

**ESSAYS ON INTERNATIONAL PORTFOLIO CHOICES
AND CAPITAL FLOWS**

Ning Zhang

**A Thesis Submitted for the Degree of PhD
at the
University of St Andrews**



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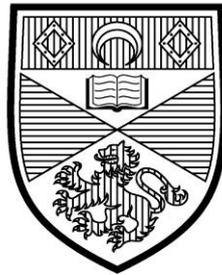
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Essays on International Portfolio Choices and Capital Flows

Ning Zhang



University of
St Andrews

This thesis is submitted in partial fulfilment for the degree of PhD
at the
University of St Andrews

October 2015

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Abstract

The goal of this thesis is to study the international portfolio choices of countries in an asymmetric world. In practice, this corresponds to the salient facts of country portfolios and the underlying structural asymmetries between developing and developed countries in a financially integrated world. In the three main chapters of the thesis, frameworks are developed to advance our understanding of the way various country asymmetries contribute to the emergence of these persistent phenomena in international capital markets.

The first essay studies the question of why developing countries experience net equity inflows and bond outflows while developed countries experience net equity outflows and bond inflows, the so-called ‘two-way capital flows’. The analysis is based on an open-economy New Keynesian model of endogenous country portfolios with representative agents in each country. The model is so general that it allows one to perform an assessment of the roles of a long list of country asymmetries in determining the pattern of two-way capital flows.

While steady-state net country portfolios are zero in the first essay, the second and third essays consider the situations where this is not true. The second essay presents an *OLG* model of an endowment economy with a country asymmetry in households’ patience. Global imbalances in net positions emerge. Gross portfolio positions are obtained as the sum of standard self-hedging and, moreover, the hedging due to external imbalances. The valuation effects of external adjustments between creditor and debtor countries are rationalized.

By introducing non-tradable risks, the third essay models a production *OLG* economy with a country asymmetry in wealth division. Global imbalances in net positions again arise. Gross portfolio positions are composed of self-hedging, hedging of non-tradable income and hedging of external interest payments, which accounts for the reality of asymmetric asset home bias, i.e. although assets are locally biased everywhere, the pattern is more pronounced in creditor countries.

JEL classifications: E44; F21; F32; F36; F37; F41;

Key words: Financial globalization; Country portfolios; Two-way capital flows; Global imbalances; External adjustments; Asset home bias

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Chapter 1

Introduction

Over the last few decades, the world has witnessed dramatic increases in the scale of cross-border asset transactions. This constitutes one of the most prominent recent features of the world economy, i.e. financial globalization. According to the latest updated dataset of Lane and Milesi-Ferretti (2007), the index of international financial integration, defined as the sum of cross-border financial external claims and liabilities scaled by annual *GDP*, for developed countries as a group has increased by a factor of over 6, from 68.4% in 1980 to over 438.2% in 2007. The index for developing countries has been rising steadily as well in the meantime. It doubled from 34.9% in 1980 to 73.3% in 2007. By Gourinchas and Rey's (2013) computation, the sum of external assets and liabilities scaled by world *GDP* for *G8* economies increased substantially from 75% in the 1990s to 210% at its peak in 2007 while the same measure for the so-called *BRICs* (Brazil, Russia, India and China), four large and fast growing emerging countries, has increased tenfold, from 2% in the 1990s to 20% in 2007.

Almost during the same period, the world has entered an era of so-called 'global imbalances', capital has flowed from the poor south to the rich north, notably, the United States. From the 1980s, the United States started to have a current account deficit and the size of the deficit grew throughout the process of global financial integration. In 1987, the *U.S.* external debt was 4% of its *GDP*. This ratio rose to 10.3% in 1997, 17% in 2007 and about 21% in 2011 (with an average current account deficit of over \$600 billion per year in the period of 2005 – 2011). Corresponding to the worsening of *U.S.* position, developing

countries have seen a striking improvement in external balances since the 1990s (Lane and Milesi-Ferretti, 2007). In particular, China, one star of the emerging economies in terms of growth performance, is a main contributor to the global imbalances from the side of the South. It enjoyed an average current account surplus of over \$250 billion per year in the period 2005 – 2011 and is the largest holder of foreign reserve in the world with more than \$3 trillion by the end of 2011, mostly in *U.S.* government bonds (Wang et al. 2015).

Rapid development of financial globalization raises new questions in international macroeconomics. Some patterns about both gross and net cross-border asset trade among countries, especially those between advanced and less developed economies, have been arising and evolving into a set of persistent and puzzling phenomena. Closely associated with the above two facts, i.e. massive expansion of gross flows and positions and sustained current account global imbalances, three other stylized facts in international financial markets can be identified as follows.

Firstly, two-way capital flows. There is a large heterogeneity in asset composition of the external balance sheet of countries. Although as a whole, developed countries receive net capital inflows, they are actually net exporters of equity capital. The net capital inflows into these countries are mainly in terms of trade in international bonds. In contrast, while experiencing net capital outflows, developing countries are in fact net importers of equity capital. And many of them are at the same time holding net assets in terms of safe securities such as government bonds. If we use the *U.S.* and China to represent the developed and developing countries respectively, then the pattern of two-way capital flows can be summarized in the form of an investment strategy of ‘long equity, short bond’ in developed countries and the strategy of ‘short equity, long bond’ in developing countries (Lane and Milesi-Ferretti, 2007).

Secondly, valuation effects of external adjustments. Unlike in a world with only cross-border commodity trade, holding large-scale external assets exposes countries to potentially large capital gains and losses as asset prices fluctuate. This poses a challenge to the traditional view of external current account adjustment. Empirical studies show that the effects are economically sizable (Tille, 2003, Higgins et al., 2006, Gourinchas and Rey, 2007, Gourinchas, 2008). For example, Gourinchas and Rey (2007) estimate that valuation effects contribute

about 30% percent of the cyclical external current account adjustments in the *U.S.* More importantly, against the background of global imbalances, the effects imply revisions to the wealth of debtor and creditor countries along different directions. To be specific, for instance, when debtor countries enjoy capital gains creditor countries suffer from capital losses. It can be calculated that, between 1970 and 2010, the effects from which the *U.S.* benefits are equivalent to an additional current account surplus of about 2% of output every year. On the contrary, the *BRICs* economies experience significant valuation losses since 2010 which amount to, cumulatively, around 10% of output for Brazil and China, 25% for India and 40% for Russia (Gourinchas and Rey, 2013).

Thirdly, asymmetric (equity) asset home bias. Asset home bias describes the case where local assets account for a disproportionately high share in country portfolios. This would be understandable in a more isolated world. With the on-going accelerated pace of financial globalization, despite the large reduction in the trade costs of international assets, investors seem to be still reluctant to diversify their asset choices in most countries. Moreover, this tendency is much more significant in developing countries than in developed countries.

See for instance the data shown in Table 1.1 for total market capitalisation for OECD countries and the value of equity assets held locally for each country in 2005 (Sercu and Vanpee, 2007). Using a standard classification of countries into developed and developing categories it is found that on average 70% of equity is held locally in developed countries while the corresponding figure for developing countries is 84%. Coeurdacier and Rey (2012) using a different measure of asset home bias (which is defined as the difference between 1 and the ratio of the share of foreign equity in a country's portfolio over that in world portfolio) report that in developed countries their measure is close to 0.7 while the same measure in developing countries is more than 20 percent higher, at 0.9.

This thesis is motivated by the observation of these stylized facts. These facts are all well documented in the literature (for instance in Lane and Milesi-Ferretti, 2007, Gourinchas and Rey, 2013, Coeurdacier and Rey, 2012 etc.) but have not yet been fully analysed in theoretical terms. This thesis exploits advances in the methodology of portfolio computations to allow these facts to be analysed theoretically.

Previously, in most international macroeconomic models, gross portfolios

Country	Domestic capitalisation	Domestic holding	Home bias
Argentina	47,590	45,619	95.86%
Australia	804,015	645,679	80.31%
Austria	126,309	89,662	70.99%
Belgium	286,326	200,297	69.95%
Brazil	474,647	374,941	78.99%
Canada	1,482,185	1,185,688	80.00%
Chile	136,493	130,551	95.65%
Colombia	50,501	49,315	97.65%
Czech Republic	53,798	47,249	87.83%
Denmark	187,161	147,868	79.01%
Egypt	79,509	74,996	94.32%
Finland	228,266	111,225	48.73%
France	1,769,569	1,169,497	66.09%
Germany	1,221,106	713,687	58.45%
Greece	145,121	117,117	80.70%
HongKong	1,054,999	935,765	88.70%
Hungary	32,576	19,279	59.18%
India	1,069,046	968,242	90.57%
Indonesia	81,428	64,153	78.78%
Israel	122,578	86,714	70.74%
Italy	798,073	555,177	69.56%
Japan	5,542,716	4,613,580	83.24%
Korea	718,011	530,508	73.89%
Malaysia	180,518	157,278	87.13%
Mexico	239,128	163,750	68.48%
Netherland	575,843	226,685	39.37%
New Zealand	40,593	32,398	79.81%
Norway	190,952	135,971	71.21%
Philippines	39,818	34,122	85.69%
Poland	93,602	78,848	84.24%
Portugal	75,066	55,352	73.74%
Russia	527,022	478,894	90.87%
Singapore	257,341	202,370	78.64%
South Africa	549,310	490,436	89.28%
Spain	959,910	774,475	80.68%
Sweden	420,953	296,161	70.35%
Switzerland	935,448	534,563	57.15%
Thailand	123,885	98,139	79.22%
Turkey	161,538	135,034	83.59%
United Kingdom	3,058,182	1,840,956	60.20%
United States	17,000,805	15,336,311	90.21%
Venezuela	7,316	6,729	91.98%

Table 1.1: Asset home bias across countries (USD million)

played a negligible role. Because it is not easy to obtain the solution for portfolio choices, asset structures are usually assumed such that either only one non-contingent bond is traded internationally or a full set of Arrow-Debreu securities are available. These modelling strategies proved to be useful and sufficient in many cases. For example, the former strategy can be used if one interprets global net imbalances as the situation where one group of countries are borrowing while the other group of countries are lending. However, when the analysis of many assets is required, as it is the case of valuation effects under global imbalances, these simple modelling strategies are no longer adequate. Similarly, although assuming a complete asset market avoids one's worries about portfolio allocations (it is proper to do so when some other aspects than portfolio allocations is the focus of research), it is obviously of limited use for modelling the above listed stylized facts. No matter which one of the above stylized facts is to be explored, a reliable method for obtaining the solution of gross asset positions is vital.

Based on perturbation methods (Judd, 1998), Devereux and Sutherland (2010) show that to solve for the zero-order (steady-state) portfolios, a second-order approximation of the first order condition of optimal portfolios is needed, while the other variables need to be approximated to first-order accuracy. For a general class of international macroeconomic models, Devereux and Sutherland (2011) derive a formula which can be used to yield portfolio holdings in conjunction with the solution of standard first-order approximation of a model. The method is applicable no matter whether the asset market is complete or incomplete and how many assets are present in the model. It thus provides a powerful tool for our analysis of the above listed stylized facts of country portfolios. In all three core chapters of the thesis, we rely on the techniques in Devereux and Sutherland (2010, 2011) to compute optimal gross portfolios.

For our purpose, the analysis in all of these three chapters is based on models consisting of two different large countries, i.e. a developing country and a developed country. Instead of building an all-embracing model where the above listed stylized facts can be taken into account altogether, we take a step-by-step approach in explaining these facts in this thesis. All three chapters analyse financial globalisation in the sense that they provide an analysis of the importance of gross asset positions. Each chapter then analyses different two-country models

with emphasis being put on different aspects of the above stylised facts against the background of financial globalisation. To put it briefly, Chapter 2 looks at the determination of gross asset positions when there are two-way capital flows. Chapter 3 explores the question of how the presence of the *NFA* global imbalances affects the determination of gross asset positions and how these resulting positions can be used to shed light on the valuation effects of external adjustments between debtor and creditor countries. Chapter 4 is devoted to the analysis of the emergence of asymmetric asset home bias in which process solving gross asset positions is also inevitable. As a road map, the topics and questions that are covered in each of these three core chapters are summarised below.

Chapter 2 focuses on two-way capital flows. To allow attention to be focused on this specific question, it is assumed that there are no steady state net foreign asset imbalances between the two countries in the model. Chapter 2 therefore specifically excludes analysis of global imbalances in net foreign assets. Even though steady state net foreign asset positions are assumed to be fixed at zero, the subcategories of external balance sheets are not necessarily balanced. As seen from the description of the stylised facts above, two-way capital flows require that the developing country holds a negative net position in equities and a positive net position in bonds while the developed country holds a positive net position in equities and a negative net position in bonds. For this to emerge, on the one hand, we assume that both countries can issue equities and bonds. On the other hand, while both modelled following the New Keynesian approach in the same way, the two countries are assumed to differ in certain aspects which are in turn captured by asymmetric calibration of the associated structural parameters. A long list of country asymmetries is considered in this chapter. These include those related to economic structure, severity of economic frictions and monetary policy stances, etc. The gross positions of both equities and bonds in the model are computed for each asymmetry. And the resulting pattern of the gross positions are examined, through which process the roles played by different asymmetries in generating two-way capital flows individually are assessed both qualitatively and quantitatively. In addition, we also assess the composite effect of country asymmetries on two-way capital flows through a fully asymmetric calibration of the model. By adjusting the number of available assets in the model, this chapter also assesses the effect of the asymmetry associated with the structure

of international financial market on two-way capital flows.

Chapters 3 and 4 shift the focus away from two-way capital flows and concentrate instead on the analysis of gross asset positions with the presence of global net foreign asset imbalances. This requires that we have models which are capable of explaining the emergence of net foreign asset imbalances and, simultaneously, yielding solutions for gross asset positions. Nevertheless, as will be demonstrated in Chapter 3, within a model with representative agents, the presence of net foreign asset imbalances makes the model non-stationary. In other words, for these individual variables there is no steady state around which, as a standard procedure to solve such a model, we can linearize the model. For this reason, the framework with representative agents is not suitable for our purpose. Therefore, unlike Chapter 2, the analyses in Chapter 3 and 4 are thus built on a model with overlapping generations (*OLG*) which overcomes the non-stationarity problem. Besides, to not obscure the key implication of introducing net foreign asset global imbalances, we simplify the structure of financial markets by assuming that asset trade is restricted to trade in equities in these two chapters.

Chapter 3 aims at analysing the valuation effects of external adjustments between debtor and creditor countries. To make our discussion simpler, we develop an endowment *OLG* model with cross-country differences in the degree of patience (i.e. differences in household discount rates). The reason why this structural country difference leads to global net foreign asset imbalances in the model is explained. And the solution for gross positions in equities is obtained. We also examine the hedging motives that support these gross positions in order to understand the way in which these positions are structured and how this differs from that in a model of steady state zero *NFA* positions. As the most important goal of this chapter, the processes of external adjustments between the two countries, especially the asymmetric valuation effects in this process, are analysed. This chapter therefore provides a framework for analysing *NFA* imbalances and the external adjustments under the *NFA* imbalances against the background of financial globalisation.

Chapter 4 seeks to explain the home bias of gross asset positions under global net foreign asset imbalances. It extends the model of Chapter 3 to a world with production. This on the one hand allows a broader array of cross-country asym-

metries, for instance those associated with financial development and production structure, to be analysed. On the other hand, the extension of the model along this dimension has important implications for portfolio allocation because both financial development and production structure affect the share of income going to capital and therefore the share of income which can be traded on asset markets via equities. Again the model shows that global *NFA* imbalances arise. In addition, asset home bias also emerges and this bias displays an asymmetry which is consistent with the stylised facts described above. Chapter 4 therefore provides an analysis of *NFA* imbalances and asymmetric asset home bias against the background of financial globalisation.

Focusing on net, gross positions and asset composition of external balance sheet of countries, the three core chapters of this thesis constitute a connected and complete treatise on country portfolio allocations. In our last chapter, Chapter 5, we will conclude the whole thesis by summarizing our findings throughout and their further implications.

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Chapter 2

Portfolio choices in a general model of representative agents: Two-way capital flows between developing and developed countries

In this chapter, we focus on issues associated with the asset composition of country balance sheets. In particular, the differences between the asset holdings of developed and emerging market countries. We aim to shed light on the mechanisms that determine how country asymmetries affect the hedging properties of different types of assets as well as the amount of risk that countries are exposed to. This in turn will shed light on asymmetries in the gross portfolio holdings of countries and will allow an analysis of ‘two-way capital flows’, i.e. the situation where developed countries are net holders of emerging market equity and emerging market countries are net holders of developed country bonds. For the purposes of this analysis, we ignore imbalances in net portfolios in this chapter, i.e. the steady state net foreign asset positions of each country in the model is assumed to be fixed at zero. In chapter 3 and 4, we will relax this assumption by introducing global imbalances into the model of country portfolios.

2.1 Introduction

In international macroeconomics, the so-called two-way capital flows between developing and developed countries is an interesting phenomenon, i.e. net bond asset flows towards developing countries while net equity asset flows towards developed countries as a whole. (See Lane and Milesi-Ferretti, 2001, 2007*a*, 2007*b*, Ju and Wei, 2010.) What is the reason for this phenomenon? Why do equity and bond assets flow in such ways rather than the other way around? Are there any casual links between the stage of development of a country and their preference over different types of international assets? If there are, what are they and how do they work? This chapter seeks to answer these questions.

There has been an increasingly large literature studying net capital flows between developing and developed worlds since 1990 when Lucas (1990) proposed the famous question of why does capital not flow from developed countries to developing countries. Or even though it does why is this flow not stronger than observed. Based on standard neoclassical models, capital tends to flow to where it is able to yield a higher return. And the most basic reason for a differing return in such models is the degree of capital scarcity. Since developing countries are usually capital scarce in comparison to developed countries, the model predicts that net capital should flow from the latter to the former. The puzzle might not have gained so much attention if it was just a problem of size in a world of balanced international payments. In a world featuring global imbalances, as has emerged since 1990, this becomes even more puzzling because net capital actually flows the opposite way to that predicted by the neoclassical model.

Various theories have been proposed to explain this puzzling fact. Explanations include policy misalignments (Obstfeld and Rogoff, 2007, Summers, 2004, Blanchard et al., 2005, etc.), difference in productivity growth (Hunt et al., 2005, Engle and Rogers, 2006), demographic dynamics (Henriksen, 2005, Attanasio et al., 2006), volatility of the business cycle (Fogli and Perri, 2006) and a global savings glut (Bernanke, 2005) etc. In particular, one strand of the literature emphasizes the importance of financial underdevelopment of developing countries in reconciling the facts. According to these studies, various financial frictions, for instance lack of enforceability of financial contracts (Mendoza et al., 2009), incapability in supplying a sufficient asset stock (Caballero et al., 2009) or/and

in insuring away idiosyncratic risk (Angeletos and Panousi, 2011) etc., can distort the decisions of saving and investment in emerging markets, which in turn results in both a lower interest rate and a lower capital stock in autarky. While saving cannot be effectively channelled to investment domestically due to these financial frictions, under financial integration, excess saving must find its way to developed countries in the form of a net capital flow.

There is also an expanding literature on ‘two-way capital flows’ (i.e. where bonds and equity flows in opposite directions). Most of this literature also focuses exploring the effects of financial distortions on the choices of different types of asset. Ju and Wei (2010) attribute the major reason to financial market imperfections and related institutions such as property rights protection. The mechanism of financial capital flowing out while investment arriving in the form of *FDI* can serve as a nice vehicle bypassing the adverse effect of an inefficient financial system within developing countries. Hagen and Zhang (2011) model financial development as an endowment fixed in the short run. With the comparative advantage of providing financial service, developed countries will find it optimal to import financial capital and export *FDI* while the developing countries follow the opposite pattern. Wang et al. (2015) show that the common presence of underdevelopment factors in the credit market of developing countries can lower the rate of return of financial capital while raise that of fixed capital at the same time. So under capital liberalization, financial capital flows out while the fixed capital flows in.

Rather than mainly focusing on the return and mobility aspects of assets in the above literature, another strand of literature such as Devereux and Sutherland (2010, 2011) and Tille and Wincoop (2010) pay attention to different risk characteristics of international assets and the role they play in determining capital flows. The asset holdings of a country are determined because all assets have different risk characteristics and thus satisfy specific demands of households in different countries for risk-hedging devices. This approach allows for the analysis of many other potential factors in addition to financial frictions that behind net capital flows. This current chapter of this thesis falls in the category of this literature in explaining two-way capital flows. However, Devereux and Sutherland (2010, 2011) focus on methodological usefulness while Tille and Wincoop (2010) on (both net and gross) portfolio dynamics in a world of two symmetric

countries. In terms of two-way capital flows, asymmetries must be involved.

The analytical framework in this chapter is a model of a two-country world. Two types of assets, equity and bond, are assumed to be present. In separation, each country can be described by a medium-scale full-fledged model of the New Keynesian approach. So as a whole, the model of the two economies also represents an extension of the literature such as Woodford (2003), Gali (2008), Christiano et al. (2010), etc. to the context of international economy with endogenous portfolio choices. Specifically, the environment in each country is very close to that of Smets and Wouters (2007). For our purpose of distinguishing developing and developed country, we assume different values of structural parameters for them. These parameters capture various aspects in which the two economies may differ, including those of economic structure, policy stance, severity of various (real and nominal) frictions and properties of economic shocks, etc.. The studies in the literature employing econometric techniques to estimate the *DSGE* models of developed (for instance Smets and Wouters, 2003, 2007, etc.) and developing countries (for instance Sun and Sen, 2012, Dai, 2012 and Miao and Peng 2012, etc.) provide us with these parameter values of empirical relevance. Given the presence of the country asymmetries, optimal portfolio choices are computed and then assessed from the perspective of conforming to or contradicting the pattern of two-way capital flows. Through this process, we uncover which asymmetries matter and to what extent they matter.

To summarize our findings, firstly, we find that the asymmetries associated with country's industrial structure, severity of nominal rigidities, trade openness, consumption home bias, investment adjustment frictions, monetary policy stance, market competitiveness and pricing strategy of international trade, etc. can cause the two-way capital flows between developing and developed countries. Secondly, among these factors, those from the real side of economy are more important than those from the nominal side. Lastly, we simulate the model with fully asymmetric parameter values and find that it yields a portfolio allocation that are broadly consistent with the pattern of two-way capital flows. Besides, if we take into account of the situation where international bonds can only be issued by the developed country (as it is often the case in reality), this result still holds.

This work is closely related to Devereux and Sutherland (2009). The latter

considers asymmetry in asset market structure and finds that under the pattern of two-way capital flows the economies achieve a relatively high level of international risk-sharing, which supplies evidence in support of the emergence of the pattern. We follow a similar idea in this chapter, however, with substantial extension of the model and analysis. This, on the other hand, explains why we need such a general framework of New Keynesian approach (with each economy being modelled with rich features) in this chapter. Non-trivial monetary policy is present so that bond assets' return can be defined while many frictions, price/wage rigidities and costly investment adjustments for instance, are assumed here so that a long list of asymmetries associated with these features can be examined in the analysis. The work is also linked to Devereux et al. (2014) when it comes to decomposing the hedging properties of assets into correlation and variability effects which sheds light on the machinery of each asymmetry. With the presence of the central role of differing hedging properties of different types of asset in the model, it also connects to the literature on (symmetric) asset home bias in international macroeconomics. Coeurdacier and Rey (2012) give a survey of the literature on this topic.

The rest of the chapter is structured as follows. Section 2.2 presents the model. Section 2.3 discusses the determination, representation and interpretation of optimal country portfolios in the general model. Section 2.4 simulates the model symmetrically. Section 2.5 simulates the model asymmetrically and assesses country asymmetries' impact on the pattern of two-way capital flows. Section 2.6 concludes.

2.2 Model

The model assumes a world consisting of two countries, Home and Foreign. For the reader's convenience, a figure, Figure 2.1, is employed to summarize the economic structure of the two countries. At the top of the figure is a diagram of resource flows while on the lower half are some key points of information. The two countries are the same in terms of economic structure, which is reflected by the fact that the flows in the foreign country are drawn to be a mirror image of those in the home country. As shown in the diagram, each economy consists of five sectors. From left to right, they are the sector of households, labour union,

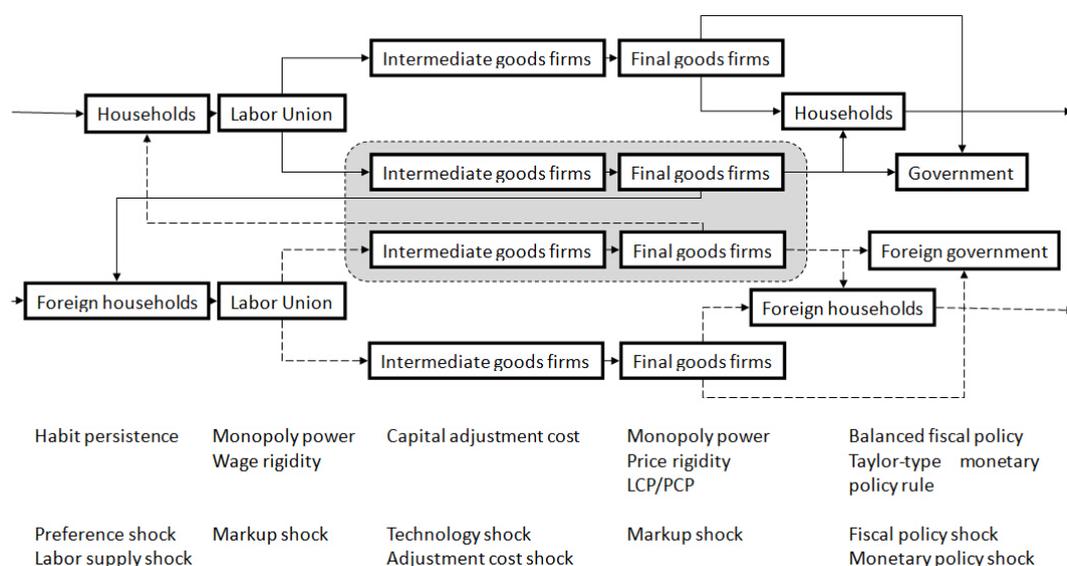


Figure 2.1: The structure of the model

intermediate goods sector, final goods sector and government. The lines linking sectors represent resource flows with the arrows showing the direction of flow. In each economy, households consume final goods from both home and foreign countries. They supply, domestically, their labour to labour unions for wages and capital to intermediate goods firms for capital rental. The labour unions distribute the labour supplies. And the intermediate goods firms combine the labour and capital collected to produce intermediate goods whose usefulness is only to be sold to the final goods sector. The firms in the final goods sector produce the final goods which are then ready for use for consumption and investment.

Following the literature, the intermediate and final goods firms are further divided into two parallel sectors of traded and non-traded goods production in both countries. In the diagram, this is reflected by the fact that the traded goods sectors are circled in a shadowed area. The traded and non-traded goods sectors are different such that the final goods produced by non-traded sectors can only be sold to domestic households while the final goods produced by the traded sector can be sold to both domestic and foreign households. There is one public sector, government, in the economy. They tax and consume on the one hand and implement fiscal and monetary policies according to rules on the other hand.

On the lower half of the figure, the first row lists the frictions embedded in the private sector and the policy rules adopted by the governments while the second row lists the shocks that are present. Being put forward without explanation, they are gathered here to give a better general description of the whole model and will be explained in more detail below. In what follows, the complete behaviours of each sector will be specified. However, because the two economies have the same structure, we will focus on the case of the home country. As a convention, when it is necessary to mention foreign country variables, an asterisk is used.

2.2.1 Households

Assume a continuum of household $z \in (0, 1)$. The representative household z is an intertemporal optimizer whose objective is to maximize the following utility function:

$$E_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{C_{X_{t+i}}^{1-\rho}(z)}{1-\rho} - \chi_{t+i} \frac{L_{t+i}^\mu}{\mu} \right\} \quad (2.1)$$

The function is an expected summation of an infinite series of single period utility. The latter equals the utility from consumption of a composite good C , $\frac{C_{X_{t+i}}^{1-\rho}(z)}{1-\rho}$, less the disutility from hours worked, $\chi_{t+i} \frac{L_{t+i}^\mu}{\mu}$. β , ρ and μ are respectively the discount factor, the risk aversion parameter (or inverse of the elasticity of intertemporal substitution) and the elasticity of labour supply. χ represents a weight between consumption and working hours. It is assumed to be a labour supply shock following the process $\widehat{\chi}_t = \delta_\chi \widehat{\chi}_{t-1} + \varepsilon_{xt}$ where a hat over a variable indicates a log-deviation from the steady state. Here if ε_{xt} is realized to be positive, there is negative shock to the labour supply.

The household z faces two restrictions when maximizing the above utility function. First, there is an (external) habit formation process

$$C_{X_{t+i}}(z) = C_{t+i}(z) - hC_{t+i-1} \quad (2.2)$$

where h is the degree of habit persistence.

Second, the household should meet the intertemporal budget constraint as follows:

$$F_t = \sum_{i=1}^4 r_{it} \alpha_{it-1} + \frac{w_t}{P_t} L_t - C_t(z) + \Pi_t + \Theta_t - T_t \quad (2.3)$$

where F_t is the net wealth of households at the end of time t . In the model of representative agents, it also denotes per capita net foreign asset (NFA) of the country. We assume that both the home and foreign countries issue equities and bonds. So there are $2 \times 2 = 4$ assets in total in the model. To understand the budget constraint, note that we denote the households' holding of asset i at the end of time t as α_{it} , so $F_t = \sum_{i=1}^4 \alpha_{it}$. We further denote the gross rate of return for asset i during period t as r_{it} , so the total return by holding the time- $(t-1)$ portfolio to the end of time t is given by $\sum_{i=1}^4 r_{it} \alpha_{it-1}$ which explains the first term on the right hand side of Eq.(2.3). For the rest of the terms on the right hand side, w_t is the nominal wage received by households. P_t is home country CPI , i.e. price index of composite good C . L_t is labour supply so $\frac{w_t}{P_t} L_t$ is labour income. We assume that households own firms and the labour unions. Π_t and Θ_t in the equation denote the profits of firms and labour unions that are received by households. $C_t(z)$ and T_t are households' spending on consumptions and taxation. So the budget constraint states that the amount of net total wealth each period is given by the sum of the gross return by holding existing portfolio and the newly earned saving.

The households' choice variables include the levels of consumption C , labour supply L and portfolio holdings α_i s. The first-order conditions associated with optimal C , L and α_i s are respectively:

$$\Omega_{t+i} = \beta^i C_{Xt+i}^{-\rho} \quad (2.4)$$

$$w_t = \chi_t L_t^{\mu-1} P_t \frac{1}{\Omega_t} = \chi_t L_t^{\mu-1} P_t C_{Xt}^{\rho} \quad (2.5)$$

$$C_{Xt}^{-\rho} = \beta E_t [C_{Xt+1}^{-\rho} r_{it+1}] \quad (2.6)$$

where Ω_{t+i} are multipliers for budget constraints at time $t+i$. Eqs.(2.4) and (2.5) are familiar intertemporal and intratemporal optimal conditions which define optimal C and L . Eq.(2.6) determines the optimal portfolio choices α_i . To understand it, it asserts that at the optimum, the marginal loss of utility by forgoing consumption (and investing in an asset) today should be equal to the marginal gain of utility by reaping the asset return tomorrow after discounting.

Once C is determined, following the literature, we assume the composite good is made up of non-traded and traded goods by the Dixit-Stiglitz aggregation

relation as follows:

$$C = \left[\kappa^{\frac{1}{\phi}} C_N^{\frac{\phi-1}{\phi}} + (1 - \kappa)^{\frac{1}{\phi}} C_T^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \quad (2.7)$$

where C_N and C_T are consumptions of non-traded and traded goods. Their weights in the basket are respectively κ and $(1 - \kappa)$. ϕ is the elasticity of substitution between the two types of good.

Investment goods are assumed to be aggregated in the same way, so

$$I = \left[\kappa^{\frac{1}{\phi}} I_N^{\frac{\phi-1}{\phi}} + (1 - \kappa)^{\frac{1}{\phi}} I_T^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \quad (2.8)$$

Given the aggregation relations of spending above, the demands for non-traded and traded goods in the home country are given by

$$D_N = \kappa (C + I) \left[\frac{P_N}{P} \right]^{-\phi} \quad (2.9)$$

$$D_T = (1 - \kappa) (C + I) \left[\frac{P_T}{P} \right]^{-\phi} \quad (2.10)$$

where P_T and P_N denote price indices for traded and non-traded goods. Moreover, the price index of the composite good at home P is

$$P = \left[\kappa P_N^{1-\phi} + (1 - \kappa) P_T^{1-\phi} \right]^{\frac{1}{1-\phi}} \quad (2.11)$$

Further assume that the demand for traded goods is made up of home and foreign traded goods (with subscript of H and F respectively) by the same technology with the weight and elasticity of substitution being now γ and θ :

$$C_T = \left[\gamma^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1 - \gamma)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad (2.12)$$

$$I_T = \left[\gamma^{\frac{1}{\theta}} I_H^{\frac{\theta-1}{\theta}} + (1 - \gamma)^{\frac{1}{\theta}} I_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad (2.13)$$

Combining with their foreign counterparts, it follows that the home demands of home and foreign traded goods are respectively:

$$D_H = \gamma D_T \left[\frac{P_D}{P_T} \right]^{-\theta} \quad (2.14)$$

$$D_F = (1 - \gamma) D_T \left[\frac{S^\eta P_X^*}{P_T} \right]^{-\theta} \quad (2.15)$$

and the foreign demands of home and foreign traded goods are respectively:

$$D_H^* = (1 - \gamma) D_T^* \left[\frac{S^{-\eta} P_X}{P_T^*} \right]^{-\theta} \quad (2.16)$$

$$D_F^* = \gamma D_T^* \left[\frac{P_D^*}{P_T^*} \right]^{-\theta} \quad (2.17)$$

where P_D and P_X are prices of home traded goods for home and foreign buyers. P_D^* and P_X^* are prices of foreign traded goods for foreign and home buyers. Note that in Eqs.(2.15) and (2.16), prices of exports P_X^* and P_X are converted to local terms if they are not set through local currency pricing (*LCP*) but rather the producer currency pricing (*PCP*). The nominal exchange rate S , defined as the price of foreign currency in terms of home currency, is thus involved in the above equations. Note we use a switch parameter of different pricing strategies η here. It takes the value of 1 in the *PCP* case or 0 in the *LCP* case.

The price index of the home traded goods is thus

$$P_T = \left[\gamma P_D^{1-\theta} + (1 - \gamma) (S^\eta P_X^*)^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (2.18)$$

The price index of the foreign traded goods P_T^* has a similar expression.

2.2.2 Labour unions

The representative labour union z buys labour from households and sells it to intermediate goods producers. Their problem is to maximize the following profit function

$$E_t \sum_{i=0}^{\infty} \Omega_{t+i} \Theta_{t+i} \quad (2.19)$$

with subject to

$$\Omega_{t+i} = \beta^i C_{Xt+i}^{-\rho} \quad (2.20)$$

$$L_t(z) = L_t \left(\frac{w_t(z)}{W_t} \right)^{-\xi} \quad (2.21)$$

$$\Theta_t = L_t(z) \frac{w_t(z)}{P_t} - L_t(z) \frac{w_t}{P_t} \quad (2.22)$$

We assume that they use the same discount factor as the one used by households, which leads to Eq.(2.20). $w(z)$ and W denote respectively the optimal (nominal) wage which is set by z and the aggregate wage index of labour sold to intermediate goods sector. With a constant elasticity of substitution between different types of labour supply ξ , the labour amount sold by the labour union is given by $L_t(z)$ by Eq.(2.21). Using w_t to represent the nominal wage paid by the labour union to households, we obtain the labour union's period profit function, i.e. Eq.(2.22).

This defines the problem of how $w_t(z)$ is chosen optimally. Moreover, we assume that the process of wage setting suffers from a rigidity friction. Wages adjust infrequently through a Calvo-type contract. Each time only a fraction of all wages $(1 - \varsigma)$ can be reset and the rest of wages ς are indexed to past inflation automatically with an indexation degree of ϖ .

To solve the labour union's problem, note that the related Lagrangian equation is:

$$E_t \sum_{i=0}^{\infty} \Omega_{t+i} \varsigma^i \left\{ \begin{array}{l} L_{t+i} \left[\frac{w_t(z)}{W_{t+i}} \left(\frac{W_{t+i-1}}{W_{t-1}} \right)^{\varpi} \right]^{1-\xi} \frac{W_{t+i}}{P_{t+i}} \\ -L_{t+i} \left[\frac{w_t(z)}{W_{t+i}} \left(\frac{W_{t+i-1}}{W_{t-1}} \right)^{\varpi} \right]^{-\xi} \frac{w_{t+i}}{P_{t+i}} \end{array} \right\} \quad (2.23)$$

By the associated first-order condition, the optimal wage rate set at time t can be obtained as:

$$w_t(z) = \frac{\xi}{\xi - 1} \frac{E_t \sum_{i=0}^{\infty} \Omega_{t+i} \varsigma^i L_{t+i} \frac{W_{t+i}^{\xi}}{P_{t+i}} \left[\left(\frac{W_{t+i-1}}{W_{t-1}} \right)^{\varpi} \right]^{-\xi} w_{t+i}}{E_t \sum_{i=0}^{\infty} \Omega_{t+i} \varsigma^i L_{t+i} \frac{W_{t+i}^{\xi}}{P_{t+i}} \left[\left(\frac{W_{t+i-1}}{W_{t-1}} \right)^{\varpi} \right]^{1-\xi}} \quad (2.24)$$

from which it is clear that the optimal wage is a mark-up over a weighted average of future marginal cost of labour w_{t+i} . The weight is affected by the degree of wage rigidity ς and other variables. The stronger the degree of wage rigidity, i.e. a high ς , the less is the importance of the current marginal cost comparing to the future marginal cost. The mark-up factor $\frac{\xi}{\xi-1}$ is a function of the elasticity of labour substitution ξ . The lower is the substitution rate ξ , the lower is the market competitiveness and the higher is the mark-up. We introduce a mark-up shock $V = \frac{\xi}{\xi-1}$ here and we assume that it follows the process $\hat{V}_t = \delta_V \hat{V}_{t-1} + \varepsilon_V$. When there is a positive realization of ε_V , there is a negative shock to market power in labour market.

Given the optimal wage $X_{wt} \equiv w_t(z)$, by aggregation, the aggregate wage

index W_t is given by:

$$W_t = \left\{ \varsigma \left[W_{t-1} \left(\frac{W_{t-1}}{W_{t-2}} \right)^\varpi \right]^{1-\xi} + (1-\varsigma) X_{wt}^{1-\xi} \right\}^{\frac{1}{1-\xi}} \quad (2.25)$$

2.2.3 Intermediate goods firms

As mentioned before, there are two parallel intermediate goods sectors within each country. In either sector, the firms only supply intermediate goods to final goods firms of the same sector. Except for this difference, the structure of the two intermediate goods sectors is the same. So in this subsection, unless it is necessary, we only discuss the behaviour of the traded sector. The related equations for non-traded sector are similar.

The intermediate firms buy labour and capital and combine them to produce the intermediate goods. For a representative firm z , its problem is to maximize its profit:

$$E_t \sum_{i=0}^{\infty} \Omega_{t+i} \Pi_{Mt+i} \quad (2.26)$$

with subject to

$$\Pi_{Mt+i} = \frac{q_t}{P_t} Y_t - \frac{W_t}{P_t} L_t - I_t - \frac{\psi (\varepsilon_t I_t - \bar{I})^2}{2\bar{I}} \quad (2.27)$$

$$K_{t+1} = I_t + (1 - \delta) K_t \quad (2.28)$$

$$Y_t = A_t K_{t-1}^{1-a} L_t^a \quad (2.29)$$

The production function is assumed to be of the Cobb-Douglas form, Eq.(2.29). The share of labour L and capital K in the output are respectively a and $1 - a$. The factors of technology or efficiency enter the function through variable A . Following the literature (for instance Corsetti et al., 2008 and Devereux et al. 2014), the exogenous state vector of technology $\hat{A} \equiv [\hat{A}_T \hat{A}_N]$ are assumed to evolve according to

$$\hat{A}_{Tt} = \delta_{TT1} \hat{A}_{Tt-1} + \delta_{TT2} \hat{A}_{Tt-1}^* + \delta_{TN1} \hat{A}_{Nt-1} + \delta_{TN2} \hat{A}_{Nt-1}^* + \varepsilon_T \quad (2.30)$$

$$\hat{A}_{Nt} = \delta_{NT1} \hat{A}_{Tt-1} + \delta_{NT2} \hat{A}_{Tt-1}^* + \delta_{NN1} \hat{A}_{Nt-1} + \delta_{NN2} \hat{A}_{Nt-1}^* + \varepsilon_N \quad (2.31)$$

where $[\varepsilon_T \varepsilon_N]$ are disturbances to technology.

Eq.(2.28) is the standard capital accumulation equation. Capital at the end of time t , K_{t+1} , equals the sum of the investment this period, I_t , and the depreciation-adjusted capital stock, $(1 - \delta) K_t$. The capital depreciation rate is δ .

Eq.(2.27) gives the profit function for the intermediate goods firm. q is the price of intermediate goods. The first term of the equation represents the income by selling the goods. The second and third terms on the right hand side of the equation represent the cost of the labour and capital inputs respectively. We assume a cost of investment adjustment, i.e. $\frac{\psi(\varepsilon_t I_t - \bar{I})^2}{2\bar{I}}$. The cost function is set to be a quadratic form mainly out of tractability. Moreover, it also implies that both accumulation and decumulation of capital will incur adjustment cost and the cost is marginally increasing. The parameter ψ is used to govern the degree of the friction. We assume there is a shock variable ε_t that affects investment-adjustment cost which follows the process of $\hat{\varepsilon}_t = \delta_\varepsilon \hat{\varepsilon}_{t-1} + \varepsilon_\varepsilon$.

The choice variables for intermediate goods firm are labour demand L , investment I and capital stock K . The associated first-order conditions are:

$$MPL_t = \frac{W_t}{q_t} \quad (2.32)$$

$$\Psi_t = \left[1 + \frac{\psi(\varepsilon_t I_t - \bar{I})}{\bar{I}} \varepsilon_t \right] \quad (2.33)$$

$$\Omega_t \Psi_t = E_t \Omega_{t+1} \left[\frac{q_{t+1}}{P_{t+1}} MPK_{t+1} + (1 - \delta) \Psi_{t+1} \right] = E_t \Omega_{t+1} R_{K_{t+1}} \quad (2.34)$$

The optimal L is determined by Eq.(2.32). The condition states that at the optimum the marginal product of labour should be equal to the real wage, which should be familiar. Eq.(2.33) is a type of Tobin's Q equation where the price of the investment goods is set to be the same as the price of the final goods which is normalized to 1. The Ψ on the left hand side of the equation is the multiplier associated with the constraint of Eq.(2.28). It also stands for the marginal product of investment. In equilibrium, it should be equal to the marginal cost of investment on the right hand side. This equation ties down the optimal investment I_t . Eq.(2.34) determines the optimal capital stock K_t . It balances the intertemporal use of capital. Existing capital can either be used today or be

invested as capital tomorrow. At optimum, there should be no difference between the marginal benefits of the two different uses.

2.2.4 Final goods firms

The final goods sector is also divided into traded and non-traded sectors. As before, in this subsection, we only consider the traded sector. The equations for the non-traded sector are similar. In addition, because the firms in the traded sector have to set the price for exports, this again involves different pricing strategies, i.e. whether *PCP* or *LCP* is adopted. In what follows, as before, this is represented by the cases of $\eta = 1$ for *PCP* and $\eta = 0$ for *LCP*.

The structure of the problem of the final goods sector is similar to that of the labour unions. The firms buy intermediate goods from the intermediate goods sector, transform them into final goods and sell the goods to domestic and foreign buyers. The goods have some degree of heterogeneity so firms have power to set prices. However, the prices cannot change every period. The change is subject to a Calvo-type price rigidity.

A representative firm z chooses $p_{Dt}(z)$ and $p_{Xt}(z)$ to maximize the profit function:

$$E_t \sum_{i=0}^{\infty} \Omega_{t+i} \Pi_{Ft+i} \quad (2.35)$$

subject to

$$\Pi_{Ft} = y_{1t}(z) \frac{p_{Dt}(z)}{P_{Dt}} \frac{P_{Dt}}{P_t} + y_{2t}(z) \frac{p_{Xt}(z)}{P_{Xt}} \frac{S_t^{1-\eta} P_{Xt}}{P_t} - y_{1t}(z) \frac{q_{Tt}}{P_t} - y_{2t}(z) \frac{q_{Tt}}{P_t} \quad (2.36)$$

$$y_{1t}(z) = D_t \left[\frac{p_{Dt}(z)}{P_{Dt}} \right]^{-\varphi} \quad (2.37)$$

$$y_{2t}(z) = D_t^* \left[\frac{p_{Xt}(z)}{P_{Xt}} \right]^{-\varphi} \quad (2.38)$$

$p_{Dt}(z)$ and $p_{Xt}(z)$ are the prices of home traded goods for home and foreign buyers respectively. With the assumptions of a constant elasticity of substitution φ , the demand for z 's goods from home and foreign countries $y_{1t}(z)$ and $y_{2t}(z)$ are given by Eqs.(2.37) and (2.38). So the first two terms on the right hand side of Eq.(2.36) are the related income by selling final goods while the last two terms

are the costs of buying intermediate goods. By taking the difference of the two, Eq.(2.36) gives the profit of firm z at period t .

We assume that the degree of price rigidity and price indexation are given by λ and ω respectively, the related Lagrangian equation of the final goods firm's problem can be set up following the same logic as in Eq.(2.23). The associated first-order conditions lead to the optimal $p_{Dt}(z)$

$$p_{Dt}(z) = \frac{\varphi}{1-\varphi} \frac{E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i \frac{D_{t+i}}{P_{t+i}} P_{Dt+i}^{\varphi} \left[\left(\frac{P_{Dt+i-1}}{P_{Dt-1}} \right)^{\omega} \right]^{-\varphi} q_{Tt+i}}{E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i \frac{D_{t+i}}{P_{t+i}} P_{Dt+i}^{\varphi} \left[\left(\frac{P_{Dt+i-1}}{P_{Dt-1}} \right)^{\omega} \right]^{1-\varphi}} \quad (2.39)$$

and the optimal $p_{Xt}(z)$

$$p_{Xt}(z) = \frac{\varphi}{1-\varphi} \frac{E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i \frac{D_{t+i}^*}{P_{t+i}} P_{Xt+i}^{\varphi} \left[\left(\frac{P_{Xt+i-1}}{P_{Xt-1}} \right)^{\omega} \right]^{-\varphi} \frac{q_{Tt+i}}{S_{t+i}^{1-\eta}}}{E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i \frac{D_{t+i}^*}{P_{t+i}} P_{Xt+i}^{\varphi} \left[\left(\frac{P_{Xt+i-1}}{P_{Xt-1}} \right)^{\omega} \right]^{1-\varphi}} \quad (2.40)$$

As before, the optimal prices under the nominal rigidity are markups over weighted average of the current and future marginal costs q_{Tt+i} and $\frac{q_{Tt+i}}{S_{t+i}^{1-\eta}}$. The weight over time is affected by how serious is the price rigidity, i.e. λ . And the markup is mainly controlled by the degree of the market competitiveness i.e. φ . As in the case of the labour union, we assume $V = \frac{\varphi}{1-\varphi}$ is a price markup shock and assume that it follows the process of $\hat{V}_t = \delta_V \hat{V}_{t-1} + \varepsilon_V$.

2.2.5 Government

The government implements both fiscal and monetary policies. The fiscal policy is assumed to be aimed at a balanced budget. So we have the following rule

$$P_{Gt}G_t = P_tT_t \quad (2.41)$$

As for the scale of government, we assume that the total expenditure of government in the steady state amounts to a fixed proportion of the total output in steady state. Parameter g governs the ratio:

$$G = gY \quad (2.42)$$

where for G and Y the time subscript t is dropped to indicate a steady state value of them.

Government spending is assumed to be subject to a fiscal policy shock:

$$\hat{G}_t = \delta_G \hat{G}_{t-1} + \varepsilon_{Gt} \quad (2.43)$$

We further assume that the government buys both traded and non-traded goods. And the shares are consistent with that of private spending, i.e. a constant proportion of the total expenditure κ goes to non-traded goods and the remaining proportion $1 - \kappa$ goes to traded goods. We assume that the government only buys domestic traded goods. So we have:

$$G_{Nt} = \kappa G_t \quad (2.44)$$

$$G_{Ht} = (1 - \kappa) G_t \quad (2.45)$$

Monetary policy follows a standard Taylor-type rule. By assumption, the deviation of the chosen interest rate from its steady state can be broken down into terms of interest rate smoothing, inflation feedback, output gap feedback and monetary shock respectively. In particular, the rule takes the form:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\delta_R} \left[\left(\frac{\pi_t}{\pi} \right)^{\delta_\pi} \left(\frac{Y_t}{Y} \right)^{\delta_Y} \right]^{1-\delta_R} r_{rt} \quad (2.46)$$

where $\frac{R_t}{R}$ denotes the deviation of the interest rate from its steady state. δ_R is the degree of interest rate smoothing. δ_π and δ_Y are respectively feedback parameters of inflation and output gap. And r_{rt} stands for a monetary shock which follows the process of $\hat{r}_{rt} = \delta_{rr} \hat{r}_{rt-1} + \varepsilon_{rrt}$.

2.2.6 Financial markets

In this subsection, let us define the rate of return for the assets available in the international financial market. As mentioned, both countries can issue equities and nominal bond. For home and foreign equities, we assume that they represent claims on the profit made by the firms in the issuer country. The gross (real) rate of return for home and foreign equities are thus given by:

$$r_{1t} = \frac{\Pi_t + Z_{1t}}{Z_{1t-1}} \quad (2.47)$$

$$r_{2t} = \frac{\Pi_t^* \cdot Q_t + Z_{2t}}{Z_{2t-1}} \quad (2.48)$$

where $\Pi_t = \Pi_{Mt} + \Pi_{Ft} + \Theta_t$ and $\Pi_t^* = \Pi_{Mt}^* + \Pi_{Ft}^* + \Theta_t^*$ are the total profits of firms, i.e. the profits belonging to intermediate and final goods firms of both traded and non-traded goods sectors plus labour unions, in the two countries. Z_{1t} and Z_{2t} are the real prices of home and foreign equities. $Q_t = (S_t \cdot P_t^*) / P_t$ in Eq.(2.48) is the real exchange rate representing the price of foreign consumption basket in terms of home consumption basket. The rate of return of the foreign equity r_{2t} is defined in terms of home basket and is comparable to r_{1t} .

For the home and foreign bonds, we assume that they represent claims on one unit of currency per period in the issuer country. The gross (real) rates of return for them are thus given by:

$$r_{3t} = \frac{1/P_t + Z_{3t}}{Z_{1t-1}} \quad (2.49)$$

$$r_{4t} = \frac{(1/P_t^*) \cdot Q_t + Z_{4t}}{Z_{4t-1}} \quad (2.50)$$

where $1/P_t$ and $(1/P_t^*) \cdot Q$ denote real payoffs of one unit of home and foreign bonds. Again, Q is used to convert the foreign payoff into terms of the home consumption basket.

2.2.7 Market clearing

In equilibrium, all markets should clear. These include market clearing in the goods market, the labour market and asset markets.

In the goods market, for the non-traded sector, we should have

$$Y_{Nt} = D_{Nt} + \kappa G_t \quad (2.50)$$

where D_N is the private demand for home non-tradables and κG is the public spending on them. Note that as explained there is no demand coming from the foreign country for home non-tradables.

For the traded sector, we have

$$Y_{Tt} = D_{Ht} + D_{Ht}^* + (1 - \kappa) G_t \quad (2.51)$$

where D_{H1} and D_{H2} are the private demands for home tradables from the home and foreign countries, whose formulae are given by Eqs.(2.14) and (2.16), and $(1 - \kappa) G$ is the public spending on home tradables.

Aggregating the goods demands across sectors leads to the total demand for goods

$$Y_t = Y_{Nt} + Y_{Tt} \quad (2.52)$$

In the labour market, the total labour supply L is made up of that of traded sectors L_T and that of non-traded sectors L_N

$$L_t = L_{Tt} + L_{Nt} \quad (2.53)$$

In the foreign country, these conditions are similar.

In asset markets, all assets are in net supply of zero, so

$$\alpha_{it} + \alpha_{it}^* = 0 \quad (2.54)$$

for $i = 1, 2, 3, 4$. Note that i is an index of assets and the α s with asterisk are foreign holdings. By the market clearing conditions of assets, once (steady-state) asset holdings of home country are obtained, those of foreign country are simply $\alpha_i^* = -\alpha_i$. So in what follows, we only focus on the solutions of home portfolio choices, i.e. the α_i s.

2.3 Optimal portfolios in the general model

After specifying the details of the model, in this section, we are ready to discuss the determination of the optimal portfolios, i.e. the α_i s. We first derive the optimality condition that can be used to tie down the α_i s from the Euler equations. It turns out that the α_i s are determined by first-order behaviour of the cross-country consumption differential and asset excess returns. We approximate the budget constraints of the two countries and apply them to the optimality condition to yield α_i s as variance-covariance ratios. The ‘correlation’ and ‘variability’ effects are defined and derived following the literature, which provide useful hints about the way of how the optimal portfolios are structured.

2.3.1 Optimality condition

As noted in the previous section, the optimal portfolio choices are determined by equation set (2.6) and its foreign counterpart. In the home country, Eq.(2.6)

gives us the following three restrictions that need to be satisfied:

$$E [C_{Xt+1}^{-\rho} r_{it+1}] = E [C_{Xt+1}^{-\rho} r_{4t+1}] \quad (3.1)$$

for $i = 1, 2, 3$. Following Devereux and Sutherland (2011) (and also Tille and Wincoop, 2010), to obtain the zero-order α_i s, at least second-order approximations of the portfolio conditions are required. So we approximate the above conditions in a standard way up to second-order accuracy. Combined with the foreign approximated conditions, we can arrive at the following covariance condition

$$E \left[\left(\hat{C}_{Xt+1} - \hat{C}_{Xt+1}^* - \hat{Q}_{t+1}/\rho \right) \hat{r}_{ixt+1} \right] = 0 + O(\epsilon^3) \quad (3.2)$$

where, except for \hat{r}_{ixt+1} which is defined as $(\hat{r}_{it+1} - \hat{r}_{4t+1})$, all other variables with hats represent log deviations from their steady states. For example, $\hat{C}_{Xt+1} = \log[(C_{Xt+1} - C_X)/C_X]$ where C_X is steady-state C_{Xt} . \hat{C}_{Xt+1}^* and \hat{Q}_{t+1} are defined similarly.

Eq.(3.2) can serve as the condition to tie down the α_i s for $i = 1, 2, 3$. Note by this equation, the α_i s are determined by two first-order behaviours. There are $\hat{C}_{Xt+1}^D = \left(\hat{C}_{Xt+1} - \hat{C}_{Xt+1}^* - \hat{Q}_{t+1}/\rho \right)$, which is referred to as the cross-country consumption differential (with habit formation), and \hat{r}_{ixt+1} , which is referred to as the excess returns of asset i over asset 4 which is the numeraire asset in the model. At the optimum, the α_i s are chosen so that the covariance between the two is zero, or the two are orthogonal, which indicates the optimal portfolios as hedging vehicles smoothing relative consumption fluctuations through generating relative asset returns.

Once α_1 to α_3 are derived from Eq.(3.2), α_4 can be obtained by the fact of $\alpha_4 = F - (\alpha_1 + \alpha_2 + \alpha_3)$ where F is steady-state NFA in the home country. Because in this chapter, we assume that the steady state autarky interest rates are equalized across countries $r = r^* = \frac{1}{\beta}$. There is no reason for capital flows to particular country in net terms. Steady state net foreign assets in equilibrium is thus zero, i.e. $F = 0$.

2.3.2 Approximating budget constraints

Obviously, \hat{C}_{Xt}^D is endogenous and it depends on the optimal portfolio α_i s in the model. Most basically, consumptions link to portfolios through budget con-

straints. By writing out the links between them, we can establish expressions of portfolios explicitly instead of implicitly as in Eq.(3.2). This procedure is usually very useful in providing us with intuitions on which kind of motive drives the emergence of the observed portfolios, i.e. the motive to hedge away certain income risks. In this subsection, we obtain the links by approximating the budget constraints of countries. In the next subsection, we derive the portfolios as a variance-covariance ratio representing them explicitly.

Let us start with the home budget constraint, Eq.(2.3), which can be rewritten as

$$F_t = \alpha'_{t-1} r_{xt} + r_4 F_{t-1} + Y_{ct} - C_t \quad (3.3)$$

where we define portfolio vector $\alpha'_{t-1} = [\alpha_{1t-1} \ \alpha_{2t-1} \ \alpha_{3t-1}]$, excess vector $r_{xt} = [r_{1xt} \ r_{2xt} \ r_{3xt}]'$ and disposable income $Y_{ct} = \frac{w_t}{P_t} L_t + \Pi_t + \Theta_t - T_t$.

First-order approximating the equation around the steady-state yields

$$\hat{C}_t = \hat{Y}_{ct} + \frac{1}{c} \tilde{\alpha}' \hat{r}_{xt} + \frac{1}{c} \frac{1}{\beta} \hat{F}_{t-1} - \frac{1}{c} \hat{F}_t \quad (3.4)$$

where $\hat{Y}_{ct} = \log[(Y_{ct} - Y_c)/Y_c]$ and $\hat{C}_t = \log[(C_t - C)/C]$. Because in steady state, $F = 0$, \hat{F}_t is defined here as deviation of F_t from its steady state (of zero) as a percentage of equilibrium income Y instead of F , i.e. $\hat{F}_t = \log[F_t/Y]$. Besides, we define $\tilde{\alpha}' = \frac{1}{\beta Y} \alpha'$ and $\hat{r}_{xt} = [r_{1xt} \ r_{2xt} \ r_{3xt}]'$. c is the steady-state ratio of consumption to income $c = C/Y$.

The budget constraint in the foreign country is

$$\frac{F_t^*}{Q_t} = \frac{1}{Q_t} (\alpha'^*_{t-1} r_{xt} + r_4 F_{t-1}^*) + Y_{ct}^* - C_t^* \quad (3.5)$$

Note that exchange rate appears in the constraint because all asset returns are in terms of the home consumption basket while foreign consumption and disposable income are in terms of foreign consumption basket.

Similarly, approximating this constraint yields

$$\hat{C}_t^* = \hat{Y}_{ct}^* + \frac{1}{c^* Q} \tilde{\alpha}'^* \hat{r}_{xt} + \frac{1}{c^* Q} \frac{1}{\beta} \hat{F}_{t-1}^* - \frac{1}{c^* Q} \hat{F}_t^* \quad (3.6)$$

where variables are defined analogously.

