

NECESSITISM, CONTINGENTISM AND THEORY EQUIVALENCE

Bruno Jacinto

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Necessitism, Contingentism and Theory Equivalence

Bruno Jacinto



University of
St Andrews

This thesis is submitted in partial fulfilment for the degree of
PhD
at the
University of St Andrews

January 22, 2016

Declaration of Authorship

I, Bruno Jacinto, hereby certify that this thesis, which is approximately 80,000 words in length, has been written by me, and that it is the record of work carried out by me, and that it has not been submitted in any previous application for a higher degree.

I was admitted as a research student in January 2012 and as a candidate for the degree of Doctor of Philosophy in January 2012; the higher study for which this is a record was carried out in the University of St Andrews between 2012 and 2016.

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It is therefore surprising that there is a proof of my necessary existence, a proof that generalizes to everything whatsoever. (...) A first reaction is that a 'proof' of such an outrageous conclusion must contain some dreadful fallacy. Yet the proof does not collapse under scrutiny. Further reflection suggests that, suitably interpreted, it may be sound. So interpreted, the conclusion is not outrageous, although it may not be the view you first thought of.

– Timothy Williamson, 'Necessary Existents'

The lesson is that whether 'we' may take a philosopher at his word depends crucially on who 'we' are, and what philosophical premisses we ourselves argue from. That is distressing. It would be nice to arrive at a non-partisan consensus about what the several parties say, before we go on to take sides in the argument. And it would be nice to do this in our own words, translating all parties into a common language, rather than by force of direct quotation. We can go some distance by giving the utmost benefit of doubt. We should be at least as generous as conscience will allow in letting things bear names we think that they do not very well deserve, especially when we report a position according to which there is no better deserver of the name to be had. But there is a limit to generosity. When we must quietly go along with (what we take to be) someone's mis-speaking in order to give a non-partisan report of his position, the price is too high. For then the advantage of common language is already forsaken.

– David Lewis, 'Noneism or Allism?'

Abstract

Two main questions are addressed in this dissertation, namely:

1. What is the correct higher-order modal theory;
2. What does it take for theories to be equivalent.

The whole dissertation consists of an extended argument in defence of the joint truth of two higher-order modal theories, namely, *Plantingan Moderate Contingentism*, a higher-order necessitist theory advocated by [Plantinga \(1974\)](#) and committed to the contingent being of some individuals, and *Williamsonian Thorough Necessitism*, a higher-order necessitist theory advocated by [Williamson \(2013\)](#) and committed to the necessary being of every possible individual.

The case for the truth of these two theories relies on defences of the following metaphysical theses: i) *Thorough Serious Actualism*, according to which no things could have been related and yet be nothing, ii) *Higher-Order Necessitism*, according to which necessarily, every higher-order entity is necessarily something. It is shown that Thorough Serious Actualism and Higher-Order Necessitism are both implicit commitments of very weak logical theories.

Prima facie, Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are jointly inconsistent. The argument for their joint truth thus relies also on showing i) their equivalence, and ii) that the dispute between Plantingans and Williamsons is merely verbal. The case for i) and ii) relies on the *Synonymy Account*, an account of theory equivalence developed and defended in the dissertation. According to the account, theories are equivalent just in case they have the same structure of entailments and commitments, and the occupiers of the places in that structure are the same propositions. An immediate consequence of the Synonymy Account is that proponents of synonymous theories are engaged in merely verbal disputes. The Synonymy Account is also applied to the debate between noneists and Quineans, revealing that what is in question in that debate is what are the expressive resources available to describe the world.

Acknowledgments

My philosophical explorations started with my BA in philosophy. Therefore, it is only appropriate to begin by thanking my teachers from back then. Many thanks in particular to João Branquinho, Adriana Silva Graça, Adriana Serrão, Maria Leonor Xavier and António Zilhão. Special thanks to the late Professor Manuel Lourenço. His generosity and philosophical acumen have had a profound impact not only in my own philosophical development but also in that of the younger generations of Portuguese philosophers.

During my BA I had the opportunity to have an enormous amount of (often heated!) philosophical discussions on the most varied topics with my dear friends Francisco Gouveia, José Mestre, Ricardo Miguel and Josiano Nereu. Our philosophical discussions continue to take place nowadays, whenever we have the opportunity to meet. Many thanks to Francisco, José, Ricardo and Josiano for continuously sparking my curiosity and forcing me to reevaluate my presuppositions. Many thanks also to my dear friend Carla Simões for her continuous incentive and support.

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Glancing through the dissertations of previous Archeans it is notorious how often Arché is praised for its unique atmosphere. I absolutely agree with them. Arché is a truly amazing place for doing philosophy. Philosophical discussion is a constant and few topics are out of bounds. Moreover, discussion takes place in the best of ways. It is both serious and friendly, conducive to collaboration

and novel ideas. In what follows I want to thank the Archeans and visitors with whom I have had the opportunity to interact and learn from.

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For Ana and Elisa.
For Inês. And for the Wee One.

1

Introduction

1.1 Questions

1.1.1 What is the Correct Higher-Order Modal Theory?

The present dissertation lies on the intersection between metaphysics and philosophical logic. Consider a language containing only the propositional connectives, modal and actuality operators, first- and higher-order quantifiers, and identity. What is the true and most comprehensive theory formulated in this language? What is the correct theory of higher-order quantification, modality, identity and their interaction? What is, in this sense, the correct higher-order modal logic? This is the question that is directly addressed by the dissertation.¹

The following are some of the relevant theses concerning the interaction between metaphysical modality and quantification:²

Necessitism. Necessarily, every individual is necessarily something.

Higher-Order Necessitism. Necessarily, every higher-order entity is necessarily something.

Contingentism. Possibly, some individual is possibly nothing.

Higher-Order Contingentism. Possibly, some higher-order entity is possibly nothing.

Part of the interest in the interaction between metaphysical modality, quantification and identity stems from the fact that some theoretical considerations favour the truth of Necessitism and Higher-Order Necessitism, despite the fact that: i) common-sense favours Contingentism and ii) theoretical considerations of a different sort favour Higher-Order Contingentism.

¹Here, what is meant with 'correct' may be cashed out as follows. A theory formulated in a language L is correct if and only if the sentences to whose truth it is committed and arguments that it takes to be valid are all and only the true sentences of the language and all and only the valid arguments of the language.

²An important caveat. The formulation of Higher-Order Necessitism here given presupposes a certain view on the identity conditions of higher-order entities, namely, that necessarily, higher-order entities P and Q are the same if and only if P and Q are necessarily coextensive. As will be explained below, identity, for the case of higher-order entities, is being used as shorthand for necessary coextensiveness. That is, whenever it is said, for instance, that x is identical to y (with x and y of type $\langle t_1, \dots, t_n \rangle$), what is meant is that necessarily, for all things z^1, \dots, z^n (of types, respectively, t_1, \dots, t_n), x is true of $z^1 \dots z^n$ if and only if y is also true of $z^1 \dots z^n$. Even though I have sympathy for the view that the identity conditions for properties is given in terms of their necessary coextensiveness, this view will play no role in the arguments to be presented in the dissertation, and so its truth is assumed nowhere.

Theories accounting for the interaction between metaphysical modality, quantification and identity are usefully grouped according to whether, according to them, Necessitism or Contingentism is true, and whether Higher-Order Necessitism or Higher-Order Contingentism is true. The theories of Adams (1981), Fine (1977), Plantinga (1976) and Stalnaker (2012), to name just a few, are all contingentist, whereas the theories of Linsky & Zalta (1994) and Williamson (1998, 2013) are all necessitist.

Williamson is not only a necessitist but also a higher-order necessitist. Moreover, even though Linsky and Zalta are concerned only with the Necessitism–Contingentism debate (and not with which one of Higher-Order Necessitism and Higher-Order Contingentism is true), it is reasonable to think that their reasons for adopting Necessitism carry over as reasons for adopting Higher-Order Necessitism.³

Among contingentists, Plantinga is, arguably, the most notable proponent of Higher-Order Necessitism. Adams, Fine and Stalnaker are all higher-order contingentists. The following groupings emerge:⁴

	Higher-order necessitism	Higher-order contingentism
Necessitism	<i>Thorough Necessitism:</i> Linsky and Zalta, Williamson	<i>Moderate Necessitism:</i> ?
Contingentism	<i>Moderate Contingentism:</i> Plantinga	<i>Thorough Contingentism:</i> Adams, Fine, Stalnaker

I will call a theory *thoroughly necessitist* just in case it is committed to the truth of both first- and higher-order necessitism, and *thoroughly contingentist* just in case it is committed to the truth of both first- and higher-order contingentism. Moreover, I will say that a theory is *moderately contingentist* just in case it is committed the joint truth of contingentism and higher-order necessitism. And I will say that a theory is *moderately necessitist* just in case it is committed to the joint truth of necessitism and higher-order contingentism.

The dissertation addresses the question whether thoroughly necessitist, moderately contingentist, moderately necessitist or thoroughly contingentist theories, or variations thereof, are correct – or at least are closer to the correct theory when compared to the existing rivals.

One quick caveat. Consider the following theses:

³Some of the reasons advocated by Linsky and Zalta in favour of Necessitism is that Necessitism is a theorem of the logic that results from combining propositional modal logic and classical first-order logic in the simplest way, the Simplest Quantified Modal Logic. Similarly, Higher-Order Necessitism turns out to be a theorem of the system resulting from combining propositional modal logic with classical higher-order logic in the simplest way.

⁴It is striking that there is an unoccupied camp, given how philosophers are prone to test the limits of different positions. First-order necessitists advocating Aristotelian views on properties, such as the view that properties are something only if instantiated, would occupy this camp, as long as they were committed to the plausible view that there could have been instantiated properties that could have been uninstantiated. There are, of course, many other ways of being both a first-order necessitist and a higher-order contingentist. Arguably, one of the reasons why the camp is presently unoccupied is that the current reasons for Necessitism, having to do with the simplicity and elegance of the resulting logics, are also present in the higher-order case.

Actualism. Every individual is actually something.

Higher-Order Actualism. Every higher-order entity is actually something.

Let Thorough Actualism consist in the conjunction of Actualism with Higher-Order Actualism. Thorough Actualism is close to a truism (even though its necessitation is not). Yet, one of the most famous theories in the metaphysics of modality, Lewis's Extreme Realism, is committed to the falsity of Actualism. Moreover, Lewis (1986, p. 97-101) has offered one important argument aimed at showing that Actualism is anything but a truism.

Yet, in this dissertation the truth of Thorough Actualism will be presupposed. I will not offer a careful defence of Thorough Actualism here. Suffice it to say that I find very plausible a conception of possible worlds, the Kripke-Stalnaker conception (Kripke, 1980), (Stalnaker, 1976), according to which Thorough Actualism comes out as true. On this conception, possible worlds are (maximal) possible states of the world, (maximal) ways things might have been. Of these ways things might have been, only one obtains. The others could have obtained but do not.⁵

Let 'Worldy', stand for the way things might have been that obtains. On the Kripke-Stalnaker conception, it is very natural to adopt the following take on the truth of sentences prefixed with 'actually', when this operator is given a rigid reading:

(1) Necessarily: actually, p if and only, if Worldy had obtained, then p .

Of course, 'actually' could be used differently. But given a commitment to the Kripke-Stalnaker conception, (1) captures one way in which 'actually' may be used. This is thus the way that 'actually' is presently being used.

Moreover, the following turns out to be true on the Kripke-Stalnaker conception:

(2) p if and only if, if Worldy had obtained, then p .

The reason why (2) is true on the Kripke-Stalnaker conception is simply that Worldy turns out to be that possible world that obtains. This means that (2) is true, even if contingently.

From (1) and (2) together it follows that

(3) p if and only if actually, p .

So, take any entity x that is something. It follows from (3) that x is actually something. Thus, every entity that is something is actually something. But every entity is something (i.e., every entity is some entity). Therefore, every entity is actually something. Thorough Actualism is true.

1.1.2 What does it take for theories to be equivalent?

The dissertation also addresses a subsidiary question, namely, what does it take for theories to be equivalent. The notion of theory equivalence in question is one concerned with what theories say,

⁵Arguably, this is the conception of possible worlds that is favoured by unreflective common sense, as argued in (Stalnaker, 1976).

not with the means by which they say it. That is, it is a notion of equivalence concerned with the relationship between theories and the world. On this notion, equivalent theories are theories whose truth requires the same thing of reality.

The question what does it take for theories to be equivalent would already be worthy of a whole dissertation addressing it. The reason why it is a subsidiary question in the present dissertation is that it is addressed in the interest of answering the question what is the correct higher-order modal theory. More precisely, it is argued in the dissertation that two theories currently on offer – theories that are, arguably, the best higher-order modal theories currently available – commonly thought to be jointly inconsistent turn out to be equivalent.

The two main views on the nature of theories, namely, the *syntactic* view, and the *semantic* view, naturally give rise to two views on theory equivalence. According to the syntactic view on the nature of theories, a theory consists in (or is adequately represented by) a set of sentences of some formal language.⁶ The semantic view has it that a theory consists in nothing but a collection of models, where these are understood as nonlinguistic entities.⁷

Regardless of whether the syntactic and semantic views are right *qua* views on the nature of theories, they offer natural accounts of theory equivalence. According to the syntactic account two theories are equivalent if and only if they consist in the same set of sentences of some formal language. And according to the semantic account two theories are equivalent if and only if they consist in the same set of models. Both the syntactic and semantic accounts are problematic, for reasons already discussed in the literature.⁸ The dissertation thus offers a novel account of theory equivalence, the *Synonymy Account*, and applies this account to the debate concerning the correct higher-order modal theory.

⁶The *received view*, put forward by Carnap (1956), Feigl (1970) and Hempel (1965), imposes the stronger constraint according to which theories contain only theoretical terms, which are connected to observational terms via correspondence rules. These contain both theoretical and observational terms. Here, the interest is not in the received view but solely in the weaker, syntactic view. For a recent defence of the received view and its history, see (Lutz, 2012).

⁷Different proponents of the semantic view are van Fraassen (1980), Giere (1988), Suppe (1989) and Suppes (2002). Some of these take theories to be set-theoretic predicates, whereas others take theories to be collections of state spaces, and even others allow models to consist of somewhat more concrete entities, such as planets and animals.

⁸According to the syntactic account there are no two theories that are both equivalent and (non-trivial) notational variants of one another. However, this is not right. It is not because ‘ \neg ’ is used for negation instead of ‘ \sim ’ and ‘ \wedge ’ is used for conjunction instead of ‘ $\&$ ’ that we thereby happen to have two non-equivalent theories. See fn. ??ch. 4]footnote:account-notational-variant for the account of notational variant being presently used.

According to the second account there are no two theories that are both equivalent and yet consist in different collections of models. But consider the collection of models consisting in all partially ordered sets such that every pair of elements has both a least upper bound and a greatest lower bound and the collection of models consisting in all algebraic structures that satisfy the commutative, associative and absorption laws. These are different collections and yet the theories that correspond to the two collections of models are equivalent, corresponding to the theory of lattices. These considerations are spelled out in a bit more detail in ch.4, p. 103.

1.2 Main Theses

1.2.1 Overgeneration of the Propositional Functions Account

Four main theses are defended in the dissertation. The first of these theses concerns a recent account of the semantics of first-order modal logic, proposed in (Stalnaker, 2012).

Stalnaker proposes an account of the semantics of first-order modal logic that I have dubbed the *Propositional Functions Account*. The propositional functions account is proposed partly in order to address a challenge facing theorists with thoroughly contingentist commitments. The challenge is to provide a compositional semantics for first-order modal languages consistent with the typical reasons for endorsing Thorough Contingentism.

Thus, Stalnaker intends the Propositional Functions Account to play an important dialectical role in the defence of his preferred higher-order modal theory. The existence of the account is intended to show that there is a compositional semantics for first-order modal predicate languages consistent with the motivations for his higher-order modal theory. Pace Stalnaker, it is shown in the dissertation that, from his own standpoint, the Propositional Functions Account overgenerates, in a sense to be explained, and for this reason is inconsistent with Stalnaker's higher-order modal theory.

Consider the following theses:

Thorough Serious Actualism. Necessarily, for every relation R , of any type, no things could have been R -related and yet been nothing.

Necessity of Something. Necessarily, there is some individual.

Thorough Serious Actualism is defended in §3.3, and independently supported by thorough contingentists such as Adams and Stalnaker (but not by Fine).

The thesis of the Necessity of Something is implied by claims to which many adhere. For instance, it is implied by widely accepted claims such as i) at least one number is a necessary being, and ii) at least one set is a necessary being (e.g., the empty set).

Say that a property is an *haecceity* of x if and only if it is the property of being x , and that it is an haecceity just in case it is possible that it is the haecceity of some x . Also, say that a proposition is an attribution of being to x just in case it is the proposition that x is something, and that it is an attribution of being if and only if it is possible that there is some x such that the proposition is an attribution of being to x . Consider the following theses:

Haecceity Necessitism. Necessarily, every haecceity is necessarily something.

Attributions of Being–Necessitism. Necessarily, every attribution of being is necessarily something.

The theses of Haecceity Necessitism and Attributions of Being–Necessitism are both consistent with Higher-Order Contingentism. However, these theses are *inconsistent* with some of the main motivations for adopting Higher-Order Contingentism. The reason is that haecceities and attributions

of being are offered by higher-order contingentists as paradigmatic examples of higher-order entities whose being is contingent, if the being of any higher-order entities is.

The first of the main claims defended in the dissertation is the following:

Overgeneration of the Propositional Functions Account. The Propositional Functions Account, together with Thorough Serious Actualism and the Necessity of Something, imply i) the Necessary Being of Haecceities, and ii) Attributions of Being-Necessitism.

In addition, it is shown that natural ways of improving on the Propositional Functions Account all lead to consequences that are undesirable from the standpoint of thorough contingentists committed to Thorough Serious Actualism.

The Overgeneration of the Propositional Functions Account implies the inconsistency between Stalnaker's higher-order modal theory and the account, pace Stalnaker, since he is committed to the falsity of both Haecceity Necessitism and Attributions of Being-Necessitism.

1.2.2 Higher-Order Necessitism

The overgeneration of the Propositional Functions Account does not suffice to establish the truth of Higher-Order Necessitism. Yet, it puts pressure on thoroughly contingentist higher-order modal theories, such as Adams's and Stalnaker's, given the defence of Thorough Serious Actualism offered in the dissertation. Higher-Order Contingentists are thus faced with the challenge of offering plausible alternative compositional accounts of the semantics of first-order modal languages, ones consistent with their theories. The fact that such accounts have yet to be offered counts against thoroughly contingentist higher-order modal theories, and so in favour of Higher-Order Necessitism.

Higher-Order Necessitism is the second of the main theses defended in the dissertation. Besides the fact that the truth of the thesis is favoured by the absence of satisfactory semantic accounts of first-order modal languages consistent with Higher-Order Contingentism, a direct deductive arguments in its defence are also offered in the dissertation, in §3.8. The arguments offered are analogous to the arguments offered for a thesis that consists in an instance of Higher-Order Necessitism, namely, the following:

Propositional Necessitism. Necessarily, every proposition is necessarily something.

Offering a defence of Propositional Necessitism is the main aim of chapter 3. Given the similarities between the arguments for Propositional Necessitism and Higher-Order Necessitism, the defence of Propositional Necessitism is easily transposed to a defence of Higher-Order Necessitism. Finally, schematic versions of the arguments for Higher-Order Necessitism to be offered turn out to support the following comprehension principle for higher-order modal logic:

$\hat{\text{Comp}}$. The relation that holds between entities x^1, \dots, x^n such that φ is necessarily something.

The result of prefixing any instance of $\hat{\text{Comp}}$ with any sequence of universal quantifiers of any type (binding parameters in φ) and necessity operators, in any order, is also an instance of $\hat{\text{Comp}}$. Principle

$\hat{\text{Comp}}$ is strictly stronger than Higher-Order Necessitism. A principle equivalent to $\hat{\text{Comp}}$ is defended in (Williamson, 2013, ch. 6). Whereas Williamson's case for $\hat{\text{Comp}}$ is abductive, the arguments for $\hat{\text{Comp}}$ presented in this dissertation are deductive.

Arguably, the arguments offered reveal that logics traditionally thought to be very weak are already committed to the theses of Propositional Necessitism, Higher-Order Necessitism, and indeed $\hat{\text{Comp}}$.

1.2.3 Theory Equivalence is Theory Synonymy

The third of the main theses defended in the dissertation is a thesis on what it takes for two theories to be equivalent. Roughly, say that formulations T_1 and T_2 of two theories, given in, respectively, languages L_{T_1} and L_{T_2} have the same *theoretical structure* under functions $f : L_{T_1} \rightarrow L_{T_2}$ and $g : L_{T_2} \rightarrow L_{T_1}$ if and only if f and g both preserve the structure of entailments and commitments of the theories. Also, say that a translation f from L_{T_1} to L_{T_2} is *deeply correct* if and only if, for every sentence φ of L_{T_1} , the proposition that is expressed by φ according to the proponents of T_1 is the same as the proposition expressed by $f(\varphi)$ according to the proponents of T_2 . Finally, say that formulations T_1 and T_2 are *synonymous* just in case there are deeply correct translations $f : L_{T_1} \rightarrow L_{T_2}$ and $g : L_{T_2} \rightarrow L_{T_1}$ such that formulations T_1 and T_2 have the same theoretical structure under f and g . The following thesis is defended in the dissertation:

Theory Equivalence is Theory Synonymy. Two theories are equivalent if and only if they have Synonymous formulations.

The Synonymy Account of theory equivalence is developed and defended in chapter 4. Part of the case for the correctness of the Synonymy Account relies on showing that it enables a more nuanced diagnostic of the debate between noneists and Quineans.

1.2.4 Equivalence

The last of the main thesis defended in the dissertation is the following:

Equivalence. Williamsonian Thorough Necessitism and Plantingan Moderate Contingentism are equivalent.

Williamsonian Thorough Necessitism is a theory containing the main elements of the higher-order modal theory defended by Williamson. Similarly, Plantingan Moderate Contingentism is a theory containing the main elements of the higher-order modal theory defended by Plantinga. One corollary of the equivalence between the theories is that there is a sense in which one need not decide between them. Rather, proponents of both theories have the same commitments, even though they have chosen different means to express them. Proponents of the two theories are involved in a *verbal dispute*.

Say that a theory is *informally sound* if and only if the arguments that are valid according to the theory are indeed valid, and the sentences to whose truth the theory is committed are indeed true. Also, say that a theory is *informally complete* if and only if the arguments that are valid in the language

of the theory are valid according to the theory, and the sentences that are true in the language of the theory are commitments of the theory.

Given the defences of Higher-Order Necessitism and $\hat{\text{Comp}}$, it is natural to conjecture that one of Williamsonian Thorough Necessitism and Plantingan Moderate Contingentism is informally sound. Moreover, given the equivalence between Williamsonian Thorough Necessitism and Plantingan Moderate Contingentism, it is natural to conjecture that *both* are informally sound, provided that they are understood according to how they are using their language.

The informal soundness of the theories does not imply that they are correct (according to how the proponents of each theory uses that theory's language), since Williamsonian Thorough Necessitism and Plantingan Moderate Contingentism are *informally incomplete*. Yet, the truth of the conjectures implies that Williamsonian Thorough Necessitism and Plantingan Moderate Contingentism are equally good approximations to the correct higher-order modal theory.

1.3 Higher-Order Quantification

One major presupposition of the dissertation is that higher-order resources are legitimate. In this section the higher-order modal language in which the theories under consideration are formulated is presented. In the next section a brief defence of higher-order resources is offered.

1.3.1 The Language

The basic higher-order language for which the higher-order theories are formulated is the language ML_P , offered in (Gallin, 1975, ch. 3). This language is defined in terms of a hierarchy of types, which I will first introduce this hierarchy, and then proceed to present language ML_P .

This hierarchy contains just one *basic* type, e . The remaining types are defined recursively. The only rule is that, for every natural number (including 0) if t_1, \dots, t_n are all types, then $\langle t_1, \dots, t_n \rangle$ is also a type. The type e , the basic type of the hierarchy, corresponds to the category of *individuals* (i.e., of *particulars*). Entities of this category, are things such as Obama and Mars. Propositions, properties of individuals, relations between propositions and individuals, and the like do not belong to this category. So, where t_1, \dots, t_n are types corresponding to the categories $c(t_1), \dots, c(t_n)$ of entities, the type $\langle t_1, \dots, t_n \rangle$ corresponds to the category of relations between entities of categories $c(t_1), \dots, c(t_n)$. For instance, the type $\langle e \rangle$ corresponds to the category of properties of particulars, the type $\langle e, e \rangle$ corresponds to the category of binary relations between particulars, and the type $\langle \langle e \rangle \rangle$ corresponds to the category of properties of properties of particulars. Note that the type $\langle \rangle$ corresponds to the category of propositions. The idea is that propositions are just one kind of relation, namely, 0-ary relations. From now onwards I will be informally calling entities of category $c(t)$ 'entities of type t '. So, individuals are entities of type e , properties are entities of type $\langle e \rangle$, propositions are entities of type $\langle \rangle$, etc.

The language ML_P contains the usual truth-functional connectives, ' \neg ', ' \wedge ', ' \vee ', ' \rightarrow ' and ' \leftrightarrow '. Besides these, ML_T contains, for each type in Gallin's hierarchy, a stock of variables ' x_t^1 ', ' x_t^2 ', \dots of

type t , and constants $\ulcorner c_t^1 \urcorner, \ulcorner c_t^2 \urcorner, \dots$ — except the set of constants of type $\langle e, e \rangle$, which also contains the constant $\ulcorner =_{\langle e, e \rangle} \urcorner$. The language also contains the quantifiers $\ulcorner \forall \urcorner$ and $\ulcorner \exists \urcorner$, and the modal operators $\ulcorner \Box \urcorner$ and $\ulcorner \Diamond \urcorner$. The set of formulas of the language is defined recursively. If s^1, \dots, s^n are expressions of type t_1, \dots, t_n , and s^0 is an expression of type $\langle t_1, \dots, t_n \rangle$, then $\ulcorner s^0 s^1 \dots s^n \urcorner$ is a formula. If φ and ψ are both formulas, then $\ulcorner \neg \varphi \urcorner, \ulcorner \Box \varphi \urcorner$ and $\ulcorner \varphi \rightarrow \psi \urcorner$ are formulas. If v is a variable of type t , and φ is a formula, then $\ulcorner \forall v \varphi \urcorner$ and $\ulcorner \exists v \varphi \urcorner$ are formulae. Nothing else is a formula. Mention of types will be omitted whenever context allows. Also, I will typically appeal to $\ulcorner x \urcorner, \ulcorner y \urcorner, \ulcorner z \urcorner$ and $\ulcorner u \urcorner$ rather than $\ulcorner x^1 \urcorner, \ulcorner x^2 \urcorner, \ulcorner x^3 \urcorner$ and $\ulcorner x^4 \urcorner$.

Slight variations of ML_P will also be considered. The addition of an actuality operator will prove useful in several contexts. The language $\text{ML}_P^\textcircled{\text{A}}$ results from language ML_P by adding to it the clause that, if φ is a formula, then $\ulcorner @\varphi \urcorner$ is also a formula. The variable-binding operator, $\ulcorner \ulcorner \urcorner$, will be used in chapter 3 in the context of arguments formulated in a higher-order modal language. The language that results from adding $\ulcorner \ulcorner \urcorner$ to ML_P is the language $\hat{\text{ML}}_P$. The addition of this operator to the language requires that complex expressions be defined simultaneously. If s is of type $\langle t_1, \dots, t_n \rangle$, and s^1, \dots, s^n are expressions of types t_1, \dots, t_n , then $\ulcorner s s^1 \dots s^n \urcorner$ is an expression of type $\langle \rangle$. If φ, ψ are expressions of type $\langle \rangle$, then $\ulcorner \neg \varphi \urcorner, \ulcorner \varphi \wedge \psi \urcorner, \ulcorner \Box \varphi \urcorner, \ulcorner @\varphi \urcorner$ and $\ulcorner \forall v_t \varphi \urcorner$ are expressions of type $\langle \rangle$. If φ is of type $\langle \rangle$ and v^1, \dots, v^n are variables of types, respectively, t_1, \dots, t_n , then $\ulcorner \hat{v}^1 \dots \hat{v}^n(\varphi) \urcorner$ is a complex expression of type $\langle t_1, \dots, t_n \rangle$.

1.3.2 Identity Between Higher-Order Entities

The theses of Higher-Order Necessitism and Higher-Order Contingentism were formulated in terms of identity and existential quantification. What are the identity conditions for higher-order entities? According to one proposed criterion, higher-order entities are the same if and only if they are i) necessarily coextensive, and ii) something. That is, let P and Q be entities of type $\langle t_1, \dots, t_n \rangle$. Then, according to the proposed criterion, necessarily, P and Q are identical if and only if both i) $\Box \forall x_{t_1}^1 \dots \forall x_{t_n}^n (Px^1 \dots x^n \leftrightarrow Qx^1 \dots x^n)$ and ii) $\exists X_{\langle t_1, \dots, t_n \rangle} \Box \forall x_{t_1}^1 \dots \forall x_{t_n}^n (Px^1 \dots x^n \leftrightarrow Xx^1 \dots x^n)$.⁹

In what follows I remain neutral, for the most part, on what the identity conditions between higher-order entities are.¹⁰ The identity symbol will be used as a mere shorthand for necessary coextensiveness. That is, $P_{\langle t_1, \dots, t_n \rangle} = Q_{\langle t_1, \dots, t_n \rangle} :=$

⁹Let *Higher-Order Serious Actualism* be the thesis that necessarily, no higher-order entities could have been related in circumstances in which one of them was nothing. The account of higher-order identity given here presupposes the truth of Higher-Order Serious Actualism twice over. Clause i) presupposes the truth of Higher-Order Serious Actualism since otherwise it could have been that $\Box \forall x_{t_1}^1 (Px \leftrightarrow Qx)$ and yet P and Q differed in that there is at least one world w such that it is true at w that x is a P and it is not true at w that x is a Q , where x is an individual that is nothing at w . Clause ii) presupposes the truth of Higher-Order Serious Actualism in that it requires that for higher-order entities to be identical they have to be something. A defence of Thorough Serious Actualism is offered in §3.3. That argument does not assume the account of higher-order identity being presented. Instead, it treats higher-order identity as a primitive.

¹⁰The exception is the take on propositions adopted in chapters 4 and 5. The Synonymy Account, defended in chapter 4, presupposes that propositions are the same if and only if they entail and are entailed by all the same propositions. Given certain auxiliary assumptions, this implies that propositions are the same if and only if they are mutually entailing.

$$:= \Box \forall x_{t_1}^1 \dots \forall x_{t_n}^n (Px^1 \dots x^n \leftrightarrow Qx^1 \dots x^n) \wedge \\ \wedge \exists X_{\langle t_1, \dots, t_n \rangle} \Box \forall x_{t_1}^1 \dots \forall x_{t_n}^n (Px^1 \dots x^n \leftrightarrow Xx^1 \dots x^n)$$

Thus, it turns out that $\exists X_{\langle t_1, \dots, t_n \rangle} (P_{\langle t_1, \dots, t_n \rangle} = X)$ is equivalent to:

$$\exists X_{\langle t_1, \dots, t_n \rangle} \Box \forall x_{t_1}^1 \dots \forall x_{t_n}^n (Px^1 \dots x^n \leftrightarrow Xx^1 \dots x^n)$$

Similarly, the English phrase ‘ P is something’ is itself used as shorthand for ‘there is something necessarily coextensive with P ’. Thus, the higher-order modal theses presented so far are appropriately regimented as follows:

Necessitism.

- $\Box \forall x_e \Box \exists y_e (x_e = y_e)$.

Higher-Order Necessitism.

