full characterization of stable coalitional structures for this special case, and its generalization to what we call three-way clustered societies, that is, societies with an arbitrary number of types but whose members are clustered into at most three distinct coalitions of agents whose productivities are "similar" within each cluster and yet "sufficiently differentiated" across them.

The detailed study of three-way clustered societies is complex, but its essential features can be grasped by a close examination of the case where there are only agents with three types and $v = n/2$: that is where the number of agents in society allows for the possibility of just forming two coalitions. In that case, the conditions under which non-segregated coalitions necessarily arise as part of the unique core stable organizational structure become transparent. Interestingly, these cases correspond to situations where the predominant type in society is the middle one, while high and low types are relatively few. More in general, the cases where non-segregation must be expected are also identified.

Our model is definitely simple, yet complicated enough to analyze. Hence, we offer it as a first step, that we hope provides robust enough insights, and may be used as a starting point for further developments. As a first test of its robustness we present a variant of it, where the effort that agents contribute to coalitions may vary, depending on the reward they expect to get.

In this new set up meritocracy is easier to sustain, and this works in favor of the stability of segregated organizations. Yet, we can still identify a family of societies admitting core stable structures where non segregation necessarily arises.

Our work contributes to the literature on the endogenous formation of institutions, local public goods, clubs, and sorting (Tiebout (1959), Schelling (1969), Caplin and Nalebuff (1994), Ellickson et al (1999), Piccione and Razin (2009) and Morelli and Park (2014)). Piccione and Razin (2009) and Morelli and Park (2014) share our main general motivation: examining coalition formation

[2] People are classified by social status into the elite, the middle and the lower class; countries are classified into developed, developing and less developed, etc..

jointly with endogenous institutions of distribution in societies where individuals are vertically differentiated in productivities. Both of these papers assume more complex production possibilities and coalition externalities than us; their coalition formation games are not hedonic. What distinguishes our formulations is the assumption that coalitions are committed to a majoritarian decision, which combined with very simple production assumptions delivers an hedonic game. By choosing a framework of maximal simplicity and using the very fundamental concept of core stability, we hope to signal that the difficulties and features that arise here are deeply rooted in the analysis of the coalition formation problem.

The paper is organized as follows. After this introduction, Section 2 presents the basic model and discusses a variety of examples that announce the main messages of the paper. Sections 3, 4 and 5 discuss the existence of stable organizational structures, and their characteristics under different situations. In Section 3 we emphasize sufficient conditions and their consequences for the case where productive organizations must be large relative to society's size. Section 4 is devoted to three-type societies, and Section 5 extends its results to the case of three clustered societies. Section 6 introduces the possibility of variable effort levels, and Section 8 concludes with some final remarks.

2. The basic model and its derived organizational structures

In this section we present our basic model, and we then illustrate, by means of examples, the richness of implications that arise from it, regarding the variety of organizational structures that may occur in stable societies, their sensitivity to different distributions of productivities, and the role of voting as a possible source of instability. The examples are also used to provide an overview of some of the formal results that will be presented in subsequent sections, on the existence and characterization of stable organizational structures.

Let $N = \{1, 2, ..., n\}$ be a set of n individuals characterized by their individual potential productivities $\lambda = (\lambda_1, ..., \lambda_n)$ with $\lambda_1 \geq ... \geq \lambda_n \geq 0$. Non-empty subsets of N are called *coalitions*. Individuals can only become productive if they work within a coalition $G \subseteq N$ of size at least v. Coalitions of smaller size produce nothing, while coalitions of size v or larger produce the sum of their members' productivities. A *society* is represented by a triple (N, λ, v).

We refer to a coalition of cardinality less than v as being *unproductive*. The top set $T = \{1, ..., v\}$ contains the first v agents in terms of productivity. Similarly for any $G \subset N$, $T(G)$ denotes the first v agents in G.

We denote the average productivity of a coalition $G \subseteq N$ by $\bar{\lambda}_G$, and by λ_G the vector of productivities of the agents in G.

If a productive coalition is formed, its total production must be distributed among the agents of the coalition. Agents prefer to get a higher than a lower pay. Lexicographically, if they must choose among organizations that will pay them the same, they prefer those whose average productivity is higher.

Productive coalitions internally decide, by majority voting, whether to distribute their product in an egalitarian or in a meritocratic manner. That is, whether all agents in the organization G get the same reward, $\bar{\lambda}_G$, or each one is rewarded by its productivity, λ_i. There is no way to commit a priori to any of these two principles. A majority in coalition G will favor meritocracy if the productivity of the median, $\lambda_{m(G)}$, is greater than $\bar{\lambda}_G$. Otherwise, the majority will be for egalitarianism. Ties are broken in the following way: if there are more than one median agent, ties are broken in favor of the agent with the highest productivity. If the productivity of the median agent is equal to the mean productivity, we consider that the coalition is meritocratic. These tie breaking rules are just a convention and inconsequential for the results.

We assume that all agents are fully informed about the characteristics of all others. This is a strong but very standard assumption, especially in the theory of cooperative games. Knowing the productivities of their potential partners, they can anticipate what reward they would get if joining any given coalition. Thus, they will play a hedonic game (Drèze and Greenberg (1980)), where outcomes are partitions of agents into coalitions. A natural prediction is that stable partitions will arise from playing these games. The following definitions formalize the stability concept that we use in this paper.

Definition 1. *Given a society (N, λ, v), an organizational structure is a partition of N denoted by π. Two organizational structures, π and π', are equivalent if for all $G \in \pi$ there is $G' \in \pi'$ such that $\lambda_G = \lambda_{G'}$ and viceversa. A coalition G is segregated if given i and j in G with $\lambda_i < \lambda_j$, and $k \in N$ such that $\lambda_i \leq \lambda_k \leq \lambda_j$, $k \in G$. An organizational structure is segregated if it contains at least two coalitions and all the coalitions in the partition are segregated.*

Definition 2. *An organizational structure is blocked by a coalition G if all members in G are strictly better off in G than in the coalition they are assigned in the organizational structure. An organizational structure is core stable if there are no coalitions that block it.*

We concentrate on the concept of core stability, rather than resorting to the often used concept of Nash stability, and this has interesting implications regarding interpretation . Models of coalition formation often assume implicitly that coalitions, when forming, may be conditioned by some exclusion rules, indicating under what conditions can agents enter the coalitions they would like to join. The role of such rules has been analyzed in Jehiel and Scotchmer (2001), who present a variety of them, ranging from no exclusion to unanimity requirements for admission. In our world, stability arises when coalitions of agents voluntarily agree to form, and they need not admit anyone who would decrease their level of satisfaction. Thus, informally, we interpret the use of core stability to be closely connected with the assumption of unanimity, while the Nash approach would rather reflect a world with no exclusion rules.

Now, as promised in the introduction, let us discuss our main assumptions.

Our assumption that productive coalitions must reach a minimum size v is a very simple way to introduce positive group externalities in production. It is indeed a drastic, very stark specification, and one can think of others. An alternative specification that would still leave us within the realm of hedonic games would come from assuming that a coalition is productive only if the sum of its member's productivities reaches a minimal thershold. A richer one would condition the required size for a coalition to become productive on the characteristics of the rest of coalitions with which they must co-exist. This would certainly lead to a more complex game, beyond the hedonic framework. These and other forms of externalities in production would be worth studying, and possibly fit some applications better than our present formulation. But since our assumption delivers a tractable model and has meaning, we take it as a significant first step.

Our assumption that preferences are essentially based on individual rewards but are lexicographically complemented by the aggregate productivity of the group is again a very simple form of introducing an externality, this time on the valuation of outcomes. The specification has content: we may think of the lexicographic component as a measure of the prestige associated with joining a coalition. And its introduction still allows for the model to be tractable. It is important to point out that our results are robust to the natural change in the specification of preferences that would just drop the lexicographic component. That would not change our main conclusions, though existence proofs would be easier and some of our uniqueness results would not be so sharp, due to the increase of stability that these less discriminating preferences would bring with them. Our text includes comments on the potential consequences of dropping this assumption in any instance where it plays a significant role[3].

Another assumption is that agents will be equally productive regardless of the reward they get. In Section 6 we present a model showing that our results are robust, even if agents can condition their effort to their pay.

We now present different examples that will illustrate the richness of implications arising from the fact that agents do vote on distributional issues.

Our first example shows two important and independent points. The first one is that in a core stable organizational structure different reward systems may coexist. The second one is that a core stable organizational structure may be non-segregated. The example shows how people that are diverse may find an advantage to get together for distributional reasons.

Example 1. *A society with stable non-segregated organizations where different reward systems coexist.*
Let $N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $v = 5$, *and* $\lambda = (100, 100, 75, 75, 75, 75, 75, 75, 75, 45)$. *Let* $G_1 = \{1, 3, 4, 5, 10\}$ *and* $G_2 = \{2, 6, 7, 8, 9\}$. *Note that* G_1 *is meritocratic and* G_2 *is egalitarian. Let us see that the organizational structure* $\pi = (G_1, G_2)$ *is core stable. Note that the medium type agents in* G_2 *can only improve if a high type is added to the coalition or if a medium type is substituted by a high type. But since the other high type not in* G_2 *is already in a meritocratic coalition, he does not have incentives to form the potential blocking coalition. The two high types cannot be together in a meritocratic coalition, and any other agent needs high types to improve. That implies that* π *is a core stable organizational structure. Note that high and medium productivity agents are split between the two coalitions. Any other core stable organizational structure is equivalent to this one.*

Notice, for further reference, that this is an example of a society with three types of individuals, defined by three different productivity levels, and whose numbers allow to form exactly two productive coalitions. We'll see in Section 4 that for societies with these characteristics, stability is always guaranteed (Proposition 4) and that in fact this is a special case where the core stable organizational structures are necessarily non-segregated and unique (Proposition 5).

At this point, we want to emphasize that in our model it is not only possible, but even necessary in many cases, to have non-segregated coalitions in stable societies. This is in sharp contrast with

[3] See footnotes 9 to 14 in the text, and Example 8 in the Appendix.

the results that one would obtain if one of our two distributional rules was imposed, or, in the other extreme, if the choice of rewards was fully open to negotiation. If agents were forced to adopt a fixed distributional rule, either meritocracy or egalitarianism, the stable organizational structure would be segregated: the one where the v most productive agents get together, then the next v most productive form a second coalition, and so on, thus eventually leaving some agents out of any productive coalition. Segregation would also be the consequence of stability in the polar case where agents could freely bargain how to distribute the gains from cooperation. Whenever a (core) stable allocation of gains exists, it must be one where each agent gets her productivity. This forces all stable societies to be segregated[4]. For example, when $n = kv$ for some integer k, the unique stable structure would again be the one we just described, under a meritocratic reward scheme.

By contrast, in our model, the ability of societies to vote between our two distributional criteria gives rise to the possibility of non-segregated stable organizational structures. Even more: non-segregation may become necessary for stability in some cases we shall be able to pinpoint[5].

Unfortunately, existence issues arise, both under free bargaining and in our case. When there are no constraints on the possible rewards to agents, guaranteeing their productivity to the highest productive agents may require to leave some of the lowest in an unproductive coalition. Then those individuals have reservation value zero and can be offered low rewards to form blocking coalitions. As we shall see, instability may also be unavoidable in our model for some societies. But, as shown in the examples that follow (Examples 2 and 3), it will arise in subtler ways than in the case of bargaining. Among other differences, in our case stability may be compatible with some agents being left into unproductive coalitions (Example 4).

Example 2. *A society with no core stable organizational structures.*

Let $N = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $v = 5$, and $\lambda = (100, 100, 75, 75, 75, 75, 75, 75, 30)$. In any organizational structure there will be at most one productive coalition. The egalitarian coalition that contains the two high type and three medium type agents dominates any organizational structure containing some other productive egalitatarian coalition. In turn, if that maximal egalitarian coalition is part of an organizational structure, this will be dominated by the meritocratic coalition

[4] Our assumption that, under equal rewards, individuals lexicographically favor coalitions of greater mean is important here. Otherwise, any structure where all individuals are in some productive coalition and each gets his productivity would be stable as well.