Notice that in a two-country world we have $F_t^* = -F_t$ so

$$\hat{F}_t^* = -\frac{Y}{Y^*} \hat{F}_t$$

By the conditions of asset market clearing, we have

$$\tilde{\alpha}^* = \frac{\alpha^*}{\beta Y^*} = \frac{-\alpha Y}{\beta Y Y^*} = -\frac{Y}{Y^*} \tilde{\alpha}$$

Making use of these facts, we can rewrite Eq.(3.6) as

$$\hat{C}_t^* = \hat{Y}_{ct}^* - \frac{Y}{Y^*} \frac{1}{c^* Q} \tilde{\alpha}' \hat{r}_{xt} - \frac{Y}{Y^*} \frac{1}{c^* Q} \frac{1}{\beta} \hat{F}_{t-1} + \frac{Y}{Y^*} \frac{1}{c^* Q} \hat{F}_t \quad (3.7)$$

2.3.3 Variance-covariance representation of portfolios

In this subsection, we represent $\tilde{\alpha}$ as a variance-covariance ratio. For convenience, approximated home and foreign budget constraints that were obtained above are put together as follows

$$\hat{C}_t = \hat{Y}_{ct} + \frac{1}{c} \tilde{\alpha}' \hat{r}_{xt} + \frac{1}{c} \frac{1}{\beta} \hat{F}_{t-1} - \frac{1}{c} \hat{F}_t$$

$$\hat{C}_t^* = \hat{Y}_{ct}^* - \frac{Y}{Y^*} \frac{1}{c^* Q} \tilde{\alpha}' \hat{r}_{xt} - \frac{Y}{Y^*} \frac{1}{c^* Q} \frac{1}{\beta} \hat{F}_{t-1} + \frac{Y}{Y^*} \frac{1}{c^* Q} \hat{F}_t$$

According to Eq.(2.2), i.e. $C_{Xt+1} = C_{t+1} - hC_t$, we have

$$(1-h) \hat{C}_{Xt+1} = \hat{C}_{t+1} - h\hat{C}_t$$

which can be used to rewrite \hat{C}_{Xt+1}^D as

$$\hat{C}_{Xt+1}^D = \frac{1}{1-h} \left(\hat{C}_{t+1} - h\hat{C}_t \right) - \frac{1}{1-h^*} \left(\hat{C}_{t+1}^* - h\hat{C}_t^* \right) - \frac{1}{\rho} \hat{Q}_{t+1}$$

With the expressions of consumption behaviours above, it follows that

$$\begin{aligned} \sum_{i=0}^{\infty} \beta^i \hat{C}_{Xt+1+i}^D &= \sum_{i=0}^{\infty} \beta^i \left[\frac{1}{1-h} \left(\hat{C}_{t+1+i} - h\hat{C}_{t+i} \right) - \frac{1}{1-h^*} \left(\hat{C}_{t+1+i}^* - h\hat{C}_{t+i}^* \right) \right. \\ &\quad \left. - \frac{1}{\rho} \hat{Q}_{t+1+i} \right] \\ &= \sum_{i=0}^{\infty} \beta^i \left[\frac{1}{1-h} \left(\hat{Y}_{ct+1+i} - h\hat{Y}_{ct+i} \right) - \frac{1}{1-h^*} \left(\hat{Y}_{ct+1+i}^* - h\hat{Y}_{ct+i}^* \right) \right] \\ &\quad + \sum_{i=0}^{\infty} \beta^i \left[\tau_1 \cdot 2\tilde{\alpha}' \hat{r}_{xt+1+i} - \tau_2 \cdot 2\tilde{\alpha}' \hat{r}_{xt+i} - \frac{1}{\rho} \hat{Q}_{t+1+i} \right] + t.i. \end{aligned}$$

where

$$\tau_1 = \left[\frac{1}{1-h} \frac{1}{2c} + \frac{1}{1-h^*} \frac{Y}{Y^*} \frac{1}{2c^* Q} \right]$$

$$\tau_2 = \left[\frac{h}{1-h} \frac{1}{2c} + \frac{h^*}{1-h^*} \frac{Y}{Y^*} \frac{1}{2c^*Q} \right]$$

and *t.i.* denotes terms of irrelevance (whose covariance with \hat{r}_{xt+1} is 0). The summation is equivalent to

$$\begin{aligned} \sum_{i=0}^{\infty} \beta^i \hat{C}_{Xt+1+i}^D &= \frac{1}{(1-\beta)} \hat{C}_{Xt+1}^D \\ &= \sum_{i=0}^{\infty} \beta^i \left[\frac{1-\beta h}{1-h} \hat{Y}_{ct+1+i} - \frac{1-\beta h^*}{1-h^*} \hat{Y}_{ct+1+i}^* + \tau \cdot 2\tilde{\alpha}' \hat{r}_{xt+1+i} - \frac{1}{\rho} \hat{Q}_{t+1+i} \right] + t.i. \end{aligned}$$

or

$$\hat{C}_{Xt+1}^D = (1-\beta) (\Gamma_{yt+1} + \tau \cdot 2\tilde{\alpha}' \hat{r}_{xt+1} + t.i.) \quad (3.8)$$

where

$$\Gamma_{yt+1} = \sum_{i=0}^{\infty} \beta^i \left[\frac{1-\beta h}{1-h} \hat{Y}_{ct+1+i} - \frac{1-\beta h^*}{1-h^*} \hat{Y}_{ct+1+i}^* - \frac{1}{\rho} \hat{Q}_{t+1+i} \right] \quad (3.9)$$

denotes the sum of discounted expected fluctuations in relative disposable incomes and

$$\tau = \tau_1 - \beta\tau_2 \quad (3.10)$$

denotes a wedge whose value depends on the severity of the habit friction and the degree of country differences in the general model.

Putting Eq.(3.8) back into Eq.(3.2) leads to

$$E_t \{ \hat{r}_{xt+1} (\Gamma_{yt+1} + \tau \cdot 2\tilde{\alpha}' \hat{r}_{xt+1}) \} = 0$$

or

$$\tilde{\alpha}_i = -\frac{1}{2\tau} \frac{cov(\zeta_{yt+1}, \hat{r}_{xt+1})}{var(\hat{r}_{xt+1})} \quad (3.11)$$

where $\tilde{\alpha}_i$ for $i = 1, 2, 3$ is element of $\tilde{\alpha}$. $\zeta_{yt+1} = \Gamma_{yt+1} - E_t \Gamma_{yt+1}$ is the sum of discounted expected innovations in relative disposable incomes while $\hat{r}_{xt+1} = \hat{r}_{xt+1} - E_t \hat{r}_{xt+1}$ is the innovations in excess return of assets¹. Eq.(3.11) states

¹Note that $E_t \hat{r}_{xt+1} = 0$ is derived from the first-order approximation of Eq.(3.1). Both Devereux and Sutherland (2011) and Tille and Wincoop (2010) also share this property. Later on in Eq.(3.27) of next chapter, we show in more detail how this can be the case in a similar context.

that the optimal portfolios $\tilde{\alpha}$ depends on how the innovations in discounted expected relative disposable incomes co-vary with that in excess return of assets. The equation coincides with Eq.(24) of Devereux et al. (2014) if we ignore the presence of τ .² While in Devereux et al. (2014), τ collapses (into $1/C$) because the two countries are entirely symmetric and they do not consider the situation where households form habits, in the current model we are interested in the portfolio choices in an asymmetric world. And to consider possible asymmetry in habit persistence between developing and developed countries and its effects on portfolio choices, habit formation is taken into account. So τ emerges as one measure of how $\tilde{\alpha}$ differs in the asymmetric model from that in a symmetric model. While τ has a multiplicative effect on the size of portfolio holdings, the fundamental force underlying the determination of $\tilde{\alpha}$ is essentially the same as that in the symmetric model, i.e. households' motive to hedge against those risks that disturb their desired smooth schedule of relative consumption. Eq.(3.11) makes sense given that relative consumption is always supported by relative disposable income.

2.3.4 Correlation and variability effects

We now define and derive the 'correlation' and 'variability' effects. These effects provide a useful decomposition of the portfolio expressions which will be used in the analysis reported below. By Eq.(3.11), τ is the same across $\tilde{\alpha}_i$ s, i.e. elements in $\tilde{\alpha}$. If there are differences among the $\tilde{\alpha}_i$ s they must come from the differences among the variance-covariance ratios. The correlation and variability effects will provide some clues about the causes of these differences across assets.

Note that the $\tilde{\alpha}_i$ s in Eq.(3.11) can be re-written as

$$\tilde{\alpha}_1 = -\frac{1}{2\tau} \text{corr}(\zeta_{yt+1}, \hat{r}_{1xt+1} | \hat{r}_{2xt+1}, \hat{r}_{3xt+1}) \frac{\text{StD}(\zeta_{yt+1} | \hat{r}_{2xt+1}, \hat{r}_{3xt+1})}{\text{StD}(\hat{r}_{1xt+1} | \hat{r}_{2xt+1}, \hat{r}_{3xt+1})} \quad (3.12)$$

$$\tilde{\alpha}_2 = -\frac{1}{2\tau} \text{corr}(\zeta_{yt+1}, \hat{r}_{2xt+1} | \hat{r}_{1xt+1}, \hat{r}_{3xt+1}) \frac{\text{StD}(\zeta_{yt+1} | \hat{r}_{1xt+1}, \hat{r}_{3xt+1})}{\text{StD}(\hat{r}_{2xt+1} | \hat{r}_{1xt+1}, \hat{r}_{3xt+1})} \quad (3.13)$$

²Except that ζ_{yt+1} is also defined in a slightly different way. Specifically, in their paper, ζ_{yt+1} is multiplied by steady-state consumption C which is equalized across countries in their model. The degree of asymmetry in the model of this chapter is instead reflected in τ here.

$$\tilde{\alpha}_3 = -\frac{1}{2\tau} \text{corr}(\zeta_{yt+1}, \hat{r}_{3xt+1} | \hat{r}_{1xt+1}, \hat{r}_{2xt+1}) \frac{\text{StD}(\zeta_{yt+1} | \hat{r}_{1xt+1}, \hat{r}_{2xt+1})}{\text{StD}(\hat{r}_{3xt+1} | \hat{r}_{1xt+1}, \hat{r}_{2xt+1})} \quad (3.14)$$

According to above formulae, the signs of asset holdings are determined by the correlation between relative disposable income and the excess return of the asset conditional on the excess returns of other assets, i.e. $\text{corr}(\zeta_{yt+1}, \hat{r}_{ixt+1} | \hat{r}_{-ixt+1})$ where to ease notation we define \hat{r}_{-ixt+1} as a vector consisting of all elements of \hat{r}_{xt+1} except for \hat{r}_{ixt+1} . In other words, the short or long positions of asset holdings depend on the (conditional) hedging properties of related assets. Suppose for asset i , given the presence of the other assets, its excess return co-moves negatively with the relative disposable income, so $\text{corr}(\zeta_{yt+1}, \hat{r}_{ixt+1} | \hat{r}_{-ixt+1}) < 0$. This means after a shock, households' relative income moves in one direction while the asset yields returns that move in the offsetting direction. The asset is able to stabilize households' relative consumption. In this sense the asset is deemed as a good hedge and will be held in long position. Otherwise, if its excess return co-move positively with the relative incomes $\text{corr}(\zeta_{yt+1}, \hat{r}_{ixt+1} | \hat{r}_{-ixt+1}) > 0$, holding the asset would exaggerate the effects of the risks. This means that in order to provide a good hedge the asset will be held in a short position by households.

Coming back to our model, $\tilde{\alpha}_1$ and $\tilde{\alpha}_3$ are gross holdings of home assets which are supplied by the home country by default, so they are expected to be negative. That is to say, the two associated correlations are expected to be positive. $\tilde{\alpha}_2$ and $\tilde{\alpha}_4$ are gross (and also net) holdings of foreign assets, so they are expected to be positive. That is to say, the two associated correlations are expected to be negative. ($\tilde{\alpha}_4$'s expression can be obtained if another asset, say asset 2, is chosen as the numeraire asset. The representation is analogous. Note that the choice of numeraire asset does not matter in the sense that they all yield the same portfolio solutions $\tilde{\alpha}$.)

The size of asset holdings are determined by both $\text{corr}(\zeta_{yt+1}, \hat{r}_{ixt+1} | \hat{r}_{-ixt+1})$ and the ratio of $\frac{\text{StD}(\zeta_{yt+1} | \hat{r}_{-ixt+1})}{\text{StD}(\hat{r}_{ixt+1} | \hat{r}_{-ixt+1})}$. Following the literature, from now on, we refer them respectively as the 'correlation' and 'variability' effects. The two effects have very intuitive interpretations when it comes to affecting the size of $\tilde{\alpha}_i$ s.

The size of the $\tilde{\alpha}_i$ s positively depend on the correlation effect. This is because the higher is the conditional correlation (in absolute value), the closer is the co-movement between the relative disposable income and excess return, the more

significant is the role of asset in serving as a good hedge against risks. So the households desire to hold a more substantial amount of it, positively or negatively. The effect can be thought of as a quality effect, i.e. the assets which are more efficient in hedging (or exaggerating) risks will be bought (or sold) more. The correlation effect measures how relevant are the assets. The more relevant they are in risk-hedging, the more important they are in portfolios.

The size of the $\tilde{\alpha}_i$ s also depends positively on the variability effect as well. Note that the latter is the ratio of the conditional standard deviation of relative disposable income to that of excess return. It tells us how much the volatility of the relative disposable income is relative to that of the excess return. While the former volatility provides us with a measure of total amount of risks to be hedged against, the latter provides a measure of the amount of hedging that is made available by holding one unit of certain asset. A higher value of the ratio implies that more units of the asset is required. So the effect can be thought of as a quantity effect, i.e. more income volatility requires more units of hedging.

In the case of two-way capital flows, the developing country imports equities while exports bonds in net terms. If we define the net holding of equities and bonds as, respectively, $\tilde{\alpha}_E = \tilde{\alpha}_1 + \tilde{\alpha}_2$ and $\tilde{\alpha}_B = \tilde{\alpha}_3 + \tilde{\alpha}_4$, then two-way capital flows implies $\tilde{\alpha}_E < 0$ and $\tilde{\alpha}_B > 0$. Because $\tilde{\alpha}_1$ and $\tilde{\alpha}_3$ have negative signs, so they are equivalent to the pattern of $|\tilde{\alpha}_1| > |\tilde{\alpha}_2|$ and $|\tilde{\alpha}_3| < |\tilde{\alpha}_4|$ in optimal portfolios, i.e. the size of $\tilde{\alpha}_1$ is larger than that of $\tilde{\alpha}_2$ while the size of $\tilde{\alpha}_3$ is less than that of $\tilde{\alpha}_4$. Applying the above analysis, we know that this pattern can be the result of a certain combination of correlation and variability effects. As a central analysis of this chapter, in Section 2.5 we will assess the effect of various asymmetries between countries in generating the two-way capital flows. The correlation and variability effects we define here will provide useful devices in order to understand the findings there.

To end this section, we have to remind that neither Eq(3.11) nor Eqs.(3.12 – 14) are full reduced forms because as the determinants of $\tilde{\alpha}$ in Eq.(3.2), the second moments in these formulae are also in themselves depending on $\tilde{\alpha}$. In other words, both Eq.(3.2) and Eqs.(3.11 – 14) indicates $\tilde{\alpha}$ as a fixed point except that the former defines it implicitly while the latter explicitly and thus provide intuitions for the results. To sum up, we apply Devereux and Sutherland (2011)'s method to Eq.(3.2) to obtain $\tilde{\alpha}$ and make use of this $\tilde{\alpha}$ and Eqs.(3.12) to decom-

pose portfolios into correlation and variability effects. In the sections below, we analyse the model numerically.

2.4 Model simulation: Symmetric case

We will compute the numerical solution of equilibrium portfolios by simulating the model. As a benchmark, the two countries are firstly calibrated symmetrically in this section. We choose parameter values at their standard levels of calibration in the literature which are basically descriptions of advanced economies or/and from the estimates that are based on *U.S.* data. So we will see what the portfolios will look like without country asymmetry. In the next section, we will take into account the existence of a developing country by considering asymmetric simulations.

2.4.1 Parameterization

The frequency is assumed to be quarterly which is consistent with the literature on business cycles. The discount factor β is set at 0.99 which implies an annual interest rate of 4 percent. The elasticity of substitution between home and foreign traded goods is set at $\theta = 1.5$ which conforms to that of Backus et al. (1994). As for the values of the share of home traded goods in traded consumption basket γ , the share of nontraded goods in the total consumption basket κ and the elasticity of substitution between traded and nontraded goods ϕ , we choose them based on an average of values used in Benigno and Thoenissen (2008), Corsetti et al. (2008) and Stockman and Tesar (1995). The elasticity of substitution among individual final goods is set at 10 which implies an approximate 10 percent price mark-up over marginal cost.

For the production technology, the labour share of income a is calibrated to approximately $2/3$ which is common in the literature and consistent with *U.S.* data. Based on the same grounds, the share of government spending in total expenditure g is assumed to be 0.18. The depreciation rate of capital δ is set at 0.025 implying an annual depreciation rate of capital of 10%. The coefficient of investment adjustment cost ψ is chosen as 0.25 so that the variance of total investment is approximately 3 times the variance of *GDP* which is consistent

with *U.S.* data.

The values for the remaining parameters come from the median estimates by Smets and Wouters (2007) based on the data of the *U.S.* economy. These parameters include those related to preference (such as risk aversion ρ , labour supply elasticity μ and habit persistence h), Calvo price-setting, the monetary policy rule and structural shocks. Note by the parameter values, the *U.S.* households feature a persistent habit formation with $h = 0.7$. The price and wage adjust infrequently and the average duration of a price is about 3 quarters, $\lambda = 0.66$ and $\varsigma = 0.7$. In addition, the price and wage index to previous levels to some degree and the degree of wage indexation is higher than that of price, $\omega = 0.24$ while $\varpi = 0.58$. The interest rate is highly persistent with a persistence of $\delta_R = 0.81$. The related feedback coefficients of monetary policy with regard to inflation and output gap are respectively 2 and 0.1. The table 2.1 lists all values of parameters used in the benchmark calibration.

2.4.2 Symmetric case: Benchmark

Table 2.2 reports the result for equilibrium portfolios (divided by βY) under the benchmark calibration. The home households' holdings of home and foreign equity are -2.2985 and 2.2985 (times of steady-state income) while their holdings of home and foreign bonds are -0.7756 and 0.7756 (times of steady-state income). The home demands of home assets $\tilde{\alpha}_1$ and $\tilde{\alpha}_3$ are negative reflecting the fact that the home country is net supplier of home assets. The home demands of foreign assets $\tilde{\alpha}_2$ and $\tilde{\alpha}_4$ are positive reflecting the fact that the home country is net demander of foreign assets. The home net holdings of equities and bonds are both equal to zero $\tilde{\alpha}_E = \tilde{\alpha}_B = 0$ because the two countries are the same. In the light of portfolio decomposition, the symmetry of the countries implies that the correlation and variability effects of the same type of assets across countries are also equal to each other in absolute value. (The correlation effects have opposite signs because of different country identity.) As references, the value of τ here is 1.6818. The correlation and variability effects associated with $\tilde{\alpha}_1$ are respectively 0.1763 and 43.8628 while those associated with $\tilde{\alpha}_2$ are -0.1763 and 43.8628. For

Description	Variable values
Calvo price rigidity parameter	$\lambda = 0.66$
Calvo wage rigidity parameter	$\varsigma = 0.70$
Price indexation	$\omega = 0.24$
Wage indexation	$\varpi = 0.58$
Discount factor	$\beta = 0.99$
Habit persistence	$h = 0.70$
Risk aversion coefficient	$\rho = 1.38$
Labour supply elasticity	$\mu = 2.83$
Share of home traded goods in traded basket	$\gamma = 0.58$
Share of nontraded goods in consumption	$\kappa = 0.40$
Substitutability between traded goods	$\theta = 1.50$
Substitutability between traded and nontraded goods	$\phi = 0.45$
Substitutability among individual goods	$\varphi = 10$
Labour share of income in traded goods sector	$a_T = 0.67$
Labour share of income in nontraded goods sector	$a_N = 0.67$
Capital depreciation rate	$\delta = 0.025$
Investment adjustment cost	$\psi = 0.25$
Share of government spending	$g = 0.18$
Interest rate smoothing factor in Taylor rule	$\delta_R = 0.81$
Inflation feedback in Taylor rule	$\delta_\pi = 2$
Output feedback in Taylor rule	$\delta_Y = 0.1$
Pricing strategy	$\eta = 0$
Persistence of technology shock in traded sector	$\delta_{TT1} = 0.95, \delta_{TT2} = 0$
Variance of technology shock in traded sector	$\sigma_T = 0.0045$
Persistence of technology shock in non-traded sector	$\delta_{NN1} = 0.95, \delta_{NN2} = 0$
Variance of technology shock in non-traded sector	$\sigma_N = 0.0045$
Cross terms of technology shocks	$\delta_{TN1} = \delta_{TN2} = 0.60$ $\delta_{NT1} = \delta_{NT2} = 0$
Monetary policy shock	$\delta_{rr} = 0.15, \sigma_{rr} = 0.0024$
Government spending shock	$\delta_G = 0.97, \sigma_G = 0.0053$
Mark-up shock	$\delta_V = 0.89, \sigma_V = 0.002$
Labour supply shock	$\delta_\chi = 0.90, \sigma_\chi = 0.025$
Investment adjustment cost shock	$\delta_\psi = 0.71, \sigma_\psi = 0.0045$

Table 2.1: Parameter values: Symmetric case

Assets menu	Optimal portfolio choices
Home equity	$\tilde{\alpha}_1 = -2.2985$
Foreign equity	$\tilde{\alpha}_2 = 2.2985$
Home bond	$\tilde{\alpha}_3 = -0.7756$
Foreign bond	$\tilde{\alpha}_4 = 0.7756$

Table 2.2: Optimal portfolio choices: Symmetric case

bond assets, the two effects associated with $\tilde{\alpha}_3$ are respectively 0.4156 and 6.2776 while those associated with $\tilde{\alpha}_4$ are -0.4156 and 6.2776. One can verify that these values are consistent with the optimal portfolios via Eqs.(3.12 – 14). It also follows by inspection of the effects that the (conditional) correlation between the innovation in the equity excess return and that of relative disposable income is relatively low while the correlation between the innovation in the bond excess return and that of relative disposable income is relatively high. The bond assets' return moves more closely with relative disposable income in the model. According to the analysis in the last section, more sizable bond positions should be held in optimal portfolios due to the relative correlation effect. In contrast, the (conditional) variability effect belonging to equity assets is relatively high while that belonging to bond assets is relatively low. Due to this relative variability effect, however, more sizable equity positions should be held in optimal portfolios. It turns out that the relative variability effect dominates the correlation effect, so in the end we observe that the size of equity positions outweighs that of bond positions.

The key information conveyed by the benchmark calibration is that the pattern of two-way capital flows cannot arise in a symmetric model. There must be some asymmetries between the two countries which make this happen. By design, our model is general enough to allow for assessments of various asymmetries' impact on the capital flows. The next section is thus dedicated to such assessments in which course the result of the symmetric simulation in this section is always used as a comparison.

2.5 The two-way capital flows: developing vs developed countries

Now we turn to consider asymmetric situations in this section. The integration of developing country into the world economy is considered. To distinguish, in what follows, the home country is viewed as developing country while the foreign country is viewed as developed country. Because it is very likely the case that between the two types of countries various asymmetries coexist at the same time, we take two steps to investigate their impacts. First of all, we consider the individual effect of each asymmetry on net portfolio positions and two-way capital flows. Through the exercise, we will know whether the asymmetry considered matters for the emergence of the pattern of two-way capital flows. Moreover, if we find that an asymmetry does generate a two-way capital flow we also examine the question of in which direction the asymmetry plays its role (i.e. does it cause equity capital to flow to or from the developing country). The correlation and variability effects will also be traced during the course of the analysis in order to uncover the main channels in operation. After checking these individual effects, we put all asymmetries together into the same picture. By picking different sets of parameter values for the two countries, we simulate a fully asymmetric model mimicking a world of developing and developed countries that differ along multiple dimensions. We will thus check the composite effect of all asymmetries on portfolio choices.

2.5.1 Asymmetric cases: Single factors

To separate the effects of the asymmetries from each other, in this subsection, we examine them one by one. The process is as follows. We treat the foreign country as a control group and fix all foreign country parameter values at the benchmark levels. For each asymmetry, in the home country, we change the value of the associated parameter over a range around the benchmark value. Our target is to see how the net foreign equity and bond positions, $\tilde{\alpha}_E$ and $\tilde{\alpha}_B$, respond to such changes.

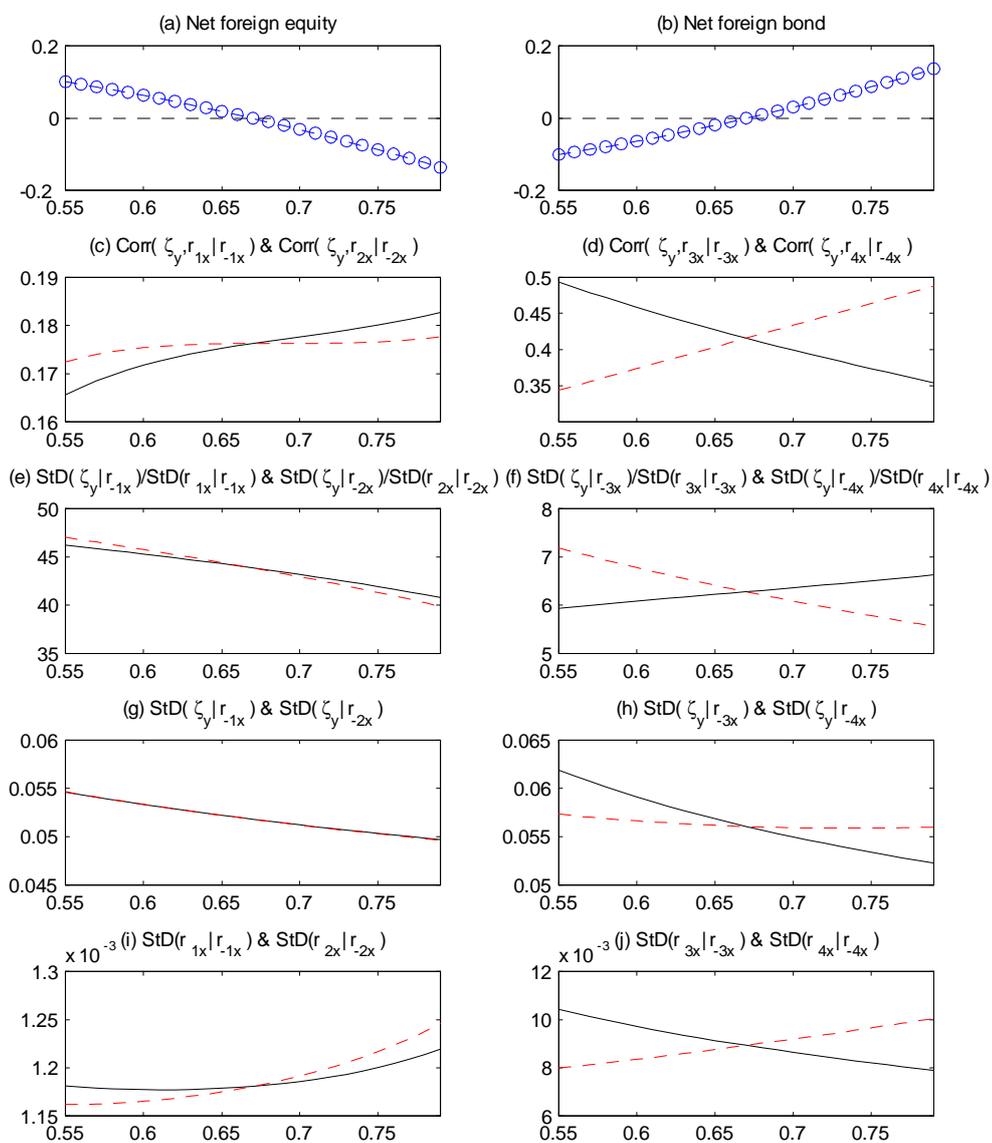


Figure 2.2: Labour intensity of technology a_T and a_N

Labour intensity

As the first experiment, we look at labour intensity of technology. The parameter characterizing this aspect is a . In the experiment, the foreign labour share a^* is fixed at the standard value of 0.67 while the home share a ranges from 0.55 to 0.79. The results are depicted as Figure 2.2. In this figure, panels (a) and (b) demonstrate the variations in $\tilde{\alpha}_E$ and $\tilde{\alpha}_B$ respectively. At the horizontal middle, $\tilde{\alpha}_E$ and $\tilde{\alpha}_B$ are both equal to zero which corresponds to the benchmark case of $a = a^* = 0.67$. To the right hand side of the point, $a > a^*$. We observe $\tilde{\alpha}_E < 0$ and $\tilde{\alpha}_B > 0$. That is to say, when the labour share is higher in the home country than in the foreign country, the home country holds a negative net equity position and a positive net bond position, i.e. there are two-way capital flows in the form observed for developing countries. Moreover, as the magnitude of the asymmetry grows, i.e. when a is much higher than a^* , the pattern in capital flows become more significant, $\tilde{\alpha}_E$ and $\tilde{\alpha}_B$ both increase in absolute size.

To explore why this is the case, we decompose the portfolios into associated correlation and variability effects, whose results are documented in the remaining panels of the figure. Since we will present the results of other asymmetries in the same way, some explanation on how to read these figures will be useful. Panels (c) and (e) report the correlation and variability effects for equities (in absolute value), i.e. $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$. Panels (d) and (f) do the same for bonds, i.e. $\tilde{\alpha}_3$ and $\tilde{\alpha}_4$. Because the variability effect is a ratio between two volatilities, the latter are also displayed as bottom panels, i.e. in panels (g) and (i) are the conditional volatility of relative disposable income and that of the excess return belonging to the two equities while in panels (h) and (j) are conditional volatility of relative disposable income and that of the excess return belonging to the two bonds. In all these panels, solid lines are used for home assets while dashed lines for foreign assets.

According to (c) and (d), $\tilde{\alpha}_E$ decreases because both the correlation and variability effects associated with $\tilde{\alpha}_1$ are higher than those of $\tilde{\alpha}_2$. As is shown, as a increases, the correlation effects of both equities increase, which implies an enhancement of equities' role as a good hedge against income risks. However, the increase in the correlation effect for home equity is more significant. On the other hand, the variability effects of both equities decrease, which implies

lower gross positions are required to hedge against risks. (This is in turn due to a decrease in the volatility of relative incomes while there is an increase in the volatility of asset returns based on the facts in panels (g) and (i)). However, the decrease in the variability effect of the home equity is less significant. Both facts point to a relative rise in the size of $\tilde{\alpha}_1$ which favours presence of a negative $\tilde{\alpha}_E$.

For $\tilde{\alpha}_B$, we look at panels (d) and (f). As a increases, the correlation effect of $\tilde{\alpha}_3$ decreases while that of $\tilde{\alpha}_4$ increases, which favours presence of a positive $\tilde{\alpha}_B$. On the other hand, the variability effect of $\tilde{\alpha}_3$ increases while that of $\tilde{\alpha}_4$ decreases. (Based on the facts in panels (h) and (j), the rise in $\tilde{\alpha}_3$ is because the associated volatility of relative income decreases less than that of the asset return while the decline in $\tilde{\alpha}_4$ is because the associated volatility of relative income increases less than that of the asset return.) So the change in the variability effect favours the presence of a negative $\tilde{\alpha}_B$ instead. It turns out that in the race between the two effects the former one wins out and $\tilde{\alpha}_B$ becomes positive.

Nominal rigidity

We consider both price and wage rigidities in this subsection.

First, for the degree of price stickiness λ , we set the foreign value at the standard value of 0.66 while we vary the home value from 0.54 to 0.78. As is shown in Figure 2.3, on either side of the middle point of $\lambda = \lambda^* = 0.66$, the pattern of two-way capital flows emerges with $\tilde{\alpha}_E < 0$ while $\tilde{\alpha}_B > 0$, so the home country has a negative net position in equities and a positive net position in bonds in the way observed in the data for developing countries.

It is rather surprising that, in the case illustrated in Figure 2.3, the direction of the asymmetry in price rigidity appears to be unimportant in generating an outcome with $\tilde{\alpha}_E < 0$ while $\tilde{\alpha}_B > 0$. To test the sensitivity of this result, we conduct further experiments in which λ^* (i.e. the foreign degree of price rigidity) is different from 0.66. These experiments are illustrated in Figures 2.4 and 2.5. These figures show the effects of varying λ on $\tilde{\alpha}_E$ and $\tilde{\alpha}_B$ for a high value of λ^* (Figure 2.4) and a low value of λ^* (Figure 2.5). By Smets and Wouters' estimation, the value of λ lies in a confidence interval of 0.56 and 0.74 so we use these two extremes as values for λ^* . These two figures show that in general the effects of λ and λ^* on $\tilde{\alpha}_E$ and $\tilde{\alpha}_B$ are quite complicated. Both figures show that

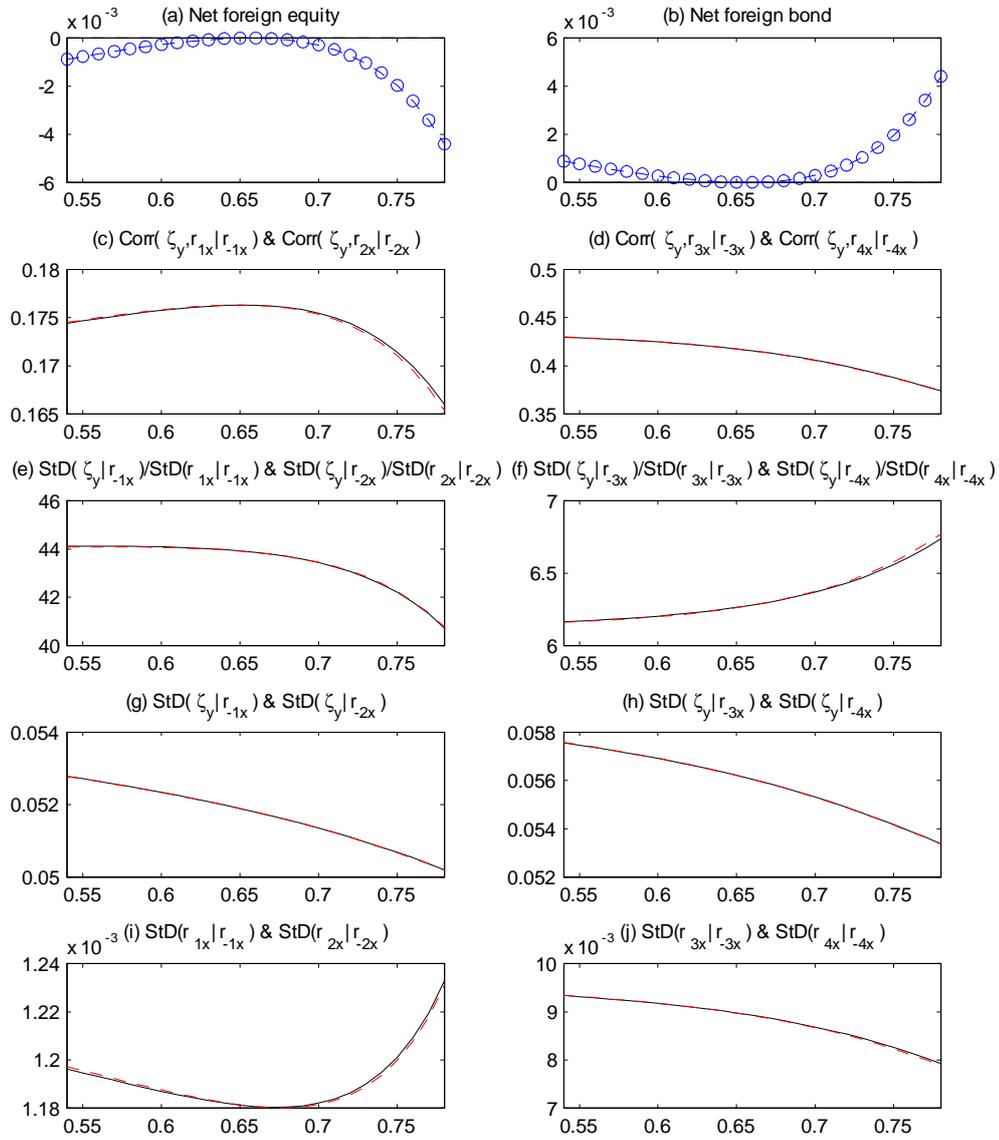
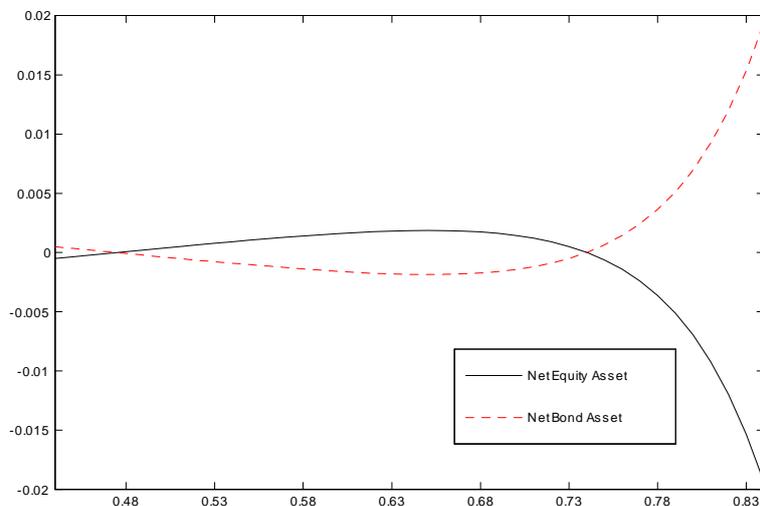


Figure 2.3: Nominal (price) rigidity λ

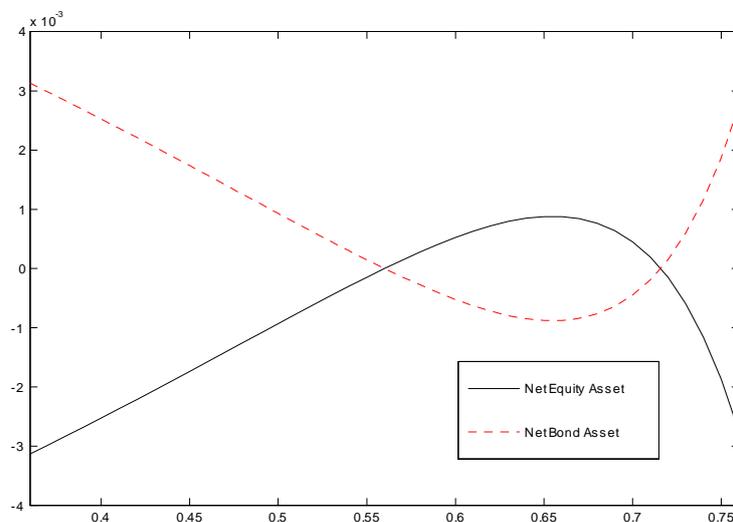
Figure 2.4: Price rigidity: High λ^*

the plots for $\tilde{\alpha}_E$ and $\tilde{\alpha}_B$ cross at two values of λ . For either high values of λ or low values of λ the pattern of two-way capital flows is observed with $\tilde{\alpha}_E < 0$ and $\tilde{\alpha}_B > 0$. But for intermediate values of λ the opposite result emerges.

From the last paragraph, the impact of asymmetry in λ on two-way capital flows is in general complicated in terms of signs. However, in terms of magnitude, it turns out that the asymmetry in λ is always a factor of little importance. The sizes of $\tilde{\alpha}_E$ and $\tilde{\alpha}_B$ under asymmetric cases are generally below 0.01. This is consistent with the results of the decomposition into correlation and variability effects. The other panels in Figure 2.3 show that the conditional second moments that are associated with home and foreign assets are generally very similar regardless of the value of λ .

Turning now to the degree of wage stickiness ς , we set ς^* at the standard value of 0.7 while we vary ς from 0.58 to 0.82. The result is shown in Figure 2.6. It is obvious that when $\varsigma > \varsigma^*$, $\tilde{\alpha}_E < 0$ and $\tilde{\alpha}_B > 0$. The more severe is the problem of wage stickiness in the home country, the more significant is the pattern of two-way capital flows in the model. For different foreign values, the result is robust.

When $\varsigma > \varsigma^*$, a rise in ς increases the correlation effect of equities to approx-

Figure 2.5: Price rigidity: low λ^*

imately the same degree (see panel (c)). It also increases the variability effect, however, with that belonging to $\tilde{\alpha}_1$ more significantly according to panel (e). (By panel (i), this is in turn because the excess return of the home equity becomes relatively less volatile.) This leads to $\tilde{\alpha}_E < 0$.

A rise in ζ moves the correlation and variability effects of bonds as well. While the correlation effect associated with $\tilde{\alpha}_3$ is higher than that of $\tilde{\alpha}_4$, its variability effect is lower than that of $\tilde{\alpha}_4$. It turns out that the correlation effect dominates the variability effect so $\tilde{\alpha}_B > 0$.

As was seen with the asymmetry in price stickiness, the pattern of two-way capital flows is insensitive to the asymmetry in wage stickiness, with the sizes of $\tilde{\alpha}_E$ and $\tilde{\alpha}_B$ under asymmetric calibrations being generally below 0.01 (in panels (a) and (b)) so we can conclude that asymmetries in the degree of both wage and price stickiness are of little importance in generating large two-way capital flows.

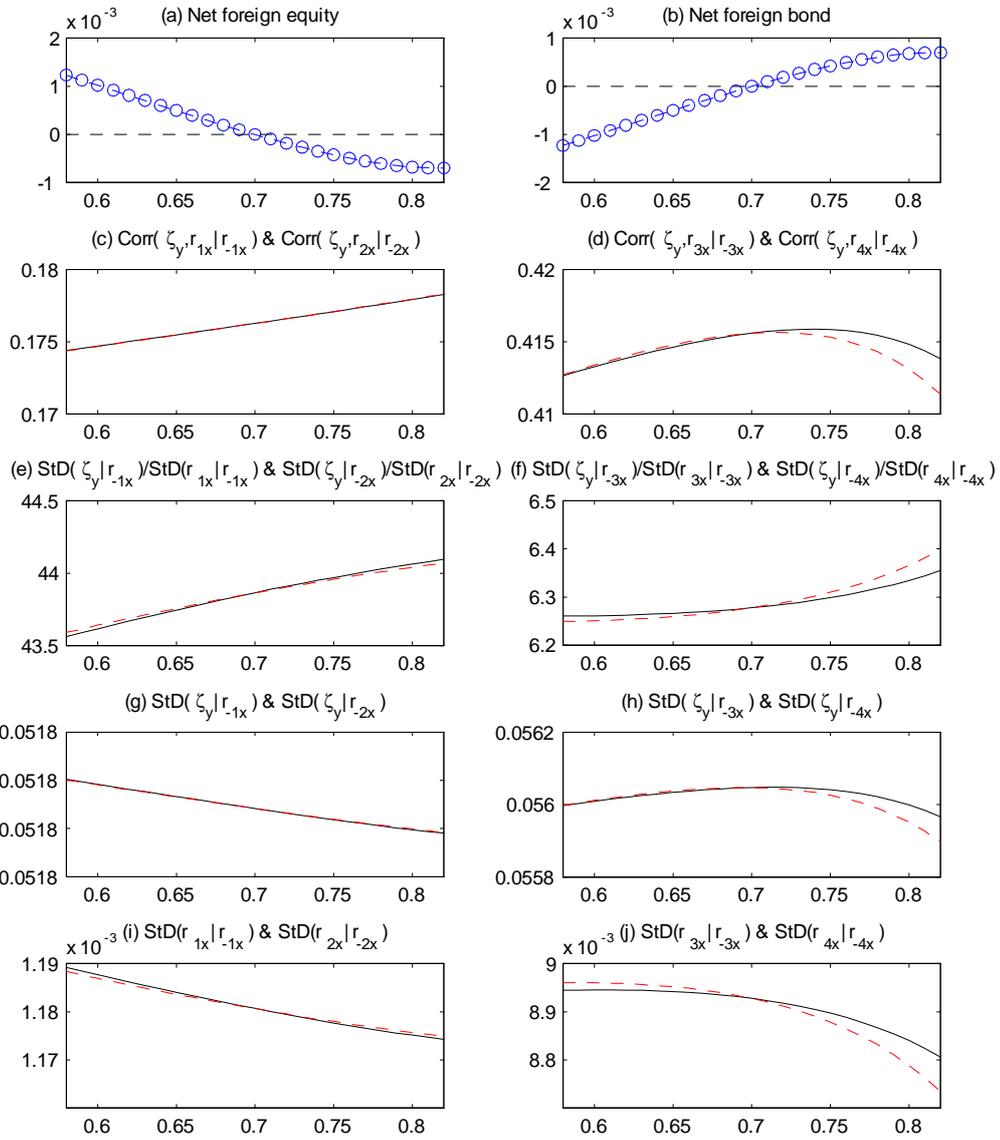


Figure 2.6: Nominal (wage) rigidity ς

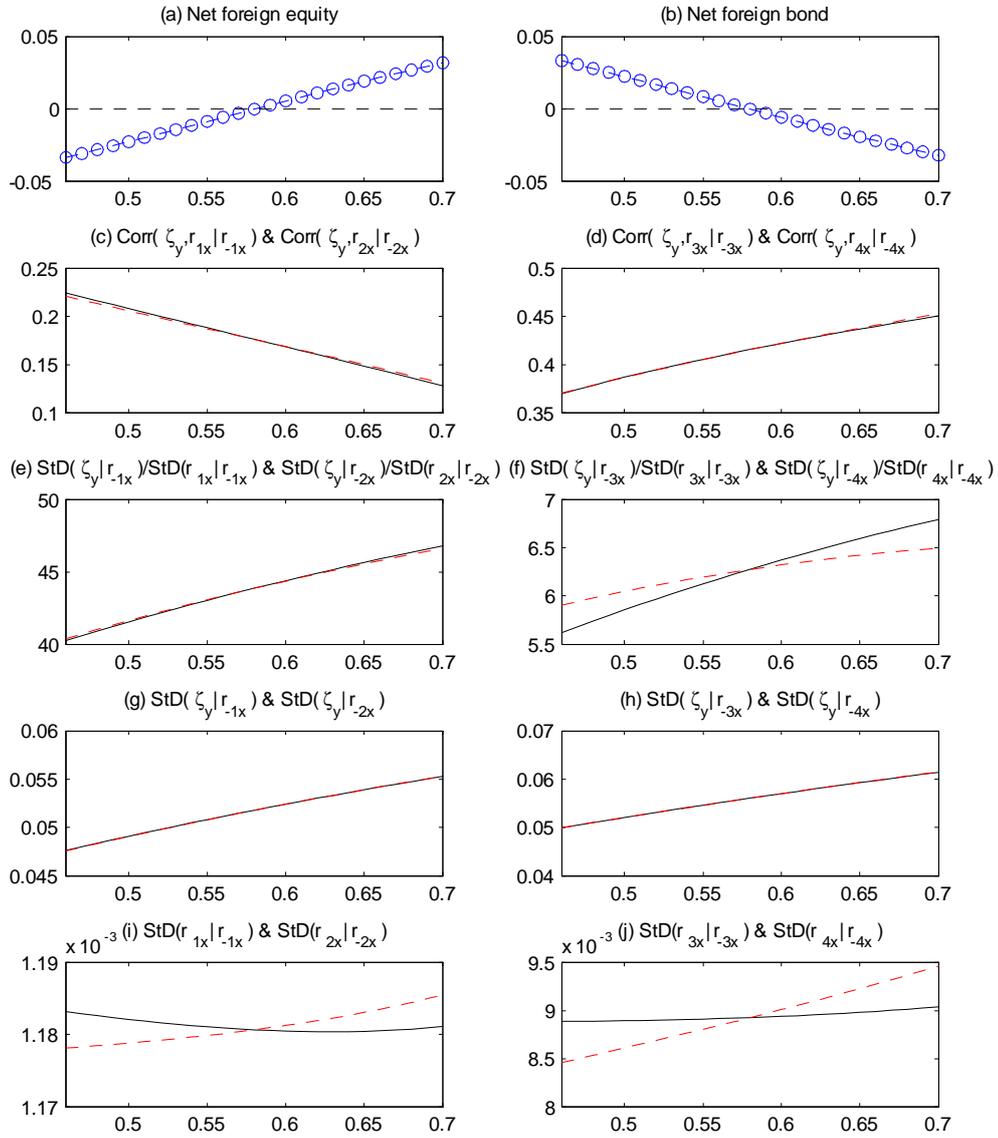


Figure 2.7: Home good bias γ

Home good bias

The parameter that determines the steady state share of home traded goods in the traded consumption basket, γ , governs the severity of home good bias. The higher is the value of γ , the more severe is home good bias. We set γ^* at the standard value of 0.58 and vary γ from 0.46 to 0.70. Figure 2.7 reports the results for this experiment. From panels (a) and (b), when $\gamma < \gamma^*$, we obtain $\tilde{\alpha}_E < 0$ and $\tilde{\alpha}_B > 0$. So a less severe home good bias in the home country will lead to two-way capital flows, with the home country holding a net negative position in equities and a net positive position in bonds (as observed in the data for developing countries).

Panel (c) tells us that when $\gamma < \gamma^*$, the relative return on home equity is more closely correlated with relative income than that of the foreign equity, which implies a relatively large absolute position of $\tilde{\alpha}_1$. This is the reason for a negative $\tilde{\alpha}_E$. By panel (e), the relative variability effect actually works in the other direction. When $\gamma < \gamma^*$, the relative returns conditional on r_{-1x} and r_{-2x} have the same volatility (panel (g)), but because the excess return of home equity has a relatively high volatility compared to that of the foreign equity (panel (i)), the variability effect is lower (panel (e)), which entails a relatively small position of $\tilde{\alpha}_1$. This partially offsets the relative correlation effect.

For bond positions, when $\gamma < \gamma^*$, the relative variability effect between home and foreign assets are similar to that of equity assets. The variability effect associated with home bond is relatively low (panel (f)), which entails a relatively small position of $\tilde{\alpha}_3$ (and a relatively large position of $\tilde{\alpha}_4$ correspondingly). This is the reason for a positive $\tilde{\alpha}_B$. The relative correlation effects between $\tilde{\alpha}_3$ and $\tilde{\alpha}_4$ are approximately zero, i.e. the lines representing the two effects overlap each other (panel (d)).

Trade openness

Trade openness can be represented by the share of nontraded goods in the total consumption basket, which is determined by the parameter κ . The higher is the value of κ , the less open is trade in the country. We set κ^* at the standard value of 0.4 and vary κ from 0.28 to 0.52. As is shown in the Figure 2.8, the result is that as the home country has a smaller share of nontraded goods in the

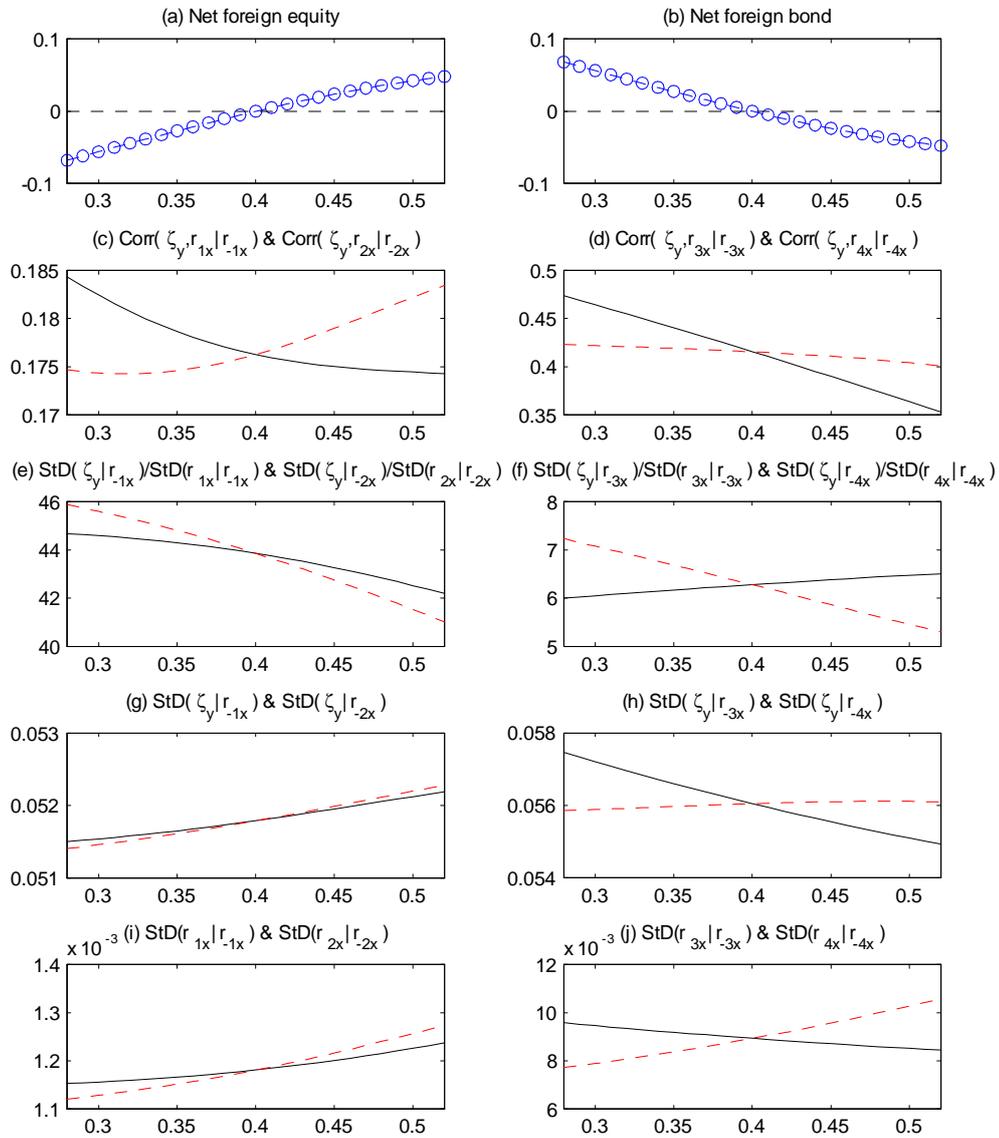


Figure 2.8: Trade openness κ

consumption basket, the more pronounced are two-way capital flows (panels (a) and (b)) i.e. where $\tilde{\alpha}_E < 0$ and $\tilde{\alpha}_B > 0$. The pattern therefore resembles that of home bias shown above.

In terms of decomposition into correlation and variability effects, we observe that, when $\kappa < \kappa^*$, the correlation effect associated with $\tilde{\alpha}_1$ is always higher than that of $\tilde{\alpha}_2$ (panel (c)). This gives rise to a large position of $\tilde{\alpha}_1$ and a negative $\tilde{\alpha}_E$. In addition, both the conditional volatility of relative income and that of the excess return associated with home equity are higher than those associated with foreign equity (panels (g) and (i)). But the volatility of the excess return rises more than that of relative income. This generates a lower variability effect of $\tilde{\alpha}_1$ compared to that of $\tilde{\alpha}_2$ (panel (e)), which partially offsets the relative correlation effect.

For bond positions, when $\kappa < \kappa^*$, the correlation effect associated with $\tilde{\alpha}_3$ is always greater than that associated with $\tilde{\alpha}_4$ (panel (d)), which implies a relatively large position of $\tilde{\alpha}_3$ and a negative $\tilde{\alpha}_B$. However, the variability effect associated with $\tilde{\alpha}_3$ is always below that associated with $\tilde{\alpha}_4$ (panel (f)), which, in contrast, implies a relatively small position of $\tilde{\alpha}_3$ and a positive $\tilde{\alpha}_B$. The importance of the relative variability effect quantitatively outweighs that of the relative correlation effect. This justifies the presence of a positive $\tilde{\alpha}_B$.

Household preferences

In this subsection, we consider asymmetries associated with two parameters of households' preferences. These are the elasticity of substitution between home and foreign tradables, θ , and the elasticity of substitution between tradables and non-tradables, ϕ .

For the former, θ^* is set at 1.5 while θ ranges from 1 to 2. The result is shown in Figure 2.9. It is obvious that there is no effect of this asymmetry on $\tilde{\alpha}_E$ and $\tilde{\alpha}_B$. The asymmetry associated with θ seems to be an irrelevant factor when it comes to two-way capital flows.

For the latter parameter, ϕ^* is set at 0.45 while ϕ ranges from 0.33 to 0.57. It is clear from Figure 2.10 that as ϕ increases, the portfolio pattern displays two-way capital flows where the home country has a negative net position in equities and a positive net position in bonds in line with observed data on developing

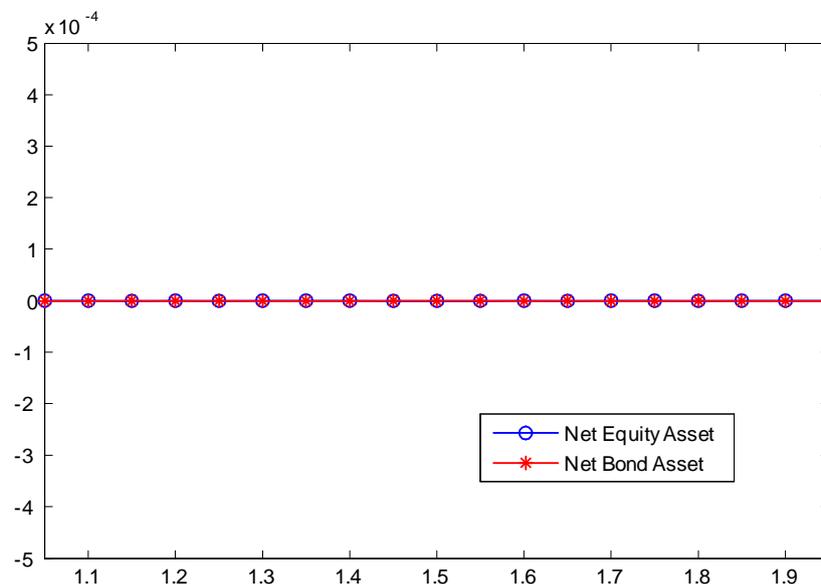


Figure 2.9: Substitutability between home and foreign tradables θ

countries. The higher is ϕ relative to ϕ^* , the more significant are the two-way capital flows.

When $\phi > \phi^*$, the correlation effect associated with $\tilde{\alpha}_1$ is greater than that associated with $\tilde{\alpha}_2$ (panel (c)), which implies that home equity as a hedge against income risks is relatively superior to foreign equity. This tends to generate a negative $\tilde{\alpha}_E$. However, the variability effect associated with $\tilde{\alpha}_1$ is less than that associated with $\tilde{\alpha}_2$ (panel (e)), which implies, given the presence the other assets, it requires a relatively smaller $\tilde{\alpha}_1$ to hedge against the related income risks. This tends to generate a positive $\tilde{\alpha}_E$. It turns out that the relative correlation effect is more important, so $\tilde{\alpha}_E < 0$ is observed.

For bond positions, when $\phi > \phi^*$, the correlation effect associated with $\tilde{\alpha}_3$ is below that associated with $\tilde{\alpha}_4$ (panel (d)) which tends to generate a positive $\tilde{\alpha}_B$. Moreover, the variability effect associated with $\tilde{\alpha}_3$ is below that associated with $\tilde{\alpha}_4$ (panel (f)), which tends to reinforce the relative correlation effect in generating a positive $\tilde{\alpha}_B$.

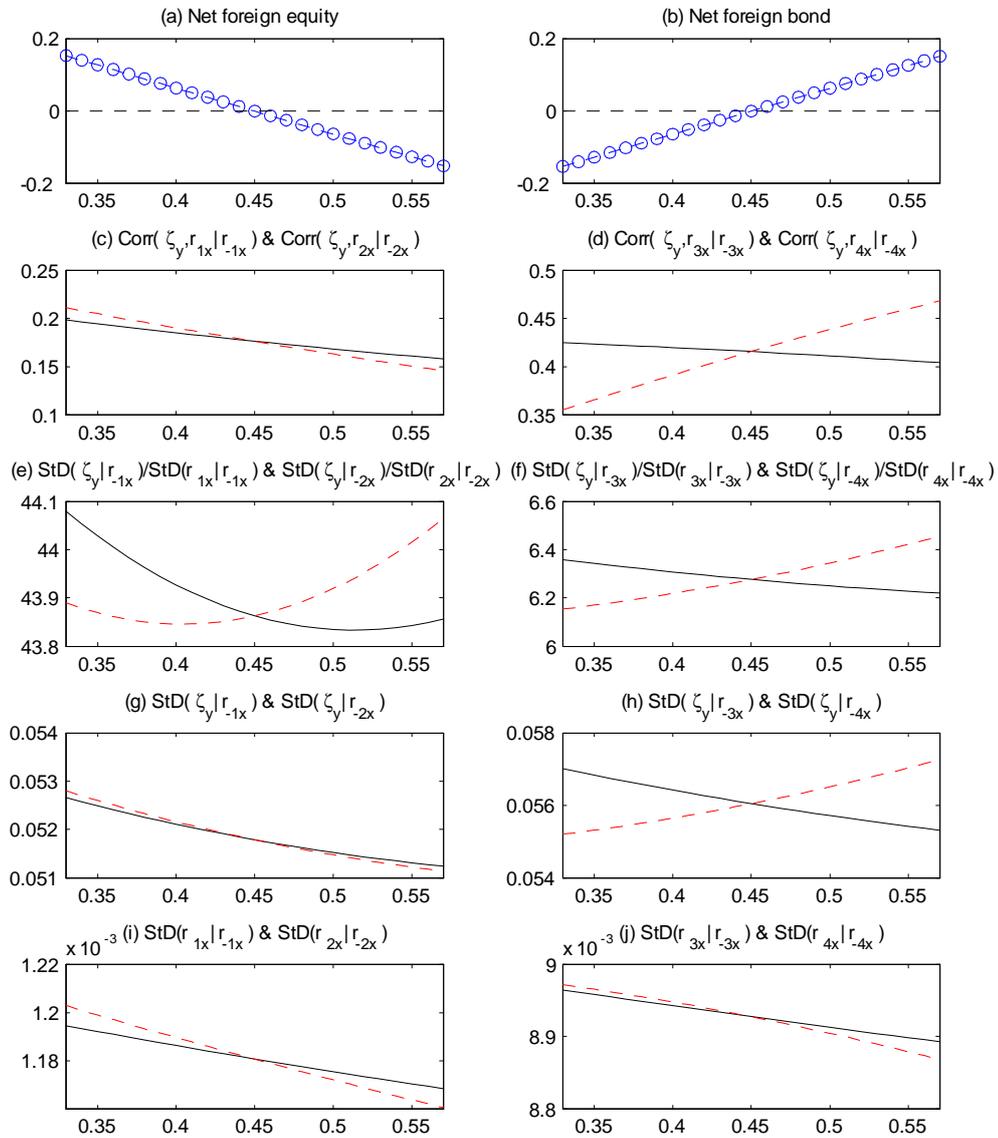


Figure 2.10: Substitutability between tradables and non-tradables ϕ

Capital adjustment costs

It is possible that marginal costs of capital adjustment in developing and developed countries are not equal. What is the consequence of this asymmetry on country portfolios? In our model, this can be determined by manipulating the parameter ψ . We set ψ^* at its benchmark level of 0.25 and vary ψ from 0.13 to 0.37. The results are displayed in Figure 2.11. By panels (a) and (b), when $\psi < \psi^*$, we have $\tilde{\alpha}_E < 0$ and $\tilde{\alpha}_B > 0$. Thus the lower is ψ relative to ψ^* , the more significant is the pattern of two-way capital flows (with the home country holdings a negative net position in equities and a positive net position in bonds).