- $\Box \forall X_{\langle t_1, \dots, t_n \rangle} \Box \exists Y_{\langle t_1, \dots, t_n \rangle} \Box \forall x_{t_1}^1 \dots \forall x_{t_n}^n (Xx^1 \dots x^n \leftrightarrow Yx^1 \dots x^n)$.

Contingentism.

- $\Diamond \exists x_e \Diamond \exists y_e \neg (x_e = y_e)$.

Higher-Order Contingentism.

- $\Diamond \exists X_{\langle t_1, \dots, t_n \rangle} \Diamond \exists Y_{\langle t_1, \dots, t_n \rangle} \Diamond \neg \forall x_{t_1}^1 \dots \forall x_{t_n}^n (Xx^1 \dots x^n \leftrightarrow Yx^1 \dots x^n)$.

Thorough Serious Actualism.

- $\Box \forall Y_{\langle t_1, \dots, t_n \rangle} \Box \forall x_{t_1}^1 \dots \Box \forall x_{t_n}^n \Box (Yx^1 \dots x^n \rightarrow (\exists z_{t_1} (x^1 = z_{t_1}) \wedge \dots \wedge \exists z_{t_n} (x^n = z_{t_n})))$.

Necessity of Something.

- $\Box \exists x_e \exists y_e (x_e = y_e)$.

Haecceity Necessitism.

- $\Box \forall X_{\langle e \rangle} (\Diamond \exists z_e \Box \forall u_e (Xu_e \leftrightarrow u = z) \rightarrow \Box \exists Y_{\langle e \rangle} (X = Y))$.

Attributions of Being–Necessitism.

- $\Box \forall X_{\langle \rangle} (\Diamond \exists z_e \Box (X \leftrightarrow \exists u_e (u = z)) \rightarrow \Box \exists Y_{\langle \rangle} \Box (X \leftrightarrow Y))$.

1.3.3 Neutral Higher-Order Modal Logic

The arguments that will be presented require an appeal to a logic neutral between the different higher-order modal theories on offer. A model-theoretic characterisation of one such neutral logic is offered in §1.6.

The logic given has as its modal propositional fragment the very weak logical system **K**. In general, the stronger modal logic **S5** is presupposed in the dissertation. The exception is chapter 3, the arguments presented there will rely on propositional modal logics weaker than **S5**, namely, the logics **K** and **KD**.

In the dissertation’s [appendix A](#) several first- and second-order modal logics incorporating the assumption that (zero- and first-order) constants are *strongly Millian* are offered, where a constant is strongly Millian just in case it is guaranteed to have a semantic value in the actual world. It is shown that once this metalinguistic presupposition is in place contingentists have available the full power of

classical quantification theory, in a sense made precise in the appendix. The logics are characterised both axiomatically and model-theoretically, with proofs of soundness and completeness being given.

As mentioned, the dissertation presupposes the legitimacy of higher-order quantification. In the next section an objection to the legitimacy of higher-order quantification is considered. The defence is primarily of the legitimacy of propositional quantification, even though it is extendable to remaining forms of higher-order quantification.

1.4 A Defence of Higher-Order Resources

One of the claims that will be discussed later on is the following:

- (4) Necessarily, if Obama is a president, then it is true that Obama is a president.

This claim is clearly an instance of a general principle about truth. Without the resources of propositional quantification, the principle in question cannot be appropriately captured. Without such resources, one would have to appeal to a schematic presentation of the principle, such as the following:

- (5) Necessarily, if φ , then it is true that φ .

Here, φ is a metalinguistic variable which may be substituted by sentences of the language in question. A commitment to the truth of schema (5) is nothing but a commitment to the truth of every formula that is the result of substituting φ by some sentence of English (or some sublanguage thereof).

The problem is that a schematic formulation of the principle does not have the required generality. Some propositions are not expressible in English. Even if all propositions were expressible in English, English might not contain sentences expressing every possible proposition. But the appropriate generalisation of (5) needs to be applicable to every possible proposition, as we shall see in §3.4.

Once propositional quantification is available, this problem vanishes. The appropriate generalisation of (5) consists in the following claim (where ‘ T ’ is a predicate of type $\langle\langle\rangle\rangle$ standing for the property of truth, as applicable to *propositions*):

Truth Introduction.

1. Necessarily, for every p , necessarily, if p , then it is true that p .
2. $\Box\forall p\Box(p \rightarrow Tp)$

Yet, some find the appeal to higher-order resources suspect.¹¹ Consider the following thesis:

Meaningless Propositional Quantification. If the propositional and higher-order quantifiers are meaningful, then the meanings of propositional and higher-order quantifications are compositionally specifiable in English or in some other natural language.

¹¹Quine (1986) rejects the legitimacy of both propositional and higher-order quantification, whereas Richard (2013) argues against the legitimacy of propositional quantification.

Suppose that it is true, as it appears to be, that there is no compositional specification, in any natural language, of the meanings of propositional and higher-order quantifications. Assume that Meaningless Propositional Quantification is true. Then propositional and higher-order quantifiers have no meaning. A fortiori, my appeal to the expressive resources of propositional and higher-order quantification is illegitimate. In what follows I will be focusing on the case of propositional quantification. But what will be said can easily be transposed to the case of other higher-order resources.

Meaningless Propositional Quantification is motivated by the following view on how artificial languages are endowed with meaning:

Meaningfulness for Artificial Languages. The sentences and subsentential expressions of any artificial language L are endowed with meaning partly via compositional specifications of their meanings in some other meaningful language.

Arguably, Richard's (2013, pp. 139-142) objection to the legitimacy of propositional quantification is based on something like Meaningfulness for Artificial Languages. Richard argues that Prior's account of propositional quantification (an account similar to the one developed here) is mysterious on the grounds that it 'provides no hope of finding a systematic account of their truth-conditions' (Richard, 2013, p. 140), and proposes that it be abandoned for this reason. But later on Richard does mention the possibility of a compositional semantics for propositional quantification, albeit one not 'of a traditional sort' (Richard, 2013, p. 141). Given the direction of the discussion, it is reasonable to assume that he disavows the legitimacy of such untraditional compositional semantics. But why is it illegitimate? Note that the reason cannot just be that this semantics would have to employ the expressive resources of propositional quantification. After all, the typical compositional semantic accounts of objectual quantification themselves appeal to the expressive resources of objectual quantification. It would appear that the untraditional semantics is illegitimate because it *outstrips*, in a certain sense, the expressive resources of natural language. The thesis of Meaningfulness for Artificial Languages makes precise the sense in which this is so.

If Meaningfulness for Artificial Languages is true, then the sentence ' $\forall x(x = x)$ ' has some meaning only if it has been compositionally specified in some other meaningful language. Note that it is not claimed that such specification is sufficient for the sentence ' $\forall x(x = x)$ ' to have its meaning. Other conditions are certainly required.

Assume for the moment that Meaningfulness for Artificial Languages is true. In such case specifications of the meanings of the sentences of an artificial language have to bottom out in some natural language. Otherwise, there will be a vicious infinite descent. That is, otherwise, an artificial language L_1 is meaningful only if the meanings of its sentences are specified in an artificial language L_2 , and L_2 is meaningful only if the meanings of its sentences are specified in an artificial language L_3 , and so on. In such case none of the languages in the chain have been endowed with meaning.

Consider any meaning-conferring chain, and let L be its end language. The meanings of the sentences of every language preceding L must be compositionally specifiable in L , since compositional

specificability is a transitive relation. So, the meanings of artificial languages with propositional quantification must be compositionally specificable in some natural language. That is, if Meaningfulness of Artificial Languages is true, then so is Meaningless Propositional Quantification.

But why think that Meaningfulness of Artificial Languages is true? First, note that it is possible for speakers of a natural language to augment the expressive resources of their language without any appeal to other languages. That is, for instance, English speakers can augment English's expressive resources without resorting to other languages from which those resources are borrowed. So, likewise, it is possible to augment the expressive resources of an already meaningful artificial language without those resources being borrowed from any other language. For example, suppose that an artificial language L is endowed with meaning via specifications of the meanings of its sentences in English. Just as it is possible for English speakers to augment English's expressive resources without resorting to any other language, it is also possible for users of L to augment L 's expressive resources without those resources being borrowed from any other language.

We have just seen that it is possible to endow sentences and subsentential expressions of extensions of natural languages with meanings even when the sentences' meanings cannot be specified in any natural language. So, likewise, it should be possible to endow sentences and subsentential expressions of artificial languages with meanings even when the meanings of some sentences cannot be specified in any other natural language.

One way in which this can be done is by *indicating* the meanings of those sentences. *Specification* is a form of indication. That is, one way in which the meanings of sentences of a language may be indicated is by specifying sentences of some other language having the same meanings. But specification is not the only form of indication. Another way in which the meanings of sentences may be indicated is by appealing to background common knowledge, use of contextually salient features, analogies, etc.

Suppose one indicates the meanings of sentences of a source language by specifying their meanings in some already meaningful target language. Then mastery of the target language will guarantee at least some grasp of these meanings. In other cases of indication one has to secure that one's interlocutors grasp those meanings and that these meanings get assigned to the right sentences of the source language. As with other forms of sharing knowledge, some of the work in figuring out the knowledge being shared may be left to one's interlocutor.

For instance, when imparting mathematical knowledge it is not uncommon to leave some facts unsaid, letting the student do some of the work. And often one of the cues given to the student is that there are certain structural similarities between his current topic of study and something else which he has previously encountered. That is, the student is directed to the requisite piece of knowledge by being made aware of the fact that his current topic of study is analogous to something else which he has previously studied. Relatedly, the meanings of sentences of artificial languages can be indicated also via analogy. Analogy may be used, for instance, in those cases in which one wants to indicate the meanings of some sentences to those who do not yet have a grasp of their meanings.

I am not claiming that one endows the sentences of propositionally quantified languages with meanings simply by indicating them. Similarly, the proponent of Meaningfulness of Artificial Languages does not claim that the sentences of artificial languages are endowed with meanings solely in virtue of specifications of these in some meaningful language. Use, in a broad sense, must play some additional role. Furthermore, my claim is a conditional one. If specifications of meanings, together with use, suffice to endow the sentences of artificial languages with meaning, then indications of meanings, together with use, suffice to endow the sentences of artificial languages with meaning.

The meanings of sentences of artificial languages with propositional quantifiers can be indicated by appealing to structural similarities between objectual and propositional quantification. Quine (1960) says that individual variables are best understood as ‘abstractive pronouns’. As he puts it (Quine, 1960, p. 343), an individual variable is ‘a device for marking positions in a sentence, with a view to abstracting the rest of the sentence as predicate’. Quine is here alluding to two aspects of individual variables that work together: i) an individual variable marks positions in a sentence; ii) by marking positions in a sentence, the rest of the sentence can be abstracted as a predicate.

A propositional variable, just like an individual variable, marks positions in a sentence. But the positions in a sentence that it can mark are not those that can be marked by an individual variable. An individual variable can only mark the positions of individual constants. These positions correspond, in English readings of formulas, to the positions occupied by pronouns.¹² Whereas individual variables can only mark the positions of individual constants, propositional variables can only mark the positions of formulas. These positions correspond, in English readings, to the positions of sentences.

What does it take for an individual variable to play an *abstractive* role? Occurrences of an individual variable ‘ x ’ in a formula S play an abstractive role because, if ‘ x ’ had a referent, then the truth of S relative to a world would depend on what the referent of ‘ x ’ was. A formula S containing some occurrences of an individual variable ‘ x ’ behaves as a predicate because the truth-value of S relative to a world depends on the referent of ‘ x ’ (or would depend on the referent of ‘ x ’, if ‘ x ’ had a referent).

Likewise, propositional variables play an abstractive role. Occurrences of a propositional variable ‘ p ’ in a formula S play an abstractive role because, if ‘ p ’ had a meaning, then the truth of S relative to a world would depend on what the meaning of ‘ p ’ was. By analogy with the case of individual variables it can be seen that a formula containing some occurrences of a propositional variable behaves as a predicate. A formula S containing some occurrences of a propositional variable ‘ p ’ behaves as a predicate because the truth-value of S relative to a world depends on the meaning of ‘ p ’ (or would depend on the meaning of p , if p had a meaning).

Before proceeding, let me note that when speaking of the meanings of sentences I seem to be talking about something that could be referred to by a name, or by an individual constant. But my talk of meanings has been mere *façon de parler*. Sentences, unlike names and individual constants, do not refer to anything. At this point analogy is required. Propositional variables play an abstractive

¹²For instance, the formula ‘ $\forall x(Nx \rightarrow (x + 1 = x + (3 - 2)))$ ’ has as one of its readings the sentence ‘for every thing, if it is a number, then the result of adding 1 to it is identical to the result of adding 3 - 2 to it’. The position occupied by ‘it’ in the English sentence corresponds to the position occupied by x in ‘ $Nx \rightarrow (x + 1 = x + (3 - 2))$ ’.

role similar to that of individual variables. But this abstractive role cannot be fully stated (in English), since English does not have the expressive resources to do so, precisely because it lacks propositional quantifiers. Nonetheless, the structural similarities between the abstractive roles of individual variables and of propositional variables are there, and can be appreciated. Still, the interlocutor has to do some of the work.

What are, then, the meanings of propositional quantifications? Once more, the place to start is objectual quantification. Consider the formula ' $x = x \vee \neg x = x$ '. When ' m ', an individual constant referring to Mars, replaces ' x ' in ' $x = x \vee \neg x = x$ ', the result is the true formula ' $m = m \vee \neg m = m$ '. The same holds for every individual constant of the language that has a referent. Even if one augmented the language with a novel stock of individual constants, replacing ' x ' with any individual constant c would result in a true sentence, provided that c had a referent. Moreover, this would have been no accident, since for every thing, if x had it as a referent, then ' $x = x \vee \neg x = x$ ' would have been true. ' $\forall x(x = x)$ ' is true if and only if for every thing, if x had it as a referent, then ' $x = x \vee \neg x = x$ ' would have been true. More generally: ' $\forall x(\varphi)$ ' is true at a world w if and only if for every thing that is something at w , if ' x ' had it as a referent, then φ would have been true at w .

Consider now the formula ' $p \vee \neg p$ '. When ' $m = m$ ' replaces ' p ', then the resulting formula, ' $m = m \vee \neg m = m$ ' is true. The same holds for every other sentence that has a meaning. Furthermore, consider any extension of the language, and any sentence S of this extension that has a meaning. The result of replacing ' p ' by S in ' $p \vee \neg p$ ' would result in a true sentence. Moreover, this would have been no accident, since . . . ' $\forall p(p \vee \neg p)$ ' is true if and only if . . . More generally: ' $\forall p(\varphi)$ ' is true at a world w if and only if . . .

The dots cannot be appropriately filled by appealing only to first-order resources. But my interlocutor should be able to figure out what I am getting at. The most that I can say is the following: ' $\forall p(p \vee \neg p)$ ' is true if and only if for every proposition, if ' p ' had it as its meaning, then ' $p \vee \neg p$ ' would have been true; and ' $\forall p(\varphi)$ ' is true if and only if for every proposition, if p had it as its meaning, then φ would have been true. But this is mere *façon de parler*. What I have literally said is false, since meanings are in the range of first-order quantifiers. Nonetheless, I will have communicated something true, provided that my interlocutor understood the analogy between objectual and propositional quantification.

I have shown how the meanings of propositionally quantified sentences may be indicated. If specifications of the meanings of propositionally quantified sentences, together with use, suffice to endow them with meaning, then indications of the meanings of sentences, together with use, suffice to endow them with meanings. Given the dialectically neutral assumption that specifications of the meanings of propositionally quantified sentences, together with use, suffice to endow these sentences with meaning, it follows that use, together with indications of meanings on the lines of the ones given, suffice to endow propositionally quantified sentences with meaning.

To conclude, propositional quantification is a legitimate form of quantification. The meanings of sentences exhibiting other forms of higher-order quantification may be indicated in similar ways,

and so these forms of quantification are also legitimate. More complete defences of the legitimacy of propositional quantification are given in (Prior, 1971) and (Grover, 1972).¹³

The present defence of propositional quantification adds to those discussions the observation that one of the ways in which we can augment our expressive resources with propositional quantifiers appeals to processes that are quite familiar, namely, reasoning by analogy. All the proponents of propositional quantification appeal to this form of reasoning at some point in their exposition of propositional quantification. My view is that reasoning by analogy plays an important role in a general account of how the expressive resources of a language can be extended from within that language.

In what follows, I will, for the most part, continue to speak of propositions, as if sentences had as their meanings things that are possible values of individual variables. But it should by now be clear what is meant with such talk. Propositions are “higher-order entities”. Roughly, they are the entities in the range of propositional variables.

1.5 Overview of the Dissertation

In chapter 2 the thesis of the Overgeneration of the Propositional Functions Account is defended. A subsidiary aim of the chapter is to show that natural ways of improving on the Propositional Functions Account all lead to consequences that are undesirable from the standpoint of thorough contingentists committed to Thorough Serious Actualism.

The main aim of chapter 3 is to present a defence of Propositional Necessitism. The defence of Propositional Necessitism crucially depends on the truth of Thorough Serious Actualism. For this reason a defence of Thorough Serious Actualism is offered in chapter 3. A defence of Propositional Necessitism is offered in this chapter, and it is also shown in chapter 3 that the main arguments offered for Propositional Necessitism have analogue arguments establishing the truth of *Property Necessitism* (the thesis that every property is necessarily something), Higher-Order Necessitism, and indeed of the stronger comprehension principle $\hat{\text{Comp}}$.

At this point in the dissertation higher-order resources will have been vindicated. Moreover, it will have been shown that the Propositional Functions Account is inconsistent with the intuitions underlying thoroughly necessitist theories committed to Thorough Serious Actualism. The truth of Thorough Serious Actualism will also have been defended, showing that the elegance and plausibility of the Propositional Functions Account constitutes abductive evidence in favour of Thorough Higher-Order Necessitism. Finally, besides such indirect evidence for Higher-Order Necessitism, a more direct argument in its defence will have been presented.

In the fourth chapter the Synonymy Account of theory equivalence is presented and defended. The defence of the account proceeds in three steps. Some desiderata on a correct theory of equivalence are extracted from the literature on the debate between noneists and Quineans. Regardless of the status of that debate, the literature reveals some desiderata that an account of theory equivalence

¹³The legitimacy of higher-order quantification is recently defended in (Williamson, 2013, §5.9).

should be able to satisfy. The first step in the defence of the Synonymy Account consists in showing that the account satisfies the desiderata. Afterwards, the account is applied to the debate between Quineans and noneists, revealing that it leads to a deeper understanding of this and other debates. Finally, some objections to the Synonymy Account are considered and replied to.

The Synonymy Account is then applied to the question what is the correct higher-order modal theory. In previous chapters the truth of Higher-Order Necessitism was defended. In chapter 5 it is argued that the two main candidate higher-order necessitist theories, namely, Williamsonian Thorough Necessitism and Plantingan Moderate Contingentism, are equivalent. It is shown how sense can be made of this result, and consequences of the equivalence between the two theories are drawn. One particularly significant consequence is that the dispute between the proponents of the two theories is merely verbal. The chapter closes with a natural conjecture, namely, that the theories are both informally sound when understood in the way intended by their proponents.

The conclusion sums up the findings of the dissertation. A salient direction for future research consists in further developing the foundations of the Synonymy Account of Theory Equivalence, and to apply it to debates in metaphysics and other areas.

Finally, in appendices 1.6 and A logical tools for engaging in the debate concerning what is the correct higher-order modal theory are presented. In particular, logics for *strongly Millian* constants are offered in appendix A, where a constant is strongly Millian just in case its semantic value is actually something. Axiomatic and model-theoretic characterisations of several first- and second-order modal strongly Millian logics are offered, and accompanying completeness results are given. It is shown that once the metalinguistic presupposition that the constants of the language are strongly Millian is in place contingentists have available the full power of classical quantification theory.

1.6 Appendix

A model-theoretic characterisation of the neutral higher-order modal logic presupposed in the dissertation is here given. The first step of this characterisation is the notion of a **K**-neutral model structure:

Definition (K-Neutral Model Structure.). A **K**-neutral model structure is a quintuple $\langle W, \odot, R, d, D \rangle$, where:

1. W is a non-empty set;
2. R is relation on $W \times W$ (i.e., $R \subseteq W \times W$);
3. $\odot \in W$;
4. $R \subseteq W \times W$;
5. d is a function with domain W and range the set-theoretic hierarchy such that $\bigcup_{w \in W} d(w) \neq \emptyset$;
6. Let T be the set of types, as these are defined in 1.3.1 (and in (Gallin, 1975, ch. 3)). D is any function with domain $W \times T$ and range the set-theoretic hierarchy such that:
 - (a) $D_e(w) = d(w)$;
 - (b) $D_{\langle t_1, \dots, t_n \rangle}(w) \subseteq (\mathcal{P}(\bigcup_{w \in W} D_{t_1}(w) \times \dots \times \bigcup_{w \in W} D_{t_n}(w)))^W$; ¹⁴
 - (c) $\bigcup_{w \in W} D_t(w) \neq \emptyset$, for all $t \in T$.

According to the usual glosses, W is or represents the set of all possible worlds, \odot is or represents the actual world, R is or represents the relation that obtains between two worlds when the second is possible from the standpoint of the first, $D_t(w)$ is or represents the domain of entities of type t that are something at w .

Let me now turn to **K**-neutral models:

Definition (K-Neutral Model.). A **K**-neutral model based on a **K**-neutral model structure $\langle W, \odot, R, d, D \rangle$ is a sextuple $\langle W, \odot, R, d, D, V \rangle$, where V is a function such that for any constant s of type t , $V(s) \in \bigcup_{w \in W} D_t(w)$. In particular, $V(=)(w) = \{\langle d, d \rangle : d \in D_e(w)\}$.

A variable assignment g of a **K**-neutral model M is any function g from the set of variables to $\bigcup_{t \in T} \bigcup_{w \in W} D_t(w)$ such that, for each variable v of type t , $g(v) \in \bigcup_{w \in W} D_t(w)$. Where g is a variable-assignment, $g[v/f]$ is a function just like g except that it assigns f to the variable v .

The function Val is defined as follows:

Definition (Val Function.).

1. $Val^g(s) = V(s)$ for all constants s of type e ;
2. $Val_w^g(s) = V(s)(w)$ for all constants s of any type other than e ;
3. $Val_w^g(v) = g(v)(w)$ for all variables v , of any type;

¹⁴Here and throughout the dissertation the expression $\ulcorner X^W \urcorner$ is used to denote the collection of all functions with domain W and codomain X .

4. $Val_w^g(s_{\langle t_1, \dots, t_n \rangle} s_{t_1}^1 \dots s_{t_n}^n) = \{\emptyset : \langle Val^g(s^1) \dots Val^g(s^n) \rangle \in Val^g(s)(w)\};$
5. $Val_w^g(\neg\varphi) = \{\emptyset\} - Val_w^g(\varphi);$
6. $Val_w^g(\varphi \wedge \chi) = Val_w^g(\varphi) \cap Val_w^g(\psi);$
7. $Val_w^g(\Box\varphi) = \bigcap_{w \in W} Val_w^g(\varphi);$
8. $Val_w^g(@\varphi) = Val_{\odot}^g(\varphi);$
9. $Val_w^g(\forall v_t \varphi) = \bigcap_{f \in D_t(w)} Val_w^{g[v_t/f]}(\varphi);$
10. $Val_w^g(\hat{v}_{t_1}^1 \dots \hat{v}_{t_n}^n(\varphi)) =$
 $= \{\langle f^1, \dots, f^n \rangle \in \bigcup_{w \in W} D_{t_1}(w) \times \dots \times \bigcup_{w \in W} D_{t_n}(w) : Val_w^{g[v^1/f^1, \dots, v^n/f^n]}(\varphi)\}.$

Note that, for each expression ζ (of any type other than e), $Val^g(\zeta)$ is used to denote a function f with domain W and such that $f(w) = Val_w^g(\zeta)$.

Definition (Truth). An expression φ of type $\langle \rangle$ is true in a **K**-neutral model M relative to variable-assignment g and world w , $M, w, g \models_{\mathbf{K}n} \varphi$, if and only if $Val_w^g(\varphi) = \{\emptyset\}$.

Definition (K-Neutral Validity). An argument with premises Γ and conclusion φ is **K**-neutrally valid, $\Gamma \models_{\mathbf{K}n} \varphi$, if and only if there is no **K**-neutral model $M = \langle W, \odot, R, d, D, V \rangle$, variable-assignment g of M and $w \in W$ such that $M, w, g \models_{\mathbf{K}n} \gamma$ for all $\gamma \in \Gamma$ and $M, w, g \not\models_{\mathbf{K}n} \varphi$.

Let a **K**-neutral inhabited model structure $M = \langle W, \odot, R, d, D, V \rangle$ be a **S5**-neutral inhabited model structure if and only if $R = W \times W$. Also, let a **S5**-neutral model be any **K**-neutral model based on an **S5**-neutral inhabited model structure.

Definition (S5-Neutral Validity). An argument with premises Γ and conclusion φ is **S5**-neutrally valid, $\Gamma \models_{\mathbf{S5}n} \varphi$, if and only if there is no **S5**-neutral model $M = \langle W, \odot, R, d, D, V \rangle$, variable-assignment g of M and $w \in W$ such that $M, w, g \models_{\mathbf{K}n} \gamma$ for all $\gamma \in \Gamma$ and $M, w, g \not\models_{\mathbf{K}n} \varphi$.

For the most part of the dissertation **S5**-neutral validity is the canon of validity underlying the arguments given. The exception is chapter 3, where the canon of validity is **K**-neutral validity.

2

Thorough Contingentism and the Propositional Functions Account

2.1 Introduction

The classic compositional accounts of the semantics of first-order modal languages are inconsistent with the conjunction of Thorough Actualism with common intuitions advanced in support of Thorough Contingentism. A major challenge facing thorough contingentists is thus to offer a satisfactory, compositional account of the semantics of these languages that is consistent with their commitments.

Recently, [Stalnaker \(2012\)](#) has offered an account, which I will be calling the ‘Propositional Functions Account’, that he takes to meet this challenge. The present chapter has three aims. The first aim consists in offering a more detailed characterisation of the Propositional Functions Account. The second aim is to present some unforeseen consequences of the Propositional Functions Account, and to show that these consequences, in conjunction with Thorough Serious Actualism, are inconsistent with the intuitions underlying higher-order contingentist theories (and a fortiori, these consequences are also inconsistent with the intuitions underlying thoroughly contingentist theories). In particular, the Propositional Functions Account turns out to be inconsistent with Stalnaker’s own higher-order modal theory. The third and final aim is to show that easy fixes to the Propositional Functions Account yield unsatisfactory accounts of the semantics of first-order modal languages.

The inconsistency between i) the Propositional Functions Account, ii) Thorough Serious Actualism, and iii) the intuitions underlying higher-order contingentist theories turns out to have important consequences. Assuming the defence of Thorough Serious Actualism offered in [chapter 3](#) is successful, the Propositional Functions Account is inconsistent with the thoroughly contingentist theories worth considering, namely, the thoroughly contingentist theories committed to Thorough Serious Actualism. It thus remains to be seen whether proponents of thoroughly contingentist and thoroughly seriously actualist theories are able to meet the challenge of offering a compositional semantics of first-order modal languages consistent with their commitments.

Moreover, the attractiveness of the Propositional Functions Account itself counts as a pro tanto reason for higher-order necessitist theories, since the Propositional Functions Account is inconsistent

with the intuitions for higher-order contingentism.

The chapter is structured as follows. In the second section the classic compositional semantics for first-order modal languages (inspired in the model-theory proposed in (Kripke, 1963)) are presented and shown to be inconsistent with the conjunction of i) intuitions driving thorough contingentism – namely, i.a) the thesis that there could have been something that is actually nothing, and i.b) the thesis that there could have been something such that the property that would have been its haecceity is actually nothing –, and ii) Thorough Serious Actualism, the thesis that every entity is actually something.

Then, the Propositional Functions Account is characterised in some detail in §2.3. Firstly, a rough sketch of the account is offered. Afterwards, a detailed presentation is provided. Finally, it is shown how the Propositional Functions Account can be modelled via the Kripkean model-theory for first-order modal languages.

In §2.4 it is shown that, from the standpoint of thorough contingentists committed to Thorough Serious Actualism, the Propositional Functions Account *overgenerates*, in the sense that it is inconsistent with intuitions underlying support for thorough contingentism once it is considered in conjunction with Thorough Serious Actualism. Finally, it is shown in §2.5 that different proposals for ways of fixing the Propositional Functions Account all turn out to yield unsatisfactory accounts of the semantics of first-order modal languages.

2.2 Compositional Semantics and Thorough Contingentism

In the most common first-order modal languages the quantifiers perform a double duty as devices of generality and as variable-binding operators. A quantified formula ψ is obtained by attaching a variable v to a quantifier Q , thus obtaining a variable-binding expression Qv , and attaching to this variable binding expression a formula φ , thus obtaining the quantified formula $\psi = Qv\varphi$.

The focus of the present chapter will be on slightly different first-order modal languages. In these languages the the quantifiers are simply devices of generality. Each quantifier Q attaches directly to a unary (simple or complex) predicate P to form a quantified formula QP . In addition, the languages contain a variable-binding operator, $\hat{\cdot}$, with \hat{v} being a variable-binding expression which attaches to a formula φ to form a complex unary predicate $\hat{v}(\varphi)$, intended to express the property of being a v such that φ .¹ The reason for focusing on such languages is that these are the first-order modal languages for which Stalnaker has originally proposed the account. The focus on these languages should be unproblematic. The distinction between quantification and variable-binding is both syntactically and semantically perspicuous.

Briefly, besides the universal quantifier and the variable-binding operator, $\hat{\cdot}$, the first-order modal languages considered contain a stock of denumerably many individual variables, at most denumerably many individual constants and at most denumerably many (atomic) n -ary predicates, with $=_2$ a binary

¹(Stalnaker, 1977) is a relevant discussion of the advantages of distinguishing between variable-binding and quantification.

(atomic) predicate. The languages under consideration also contain the boolean connectives \neg and \wedge , and the modal operators \Box and $\@$ (the remaining boolean connectives and the possibility operator are all defined in the usual manner). The set of formulae and unary predicates are the smallest sets such that:

1. If s^1, \dots, s^n are terms (i.e., individual constant or variable) and ζ_n is an n -ary predicate, then $\zeta_n s^1, \dots, s^n$ is a formula;
2. If φ is a formula, then $\neg\varphi$ is a formula;
3. If φ and ψ are formulae, then $\varphi \wedge \psi$ is a formula;
4. If φ is a formula, then $\Box\varphi$ is a formula;
5. If φ is a formula, then $\@\varphi$ is a formula;
6. If ζ is a unary predicate, then $\forall\zeta$ is a formula;
7. Atomic unary predicates are unary predicates;
8. If φ is a formula, then $\hat{v}(\varphi)$ is a (complex) unary predicate.

I will call any such language a $\hat{M}\hat{L}$ -language. The standard accounts of the compositional semantics for $\hat{M}\hat{L}$ -languages are directly inspired on the Kripkean model-theoretic semantics for these languages. I will thus begin by offering a brief description of the Kripkean model-theoretic semantics for $\hat{M}\hat{L}$ -languages.

An inhabited model structure ('model structure', for short) consists of a triple $IS = \langle W, \odot, D \rangle$, where $\odot \in W$ and D is a function with domain W and range some set in the set-theoretic hierarchy (perhaps enriched with urelements), and such that $\bigcup_{w \in W} D(w) \neq \emptyset$.²

A model is a pair $M = \langle IS, V \rangle$, where IS is an inhabited model structure, the inhabited model structure of M , and V , the valuation function of M , is a function such that:

1. For each individual constant s , $V(s) \in \bigcup_{w \in W} D(w)$;
2. For each n -ary predicate ζ , $V(\zeta) \in (\mathcal{P}(\bigcup_{w \in W} D(w))^n)^W$, for each natural number other than zero;
 - In particular, $V(=)$ is a function with domain W and such that $V(=)(w) = \{\langle o, o \rangle : o \in \bigcup_{w \in W} D(w)\}$
3. For each 0-ary predicate ζ , $V(\zeta) \subseteq W$.