[5] In Bogomolnaia et al (2008) non-segregated groups also arise from the combination of voting and group formation. In their model agents in a group decide by vote the location of a public good, but share its cost equally.

formed by one high, three medium and the one low type agents. Notice however that all productive coalitions that contain both high type agents are egalitarian. Therefore, organizational structures with a productive meritocratic coalition must leave at least one high type in the unproductuive coalition. But then, this structure will be blocked by the egalitarian coalition formed by one high and four medium type agents. Thus, there is no core stable organizational structure.

For further reference, notice that in this example it is only possible to form one productive organization, at most. Thus, we are in a world where the minimal size of productive organizations is large relative to the overall population. This case is examined in Section 3, and there we provide a necessary and sufficient condition for existence of stable organizational structures. This condition requires societies to satisfy the weak top coalition property, a sufficient condition for stability under any general hedonic games that turns out to be also necessary in this case.

In the following example we show that the issue of existence may also arise in societies allowing for larger numbers of potential coalitions, and even if a priori there is no need to leave any agent outside of a productive coalition.

Example 3. *A society with no core stable organizational structure,* $n = 3v$*.*
Let $N = \{1, ..., 9\}$, $\lambda = (100, 75, 75, 75, 25, 25, 25, 25, 25)$, $v = 3$. *In order to prove that no organizational structure is stable, it is enough to show that, in a stable structure, the high type productivity agent cannot belong to an unproductive coalition, cannot be part of an egalitarian coalition, and cannot be part of a meritocratic coalition. Clearly, if the high productivity agent is in unproductive coalition, no matter how the other agents are organized, medium type agents will always prefer to form a productive coalition with the high type one. If the high type is in an egalitarian coalition, it has to be the one with the greatest mean that leaves behind one of the medium type agents. The rest of the society has to be organized in a stable way, which implies an egalitarian coalition with productivities* $(75, 25, 25)$ *and a meritocratic coalition with productivities* $(25, 25, 25)$*. The high type agent, together with the medium type agent in the second coalition and a low type agent in the third coalition, can form a meritocratic coalition that blocks that organization. Finally, if the high type is in a meritocratic coalition, this coalition contains medium type agents, but independently of how the rest of agents are organized, the coalition of medium type agents blocks that organization.*

Once more, for further reference, observe that we are again in a case with only three types of agents, as in Example 1. However, the size of society now allows for more than two productive coalitions to form, whereas in Example 1 only two coalitions at most could arise. As we shall see in subsection 4.2, this larger relative size of society does no longer guarantee that stability holds. More specifically, our example here fails to satisfy a condition that we identify in subsection 4.2 and Proposition 5, as being necessary and sufficient for the existence of stable organizational structures in general, three type societies[6].

The next example shows that, unlike in the case where individuals could freely bargain for their rewards, instability is not necessarily associated with the existence of agents who are left out of productive coalitions. It also shows that even in the event where several coalitions of minimal productive size could form, stability may generate the emergence of larger coalitions.

Example 4. *A case where $n = kv$, and yet no partition of agents into coalitions of size v can achieve stability.*
Let $N = \{1, 2, 3, 4, 5, 6\}$, $v = 3$, and $\lambda = (50, 40, 40, 35, 25, 10)$. Let (P, U) be an organizational structure where $P = \{1, 2, 3, 5\}$ and it is meritocratic and $U = \{4, 6\}$ and it is an unproductive coalition. (P, U) cannot be blocked because P is the meritocratic coalition with the highest mean and the only agent that could improve without using anyone from P is agent 4 but $\{4, 5, 6\}$ is meritocratic. The egalitarian coalition with the greatest mean is $E = \{1, 2, 3\}$, $N \backslash E$ is meritocratic. The organization $(E, N \backslash E)$ is blocked by $G = \{1, 4, 6\}$ which is a meritocratic coalition with a greatest mean than $N \backslash E$. Any organization with two meritocratic coalitions or one meritocratic and one unproductive coalition is blocked by P, any organization with two egalitarian coalitions or one egalitarian and one unproductive coalition is blocked by E. Any organization where the most productive agent is part of an egalitarian coalition different from E and the other coalition is meritocratic is blocked by E. It can be checked that any other organization is blocked by P. Thus, (P, U) is the unique core stable organizational structure.

The examples that follow are intended to show that our model is amenable to perform some comparative static analysis. Before we introduce them, let us clearly state that this type of exercise is well grounded, because in the sections that follow we shall identify conditions guaranteeing that core stable equilibria are "almost unique", in a well defined sense. Our examples below conform to society characteristics implying almost uniqueness, as it was also the case in our Example 1.

We first remark that the issue of stability is related to the size of minimal productive coalitions in a non-trivial manner.

[6] This condition requires that societies be structured, according to Definition 6 in subsection 4.2.

Example 5. *Changes in v can be either stabilizing or de-stabilizing.*
Let $N = \{1, 2, ..., 7\}$ and $\lambda = (100, 84, 84, 84, 84, 60, 60)$. Suppose that initially $v = 4$.
Note that medium type agents can form a coalition by their own with a payoff of 84. The egalitarian coalition with the greatest mean is blocked by the meritocratic coalition containing the high, one medium and two low type agents. Any meritocratic G with the high type is blocked by the four medium agents together. No organizational structure is stable.
But, if $v = 3$, (G_1, G_2, U), with $G_1 = \{1, 2, 6\}$ and $G_2 = \{3, 4, 5\}$ both meritocratic and $U = \{7\}$ unproductive is a core stable organizational structure, because the high type is in a meritocratic coalition and he cannot increase the mean above 84 while keeping meritocracy.

Our last example is suggestive of a variety of applications that might derive from our model, if embedded in a more general setting. Let us first exhibit the example and then comment on its possible implications.

Example 6. *Changes in v and n can modify the distributional criteria in stable organizational structures.*
Let $N = \{1, ..., 14\}$, $v = 7$, $\lambda = (10, 10, 7, 7, 7, 7, 1, 1, 1, 1, 1, 1, 1, 1)$.
The organization structure $(T, N\backslash T)$, where the top set T is meritocratic and $N\backslash T$ is egalitarian is core stable. Assume now that the size of coalitions, and the set of potential participants must be reduced to $v' = 5$ and $N' = \{1, ..., 10\}$. It would seem natural to fire the two worse people of each coalition, so that the productivities in this new society are $\lambda' = (10, 10, 7, 7, 7, 1, 1, 1, 1, 1)$. The organization $(T', N\backslash T')$, remains core stable for (N', v', λ'). Yet, in that case, the top coalition in this organization becomes egalitarian. Whereas, if the first organization would have fired two of the medium productivity agents, rather than the two low ones, the core stable partition of the resulting smaller society would still be meritocratic.

This example has been chosen to identify the potential consequences of changes in parameter v that may be interpreted as budget cuts. We do not want to exaggerate the importance of the example, but notice that it could become the starting point of a study regarding the ability of societies to compete in a larger world. What happens in the example is that the best coalition may end up shifting from meritocracy to egalitarianism at equilibrium. We have not modeled external competition for high level individuals, but we could assume that the most productive agents are likely to get outside options involving rewards higher than average. Under these unmodeled but reasonable assumption, our example is a warning that changes that make viable smaller coalitions may have a high decapitalizing effect in societies where distributional decisions are made by the majority.

Similar and apparently anomalous phenomena would arise as the potential result of other parametric changes. In the same example, if the low type members would upgrade their qualifications close to the medium type, say from 1 to 6, meritocracy would also be lost in stable organizational structures.

3. Sufficient conditions for core stability, and their necessity when organizations must be "large"

Simple sufficient conditions assuring the existence of core stable organizational structures are easy to describe. For any distribution of productivities guaranteeing that segregated coalitions are meritocratic, any organization of society into segregated coalitions of minimal size is core stable. This is the case for example, under a uniform or concave distribution[7], that is, when for any three consecutive agents i, j, k with $\lambda_i \leq \lambda_j \leq \lambda_k$, $\lambda_k - \lambda_j \leq \lambda_j - \lambda_i$. Other environments where the existence of core stable organizational structures is guaranteed are those where all agents have the same productivity (one type societies), or are divided into two sets, the set H of n_H identical individuals of high type and a set L of $n - n_H$ identical individuals of low type (two type societies). Existence in the first case is trivial. In the second case, if $n_H \geq v$, the organizational structure (H, L) is trivially core stable. If $n_H < v$, the reader may check that the organizational structure $(T, N \backslash T)$ is also core stable. In the following section we shall discuss the much richer case where agents come in three different types, and show that existence issues become challenging then.

We now turn attention to a more general condition, that is in fact sufficient for existence of core stable organizational structures in general hedonic games: *the weak top coalition property* (Banerjee et al, 2001). We begin by proving that identifying weak top coalitions in our model, when they exist, is an easy task (Proposition 1). In addition to its intrinsic interest, this result is used in subsequent sections, when searching for potential candidates to form core stable organizational structures. We then show that the weak top coalition property has additional bite in societies where $n < 2v$, and thus only one productive coalition can be formed, at most. For these simple societies requiring large minimal size organizations relative to total population, the weak top coalition property is necessary

[7] In the dual case, where all segregated groups are egalitarian, the organization of society into segregated groups of minimal size does no longer assure core stability.

and sufficient for core stable organizational structures to exist (Proposition 2). Finally, we identify those societies that are sufficiently small for existence to be guaranteed in any case (Proposition 3).

Definition 3. *A coalition $W \subseteq G \subseteq N$, is a weak top coalition of G if it has an ordered partition $(S_1, ..., S_l)$ such that (i) any agent in S_1 weakly prefers W to any subset of G, and (ii) for any $k > 1$, any agent in S_k needs cooperation of at least one agent in $\cup_{m<k} S_m$ in order to form a strictly better coalition than W. A game satisfies the weak top coalition property if for any coalition $G \subseteq N$, there exists a weak top coalition W of G.*

If the weak top coalition property is satisfied, a core stable organizational structure, $(G_1, ..., G_m)$ always exists and can be constructed by sequentially selecting weak top coalitions from the population: G_1 is the weak top coalition of N, G_2 the weak top coalition of $N \backslash G_1$, and so on [8].

We can now show that in our model, weak top coalitions, if they exist, must have a very specific and simple form. This fact will greatly simplify our discussion of stability, and is therefore an important step to be repeatedly used in our proofs.

Before discussing the form of weak top coalitions, let's introduce the notion of a congruent coalition (Le Breton et al (2008)).

Definition 4. *A coalition $C \subseteq G \subseteq N$, is a congruent coalition of G if for all $i \in C$, and for all $S \subset G$ such that S is a strictly better coalition than C for i, there is an agent $j \in S \cap C$ such that C is a strictly better coalition than S for this agent j.*

Note that any weak top coalition of G is a congruent coalition of G.

We can now state our characterization result for weak top coalitions of G.

Proposition 1. *Let $M_+(G)$ be the set of meritocratic coalitions of G with the greatest mean, and let $E_+(G)$ be the set of egalitarian coalitions of G with the greatest mean. A coalition W is weak top coalition of G if and only if it is a congruent coalition of G, and either belongs to $M_+(G)$ or to $E_+(G)$.*[9]

[8] Stronger conditions can be found in the literature that guarantee core stable organizational structures. For example, the Top Group Property (TGP), requires that any group G of agents contains a subgroup that is the best subset of G for all of its members (Banerjee et al, 2001). The TGP is a relaxation of the common ranking property introduced by Farrell and Scotchmer (1988). Under those conditions the core is nonempty and it has a unique element.

[9] If we drop the lexicographic assumption on preferences among equal reward coalitions, the set of potential weak top coalitions will still include $E_+(G)$ and now will be enlarged to any congruent meritocratic group, in addition to $E_+(G)$.

The proof is presented in the Appendix.

Consider next societies where organizations must be relatively large so that only one productive coalition can be organized, i.e. $v > n/2$. In these societies the weak top coalition property is necessary and sufficient for the existence of core stable organizational structures.

Proposition 2. *A society where $v > n/2$ has core stable organizational structures if and only if N has a weak top coalition.*[10]

Proof. Sufficiency is clear: just partition the society into the weak top coalition of N and leave the other agents together in an unproductive coalition.