By panel (c), $\tilde{\alpha}_E$ is negative mainly because the correlation effect associated with $\tilde{\alpha}_1$ is above that associated with $\tilde{\alpha}_2$. The variability effect associated with $\tilde{\alpha}_1$ is, by panel (e) however, below that associated with $\tilde{\alpha}_2$. This is in turn because even though the conditional relative income and excess return of home equity both are more volatile than that of foreign equity, the volatility in excess return dominates.

By panel (d) and (f), the correlation and variability effect associated with $\tilde{\alpha}_3$ are both below that associated with $\tilde{\alpha}_4$. They combine to lower the size of $\tilde{\alpha}_3$ comparing to that of $\tilde{\alpha}_4$, which explains why $\tilde{\alpha}_B$ is positive. Besides, by panels (h) and (j), the relatively low variability effect of the home bond is due to the relative low volatility of disposable income and relative high volatility of excess return when $\psi < \psi^*$.

Monetary policy

Monetary policies in developing and developed countries may be conducted in different ways. In this subsection, we explore the possibility that they put different weights on inflation and output gap stabilization. This is captured by asymmetries associated with the two feedback coefficients of Taylor rule in the model, i.e. inflation feedback coefficient δ_π and output gap feedback coefficient δ_y respectively.

For the former, we set δ_π^* at the benchmark level of 2 and vary δ_π from 1.1 to 2.8. We plot the results in Figure 2.12. By panels (a) and (b), it is obvious that when $\delta_\pi < \delta_\pi^*$, we have $\tilde{\alpha}_E < 0$ and $\tilde{\alpha}_B > 0$, i.e. if the home country puts relatively less weight on inflation stabilization when conducting monetary

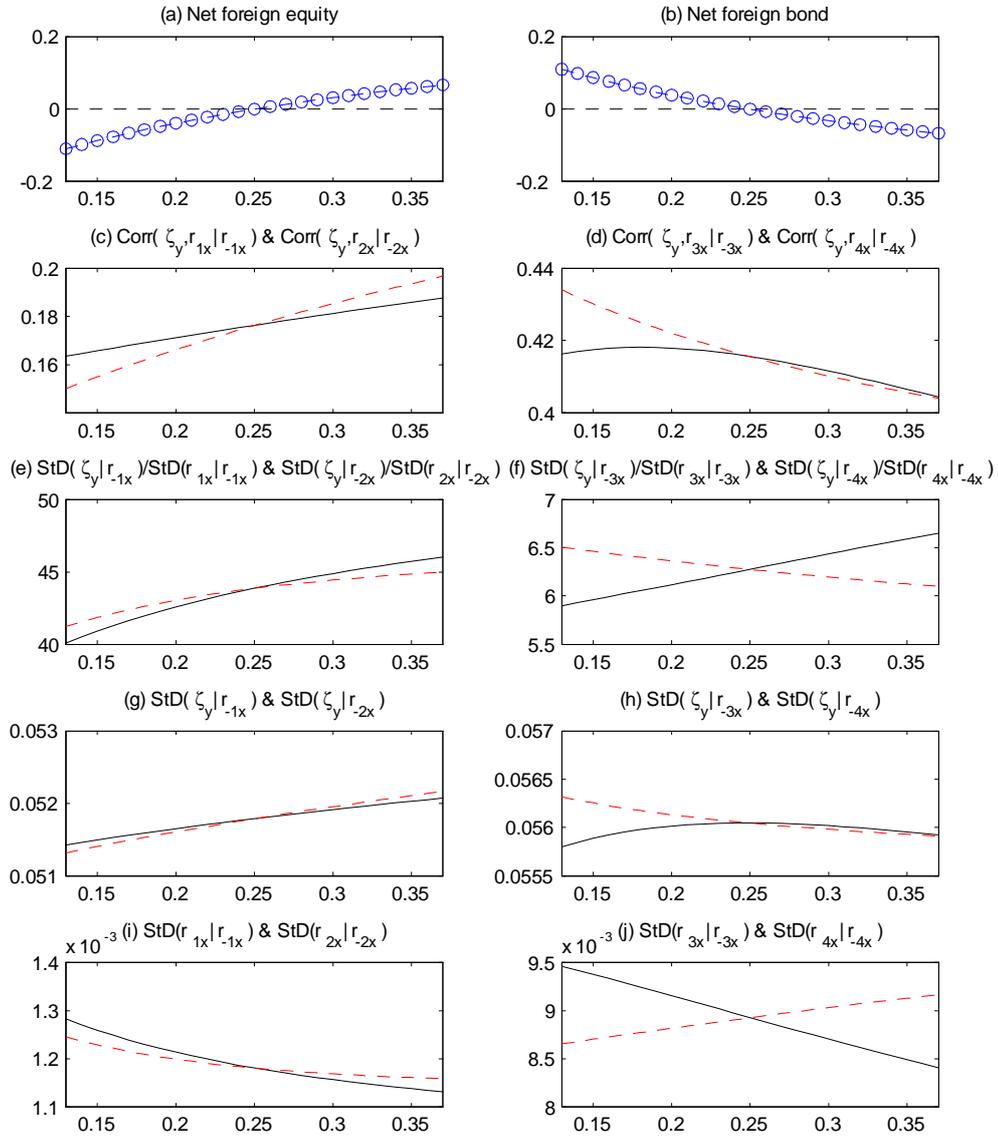


Figure 2.11: Capital adjustment costs ψ

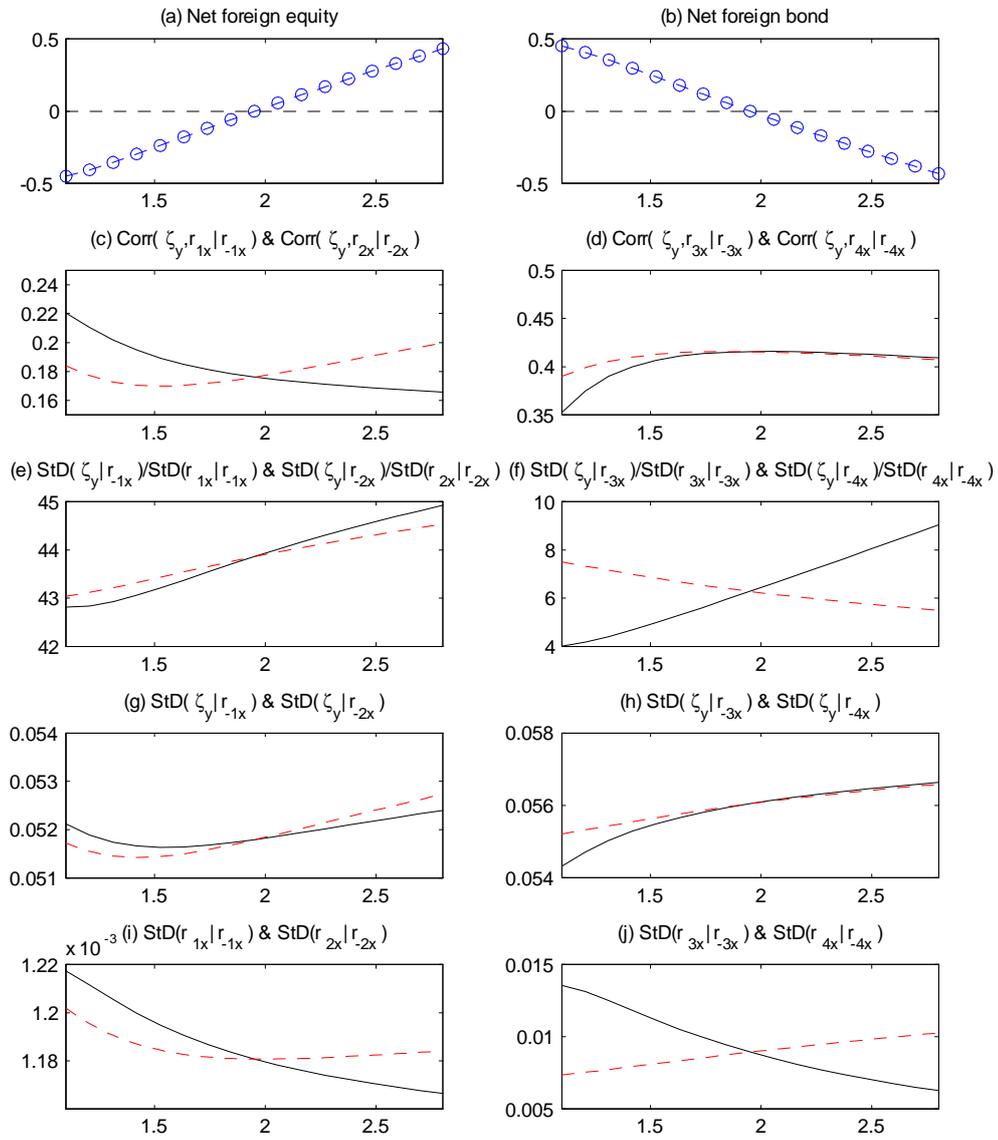


Figure 2.12: Monetary policy: Inflation feedback δ_π

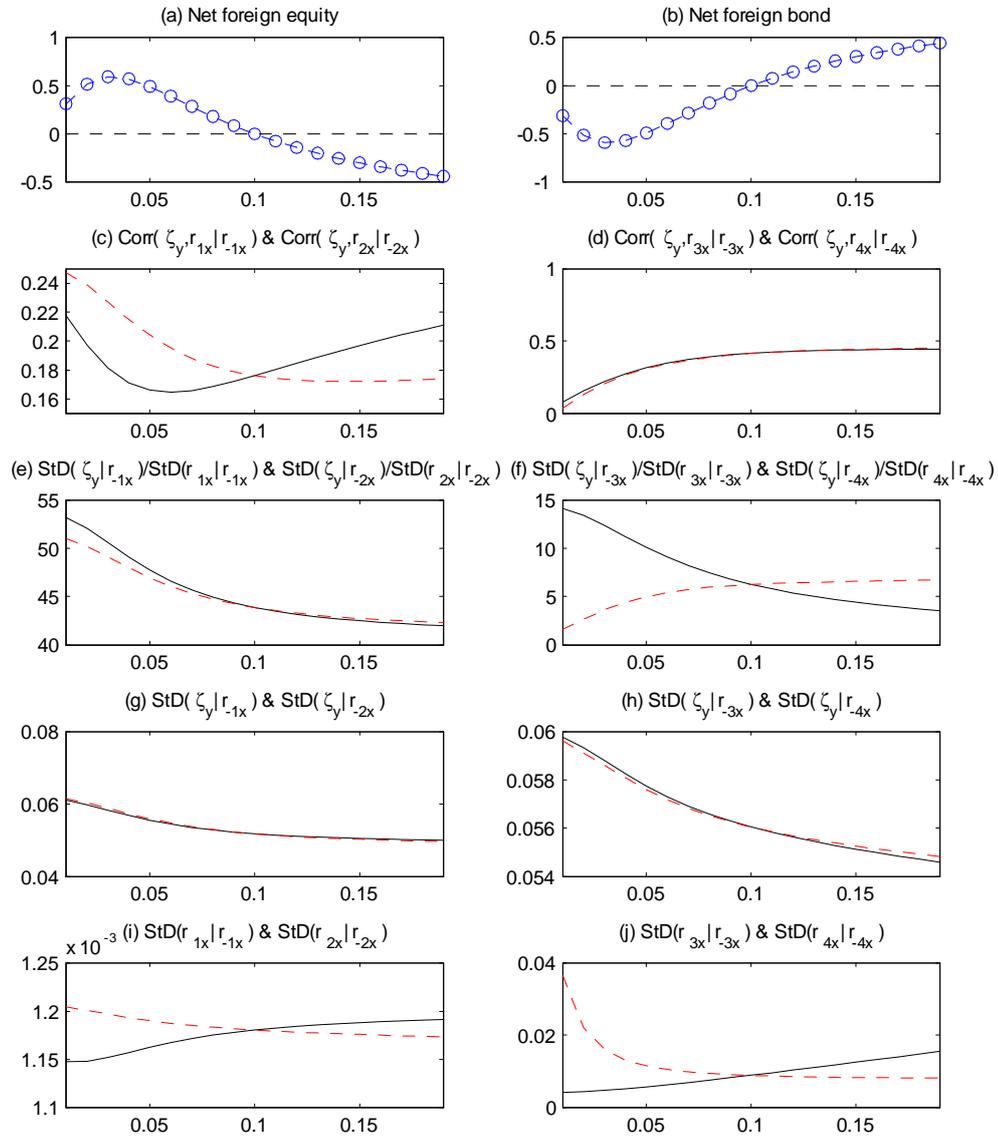


Figure 2.13: Monetary policy: Output gap feedback δ_y

policy, there tends to be a two-way capital flows with the home country holding a negative net position in equities and a positive net position in bonds.

When $\delta_\pi < \delta_\pi^*$, by panel (c), the correlation effect associated with $\tilde{\alpha}_1$ is above that associated with $\tilde{\alpha}_2$, which involves a relatively large negative position in home equity and thus a negative net equity position. However, there is a minor conflicting effect from the variability effect. By panel (e), the variability effect associated with $\tilde{\alpha}_1$ is below that associated with $\tilde{\alpha}_2$, partially offsetting the correlation effect.

By panel (d), when $\delta_\pi < \delta_\pi^*$, both the correlation and variability effects associated with the home bond are lower than those associated with the foreign bond. This means the position in the home bond should be smaller than that of foreign bond, which explains a positive net bond position.

For the asymmetry in δ_y , we set δ_y^* at the benchmark level of 0.1 and change the home value from 0.01 to 0.19. As shown in Figure 2.13, it turns out that when $\delta_y > \delta_y^*$, $\tilde{\alpha}_E < 0$ and $\tilde{\alpha}_B > 0$, i.e. if the monetary policy in home country reacts more to the output gap than in foreign country, there tends to be a two-way capital flow between the two countries, with the home country holding a net negative position in equities and a net positive position in bonds.

When $\delta_y > \delta_y^*$, the correlation effect associated with the home equity is well above that associated with the foreign equity (panel (c)) while the variability effect associated with the home equity is slightly below that associated with the foreign equity (panel (e)), so in total, the position of the home equity will exceeds that of the foreign equity which leads to a negative $\tilde{\alpha}_E$.

For bond assets however, when $\delta_y > \delta_y^*$, both the correlation and variability effects associated with the home bond are below those associated with the foreign bond (panels (d) and (f)). The emphasis on output stabilization in the developing country at the same time (relatively) undermines the relevance of the home bond in risk hedging and the risk amount to be hedged against by it, which implies a smaller position in the home bond compared to that of the foreign bond and thus a positive $\tilde{\alpha}_B$.

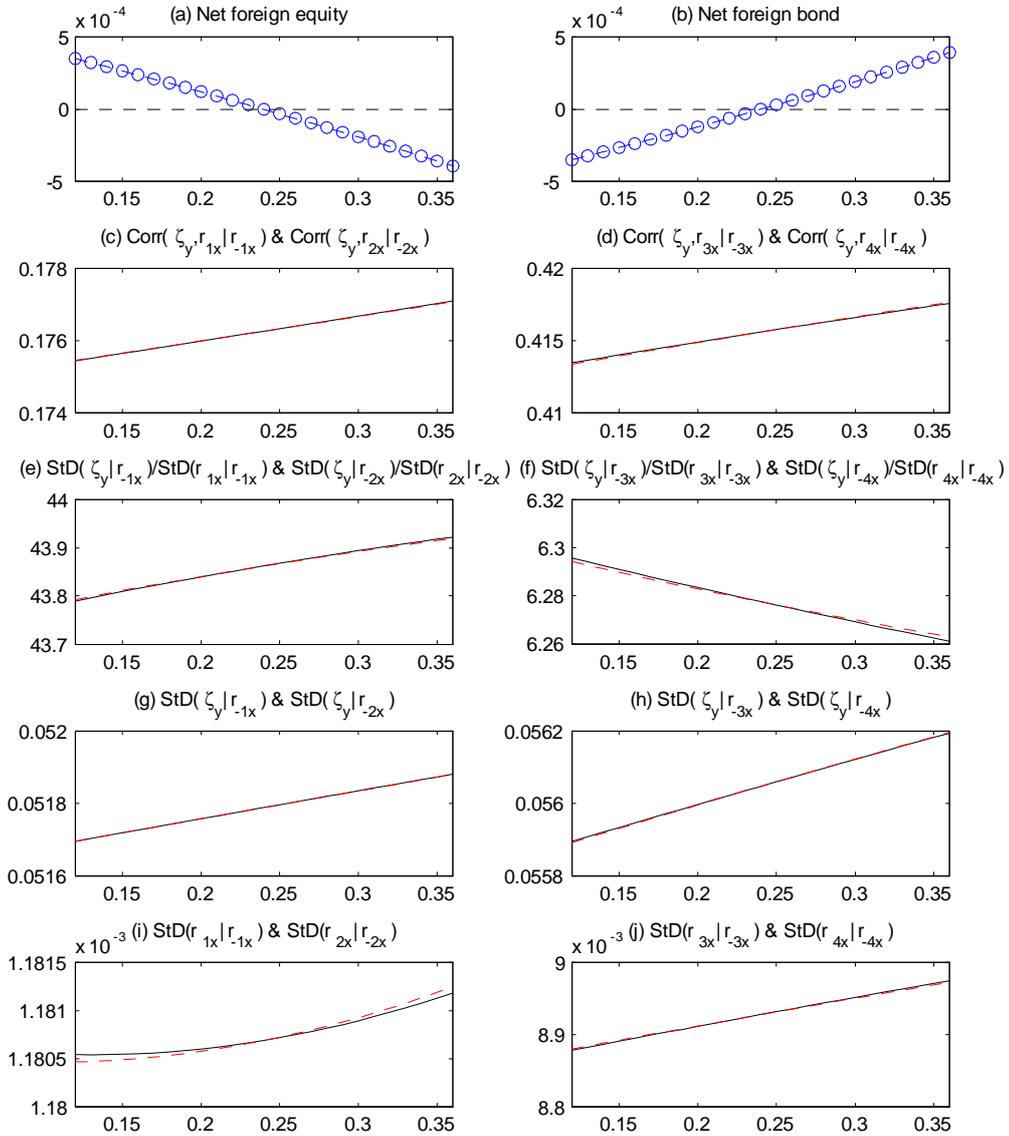


Figure 2.14: Price indexation ω

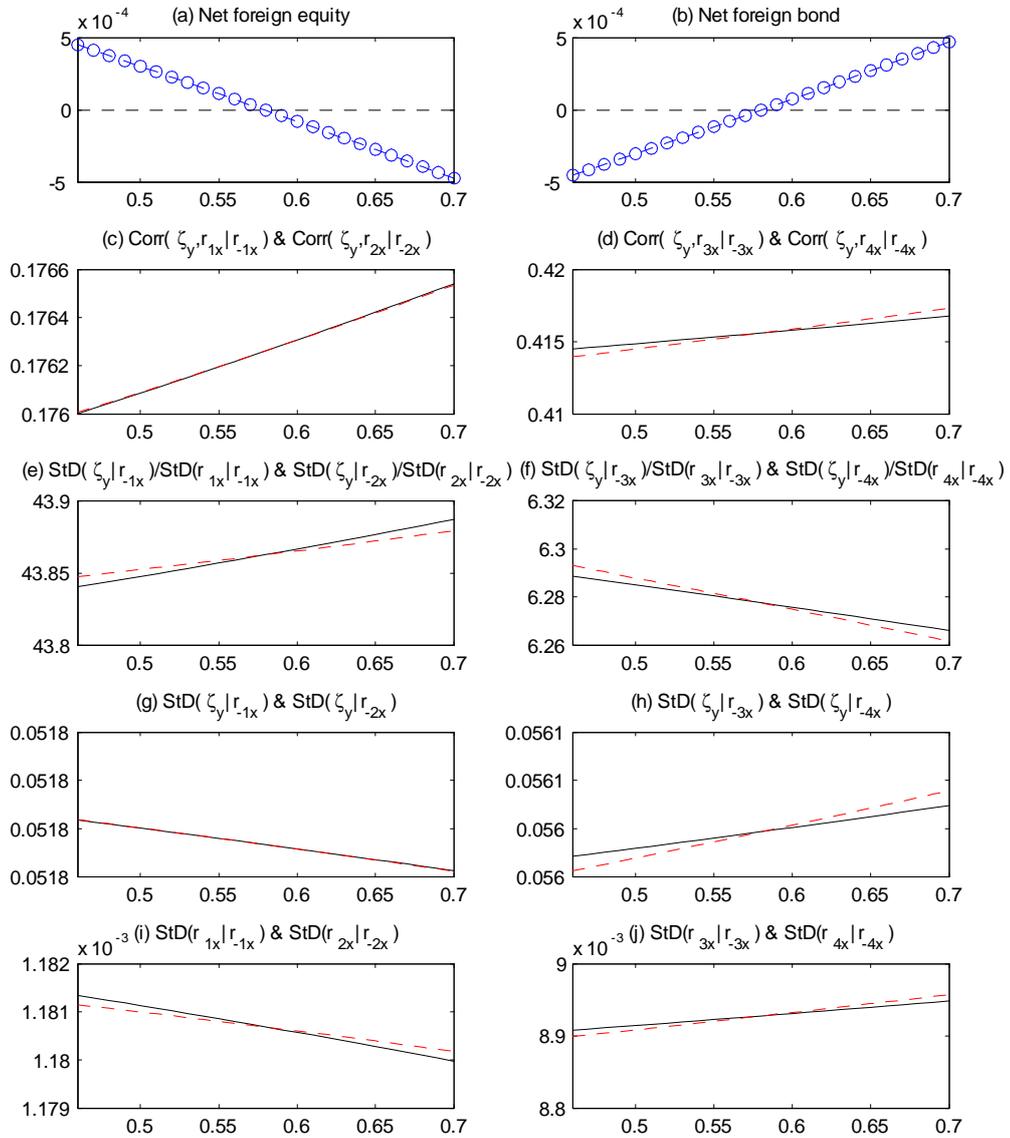


Figure 2.15: Wage indexation ϖ

Price/Wage indexation

We turn to asymmetries in price and wage indexation across countries in this subsection. For price indexation, we set ω^* at the standard value of 0.24 and change the home value from 0.12 to 0.36 while for wage indexation, we set ϖ^* at the standard value of 0.58 and change the home value from 0.46 to 0.70. As is shown in Figure 2.14 and 2.15, when $\omega > \omega^*$ or/and $\varpi > \varpi^*$, then $\tilde{\alpha}_E < 0$ and $\tilde{\alpha}_B > 0$, so there is a two-way capital flow with the home country holding a negative net position in equities and a positive net position in bonds. The results tend to suggest that a high degree of price and wage indexation in developing countries is consistent with the emergence of two-way capital flows between the two groups of countries. However, as in the case of asymmetries in the degree of price and wage rigidity (λ and ς), the asymmetries in ω and ϖ have a very small effect on net equity and bond positions.

Habit formation

The degree of habit formation is governed by the parameter h . To assess the effect of asymmetry in h on two-way capital flows, we set h^* at the benchmark value of 0.7 and vary the value of h from 0.58 to 0.82. Figure 2.16 plots the result. It is clear from panels (a) and (b) that when $h < h^*$, we have $\tilde{\alpha}_E < 0$ and $\tilde{\alpha}_B > 0$, i.e. if the home households have a lower degree of habit formation than foreign households, this will result in two-way capital flows with the home country holding a negative net position in equities and a positive net position in bonds.

When $h < h^*$, we have $\tilde{\alpha}_E < 0$ because both the correlation and variability effect associated with the home equity are above those associated with the foreign equity (panels (c) and (e)). For bond positions, when $h < h^*$, we have $\tilde{\alpha}_B > 0$ because on the one hand, the correlation effect associated with the home bond is relatively lower than that of the foreign bond, on the other hand, the variability effect associated with it is relatively higher but the correlation effect dominates.

Market competitiveness

We can use the parameter of φ to represent the degree of competitiveness in an economy. The lower is φ , the lower is the substitutability between varieties and

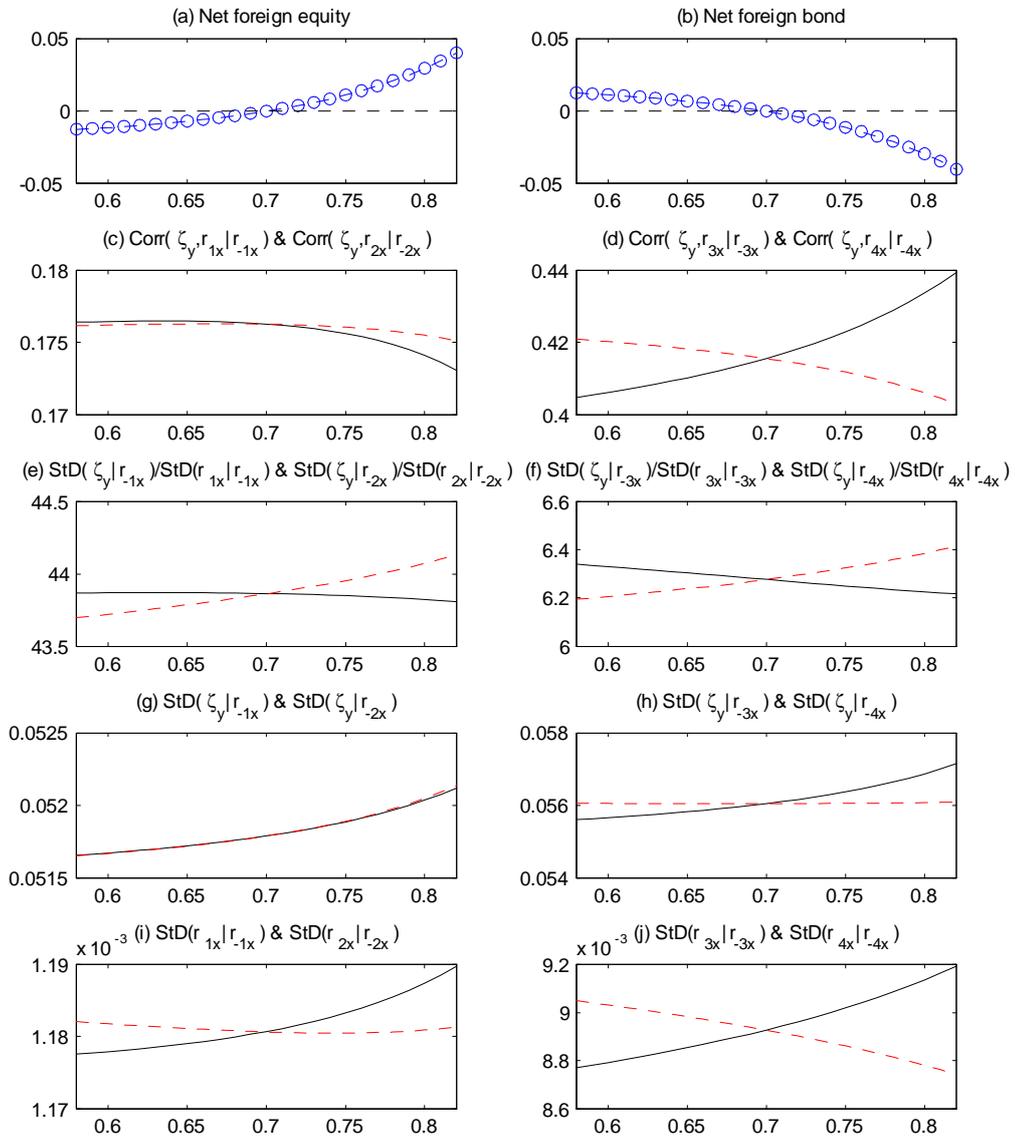


Figure 2.16: Habit formation h

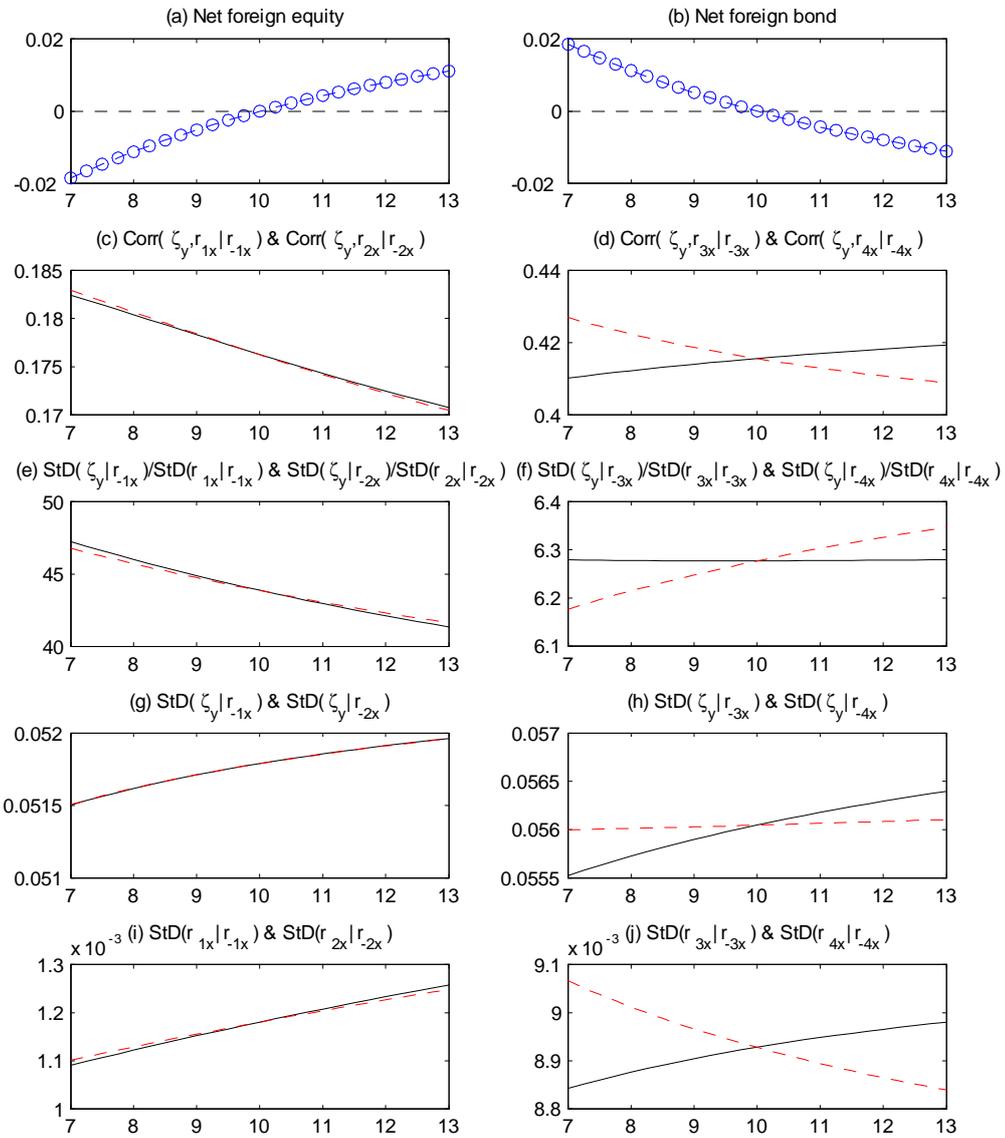


Figure 2.17: Market competitiveness φ

so the more power firms have when setting prices. Also note that the optimal price of final good is a mark-up over associated marginal cost of production, $\frac{\varphi}{\varphi-1}$, so the lower is φ the higher is the mark-up. In other words, the lower is φ , the lower is the degree of market competitiveness.

To check the effect of the asymmetry associated with φ on two-way capital flows, we set the value of φ^* at 10 as in the benchmark calibration and vary the value of φ from 7 to 13 which corresponds to a price mark-up from about 8.3% to 16.7% in economy. As is shown in Figure 2.17, two-way capital flows arise if φ is less than φ^* , i.e. the home market is less competitive than the foreign market.

When $\varphi < \varphi^*$, the correlation effect associated with the home equity is below that associated with the foreign equity while the variability effect associated with the home equity is above that associated with the foreign equity (panels (c) and (e)). The difference in variability effect is quantitatively more important, so the gross position in the home equity is relatively large (in absolute value) and $\tilde{\alpha}_E < 0$. For bond assets, the correlation effect associated with the home bond is also below that associated with the foreign bond while the variability effect associated with the home bond is above that associated with the foreign bond (panels (d) and (f)). However, the difference in correlation effect is quantitatively more important, so the gross position in the home bond is relatively small (in absolute value) and $\tilde{\alpha}_B > 0$.

Pricing strategy

Different pricing strategies, *PCP* or *LCP*, have different implications for behaviour of import prices. So it is worthwhile to check the cases in which the developing and developed countries price products according to different strategies. Because, without loss of generality, the home country is viewed as developing country and the currencies used in international transactions are usually those of developed country, it is natural to believe that the firms in the home country use *LCP* while the firms in the foreign country use *PCP*. Based on this belief, in what follows we consider two experiments. First, suppose the firms in the home country all set prices of tradables according to *LCP* and the foreign country's pricing strategy stands at different position between perfect *LCP* and *PCP*, one can interpret this as such that some foreign firms adopt *LCP* while others adopt

PCP. Second, suppose conversely that the firms in foreign country all set prices of tradables according to *PCP* and the home country's pricing strategy stands at different positions between perfect *LCP* and *PCP*, again one can interpret this as such that some home firms adopt *LCP* while others adopt *PCP*.

For the former experiment, we set $\eta = 0$ and vary value of η^* from 0 to 1. The result is displayed in Figure 2.18. Note that the symmetric benchmark corresponds to the allocation at the left-hand side in all panels in the figure. It is obvious from the figure, as the foreign country's choice of pricing strategy approaches *PCP*, the portfolio allocations exhibit two-way capital flows, i.e. when $0 = \eta < \eta^*$, we have $\tilde{\alpha}_E < 0$ while $\tilde{\alpha}_B > 0$ (panels (a) and (b)), so the home country has a net negative holding of equities and a net positive holding of bonds. Further investigation shows that when $0 = \eta < \eta^*$, the correlation effects associated with home assets are roughly the same as those associated with foreign assets, however, the variability effect associated with the home equity is above that associated with the foreign equity (panel (e)) while the variability effect associated with the home bond is below that associated with the foreign bond (panel (f)), so the net equity position is negative while the net bond position is positive.

For the second experiment, we set η^* at 1 and vary the value of η from 0 to 1. The result is displayed in Figure 2.19. Note that the allocation at the right-hand side in all panels in the figure corresponds to a symmetric case. By panels (a) and (b), as the home country's choice of pricing strategy approaches *LCP*, the portfolio allocations always exhibit two-way capital flows, i.e. when $\eta < \eta^* = 1$, we have $\tilde{\alpha}_E < 0$ while $\tilde{\alpha}_B > 0$, so the home country has a negative net holding of equities and a positive net holding of bonds. Further investigation shows that when $\eta < \eta^* = 1$, as in the previous experiment, the correlation effects of home and foreign assets are roughly the same, however, the variability effect associated with the home equity is above that associated with the foreign equity (panel (e)) while the variability effect associated with the home bond is below that associated with the foreign bond (panel (f)), so the net equity position is negative while the net bond position is positive.

So to sum up, if the home country has a lower η compared to the foreign country, i.e. the developing country's pricing strategy is relatively close to *LCP* while the developed country's strategy is relatively close to *PCP*, two-way capital

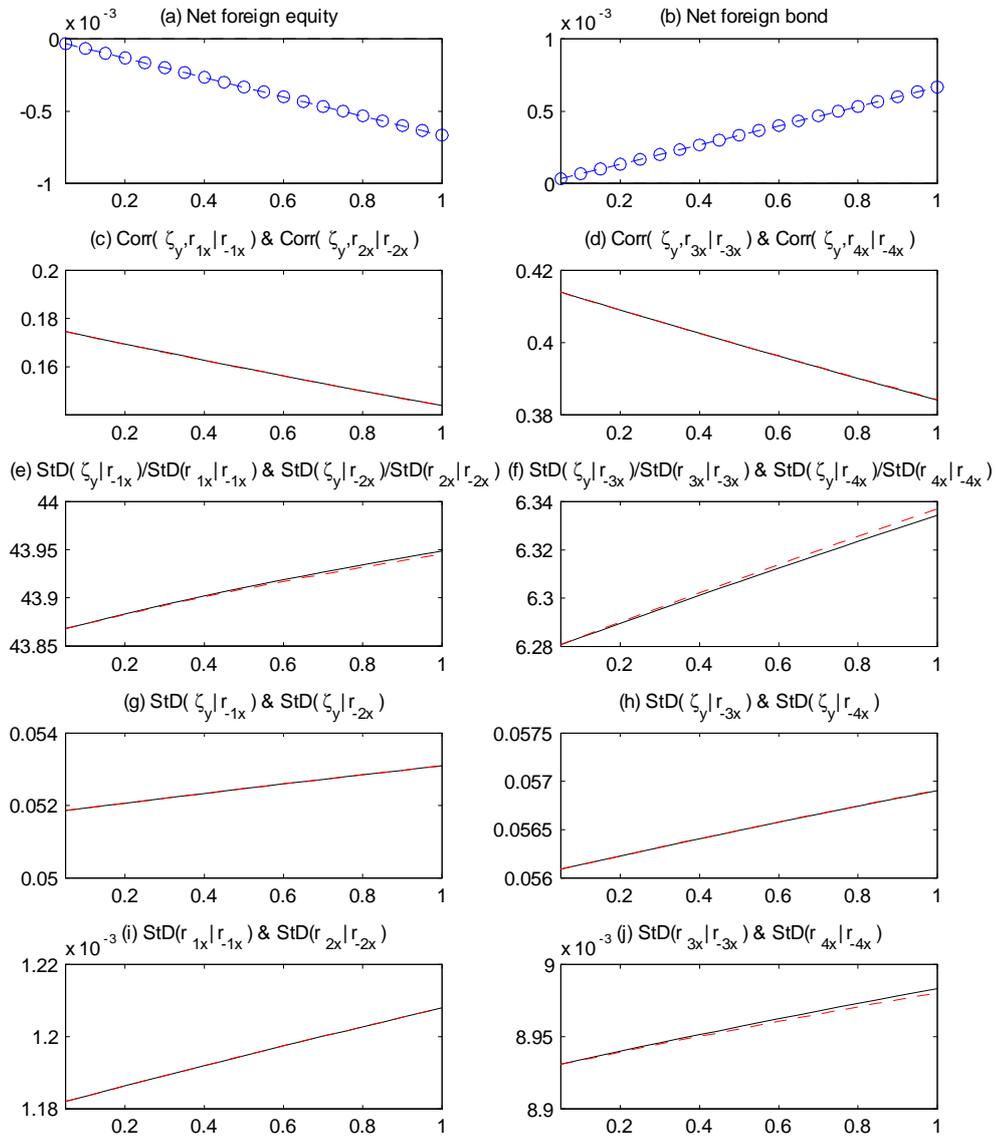


Figure 2.18: Pricing strategy: Foreign country moving to PCP η^*

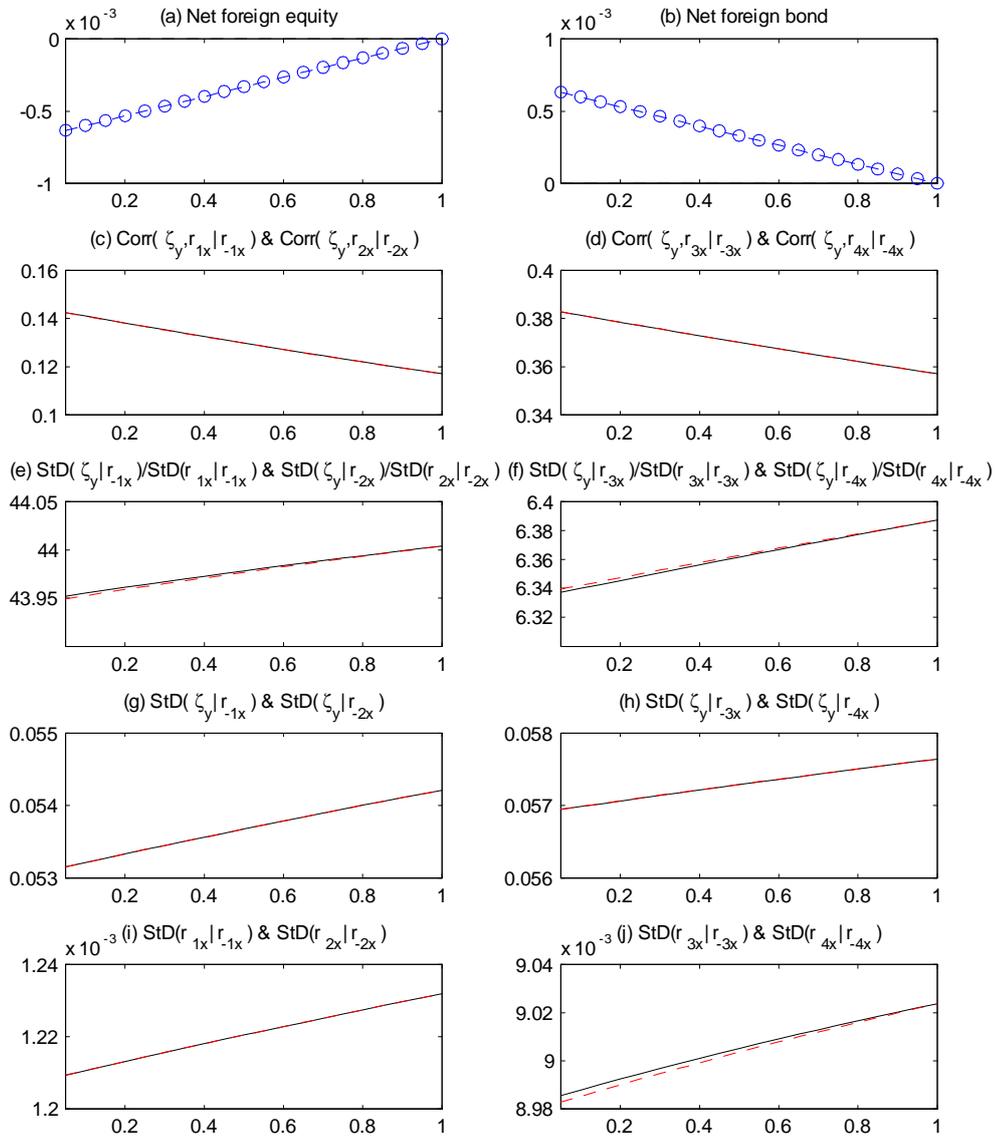


Figure 2.19: Price strategy: Home country moving to LCP η

flows arise. However, again we have to notice that the magnitude of the effect that the asymmetry has on net positions is very small. It turns out that it is always below 0.001 so we also view the asymmetry as a minor factor in affecting the pattern of two-way capital flows.

Short summary

In this section, we have examined the various asymmetries' role in generating two-way capital flows between the two types of countries. We have obtained at least two sets of result. The first set of result concerns the question of which direction the asymmetries impact the pattern of capital flows. And we have found that the following facts are candidates in favour of the emergence of two-way capital flows (which are consistent with observed data for developing countries). Compared to a developed country, in a developing country, if firms use more labour intensive technology $a > a^*$; it is less costly for them to adjust investment $\psi < \psi^*$; when setting prices for products and labour (given that both countries feature high nominal rigidity) they are confronted with more frictions $\lambda > \lambda^*$ and/or $\varsigma > \varsigma^*$; in the traded sector, firms set the prices through *LCP* more often, $\eta < \eta^*$; households consume more traded goods $\kappa < \kappa^*$ and imports $\gamma < \gamma^*$; traded and non-traded goods are more substitutable $\phi > \phi^*$; there is less persistent habit formation $h < h^*$; the monetary authority responds more intensely to the output gap while less so to inflation, $\delta_y > \delta_y^*$ and/or $\delta_\pi < \delta_\pi^*$; the market in developing country is less competitive $\varphi < \varphi^*$; and the degree of price/wage indexation is higher $\omega > \omega^*$ and/or $\varpi > \varpi^*$. The second set of results concerns the magnitude of the effects of asymmetries on two-way capital flows. While some asymmetries that we mentioned do affect the pattern of capital flows, they are not that important because the magnitude of their effects is relatively low. These include the asymmetries associated with nominal rigidities, the degree of price/wage indexation and pricing strategy. So we see that the pattern of two-way capital flows is more likely driven by asymmetries in real factors instead of asymmetries in nominal factors. In addition, we found that not all asymmetries in the model are relevant for the question of two-way capital flows. For instance, asymmetry in the substitutability between home and foreign traded goods has no effect on the pattern of two-way capital flows.

Description	Variable values
Calvo price rigidity parameter	$\lambda = 0.95, \lambda^* = 0.66$
Calvo wage rigidity parameter	$\varsigma = 0.79, \varsigma^* = 0.70$
Price indexation	$\omega = 0.97, \omega^* = 0.24$
Wage indexation	$\varpi = 0.61, \varpi^* = 0.58$
Discount factor	$\beta = \beta^* = 0.99$
Habit persistence	$h = 0.81, h^* = 0.70$
Risk aversion coefficient	$\rho = \rho^* = 1.38$
Labour supply elasticity	$\mu = \mu^* = 2.83$
Share of home traded goods in traded basket	$\gamma = \gamma^* = 0.58$
Share of nontraded goods in consumption	$\kappa = \kappa^* = 0.40$
Substitutability between Home and Foreign tradables	$\theta = \theta^* = 1.50$
Substitutability between nontraded and traded goods	$\phi = \phi^* = 0.45$
Substitutability between individual goods	$\varphi = \varphi^* = 10$
Labour share of income in traded goods sector	$a_T = 0.5, a_T^* = 0.67$
Labour share of income in nontraded goods sector	$a_N = 0.5, a_N^* = 0.67$
Capital depreciation rate	$\delta = \delta^* = 0.025$
Investment adjustment cost	$\psi = \psi^* = 0.25$
Share of government spending	$g = g^* = 0.18$
Interest rate smoothing factor in Taylor rule	$\delta_R = 0.98, \delta_R^* = 0.81$
Inflation feedback in Taylor rule	$\delta_\pi = 1.67, \delta_\pi^* = 2$
Output feedback in Taylor rule	$\delta_Y = 0.15, \delta_Y^* = 0.1$
Pricing strategies	$\eta = 0, \eta^* = 1$
Persistence of technology shock in traded sector	$\delta_{TT1} = 0.93, \delta_{TT2} = 0$ $\delta_{TT1}^* = 0, \delta_{TT2}^* = 0.95$
Variance of technology shock in traded sector	$\sigma_T = 0.0277, \sigma_T^* = 0.0045$
Technology shock in non-traded sector	$\delta_{NN1} = 0.93, \delta_{NN2} = 0$ $\delta_{NN1}^* = 0, \delta_{NN2}^* = 0.95$
Variance of technology shock in non-traded sector	$\sigma_N = 0.0277, \sigma_N^* = 0.0045$
Cross terms of technology shocks	$\delta_{TN1} = \delta_{TN2} = 0.60,$ $\delta_{NT1} = \delta_{NT2} = 0$ $\delta_{TN1}^* = \delta_{TN2}^* = 0.60,$ $\delta_{NT1}^* = \delta_{NT2}^* = 0$
Monetary policy shock	$\delta_{rr} = 0.15, \sigma_{rr} = 0.0015$ $\delta_{rr}^* = 0.15, \sigma_{rr}^* = 0.0024$
Government spending shock	$\delta_G = 0.90, \sigma_G = 0.0877$ $\delta_G^* = 0.97, \sigma_G^* = 0.0053$
Mark-up shock	$\delta_V = 0.89, \sigma_V = 0.05$ $\delta_V^* = 0.89, \sigma_V^* = 0.002$
Labour supply shock	$\delta_\chi = 0.90, \sigma_\chi = 0.025$ $\delta_\chi^* = 0.90, \sigma_\chi^* = 0.025$
Investment adjustment cost shock	$\delta_\psi = 0.78, \sigma_\psi = 0.0128$ $\delta_\psi^* = 0.71, \sigma_\psi^* = 0.0045$

Table 2.3: Parameter values: Asymmetric case

2.5.2 A fully asymmetric simulation

After investigating the effect of each asymmetry on the pattern of country portfolios, we will now undertake another exercise, i.e. taking into account all asymmetries at the same time. In this section, we simulate the model in a fully asymmetric way. This will yield steady-state portfolios allowing us to assess the composite effect of coexistence of multiple asymmetries.

Following our convention, the home and foreign countries are labelled as developing and developed country respectively. Our strategy is to choose parameter values for the home country from the estimates based on the data of developing countries, especially China, (if they are available) while choosing parameter values for the foreign country from the estimates based on the data of developed countries, especially *U.S.*. The task of choosing parameter values for the foreign country is already done in the symmetric simulation. Now we describe how we choose parameter values for the home country.

In the model, the value of many parameters in the foreign country is obtained from Smets and Wouters (2007). That paper estimates a New Keynesian model of the *U.S.* economy. Recently, there are many studies applying the framework to emerging markets, in particular China, and these provide us with estimates of parameters in the context of developing countries. The main contributions to this empirical literature include Mehrotra et al. (2011), Sun and Sen (2012), Dai (2012) and Miao and Peng (2012) among others. In the following exercise, we mainly rely on Sun and Sen's (2012) estimation in choosing parameter values. These parameters include the degrees of price/wage stickiness λ and ς , the degrees of price/wage indexation ω and ϖ , habit persistence h , the feedback coefficients in monetary policy δ_R , δ_π and δ_y and the persistence and volatility of various shocks.

The discount factors are assumed to be equalized to be consistent with our assumptions of equalized autarky interest rates across countries and $F = 0$ in steady state. For the same reason, the values of the parameters appearing in households' preference are assumed to be in line with the benchmark. These include ρ and μ . Mehrotra et al (2011) estimates the elasticity of investment

with respect to the current price of installed capital in China, $1/\psi$, and finds it is very close to that found in the *U.S.* by Christiano et al. (2005), which make us to choose $\psi = \psi^*$. Based on Miao and Peng's (2012) estimation, the values of g and δ are also the same as their foreign counterparts. For the choice of labour share of production a , there is a wide spectrum. According to Chinese data (that reported in china statistical yearbook), the labour income share is at around 0.5 which is much lower than that in the *U.S.*. However, the current literature suggests that the real share in China should be higher than this and view the reported level as puzzling. Based on the literature, the reason for a reported low a are possibly due to measurement problems (Golin 2002) or/and misallocation frictions (Hsieh and Klelow 2009). Na (2015) estimates an average labour share for emerging countries of 0.7. For our simulation, because a^* is chosen based on reported share we also use the reported level of $a = 0.5$. Note that according to our analysis in the last section, a higher a tends to strengthen the pattern of two-way capital flows.

It is another challenge to obtain the estimates of the parameters that associated with open economy for the developing country. These include the share of traded goods in all tradables, γ , the share of non-traded goods in the consumption basket, κ , and the elasticity of substitution between home and foreign traded goods, θ , and that between traded and non-traded goods, ϕ . Schmitt-Grohe and Uribe (2015), using data from 38 poor and emerging countries, calibrate κ and ϕ at 0.44 and 0.5 which are still within the range of the parameter estimation for developed countries. Some literature, such as Laxton et al. (2010) and Prasad and Zhang (2015), use the same value of these parameters for the different types of country. Due to the lack of accurate estimate for these parameters for developing countries and the fact that (to our knowledge) no evidence shows a significant difference between these estimates in developing and developed countries, we follow the approach of Laxton et al. (2010) and Prasad and Zhang (2015) to be on the safe side in our simulation. (We also assume that the elasticity of substitution among individual goods φ are the same across countries.) For simplicity, we assume that the firms in the home country use *LCP* to price their exports while those in the foreign country use *PCP*, so $\eta = 0$ and $\eta^* = 1$.

Given that the differences between the two countries are specified by the parameter values as in Table 2.3, the result of fully asymmetric simulation of the

Assets menu	Optimal portfolio choices
Home equity	$\tilde{\alpha}_1 = -0.4177$
Foreign equity	$\tilde{\alpha}_2 = 0.1341$
Net equity asset	$\tilde{\alpha}_E = -0.2836$
Home bond	$\tilde{\alpha}_3 = -0.0739$
Foreign bond	$\tilde{\alpha}_4 = 0.3575$
Net bond asset	$\tilde{\alpha}_B = 0.2836$

Table 2.4: Optimal portfolio choices: Fully asymmetric case

model is documented in Table 2.4. According to the results, the home country sells the home equity to the amount of 0.42 (multiplied by βY) while it buys the foreign equity to the amount of 0.13, which results in a negative net position of equity, $\tilde{\alpha}_E = -0.28 < 0$. On the other hand, the home country also sells the home bond to the amount of 0.07 while it buys the foreign bond to the amount of 0.36, which results in a positive net position in bonds, $\tilde{\alpha}_B = 0.28 > 0$. By the condition of asset market clearing, the short position of an asset at home is a long position of the asset in the foreign country, $\tilde{\alpha}_i = -\tilde{\alpha}_i^*$. This leads to the fact that in foreign country we must have $\tilde{\alpha}_E^* > 0$ and $\tilde{\alpha}_B^* < 0$. Putting these facts together, we observe that the optimal portfolio allocations between the two asymmetric countries can be just described by the pattern of two-way capital flows, i.e. the home (developing) country ends up with a negative net position in equities and a positive net position in bonds while the foreign (developed) country ends up with a positive net position in equities and a negative net position in bonds.

2.5.3 Asymmetry in asset menu

In our model, we assume that both countries can issue equities and bonds. However, due to financial underdevelopment and high risk of default in emerging markets, international bonds that are frequently transacted are those issued by advanced economies. In this subsection, let us consider the situation where the asset menu offered by the two countries in international financial market is asymmetric. Specifically, suppose that the home (developing) country can only issue

Assets menu	Optimal portfolio choices
Home equity	$\tilde{\alpha}_1 = -0.4442$
Foreign equity	$\tilde{\alpha}_2 = 0.1081$
Net equity asset	$\tilde{\alpha}_E = -0.3361$
Foreign bond	$\tilde{\alpha}_4 = 0.3361$
Net bond asset	$\tilde{\alpha}_B = 0.3361$

Table 2.5: Optimal portfolio choices: Asymmetry in asset menu

home equity while the developed country can issue both foreign equity and a bond. The specification of other aspects of the model is the same as before. Under the current fully asymmetric parameterization, the optimal country portfolios is computed and displayed in Table 2.5. By this result, the home country sells the home equity to the amount of 0.44 (multiplied by βY) and buys the foreign equity to the amount of 0.11, which, again, implies a negative net position of equity $\tilde{\alpha}_E = -0.34 < 0$. At the same time, the home country buys the foreign bond to the amount of 0.34. The home bond being absent, this also implies the net position in bonds of the same volume $\tilde{\alpha}_B = 0.34 > 0$. By the same argument, the reverse pattern of net asset positions will be seen in the foreign country. As in the last subsection, with the asymmetric asset menu, a two-way capital flow between the two countries persists. Moreover, because net positions of equities and bonds are both higher (in absolute value) than before, the asymmetry in fact strengthens the pattern of capital flows.

2.6 Conclusion

There is a noticeable heterogeneity in country's asset composition of the gross flows and positions. In the literature, this is documented as the pattern of "short equity, long bond" in (many) developing countries and "long equity, short bond" in developed countries. We present an international macroeconomic model of both equity and bond portfolios in this chapter. It shows that the presence of a selection of empirically relevant asymmetries between two countries can generate such a pattern of capital flows.

We find that these asymmetries include those related to industrial structure, severity of nominal rigidities, trade openness, consumption home bias, investment adjustment frictions, monetary policy stance, market competitiveness and pricing strategy of international trade, etc. In particular, for the two-way capital flows to happen, it is found that this can be the case if the developing country relies on more labour intensive technology to produce, or/and is more dependent on international trade, or/and features less local goods preference, or/and faces a relative low cost of investment adjustment, or/and is less focused on inflation stabilization while more focused on stabilization of the output gap when conduct monetary policy, or/and has a less competitive goods market. We also find that the factors from the real side of economy are more important than those from the nominal side. With the help of other empirical studies' results of parameter estimation, the fully simulated model yields optimal portfolio holdings that are broadly consistent with the pattern of two-way capital flows. Moreover, if we assume that international bonds can only be issued by the developed country (as it is often the case in reality) the result is strengthened.

The chapter highlights the role of correlation and variability effects in understanding the size of gross positions of certain types of asset which have particular importance in driving two-way capital flows. The correlation effect reflects how relevant the asset is in hedging risks while the variability effect reflects how much the amount of risk exposure is for the asset to hedge against. It turns out that the size of portfolio holdings are increasing in both of the effects.

The contribution of this work is at least threefold. Firstly, we use an open economy model with full-fledged New Keynesian features and endogenous portfolio choices. This framework is very general and obviously convenient to be modified for the purposes of understanding many other international macroeconomic issues where the presence of distinct country portfolios is required. Secondly, we identify a selection of factors that matter in accounting for heterogeneous asset composition. This is not only useful for explaining the two-way capital flows between developing and developed countries. As an example, the patterns of international capital flows within the group of developed countries or that of developing countries can be explored. Lastly, we make use of the recent estimation of structural parameter values that are based on the data of the *U.S.* and China when simulating our model. The results contribute to the related literature on

emerging markets, especially China.

Appendix

2.A Price setting in the final goods sector (LCP)

In this section, we show how optimal prices are chosen in the final goods sector. The case of the traded sector is considered while the case of non-traded sector can be obtained similarly. Besides, the case of *LCP* is considered while the case of *PCP* can be obtained by removing S from the profit function and then following similar derivations.

The firms' problem has been described by Eqs.(2.35 – 38) in the main text. The related Lagrangian function for the problem is

$$\begin{aligned} \Lambda_t = & E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i \left\{ \begin{array}{l} D_{t+i} \left[\frac{p_{Dt}(z)}{P_{Dt+i}} \left(\frac{P_{Dt+i-1}}{P_{Dt-1}} \right)^\omega \right]^{1-\varphi} \frac{P_{Dt+i}}{P_{t+i}} \\ - D_{t+i} \left[\frac{p_{Dt}(z)}{P_{Dt+i}} \left(\frac{P_{Dt+i-1}}{P_{Dt-1}} \right)^\omega \right]^{-\varphi} \frac{q_{Tt+i}}{P_{t+i}} \end{array} \right\} \\ & + E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i \left\{ \begin{array}{l} D_{t+i}^* \left[\frac{p_{Xt}(z)}{P_{Xt+i}} \left(\frac{P_{Xt+i-1}}{P_{Xt-1}} \right)^\omega \right]^{1-\varphi} \frac{S_{t+i} P_{Xt+i}}{P_{t+i}} \\ - D_{t+i}^* \left[\frac{p_{Xt}(z)}{P_{Xt+i}} \left(\frac{P_{Xt+i-1}}{P_{Xt-1}} \right)^\omega \right]^{-\varphi} \frac{q_{Tt+i}}{P_{t+i}} \end{array} \right\} \end{aligned}$$

First-order condition with respect to $p_{Dt}(z)$ is

$$\begin{aligned} \frac{\partial \Lambda_t}{\partial p_{Dt}(z)} &= E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i (1 - \varphi) \frac{D_{t+i}}{P_{t+i}} \left[\frac{p_{Dt}(z)}{P_{Dt+i}} \left(\frac{P_{Dt+i-1}}{P_{Dt-1}} \right)^\omega \right]^{-\varphi} \left(\frac{P_{Dt+i-1}}{P_{Dt-1}} \right)^\omega \\ &+ E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i \varphi \frac{D_{t+i}}{P_{Dt+i}} \left[\frac{p_{Dt}(z)}{P_{Dt+i}} \left(\frac{P_{Dt+i-1}}{P_{Dt-1}} \right)^\omega \right]^{-\varphi} \frac{q_{Tt+i}}{P_{t+i}} \left(\frac{P_{Dt+i-1}}{P_{Dt-1}} \right)^\omega \\ &= 0 \end{aligned}$$

so

$$E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i \frac{D_{t+i}}{P_{t+i}} \left(\frac{P_{Dt+i-1}}{P_{Dt-1}} \right)^\omega \left\{ \begin{array}{l} (\varphi - 1) \left[\frac{p_{Dt}(z)}{P_{Dt+i}} \left(\frac{P_{Dt+i-1}}{P_{Dt-1}} \right)^\omega \right]^{-\varphi} - \\ \frac{\varphi}{P_{Dt+i}} \left[\frac{p_{Dt}(z)}{P_{Dt+i}} \left(\frac{P_{Dt+i-1}}{P_{Dt-1}} \right)^\omega \right]^{-\varphi-1} q_{Tt+i} \end{array} \right\} = 0$$

so

$$E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i \frac{D_{t+i}}{P_{t+i}} P_{Dt+i}^\varphi \left[\left(\frac{P_{Dt+i-1}}{P_{Dt-1}} \right)^\omega \right]^{1-\varphi} (\varphi - 1) \left\{ \begin{array}{l} p_{Dt}(z) - \\ \left(\frac{P_{Dt+i-1}}{P_{Dt-1}} \right)^{-\omega} \frac{\varphi}{\varphi-1} q_{Tt+i} \end{array} \right\} = 0$$

Rearranging the equation, one obtains Eq.(2.39).