The function $V(\cdot)$ is, or represents, a function assigning, to the individual constants and simple n -ary predicates of the language entities which are, or represent, their semantic values.

Let a variable-assignment g over an inhabited model structure IS be a function mapping each

²In (Kripke, 1963) quantificational model structures (here called *inhabited model structures*) also contain an accessibility relation between the elements in W . Here, only the simplest case is considered, the one where inhabited model structures possess no accessibility relation. The class of models based on these inhabited model structures determines the propositional modal logic **S5**.

Even though the system **S5** is not uncontroversial (see, e.g., (Salmon, 1989)), according to Williamson (2013, p. 44), '(...) most metaphysicians accept **S5** as the propositional modal logic of metaphysical modality (...): The assumption of **S5** should thus be relatively unproblematic in the present context. Moreover, the results here presented would carry over to weaker modal systems.

variable to $\bigcup_{w \in W} D(w)$. Given a variable-assignment g , let $g[v/d]$ be a function just like g except that it assigns to the variable v the object $d \in \bigcup_{w \in W} D(w)$.

Where V is the valuation function of M and g is a variable-assignment over the inhabited model structure of M , the function V^g is defined as follows:

1. $V^g(s) = V(s)$
2. $V^g(v) = g(v)$
3. $V^g(\zeta) = V(\zeta)$
4. $V^g(\zeta t^1 \dots t^n) = \{w \in W : \langle V^g(t^1), \dots, V^g(t^n) \rangle \in V^g(\zeta)(w)\}$
5. $V^g(\neg\psi) = W - V^g(\psi)$
6. $V^g(\psi \wedge \chi) = V^g(\psi) \cap V^g(\chi)$
7. $V^g(\Box\psi) = W$ if $V^g(\psi) = W$; otherwise, $V^g(\Box\psi) = \emptyset$
8. $V^g(@\psi) = W$ if $\odot \in V^g(\psi)$; otherwise, $V^g(@\psi) = \emptyset$
9. $V^g(\hat{v}(\psi))(w) = \{d \in \bigcup_{w \in W} D(w) : V^{g[v/d]}(\psi)(w) = \emptyset\}$
10. $V^g(\forall\hat{v}(\psi)) = \{w \in W : V^g(\hat{v}(\psi))(w) = d(w)\}$

A formula φ is true in a model M relative to a variable-assignment g if and only if $\odot \in V^g(\varphi)$. A formula is true in M if and only if for every variable-assignment g of M , $\odot \in V^g(\varphi)(\odot)$.

2.2.1 The Literal Account

According to the usual gloss on the Kripkean model-theory, the set W is, or represents, the set of all possible worlds, and \odot is, or represents, the actual world. Also, for each $w \in W$, $D(w)$ is, or represents, the set of all individuals that are something at w , and the value of an expression φ relative to a variable-assignment g , $V^g(\varphi)$, is, or represents, the semantic value of φ relative to g . The first classic account of the semantics of first-order modal languages, to which I will be calling the ‘Literal Account’, arises by taking this gloss on the Kripkean model-theory somewhat literally. According to the Literal Account, there is a model structure that is the *intended model structure* for the language. Moreover, on the Literal Account, what it is for a model structure IS to be the *intended model structure*, is for IS to be such that:³

1. W is in fact the set of all possible worlds;
2. For each world $w \in W$, $D(w)$ is in fact the set of individuals that are something at w ;
3. \odot is the actual world;

³In general, theorists committed to the existence of an intended model structure and to a literal conception of what it is for a model structure to be intended are faced with an immediate problem, since no set contains everything, by Cantor’s theorem. These problems are widely discussed in the literature on absolute generality. Modal theorists with these commitments have the option to appeal to the solutions discussed in the absolute generality literature. The natural option is to rework the model-theory for modal logic, substituting quantification over sets by plural quantification, or alternatively by higher-order quantification.

Note that the modal theorist adopting the stance on the model-theory for modal logic described in §2.3 is faced with no such problem. Once this stance is adopted, it is natural to regard all the model structures and models described in §2.2 as elements of the von Neumann hierarchy of sets, with these elements playing a purely instrumental role.

Also, there is a model M based on the intended model structure IS that is the *intended model*. On the Literal Account the semantic value of an expression φ relative to a variable-assignment g is obtained from the value of φ relative to g in the intended model as follows:

1. The semantic value of an individual constant relative to a variable-assignment g is its value relative to g , i.e., an element of $\bigcup_{w \in W} D(w)$;
2. The semantic value of an n -ary predicate ζ relative to g is a relation that obtains, at each world w , of all and only those n -tuples of individuals that belong to the value of ζ , relative to g , at w .
3. The semantic value of a sentence φ is a proposition that is true at a world w relative to g if and only if w belongs to the value of φ relative to g .

Finally, on the Literal Account, a sentence (i.e., a closed formula) is true if and only if its semantic value is true at the actual world relative to any variable-assignment. That is, a sentence is true if and only if the actual world belongs to the value of the sentence in the intended model relative to any variable-assignment.

The recursive clauses in the definition of the value function of the intended model directly yield a compositional account of the semantic values of all the expressions of the language. For instance, the semantic value, relative to a variable-assignment g of $x = a$ is true at a world w if and only if x is mapped to an individual o that bears to the semantic value of a , at world w , the relation that is the semantic value of $=$, and so if and only if o is identical to the semantic value of a at world w . The semantic value relative to variable-assignment g of the complex predicate $\hat{x}(x = a)$ consists in the property that is instantiated at a world w by individual d if and only if the semantic value relative to the variable-assignment $g[x/d]$ of $x = a$ is true at w .

The semantic value relative to a variable-assignment g of $\exists \hat{x}(x = a)$ is a proposition true at a world w if and only if the property that is the semantic value relative to g of $\hat{x}(x = a)$ is instantiated at w . Note that, according to the Literal Account, this is so if and only if there is some individual d such that the semantic value relative to $g[x/d]$ of $x = a$ is true at w .

2.2.2 Thorough Contingentism, Thorough Actualism and the Literal Account

Consider the following thesis:

Aliens. There could have been some individual that actually is nothing.

The thesis of Aliens enjoys support from unreflective common sense. For instance, there could have been something that would have been a seventh son of Kripke even though, actually, it is nothing. Also, note that contingentists who reject the truth of Aliens incur the burden of spelling out why it is that there could have been some thing that is nothing in other possibilities, even though there could not have been some thing that is nothing in the actual world. It is clear that the actual world is special, in that it is the world that obtains. But this observation does not suffice to show why one should think that the actual world is special in that there could not have been something that is nothing in the

actual world. Consequently, many thorough contingentists take Aliens to be one of the underlying motivations for Contingentism.

Recall the thesis of Actualism, according to which every individual is actually something. The conjunction of the Literal Account, Aliens and Actualism turns out to be inconsistent. Suppose that Aliens is true. Then, there could have been an individual, call it o , such that actually, o is nothing. So, the thesis of Aliens is true only if the following sentence is true:

$$(1) \quad \diamond \exists \hat{x}(x = o)$$

According to the Literal Account, (1) is true if and only if there is some individual, namely, o , that belongs to the domain of some possible world. From Actualism it follows that actually, o is something. But this contradicts the claim that actually o is nothing. So, Aliens, the Literal Account and Actualism together imply a contradiction, namely, that actually, o is something and o is nothing. That is, Aliens, the Literal Account and Actualism are jointly inconsistent.

Arguably, a thoroughly contingentist theory committed to the truth of Aliens is, all things being equal, preferable to a thoroughly contingentist theory committed to the falsehood of Aliens. Thus, the fact that the Literal Account implies, in conjunction with Actualism, the falsehood of Aliens makes it unattractive from the standpoint of thoroughly contingentist actualists.

2.2.3 The Haecceities Account

The other classic account of the semantics of $\text{ML}^{\hat{L}}$ -languages is what I will be calling the *Haecceities Account*. This account has been proposed by Plantinga (1974) and developed by Jager (1982). Just as the Literal Account, the Haecceities Account is based on the Kripkean model-theoretic semantics, and it also appeals to the idea that there is a distinguished, ‘intended’ model structure and a distinguished, ‘intended’ model. However, proponents of the Literal Account and of the Haecceities Account turn out to mean different things by ‘intended’.

Say that a property is an *haecceity of an individual x* if and only if it is the property of being x , and that a property is an *haecceity* if and only if it could have been the haecceity of some individual. According to the Haecceities Account, the intended model structure is one in which:

1. The set W is the set of all possible worlds;
2. The function D assigns to each world w the set $D(w)$ of all *haecceities* that are instantiated at w ;
3. \odot is the actual world.

Moreover, according to the Haecceities Account, the semantic value of an expression φ relative to g is obtained from the value of φ relative to g in the intended model as follows:

1. The semantic value of an individual constant relative to a variable-assignment g is an *haecceity*, an element of $\bigcup_{w \in W} D(w)$;

2. The semantic value relative to g of an n -ary predicate ζ is a relation that is jointly instantiated, at each world w , with all and only those n -tuples of haecceities that belong to the value of ζ , relative to g , at w .
3. The semantic value of a sentence φ is a proposition that is true at a world w relative to g if and only if w belongs to the value of φ relative to g .

Finally, on the Haecceities Account, a sentence is true if and only if its semantic value is true at the actual world relative to any variable-assignment. That is, a sentence is true if and only if the actual world belongs to the value of the sentence in the intended model relative to any variable-assignment.

The recursive clauses in the definition of the value function of the intended model directly yield a compositional account of the semantic values of all the expressions of the language. For instance, the semantic value, relative to a variable-assignment g of $x = a$ is true at a world w if and only if x is mapped to an haecceity h and the identity relation is jointly instantiated at w with the pair consisting of h and the haecceity that is the semantic value of a . The semantic value relative to variable-assignment g of the complex predicate $\hat{x}(x = a)$ consists in the property that is coinstantiated at a world w with haecceity h if and only if the proposition that is the semantic value relative to the variable-assignment $g[x/h]$ of $x = a$ is true at w .

The semantic value relative to a variable-assignment g of $\exists \hat{x}(x = a)$ is a proposition true at a world w if and only if the property that is the semantic value relative to g of $\hat{x}(x = a)$ is instantiated at w . Note that, according to the Haecceities Account, this is so if and only if there is some haecceity h such that the proposition that is the semantic value relative to $g[x/h]$ of $x = a$ is true at w .

The main difference between the Literal and the Haecceities accounts is the following. Whereas the domain of each world of the Literal Account's intended model structure consists of individuals, the domain of each world of the Haecceities Account's intended model structure consists of haecceities. Since the domains of the Haecceities Account's intended model structure consist of haecceities rather than individuals, the thesis of Aliens turns out not to be inconsistent with the conjunction of the Haecceities Account and Actualism, contrary to what was seen to be the case with the Literal Account.

2.2.4 Thorough Contingentism, Thorough Actualism and the Haecceities Account

Consider the following thesis:

No Actual Haecceity. There could have been something such that actually nothing is possibly its haecceity.

Arguably, contrary to what was the case with the thesis of Aliens, No Actual Haecceity is not supported by unreflective common sense (it is not that unreflective common sense is *opposed* to its truth; rather, it has no particular stance towards the truth of No Actual Haecceity). Yet, the thesis is a consequence of theses supported by different contingentists, when these theses are conjoined with the thesis of Aliens.

To begin with, on an Aristotelian view on properties, according to which properties are something only if they are instantiated, the thesis of No Actual Haecceity is an immediate consequence of Aliens. If there could be some x that actually is nothing, then actually the property of being x is uninstantiated. Therefore, according to the Aristotelian view, actually there is no property of being x .

The classic theoretical commitments adduced by thorough contingentists unsympathetic to an Aristotelian view of properties also turn out to imply the truth of No Actual Haecceity. One such commitment is to the view that some higher-order entities bear particularly strong links to their instances, and so ontologically depend on them. Haecceities are pointed out as paradigmatic cases of higher-order entities of this kind, since i) they are instantiated whenever the things that they are haecceities of are something, and ii) they are uninstantiated if the things that they are haecceities of are nothing. In conjunction with Aliens, the view that Haecceities ontologically depend on their instances implies No Actual Haecceity.

A different theoretical commitment of (some) thorough contingentists is to the claim that necessarily, if P is a nonqualitative property, then necessarily, P is something if and only if P 's application conditions are specifiable solely in terms of individuals and qualitative properties (that are all something).⁴ To proponents of the thesis of Aliens, it is intuitively plausible that there could be some x such that the application conditions of the property of being x actually are not specifiable in terms of individuals and qualitative properties that are all something. For instance, according to typical thorough contingentists there could have been many seventh sons of Kripke, all of which are actually nothing.

These thorough contingentists find it plausible to think that the application conditions of the haecceities of the merely possible seventh sons of Kripke actually are not specifiable in terms of individuals and qualitative properties that are all something. According to them, one cannot distinguish one of the possible seventh sons of Kripke from all the other possible ones solely in terms of the individuals and qualitative properties that are actually something. Since the application conditions of the possible haecceity of at least one of the possible seventh sons of Kripke is not specifiable solely in terms of the individuals and qualitative properties that are actually something, then its haecceity is actually nothing, and so No Actual Haecceity is true.

Finally, even without delving into the particulars of the theoretical commitments of thorough contingentists, haecceities of merely possibles are some of the typical examples given by them of higher-order entities that could have been something despite actually being nothing, and thus of higher-order entities witnessing the truth of thorough contingentism. Therefore, to most thorough contingentists a compositional semantics of first-order modal languages will be satisfactory only if it is consistent with the thesis of No Actual Haecceity.

Now, suppose that No Actual Haecceity is true. Then, there is or could have been some individual, call it o , such that actually o 's haecceity is nothing. So, the thesis of No Actual Haecceity is true only if the following sentence is:

⁴A view on the being of nonqualitative properties such as the one just offered can be found in [Fine \(1985, p. 189-191\)](#).

$$(2) \quad \diamond \exists \hat{x}(x = o)$$

According to the Haecceities Account, (2) is true only if there is some haecceity h such that h is coinstantiated with o 's haecceity at some possible world. But if some haecceity is coinstantiated with another one, then the two haecceities are the same. That is, necessarily, if the property of being x is coinstantiated with the property of being y , then x is identical to y , and so the property of being x is identical to the property of being y . Therefore, according to the Haecceities Account, the haecceity of o is something.

Finally, recall the thesis of Thorough Actualism, according to which every entity is *actually* something. It follows from Thorough Actualism and the thesis that the haecceity of o is something that the haecceity of o is actually something. But this contradicts the claim that actually o 's haecceity is nothing. So, No Actual Haecceity, the Haecceities Account and Thorough Actualism together imply a contradiction, namely that actually, o 's haecceity is something and o 's haecceity is nothing. That is, No Actual Haecceity, the Haecceities Account and Thorough Actualism are jointly inconsistent.

In this section the two classic accounts of the semantics of first-order modal languages were presented, namely, the Literal and the Haecceities Accounts. It was shown that typical contingentists accept the truth of Aliens and No Actual Haecceity. It was also shown that the Literal Account, Aliens and Actualism are jointly inconsistent, and that the Haecceities Account, No Actual Haecceities and Thorough Actualism are jointly inconsistent.

This presents typical thorough contingentists with a dilemma, namely, to reject both of the classic accounts of the semantics of first-order modal languages, or else to reject Thorough Actualism. The challenge facing proponents of both Thorough Actualism and Thorough Contingentism is thus that of offering a satisfactory alternative account of the semantics of first-order modal languages.

2.3 The Propositional Functions Account

[Stalnaker \(2012\)](#) proposes the Propositional Functions Account with an eye towards meeting the challenge of offering a compositional semantics for first-order modal languages consistent with his own thoroughly actualist and thoroughly contingentist higher-order modal theory. He offers the following remark on the Propositional Functions Account ([Stalnaker, 2012](#), p. 147):

'We can talk with a clear conscience, in the metalanguage, about a domain of possible individuals because we have shown how to reconcile that talk with more austere ontological commitments and how to do the compositional semantics in a way that assigns as values only properties, relations, and functions that actually exist, according to the metaphysics that is presupposed.'

The quote refers to two related aspects in which the success of the Kripkean model-theory for first-order modal languages has been taken to pose a challenge to thorough contingentists committed to thorough actualism.

The first aspect has to do with what [Fine \(1985\)](#) has called ‘possibilist discourse’. As previously seen, the classic account that is more naturally extracted from the Kripkean model-theoretic semantics for first-order modal languages, the Literal account, is consistent with the thesis of Aliens only if *possibilism* is true, where possibilism is the negation of actualism, i.e., possibilism is the thesis that there are individuals that are actually nothing. Thus, a contingentist understanding of the Literal Account requires regarding it as quantifying over merely possible individuals.

Briefly, Stalnaker presents a different picture of how to understand the Kripkean model-theory for first-order modal languages and in particular the notion of an intended model. According to this picture, for a model to be intended is not for it to consist of ‘modal reality’. Instead, an intended model is understood as a *representation* of certain features of reality that the theorist is trying to capture (and so, more than one model may be intended, if the theorist is using more than one model to represent the phenomena in which he is interested).

In the present case, the interest is in models that represent the semantic values of the different expressions of the language, as well as the relationships that obtain between them. Not every element of an intended model is taken to be representationally significant. For instance, the set $\bigcup_{w \in W} d(w)$ does not consist, nor represents, the set of all possible individuals. The elements in $\bigcup_{w \in W} d(w)$ (except those in $d(\odot)$) are representationally insignificant. These elements are required to give structure to the set-theoretic constructs representing higher-order entities such as propositions, properties, relations, etc., and to represent the relations that obtain between these entities.

As an example, properties are represented by certain functions with domain W and which have subsets of $\bigcup_{w \in W} d(w)$ as values. When two functions f and f' that represent properties are such that, for every $w \in W$, $f(w) \subseteq f'(w)$, this represents the fact that necessarily, whatever has the property represented by f also has the property represented by f' . Given such an account of what it takes for a model to be intended, apparent quantification over merely possible individuals is unproblematic, since it is *merely* apparent.⁵

The second aspect in which the success of the model-theory for first-order modal languages has been taken to pose a challenge to thorough contingentists committed to thorough actualism is related to the first. Whereas the Literal Account is committed to there being mere possibilia, if consistent with the truth of Aliens, the other account based on the Kripkean model-theory, the Haecceities Account, is itself committed to ‘ontologically extravagant’ entities, namely, haecceities of merely possible individuals. Thus, the ‘metaphysics that is presupposed’ is a thoroughly contingentist metaphysics.

Stalnaker claims to have shown ‘how to do the compositional semantics in a way that assigns as values only properties, relations, and functions that actually exist, according to the metaphysics that is presupposed’ since he takes the Propositional Functions Account, proposed in ([Stalnaker, 2012](#),

⁵Stalnaker also proposes the addition of certain classes of functions to models for first-order modal languages. The purpose of these functions is that of distinguishing the elements of the model that are representationally significant from the elements of the model that are not representationally significant. See ([Stalnaker, 2012](#), chs. 1-3 and Appendices A and C).

appendix B), to assign as semantic values only entities that are actually something according to a thoroughly contingentist metaphysics. In this section Stalnaker's Propositional Functions Account is presented in some detail. In the next section some results delivered by the Propositional Functions Account are presented. These results show that, pace Stalnaker, the Propositional Functions Account is, 'ontologically extravagant', in the sense of Stalnaker, favouring Higher-Order Necessitism (when conjoined with Thorough Serious Actualism, as shall be seen).

2.3.1 A Sketch of the Propositional Functions Account

Unsurprisingly, according to the Propositional Functions Account individuals are the semantic values of individual constants and n -ary relations are the semantic value of n -ary predicate letters. Roughly, closed complex predicates also have properties as their semantic values, and closed formulas have propositions as their semantic values.⁶

A distinctive feature of the Propositional Functions Account is its take on the semantic values of open formulas and open complex predicates. According to the account, the semantic value of an open formula is a n^{th} -level *propositional function*, for some natural number n , and the semantic value of an open complex predicate is a n^{th} -level *property function*, for some natural number n . The notions of a n^{th} -level propositional function and property function are defined recursively:⁷

- A 0^{th} -level propositional function is just a proposition;
- A 0^{th} -level property function is just a property;
- A $(n + 1)^{\text{th}}$ -level propositional function f is a relation between individuals and n^{th} -level propositional functions such that necessarily, for every individual x , there is one and only one n^{th} -level propositional function g such that f relates x to g (i.e., $f(x) = g$);
- A $(n + 1)^{\text{th}}$ -level property function is a relation f between individuals x and n^{th} -level property functions such that necessarily, for every individual x , there is one and only one n^{th} -level property function g such that f relates x to g (i.e., $f(x) = g$).

The assumption of Thorough Serious Actualism agrees with Stalnaker's own presentation of the Propositional Functions Account. Moreover, the thesis is explicitly endorsed by Stalnaker. The main aim of the present chapter is that of showing that the Propositional Functions Account, in conjunction with Thorough Serious Actualism, has consequences that favour Higher-Order Necessitism.

I will now turn to some examples. Let a be an individual constant, P be a unary predicate letter and Q be a binary predicate letter, with semantics values, respectively, Michael Jordan (the basketball

⁶This is not, strictly speaking, correct. As will be shown later on, certain closed complex predicates do not have properties as their semantic values, and certain closed formulas do not have propositions as their semantic values.

⁷Recall the thesis of Thorough Serious Actualism, presented in chapter 1, according to which necessarily, if (0^{th} - or higher-order) entities are related, then they are all something. The characterisation of n^{th} -level propositional functions offered in the text presupposes the truth of Thorough Serious Actualism. On a characterisation independent of this assumption, a $(n + 1)^{\text{th}}$ -level propositional function f is a relation between individuals and n^{th} -level propositional functions such that necessarily, for every individual x , it is possible that there is a n^{th} -level propositional function g such that necessarily $f(x) = g$. A $(n + 1)^{\text{th}}$ -level property function f is a relation between individuals and n^{th} -level property functions such that necessarily, for every individual x , it is possible that there is a n^{th} -level property function g such that necessarily $f(x) = g$.

player), the property *being tall*, and the relation *being a father of*. Then, according to the Propositional Functions Account:

- The semantic value of the open formula Px , $\llbracket Px \rrbracket$, is a 1^{st} -level propositional function:
 - It is that 1^{st} -level propositional function which necessarily, for every individual x , maps x to the proposition that x is tall.
- $\llbracket Qax \rrbracket$ is a 1^{st} -level propositional function:
 - Necessarily, for every x , $\llbracket Qax \rrbracket$ maps x to the proposition that Jordan is a father of x .
- $\llbracket \wedge \rrbracket$ is a function from n^{th} -level propositional functions to n^{th} -level propositional functions:
 - $\llbracket \wedge \rrbracket$ maps $\llbracket Px \rrbracket$ and $\llbracket Qax \rrbracket$ to $\llbracket Px \wedge Qax \rrbracket$, a 1^{st} -level propositional function, which necessarily, for every x , maps x to the proposition that x is tall and Jordan is a father of x .
- $\llbracket \hat{x} \rrbracket$ is a function from $(n + 1)^{th}$ -level propositional functions to n^{th} -level property functions:
 - $\llbracket \hat{x} \rrbracket$ maps $\llbracket Px \wedge Qax \rrbracket$ to $\llbracket \hat{x}(Px \wedge Qax) \rrbracket$, a property which necessarily, for every x , holds of x if and only if x is mapped by $\llbracket Px \wedge Qax \rrbracket$ to a true proposition.
- $\llbracket \forall \rrbracket$ is a function which maps n^{th} -level property functions to n^{th} -level propositional functions:
 - $\llbracket \forall \rrbracket$ maps $\llbracket \hat{x}(Px \wedge Qax) \rrbracket$ to $\llbracket \forall \hat{x}(Px \wedge Qax) \rrbracket$, a proposition which, necessarily, obtains if and only if the property $\llbracket \hat{x}(Px \wedge Qax) \rrbracket$ is instantiated by everything.
- $\llbracket \square \rrbracket$ is a function from n^{th} -level propositional functions to n^{th} -level propositional functions:
 - $\llbracket \square \rrbracket$ maps $\llbracket \forall \hat{x}(Px \wedge Qax) \rrbracket$ to $\llbracket \square \forall \hat{x}(Px \wedge Qax) \rrbracket$, a proposition which, necessarily, obtains if and only if the proposition $\llbracket \forall \hat{x}(Px \wedge Qax) \rrbracket$ necessarily obtains.

2.3.2 Ambiguities and Types

If the Propositional Functions Account turns out to be the correct semantic account of first-order modal languages, then these languages happen to be ambiguous in unexpected ways. Consider the formula Pa . Taken on its own, this expression has as its semantic value a proposition, namely, the proposition that Jordan is tall. However, in the context of the formula $\forall \hat{x}(Pa)$, the expression Pa has as its semantic value a 1^{st} -level propositional function, namely, that propositional function which necessarily, for every x , maps x to the proposition that Jordan is tall.

Moreover, in the context of the formula $\forall \hat{x}(\exists \hat{y}(Pa \wedge Qxy))$ the semantic value of Pa turns out to be a 2^{nd} -level propositional function. Namely, it is that 2^{nd} -level propositional function f which necessarily, for every y , maps y to that propositional function g which necessarily, for every x , maps x to the proposition that Jordan is tall.

A second kind of ambiguity is illustrated by considering the open formula Qxy when embedded in, respectively, the formulas $\exists \hat{y}(\exists \hat{x}(Qxy))$ and $\exists \hat{x}(\exists \hat{y}(Qxy))$. In the context of the first closed formula, $\exists \hat{y}(\exists \hat{x}(Qxy))$, the formula Qxy has as its semantic value that 2^{nd} -level propositional function f which necessarily, for every x , maps x to that propositional function g which necessarily, for every y , maps y to the proposition that y is a father of x . In the context of the second closed formula, $\exists \hat{x}(\exists \hat{y}(Qxy))$, the formula Qxy has as its semantic value that 2^{nd} -level propositional function f which necessarily, for every x , maps x to that propositional function g which necessarily, for every y ,

maps y , to the proposition that x (not y) is a father of y (not of x).

Thus, in an extended sense of ‘open formula’, according to which open formulas are those expressions of the language that have as semantic values propositional functions, the formula ‘ Pa ’ may itself be considered an open formula. When in the context of a formula such as ‘ $\forall \hat{x}(Pa)$ ’, the formula ‘ Pa ’ does not have as its semantic value a proposition. Instead, its semantic value is a 1st-level propositional function.

On Stalnaker’s own account, the ambiguities just noted are resolved *in situ*. The same expression has different semantic values depending on the larger linguistic context in which it occurs. Here, instead of adopting the *in situ* strategy, the Propositional Functions Account is given for languages stripped of ambiguities of the kind in question. By focusing on languages without these ambiguities the consequences of the Propositional Functions Account become clearer.⁸ To the languages for which the Propositional Functions Account is here given I will call TML \hat{L} -languages. The ambiguities noted in ML \hat{L} -languages are resolved in two ways.

In order to account for the fact that expressions such as Qxy can express different propositional functions, the ordering of the variables of the language will be exploited. The two propositional functions that were the possible semantic values of Qxy in the context of, respectively, the formulas $\exists \hat{y}(\exists \hat{x}(Qxy))$ and $\exists \hat{x}(\exists \hat{y}(Qxy))$, are the semantic values of, respectively, the formulas Qx^1x^2 and Qx^2x^1 .

In order to account for the fact that an expression such as Pa may express propositional functions of different levels, types are added to the language. The type-hierarchy adopted is the one presented in (Gallin, 1975, p. 68). The variables of the language are typed with e , the type of individuals. The variables of the language are thus x_e^1, x_e^2, \dots . Similarly, the individual constants of the language are typed with the type e , and the n -ary predicates of the language are typed with the type $\langle e^1, \dots, e^n \rangle$, the type of n -ary relations. So, whereas before a was an individual constant, now a_e is an individual constant, and whereas before P and Q were, respectively, a unary predicate and a binary predicate, now $P_{\langle e \rangle}$ and $Q_{\langle e, e \rangle}$ are, respectively, a unary predicate and a binary predicate.

Let U and S be the following subsets of the set P of all types:

- U is the smallest set such that $\langle \rangle$ belongs to U , and if τ belongs to U , then $\langle e, \tau \rangle$ belongs to U ;
- S is the smallest set such that $\langle e \rangle$ belongs to S , and if τ belongs to S , then $\langle e, \tau \rangle$ belongs to S .

The sets U and S are, respectively, the sets of types of propositional functions and of property functions. Thus, $\langle \rangle$ is the type of propositions, $\langle e, \langle \rangle \rangle$ is the type of 1st-level propositional functions, $\langle e, \langle e, \langle \rangle \rangle \rangle$ is the type of 2nd-level propositional functions, and so on. Moreover, $\langle e \rangle$ is the type of properties, $\langle e, \langle e \rangle \rangle$ is the type of 1st-level property functions, $\langle e, \langle e, \langle e \rangle \rangle \rangle$ is the type of 2nd-level property functions, and so on.

Each one of the types in, respectively, U and S , is abbreviated as follows:

1. For each natural number n , $\langle n, 0 \rangle$ is an abbreviation of the type of n^{th} -level propositional

⁸This is not to say that there are no reasons to prefer the *in situ* strategy. On the contrary, the *in situ* strategy is conservative with respect to the present usage of first-order modal languages.

functions;

2. For each natural number n , $\langle n, 1 \rangle$ is an abbreviation of the type of n^{th} -level property functions. This means that $\langle 0, 0 \rangle$ is the type of propositions, $\langle 0, 1 \rangle$, and in general, $\langle 0, n \rangle$ is the type of n -ary relations. Furthermore, $\langle n, 0 \rangle$ is the type of n^{th} -level propositional functions, and $\langle n, 1 \rangle$ is the type of n^{th} -level property functions.

The set of complex expressions of TML^\wedge -languages is now defined:

1. Every variable of type τ is a term of type τ ;
2. Every constant of type τ is a term of type τ ;
3. If s is a term of type $\langle i, n \rangle$, s^1, \dots, s^n are terms of type e , and k is the highest index of all the variables occurring free in s, s^1, \dots, s^n , then, for each natural number m , $(ss^1 \dots s^n)_{\langle \max(i, k) + m, 0 \rangle}$ is a term of type $\langle \max(i, k) + m, 0 \rangle$;
4. φ is a term of type $\langle n, 0 \rangle$, then $(\neg\varphi)_{\langle n, 0 \rangle}$, $(@\varphi)_{\langle n, 0 \rangle}$, $(\Box\varphi)_{\langle n, 0 \rangle}$ are terms of type $\langle n, 0 \rangle$;
5. If φ, ψ are terms of type $\langle n, 0 \rangle$, then $(\varphi \wedge \psi)_{\langle n, 0 \rangle}$ is a term of type $\langle n, 0 \rangle$;
6. If φ is a term of type $\langle n + 1, 0 \rangle$, then $\hat{x}_e^{n+1}(\varphi)_{\langle n, 1 \rangle}$ is a term of type $\langle n, 1 \rangle$;
7. If s is a term of type $\langle n, 1 \rangle$, then $(\forall s)_{\langle n, 0 \rangle}$ is a term of type $\langle n, 0 \rangle$.

For instance:

- $(Q_{\langle 0, 2 \rangle} x_e^1 a_e)_{\langle 1, 0 \rangle}$ is a formula whose type is that of a 1^{st} -level propositional function;
- $(Q_{\langle 0, 2 \rangle} x_e^1 a_e)_{\langle 2, 0 \rangle}$ is a formula whose type is that of a 2^{nd} -level propositional function;
- $\hat{x}_e^2((Q_{\langle 0, 2 \rangle} x_e^1 x_e^2)_{\langle 2, 0 \rangle})_{\langle 1, 1 \rangle}$ is a term with the type of a 1^{st} -level property function;
- $(\exists \hat{x}_e^2((Q_{\langle 0, 2 \rangle} x_e^1 x_e^2)_{\langle 2, 0 \rangle}))_{\langle 1, 1 \rangle}{}_{\langle 1, 0 \rangle}$ is a term whose type is that of a 1^{st} -level propositional function.