Necessity follows from the fact that if a partition $\pi = (P, N \backslash P)$ is in the core, $P \in M_+(N)$ or $P \in E_+(N)$. Since π cannot be blocked, there is no coalition $S \subseteq N$ such that all $i \in S \cap P$ are better off in S than in P. Thus, P is congruent and by Proposition 1 it is a weak top coalition of N. ■

A direct application of Proposition 2 is the following.

Proposition 3. *In societies where $v > 3n/2$ a weak top coalition of N always exists. Therefore, there are always core stable organizational structures.*

Proof. Let $T = \{1, ..., v\}$. If the top set T is meritocratic, it is trivially a weak top coalition of N and thus the core is not empty. Let us see that if T is egalitarian it is also a weak top coalition of N. Note first that all agents with productivity below the mean are in their best coalition. Only agents above the mean could improve. But, since the coalition is egalitarian, the mean is above the median and thus the coalition that can improve has a cardinality smaller than $v/2$. But the unproductive coalition $I = \{v + 1, .., n\}$ also has a cardinality smaller that $v/2$. Thus, there is no way of forming a coalition that can improve upon T. ■

Note, however, that existence of core stable organizational structures is not guaranteed when $n/2 < v < 3n/2$ as we have shown in Example 2, where neither the meritocratic coalition with the greatest mean (the coalition of the medium productivity agents), nor any of the egalitarian coalitions with the greatest mean (the two high plus three medium productivity agents) are weak

[10] Proposition 2 still holds if we drop the lexicographic assumption on preferences among equal reward coalitions. Since more weak top coalitions can exist, as remarked in the previous note, the set of stable organizational structures may be enlarged.

top coalitions of N.[11]

Finally, let us make clear that when $v \geq n/2$, the weak top coalition property is not a necessary condition for the existence of core stable organizational structures. This can be checked in Example 1.

4. Three-type societies

In this section we begin to study the benchmark case where agents can be classified into three classes. As we already remarked in the introduction, the study of such cases is standard and productive in many contexts. What we add here is that agents within each class have exactly the same productivity level, which we identify with their type. In Section 5 we extend r esults to the case where the three classes can still be clearly identified and yet productivities can differ across individuals within each class.

Formally, in a three type society, (N, λ, v), a generic type is denoted by j, $j \in \{h, m, l\}$, and productivities are $\lambda_h > \lambda_m > \lambda_l$. We denote by H, M, and L the sets of all high, medium and low type agents respectively, and by n_H, n_M and n_L the cardinality of these sets. The order of individuals of the same type is arbitrary and will have no effect on our results. Note that because of this arbitrariness, any two organizational structures which only differ in the numbering of individuals of the same type are equivalent. In what follows when we refer to uniqueness of core stable organizational structures, we mean that they are all equivalent.

In Section 4.2 we present a general, necessary and sufficient condition for the existence of stable organizational structures for three type societies. Before that, in Section 4.1, we analyze the special case where $v = n/2$. This case is interesting for several reasons. One is that, in that case, existence is always guaranteed. Moreover, we can then identify and characterize those societies where non segregation is not only possible but in fact is required for stability, and the line of proof for our characterization result in this admittedly very special case already contains the main features of the proof for the more general case we consider right after, while avoiding some complications.

[11] This non existence problem will not be alleviated by dropping the lexicographic assumption on preferences among equal reward coalitions. See Example 2, where that assumption plays no role.

4.1. The case of three types and $v = n/2$

Our initial purpose in this section is to prove that in this case stable organizational structures will always exist. Remember that in Section 3 we already proved that, should there only be one or two types in society, existence is guaranteed. So, we must just prove it for the non-degenerate case where there is at least one agent of each type. To do so, it is useful to concentrate on the segregated partition $(T, N/T)$ where a coalition of most productive agents of size v is formed, and the rest of agents gather together in a second coalition. We shall prove that either this structure is in the core, or else a different organizational structure will be core stable, unique and include a non-segregated coalition.

To distinguish between these two cases, let us classify societies by introducing the distributional characteristics of productivities that will mark the difference between their stable structures. The definition that follows has technical consequences, but we want to emphasize that it covers situations that will plausibly apply in many applications: it requires that the bulk of population be of a medium type, with a few highly productive agents and also some low productivity agents, and imposes some additional constraints on the ability to form meritocratic coalitions involving the three types.

Definition 5. *A society is maximally mixed meritocratic if* $n_H < v/2$, $n_L \leq v/2$, *and* $(\lambda_h + \lambda_m + n_L\lambda_l)/(n_L + 2) \leq \lambda_m$.

In maximally mixed meritocratic societies we can always construct a meritocratic coalition of cardinality v that contains agents of all three types, all agents of the low type and the highest number of high types allowing for all the preceding characteristics to hold. We call this a *maximally mixed meritocratic coalition*, and denote it by $M3$. This coalition can be constructed as follows. Start with all n_L low types, one medium type and one high type. This starting coalition may not be productive, but the mean of its $\lambda's$ is below λ_m. Next add as many high types as possible while keeping the mean of the $\lambda's$ below λ_m. And finally, if the coalition is not yet productive, fill the set with medium types until reaching size v. Note that $N\backslash M3$ is either an egalitarian coalition with high and medium type agents, or a meritocratic coalition with only medium type agents. Remark that an organizational structure that contains a coalition with the characteristics of $M3$ is non-segregated.

Proposition 4. *(a) In three type societies where $v = n/2$, stable organizational structures always exist.*
(b) If societies are maximally mixed meritocratic, then the structure $(M3, N/M3)$, where $M3$ is non-segregated, is the only stable one.
(c) If societies are not maximally mixed meritocratic, then the segregated organizational structure $(T, N/T)$ is stable, and there is at most one another stable structure. [12]

The proof of of Proposition 4 is in the Appendix. The reader may want to read that proof carefully before going to the more general one of Proposition 5, because, as already pointed out, this one is a good and simpler introduction to the main ideas that also appear in the latter.

Let us highlight the features of stable structures in our special case, since they will basically extend when we allow for $n > 2v$.

One first lesson refers to segregation. For societies that are maximally mixed meritocratic, stability implies non-segregation, as proven in Propositions 4. For societies that are not, we can assert for sure that stability holds for the segregated structure $(T, N\backslash T)$, but this is sometimes compatible with the existence of a second stable structure which may be non-segregated.

A second set of remarks refer to the combinations of reward schemes that are compatible within core stable organizational structures. In societies that are maximally mixed meritocratic, at least one of the coalitions in a stable structure must be meritocratic, while the second coalition may adopt meritocracy or egalitarianism. In societies where $(T, N\backslash T)$ is stable, each one of the two sets can adopt any of the two distributional criteria. Moreover, note that in this case the resulting distributional criteria are determined by the number of agents of each type that belong to each of the two sets, and not on the exact values of their productivities.

The (almost) uniqueness results in the present section provides the grounds for the use of comparative statics that we have discussed in the Examples of Section 2.

4.2. Three-type societies: the general case.

In this subsection we discuss the characteristics of three-type societies where core stable organizational structures exist and also the form that these structures take under different conditions.

[12] This is a case where the lexicographic assumption on preferences among equal reward coalitions has some consequences. All statements remain true, except that the non-segregated organizational structure described in (b) is no longer the unique stable one: the segregated organizational structure with minimal size productive coalitions would also be.

We shall distinguish between two sets of societies, that we call structured and unstructured, and prove that the limits between the two indeed determine whether or not core stability can be attained. We can prove that core stable organizational structures will exist in a society if and only if it is structured.

The reader will appreciate that many of the ideas that arose in the preceding subsection do come back, but with some additional complications that were avoided in the case where only two productive coalitions could be formed.

Since the definition of a structured society is complex, we start by describing its characteristics from two different perspectives.

First, regarding the type of coalitions that may be part of core stable structures. We will prove that such structures must either contain the top set T or some meritocratic coalition G with high type agents. Although this does not provide a full description of the whole structure, it points at a salient coalition in it. We'll say that core stable partitions must be structured around T or around some meritocratic G, meaning that one of these sets has to be part of the partition and that the rest of society must be able to accommodate the further requirements imposed by overall stability. As a result, stability requires in all cases that some of the high type agents are part of a coalition where they get their best possible treatment. They will either be all part of the best egalitarian coalition, when no stable partition can be structured around any meritocratic coalition containing high types, or else some of them will manage to structure a stable organization around a meritocratic coalition, where they get paid their full productivity, even if sometimes at the expense of other high type agents.

Second, we can look at the requirements that separate these two types of societies. In order to be unstructured, a society must have a rather special distribution of types. In particular, it must satisfy at least the following requirements: (i) It must be that the number of high type agents is less than $v/2$. Otherwise, they could form a meritocratic coalition including all of them, and let the remaining members of society, which will now be of at most two types, to organize in a stable manner. (ii) In addition, unstructured societies must contain a number of middle types that is bounded above and below, so that $v \leq n_H + n_M < 2v$. This is because a very small middle class, when coupled with a small high class, cannot de-stabilize a partition structured around T, while a large enough middle class will leave room for T to structure a stable partition again, this time thanks to the fact that the remaining middle type agents not in T will be able to achieve the highest mean meritocratic coalition, the one formed by medium type agents alone. In the case $n_H + n_M < 3v/2$, unstructured societies must contain a "sufficient" number of low types to allow high type agents to challenge a partition structured around T with a meritocratic coalition. Finally, (iii) unstructured societies are not able to satisfy medium type agents. Any partition structured around a meritocratic coalition G with high type agents can always be challenged by some of the medium type agents.

We will write $H(G)$, $M(G)$, and $L(G)$ to denote respectively the high, medium and low type agents in G. The formal definition of a structured society is as follows.

Definition 6. *A three type society is structured if at least one of the following three conditions holds:*

1. N has a weak top coalition.

2. Either $(n_H + n_M)\frac{1}{2} \geq v$, or $(n_H + n_M)\frac{2}{3} < v$ and for all G meritocratic such that all $i \in G \cap T$ are better off in G than in T, $0 \leq n - 2v < \#L(G)$.

3. There exists a meritocratic coalition G_1 with $G_1 \cap H \neq \emptyset$ and $\#(N \backslash G_1) \geq v$ such that:

(a) $\bar{\lambda}_{G_1} \geq \bar{\lambda}_G$ for all meritocratic coalition $G \subset (G_1 \cup H(G_2) \cup G_3)$ where $G_2 = T(N \backslash G_1)$ and $G_3 = N \backslash (G_1 \cup G_2)$, and

(b) Either $\#(H \cup M) \backslash G_1 = v$ or $\#(M \cup H(G_2)) < v$, $M \subset G_1$ and $\bar{\lambda}_{T(M \cup (N \backslash G_1))} < \lambda_m$.

A three type society is unstructured if it is not structured, that is, if none of the above conditions holds.

Note that condition 1 is a limited version of the weak top coalition condition. Recall that the latter is a sufficient condition for the existence of core stable structures in general hedonic games. Here we only need to require the existence of a weak top coalition of N, the set of all agents. Also remark that, in view of Proposition 1, this condition is an easy one to check. Given its transparency, we do not elaborate any further regarding it. Condition 2, then, specifies that a society may still be structured, this time around the top set T, in the absence of a weak top coalition for N, provided the set of middle productivity agents is "small enough" or "large enough", in the sense of point (ii) in our preceding discussion. Notice that these cases essentially extend the ideas we discussed when $v = n/2$, for the case where the segregated partition is stable. Similarly, though with some added complication, condition 3 provides conditions for the existence of a stable organizational structure around a non-segregated coalition, in the spirit of the maximally mixed meritocratic societies discussed in subsection 4.1.

Thus, the definition of a structured society is "nested" in the following sense: Condition 1 is a sufficient condition for existence of stable organizational structures. If condition 1 does not hold, condition 2 is sufficient for the existence of stable organizational structures, and finally, if neither condition 1 nor condition 2 hold, condition 3 is sufficient for the existence of stable organizational structures. Furthermore, if none of the conditions hold the core is empty. The following proposition formally states these results.

Proposition 5. *There exist core stable organizational structures for a three type society if and only if the society is structured.*[13]

Proof. *Part 1: Structured societies have core stable organizational structures.*
For each condition assuring a structured society we describe how to construct a core stable organizational structure.