Similarly, first-order condition with respect to $p_{X_t}(z)$, $\frac{\partial \Lambda_t}{\partial p_{D_t}(z)} = 0$, leads to Eq.(2.40). We omit the derivations here.

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Chapter 3

Portfolio choices in an endowment OLG model: External adjustments under global imbalances

Observed net portfolio positions across countries can deviate from zero persistently so that some countries appear to be creditors in steady state while others appear to be debtors in steady state. It must therefore be the case that underlying country asymmetries are such that there is a permanent imbalance in the schedules of total saving and investment. In this chapter, we consider one of such asymmetry, namely a difference in the discount factor between countries. In consequence, net country portfolios are non-trivial in the model of this chapter. To avoid some technical problem intrinsic to a model with the presence of non-trivial net portfolios, we abandon the representative-agent framework used in the previous chapter and turn to an *OLG* framework from now on. In addition, to reduce the complexity of the model and focus our analysis only on how to merge net and gross portfolios into the same model, in contrast to Chapter 2, we simplify the structure of asset market by assuming that only one type of asset is present and we focus on an endowment economy with a single good. In Chapter 4 the model will be extended to an economy with production and multiple goods.

3.1 Introduction

External adjustments in debtor and creditor countries have long been an important issue in international macroeconomics. Over the last two decades, the study of this question has gained new impetus following developments in both trade and financial globalization. In a world with large current account imbalances and large net foreign asset (*NFA*) imbalances, the adjustment dynamics of *NFA* is very important to understand from both a theoretical and a practical policy point of view.

Traditional open economy macroeconomic analysis has focused on models with trade in a single asset. This type of framework means that attention is confined to terms of trade effects or intertemporal effects related to consumption and saving. But with accelerating financial globalisation and the growth of gross portfolios it has become clear that portfolio valuation effects can play a big role in *NFA* dynamics. Traditional open economy macro models with trade in a single asset cannot be used to analyse these effects.

Recent developments in the analysis of country portfolios (see for instance Devereux and Sutherland, 2010, Tille and Wincoop, 2010 and Ghironi et al., 2015, etc.) have focused attention on the determination of country portfolios and have allowed greater understanding of portfolio valuation effects and their role in *NFA* adjustment. However, an important limitation of this new literature is that it is usually based on a modelling framework where steady state *NFA* positions are assumed to be zero. Indeed, the modelling framework adopted in this literature does not provide any way to explain or endogenously determine the steady state *NFA* position. The analysis of portfolio valuation effects is therefore conducted in isolation from any attempt to explain or analyse the country asymmetries which give rise to the *NFA* imbalances which make the question of *NFA* adjustment such an important issue in the first place.

There are two well-developed lines of research in the current literature which investigate external adjustment. One attempts to explain why steady state global imbalances exist due to structural asymmetries between the north and south while the other stresses how international portfolio valuation effects work alongside trade effects in leading to adjustments in net external assets. However, any truly satisfactory analysis of *NFA* adjustment dynamics must start with a

framework which captures the underlying causes of net imbalances on the one hand, i.e. a model which is based on some underlying structural asymmetry which causes steady state *NFA* imbalances. On the other hand, the model must then also incorporate portfolio allocation in a multi-asset world in order to capture the determination of gross positions which lead to valuation effects. The existing literature emphasizes either one approach without the other or, when accommodating the two, uses an approach which lacks microeconomic foundations (see for example, Obstfeld 2004, Blanchard et al., 2005 and Gourinchas and Rey, 2007, etc. Gourinchas and Rey, 2013 surveys the related literature.) This chapter aims to provide a theoretical framework integrating both the structural asymmetry approach to global imbalances and the portfolio approach to external adjustments in a model with fully developed microeconomic foundations.

For this purpose, and not to incur too much complexity, in this chapter we assume the following three constructs in a simple two-country endowment economy. First, for emergence of global imbalances in such a model, this chapter follows Buiter (1981) in assuming a different degree of patience in the two countries. Second, we introduce international portfolio choices by allowing trade in two equity-style assets to hedge against endowment shocks. Third, to induce stationarity in this asymmetric model, an *OLG* structure is assumed à la Weil (1989).

Under these assumptions, our analysis proceeds with the following logic. Countries are differentiated by a differing value of the discount factor $\beta < \beta^*$. This structural country difference gives rise to steady state *NFA* imbalances. So non-trivial net positions of country portfolios can be tied down. Households consume their permanent income, i.e. an average of their life-time resource. With the presence of the asymmetry in the model, their life-time resource is composed of two parts. The first part is the *GDP* income (just as it is in a symmetric model). The second part is the interest payments that arise from *NFA* imbalances. So consumption depends on the country asymmetry and *NFA* imbalances. International assets are vehicles for smoothing (cross-country) consumption volatilities. So portfolio choices will in general depend on the country asymmetry and *NFA* imbalances as well. The solution of the model ties down gross positions of country portfolios and can thus make the connection between the determination of *NFA* positions and gross positions. With both the net and gross country port-

folios in hand, the economic responses and especially the external adjustments of countries under global imbalances can be examined.

The findings revolve around the answers to the following questions. (1) How are the optimal portfolio holdings determined in an asymmetric model? (2) What would the pattern of the portfolio allocations look like in such a case? (3) And how do *NFA* positions adjust in response to shocks in both debtor and creditor countries?

In answer to the first question, the (gross) portfolio holdings are composed of diversification term and imbalance terms. While only the diversification term is present in symmetric models to hedge against *GDP* income risks, the imbalance terms emerges in our asymmetric model to hedge against additional risks. These risks include those related to interest payments (corresponding to second part of the wealth effect in consumption) and those related to differing consumption averaging (the composition effect in consumption). The properties of all these terms are analysed qualitatively and quantitatively in this chapter.

In answer to the second question, due to the presence of the asymmetry, the portfolio allocation is found to depart from the Lucas (1982) benchmark result of full diversification. The result of numerical experiments shows that under our assumptions, the optimal portfolio allocation under global imbalances always and in all countries exhibits an asset home bias. Moreover, the degree of the bias positively depends on the severity of the asymmetry, i.e. the asset home bias deepens as the gap between the two β 's widens. When the asymmetry is removed, the bias in portfolios disappears and we return to the benchmark of full diversification.

In answer to the third question, and to understand the contribution of the model to understanding *NFA* dynamics, consider in more detail the shortcomings of the approaches taken in the existing literature. In a model which focuses on trade in one asset, a temporary positive supply shock to a country (for example) makes households save more and tends to raise the net external wealth of the country, i.e. the shock creates a trade balance effect. Besides the trade balance effect, the current interest rate falls, which tends to decrease the net interest income of a creditor country and decrease the interest payments of a debtor country, i.e. this is often referred to as an intertemporal terms-of-trade effect. However, a portfolio valuation effect is absent in this traditional approach

because one cannot distinguish gross portfolios from net portfolios. In the models with portfolio choices yet without steady state *NFA* imbalances, country external adjustments take place through the trade balance effect and a valuation effect. Considering again the example of a positive home supply shock, in addition to the expansionary trade balance effect, given that now the home asset return will be relatively higher than the foreign asset return, a negative gross holding of the home asset means that wealth will transfer from the home to foreign country, i.e. there is a valuation effect. However, the intertemporal terms-of-trade effect mentioned above is absent here because net portfolios are zero in these models of existing portfolio literature. The model in this chapter thus constitutes a very general and consistent framework which captures all three channels of external adjustment and is thus more suitable for analysing external dynamics of nations and evaluating the implications of related shocks and policies in a world of financial integration and global imbalances.

The work in this chapter is related to the following strands of the existing literature. (1) The vast literature on global imbalances, especially Buiter (1981) which, as mentioned, uses the asymmetric degree of patience across countries to justify the emergence of non-trivial net foreign assets. In the introduction of Chapter 2, we listed many other popular explanations along this line. However, these contributions share one common feature, i.e. while focusing on the determination of net positions of external assets they do not touch on the issue of how gross positions of country portfolios are determined. Our framework overcomes this problem. (2) The literature on the determination of gross portfolios, most of which aims to reconcile the puzzle of equity home bias with various types of hedging motive, e.g. those following Baxter and Jermann (1997). However, this literature only considers symmetric models (see Coeurdacier and Rey, 2012 for a survey) while we introduce a country asymmetry which explains asymmetric gross portfolio holdings across countries (and a home bias as a motive to hedge against the risks associated with global imbalances.) (3) The literature on the portfolio approach to external adjustments. For example, Devereux and Sutherland (2010), Tille and Wincoop (2010) and Ghironi et al. (2015) study the process of external adjustments between two identical countries and highlight the role of valuation effects in the process. They find that because countries all hold sizable cross-border asset positions, they experience large capital gains/losses af-

ter shocks on top of traditional trade balance effect. But due to the symmetry between countries, the terms-of-trade effect is missing from the analysis. The current work generalizes this approach to asymmetric situations.

The remainder of this chapter is structured as follows. Section 3.2 presents the model. Section 3.3 derives per capita steady states and key model equations in their linearized form. In section 3.4, we state portfolio optimality conditions and discuss how we compute optimal portfolios. Section 3.5, 3.6 and 3.7 analyse the behaviour of the consumption differential, optimal portfolio choices and external adjustments. The model is simulated and analysed numerically in section 3.8. Section 3.9 concludes.

3.2 Model assumptions

The model is essentially the one of Weil (1989) extended to a two-country world and modified to encompass international portfolios. To be specific, this is a one-good, two-country endowment economy. To focus only on the influence of asymmetric patience, I assume all the other aspects of the two countries are the same. In each country, at time $t = 0$, the population is normalized to 1. Afterwards new households are born in each period and the population grows at a net rate of n and no one dies. The households born in different periods are of the same structure. A household born at time v maximizes an additive log utility function of the following form in period t

$$U_t^v = E_t \sum_{s=t}^{\infty} \beta^{s-t} \log(c_s^v) \quad (2.1)$$

where E_t , β and c_s^v are the expectation operator, the discount factor and individual consumption respectively. Superscripts are used to denote vintage and subscripts for time.

The period-to-period budget constraint for vintage v household at time t is

$$\alpha_{1t+1}^v + \alpha_{2t+1}^v = \alpha_{1t}^v r_{1t} + \alpha_{2t}^v r_{2t} + y_t^v - c_t^v \quad (2.2)$$

where α_{1t+1}^v and α_{2t+1}^v denote the household's gross holdings of the two assets at the end of time t . r_{1t} and r_{2t} are gross rates of return of assets from time $t - 1$ to t . The individual endowment and consumption are y_t^v and c_t^v so $(y_t^v - c_t^v)$ is net

saving within period t . Eq.(2.2) simply states that net wealth of the household at the end of time t equals the portfolio returns from the last period plus the net saving generated this period.

For individual households, the first order condition for optimal choice of c_{t+i}^v is

$$\lambda_{t+i} = E_t \left[\beta^i (c_{t+i}^v)^{-1} \right] \quad (2.3)$$

where λ_{t+i} is the Lagrangian multiplier associated with time- $(t+i)$ budget constraint.

The first order conditions for optimal choice of α_{1t+1}^v and α_{2t+1}^v are

$$(c_t^v)^{-1} = \beta E_t \left[(c_{t+1}^v)^{-1} r_{1t+1} \right] \quad (2.4)$$

$$(c_t^v)^{-1} = \beta E_t \left[(c_{t+1}^v)^{-1} r_{2t+1} \right] \quad (2.5)$$

Similar conditions also apply for the foreign country.

Without loss of generality, we assume that the home country is less patient so $\beta < \beta^*$ for emergence of global imbalances. An asterisk is used to denote foreign (except for asset-related) variables.

In addition, we assume that the two assets are respectively home and foreign equities representing claims on endowment income of issuing country. So r_{1t} and r_{2t} are defined as

$$r_{1t} = \frac{y_t + z_{1t}}{z_{1t-1}} \quad (2.6)$$

$$r_{2t} = \frac{y_t^* + z_{2t}}{z_{2t-1}} \quad (2.7)$$

where z_{1t} and z_{2t} denote equity prices.

The uncertainty in the model comes from shocks to endowment income. We assume that $y_t^v = y_t$, $y_t^{v*} = y_t^*$ for all v and t for simplicity where y_t s represent per capita aggregate incomes. The processes that the y_t s follow are $AR(1)$ as follows

$$\log (y_t / \bar{y}) = \mu \log (y_{t-1} / \bar{y}) + \varepsilon_t \quad (2.8)$$

$$\log (y_t^* / \bar{y}^*) = \mu \log (y_{t-1}^* / \bar{y}^*) + \varepsilon_t^* \quad (2.9)$$

For a per capita aggregate variable in this chapter, we use a overbar (with time subscript being dropped) to denote its steady-state value. So \bar{y} and \bar{y}^* in above

equations denote steady-state aggregate incomes. We assume that $0 \leq \mu \leq 1$ and ε and ε^* are zero-mean *i.i.d* shocks with $\text{var}(\varepsilon) = \text{var}(\varepsilon^*) = \sigma^2$ and $\text{cov}(\varepsilon\varepsilon^*) = 0$.

3.3 Non-stochastic aggregate steady state

In this section, we show that because $\beta < \beta^*$ individual consumption is tilted downwards in the home country and upwards in the foreign country along the perfect foresight optimal path. So there is no steady state for individual variables (other than consumption of new-borns). However, with the assumed demographic structure, aggregate steady states are shown to be available. So analysis in the following sections will all revolve around per capita variables. For this reason, in this section we also derive and linearize per capita Euler equations, country budget constraints and the world resource constraint and compare them to those of a symmetric model.

3.3.1 Non-stationary individual variables

To see the non-stationarity of individual variables, we can check individual Euler equations Eqs.(2.4) and (2.5). Because $\beta < \beta^*$, the steady-state international interest rate r lies between two extreme autarky interest rates $\frac{1}{\beta^*} < r < \frac{1}{\beta}$ (shown below). For example for the home country we have $r\beta < 1$. By Euler equations \bar{c}_t^v is decreasing over time along the perfect foresight optimal path

$$\bar{c}_{t+1}^v = r\beta\bar{c}_t^v \tag{3.1}$$

Note that for a individual variable in this chapter, we use a overbar to denote its value under the non-stochastic perfect foresight.

In the home country households consume less and less as they grow older. The intuition is that since they value consuming now more than in the future in comparison to the foreign households, it is optimal for them to shift resources from the future to the present given the life-time budget constraint to be respected. Conversely, we have $r\beta^* > 1$ for the foreign country. So \bar{c}_t^{*v} is tilted upwards (toward the future).

\bar{c}_t^v is not stable, so, from the budget constraint, gross and net individual portfolios are not stable either.

To derive the consumption of new-borns, first we define net wealth of household as $w_{t+1}^v = \alpha_{1t+1}^v + \alpha_{2t+1}^v$ so Eq.(2.2) can be rewritten as

$$w_{t+1}^v = w_t^v r_{2t} + \alpha_{1t}^v r_{xt} + y_t^v - c_t^v \quad (3.2)$$

where $r_{xt} = r_{1t} - r_{2t}$ defines the excess return of asset 1 over asset 2.

By iterating forward this constraint and imposing a transversality condition

$$\lim_{T \rightarrow \infty} \frac{1}{\prod_{i=1}^{\infty} r_{2t+i}} w_{t+T}^v = 0 \quad (3.3)$$

we obtain a life-time budget constraint which in turn can be used to combine with an income normalization

$$\bar{y}_t^v = \bar{y} = 1 \quad (3.4)$$

and Euler equations to obtain the consumption function as follows

$$\bar{c}_t^v = (1 - \beta) \left[\bar{\alpha}_{1t}^v \bar{r}_{1t} + \bar{\alpha}_{2t}^v \bar{r}_{2t} + \sum_{i=0}^{\infty} \frac{1}{\bar{r}^i} \bar{y}_t^v \right] \quad (3.5)$$

Following Weil (1989), we assume that households are born with no wealth, i.e. $\alpha_{1t}^t = \alpha_{2t}^t = 0$, so at the time they are born they only consume a fraction of the discounted sum of their life-time endowments.

$$\bar{c}_t^t \equiv \bar{c}_t^n = \frac{(1 - \beta) r}{(r - 1)} \bar{y} \quad (3.6)$$

where we introduce c_t^n with superscript n to denote new-borns' consumption.

3.3.2 Net foreign assets, w

Non-stationarity of individual variables in the model poses a difficulty in attempting, as is usually done when dealing with representative-agents models, to use the individual variables and their steady states to represent the country-level variables and steady states. This is for two reasons. First, generally, individual variables do not coincide with country level variables in this model. Second, they do not even have steady states around which we can perform standard approximation procedures. So we will instead focus on the country per capita average of

individual variables which, by definition, is free of the first problem and, by Weil (1989), enjoys a steady state because of the assumed *OLG* structure. We will first show how steady state per capita net external positions, w , in the two countries are determined in this subsection. The determination of the steady-state international interest rate, per capita consumptions and key model equations are studied subsequently.

According to the assumption regarding population growth, per capita aggregation of a variable, for example c_t^v , can be conducted according to

$$c_t = \frac{c_t^0 + nc_t^1 + n(1+n)c_t^2 + \dots + n(1+n)^{t-1}c_t^t}{(1+n)^t} \quad (3.7)$$

where c_t without superscript v denotes per capita aggregate consumption at time t . To understand Eq.(3.7), note that at time t , the total population is $(1+n)^t$ within which the number of vintage 0 households is 1, the number of vintage 1 households is $(1+n) - 1 = n$, the number of vintage 2 households is $(1+n)^2 - (1+n) = n(1+n)$ and so on.

To find steady-state w , first use Eq.(3.7) to aggregate the budget constraint Eq.(3.1)

$$(1+n)w_{t+1} = r_{2t}w_t + \alpha_{1t}r_{xt} + y_t - c_t \quad (3.8)$$

while the terms on the right hand side (*RHS*) are obvious, the term on the left hand side (*LHS*) is

$$\begin{aligned} & \frac{w_{t+1}^0 + nw_{t+1}^1 + n(1+n)w_{t+1}^2 + \dots + n(1+n)^{t-1}w_{t+1}^t}{(1+n)^t} \\ = & (1+n) \frac{w_{t+1}^0 + nw_{t+1}^1 + n(1+n)w_{t+1}^2 + \dots + n(1+n)^t w_{t+1}^{t+1}}{(1+n)^{t+1}} \\ = & (1+n)w_{t+1} \end{aligned} \quad (3.9)$$

where the assumption that $\alpha_{1t}^t = \alpha_{2t}^t = 0$ so $w_t^t = 0$ is used in the first equality and the aggregation relation is used in the second equality. Divided by $(1+n)$, the constraint becomes

$$w_{t+1} = \frac{r_{2t}w_t + \alpha_{1t}r_{xt} + y_t - c_t}{(1+n)} \quad (3.10)$$

Then, by aggregating the consumption function Eq.(3.5) we obtain

$$c_t = (1-\beta) \left[r_{2t}w_t + \alpha_{1t}r_{xt} + \sum_{i=0}^{\infty} \frac{1}{r^i} y_t \right] \quad (3.11)$$

which can be used to substitute into Eq.(3.10) to eliminate c_t and simplified to yield the law of motion of w

$$w_{t+1} = \frac{r\beta}{(1+n)}w_t + \frac{r\beta - 1}{(1+n)(r-1)}y_t \quad (3.12)$$

Assuming the following stability condition

$$(1+n) > r\beta \quad (3.13)$$

steady-state w can be obtained as

$$\bar{w} = \bar{\alpha}_1 + \bar{\alpha}_2 = \phi_{wy}\bar{y} \quad (3.14)$$

where

$$\phi_{wy} = \frac{r\beta - 1}{((1+n) - r\beta)(r-1)} \quad (3.15)$$

denotes the steady-state ratio of net wealth to endowment. Note that for the impatient country (the home country here) we have $(r\beta - 1) < 0$. The wealth ratio ϕ_{wy} is negative, so the country will be in a position of net debt. In contrast, we have $(r\beta^* - 1) > 0$ for the patient country. So the foreign country will be in the position of net credit, i.e.

$$\bar{w}^* = \bar{\alpha}_1^* + \bar{\alpha}_2^* = \phi_{wy}^*\bar{y}^* \quad (3.14^*)$$

where

$$\phi_{wy}^* = \frac{r\beta^* - 1}{((1+n) - r\beta^*)(r-1)} \quad (3.15^*)$$

These results confirm the intuition that the impatient country consumes by borrowing while the patient country saves by lending.

3.3.3 International interest rate, r

The international interest rate in the steady state is determined by international asset market clearing conditions. To obtain the latter, we observe that the net asset holdings in the two countries are as in Table 3.1.

	Home holdings	Foreign holdings
Asset 1-Home equity	$\alpha_1 + z_1$	α_1^*
Asset 2-Foreign equity	$\alpha_2 = w - \alpha_1$	$\alpha_2^* + z_2 = w^* - \alpha_1^* + z_2$

Table 3.1: Net asset holdings across countries

Two facts are used here. First, the home country is assumed to be the default owner of home equity whose amount is normalized to 1. So the home country's net holding of the home equity is given by $z_1 + \alpha_1$ where z_1 is the home equity price. Second, the definition of $w = \alpha_1 + \alpha_2$ so the home holding of the foreign equity is given by $w - \alpha_1$. Foreign holdings are obtained following the same logic.

Market clearing of the two assets implies

$$z_1 + \alpha_1 + \alpha_1^* = z_1 \quad (3.16)$$

$$w - \alpha_1 + z_2 + w^* - \alpha_1^* = z_2 \quad (3.17)$$

or

$$\alpha_1^* = -\alpha_1 \quad (3.18)$$

$$w^* = -w \quad (3.19)$$

Eq.(3.18) implies only α_1 is important in solving portfolio allocations of two countries by Table 3.1. The other three α s can be linked to α_1 by w and z . So I denote $\alpha \equiv \alpha_1$ to represent portfolios from now on and focus on solving for α in Section 3.6.

Eq.(3.19) simply states that the home deficit and the foreign surplus are two sides of the same coin. This condition can be used to determine the steady-state interest rate as follows

$$r = \frac{(2+n)(\beta + \beta^*) - \sqrt{(2+n)^2(\beta + \beta^*)^2 - 16(1+n)\beta\beta^*}}{4\beta\beta^*} \quad (3.20)$$

One can verify that Eq.(3.20) confirms our previous assertion that the value of r lies between $\frac{1}{\beta^*}$ and $\frac{1}{\beta}$. In addition, r is decreasing in the β s and increasing in n .

3.3.4 Consumption, c

Substituting the steady-state wealth ratio ϕ_{wy} back into the steady-state version of Eq.(3.8), we get the related steady-state consumption ratio to endowment

$$\phi_{cy} = 1 + (r - (1 + n)) \phi_{wy} \quad (3.21)$$

For the steady state to be dynamically efficient, we assume $r > (1 + n)$. In this case, $\phi_{wy} < 0$ and so $\phi_{cy} < 1$ for the home country. For the foreign country, because $\phi_{wy}^* > 0$ by (3.15*), Eq.(3.21) implies $\phi_{cy}^* > 1$. This point can also be seen by noting that the following resource constraint always holds

$$c + c^* = y + y^* \quad (3.22)$$

The relation between ϕ_{cy} and ϕ_{cy}^* is thus

$$\phi_{cy} + \phi_{cy}^* = 2 \quad (3.23)$$

So when $\phi_{cy} < 1$ it must be that $\phi_{cy}^* > 1$.

It is useful to compare the current case of an asymmetric world with that of a symmetric one in terms of the value of ϕ_{wy} and ϕ_{cy} we found above. Now we have $\phi_{wy} < 0$ and $\phi_{cy} < 1$ for the home country and $\phi_{wy}^* > 0$ and $\phi_{cy}^* > 1$ for the foreign country, which implies the impatient country consumes less than its average income and runs a trade deficit in the steady state while the patient country consumes more than its average income and runs a trade surplus in the steady state. The emergence of these unbalanced ratios is completely due to the existence of our assumption of asymmetry in β . If the two countries are the same, i.e. $\beta = \beta^*$, we end up with $\phi_{wy} = \phi_{wy}^* = 0$ and $\phi_{cy} = \phi_{cy}^* = 1$, i.e. global imbalances disappear and the two countries eat their average resources, neither more nor less.

For later use, we derive the relation between \bar{c} and \bar{c}^n by making use of Eq.(3.21) and (3.6). That is

$$\bar{c} = \frac{n}{(1 + n - r\beta)} \bar{c}^n \quad (3.24)$$

This relation can also be obtained from the aggregation relation Eq.(3.7).

3.3.5 Euler equations, budget and resource constraints

To find the per capita Euler equations corresponding to Eqs.(2.4 – 5), we approximate the individual equations first and then aggregate them. As explained, because there are no steady states around which we can approximate individual consumptions, the approximation is conducted around the levels along their perfect foresight optimal path Eq.(3.1). The individual Euler equations after approximation are

$$\hat{c}_t^v = E_t \hat{c}_{t+1}^v - E_t \hat{r}_{1t+1} \quad (3.25)$$

$$\hat{c}_t^v = E_t \hat{c}_{t+1}^v - E_t \hat{r}_{2t+1} \quad (3.26)$$

A few words on notations are in order. In the model, all individual variables with hats denote percentage deviation of variables from their levels along the optimal path while aggregate variables with hats denote that of variables from their steady states. So in the above equations, $\hat{c}_t^v = \frac{c_t^v - \bar{c}_t^v}{\bar{c}_t^v}$ and $\hat{r} = \frac{r - \bar{r}}{\bar{r}}$. (To ease notation, \bar{r} is replaced by r in all other places in this chapter).

One result from the two equations above is that expected asset returns are equalized in equilibrium

$$E_t \hat{r}_{1t+1} = E_t \hat{r}_{2t+1} \equiv E_t \hat{r}_{t+1} \quad (3.27)$$

To aggregate the approximated equations, we also approximate the aggregation relation

$$\hat{c}_t = \frac{1}{(1+n)^t \bar{c}} \left[\begin{array}{l} \bar{c}_t^0 \hat{c}_t^0 + n \bar{c}_t^1 \hat{c}_t^1 + n(1+n) \bar{c}_t^2 \hat{c}_t^2 + \dots \\ + n(1+n)^{t-2} \bar{c}_t^{t-1} \hat{c}_t^{t-1} + n(1+n)^{t-1} \bar{c}_t^t \hat{c}_t^t \end{array} \right] \quad (3.28)$$

which links the per capita consumption deviation \hat{c}_t to the individual consumption deviation \hat{c}_t^v .

In period $t+1$, Eq.(3.28) is

$$E_t \hat{c}_{t+1} = \frac{1}{(1+n)^{t+1} \bar{c}} \left[\begin{array}{l} \bar{c}_{t+1}^0 E_t \hat{c}_{t+1}^0 + n \bar{c}_{t+1}^1 E_t \hat{c}_{t+1}^1 + \dots + \\ n(1+n)^{t-1} \bar{c}_{t+1}^t E_t \hat{c}_{t+1}^t + n(1+n)^t \bar{c}_{t+1}^{t+1} E_t \hat{c}_{t+1}^{t+1} \end{array} \right] \quad (3.29)$$

The above two equations are put together because on the *RHS* of these equations are \hat{c}_t^v and \hat{c}_{t+1}^v which are linked by the individual Euler equations given by Eqs.(3.25) and (3.26). At the same time, on the *LHS* of the equations are \hat{c}_t and \hat{c}_{t+1} which are linked by the per capita Euler equations to be established.

Before proceeding, it is useful to define some steady-state (weighted average) consumption shares. We label all generations other than that of the new-born as the old vintage in every period. So at any time, for example time $t + 1$, total consumption can be divided into the consumption by the old and that by the new-born

$$\bar{c} = \underbrace{\frac{1}{(1+n)^{t+1}} \left[\bar{c}_{t+1}^0 + n\bar{c}_{t+1}^1 + n(1+n)\bar{c}_{t+1}^2 + \dots + n(1+n)^{t-1}\bar{c}_{t+1}^t \right]}_{\text{Weighted average consumption of the old}} + \underbrace{\frac{n}{(1+n)} \bar{c}_{t+1}^{t+1}}_{\text{Weighted average consumption of the newborn}} \quad (3.30)$$

Because \bar{c} and \bar{c}_{t+1}^{t+1} are constant, so for different t steady-state (weighted average) consumption of the old should also be constant. Dividing the two sides of Eq.(3.30) by \bar{c} and making use of the relation between \bar{c} and \bar{c}_{t+1}^{t+1} , i.e. Eq.(3.24), we obtain the following consumption shares of the old and the new-born respectively

$$\frac{1}{(1+n)^{t+1} \bar{c}} \left[\bar{c}_{t+1}^0 + n\bar{c}_{t+1}^1 + n(1+n)\bar{c}_{t+1}^2 + \dots + n(1+n)^{t-1}\bar{c}_{t+1}^t \right] = \tau \quad (3.31)$$

$$\frac{n}{(1+n)} \bar{c}_{t+1}^{t+1} = (1 - \tau) \quad (3.32)$$

where

$$\tau \equiv \frac{r\beta}{(1+n)} < 1 \quad (3.33)$$

Eq.(3.30) can be rewritten as

$$1 = \underbrace{\tau}_{\text{Consumption share due to the old}} + \underbrace{(1 - \tau)}_{\text{Consumption share due to the newborn}} \quad (3.34)$$

Now let us return to the aggregation problem. We take aggregating Eq.(3.25) for example. The method is to impose it on the approximated aggregation relation, i.e. substitute $E_t \hat{c}_{t+1}^v = \hat{c}_t^v + E_t \hat{r}_{1t+1}$ into the *RHS* of Eq.(3.29) and then simplify the equation in which course the above steady state consumption shares of the old and the new-born can be used.

The resulting per capita Euler equation is thus

$$E_t \hat{c}_{t+1} = \tau \hat{c}_t + (1 - \tau) E_t \hat{c}_{t+1}^n + \tau E_t \hat{r}_{1t+1} \quad (3.35)$$

The foreign counterpart of Eq.(3.25) can be aggregated similarly which leads to

$$E_t \hat{c}_{t+1}^* = \tau^* \hat{c}_t^* + (1 - \tau^*) E_t \hat{c}_{t+1}^{n*} + \tau^* E_t \hat{r}_{1t+1} \quad (3.35^*)$$

where τ^* is also defined similarly

$$\tau^* \equiv \frac{r\beta^*}{(1+n)} < 1 \quad (3.33^*)$$

For comparison, the counterpart of the pair of Eq.(3.35) (3.35*) in a symmetric model with $\beta = \beta^*$ is

$$E_t \hat{c}'_{t+1} = \hat{c}'_t + E_t \hat{r}'_{1t+1} \quad (3.35')$$

for both countries. So the differences between the Euler equations of the model with and without global imbalance are:

1. From one period to another there is always a new generation coming in. We have a new term \hat{c}^n in the Euler equation reflecting this fact. The parameter τ determines the relative importance of consumption of different population groups. The higher the growth rate n , the more important is the new-born's consumption relative to the old's, i.e. the first two terms of the *RHS* of Eqs.(3.35) (3.35*). While n is the same across countries, the new-born's consumption in the home country is relatively more important ($(1 - \tau) > (1 - \tau^*)$) than that in the foreign country due to the fact that $\beta < \beta^*$.

2. As the coefficient of $E_t \hat{r}_{t+1}$, τ gains another interpretation, i.e. the consumption tilting factor under the assumptions of global imbalances $r\beta \neq 1$ and population growth $n > 0$. The tilting effect is dampened ($\tau, \tau^* < 1$) comparing to a symmetric model because only the existing population (which accounts for only a fraction of aggregate consumption now) tilts consumption each time in response to a variation in $E_t \hat{r}_{t+1}$, i.e. the third term of the *RHS* of Eq.(3.35) (3.35*).

The per capita country budget constraint and world resource constraint can be obtained from Eq.(3.8) and (3.22). We linearize them to yield

$$(1+n)\phi_{wy}\hat{w}_{t+1} = r\phi_{wy}\hat{w}_t + r\phi_{wy}\hat{r}_{2t} + r\phi_{\alpha y}\hat{r}_{xt} + \hat{y}_t - \phi_{cy}\hat{c}_t \quad (3.36)$$

$$\phi_{cy}\hat{c}_t + \phi_{cy}^*\hat{c}_t = \hat{y}_t + \hat{y}_t^* \quad (3.37)$$

Compare these to their counterparts in a symmetric model (\hat{w}'_t is defined as w'_t/\bar{y}' since $\bar{w}' = 0$) which are as follows

$$\hat{w}'_{t+1} = r'\hat{w}'_t + r'\phi'_{\alpha y}\hat{r}'_{xt} + \hat{y}'_t - \hat{c}'_t \quad (3.36')$$

$$\hat{c}'_t + \hat{c}'_{t^*} = \hat{y}'_t + \hat{y}'_{t^*} \quad (3.37')$$

The main differences between the budget and resource constraints of the model with and without global imbalances are

1. With the existence of population growth, we have $(1+n)$ in the budget constraint when being aggregated.
2. Because steady-state w is non-zero, variation in its return emerges as the term $r\phi_{wy}\hat{r}_{2t}$ in contrast to the case of the symmetric model.
3. Unbalanced consumption ratios appear in the budget and resource constraints, i.e. $\phi_{cy} < 1$ and $\phi_{cy}^* > 1$ as opposed to the case of symmetric case where $\phi'_{cy} = \phi^*_{cy} = 1$.

We will show in subsequent sections that the above differences in Euler equations, budget and resource constraints between the models with and without global imbalances, lead to important revision of risks, optimal portfolio allocations and the external adjustment process for both debtor and creditor countries. The full description of the asymmetric and symmetric models (the symmetric portfolio model is essentially the one in Devereux and Sutherland, 2010) can be found in Section 3.A of the Appendix.

3.4 Portfolio optimality condition

In this section, we derive the optimality condition used to determine per capita country portfolios. By the condition, two determinants of per capita portfolios turn out to be respectively the first-order behaviour of the consumption differential, \hat{c}_t^D , and that of the excess return, \hat{r}_{xt} . At the end of this section, we explain how we compute the optimal portfolios in the model.

3.4.1 Individual conditions

Households' decisions on portfolio choices are made individually. So we consider individual portfolio conditions first. $\bar{\alpha}^v$ is determined by Eqs.(2.4) and (2.5).

Combining the two yields the condition

$$E_{t-1} [(c_t^v)^{-1} r_{1t}] = E_{t-1} [(c_t^v)^{-1} r_{2t}] \quad (4.1)$$

Following Devereux and Sutherland (2011) and Tille and Wincoop (2010), to pin down zero-order component of the portfolio, the above condition should be approximated to at least second-order accuracy, which is

$$E_{t-1} [\hat{c}_t^v \hat{r}_{xt}] = E_{t-1} \left[\frac{1}{2} \hat{r}_{xt}^{(2)} \right] + O(\varepsilon^3) \quad (4.2)$$

where $\hat{r}_{xt} \equiv \hat{r}_{1t} - \hat{r}_{2t}$, $\hat{r}_{xt}^{(2)} \equiv \hat{r}_{1t}^2 - \hat{r}_{2t}^2$ which denotes the second-order term of \hat{r}_{xt} .

For the foreign country, a similar condition is obtained as follows

$$E_{t-1} [\hat{c}_t^{v*} \hat{r}_{xt}] = E_{t-1} \left[\frac{1}{2} \hat{r}_{xt}^{(2)} \right] + O(\varepsilon^3) \quad (4.3)$$

Combining Eqs.(4.2) and (4.3) leads to

$$E_{t-1} [(\hat{c}_t^v - \hat{c}_t^{v*}) \hat{r}_{xt}] = 0 \quad (4.4)$$

which can serve as the condition determining $\bar{\alpha}^v$.

3.4.2 Aggregate conditions

What matters for our analysis is the per capita aggregate portfolio, $\bar{\alpha}$, rather than individual $\bar{\alpha}^v$ s, which implies we need a per capita aggregate condition. This can be achieved by aggregating individual conditions derived above. In what follows, we first aggregate Eqs.(4.2) and (4.3) and then combine the two resulting conditions to obtain the final per capita condition.

To aggregate Eq.(4.2), remember the first-order approximation of the aggregation relation is given by Eq.(3.28) or

$$E_{t-1} \hat{c}_t = \frac{1}{(1+n)^t \bar{c}} \left[\begin{array}{l} \bar{c}_t^0 E_{t-1} \hat{c}_t^0 + n \bar{c}_t^1 E_{t-1} \hat{c}_t^1 + n(1+n) \bar{c}_t^2 E_{t-1} \hat{c}_t^2 + \dots \\ + n(1+n)^{t-2} \bar{c}_t^{t-1} E_{t-1} \hat{c}_t^{t-1} + n(1+n)^{t-1} \bar{c}_t^t E_{t-1} \hat{c}_t^t \end{array} \right] \quad (4.5)$$

Multiplying \hat{r}_{xt} to both sides of the equations yields

$$E_{t-1} [\hat{c}_t \hat{r}_{xt}] = E_{t-1} \left[\frac{1}{2} \hat{r}_{xt}^{(2)} \right] + O(\varepsilon^3) \quad (4.6)$$

By the same token, aggregating Eq.(4.3) yields

$$E_{t-1} [\hat{c}_t^* \hat{r}_{xt}] = E_{t-1} \left[\frac{1}{2} \hat{r}_{xt}^{(2)} \right] + O(\varepsilon^3) \quad (4.7)$$

Combining the above two equations gives us the following condition

$$E_{t-1} [\hat{c}_t^D \hat{r}_{xt}] = 0 \quad (4.8)$$

where $\hat{c}_t^D \equiv (\hat{c}_t - \hat{c}_t^*)$ is defined as the (per capita) consumption differential between countries. Given individual conditions Eqs.(4.2) and (4.3) being always satisfied, the above aggregate condition should be always satisfied as well. We used Eq.(4.8) as the key condition to tie down $\bar{\alpha}$ in this chapter.

3.4.3 Portfolio solution as a fixed point

By Eq.(4.8), the optimal portfolio, $\bar{\alpha}$, is determined by \hat{c}_t^D and \hat{r}_{xt} . Usually, for a model that is already approximated up to first-order accuracy, the first-order behaviour of the model can be easily obtained by solving a multi-variable linear system. However, with $\bar{\alpha}$ not yet known, this behaviour generally depends on $\bar{\alpha}$. Representing this behaviour as functions of $\bar{\alpha}$, we can then rely on Eq.(4.8) to translate the problem into the one of solving an equation in $\bar{\alpha}$. In other words, in the optimality condition Eq.(4.8), two determinants of $\bar{\alpha}$ are themselves endogenous functions of $\bar{\alpha}$, in which sense the solution of $\bar{\alpha}$ is viewed as a fixed point.

In the current model the solution process is less difficult than the fully general case because \hat{r}_{xt} does not depend on the portfolio. According to the equations defining asset returns, i.e. Eq.(2.6 – 7), \hat{r}_{xt} is simply given by

$$\hat{r}_{xt} = \frac{(r-1)}{(r-\mu)} (\hat{y}_t - \hat{y}_t^*) - (\hat{z}_{1t-1} - \hat{z}_{2t-1}) \quad (4.9)$$

For our purpose, we consider the case where the model is in a steady state at time $t-1$. So \hat{r}_{xt} is

$$\hat{r}_{xt} = \frac{(r-1)}{(r-\mu)} (\varepsilon_t - \varepsilon_t^*) \quad (4.10)$$

Because ε_t and ε_t^* are *i.i.d.* shocks, by this formula, \hat{r}_{xt} is also an *i.i.d.* shock.

Notice that by the budget constraint Eq.(3.36), the portfolio enters the model through a term ξ_t

$$\xi_t \equiv \tilde{\alpha} \hat{r}_{xt} \quad (4.11)$$

where a re-scaled optimal portfolio $\tilde{\alpha}$ is defined as

$$\tilde{\alpha} \equiv (r/\bar{y})\bar{\alpha} \quad (4.12)$$

So ξ_t can be viewed as an *i.i.d.* shock as well.

Our strategy to solve for $\bar{\alpha}$ is, firstly, to suppress $\bar{\alpha}$ and replace it with an *i.i.d.* shock ξ_t . So without loss of generality, the solution of model variables, specifically for instance \hat{c}_t^D , will take the form of

$$\hat{c}_t^D = \chi_{cw}^D \hat{w}_t + \chi_{cz1}^D \hat{z}_{1t-1} + \chi_{cz2}^D \hat{z}_{2t-1} + \chi_{ce1}^D \hat{y}_t + \chi_{ce2}^D \hat{y}_t^* + \chi_{cx}^D \xi_t \quad (4.13)$$

in which the responses of \hat{c}_t^D to 5 state variables $\{\hat{w}_t, \hat{z}_{1t-1}, \hat{z}_{2t-1}, \hat{y}_{t-1}, \hat{y}_{t-1}^*\}$ are given respectively by $\{\chi_{cw}^D, \chi_{cz1}^D, \chi_{cz2}^D, \mu\chi_{ce1}^D, \mu\chi_{ce2}^D\}$ and the responses to 3 shocks $\{\varepsilon_t, \varepsilon_t^*, \xi_t\}$ are given respectively by $\{\chi_{ce1}^D, \chi_{ce2}^D, \chi_{cx}^D\}$.

Secondly, by replacing ξ_t with Eq.(4.11) (and \hat{r}_{xt} with Eq.(4.10)), we obtain the following expression for \hat{c}_t^D

$$\hat{c}_t^D = \zeta_{ce1}^D \varepsilon_t + \zeta_{ce2}^D \varepsilon_t^* + t.i. \quad (4.14)$$

where

$$\zeta_{ce1}^D = \chi_{ce1}^D + \frac{(r-1)}{(r-\mu)} \chi_{cx}^D \tilde{\alpha} \quad (4.15)$$

$$\zeta_{ce2}^D = \chi_{ce2}^D - \frac{(r-1)}{(r-\mu)} \chi_{cx}^D \tilde{\alpha} \quad (4.16)$$

and *t.i.* denotes terms of irrelevance whose covariance with ε_t and ε_t^* is zero.

Lastly, substituting for \hat{c}_t^D and \hat{r}_{xt} using Eqs.(4.14) and (4.10) into Eq.(4.8) it is possible to obtain an equation in $\tilde{\alpha}$ which can be easily solved for.

The above procedures are actually an implementation of those described in Devereux and Sutherland (2011). We follow them when we derive expressions of $\tilde{\alpha}$ (and $\bar{\alpha}$) in our model using the software Dynare and Mathematica. In Dynare, we parameterize the model first and following the above procedures derive numerical solution for $\tilde{\alpha}$. It is convenient to generate model dynamics with the help of the software, which makes our work in Section 3.8 easier. Nevertheless,

all solutions to variable responses and $\tilde{\alpha}$ are in numerical form. As a supplement, we employ Mathematica to derive analytical solution of $\tilde{\alpha}$. The key idea is still to obtain responses of endogenous variables as function of $\tilde{\alpha}$, in which process the method of undetermined coefficients is used. In Sections 3.B and 3.C of the Appendix, we provide more details on this. Using either Dynare or Mathematica, we arrive at the same result of $\tilde{\alpha}$, which is verified through parameterising the $\tilde{\alpha}$ that obtained by Mathematica.

With $\tilde{\alpha}$ having being solved for, it is natural then to analyse its main determinants. Eq.(4.8) tells us that $\tilde{\alpha}$ as a whole can be viewed as the hedging of risk of the consumption differential \hat{c}_t^D , i.e. it is driven by households' desire to smooth volatile relative consumptions by investing in the assets which have volatile excess returns. However, the interpretation does not reveal much about how $\tilde{\alpha}$ is composed. In fact, as the outcome of (relative) disposable income, (relative) consumption is composed of a series underlying income risks. From this perspective, the final $\tilde{\alpha}$ must also be composed of a series hedging components corresponding to these underlying income risks. In the next section, we examine the question of which income risks make up \hat{c}_t^D 's behaviour in our model and the associated difference with the symmetric model. Then in Section 3.6, we explore what makes up $\bar{\alpha}$ under global imbalances and the associated difference with the symmetric model.

3.5 Consumption differential

In response to a shock, fluctuations in current consumption are made up of two effects from the point of view of consumption smoothing. The first is the effect due to the change in total consumption or life-time resource (a wealth effect). The second is the effect due to the change in the optimal ratio of current consumption to overall consumption (a composition effect). In this section, we use respectively the budget constraints and Euler equations to decompose these two effects.

3.5.1 Wealth effect

We start from the budget constraints and decompose the wealth effect. It is convenient to copy Eq.(3.36) here

$$(1+n)\phi_{wy}\hat{w}_{t+1} = r\phi_{wy}\hat{w}_t + r\phi_{wy}\hat{r}_{2t} + \hat{\xi}_t + \hat{y}_t - \phi_{cy}\hat{c}_t \quad (5.1)$$

and rearrange to yield

$$\hat{c}_t = -\frac{(1+n)\phi_{wy}}{\phi_{cy}}\hat{w}_{t+1} + \frac{r\phi_{wy}}{\phi_{cy}}\hat{w}_t + \frac{r\phi_{wy}}{\phi_{cy}}\hat{r}_{2t} + \frac{1}{\phi_{cy}}\hat{\xi}_t + \frac{1}{\phi_{cy}}\hat{y}_t \quad (5.2)$$

Lead the time period forward to yield

$$\hat{c}_{t+1} = -\frac{(1+n)\phi_{wy}}{\phi_{cy}}\hat{w}_{t+2} + \frac{r\phi_{wy}}{\phi_{cy}}\hat{w}_{t+1} + \frac{r\phi_{wy}}{\phi_{cy}}\hat{r}_{2t+1} + 0 + \frac{1}{\phi_{cy}}\hat{y}_{t+1} \quad (5.3)$$

$$\hat{c}_{t+2} = -\frac{(1+n)\phi_{wy}}{\phi_{cy}}\hat{w}_{t+3} + \frac{r\phi_{wy}}{\phi_{cy}}\hat{w}_{t+2} + \frac{r\phi_{wy}}{\phi_{cy}}\hat{r}_{2t+2} + 0 + \frac{1}{\phi_{cy}}\hat{y}_{t+2} \quad (5.4)$$

...

We define Σ_t^c as the discounted sum of consumption in the home country (with discount factor $\frac{(1+n)}{r}$) and Σ_{t+1}^{rn} as the discounted sum of future interest rates (with discount factor $\frac{(1+n)}{r}$)

$$\Sigma_t^c \equiv \hat{c}_t + \frac{(1+n)}{r}\hat{c}_{t+1} + \frac{(1+n)^2}{r^2}\hat{c}_{t+2} + \dots \quad (5.5)$$

$$\Sigma_{t+1}^{rn} \equiv \hat{r}_{t+1} + \frac{(1+n)}{r}\hat{r}_{t+2} + \frac{(1+n)^2}{r^2}\hat{r}_{t+3} + \dots \quad (5.6)$$

Using the above budget constraints and imposing a transversality condition, we obtain

$$\underbrace{\Sigma_t^c}_{\text{Total wealth effect}} = \underbrace{\frac{r\phi_{wy}}{\phi_{cy}} \left[\hat{r}_{2t} + \frac{(1+n)}{r}\Sigma_{t+1}^{rn} \right]}_{\text{Net portfolio return}} + \underbrace{\frac{1}{\phi_{cy}}\hat{\xi}_t}_{\text{Gross portfolio return}} + \underbrace{\frac{r}{r-\mu(1+n)}\frac{1}{\phi_{cy}}\hat{y}_t}_{\text{Total endowment}} \quad (5.7)$$

Similarly, we can obtain Σ_t^{c*} for the foreign country.

$$\underbrace{\Sigma_t^{c*}}_{\text{Total wealth effect}} = \underbrace{-\frac{r\phi_{wy}}{\phi_{cy}^*} \left[\hat{r}_{2t} + \frac{(1+n)}{r} \Sigma_{t+1}^{rn} \right]}_{\text{Net portfolio return}} \underbrace{-\frac{1}{\phi_{cy}^*} \hat{\xi}_t}_{\text{Gross portfolio return}} + \underbrace{\frac{r}{r-\mu(1+n)} \frac{1}{\phi_{cy}^*} \hat{y}_t^*}_{\text{Total endowment}} \quad (5.8)$$

The analogues of the above equations in the symmetric case by the budget constraint Eq.(3.36') are

$$\Sigma_t^{c'} = \underbrace{\hat{\xi}_t'}_{\text{Gross portfolio return}} + \underbrace{\frac{r'}{r'-\mu} \hat{y}_t}_{\text{Total endowment}} \quad (5.7')$$

$$\Sigma_t^{c*'} = \underbrace{-\hat{\xi}_t'}_{\text{Gross portfolio return}} + \underbrace{\frac{r'}{r'-\mu} \hat{y}_t^*}_{\text{Total endowment}} \quad (5.8')$$

where $\hat{\xi}_t' \equiv r' \phi'_{\alpha y} \hat{r}_{xt}$. Notice that Eqs.(5.7') and (5.8') can also be obtained by simply imposing the facts that $n = 0$, $\beta^* = \beta$, $\phi'_{wy} = 0$, $\phi'_{cy} = \phi_{cy}^* = 1$ in Eqs.(5.7) and (5.8) and adjusting the meanings of Σ_t^c and r by Eqs.(5.5) and (3.20) according to

$$\Sigma_t^{c'} \equiv \hat{c}_t' + \frac{1}{r'} \hat{c}_{t+1}' + \frac{1}{r'^2} \hat{c}_{t+2}' + \dots \quad (5.5')$$

$$r' = \frac{1}{\beta} \quad (3.20')$$

Eqs.(5.7) and (5.8) describe the movement in the discounted sum of consumption after shocks for the two countries. By these equations, after a shock, the total resource that a country can enjoy changes through three channels. First, the endowments change. Second, the current and future returns to net portfolio w change. Third, the return to the gross portfolio α changes.

Comparing the result to the symmetric case, the net portfolio return emerges to be a new channel in the asymmetric case as the result of $\phi_{wy} \neq 0$. Moreover, in the asymmetric case the other two effects are not symmetric for the two countries due to the unequal consumption ratios $\phi_{cy} \neq \phi_{cy}^*$.

3.5.2 Composition effect

Now, let us turn to the Euler equations and decompose the composition effect. We copy Eq.(3.35)

$$\hat{c}_{t+1} = \tau \hat{c}_t + (1 - \tau) \hat{c}_{t+1}^n + \tau \hat{r}_{2t+1} \quad (5.9)$$

Use the discount factor $\frac{(1+n)}{r}$ to aggregate Eq.(5.9) for $s = t, t + 1, \dots$ to yield

$$\hat{c}_t = \underbrace{(1 - \beta) \Sigma_t^c}_{\text{Average wealth effect}} - \underbrace{\frac{(1 - \tau)(1 + n)}{(r - \mu(1 + n))} \hat{c}_{t+1}^n}_{\text{Newborn's consumption effect}} - \underbrace{\beta \Sigma_{t+1}^{rn}}_{\text{Interest rate tilting effect}} \quad (5.10)$$

Composition effect

Similarly, for the foreign country

$$\hat{c}_t^* = \underbrace{(1 - \beta^*) \Sigma_t^{c*}}_{\text{Average wealth effect}} - \underbrace{\frac{(1 - \tau^*)(1 + n)}{(r - \mu(1 + n))} \hat{c}_{t+1}^{n*}}_{\text{Newborn's consumption effect}} - \underbrace{\beta^* \Sigma_{t+1}^{rn}}_{\text{Interest rate tilting effect}} \quad (5.11)$$

Composition effect

Note that Σ_{t+1}^{rn} is the same across countries by Eq.(3.27).

The analogues of above two equations in the symmetric case can be found to be

$$\hat{c}'_t = \underbrace{(1 - \beta) \Sigma_t^{c'}}_{\text{Average wealth effect}} - \underbrace{\beta \Sigma_{t+1}^{r'}}_{\text{Interest rate tilting or Composition effect}} \quad (5.10')$$

$$\hat{c}^{*'}_t = \underbrace{(1 - \beta) \Sigma_t^{c^{*'}}}_{\text{Average wealth effect}} - \underbrace{\beta \Sigma_{t+1}^{r'}}_{\text{Interest rate tilting or Composition effect}} \quad (5.11')$$

As before, Eqs.(5.10') and (5.11') can also be obtained by imposing $n = 0$, $\beta^* = \beta$, $\tau = \tau^* = 1$ in Eqs.(5.10) and (5.11) and replacing Σ_t^c , r , Σ_{t+1}^{rn} with $\Sigma_t^{c'}$, r' , $\Sigma_{t+1}^{r'}$ where $\Sigma_{t+1}^{r'}$ is defined as

$$\Sigma_{t+1}^{r'} \equiv \hat{r}_{t+1} + \frac{1}{\bar{r}'} \hat{r}_{t+2} + \frac{1}{\bar{r}'^2} \hat{r}_{t+3} + \dots$$

Eqs.(5.10) to (5.11') describe the movements in date- t consumption for the two countries. Due to the existence of the interest rate tilting effect, the movements in consumption are not flat even in the symmetric case. Thus the effect on \hat{c}_t is based on the average total wealth effect and adjusted by the interest-rate-tilting effect. However, two places distinguish the asymmetric model from

the symmetric one. First, in the asymmetric model the tilting effects differ in their strengths in the two countries while in the symmetric model their strengths are the same. Second, in addition to the interest rate tilting effect, because we assumed the entry of new generation, we have the new new-born's consumption effect as shown in the equations.

3.5.3 Country difference

Take the difference between \hat{c}_t and \hat{c}_t^* to yield

$$\begin{aligned}
 \hat{c}_t^D = & \underbrace{(1 - \beta) \Sigma_t^c - (1 - \beta^*) \Sigma_t^{c*}}_{\text{Difference in average total consumptions}} \\
 & - \underbrace{\frac{(1 + n)}{(\bar{r} - \mu(1 + n))} [(1 - \tau) \hat{c}_{t+1}^n - (1 - \tau^*) \hat{c}_{t+1}^{n*}]}_{\text{Difference in average total adjustments due to newborn's consumption}} \\
 & - \underbrace{(\beta - \beta^*) \Sigma_{t+1}^{rn}}_{\text{Difference in average total interest rate tilting effects}} \tag{5.12}
 \end{aligned}$$

and for the symmetric case

$$\hat{c}_t^{D'} = \underbrace{(1 - \beta) (\Sigma_t^{c'} - \Sigma_t^{c*'})}_{\text{Difference in average total consumptions}} \tag{5.12'}$$

For the asymmetric case, because all the weights involved differ in size, all sub effects are retained. For the symmetric case, however, because the interest-rate-tilting effects work in the same way and to the same strength in both countries, they disappear when we take the difference.

We use Eqs.(5.7) and (5.8) to expand the Σ_t^c and Σ_t^{c*} above to finally obtain

$$\begin{aligned}
 \hat{c}_t^D = & \underbrace{\phi^{\beta g} \hat{\xi}_t}_{\text{Gross portfolio return}} + \underbrace{\frac{r}{r - \mu(1+n)} \left[\frac{1 - \beta}{\phi_{cy}} \hat{y}_t - \frac{1 - \beta^*}{\phi_{cy}^*} \hat{y}_t^* \right]}_{\hat{c}_t^D[1] \equiv \text{Endowment effect}} \\
 & + \underbrace{\phi^{\beta g} r \phi_{wy} \hat{r}_{2t}}_{\hat{c}_t^D[2] \equiv \text{Current net portfolio return}} + \underbrace{\frac{(1+n)}{r} \phi^{\beta g} r \phi_{wy} \Sigma_{t+1}^{rn}}_{\hat{c}_t^D[3] \equiv \text{Future net portfolio return}} \\
 & - \underbrace{\frac{(1+n)}{(r - \mu(1+n))} \left[(1 - \tau) \hat{c}_{t+1}^n - (1 - \tau^*) \hat{c}_{t+1}^{n*} \right]}_{\hat{c}_t^D[4] \equiv \text{Newborn's consumption effect}} \\
 & - \underbrace{(\beta - \beta^*) \Sigma_{t+1}^{rn}}_{\hat{c}_t^D[5] \equiv \text{Interest rate tilting effect}} \tag{5.13}
 \end{aligned}$$

where

$$\phi^{\beta g} \equiv \frac{(1 - \beta)}{\phi_{cy}} + \frac{(1 - \beta^*)}{\phi_{cy}^*} \tag{5.14}$$

For the symmetric case

$$\hat{c}_t^{D'} = \underbrace{\phi^{\beta g'} \hat{\xi}_t'}_{\text{Gross portfolio effect}} + \underbrace{\frac{(1 - \beta') r'}{(r' - \mu)} (\hat{y}_t - \hat{y}_t^*)}_{\text{Endowment effect}} \tag{5.13'}$$

where

$$\phi^{\beta g'} = 2(1 - \beta') \tag{5.14'}$$

In summary, by Eq.(5.13), \hat{c}_t^D consists of five risk factors. They are:

1. The endowment effect. Endowments move directly because of shocks. This is the only risk term of \hat{c}_t^D in a symmetric model.

2. The current and future net portfolio return effects. These effects emerge because in an asymmetric model, the steady-state net portfolio is non-zero. The net portfolio return effect collapses into zero when the steady-state net portfolio goes to zero as it does in the symmetric model.

When endowments change, the extra consumption resources into the infinite future for a country also change. The endowment effect and the current and future net portfolio return effects taken together constitute this wealth effect.

3. The new-born's consumption effect. This effect captures the movements in new-born's consumption. As explained above, this effect emerges because we assume the entry of new generation which is absent from the symmetric model.

4. The interest-rate-tilting effect. This effect captures households' behaviour in shifting consumption across time to take advantage of intertemporal opportunities. This effect only emerges in the asymmetric model because in the symmetric model the tilting effects are equal in the two countries so it does not have a role in the country difference.

For the current model, the new-born's consumption effect and the interest-rate-tilting effect taken together constitute the composition effect. They dictate how far the result deviates from the situation where desired per capita consumption is smoothed over time.

3.6 Steady-state portfolio, $\bar{\alpha}$

Substituting \hat{c}_t^D from Eq.(5.13) into Eq.(4.8), we obtain the steady-state international portfolio, $\bar{\alpha}$.

$$\bar{\alpha} = \underbrace{-\frac{1}{\phi^{\beta g r}} \frac{\text{cov}(\hat{c}_t^D [1], \hat{r}_{xt})}{\text{var}(\hat{r}_{xt})}}_{\bar{\alpha}[1]} \underbrace{-\frac{1}{\phi^{\beta g r}} \frac{\text{cov}(\hat{c}_t^D [2], \hat{r}_{xt})}{\text{var}(\hat{r}_{xt})}}_{\bar{\alpha}[2]} \underbrace{-\frac{1}{\phi^{\beta g r}} \frac{\text{cov}(\hat{c}_t^D [3], \hat{r}_{xt})}{\text{var}(\hat{r}_{xt})}}_{\bar{\alpha}[3]} + \underbrace{\frac{1}{\phi^{\beta g r}} \frac{\text{cov}(\hat{c}_t^D [4], \hat{r}_{xt})}{\text{var}(\hat{r}_{xt})}}_{\bar{\alpha}[4]} + \underbrace{\frac{1}{\phi^{\beta g r}} \frac{\text{cov}(\hat{c}_t^D [5], \hat{r}_{xt})}{\text{var}(\hat{r}_{xt})}}_{\bar{\alpha}[5]} \quad (6.1)$$

Eq.(6.1) expresses the optimal portfolio as the sum of a series of variance-covariance ratios. Following the explanation in Chapter 2 and at the end of Section 3.4 of this chapter, each of these ratios represents a hedging of the different income risks in \hat{c}_t^D . \hat{c}_t^D is composed of five income risks, so $\bar{\alpha}$ is composed of 5 corresponding components. We now discuss each of these components in turn.

Diversification term The first term in Eq.(6.1) is

$$\bar{\alpha} [1] = -\frac{(r - \mu)}{2(r - 1)(r - \mu(1 + n))} < 0 \quad (6.2)$$

where $\bar{\alpha} [1]$ is defined as the diversification term (or self-hedging term). The term reflects households' motive to hedge against the risk of \hat{c}_t^D that is directly associated with the fluctuation in endowments, i.e. $\hat{c}_t^D [1]$ or the endowment effect of Eq.(5.13) defined in the previous section.