Note that there is no formula corresponding to the string $(Q_{\langle 0, 2 \rangle} x_e^2 a_e)_{\langle 1, 0 \rangle}$. That is, if the variable with the highest index occurring in the formula is the variable x_e^n , then the formula has at least the type of a n^{th} -level propositional function. The string $(Q_{\langle 0, 2 \rangle} x_e^2 a_e)_{\langle 1, 0 \rangle}$ violates this constraint, since the variable with the highest index occurring in it is x_e^2 , whereas this string is labelled with the type $\langle 1, 0 \rangle$. In order to count as a formula, the string would have to be labelled with a type $\langle n, 0 \rangle$, for $n > 1$. Note also that $\hat{x}_e^2((Q_{\langle 0, 2 \rangle} x_e^1 x_e^3)_{\langle 3, 0 \rangle})_{\langle 2, 1 \rangle}$ is not a term, since the variable being bound is not x_e^3 , contrary to what is required by clause 7.

To those interested in the minutiae of the Propositional Functions Account, in the remainder of §2.3 the Propositional Functions Account is presented in more detail and it is shown how the Kripkean model-theory may be used to model the account. Others may want to skip ahead to §2.4.

2.3.3 The Propositional Functions Account

The semantic value of an expression of type e is an individual (that is actually something), and the semantic value of a constant of type $\langle 0, n \rangle$ is a n -ary relation. In particular, the semantic value of $=_{\langle 0, 2 \rangle}$ is, as expected, the identity relation. Let $\llbracket \cdot \rrbracket_{\langle z_1, \dots, z_n \rangle}$ denote a function which, when applied to an expression φ of a TML^\wedge -language, maps φ to its semantic value, except that if φ is a variable x_e^i ,

$1 \leq i \leq n$, $\llbracket \varphi \rrbracket_{\langle z_1, \dots, z_n \rangle}$ is z_i , where n is any natural number. Here is a specification of the semantic values of the remaining terms of the language (except for the variables, which have no semantic value whatsoever):⁹

1. $\llbracket (ss^1 \dots s^n)_{\langle \max(i,k)+m, 0 \rangle} \rrbracket = f_0$, where f_0 is that $(\max(i, k) + m)^{th}$ -level propositional function which is such that, necessarily, for every y_1 , for every $((\max(i, k) + m) - 1)^{th}$ -level propositional function f_1 and every $(i - 1)^{th}$ -level property function f_1^* , $f_0(y_1) = f_1$ if and only if $\llbracket s \rrbracket(y_1) = f_1^*$ and, necessarily, for every y_2 , for every $((\max(i, k) + m) - 2)^{th}$ -level propositional function f_2 and every $(i - 2)^{th}$ -level property function f_2^* , $f_1(y_2) = f_2$ if and only if $f_1^*(y_2) = f_2^*$ and, necessarily, . . . and, necessarily, for every y_i , for every $((\max(i, k) + m) - i)^{th}$ -level propositional function f_i , for every 0^{th} -level property function f_i^* , $f_{(i-1)}(y_i) = f_i$ if and only if $f_{(i-1)}^*(y_i) = f_i^*$ and, necessarily, for $y_{(i+1)}$, for every $((\max(i, k) + m) - (i + 1))^{th}$ -level propositional function $f_{(i+1)}$, $f_i(y_{(i+1)}) = f_{(i+1)}$ if and only if necessarily, for every $y_{(i+2)}$, for every $((\max(i, k) + m) - (i + 2))^{th}$ -level propositional function f_{i+2} , $f_{(i+1)}(y_{(i+2)}) = f_{(i+2)}$ and . . . and necessarily, for every $y_{(\max(i,k)+m)}$, for every 0^{th} -level propositional function $f_{(\max(i,k)+m)}$, $f_{(\max(i,k)+m)}$ obtains if and only if f_i^* holds between $\llbracket s^1 \rrbracket_{\langle y_1, \dots, y_{(\max(i,k)+m)} \rangle}$ and . . . and $\llbracket s^n \rrbracket_{\langle y_1, \dots, y_{(\max(i,k)+m)} \rangle}$.
2. $\llbracket (\neg\varphi)_{\langle n, 0 \rangle} \rrbracket = f_0$, where f_0 is that n^{th} -level propositional function which is such that, necessarily, for every y_1 , for every $(n - 1)^{th}$ -level propositional function f_1 , for every $(n - 1)^{th}$ -level propositional function f_1^* , $f_0(y_1) = f_1$ if and only if $\llbracket \varphi \rrbracket(y_1) = f_1^*$ and, necessarily, . . . and, necessarily, for every y_n , necessarily, for every 0^{th} -level propositional function f_n , necessarily, for every 0^{th} -level propositional function f_n^* , necessarily, $f_{(n-1)}(y_n) = f_n$ if and only if $f_{n-1}^*(y_n) = f_n^*$ and, necessarily, f_n obtains if and only if it is not the case that f_n^* obtains;
3. $\llbracket (\Box\varphi)_{\langle n, 0 \rangle} \rrbracket = f_0$, where f_0 is that n^{th} -level propositional function which is such that, necessarily, for every y_1 , for every $(n - 1)^{th}$ -level propositional function f_1 , for every $(n - 1)^{th}$ -level propositional function f_1^* , $f_0(y_1) = f_1$ if and only if $\llbracket \varphi \rrbracket(y_1) = f_1^*$ and . . . and, necessarily, for every y_n , for every 0^{th} -level propositional function f_n , for every 0^{th} -level propositional function f_n^* , $f_{(n-1)}(y_n) = f_n$ if and only if $f_{n-1}^*(y_n) = f_n^*$ and, necessarily, f_n obtains if and only if necessarily, f_n^* obtains;
4. $\llbracket (@\varphi)_{\langle n, 0 \rangle} \rrbracket = f_0$, where f_0 is that n^{th} -level propositional function which is such that, necessarily, for every y_1 , for every $(n - 1)^{th}$ -level propositional function f_1 , for every $(n - 1)^{th}$ -level propositional function f_1^* , $f_0(y_1) = f_1$ if and only if $\llbracket \varphi \rrbracket(y_1) = f_1^*$ and . . . and, necessarily, for every y_n , for every 0^{th} -level propositional function f_n , for every 0^{th} -level propositional function f_n^* , $f_{(n-1)}(y_n) = f_n$ if and only if $f_{n-1}^*(y_n) = f_n^*$ and, necessarily, f_n obtains if and only if actually, f_n^* obtains;
5. $\llbracket (\varphi \wedge \psi)_{\langle n, 0 \rangle} \rrbracket = f_0$, where f_0 is that n^{th} -level propositional function which is such that, neces-

⁹Hopefully, it will be clear that the Propositional Functions Account could have been offered in the higher-order modal language ML_P (enriched with some extra primitives) presented in chapter 1, even if it was there given in what might be called 'logical English'.

sarily, for every y_1 , for every $(n-1)^{th}$ -level propositional function f_1 , for every $(n-1)^{th}$ -level propositional function f_1^* , for every $(n-1)^{th}$ -level propositional function f_1' , $f_0(y_1) = f_1$ if and only if $\llbracket \varphi \rrbracket(y_1) = f_1^*$ and $\llbracket \psi \rrbracket(y_1) = f_1'$ and, necessarily, \dots and, necessarily, for every y_n , necessarily, for every 0^{th} -level propositional function f_n , necessarily, for every 0^{th} -level propositional function f_n^* , necessarily, for every 0^{th} -level propositional function f_n' , necessarily, $f_{(n-1)}(y_n) = f_n$ if and only if $f_{(n-1)}^*(y_n) = f_n^*$ and $f_{(n-1)}'(y_n) = f_n'$ and, necessarily, f_n obtains if and only if f_n^* and f_n' both obtain;

6. $\llbracket \hat{x}_e^{n+1}(\varphi)_{\langle n,1 \rangle} \rrbracket = f_0$, where f_0 is that n^{th} -level property function such that, necessarily, for every y_1 , for every $(n-1)^{th}$ -level property function f_1 , for every n^{th} -level propositional function f_1^* , $f_0(y_1) = f_1$ if and only if $\llbracket \varphi \rrbracket(y_1) = f_1^*$ and \dots and, necessarily, for every y_n , for every 0^{th} -level property function f_n , for every 1^{st} -level propositional function f_n^* , $f_{n-1}(y_n) = f_n$ if and only if $f_{n-1}^*(y_n) = f_n^*$ and, necessarily, for every y_{n+1} , for every 0^{th} -level propositional function f_{n+1}^* , necessarily, f_n holds of y_{n+1} if and only if $f_n^*(y_{n+1}) = f_{n+1}^*$ and f_{n+1}^* obtains.
7. $\llbracket (\forall s)_{\langle n,0 \rangle} \rrbracket = f_0$, where f_0 is that n^{th} -level propositional function such that, necessarily, for every y_1 , for every $(n-1)^{th}$ -level propositional function f_1 , for every $(n-1)^{th}$ -level property function f_1^* , $f_0(x_1) = f_1$ if and only if $\llbracket s \rrbracket(y_1) = f_1^*$ and, \dots and, necessarily, for every y_n , for every 0^{th} -level propositional function f_n , for every 0^{th} -level property function f_n^* , necessarily, $f_{(n-1)}(y_n) = f_n$ and $f_{(n-1)}^*(y_n) = f_n^*$ and, necessarily, f_n obtains if and only if f_n^* holds of everything.

Contrary to the Literal account, the Propositional Functions Account is consistent with the conjunction of Aliens and Actualism. Prima facie, the account is also consistent with the conjunction of No Actual Haecceity and Thorough Actualism. However, this is not so. As shall be seen, the Propositional Functions Account is committed to claims that imply the falsehood of No Actual Haecceity, for instance, the claim that necessarily every haecceity is necessarily something. Before presenting these problematic consequences of the Propositional Functions Account, it will be shown how the Kripkean model-theory may be used to provide a model (i.e., a representation) of what is, according to the account, the semantics of first-order modal languages.

2.3.4 Modelling the Account

Recall Stalnaker's views of what it takes for a model to be intended. The intended model does not consist of 'modal reality'. Instead, it is a representation of certain features of reality that the theorist aims to capture. For the purposes of the Propositional Functions Account, the relevant features are the semantic values of the different expressions of the language, and the relationships between these. This representational use of the model-theoretic semantics requires that a particular class of set-theoretic entities be singled out to do the job of representing the semantic values of the different expressions of the language.

The admissible semantic values of the constants of type e of the language are represented by

elements in $d(\odot)$. The admissible semantic values of expressions of type $\langle e^1, \dots, e^n \rangle$, for each natural number n , consist of elements of the set of functions f with domain W and such that for every $w \in W$, $f(w) \subseteq (d(w))^n$. Contrary to what was the case in the model-theoretic semantics specified in §2.2, only elements in this set are considered, since otherwise certain formulas contradicting Thorough Serious Actualism would be true in the models for the language. The sets of entities that represent the semantic values of the remaining expressions of types $\langle n, 0 \rangle$ and $\langle n, 1 \rangle$ are defined in a similar fashion, as we shall see.

The relevant class of models is now defined in more detail.

Definition 1 (PF-Models). A PF-model based on an inhabited model structure $IS = \langle W, \odot, D \rangle$ is a pair $M = \langle IS, V \rangle$, where V is a valuation function assigning a value to each individual constant and n -ary relation letter in the following way:

1. For every (atomic) expression s of type e , $V(s) \in D(\odot)$
2. For every atomic expression s of type $\langle 0, n \rangle$, for every natural number n , $V(s)$, is a function with domain W and such that, for every $w \in W$, $V(s)(w) \subseteq (d(w))^n$.

The next step is to extend the definition of value to the remaining expressions of the language. In order to do so, it is useful to first define a hierarchy of ‘domains’ of n -ary relation functions:

Definition 2 (Domains of n -ary Relation Functions).

- $D_{\langle 0, n \rangle} = \{f \in ((\bigcup_{w \in W} D(w))^n)^W : f(w) \subseteq (D(w))^n\}$
- $D_{\langle m+1, n \rangle} = \{f \in (\bigcup_{w \in W} D(w) \times D_{\langle m, n \rangle})^W : f(w) = \{\langle o, g \rangle : o \in D(w)\}\}$

The values of expressions of type $\langle n, 0 \rangle$ and $\langle n, 1 \rangle$ – that is, of expressions whose type is, respectively, that of n -ary propositional functions and that of n -ary property functions – belong, respectively, to the sets $D_{\langle n, 0 \rangle}$ and $D_{\langle n, 1 \rangle}$.

Now, let \vec{o}_n be shorthand for the sequence o_1, \dots, o_n of meta-variables. Also, let $V^{(o_1/\bar{o}_1, \dots, o_n/\bar{o}_n)}$ extend the original valuation V by assigning, for each $1 \leq i \leq n$, the individual constant \bar{o}_i to the element $o_i \in \bigcup_{w \in W} S(w)$, where \bar{o}_i is not in language. Let $\vec{\bar{o}}_n$ be shorthand for the sequence $\bar{o}_1, \dots, \bar{o}_n$ and $V^{(\vec{\bar{o}}_n/\vec{o}_n)}$ be shorthand for $V^{(o_1/\bar{o}_1, \dots, o_n/\bar{o}_n)}$. Finally, let $s_{t'}^t$ be the result of substituting t' for t in s , and $s_{\vec{t}'}^{\vec{t}}$ be the result of substituting t'_1 for t_1, \dots, t'_n for t_n in term s .

The value of the typed formulas and complex predicate of the language is defined as follows – note that the definition of value is not the usual one, since it does not appeal to variable-assignments:

Definition 3 (Value of a typed formula and complex predicate).

1. $V((ss^1 \dots s^n)_{\langle u, 0 \rangle}) = f \in D_{\langle u, 0 \rangle}$ such that,

$$\text{for every } w_j \in W, o_j \in d(w_j), 1 \leq j \leq u : f(w_1)(o_1) \dots (w_u)(o_u) = h$$

such that:

$$h = \{w \in W : \langle V^{(\vec{\bar{o}}_u/\vec{o}_u)}((s^1)_{\vec{o}_u}^{\vec{x}_u}), \dots, V^{(\vec{\bar{o}}_u/\vec{o}_u)}((s^n)_{\vec{o}_u}^{\vec{x}_u}) \rangle \in V(s)(w_1)(o_1) \dots (w_i)(o_i)(w)\}$$

2. $V((\neg\varphi)_{\langle n,0 \rangle}) = f \in D_{\langle n,0 \rangle}$ such that,

$$\text{for every } w_j \in W, o_j \in d(w_j), 1 \leq j \leq n : f(w_1)(o_1) \dots (w_n)(o_n) = h$$

such that:

$$h = W - V(\varphi)(w_1)(o_1) \dots (w_n)(o_n)$$

3. $V((\Box\varphi)_{\langle n,0 \rangle}) = f \in D_{\langle n,0 \rangle}$ such that,

$$\text{for every } w_j \in W, o_j \in d(w_j), 1 \leq j \leq n : f(w_1)(o_1) \dots (w_n)(o_n) = h$$

such that,

$$h = \{w \in W : V(\varphi)(w_1)(o_1) \dots (w_n)(o_n) = W\}$$

4. $V((\@ \varphi)_{\langle n,0 \rangle}) = f \in D_{\langle n,0 \rangle}$ such that,

$$\text{for every } w_j \in W, o_j \in d(w_j), 1 \leq j \leq n : f(w_1)(o_1) \dots (w_n)(o_n) = h$$

such that

$$h = \{w \in W : \odot \in V(\varphi)(w_1)(o_1) \dots (w_n)(o_n)\}$$

5. $V((\varphi \wedge \psi)_{\langle n,0 \rangle}) = f \in D_{\langle n,0 \rangle}$ such that,

$$\text{for every } w_j \in W, o_j \in d(w_j), 1 \leq j \leq n : f(w_1)(o_1) \dots (w_n)(o_n) = h$$

such that:

$$h = V(\varphi)(w_1)(o_1) \dots (w_n)(o_n) \cap V(\psi)(w_1)(o_1) \dots (w_n)(o_n)$$

6. $V(\hat{x}_e^{n+1}(\varphi)_{\langle n,1 \rangle}) = f \in D_{\langle n,1 \rangle}$ such that,

$$\text{for every } w_j \in W, o_j \in D(w_j), 1 \leq j \leq n : f(w_1)(o_1) \dots (w_n)(o_n) = h$$

such that, for every $w \in W$:

$$h(w) = \{o \in D(w) : w \in V(\varphi)(w_1)(o_1) \dots (w_n)(o_n)(w)(o)\}$$

7. $V((\forall s)_{\langle n,0 \rangle}) = f \in D_{\langle n,0 \rangle}$ such that,

$$\text{for every } w_j \in W, o_j \in D(w_j), 1 \leq j \leq n : f(w_1)(o_1) \dots (w_n)(o_n) = h$$

such that:

$$h = \{w \in W : V(s)(w_1)(o_1) \dots (w_n)(o_n)(w) = d(w)\}$$

Finally, a term φ of type $\langle \rangle$ is true in a model if and only if $\odot \in V(\varphi)$.

This concludes the exposition of the Propositional Functions Account. We are now in a position to show why the account is not austere, pace Stalnaker, instead favouring higher-order necessitism.

2.4 Overgeneration of the Propositional Functions Account

Before turning to the case for the Overgeneration of the Propositional Functions Account, I will briefly offer some comments on its virtues. The Propositional Functions Account is an elegant account of the semantics of first-order modal languages. Not only is the account consistent with the conjunction of Aliens and Contingentism – contrary to what was the case with the Literal Account – it also avoids certain somewhat puzzling features of the Haecceities Account. Whereas according to the Haecceities Account the semantic value of an individual constant consists of an haecceity, according to the Propositional Functions Account the semantic value of an individual constant consists of an individual (that is actually something). The latter is, arguably, a more natural view.

These are advantages of the Propositional Functions Account from the standpoint of Thorough Contingentists committed to Thorough Actualism. There is yet another advantage of the Propositional Functions Account over the classic accounts that is orthogonal to the question whether any of these theses is true. Contrary to the other accounts, the Propositional Functions Account does not require an appeal to a notion of semantic value relativised to variable-assignments. Variable-assignments turn out to be, on the Propositional Functions Account, relics of the model-theoretic formalism used to model the semantics of quantified expressions. These relics should not be reflected in an account of the real semantics of quantified expressions. Arguably, these features of the Propositional Functions Account make it more attractive in comparison to the classic accounts. Arguably, the availability of the Propositional Functions Account reveals that the classical accounts confuse the elements of models with the things that they represent.

In this section it will be shown that, despite the advantages of the Propositional Functions Account over the classic accounts, the Propositional Functions Account *overgenerates* from the standpoint of Higher-Order Contingentists committed to Thorough Serious Actualism. To explain what precisely is meant with the overgeneration claim, let me introduce some notions and theses. Say that a proposition is an *attribution of being to x* just in case it is the proposition that x is something, and that it is an *attribution of being* (simpliciter) just in case it is possible that there is some x such that it is an attribution of being to x . Consider the following theses:

Necessity of Being. Necessarily, there is some individual.

Haecceity Necessitism. Necessarily, every haecceity is necessarily something.

Attributions of Being–Necessitism. Necessarily, every attribution of being is necessarily something.

The Propositional Functions Account overgenerates from the standpoint of proponents of Higher-Order Contingentism committed to Thorough Serious Actualism in the following sense:

Overgeneration of the Propositional Functions Account. The Propositional Functions Account, together with Thorough Serious Actualism and Necessity of Being, implies both i) Haecceity Necessitism, and ii) Attributions of Being–Necessitism.

Let me start by showing that the Propositional Functions Account, Thorough Serious Actualism and Necessity of Being together imply that Jordan's haecceity is necessarily something, and that the attribution of being to Jordan is necessarily something.

Consider the following expressions:

$$(3) \quad \hat{y}_e(a = y)_{\langle 1,1 \rangle}$$

$$(4) \quad (\exists \hat{y}_e(a = y))_{\langle 1,0 \rangle}.$$

Note that the expressions $\hat{y}_e(a = y)_{\langle 1,1 \rangle}$ and $(\exists \hat{y}_e(a = y))_{\langle 1,0 \rangle}$ are used, for instance, in formulating the claim that $(\Box \forall \hat{x}_e(\Box(Qax \rightarrow (\exists \hat{y}_e(a = y) \wedge \exists \hat{y}_e(x = y))))))_{\langle 0,0 \rangle}$, i.e., the claim that necessarily, for every individual x , necessarily, if Michael Jordan is a father of x , then Michael Jordan is something and x is something. Here, x is being used for the variable x_e^1 , y is being used for the expression x_e^2 , and z for x_e^3 .

According to the Propositional Functions Account the semantic values of (3) and (4) are, respectively, i) a first-level property function which necessarily, for every individual y , maps y to the property of being Jordan, and ii) a propositional function which necessarily, for every individual y , maps y to the proposition that Jordan is something. From the thesis of Necessary Being and i) it follows that a) necessarily, some individual is mapped to the property of being Jordan – and so, necessarily, some individual is related to the property of being Jordan; and from the thesis of Necessary being and ii) it follows that b) Necessarily, some individual is mapped to the proposition that Jordan is something – and so necessarily, some individual is related to the proposition that Jordan is something.

Finally Thorough Serious Serious Actualism and a) together imply that necessarily, the property of being Jordan is something. Moreover, Thorough Serious Actualism and b) together imply that the proposition that Jordan is something is something.

These consequences are generalisable. Consider the following expression:

$$(5) \quad \hat{z}_e(z = x_e)_{\langle 2,1 \rangle}.$$

According to the Propositional Functions Account, the semantic value of $\hat{z}_e(z = x_e)_{\langle 2,1 \rangle}$ is that 2^{nd} -level property function f which necessarily, for every x , maps x to that 1^{st} -level property function which necessarily, for every y , maps y to the property of being x .

From the thesis of Thorough Serious Actualism it follows that necessarily, for every x , there is a 1^{st} -level property function g which necessarily, for every y , maps y to the property of being x . From the thesis of Necessity of Being it follows that i) necessarily, for every x , there is a 1^{st} -level property function g which necessarily, maps some y to the property of being x . Thorough Serious Actualism and i) together imply that necessarily, for every x , necessarily, the property of being x is something. That is, Thorough Serious Actualism and i) together imply Haecceity Necessitism.

Similarly, consider the expression

$$(6) \quad (\exists \hat{z}_e(z = x_e))_{\langle 2,0 \rangle}$$

According to the Propositional Functions Account, the semantic value of $(\exists z_e(z = x_e))_{(2,0)}$ is that 2^{nd} -level propositional function f which necessarily, for every x , maps x to that 1^{st} -level propositional function which necessarily, for every y , maps y to the proposition that x is something.

From the thesis of Thorough Serious Actualism it follows that necessarily, for every x , there is a 1^{st} -level propositional function which necessarily, for every y , maps y to the proposition that x is something. From the thesis of Necessity of Being it follows that ii) necessarily, for every x , there is a 1^{st} -level propositional function which necessarily, maps some y to the proposition that x is something. Thorough Serious Actualism and ii) together imply that necessarily, for every x , necessarily, the proposition that x is something is itself something. That is, Thorough Actualism and ii) together imply Attributions of Being–Necessitism.

Hence, the Propositional Functions Account overgenerates from the standpoint of proponents of Higher-Order Contingentism.

How significant is this result? To begin with, Stalnaker's own higher-order modal theory is committed to Thorough Serious Actualism, as well as to the negation of Haecceity Necessitism and of Attributions of Being–Necessitism. Arguably, Stalnaker is also committed to the necessary being of at least some entities, such as mathematical entities and other abstract objects. Thus, the Overgeneration of the Propositional Functions Account reveals that Stalnaker's own higher-order modal theory is inconsistent with the Propositional Functions Account. Thus, he cannot hope to appeal to it in order to address the challenge of offering a satisfactory account of the semantics of first-order modal languages consistent with his higher-order modal theory.

The significance of the overgeneration of the Propositional Functions Account goes beyond Stalnaker's own higher-order modal theory. First, note that typical higher-order contingentists should be at least as opposed to the truth of Haecceity Necessitism as they are to the truth of the negation of No Actual Haecceity, since Haecceity Necessitism implies the falsehood of No Actual Haecceity.

Indeed, higher-order contingentists such as Adams, Fine, Prior and Stalnaker all reject the truth of the conjunction of Haecceity Necessitism and Attributions of Being–Necessitism. Moreover, it is difficult to see how some higher-order entities may fail to be something, while at the same time it is necessary that all haecceities are necessarily something, and that all attributions of being are necessarily something. Arguably, the conjunction of Haecceity Necessitism and Attributions of Being–Necessitism is true only if Higher-Order Necessitism is itself true.

Second, the thesis of Necessity of Being is rather plausible. For instance, the thesis is a direct consequence of the view that there is at least one necessary being. But according to many, things such as the empty set, the number one, and other mathematical entities are all necessary beings.

Finally, in chapter 3 a defence of Thorough Serious Actualism is offered, a defence that I will assume here to be successful.

Given this information, the plausible higher-order contingentist theories are committed to the Necessity of Being and to Thorough Serious Actualism, and to the falsehood of Haecceity Necessitism and Attributions of Being–Necessitism. The Overgeneration of the Propositional Functions Account

shows that proponents of plausible higher-order contingentist theories will not find in the Propositional Functions Account an account of the semantics of first-order modal languages consistent with their commitments.

Now, the Propositional Functions Account appears to be independently attractive, as was previously shown, in the first paragraphs of the present section. Assuming the truth of Thorough Serious Actualism and of Necessity of Being, the independent attractiveness of the Propositional Functions Account constitutes a pro tanto reason in favour of Haecceity Necessitism and Attributions of Being–Necessitism. So, the attractiveness of the Propositional Functions Account constitutes a pro tanto reason in favour of Higher-Order Necessitism.¹⁰

2.5 Overgeneration of Alternative Proposals

An obvious way of defeating the support for Higher-Order Necessitism given by the attractiveness of the Propositional Functions account consists in finding an alternative account of the semantics of first-order modal languages as attractive as the Propositional Functions Account, and which does not imply theses favouring the truth of Higher-Order Necessitism.

In this section I consider some natural ways of ‘tweaking’ the Propositional Functions Account with the aim of avoiding a commitment to these theses. It is shown that all of the ways considered turn out to be unsatisfactory, assigning the wrong semantic values to some of the expressions of the language.

2.5.1 No Middle Men

The first proposal for ‘amending’ the Propositional Functions Account consists in adopting the view that complex expressions whose semantic values are properties are determined as a function not of propositional functions, but instead of other properties. According to the present proposal, associated with each expression is a *construction tree*. The initial nodes of a construction tree contain primitive expressions of the language. The other nodes of the tree are the result of applying syntactic operations to its earlier nodes. Each of these syntactic operations have as their semantic values operations on *relations* (the semantic values of predicates).

¹⁰One route to this conclusion is via the assumption, abductively grounded, that the conjunction of Haecceity Necessitism and Attributions of Being–Necessitism is true only if Higher-Order Necessitism is itself true.

A different route for the same conclusion is the following. Note that, for any formula $\varphi_{\langle 2,0 \rangle}$, $\hat{x}_e(\varphi)_{\langle 1,1 \rangle}$ is a well-formed complex predicate. Together, Thorough Serious Actualism, the Necessity of Being and the Propositional Functions Account imply that, for any formula $\varphi_{\langle 2,0 \rangle}$, the property that is the semantic value of $\hat{x}_e(\varphi)_{\langle 1,1 \rangle}$ is necessarily something. That is, every expressible property (of individuals) is necessarily something. Moreover, consider the result of extending TML-languages with higher-order resources, and in particular the ability to bind sequences of variables of different types. Together, Thorough Serious Actualism, the Necessity of Being and the natural extension of the Propositional Functions Account to such languages imply that, for any expression of type $\varphi_{\langle t_1, \dots, \langle t_n, \langle e, 0 \rangle \rangle \rangle}$, the semantic value of $\hat{x}_{t_1}^1 \dots \hat{x}_{t_n}^n(\varphi)_{\langle 1, t \rangle}$ is necessarily something, where t_1, \dots, t_n are any types in Gallin’s type-hierarchy, and $t = \langle t_1, \dots, t_n \rangle$. That is, every expressible relation of any type is necessarily something. This result does not imply Higher-Order Necessitism. Yet, it strongly favours it. Arguably, the best explanation for the fact that every expressible relation of any type is necessarily something is that necessarily, every higher-order entity is necessarily something.

These syntactic operations have operations on relations as their semantic values. To the syntactic operation $pred_1$ corresponds the operation $Pred_1(\cdot)$ which takes as an argument the semantic value of the 1-ary predicate letter s (the first argument of the operation $pred_1$), and maps it to itself. Thus, the semantic values of $\hat{x}(Px)$ and $\hat{y}(Ry)$ are, respectively, the properties of being tall and being a basketball player.

To the operation $cond(\cdot)$ corresponds an operation, $Cond(\cdot)$, which maps the n -ary relation that is the semantic value of $\hat{v}^1 \dots v^n(\varphi)$ and the m -ary relation that is the semantic value of $\hat{v}'^1 \dots v'^m(\psi)$, and maps them to the semantic value of $cond(\hat{v}^1 \dots v^n(\varphi), v'^1 \dots v'^m(\psi))$. This is the $(n + m)$ -ary relation that holds of $v^1, \dots, v^n, v'^1, \dots, v'^m$ if and only if $\varphi \rightarrow \psi$. Thus, the semantic value of $\hat{xy}(Px \rightarrow Qy)$ is the relation that holds of x and y if and only if, if x is tall, then y is a basketball player.

To the operation $refl_{1,2}(\cdot)$ corresponds the function $Ref_{1,2}(\cdot)$. This function maps the relation that holds of x and y if and only if, if x is tall, then y is a basketball player to the property of being an x such that, if x is tall, then x is a basketball player.

Finally, to the operation $nec(\cdot)$ corresponds the function $Nec(\cdot)$. This functions maps the property of being an x such that, if x is tall, then x is a basketball player to the property of being an x such that necessarily, if x is tall, then x is a basketball player. This property is the semantic value of $\hat{x}(\Box(Px \rightarrow Rx))$.

One of the uses of complex predicates has been in regimenting essentialist theses. For instance, by appealing to quantified first-order modal languages containing devices for forming complex predicates it is possible to have predicates corresponding to the natural language predicates such as expressing properties such as the property of being essentially a man, assuming that to be essentially a man is to be an individual x such that necessarily, x is a man if x is something. Let M express the property of being a man, and E express the property of being something. The property of being essentially a man is the semantic value of the complex predicate $\hat{x}(\Box(Ex \rightarrow Mx))$.

One problem for the no middle men proposal, the one in which we will be focusing here, is that the complex predicates $\hat{x}(\Box(Ex \rightarrow Mx))$ and of $\hat{x}(\Box(Mx))$ — intended to express, respectively, the property of being essentially a man and the property of being necessarily a man — turn out to have the same satisfaction conditions according to the no middle men proposal, despite the fact that they have different satisfaction conditions, assuming that some men could have been nothing. If some men could have been nothing, then there is at least one x such that i) x has the property of being necessarily a man if something (assuming that no men could have been something and not a man), and yet ii) x does not have the property of being necessarily a man, since being necessarily a man implies being necessarily something, and x could have been nothing.

Consider the syntactic construction trees for these two expressions:

man. But the property of being a man is the semantic value of $\hat{x}(Mx)$. Hence, necessarily, for every individual y , necessarily, y instantiates the property that is the semantic value of $\hat{x}(Ex \rightarrow Mx)$ if and only if y instantiates the property that is the semantic value of $\hat{x}(Mx)$.