(i) Suppose condition 1 holds, i.e. there exist weak top coalitions in N. We first argue that there will always be one weak top coalition W such that $N \backslash W$ contains only two types. This is because

- if $n_H \geq v$, then H is weak top (in fact top), and therefore $N \backslash H$ contains two types of agents, medium and low.

- if $n_H < v$, and T is meritocratic, T is weak top and $N \backslash T$ contains at most two types of agents, medium and low.

- if $n_H < v$, T is egalitarian and weak top, then $N \backslash T$ contains at most two types of agents, medium and low.

- if $n_H < v$, T is egalitarian but not weak top, then any weak top coalition W must be meritocratic with highest mean. W must contain some high type agents, because all agents in a meritocratic coalition without high type agents will gain from adding one high type, whether this enlarged set is egalitarian or meritocratic. In addition, W must contain all medium type agents, because if one of them was left out, adding that agent would increase the coalition mean while keeping meritocracy. Then $N \backslash W$ contains at most two types of agents, high and low.

Let us now construct a core stable structure. Take a weak top coalition W such that $N \backslash W$ contains only two types. We have just shown that this is always possible. Let W be one of the coalitions in the organizational structure. Note that because $N \backslash W$ it is composed of only two

[13] A similar result could be proved if we relaxed the lexicographic assumption on preferences among equal reward coalitions. It would involve a weakening of our present definition of structured societies, since it is clear that without the assumption more stable organizational structures may exist.

types, it has a core stable organizational structure; combining this structure with W we obtain a core stable structure for our initial society.

(ii) Suppose that condition 1 does not hold but condition 2 holds. Since condition 1 does not hold, $T = \{1, .., v\}$ is egalitarian, thus $n_H < v/2$.

If $n_H + n_M \geq 2v$, high and medium types alone can form two productive coalitions. Let $G_1 = T$, $G_2 = M \backslash T$, and $G_3 = L$. Clearly (G_1, G_2, G_3) is a core stable organizational structure.

If $n_H + n_M < 2v$, then $n_H + n_M < 3v/2$ and $0 \leq n - 2v < \#L(G)$ for every meritocratic coalition G such that all $i \in G \cap T$ are better off in G than in T. Let $G_1 = T$, $G_2 = \{v + 1, .., 2v\}$ (G_2 is an egalitarian coalition given that $n_H + n_M < 3v/2$), and $G_3 = N \backslash (G_1 \cup G_2)$ a coalition of low types. Again (G_1, G_2, G_3) is a core stable organizational structure. This is because the potential blocking coalition of this structure is a meritocratic coalition G that contains low type agents. But since low type agents in G_2 are in an egalitarian coalition, they cannot be part of the blocking, and since $n - 2v < \#L(G)$, for any of those potential meritocratic coalitions blocking π, low type agents in G_3 are not enough to form the potential blocking coalition G.

(iii) Last, suppose that condition 1 and 2 fail but condition 3 holds.

First of all note that, because of the failure of 1 and 2, $n_H < v/2$ and $n_H + n_M < 2v$.

Second, because 3 holds, there exists a meritocratic coalition G_1 with $G_1 \cap H \neq \emptyset$ and $\#(N \backslash G_1) \geq v$ satisfying (a) and (b). Let $\pi = (G_1, G_2, G_3)$ where $G_2 = T(N \backslash G_1)$ and $G_3 = N \backslash (G_1 \cup G_2)$.

If $\#(H \cup M) \backslash G_1 = v$, G_2 is either an egalitarian coalition with high and medium types or just a meritocratic coalition with medium type agents if all high type agents are in G_1, and G_3 is a coalition of low types. If $\#(H \cup M) \backslash G_1 \neq v$, all the medium type agents are in G_1, G_2 is an egalitarian coalition with high and low types and G_3 is a coalition of low type agents if any. In both cases, conditions a and b guarantee that π cannot be blocked.

Part 2: Unstructured societies have no core stable organizational structures.

Assume that neither 1 nor 2 nor 3 hold and that a core stable organization structure π exists. Let $G \in \pi$ such that $G \cap H \neq \emptyset$. We show that G cannot be meritocratic, nor egalitarian, nor unproductive, which is a contradiction.

(i) Assume G is meritocratic.

Since condition 1 does not hold, there are no weak top coalitions in N. Then $n_H < v/2$, because otherwise the top set T would be a meritocratic coalition and it would be a weak top coalition of N.

Thus, if G is a meritocratic coalition it must include three types of agents. Since there is no weak top coalition, $\#N\backslash G \geq v$, because otherwise, if the remaining agents are in an unproductive coalition, π can be blocked. Apart from G, no other productive coalition $G' \in \pi$ with three types can be meritocratic. Otherwise the medium type agents in the coalition with lower average productivity can switch to that other coalition. This generates a meritocratic new coalition with a greater average productivity that blocks π. So, if π contains another productive coalition G' with three types, that G' must be egalitarian and it must contain all the high type agents in $(H \cup M)\backslash G$. If $\overline{\lambda}_{G'} > \lambda_m$, replacing a low type in G' by one of the medium types in G increases the average and keeps egalitarianism, and this later coalition blocks π. But if $\bar{\lambda}_{G'} \leq \lambda_m$ we contradict that π is core stable as well - since switching one of the medium types from G' to G increases the average in G and keeps meritocracy. Thus, agents in $N\backslash G$ can only be organized in two-types coalitions, and the high types in $N\backslash G$ are in an egalitarian coalition. Note also that medium type agents cannot be in a coalition with just low type agents, because by joining G they increase the mean while keeping meritocracy, and this new coalition will block π. Thus, π contains $G_2 = T(N\backslash G)$, which is either egalitarian with high and medium types, or meritocratic with just medium type agents (if all high type agents are in G), or egalitarian with high and low types if G contains all the medium agents. In any case, the remaining agents, $N\backslash(G \cup G_2)$ are low type agents.

Since condition 3 does not hold, either (a) or (b) fails.

If (a) fails, a meritocratic coalition $G' \subset (G \cup H(G_2) \cup G_3)$ where $G_2 = T(N\backslash G)$ and $G_3 = N\backslash(G \cup G_2)$ exists with $\bar{\lambda}_{G'} > \bar{\lambda}_G$. Since only high type agents in G_2 are potentially part of this meritocratic coalition, G' blocks π.

If (b) fails, then $\#(H \cup M)\backslash G \neq v$. Since, as we argue above, π cannot place medium type agents in a coalition with just low type agents, then $\#(H \cup M)\backslash G < v$. Thus, all medium type agents are in G, and π organizes $N\backslash G$ with an egalitarian coalition with high and low types and a coalition of low type agents alone. If $\#(M \cup H(G_2)) \geq v$, then the coalition of cardinality v with high types not in G and medium type agents is egalitarian (or meritocratic if only contains medium type agents) and blocks π. If $\#(M \cup H(G_2)) < v$, the average productivity of $T(M \cup (N\backslash G))$ is greater than λ_m, which implies that $T(M \cup (N\backslash G))$ is an egalitarian coalition which blocks π.

Because of all the above points, high type agents cannot be in a meritocratic coalition.

(ii) Assume next that G is egalitarian.

Then, since there are no weak top coalitions and high type agents cannot be in a meritocratic coalition, it must be that $G = T$. Since condition 2 does not hold, $n_H + n_M < 2v$ and either $n_H + n_M \geq 3v/2$ or there exist a meritocratic coalition G^* such that all $i \in G^* \cap T$ are better off in G^* than in T and $n - 2v \geq \#L(G^*)$.

In the first case, any organizational structure containing the top set T, where agents in $N \backslash T$ are organized in a stable way, is such that $T(N \backslash T)$ is a meritocratic coalition with medium and low types, and the remaining agents are just low type agents. Since T is not weak top, a meritocratic coalition G' exist such that all $i \in G' \cap T$ are better off in G' than in T. This meritocratic coalition contains high type agents in T and medium and low types in $N \backslash T$. Medium type agents in $N \backslash T$ are in a meritocratic coalition and low type agents are also in meritocratic coalitions or alone. Then G' blocks π because (1) high type agents in $G' \cap T$ are better off in G' than in T, and (2) medium and low types in G' are better off than in their respective coalitions because G' has a greater mean.

In the second case, $T(N \backslash T)$ is egalitarian, and the low agents in $T(N \backslash T)$ cannot be used to block π with a meritocratic coalition. But, since condition 2 fails, then a meritocratic coalition can be constructed that blocks π. This is because the remaining low types not in T neither in $T(N \backslash T)$ are enough to construct G^*.

(iii) To conclude, assume G is unproductive.

Given that $h \in G$ is very welcome in any coalition, T blocks π.

Hence, there are no core stable organizational structures. ■

Remark 1. *(a) Note that when $n < 2v$, conditions 2 and 3 in the definition of a structured society never hold because they involve restrictions that only apply when more than one coalition can form. Hence, if $n < 2v$ a society is structured if and only if N has weak top coalitions. This remark leads us directly to the necessary and sufficient condition for the existence of core stable organizational structures that we already discussed in Proposition 2.*

(b) Also note that, in a structured society that fails to satisfy conditions 1 and 2, all stable organization structures are non-segregated[14]. They are structured around a meritocratic coalition G that may or may not contain all high type agents. If G leaves some high type agents out, these must be organized in an egalitarian coalition. If G contains all the high type agents, there must be enough medium type agents out of G to form a productive coalition by themselves. These conditions are the analogue of the maximally mixed meritocratic property for general three-type societies.

[14] If we relaxed the lexicographic assumption on preferences among equal reward coalitions, there still exist societies where all stable organizational structures are non segregated, as shown by Example 8 in the Appendix.

5. Three way clustered societies

In this section we extend our analysis of societies that can be divided into three classes to a much more general case than the one we just considered. We now allow for agents within a class (or cluster) to have different productivities, provided the agents in each class are sufficiently similar, relative to that of agents in other classes, in terms that are made precise in the definition that follows. With some adjustments, we provide a new definition of structured societies within this larger context, and prove that being structured in the extended sense is very much related to the existence of stable organizational structures, which again can be of different forms depending on distributional characteristics. This extension proves that our previous results, based on a simplified model are robust, even if we do not get a full characterization result as we did before.

Definition 7. *A society $\mathcal{S} = (N, \lambda, v)$ is three clustered if there exists a partition of N into three coalitions $\{H, M, L\}$ (clusters)[15] with the following properties:*
C1. For all $h \in H$, $m \in M$, and $l \in L$, $\lambda_h > \lambda_m > \lambda_l$.
C2. For any $J \in \{H, M, L\}$, all segregated productive subcoalitions of J are meritocratic.
C3. For any J, $J' \in \{H, M, L\}$, $J \neq J'$ such that $\lambda_i < \lambda_j$ for all $i \in J$, $j \in J'$, and for any $S_J \subseteq J$ and $S_{J'} \subseteq J'$ $\lambda_i < \bar{\lambda}_{S_J \cup S_{J'}} < \lambda_j$ for all $i \in S_J$, for all $j \in S_{J'}$.
C4. For all $S_H \subseteq H$, $S_L \subseteq L$ and $j \in M$ and $S_M \subseteq M$, if $\bar{\lambda}_{S_H \cup \{j\} \cup S_L} < \lambda_j$ (resp $> \lambda_j$), then $\bar{\lambda}_{S_H \cup S_M \cup S_L} < \lambda_i$ (resp $> \lambda_i$) for all $i \in S_M$.

Condition $C1$ just requires that clusters must be formed by agents whose productivities are correlative in the natural order, and thus allows to properly speak about the high, the medium and the low cluster. All the agents with the same productivity must belong to the same cluster. Condition $C2$ is an intracluster condition. It always holds if for example productivities of the agents in a cluster are uniformly distributed or have a concave distribution, that is, for any three consecutive agents $i, j, k \in J$ with $\lambda_i \leq \lambda_j \leq \lambda_k$, $\lambda_k - \lambda_j \leq \lambda_j - \lambda_i$. Conditions $C3$ and $C4$ are intercluster conditions. Condition $C3$ requires that there should be enough "distance" between any two clusters. Condition $C4$ requires that the average of productivities for any set containing elements of the three clusters should be "strictly between" clusters. That is, either it belongs to the interval $(\min_{j \in S_H} \lambda_j, \max_{j \in S_M} \lambda_j)$ or to the interval $(\min_{j \in S_M} \lambda_j, \max_{j \in S_L} \lambda_j)$.