To better understand $\bar{\alpha} [1]$, note that in symmetric model, it follows that

$$\bar{\alpha}' = \bar{\alpha}' [1] = -\frac{1}{2(r' - 1)} < 0 \quad (6.2')$$

In this case the endowment effect is the only risk to be hedged, so the diversification term is the only term in the portfolio in the symmetric model. Note that steady-state equity prices are given by $\bar{z}_1 = \bar{z}_2 = \frac{1}{r-1}$, so $\bar{\alpha}_1 = -\frac{\bar{z}}{2}$. Recall net portfolio holdings across countries are given in Table 3.1, we thus have $\bar{z} + \bar{\alpha}_1 = \frac{\bar{z}}{2}$ and $\bar{\alpha}_2 = \bar{w} - \bar{\alpha}_1 = \frac{\bar{z}}{2}$ for the home country and $\bar{\alpha}_1^* = -\bar{\alpha}_1 = \frac{\bar{z}}{2}$ and $\bar{z} + \bar{\alpha}_2^* = \bar{z} + \bar{w} - \bar{\alpha}_1^* = \frac{\bar{z}}{2}$ for the foreign country. That is, both countries hold the world portfolio. We arrive at the classic conclusion that country portfolios should be fully diversified in the symmetric model.

Comparing Eqs.(6.2) to Eq.(6.2') we can see that the diversification term in the asymmetric model is the same as that in symmetric models except for the inclusion of a wedge of $\frac{(r-\mu)}{(r-\mu(1+n))}$. This result has nothing to do with country asymmetry here but is due to our assumption of the entrance of a new cohort in each period. Abstracting from the assumption by imposing $n = 0$, we get back to the symmetric situation, $\bar{\alpha} [1] = \bar{\alpha}' [1]$. Otherwise with $1 < (1+n) < r$, $\frac{(r-\mu)}{(r-\mu(1+n))} > 1$. So $\bar{\alpha} [1]$ is larger than $\bar{\alpha}' [1]$ in absolute value.

Hedging the current net portfolio return The second term in Eq.(6.1) is

$$\bar{\alpha} [2] = -\frac{(r-\mu)(\zeta_{re1} - \zeta_{re2})\phi_{wy}}{2(r-1)} \quad (6.3)$$

where ζ_{re1} and ζ_{re2} are responses of \hat{r}_{2t} to home and foreign shocks $\{\varepsilon_t, \varepsilon_t^*\}$. According to the asset pricing relations, a positive shock in either country increases rates of return for both the asset in the other country (through inducing lower expected future interest rates and thus higher capital gains today) and the asset domestically (through both the channel of higher capital gains and dividend payments). So ζ_{re1} and ζ_{re2} are both positive and it is very likely that $\zeta_{re1} < \zeta_{re2}$.

This term reflects households' motive to hedge against the risk associated with fluctuations in the returns to the current net portfolio when endowments are shocked, i.e. the current net portfolio effect of \hat{c}_t^D of Eq.(5.13) in the previous section. This risk is absent from the symmetric model, so $\bar{\alpha}$ [2] does not exist in symmetric models. This can be also seen by noting that in Eq.(6.3) when $\phi_{wy} = 0$, $\bar{\alpha}$ [2] = 0.

The sign of $\bar{\alpha}$ [2] depends on the relative magnitude of ζ_{re1} and ζ_{re2} . If as conjectured, $\zeta_{re1} < \zeta_{re2}$, then $\bar{\alpha}$ [2] is negative. To understand, note that the home country is a debtor country and has to pay interest on its steady-state net foreign liability. Home and foreign (positive) shocks both boost the current interest rate and so also interest payments to the foreign country. When $\zeta_{re1} < \zeta_{re2}$, the increase in interest payments is smaller in response to the home shock, i.e. when home consumption is high the excess return is also high so asset 1 is a bad hedge against the risk of this income stream. The home country therefore chooses to short asset 1. Otherwise, if $\zeta_{re1} > \zeta_{re2}$, by the opposite argument, asset 1 would be a good hedge against the risk from the net portfolio return which would in turn result in a long position.

Hedging future net portfolio returns The third term in Eq.(6.1) is

$$\bar{\alpha} [3] = - \frac{(r - \mu) (\zeta_{sre1} - \zeta_{sre2}) (1 + n) \phi_{wy}}{2(r - 1) r} \quad (6.4)$$

where ζ_{sre1} and ζ_{sre2} are the responses of Σ_{t+1}^n , defined in Eq.(5.6), to home and foreign shocks $\{\varepsilon_t, \varepsilon_t^*\}$. Because a positive shock in either country tends to reduce expected future interest rates, ζ_{sre1} and ζ_{sre2} are both negative.

This term reflects households' motive to hedge against the risk associated with fluctuations in the returns to the net portfolio in subsequent periods when endowments are shocked. The risk corresponds to the future net portfolio effect of \hat{c}_t^D , i.e. \hat{c}_t^D [3] of Eq.(5.13) in the previous section. This risk is absent from symmetric models, so this hedging term does not exist in symmetric models. This can be also seen by noting that in Eq.(6.4) when $\phi_{wy} = 0$, $\bar{\alpha}'$ [3] = 0.

The sign of $\bar{\alpha}$ [3] depends on the relative magnitude of ζ_{sre1} and ζ_{sre2} . If $\zeta_{sre1} > \zeta_{sre2}$, then $\bar{\alpha}$ [3] is positive. To understand, note that the home country also has to pay interest on its net foreign liability in subsequent periods after

shocks. Home and foreign (positive) shocks both depress expected future interest rates and so interest payments to the foreign country in the future are depressed. When $\zeta_{sre1} > \zeta_{sre2}$, the home shock induces a relatively smaller decrease in interest payments (so consumption is low) while the excess return on asset 1 is high. So asset 1 is a good hedge against future portfolio return risk. The home country therefore chooses a long position in asset 1. Otherwise, if $\zeta_{sre1} < \zeta_{sre2}$, asset 1 would be shorted as a bad hedge against future portfolio return risk.

Hedging risk from new-born's consumption The fourth term in Eq.(6.1) is

$$\bar{\alpha} [4] = \frac{(1+n)\mu(2-\tau-\tau^*)}{2\phi^{\beta g} r(r-\mu(1+n))} > 0 \quad (6.5)$$

The fourth term reflects hedging of the country as a whole against the risk associated with the fluctuations in new-born's consumption. This risk corresponds to the new-born's consumption effect of \hat{c}_t^D , i.e. $\hat{c}_t^D [4]$ of Eq.(5.13) in the previous section. The term is absent from a symmetric model. To see this, note that in a symmetric world we have $\tau = \tau^* = 1$ in Eq.(6.5) so this term disappears. Because τ and τ^* are both less than 1, $\bar{\alpha} [4]$ is positive.

To understand this, note that the Euler equation in this model, i.e. $\hat{c}_{t+1} = \tau\hat{c}_t + (1-\tau)\hat{c}_{t+1}^n + \tau\hat{r}_{t+1}$, is different from that in an *OLG*-free symmetric model, i.e. $\hat{c}_{t+1} = \hat{c}_t + \hat{r}_{t+1}$. Besides the interest tilting effect, \hat{r}_{t+1} , the future consumption \hat{c}_{t+1} is equal to a weighted average of the current consumption and expected future consumption of new generation. To obtain the current consumption, it is necessary to adjust the average total resource of the whole country (at the per capita level) by deducting the consumption of yet unborn generations (see Eqs.(5.10) and (5.11)). According to the Euler equation, the deduction is more responsive to the home shock than to the foreign shock due to the fact that $(1-\tau) > (1-\tau^*)$. In other words, when, for instance a positive shock in the home country takes place, (because the deduction of new-born's consumption is relatively high) the home consumption is low while the excess return on asset 1 is high. So asset 1 is a good hedge against the new-born consumption risk which explains a positive value of $\bar{\alpha} [4]$.

Hedging risk from differing consumption tilting The fifth term in Eq.(6.1) is

$$\bar{\alpha} [5] = \frac{(r - \mu) (\zeta_{sre1} - \zeta_{sre2}) (1 + n) (\tau - \tau^*)}{2\phi^{\beta g} r^2 (r - 1)} \quad (6.6)$$

The fifth term reflects households' motive to hedge against the risk associated with the country difference in shifting consumptions across time in response to variations in expected future interest rates after shocks. The risk corresponds to the interest rate tilting effect in the last section, i.e. $\hat{c}_t^D [5]$ of (5.13) which is absent from symmetric models. This point can be also seen by noting that $\bar{\alpha} [5] = 0$ if $\tau = \tau^*$.

As before, the sign of this term also depends on relative magnitude of two ζ s in the expression. If $\zeta_{sre1} > \zeta_{sre2}$, then we would have $\bar{\alpha} [5] < 0$. The explanation for this is as follows. Lower expected future interest rates after positive shocks tilt consumption to the present. So besides adjusting to the new-born's consumptions, the average total resource should be further adjusted by adding this interest tilting effect to obtain the current consumption. By the asymmetry and Eq.(5.12), the addition to the current consumption (or the tilting effect) is less at home ($\tau < \tau^*$). However, when $\zeta_{sre1} < \zeta_{sre2}$, the home shock depresses interest rates less in absolute value. So the relatively lower tilting at home becomes less important (current consumption is high) when the excess return on asset 1 is high. So asset 1 is a bad hedge against the risk associated with consumption tilting. The home country therefore chooses to short asset 1. Otherwise, if $\zeta_{sre1} > \zeta_{sre2}$, the asset 1 will be a good hedge and the home country will choose a long position.

Asset home bias As explained before, once the home (gross) holding of domestic asset $\bar{\alpha}$ is known, net asset holdings across countries are determined by market clearing conditions as shown in Table 3.1. Specifically, here, holdings of asset 1 and 2 in the foreign country are respectively $-\bar{\alpha}$ and $\bar{z}_2 - \bar{w} + \bar{\alpha}$. And holdings of asset 1 and 2 in the home country is $\bar{z}_1 + \bar{\alpha}$ and $\bar{w} - \bar{\alpha}$. The condition for the emergence of asset home bias, i.e. the proportion of domestic asset holding in country portfolio is more than that in world portfolio, is equivalent to

$$\bar{z}_1 + \bar{\alpha} > \bar{w} - \bar{\alpha} \quad (6.14)$$

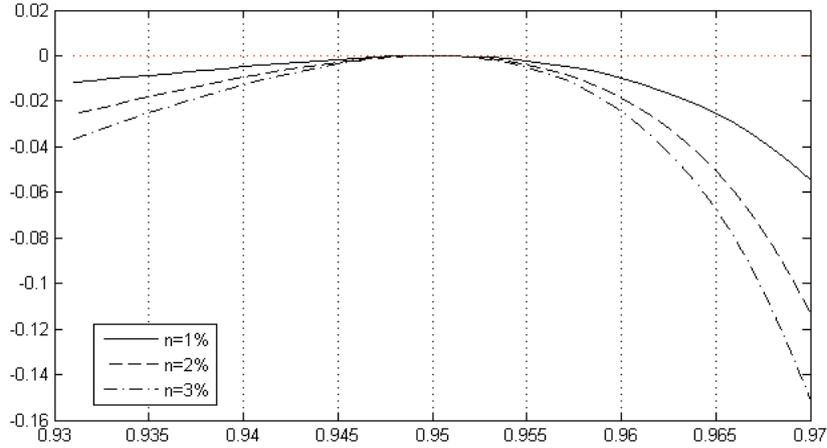


Figure 3.1: Testing asset home bias under global imbalances

In symmetric models where $\bar{w} = 0$, (because $\bar{\alpha} < 0$) the condition is

$$\Gamma = |\alpha| - \frac{1}{2(r-1)} < 0 \quad (6.15)$$

However, in the current asymmetric model with $\bar{w} \neq 0$, inequality (6.14) leads to a slightly different condition from inequality (6.15):

$$\Gamma = |\alpha| - \frac{1}{2(r-1)} \left[1 + \frac{(1-r\beta)}{(1+n-r\beta)} \right] < 0 \quad (6.16)$$

Numerical experiments suggest that this condition always holds given that the country asymmetry ($\beta \neq \beta^*$) exists and the stability and dynamic efficiency conditions ($r > (1+n) > r\beta^*$, $r\beta$) are satisfied.

Figure 3.1 is an illustration of the test of inequality (6.16). In the figure, β^* is set at 0.95 and β ranges from 0.93 to 0.97. We plot Γ for the cases of $n = 1\%$, 2% , 3% (while set σ^2 at 1)¹. (The choices of n are to make sure the stability and dynamic efficiency conditions are satisfied.) As is shown in the figure, Γ is

¹Note that the solution of portfolio α does not depend on the value of σ^2 . This is because when α is computed as a covariance-variance ratio, σ^2 appears in the numerator and denominator at the same time and therefore drops out in our model. This is also the case for the next chapter, i.e. chapter 4. To see how this is taking place, one is referred to Devereux and Sutherland (2011) where a much simpler set-up and thus solution of α are provided. In the calibration here, σ^2 can be specified to be any positive value.

always negative given $\beta \neq \beta^*$. Similar to this, further tests where the value of β^* moves are conducted and the results of $\Gamma < 0$ are very robust.

A caveat on the above discussion of asset home bias should be highlighted in anticipation of the analyses in Chapter 4. The portfolio allocation of this model exhibits asset home bias in the sense that in the portfolio of individual countries the role of domestic assets outweighs that of foreign assets. As described in Chapter 1, however, asset home bias in reality is so pronounced that in addition to this type of bias we also observe that, across countries, domestic assets are mostly held by domestic investors. In other words, to account for the biased pattern in the empirical relevance, condition Eq.(6.14) should be replaced by the following conditions for the two countries

$$\frac{\bar{z}_1 + \bar{\alpha}}{\bar{z}_1} > \frac{1}{2} \quad (6.17)$$

$$\frac{\bar{z}_2 - \bar{w} + \bar{\alpha}}{\bar{z}_2} > \frac{1}{2} \quad (6.18)$$

which state that in either country the domestic ownership of assets exceeds the critical level of one half. A tricky fact about the two characterisations of asset home bias is that they are not always consistent with each other. To be specific, Eqs.(6.17) and (6.18) are stronger conditions than Eq.(6.14) in defining the pattern of asset home bias. When both Eqs.(6.17) and Eq.(6.18) hold, Eq.(6.14) and the resulting Eq.(6.16) must also be satisfied. However, when Eq.(6.14) and Eq.(6.16) hold, Eqs.(6.17) and Eq.(6.18) are not necessarily true as is the case of this model. This point can be seen much more clearly once we proceed to Section 3.8 of this chapter where the portfolio allocation of this model is depicted in a figure, Figure 3.5. However, the weak definition is adopted here in order to provide a description of the portfolio allocation in this model. In Chapter 4 where the empirically relevant asset home bias is to be taken up as the central theme of analysis, we will adopt the stronger measure of asset home bias in the form of the domestic ownership of assets, i.e. the fractions (defined as π and π^* in Chapter 4) on the left hand side of Eqs.(6.17) and (6.18).

3.7 Net portfolio dynamics, \hat{w}

In this section, we study the dynamics of net foreign assets from a theoretical point of view. We show that in addition to the trade channel, the current model generates an intertemporal terms-of-trade effect on the net portfolio and valuation effects on gross portfolios. Following the literature, the intertemporal terms-of-trade effect is defined as the effect working through the change in the return of numeraire asset, while the valuation effects are defined as the effects working through the change in the relative excess returns of the other assets.

The external budget constraints in the two countries are

$$\hat{w}_{t+1} = \frac{r}{1+n} \hat{w}_t + \frac{r\phi_{wy}}{1+n} \hat{r}_{2t} + \frac{r\phi_{\alpha y}}{1+n} \hat{r}_{xt} + \frac{1}{1+n} [\hat{y}_t - \phi_{cy} \hat{c}_t] \quad (7.1)$$

$$\hat{w}_{t+1} = \frac{r}{1+n} \hat{w}_t + \frac{r\phi_{wy}}{1+n} \hat{r}_{2t} + \frac{r\phi_{\alpha y}}{1+n} \hat{r}_{xt} - \frac{1}{1+n} [\hat{y}_t^* - \phi_{cy}^* \hat{c}_t^*] \quad (7.2)$$

Combining them yields

$$\hat{w}_{t+1} = \frac{r}{1+n} \hat{w}_t + \underbrace{\frac{r\phi_{wy}}{(1+n)} \hat{r}_{2t}}_{TT} + \underbrace{\frac{r\phi_{\alpha y}}{(1+n)} \hat{r}_{xt}}_{VAL} + \underbrace{\frac{1}{(1+n)\phi^g} \left[\frac{\hat{y}_t}{\phi_{cy}} - \frac{\hat{y}_t^*}{\phi_{cy}^*} - \hat{c}_t^D \right]}_{TB} \quad (7.3)$$

where

$$\phi^g = \frac{1}{\phi_{cy}} + \frac{1}{\phi_{cy}^*} \quad (7.4)$$

Eq.(7.3) shows how the trade balance effect TB , intertemporal terms-of-trade effect TT and valuation effect VAL are all nested in a unified framework in this chapter. From a glance at the equations, it follows that the existence of TT effect depends on whether in a model there is non-trivial net portfolio, i.e. global imbalances, while the existence of the VAL effect depends on whether there are portfolio choices. By incorporating both of these two ingredients, adjustments in w take place through all these three channels here. Let us consider them one by one.

3.7.1 Trade balance effect

The TB effect will survive even in a portfolio-free symmetric model. The term remains in the current general framework, however, with a different composition

and implication. By expanding \hat{c}_t^D in Eq.(7.3) with Eq.(4.13), we obtain the following expression for TB which decomposes it into three components on the right hand side.

$$TB = \frac{1}{(1+n)\phi^g} \left[\frac{\hat{y}_t}{\phi_{cy}} - \frac{\hat{y}_t^*}{\phi_{cy}^*} - (\chi_{ce1}^D \hat{y}_t + \chi_{ce2}^D \hat{y}_t^*) \right] - \underbrace{\frac{\chi_{cx}^D \tilde{\alpha} [1]}{(1+n)\phi^g} \hat{r}_{xt}}_{VAL^c[1]} - \underbrace{\frac{\chi_{cx}^D \sum_{i=2}^5 \tilde{\alpha} [i]}{(1+n)\phi^g} \hat{r}_{xt}}_{VAL^c[2]} \quad (7.5)$$

The first term denotes the conventional trade balance effect as documented in standard textbooks of international macroeconomics. To understand, in a portfolio-free symmetric model, it read $TB = \frac{1}{2} [\hat{y}_t^D - \hat{c}_t^D]$. It should be familiar that when a country is experiencing a temporary increase in relative income or a temporary decrease in relative expenditure, there tends to be a current account surplus for that country. The TB effect here shares the same interpretation. However, because of the presence of the country asymmetry and *OLG* structure in the model, we observe many terms in the ϕ s and n in the current expression. Besides, with the presence of portfolio choices in the model, we extract \hat{c}_t^D 's direct response to exogenous shocks, i.e. $\chi_{ce1}^D \hat{y}_t + \chi_{ce2}^D \hat{y}_t^*$, to represent the whole behaviour of \hat{c}_t^D in order to obtain the corresponding conventionally-defined trade balance.

The latter two terms in Eq.(7.5) represent (indirect) valuation effects working through consumption on the trade balance effect which when taken together are denoted by VAL^c . These terms exist because portfolio returns will feedback on income and consumption after shocks. Let us now explain how they work. For instance, when the home country receives a positive endowment shock, \hat{y}_t increases. As a result, the return on the home asset exceeds that on the foreign asset, i.e. \hat{r}_{xt} increases. With a negative gross holding of home asset $\tilde{\alpha}$, this implies a wealth transfer from the home country to the foreign country in terms of capital gains. (Imagine that the return to external wealth of the foreign country, \hat{r}_{1t} , is relatively higher while the return to external wealth of the home country, \hat{r}_{2t} , is relative lower.) This lowers home households' income and also consumption. In other words, when relative income increases due to a positive shock, relative consumption increases as well, with the presence of optimal portfolios however, not as much as that implied previously, i.e. $\chi_{ce1}^D \hat{y}_t + \chi_{ce2}^D \hat{y}_t^*$.

Correspondingly, the improvement in the trade balance is more than that defined conventionally, i.e. $VAL^c > 0$.

According to our categorization of portfolio components into different hedging, we know that the diversification term $\tilde{\alpha}[1]$ always exists in $\tilde{\alpha}$ while the imbalance terms $\tilde{\alpha}[i]$, $i = 2, 3, 4, 5$, emerge only in an asymmetric model. The two types of VAL^c can thus be distinguished. $VAL^c[1]$ captures the indirect valuation effect that works through $\tilde{\alpha}[1]$. Replacing $\tilde{\alpha}[1]$ with $\tilde{\alpha}'[1]$, Eq.(6.2'), one can obtain the representation of the effect between two symmetric countries, the same expression as the one in Eq.(27) of Devereux and Sutherland (2010). $VAL^c[2]$, however, captures the indirect valuation effects that work through $\tilde{\alpha}[2]$, which can only be obtained in the model with global imbalances.

3.7.2 Terms of trade effect

The TT effect emerges only when we take into account country asymmetries (that are able to generate external imbalances). In Eq. (7.3), this can be seen by noticing that if $\phi_{wy} = 0$, TT becomes 0.

The presence of the TT effect reflects the fact that when incomes are shocked, interest rates will move as well, i.e. the terms on which consumers trade current and future consumptions changes. If net external positions of a country in steady state are non-zero, this gives rise to movements in the interest payments or revenues on net external positions. In a symmetric model, because the steady-state net portfolio is zero, this term disappears. In an asymmetric model, the direction of the effect relies on whether the country runs a net external deficit or surplus. When it is the former case, as is the case of the home country here, because (for instance) a positive domestic shock favours improvement in the trade balance on the one hand, on the other hand, the interest rate on the external net position is also driven up, the TT effect worsens and so works in the opposite direction to that of trade balance. If the TT effect is sufficiently large that it dominates the trade balance effect, positive shocks to income would even lead to a deterioration of net external wealth as a whole rather than improving it (Bhagwati, 1958 and Chapter 1 of Obstfeld and Rogoff, 1996). In comparison, for a country running a steady-state current account surplus, as is the case of foreign country here, a positive domestic shock causes a TT effect which reinforces the TB effect by

improving net external wealth of the country.

3.7.3 Valuation effect

The *VAL* effect emerges only when we take into account international portfolios. In Eq.(7.3), this can be seen by noticing that if only a single asset is present, \hat{r}_{xt} does not make any sense.

As explained before, following a positive domestic shock, the excess return of the home asset increases. The home country possessing negative gross external position in the home asset will suffer a wealth transfer to the foreign country. The *VAL* effect captures this effect. It revises \hat{w}_{t+1} downward. By the same token, when a negative domestic shock takes place, the opposite happens, i.e. the *TB* effect is negative while the *VAL* effect is positive. So the valuation effect always works in the opposite direction to the trade effect. In the literature and empirical evidence, this is documented as a stabilizing valuation effect.

By expanding $\tilde{\alpha}$ in Eq.(7.3)

$$VAL = \underbrace{\frac{\tilde{\alpha} [1]}{(1+n)} \hat{r}_{xt}}_{VAL[1]} + \underbrace{\frac{\sum_{i=2}^5 \tilde{\alpha} [i]}{(1+n)} \hat{r}_{xt}}_{VAL[2]} \quad (7.6)$$

we observe the difference that is made by the assumption of a country asymmetry. As in the case of VAL^c , because the steady-state portfolio, $\bar{\alpha}$, is made up of diversification and imbalance terms, *VAL* is made up of *VAL* [1] and *VAL* [2] which denote respectively the (direct) valuation effect through diversification term of $\tilde{\alpha}$ and that through imbalance terms of $\tilde{\alpha}$. Again, by replacing $\tilde{\alpha} [1]$ with its symmetric analogy $\tilde{\alpha}' [1]$, we recover the *VAL* effect between two symmetric countries, i.e. Eq.(26) of Devereux and Sutherland (2010). With the presence of a country asymmetry in our model, not only *VAL* [1] differs, *VAL* [2] emerges to reflect that the gross position of country is altered to meet the need to hedge against additional risks.

3.7.4 The intertemporal external constraint

By assuming a no-Ponzi-game condition, we can iterate Eq.(7.3) forward to obtain

$$\hat{w}_t = - \sum_{j=0}^{+\infty} \left[\frac{1+n}{r} \right]^{j+1} \{TT_{t+j} + VAL_{t+j} + TB_{t+j}\} \quad (7.7)$$

by which we have the following result.

External adjustments of countries in the world of global imbalance work through three channels, i.e. the trade balance effect, TB , the terms of trade effect, TT , and the valuation effect VAL . The presence of the TT effect is associated with the assumption of a country asymmetry. The presence of the VAL effect is associated with the assumption of portfolio choices.

Eq.(7.7) can be read in parallel with the similar equations in literature, for example Eq.(9) in Gourinchas and Rey (2007) and Eq.(3) in Blanchard et al. (2005). Note that because the net and gross portfolio positions are determined endogenously in this model, the terms of trade and valuation effects generated here in Eq.(7.7) are fully endogenous and micro-founded in contrast to these two other papers.

3.8 Model simulation

To illustrate the quantitative implications of the model, we perform a simulation exercise in this section. It is not intended to be a serious guide to any real issues of the current world, so the calibration is not based on empirical description of specific countries. The idea when we choose them is to have an autarky interest rate of 3% per period in the foreign country and in the home country it is 1% higher. The growth rate of population is chosen at 1%. Table 3.2 lists all parameter values we use. The resulting steady-state net foreign asset, international interest rate and consumption ratios in two countries are also reported at the foot of this table.

Description	Value
Home discount factor	$\beta = 0.96$
Foreign discount factor	$\beta^* = 0.97$
Population growth rate	$n = 0.01$
Persistence of endowment shocks	$\mu = 0.95$
Parameter in home Euler equation	$\tau = 0.983$
Parameter in foreign Euler equation	$\tau^* = 0.993$
Steady-state net foreign asset	$\phi_{wy} = -12.47$
International interest rate	$r = 1.034$
Home consumption ratio to endowment	$\phi_{cy} = 0.70$
Foreign consumption ratio to endowment	$\phi_{cy}^* = 1.30$

Table 3.2: Parameter values and Steady states

3.8.1 Per capita steady states

According to the results in Table 3.2, the β and β^* we chose imply autarky interest rates of 4.17% and 3.09% in the two countries. The steady state r lies in between at about 3.4%. The related stability and dynamic efficiency conditions ($\tau, \tau^* < 1$ and $r > (1 + n)$) are both satisfied. Net foreign asset proves to be unrealistically large in response to even such a small gap between β and β^* . A one percent difference between the β s results in a volume of home net foreign asset at the level of around 12.5 times average GDP . This is mainly because we do not allow for any frictions such as trade costs in the model, which is not a realistic assumption. This tends to exaggerate steady-state w in comparison to y given that the former is a stock variable while the latter is a flow variable. However, for our main purpose in this chapter, the strategy in abstracting from such complexities is more suitable.

Figure 3.2 plots the profile of individual consumption c^v across vintages in the home country. In the figure, the decreasing line is the profile of individual consumptions in the home country while the dark dot denotes steady-state aggregate per capita consumption in the country. c^v is higher than average GDP for young vintages while lower for vintages beyond a certain age. As explained in Section 3.2, this is because in an impatient country, households of any vintage consume more than their permanent income when they were born and then consume less and less afterwards. Even though different vintages enjoy different

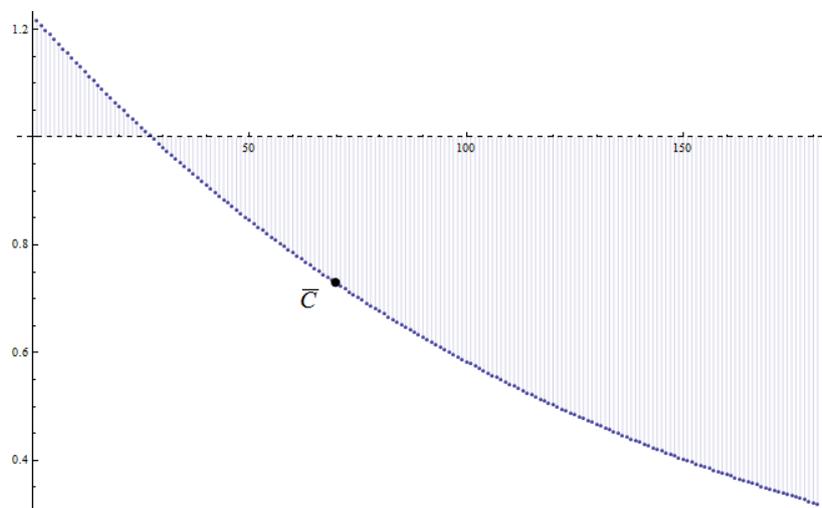


Figure 3.2: Consumption aggregation

level of individual consumptions, our assumption of population growth $n > 0$ make sure that when we aggregate c^v into per capita consumption c , young vintages always account for higher weights while old vintages account for less and less weights. As a result, when a vintage gets sufficient old, the role of its consumption becomes so small that to be negligible, which justifies that it is no need to explicitly assume death in the model. After aggregation, the per capita consumption finds its steady-state level at 0.7 which is lower than average income of 1. The converse is true for the foreign country where the profile of individual consumption is upward sloping. Aggregate per capita consumption c^* is higher than 1 in steady state at around 1.3. The simulation observations here are consistent with our theoretical findings in Section 3.3.

3.8.2 Behaviour of model variables (with and without a country asymmetry)

In this section, we present the results of first-order dynamics of the model. To better see how the presence of the country asymmetry and global imbalances matter, we first report the results of the model in a symmetric case.

Figure 3.3 plots the case where the model is symmetrically parameterized such that $\beta = \beta^* = 0.97$. This figure shows the first-order behaviour of the model

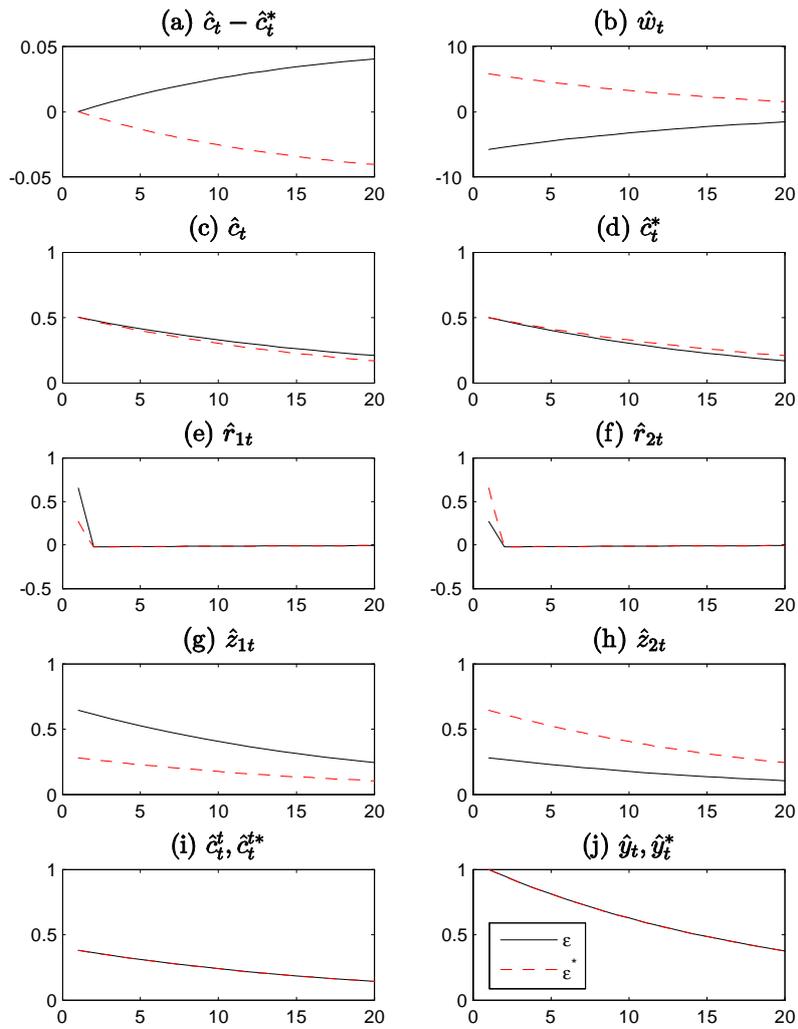


Figure 3.3: IRFs of the model to home (solid line) and foreign (dashed line) shocks: Symmetric case

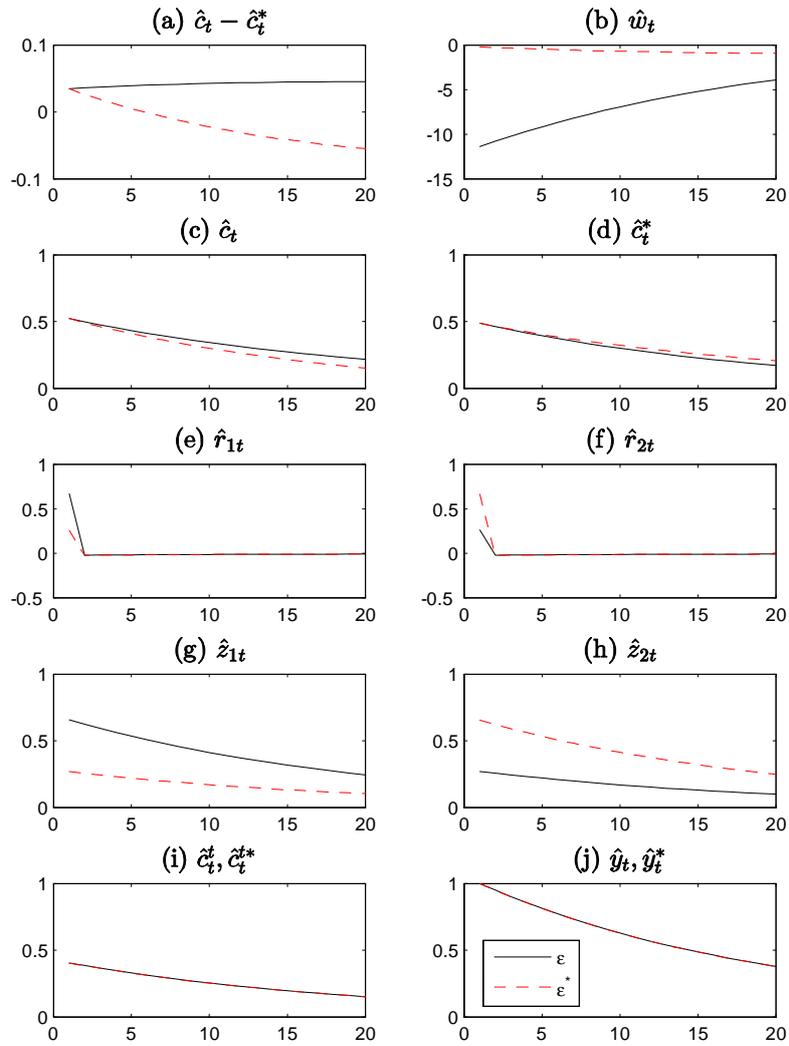


Figure 3.4: IRFs of the model to home (solid line) and foreign (dashed line) shocks: Asymmetric case

after a 1 percent positive shock to home (solid line) or/and foreign (dashed line) endowments. Endowments are shown in Panel (j). Panels (a) to (h) are thus respectively the response of the consumption differential, home *NFA* (relative to \bar{y}), home and foreign consumption, home and foreign asset returns, and home and foreign asset prices. Panel (i) shows responses of new-born's consumption in the two countries. We use this to represent the case of a symmetric model even though in the literature, these models feature representative agents. Except for inclusion of new-born's consumption, the dynamics of the model is similar to a standard model without overlapping generations.

Let us look at the solid lines first. In response to a positive shock to the home endowment, consumption in both country rises as shown in Panels (c) and (d). With the shock decaying, consumption decrease gradually towards the steady state. With current consumption being higher than future consumption, expected future interest rates are driven down according to the Euler equations. That is \hat{r}_1 and \hat{r}_2 are equal and both below 0 from the second period onwards as seen in Panels (e) and (f). The sum of the discounted expected future interest rates, Σ^{rn} , is thus also negative which pushes up asset prices \hat{z}_1 and \hat{z}_2 . Given that the shock hits the home country, higher expected future dividend drives up the price of the home asset \hat{z}_1 further, i.e. $\hat{z}_1 > \hat{z}_2$, as shown in Panels (g) and (h). As there are capital gains, the higher price of the foreign equity implies a higher current rate of return to the foreign equity which explains the first period increase in \hat{r}_{2t} in Panel (f). By the same token, \hat{r}_{1t} will be also higher due to a higher \hat{z}_1 . On top of this, a higher current dividend payment means \hat{r}_{1t} increase even further, i.e. $\hat{r}_{1t} > \hat{r}_{2t}$ or $\hat{r}_{xt} > 0$. With gross external positions across countries in the model, the rise in \hat{r}_{xt} implies a wealth transfer from the home country to the foreign country, i.e. a negative *VAL* effect. This effect is so big that it exceeds that of the initial trade surplus. So the net foreign asset position of the home country, \hat{w}_t , declines, as shown in Panel (b). Lastly, as shown in Panel (i), new-born's consumption shows a persistent increase, consistent with Eq.(3.6).

Now let us look at the dashed lines which describe the model responses after a positive foreign shock. It is obvious that the dynamics are symmetric to above dynamics in the sense that the responses of home (foreign) variables now is just the responses of foreign (home) variables, which is easy to understand because

the home country in this case steps into the foreign country's previous shoes.

Now let us turn to the asymmetric case where $\beta < \beta^*$. Figure 3.4 depicts the corresponding dynamics. The panels represent responses of variables in the same sequence as in Figure 3.3.

We find that, except for \hat{w}_t , the responses of variables does not change very much when the asymmetry is introduced. They are qualitatively the same but with quantitative differences reflecting the introduction of the country asymmetry. The symmetry between the responses to the two shocks (i.e. the solid and dashed lines) that exists in Figure 3.3 therefore breaks down.

With preservation of the qualitative properties of the variable responses, importantly, as previously, a positive home shock still raises the rate of return of the home asset more than that of foreign asset. In terms of our previous notation, this implies $0 < \zeta_{re1} < \zeta_{re2}$. The sum of discounted expected future interest rates on impact is depressed and responds to home and foreign shocks to the same degree, i.e. $0 > \zeta_{sre1} = \zeta_{sre2}$.

As for \hat{w}_t , a positive home shock still involves a positive trade balance effect and a negative valuation effect due to, as explained, the facts of $\hat{r}_{xt} > 0$ and a negative external gross position of the home equity by the home country. In addition, under the asymmetric parameterization $\beta < \beta^*$, the home country features a large negative net external position. The attendant rise in \hat{r}_2 after the shock also burdens the home country's interest payments to the foreign country. A negative terms-of-trade effect thus emerges, which reinforces the valuation effect in worsening the net external balance. \hat{w}_t (the solid line in Figure 3.4) declines further in comparison to that in Figure 3.3. Following similar logic, a positive foreign shock involves a negative trade balance effect and a positive valuation effect in the home country. And because the shock hits the foreign country endowment, \hat{r}_{2t} increases even more significantly than in the last case. Given the steady-state external net position in the home country, the total interest payments rises substantially, i.e. a negative terms-of-trade effect emerges again. \hat{w}_t (the dashed line in Figure 3.4) is lower than in the symmetric case (the dashed line in Figure 3.3).

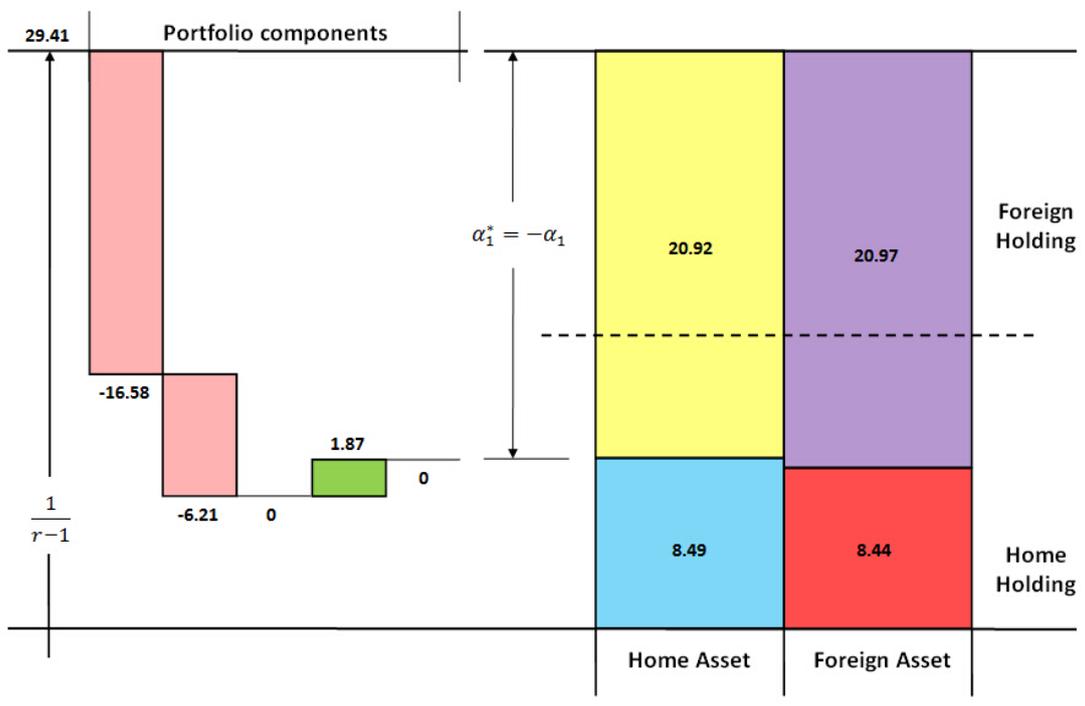


Figure 3.5: Steady-state international portfolios under global imbalances

3.8.3 International portfolios

In this section, we focus on the results associated with optimal portfolios given the assumed parameterization. Using the method we discussed in Section 3.4, $\bar{\alpha}$, the optimal gross holding of home equity by the home country, is computed to be -20.92 . Figure 3.5 plots its 5 components, as listed and discussed in Section 3.6, and the related net asset allocations in each country.

The left half of the figure shows the 5 components of $\bar{\alpha}$ with 5 bars. From left to right, they are the portfolio diversification term, the term representing hedging of the current net portfolio return, the term representing hedging of future net portfolio returns, the term representing hedging of new-born's consumption and the term representing hedging of consumption tilting. To represent their signs, negative components are accumulated from the top line (whose height denotes the value of the assets stock in steady state, i.e. \bar{z}) downwards while positive components are accumulated the other way around. The area of the bars corresponds to their sizes.

For the first and longest bar to the left, as explained, because home endowment income always moves in the same direction as the home asset return, as a bad hedge against the risk, the home asset will be shorted by home households, i.e. a negative diversification term $\bar{\alpha}[1] = -12.58 < 0$.

The second bar represents hedging of the current net portfolio return, $\bar{\alpha}[2]$. We know that $0 < \zeta_{re1} < \zeta_{re2}$ from the model dynamics described above, so according to our discussion in Section 3.6, because asset 2's return increases as endowments increase in either country, but it rises less in response to a home shock than to a foreign shock, there is a smaller increase in interest payments on the net external position at home. The home equity return is therefore positively correlated with the home net portfolio current returns. Home households therefore further short the home asset $\bar{\alpha}[2] = -6.21 < 0$.

We also see from last subsection that the sum of all discounted expected future asset returns declines as endowment increases in either country. And it responds to the two shocks in the same way ($\zeta_{sre1} = \zeta_{sre2}$), which implies that no risk arises associated with future net portfolio returns. The term representing hedging of future net portfolio returns, $\bar{\alpha}[3]$, and that of consumption tilting, $\bar{\alpha}[5]$, are thus both zero, which are represented by the fact that the third and

fifth bars collapse into two lines $\bar{\alpha} [3] = \bar{\alpha} [5] = 0$.

Lastly, as explained in Section 3.6, the term representing the hedging of new-born's consumption is always positive. This is verified by our result that $\bar{\alpha} [4] = 1.87 > 0$ (the fourth bar in the figure).

Let us compare the sizes of these components. Because self-hedging is linked to the risk associated with total *GDP* which is the most important source of risk, the diversification term is the largest component. Under global imbalances, the hedging of the current net portfolio return is linked to the risk associated with the income difference between *GNP* and *GDP*, which should be secondary comparing to total *GDP*, the hedging is thus less substantial. However, with the very large external net position here, the hedging of the net portfolio return is still considerable. Lastly, as an adjustment whose emergence is due to a small growth rate of population, the term representing the hedging of new-born's consumption is small.

Now let us turn to the pattern of net portfolio allocations associated with the level of $\bar{\alpha}$. The right half of figure shows this in more detail. There are two wide grey columns representing, from left to right, respectively home and foreign equity supplies. The height of both of these columns is steady state value of asset stocks which are equal to $\bar{z} = 1/(r - 1) = 29.41$. The values are divided by two solid lines so that there are four cells, i.e. upper left, bottom left, bottom right and upper right anti-clockwise. The starting axes for home and foreign holdings are respectively the bottom and the top lines. So the area of the upper left cell represents the foreign holding of the home asset which equals $\alpha_1^* = -\alpha_1 = 20.92$. (Note that foreign buying is home selling of the home equity). The home (net) holding of home asset is thus the sum of its endowment and gross external holding, i.e. $\bar{z} + \alpha_1 = 8.49$, which explains the area of the bottom left cell. The area of the bottom right cell, i.e. the home holding of the foreign asset, is given by $\alpha_2 = \bar{w} - \alpha_1 = 8.44$. Lastly, the area of the upper right cell is the foreign holding of the foreign asset, $\bar{z} - \alpha_2 = 20.97$.

To compare the allocation to that in a symmetric model, we draw a dashed line in the middle of the two columns. It divides the two columns into four cells of the same area which corresponds to the situation of fully diversified portfolio allocation when the two countries are identical. Inspecting the figure, we find that the current model deviates from the symmetric case (benchmark) in two

ways. First, the steady-state net foreign asset at home is negative. The two solid lines move from the benchmark down to create an area which represents the home country net foreign asset position, i.e. the area above the solid lines but below the dashed line. Second, the net portfolio allocations under global imbalances exhibit asset home bias, i.e. the home holding of the home asset is larger than the home holding of the foreign asset even though the two assets are supplied equally in the world portfolio. The left solid line is higher than the right one. Further experiments show that, as the asymmetry between countries becomes more severe, the two solid lines move downwards (i.e. the global *NFA* imbalance is exacerbated) and the gap between the two widens (i.e. asset home bias deepens).

3.8.4 Risk-sharing and external adjustments (with and without country portfolios)

In Figure 3.6 we plot the dynamics of the consumption differential, \hat{c}^D , and that of the components of net foreign assets, \hat{w} , after a home shock in the cases with (solid line) and without (dashed line) holdings of international portfolios. By ‘without portfolio’, we mean that the condition of $\alpha = 0$ is imposed. We come up with this hypothetical case and compare it to the case with portfolios in order to highlight the role of the presence of portfolio choices in giving rise to valuation effects in a model. Panel (*j*) shows that endowment increases by 1%. Panels (*a*) to (*h*) depict responses of the consumption differential, home and foreign consumption, home *NFA*, the trade balance effect, the terms of trade effect, the valuation effect and interest income (which is negative in the home country) on *NFA*.

Let us first focus on Panels (*a*) to (*d*). In the case without portfolio holdings, most of the effect of an increase in home endowment on consumption will be on home consumption because the shock affects consumption mainly through the increased *GDP* (see the high dashed line in Panel (*b*) compared to the low dashed line in Panel (*c*)). The gap between home and foreign consumption is thus very large at around 0.73 as shown in Panel (*a*). However, if optimal portfolios are in place, the rise in the home endowment affects consumption not only through affecting *GDP*, but also through affecting asset returns and therefore portfolio

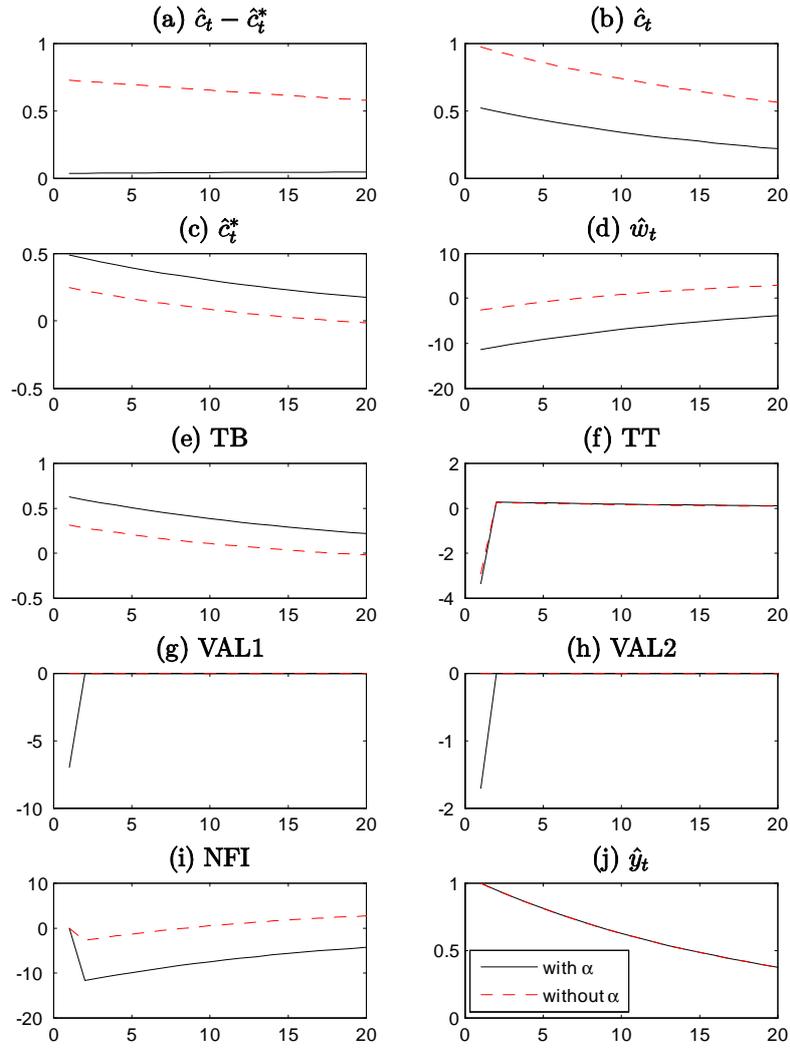


Figure 3.6: IRFs to home shock when with (solid line) and without (dashed line) portfolios

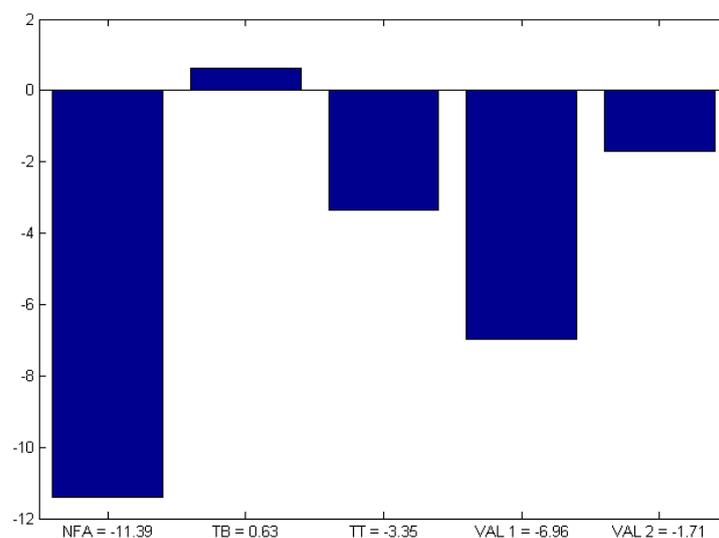


Figure 3.7: Decomposition of the change in NFA in response to 1% increase in \hat{y}

returns. As previously described, the excess return on home equity is driven up by the home shock. The fact that home country holds a negative external gross position in home equity while the foreign country holds a positive position implies a wealth transfer from the home to the foreign country in the form of a portfolio excess return. Net external wealth, \hat{w} , is lower than in the case without portfolios (see Panel (d)). Home consumption is depressed (to the low solid line in Panel (b) at the level of around 0.5) at the same time as foreign consumption is elevated (to the high solid line in Panel (c) also at the level of around 0.5). The consumption differential across countries is thus narrowed down substantially. As shown by the solid line in Panel (a), with optimal portfolios, \hat{c}^D after the shock is near 0 indicating a significantly improved level of risk-sharing across countries.

We discussed the roles of trade and valuation channels in affecting a country's external wealth and consumption in Section 3.7. To better see the dynamics of each component of \hat{w} under the current parameterization, we plot them in the remaining panels of Figure 3.6. Panel (e) shows that the positive home shock raises the trade balance, which is because when GDP increases, consumption increases

but less than one for one. Without portfolio choices, this is represented by the dashed line in this panel. If countries hold optimal portfolios, home consumption is lower due to the negative indirect valuation effect through consumption, $VAL^c < 0$, and therefore the trade balance can be higher. This is represented by the solid line above the dashed line in Panel (e). Now for Panel (f), again look at the dashed line for the case of without portfolio choices first. Because \hat{r}_2 is driven up immediately after the shock, so the interest payments on net foreign liabilities, i.e. the TT effect, is lower in the first period. In subsequent periods, \hat{r}_{t+i} is negative but close to 0. So the interest payments on the negative position of net foreign assets become positive but also close to 0. Looking at the solid line in the panel, the inclusion of portfolio choice, $\bar{\alpha}$, has no significant impact on the TT effect because, as explained in Section 3.7, the TT effect is mainly linked to the net instead of the gross portfolio position. Panels (g) and (h) plot respectively the valuation effects on the diversification term of $\bar{\alpha}$ and that on imbalance terms, i.e. VAL [1] and VAL [2] for the cases with and without portfolio choices. When home endowment increases, a substantial rise in the excess return \hat{r}_x leads to a large fall in both of these effects on impact. Because expected future excess returns are 0 in this model, we do not observe any valuation effects in subsequent periods. Since the presence of the VAL effect is linked to gross portfolio positions, when $\bar{\alpha} = 0$, $VAL = 0$.

To see clearer the magnitude of each effect, we depict the sizes of all effects in Figure 3.7. A one percent increase in the home GDP leads to 0.63 percent increase in the trade balance, i.e. the second bar from the left hand side which shows the TB effect. In the case without the country asymmetry and portfolio choices, this constitutes all of the adjustment in net foreign asset (in the period of the shock), so \hat{w} improves. However, with the presence of the country asymmetry, a negative external deficit implies higher interest payments in the home country after the shock. So \hat{w} decreases through the negative terms-of-trade effect, i.e. the third bar which shows the TT effect in the figure, which is 3.35%. Moreover, if portfolios are optimally chosen, then \hat{w} decreases further as the valuation effect, VAL , contributes another 8.67% adjustments downwards, i.e. the fourth and fifth bar in the figure. While the valuation effects always work in the opposite way to the trade balance effects, the direction of the terms-of-trade effect depends on whether the country is a debtor or a creditor in the steady state. With the

home country is a debtor, the assumed positive supply shock in the home country implies that both TT and VAL are negative. And because both the steady state net foreign asset position and gross external position are very large, these two effects are also very large which totally offset the effect of initial TB effect and result in the substantial deterioration of the net foreign asset position in the home country.

3.9 Conclusion

While the traditional single-bond model of global imbalances falls short of including portfolio choices, the recent portfolio approach to external adjustments only considers symmetric situations. In this chapter, we construct an asymmetric model of a two-country economy to study optimal international portfolio choices and external adjustments under global current account imbalances. Country asymmetries create new channels through which consumption fluctuates and thus new risks to be hedged against by holding international portfolios. Specifically, because the steady-state net external asset positions are non-zero in this case, countries are exposed to the risks associated with international interest payments in addition to GDP shocks. This gives rise to the hedging of the net portfolio return in the optimal (gross) portfolio holdings in addition to otherwise only the standard diversification term. Numerical experiments suggest that the portfolio allocations implied always and in all countries exhibit equity home bias. What is important is that, because the model features both global imbalances and international portfolio choices, the resulting processes of external adjustment of the country nest all possible trade and financial channels, i.e. the terms-of-trade effects on the net portfolio and the valuation effects on gross portfolios emerge simultaneously in addition to the traditional trade balance effects.

The model is fully optimizing and micro-founded in accounting for both net and gross portfolio positions. As a bridge between the literature on global imbalances and that on portfolio choices in international macroeconomics, the model, obviously, allows for meaningful extensions along these two dimensions.

For the former, a different story of global imbalances than the one adopted here can be told. As alluded in the introduction of chapter 2, one common feature of these works is stressing the role of specific asymmetries or frictions in

creating a low autarky interest rate in the developing country while a relatively high autarky interest rate in developed country as the key driver of global imbalances (Gourinchas and Rey, 2013). So even though only the asymmetry in the degree of patience is modelled here, the framework developed in this chapter is representative and can in fact be tailored for potential use with alternative explanation of non-trivial net portfolios along this line. In Chapter 4, we will provide such an example where an asymmetry associated with wealth division across countries is used. It proves that the key implications of global imbalances on gross portfolios we found in this chapter carry over and they are important for the analysis in the next chapter.

For the latter, as mentioned in the introduction of this chapter, the existing literature on gross portfolios choices mostly focuses on explaining (symmetric) asset home bias with additional hedging motives among which the most important two are the hedging of labour income and exchange-rate risks. In the next chapter, we will also take into account these hedging motives in an extended variant of the current asymmetric model.

Appendix

3.A The models with and without global imbalances

Log-linearized model equations for the current asymmetric and comparison symmetric models are given in this section.

Based on the assumptions described in Section 3.2, the asymmetric model consists of the following 11 equations

$$\hat{r}_{1t} = \left(1 - \frac{1}{r}\right) \hat{y}_t + \frac{1}{r} \hat{z}_{1t} - \hat{z}_{1t-1} \quad (\text{A.1})$$

$$\hat{r}_{2t} = \left(1 - \frac{1}{r}\right) \hat{y}_t^* + \frac{1}{r} \hat{z}_{2t} - \hat{z}_{2t-1} \quad (\text{A.2})$$

$$(1+n) \phi_{wy} \hat{w}_{t+1} = r \phi_{wy} \hat{w}_t + r \phi_{wy} \hat{r}_{2t} + r \phi_{\alpha y} \hat{r}_{xt} + \hat{y}_t - \phi_{cy} \hat{c}_t \quad (\text{A.3})$$

$$E_t \hat{c}_{t+1} = \tau \hat{c}_t + (1-\tau) E_t \hat{c}_{t+1}^n + \tau E_t \hat{r}_{1t+1} \quad (\text{A.4})$$

$$E_t \hat{c}_{t+1} = \tau \hat{c}_t + (1-\tau) E_t \hat{c}_{t+1}^n + \tau E_t \hat{r}_{2t+1} \quad (\text{A.5})$$

$$E_t \hat{c}_{t+1}^* = \tau^* \hat{c}_t^* + (1-\tau^*) E_t \hat{c}_{t+1}^{n*} + \tau^* E_t \hat{r}_{2t+1} \quad (\text{A.6})$$

$$\phi_{cy} \hat{c}_t + \phi_{cy}^* \hat{c}_t^* = \hat{y}_t + \hat{y}_t^* \quad (\text{A.7})$$

$$\hat{y}_t = \mu \hat{y}_{t-1} + \varepsilon_t \quad (\text{A.8})$$

$$\hat{y}_t^* = \mu \hat{y}_{t-1}^* + \varepsilon_t^* \quad (\text{A.9})$$

$$\hat{c}_t^n = \frac{r-1}{r-\mu} \hat{y}_t \quad (\text{A.10})$$

$$\hat{c}_t^{n*} = \frac{r-1}{r-\mu} \hat{y}_t^* \quad (\text{A.11})$$

Approximations are conducted around the per capita steady states. The approximation method is standard. So here we only explain for above equations where they come from. Equations (A.1) and (A.2) come from (2.6) and (2.7) which define the rates of return for the two assets. Eq.(A.3) is the per capita intertemporal budget constraint for the home country, i.e. Eq.(3.36). Eq.(A.4 – 6) are aggregated per capita Euler equations, i.e. Eq.(3.35) and the

like. There should have been four such equations in total yet one of them is redundant. Eq.(A.7) is the resource constraint for the whole world, i.e. Eq.(3.37). Combining it with (A.3) implies the budget constraint of the foreign country. Equations (A.8 – 9) are the processes of shocks in the model, Eq.(2.8 – 9). The last two equations (A.10 – 11) approximate the new-born's consumption around the steady state, i.e. Eq.(3.5) and its foreign analogue to describe c_t^n and \hat{c}_t^{n*} .