Thus, given the assumption of Thorough Serious Actualism, the predicates $\hat{x}(Ex \rightarrow Mx)$ and $\hat{x}(Mx)$ have the same satisfaction conditions. But then, it follows from claims i) and ii) that the complex predicates $\hat{x}(\Box(Ex \rightarrow Mx))$ and $\hat{x}(\Box(Mx))$ have the same satisfaction conditions.

Insofar as the properties of being necessarily a man and being essentially a man are different, the no middle men proposal has the troublesome consequence of removing the ability to use the language of complex predication to define one of these properties in terms of the other in the natural way. From the standpoint of the Propositional Functions Account, the problem with the no middle men proposal is that it generates the semantic values of $\hat{x}(\Box(Ex \rightarrow Mx))$ and $\hat{x}(\Box(Mx))$ in terms of the semantic values of $\hat{x}(Ex \rightarrow Mx)$ and $\hat{x}(Mx)$, predicates with the same satisfaction conditions.

Instead, according to the Propositional Functions Account, the semantic values of the two predicates are generated in terms of the propositional functions that are the semantic values of $(Ex \rightarrow Mx)$ and (Mx) . Crucially, these propositional functions are not necessarily coextensive. The propositional function $\llbracket Ex \rightarrow Mx \rrbracket$ maps every possible individual x to the proposition that if x is something, then x is a man, whereas the propositional function $\llbracket Mx \rrbracket$ maps every possible individual x to the proposition that x is a man. The propositions that if x is something, then it is a man, and the proposition that x is a man are true at different possibilities. Whereas the former proposition is true at those possible worlds in which x is nothing, the latter proposition is false at any such possible world.

The upshot is that the no middle men proposal does violence to the intended interpretation of first-order modal languages with complex predicates, and so is unsatisfactory.

2.5.2 Partial Functions

A different route available to thorough contingentists consists in thinking that the mistake with the Propositional Functions Account has been that of thinking that propositional functions must be *total*. An alternative option is to take propositional functions to be partial, defined only for some individuals.

For instance, according to this proposal the second-level propositional function f that is the semantic value of $(a = x)_{\langle 1,0 \rangle}$ is a relation that necessarily, for every individual x , obtains between x and a proposition h if and only if h is the proposition that x is identical to a . Thus, if the proposition that x is identical to a does not exist, then the propositional function f does not relate x to any proposition whatsoever. The property that is the semantic value of $\hat{x}(a = x)_{\langle 0,1 \rangle}$ is determined in the same way, as a function of the propositional function that is the semantic value of $(a = x)_{\langle 1,0 \rangle}$. If f does not relate an individual x to any proposition, then x is not in the extension of the property that is the semantic value of $\hat{x}(a = x)_{\langle 0,1 \rangle}$. Call this proposal the *partial functions proposal*.

The partial functions proposal comes with its own problems. As will be shown, the fact that it is possible that there are some individuals for which a propositional function is undefined has the consequence that the recursive clauses of the account of semantic value do not assign semantic values

to expressions that ought to have a semantic value.

Higher-order contingentists are sympathetic to the view that Attributions of Being–Necessitism is false, and in particular that there are some individuals x such that the proposition that x is something is itself something only contingently. As in the previous section, let $E_{\langle 0,1 \rangle}$ be a 1-ary predicate letter whose semantic value is the property of being something. Consider the expression $(Ea)_{\langle 1,0 \rangle}$, and let f be the propositional function that is the semantic value of this expression. Let w be some counterfactual possibility such that the proposition that Michael Jordan is something is nothing at w , and at which the empty set is something. Since the proposition that Jordan is something is nothing at w , it is not the case that the propositional function f relates the empty set to a proposition at the relevant counterfactual possibility.

How is the property that is the semantic value of $\hat{x}(Ea)_{\langle 0,1 \rangle}$ determined in terms of the propositional function f ? The two natural options available are: i) necessarily, for every individual x , x has the property if and only if f maps x to a proposition and that proposition is true; ii) necessarily, for every individual x , x has the property if and only if either f maps x to a proposition and that proposition is true, or f maps x to no proposition whatsoever.

If option i) is adopted, then it is not the case that the empty set has the property that is the semantic value of $\hat{x}(Ea)_{\langle 0,1 \rangle}$ at w , since the function f that is the semantic value of $(Ea)_{\langle 1,0 \rangle}$ does not relate the empty set to any proposition whatsoever at w . If option ii) is adopted, then the empty set does have, at w , the property that is the semantic value of $\hat{x}(Ea)_{\langle 0,1 \rangle}$. Option i) is the one that delivers the right result in the present case. The intended semantic value of $\hat{x}(Ea)_{\langle 0,1 \rangle}$ is the property of being such that Michael Jordan is something, that property that necessarily, for every individual holds of that individual if and only if Michael Jordan is something. Since Jordan is nothing at w , the empty set does not have the property of being such that Jordan is something.

Let us thus adopt option i). Consider now the propositional function $(\neg(Ea))_{\langle 1,0 \rangle}$. By the recursive clause for negated expressions, the propositional function g that is the semantic value of the expression $(\neg Ea)_{\langle 1,0 \rangle}$ also does not relate the empty set to any proposition whatsoever. But then the empty set also does not have, at w , the property that is the semantic value of $\hat{x}(\neg Ea)_{\langle 0,1 \rangle}$. This, however, is the wrong result. The intended semantic value of $\hat{x}(\neg Ea)_{\langle 0,1 \rangle}$ is the property of being such that Jordan is nothing. Insofar as Jordan is nothing at w , the empty set has, at w , the property of being such that Jordan is nothing at w . Since the empty set does not have, at w , the property that is the semantic value of $\hat{x}(\neg Ea)_{\langle 0,1 \rangle}$ according to the the partial functions proposal, the semantic value of $\hat{x}(\neg Ea)_{\langle 0,1 \rangle}$ according to the partial functions proposal is not its real semantic value.

Note also that if option i) is adopted then the semantic values of $\hat{x}(Ea)_{\langle 0,1 \rangle}$ and $\hat{x}(\neg Ea)_{\langle 0,1 \rangle}$ turn out not to be exhaustive properties. In addition, the adoption of option i) forces the rejection of certain plausible principles of first-order modal logic. Let b have as its semantic value the empty set. For instance, the adoption of option i) requires the rejection of the following claim:

$$(9) \quad \Box(\hat{x}(\neg(Ea))b \leftrightarrow (\neg Ea \wedge Eb))_{\langle 0,0 \rangle}$$

Even though it is the case that the empty set is something at w and Michael Jordan is nothing at w (let us assume, since the proposition that Michael Jordan is something is itself nothing at w), it is not the case that, at w , the empty set has the property that is the semantic value of $\hat{x}(\neg(Ea))$. But (9) is a principle valid in fairly minimal first-order modal logics, such as the one offered in Stalnaker (1994).

Before proceeding, it is important to make it clear that this result is not intended to show that the partial functions proposal is contradictory. Instead, the argument shows that the property that is delivered by the partial functions account as the semantic value of $\hat{x}(\neg Ea)_{(0,1)}$ is not the property that in fact is the semantic value of this expression. Since the partial functions account is unable to deliver the right semantic values of some of the expressions of the language, it is unsatisfactory.

2.5.3 Other Proposals

Let me quickly mention two other proposals. The first of these proposals consists in adopting the view that the semantic value of expressions of the type of $(n + 1)^{th}$ -level propositional functions to consist in $(n + 1)^{th}$ -level *coarse-grained* propositional functions. Briefly, a 0^{th} -level *coarse-grained* propositional function consists in either the necessary proposition (say, in the proposition that $\emptyset = \emptyset$) or in the impossible proposition (say, in the proposition that $\emptyset \neq \emptyset$). A $(n + 1)^{th}$ -level *coarse-grained* propositional function is a relation between individuals and n^{th} -level *coarse-grained* propositional functions such that each $(n + 1)^{th}$ -level propositional function f is such that necessarily, for every individual x , there is one and only one n^{th} -level *coarse-grained* propositional function g such that f relates x to g (i.e., $f(x) = g$). Call this proposal the *coarse-grained propositional functions proposal*.

Consider once more the expression $(\exists \hat{z}(z = x))_{(2,0)}$. As seen in §2.4, the semantic value of this expression is, according to the Propositional Functions Account, a second-level propositional function f which necessarily, for every x , maps x to the first-level propositional function g which necessarily, for every y , maps y to the proposition that x is something.

Furthermore, as also seen in §2.4, the existence of such propositional function implies that necessarily, for every individual x , the proposition that x is something is necessarily something. The ‘coarse-grained propositional functions’ proposal is designed to avoid consequences such as this one.

According to this proposal, the semantic value of $(\exists \hat{z}(z = x))_{(2,0)}$ is not the second-level propositional function previously described. It is, instead, the second-level *coarse-grained* propositional function f^* which necessarily, for every x , maps x to that first-level *coarse-grained* propositional function g^* which necessarily, for every y , maps y to the necessary proposition if and only if x is something, and otherwise maps y to the impossible proposition.

This solution blocks the problematic consequence of the Propositional Functions Account. What is concluded is that either the necessary proposition is something in circumstances in which x is something, or else the impossible proposition is something in circumstances in which x is nothing. Since it is plausible to think that the necessary and the impossible propositions are necessarily something anyway, this consequence of the coarse-grained propositional functions proposal is unproblematic.

Still, the coarse-grained propositional functions proposal leads to problematic consequences of its

own. In a nutshell, the coarse-grained propositional functions proposal makes propositional functions ‘too coarse-grained’. Consider the expressions $(\exists\hat{y}(y = x))_{\langle 1,0 \rangle}$ and $(\Box(\exists\hat{y}(y = x)))_{\langle 1,0 \rangle}$. Let h be the first-level coarse-grained propositional function that is the semantic value of $(\exists\hat{y}(y = x))_{\langle 1,0 \rangle}$ and h^* be the first-level coarse-grained propositional function that is the semantic value of $(\Box(\exists\hat{y}(y = x)))_{\langle 1,0 \rangle}$.

According to the coarse-grained propositional functions proposal, the first-level coarse-grained propositional function h is that first-level coarse-grained propositional function which necessarily, for every x , maps x to the necessary proposition if and only if x is something. Thus, h (actually) maps Michael Jordan to the necessary proposition, since Michael Jordan (actually) is something.

According to the semantic clause for necessitated expressions, h^* is that first-level coarse-grained propositional function which necessarily, for every x , maps x to the proposition that obtains if and only if the proposition to which x is mapped to by h necessarily obtains. Thus, h^* also maps Michael Jordan to the necessary proposition. Therefore, Michael Jordan instantiates the property that is the semantic value of $\hat{x}(\Box(\exists\hat{y}(y = x)))_{\langle 0,1 \rangle}$, since the semantic value of $\hat{x}(\Box(\exists\hat{y}(y = x)))_{\langle 0,1 \rangle}$ is that property which necessarily, for every individual x , holds of x if and only if h^* maps x to a true proposition.

This means that the semantic value of $\hat{x}(\Box(\exists\hat{y}(y = x)))_{\langle 0,1 \rangle}$ is not the one that is intended, namely, the property of being necessarily something, since it is not the case that Michael Jordan is necessarily something (from the standpoint of thorough contingentists), even though he instantiates the property that is the semantic value of $\hat{x}(\Box(\exists\hat{y}(y = x)))_{\langle 0,1 \rangle}$ according to the propositional functions account.

More generally, the problem with the coarse-grained propositional functions account is that by having one of the necessary or the impossible propositions as 0-ary propositional functions, it is these propositions, rather than more fine-grained proposition, that contain the information to be used in the semantic composition. Since Jordan is something, the propositional function h maps him to the necessary proposition. But then, h^* also maps him to the necessary proposition – independently of whether he is in fact necessarily something. The upshot is that the coarse-grained propositional functions proposal is also unsatisfactory, assigning the wrong semantic values to some of the expressions of the language.

The second proposal consists in resorting to more familiar first-order languages without any dedicated variable-binding operators, and in which the quantifier \exists attaches directly to variables v in order to form an expression $\exists v$ which maps $(n + 1)^{th}$ -level propositional functions to n^{th} -level propositional functions.

The problem with this proposal is that it does not really solve one of the problems noted in §2.4, namely, the one involving the expression $(Ey)_{\langle 2,0 \rangle}$. This expression still turns out to have as its semantic value a second-level propositional function whose being implies that necessarily, for every individual x , the proposition that x is something is necessarily something. That is, the resulting account is still committed to Attributions of Being–Necessitism. Thus, from the standpoint of higher-order contingentists, this proposal is also unsatisfactory.

2.6 Conclusion

In this chapter a detailed presentation of the Propositional Functions Account of the semantics of first-order modal languages, proposed in (Stalnaker, 2012), has been offered. It was shown that the Propositional Functions Account, together with Thorough Serious Actualism and Necessity of Being, implies both Haecceity Necessitism, and Attributions of Being–Necessitism.

This result reveals that i) the Propositional Functions Account is inconsistent with Stalnaker's higher-order modal theory, contrary to Stalnaker's own views on the matter; and ii) the attractiveness of the Propositional Functions Account constitutes a reason in favour of higher-order necessitism.

Finally, some natural ways of amending the Propositional Functions Account were surveyed with the aim of finding a satisfactory account of first-order modal languages not opposed to Thorough Contingentism. All of these alternative proposals were seen to lead to problems of their own, assigning incorrect semantic values to some of the expressions of first-order modal languages.

The overall conclusion is that Thorough Contingentists are still faced with the challenge of offering a satisfactory account of the semantics of first-order modal languages. The Propositional Functions Account is not satisfactory from their standpoint.

In the next chapter I will continue to look at the prospects of Higher-Order Necessitism. I will offer a principled defence of Propositional Necessitism, the thesis that necessarily, every proposition is necessarily something. Moreover, I will show that this defence extends to a defence of Higher-Order Necessitism.

3

Propositions as Necessary Beings

3.1 Introduction

Different theorists take propositions to fulfil different, albeit related job descriptions. In this chapter the focus is on propositions understood as follows: i) higher-order entities – roughly, the semantic values of 0-ary predicates – entities of type $\langle \rangle$, on the type hierarchy presented in chapter 1 ; ii) shareable objects of the attitudes, i.e., of mental states such as believing, desiring, asserting, doubting, assuming, etc.; iii) bearers of truth, falsity and of alethic modalities, iv) relata of logical consequence.¹

Recall *Propositional Necessitism*, the thesis that necessarily, every proposition is necessarily something. The main aim of the present chapter is to defend Propositional Necessitism. Recent proponents of Propositional Necessitism are [Plantinga \(1976\)](#) and [Williamson \(2013\)](#), whereas Propositional Contingentism, its contradictory, has been advocated by, among others, [Adams \(1981\)](#), [Fine \(1977\)](#), [Prior \(1957\)](#) and [Stalnaker \(2012\)](#).

The defence of Propositional Necessitism to be offered may be divided in two steps. One of these steps consists in providing positive arguments for the truth of Propositional Necessitism. An interesting feature of these arguments is that their weakest assumption consists in the claim that the very weak propositional modal logic **KD** is sound for metaphysical modality. Thus, the positive arguments show that very weak propositional modal logics are already committed to Propositional Necessitism. Another interesting feature of the defence concerns the more general thesis of Higher-Order Necessitism. Not only is it the case that the truth of Higher-Order Necessitism is favoured by the truth of Propositional Necessitism, the defence of Propositional Necessitism to be offered is extendable to a defence of Higher-Order Necessitism.

The other step consists in a defence of Propositional Necessitism against a well-known objection. Briefly, the objection departs from the fact that, given plausible auxiliary assumptions, Propositional Necessitism is inconsistent with the conjunction of Contingentism and the *Classical Conception of*

¹The present way of understanding propositions is thus close to the one present in ([McGrath, 2014](#)). The differences are the following: i) McGrath does not fix the meaning of ‘proposition’ by reference to alethic modalities and entailment. He does, however, acknowledge that ‘If there are propositions, they would appear to be good candidates for being the bearers of alethic modal properties (necessary and possible truth), as well as the relata of entailment’; ii) McGrath takes propositions to be the *primary* bearers of truth and falsity, whereas I am leaving it open whether this is so. See §3.7.

propositions (a view on the nature of propositions that will be presented in section §3.2). Since Contingentism is supported by unreflective common sense and the Classical Account is the received view on the nature of propositions, the joint inconsistency of Propositional Necessitism, Contingentism and the Classical Conception poses a challenge to any defence of Propositional Necessitism. According to proponents of the objection, the inconsistency shows the falsehood of Propositional Necessitism.

I will argue that, independently of the truth of Contingentism, the objection is unsuccessful because the Classical Conception of propositions is false. Moreover, it will be shown that a much attacked commitment of the Classical Conception, one driving many theorists, including myself, to reject it, is the commitment responsible for the joint inconsistency of Propositional Necessitism, Contingentism and the Classical Conception.

The chapter has two subsidiary aims. The first of these has already been mentioned, namely, to extend the positive arguments for Propositional Necessitism to arguments for Higher-Order Necessitism. The other subsidiary aim is to offer a defence of Thorough Serious Actualism. The reason for such a defence is that the truth of this thesis is one of the assumptions common to the different arguments for Propositional Necessitism.

The chapter is structured as follows. In §2 the Classical Conception of propositions is presented, and it is shown to be inconsistent with the conjunction of Propositional Necessitism and Contingentism.

Afterwards, in §3.3, a defence of Thorough Serious Actualism is offered. *Serious Actualism* is a special case of Thorough Serious Actualism, one applying only to first-order relations (i.e., relations between individuals):

Serious Actualism Necessarily, for every relation R between individuals, no individuals could have been R -related and yet have been nothing.

I begin by addressing an objection to Serious Actualism put forward by Salmon (1987). A positive argument for the truth of Thorough Serious Actualism is then offered, one which depends on very minimal assumptions.

Then, I offer some arguments for Propositional Necessitism. All the arguments appeal to generalisations of fairly weak principles of propositional modal logic and to the assumption that the modal operators have as semantic values properties of propositions.

Some objections to the positive arguments for Propositional Necessitism are discussed in §5. One interesting aspect of the arguments plays central stage in some of the discussion in this section, namely, the fact that the arguments are very similar to the argument offered by Plantinga (1983) against *Existentialism* – where Existentialism is the thesis that no proposition about a thing could have been something while the thing that it was about was nothing.

Plantinga's argument has been rejected on the grounds that it conflates two notions of truth relative to a world. One of the things shown in §5 is that the distinction between these two notions, by itself, does not afford the resources required to resist the arguments for Propositional Necessitism.

Then, in §6 it is argued that the Classical Conception of propositions is false, and so that the fact that the account is inconsistent with the conjunction of Contingentism and Propositional Necessitism does not support the conclusion that Propositional Necessitism is false.

In §7 I take a look back at the arguments for Propositional Necessitism that have been offered, and make a case for the claim that the lesson to take from them is that propositional modal logic is already committed to the truth of Propositional Necessitism.

Finally, in §8 I show how the arguments for Propositional Necessitism are extendible to arguments for Higher-Order Necessitism.

3.2 The Classical Conception of Propositions

King and Soames have characterised as the *Classical Conception* of propositions the view, common to the theories of Frege, the early Russell and more recent possible worlds' semantics that propositions are mind-independent, abstract entities that are intrinsically and essentially representational and thus are the primary bearers of truth and falsity.² Relevant for the present purposes is the fact that according to the Classical Conception of propositions these are intrinsically and essentially representational.

Pictures, sculptures and sentences represent things as being one way or another. In this sense, pictures, sculptures and sentences may be said to be 'about' things. According to the Classical Conception propositions are also about things, representing them as being one way or another. In effect, according to the Classical Conception pictures, sculptures and sentences are representational in virtue of having intrinsically and essentially representational entities — propositions — as their contents.

Moreover, according to the Classical Conception propositions are *intrinsically* and *essentially* representational. To represent things as being a certain way is part of the nature of propositions. Propositions contrast with pictures, sculptures and sentences in this respect. Since pictures, sculptures and sentences represent things as being a certain way *in virtue of* the cognitive activities of agents, they are not intrinsically representational. They are also not essentially representational, since the cognitive activities of agents may fail to endow them with representational powers.

The following are commitments of the Classical Conception of propositions:³

Aboutness. Some propositions are about individuals.

²See, e.g., the texts in (King et al., 2014) for a presentation of the classical conception (in particular ch.3).

³One may want to distinguish between *direct representation* and *indirect representation*. A direct representation of an object as being a certain way is a representation of an object as being a certain way but not qua holder of a certain property, whereas an indirect representation of an object is representation of an object as being a certain way qua holder of a certain property. For instance, if things are represented as being such that there is a thing that is the president of the United States and a politician, the representation is about Obama, but only indirectly. On the other hand, if things are represented as being such that Obama is a politician then, the representation in question is *directly about* Obama. Correspondingly, two notions of aboutness may be distinguished, namely, *direct aboutness* and *indirect aboutness*. The proposition that Obama is a politician is directly about Obama, whereas the proposition that the president of the United States is a politician is indirectly about Obama. The intended notion of aboutness is that of *direct aboutness*. See (Glick, Forthcoming) for a discussion and account of singular propositions appealing to the distinction between direct and indirect aboutness.

Essential Aboutness. Necessarily, if a proposition is about something, then it is essential to the proposition that it be (intrinsically) about the things that it is (intrinsically) about.

Now, consider the following claims:

About Contingents. There could have been some proposition that could have been about an individual that could have been nothing.

Thorough Serious Actualism. Necessarily, no things could have been related and yet have been nothing.

Together, Thorough Serious Actualism and Essential Aboutness imply a thesis which Plantinga has called *Existentialism*:⁴

Existentialism. There could not have been a proposition p about some x such that p could have been something and yet x was nothing.

The argument from Essential Aboutness and Thorough Serious Actualism to Existentialism goes as follows. Suppose p is about x . Then, it is essential to p that it be about x , and so necessarily, if p is something then p is about x . Moreover, by Thorough Serious Actualism, necessarily, if p and x are related, then both of them are something. Since it is essential to p that it be about x , it follows that necessarily, if p is something then x is something. A fortiori, there could not have been a proposition p about some x such that p could have been something and yet x was nothing. That is, Existentialism is true.

The theses of Existentialism and About Contingents together imply that there could have been a proposition that could have been nothing. So, Essential Aboutness, About Contingents and Thorough Serious Actualism together imply the falsehood of Propositional Necessitism.

As just seen, Essential Aboutness is a commitment of the Classical Conception. In addition, About Contingents is a typical commitment of those theorists endorsing both Contingentism and the Classical Conception of propositions. One way to see this is by noting that Contingentism implies About Contingents when conjoined with the following thesis:

Plenitudinous Aboutness. Necessarily, for every individual there could have been a proposition about it.

Recall the notions of an *attribution of being* to an individual and the notion of an attribution of being, introduced in chapter 2. An attribution of being to an individual x consists in the proposition that x is something, and an attribution of being consists in a proposition that is possibly an attribution of being to something.

Plenitudinous Aboutness is a consequence of the following claims: i) an attribution of being to x is a proposition about x ; and ii) necessarily, for every individual x , the attribution of being to x is something. Claim i) seems intuitively true, provided that there are propositions about individuals.

⁴See (Plantinga, 1983).

That is, if there are propositions about individuals, then, certainly, attributions of being are about the things that they attribute being to. Moreover, it is also intuitively plausible that things could not have been in such a way that there was an individual and yet there was no attribution of being to x .

Other routes for Plenitudinous Aboutness are also available. If there are any propositions about individuals, then propositions attributing properties to individuals are about individuals, being about the individuals that they attribute properties to. And it is intuitively plausible that necessarily, for every individual, there could have been a proposition that consists in the attribution of some property to it.

Thus, the Classical Conception of propositions, together with Contingentism, Plenitudinous Aboutness and Thorough Serious Actualism, implies the falsehood of Propositional Necessitism. This poses a challenge to any defence of Propositional Necessitism, since the Classical Conception is the received view on the nature of propositions, Contingentism is supported by unreflective common sense, Plenitudinous Aboutness is very plausible on the assumption that the Classical Conception of propositions is true, and Thorough Serious Actualism is intuitively appealing, and there is a compelling case to be made for its truth, as shown in §3.3

The challenge posed to the truth of Propositional Necessitism by the conjunction of Contingentism with the Classical Conception of Propositions will be answered in §3.7, after the presentation and discussion of the arguments for Propositional Necessitism. The answer offered there consists in showing that Essential Aboutness is false (and so, that the Classical Conception is itself false).⁵ I will sketch how I think the Classical Conception should be revised in §3.7

The Classical Conception thus conflicts with Propositional Necessitism. The main aim of this chapter is to present a defence of this thesis. Before doing so a defence of Thorough Serious Actualism is offered. Thorough Serious Actualism turns out to be an assumption common to all the arguments for Propositional Necessitism that will be offered.

3.3 A Defence of Thorough Serious Actualism

Thorough Serious Actualism enjoys support from unreflective common sense. After all, how could things have been related while at least one of them was nothing? Yet, there are objections to the truth of Serious Actualism, and so, a fortiori, to the truth of Thorough Serious Actualism.

The section begins with the presentation and discussion of a worry with any defence of Serious Actualism, namely, that such defence will exclude noneist theories from the outset, despite the fact that some take noneist theories to offer the appropriate solutions to several philosophical puzzles. Afterwards, an influential objection by Salmon (1987) to Serious Actualism is presented, and replies to the objection on behalf of Serious Actualism are offered. Then, a positive argument for Thorough Serious Actualism is presented. Finally, the positive argument for Serious Actualism is shown to reveal

⁵This makes the points in this chapter part of the recent criticisms that have been offered to the Classical Conception. For instance, (King et al., 2014) is motivated by dissatisfaction with the Classical Conception of propositions.

a tension in Salmon's views: he appears to accept the truth of all the premises of the argument for Thorough Serious Actualism and yet rejects the arguments' conclusion.

3.3.1 Serious Actualism and Noneism

Noneism consists in the following metaphysical thesis:

Noneism. Some things do not exist.

According to noneists, appropriate solutions to puzzles concerning the intentionality of thought, fictional discourse, discourse about time, what there could have been and there could not have been, etc. all imply the truth of Noneism.

Noneists advocate the *Principle of Independence of Being from So-Being*. According to this principle, the nature of a thing is independent of its existence.⁶ Even though it is not completely clear what is meant with 'independence', and so what is the exact content of the Principle of Independence, it is commonly taken to imply the following claim:⁷

Accidental Existence. There could have been some things that could have had properties while not existing.

Not only is Accidental Existence a consequence of the Principle of Independence, it is a consequence of noneist theories.

Prima facie, Accidental Existence consists in the negation of Serious Actualism, in which case Noneism (or the bulk of noneist theories) are inconsistent with Serious Actualism. This would mean that an appropriate defence of Serious Actualism would require taking a stance on the Noneism-Allism debate, and offering a defence of Allism (where Allism is the contradictory of Noneism, i.e., Allism is the thesis that everything exists). Reasonable considerations in favour of Noneism would turn out to count against Serious Actualism.

However, Accidental Existence is not inconsistent with Serious Actualism, and thus a defence of Serious Actualism does not require taking a stance on the Noneism-Allism debate. Let me briefly explain why.

Noneists distinguish between *neutral* and *loaded* quantification. Neutral quantification is quantification over everything, unrestrictedly. Loaded quantification is quantification restricted to what exists. As mentioned in chapter 1, the quantifiers are here being understood as being *unrestricted*. This means that Serious Actualism, as the principle is here understood, is a principle concerned with what holds of everything.

Let 'E' be a first-order predicate of type $\langle e \rangle$ standing for the property of existence. Consider the following two claims:

⁶The Principle of Independence was borrowed by Meinong from Mally, his student. See (Meinong, 1960, fn. 7).

⁷Lambert (1983) fleshes out independence in terms of the invalidity of a certain argument. According to him, the principle of independence states that the argument 'there are nuclear properties P_1, P_2, \dots such that the set of P_1, P_2, \dots attaches to s ; So, s has being' is invalid. From this reading of the Principle of Independence it follows that there could have been some things that could have had properties while not existing if it is assumed that an argument is invalid only if it is possible for its premises to be true and its conclusion to be false.

- (1) a. Possibly there is some property P and possibly there is some thing x such that possibly x is a P and x is nothing.
 b. $\Diamond\exists P\Diamond\exists x\Diamond(Px \wedge \neg\exists y(y = x))$.
- (2) a. Possibly there is some property P and possibly there is some thing x such that possibly x is a P and x does not exist.
 b. $\Diamond\exists P\Diamond\exists x\Diamond(Px \wedge \neg Ex)$.

Claim (1) is inconsistent with Serious Actualism, whereas claim (2) is consistent with Serious Actualism. But it is (2) that is a consequence of the Principle of Independence. The Principle of Independence does not imply (1). So, the Principle of Independence is not inconsistent with Serious Actualism. Thus, there is no objection to Serious Actualism from the assumption of the truth of the Principle of Independence.

Does an unrestricted understanding of the quantifiers imply that Propositional Necessitism is trivially true? No, since the fact that quantification is unrestricted does not mean that there could not have been propositions that there actually aren't.

A comparison with noneist theories may be helpful. Some noneists are committed to Necessitism, i.e., they hold that necessarily every thing is necessarily something.⁸ One may think that, since the quantifiers are unrestricted, the view that necessarily every thing is necessarily something is trivial. But it isn't.

For instance, noneists committed to Necessitism are troubled with objections that do not trouble other noneists. One problem for noneists endorsing Necessitism is that they cannot make sense of the fact that fictional characters are created. The intuition that fictional characters are created could be respected by endorsing the view that fictional characters are nothing in at least some circumstances in which their creators are nothing. Note that such view *would not* require a rejection of the claim that necessarily, every fictional character is such that it does not exist. What would be required by this view would be a commitment to the claim that there could be fictional characters that could have been nothing in some of the circumstances in which they do not exist.

Similarly, suppose that every proposition is such that it does not exist. Still, to some it will seem reasonable to think that if the things that propositions are about had been nothing, then the propositions would themselves be nothing. For instance, in circumstances in which Sherlock Holmes is nothing any proposition about Sherlock Holmes is itself nothing. Propositional Necessitism excludes cases such as this. Hence, regardless of whether Propositional Necessitism is true or not, the thesis is not trivial even when the quantifiers are understood unrestrictedly.

To reiterate, Serious Actualism is not inconsistent with Accidental Existence. Thus, a defence of Serious Actualism does not require a defence of Allism. Which of Noneism-Allism is true? In chapter 4 I query whether some noneist theories turn out to be equivalent to some allist theories, and thus,

⁸See, e.g., (Priest, 2005, §§1-2). Necessitism turns out to be a logical validity on the semantics for identity and necessity offered by Priest.

whether proponents of these theories are involved in a verbal dispute.

3.3.2 Serious Actualism and Noman

A more troublesome objection to Serious Actualism has been put forward by Salmon (1987). The objection relies on the description of a scenario that Salmon takes to witness some claims concerning naming and reference, claims which, if true, establish the falsehood of Serious Actualism.

On Salmon's scenario, 'Ovum' and 'Sperm' are names for, respectively, a particular ovum of Salmon's mother, and a particular sperm of his father. Moreover, on Salmon's scenario Ovum and Sperm have not and will not unite, even though they might. Salmon assumes that the following speech act would succeed in fixing the reference of 'Noman': let 'Noman' be the thing that could have resulted from the union of Ovum and Sperm.

Salmon assumes that there could have been something resulting from the union of Ovum and Sperm and that there could not have been more than one thing resulting from the union of Ovum with Sperm. He takes his naming act to successfully fix the referent of 'Noman' insofar as only one possible thing could have resulted from the union of Ovum and Sperm

Given the description of the scenario, Salmon holds that 'Noman' has a referent, namely Noman, and so that Noman has the property of being the referent of 'Noman'. Since Ovum and Sperm have not actually united, Noman is actually nothing. And since Ovum and Sperm could have united, Noman could have been something. So, Salmon holds that he has successfully described a case, a possible case, in which something, namely, Noman, has a property, namely, the property of being the referent of Noman, despite the fact that actually, Noman is nothing. If Salmon is right and the scenario he has described is possible, then it constitutes a counterexample to Serious Actualism.