[15] When this does not lead to confusion and in order to avoid repetitions we may sometimes refer to those agents belonging to the same cluster as being of the same type. Notice however that unlike in the preceding section this loose way to speak does to imply that two members of a cluster are identical.

The following notation will be useful in what follows. Given a society (N, λ, v) and any set $G \subseteq N$ of cardinality n_G, k_G denotes the maximal number of productive coalitions of size v in G and $r_G = n_G - k_G v$. Subsets of G are denoted S_G. The partition of the first $k_G v$ elements of G into k_G segregated minimal size productive coalitions is denoted by $(S_G^1 ... S_G^{k_G})$: that is, $S_G^1 = T(G)$ and $S_G^k = T(G \backslash \cup_{k=1}^{k-1} S_G^q)$.

Remark 2. *Our definition allows for three clustered societies which are degenerate in the sense that some of the clusters may be empty. In these cases, it is easy to prove that core stable organizational structures exist. When only one cluster in non empty, only the intracluster condition $C2$ is operative. And then, the segregated partition of the $k_N v$ most productive agents into k_N meritocratic coalitions of size v, along with an unproductive coalition formed by the r_N less productive agents is trivially core stable.*
In societies with two non-empty clusters, say H and L, let R_H be the set that contains the last r_H agents in cluster H and at most the $v - r_H$ most productive agents in cluster L. If $n_L < v - r_H$, R_H is an unproductive coalition and the structure $(\{S_H^k\}_{k=1}^{k_H}, R_H)$ is core stable. If $n_L \geq v - r_H$, let $\hat{L}$ be the remaining agents in the low cluster, that is, $\hat{L} = L \backslash R_H$. Then $(\{S_H^q\}_{q=1}^{k_H}, R_H, \{S_{\hat{L}}^q\}_{q=1}^{k_{\hat{L}}}, U)$, where U is an unproductive coalition formed by the $r_{\hat{L}}$ less productive agents is a core stable structure.

We now turn to the non degenerate case with three non empty clusters. Our first result refers to the distribution of agents from the high cluster within any core stable organization.

Proposition 6. *If a three cluster society $\mathcal{S}$ has a core stable structure, then at most $v - 1$ agents in H belong to coalitions containing agents from other clusters.*

Proof. Let $\hat{\pi}$ be a core stable organizational structure for society $\mathcal{S}$. Denote by S_H the subcoalition of the high cluster whose agents are assigned in $\hat{\pi}$ to coalitions containing individuals from other clusters. Refer to coalitions containing agents from at least two clusters as mixed coalitions. Assume that $\#S_H \geq v$. If all mixed coalitions containing agents from S_H are egalitarian, by condition $C3$ the high type members receive a payoff below their productivity. In this case, the coalition S_H, which has a greater average, will block $\hat{\pi}$ independently of its regime. If some of the mixed coalitions containing agents from S_H are meritocratic, we distinguish two cases:
(i) Suppose that there is at least a productive subcoalition of S_H which is meritocratic. Then, this subcoalition constitutes a blocking coalition of $\hat{\pi}$, because it is meritocratic and has a greater mean that any of the other coalitions in $\hat{\pi}$ containing agents from S_H.

(ii) Suppose all productive subcoalitions of S_H are egalitarian. Consider the meritocratic coalition in $\hat{\pi}$ containing agents from S_H with the greatest mean. Call this coalition G. Let $j \in G$ be the agent in G not in S_H with the greatest productivity in G. Form the coalition $G' = S_H \cup \{j\}$. The coalition G' is meritocratic because agents in S_H form a majority and, by $C3$, the average of the coalition is between the productivity of the less productive agent in S_H and λ_j. If $G' \neq G$, then G' is a blocking coalition of $\hat{\pi}$. If $G' = G$, suppose first that some agents of the high cluster not in S_H are organized in an egalitarian coalition. This implies that some of those agents are receiving less than their productivity. Add those agents to G'. The new coalition is meritocratic with a greater mean than G', and will block $\hat{\pi}$. Suppose now that all agents outside S_H are organized in meritocratic coalitions. Since S_H form an egalitarian coalition, it is non-segregated nor are some of the coalitions with high types outside S_H. Order the coalitions in $H \backslash S_H$ so that the first one is the one that contains the highest productivity agent, the second the one which contain the highest productivity agent among the remaining agents, and so on. Consider the first coalition in this order which is non-segregated and let i be the agent with the greatest productivity in that coalition. Form the segregated productive coalition of cardinality v that contains i as the highest productivity agent. Note that to form this coalition we could use agents in S_H. Clearly this new meritocratic coalition will block $\hat{\pi}$.

All the above arguments imply that at most $v - 1$ agents in H belong to coalitions containing agents from other clusters. ■

In view of Proposition 6 it is important to understand the characteristics of core stable organizational structures in societies with at most $v - 1$ agents in the high cluster.

We first define a condition that is necessary and sufficient for the existence of core stable organizational structures for such societies. It is a natural extension of our previous notion of structured societies.

Definition 8. *A non degenerate three clustered society with $n_H < v$ is structured if the following holds:*

1. N has a weak top coalition.

2. For all G meritocratic such that all $i \in G \cap T$ are better off in G than in T, either the society $(N \backslash T, \lambda_{N \backslash T}, v)$ has a core stable structure, π_1, such that $\#\{i \in M \backslash T \mid (\text{payoff of } i \text{ in } \pi_1) < \lambda_i\} < \#M(G)$, or $\#\{i \in L \backslash T \mid (\text{payoff of } i \text{ in } \pi_1) \leq \lambda_i\} < \#L(G)$.

3. There exists a meritocratic coalition G_1 with $G_1 \cap H \neq \emptyset$ and $\#(N \backslash G_1) \geq v$ such that:

(a) $\bar{\lambda}_{G_1} \geq \bar{\lambda}_G$ *for all meritocratic coalitions* $G \subset (G_1 \cup H(G_2) \cup G_3)$ *where* $G_2 = T(N \backslash G_1)$ *and* $G_3 = L \backslash (G_1 \cup G_2)$.

(b) Either the society $((H \cup M) \backslash G_1, \lambda_{H \cup M) \backslash G_1}, v)$ *has a core stable structure with segregated coalitions all of them productive, or* $\#(M \cup H(G_2)) < v$, $M \subset G_1$ *and* $\bar{\lambda}_{T(M \cup (N \backslash G_1))} < \lambda_m$.

Proposition 7. *A non degenerate three clustered society with* $n_H < v$ *has a core stable organizational structure if and only if it is structured.*

The proof is similar to that of Proposition 5 and is presented in the Appendix.

Finally, we provide two results regarding core stability in societies with v or more agents in the high cluster. One is a necessary condition and the other a sufficient condition for existence. Both are based on our previous results.

For this purpose, we introduce some additional notation.

Given a non degenerated three cluster society $\mathcal{S}$, let $\mathbf{C}^H$ be the set of core stable structures for (H, λ_H, v), the subsociety formed by the high cluster agents. For any $\pi \in \mathbf{C}^H$, let U^π be the set of unproductive agents in π and let $\mathcal{S}^\pi = (U^\pi \cup M \cup L, \lambda_{U^\pi \cup M \cup L}, v)$. That is: we take those high type agents, U^π, that would be in an unproductive coalition within a stable organization π of the high cluster, and consider the subsociety, $\mathcal{S}^\pi$, that they would form along with agents in the medium and low clusters.

Proposition 8. *If a three cluster society* $\mathcal{S}$ *has a core stable structure, then there exists* $\pi \in \mathbf{C}^H$ *such that subsociety* $\mathcal{S}^\pi$ *is structured.*

Proof. The proof is a direct consequence of Propositions 6 and 7 and the fact that subpartitions of a stable organization must be stable within their subsociety. ■

Recall that by Remark 2 the subsociety (H, λ_H, v) has at least one core stable structure, namely the segregated partition. Denote it by $\pi^s = (\{S_H^k\}_{k=1}^{k_H}, R_H)$. With this notation, the sufficient condition reads as follows.

Proposition 9. *Consider a three clustered society* $\mathcal{S}$. *If the subsociety* $\mathcal{S}^{\pi^s} = (R_H \cup M \cup L, \lambda_{R_H \cup M \cup L}, v)$ *is structured then* $\mathcal{S}$ *has a core stable organization.*

Proof. Take $\pi^s \in \mathbf{C}^H$, that is $\pi^s = (\{S_H^k\}_{k=1}^{k_H}, R_H)$, and let $\pi(\mathcal{S}^{\pi^S})$ be a core stable structure of $\mathcal{S}^{\pi^c}$. Let us see that $\pi(\mathcal{S}) = (\{S_H^k\}_{k=1}^{k_H}, \pi(\mathcal{S}^{\pi^c}))$ is a core stable structure of $\mathcal{S}$. If a set G blocks

$\pi(\mathcal{S})$ it must contain agents from $H \backslash R_H$ and agents from $M \cup L$. But, given conditions $C3$ and $C4$, the high type agents in a mixed coalition are always worse off than in a meritocratic coalition with just high type agents (as they are in $\{S_H^k\}_{k=1}^{k_H}$). This holds because the average of productivities in a mixed coalition is always smaller than the productivity of the less productive agent in the high type cluster. Thus, $\pi(\mathcal{S})$ is core stable. ■

For this general case we do not reach a full characterization result. There is some gap between the necessary and sufficient condition for existence. The necessary condition is not sufficient because the productive coalitions in the core stable partition of the high cluster may not match consistently with the core stable partition of the rest of society to form an overall stable organization. The sufficient condition is not necessary because the segregated partition of the high cluster need not be the only form to organize those agents within a core stable organization of the whole society.

6. Endogenous Effort

Our model has assumed that agents contributions to production are independent of the reward system. This is consistent with our basic purpose in this paper, which is to analyze the consequences of voting for one of two distributional criteria when neither undermines productive efficiency. But we believe that, in fact, reward systems will affect effort whenever effort is costly and agents are allowed to choose how much to contribute to the coalitions they join. In this section we present a simple model where individual effort decisions are strategic, and agents are still allowed to vote between meritocracy and egalitarianism. Clearly, in such a model, the decision to join a meritocratic coalition will become favored by the fact that, under this reward scheme, the most productive workers will be willing to exert more effort. We can show that, even within this more elaborate version of the model, our basic conclusion that different regimes can coexist at equilibrium still holds. Hence, we can interpret our basic model as one that gives the most advantage to the emergence of egalitarianism, but whose main results persist after the productive benefits of meritocracy are taken into account.

Here is the model. Given a society (N, λ, v) we assume that a coalition G with cardinality $g \geq v$, produces $\sum_{i \in G} \lambda_i e_i$, where e_i is the voluntary effort of agent i, which has (individually incurred) cost $\frac{1}{2} e_i^2$. That is, if agent i exerts effort e_i she obtains:

$$\pi_i^M = \lambda_i e_i - \frac{1}{2} e_i^2, \text{ if } G \text{ is meritocratic, and}$$

$$\pi_i^E = \frac{\sum_{j \in G} \lambda_j e_j}{g} - \frac{1}{2} e_i^2, \text{ if } G \text{ is egalitarian.}$$

We assume that for each coalition G and each reward regime agents choose efforts simultaneously and non-cooperatively. Hence endogenous efforts will be determined by the unique Nash equilibrium of this non cooperative game. It is easy to check that the Nash equilibrium choice of efforts are as follows.