To see the implication of global imbalances in the current model, we recast here another model of a parallel world where there is no asymmetry. It is an easy task. We get rid of the global imbalance by assuming $\beta = \beta^*$. And to be consistent with the past literature on two symmetric countries, we also take away the *OLG* structure. It is obvious that the steady states are $\bar{c} = \bar{y}$, $\bar{c}^* = \bar{y}^*$, $\bar{w} = 0$ and $\bar{r} = \frac{1}{\beta}$. Approximating the model around the steady states yields the following system

$$\hat{r}_{1t} = \left(1 - \frac{1}{r}\right) \hat{y}_t + \frac{1}{r} \hat{z}_{1t} - \hat{z}_{1t-1} \quad (\text{A.1}')$$

$$\hat{r}_{2t} = \left(1 - \frac{1}{r}\right) \hat{y}_t^* + \frac{1}{r} \hat{z}_{2t} - \hat{z}_{2t-1} \quad (\text{A.2}')$$

$$\hat{w}_{t+1} = r\hat{w}_t + r\phi_{\alpha y} \hat{r}_{xt} + \hat{y}_t - \hat{c}_t \quad (\text{A.3}')$$

$$E_t \hat{c}_{t+1} = \hat{c}_t + E_t \hat{r}_{1t+1} \quad (\text{A.4}')$$

$$E_t \hat{c}_{t+1} = \hat{c}_t + E_t \hat{r}_{2t+1} \quad (\text{A.5}')$$

$$E_t \hat{c}_{t+1}^* = \hat{c}_t^* + E_t \hat{r}_{2t+1} \quad (\text{A.6}')$$

$$\hat{c}_t + \hat{c}_t^* = \hat{y}_t + \hat{y}_t^* \quad (\text{A.7}')$$

$$\hat{y}_t = \mu \hat{y}_{t-1} + \varepsilon_t \quad (\text{A.8}')$$

$$\hat{y}_t^* = \mu \hat{y}_{t-1}^* + \varepsilon_t^* \quad (\text{A.9}')$$

As in the asymmetric model, Eq.(A.1' – 2') define the rates of return on assets while (A.3') is the home budget constraint. Eq.(A.4' – 6') are Euler equations. (A.7') is the resource constraint and (A.8' – 9') are shocks to endowments. Note that abstracting from the *OLG* structure, the model is of representative agent form. So there is no difference between the individual and per capita variables.

The difference between the two models is compared in the main text. In addition, remember that all per capita variables with a hat are defined as proportional

deviations from their steady states, i.e. $\hat{x} = \frac{x-\bar{x}}{\bar{x}}$. However, for symmetric models, $\bar{w} = 0$ so \bar{w} cannot be used to normalize the deviation of w into percentage changes. Following the past literature, in Eq.(A.3'), \hat{w} is defined as the change in w relative to \bar{y} , i.e. $\hat{w} \equiv \frac{w}{\bar{y}}$.

3.B The MUC (General steps)

We solve the model by the method of undetermined coefficients (MUC). We do not solve the model all in once but rather in steps, i.e. first solve for the behaviour of expected future variables and then those of current variables to obtain simpler analytical solutions to (both current and especially expected future) variables (in terms of variable's elasticities to shocks, i.e. χ s below) which are very useful for latter comparisons of the magnitudes of variable responses to home and foreign shocks. This section gives the general description of these steps.

According to the Euler equations for home and foreign countries (A.4 – 6), expected asset returns \hat{r}_{t+1} are equalized across assets. This means that $\hat{r}_{xt+1} = 0$. If we look at the budget constraints (A.3) for time t and $t+i$, they are different in the fact that for the latter, \hat{r}_{xt} disappears. The solution process can thus be separated into the following three steps. That is:

First, solve the system of expected future (or time $t+1$) variables by MUC.

Second, use the \hat{r}_{t+1} obtained, asset pricing equation Eq.(A.2) and an assumed solution to current variables to obtain \hat{r}_{2t} with coefficients to be determined.

Third, substitute this \hat{r}_{2t} into the system of current (or time t) variables and solve it with MUC.

Note we have five state variables $[\hat{w}_t, \hat{z}_{1t-1}, \hat{z}_{2t-1}, \hat{y}_{1t-1}, \hat{y}_{2t-1}]'$ and six free variables $[\hat{c}_t, \hat{c}_t^*, \hat{r}_{1t}, \hat{r}_{2t}, \hat{c}_t^n, \hat{c}_t^{n*}]'$.

3.B.1 The expected future variables (consumption and asset return)

Let us first focus on the sub-system consisting of Eq.(A3) and (A5 – 7) to solve for $[\hat{w}_{t+2}, \hat{c}_{t+1}, \hat{c}_{t+1}^*, \hat{r}_{2t+1}]'$. Lead one period forward to obtain

$$\hat{w}_{t+2} = J_1 \hat{w}_{t+1} + J_2 \hat{r}_{2t+1} + J_4 \hat{y}_{t+1} + J_5 \hat{c}_{t+1} \quad (\text{B.1})$$

$$\hat{c}_{t+2} = K_1 \hat{c}_{t+1} + K_2 \hat{y}_{t+2} + K_3 \hat{r}_{t+2} \quad (\text{B.2})$$

$$\hat{c}_{t+2}^* = K_1^* \hat{c}_{t+1}^* + K_2^* \hat{y}_{t+2}^* + K_3^* \hat{r}_{t+2}^* \quad (\text{B.3})$$

$$\bar{\phi}_{cy} \hat{c}_{t+1} + \bar{\phi}_{cy}^* \hat{c}_{t+1}^* = \hat{y}_{t+1} + \hat{y}_{t+1}^* \quad (\text{B.4})$$

This is a system of four expected future variables $[\hat{w}_{t+2}, \hat{c}_{t+1}, \hat{c}_{t+1}^*, \hat{r}_{t+1}]'$ driven by only net wealth and two shocks $[\hat{w}_{t+1}, \hat{y}_{t+1}, \hat{y}_{t+1}^*]'$. (Note that \hat{w}_{t+1} is determined at time t i.e. a predetermined variable and by (A.8 – 9) that \hat{y}_{t+2} and \hat{y}_{t+2}^* can be replaced by $\mu \hat{y}_{t+1}$ and $\mu \hat{y}_{t+1}^*$.) We have four unknowns and four equations. Suppose that the solution of the system is of the following form with all χ s being the coefficients to be determined.

$$\hat{w}_{t+2} = \chi_{wwp} \hat{w}_{t+1} + \chi_{wy1p} \hat{y}_{t+1} + \chi_{wy2p} \hat{y}_{t+1}^* \quad (\text{B.5})$$

$$\hat{c}_{t+1} = \chi_{cwp} \hat{w}_{t+1} + \chi_{cy1p} \hat{y}_{t+1} + \chi_{cy2p} \hat{y}_{t+1}^* \quad (\text{B.6})$$

$$\hat{c}_{t+1}^* = \chi_{cwp}^* \hat{w}_{t+1} + \chi_{cy1p}^* \hat{y}_{t+1} + \chi_{cy2p}^* \hat{y}_{t+1}^* \quad (\text{B.7})$$

$$\hat{r}_{t+1} = \chi_{rwp} \hat{w}_{t+1} + \chi_{ry1p} \hat{y}_{t+1} + \chi_{ry2p} \hat{y}_{t+1}^* \quad (\text{B.8})$$

Following the process of MUC, we can obtain solutions for $[\hat{w}_{t+2}, \hat{c}_{t+1}, \hat{c}_{t+1}^*, \hat{r}_{t+1}]'$ by pinning down the 12 χ s above. They will be referred as elasticities/responses of future variables.

3.B.2 The current asset returns

Now we can use Eq.(A.1 – 2) to solve for the behaviour of current asset returns. In this description the focus will be placed on asset 2's price and return rate, i.e. \hat{z}_{2t} and \hat{r}_{2t} . The derivations and results for asset 1 are similar.

By (A.2), we have

$$\hat{z}_{2t} = \frac{(r-1)\mu}{(r-\mu)} \hat{y}_t^* - \left(\hat{r}_{t+1} + \frac{1}{r} \hat{r}_{t+2} + \frac{1}{r^2} \hat{r}_{t+2} + \dots \right) \quad (\text{B.9})$$

Define

$$\Sigma_{t+1}^r \equiv \left(\hat{r}_{t+1} + \frac{1}{r} \hat{r}_{t+2} + \frac{1}{r^2} \hat{r}_{t+2} + \dots \right) \quad (\text{B.10})$$

as the discounted sum of future asset returns at time t (with the discount factor r). Making use of the solutions for \hat{w}_{t+i} and \hat{r}_{t+i} as in the last subsection, i.e.

(B.5) and (B.8), one can show that Σ_{t+1}^r is equal to (Appendix C.2)

$$\begin{aligned} \Sigma_{t+1}^r &= \underbrace{\frac{r\chi_{rwp}}{r - \chi_{wvp}} \hat{w}_{t+1}}_{\chi_{swp}} \\ &+ \underbrace{\left[\frac{r\chi_{ry1p}}{r - \mu} + \frac{\chi_{rwp}\chi_{wy1p}}{r - \chi_{wvp}} \frac{r}{r - \mu} \right]}_{\equiv \chi_{sy1p}} \hat{y}_{t+1} \\ &+ \underbrace{\left[\frac{r\chi_{ry2p}}{r - \mu} + \frac{\chi_{rwp}\chi_{wy2p}}{r - \chi_{wvp}} \frac{r}{r - \mu} \right]}_{\equiv \chi_{sy2p}} \hat{y}_{t+1}^* \end{aligned} \quad (B.11)$$

where we define future elasticities of Σ_{t+1}^r , i.e. χ_{swp} , χ_{sy1p} and χ_{sy2p} as functions of the future elasticities of \hat{w}_{t+2} and \hat{r}_{t+1} found in the last subsection. So they are all known.

Replace \hat{w}_{t+1} with a proposed solution as in the next subsection, i.e. (B.24) and use $\hat{y}_{t+1} = \mu \hat{y}_t$ to yield

$$\begin{aligned} \Sigma_{t+1}^r &= \underbrace{\chi_{swp}\chi_{wx}}_{\equiv \chi_{sx}} \hat{\xi}_t + \underbrace{\chi_{swp}\chi_{ww}}_{\equiv \chi_{sw}} \hat{w}_t + \underbrace{\chi_{swp}\chi_{wz2}}_{\equiv \chi_{sz2}} \hat{z}_{2t-1} \\ &+ \underbrace{[\chi_{swp}\chi_{we1} + \mu\chi_{sy1p}]}_{\equiv \chi_{se1}} \hat{y}_t + \underbrace{[\chi_{swp}\chi_{we2} + \mu\chi_{sy2p}]}_{\equiv \chi_{se2}} \hat{y}_t^* \end{aligned} \quad (B.12)$$

where we define another five coefficients, i.e. $[\chi_{sx}, \chi_{sw}, \chi_{sz2}, \chi_{se1}, \chi_{se2}]'$ as functions of $[\chi_{wx}, \chi_{ww}, \chi_{wz2}, \chi_{we1}, \chi_{we2}]'$ in addition to the future elasticities. Because the χ s in the former vector are to be determined in the next subsection, the χ s in the latter vector are also coefficients to be determined.

Substituting (B.12) into (B.9) we get

$$\hat{z}_{2t} = \left[\frac{(r-1)\mu}{(r-\mu)} - \chi_{se2} \right] \hat{y}_t^* - \chi_{sx} \hat{\xi}_t - \chi_{sw} \hat{w}_t - \chi_{sz2} \hat{z}_{2t-1} \quad (B.13)$$

And by Eq.(A.2)

$$\hat{r}_{2t} = \chi_{rx} \hat{\xi}_t + \chi_{rw} \hat{w}_t + \chi_{rz2} \hat{z}_{2t-1} + \chi_{re1} \hat{y}_t + \chi_{re2} \hat{y}_t^* \quad (B.14)$$

where

$$\chi_{rx} = -\frac{1}{r} \chi_{sx} = -\frac{\chi_{rwp}\chi_{wx}}{r - \chi_{wvp}} \quad (B.15)$$

$$\chi_{rw} = -\frac{1}{r}\chi_{sw} = -\frac{\chi_{rwp}\chi_{ww}}{r - \chi_{wvp}} \quad (\text{B.16})$$

$$\chi_{rz2} = -\frac{1}{r}\chi_{sz2} - 1 = -\left[1 + \frac{\chi_{rwp}\chi_{wz2}}{r - \chi_{wvp}}\right] \quad (\text{B.17})$$

$$\begin{aligned} \chi_{re1} &= -\frac{1}{r}\chi_{se1} \\ &= -\frac{1}{r}[\chi_{swp}\chi_{we1} + \mu\chi_{sy1p}] \end{aligned} \quad (\text{B.18})$$

$$\begin{aligned} \chi_{re2} &= \frac{(r-1)}{(r-\mu)} - \frac{1}{r}\chi_{se2} \\ &= \frac{(r-1)}{(r-\mu)} - \frac{1}{r}[\chi_{swp}\chi_{we2} + \mu\chi_{sy2p}] \end{aligned} \quad (\text{B.19})$$

Note that the above expressions are not yet known with $[\chi_{wx}, \chi_{ww}, \chi_{wz2}, \chi_{we1}, \chi_{we2}]'$ to be determined in Eq.(B.24) in the next subsection.

\hat{r}_{1t} can be obtained in a similar way to (B.14). Subtracting \hat{r}_{2t} from \hat{r}_{1t} yields

$$\hat{r}_{xt} = \frac{(r-1)}{(r-\mu)}(\hat{y}_t - \hat{y}_t^*) - (\hat{z}_{1t-1} - \hat{z}_{2t-1}) \quad (\text{B.20})$$

3.B.3 The current variables (consumption and asset returns)

Substitute \hat{r}_{2t} into Eq.(A.3) and defining J' properly to yield

$$\hat{w}_{t+1} = J'_1\hat{w}_t + J'_3\hat{\xi}_t + J'_4\hat{y}_t + J'_5\hat{c}_t + J'_6\hat{y}_t^* + J'_7\hat{z}_{2t-1} \quad (\text{B.21})$$

This can be combined with Eq.(A.5) and (A.6)

$$\hat{c}_{t+1} = K_1\hat{c}_t + K_2\hat{y}_{t+1} + K_3\hat{r}_{t+1} \quad (\text{B.22})$$

$$\hat{c}_{t+1}^* = K_1^*\hat{c}_t^* + K_2^*\hat{y}_{t+1}^* + K_3^*\hat{r}_{t+1} \quad (\text{B.23})$$

to form a system for current variables. Observing the structure of the system, we propose the solutions

$$\hat{w}_{t+1} = \chi_{ww}\hat{w}_t + \chi_{wz2}\hat{z}_{2t-1} + \chi_{wx}\hat{\xi}_t + \chi_{we1}\hat{y}_t + \chi_{we2}\hat{y}_t^* \quad (\text{B.24})$$

$$\hat{c}_t = \chi_{cw}\hat{w}_t + \chi_{cz2}\hat{z}_{2t-1} + \chi_{cx}\hat{\xi}_t + \chi_{ce1}\hat{y}_t + \chi_{ce2}\hat{y}_t^* \quad (\text{B.25})$$

$$\hat{c}_t^* = \chi_{cw}^*\hat{w}_t + \chi_{cz2}^*\hat{z}_{2t-1} + \chi_{cx}^*\hat{\xi}_t + \chi_{ce1}\hat{y}_t + \chi_{ce2}\hat{y}_t^* \quad (\text{B.26})$$

with all the χ s as coefficients to be determined. Again, these 15 coefficients are solved by MUC.

With $[\chi_{wx}, \chi_{ww}, \chi_{wz2}, \chi_{we1}, \chi_{we2}]'$ being solved, they can be substituted back into the undetermined expressions in the previous subsection.

Now the first-order behaviour of all relevant current and expected future variables of $[\hat{w}, \hat{c}, \hat{c}^*, \hat{r}]'$ is solved. The other variables can be easily obtained from the solution of $[\hat{w}, \hat{c}, \hat{c}^*, \hat{r}]'$ and the other equation by substitution.

By Eq.(B.25) and (B.26), \hat{c}_t^D can be obtained as

$$\hat{c}_t^D = \chi_{cx}^D\hat{\xi}_t + \chi_{cy1}^D\hat{y}_t + \chi_{cy2}^D\hat{y}_t^* \quad (\text{B.27})$$

where $\chi^D \equiv \chi - \chi^*$ and the conditions $\hat{w}_t = 0$ and $\hat{z}_{2t-1} = 0$ are imposed.

3.C The MUC (Other details)

This section consists of another three corresponding subsections to those in the last section with each elaborating further details on the MUC when coding them in Mathematica.

3.C.1 The system of (expected) future variables

The corresponding J , K and K^* matrix are defined as follows

$$J_1 = J_2 = \frac{r}{(1+n)}, J_3 = \frac{1}{(1+n)\phi_{wy}} \quad (\text{C.1})$$

$$J_4 = \frac{1}{(1+n)\phi_{wy}}, J_5 = -\frac{\phi_{cy}}{(1+n)\phi_{wy}} \quad (\text{C.2})$$

$$K_1 = K_3 = \frac{r\beta}{(1+n)}, K_2 = \frac{(1+n) - r\beta}{(1+n)} \frac{(r-1)}{(r-\mu)} \quad (\text{C.3})$$

$$K_1^* = K_3^* = \frac{r\beta^*}{(1+n)}, K_2^* = \frac{(1+n) - r\beta^*}{(1+n)} \frac{(r-1)}{(r-\mu)} \quad (\text{C.4})$$

The solution of the system is assumed to be of the form of

$$\hat{w}_{t+2} = \chi_{wwp}\hat{w}_{t+1} + \chi_{wy1p}\hat{y}_{t+1} + \chi_{wy2p}\hat{y}_{t+1}^* \quad (\text{C.5})$$

$$\hat{c}_{t+1} = \chi_{cwp}\hat{w}_{t+1} + \chi_{cy1p}\hat{y}_{t+1} + \chi_{cy2p}\hat{y}_{t+1}^* \quad (C.6)$$

$$\hat{c}_{t+1}^* = \chi_{cwp}^*\hat{w}_{t+1} + \chi_{cy1p}^*\hat{y}_{t+1} + \chi_{cy2p}^*\hat{y}_{t+1}^* \quad (C.7)$$

$$\hat{r}_{t+1} = \chi_{rwp}\hat{w}_{t+1} + \chi_{ry1p}\hat{y}_{t+1} + \chi_{ry2p}\hat{y}_{t+1}^* \quad (C.8)$$

with 12 χ s to be determined. 12 equations are needed.

Substitute (B.6) and (B.8) into Eq.(B.1) to obtain the first three equations

$$\chi_{wvp} = J_1 + J_2\chi_{rwp} + J_5\chi_{cwp} \quad (C.9)$$

$$\chi_{wy1p} = J_4 + J_2\chi_{ry1p} + J_5\chi_{cy1p} \quad (C.10)$$

$$\chi_{wy2p} = J_2\chi_{ry2p} + J_5\chi_{cy2p} \quad (C.11)$$

Substitute (B.6) and (B.8) into Eq.(B.2) to obtain another three equations

$$\chi_{cwp}\chi_{wvp} = K_1\chi_{cwp} + K_3\chi_{rwp}\chi_{wvp} \quad (C.12)$$

$$\chi_{cwp}\chi_{wy1p} + \mu\chi_{cy1p} = K_1\chi_{cy1p} + K_2\mu + K_3(\chi_{rwp}\chi_{wy1p} + \mu\chi_{ry1p}) \quad (C.13)$$

$$\chi_{cwp}\chi_{wy2p} + \mu\chi_{cy2p} = K_1\chi_{cy2p} + K_3(\chi_{rwp}\chi_{wy2p} + \mu\chi_{ry2p}) \quad (C.14)$$

Substitute (B.7) and (B.8) into Eq.(B.3) to obtain the third three equations

$$\chi_{cwp}^*\chi_{wvp} = K_1^*\chi_{cwp}^* + K_3^*\chi_{rwp}\chi_{wvp} \quad (C.15)$$

$$\chi_{cwp}^*\chi_{wy1p} + \mu\chi_{cy1p}^* = K_1^*\chi_{cy1p}^* + K_3^*(\chi_{rwp}\chi_{wy1p} + \mu\chi_{ry1p}) \quad (C.16)$$

$$\chi_{cwp}^*\chi_{wy2p} + \mu\chi_{cy2p}^* = K_1^*\chi_{cy2p}^* + K_2^*\mu + K_3^*(\chi_{rwp}\chi_{wy2p} + \mu\chi_{ry2p}) \quad (C.17)$$

Substitute (B.6) and (B.7) into Eq.(B.4) to obtain the last three equations

$$\phi_{cy}\chi_{cwp} + \phi_{cy}^*\chi_{cwp}^* = 0 \quad (C.18)$$

$$\phi_{cy}\chi_{cy1p} + \phi_{cy}^*\chi_{cy1p}^* = 1 \quad (C.19)$$

$$\phi_{cy}\chi_{cy2p} + \phi_{cy}^*\chi_{cy2p}^* = 1 \quad (C.20)$$

3.C.2 Solutions to Σ_{t+1}^r and Σ_{t+1}^{rn}

In this subsection, we derive Eq.(B.11) for Σ_{t+1}^r (defined in (B.10)) and Eq.(5.16) for Σ_{t+1}^{rn} (defined in (5.6)).

By the solution of \hat{r}_{t+1} , i.e. Eq.(B.8)

$$\hat{r}_{t+1} = \chi_{rwp}\hat{w}_{t+1} + \chi_{ry1p}\hat{y}_{t+1} + \chi_{ry2p}\hat{y}_{t+1}^*$$

and Eq.(A.9), one can obtain

$$\begin{aligned} \hat{r}_{t+2} &= \chi_{rwp}\hat{w}_{t+2} + \chi_{ry1p}\hat{y}_{t+2} + \chi_{ry2p}\hat{y}_{t+2}^* & (C.21) \\ &= \chi_{rwp}(\chi_{wzp}\hat{w}_{t+1} + \chi_{wy1p}\hat{y}_{t+1} + \chi_{wy2p}\hat{y}_{t+1}^*) + \chi_{ry1p}\hat{y}_{t+2} + \chi_{ry2p}\hat{y}_{t+2}^* \\ &= \chi_{rwp}\chi_{wzp}\hat{w}_{t+1} + (\chi_{rwp}\chi_{wy1p} + \mu\chi_{ry1p})\hat{y}_{t+1} \\ &\quad + (\chi_{rwp}\chi_{wy2p} + \mu\chi_{ry2p})\hat{y}_{t+1}^* \end{aligned}$$

$$\begin{aligned} \hat{r}_{t+3} &= \chi_{rwp}\hat{w}_{t+3} + \chi_{ry1p}\hat{y}_{t+3} + \chi_{ry2p}\hat{y}_{t+3}^* & (C.22) \\ &= \chi_{rwp}(\chi_{rwp}\hat{w}_{t+2} + \chi_{ry1p}\hat{y}_{t+2} + \chi_{ry2p}\hat{y}_{t+2}^*) + \chi_{ry1p}\hat{y}_{t+3} + \chi_{ry2p}\hat{y}_{t+3}^* \\ &= \chi_{rwp}(\chi_{rwp}(\chi_{wzp}\hat{w}_{t+1} + \chi_{wy1p}\hat{y}_{t+1} + \chi_{wy2p}\hat{y}_{t+1}^*) + \chi_{ry1p}\hat{y}_{t+2} + \chi_{ry2p}\hat{y}_{t+2}^*) \\ &\quad + \chi_{ry1p}\hat{y}_{t+3} + \chi_{ry2p}\hat{y}_{t+3}^* \\ &= \chi_{rwp}\chi_{wzp}^2\hat{w}_{t+1} \\ &\quad + (\chi_{wzp}\chi_{rwp}\chi_{wy1p} + \mu\chi_{rwp}\chi_{wy1p} + \mu^2\chi_{ry1p})\hat{y}_t \\ &\quad + (\chi_{wzp}\chi_{rwp}\chi_{wy2p} + \mu\chi_{rwp}\chi_{wy2p} + \mu^2\chi_{ry2p})\hat{y}_t^* \\ &\quad \dots \end{aligned}$$

by which the regularity in \hat{r}_{t+i} is revealed.

To ease notation, define

$$\Sigma^{\chi_{wzp}} \equiv 1 + \frac{\chi_{wzp}}{r} + \frac{\chi_{wzp}^2}{r^2} + \dots = \frac{r}{r - \chi_{wzp}} \quad (C.23)$$

$$\Sigma^\mu \equiv 1 + \frac{\mu}{r} + \frac{\mu^2}{r^2} + \dots = \frac{r}{r - \mu} \quad (C.24)$$

For proof of Eq.(B.11), by \hat{r}_{t+i} obtained above and (B.10), Σ_{t+1}^r equals

$$\begin{aligned}
 & \chi_{rwp} \Sigma^{\chi_{wvp}} \hat{w}_t & (C.25) \\
 & + [\chi_{ry1p} \Sigma^\mu + \frac{1}{r} \chi_{rwp} \chi_{wy1p} \Sigma^\mu + \frac{1}{r^2} \chi_{rwp} \chi_{wvp} \chi_{wy1p} \Sigma^\mu \\
 & + \frac{1}{r^3} \chi_{rwp} \chi_{wvp}^2 \chi_{wy1p} \Sigma^\mu + \dots] \hat{y}_t \\
 & + [\chi_{ry2p} \Sigma^\mu + \frac{1}{r} \chi_{rwp} \chi_{wy2p} \Sigma^\mu + \frac{1}{r^2} \chi_{rwp} \chi_{wvp} \chi_{wy2p} \Sigma^\mu \\
 & + \frac{1}{r^3} \chi_{rwp} \chi_{wvp}^2 \chi_{wy2p} \Sigma^\mu + \dots] \hat{y}_t^*
 \end{aligned}$$

Note that the infinite sums in the two brackets can be further simplified to

$$\begin{aligned}
 & \chi_{ry1p} \Sigma^\mu + \frac{1}{r} \chi_{rwp} \chi_{wy1p} \Sigma^\mu \Sigma^{\chi_{wvp}} \\
 & \chi_{ry2p} \Sigma^\mu + \frac{1}{r} \chi_{rwp} \chi_{wy2p} \Sigma^\mu \Sigma^{\chi_{wvp}}
 \end{aligned}$$

Replacing $\Sigma^{\chi_{wvp}}$ and Σ^μ with Eq.(C.23) and (C.24), one obtains Eq.(B.11). Because Σ_{t+1}^r and Σ_{t+1}^{rn} differ only in the discount factor, if replacing r with $\frac{r}{(1+n)}$, one obtains

$$\begin{aligned}
 \Sigma_{t+1}^{rn} &= \underbrace{\frac{r \chi_{rwp}}{r - \chi_{wvp} (1+n)}}_{\chi_{srwp}} \hat{w}_{t+1} & (C.26) \\
 & + \underbrace{\left[\frac{r \chi_{ry1p}}{r - \mu (1+n)} + \frac{\chi_{rwp} \chi_{wy1p}}{r - \chi_{wvp} (1+n)} \frac{r (1+n)}{r - \mu (1+n)} \right]}_{\equiv \chi_{sry1p}} \hat{y}_{t+1} \\
 & + \underbrace{\left[\frac{r \chi_{ry2p}}{r - \mu (1+n)} + \frac{\chi_{rwp} \chi_{wy2p}}{r - \chi_{wvp} (1+n)} \frac{r (1+n)}{r - \mu (1+n)} \right]}_{\equiv \chi_{sry2p}} \hat{y}_{t+1}^*
 \end{aligned}$$

$$\begin{aligned}
 \underbrace{\Sigma_{t+1}^{rn}}_{\text{Future asset return effect}} &= \underbrace{\chi_{srwp} \chi_{wx}}_{\equiv \chi_{srx}} \hat{r}_{xt} + \underbrace{\chi_{srwp} \chi_{wv}}_{\equiv \chi_{srw}} \hat{w}_t + \underbrace{\chi_{srwp} \chi_{wz}}_{\equiv \chi_{srz}} \hat{z}_{2t-1} & (C.27) \\
 & + \underbrace{[\chi_{srwp} \chi_{we1} + \mu \chi_{sry1p}]}_{\equiv \chi_{sre1}} \hat{y}_t + \underbrace{[\chi_{srwp} \chi_{we2} + \mu \chi_{sry2p}]}_{\equiv \chi_{sre2}} \hat{y}_t^*
 \end{aligned}$$

3.C.3 The system of current variables

Following subsection (B.3), the system for current variables is

$$\hat{w}_{t+1} = J'_1 \hat{w}_t + J'_3 \hat{r}_{xt} + J'_4 \hat{y}_t + J'_5 \hat{c}_t + J'_6 \hat{y}_t^* + J'_7 \hat{z}_{2t-1} \quad (C.28)$$

$$\hat{c}_{t+1} = K_1 \hat{c}_t + K_2 \hat{y}_{t+1} + K_3 \hat{r}_{t+1} \quad (C.29)$$

$$\hat{c}_{t+1}^* = K_1^* \hat{c}_t^* + K_2^* \hat{y}_{t+1}^* + K_3^* \hat{r}_{t+1} \quad (C.30)$$

where

$$J'_1 = J_1 + J_2 \chi_{rw} = J_1 - J_2 \frac{\chi_{rwp} \chi_{ww}}{r - \chi_{wvp}} \quad (C.31)$$

$$J'_3 = J_3 + J_2 \chi_{rx} = J_3 - J_2 \frac{\chi_{rwp} \chi_{wx}}{r - \chi_{wvp}} \quad (C.32)$$

$$J'_4 = J_4 + J_2 \chi_{re1} = J_4 - J_2 \left[\frac{\chi_{ry1p}}{r - \mu} + \frac{\chi_{rwp} \chi_{wy1p}}{(r - \chi_{wvp})(r - \mu)} \right] \quad (C.33)$$

$$J'_5 = J_5 \quad (C.34)$$

$$J'_6 = J_2 \chi_{re2} = J_2 \frac{(r-1)}{(r-\mu)} - J_2 \left[\frac{\chi_{ry2p}}{r - \mu} + \frac{\chi_{rwp} \chi_{wy2p}}{(r - \chi_{wvp})(r - \mu)} \right] \quad (C.35)$$

$$J'_7 = -J_2 \left[1 + \frac{\chi_{rwp} \chi_{wz2}}{r - \chi_{wvp}} \right] \quad (C.36)$$

Supposed the solutions are

$$\hat{w}_{t+1} = \chi_{ww} \hat{w}_t + \chi_{wz2} \hat{z}_{2t-1} + \chi_{wx} \hat{r}_{xt} + \chi_{we1} \hat{y}_t + \chi_{we2} \hat{y}_t^* \quad (C.37)$$

$$\hat{c}_t = \chi_{cw} \hat{w}_t + \chi_{cz2} \hat{z}_{2t-1} + \chi_{cx} \hat{r}_{xt} + \chi_{ce1} \hat{y}_t + \chi_{ce2} \hat{y}_t^* \quad (C.38)$$

$$\hat{c}_t^* = \chi_{cw}^* \hat{w}_t + \chi_{cz2}^* \hat{z}_{2t-1} + \chi_{cx}^* \hat{r}_{xt} + \chi_{ce1}^* \hat{y}_t + \chi_{ce2}^* \hat{y}_t^* \quad (C.39)$$

with here another 12 χ s are coefficients to be determined.

Substitute (C.38) into Eq.(C.28) to obtain the first five equations

$$\chi_{ww} = J'_1 + J'_5 \chi_{cw} \quad (C.40)$$

$$\chi_{wz2} = J'_7 + J'_5 \chi_{cz2} \quad (C.41)$$

$$\chi_{wx} = J'_3 + J'_5 \chi_{cx} \quad (C.42)$$

$$\chi_{we1} = J'_4 + J'_5 \chi_{ce1} \quad (C.43)$$

$$\chi_{we2} = J'_6 + J'_5 \chi_{ce2} \quad (C.44)$$

Substitute (C.38) into Eq.(C.29) to obtain another five equations

$$\chi_{cwp} \chi_{ww} = K_1 \chi_{cw} + K_3 \chi_{rwp} \chi_{ww} \quad (C.45)$$

$$\chi_{cwp} \chi_{wz2} = K_1 \chi_{cz2} + K_3 \chi_{rwp} \chi_{wz2} \quad (C.46)$$

$$\chi_{cwp} \chi_{wx} = K_1 \chi_{cx} + K_3 \chi_{rwp} \chi_{wx} \quad (C.47)$$

$$\chi_{cwp} \chi_{wy1} + \mu \chi_{cy1p} = K_1 \chi_{cy1} + \mu K_2 + K_3 \chi_{rwp} \chi_{wy1} + K_3 \mu \chi_{ry1p} \quad (C.48)$$

$$\chi_{cwp} \chi_{wy2} + \mu \chi_{cy2p} = K_1 \chi_{cy2} + K_3 \chi_{rwp} \chi_{wy2} + K_3 \mu \chi_{ry2p} \quad (C.49)$$

Substitute (C.39) into Eq.(C.30) to obtain the last five equations

$$\chi_{cwp}^* \chi_{ww} = K_1^* \chi_{cw}^* + K_3^* \chi_{rwp} \chi_{ww} \quad (C.50)$$

$$\chi_{cwp}^* \chi_{wz2} = K_1^* \chi_{cz2}^* + K_3^* \chi_{rwp} \chi_{wz2} \quad (C.51)$$

$$\chi_{cwp}^* \chi_{wx} = K_1^* \chi_{cx}^* + K_3^* \chi_{rwp} \chi_{wx} \quad (C.52)$$

$$\chi_{cwp}^* \chi_{wy1} + \mu \chi_{cy1p}^* = K_1^* \chi_{cy1}^* + K_3^* \chi_{rwp} \chi_{wy1} + K_3^* \mu \chi_{ry1p} \quad (C.53)$$

$$\chi_{cwp}^* \chi_{wy2} + \mu \chi_{cy2p}^* = K_1^* \chi_{cy2}^* + \mu K_2^* + K_3^* \chi_{rwp} \chi_{wy2} + K_3^* \mu \chi_{ry2p} \quad (C.54)$$

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Chapter 4

Portfolio choices in a production OLG model: Asymmetric asset home bias

In Chapter 3, we showed how analysis of both (non-trivial) net and gross portfolios can be integrated into the same model. This represents an important step toward our goal of understanding the empirical pattern of portfolio choices between developing and developed countries from a theoretical point of view. However, because we ignore the importance of other income risks, firstly, the size of gross external positions in the model is close to half of the steady state asset supplies. This is still too big compared to the data. In reality, a much smaller proportion of domestic assets is held by overseas investors. Secondly, the size of net external positions is many times the size of annual *GDP*. This is also too big compared to the data. In reality, a net portfolio of this size can only be seen in very rare cases of small countries/areas with a special status as international financial centre, for instance Hong Kong and Singapore. As a result of these facts, as shown in Section 3.8, the valuation and terms-of-trade effects generated are unrealistically high, which prevents the simple *OLG* model in Chapter 3 from being a proper framework to think about practical issues. We overcome these drawbacks of the framework in this chapter by generating gross positions with a more (home) biased pattern as well as a net position with a more realistic magnitude through including hedging motives of additional risks,

i.e. the risks associated with labour income and exchange rate fluctuations.

4.1 Introduction

Asset home bias describes the phenomenon across countries where foreign ownership of assets is limited and assets are mostly held domestically (French and Poterba, 1991, Cooper and Kaplanis, 1994 and Tesar and Werner, 1995, etc.) According to the basic international capital asset pricing model (*CAPM*), however, countries are expected to hold a world market portfolio. Because in reality no asset of a single country amounts to more than half of world capitalisation, this implies that assets should actually be mostly held by overseas investors across countries. Countries' reluctance to reap the obvious international diversification gains by diversifying their portfolios makes the pattern a well-known puzzle in international finance. The analysis in this chapter sheds light on the pattern of asset home bias, however, in terms of its incarnation between developing and developed countries.

As documented in the first chapter, a striking fact about the gross portfolio holdings under financial globalisation is that even though asset home bias is prevalent across countries, the degree of the bias is much more significant in developing countries than in developed countries. In other words, like the pattern of net external portfolios, the pattern of gross external portfolios between them is also asymmetric. In this chapter, we argue that the two patterns are actually inter-linked. As a background to model these facts, from the literature on the determination of gross portfolios, introducing non-tradable income risks can account for the emergence of symmetric asset home bias (Heathcote and Perri, 2013), while from the literature on the determination of non-zero net portfolios, the unequal amount of the non-tradable income risks across countries can account for the emergence of global imbalances (Caballero et al., 2008). However, the two strands of literature do not formally interact in the sense that the former does not deal with asymmetric cases while the latter does not explicitly solve for the gross positions that are consistent with the net position obtained. The model in Chapter 3, with an *OLG* structure, resolves the methodological problem associated with the analysis of the net and gross portfolio positions. Based on the results of the above two strands of literature, the model in this chapter assumes

differing amounts of the non-tradable income risks across countries to allow for the interaction of non-trivial net positions and biased gross positions, from which the pattern of asymmetric asset home bias is found to arise endogenously.

The model of this chapter is similar to that in Chapter 3, i.e. a model of a two-country open economy with two equity assets available in financial market to hedge against two exogenous (technological) shocks. As shown in the previous chapter, the assumption of fully capitalizable output in a model (with a country asymmetry) gives rise to nearly fully-diversified rather than significant biased portfolios. So instead, in this chapter we assume that the wealth in the two countries is divided into financial and human (labour) wealth. And only the financial wealth can be capitalized. Moreover, wealth divisions in the two countries are assumed to be different so that in the home, i.e. developed, country, financial wealth accounts for a relatively large proportion of total wealth while, accordingly, human wealth accounts for the remaining relatively small proportion.

The cross-country asymmetry in the degree of capitalization can be caused by various reasons. In the literature, Caballero et al. (2008) attributes it to financial development while Jin (2012) attributes it to industrial structure, both of which are easy to understand. For the former, many factors which affect the pledgeability of future income streams can influence how much financial assets is finally formed in an economy. Potential candidates are those related to laws (especially those associated with protection of property rights), quality of governance and corporate management, etc, all of which tend to be relatively poor in developing countries and therefore result in a relatively low share of financial wealth in these countries. For the latter, the technological difference across countries simply explains it (Jin, 2012). Imagine an economy mainly relying on tea gathering, the country would never feature a large share of capital (and a low share of labour) in income. Anyway, in either case, the fact that income shares differ across countries implies that in the home country the asset supply is relatively high and asset demand is relatively low while the opposite is true in the foreign country, which, in a financially integrated world, will generate current account global imbalances. To be specific, with net excess supply of assets, i.e. net capital inflows, the home country will be a debtor in steady state, while, with net excess demand of assets, i.e. net capital outflows, the foreign country will be

a net creditor in steady state.

On the other hand, the assumption of asymmetric wealth division has implications for portfolio diversification as well. This is because portfolio choice crucially depends on the relationship between the two income streams. If financial and labour incomes co-move positively, the portfolio should be biased toward foreign assets because holding more domestic assets exaggerates the effect of risks (Baxter and Jermann, 1997). If however, the two income streams co-move negatively, the portfolio should be biased toward local assets because holding more of them hedges the effect of labour income risk. Given the fact that the purpose of the current chapter is not to show why we have asset home bias but rather how it is different in asymmetric models, admittedly, to retain the result of the home-biased portfolio in our model, one shortcut is to assume exogenously negative correlation between the two types of incomes. So a modified endowment *OLG* model of Chapter 3 can be used to achieve the result of this asset home bias with the assumption of exogenous forces.

Heathcote and Perri (2013) show that, in a two good production open economy, financial and labour incomes will move in opposite directions due to the automatic responses of the real exchange rate and investment. So even when there is no asymmetry in wealth divisions, portfolios are (symmetrically) biased towards local assets by endogenous forces. For this reason, instead of imposing shocks with a negative correlation, we adopt the same environment by extending the simple *OLG* model to the context of multiple goods with capital accumulation and production. Under our assumption of country asymmetry in wealth division, we find that the symmetry in the pattern of asset home bias is broken. While local assets are held disproportionately in both countries, the degree of bias is higher in the foreign (developing) country than in the home (developed) country, which is consistent with the stylised fact.

To see the mechanics underlying the result, we decompose the optimal portfolio position as in Chapter 3. It turns out that the gross holding of local assets of countries is composed of four terms, i.e. the self-hedging, the hedging of labour income, the hedging of external interest payments and an adjustment term due to the *OLG* structure. While the first and fourth terms are both unambiguously negative so as to cause agents to diversify portfolio toward overseas assets, the hedging of labour income, as expected, is unambiguously positive which biases

portfolios locally. Moreover, the existence of the country asymmetry is reflected on the magnitude of these terms in both countries, however, in the same way. To put it differently, although being important in creating asset home bias worldwide, the first, second and fourth hedging terms are not important when it comes to the asymmetry associated with the bias.

Instead, the reason that the bias is more significant in the home country is related to the hedging of external interest payments. We find that this hedging term has a different sign across countries depending on the debtor or creditor status of country, i.e. it is negative in the home country and positive in the foreign country. This tends to mitigate the degree of the asset bias in the home country while exacerbates that in the foreign country.

This chapter is organized as follows. In section 4.2, we describe the model environment. Section 4.3 presents the steady states of the model. How steady state global imbalances arise in the model is also demonstrated in this section. Section 4.4 is used to yield the optimal conditions for portfolios. One key determinant of the optimal portfolio, the behaviour of the consumption differential, is analysed in Section 4.5. Section 4.6 presents the optimal portfolio as the sum of hedging components with their properties and relevance to the pattern of asymmetric asset home bias being discussed. We carry out a benchmark simulation of the model in Section 4.7 and alternative simulations to assess the robustness of our result in Section 4.8. Section 4.9 concludes.

4.2 Model

This is a world consisting of two countries. Each country is populated by the infinitely lived *OLG* households of identical structure to that in the last chapter (following Weil, 1989). Recall that a population of measure 1 is born at time $t = 0$ in each country and then grows at a net rate of n . No one dies so at time $t \geq 1$ the number of new-born is $(1 + n)^t - (1 + n)^{t-1} = n(1 + n)^{t-1}$. So in any period, the per capita aggregate of any variable x can be obtained by

$$x_t = \frac{x_t^0 + nx_t^1 + n(1+n)x_t^2 \dots + n(1+n)^{t-1}x_t^t}{(1+n)^t}$$

where the superscript v and subscript t of x_t^v denote vintage and time respectively.

The economic structure of home and foreign countries follows Backus, Kehoe and Kydland (1994) and Heathcote and Perri (2013). In each country, households make decisions on consumption and labour supply. Domestic capital and labour cannot be traded internationally. They are used within the border by firms to produce a country-specific intermediate good. The two intermediate goods are then traded internationally to compose the final goods that are ready for the use of consumption and investment in the two countries. In terms of financial markets, two equity-style assets are traded respectively representing claims on the profit made by the intermediate-good producers in either country. The risk in the world economy comes from stochastic shocks to firms' production technology. We lay out the whole model now.

4.2.1 Households' problem

For households of vintage v , their life-time utility function at time t is assumed to have the following form

$$U_t^v = \sum_{i=0}^{\infty} \beta^i [\log(c_{t+i}^v) + \gamma \log(1 - h_{t+i}^v)]$$

where c_t^v and h_t^v denote individual consumption and labour supply. β and γ are respectively the discount factor and the weight controlling the relative importance between consumption and leisure.

The budget constraint facing the individual is

$$\alpha_{1t+1}^v + \alpha_{2t+1}^v = r_{1t}\alpha_{1t}^v + r_{2t}\alpha_{2t}^v + l_t^v - c_t^v$$

for all t where l_t^v denotes generation v 's labour or human income during period t . It is given by the product of labour supply h_t^v and real wage (nominal wage g_t over CPI , p_t).

$$l_t^v = \frac{g_t}{p_t} h_t^v$$

α_t^v denotes generation v 's net holding of a particular asset at the end of period $t-1$. We label the home equity asset 1 and the foreign equity asset 2. So α_{1t}^v and α_{2t}^v are respectively their holding of home and foreign assets. r_{1t} and r_{2t} denote the two assets' gross rate of return which we will define later. So the budget constraint states that households can save by investing in the two assets.

It is useful to define the gross wealth as the sum of all holdings across asset

$$w_t^v = \alpha_{1t}^v + \alpha_{2t}^v$$

so above constraint can also be written as

$$w_{t+1}^v = r_{2t}w_t^v + \alpha_{1t}^v r_{xt} + l_t^v - c_t^v$$

where $r_{xt} = r_{1t} - r_{2t}$ is the excess return of asset 1 over asset 2.

The household's problem is to choose optimal c_t^v , l_t^v , α_t^v s to maximize their life-time utility, U_t^v , subject to all intertemporal budget constraints. Their behaviour can thus be described by the following first-order conditions

$$\begin{aligned}\lambda_t &= (c_t^v)^{-1} \\ (c_t^v)^{-1} &= \beta E_t \left[r_{1t+1} (c_{t+1}^v)^{-1} \right] \\ (c_t^v)^{-1} &= \beta E_t \left[r_{2t+1} (c_{t+1}^v)^{-1} \right] \\ h_t^v &= 1 - \gamma \frac{p_t}{g_t} c_t^v\end{aligned}$$

where λ_t is the Lagrangian multiplier associated with time- t budget constraint.

Foreign households maximize the utility function of the same form. However, their budget constraints read

$$s_t (\alpha_{1t+1}^{*v} + \alpha_{2t+1}^{*v}) = s_t (r_{1t} \alpha_{1t}^{*v} + r_{2t} \alpha_{2t}^{*v}) + l_t^{*v} - c_t^{*v}$$

where s_t denotes the real exchange rate at time t . That is the price of the home consumption basket in terms of the foreign consumption basket. It appears in the constraint because we adopt the following convention. Apart from the asset-related variables (including foreign asset holding α_t^{*v} and return r_{2t}) which are denoted in terms of the home country consumption basket, all the other variables are in terms of the local consumption basket. The related first-order conditions are obtained similar to those of home country.

For the reasons we elaborated in the last chapter, even though households' optimization is undertaken at the individual level, we care about the model's per capita outcome (in other words, aggregate behaviour). By the demographic assumptions we have made at the beginning of this section, apart from the Euler

equations, all the other equations above are linear relations which means they can be easily aggregated into the related per capita version by our aforementioned method of population-weighted averaging. For example, per capita labour income is just the product of real wage and per capita labour supply

$$l_t = \frac{g_t}{p_t} h_t$$

Per capita budget constraints for the two countries are

$$(1+n)(\alpha_{1t+1} + \alpha_{2t+1}) = r_{1t}\alpha_{1t} + r_{2t}\alpha_{2t} + l_t - c_t$$

$$s_t(1+n)(\alpha_{1t+1}^* + \alpha_{2t+1}^*) = s_t(r_{1t}\alpha_{1t}^* + r_{2t}\alpha_{2t}^*) + l_t^* - c_t^*$$

where $(1+n)$ emerges because we assume new generations are born with no assets so $\alpha_t^t = 0$ following Weil (1989). This is also familiar from Section 3.3.2 of Chapter 3.

Aggregating the Euler equations is a little bit more difficult. Section 3.3.5 of Chapter 3 described how we deal with them in more detail. To put it simply, because to solve the model we still use the standard method of log-linearization around the (per capita) steady states, we do not need the model in exact form if we can obtain their linearized form. So for the Euler equations, rather than to obtain the exact (per capita) form and then linearize them, we linearize them first and then aggregate them. The resulting linearized equations are omitted here and can be found in Section 4.B of the Appendix to this chapter.

To end this subsection, let us define asset returns r_{1t} and r_{2t}

$$r_{1t}z_{1t} = d_t + (1+n)z_{1t+1}$$

$$r_{2t}z_{2t} = \frac{d_t^*}{s_t} + (1+n)z_{2t+1}$$

where z_{1t} and z_{2t} denote the prices of asset 1 and 2 (in terms of the home consumption basket) at the end of period $t-1$. d_t is the dividend paid by the home firm while d^* is that paid by the foreign firm. s_t converts the latter into the home consumption basket. Note the asset returns are defined at the per capita aggregate level. (In fact all the aforementioned variables are per capita variables as is indicated by the absence of the superscript v .)

4.2.2 Firm's problem

We assume the firm uses the familiar Cobb-Douglas production function to produce intermediate good x

$$x_t = e^{\varepsilon_t} (k_t)^\delta (h_t)^{1-\delta}$$

$$x_t^* = e^{\varepsilon_t^*} (k_t^*)^{\delta^*} (h_t^*)^{1-\delta^*}$$

Here ε_t and ε_t^* represent technology shocks which follow the following $AR(1)$ processes

$$\varepsilon_t = \mu\varepsilon_{t-1} + \epsilon_t$$

$$\varepsilon_t^* = \mu\varepsilon_{t-1}^* + \epsilon_t^*$$

where $0 < \mu < 1$. The innovations ϵ and ϵ^* are zero-mean *i.i.d* processes with the property of $var(\epsilon) = var(\epsilon^*) = \sigma^2$ and $cov(\epsilon\epsilon^*) = 0$.

In the production function, δ and δ^* represent the shares of financial (non-human) wealth in the total wealth. $(1 - \delta)$ and $(1 - \delta^*)$ represent the shares of labour (human) wealth. We assume $\delta > \delta^*$ and so $(1 - \delta) < (1 - \delta^*)$. The fact that the share of financial wealth is relatively higher in the home, in this chapter also developed, country can be interpreted in different ways. At the most basic level, the δ s denote the capital shares in production. So the asymmetry reflects many factors underlying the determination of differences in industrial structure across countries among which the most important one is technological difference, i.e. the home country uses a more capital-intensive technology than the foreign country as argued by Jin (2012). Besides, as explained by Caballero et al. (2008), the share of financial wealth crucially depends on many factors determining one country's ability in capitalizing future income or, in their words, generating a storage of value. These factors include those of financial development and social institutions. The question of which particular one or mix of these factors is most likely to be the real reason leading to the asymmetry in δ is interesting. However, for the purpose of the current work, this is not the focus. So here we just refer to the δ s as wealth division parameters. But one should understand that they actually summarize much deeper information about the differences between the two countries.

Given the production function, the firm maximizes the sum of the present value of all future dividends

$$\sum_{i=0}^{\infty} \Omega^i d_{t+i}$$

where Ω denotes the discount factor

$$\Omega^i = \beta^i c_{t+i}^{-1}$$

and d_t denotes the dividend, which is defined as the difference between the revenue and labour cost and investment

$$d_t = \frac{q_t}{p_t} x_t - l_t - i_t$$

Here q_t denotes the price of the home intermediate good. q_t/p_t is thus the price in terms of home final good.

Corresponding to the *OLG* structure in the model, investment is given by

$$i_t = (1 + n) k_{t+1} - k_t$$

The first order condition of optimal choices of labour and capital demand can thus be obtained as follows

$$MPL_t = \frac{g_t}{q_t}$$

$$r_{kt} = \frac{q_t}{p_t} MPK_t + 1$$

$$\Omega_t = E_t [\Omega_{t+1} r_{kt+1}]$$

The two intermediate goods are then combined to form final goods y and y^* . The following formulae describe how the final goods are aggregated.

$$y_t = \left[\kappa^{\frac{1}{\phi}} (x_{ht})^{\frac{\phi-1}{\phi}} + (1 - \kappa)^{\frac{1}{\phi}} (x_{ft})^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}$$

$$y_t^* = \left[(1 - \kappa)^{\frac{1}{\phi}} (x_{ht}^*)^{\frac{\phi-1}{\phi}} + \kappa^{\frac{1}{\phi}} (x_{ft}^*)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}$$

In these formulas, x_{ht} and x_{ft} denote home demands for home and foreign intermediate goods. And x_{ht}^* and x_{ft}^* denote the foreign demands. Following the literature, we assume that households in each country are bias towards local

	Home holdings	Foreign holdings
Asset 1-Home equity	α_1	α_1^*
Asset 2-Foreign equity	$\alpha_2 = w - \alpha_1$	$\alpha_2^* = w^* - \alpha_1^*$

Table 4.1: Net asset holdings across countries

products. So the degree of local preference κ satisfies $1/2 < \kappa < 1$. ϕ denotes the elasticity of substitution between the home and foreign intermediate goods.

Given the above aggregation relation of the final goods, their related consumption-based price indices can be obtained as

$$p_t = \left[\kappa (q_t)^{1-\phi} + (1-\kappa) \left(\frac{q_t^*}{s_t} \right)^{1-\phi} \right]^{\frac{1}{1-\phi}}$$

$$p_t^* = \left[(1-\kappa) (s_t q_t)^{1-\phi} + \kappa (q_t^*)^{1-\phi} \right]^{\frac{1}{1-\phi}}$$

where q_t^* is price of the foreign good. The law of one price holds for the two internationally traded goods so the foreign price of the home good is given by $s_t q_t$ and the home price of the foreign good is given by $\frac{q_t^*}{s_t}$.

The demands for the intermediate goods are respectively

$$x_{ht} = \kappa \left(\frac{q_t}{p_t} \right)^{-\phi} y_t$$

$$x_{ft} = (1-\kappa) \left(\frac{q_t^*}{s_t p_t} \right)^{-\phi} y_t$$

$$x_{ht}^* = (1-\kappa) \left(\frac{s_t q_t}{p_t^*} \right)^{-\phi} y_t^*$$

$$x_{ft}^* = \kappa \left(\frac{q_t^*}{p_t^*} \right)^{-\phi} y_t^*$$

4.2.3 Market clearing

In equilibrium, all markets should clear including asset and good markets. Consider the asset market first. Remember the portfolio allocation is such as in Table 4.1.

Asset market clearing requires that the total asset demands meets total asset supply, i.e.

$$\alpha_{1t} + \alpha_{1t}^* = z_{1t}$$

$$\alpha_{2t} + \alpha_{2t}^* = z_{2t}$$

which are equivalent to

$$\alpha_{1t} = z_{1t} - \alpha_{1t}^*$$

$$w_t - z_{1t} = -(w_t^* - z_{2t})$$

While the interpretation of the first formula is obvious, the second formula states that the net asset demands, i.e. the difference between gross asset demand w_t and gross asset supply z_t , of home and foreign countries are of the same size but opposite sign. So when the home country has net capital inflows $w_t - z_{1t} < 0$, the foreign country must have net capital outflows $w_t^* - z_{2t} > 0$ of the same magnitude. Denote f_t as the net foreign asset at the end of time $t - 1$, so the above formula can be rewritten as

$$f_t = -f_t^*$$

Note that the presence of the asset market clearing conditions also implies we only need to work out any one of the four asset holdings and then the world portfolio allocation can be obtained. When computing the optimal portfolios for the model later in this chapter, we choose to first tie down $\alpha_{1t} - z_{1t} = -\alpha_{1t}^*$, i.e. home gross holding of the home asset.

For goods markets, intermediate goods market clearing requires

$$x_{ht} + x_{ht}^* = x_t$$

$$x_{ft} + x_{ft}^* = x_t^*$$

while final goods market clearing requires

$$c_t + i_t = y_t$$

$$c_t^* + i_t^* = y_t^*$$

4.3 Non-stochastic steady states

In this section, we discuss steady states and how global imbalances emerge as a consequence of the asymmetry in δ . We focus on the case of the home country. The foreign country's situation is similar. Remember, we assumed that the asymmetry in δ is the only structural difference between the two countries, so, to obtain the foreign country analogies we need simply to replace δ with δ^* in the expressions which follow.

GDP normalization When we say they are two otherwise identical countries except for $\delta > \delta^*$, we implicitly assume that the sizes of the two countries are also the same. Given the populations are already the same as a result of our assumptions on demographics, we now impose that the two countries have the same level of income. To be specific, we normalize the *GDPs* at the two countries to 1.¹ By the production approach to *GDP* it follows that

$$p_t \cdot gdp_t = q_t \cdot [x_{ht} + x_{ht}^*] = q_t x_t$$

$$p_t^* \cdot gdp_t^* = q_t^* \cdot [x_{ft} + x_{ft}^*] = q_t^* x_t^*$$

So

$$\frac{q_t}{p_t} x_t = gdp_t \equiv 1$$

$$\frac{q_t^*}{p_t^*} x_t^* = gdp_t^* \equiv 1$$

These formulas hold in steady state leading to

$$\frac{q}{p} x = \frac{q^*}{p^*} x^* = 1$$

where we drop the time subscripts to represent its steady state.

Capital stock k and Investment i By $\Omega_t = E_t[\Omega_{t+1} r_{kt+1}]$ and the Euler equations, the steady-state return to capital, r_k , is in line with the steady-state return to financial assets r

$$r_k = r$$

¹In order to do so, in our model, the parameter of γ in the utility function is allowed to be adjusted automatically.

By $r_{kt} = \frac{q_t}{p_t} MPK_t + 1$, the marginal product of capital is given by

$$MPK = \frac{p}{q} (r - 1)$$

where p and q are to be determined below.

By the production function of the intermediate good x , $MPK_t = \delta \frac{x_t}{k_t}$, so the capital stock is given by

$$k = \frac{\delta}{(r - 1)} \frac{q}{p} x = \frac{\delta}{(r - 1)}$$

By law of motion of the investment $i_t = (1 + n) k_{t+1} - k_t$, investment is given by

$$i = nk = \frac{n\delta}{(r - 1)}$$

Labour income l and dividend d By $MPL_t = (1 - \delta) \frac{x_t}{h_t}$ and $MPL_t = \frac{q_t}{p_t}$, labour income satisfies

$$l = \frac{g}{p} h = \frac{q}{p} \frac{g}{q} h = \frac{q}{p} (1 - \delta) x = (1 - \delta)$$

By $d_t = \frac{q_t}{p_t} x_t - l_t - i_t$, the dividend is given by

$$d = \frac{(r - \tilde{n})}{(r - 1)} \delta$$

Asset demand w and asset supply z With infinitely lived *OLG* households, the derivation from which the steady-state asset demand, w , is determined is similar to that in Chapter 3. By the individual budget constraint and Euler equations, the optimal individual consumption is given by

$$c_t^v = (1 - \beta) \left[r w_t^v + \sum_{i=0}^{\infty} \frac{1}{r^i} l_t^v \right]$$

Aggregating c_t^v over v to yield the consumption function in per capita form

$$c_t = (1 - \beta) \left[r w_t + \sum_{i=0}^{\infty} \frac{1}{r^i} l_t \right]$$

Substitute this into the aggregate budget constraint $(1 + n) w_{t+1} = w_t r_{2t} + \alpha_{1t} r_{xt} + l_t - c_t$ to yield the law of motion of w_t

$$w_{t+1} = \frac{r\beta}{(1 + n)} w_t + \frac{(r\beta - 1)}{(1 + n)(r - 1)} l_t$$

Impose a stability condition

$$\tau \equiv \frac{r\beta}{(1+n)} < 1$$

and make use of the steady-state labour income l to solve for steady-state asset demand

$$w = \frac{(r\beta - 1)}{(\tilde{n} - r\beta)(r - 1)} l = \frac{(r\beta - 1)(1 - \delta)}{(\tilde{n} - r\beta)(r - 1)}$$

where $\tilde{n} \equiv (1 + n) > r\beta > 1$ denotes the gross growth rate of population.

By $r_{1t}z_{1t} = d_t + (1 + n)z_{1t+1}$, asset supply is consistent with capital stock in steady state

$$z = \frac{d}{(r - \tilde{n})} = \frac{\delta}{(r - 1)} = k$$

Net foreign assets, f Net foreign assets, f , is the net asset demand and can be obtained by subtracting the asset supply z from w

$$f = w - z = \left[\frac{(r\beta - 1)(1 - \delta)}{(\tilde{n} - r\beta)(r - 1)} - \frac{\delta}{(r - 1)} \right]$$

$w = \frac{(r\beta - 1)(1 - \delta)}{(\tilde{n} - r\beta)(r - 1)}$ can be proved to be a positive function of r

$$\frac{\partial w}{\partial r} = \frac{(1 - \delta)}{(\tilde{n} - r\beta)(r - 1)} \left[\frac{\beta(\tilde{n} - 1)}{\tilde{n} - r\beta} - \frac{r\beta - 1}{r - 1} \right] > 0$$

While the fraction outside the bracket is obviously positive, one can show that the expression in the bracket is positive when we have $(r\beta - 1)^2 + n(1 - \beta) > 0$ which always holds.

In addition it follows that $z = \frac{\delta}{r - 1}$ is a negative function of r

$$\frac{\partial z}{\partial r} = -\frac{\delta}{(r - 1)^2} < 0$$

Interest rate r In autarky, the condition that the home asset demand equals the supply $w = z$ or net foreign asset equals zero $f = 0$ determines the steady-state interest rate

$$r^a = \frac{1 + \delta n}{\beta}$$

From this expression, r^a positively depends on time preference $\frac{1}{\beta}$, population growth rate n and parameter δ . Therefore, in a two country open economy, the

foreign country, having a low $\delta^* < \delta$, would have a low autarky interest rate $r^{a*} < r^a$.

The international interest rate is determined by the condition of asset market clearing $w_t - z_{1t} = -(w_t^* - z_{2t})$ or

$$f + f^* = 0$$

It thus lies between the two extreme autarky interest rates $r^{a*} < r < r^a$.

From the Euler equations in the two countries it follows that $r_1 = r_2 = r$ so

$$r_x = 0$$

And given the fact that $r^{a*} < r < r^a$, we have

$$f < 0$$

and

$$f^* = -f > 0$$

In other words, net capital flows from the foreign to home country.