There are two lines of reply available to serious actualists. The first of these consists in accepting that, in Salmon's scenario, Noman has the property of being the referent of 'Noman'. Yet, the fact that Noman has a property in Salmon's scenario is unproblematic because, in the scenario, Noman is something. Even though he has not resulted from the union of Ovum and Sperm, he still could have resulted from the union of Ovum and Sperm.

Linsky and Zalta's and Williamson's Necessitism would fit appropriately with this reply. Noman would, according to their necessitist theories, be something that is neither concrete nor abstract, and would have been concrete had he resulted from the union of Ovum and Sperm. In those possibilities in which Noman is nonconcrete, most of his properties are modal properties, such as the property of possibly resulting from the union of Ovum and Sperm.

The second line of reply rejects that Noman is something in the scenario described by Salmon. This line of reply implies contingentism, and fits with unreflective common sense. I see two reasonable strategies for developing this line.

The first strategy consists in rejecting the claim that 'Noman' is a genuine proper name. According to this option, 'Noman' is shorthand for the definite description 'the thing that results from the union of Ovum and Sperm'. Those following this line incur the burden of spelling out why it is that Salmon

has not succeeded in introducing the proper name 'Noman' into our language. That is, why is it that, instead, 'Noman' must be understood as nothing but a shorthand for a definite description.

I am more attracted to a different (albeit related) strategy. Recently, some compelling linguistic evidence has been gathered in support of *Predicativism*, the view according to which what we tend to call proper names are really predicates.⁹ More precisely, according to Predicativism, the semantic value of a name is the same kind of thing that is the semantic value of a predicate. On the view in question, names are count nouns. More relevant to the present discussion is the predicativists' commitment to the view that names do not have referents. Instead, names are *true of* their bearers.

This is not the place to offer a defence of Predicativism. Let me just point out that among the data to which predicativists appeal is the fact that names sometimes do occur as count nouns. Some examples are the following:¹⁰

- (3) Every Sarah I've met sometimes works as a babysitter.
- (4) Sarahs from Alaska are usually scary.
- (5) Some Alfreds are crazy; some are sane.

Predicativists incur the burden of explaining how names can compose with predicates to yield truth-values, since predicates in general are unable to do so on their own – for instance, 'dog is an animal' is ill-formed to begin with, and the meanings of "dog" and 'is an animal' do not compose to yield a truth-value. My preferred view on these matters is *The-Predicativism*. According to The-Predicativism, names, when occurring in subject position, are accompanied by an unpronounced definite article. So, the following sentences pairs have the same syntactic form:¹¹

- (6) a. The table is tall.
b. \emptyset_{the} Maria is tall.
- (7) a. Bears from the North of Alaska are usually scary.
b. Sarahs from \emptyset_{the} Alaska are usually scary.

Hopefully, this suffices as an explanation of Predicativism. If Predicativism is true, then it is *false* that names refer. Thus, it is false that things have the property of being referred to by names, and so it is false that there is anything that has the property of being the referent of 'Noman'.

I find this line of reply to Salmon's objection to Serious Actualism preferable to the others that have been discussed. Whereas the other replies are guided by an attempt to make Serious Actualism compatible with at least some aspects of the scenario described by Salmon, the present reply to Salmon's objection appeals to independent evidence, of a linguistic nature. Arguably, the evidence favours a theory on the syntax and semantics of names, Predicativism, that implies that names do not

⁹Predicativism is defended by, among others, Burge (1973), Elugardo (2002), Fara (2015) and Sawyer (2009).

¹⁰These examples are directly taken from (Fara, 2015, p. 61)

¹¹These examples are again taken directly from (Fara, 2015, p. 71).

refer. A fortiori, ‘Noman’ has no referent, and there is nothing that has the property of being the referent of ‘Noman’.

It might be thought that this is a problematic reply. Since the thing that is the result of the union of Ovum and Sperm is called Noman, does it not instantiate the property of being Noman? The correct predicativist reply from the standpoint of contingentists is, I think, that it doesn’t. At most, in Salmon’s scenario the semantic value of ‘Noman’ is successfully fixed as being a certain property, a property that nothing has in the scenario, even though one thing could have had it. Nothing is \emptyset_a Noman. Rather, there could have been something that was \emptyset_{the} Noman, which would then have been the result of the union of Ovum and Sperm.¹²

Some ways of rejecting Salmon’s objection to Serious Actualism have been surveyed. In what follows a positive argument for Thorough Serious Actualism will be offered. Afterwards, it will be shown that the argument reveals some tensions in Salmon’s thought.

3.3.3 The Argument for Thorough Serious Actualism

I will call the argument for Thorough Serious Actualism that will be offered the *instantiation argument*. The Instantiation Argument relies on the following assumptions:¹³

Premises of the Argument for Thorough Serious Actualism

Relatedness Implies Identity.

1. Necessarily, for every relation X , it is impossible that some things possibly stand in X and one of them is not identical to itself.
2. $\Box \forall X_{\langle t_1, \dots, t_n \rangle} \Box \forall x_{t_1}^1 \dots \Box \forall x_{t_n}^n \Box (Xx^1 \dots x^n \rightarrow (x^1 = x^1 \wedge \dots \wedge x^n = x^n))$.

Identity Implies Being Identical.

1. Necessarily, if a thing is identical to itself, then it has the property of being identical to itself.
2. $\Box \forall x_t \Box (x = x \rightarrow \hat{y}(y = x)_{\langle t \rangle} x)$.

Being Identical Implies Instantiation.

1. Necessarily, if a thing has the property of being identical to itself, then the property of being identical to it is instantiated.

¹²From the standpoint of necessitist predicativists, the correct reply is that Noman does instantiate the property of being Noman. Thus, from their standpoint, the property of being Noman is instantiated, and so, contra Salmon, something is \emptyset_a Noman (by the thesis of Instantiation is Equivalent to Being Something, one of the premises of the argument for Thorough Serious Actualism – an argument offered in §3.3.3).

¹³The arguments given throughout this chapter are appropriately regimented in the language $M\hat{L}_p$, presented in §1.3.1. One important caveat concerning the argument for Thorough Serious Actualism is that the notion of higher-order identity that it appeals to is a primitive notion, not the defined notion introduced in §1.3.1. The reason for this is that the defined notion *presupposes* the truth of Higher-Order Serious Actualism (see fn. 9 of chapter 1). The thesis that if a higher-order entity is something, then there is some higher-order entity necessarily coextensive with it is very plausible, if not a truism. This claim and the version of Thorough Serious Actualism having identity as an undefined primitive together imply the version of Thorough Serious Actualism in which identity is a defined notion, defined in terms of necessary coextensiveness. Note that there is an ambiguity here. The expression for identity qua primitive notion is also being used for the defined notion. But this is unproblematic. The only case in which the identity symbol will stand for the primitive notion is in the Instantiation Argument for Thorough Serious Actualism. Of course, if identity between higher-order entities does boil down to necessary coextensiveness, then there turns out to be no ambiguity. In the dissertation I remain open with respect to whether identity between higher-order entities consists in nothing but necessary coextensiveness.

$$2. \quad \Box \forall x_t \Box (\hat{y}(y = x)_{\langle t \rangle} x \rightarrow I_{\langle \langle t \rangle \rangle} \hat{y}(y = x)).$$

Instantiation is Equivalent to Being Something.

1. Necessarily, the property of being identical to a thing is instantiated if and only if something is identical to that thing.
2. $\Box \forall x_t \Box (\exists y(y = x) \leftrightarrow I \hat{y}(y = x)).$

The argument goes as follows. Take any two (possible) things, say, Obama and Mars, and any relation between individuals, say, the relation of being 24×10^{-6} light years distant. By Relatedness Implies Identity, necessarily, if Obama is 24×10^{-6} light years distant from Mars, then Obama is identical to Obama and Mars is identical to Mars. By Identity Implies Being Identical, necessarily, if Obama is 24×10^{-6} light years distant from Mars, then Obama has the property of being identical to Obama and Mars has the property of being identical to Mars. From the Being identical Implies Instantiation assumption, it follows that necessarily, if Obama is 24×10^{-6} light years distant from Mars, then the property of being identical to Obama is instantiated, and that the property of being identical to Mars is instantiated. Finally, by Instantiation is Equivalent to Being Something, necessarily, if Obama is 24×10^{-6} light years distant from Mars, then Obama is (identical to) something and that Mars is (identical to) something. But Obama, Mars and the relation of being 24×10^{-6} light years distant were picked arbitrarily. So, Serious Actualism follows.

No doubt there will be theorists rejecting one or more premises of the above argument. Still, the availability of the argument burdens opponents of Thorough Serious Actualism with identifying the premises that they reject and arguing for their falsity in a non ad hoc manner. I will now consider what premises of the argument for Thorough Serious Actualism would be rejected by Salmon, and why.

The discussion will reveal some of the considerations underlying support for each of the premises. It will also reveal an inconsistency in Salmon's thought.

3.3.4 Noman and the Argument for Thorough Serious Actualism

Salmon infers that Noman has the property of being referred to by 'Noman' from the (putative) fact that 'Noman' refers to Noman. So, arguably, he would also not be averse to inferring that if a thing is identical to itself, then it has the property of being identical to itself. It is thus reasonable to think that Salmon would accept the thesis of Identity Implies Being Identical. This is a charitable interpretation of Salmon, since otherwise Salmon would be in the difficult position of having to explain why it is that from the fact that 'Noman' refers to Noman it follows that Noman has the property of being referred to, even though from the fact that a thing is identical to itself it does not follow that it has the property of being identical to itself.

It is also reasonable to think that Salmon would accept the premises Being Identical Implies Instantiation and Instantiation Is Equivalent to Being Something. In (Salmon, 1987, p. 64) he says the following:

'The sense or content of the second-order predicate (quantifier) 'something' is the property of classes of individuals of not being empty, the property of having at least one

element.'

Arguably, this shows that Salmon acknowledges that there is such a thing as the second-order property of being instantiated, and that Salmon takes this second-order property to be the one captured by the quantifier '∃'.

Since Salmon takes the second-order property of being instantiated to be the one captured by the quantifier '∃', he is committed to Instantiation is Equivalent to Being Something.

Moreover, suppose that x is identical to itself. By the thesis Identity Implies Being Identical, it follows that x has the property of being identical to itself. Since Salmon takes the second-order property of being instantiated to be the one captured by the quantifier '∃', he is committed to the being of this higher-order property. But then, the property of being identical to x has the higher-order property of being instantiated, since x has the property of being identical to x . Thus, necessarily, if x is identical to itself, then the property of being identical to x has the property of being instantiated. Therefore, Salmon is committed to Being Identical Implies Instantiation.

So, Salmon must reject the thesis of Relatedness Implies Identity, or else be committed to the truth of Serious Actualism. A rejection by Salmon of Relatedness Implies Identity would require Salmon to understand the scenario concerning 'Noman' as one in which it is not the case that Noman is identical to Noman, despite the fact that Noman has the property of being the referent of 'Noman'.

As just seen, Salmon is committed to Instantiation is Equivalent to Being Something. Moreover, he accepts that Noman has the property of being the referent of 'Noman'. Therefore, Salmon is committed to the property of being the referent of 'Noman' being instantiated. That is, Salmon is committed to something being the referent of 'Noman'.

Since Salmon refers approvingly to free quantified logic, it is reasonable to assume that he accepts the following theorems of free quantified logic:¹⁴

$$(8) \quad \forall x(x = x)$$

$$(9) \quad (\forall x(x = x) \wedge \exists x(\varphi)) \rightarrow \exists x(\varphi \wedge x = x)$$

From (8), (9) and the claim that something is the referent of 'Noman' it follows that something is the referent of 'Noman' and it is self-identical.

So, Salmon is committed to there being something that is the referent of 'Noman' and that thing being identical to itself. Given how Salmon's scenario is described, it is clear that he accepts that if anything is the referent of 'Noman', then Noman is the referent of Noman. Thus, Salmon is committed to Noman being identical to Noman after all.

Hence, Salmon appears to be committed to all the premises of the argument for Thorough Serious Actualism. Since Salmon rejects the truth of Serious Actualism, he has inconsistent commitments.

It is worth pointing out that Salmon explicitly disavows any commitment to the claim that something

¹⁴See (Salmon, 1987, p. 92).

is the referent of 'Noman'.¹⁵ But the fact that he disavows any such commitment does not suffice for this claim not to be a commitment of his. After all, according to Salmon, Noman has the property of being the referent of 'Noman', and he takes '∃' to consist in the property of being instantiated. Surely, if Noman has the property of being the referent of 'Noman', the property of being the referent of 'Noman' is instantiated.

It might be helpful to compare Salmon's views to those of noneists, since Salmon talks mostly in terms of 'exists' (instead of appealing to the existential quantifier). Noneists do not account for the property of existence in terms of the existential (particular) quantifier. According to them, the property of existence is not necessarily coextensive with the property of being something. Thus, noneists are free to hold that the property of being the referent of 'Noman' is instantiated – and thus that *something* is the referent of 'Noman' –, while simultaneously rejecting the claim that the referent of 'Noman' exists.

This option is unavailable to Salmon because he explicitly takes 'exists' to be definable in terms of the existential quantifier, and understands the existential quantifier as the second-order property of not being empty (as previously mentioned). Since Salmon accepts that Noman has the property of being the referent of 'Noman', he is committed to the nonemptiness of the property of being the referent of 'Noman'. A fortiori, something is the referent of 'Noman'.

Arguably, Salmon's intuitions concerning the nonbeing of the referent of 'Noman' are guided by the view that Noman does not exist. He takes Noman to be nothing because i) he takes Noman not to exist, and ii) he takes existence to be captured in terms of the existential quantifier (and identity). On the other hand, he is committed to something being the referent of 'Noman' because he accepts the claim that iii) Noman has the property of being the referent of 'Noman', and thus is committed to the claim that the property of being the referent of 'Noman' is instantiated. Consistency can be achieved by revising one of i)-iii). None of the revisions would, by itself, lead to a theory inconsistent with Serious Actualism.

Summing up, in this section I began by showing that, despite appearances, Noneism and the Principle of Independence are consistent with Serious Actualism. Properly understood, Serious Actualism turns out to be advocated by the generality of noneists.

Afterwards, Salmon's objection to Serious Actualism was considered. Possible replies to the objection on behalf of serious actualists were pointed out. My preferred reply consists in rejecting the view that 'Noman' has any referent whatsoever, because *Predicativism* about names, a view for which there is much independent support, and which I endorse, does not support the view that, in general, names refer. I noted that from the standpoint of contingentist predicativists endorsing serious actualism the correct verdict is that the property that is the semantic value of 'Noman' is not true of anything, even though it could have been true of something.

Then, I offered a positive argument in defence of Serious Actualism. Finally, I discussed how

¹⁵See (Salmon, 1987, p. 94).

Salmon might try to reject the argument's cogency. It was shown that Salmon appears to be committed to the truth of all the premises of the argument, and thus that he has inconsistent commitments.

3.4 Arguments for Propositional Necessitism

One strategy for arguing for Propositional Necessitism starts with a defence of a particular conception of propositions, their nature and identity conditions, presenting a case for view that entities whose nature and identity conditions are in accordance with the conception of propositions in question are necessary beings.

A different strategy relies on an appeal to features that propositions are accepted to have, showing that well established theories about those features imply that the things that have them are necessary beings. The arguments that I will be advancing here in defence of Propositional Necessitism fall under the second strategy. Thus, they do not rely on more controversial assumptions such as the assumption that propositions are structured complexes, or sets of possible worlds, or what not.

Assuming that the arguments for Propositional Necessitism that I will be presenting are sound, the truth of this thesis imposes constraints on the correctness of accounts of the nature and identity conditions of propositions. If those accounts are inconsistent with the necessary being of propositions, then they should be rejected.

3.4.1 A Blocked Route?

As I mentioned in the previous section, the view that propositions are abstract is part of the Classical Conception of propositions. Assuming that propositions are indeed abstract, there appears to be a route available for the truth of Propositional Necessitism. Abstract entities are typically assumed to have necessary being. After all, the main examples (if not the only examples) available of necessary beings, if indeed there are any necessary beings, consist of abstract entities, namely, numbers, sets and other mathematical entities. Since Propositional Necessitism follows from the claims that propositions are abstract and that abstract entities have necessary being, this is an easy route to the necessary being of propositions.

The view that all abstract entities have necessary being has been resisted. One common objection is that some impure sets are abstract, and yet have contingent being. The objection is based on the view that the members of sets are essential to them: necessarily, a set could not have been something and failed to have had some of its members.¹⁶

Consider the unit set $\{x : x \text{ is Obama}\}$. According to the objection, it is an essential property of this set that Obama belongs to it. That is, necessarily, if the set is something, then Obama belongs to it. This implies that necessarily, if this set is something, then Obama stands in the membership relation to it. By Serious Actualism it follows that necessarily, if $\{x : x \text{ is Obama}\}$ is something, then Obama is also something. Since Obama could have been nothing, it follows that $\{x : x \text{ is Obama}\}$

¹⁶An argument close to the one to be given for the contingency of sets on their members is articulated in (Fine, 1977, pp. 141-142).

could also have been nothing. So, some abstract things could have been nothing, for instance, the set $\{x : x \text{ is Obama}\}$.

Even though I find the essentiality of membership plausible, this is not the place to offer a defence of its truth. After all, the main aim of the present chapter is to offer a defence of Propositional Necessitism, and the essentiality of membership has been mentioned because it is an assumption of an argument for Propositional Necessitism.¹⁷

For the present purposes, what is relevant is that there is good reason to question the soundness of the argument from the abstractness of propositions to their necessary being. The arguments for Propositional Necessitism to be offered do not appeal to the claim that abstract entities are necessary beings.

3.4.2 The Truth-Values Argument

The first argument for Propositional Necessitism that will be offered is the *Truth-Values Argument*. This argument is not the main argument for Propositional Necessitism to be presented. The reason is that, on its own, its cogency can be resisted. The reason for presenting it anyway is that it has strong similarities to the stronger arguments for Propositional Necessitism yet to be offered, and to an argument of Plantinga's that will be discussed in §3.5.

I will begin by offering an argument for an instance of Propositional Necessitism. The argument's conclusion is the claim that the proposition that Obama is a president is necessarily something. The proposition that Obama is a president thus takes the role of an arbitrary proposition that is possibly something (note that if necessarily there are no propositions, then Propositional Necessitism is true; a counterexample to the truth of Propositional Necessitism requires that there could have been some proposition that could have been nothing).

Afterwards the premises of the argument for the necessary being of the proposition that Obama is a president will be generalised to the premises of the Truth-Values Argument for Propositional Necessitism.

¹⁷The thesis that the members of a set are essential to it does not follow from the axioms of ZFC. Yet, those axioms do preclude some natural alternative conceptions of sets on which the members of a set are not essential to it.

One such conception is an *intensional* conception of set. According to this conception, it is of the nature of sets to be the extension of (at least some) properties. For instance, according to this conception the set $\{x : x \text{ is a man}\}$ could have had more members than it actually has, since there could have been more men than the ones there actually are.

On the intensional conception of set the argument for the nonbeing of the set $\{x : x \text{ is Obama}\}$ in circumstances in which Obama is nothing would fail. Since in such circumstances nothing is Obama, the set $\{x : x \text{ is Obama}\}$ is empty at that world. Yet, the set is still something.

Let me offer an argument against the intensional conception. Suppose, absurdly, that the set $\{x : x \text{ is a man}\}$ could have had more members than the ones it actually has. Let h be an enumeration of all the men, and suppose that the cardinality of $\{x : x \text{ is a man}\}$ is n . Thus, $\{x : x \text{ is a man}\} = \{x : x = h(1) \text{ or } x = h(2) \text{ or } \dots \text{ or } x = h(n)\}$, by the axiom of extensionality. Consider now a circumstance w at which $\{x : x \text{ is a man}\}$ has more members than it actually has. Since $\{x : x \text{ is a man}\} = \{x : x = h(1) \text{ or } x = h(2) \text{ or } \dots \text{ or } x = h(n)\}$, it follows that, at w , $\{x : x = h(1) \text{ or } x = h(2) \text{ or } \dots \text{ or } x = h(n)\}$ has more members than it actually has. Thus, at w , there is some $o \in \{x : x = h(1) \text{ or } x = h(2) \text{ or } \dots \text{ or } x = h(n)\}$ such that $o \neq h(i)$, for all i such $1 \leq i \leq n$. But in such case, o does not satisfy the condition of being an x such that $x = h(1) \text{ or } x = h(2) \text{ or } \dots \text{ or } x = h(n)$. So o does not belong to the set $\{x : x = h(1) \text{ or } x = h(2) \text{ or } \dots \text{ or } x = h(n)\}$. Contradiction. Hence, it is not the case that $\{x : x \text{ is a man}\}$ could have had more members than the ones it actually has.

Let p stand for the proposition that Obama is a president, assuming that this proposition is possibly something. It will be argued that the following claim is true:

(NecPropObama)

1. Necessarily, the proposition that Obama is a president is something.
2. $\Box(\exists q(p = q))$.

The premises of the argument for the necessary being of the proposition that Obama is a president are the following:

(P1-TV_{Ai})

1. Necessarily, Obama is a president or Obama is not a president..
2. $\Box(p \vee \neg p)$.

(P2-TV_{Ai})

1. Necessarily, if Obama is a president, then it is true that Obama is a president.
2. $\Box(p \rightarrow Tp)$.

(P3-TV_{Ai})

1. Necessarily, if Obama is not a president, then it is false that Obama is a president.
2. $\Box(\neg p \rightarrow Fp)$

(P4-TV_{Ai})

1. Necessarily, if it is true that Obama is a president, then the proposition that Obama is a president is something.
2. $\Box(Tp \rightarrow \exists q(p = q))$

(P5-TV_{Ai})

1. Necessarily, if it is false that Obama is a president, then the proposition that Obama is a president is something.
2. $\Box(Fp \rightarrow \exists q(p = q))$.

One important remark is that ‘it is true that’ and ‘it is false that’ are here understood as properties of propositions, i.e., as standing for entities of type $\langle\langle\rangle\rangle$. Thus, they do not stand for properties of sentences, nor of any other individuals. That is, they do not stand for entities of type $\langle e \rangle$, since on the typology of entities being presupposed, propositions are not individuals, but instead higher-order entities.¹⁸ The argument from (P1-TV_{Ai})-(P5-TV_{Ai}) to (NecPropObama) goes as follows.

Premises (P1-TV_{Ai}), (P2-TV_{Ai}) and (P3-TV_{Ai}) together imply:

- (10) 1. Necessarily, it is true that Obama is a president or it is false that Obama is a president.
2. $\Box(Tp \vee Fp)$

¹⁸A defence of higher-order resources has been offered in chapter 1.

Moreover, (10), (P4-TVAi) and (P5-TVAi) together imply (NecPropObama).

The premises of the Truth-Values Argument are the following:¹⁹

Premises of the Truth-Values Argument

(P1-TVA) Excluded Middle .

1. Necessarily, for every p , necessarily, p or $\neg p$.
2. $\Box \forall p \Box (p \vee \neg p)$

(P2-TVA) Truth Introduction.

1. Necessarily, for every p , necessarily, if p , then it is true that p .
2. $\Box \forall p \Box (p \rightarrow Tp)$

(P3-TVA) Falsity Introduction .

1. Necessarily, for every p , necessarily, if $\neg p$ then p has the property of being false.
2. $\Box \forall p \Box (\neg p \rightarrow Fp)$

(P4-TVA) Thorough Serious Actualism.

Premise (P1-TVAi) is an instance of Excluded Middle, (P2-TVAi) is an instance of Truth Introduction, (P3-TVAi) is an instance of Falsity Introduction. Finally, (P4-TVAi) and (P5-TVAi) are both instances of Thorough Serious Actualism. It should be clear that the truth of Propositional Necessitism follows from (P1-TVA) - (P4-TVA), given the reasoning presented in the argument for the truth of (NecPropObama).

The weakest assumption of the Truth-Values Argument, in the sense of being the least controversial, is Excluded Middle. Every instance of the schema $\varphi \vee \neg \varphi$ is a propositional tautology, and thus every instance of the schema $\Box(\varphi \vee \neg \varphi)$ is a theorem of the very weak propositional modal logic **K**. Moreover, the fact that every instance of $\Box(\varphi \vee \neg \varphi)$ is a theorem of **K** is no accident owing to the lack of expressive resources of **K**. Instances of the schema are true no matter what possible proposition turns out to be the semantic value of φ . This means that necessarily, for every p , necessarily, p or $\neg p$. That is, Excluded Middle is true.

Thorough Serious Actualism was defended in §3.3. Discussion of Truth Introduction and Falsity Introduction will be left for §3.5. Suffice it to say for now that I think that propositional contingentists

¹⁹The argument does not require the full strength of Thorough Serious Actualism. Rather, the following thesis would suffice:

Serious Actualism_{<{}>}

1. Necessarily, for every property X of propositions, necessarily, for every proposition p , necessarily, if p has X , then p is something.
2. $\Box \forall X \Box \forall p \Box (Xp \rightarrow \exists q(p = q))$.

The same remark applies to the remaining arguments to be considered in this section.

The reason for appealing to the full strength of Thorough Serious Actualism concerns the fact that later it will be queried what would the status be if modal expressions were seen as being analysable in terms of truth at a world, instead of constituting themselves predications of properties to propositions. It will be shown that the Possibility Argument, yet to be presented, is still valid under such understanding of modal expressions. Yet, this would not be so if the Possibility Argument were formulated in terms of Serious Actualism_{<{}>} instead of being formulated in terms of Thorough Serious Actualism.

have interesting objections to Truth Introduction and Falsity Introduction. Given the other arguments for Propositional Necessitism, these objections only have a local impact, since they are not applicable to the remaining arguments.

3.4.3 The Possibility Or Impossibility Argument

Whereas the previous argument appealed to principles governing truth and falsity, the argument to be presented, the *Possibility Or Impossibility Argument*, appeals to principles governing alethic modalities. As before, I will start by presenting an argument for the necessary being of the proposition that Obama is a president. Afterwards, this argument will be generalised to the premises of the Possibility Or Impossibility Argument for Propositional Necessitism.

The premises of the argument for the necessary being of the proposition that Obama is a president are the following:

(P1-PIAi)

1. Necessarily, it is possible that Obama is a president or it is impossible that Obama is a president.
2. $\Box(\Diamond p \vee \blacksquare p)$

(P2-PIAi)

1. Necessarily, if it is possible that Obama is a president, then the proposition that Obama is a president is something.
2. $\Box(\Diamond p \rightarrow \exists q(p = q))$

(P3-PIAi)

1. Necessarily, if it is impossible that Obama is a president, then the proposition that Obama is a president is something.
2. $\Box(\blacksquare p \rightarrow \exists q(p = q))$

The symbol '■' is the formal language analogue of 'it is impossible that'. One important remark is that 'it is possible that', 'it is impossible that' and 'it is necessary that' are here understood as properties of propositions, i.e., as standing for entities of type $\langle\langle\rangle\rangle$. Thus, they do not stand for properties of sentences, nor of any other individuals.

The truth of (NecPropObama) is an immediate consequence of premises (P1-PIAi), (P2-PIAi) and (P3-PIAi). The Possibility Or Impossibility Argument has two premises, namely:

Premises of the Possibility Or Impossibility Argument:

(P1-PIA) Possibility Or Impossibility.

1. Necessarily, for every p , necessarily, it is possible that p or it is impossible that p .
2. $\Box\forall p\Box(\Diamond p \vee \blacksquare p)$

(P2-PIA) Thorough Serious Actualism.

Premise (P1-PIAi) is an instance of Possibility Or Impossibility. Premises (P2-PIAi) and (P3-PIAi) are both instances of Thorough Serious Actualism.

It should be clear that Propositional Necessitism follows from Possibility Or Impossibility together with Thorough Serious Actualism, given the reasoning of the argument from (P1-PIAi), (P2-PIAi) and (P3-PIAi) to (NecPropObama).

Thorough Serious Actualism was defended in §3.3. Let me turn to the thesis of Possibility Or Impossibility.

Propositional modal logic contains no operator for impossibility. But it is not difficult to see how one may be added to it. The following axiom-schema is added to whatever system one is interested on:

Axiom Schema I. $\blacksquare\varphi \leftrightarrow \neg\Diamond\varphi$.

Adding axiom schema **I** to the propositional modal logic **K** yields the system **K+I**. All instances of the schema $\Box(\Diamond\varphi \vee \blacksquare\varphi)$ are theorems of **K+I**. To see why, note that every instance Excluded Middle, and thus of the schema $\Box(\Diamond\varphi \vee \neg\Diamond\varphi)$, is a theorem of **K**. From $\Box(\Diamond\varphi \vee \neg\Diamond\varphi)$ and **I** it straightforwardly follows that $\Box(\Diamond\varphi \vee \blacksquare\varphi)$, by reasoning valid in **K**.

As in the discussion of Excluded Middle, the truth of every instance of the schema $\Box(\Diamond\varphi \vee \blacksquare\varphi)$ is not a result of a poverty of expressive resources. So, necessarily, for every p , it is possible that p or it is impossible that p . That is, Possibility Or Impossibility is true.

Arguably, this is the simplest of the arguments to be offered from facts concerning metaphysical modalities and Thorough Serious Actualism to Propositional Necessitism, in that it does not appeal to considerations of any other nature. It will be helpful to see two other arguments from facts concerning metaphysical modalities and Thorough Serious Actualism to Propositional Necessitism. The distinctive feature of these arguments is that they appeal to an extra assumption about propositions, namely, that if a proposition is something, then its contradictory is also something. This extra assumption also enables the formulation of an alternative to the Truth-Values Argument. The alternatives to the Truth-Values and the Possibility or Impossibility arguments are presented in what follows.

3.4.4 Alternative Arguments: The Truth Argument

Some will find the Truth-Values Argument and the Possibility or Impossibility Argument objectionable on the grounds that there are no properties of falsity or impossibility. To say that a proposition p is false is not really to attribute a property to it. Rather it is to say that it is not true that p . Similarly, to say that a proposition p is impossible is not really to attribute a property to it. Rather, it is to say that it is not possible that p .

Alternative arguments will now offered. These arguments do not appeal to theses about the properties of falsity or of impossibility. Instead, the alternative arguments all appeal to the following thesis:

Contradictoriness.

1. Necessarily, for every proposition p , $\neg p$ is something.
2. $\Box \forall p \exists q (\neg p = q)$.

The alternative to the Truth-Values Argument is the *Truth Argument*. Its premises are the following:

Premises of the Truth Argument

(P1-TA) Excluded Middle.

(P2-TA) Truth Introduction.

(P3-TA) Thorough Serious Actualism.

(P4-TA) Contradictoriness.

As before, I will focus on showing that instances of these premises imply (NecPropObama). It will be clear from the argument offered that (P1-TA), (P2-TA), (P3-TA) and (P4-TA) together imply Propositional Necessitism.

The instances of (P1-TA) - (P4-TA) required to establish the truth of (NecPropObama) are the following:

(P1-TAi)

1. Necessarily, Obama is a president or Obama is not a president.
2. $\Box (p \vee \neg p)$.

(P2-TAi)

1. Necessarily, if Obama is a president, then it is true that Obama is a president.
2. $\Box (p \rightarrow Tp)$.

(P3-TAi)

1. Necessarily, if Obama is not a president, then it is true that that Obama is not a president.
2. $\Box (\neg p \rightarrow T\neg p)$.

(P4-TAi)

1. Necessarily, if it is true that Obama is a president, then the proposition that Obama is a president is something.
2. $\Box (Tp \rightarrow \exists q (p = q))$.

(P5-TAi)

1. Necessarily, if it is true that Obama is not a president, then the proposition that Obama is not a president is something.
2. $\Box (T\neg p \rightarrow \exists q (\neg p = q))$.