In an egalitarian coalition individuals have strong incentives to free ride; they exert effort only in a fraction $1/g$ of their productivity $e_i^E = \lambda_i / g$. Hence, the payoffs from membership in egalitarian coalition G are

$$\pi_i^E = \frac{\sum_{j \in G} \lambda_j^2}{g^2} - \frac{\lambda_i^2}{2g^2}.$$

On the other hand, in a meritocratic coalition individuals exert effort equal to their productivity $e_i^M = \lambda_i$. Hence, the payoffs from membership in any meritocratic coalition are

$$\pi_i^M = \frac{\lambda_i^2}{2}.$$

Preferences regarding meritocracy and egalitarianism inside each productive coalition are a bit more complex than in the baseline model. Agent $i \in G$ prefers meritocracy rather than egalitarianism if and only if $\pi_i^M \geq \pi_i^E$, or equivalently,

$$\frac{\lambda_i^2}{2} \geq \frac{\sum_{j \in G} \lambda_j^2}{g^2 + 1}. \tag{6.1}$$

Additionally, only if necessary to compare two coalitions with the same regime and identical payoff; the lexicographic preference for greater per capita production applies.

Consider now the vote inside each coalition. Note that if the median member of coalition G prefers meritocracy to egalitarianism, then all agents with a greater productivity share this preference. Hence, the median productivity member of each coalition remains decisive voter of the coalition in the present set up.

Meritocracy prevails more often than in the baseline model, since condition (6.1) may hold for a median with productivity $\lambda_{m(G)} \leq \overline{\lambda}_G$. However the qualitative results of our baseline model are robust. In particular, scenarios where stable organizations structures deliver non-segregated coalitions and heterogeneous distribution regimes still exist.

For three-type societies where $n = 2v$ the conditions analogous to the "maximally mixed meritocratic societies" that deliver a non-segregated structure in the core are the following:

1. $n_H < v/2$, $n_L \leq v/2$, and $(\lambda_M/\lambda_H)^2 < (2n_H)/((v-1)^2+2n_H)$, i.e., T is egalitarian and $N \backslash T$ is meritocratic, and

2. $\lambda_M^2/2 \geq (\lambda_H^2 + \lambda_M^2 + n_L\lambda_L^2)/((n_L+2)^2+1)$, i.e. T is not weak top.

Example 7 is a society where the stable organization is non-segregated and different coalitions select different regimes.

Example 7. *Endogenous effort and a society with stable non-segregated organizations and different regimes.*
Let $N = \{1, ..., 14\}$, $\lambda = (13, 13, 13, \sqrt{8}, \sqrt{8}, \sqrt{8}, \sqrt{8}, \sqrt{8}, \sqrt{8}, \sqrt{8}, \sqrt{8}, 0, 0, 0)$, $v = 7$.
T *is egalitarian (because* $\lambda_M^2/2 = 4 < 7.1 = (\sum_{j \in T} \lambda_j^2)/(g^2+1)$*);* $N \backslash T$ *is meritocratic (because* $\lambda_M^2/2 = 4 > 3.2 = (\sum_{j \in T} \lambda_j^2)/(g^2+1)$*) with an average production equal to* 4.5714*. But* $(T, N \backslash T)$ *is not stable because the coalition of agents with productivities* $(13, \sqrt{8}, \sqrt{8}, \sqrt{8}, 0, 0, 0)$ *(where the three medium type agents are from* $N \backslash T$*) is a meritocratic coalition with an average production of* 27.571 *that blocks* $(T, N \backslash T)$*.*
The structure $\{(1, 4, 5, 6, 12, 13, 14), (2, 3, 7, 8, 9, 10, 11)\}$ *where the first coalition is meritocratic and the second is egalitarian is (uniquely) stable.*

7. Concluding Remarks

We have presented a very simple model of coalition formation where people are driven to cooperate by a minimal size requirement, and choose their reward schemes by majority. This model is able to generate a variety of interesting stylized facts that are under examination in different strands of literature, through more complex formulations.

Societies in equilibrium can generate non-segregated partitions, where agents of several types do mix even if they could have joined individuals of the same or the closest type. This is a direct consequence of the fact that coalitions choose among two rewards schemes by vote, and it is a phenomenon that would not occur if agents were forced to use one of the two, or else allowed to freely bargain on the distribution of their joint output. Moreover, non-segregation may arise under meritocracy or under egalitarianism, and these two regimes can coexist. The possibility of non-segregation, which may arise in different cases, does become a requirement for stability in well-defined cases, according to our characterization results. These cases are simple to interpret when agents are only of three types and at most two coalitions can be formed. Then, non-segregation is required for the stability of societies with an abundant middle class and a small number of high and low-productivity agents. In more general cases, the conditions under which segregation in at last some of the coalitions become necessary to achieve stability are still spelled out in our results.

Let us comment on the sensitivity of our results to alternative specifications of the model. We have already shown, by means of the example, in Section 6 that allowing agents to condition their effort within coalitions to their expected rewards does not alter our conclusions regarding non-segregation and diversity of regimes. We have also tested, in further work, that the characteristics of our discrete coalition formation model can be retained by an alternative formulation involving a continuum of agents, a familiar assumption in the public economics literature.

An important assumption in our model that deserves comment is that agents must chose between only two reward systems. Our reward systems can be seen as resulting from a model of tax choice where a proportional tax t is levied and its proceeds are equally distributed: egalitarianism corresponds to the case $t = 1$ and meritocracy arises when $t = 0$, since voters will always favor one of these two extreme cases as their best choice. Our implicit assumption that taxes are proportional and its proceeds are equally redistributed would certainly be worth relaxing by considering a larger variety of reward schemes. In further research we are exploring an extension of our model that will link our work more directly to the literature on optimal taxation and mobility (Mirrlees (1971, 1981), Epple and Romer (1991), Puy (2007), Morelli et al (2012), Bierbrauer et al (2013)).

References

[1] Alcalde, J. and Romero-Medina A., "Coalition Formation and Stability", *Social Choice and Welfare,* (2006) 27, 365-375.

[2] Banerjee, S. , H. Konishi and T. Sonmez, "Core in a simple coalition formation game", *Social Choice and Welfare* (2001),18, 135-153.

[3] Bierbrauer, F., C. Brett, and J. Weymark, "Strategic nonlinear income tax competition with perfect labor mobility", *Games and Economic Behavior* (2013), 292-311.

[4] Bogomolnaia, A. and M.O. Jackson. "The stability of hedonic coalition structures", *Games and Economics Behavior* (2002), 38, 201-230.

[5] Bogomolnaia, A., M. Le Breton, A. Savvateev and S.Weber. "Stability of jurisdiction structures under the equal share and median rules", *Economic Theory* (2008), 34, 525-543.

[6] Caplin, A., and B. Nalebuff, "Competition Among Institutions", *Journal of Economic Theory* (1997),72, 306-342.

[7] Drèze, J.H., and J. Greenberg, "Hedonic Coalitions: Optimality and Stability", *Econometrica* (1989), 48, 987-1003.

[8] Ellickson, B., B. Grodal, S, Scotchmer and W. Zame, "Clubs and the Markets", *Econometrica* (1999), 67, 1185-1218.

[9] Epple, D. and T. Romer, "Mobility and redistribution", *Journal of Political Economy* (1991), 99, 828–858.99, 828–858.

[10] Farrell, J., and S. Scotchmer, "Partnerships", *Quarterly Journal of Economics* (1988), 103, 279-297.

[11] Iehlé, V. "The core-partition of a hedonic game", *Mathematical Social Sciences* (2007), 54(2), 176-185.

[12] Jehiel, P., and S. Scotchmer."Constitutional Rules of Exclusion in Jurisdiction Formation", *Review of Economic Studies* (2001), 68, 393-413.

[13] Le Breton, M., I. Ortuño-Ortin, S. Weber. "Gamson's Law and Hedonic Games", *Social Choice and Welfare,* (2008), 30, 57-67.

[14] Mirrlees, J. "An exploration in the theory of optimal income taxation", *Review of Economic Studies* (1971), 38, 175-208

[15] Mirrlees, J. "Migration and optimal income taxes", *Journal of Public Economics*, (1981), 18, 319-341.

[16] Morelli, M., and I. U.Park, "Internal Hierarchy and Stable Colaition Structures", (2014) *mimeo.*

[17] Morelli, M., H. Yang, and L.Ye, "Comepetitive Nonlinear Taxation and Constitutional Choice", *American Economic Journal: Microeconomics* (2012), 142-175.

[18] Piccione, M. and R. Razin, "Coalition Formation under power relations", *Theoretical Economics*, (2009), 4(1), 1-15.

[19] Papai, S. "Hedonic Coalition Formation and Individual Preferences", Mimeo (2011).

[20] Pycia, M. "Stability and Preference Alignment in Matching and Coalition Formation", *Econometrica* (2012), 80, 1, 323-362.

[21] Puy, M.S. "Skill Distributions and the Compatibility between Mobility and Redistribution Economies", *Regional Science and Urban Economics* (2007), 37, 3, 345-362.

[22] Schelling, Thomas C. "Models of segregation", *American Economic Review* (1969), 59, 488-493.

[23] Tiebout, C. "A Pure Theory of Local Expenditures", *Journal of Political Economy* (1959), 416-424.

8. Appendix

Proof of Proposition 1**.** The proof consists of two parts.

Part 1: Weak top coalitions of G are congruent coalitions of G and must belong to either $M_+(G)$ or to $E_+(G)$.

If W is a weak top coalition of G then it is a congruent coalition of G.

Next we show that if G has a weak top coalition, W, then $\bar{\lambda}_W \geq \bar{\lambda}_S$ for all $S \subseteq G \backslash W$. Suppose on the contrary that there is a coalition $S \subseteq G \backslash W$ such that $\bar{\lambda}_W < \bar{\lambda}_S$. Suppose first that there is an agent $i \in W$ such that $\bar{\lambda}_W \leq \lambda_i < \bar{\lambda}_S$. Let $S' = S \cup \{i\}$. Since $\lambda_i < \bar{\lambda}_S$, the mean productivity of coalition S' will be bigger than the productivity of i, $\lambda_i < \bar{\lambda}_{S'}$. Thus, agent i, independently of the regime will be better off in S' than in W, in contradiction with W being a weak top coalition. If there is no agent $i \in W$ such that $\bar{\lambda}_W \leq \lambda_i < \bar{\lambda}_S$, we distinguish two cases: in the first one we suppose that W is egalitarian and in the second we suppose that W is meritocratic.

If W is egalitarian, since no agent $i \in W$ exists such that $\bar{\lambda}_W \leq \lambda_i < \bar{\lambda}_S$, then an agent $i \in W$ exists such that $\bar{\lambda}_S \leq \lambda_i$. Let $S' = S \cup \{i\}$, note first that $\bar{\lambda}_W < \bar{\lambda}_{S \cup \{i\}} \leq \lambda_i$. So, independently of the regime of S', agent i will be better off in S' than in W, in contradiction with W being a weak top coalition.

If W is meritocratic, since no agent $i \in W$ exists such that $\bar{\lambda}_W \leq \lambda_i < \bar{\lambda}_S$, the median productivity of W is above $\bar{\lambda}_S$. Let $\lambda_{med}(W)$ be this median productivity. Let $i \in W$ such that $\lambda_i < \bar{\lambda}_W$. Suppose first that there is an agent $j \in S$ such that $\lambda_i < \lambda_j \leq \lambda_{med}(W)$. Let $W' = (W \backslash \{i\}) \cup \{j\}$. Note that since the productivities of agents i and j are both below the median productivity of W, replacing in W agent i by agent j does not change the median but increases the average. Thus, all agents in $W' \cap W$ are better off in W' than in W, in contradiction with W being a weak top coalition. Finally, if there is no an agent $j \in S$ such that $\lambda_i < \lambda_j \leq \lambda_{med}(W)$, then there is an agent $j \in S$ such that $\lambda_j < \lambda_i < \bar{\lambda}_W$. Let $S' = (S \backslash \{j\}) \cup \{i\}$, $\bar{\lambda}_{S'} > \bar{\lambda}_S > \bar{\lambda}_W > \lambda_i$. Thus, independently of the regime of S', agent i will be better off in S' than in W, in contradiction with W being a weak top coalition.

Suppose now that the weak top coalition is meritocratic but does not belong to $M_+(G)$. Note first that $W \cap M_+ = \emptyset$ for all $M_+ \in M_+(G)$, because otherwise, all agents in $W \cap M_+$ would strictly prefer M_+ to W contradicting that W is a weak top coalition. Since $W \cap M_+ = \emptyset$, our previous reasoning applies, and therefore $\bar{\lambda}_W \geq \bar{\lambda}_M$. But then $W \in M_+(G)$, a contradiction. The same argument applies if W is an egalitarian coalition.