In Figure 4.1, we cast the result of steady-state net foreign asset global imbalances into a familiar Metzler diagram. Asset demand and supply, which are measured on the horizontal axes, for the foreign and home countries are displayed in the left and right panels of the diagram respectively. The two asset demand schedules, w and w^* , are increasing functions of r (which is measured on the vertical axes). So they are positively sloped. The two asset supply schedules, z_1 and z_2 , are decreasing functions of r so they are negatively sloped. The autarky interest rate in each country is determined by the intersection of asset demand and supply schedules of the individual countries. Because $\delta > \delta^*$, we find that in the foreign country, asset demand is relatively high, so $w^* > w$, while asset supply is relatively low, so $z_2 < z_1$, for a given level of r , which results in a lower autarky interest rate in the foreign country than in the home country $r^{a*} < r^a$. In a financially integrated world, the net (excess) asset demand in the foreign country thus translates into an accumulation of current account surpluses in that country, which is consistent with the fact that net capital flows out to where the capital return is relative high. The process keeps going to the point when the

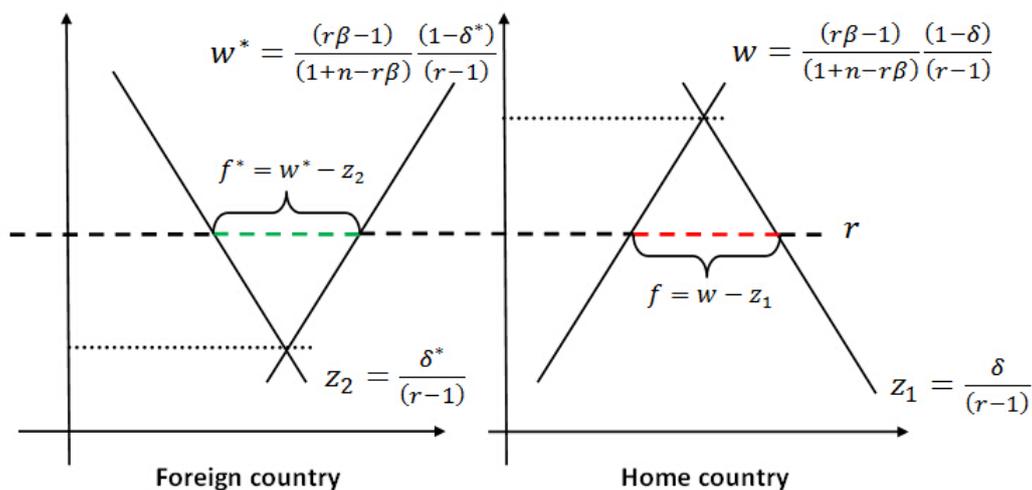


Figure 4.1: Interpret the global NFA imbalances in the model under a Metzler diagram

net asset demand in the foreign country is fully satisfied by the net (excess) asset supply in the home country and interest rates are equalized, at which point the equilibrium under financial integration is attained. Under this equilibrium, the foreign country possess net external claims while the home country has net external liabilities, i.e. net foreign asset global imbalances emerge.

Consumption and final good demand Knowing w , from the budget constraint $(1+n)w_{t+1} = w_t r_{2t} + \alpha_{1t} r_{xt} + l_t - c_t$, we can obtain the steady-state consumption, c ,

$$\begin{aligned}
 c &= l + (r - \tilde{n})w \\
 &= (1 - \delta) + (r - \tilde{n})(z + f) \\
 &= (1 - \delta) + d + (r - \tilde{n})f \\
 &= 1 - i + (r - \tilde{n})f
 \end{aligned}$$

from which the final good demand, y , is given by

$$y = c + i = 1 + (r - \tilde{n})f$$

One country's total spending (expenditure on consumption c and investment i) is restricted by its total income or GNP , with the possibility of external imbalances that is GDP ($\equiv 1$) plus net foreign asset income $(r - \tilde{n})f$. Obviously, net foreign asset income can be positive or negative depending on the sign of the country's net external position f .

As in Chapter 3, we assume a condition of global efficiency:

$$r > \tilde{n}$$

so the debtor (creditor) country's disposable expenditure is less (more) than their average GDP , i.e. when net foreign asset income is negative (positive) or $f < 0$ (> 0) we have $(c + i) > 1$ (< 1).

Knowing y and y^* , we obtain the intermediate goods demands as below

$$\begin{aligned} x_h &= \kappa \left(\frac{q}{p} \right)^{-\phi} y \\ x_f &= (1 - \kappa) \left(\frac{q^*}{sp} \right)^{-\phi} y \\ x_h^* &= (1 - \kappa) \left(\frac{sq}{p^*} \right)^{-\phi} y^* \\ x_f^* &= \kappa \left(\frac{q^*}{p^*} \right)^{-\phi} y^* \end{aligned}$$

The new-born's consumption can be obtained from $c_t^n = (1 - \beta) \sum_{i=0}^{\infty} \frac{1}{r^i} l_t$, which yields

$$c^n = \frac{(r - r\beta)}{(r - 1)} l$$

or from the aggregation relation (in steady state)

$$c^n = \frac{(1 + n - r\beta)}{n} c$$

One can verify that these two relations are equivalent.

Employment h and wage g By the equations $h_t = 1 - \gamma \frac{p_t}{g_t} c_t$, $MPL_t = (1 - \delta) \frac{x_t}{h_t}$ and $MPL_t = \frac{g_t}{q_t}$, we have

$$\gamma \frac{c}{(1 - h)} = \frac{g}{p} = \frac{g q}{q p} = MPL \frac{q}{p} = \frac{(1 - \delta)}{h}$$

Given c , we solve the equation $\gamma \frac{c}{(1-h)} = \frac{(1-\delta)}{h}$ to obtain steady state working hours

$$h = \frac{(1-\delta)}{\gamma cp/q + (1-\delta)}$$

In addition, the wage rate is

$$g = \frac{(1-\delta)q}{h}$$

Prices indices The determination of the price system boils down to the determination of the prices of the two intermediate goods p and p^* if one notices that the prices of final goods p and p^* and the real exchange rate s relate to p and p^* through

$$p = \left[\kappa (q)^{1-\phi} + (1-\kappa) \left(\frac{q^*}{s} \right)^{1-\phi} \right]^{\frac{1}{1-\phi}}$$

$$p^* = \left[(1-\kappa) (sq)^{1-\phi} + \kappa (q^*)^{1-\phi} \right]^{\frac{1}{1-\phi}}$$

$$s = \frac{p}{p^*}$$

q and q^* clear the markets of two intermediate goods, so they must satisfy

$$\kappa \left(\frac{q}{p} \right)^{-\phi} y + (1-\kappa) \left(\frac{sq}{p^*} \right)^{-\phi} y^* = x = \frac{p}{q}$$

$$(1-\kappa) \left(\frac{q^*}{sp} \right)^{-\phi} y + \kappa \left(\frac{q^*}{p^*} \right)^{-\phi} y^* = x^* = \frac{p^*}{q^*}$$

Given the two countries being asymmetric, the price vector $[q \ q^* \ p \ p^* \ s]$ is generally asymmetric as well, i.e. it deviates from $[1 \ 1 \ 1 \ 1 \ 1]$.

4.4 Portfolio optimality condition

We derive the condition for optimal portfolios in this section. As in Section 3.4 of Chapter 3, with the *OLG* structure being present, the condition can be obtained through three steps.

First, optimization of portfolio choices is undertaken at the level of individual households. Individual portfolio choices α_i^v must satisfy the related first-order

conditions. Following Devereux and Sutherland (2011), we approximate these first-order conditions up to second-order accuracy along their optimal paths (see Chapter 3 for the definition of the optimal path of individual variables). This yields

$$E_t \left[\hat{r}_{xt} + \frac{1}{2} \hat{r}_{xt}^{(2)} - \hat{c}_t^v \hat{r}_{xt} \right] = 0 + O(\varepsilon^3)$$

$$E_t \left[\hat{r}_{xt} + \frac{1}{2} \hat{r}_{xt}^{(2)} - \hat{c}_t^{v*} \hat{r}_{xt} + \hat{s}_t \hat{r}_{xt} \right] = 0 + O(\varepsilon^3)$$

where $\hat{r}_{xt}^{(2)} = \hat{r}_{1t}^2 - \hat{r}_{2t}^2$. These conditions can be referred to as individual portfolio conditions.

Second, imposing these individual conditions on the (first-order approximated) per capita aggregation relation. This yields

$$E_t \left[\hat{r}_{xt} + \frac{1}{2} \hat{r}_{xt}^{(2)} - \hat{c}_t \hat{r}_{xt} \right] = 0 + O(\varepsilon^3)$$

$$E_t \left[\hat{r}_{xt} + \frac{1}{2} \hat{r}_{xt}^{(2)} - \hat{c}_t^* \hat{r}_{xt} + \hat{s}_t \hat{r}_{xt} \right] = 0 + O(\varepsilon^3)$$

which can be referred to as (per capita) aggregate portfolio conditions.

Third, combine the above two conditions to yield the following condition

$$E_t [(\hat{c}_t - \hat{c}_t^* + \hat{s}_t) \hat{r}_{xt}] = 0$$

which serves as the condition to tie down α .

By this condition, households choose the optimal portfolio to achieve optimal sharing of income risks. Depending on the income risks contained in the cross-country consumption differential $cd_t \equiv (\hat{c}_t - \hat{c}_t^* + \hat{s}_t)$, the optimal portfolio α is built up as the result of a series of corresponding hedging motives. To analyse this, as in Chapter 3, in the next section we will first analyse the risk components of the consumption differential. Then in Section 4.6, we turn to the analysis of the components of the optimal portfolio in the current model.

4.5 Consumption differential, cd_t

In this section, we identify all components of the income stream that impact on the consumption differential $cd_t \equiv (\hat{c}_t - \hat{c}_t^* + \hat{s}_t)$. As in Chapter 3, we fulfil the task through analysing the wealth and composition effects of consumption.

4.5.1 Wealth effect, Σ_t^c and Σ_t^{c*}

After a shock, total resources that one country can use (over its infinite horizon) will change. The amount of the change in total resources can be obtained by aggregating the country's intertemporal budget constraints.

For the home country, the period budget constraint is

$$\tilde{n}f_{t+1} = r_{1t}f_t + d_t - \alpha_{2t}r_{xt} + l_t - c_t$$

Approximate to yield

$$\tilde{n}f\hat{f}_{t+1} = rf\hat{f}_t + rf\hat{r}_{1t} + d\hat{d}_t + l\hat{l}_t - \alpha_2r\hat{r}_{xt} - c\hat{c}_t$$

That is

$$\hat{c}_t = -\frac{\tilde{n}f}{c}\hat{f}_{t+1} + \frac{rf}{c}\hat{f}_t + \frac{rf}{c}\hat{r}_{1t} + \frac{d}{c}\hat{d}_t + \frac{l}{c}\hat{l}_t - \frac{\alpha_2r}{c}\hat{r}_{xt}$$

So we have total consumption

$$\Sigma_t^c = \frac{rf}{c} \left[\hat{r}_{1t} + \frac{\tilde{n}}{r}\Sigma_{t+1}^{rn} \right] + \frac{d}{c}\Sigma_t^d + \frac{l}{c}\Sigma_t^l - \frac{\alpha_2r}{c}\hat{r}_{xt}$$

where $\Sigma_t^c = \Sigma_{i=0}^{\infty} \left[\frac{\tilde{n}}{r} \right]^i \hat{c}_{t+i}$, $\Sigma_{t+1}^{rn} = \Sigma_{i=0}^{\infty} \left[\frac{\tilde{n}}{r} \right]^i \hat{r}_{t+1+i}$, $\Sigma_t^d = \Sigma_{i=0}^{\infty} \left[\frac{\tilde{n}}{r} \right]^i \hat{d}_{t+i}$ and $\Sigma_t^l = \Sigma_{i=0}^{\infty} \left[\frac{\tilde{n}}{r} \right]^i \hat{l}_{t+i}$.

The above equation can be re-written as follows

$$\Sigma_t^c = \frac{rf}{c} \left[\hat{r}_{2t} + \frac{\tilde{n}}{r}\Sigma_{t+1}^{rn} \right] + \frac{d}{c}\Sigma_t^d + \frac{l}{c}\Sigma_t^l - \frac{\alpha_2r}{c}\hat{r}_{xt} + \frac{rf}{c}\hat{r}_{xt}$$

i.e.

$$\Sigma_t^c = \frac{rf}{c} \left[\hat{r}_{2t} + \frac{\tilde{n}}{r}\Sigma_{t+1}^{rn} \right] + \frac{d}{c}\Sigma_t^d + \frac{l}{c}\Sigma_t^l + \frac{(\alpha_1 - z_1)r}{c}\hat{r}_{xt}$$

because $(f - \alpha_2) = (w - z_1) - \alpha_2 = (w - \alpha_2) - z_1 = (\alpha_1 - z_1)$. In this equation, home total consumption is composed of, from left to right respectively, total external interest payments, total financial wealth, total labour wealth and unanticipated portfolio valuation effects.

Similarly, for the foreign country its budget constraint is

$$\tilde{n}f_{t+1}^* = r_{2t}f_t^* + \frac{d_t^*}{s_t} + (z_{1t} - \alpha_{1t})r_{xt} + \frac{l_t^*}{s_t} - \frac{c_t^*}{s_t}$$

which can be approximated to yield

$$-\tilde{n}f\hat{f}_{t+1}^* = -rf\hat{f}_t^* - rf\hat{r}_{2t}^* + \frac{d^*}{s} \left(\hat{d}_t^* - \hat{s}_t \right) + \frac{l^*}{s} \left(\hat{l}_t^* - \hat{s}_t \right) + (z_1 - \alpha_1)r\hat{r}_{xt} - \frac{c^*}{s} \left(\hat{c}_t^* - \hat{s}_t \right)$$

where we use the facts that $f^* = -f$ and $\hat{f}_t = \hat{f}_t^*$.

So we have

$$\Sigma_t^{c^*} = -\frac{rfs}{c^*} \left[\hat{r}_{2t} + \frac{\tilde{n}}{r} \Sigma_{t+1}^{rn} \right] + \frac{d^*}{c^*} \Sigma_t^{d^*} + \frac{l^*}{c^*} \Sigma_t^{l^*} - \frac{(\alpha_1 - z_1)rs}{c^*} \hat{r}_{xt}$$

where we define $\Sigma_t^{c^*} = \Sigma_{i=0}^{\infty} \left[\frac{\tilde{n}}{r} \right]^i [\hat{c}_{t+i}^* - \hat{s}_{t+i}]$, $\Sigma_t^{d^*} = \Sigma_{i=0}^{\infty} \left[\frac{\tilde{n}}{r} \right]^i [\hat{d}_{t+i}^* - \hat{s}_{t+i}]$ and $\Sigma_t^l = \Sigma_{i=0}^{\infty} \left[\frac{\tilde{n}}{r} \right]^i [\hat{l}_{t+i}^* - \hat{s}_{t+i}]$. Foreign total consumption is also composed of the above four income resources. However, notice that because the world consists of the two countries, the interest payments and valuation effects at home and abroad have the opposite sign.

4.5.2 Composition effect, \hat{c}_t and \hat{c}_t^*

How the lifetime resources are distributed across time in one country is regulated by the related Euler equations. As in Chapter 3, we first approximate the individual Euler equations and then aggregated them to obtain the following (home country) per capita Euler equation

$$\hat{c}_{t+1} = \tau \hat{c}_t + (1 - \tau) \hat{c}_{t+1}^n + \tau \hat{r}_{t+1}$$

from which we derive

$$\begin{aligned} \Sigma_t^c &= \hat{c}_t + \frac{\tilde{n}}{r} [\tau \hat{c}_t + (1 - \tau) \hat{c}_{t+1}^n + \tau \hat{r}_{t+1}] \\ &\quad + \left[\frac{\tilde{n}}{r} \right]^2 [\tau \hat{c}_{t+1} + (1 - \tau) \hat{c}_{t+2}^n + \tau \hat{r}_{t+2}] + \dots \\ &= \hat{c}_t + \frac{\tau \tilde{n}}{r} \Sigma_t^c + \frac{(1 - \tau) \tilde{n}}{r} \Sigma_{t+1}^{cn} + \frac{\tau \tilde{n}}{r} \Sigma_{t+1}^{rn} \end{aligned}$$

where $\Sigma_{t+1}^{cn} = \Sigma_{i=0}^{\infty} \left[\frac{\tilde{n}}{r} \right]^i \hat{c}_{t+1+i}^n$.

Rearrange this so we get an expression of \hat{c}_t as follows

$$\hat{c}_t = \frac{r - \tau \tilde{n}}{r} \Sigma_t^c - \frac{(1 - \tau) \tilde{n}}{r} \Sigma_{t+1}^{cn} - \frac{\tau \tilde{n}}{r} \Sigma_{t+1}^{rn}$$

By this expression, current consumption in the home country is given by an average total consumption minus the consumption of the yet unborn and the interest rate tilting effect of consumption. Of the latter two terms, the tilting

effect is familiar while the latter arises due to our assumption of new generations being born each period.

Similarly, aggregating the foreign Euler equation yields

$$\hat{c}_{t+1}^* = \tau \hat{c}_t^* + (1 - \tau) \hat{c}_{t+1}^{n*} + \tau [\hat{r}_{t+1} + \hat{s}_{t+1} - \hat{s}_t]$$

which in turn leads to the following expression for \hat{c}_t^*

$$\begin{aligned} \hat{c}_t^* = & \frac{r - \tau \tilde{n}}{r} \Sigma_t^{c*} + \frac{r - \tau \tilde{n}}{r} \Sigma_t^s - \frac{(1 - \tau) \tilde{n}}{r} \Sigma_{t+1}^{cn*} - \frac{(1 - \tau) \tilde{n}}{r} \Sigma_{t+1}^s \\ & - \frac{\tau \tilde{n}}{r} \Sigma_{t+1}^{rn} - \frac{\tau \tilde{n}}{r} \Sigma_{t+1}^s + \frac{\tau \tilde{n}}{r} \Sigma_t^s \end{aligned}$$

where $\Sigma_t^s = \sum_{i=0}^{\infty} \left[\frac{\tilde{n}}{r} \right]^i \hat{s}_{t+i}$ and $\Sigma_{t+1}^{cn*} = \sum_{i=0}^{\infty} \left[\frac{\tilde{n}}{r} \right]^i [\hat{c}_{t+1+i}^n - \hat{s}_{t+1+i}]$. The explanation for this expression is similar to that for the home country.

4.5.3 Country difference, cd_t

The consumption differential ($\hat{c}_t - \hat{c}_t^* + \rho \hat{s}_t$) equals

$$\frac{(r - \tau \tilde{n})}{r} [\Sigma_t^c - \Sigma_t^{c*}] - \frac{(1 - \tau) \tilde{n}}{r} [\Sigma_{t+1}^{cn} - \Sigma_{t+1}^{cn*}]$$

After substituting Σ_t^c and Σ_t^{c*} into above equation, we obtain

$$\frac{(r - \tau \tilde{n})}{r} \left[\left(\frac{1}{c} + \frac{s}{c^*} \right) \tilde{\alpha} \hat{r}_{xt} + \Delta d_t + \Delta l_t + \left(\frac{1}{c} + \frac{s}{c^*} \right) r f \Sigma_{2t}^{rn} \right] - \frac{(1 - \tau) \tilde{n}}{r} \Delta c_{t+1}^n$$

where

$$\Delta d_t = \frac{d}{c} \Sigma_t^d - \frac{d^*}{c^*} \Sigma_t^{d*}$$

$$\Delta l_t = \frac{l}{c} \Sigma_t^l - \frac{l^*}{c^*} \Sigma_t^{l*}$$

$$\Sigma_{2t}^{rn} = \hat{r}_{2t} + \frac{\tilde{n}}{r} \Sigma_{t+1}^{rn}$$

$$\Delta c_{t+1}^n = \Sigma_{t+1}^{cn} - \Sigma_{t+1}^{cn*}$$

respectively represents the home country's (changes in) relative financial income, relative labour income, external interest payments and the relative consumption of new-borns.

According to the analysis in this section, we find that the relative consumption, cd_t , in the model is composed of relative financial income, relative labour income, external interest payments, relative consumption of new-borns and excess return by holding the optimal portfolio $\tilde{\alpha}$.

4.6 Steady-state $\tilde{\alpha}$ and Asymmetric asset home bias

In this section, we represent the steady-state portfolio $\tilde{\alpha}$ as the sum of a series of hedging terms by which process the portfolio solution gains an intuitive interpretation. We compare our results with other results in the related literature to show their connections and, more importantly, to demonstrate the innovative aspects of the optimal portfolio in our model. We explore the role of the country asymmetry in accounting for asymmetric asset home bias through generating global imbalances and the attendant hedging motive of external interest payments in the optimal portfolio.

Substituting the expression for cd_t obtained in the previous section into the optimal portfolio condition, we can obtain the solution for $\tilde{\alpha}$ as follows

$$\tilde{\alpha} = \underbrace{-\frac{cc^*}{cs + c^*} \frac{\text{cov}(\Delta d_t, \hat{r}_{xt})}{\text{var}(\hat{r}_{xt})}}_{\text{Self-hedging (-)}} - \underbrace{\frac{cc^*}{cs + c^*} \frac{\text{cov}(\Delta l_t, \hat{r}_{xt})}{\text{var}(\hat{r}_{xt})}}_{\text{Hedging labour income (+)}} + \underbrace{-r f \frac{\text{cov}(\Sigma_{2t}^{rn}, \hat{r}_{xt})}{\text{var}(\hat{r}_{xt})}}_{\text{Hedging interest payment (-)}} + \underbrace{\frac{cc^*}{cs + c^*} \frac{(1 - \tau) \tilde{n} \text{cov}(\Delta c_{t+1}^n, \hat{r}_{xt})}{(r - \tau \tilde{n}) \text{var}(\hat{r}_{xt})}}_{\text{Hedging newborn's consumption (-)}}$$

Recall that $\tilde{\alpha} = (\alpha_1 - z_1)r$ denotes home gross holding of the home asset (multiplied by steady state r). Because the home country is the default supplier of the home asset, a realistic $\tilde{\alpha}$ should be negative and it should satisfy $-rz_1 < \tilde{\alpha} < 0$. In absolute value, $\tilde{\alpha}$ is viewed as the gross external liability of the home country. As the other side of the same coin, the absolute value of $\tilde{\alpha}$ is also viewed by the foreign country as its gross external asset stock.

On the right hand side of the above expression for $\tilde{\alpha}$, in turn the four terms can be interpreted respectively as follows:

1. The hedging of financial income risk (self-hedging). By the asset pricing relation, the (relative) rate of return on an asset is an increasing function of (relative) return on physical capital (financial income), i.e. the (relative) dividend. In other words, $\text{cov}(\Delta d_t, \hat{r}_{xt})$ is positive. So the home asset is a bad hedge against the risk associated with the relative dividend income. Self-hedging is therefore negative.

Our model features wealth division between financial and labour incomes and the associated country asymmetry. In steady state, capitalizable income only accounts for a fraction, $\frac{(r-\tilde{n})}{(r-1)}\delta$, rather than all of *GDP*. This implies that the self-hedging also only represents a fraction instead of all of $\tilde{\alpha}$. With the presence of the country asymmetry, i.e. $\delta \neq \delta^*$, it follows that in general the values of steady state consumption and the real exchange rate deviate from their value in symmetric models, i.e. $c \neq c^*$ and $s \neq 1$. This implies that in general the value of the coefficient $\frac{cc^*}{cs+c^*}$ in the above formula for $\tilde{\alpha}$ is different from that in a symmetric model, $\frac{1}{2}$.

Suppose that we assume all wealth is capitalizable (with no production and investment), the relative dividend income, Δd_t , thus represents the whole source of risk in the consumption differential. According to the asset pricing relation, Δd_t will be given by $\frac{r}{(r-1)}\hat{r}_{xt}$. Besides, if we also abstract from the country asymmetry (and the *OLG* structure), steady state levels of consumption and real exchange rate will be given by $c = c^* = 1$ and $s = 1$. So we have $\tilde{\alpha} = -\frac{r}{2(r-1)}$ and $\alpha = \frac{1}{2}z_1$, i.e. in equilibrium both of the two countries hold half the stock of the local and overseas assets, and we thus return to Lucas's (1982) result of full diversification.

2. The hedging of labour income risk. The sign of this term depends on how labour income co-moves with the financial income. If the two income streams co-move positively, (and because the excess return always co-moves positively with the relative dividend, the excess return of the home asset will co-move positively with relative labour income as well) the home asset is a bad hedge against home labour income risk. The hedging term for labour income risk will therefore be negative. Together with a negative term for self-hedging, this implies that portfolio allocation exhibits a foreign bias (Baxtor and Jermann, 1997). If, however, the two income streams co-move negatively, by similar logic, the home asset will be a good hedge against home labour income risk. The hedging term for labour income risk will thus be positive. This offsets the effect of a negative self-hedging and reduces the share of overseas asset in the home country's portfolio. The portfolio allocation can thus exhibit home bias.

By following Heathcote and Perri (2013) in assuming a world with the production of two imperfectly substitutable goods and a home bias in commodity trade, the financial and labour incomes in the model are in fact negatively corre-

lated. For a brief explanation, in such an environment, the cheaper home good in response to a positive shock to home productivity can very effectively stimulate (relative) investment in the home country. The rise in (relative) investment, however, tends to reduce the (relative) dividend by the fact that $d_t = \frac{q_t}{p_t}x_t - l_t - i_t$. Given that labour income always increases in response to positive supply shocks, it follows that the dividend (which declines) and labour income (which rises) move in opposite direction, i.e. $cov(\Delta l_t, \hat{r}_{xt}) < 0$ which therefore leads to a positive hedging term for labour income risk. Note that due to the presence of the country asymmetry, this hedging term also involves a generally asymmetric coefficient of $\frac{cc^*}{cs+c^*} \neq \frac{1}{2}$.

3. The hedging of interest payment risk. As demonstrated in Section 4.3, the country asymmetry in the model generates non-trivial net external positions in steady state. As a result, the income and consumption of the two countries contain non-trivial international interest payments (Section 4.5). The third term in $\tilde{\alpha}$ captures households' motives to hedge against the risk associated with this income stream.

To find the sign of this hedging term, notice that in the model, the home country is a debtor with a steady state negative net external position, $f < 0$. It thus has to pay external interest payments. Let us consider when the home asset's excess return is high. Recall from the described variable impulse above, for the country which experiences positive productivity shock, unlike the endowment economy in Chapter 3, the return to that country's asset will be relatively low instead of being relatively high due to the response of investment. In other words, when the home asset's excess return is high, the foreign (instead of the home) country's productivity is hit by a positive shock. The amount of interest payments is relatively low so the home country's disposable income and therefore consumption is also high. This indicates that the home asset is a bad hedge against interest payment risk for the home country. The hedging term for interest payments is therefore negative for the home country. That is to say, it biases the portfolio toward the foreign asset, which tends to reduce the degree of asset home bias for the home country.

4. The hedging of new-born's consumption. The emergence of this term is due to the consumption of the yet unborn future generations in the model. Recall that in Section 4.5, we obtain the (per capita) current consumption by averaging

the sum of all future consumption with a deduction for the consumption of yet unborn future generations. The hedging then corresponds to the risk associated with this deduction. Note that this hedging also appears in Chapter 3 in the form of Eq.(6.5), they emerge for the same reason.

To analyse its properties, consider when a positive shock hits home country productivity, the consumption of all future generations goes up and the required downward adjustment of current consumption is high. Therefore current consumption is low. At the same time, due to the response of investment, the positive shock depresses the relative dividend and thus the excess return of the home asset. That is, when home country relative consumption is low, the excess return on the home asset is also low. The home asset is thus a bad hedge against the risk from new-born consumption. The hedging of new-born consumption is therefore negative. Note that the sign of this hedging term is positive in Chapter 3. The change in sign in this chapter occurs because, unlike in the previous chapter, the positive supply shock in the home country decreases, instead of increases, the relative excess return of the home asset in the current model.

Having obtained $\tilde{\alpha}$, in Section 4.C of the Appendix to this chapter we show how the analogous portfolio holding in the foreign country $\tilde{\alpha}^* \equiv (\alpha_2^* - z_2) r$ can be derived from the above expression from the fact of $\tilde{\alpha}^* = \tilde{\alpha} + f^*$. It follows that

$$\begin{aligned} \tilde{\alpha}^* = & \underbrace{\frac{cc^*}{cs + c^*} \frac{\text{cov}(\Delta d_t^*, \hat{r}_{xt}^*)}{\text{var}(\hat{r}_{xt}^*)}}_{\text{Self-hedging (-)}} - \underbrace{\frac{cc^*}{cs + c^*} \frac{\text{cov}(\Delta l_t^*, \hat{r}_{xt}^*)}{\text{var}(\hat{r}_{xt}^*)}}_{\text{Hedging labour income (+)}} \\ & - \underbrace{r f^* \frac{\text{cov}(\Sigma_{1t}^{rn}, \hat{r}_{xt}^*)}{\text{var}(\hat{r}_{xt}^*)}}_{\text{Hedging interest payment (+)}} + \underbrace{\frac{cc^*}{cs + c^*} \frac{(1 - \tau) \tilde{n} \text{cov}(\Delta c_{t+1}^{n*}, \hat{r}_{xt}^*)}{(r - \tau \tilde{n}) \text{var}(\hat{r}_{xt}^*)}}_{\text{Hedging newborn's consumption (-)}} \end{aligned}$$

where the notions of certain variables are redefined from the perspective of the foreign country, i.e. $\Delta d_t^* = -\Delta d_t$, $\Delta l_t^* = -\Delta l_t$, $\hat{r}_{xt}^* = -\hat{r}_{xt}$ respectively denote the relative dividend, labour income and excess return of the foreign asset over the home asset. $\Sigma_{1t}^{rn} = \hat{r}_{1t} + \frac{\tilde{n}}{r} \Sigma_{t+1}^{rn}$ denotes the sum of present value of all asset 1's future rates of return (asset 1 is the overseas asset for the foreign country).

The four hedging terms in $\tilde{\alpha}^*$ have the same meaning as those in $\tilde{\alpha}$. Moreover, except for the hedging of interest payments, the other three terms have the same sign and size as their analogues in the expression for the home portfolio hold-

ing. To verify, notice that the involved covariances $cov(\Delta d_t^*, \hat{r}_{xt}^*)$, $cov(\Delta l_t^*, \hat{r}_{xt}^*)$, $cov(\Delta c_{t+1}^{n*}, \hat{r}_{xt}^*)$ and variance $var(\hat{r}_{xt}^*)$ have the same value as their counterparts in $\tilde{\alpha}$'s expression. In other words, in the foreign country, the local asset (asset 2) is also a bad hedge for financial income and new-born's consumption while a good hedge for labour income, so the foreign holding of the foreign asset involves the same short position due to self-hedging, the same long position due to the hedging of labour income and the same short position due to the hedging of new-born's consumption to those in the expression for the home holding of home asset, $\tilde{\alpha}$.

However, the hedging of interest payments in the foreign country works in the opposite direction to that for the home country. To see this, notice that in the model, the foreign country is a creditor with a steady state positive net external position, $f^* = -f > 0$. The foreign country therefore receives international interest revenues. And when the foreign asset's excess return is high, i.e. the home country is hit by a positive productivity shock, the amount of the revenues is relatively low and so the foreign country's disposable income and consumption will be low, which indicates that the local asset as a good hedge against this risk. The hedging term for interest payments is thus positive in the foreign country. That is to say, it biases the portfolio towards the foreign asset, which tends to enhance the degree of asset home bias in the foreign country.

When the two countries are identical, their portfolio choices should be also identical ($f = -f^* = 0$), $\tilde{\alpha} = \tilde{\alpha}^*$. What we find above is that introducing the country asymmetry in δ breaks down this symmetry to the extent that the hedging of the return on the net external position emerges ($f = -f^* < 0$) and the hedging actually operates in the opposite direction across the two countries. It lowers the demand for the home asset in the home (debtor) country while it increases the demand for the foreign asset in the foreign (creditor) country, so $\tilde{\alpha} < \tilde{\alpha}^* < 0$. If the two countries supply the same amount of assets, then it must be the case that the home country will exhibit a relatively lower degree of asset home bias than the foreign country. Nevertheless, a relative higher δ also allows the home country to supply a relatively higher amount of the asset (Caballero et al., 2008), i.e. $z_1 > z_2 > 0$. So in response to a rise in the gap between δ and δ^* , the demand and supply effects arise simultaneously and compete in giving rise to asymmetric asset home bias in different directions. If the values of $\tilde{\alpha}$ and

Parameter	Description	Value
β	Discount factor	0.97
n	Net population growth rate	0.001
δ	Wealth division parameter in the home country	0.12
δ^*	Wealth division parameter in the foreign country	0.11
μ	Persistence of productivity shocks	0.95
ϕ	Elasticity of substitution between intermediate goods	0.9
κ	Share of local intermediate goods in final goods	0.65
τ	Parameter in stability condition	0.9991

Table 4.2: Benchmark calibration

z_1 (or/and $\tilde{\alpha}^*$ and z_2) are sufficiently low (high), then the portfolio in the home country can be less home biased than that in the foreign country. However if the opposite happens, the pattern of the relative degree of asset home bias across countries will be reversed. We argue in the following sections of this chapter that for reasonable parameterization and for the portfolios to fall into an economically meaningful interval, the asset demand effect dominates the asset supply effect, which implies that when $\delta > \delta^*$, the home country will possess a less biased portfolio than the foreign country. We show the result by performing a baseline simulation of the model in the next section. In Section 4.8, we will carry out a series of sensitivity analyses of the result.

4.7 Model simulation

In this section, we simulate the model as a benchmark to assess its quantitative performance. As in Chapter 3, the main purpose of the simulation is to verify the reasoning in the previous sections and to evaluate the model's ability in producing relevant results, i.e. a steady state of the model, impulse responses and portfolio choices, up to a reasonable order of magnitude. For this reason, it suffices if we parameterize the model with empirically plausible values. Following Chapter 3, we use a year as one period in the following simulation.

4.7.1 Parameterization

The parameter values we use in the baseline simulation is collected in Table 4.2. The discount factor in the utility function β is set at 0.97. In Chapter 3, we used a larger value of the net population growth rate, n , at 0.01. Although the current simulation is still far from a seriously data-based one, we hope to yield portfolio choices much closer to reality than the case in Chapter 3. n is chosen here so as to make sure that, firstly, it is large enough to ensure the stability of the model, i.e. the condition $\tau < 1$ is respected. And then, given this, we make it small to yield a small amount of new-born's consumption adjustments in cd_t and thus a low value of the hedging of new-born's consumption in the final $\tilde{\alpha}$. So we choose n at 0.001. Besides, together with the fact that $\beta = 0.97$, this value of n also implies a real interest rate close to 3 percent, consistent with one of the calibration targets as in Caballero et al. (2008). Also as in Caballero et al. (2008), we choose the value of the home wealth division parameter δ to be 0.12. To analyse the consequence of a shock decline in asset supply, they calibrate a δ^* of 0.08 based on the foreign country's experiences during the 1997 Asian crisis episode, which should result in a lower value than that in normal times. For this reason, we pick δ^* at a higher level but, at the same time, one percentage point lower than δ reflecting the relative underdevelopment of the foreign country. When calibrating their model with sector-specific capital intensity, Jin (2012) uses the same value for the labour intensive sector. The persistence of the productivity shock is set at 0.95 which is its median estimate by Smets and Wouters (2007). ϕ is chosen to be 0.9 following Heathcote and Perri (2013). And κ is set at 0.65 implying the local good account for 65% of the input into final goods in each country. Given the relatively low value for δ and δ^* we choose ϕ and κ here to be low relative to their typical estimates in the literature. This is to ensure that $\tilde{\alpha}$ falls in a meaningful range for home bias. We return to discuss this point in the sensitivity analysis.

Variable	Description	Value
r	Gross international interest rate	1.0311
z, k	Total asset price/supply and Capital stock	3.8645
w	Total asset demand	3.8645
f	Net foreign assets	0
c	Consumption	0.9961
i	Investment	0.0039
h	Labour supply (hours)	0.8317
l	Labour income	0.88
d	Dividend	0.1161
x	Output of intermediate goods	1
x_h	Input of local intermediate good in final good	0.65
x_f	Input of overseas intermediate good in final good	0.35
y	Final good output	1
q	Price of home intermediate good	1
q^*	Price of foreign intermediate good	1
p	Price index of home final good	1
p^*	Price index of foreign final good	1
s	Exchange rate	1
g	Wage rate	1.0581
MPK	Marginal product of capital	0.0311
MPL	Marginal product of labour	1.0581
c^n	New-born's consumption	0.8766

Table 4.3: Model steady states: Symmetric case

Variable	Description	Value
r	Gross international interest rate	1.0310
z_1, k	Home asset price/supply and Capital stock	3.8652
z_2, k^*	Foreign asset price/supply and Capital stock	3.5574
w_1	Home total asset demand	3.6828
w_2	Foreign total asset demand	3.7397
$f (= -f^*)$	Home (foreign) net foreign asset	-0.1823
c	Home consumption	0.9907
c^*	Foreign consumption	1.0019
i	Home investment	0.0039
i^*	Foreign investment	0.0035
h	Home labour supply (hours)	0.8408
h^*	Foreign labour supply (hours)	0.8461
l	Home labour income	0.8800
l^*	Foreign labour income	0.8900
d	Home dividend	0.1161
d^*	Foreign dividend	0.1065
x	Output of home intermediate good	1.0097
x^*	Output of foreign intermediate good	0.9904
x_h	Demand for x by home agents	0.6521
x_f	Demand for x^* by home agents	0.3425
x_h^*	Demand for x by foreign agents	0.3576
x_f^*	Demand for x^* by foreign agents	0.6479
y	Final good output in home country	0.9945
y^*	Final good output in foreign country	1.0054
q	Price of home intermediate good	0.9729
q^*	Price of foreign intermediate good	0.9960
p	Price index of home final good	0.9823
p^*	Price index of foreign final good	0.9864
s	Exchange rate	0.9960
g	Home wage	1.0281
g^*	Foreign wage	1.0377
MPK	Marginal product of capital in production of x	0.0313
MPK^*	Marginal product of capital in production of x^*	0.0307
MPL	Marginal product of labour in production of x	1.0568
MPL^*	Marginal product of labour in production of x^*	1.0419
c^n	Newborn's consumption in home country	0.8767
c^{n*}	Newborn's consumption in foreign country	0.8867

Table 4.4: Model steady states: Asymmetric case

4.7.2 Steady states in the two countries

Before proceeding to the asymmetric steady state, let us demonstrate the symmetric situation first. In Table 4.3, we report the steady state of the model where δ^* is set the same level as δ at 0.12. This gives us a level of r at 1.0311 which comes from the formula $r = (1 + \delta n) / \beta$ by our previous analysis. It also represents the equalized level of the autarky interest rate r^a in both countries. The force causing differing (country-specific) rates of return disappears and net external positions in steady state are zero, $f = f^* = 0$. The total asset supply, z , and demand, w , are thus both equal and given by $\delta / (r - 1) = 3.8645$. The value of the capital stock, k , is in line with that of z , so is given by 3.8645 as well. As a constant fraction of k , investment, i , is given by $nk = 0.0039$. Except for i , the remaining expenditure on $gdp = 1$ is consumption which accounts for $c = 0.9961$. Although households in the two countries both feature preference bias, the fact that they have the same wealth guarantees balanced relative prices of x and x^* , i.e. $q = q^* = 1$. This in turn implies the solution of the other price indices is unity in steady state, i.e. $p = p^* = s = 1$. Because of this, the quantities of intermediate goods and final goods are also given by 1 under the GDP normalization. Moreover, the shares of the demands for the local and overseas goods in each country's basket are divided exactly according to $\kappa = 0.65$ and $(1 - \kappa) = 0.35$. Working hours supplied by each economy, h , is obtained at 0.8317, which is linked to the above c , y and k through the optimal condition of labour supply and production function. The related wage rate is $g = 1.0581$. With all prices being 1, labour income, $l = gh/p$, amounts to 88 percent of GDP , i.e. 1 minus δ multiplied by 100% of the total income $gnp = gdp = 1$. Correspondingly, financial income amounts to the remaining 12 percent of GNP . After satisfying the need for capital accumulation, the profit of firms that is paid as dividend is given by $d = 0.1161$, consistent with z from the point of view of asset pricing. Finally, we obtain new-born consumption, c^n , at 0.8766. This is simultaneously consistent with the value of l by the fact that new generations consume the average of their life-time (discounted) labour income and that of c by the relation between c^n and c that is required by the per capita aggregation relation.

In Table 4.4, we report how the above steady state changes in an asymmetric

case when δ^* is set one percentage point lower than δ . As expected, a relatively higher δ induces a relatively higher asset supply $z_1 = 3.8652$ and lower asset demand $w_1 = 3.6828$ in the home country and a relatively lower asset supply $z_2 = 3.5574$ and higher asset demand $w_2 = 3.7397$ in the foreign country. (In other words, the total wealth of a country is about 3.7 times of GDP . As a reference, Caballero et al. (2008) uses an estimate of 4 times of country wealth to GDP for the advanced home country. According to the dataset collated by Heathcote and Perri (2013), however, this ratio averages at around 2.5 over the period 1970–2010 in the *U.S.*.) There is thus a steady state negative net external position in the home country $f < 0$ and a steady state positive net position in the foreign country $f^* > 0$. So NFA global imbalances emerge. In terms of the size of the imbalances, the net external positions are approximately 18% of the steady state GDP , i.e. $f = -f^* = 0.1823$ as is shown in the table. This is similar to what happened to *U.S.* during 1999–2010 when the percentage has an average of 18.7 according to Lane and Milesi-Ferretti's (2007) (extended) dataset. The international interest rate, r , declines slightly compared to the symmetric case and is now at 1.0310, reflecting the fact that the foreign autarky interest rate is lower than before or/and the foreign and global asset supply is reduced while demand is boosted. By the same argument as explained in the symmetric case, when z_2 becomes lower, so do k^* and i^* . Together with the fact that the foreign country maintains a higher GNP by receiving external interest revenues, i.e. $y^* = 1 + (r - \tilde{n}) f^* > y = 1 + (r - \tilde{n}) f$, a lower level of i^* in turn means a higher level of consumption c^* in the foreign country.

Consider the market clearing condition of the two intermediate goods, x and x^* , when the home country is relatively poor while the foreign country is relatively wealthy. With the same degree of preference over the local goods in the two countries, there must be an excess supply of x and an excess demand for x^* . To achieve equilibrium, the relative price of home good is thus depressed, $q < q^*$, and so is the home CPI , i.e. $p < p^*$. The home basket is thus cheaper than the foreign one, i.e. $s < 1$. (For convenience of exposition, when computing steady-state relative prices, we normalize the home price of foreign good to unity, i.e. $q^*/s = 1$, so the terms of trade of the home country, defined as the price of exports in terms of imports, is equal to $q/(q^*/s) = q$ in the model.) Steady state $GDPs$ being normalised at 1 in the two countries, the output of the cheaper

good is thus raised, i.e. $x > x^*$. Due to the fact that $\delta > \delta^*$, the demands for local and overseas goods are not distributed in the exact proportions of κ and $(1 - \kappa)$ any more.

Since the labour income share increases in the foreign country $(1 - \delta^*) > (1 - \delta)$, l^* is relatively higher now. Correspondingly, the financial income share in the foreign country decreases. Given that investments are small compared with profit, even though i^* is reduced, d^* is still lower. Again, the fall in d^* also echoes the decrease in z_2 which is the present value of discounted future dividends. A higher l^* also implies a higher c^{n*} because new-borns enjoy a higher permanent income in terms of human wealth now. Because from the per capita aggregation relation, we obtained c^{n*} as a positive function of c^* in Section 4.3, the higher c^{n*} is also consistent with the higher c^* .

4.7.3 Impulse responses

We discuss the first-order behaviour of the model around the above steady state in this subsection. Taking the case of a positive technology shock in the home country as an example, we depict the associated responses of a selection of variables in Figure 4.2.

Panel (a) of the figure shows the one unit rise in the home country productivity, ε_t . The supply shock depresses the price of the home intermediate good because the home country can produce more of it and the foreign good becomes relative scarcer, i.e. $\hat{q}_t < 0$. Due to the lower price, the demand for the home good can thus be relatively higher, i.e. $\hat{x}_t > 0$ (for both x_{ht} and x_{ht}^*), which at the same time matches the increase in the supply of the home good. Given that the home final basket is made of a relatively high proportion of the home good ($\kappa > 1/2$), the deterioration of the terms of trade translates into a depreciation of the home basket, i.e. $\hat{s}_t = \hat{p}_t - \hat{p}_t^* < 0$, as is illustrated in Panel (c).

The lower price of the home good also strongly stimulates home country investment, $\hat{i}_t > 0$, which uses the local good intensively. Relative investment rises significantly on impact, $\hat{i}_t - (\hat{i}_t^* - \hat{s}_t) > 0$, even though in this process the depreciation of the exchange rate partially offsets the effect. In Panel (d) of the figure, this is illustrated by the fact that the higher solid line is above the dashed line. Note that in order to ease comparison, all foreign variables in the figure

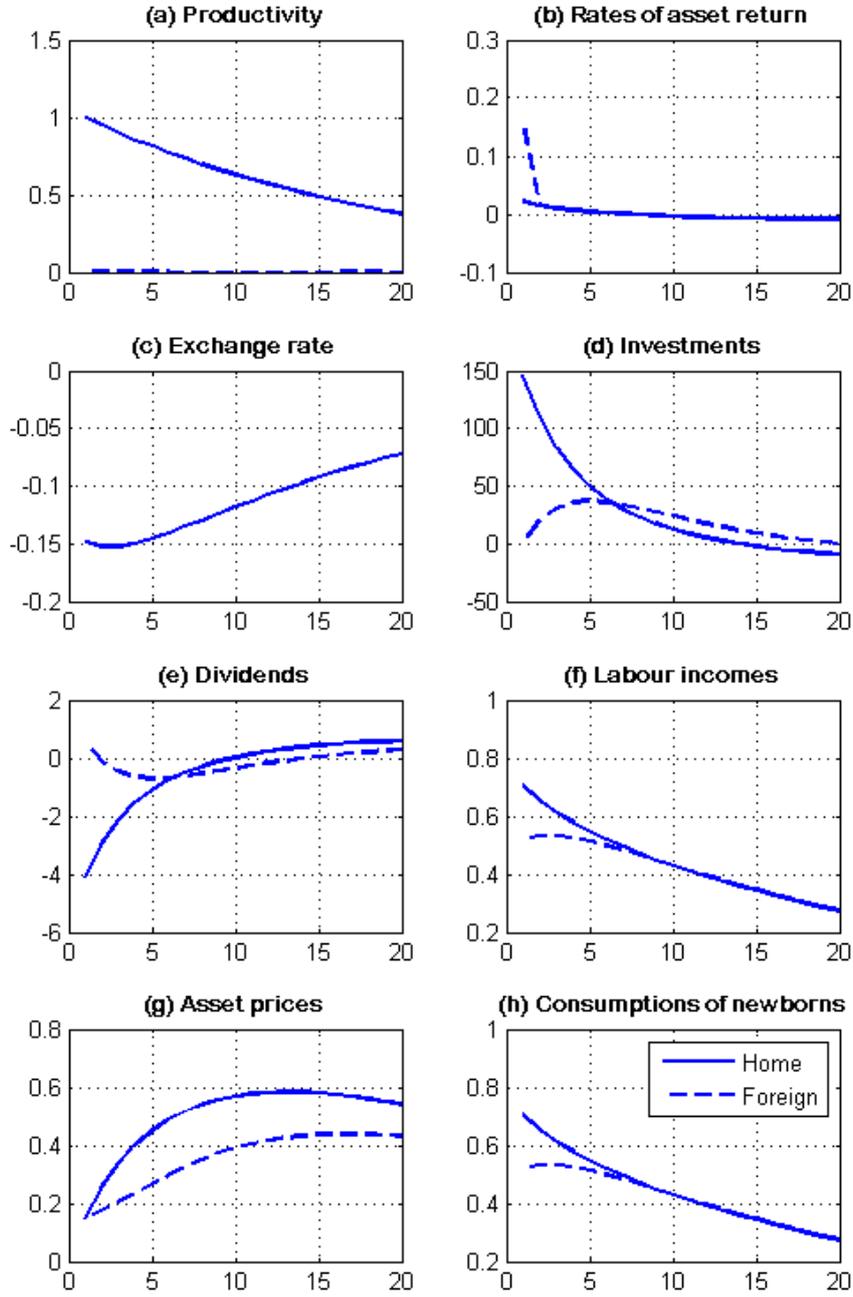


Figure 4.2: Impulse responses to a positive shock to home country productivity

have been converted into units of the home consumption basket, so the dashed line in Panel (d) represents $(\hat{i}_t^* - \hat{s}_t)$ rather than \hat{i}_t^* . While \hat{s}_t is negative, this implies that \hat{i}_t^* actually lies even lower than the dashed line appearing here.

The positive technological shock increases the marginal products of inputs, including capital and labour, in the home country. Relative labour supply and wage are higher than before. Because $l_t = \frac{q_t}{p_t} h_t$, together with a stabilizing relative price $\hat{p}_t < \hat{p}_t^*$, these lead to a higher relative labour income, i.e. $\hat{l}_t - (\hat{l}_t^* - \hat{s}_t) > 0$. This is shown in Panel (f).

In the model, final goods can be used to either consume or invest. The price of capital is thus also given by the price of final goods. The depreciation of the home consumption basket described above immediately boosts the value of the existing capital stock, with the current parameterization, so much that home capital (relative to foreign capital) goes down, i.e. $\hat{k}_t - (\hat{k}_t^* - \hat{s}_t) < 0$. This process of home capital devaluation rationalizes the initial response of asset prices as is shown in Panel (g). That is, on impact, both z_{t+1} and z_{t+1}^* jump while \hat{z}_{t+1} is very close to, but lower, than \hat{z}_{t+1}^* .

Now consider the dividend distributed by firm. Recall that the dividend is defined as the difference between the firm's revenue and its expenditure on labour employment and investment, i.e. $d_t = \frac{q_t}{p_t} x_t - l_t - i_t$. Even though x_t is higher than before, the selling price, $\frac{q_t}{p_t}$, combined with increased payments on wages and investment in fact reduces the relative home dividend, i.e. $\hat{d}_t - (\hat{d}_t^* - \hat{s}_t) < 0$. Panel (e) illustrates the decrease in relative d_t .

Asset returns are defined as the dividend d_t and capital gains z_{t+1} over the cost of asset purchase z_{t-1} . Because \hat{z}_{t+1} accounts for the largest proportion of the movement, it determines the sign of the change in rates of return. From Panel (b) of the figure, since \hat{z}_{t+1} and \hat{z}_{t+1}^* are higher, we observe that both \hat{r}_{1t} and \hat{r}_{2t} increase in response to the shock. However, because \hat{z}_{t+1} and \hat{z}_{t+1}^* are close, changes in dividends determine the relative rates of return. In Panel (b), since $\hat{d}_t - (\hat{d}_t^* - \hat{s}_t) < 0$, the rate of return for the home asset is less affected by the shock than that of the foreign asset on impact, which results in a temporary negative value for \hat{r}_{xt} . From the second period on after the shock, $\hat{r}_{1t+i} = \hat{r}_{2t+i}$ ($i \geq 1$) for the same reason as that appears in (Section 3.2 and Figure 1 of) Heathcote and Perri (2013), i.e. the optimal portfolio is free to re-structure, which sweeps away the return difference between the two assets.

Lastly, we depict the responses of new-borns' consumption in Panel (h). It is not surprising to discover that the relative consumption of new-borns is positive after the shock, i.e. $\hat{c}_t^n - (\hat{c}_t^{n*} - \hat{s}_t) > 0$, given that new generations in the home country expect a relatively higher labour income as a result of the shock.

Note that in all of the above analysis, we only consider the initial responses of the variables. While in fact the relative d_t , l_t and c_t^n will reverse their signs some periods after the shock (Panel (e), (f) and (h)), the sums of all discounted expected future relative d_t , l_t and c_t^n , i.e. the Δd_t , Δl_t and Δc^n defined in Section 4.5, still feature the same signs as those of the above initial responses due to the fact of discounting and the small magnitudes of these terms after reversal. Recognizing this point, the above analysis can be summarised as follows. In response to a positive supply shock to the home country, i.e. $\Delta x_t > 0$, the relative rate of asset returns declines, i.e. $\hat{r}_{xt} < 0$, while the relative dividend goes down so $cov(\Delta d_t, \hat{r}_{xt}) > 0$. However, relative labour income goes up so $cov(\Delta l_t, \hat{r}_{xt}) < 0$. The rate of return for both assets goes up so $cov(\hat{r}_{1t}, \hat{r}_{xt})$ and $cov(\hat{r}_{2t}, \hat{r}_{xt})$ are both negative. In addition, relative consumption of home country newborns goes up so $cov(\Delta c^n, \hat{r}_{xt}) < 0$.

Return now to the expression $\tilde{\alpha}$ in Section 4.6. These numerical results verify our analysis in that section. $\tilde{\alpha}$ is made up from negative self-hedging, positive hedging of labour income, negative hedging of interest rate payments and new-born's consumption in the home country. Below, we consider the size of these hedging terms, the optimal α s and the resulting portfolio allocations under the symmetric and asymmetric simulations.

4.7.4 Portfolio allocation

In this subsection, we look at the optimal portfolios and the related portfolio allocation across countries in the model. We show that when δ^* decreases, in comparison to the symmetric case, two results emerge. Firstly, the degree of asset home bias in both countries deepens. Secondly, the increase in the degree of asset home bias in the home country is more significant than in the foreign country. We explore the explanations for the emergence of these results through portfolio decomposition.

Table 4.5 is used for this subsection. In the table, steady-state portfolios α s for

Variable	Description	$(\delta = \delta^*)$	$(\delta > \delta^*)$
$\alpha = \tilde{\alpha}/r$	Gross holding of home asset at home	-0.7311	-0.7279
$cc^*/(cs + c^*)$	Coefficient in $\tilde{\alpha}$	0.4981	0.4991
$\frac{cc^*}{(cs+c^*)} \frac{(1-\tau)\tilde{n}}{(r-\tau\tilde{n})}$	Coefficient in $\tilde{\alpha}$	0.0142	0.0143
$cov(\Delta d_t, \hat{r}_{xt})$	Covariance between Δd_t and \hat{r}_{xt}	0.1551	0.1459
$cov(\Delta l_t, \hat{r}_{xt})$	Covariance between Δl_t and \hat{r}_{xt}	-0.0982	-0.1002
$cov(\Sigma_{2t}^{rn}, \hat{r}_{xt})$	Covariance between Σ_{2t}^{rn} and \hat{r}_{xt}	-0.0288	-0.0295
$cov(\Delta c_{t+1}^n, \hat{r}_{xt})$	Covariance between Δc_{t+1}^n and \hat{r}_{xt}	-0.0628	-0.0656
$var(\hat{r}_{xt})$	Variance of \hat{r}_{xt}	0.0388	0.0390
$std(\Delta d_t)$	Standard deviation of Δd_t	0.7877	0.7391
$std(\Delta l_t)$	Standard deviation of Δl_t	0.4987	0.5108
$std(\Sigma_{2t}^{rn})$	Standard deviation of Σ_{2t}^{rn}	0.1612	0.1655
$std(\Delta c_{t+1}^n)$	Standard deviation of Δc_{t+1}^n	0.3188	0.3367
$cor(\Delta d_t, \hat{r}_{xt})$	Correlation between Δd_t and \hat{r}_{xt}	1	0.9993
$cor(\Delta l_t, \hat{r}_{xt})$	Correlation between Δl_t and \hat{r}_{xt}	-1	-0.9930
$cor(\Sigma_{2t}^{rn}, \hat{r}_{xt})$	Correlation between Σ_{2t}^{rn} and \hat{r}_{xt}	-0.9076	-0.9015
$cor(\Delta c_{t+1}^n, \hat{r}_{xt})$	Correlation between Δc_{t+1}^n and \hat{r}_{xt}	-1	-0.9859
$vae(\Delta d_t, \hat{r}_{xt})$	$std(\Delta d_t) / std(\hat{r}_{xt})$	4	3.7408
$vae(\Delta l_t, \hat{r}_{xt})$	$std(\Delta l_t) / std(\hat{r}_{xt})$	2.5327	2.5853
$vae(\Sigma_{2t}^{rn}, \hat{r}_{xt})$	$std(\Sigma_{2t}^{rn}) / std(\hat{r}_{xt})$	0.8184	0.8378
$vae(\Delta c_{t+1}^n, \hat{r}_{xt})$	$std(\Delta c_{t+1}^n) / std(\hat{r}_{xt})$	1.6188	1.7039
$\alpha[1]$	Self-hedging	-1.9323	-1.8097
$\alpha[2]$	Hedging of labour income	1.2235	1.2428
$\alpha[3]$	Hedging of imbalance term	0	-0.1377
$\alpha[4]$	Adjustment to new-borns' consumptions	-0.0222	-0.0233
$\alpha_1 = z_1 + \alpha$	Home holding of home asset	3.1335	3.1373
$\alpha_2 = w_1 - \alpha_1$	Home holding of foreign asset	0.7311	0.5456
$\alpha_1^* = -\alpha$	Foreign holding of home asset	0.7311	0.7279
$\alpha_2^* = w_2 - \alpha_1^*$	Foreign holding of foreign asset	3.1335	3.0118
$\pi = \alpha_1/z_1$	Asset home bias in the home country	0.8108	0.8117
$\pi^* = \alpha_2^*/z_2$	Asset home bias in the foreign country	0.8108	0.8466

Table 4.5: Steady-state portfolios

the symmetric and asymmetric cases can be found on the first row. According to the previous definition, for each case, we decompose α into the self-hedging α [1], the hedging of labour income α [2], the hedging of external interest payments α [3] and the adjustment to new-born's consumption α [4] which are all shown from row 21 to 24 of the table. To trace the process of decomposition, we also report the value of the coefficients of $\frac{cc^*}{cs+c^*}$ and $\frac{cc^*}{(cs+c^*)} \frac{(1-\tau)\hat{n}}{(r-\tau\hat{n})}$, variance of \hat{r}_{xt} and associated covariances in rows 2 to 8. In addition, in rows 9 to 20 the volatility of the different components of income and the correlation and variability effects are displayed. The implied portfolio allocations across the world and the degree of asset home bias in each country are computed at the end of the table.

Let us start from examining the symmetric result. Under the symmetric case, we obtain a gross holding of the local asset of -0.7311 , which means, with the same z and w of 3.8645 across countries, each country holds the local asset to the amount of 3.1335 and holds the overseas asset to the amount of 0.7311 . This in turn leads to a domestic ownership share of the local asset of $81.08\% > 50\%$ in both countries. A symmetric asset home bias emerges. Use π to denote this degree of asset home bias, so we have $\pi = \pi^* = 81.08\%$. This is qualitatively the same result as obtained by Heathcote and Perri (2013) (except that now α includes an adjustment to newborn's consumption in addition to the other hedging), i.e. once we drop the country asymmetry in the model, the negative correlation between the financial and labour incomes implies countries overweight the local asset to the same degree. To check the relative roles of the various hedging terms, from the result of portfolio decomposition, we observe that self-hedging, α [1], is -1.9323 , a short position of the home asset up to nearly half of z_1 in magnitude. The hedging of labour income risk, α [2], is 1.2235 . This implies an increase in the net holding of the home asset which is the key portfolio component in terms of generating asset home bias. Besides, α [4] is -0.0222 and α [3] does not exist in the symmetric case.

Turning now to the asymmetric case, α is -0.7279 . With differing zs , the π s in the two countries are computed at respectively 81.17% and 84.66% . Comparing them with the previous 81.08% , we observe the following two results. Firstly, the ratios in the two countries are both higher than before. And secondly, the ratio in the home country is relative lower than that in the foreign country.

The reason for the first result lies in the change in the relative importance

of $\alpha [1]$ and $\alpha [2]$. By inspecting the distribution of $\alpha [i]$ under the asymmetric case, first of all, it is obvious that all components still have the expected signs. However, in terms of size, as the major force of portfolio component which drives the country portfolio toward the foreign asset, the self-hedging $\alpha [1]$ becomes less important. In contrast, the other components, especially the hedging of labour income $\alpha [2]$ which is the major force in driving country portfolio toward the local asset, becomes more important. For given asset supplies, this change in the relative importance of $\alpha [1]$ and $\alpha [2]$, by yielding a low absolute value of α everywhere, are enough to generate a high π in the both countries. However, in the model, asset supplies are not fixed in response to the change in δ s. In the current asymmetric simulation, a lower δ^* create a lower r internationally. Recall that $z_1 = \frac{\delta}{r-1}$, with the same δ and a lower r , the asset supply in the home country increases. This means for π to stay at the same level, $|\alpha|$ should also increase. A low $|\alpha|$ implied by the aforementioned change in relative importance of $\alpha [1]$ and $\alpha [2]$ therefore implies a higher π ($= 1 - |\alpha|/z_1$ where $|\alpha| \downarrow$ while $z_1 \uparrow$) in the home country. For the asset supply in the foreign country (recall that $z_2 = \frac{\delta^*}{r-1}$) the effect of a decrease in δ^* , as the first-order effect, outweighs that of a subsequent lower r . So z_2 decreases in response. The degree of asset home bias in the foreign country, π^* ($= 1 - |\alpha|/z_2$ where $|\alpha| \downarrow$ and $z_2 \downarrow$), will go up for sufficiently low $|\alpha|$. In summary, π and π^* are higher because the positive $\alpha [2]$ becomes sufficiently more important than the negative $\alpha [1]$, which tends to yield a less negative α across the two countries.