(P6-TAi)

1. Necessarily, the proposition that Obama is not a president is something only if the proposition that it is not the case that Obama is not a president is something.

$$2. \quad \Box(\exists q(\neg p = q) \rightarrow \exists q(\neg\neg p = q)).$$

Premise (P1-TAi) is an instance of Excluded Middle, and (P2-TAi) is an instance of Truth Introduction. Premise (P3-TAi) is an instance of Truth Introduction only if the proposition that Obama is not a president is possibly something. But this follows from i) the assumption that the proposition that Obama is a president is possibly something, and ii) Contradictoriness. Premise (P4-TAi) is an instance of Thorough Serious Actualism. Premise (P5-TAi) turns out to also be an instance of Serious Actualism, since the proposition that Obama is not a president is possibly something. Finally, premise (P6-TAi) is a consequence of Contradictoriness.

The argument for (NecPropObama) goes as follows. From (P1-TAi) and (P2-TAi) it follows that:

- (11) a. Necessarily, it is true that Obama is a president or Obama is not a president.
 b. $\Box(Tp \vee \neg p)$.

Moreover, (11) and (P4-TAi) together imply

- (12) a. Necessarily, the proposition that Obama is a president is something or Obama is not a president.
 b. $\Box(\exists q(p = q) \vee \neg p)$.

From (12) and (P3-TAi) it follows that:

- (13) a. Necessarily, the proposition that Obama is a president is something or it is true that Obama is not a president.
 b. $\Box(\exists q(p = q) \vee T\neg p)$.

Claim (13) together with (P5-TAi) implies

- (14) a. Necessarily, the proposition that Obama is a president is something or the proposition that Obama is not a president is something.
 b. $\Box(\exists q(p = q) \vee \exists q(\neg p = q))$

A relevant remark at this point is that the following claim is a consequence of using the identity symbol as shorthand for their necessary coextensiveness:

- (15) a. Necessarily, if the proposition that it is not the case that Obama is not a president is something, then the proposition that Obama is a president is something.
 b. $\Box\exists q(\neg\neg p = q) \rightarrow \exists q(p = q)$

From (P6-TAi) and (15) it follows that

- (16) a. Necessarily, if the proposition that Obama is not a president is something, then the proposition that Obama is a president is something.
 b. $\Box(\exists q(\neg p = q) \rightarrow \exists q(p = q))$.

Finally, (14) and (16) together imply (17):

- (17) a. Necessarily, the proposition that Obama is a president is something or the proposition that Obama is a president is something.
 b. $\Box(\exists q(p = q) \vee \exists q(p = q))$.

Since (17) is equivalent to (NecPropObama), (NecPropObama) follows from (P1-TAi) - (P6-TAi). It should be clear that Propositional Necessitism follows from the premises of the Truth Argument, given the argument from (P1-TAi) - (P6-TAi) to Propositional Necessitism just presented.

3.4.5 Alternative Arguments: The Possibility Or Necessity Argument

Two alternatives to the Possibility Or Impossibility Argument are considered. The first alternative to be considered, the *Possibility Or Necessity Argument*, has the following thesis as one of its assumptions:

Possibility Or Necessity.

1. Necessarily, for every proposition p , necessarily, p is possible or $\neg p$ is necessary.
2. $\Box\forall p\Box(\Diamond p \vee \Box\neg p)$.

The premises of the argument are the following:

Premises of the Possibility Or Necessity Argument

(P1-PNA) Possibility Or Necessity.

(P2-PNA) Serious Actualism.

(P3-PNA) Contradictoriness.

The Possibility Or Necessity Argument proceeds in a fashion similar to the Truth Argument, and so the details will be left out.

The thesis of Possibility Or Necessity is supported on grounds similar to the ones advanced in defence of Excluded Middle. Every instance of the schema

$$(18) \quad \Box(\Diamond\varphi \vee \Box\neg\varphi)$$

is a theorem of **K**. Moreover, this is no accident due to the lack of expressive resources of **K**. Instances of the schema are true no matter what possible proposition turns out to be the semantic value of φ . Hence, Possibility Or Necessity is true.

Despite the fact that every instance of (18) is an instance of **K**, some may be opposed to there being a property of necessity, perhaps on the grounds that if there were such a property, it would not be a natural property, since it would be defined in terms of the property of possibility. If find this objection to the Possibility or Necessity argument unappealing. For instance, proponents of such view would be faced with the burden of explaining what decides in favour of possibility to the detriment of necessity (or vice-versa). But even if they could make good sense of the objection, they would still be faced with an argument formulated solely in terms of the property of possibility, the *Possibility Argument*, to which I will now turn.

3.4.6 Alternative Arguments: The Possibility Argument

The only difference between the Possibility and the Possibility Or Necessity Argument is that the Possibility Argument has as a premise the thesis of *Seriality* instead of the thesis of Possibility Or Necessity.

Seriality.

1. Necessarily, for every p , necessarily, it is possible that p or it is possible that $\neg p$.
2. $\Box \forall p \Box (\Diamond p \vee \Diamond \neg p)$.

The premises of the Possibility Argument are thus the following:

Premises of the Possibility Argument

(P1-PA) Seriality.

(P2-PA) Serious Actualism.

(P3-PA) Contradictoriness.

The Possibility Argument proceeds in a fashion somewhat similar to the Truth Argument, and so the details will be left out. Contrary to the theses of Possibility Or Impossibility and Possibility Or Necessity, the thesis of Seriality is not supported by system **K**. Rather, it is supported by the very weak normal modal propositional logic **KD**. This logic results from adding to **K** all instances of the following schema:

Axiom schema D. $\Box \varphi \rightarrow \Diamond \varphi$

In the context of **K**, each instance $\Box \psi \rightarrow \Diamond \psi$ of axiom schema **D** turns out to be equivalent to an instance of the following schema:

$$(19) \quad \Diamond \varphi \vee \Diamond \neg \varphi$$

This means that **KD** is also the system that results from adding to **K** the schema (19).

The truth of each instance of (19) is extremely plausible given the interpretation of ‘ \Diamond ’ as meta-physical possibility. All that is required for each instance of (19) to be true is that necessarily, things could have been some way or another. Take any possible circumstance w . If, at w , things could had been some way w' , then φ would have been true at w' or $\neg \varphi$ would have been true at w' , by Excluded Middle. Thus, at w , it is possible that φ or it is possible that $\neg \varphi$.

Is it the case that, for each possible circumstance w , things could have been some way or another? Yes, since at each possibility w , things could at least have been as they are in w (this just consists in the observation that, necessarily, $p \rightarrow \Diamond p$). Let me now turn to the justification for the assumption of Contradictoriness.

Consider the following schema:

- (20) If you are consistent and believe that φ , then there is something that you do not believe, namely, that $\neg\varphi$.

Arguably, the truth of every instance of this schema is supported by unreflective common sense. The truth of every instance of this schema presupposes the truth of its universal generalisation:

- (21) For every proposition p , if you are consistent and believe that p , then there is something that you do not believe, namely, you do not believe that $\neg p$.

If (21) codifies a constraint on what it is to be consistent, as it appears to do, then its necessitation is true. Since the necessitation of (21) implies Contradictoriness, then, arguably, Contradictoriness is one of the commitments of unreflective common sense.

This argument presupposes that propositional quantification is appropriate in this context, rather than quantification over propositions understood as individuals, i.e., as entities of type e . Such understanding of propositions is a presupposition of this chapter. As mentioned in §3.1, propositions are here understood as entities of type $\langle \rangle$, and as the objects of the attitudes. If it turns out that there could be no propositions as these are here understood, then Propositional Necessitism is vacuously true.

I suspect that there are many other commitments of unreflective common sense supporting Contradictoriness. Anyway, the commitment to Contradictoriness is, on its own, independent of Propositional Necessitism. For instance, the Russellian theory of propositions, one of the theories falling under the Classical Conception, appears to imply Contradictoriness, given natural auxiliary assumptions.

According to the Russellian theory, propositions are structured entities, containing other entities as their constituents. For instance, according to standard Russellians, the proposition that Obama is a president is composed of Obama and the property of being a president.

An argument from the Russellian theory to Contradictoriness appeals to the plausible assumption that if the constituents of a structured proposition are all something, then the proposition itself is something. It is also plausible to think that the operation of negation is necessarily something. So, if a proposition p is something, then any proposition that has p and the operation of negation as its only constituents is also something. On the structured accounts of propositions, one such proposition is the proposition that $\neg p$. Thus, necessarily, for every proposition p , if p is something then $\neg p$ is something.

Thus, arguably, Contradictoriness is supported by unreflective common sense, and it is implied by some of the theories committed to the falsity of the Classical Conception of propositions. Contradictoriness is thus an assumption shared with at least some propositional contingentists.

Finally, independently of what positive support there is for Contradictoriness, the fact that the thesis is independent of Propositional Necessitism, would make it a surprising result if the best line of reply to an argument for Propositional Necessitism consisted in rejecting the truth of Contradictoriness.

This concludes the presentation of the arguments for Propositional Necessitism. In the next section objections to the arguments are considered, and replies to these objections are offered. I will be referring to the Possibility Or Impossibility Argument, Possibility Or Necessity Argument and Possibility Argument as the *modal arguments* for Propositional Necessitism.

3.5 Objections To The Arguments

3.5.1 Plantinga's Argument and the Truth-Values Argument

The Truth-Values Argument is quite similar to an argument for the falsity of Existentialism put forward by Plantinga (1983). If Plantinga's argument is cogent, then it happens to have important consequences for the status of the Classical Conception of propositions. As shown in §3.2, Existentialism is a straightforward consequence of Essential Aboutness and Thorough Serious Actualism. This means that, together, Plantinga's argument and the defence of Thorough Serious Actualism offered in this chapter constitute an argument for the falsity of the Classical Conception of Propositions.

The similarity between the Truth-Values Argument and Plantinga's argument reveals a weakness in the Truth-Values Argument. Plantinga's argument for the falsity of Existentialism has been objected to on the grounds that it ambiguates between two senses of the modalities, weak and strong senses, otherwise relying on an assumption that is not common ground between him and many existentialists.²⁰ The same objection turns out to apply to the Truth-Values Argument. For this reason it will be helpful to consider Plantinga's argument for the falsity of Existentialism, and the ambiguity objection to that argument.²¹

The premises of Plantinga's argument against Existentialism are the following:

Premises of Plantinga's Argument

(P1-PIA) Obama could have been nothing.

(P2-PIA) The proposition that Obama is nothing is about Obama.

(P3-PIA) Necessarily, if Obama is nothing, then it is true that Obama is nothing.

(P4-PIA) Necessarily, if it is true that Obama is nothing, then the proposition that Obama is nothing is something.

Premise (P1-PIA) is supported by unreflective common sense. Premise (P2-PIA) is intended to witness the claim that some propositions are about contingent beings, a claim that Plantinga takes to be common ground between him and many supporters of Existentialism. Premise (P3-PIA) is an instance of Truth Introduction, and premise (P4-PIA) is an instance of Thorough Serious Actualism.

²⁰Defences of Existentialism against Plantinga's argument similar to the one to be presented are adopted by, among others, Adams (1981), Fine (1977), Fine (1977), Speaks (2012), Stalnaker (2012).

²¹Actually, the argument to be considered is a slight reconstruction of Plantinga's. The main difference is that the argument to be presented contains Truth Introduction as one of its premises, whereas Truth Introduction is not a premise of Plantinga's Argument. Instead, a consequence of Truth Introduction plays the role of Truth Introduction in Plantinga's Argument. This formulation of the argument has been chosen to make clearer the similarities between Plantinga's Argument and the Truth-Values Argument.

Briefly, Plantinga's Argument proceeds as follows. Premises (P1-PIA) and (P3-PIA) jointly imply (22):

(22) It could have been that both Obama was nothing and the proposition that Obama is nothing was true.

Moreover, (22) and (P4-PIA) imply (23):

(23) It could have been that both Obama was nothing and the proposition that Obama is nothing was something.

Also, (23) and (P2-PIA) imply (24):

(24) There could have been a proposition p about some x such that it could have been that p was something and x was nothing.

Thus, (P1-PIA) - (P4-PIA) jointly imply the falsity of Existentialism. According to the objection to Plantinga's Argument to be considered there is no reading of 'necessity' and 'possibility' such that: i) premises (P1-PIA) and (P3-PIA) are both true, and ii) Plantinga's Argument is valid.

The two senses of the modalities are distinguishable via two notions of truth relative to a world, namely, truth *in* a world and truth *at* (or *of*) a world.²² As will be seen, virtually the same objection can be applied to the cogency of the Truth-Values Argument. Let me call this objection to both Plantinga's Argument and the Truth-Values Argument the *Truth In-Truth At Objection*. I will begin by introducing the distinction between truth in a world and truth at a world. Then, I will show how the distinction affords the resources to object to the cogency of Plantinga's argument. I will also show that, for exactly the same reasons, the distinction affords the resources to object to the cogency of the Truth-Values Argument.

One instructive way to understand the difference between the two notions is as follows. It is true *at* a possible world w that, say, Obama is a president only if a certain relation *actually* holds between the possible world w and the proposition that *Obama is a president*. However, it is true *in* a possible world w that, say, Obama is a president only if the proposition that Obama is a president would have had a certain property had world w been realised, namely, the property of being true.²³

The important difference is thus that for a proposition p to be true *in* a world w , p is required to have a property *at* w , namely, the property of being true, whereas for a proposition to be true *at* a world w , it is not required that p has any property *at* w . Put it another way, if p is true *in* w , then not only is it the case that proposition p is true *at* w . It is also the case that the proposition that p is true is

²²The truth in-truth at distinction first appears in the first and second sophisms in (Buridan, 2001, ch. 8), the basis for Prior (1969)'s distinction between possibility and possible truth.

²³Some will prefer to use 'actual' instead of 'realised', since it does not beg any questions with respect to the nature of possible worlds. On the other hand, this use of 'actual' seems to require a reading of 'actual' which is neither indexical nor rigid, whereas 'actual' is used in the dissertation mostly with its rigid sense. Since, as I see it, the relevant reading of 'actual' is, in this context, the one captured by 'is realised', I will be using 'realised' instead of 'actual' to capture this nonindexical and nonrigid reading of 'actual'.

also true at w . The following is perhaps a helpful image. If an actual proposition p is true at a world w , then p characterises w from the standpoint of the actual world. If a proposition p is true in w , then p characterises w from the standpoint of w .

Briefly, proponents of the truth in-truth at distinction have available an account of what it is for the proposition that p is true to be true at a world w . If the proposition that p is true is true at w , then it is also the case that it is true at w that p . And if p is true at w and the proposition that p is something is true at w , then the proposition that p is true is true at w . That is, the proposition that p is true is true at w if and only if it is true at w that i) p and ii) p is something. Thus, p is true in w if and only if it is true at w that i) p and ii) p is something.

Accompanying the distinction between truth in a world and truth at a world is a distinction between weak modalities and strong modalities. A proposition p is weakly necessary if and only if, for every world w , p is true at w . A proposition is strongly necessary if and only if, for every world w , p is true in w . A proposition is weakly possible if and only if, there is some world w such that p is true at w . A proposition p is strongly possible if and only if there is some world w such that p is true in w .

The Truth In-Truth At Objection relies on the thought that some propositions are true at w even though they are not true in w . These propositions are not true in w because if w were realised, then they would have been nothing. This would have been so despite the fact these propositions appropriately characterise w from the standpoint of the actual world.

Suppose, for the purposes of the example, that there could have been no proposition whatsoever, even though there actually are some propositions – and in particular the proposition that there are no propositions is actually something. Assuming that there could have been no propositions, there is a possible world w such that the proposition that there are no propositions is true at w . Yet, the proposition that there are no propositions is not true in w , since it is not the case that it is true at w that the proposition that there are no proposition is something.

The distinction between weak and strong modalities enables proponents of the Truth In-Truth At Objection to account for the intuition that all of the assumptions of Plantinga's Argument are true, while at the same contesting the argument's cogency. There are two readings of premise (P1-PIA), namely:

- (25) a. There is some possible world w such that it is true at w that Obama is nothing.
 b. There is some possible world w such that it is true in w that Obama is nothing.

The intuition that (P1-PIA) is true arises from the fact that (25-a) is indeed true, since the actual proposition that Obama is nothing is true at, or of, some possible world w . It correctly characterises some world w . Yet, it is false that there is some possible world w such that it is true in w that Obama is nothing, since, according to them, there is no possible world w such that both i) it is true at w that Obama is nothing and ii) it is true at w that the proposition that Obama is nothing is something.

Consider the following readings of (P3-PIA):

- (26) a. For every world w , if it is true in w that Obama is nothing, then it is true at w that Obama

- is nothing.
- b. For every world w , if it is true *in* w that Obama is nothing, then it is true *in* w that Obama is nothing.
 - c. For every world w , if it is true *at* w that Obama is nothing, then it is true *at* w (and *in* w) that the proposition that Obama is nothing is true.

The intuition that (P3-PIA) is true arises from the fact that (26-a) and (26-b) are indeed true. Yet, (26-c) is false. The fact that, as characterised from our world, w is a world at which Obama is nothing, does not imply that w is a world in which the proposition that Obama is nothing is itself something at it.

Since the only true reading of (P1-PIA) is, according to the objection, (25-a), consider how the argument proceeds on this reading of (P1-PIA). Premises (P1-PIA) and (P3-PIA) together imply (22). The only reading of (P3-PIA) and (22) on which the argument from (25-a) and (P3-PIA) to (22) is valid is when (P3-PIA) is understood as (26-c) and (22) is understood as follows:

- (27) There is some possible world w such that it is true *at* w that Obama is nothing and it is true *at* w (and *in* w) that the proposition that Obama is nothing is true.

But, according to the proponents of the Truth In-Truth At Objection, (26-c) is false. Thus, according to the objection, Plantinga's Argument is valid only if one of the premises is false. A fortiori, Plantinga's Argument is not cogent.

It is important to bear in mind that the Truth In-Truth At Objection is not aimed to establish the truth of Existentialism. Rather, it is aimed to show that the argument is successful only if claims that are not common ground between Plantinga and many Existentialists are assumed to be true, namely, at least one of (25-b) and (26-c). Let me now briefly show how the distinction between weak and strong modalities enables the formulation of a reply to the Truth-Values Argument accounting for the intuitions in favour of the truth of each premise, and yet on which the argument is not cogent.

The distinction between truth in a world and truth at a world shows that Excluded Middle, Truth Introduction and Falsity Introduction have four possible readings each. I will begin by focusing only on one particular instance of Excluded Middle and Truth Introduction, namely, claims (P1-TVAi) and (P2-TVAi). Claim (P1-TVAi) may be understood in one of the following ways:

- (28) Every world w is such that it is true relative to w that Obama is a president or it is true relative to w that Obama is not a president.
- a. Every world w is such that it is true *at* w that Obama is a president or it is true *at* w that Obama is not a president.
 - b. Every world w is such that it is true *in* w that Obama is a president or it is true *in* w that Obama is not a president.
 - c. Every world w is such that it is true *at* w that Obama is a president or it is true *in* w that Obama is not a president.

- d. Every world w is such that it is true in w that Obama is a president or it is true at w that Obama is not a president.

Of these readings, the objector takes (28-b) to be immediately false. After all, it requires that every possible world be such that it is true at it that the proposition that Obama is a president has the property of being true, or it is true at it that the proposition that Obama is not a president has the property of being true. Hence, by Thorough Serious Actualism, it requires that every possible world w be such that it is true at w that the proposition that Obama is a president is something, or it is true at w that the proposition that Obama is not a president is something. But the objector rejects this. Since both propositions are about Obama, none of them is something if Obama is nothing.

Moreover, according to the objector there are possible worlds w such that it is true at w that Obama is nothing. It is true at no such possible world that Obama is a president, and it is true at no such possible world that the proposition that Obama is a president is true (if it were true at w that the proposition that Obama is a president is true, then it would be true at w that the proposition that Obama is a president is something, in which case Obama would have been something and yet the proposition that Obama is a president, a proposition about Obama, would have been nothing, thus contradicting Existentialism). So, there are possible worlds w such that it is not true at w that Obama is a president and it is not true in w that Obama is not a president. So, according to the objection, (28-c) is also false. This means that the only options available are (28-a) and (28-d).

Claim (P2-TV_{Ai}) has the following readings:

- (29) a. Every world w is such that if it is true at w that Obama is a president, then it is true at w that the proposition that Obama is a president is true.
- b. Every world w is such that if it is true at w that Obama is a president, then it is true in w that the proposition that Obama is a president is true.
- c. Every world w is such that if it is true in w that Obama is a president, then it is true at w that the proposition that Obama is a president is true.
- d. Every world w is such that if it is true in w that Obama is a president, then it is true in w that the proposition that Obama is a president is true.

The objector rejects the truth of reading (29-a) because a proposition p may be true at a world w without it being true at w that p has the property of being true, since p may be nothing at w . Since (29-b) implies (29-a), the objector also rejects (29-b).

The options available are thus (29-c) and (29-d). From (28-a) and either (29-c) or (29-d) it does not follow that every world w is such that it is true at w that the proposition that Obama is a president is true or it is true at w that Obama is not a president. Thus, the reading of (P1-TV_{Ai}) as (28-a) renders the Truth-Values Argument invalid (on the assumption that (P2-TV_{Ai}) is given a true reading, i.e., either (29-c) or (29-d)). When (P1-TV_{Ai}) is understood as (28-a) and (P2-TV_{Ai}) is understood as either (29-c) or (29-d), (10) does not follow from (P1-TV_{Ai}), (P2-TV_{Ai}) and (P3-TV_{Ai}), contrary to what is required for the validity of the Truth-Values Argument.

So, the only option available is to adopt (28-d) as the true reading of Excluded Middle, and to take as true readings of Truth Introduction (29-c) or (29-d).

Finally, (P3-TV_{Ai}) has the following readings:

- (30)
- a. Every world w is such that if it is true at w that Obama is not a president, then it is true at w that the proposition that Obama is a president is false.
 - b. Every world w is such that if it is true at w that Obama is not a president, then it is true in w that the proposition that Obama is a president is false.
 - c. Every world w is such that if it is true in w that Obama is not a president, then it is true at w that the proposition that Obama is a president is false.
 - d. Every world w is such that if it is true in w that Obama is not a president, then it is true in w that the proposition that Obama is a president is false.

By reasoning similar to the one applied with respect to the readings of (P2-TV_{Ai}) it is easy to see that, according to the objector, the only available readings of (P3-TV_{Ai}) are (30-c) and (30-d).

But from (28-d), either one of (30-c) and (30-d), and either one of (29-c) and (29-d) it does not follow that every world w is such that it is true at w that the proposition that Obama is a president is true or it is true at w that the proposition that Obama is a president is false. The most one can get is that it is true at w that the proposition that Obama is a president is true or it is true at w that the Obama is not a president.

This claim is innocuous from the standpoint of propositional contingentists. What follows from it is just that every world w is such that it is true at w that the proposition that Obama is a president is something, or it is true at w that Obama is not a president. This last claim does not imply the necessary being of the proposition that Obama is a president. Moreover, it does not conflict with the truth of Existentialism.

It should be clear that the truth in-truth at distinction affords the resources for a similar objection to the Truth Argument. The overall conclusion is that the truth in-truth at distinction offers a promising way to resist the cogency of both the Truth-Values Argument and the Truth Argument, in the same way that it enables Existentialists to resist the cogency of Plantinga's Argument. In what follows I will show that, on its own, the truth in-truth at distinction does not afford propositional contingentists with the resources to resist the modal arguments for Propositional Necessitism.

3.5.2 The Truth In-Truth At Distinction and the Modal Arguments

In what follows it will be shown that one of the arguments for Propositional Necessitism previously presented, the Possibility Argument, is unscathed by the common way of understanding the modal operators in terms of the common account of truth relative to a world. This will be shown by considering, without loss of generality, particular instances of each one of the assumptions of the Possibility Argument, namely, assumptions from which (NecPropObama) follows.

Readings of the premises of this argument for (NecPropObama) will be presented in terms of

the notion of truth at a world. More specifically, only weak readings of modal expressions will be considered, ones giving rise to the following semantic account of ‘necessity’ and ‘possibility’, to which I will be calling the ‘Truth At Account’:

Truth At Account

1. Possibility
 - $\llbracket \ulcorner \Diamond \varphi \urcorner \rrbracket$ is the proposition that $\llbracket \varphi \rrbracket$ is true at some world w .
 - $\llbracket \ulcorner \Diamond \varphi \urcorner \rrbracket$ is true at a world w if and only if $\llbracket \varphi \rrbracket$ is true at some world w' accessible from w .
2. Necessity
 - $\llbracket \ulcorner \Box \varphi \urcorner \rrbracket$ is the proposition that $\llbracket \varphi \rrbracket$ is true at every world w .
 - $\llbracket \ulcorner \Box \varphi \urcorner \rrbracket$ is true at a world w if and only if $\llbracket \varphi \rrbracket$ is true at every world w' accessible from w .

The initial modal occurring in the readings of the instances of the premises of the argument for (NecPropObama), ‘it is necessary that’, will be left unanalysed, since analysing it would add extra complexity to the statements without any gains. Let ‘ $T\dots_$ ’ be a binary predicate standing for the relation *being true at*, and thus which obtains between worlds and propositions. Also, let ‘ $\exists w(\varphi)$ ’ be used as shorthand for ‘ $\exists x(Wx \wedge \varphi)$ ’.²⁴

(P1-PAi) It is necessary that it is possible that Obama is a president or it is possible that Obama is not a president.

(P1-PAi-TrAt) *Truth At Reading:*

1. It is necessary that some possible world w is such that it is true at w that Obama is a president or some possible world w is such that it is true at w that Obama is not a president.
2. $\Box(\exists w(Twp) \vee \exists w(Tw\neg p))$.

(P2-PAi) It is necessary that if it is possible that Obama is a president then the proposition that Obama is a president is something.

(P2-PAi-TrAt) *Truth At Reading:*

1. It is necessary that if some possible world w is such that it is true at w that Obama is a president, then the proposition that Obama is a president is something.
2. $\Box(\exists w(Twp) \rightarrow \exists q(p = q))$.

(P3-PAi) It is necessary that if it is possible that Obama is not a president then the proposition that Obama is not a president is something.

(P3-PAi-TrAt) *Truth At Reading:*

1. It is necessary that if some possible world w is such that it is true at w that Obama is not a president, then the proposition that Obama is not a president is something.
2. $\Box(\exists w(Tw\neg p) \rightarrow \exists q(\neg p = q))$.

²⁴I remain neutral here on the question what is the type of possible worlds, even though I am sympathetic to the view that possible worlds themselves are propositions.

(P4-PAi) It is necessary that the proposition that Obama is a president is something if and only if the proposition that Obama is not a president is something.

Briefly, (P1-PAi) is an instance of Seriality, (P2-PAi) and (P3-PAi) are instances of Thorough Serious Actualism, and (P3-PAi) is an immediate consequence of Contradictoriness.

Starting with (P1-PAi), this is a theorem of propositional modal logic, and so it seems reasonable to think that (P1-PAi-TrAt) is true, since this reading of (P1-PAi) is given in terms of the weaker understanding of the modals available, i.e., in terms of truth at a world.

As previously mentioned, all that the truth at a world w of a proposition p requires is that w and p be related, not that p would have had the property of being true had w been realised. Now, if p and w are related, then it follows that p is something (and that w is something as well) by an application of Thorough Serious Actualism, regardless of whether p would have been something had w been realised. But this means that necessarily, if p is true at some world w , then p is something, since the truth of p at some world w requires that p and some world w be related. Similarly, if $\neg p$ is true at some world w , this requires that $\neg p$ be related to some world w , and so, that $\neg p$ is something. What this shows is thus that (P2-PAi-TrAt) and (P3-PAi-TrAt) are both true.

There is also no risk of an ambiguous reading of the modalities as they occur in (P4-PAi). The only modality present in each one of the premises, the expression 'it is necessary that', may itself be given a univocal weak reading, in terms of truth at every world.

Thus, the distinction between truth in a world and truth at a world gives no reason to reject the truth of any of the premises of the argument. Moreover, the argument from (P1-PAi-TrAt) - (P4-PAi-TrAt) to (NecPropObama) may be seen to be valid straightforwardly. Thus, the Possibility Argument is itself valid, and the truth of its premises is not called into question by the availability of the distinction between truth in a world and truth at a world. So, the distinction does not afford propositional contingentists with the resources to reject the Possibility Argument.

The current state of the dialectic is the following. The distinction between truth in a world and truth at a world gives propositional contingentists the resources enabling them to resist the cogency of Plantinga's Argument and of the Truth-Values Argument. It was shown that the distinction does not, in and of itself, offer the resources to resist the cogency of the Possibility Argument for Propositional Necessitism. On the contrary, the cogency of the Possibility Argument is left unscathed by an understanding of the modalities in terms of truth at a world.

This seems to pose a dilemma to propositional contingentists. If the Truth In-Truth At Objection is pursued, then it seems that the Possibility Argument turns out to be cogent. And if the Truth In-Truth At Objection is abandoned, then a promising line of objection to Plantinga's argument and to the Truth-Values Argument is lost. Let me call this dilemma for propositional contingentists the *Truth At Dilemma*.

In what follows I will explore the prospects of a possible way out of the Truth At Dilemma. The Truth In-Truth At Objection presupposed the Truth At Account of 'necessity' and 'possibility'. One option available to propositional contingentists is to reject the Truth At Account while still maintaining

the view that modal expressions can be accounted for in terms of truth at a world. Pursuing this line requires offering a different account of ‘necessity’ and ‘possibility’. A different account is presented in what follows, and its prospects are investigated.

3.5.3 Actual Truth At a World

The account of ‘possibility’ and ‘necessity’ that will be investigated, the *Actual Truth At Account*, is closely related to the Truth At Account. The account is the following:

Actual Truth At Account

1. Possibility
 - $\llbracket \ulcorner \Diamond \varphi \urcorner \rrbracket$ is the proposition that *actually*, $\llbracket \varphi \rrbracket$ is true at some world w .
 - $\llbracket \ulcorner \Diamond \varphi \urcorner \rrbracket$ is true at a world w if and only if $\llbracket \varphi \rrbracket$ is true at some world w' accessible from the *actual world*.
2. Necessity
 - $\llbracket \ulcorner \Box \varphi \urcorner \rrbracket$ is the proposition that *actually*, $\llbracket \varphi \rrbracket$ is true at every world w .
 - $\llbracket \ulcorner \Box \varphi \urcorner \rrbracket$ is true at a world w if and only if $\llbracket \varphi \rrbracket$ is true at every world w' accessible from the *actual world*.

Note that the expression ‘actually’ is used in its rigid sense, not in an indexical sense. Let α name the actual world. Then, it is the case that actually, p if and only if it is true at α that p . The Possibility Argument turns out not to be cogent given the assumption that the Actual Truth At Account is true, as dialectically required by propositional contingentists.

Consider the reading of (P2-PAi) according to the account:

(P2-PAi-AcTrAt)

1. It is necessary that if actually some possible world w is such that it is true at w that Obama is a president, then the proposition that Obama is a president is something;
2. $\Box (@\exists w(Twp) \rightarrow \exists q(p = q))$

Contrary to what was the case with the reading of (P2-PAi-TrAt), (P2-PAi-AcTrAt) is not an instance of Thorough Serious Actualism. Suppose that it is possible that Obama is a president at an actual or counterfactual circumstance w . According to the Actual Truth At Account this means that actually, the proposition that Obama is a president is true at some world w' . If actually, the proposition that Obama is a president is true at some world w' , then actually, the proposition that Obama is a president bears the being true at relation to w' . So, by Thorough Serious Actualism, it follows that *actually*, w' and the proposition that Obama is a president are both something. However, from the fact that, at w , actually, the proposition that Obama is a president is something it does not follow that if w had been realised, then the proposition that Obama is a president would have been something. It only follows that the proposition that Obama is a president is something at the actual world.