Part 2: If a set in $M_+(G)$ or in $E_+(G)$ is a congruent coalition of G then it is a weak top coalition of G.

Suppose $M_+ \in M_+(G)$ is a congruent coalition of G. If M_+ is a segregated coalition with the best productivity agents in G, it is clearly a weak top coalition of G. If it is not of the preceding form, suppose that M_+ is not a weak top coalition of G. Since it is congruent but not weak top, there is no subcoalition of agents in M_+ for which M_+ is the best coalition. This implies that the most productive agent in G is not in M_+, and for the most productive agent in M_+ there is an egalitarian coalition E which is preferred to M_+. But then all agents in $E \cap M_+$ would be better off in E, in contradiction with M_+ being congruent.

Suppose finally that $E_+ \in E_+(G)$ is a congruent coalition of G. If E_+ is a segregated coalition with the best productivity agents in G, it is clearly a weak top coalition of G. If it is not of the preceding form, suppose that E_+ is not a weak top coalition of G. Since it is congruent but not weak top, there is no a subcoalition of agents in E_+ for which E_+ is the best coalition. But note that for the less productive agent in this coalition E_+ is always its best set, a contradiction. ■

Proof of Proposition 4

In fact, proving (b) and (c) implies (a). We start by statement (b).

(b1) To prove that $(M3, N/M3)$ is a stable organizational structure, first notice that medium type agents in $N\backslash M3$ can only improve upon if they can join an egalitarian coalition with highest mean. But such superior coalition must include high type agents from $M3$ that are not willing to join since $M3$ is meritocratic. High type agents in $N\backslash M3$ if any, could be better off joining an egalitarian coalition with greater mean or a meritocratic coalition. The first case is ruled out by the same argument as for medium type agents. The second is not possible either since, by construction, there is no other meritocratic coalition that can be formed without using other medium type agents from $N/M3$.

(b2) The proof that $(M3, N/M3)$ is the only core stable structure proceeds as follows.

- We first show that no structure with only one productive coalition can be core stable. Such productive coalition would have to be weak top. Candidates to be weak top coalitions are $G \in E_+(N)$, or $G \in M_+(N)$.

If $G \in E_+(N)$, G has size v, contradicting that the organizational structure includes only one productive coalition. In a maximally mixed meritocratic society, $n_H < v/2$, $n_L \leq v/2$, and consequently $n_M > v$, which imply that the top set T is egalitarian, $T \in E_+(N)$, and any other

$G \in E_+(N)$ is equivalent to T.

If $G \in M_+(N)$, then G contains only medium type agents. This is because any meritocratic coalition with high type agents has the mean below the productivity of the medium type agents, and $n_M > v$. But coalitions composed only of medium type agents are never weak top, because its members always prefer to add high types to their coalition.

- We now concentrate in organizational structures containing two productive coalitions $(G_1, G_2) \neq (M3, N \backslash M3)$, and prove that there will always be a coalition blocking (G_1, G_2).

(i) If G_1 and G_2 are both meritocratic, both coalitions have three types of agents or one of them three types and the other two types, medium and low. In any case, adding the medium type agents to the coalition with greater mean forms a meritocratic coalition with increased mean that blocks (G_1, G_2).

(ii) If G_1 and G_2 are both egalitarian then none of them is T, because $N \backslash T$ is meritocratic since we are in a maximaly mixed meritocratic society by assumption. Thus, T blocks (G_1, G_2).

(iii) If G_1 is meritocratic and G_2 is egalitarian, then $G_2 \neq T$, because otherwise, $G_1 = N \backslash T$ and then $M3$ blocks $(T, N \backslash T)$. The coalition G_2 cannot have three types of agents, because by replacing low types in G_2 by medium types, the mean increases while keeping egalitarianism. This new coalition will block (G_1, G_2). Thus, G_2 can only contain two types of agents. Since $n_H < v/2$, $n_L \leq v/2$, G_2 contains only high and medium types. Since G_2 is different from $N \backslash M3$ it must contain more high type agents. But then, given the construction of $M3$, we can replace medium type agents in G_1 by high type agents while keeping meritocracy and increasing the mean, and this new coalition will block (G_1, G_2).

Thus, $(M3, N \backslash M3)$ is the unique core stable organizational structure.

(c1) The existence statement in part (c) follows from the analysis of different possibilities, that we take in turn. If society is not maximally mixed meritocratic, then either $n_H \geq v/2$, or $n_L > v/2$, or $n_H < v/2$ and $n_L \leq v/2$ but $(\lambda_h + \lambda_m + n_L \lambda_l)/(n_L + 2) > \lambda_m$.

- If $n_H \geq v/2$, $T \in M_+(N)$, and therefore is a weak top coalition. Thus, $(T, N \backslash T)$ is a core stable organizational structure.

- If $n_H < v/2$, but $n_L > v/2$, T can be meritocratic (with three types) or egalitarian (with high and medium types). In the first case $T \in M_+(N)$, and therefore is a weak top coalition. Thus, $(T, N \backslash T)$ is a core stable organizational structure. In the second case, (T, B) is such that T is egalitarian and B is either egalitarian or meritocratic with just low type agents. In any of the situations (T, B) is clearly a core stable organizational structure.

- Finally, if $n_H < v/2$ and $n_L \leq v/2$ but $(\lambda_h + \lambda_m + n_L\lambda_l)/(n_L + 2) > \lambda_m$, T only contains high types and medium type agents and it is egalitarian, $N\backslash T$ contains only medium and low type agents and is meritocratic. Condition $(\lambda_h + \lambda_m + n_L\lambda_l)/(n_L + 2) > \lambda_m$ implies that high type agents cannot be part of a meritocratic coalition, thus T is a weak top coalition of N and $(T, N\backslash T)$ is a core stable organizational structure.

(c2) We distinguish three cases.

Case 1. Assume $n_H \geq v/2$.

There may exist a second core stable structure if (i) $G \in E_+(N)$, $\#G > v$, and G is a weak top coalition, or (ii) $G \in E_+(N)$, $\#G = v$, and $N\backslash G$ is also egalitarian[16].

If (i), since G is a weak top coalition, $(G, N\backslash G)$ is core stable and only G is productive.

If (ii), since both G and $N\backslash G$ are egalitarian, $(G, N\backslash G)$ is a core stable organizational structure with two productive coalitions. To see that, note that no coalition can block $(G, N\backslash G)$ because such coalition would have to be meritocratic and thus formed by agents that are receiving less than their productivity in $(G, N\backslash G)$. Given that both coalitions in $(G, N\backslash G)$ are egalitarian, those agents are the ones whose productivities are above the mean of the coalition, and since the mean is above the median, they are less than $v/2$ in each coalition. Hence they cannot form a productive coalition blocking $(G, N\backslash G)$.

Let us see that, apart from this possible second core stable structure, there can be no other.

In structures $(P, N\backslash P)$ where only P is productive, if P is not weak top, there will exist a productive coalition G such that all $i \in P \cap G$ will be better off in G than in P. Since all $i \in (N\backslash P) \cap G$ are getting zero in $N\backslash P$, they will also be better off in G. Thus, G will block $(P, N\backslash P)$. Hence, P has to be weak top, and the unique candidate in this case is the one described in (i).

Finally, let us show that any structure (G_1, G_2) with two productive coalitions different from $(T, N\backslash T)$ and the one considered in case (ii) will be unstable.

(1) If G_1 and G_2 are meritocratic, it is blocked by T which is also meritocratic.

[16] This last situation can only happen if $v_L > v/2$. To see this, note that, since T is meritocratic, high type agents have to be distributed between G and $N\backslash G$. Furthermore, let us see that all medium type agents have to be in G. If $\bar{\lambda}_G < \lambda_m$, the median agent is a low type agent, and $N\backslash G$ has to contain three types. Adding a high, a medium, and a low type agent to G from $N\backslash G$ will create a new egalitarian group of higher mean, contradicting that $G \in E_+(N)$. If $\bar{\lambda}_G \geq \lambda_m$, adding a high and a medium type to G from $N\backslash G$ will create a new egalitarian group of higher mean. Again, this contradicts that $G \in E_+(N)$. Thus, G contains all the medium type agents. Therefore, for $N\backslash G$ to be egalitarian, $v_L > v/2$.

(2) If G_1 is meritocratic and G_2 is egalitarian we distinguish two cases.

- If all the high type agents are in G_1, G_2 can only contain medium and low types, and since it is egalitarian $\bar{\lambda}_G < \lambda_m$. But then, adding a medium type agent from G_2 to G_1 creates a new meritocratic coalition of higher mean than G_1 which blocks (G_1, G_2).

- If the high type agents are split between G_1 and G_2, we can add all missing high type agents to G_1 and drop enough non high types in G_1 to create a new coalition of size v. This new coalition will still be meritocratic, have a higher mean than G_1, and block (G_1, G_2).

(3) If G_1 and G_2 are egalitarian, neither G_1 nor G_2 are in $E_+(N)$. Thus, any egalitarian coalition $G \in E_+(N)$ will block (G_1, G_2).

Case 2. Assume $n_H < v/2$ and $n_L > v/2$.

In this case, T can be either egalitarian or meritocratic.

Case 2a. Suppose first that T is meritocratic.

Since $n_H < v/2$, T has three types of agents and consequently $N\backslash T$ is the meritocratic coalition with just low types, which implies that $n_L > v$.

As in Case 1, a second core stable structure may exist if (i) $G \in E_+(N)$, $\#G > v$, and G is a weak top coalition, or (ii) $G \in E_+(N)$, $\#G = v$, and $N\backslash G$ is also egalitarian[17]. The same argument as in Case 1 applies.

Let us see that, apart from this possible second core stable structure, there can be no other.

As explained in Case 1, structures $(P, N\backslash P)$ where only P is productive are core stable if and only if P is a weak top coalition. The unique candidate in this case is the one described in (i).

Finally, let us show that any structure (G_1, G_2) with two productive coalitions different from $(T, N\backslash T)$ and the one considered in case (ii) will be unstable.

The arguments in (a) and (b) in Case 1 apply here.

In the case that G_1 is meritocratic and G_2 is egalitarian, G_2 must contain at least two types of agents. If G_2 contains high type agents, replacing a low type agent in G_1 by a high type agent will create a new meritocratic coalition G (because T is meritocratic) of higher mean than G_1 which blocks (G_1, G_2). The same kind of argument will apply if G_2 does not contains high type agents but contains medium type agents.

Case 2b**.** Suppose that T is egalitarian. Since $n_H < v/2$, T has two or three types of agents

[17] Note that since T is meritocratic, this situation can only happen if G contains all the high type agents and $v - v_H$ low type agents and it should be such that adding a medium type changes the regime. This structure only exists if $v_H = v/2 - 1$ and $v_M < v/2$.

and consequently $N\backslash T$ is either egalitarian with medium and low types or meritocratic with only low type agents. There may exist a second core stable structure if (i) $G \in M_+(N)$, $\#G > v$, and G is a weak top coalition, or (ii) if $G \in M_+(N)$, $\#G = v$, $N\backslash G$ is egalitarian, $(N\backslash G) \cap M = \emptyset$, and the mean productivity of the coalition is below λ_m.

If (i), since G is a weak top coalition, $(G, N\backslash G)$ is core stable and only G is productive.

If (ii), since the mean productivity of $N\backslash G$ is below λ_m, $(G, N\backslash G)$ is core stable. There is no possibility of blocking because a potential blocking coalition should contain medium type agents. Since they are in a meritocratic coalition with the greatest mean, they will only participate in an egalitarian coalition with mean above their productivity. But this is not possible.

Let us see that, apart from this possible second core stable structure, there can be no other.

As explained in Case 1, structures $(P, N\backslash P)$ where only P is productive are core stable if and only if P is weak top. The unique candidate in this case is the one described in (i).

Finally, let us show that any structure (G_1, G_2) with two productive coalitions different from $(T, N\backslash T)$ and the one considered in case (ii) will be unstable.

(1) If G_1 and G_2 are both egalitarian, it is blocked by T which is also egalitarian.