The reason for the second result lies in the emergence of $\alpha [3]$ s. As explained, a lower δ^* relative to δ gives rise to the steady state *NFA* global imbalances, i.e. it results in a steady state negative net external position in the home country and a steady state positive net external position in the foreign country. This expose the two countries to the risk of external interest payments in addition to the *GDP* income risk. For the foreign country, the local asset (the asset 2) does a good job in terms of hedging against this risk. But for the home country, the overseas asset (again the asset 2) is a better choice to hedge against this risk. Both of the two facts encourage the demands for the foreign asset and depress that for the home asset. The asymmetry therefore tends to decrease the home bias degree in the home country and increase the home bias degree in the foreign country. Our simulation results in $\alpha [3] = -0.1377$ and $\alpha^* [3] = f^* + \alpha [3] = 0.0446$. The

result that $\pi < \pi^*$ indicates that the effect coming from opposite hedging, i.e. $\alpha [3] < 0 < \alpha^* [3]$, exceeds that from supply side, i.e. $z_1 > z_2$.

Now, consider the underlying causes of the distribution of effects on the different $\alpha [i]$. In particular, why does $\alpha [1]$ shrink while the other hedging terms grow? Our previous analysis of the determination of the hedging terms as variance-covariance ratios provides further insights. First of all, from Table 4.5, we notice that once $\delta > \delta^*$, in comparison to the symmetric case, $var(\hat{r}_{xt})$ increases on the one hand. On the other hand, $cov(\Delta d_t, \hat{r}_{xt})$ decreases while the other covariances increase. These facts imply the changes in the relative importance of $\alpha [i]$ s. To see this more transparently, following Chapter 2, we break down the hedging terms into the correlation and variability effects. Note from the table, when $\delta = \delta^*$, except for $cor(\Sigma_{2t}^{rn}, \hat{r}_{xt})$ which is slightly lower, other correlations approach to (positive or negative) unity. Introducing a lower δ^* decreases all these correlations, which means that the collapse of the symmetry in the model weakens assets' relevance in hedging against all income risks in general. This 'quality-type' effect points to a reduction in all components of α . (See Section 2.3.4 for more detailed interpretation of the correlation effect and its role in affecting the size of portfolio holdings.) Nevertheless, note that the relative changes in the variability effects differ. The variability effect associated with the hedging of financial income risk declines while those associated with the other hedging terms rise. This is because intuitively a low δ^* lowers the global share of financial wealth. So the volatility of financial wealth accounts for a smaller proportion of the total wealth volatility in the model. Correspondingly, the volatility of the other types of wealth accounts for a remaining larger proportion of the total wealth volatility in the model. This quantity-type effect is the key reason for the fall of the relative importance of $\alpha [1]$ and the rise of that of the other hedging terms.

It is also useful to comment here on how the coefficient of $\frac{cc^*}{(cs+c^*)}$ changes. As mentioned, it reduces to $\frac{1}{2}$ in a symmetric endowment economy because $c = c^* = s = 1$. In a symmetric production economy, it decreases because consumptions account for less than total spending and are both lower than 1. In the asymmetric production economy (where the average global consumption is not fixed), consumption in the two countries is pushed up by the lower r in response to a low δ^* . Besides, the real exchange rate depreciates due to the pre-

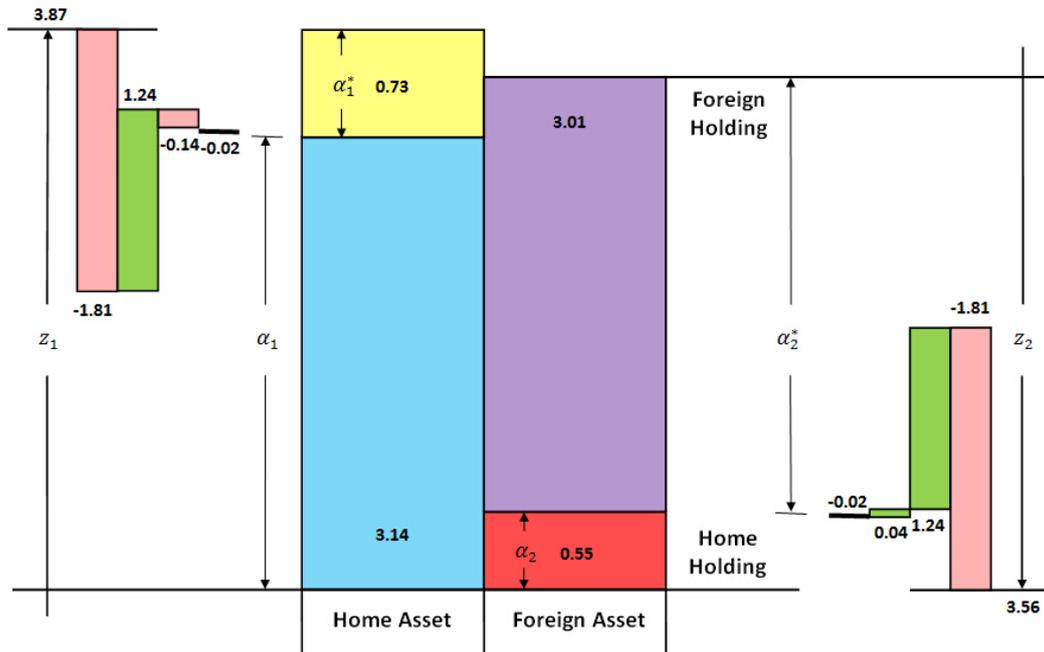


Figure 4.3: Portfolio allocation and components

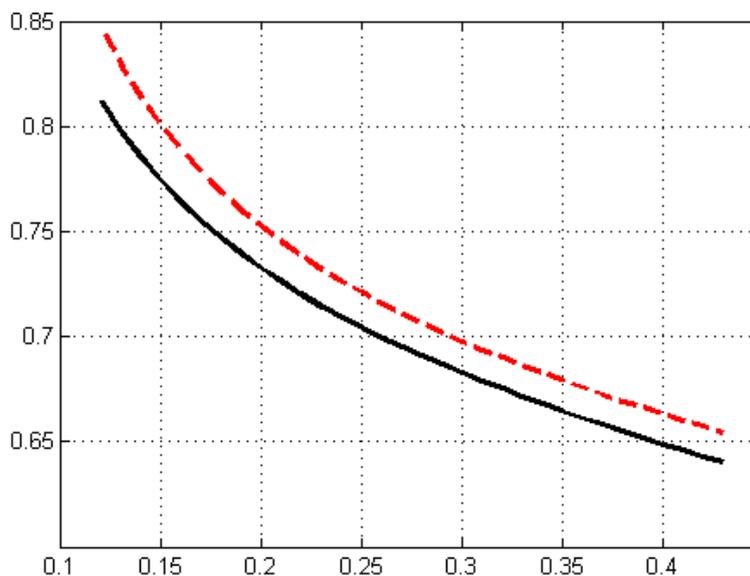
viously explained reason, so the value of this coefficient increases, which is also shown in the table.

We follow Chapter 3 by casting the results of portfolio allocations and decomposition into a diagram, i.e. Figure 4.3. In the middle of the figure, the heights of two columns represent the capital stocks, the k s, or/and total asset supplies, the z s, in the two countries. The upward vertical axis on the left hand side measures k and z_1 while the downward vertical axis on the right hand side measures k^* and z_2 . To find out how z_1 and z_2 are distributed across the two countries, for instance for z_1 , beside the left-hand-side vertical axis the four bars illustrate the hedging components and are accumulated to obtain α . From the left to the right, they are respectively the negative self-hedging, $\alpha [1]$, the positive hedging of labour income, $\alpha [2]$, the negative hedging of interest payments and the adjustment to new-born's consumption, $\alpha [3]$ and $\alpha [4]$. The net holdings of the home asset by the home and foreign countries are thus respectively $\alpha_1 = z_1 + \alpha$ and $\alpha_1^* = -\alpha$, as depicted in the figure. The same strategy also applies to the foreign asset z_2 .

With the diagram, the result of this chapter can be conveniently put into the context of the literature. In particular, on the one hand, the three results obtained by Caballero et al. (2008) are preserved here. Firstly, the home asset becomes more important in the international asset market, i.e. a higher column on the left hand side than that on the right hand side in the middle of the diagram $z_1 > z_2$. Secondly, the international real interest rate r is lower because the world asset supply is reduced while the world asset demand is driven up. Thirdly, the steady state *NFA* global imbalances emerge, i.e. the difference between the areas of α_2 (which is the gross external asset/liability of the home/foreign country) and α_1^* (which is the gross external liability/asset of the home/foreign country). What this chapter does is thus to extend their analysis to also shed light on the gross portfolio positions α s. On the other hand, our study takes the same approach as that in the literature on the determination of the gross portfolio positions, i.e. focusing on the hedging properties of assets for different types of incomes. In Heathcote and Perri (2013), the positive hedging of the labour income risk emerges as the key driving force of the symmetric asset home bias across countries. The asymmetric factor being incorporated into the framework in this chapter, the way that different hedging terms play their role in shaping portfolio holdings and thus allocations differs. The portfolios in both countries becomes more home-biased (as δ^* decreases). And moreover, the portfolio in the foreign country is relatively more home-biased than that in the home country. In the diagram, this is captured by a smaller share of area α_1 to z_1 relative to the share of area α_2^* to z_2 .

4.8 Sensitivity analysis

In the previous section, the results of the benchmark simulation provides us with an example where, when $\delta > \delta^*$, the optimal portfolio chosen in the home country is more internationally diversified than that in the foreign country, i.e. $\pi < \pi^*$. From the previous analysis, we know that this is possible only when the effect of decreasing δ^* on the portfolio choice exceeds the effect on asset supplies. So, to show that the result of Section 4.7 is not a coincidence driven by an elaborate selection of parameter values, in this section we will perform a series of robustness checks.

Figure 4.4: Asset home biases when vary δ s

4.8.1 Wealth division parameters, the δ s

As explained, our choice of the values of the δ s (δ at 0.12 while δ^* is one percent below) relies on Caballero et al.'s (2008) calibration which leads to a ratio of wealth to income lower than 4. If we take the literal meaning of the δ s, the estimated share of non-human wealth in total wealth, from the literature, this is usually about one third (for instance, Lettau 2001 estimates it to be 0.31). This is consistent with the estimates of the capital share in production in the literature. Some literature thus interprets the δ -asymmetry as the technological difference across countries (Jin, 2012). Following this approach, the values of the δ s should be higher than the ones we use in the benchmark case.

We experiment by varying δ , keeping the δ -gap constant, from the current value of 0.12 up to more than 0.4. Figure 4.4 shows the associated π (solid line) and π^* (dashed line) in the two countries. It is obvious that the portfolio in the home country is always less home-biased by the fact that $\pi < \pi^*$ when the δ is within the range shown, i.e. the dashed line lies above the solid line.

When δ increases, both lines show a lower degree of asset home bias in both

countries. This is economically sensible because, when the δ s increase, with less labour income risk to be hedged and with more hedging vehicles available, i.e. a large share of financial income and thus large asset stocks, the demands for the local assets (the good hedge for labour income) will be lower in both countries.

4.8.2 Elasticity of substitution between goods, ϕ

The quantitative literature in open economy macroeconomics usually sets the value of ϕ at around or higher than unity. For instance, Stockman and Tesar (1995) set it equal to unity while Backus et al. (1995) set it equal to 1.5. Feenstra et al. (2014) estimate the median of the “macro” elasticity to be close to (but higher than) 1 and the “micro” one to be even higher (up to 2 times larger).

One implication of the choice of value for ϕ is that, the higher is ϕ , the higher are the π s. This is because, when the two goods are more substitutable, in response to a positive shock to home country productivity, the resulting price responses (i.e. home basket depreciation) are moderated. The weakening of the stabilizing terms-of-trade effect leaves a heavier load of risk-sharing to be achieved through portfolio diversification. This requires countries to more overweight local assets. When simulating the model, given that we have already chosen very low δ s which also tends to yield high π s, we choose a low estimate of ϕ ($= 0.9$ from Heathcote and Perri, 2002) to avoid the counterfactual case of π s being greater than 1. On the other hand, because we are still interested in the case of asset home bias (thus π s still have to be above 0.5), we do not need to worry about the situation where ϕ is too low. And based on the literature, these extremely low values of ϕ are less likely to be relevant.

In Figure 4.5 we assess how π and π^* respond to ϕ varying in the region of 0.9. As expected, when ϕ increases, the π s increase. But $\pi < \pi^*$ is always observed, thus verifying the robustness of the result to the change in trade substitutability.

The gap between π and π^* increases as ϕ increases. This is because, given the δ s, asset supplies (the z s) are only affected by r whose value is very insensitive to that of ϕ . So when ϕ changes, the z s almost stay constant. But a rise in ϕ pushes up net holdings of local assets, α_1 and α_2^* , in both countries. As the country who supplies a lower amount of assets (i.e. $z_2 < z_1$), when ϕ increases, the foreign country will have a π^* growing faster than π .

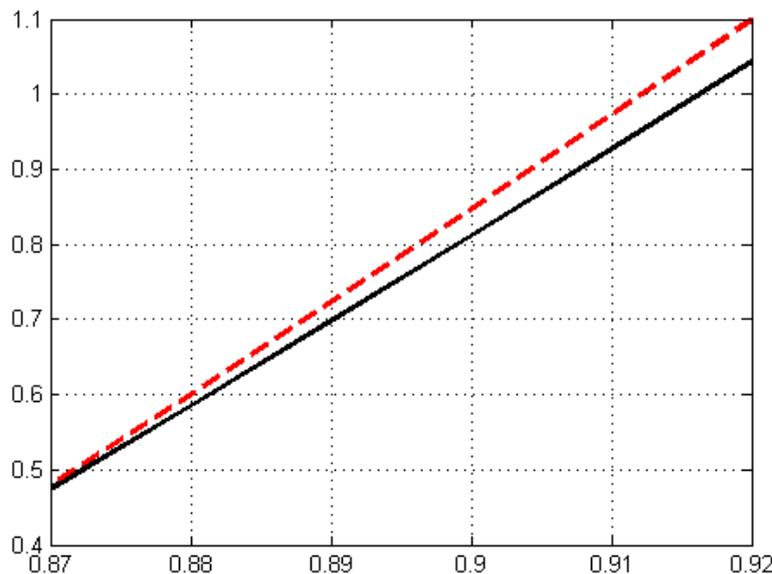
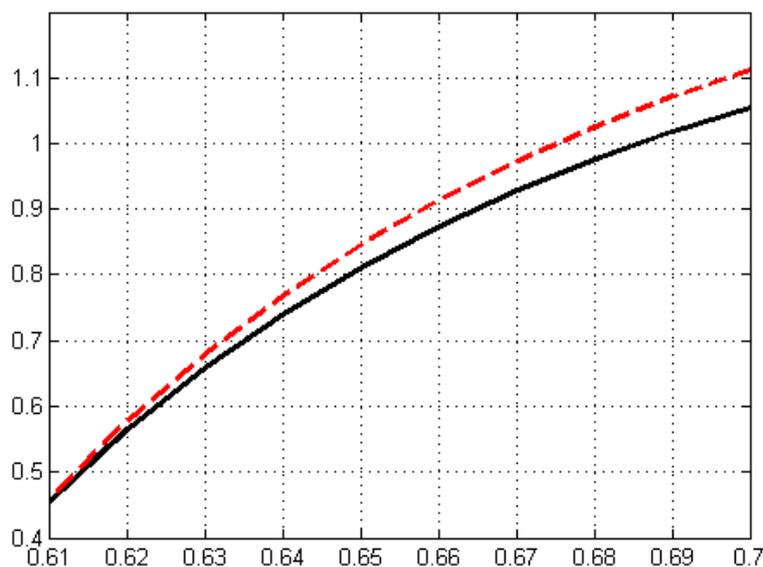


Figure 4.5: Asset home biases when vary ϕ

Figure 4.5 shows that the π s are quite sensitive to ϕ . This is due to a reinforcing effect coming from the choice of low δ s. When the δ s are set higher, the two lines in the figure becomes flatter and we obtain a wider range of ϕ yielding π s which are lower than 1. In the light of the estimated range of δ and ϕ in the literature, this allows ϕ to approach its typical value and δ to gain a more eclectic interpretation. But this comes with the sacrifice of a larger wealth income ratio.

The current study does not aim to identify which aspect of the data should be more respected in choosing parameter values for this model. Our results presented here are just an example that, even though we choose to follow Caballero et al. (2008) in setting a low δ in our benchmark simulation, we can take an alternative route by setting δ at a higher value and the resulting choice of the other parameters approach their commonly-used range. In any case, while affecting the shape of the two lines in the figure, these other parameter variations do not change the pattern of $\pi^* > \pi$ in the model.

Figure 4.6: Asset home biases when vary κ

4.8.3 Degree of local good preference, κ

We turn now to consider the effect of the degree of the trade home bias, κ , on the pattern of $\pi < \pi^*$. In Figure 4.6, we plot the π s against a range of values for κ . It is obvious that the dashed line, π^* , is always higher than the solid line, π , for the range of values of κ shown, so the main benchmark result still holds for this experiment.

As in the case of ϕ , a higher κ implies higher π s in both countries. This is because a higher κ also means a lower terms-of-trade effect takes place in response to shocks and this lowers the risk associated with income from capital. This reduces the need for self-hedging and thus induces countries to hold more of the local asset in their portfolios. To see why the terms-of-trade effect decreases, consider for instance that home country productivity receives a positive shock. The supply side effect of this shock is to cause a terms-of-trade deterioration. However, on the demand side, a higher degree of local product preference, κ , implies a higher share of spending on local goods, which tends to imply that a rise in home productivity raises home income and home demand for the home

good, which in turn counteracts the effect of the shock on the terms of trade.

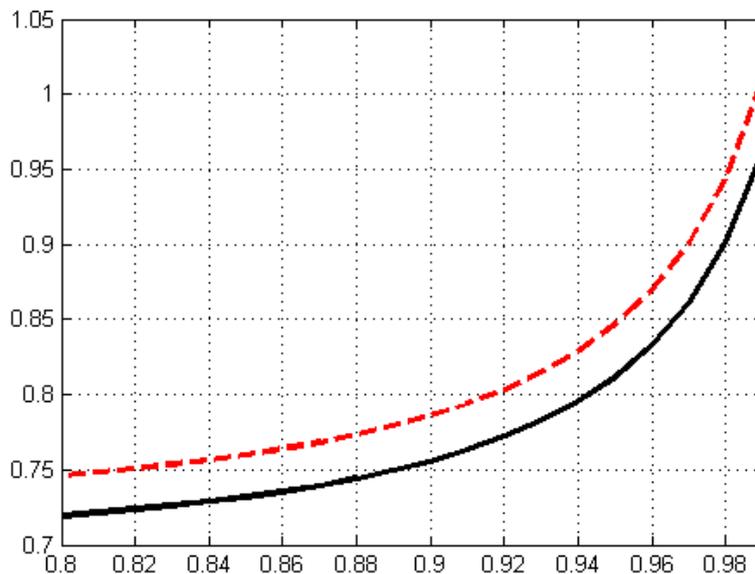
For a similar reason to that of ϕ , when κ increases, the discrepancy between π and π^* becomes larger. Given that in the literature κ is usually chosen to be around 0.85 (for example, Backus et al., 1994), the pattern of $\pi < \pi^*$ will only be strengthened relative to our benchmark if we choose a κ approaching this value.

4.8.4 Shock persistence, μ

As another experiment, now we consider the responses of portfolio allocations to different degrees of persistence in technological shocks, i.e. different values for μ . In the literature, the shock to productivity is usually assumed to be highly persistent, i.e. μ is around or higher than 0.9. For example, Devereux and Sutherland (2010) use the value of 0.9 while Backus et al. (1994), Heathcote and Perri (2002) use 0.91. We test a broad cases of μ , from 0 to 0.99, and find that π is always lower than π^* . A subset of the results is reported in Figure 4.7 where μ ranges from 0.8 to 0.99. Similar to the above results, the choice of value for μ does not alter the pattern of $\pi < \pi^*$ but only changes the level of π s in the two countries and the gap between the two π s. In particular, the π s increase as the shock becomes more persistent. This is because, the higher is μ , the more volatile are all relevant variables in determining portfolio choices (for instance \hat{r}_{xt} , Δd_t and Δl_t). Nevertheless, the degree of the increase in volatility differs across these variables. Since in the model, financial wealth accounts for a relatively lower share of total wealth, $\delta < 1/2$, Δl_t absorbs more volatility than Δd_t does. This enhances the role of hedging of labour income risk in α while reducing the role of self-hedging, which leads to a less diversified portfolio in both countries. Otherwise, if financial wealth were to account for a relatively large share of total wealth, its volatility would grow relatively fast as μ is increased, which would result in a reversed effect on the π s in response to a change in μ (but still with $\pi < \pi^*$).

4.8.5 Intertemporal elasticity of substitution, ρ

By using a logarithmic utility function in the model, we in fact assume that the intertemporal elasticity of substitution of consumption is equal to 1. To extend our framework to account for the case where the elasticity is not equal to 1, in this

Figure 4.7: Asset home biases when vary μ

subsection, we replace the benchmark utility function with the commonly-used *CRRA* function as follows

$$U_t^v = \sum_{i=0}^{\infty} \beta^i \left[\frac{(c_{t+i}^v)^{1-1/\rho}}{1-1/\rho} - \gamma \frac{(h_{t+i}^v)^{1+\eta}}{1+\eta} \right]$$

where ρ denotes the intertemporal elasticity of consumption substitution and η is the (inverse) elasticity of substitution of labour supply. The solution process of the extended model is similar to that of the current model, however, with the related optimality conditions (mainly those for labour supply h s and portfolio choices α) and the steady state being modified. The full description of how this is achieved is attached in Section 4.D of the Appendix to this chapter. In particular, we find that in the optimal portfolio, a hedging term for real exchange

rate risk emerges in addition to the existing hedging terms.

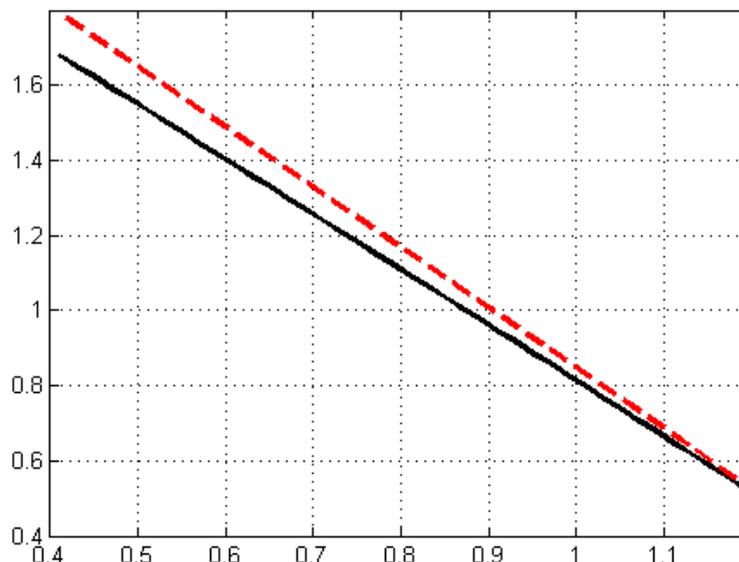
$$\begin{aligned} \tilde{\alpha} = & -\frac{cc^*}{cs+c^*} \frac{cov(\Delta d_t, \hat{r}_{xt})}{var(\hat{r}_{xt})} - \frac{cc^*}{cs+c^*} \frac{cov(\Delta l_t, \hat{r}_{xt})}{var(\hat{r}_{xt})} \\ & -rf \frac{cov(\Sigma_{2t}^{rn}, \hat{r}_{xt})}{var(\hat{r}_{xt})} + \frac{cc^*}{cs+c^*} \frac{(1-\tau)\tilde{n} cov(\Delta C_{t+1}^n, \hat{r}_{xt})}{(r-\tau\tilde{n}) var(\hat{r}_{xt})} \\ & \underbrace{-\frac{cc^*(\rho-1)}{cs+c^*} \frac{cov(\Sigma_t^s, \hat{r}_{xt})}{var(\hat{r}_{xt})}}_{\text{Hedging exchange rate}} + \underbrace{\frac{cc^*(\rho-1)(1-\tau)\tilde{n} cov(\Sigma_{t+1}^s, \hat{r}_{xt})}{cs+c^* (r-\tau\tilde{n}) var(\hat{r}_{xt})}}_{\text{Hedging newborn's consumption-related exchange rate}} \end{aligned}$$

Note that on the last line of the above formula, the first term reflects the fact that agents are exposed to real exchange rate risk and hedging against the risk arises in response. Besides, the adjustment to the new-born's consumption also contains a factor of real exchange rate risk which gives rise to hedging represented by the second term on the last line.

We simulate the model with the previous parameterization. The new parameter η is set at 1, which on the one hand is based on the existing literature (for instance Heathcote and Perri, 2013). On the other hand, when combined with $\rho = 1$, this yields a result for the π s that close to those obtained when log utility is adopted (as in the benchmark model).

In Figure 4.8, we plot the levels of π and π^* when varying ρ from about 0.4 to about 1.2. It turns out that given that the portfolios in the two countries exhibit asset home bias, the pattern of $\pi < \pi^*$ is always observed.

The figure shows that the lower is ρ , the higher are the π s. This is because the intertemporal substitution rate, ρ , controls how relative consumptions reacts to the changes in relative price. The lower is ρ , the more reluctant households are to adjust consumption in responses to a change in q . In other words, for instance when the home good becomes relatively more expensive (i.e. q increases), the more households would like to maintain the purchasing power of their income by investing in the asset yielding relatively high rate of return. In the model, the asset possessing this property (yielding higher excess return when the domestic good appreciates) is the local asset for both countries, which explains the increase in the π s. Moreover, this is actually also the explanation for a positive hedging of exchange rate when ρ is low. In this case, the hedging of the exchange rate works in the same direction as that of the hedging of labour income, thus combatting the effect of a negative self-hedging. This reinforces asset home bias.

Figure 4.8: Asset home biases when vary ρ

Again because asset supplies are insensitive to the change in ρ , when ρ decreases, the gap between π and π^* grows monotonically.

4.8.6 Country asymmetry

In all the experiments above, we keep the measure of the asymmetry constant, i.e. $\Delta\delta = \delta - \delta^* = 0.01$. It is easy to evaluate the robustness of the result of $\pi < \pi^*$ for different sizes of asymmetry across countries by simulating the model using a $\Delta\delta$ of different magnitude. And it turns out that the result persists for such experiments. Only the gap between the two π s is varying. For a given δ , increasing $\Delta\delta$ widens the gap between the π s because a larger country difference deepens global imbalances, which in turn fuels the effect of the hedging of interest payments. Again, this asymmetric effect from differing $\Delta\delta$ on portfolio choices always dominates that from relative asset supplies. When $\Delta\delta$ is large, the dominance is strong. When $\Delta\delta$ is small, it is diminishing but never disappears.

To sum up, according to the analysis in this section, we found that the result of $\pi < \pi^*$ when $\delta > \delta^*$ is robust to variants in parameter values and the

introduction of hedging motive for real exchange rate variations. It can be established that the effects of hedging of external interest payments on otherwise symmetric home-biased country portfolios usually dominates that coming from asset supplies.

4.9 Conclusion

In this chapter, we develop an asymmetric two-country *OLG* model to examine how global *NFA* imbalances and asymmetric asset home bias, the two most prominent country-portfolio phenomena in the open economy data, are at the same time driven by differences between developed and developing countries in the division of wealth between financial assets and human capital. Previously, the literature has analysed global *NFA* imbalances and asymmetric home bias as two separate issues. When they are combined together in the same model, we find that, because agents have a motive to hedge against the interest payments that are implied by non-zero net foreign positions, the portfolio allocations move away from a symmetrically home biased position. To be specific, we find that the creditor (developing) country holds domestic assets more intensively than the debtor (developed) country does. This is in line with the pattern of *NFA* imbalances and asymmetric asset home bias that is observed in the data.

One major implication of our result is that some basic facts about wealth division and net/gross portfolio phenomena between countries with different degrees of development are in fact casually connected. In particular, the importance of a country's net external balance in shaping bilateral asset holdings is highlighted, which makes this work a contribution particularly to the literature on gross portfolios (and asset home bias). Following the categorization by Coeurdacier and Rey (2012), the line of literature that focuses on hedging motives in portfolio choice in frictionless economies is extended by our work because the dispersion of home bias across countries is missing from that literature. The channel identified in this chapter also forms a supplement to the literature which attempts to explain home bias in terms of market integration and trade costs (see Tesar and Werner, 1995, Warnock, 2002). Lastly, many empirical studies consider the role of the factors associated with geography, culture and institutions (see, for instance, Portes and Rey, 2005, Chan et al., 2005, Daude and Fratzscher, 2008)

in explaining differences in country portfolios. Guided by our results reported in this chapter, the net external balance of country should be included as a control in such empirical evaluations.

As explained at the beginning of this chapter, the simple endowment *OLG* model in Chapter 3 is not a good framework to use quantitatively to think about policy-oriented problems because of the unrealistically large magnitude of both net and gross portfolios in that model. The production *OLG* model in this chapter avoids this problem and thus provides a general and useful framework suitable for the analysis of some practical issues between two large economies where realistic net and gross country portfolios are important.

Appendix

4.A Model equations in exact form

In this section, except for the utility function and Euler equations, all equations appear in their per capita form after aggregation.

4.A.1 Households' problem

Individual utility function

$$U_t^v = \sum_{i=0}^{\infty} \beta^i [\log(c_{t+i}^v) + \gamma \log(1 - h_{t+i}^v)]$$

Intertemporal budget constraints

$$(1+n)(\alpha_{1t+1} + \alpha_{2t+1}) = r_{1t}\alpha_{1t} + r_{2t}\alpha_{2t} + l_t - c_t$$

$$s_t(1+n)(\alpha_{1t+1}^* + \alpha_{2t+1}^*) = s_t(r_{1t}\alpha_{1t}^* + r_{2t}\alpha_{2t}^*) + l_t^* - c_t^*$$

Labour income

$$l_t = \frac{g_t}{p_t} h_t$$

$$l_t^* = \frac{g_t^*}{p_t^*} h_t^*$$

Asset returns and prices

$$r_{1t}z_{1t} = d_t + (1+n)z_{1t+1}$$

$$r_{2t}z_{2t} = \frac{d_t^*}{s_t} + (1+n)z_{2t+1}$$

Euler equations (optimal choices of consumption/asset holdings)

$$(c_t^v)^{-1} = \beta E_t [r_{1t+1} (c_{t+1}^v)^{-1}]$$

$$(c_t^v)^{-1} = \beta E_t [r_{2t+1} (c_{t+1}^v)^{-1}]$$

$$s_t (c_t^{v*})^{-1} = \beta E_t [s_{t+1} r_{1t+1} (c_{t+1}^{v*})^{-1}]$$

$$s_t (c_t^{v*})^{-1} = \beta E_t \left[s_{t+1} r_{t+1} (c_{t+1}^{v*})^{-1} \right]$$

Labour supply

$$h_t = 1 - \gamma \frac{p_t}{g_t} c_t$$

$$h_t^* = 1 - \gamma \frac{p_t^*}{g_t^*} c_t^*$$

New-born's consumption

$$c_t^n = (1 - \beta) l_t + \frac{1}{r} c_{t+1}^n$$

4.A.2 Firm's problem

Technology and Marginal products (country asymmetry in δ , i.e. $\delta > \delta^*$)

$$y_t = e^{\varepsilon_t} (k_t)^\delta (h_t)^{1-\delta}$$

$$y_t^* = e^{\varepsilon_t^*} (k_t^*)^{\delta^*} (h_t^*)^{1-\delta^*}$$

$$MPL_t = (1 - \delta) \frac{y_t}{h_t}$$

$$MPL_t^* = (1 - \delta^*) \frac{y_t^*}{h_t^*}$$

$$MPK_t = \delta \frac{y_t}{k_t}$$

$$MPK_t^* = \delta^* \frac{y_t^*}{k_t^*}$$

Objective function and discount factor

$$\sum_{i=0}^{\infty} \Omega^i d_{t+i}$$

$$\Omega^i = \beta^i c_{t+i}^{-1}$$

Dividend

$$d_t = \frac{q_t}{p_t} y_t - l_t - i_t$$

$$d_t^* = \frac{q_t^*}{p_t^*} y_t^* - l_t^* - i_t^*$$

Investment

$$i_t = (1 + n) k_{t+1} - k_t$$

$$i_t^* = (1 + n) k_{t+1}^* - k_t^*$$

Optimal choices of labour and capital demand

$$MPL_t = \frac{g_t}{q_t}$$

$$MPL_t^* = \frac{g_t^*}{q_t^*}$$

$$r_{kt} = \frac{q_t}{p_t} MPK_t + 1$$

$$r_{kt}^* = \frac{q_t^*}{p_t^*} MPK_t^* + 1$$

$$\Omega_t = E_t [\Omega_{t+1} r_{kt+1}]$$

$$\Omega_t^* = E_t [\Omega_{t+1}^* r_{kt+1}^*]$$

4.A.3 Market clearing

Assets market clear

$$\alpha_{1t} + \alpha_{1t}^* = z_{1t}$$

$$\alpha_{2t} + \alpha_{2t}^* = z_{2t}$$

which are equivalent to

$$\alpha_{1t} = z_{1t} - \alpha_{1t}^*$$

$$w_t - z_{1t} = -(w_t^* - z_{2t})$$

Intermediate goods market clear

$$x_{ht} + x_{ht}^* = x_t$$

$$x_{ft} + x_{ft}^* = x_t^*$$

where goods demands are given by

$$x_{ht} = \kappa \left(\frac{q_t}{p_t} \right)^{-\phi} y_t$$

$$x_{ft} = (1 - \kappa) \left(\frac{q_t^*}{s_t p_t} \right)^{-\phi} y_t$$

$$x_{ht}^* = (1 - \kappa) \left(\frac{s_t q_t}{p_t^*} \right)^{-\phi} y_t^*$$

$$x_{ft}^* = \kappa \left(\frac{q_t^*}{p_t^*} \right)^{-\phi} y_t^*$$

Final goods market clear

$$c_t + i_t = y_t$$

$$c_t^* + i_t^* = y_t^*$$

Price indices

$$p_t = \left[\kappa (q_t)^{1-\phi} + (1 - \kappa) \left(\frac{q_t^*}{s_t} \right)^{1-\phi} \right]^{\frac{1}{1-\phi}}$$

$$p_t^* = \left[(1 - \kappa) (s_t q_t)^{1-\phi} + \kappa (q_t^*)^{1-\phi} \right]^{\frac{1}{1-\phi}}$$

$$s_t = \frac{p_t}{p_t^*}$$

Defining w_t and f_t

$$w_t = \alpha_{1t} + \alpha_{2t}$$

$$w_t^* = \alpha_{1t}^* + \alpha_{2t}^*$$

$$f_t = w_t - z_{1t}$$

$$f_t^* = w_t^* - z_{2t}$$

4.B Linearized model equations

In this section, we approximate the model to first order accuracy around the steady states.

4.B.1 Households' problem

Intertemporal budget constraints at the home country

$$\tilde{n}w\hat{w}_{t+1} = rw\hat{w}_t + rw\hat{r}_{2t} + r\alpha_1\hat{r}_{xt} + (1 - \delta)\hat{l}_t - c\hat{c}_t$$

Labour income

$$\hat{l}_t = \hat{g}_t + \hat{h}_t - \hat{p}_t$$

$$\hat{l}_t^* = \hat{g}_t^* + \hat{h}_t^* - \hat{p}_t^*$$

Asset returns ($\tilde{n} \equiv (1 + n)$)

$$\hat{r}_{1t} = \left(1 - \frac{\tilde{n}}{r}\right) \hat{d}_t + \frac{\tilde{n}}{r} \hat{z}_{1t+1} - \hat{z}_{1t}$$

$$\hat{r}_{2t} = \left(1 - \frac{\tilde{n}}{r}\right) (\hat{d}_t^* - \hat{s}_t) + \frac{\tilde{n}}{r} \hat{z}_{2t+1} - \hat{z}_{2t}$$

Euler equations: Individual conditions are first approximated along the (perfect foresight) optimal path to yield, for example for $(c_t^v)^{-1} = \beta E_t [r_{1t+1} (c_{t+1}^v)^{-1}]$,

$$\hat{c}_{t+1}^v = \hat{c}_t^v + \hat{r}_{t+1}$$

Together with the approximated per capita aggregation relation (see Chapter 3 for more details), the above individual condition leads to the per capita Euler equations as below

$$\hat{c}_{t+1} = \tau \hat{c}_t + (1 - \tau) \hat{c}_{t+1}^n + \tau \hat{r}_{1t+1}$$

$$\hat{c}_{t+1} = \tau \hat{c}_t + (1 - \tau) \hat{c}_{t+1}^n + \tau \hat{r}_{2t+1}$$

$$\hat{c}_{t+1}^* = \tau \hat{c}_t^* + (1 - \tau) \hat{c}_{t+1}^{n*} + \tau [\hat{r}_{1t+1} + \hat{s}_{t+1} - \hat{s}_t]$$

$$\hat{c}_{t+1}^* = \tau \hat{c}_t^* + (1 - \tau) \hat{c}_{t+1}^{n*} + \tau [\hat{r}_{2t+1} + \hat{s}_{t+1} - \hat{s}_t]$$

Labour supply

$$h\hat{h}_t = (h - 1)(\hat{p}_t + \hat{c}_t - \hat{g}_t)$$

$$h^*\hat{h}_t^* = (h^* - 1)(\hat{p}_t^* + \hat{c}_t^* - \hat{g}_t^*)$$

New-born's consumption

$$c^n \hat{c}_t^n = (1 - \beta) \hat{l}_t + \frac{1}{r} c^n \hat{c}_{t+1}^n$$

4.B.2 Firm's problem

Technology and Marginal products

$$\begin{aligned}\hat{y}_t &= \delta \hat{k}_t + (1 - \delta) \hat{h}_t + \varepsilon_t \\ \hat{y}_t^* &= \delta^* \hat{k}_t^* + (1 - \delta^*) \hat{h}_t^* + \varepsilon_t^* \\ \widehat{MPL}_t &= \delta \hat{k}_t - \delta \hat{h}_t + \varepsilon_t \\ \widehat{MPL}_t^* &= \delta^* \hat{k}_t^* - \delta^* \hat{h}_t^* + \varepsilon_t^* \\ \widehat{MPK}_t &= (\delta - 1) \hat{k}_t + (1 - \delta) \hat{h}_t + \varepsilon_t \\ \widehat{MPK}_t^* &= (\delta^* - 1) \hat{k}_t^* + (1 - \delta^*) \hat{h}_t^* + \varepsilon_t^*\end{aligned}$$

Dividend

$$\begin{aligned}d\hat{d}_t &= \frac{qx}{p} (\hat{q}_t + \hat{y}_t - \hat{p}_t) - \hat{l}_t - \hat{i}_t \\ d\hat{d}_t^* &= \frac{q^*x^*}{p^*} (\hat{s}_t + \hat{y}_t^* - \hat{p}_t^*) - \hat{l}_t^* - \hat{i}_t^*\end{aligned}$$

Investment

$$\begin{aligned}\hat{i}_t &= \tilde{n}k\hat{k}_{t+1} - k\hat{k}_t \\ \hat{i}_t^* &= \tilde{n}k\hat{k}_{t+1}^* - k\hat{k}_t^*\end{aligned}$$

Optimal choices of labour and capital demand

$$\begin{aligned}\widehat{MPL}_t &= \hat{g}_t - \hat{q}_t \\ \widehat{MPL}_t^* &= \hat{g}_t^* - \hat{q}_t^* \\ r\hat{r}_{kt} &= (r - 1) \left[\hat{q}_t + \widehat{MPK}_t - \hat{p}_t \right] \\ r\hat{r}_{kt}^* &= (r - 1) \left[\hat{q}_t^* + \widehat{MPK}_t^* - \hat{p}_t^* \right] \\ \hat{c}_{t+1} &= \hat{c}_t + \hat{r}_{kt+1} \\ \hat{c}_{t+1}^* &= \hat{c}_t^* + \hat{r}_{kt+1}^*\end{aligned}$$

4.B.3 Market clearing

Assets market clear

$$\begin{aligned}\alpha_1 \hat{\alpha}_{1t} &= z_1 \hat{z}_{1t} - \alpha_1^* \hat{\alpha}_{1t}^* \\ w \hat{w}_t - z_1 \hat{z}_{1t} &= -(w^* \hat{w}_t^* - z_2 \hat{z}_{2t})\end{aligned}$$

Intermediate goods market clear

$$\begin{aligned}x_h \hat{x}_{ht} + x_h^* \hat{x}_{ht}^* &= x \hat{x}_t \\ x_f \hat{x}_{ft} + x_f^* \hat{x}_{ft}^* &= x^* \hat{x}_t^*\end{aligned}$$

where goods demands are given by

$$\begin{aligned}\hat{x}_{ht} &= \hat{y}_t - \phi (\hat{q}_t - \hat{p}_t) \\ \hat{x}_{ft} &= \hat{y}_t - \phi (\hat{q}_t^* - \hat{s}_t - \hat{p}_t) \\ \hat{x}_{ht}^* &= \hat{y}_t^* - \phi (\hat{s}_t + \hat{q}_t - \hat{p}_t^*) \\ \hat{x}_{ft}^* &= \hat{y}_t^* - \phi (\hat{q}_t^* - \hat{p}_t^*)\end{aligned}$$

Final goods market clear

$$\begin{aligned}c \hat{c}_t + i \hat{i}_t &= y \hat{y}_t \\ c^* \hat{c}_t^* + i^* \hat{i}_t^* &= y^* \hat{y}_t^*\end{aligned}$$

Price indices

$$\begin{aligned}\hat{p}_t &= \kappa \left(\frac{q}{p}\right)^{1-\phi} \hat{q}_t + (1-\kappa) \left(\frac{q^*}{sp}\right)^{1-\phi} (\hat{q}_t^* - \hat{s}_t) \\ \hat{p}_t^* &= (1-\kappa) \left(\frac{sq}{p^*}\right)^{1-\phi} (\hat{s}_t + \hat{q}_t) + \kappa \left(\frac{q}{p^*}\right)^{1-\phi} \hat{q}_t^*\end{aligned}$$

Real exchange rate

$$\hat{s}_t = \hat{p}_t - \hat{p}_t^*$$

Defining w_t and f_t

$$\begin{aligned}w \hat{w}_t &= \alpha_1 \hat{\alpha}_{1t} + \alpha_2 \hat{\alpha}_{2t} \\ w^* \hat{w}_t^* &= \alpha_1^* \hat{\alpha}_{1t}^* + \alpha_2^* \hat{\alpha}_{2t}^* \\ f \hat{f}_t &= w \hat{w}_t - z_1 \hat{z}_{1t} \\ f \hat{f}_t^* &= w^* \hat{w}_t^* - z_2 \hat{z}_{2t}\end{aligned}$$

4.C Derive $\tilde{\alpha}^*$ from $\tilde{\alpha}$

For the foreign country, $\tilde{\alpha}$ is negative gross external assets (multiplied by r) while $\tilde{\alpha}^*$ is negative gross external liability (multiplied by r), so net external asset (multiplied by r) \tilde{f}^* is given by their (negative) difference $-\tilde{\alpha} - (-\tilde{\alpha}^*) = \tilde{\alpha}^* - \tilde{\alpha}$. That is

$$\tilde{\alpha}^* = \tilde{f}^* + \tilde{\alpha}$$

where $\tilde{\alpha}$ is given by the expression in the main text. We define Δd_t^* , Δl_t^* , Δc_{t+1}^{n*} , \hat{r}_{xt}^* as the inverse of their analogues in the home country, so the above equation can be rewritten as

$$\begin{aligned} \tilde{\alpha}^* = \tilde{f}^* &- \frac{cc^*}{cs + c^*} \frac{cov(\Delta d_t^*, \hat{r}_{xt}^*)}{var(\hat{r}_{xt}^*)} - \frac{cc^*}{cs + c^*} \frac{cov(\Delta l_t^*, \hat{r}_{xt}^*)}{var(\hat{r}_{xt}^*)} \\ &- r f \frac{cov(\Sigma_{2t}^{rn}, \hat{r}_{xt})}{var(\hat{r}_{xt})} + \frac{cc^*}{cs + c^*} \frac{(1 - \tau) \tilde{n} cov(\Delta c_{t+1}^{n*}, \hat{r}_{xt}^*)}{(r - \tau \tilde{n}) var(\hat{r}_{xt}^*)} \end{aligned}$$

The first term in the second line can be rewritten as

$$\begin{aligned} r f^* \frac{cov(\hat{r}_{2t} + \frac{\tilde{n}}{r} \Sigma_{t+1}^{rn}, \hat{r}_{xt})}{var(\hat{r}_{xt})} &= r f^* \frac{cov(-\hat{r}_{xt} + \hat{r}_{1t} + \frac{\tilde{n}}{r} \Sigma_{t+1}^{rn}, \hat{r}_{xt})}{var(\hat{r}_{xt})} \\ &= -\tilde{f}^* - r f^* \frac{cov(\Sigma_{1t}^{rn}, \hat{r}_{xt}^*)}{var(\hat{r}_{xt}^*)} \end{aligned}$$

where we use the fact of $f = -f^*$ and define $\Sigma_{1t}^{rn} = \hat{r}_{1t} + \frac{\tilde{n}}{r} \Sigma_{t+1}^{rn}$. Substituting this back into the second to last equation, we immediately obtain the expression for $\tilde{\alpha}^*$ as in the main text.

4.D Extended model with hedging of the exchange rate

We extend the baseline model to the case where in the utility function the intertemporal substitution rate is not necessarily 1. By doing so, the extended framework also accommodates the presence of exchange rate hedging in $\tilde{\alpha}$. Note that we only document the places that the benchmark (as in the baseline model of the main text) needs to be adjusted.

Utility function Assume the following commonly-used CES utility function for vintages

$$U_t^v = \sum_{i=0}^{\infty} \beta^i \left[\frac{(c_{t+i}^v)^{1-1/\rho}}{1-1/\rho} - \gamma \frac{(h_{t+i}^v)^{1+\eta}}{1+\eta} \right]$$

where ρ denotes the intertemporal substitution rate and η the (inverse) substitution rate of labour supply. As before γ represents the relative weight between utility from consumption and disutility from working hours. The intertemporal budget constraints are the same as in the baseline model.

Euler equations The related Euler equations are

$$\begin{aligned} (c_t^v)^{-1/\rho} &= \beta E_t \left[r_{1t+1} (c_{t+1}^v)^{-1/\rho} \right] \\ (c_t^v)^{-1/\rho} &= \beta E_t \left[r_{2t+1} (c_{t+1}^v)^{-1/\rho} \right] \\ s_t (c_t^{v*})^{-1/\rho} &= \beta E_t \left[s_{t+1} r_{1t+1} (c_{t+1}^{v*})^{-1/\rho} \right] \\ s_t (c_t^{v*})^{-1/\rho} &= \beta E_t \left[s_{t+1} r_{2t+1} (c_{t+1}^{v*})^{-1/\rho} \right] \end{aligned}$$

which differ from the benchmark by the inclusion of ρ . We apply the same procedure as for the benchmark Euler equations, i.e. linearize the individual conditions and making use of the per capita aggregation relation to aggregate them into the following form, for example for line 1 and 3 with expected return rate of asset 1,

$$\begin{aligned} \hat{c}_{t+1} &= \tau \hat{c}_t + (1 - \tau) \hat{c}_{t+1}^n + \tau \rho \hat{r}_{1t+1} \\ \hat{c}_{t+1}^* &= \tau \hat{c}_t^* + (1 - \tau) \hat{c}_{t+1}^{n*} + \tau \rho [\hat{r}_{1t+1} + \hat{s}_{t+1} - \hat{s}_t] \end{aligned}$$

where τ is redefined as $\frac{(r\beta)^\rho}{(1+n)} < 1$. (As a special case, the baseline model has a $\tau = \frac{r\beta}{(1+n)}$ because $\rho = 1$.)

In addition, new-born's consumption moves according to

$$c_t^n = (1 - r^{\rho-1} \beta^\rho) l_t + \frac{1}{r} c_{t+1}^n$$

Linearizing yields

$$c_t^n \hat{c}_t^n = (1 - r^{\rho-1} \beta^\rho) \hat{l}_t + \frac{1}{r} c_t^n \hat{c}_{t+1}^n$$

Labour supply First-order conditions for optimal labour supply are

$$\gamma (h_t^v)^\eta (c_t^v)^{1/\rho} = \frac{g_t}{p_t}$$

$$\gamma^* (h_t^{v*})^\eta (c_t^{v*})^{1/\rho} = \frac{g_t^*}{p_t^*}$$

To obtain the related (linearized) per capita condition, three steps are in order (taking the case of home country as an example).

First, the relation implies $(c_t^v)^{-1/\rho} = \gamma \frac{p_t}{g_t} (h_t^v)^\eta$. Combined with the Euler equation $(c_t^v)^{-1/\rho} = \beta E_t \left[r_{1t+1} (c_{t+1}^v)^{-1/\rho} \right]$, we can obtain the relation that h_t^v must satisfy in perfect-foresight optimum

$$h_{t+1}^v = (r\beta)^{-1/\eta} h_t^v$$

That is the optimal path of h_t^v . (Remember although individual variables do not have steady states, they have an optimal path. Individual consumption c_t^v has an optimal path given by $c_{t+1}^v = (r\beta)^\rho c_t^v$. Chapter 3 has more details on the optimal paths of individual variables.)

Second, similar to that for Euler equations, linearize the optimal conditions for h_t^v along the individual optimal paths to yield

$$\eta \hat{h}_t^v + \frac{1}{\rho} \hat{c}_t^v = \hat{g}_t - \hat{p}_t$$

which also applies for the vintages of new-born each period, i.e.

$$\eta \hat{h}_t^n + \frac{1}{\rho} \hat{c}_t^n = \hat{g}_t - \hat{p}_t$$

or

$$\hat{h}_t^n = \frac{1}{\eta} \left(\hat{g}_t - \hat{p}_t - \frac{1}{\rho} \hat{c}_t^n \right)$$

where h_t^n denotes the labour supply of new-borns that are equalizing across time $h_t^t = h_{t+1}^{t+1}$. As an alternative and equivalent way, the above equation is immediately obtained if one directly linearize $\gamma (h_t^n)^\eta (c_t^n)^{1/\rho} = \frac{g_t}{p_t}$ by realizing that each period's new-born's decisions are stationary due to the assumption generations have the same structure.

Finally, according to the per capita aggregation relation for labour supply

$$h_t = \frac{h_t^0 + n h_t^1 + n(1+n) h_t^2 \dots + n(1+n)^{t-1} h_t^t}{(1+n)^t}$$

and making use of h_t^v 's optimal path in the first step, it is easy to obtain the dynamics of h_t as follows

$$h_{t+1} = \tau_l h_t + \frac{n}{\tilde{n}} h_{t+1}^n$$

where $\tau_l = \frac{(r\beta)^{-1/\eta}}{(1+n)}$. From this we can work out, first, steady state h_t^n as

$$h^n = \frac{\tilde{n}}{n} (1 - \tau_l) h$$

and second, its linearized form

$$h \hat{h}_{t+1} = \tau_l h \hat{h}_t + \frac{n}{\tilde{n}} h^n \hat{h}_{t+1}^n$$

Replacing \hat{h}_{t+1}^n with the result yielded in the second step, we finally obtain

$$h \hat{h}_{t+1} = \tau_l h \hat{h}_t + \frac{n}{\tilde{n}} \frac{h^n}{\eta} \left(\hat{g}_t - \hat{p}_t - \frac{1}{\rho} \hat{c}_t^n \right)$$

which represent h_t 's dynamics.

Optimal portfolio Following the same process as in the main text, we obtain the condition for $\tilde{\alpha}$ as

$$E_t [(\hat{c}_t - \hat{c}_t^* + \rho \hat{s}_t) \hat{r}_{xt}] = 0$$

which is similar to that of the baseline model except for the inclusion of ρ .

Following similar procedures decomposing the wealth and composition effect of $cd_t = (\hat{c}_t - \hat{c}_t^* + \rho \hat{s}_t)$ as in the main text, for the current set-up, we end up with an expression for cd_t of the following form

$$\begin{aligned} & \frac{r - \tau \tilde{n}}{r} [\Sigma_t^c - \Sigma_t^{c*}] - \frac{(1 - \tau) \tilde{n}}{r} [\Sigma_{t+1}^{cn} - \Sigma_{t+1}^{cn*}] \\ & + \frac{r - \tau \tilde{n}}{r} (\rho - 1) \Sigma_t^s - \frac{(1 - \tau) \tilde{n}}{r} (\rho - 1) \Sigma_{t+1}^s \end{aligned}$$

where the involved notations are the same as before. Due to the presence of the two terms appearing on the second line, the model generates the need to hedge against the risks associated with the exchange rate by holding $\tilde{\alpha}$. This is the case only when $\rho \neq 1$.

Substituting cd_t into the optimal condition for the portfolio, we obtain $\tilde{\alpha}$'s expression as stated in the main text.

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Chapter 5

Conclusion

The on-going process of financial globalization has been making countries across the world more and more tightly integrated through the rapid expansion of cross-border asset transactions. While financial integration has been more pronounced so far for advanced economies, developing countries are playing a more and more important role in recent years. The participation of heterogeneous countries in cross-border asset transactions gives rise to the notable stylised facts about country portfolios as identified in Chapter 1. They are, firstly, regarding the net portfolio position, in general the emerging economies have a positive position of net foreign assets while developed countries have a negative net foreign asset position. Secondly, regarding the gross portfolio position, in both groups of countries, domestic assets are mostly held domestically. And in particular, this asset home bias is more significant in developing countries than in developed countries as a whole. Associated with the presence of the large net and gross portfolio positions, valuation effects across borders arise and become an increasingly important channel affecting the process of external adjustments of a country in addition to the traditional trade channels. Thirdly, with regard to the asset composition of gross positions, it is the case from the data that in net terms, developed countries borrow in terms of safe securities, for instance bonds, and at the same time invest in risky assets, for instance portfolio equity and *FDI*, while many emerging markets on the one hand possess a large amount of foreign reserves which is invested in international bonds, on the other hand, they experience large net capital inflows in terms of equity investments.

This thesis seeks to explain these empirical facts within new theoretical frameworks. Chapter 2 focuses on accounting for the fact of two-way capital flows. A general open economy *DSGE* model is used for this purpose. We assess the effects of a long list of country asymmetries on the pattern of net equity and bond flows between countries in this chapter. The analysis shows that the following facts of underdevelopment in the developing countries can be potential roots for them to have a steady state negative net equity position and a positive net bond position and for developed countries to have a steady state positive net equity position and a negative net bond position. Developing countries are usually those using more labour intensive technology in production. The market in these countries is less competitive so that the firms have more power in pricing. The change in prices and wages is less flexible. The monetary authority is more concerned with the output gap than inflation when implementing a monetary policy rule. The degree of price and wage indexation is relatively high. Trade openness is relatively low. And when pricing exports, the firms in these countries use more often the currency of customers. Among these facts, those associated with real factors, such as the technological difference and monetary policy weights, are found to be more quantitatively relevant while those associated with nominal factors, such as nominal stickiness and pricing strategies, are of minor importance. The analysis in Chapter 2 highlights two channels through which the size of asset positions is determined, i.e. the correlation and variability effects. The former represents the relevance of an asset in risk-hedging. The latter represents the amount of risk to be hedged by that asset. The presence of the above country asymmetries alters the relative strength of the correlation and variability effects between the home and foreign assets, which generates unequal size of domestic and overseas asset holdings and in turn unbalanced net equity and bond positions across countries. The composite effect of these asymmetries is also examined and it turns out to be qualitatively consistent with empirical observed data. Moreover, in the case of asymmetric international financial markets where developing countries cannot issue bonds, we find that the degree of two-way capital flows deepens.

Chapter 3 integrates the analyses of a non-trivial net portfolio position and gross portfolio positions in the same theoretical framework. Previously, these two themes are basically dealt with separately by distinct strands of the literature.

The non-trivial net portfolio position between developing and developed countries is studied usually with no reference to the structure of financial markets and no explicit solution of gross portfolio choices, while, the determination of gross portfolio choices is mostly studied within symmetric frameworks where net portfolio position is zero in steady state. The reason for the treatment in the former strand of literature lies in the (until recently) lack of a suitable computation method of portfolio choices. And for the latter strand of literature, the reason of why it is not easy to take into account a non-zero net portfolio position and gross portfolio positions at the same time is because a model of representative agents embedding these two aspects will in general suffer from a problem of non-stationarity. In Chapter 3, we show that introducing an *OLG* structure into the model can overcome this problem. Relying on an assumption of differing degree of patience across countries, we generate a positive net foreign asset position in the developing country and a negative net foreign asset position in the developed country, i.e. global net imbalances. This creates a difference between *GNP* and *GDP* for both countries, i.e. the return on net external position. Besides, the way that consumption is distributed across time becomes asymmetric as well. So relative consumption across countries depends on not only relative *GDP* but also on the relative return on net external positions and the country difference in consumption distribution over time. According to the associated optimality condition of portfolio choice, the gross portfolio holding is determined so as to smooth the fluctuation of the relative consumption across different states, i.e. risk hedging. This change in the components of risk in the consumption differential affects the way gross portfolio positions are structured. To be specific, the gross position is composed of not only the diversification term but also the hedging term for the return on net external positions, the hedging term of new-born consumption and the hedging term of interest rate tilting. Due to presence of these additional terms, the portfolio allocation across countries becomes asymmetric. And it appears to be home biased in the sense that in the portfolio of each country, the local asset is preferred with a relatively large share. With the non-trivial net portfolio position and the asymmetric gross position being present, the external adjustments between the creditor (developing) and debtor (developed) countries work through the intertemporal terms-of-trade effect and valuation effect in addition to the traditional trade balance effect.

Chapter 4 extends the endowment *OLG* model of Chapter 3 to a production economy in order to investigate how the asymmetry associated with wealth division across countries sheds light on the emergence of both global *NFA* imbalances and asymmetric asset home bias. Various features in developing countries potentially underlie the assumed relatively low share of financial wealth and the relatively high share of labour wealth in these countries. These include technological intensity, which is associated with industrial structure and trade division, and financial underdevelopment, which is due to many deep institutional factors. As a consequence of the asymmetry, the model shows that asset demand in the developing country is relatively high and asset supply is relatively low, which depresses the autarky interest rate in the developing country and drives a net capital to flow out. A positive *NFA* position is thus formed in steady state. Correspondingly, the *NFA* position in the developed country becomes negative. Global net imbalances emerge. The distinction between financial and labour incomes gives rise to a hedging motive against labour income risk when households have to make their decisions on portfolio choices. In the two-good production open economy, the endogenous responses of investment and the real exchange rate make labour income move in the opposite direction to the return to the local asset. At the optimum, this induces a long position in local assets for both countries. Asset home bias emerges across the world. In addition, as in Chapter 3, under global net imbalances, countries are exposed to the risk associated with the return to the non-zero net external position. This results in the presence of a hedging term for net external interest payments in gross country portfolios. Moreover, the sign of the hedging in the developed country is negative, which encourage that country to hold more overseas asset. The sign of this hedging in the developing country is, however, positive, which encourage that country to hold more of the local asset. Despite an offsetting supply effect in response to the country asymmetry, both of the two above facts point to a relatively lower degree of asset home bias in the developed country compared to the developing country. To sum up, represented by the asymmetry of wealth division, some fundamental economic factors mentioned above in developing countries can be the cause of both the persistent large net external position and the significant cross-country discrepancy in the degree of home bias in portfolio holdings.

This thesis explains a set of prominent facts of country portfolios between

developing and developed countries. Besides, in this process, it provides two useful frameworks on which future works can be based. The first one is given by the model in Chapter 2. This chapter places the problem of portfolio choices in a general representative-agents model of an asymmetric two-country open economy. It is suitable for the analysis of problems where a non-trivial net portfolio position is not important. The new framework is given by the model in Chapters 3 and 4 where an *OLG* structure is employed to accommodate both non-trivial net external positions and gross portfolio choices simultaneously. Since these two aspects are both very important from a practical point of view, taking the results of Chapter 3 and 4 as a start, the next step in theoretical research will be therefore to explore the implications of merging the analysis of net and gross portfolio positions for other relevant issues in international economics. These may include, but are not restricted to, the following.

1. The analysis of gross capital flows under global imbalances. While we analysed (steady state) net capital flows in this thesis, the analysis of gross capital flows is not presented. This further analysis can be obtained through a solution for the dynamics of gross portfolio positions.
2. The analysis of policy-oriented problems, for instance optimal monetary policy and capital control problems, etc. The inclusion of portfolio choices with non-trivial *NFA* positions will inevitably affect the answers to these questions through its impact on economic volatilities, the spill-over effects of policy and international coordination problems.
3. The analysis of risk contagion and international economic crisis against the background of accelerating development of financial globalization and the previous global financial crises. All three of these lines of research represent interesting and highly topical ways to develop the analysis presented in this thesis.