(2) If G_1 and G_2 are both meritocratic, and neither G_1 nor G_2 are in $M_+(N)$, any meritocratic coalition $G \in M_+(N)$ will block (G_1, G_2). If one of them belongs to $M_+(N)$ (let us say $G_1 \in M_+(N)$), since $n_H < v/2$ and $n_L > v/2$, both G_1 and G_2 contains medium type agents. Suppose that $\bar{\lambda}_{G_1} \geq \bar{\lambda}_{G_2}$: then adding a medium type agent from G_2 to G_1 creates a new meritocratic coalition of higher mean than G_1 which blocks (G_1, G_2).

(3) If G_1 is meritocratic and G_2 is egalitarian, G_1 may contain agents of two or three types. In the first case they must be medium and low types with a majority of medium types. Thus, G_2 contains low type and high type agents and (possibly) medium types. In any case, $T \in E_+(N)$ blocks (G_1, G_2). If G_1 contains three types, G_2 can contain two or three types (with low and medium types for sure in both cases). If $\bar{\lambda}_{G_2} < \lambda_m$, adding a medium type agent from G_2 to G_1 creates a new meritocratic coalition of higher mean than G_1, which blocks (G_1, G_2). If $\bar{\lambda}_{G_2} > \lambda_m$, replacing a low type in G_2 with a medium type from G_1 creates a new egalitarian coalition of higher mean than G_2, which blocks (G_1, G_2).

Case 3. Assume that $n_H < v/2$, $n_L \leq v/2$, and $(\lambda_h + \lambda_m + n_L\lambda_l)/(n_L + 2) > \lambda_m$.

Note that the meritocratic coalition with the greatest mean in this case is M, which is not a weak top coalition. Thus, no other organizational structure with only one productive coalition can be core stable.

Let us show that any structure (G_1, G_2) with two productive coalitions different from $(T, N\backslash T)$ will be unstable.

(1) Note that G_1 and G_2 cannot be both meritocratic, since there is no meritocratic coalition that contains high type agents.

(2) If G_1 and G_2 are both egalitarian, it is blocked by T which is also egalitarian.

(3) If G_1 is meritocratic and G_2 is egalitarian, G_1 can only contain medium and low types or only medium type agents, but since this coalition is different from $N\backslash T$, G_2 must contain low type agents also. Note that since G_2 is egalitarian and low types do not constitute a majority, $\bar{\lambda}_{G_2} > \lambda_m$.

Replacing in G_2 low type agents by medium type agents from G_1 will create a new egalitarian coalition of higher mean than G_2 which blocks (G_1, G_2). ■

Proof of Proposition 7. *Part 1: Structured societies* with $n_H < v$ *have core stable organizational structures.*

For each condition assuring a structured society we describe how to construct a core stable organizational structure.

(i) Suppose condition 1 holds, i.e. there exist weak top coalitions in N. Let W be one of those weak top coalitions. Note first that $N\backslash W$ only contains agents from at most two clusters. This is because either $W = T$ and then $N\backslash T \subset M \cup L$, or W is a meritocratic coalition with agents from the three clusters. In the latter case, since W is a meritocratic coalition with maximal average productivity it is necessary that $M \subset W$ and then $N\backslash W \subset H \cup L$. Hence, by Remark 2, the two-type society $N\backslash W$ has a core stable organizational structure. The coalitions in that structure plus W constitute a core stable organizational structure for N.

(ii) Suppose that condition 1 does not hold but condition 2 does. Since condition 1 does not hold, T is egalitarian, and $(N\backslash T, \lambda_{N\backslash T}, v)$ is a two cluster society. Hence by Remark 2, $(N\backslash T, \lambda_{N\backslash T}, v)$ has a core stable organizational structure π_1. Then $\pi = \{T, \pi_1\}$ is a core stable organization of N because any coalition G potentially blocking π must be meritocratic and include agents from every cluster, and either some $i \in M \cap G$ is worse off in G than in π_1 (if $\#\{i \in M\backslash T \mid (\text{payoff of } i \text{ in } \pi_1) < \lambda_i\} < \#M(G)$), or else some $i \in L \cap G$ is worse off in G than in π_1 (if $\#\{i \in L\backslash T \mid (\text{payoff of } i \text{ in } \pi_1) < \lambda_i\} < \#L(G)$).

(iii) Last, suppose that conditions 1 and 2 fail but condition 3 holds.

There exists a meritocratic coalition G_1 with $G_1 \cap H \neq \emptyset$ and $\#(N\backslash G_1) \geq v$ satisfying a and b.

Without loss of generality suppose that $i \in G_1 \cap M$ are the agents with the lowest productivity in M (note that if this is not the case, we can always replace each of the medium type agents in G_1 by one less productive medium type agent without changing the above characteristics of G_1). Also without loss of generality, suppose that all $i \in G_1 \cap L$ are consecutive with the greater productivities in L compatible with G_1 being meritocratic. Suppose first that society $((H \cup M)\backslash G_1, \lambda_{(H\cup M)\backslash G_1}, v)$ has a core stable structure with segregated coalitions, all of them productive. Let $\pi((H \cup M)\backslash G_1)$ be this structure. Consider the following organizational structure of N : the first coalition is G_1, then all the coalitions in $\pi((H \cup M)\backslash G_1)$ and finally the core stable structure of the remaining low type agents, $\pi(L\backslash G_1)$. This structure is stable given conditions (a) and (b). Otherwise, if such core structure $\pi((H \cup M)\backslash G_1)$ does not exist, we consider the structure formed by G_1 that contains all the medium type agents (recall that since $\#(M \cup H(G_2)) < v$, medium type agents cannot form a productive coalition on their own), by $T(N\backslash G_1)$ that contains high and low types, and finally by the core stable structure of the remaining low type agents, $\pi(L\backslash(G_1 \cup T(N\backslash G_1))$. Again, conditions (a) and (b) guarantee that this is a core stable organizational structure for N.

Part 2: Unstructured societies with $n_H < v$ have no core stable organizational structures.

Assume that neither 1 nor 2 nor 3 hold and that a core stable organization structure π exists. Let $G \in \pi$ such that $G \cap H \neq \emptyset$. We show that G cannot be meritocratic, nor egalitarian, nor unproductive, which is a contradiction.

Assume G is meritocratic, let us see that the negation of conditions 1 and 3 lead to a contradiction.

Since condition 1 does not hold, there are no weak top coalitions in N. Then $n_H < v/2$, because otherwise T would be a meritocratic coalition and it would be a weak top coalition of N. Thus, if G is a meritocratic coalition it must include agents from the three clusters (by $C3$). Since there are no weak top coalitions, then $\#N\backslash G \geq v$, because otherwise, the remaining agents are in an unproductive coalition and π can be blocked. Apart from G, no other productive coalition $G' \in \pi$ with agents from the three clusters can be meritocratic. Otherwise, given $C4$, an $i \in M$ in the coalition with lower average productivity could switch to the other and increase the average productivity while keeping meritocracy, and this new coalition would block π. So, if π contains another productive coalition G' with three types, it must be egalitarian and it must contain all $i \in H\backslash G$. If $\overline{\lambda}_{G'} > \lambda_m$ for some $m \in M$, replacing an agent from L in G' by one from M in G increases the average and keeps egalitarianism, and this later coalition blocks π (given that $C4$

implies that $\bar{\lambda}_{G'} > \lambda_j$ for all $j \in G' \cap M$). But if $\bar{\lambda}_{G'} \leq \lambda_m$ for some $m \in M$, we contradict that π is core stable as well - since switching one of the agents in M from G' to G increases the average in G and keeps meritocracy. Thus, agents in $N\backslash G$ can only be organized in coalitions with agents from one or two clusters, and all $i \in H\backslash G$ are in an egalitarian coalition. Note also that an agent $i \in M \cap (N\backslash G)$ cannot be in a coalition that does not contain agents from H, because by joining G they increase the mean while keeping meritocracy, and this new coalition will block π. If $M\backslash G \neq \emptyset$, π contains $G_2 = T(N\backslash G)$ which is egalitarian with agents from H and M, or meritocratic with just agents form M (if $H \subset G$). If there are still more agents in M, they are organized in segregated meritocratic coalitions with just medium type agents. Note that they cannot be organized in egalitarian coalitions because the agents in those coalitions that receive a payoff below their productivity by joining G will increase the mean while keeping meritocracy. The rest of society is composed by agents from L. If $M\backslash G = \emptyset$, π contains $G_2 = T(N\backslash G)$ which is egalitarian with $T(N\backslash G) \subset H \cup L$, and again the remaining society is composed by agents from L. Since condition 3 does not hold, either (a) or (b) fails:

-If (a) fails, a meritocratic coalition $G' \subset (G \cup H(G_2) \cup G_3)$ where $G_2 = T(N\backslash G)$ and $G_3 = L\backslash(G_1 \cup G_2)$ exists with $\bar{\lambda}_{G'} > \bar{\lambda}_G$. Note that G' blocks π.

-If (b) fails, the society $((H \cup M)\backslash G_1, \lambda_{H \cup M)\backslash G_1}, v)$ cannot be organized in a segregated stable way with all coalitions productive for any meritocratic coalition G_1. Since, as we argued above, π cannot place $i \in M$ in coalitions without agents from H, it must be that $\#(H \cup M)\backslash G < v$. Thus, $M \subset G$, and π organizes $N\backslash G$ with an egalitarian coalition $E \subset H \cup L$ such that $H\backslash G \subset E$, and the rest of low type agents are organized in an stable way. If $\#(M \cup H(G_2)) \geq v$, then the coalition of cardinality v containing all $i \in H\backslash G$ and some medium type agents is egalitarian (or meritocratic if it only contains medium type agents) and blocks π. If $\#(M \cup H(G_2)) < v$, the average productivity of $T(M \cup (N\backslash G))$ is greater than λ_m for some $m \in M$, which implies that $T(M \cup (N\backslash G))$ is an egalitarian coalition which blocks π.

All the above points imply that a meritocratic G containing high type agents cannot be part of a core stable organizational structure of N.

Assume next that G is egalitarian. Let us see that the negation of conditions 1 and 2 leads to a contradiction.

Since there are no weak top coalitions and high type agents cannot be in a meritocratic coalition, it must be that $G = T$. Since condition 2 does not hold, any possible stable organization of the society $(N\backslash T, \lambda_{N\backslash T}, v)$ is such that $\#\{i \in M\backslash T \mid (\text{payoff of } i \text{ in } \pi_1) < \lambda_i\} \geq \#M(G)$, or $\#\{i \in L\backslash T \mid (\text{payoff of } i \text{ in } \pi_1) \leq \lambda_i\} \geq \#L(G)$. Thus, the medium and low types necessary to form the meritocratic coalition that would challenge T are available. This coalition will block π.

To conclude, assume G is unproductive. But $h \in G$ is very welcome in any meritocratic coalition (even if that changes the regime), and if there are no meritocratic coalitions, T blocks π.

Hence, there are no core stable organizational structures. ■

Example 8. *A society where all stable organizational structures are non segregated even if the lexicographic assumption on preferences among equal reward coalitions is eliminated.*

Let $N = \{1, ..., 18\}$, $v = 7$, *with three high type agents with* $\lambda_h = 100$, *seven medium type agents with* $\lambda_m = 75$, *and eight low type agents with* $\lambda_l = 64.5$. *First remark that the grand coalition is meritocratic, stable and forms a non segregated organizational structure. Moreover, notice that there exists at least another stable and non segregated organizational structure. It is the one with two productive meritocratic groups, one containing two high, four medium and five low type agents and the other containing one high, three medium and three low type agents. Finally, let us argue that segregated organizational structures are not stable. First notice that no high type can be left out of a productive coalition. Hence, any stable segregated organizational structure must contain a coalition with all high types in the same productive group. But in all segregated organizational structures, the coalition containing these three high types is egalitarian. Thus, any candidate for stability among the segregated organizational structures must contain the coalition with three high and four medium type agents, since this is the egalitarian coalition with the greatest mean. But for the same reason, it must also contain the coalition with the remaining three medium types and four low type ones, while leaving the rest of low type agents in an unproductive coalition. However, this structure is not stable because it is blocked by the meritocratic coalition formed by one high, the three medium type agents in the second coalition and the three low types in the unproductive group.*