So, once possibility is understood according to the Actual Truth At Account, there is no longer reason to hold that (P2-PAi) is true. And similarly with respect to (P3-PAi). Moreover, the objections

to Plantinga's Argument and the Truth-Values argument will still go through. In particular, truth in a world may still be defined in terms of truth at a world. Thus, the Actual Truth At Account promises to provide propositional contingentists with a way out of the Truth At Dilemma.

In what follows I will show that this is not so. But first I will quickly show that propositional contingentists have the resources required to reject the claim that the Actual Truth At Account has as an unwanted consequence the claim that it is necessary that every proposition is actually something.

To see how this worry arises, recall that from the assumption that at some world w , it is possible that Obama is a president, it follows that actually, the proposition that Obama is a president is something. Similarly, from the assumption that, at some world w , it is possible that Obama is not a president it follows that the proposition that Obama is not a president is actually something. So, the proposition that Obama is a president is actually something, or the proposition that Obama is not a president is actually something. In such case it follows, by Boolean Structure, that actually, the proposition that Obama is a president is something.

Since the proposition that Obama is a president plays, in the above argument, the role of an arbitrary proposition, it would appear that from the claim that actually, the proposition that Obama is a president is something it could be legitimately inferred that it is necessary that, for every proposition p , actually p is something. But the conclusion that it is necessary that, for every proposition p , actually, p is something is problematic for propositional contingentists.

On the one hand, the claim that it is necessary that, for every proposition p , actually p is something conflicts with the conjunction of the following claims: i) there could have been some things that actually are nothing; ii) there could have been propositions directly about those things; iii) the thesis of Existentialism. Suppose that Noman is a merely possible individual, and that the proposition that Noman is a human is about him. Since it is necessary that every proposition actually is something, it follows that the proposition that Noman is a human is actually something. But then, by Existentialism, Noman is also something, contrary to the assumption.

On the other hand, suppose that it is indeed true that it is necessary that every proposition actually is something. Then, it seems plausible to think that it is necessary that every proposition is necessarily something. Why should the actual world be special in this respect? For these reasons, if the Actual Truth At Account implies that it is necessary that every proposition actually is something, then propositional contingentists may prefer to avoid a commitment to its truth.

Proponents of the Actual Truth At Account have the resources to reject the legitimacy of the inference from the claim that actually p is something, for an arbitrary proposition p , to the claim that it is necessary that, for every proposition p , actually p is something.

The legitimacy of the inference can be resisted by adopting the view that the functions that are the semantic values of ' \diamond ' and ' \square ' have an empty extension at other worlds. They relate nothing whatsoever in worlds other than the actual world.

Once this view is adopted, the inference is illegitimate. Since actually p is something, for an arbitrary proposition p , it may be legitimately inferred that for every proposition p , actually p is

something. But *necessitation* is illegitimate. The inference of the claim that for every proposition p , actually p is something was legitimate due to a fact that holds only of actual propositions: only these are arguments of the semantic values of the modal expressions.²⁵

Call this objection to the Possibility Argument the *Truth At Objection*. I turn now to the problems with the Truth At Objection. An important initial observation is that the Truth At and the Actual Truth At accounts are mutually consistent. Thus, the Actual Truth At Objection is successful only if the following claim is true:

- (31) There is some possible proposition p and possible world w such that i) it is true at w that there is some world w' accessible from the *actual world* such that p is true at w' (and so it is true at w that $\diamond p$ according to the Actual Truth At Account) and yet ii) it is not true at w that there is some world w' accessible from w such that p is true at w' .

The truth of (31) is presupposed by the Actual Truth At Objection because otherwise it would follow that if it is true at a world w that $\diamond p$, then p is something at w , and so the Possibility Argument would indeed be valid.

The main problem with the Actual Truth At Objection is simply that (31) is false. Assuming that propositions are true at worlds, (31) is false because:

- (32) Any case in which p is not true at a world w' accessible from w is a case in which it is not true at w that $\diamond p$.

My argument for (32) will be based on showing that a certain debate on the correct logic for meta-physical modality presupposes that $\diamond p$ is judged to be false at w on those scenarios in which p is not true at a world w' accessible from w , regardless of whether p is true at a world w' accessible from the actual world. The example should make it obvious that this is a general feature, and so that (32) is true.

This will reveal that the Actual Truth At Account is based on an erroneous view of the semantics of modal expressions. I will focus on one such scenario, inspired by the arguments for the claim that axiom schema 4 of propositional modal logic, according to which if it is possible that it is possible that φ , then it is possible that φ (i.e., $\diamond\diamond\varphi \rightarrow \diamond\varphi$) has false instances.

²⁵I ultimately think that such move is unsuccessful, for reasons related to the case against the partial functions account of the semantics of first-order modal languages presented in §2.5.

In general, the problem with this strategy is that it does not have the resources to make sense of the semantic values of open formulas and what their contribution is to the semantic values of the sentences in which they occur is. For instance, what is the semantic value of $\diamond p$ in the context of the formula $\diamond\exists p(\diamond p \wedge @-\exists q(p = q))$?

The view that the semantic value of ' \diamond ' has no extension in worlds other than the actual world leaves it mysterious what the semantic value of ' $\diamond p$ ' is. The reason is that whatever possible proposition witnesses the truth of ' $\diamond\exists p(\diamond p \wedge @-\exists q(p = q))$ ', this proposition is nothing in the actual world. The problem, from the standpoint of many propositional contingentists, is that they accept the truth of the sentence. That is, they accept that there could have been true propositions that are actually nothing.

These propositional contingentists appear to be left with no satisfactory account of the semantic value of ' $\diamond p$ ' if they indeed endorse the view that the semantic value of ' \diamond ' has an extension only in the actual world.

A putative counterexample to axiom schema **4** starts by assuming the truth of the following two principles about bicycles:²⁶ i) *Tolerance*, according to which necessarily, any bicycle could have been constituted by any two thirds of its original constitution; ii) *Restriction*, according to which necessarily, no bicycle could have been constituted by one third of its original constitution.

Let me describe a scenario, the *Bicla Scenario*, of the sort envisaged by proponents of the claim that axiom schema **4** has false instances. In this scenario, *Bicla* is a bicycle. At the actual world, α , *Bicla* is constituted by frame α -Frame and wheels α -FrontWheel and α -BackWheel.

Suppose that w_1 -FrontWheel is one of the wheels of which *Bicla* could have been constituted, according to *Tolerance*. Let w_1 be a possible world witnessing the fact that *Bicla* could have been just as it is, except for having w_1 -FrontWheel as its front wheel.

Let w_2 -Frame be a frame that is something at w_1 . According to proponents of the view that axiom schema **4** has false instances there is a possible world w_2 accessible from w_1 such that *Bicla* is constituted at w_2 by w_2 -Frame, w_1 -FrontWheel and α -BackWheel. World w_2 is taken to witness the truth of *Tolerance*.

Let *Biclalternate* stand for the claim that *Bicla* is constituted by w_2 -Frame, w_1 -FrontWheel and α -BackWheel. *Biclalternate* is true at w_2 . The fact that w_2 is accessible from w_1 and *Biclalternate* is true at w_2 is taken by proponents of the view that axiom schema **4** has false instances to suffice for it to be true at w_1 that \diamond *Biclalternate*.

World w_2 is not accessible from α . If it were, then *Bicla* would have been constituted by one third of its original constitution, which would violate *Restriction*. Thus, according to *Restriction*, \diamond *Biclalternate* is false at α . Since w_1 is accessible from α and \diamond *Biclalternate* is true at w_1 , $\diamond\diamond$ *Biclalternate* is true at α . Thus, proponents of the view that axiom schema **4** has false instances take the *Bicla Scenario* to show that $\diamond\diamond$ *Biclalternate* \rightarrow \diamond *Biclalternate* is false at α .

If the Actual Truth At Objection were successful, then the thought that scenarios such as the *Bicla Scenario* constitute counterexamples to axiom schema **4** would be deeply misguided. For instance, according to the Actual Truth At Account the fact that *Biclalternate* it is true at w_2 and w_2 is accessible from w_1 is irrelevant to whether \diamond *Biclalternate* is true at w_1 . Moreover, since \diamond *Biclalternate* is false at α , there is no possible world w accessible from α such that *Biclalternate* is true at w . Hence, according to the Actual Truth At Account, \diamond *Biclalternate* is false at w_1 , and indeed at every worlds w such that w is accessible to α . A fortiori, $\diamond\diamond$ *Biclalternate* is false at w_0 . Thus, if the Actual Truth At Objection were true, then even if the *Bicla Scenario* were metaphysically possible, this would provide no counterexample to the truth of every instance axiom schema **4**.

Importantly for the present purposes, proponents and opponents of the view that there are false instances of axiom schema **4** agree that if scenarios such as the *Bicla Scenario* are metaphysically possible, then axiom schema **4** has false instances. To repeat, such widespread agreement makes no sense from the standpoint of proponents of the Actual Truth At Objection. After all, the Actual Truth

²⁶The example in the text is close the one offered in (Chandler, 1976). Besides Chandler, Salmon (2005) has defended forcefully the failure of axiom-schema **4**.

At Account implies the truth of every instance of axiom-schema 4.

Suppose that it is true that it is possible that it is possible that p , for an arbitrary proposition p . In such case, according to the Actual Truth At Account, there is some possible world w such that w is accessible from α and $\diamond p$ is true at w . So, there is some possible world w such that w is accessible from α and there is some possible world w' such that w' is accessible from α and p is true at w' . This is so if and only if there is some possible world w accessible from α such that p is true at w . So, if $\diamond\diamond p$, then $\diamond p$.²⁷

The discussion concerning the Bicla Scenario shows that the Actual Truth At Account yields the wrong semantics for ' \diamond '. If p is not true at a world w' accessible from w , then $\diamond p$ is not true at w , independently of what happens in the actual world. That is, the Actual Truth At Objection is wrongly committed to the truth of (31).

I find the predicament of proponents of the Actual Truth At Objection to be similar to that of a biologist, say, Bio, that endorses the view that all living things are composed of carbon partly on the grounds that he takes 'all' to mean all things on Earth. Just as it may be that all living things *tout court* are composed of carbon, it may very well be that all instances of axiom schema 4 are true. But just as Bio supports the claim that all living things are composed of carbon on the basis of a faulty semantics for 'all', proponents of the Actual Truth At Account support the truth of every instance of axiom schema 4 on the basis of a faulty semantics for ' \diamond '.

Read (2005, p. 321) captures the present point when referring to the contrapositive of axiom schema 4, saying that 'even when ... there is equivalence, it is misleading to say that nothing is added by prefixing 'it is necessary that'. It is a substantive thesis that necessity is idempotent (that $\Box\Box p$ is equivalent to $\Box p$). If the Actual Truth At Objection were successful, then the truth of every instance of axiom schema 4 would turn out to be non-substantive. The truth of every instance of axiom schema 4 would be consistent with scenarios like the Bicla Scenario. This reveals that the Actual Truth At Objection is unsuccessful.

3.6 The Commitments of Propositional Modal Logic

3.6.1 Modalities as Properties

The distinction between truth in a world and truth at a world was thought to offer the resources enabling the rejection of the cogency of the Possibility Argument insofar as the distinction enables a take on modal expressions as something other than properties of propositions. In the previous section it was shown that the Possibility Argument stands even once the Truth At Account of modal expressions is assumed to be true and propositions are not understood in such way.

In addition, the view that modal expressions are properties of propositions is intuitively appealing. For instance, just as being a president is one of the ways that Obama is, being possible is one of the ways that the proposition that Obama is a president is.

²⁷For the same reason, every instance of axiom-schema 5 is true, this axiom-schema stating that if it is possible that φ , then it is necessary that it is possible that $\varphi - \diamond\varphi \rightarrow \Box\diamond\varphi$.

Moreover, Kripke models for propositional modal logic have increased our understanding of modality partly because they treat ‘ \diamond ’ and ‘ \square ’ as standing for properties of propositions. It is customarily said that Kripke models have increased our understanding of modality partly by taking possibility to be relative to possible worlds. But this just means that Kripke models have increased our understanding of modality partly by treating ‘necessity’ and ‘possibility’ as properties (with Kripke models also offering a model of the modal profiles of properties and their relationships in terms of truth at a world). For instance, the extension of the property of possibility at each world w consists in the set of propositions that is true at some world accessible to w .

In general, in Kripke models properties are treated as being true of things relative to worlds. For instance, the property of existence is treated in Kripke models in such a way that it has an extension only relative to a world. Such treatment of properties increases our understanding of the relationship between properties. Are properties P and Q mutually exclusive? They are if there is no possible world in which they are coinstantiated. Otherwise, they aren’t.

A classic view on metaphysical modality is that its logic is given by **S5**. This may lead to the thought that metaphysical possibility is distinguished from other kinds of possibility in that it is not relative. Such thought may be guided by the observation that models for **S5** are often given without explicit mention of an accessibility relation (as done in this dissertation). This is a mistaken thought. The accessibility relation is there even when it is not explicitly mentioned. It is a universal accessibility relation, in that every world is accessible to every world. This means that these models treat necessity and possibility as having constant extensions at all possible worlds. It does not mean that necessity and possibility have no modal profile. Compare the case with that of the haecceity of the empty set. Just as the haecceity of the empty set is clearly a property, one that has a constant extension at all possible worlds, necessity and possibility are also properties, even if their extensions do not vary from world to world.

3.6.2 Logic is the ‘Culprit’

The option left to those wishing to resist the modal arguments for Propositional Necessitism consists in rejecting systems of propositional modal logic as weak as **KD** and **K**. For instance, [Adams \(1981\)](#) and [Prior \(1957\)](#) have both argued that the normal propositional modal logics all contain unwanted commitments.

The thought is that these systems appeal to principles, namely, the interdefinability of ‘ \square ’ and ‘ \diamond ’ and the rule of necessitation, which jointly lead to falsehoods. Let me take a detailed look at the derivation of $\square(\diamond p \vee \square \neg p)$ in **K**.

The first principle required is the following propositional tautology:

$$(33) \quad \diamond p \vee \neg \diamond p.$$

Principle (33) may be assumed to be true at every world whatsoever unproblematically. On the face of it, even if (33) is true at every world, this does not imply that p is something at every world.

By the interdefinability of ‘ \Box ’ and ‘ \Diamond ’, it follows that:

$$(34) \quad \Diamond p \vee \Box \neg p.$$

Thorough Serious Actualism together with (34) implies that $\exists q(p = q) \vee \exists q(\neg p = q)$. Moreover, $\exists q(p = q) \vee \exists q(\neg p = q)$ together with Contradictoriness implies that $\exists q(p = q)$. Thus, the inference from (33) to (34) may be regarded as unproblematic from the standpoint of propositional contingentists *provided that p is actually something*. That is, if p is actually nothing, then, the transition from (33) to (34) is not truth-preserving from the standpoint of propositional contingentists.

Thus, from the standpoint of propositional contingentists the problem at this point is that **K** sanctions the rule of necessitation, and so permits the inference of (35) from (34)

$$(35) \quad \Box(\Diamond p \vee \Box \neg p).$$

It was seen that the inference of (34) from (33) is truth-preserving only if p was actually something. Necessitation is truth-preserving only if p is something in every possible world. Since, in general, p is not something in every possible world, the inference of (35) from (34) is not truth-preserving from the standpoint of propositional contingentists.

Consider now a derivation of $\Box(\Diamond p \vee \Diamond \neg p)$ in **KT**. It starts with the following tautology, which may be assumed to hold of necessity.

$$(36) \quad p \vee \neg p$$

Now, two axioms of **KT** are

$$(37) \quad p \rightarrow \Diamond p$$

$$(38) \quad \neg p \rightarrow \Diamond \neg p$$

As in the case of the interdefinability of ‘ \Box ’ and ‘ \Diamond ’, whether (37) is true at a world w depends on whether p is something at w . If p is nothing at w , then it is false that p has the property of being possible. Similarly for (38) and $\neg p$.

From (37) and (38) it follows that

$$(39) \quad \Diamond p \vee \Diamond \neg p$$

by nonmodal reasoning. Whether (39) is true at a world w depends on whether (37) and (38) are both true at w , which in turn depends on whether p is something at w .

This means that the move to

$$(40) \quad \Box(\Diamond p \vee \Diamond \neg p)$$

is illegitimate if p is or could have been nothing.

Ultimately, I think that this is where propositional contingentists should mount their defence. That

is, I think that the modal logics **K** and **KD** do presuppose the truth of Propositional Necessitism. From the standpoint of propositional contingentists, some of the principles and inference rules of these logics are, respectively, false and not truth-preserving.

Let me call propositional contingentists that reject **K** and **KT** *ultra propositional contingentists*. The problem for ultra propositional contingentists is a lack of expressive resources for talking about what might have been. Consider, for instance, the following example:

- (41) If my parents had never met and I had been nothing, then I would have been something had my mother and father met in circumstances just like the ones in which they actually met. So, even if my parents had never met and I had been nothing, it would still have been possible that I was something.

Argument (41) strikes me as a valid argument, with perhaps a true premise. But ultra propositional contingentists must reject that this is so. If my parents had never met and I had been nothing, then the proposition that I am something would have been nothing, and so it could not have had the property of being possible. This means that contingentists must reject the conclusion of (41).

Ultra propositional contingentists should also reject the truth of the premise of (41), given the appeal to a counterfactual. The truth of the premise of (41) would require that the proposition that my parents meet in the circumstances in which they actually met and the proposition that I am something be related in at least one counterfactual circumstance in which my parents have never met and I was nothing. But, according to ultra propositional contingentists, the proposition that I am something cannot be related to any other proposition in a circumstance in which I am nothing. If the proposition that I am something is related to some other proposition in a circumstance w in which I am nothing, then the proposition is something. But, according to most ultra propositional contingentists it is not possible that I am nothing and the proposition that I am something is something.

The problem for ultra propositional contingentists is that unreflective common sense supports the soundness of argument (41). This is a valid argument with a true premise. Ultra propositional contingentists must reject this, while at the same time accounting for the intuition that (41) is sound. Arguably, they lack the resources to do so. Some of the ways that they have appealed do not work. For instance, the true in-true at distinction does not offer ultra propositional contingentists with acceptable resources. It still leads to the conclusion that (41) is a sound argument.

All this shows that propositions allow us to talk and think about what might have been *sub specie aeternitatis*. That is, propositions allow us to describe possibilities for things not only in actual but also in counterfactual circumstances. They allow us to describe possibilities for things in counterfactual circumstances without being bound by whether those things are something in those counterfactual circumstances. Insofar as propositions enable us to do so, they are necessary beings.

3.7 Propositions Are About Nothing

There is still one puzzle to be addressed. I presented in §3.2 an argument from the Classical Conception of propositions to Propositional Contingentism. Should the Classical Conception then be rejected?

The short answer is: yes. Unsurprisingly, the problem with the Classical Conception is that it is false that propositions are *intrinsically* and *essentially* representational. Several theorists have voiced their rejection of this claim. For instance, Speaks (King et al., 2014, p. 147) characterises as follows the belief on the alleged representational character of propositions that he, King and Soames all reject:

‘Here’s one thing that the three of us have in common: we all dislike the idea that propositions could be entities that are intrinsically representational – in the sense that they both are representational and would exist and be representational, even if there were no subjects around to do any representing.’

The problem for the Classical Conception is thus that it requires that there be entities that are representational without its representational properties being explained in terms of the activities of subjects doing the representing. But it is difficult to see how this can be. My view is the same as that of King, Soames, Speaks and others, namely, that there could not be any entities that were intrinsically representational. Since there could not be any entities that were intrinsically representational, the Classical Conception is false.

Moreover, I side with theorists such as Speaks and Stalnaker in thinking that propositions not only aren’t intrinsically representational, but also aren’t essentially representational. The thought that propositions are essentially representational creates, as Stalnaker (2012, p. 10) puts it, an ‘illusion of a problem’. What needs to be explained is not why propositions are essentially representational, since they are not. Rather ‘What needs to be explained is how things that express propositions – that represent the world as being some way – can express the propositions that they express’ (Stalnaker, 2012, p. 10).

Arguably, the main challenge to the position on the representational character of propositions that I am advocating is to explain how it can be that propositions have truth-conditions intrinsically and essentially, despite the fact that they are not intrinsically nor essentially about things.

Let us consider the case of properties. Just as propositions are true and false, properties are intrinsically and essentially true and false of things. But properties are not essentially representational entities. They are not about anything. For instance, the property of being red does not represent anything, and is not about anything.

Of course, there is a different sense of ‘being about’, according to which a property is indeed about things. In this sense, the property of being red is about roses, poppies, tomatoes and the other red things. But, in general, properties are not essentially ‘about’ the things that they are true of. For instance, if things had been otherwise, then redness would not have been true of the things that are actually red. More importantly, properties are not ‘about’ the things that they are true of insofar as

they represent them. They are 'about' the things that they are true of insofar as they are true of these things. Properties represent nothing, at least not intrinsically nor essentially.

If properties are true of things, and yet do not represent things, there is room to reject the view that propositions, insofar as they are true, represent the world. The analogy with the case of properties is especially fitting in the present context. The reason is that propositions are here being assumed to be 0-ary relations, i.e., things that are true or false, but not of anything. Thus, 0-ary relations are in this respect just like other relations. They are true or false despite the fact that they do not represent the world.

The objection to the view that propositions are not essentially representational can be made more precise. It is typically thought that truth depends on two things, namely on the way the world is represented to be, and on the world being that way. But then, if propositions are not essentially representational, they cannot have their truth-conditions essentially. This is quite bizarre.

I do not think that the view that propositions are not essentially representational requires giving up the view that propositions have their truth-conditions essentially. Again, the view that propositions are 0-ary relations can be helpful to disentangle these issues. Insofar as propositions are 0-ary relations, to say that a proposition p is true — i.e., to say that it is true that p — is to attribute a higher-order property to p . On the other hand, to say that a sentence is true is to attribute a first-order property to the sentence. It is indeed correct that an individual has the first-order property of being true if and only if that individual represents the world as being a certain way, and the world is that way. But it can be resisted that a proposition has the higher-order property of *being true that* just in case it represents the world as being a certain way and the world is that way. The first-order property and the higher-order property are different properties, even though they are obviously closely connected.

On my view it is incorrect to say, for instance, that for an entity to represent the world as being a certain way and the world to be that way just is for the entity to have some proposition p as its content and for p to represent the world as being that way. Rather, for an entity to represent the world as being a certain way just is for the entity to have some proposition p as its content, and for the world to be that way just is for it to be true that p .²⁸

It is often said that propositions are the primary bearers of truth and falsity. I think that talk of priority may also lead to the illusion of a problem. If sentences and the like are true or false derivatively, i.e., only insofar as their contents are intrinsically and essentially true or false, it is somewhat natural to assume that sentences and the like represent derivatively, only insofar as their contents intrinsically and essentially represent. This thought is already in tension with the view that representation is something that agents do. Moreover, I do not think that there is any interesting sense of priority here. For instance, I think that it is at best misguided to think that the truth of a sentence is, for instance, *inherited* in some way from the truth of the proposition that is its content. Rather, for the sentence to be true just is for there to be some proposition p such that the sentence has p as its content and for it

²⁸See (King et al., 2014, ch. 11) for similar considerations in defence of the view that propositions are neither intrinsically nor essentially representational.

to be true that p .

This concludes the defence of Propositional Necessitism. In the next section I will show how the arguments for Propositional Necessitism may be extended to arguments for Higher-Order Necessitism.

3.8 From Propositional Necessitism to Higher-Order Necessitism

I will begin by presenting arguments, analogous to the modal arguments, for the following thesis:

Property Necessitism.

1. Necessarily, every property of things of any type is necessarily something.
2. $\Box \forall X_{\langle t \rangle} \Box \exists Y_{\langle t \rangle} (X = Y)$.

The main presupposition of the arguments to be presented is that the quantifiers stand for higher-order properties. Let ' $\mathbb{I}x$ ' be the *none quantifier*, to be read as 'no thing x '. The argument to be given presupposes the following: i) necessarily, $\exists x_t \varphi$ if and only if the property of being a φ has the property $I_{\langle t \rangle}$ of being instantiated; ii) necessarily, $\mathbb{I}x_t \varphi$ if and only if the property of being a φ has the property $N_{\langle t \rangle}$ of not being instantiated; and iii) $\forall x_t \varphi$ if and only if the property of being a φ has the property $A_{\langle t \rangle}$ of being coextensive with the property of identity.

The thought that the quantifiers correspond to properties of properties has been given to us at our fathers' knees. Frege (1980a,b) held the view that quantifiers stand for concepts of concepts, and Russell (1905) that quantifiers stand for properties of propositional functions. The view that quantifiers are higher-order entities is also the one presupposed by generalised quantification theory, with generalised quantifiers being understood as relations between relations.²⁹

The premises of the *Something Or Nothing Argument*, an argument analogous to the Possibility Or Impossibility Argument, are the following:

Premises of the Something Or Nothing Argument

(P1-SNA) Something Or Nothing.

1. Necessarily, for every property $X_{\langle t \rangle}$, necessarily, something is an $X_{\langle t \rangle}$ or nothing is an $X_{\langle t \rangle}$.
2. $\Box \forall X_{\langle t \rangle} \Box (\exists x_t Xx \vee \mathbb{I}x_t Xx)$.

(P2-SNA) Existential Quantifier.

1. Necessarily, for every property $X_{\langle t \rangle}$, necessarily, if something is an $X_{\langle t \rangle}$, then $X_{\langle t \rangle}$ has the property of being instantiated.
2. $\Box \forall X_{\langle t \rangle} \Box (\exists x_t Xx \rightarrow I_{\langle t \rangle} X)$.

(P3-SNA) None Quantifier.

1. Necessarily, for every property $X_{\langle t \rangle}$, necessarily, if nothing is an $X_{\langle t \rangle}$, then $X_{\langle t \rangle}$ has the property of being empty.

²⁹Of course, the view that quantifiers are higher-order entities presupposes that there are, or could have been, higher-order entities. But, on its own, it does not imply that higher-order entities are necessary beings. Moreover, note that if there could not have been any higher-order entities, then Higher-Order Necessitism is trivially true. Thus, Nominalism – understood as the view that there are nor could have been any higher-order entities – does not afford a way to reject the truth of Higher-Order Necessitism.

2. $\Box \forall X_{\langle t \rangle} \Box (\mathbb{I}x_t Xx \rightarrow N_{\langle t \rangle} X)$.

(P4-SNA) Thorough Serious Actualism.

It should be clear how the argument proceeds, and so I will not go through it. The justification for the thesis of Something Or Nothing is analogous to the justification for the thesis of Possibility Or Impossibility. Even though higher-order modal languages do not typically contain the quantifier 'I', this quantifier may be added unproblematically. The logic governing it is easily obtained from that of the existential quantifier. All instances of the schema $\Box (\mathbb{I}x\varphi \leftrightarrow \neg \exists x\varphi)$ are added as axioms. All instances of the schema $\Box (\exists x\varphi \vee \neg \exists x\varphi)$ are instances of Excluded Middle, and so all instances of $\Box (\exists x\varphi \vee \neg \exists x\varphi)$ are theorems of very weak and uncontroversial higher-order modal logics. From $\Box (\mathbb{I}x\varphi \leftrightarrow \neg \exists x\varphi)$ and $\Box (\exists x\varphi \vee \neg \exists x\varphi)$ it follows that $\Box (\exists x\varphi \vee \mathbb{I}x\varphi)$. The truth of every instance of this schema reflects the fact that it holds for all possible properties. Thus, it is true that $\Box \forall X_{\langle t \rangle} \Box (\exists x_t Xx \vee \mathbb{I}x_t Xx)$.

The theses of Existential Quantifier and None Quantifier are justified by the view that quantifiers are to be understood in terms of higher-order properties. Finally, Thorough Serious Actualism has been defended in §3.3.

I will briefly present the premises of the Something or Everything Argument and of the Something Argument. These are arguments for Property Necessitism analogous to, respectively, the Possibility or Necessity Argument and the Possibility Argument:

Premises of the Something Or Everything Argument

(P1-SEA) Something Or Everything.

1. Necessarily, for every property $X_{\langle t \rangle}$, necessarily, something is an $X_{\langle t \rangle}$ or everything is not an $X_{\langle t \rangle}$.
2. $\Box \forall X_{\langle t \rangle} \Box (\exists x_t Xx \vee \forall x_t \neg Xx)$.

(P2-SEA) Existential Quantifier.

(P3-SEA) Universal Quantifier.

1. Necessarily, for every property $X_{\langle t \rangle}$, necessarily, if everything is not an $X_{\langle t \rangle}$, then $\hat{x}(\neg X_{\langle t \rangle}x)$ has the property of being coextensive with the property of identity.
2. $\Box \forall X_{\langle t \rangle} \Box (\forall x_t \neg Xx \rightarrow A_{\langle t \rangle} \hat{x}_t(\neg Xx))$.

(P4-SEA) Thorough Serious Actualism.

(P5-SEA) Contradictoriness for Properties.

1. Necessarily, for every property $X_{\langle t \rangle}$, its contradictory $\hat{x}(\neg Xx)$ is something.
2. $\Box \forall X_{\langle t \rangle} \exists Y_{\langle t \rangle} (\hat{x}_t(\neg Xx) = Y)$.

The following are the premises of the Something Argument:

Premises of the Something Argument

(P1-SA) Something.

1. Necessarily, for every property $X_{\langle t \rangle}$, necessarily, something is an $X_{\langle t \rangle}$ or something is not an $X_{\langle t \rangle}$.

2. $\Box \forall X_{\langle t \rangle} \Box (\exists x_t Xx \vee \exists x_t \neg Xx)$.

(P2-SA) Existential Quantifier.

(P3-SA) Existential Quantifier 2.

1. Necessarily, for every property $X_{\langle t \rangle}$, necessarily, if something is not an $X_{\langle t \rangle}$, then $\hat{x}(\neg X_{\langle t \rangle}x)$ has the property of instantiated.

2. $\Box \forall X_{\langle t \rangle} \Box (\exists x_t \neg Xx \rightarrow I_{\langle \langle t \rangle \rangle} \hat{x}_t(\neg Xx))$.

(P4-SA) Thorough Serious Actualism.

(P5-SA) Contradictoriness for Properties.

Both arguments proceed as expected. Note that the thesis of Something implies that, for every type t , it is necessary that, there is some entity of type t . As mentioned in §3.3, theorists committed to prima facie innocuous theses, such as the necessary being of the empty set, are immediately committed to the claim that it is necessary that there is some individual. What about the remaining types? Arguably, for every type t , it is the case that relation R is something, where R is that relation which is such that necessarily, things are R -related if and only if they are self-identical and are not self-identical. There is also that relation S that is such that necessarily, things are S -related if and only if they are self-identical. Arguably, this shows that the claim that for every type t it is necessary that there is some thing of that type is not an unpalatable consequence of the thesis of Something.

The arguments just presented may be extended to the case of relations. Let $\exists \exists$ be a quantifier just like the existential quantifier, except that it binds n variables at a time. Similarly for $\forall \forall$ and $\forall \forall$, with the appropriate changes.³⁰ Moreover, let ' $I_{\langle \langle t_1, \dots, t_n \rangle \rangle}$ ' stand for the property of relations of type $\langle t_1, \dots, t_n \rangle$ of being instantiated by a n -ary sequence of entities. Similarly for $N_{\langle \langle t_1, \dots, t_n \rangle \rangle}$ and $A_{\langle \langle t_1, \dots, t_n \rangle \rangle}$, with the appropriate changes.

According to the intended meaning of the quantifier expression $\exists \exists$, the following holds: necessarily, $\exists \exists x_{t_1}^1 \dots x_{t_n}^n \varphi$ if and only if the relation holding between $x_{t_1}^1, \dots, x_{t_n}^n$ such that φ is such that it has property $I_{\langle \langle t_1, \dots, t_n \rangle \rangle}$. Similarly for $\forall \forall$ and $\forall \forall$, with the appropriate changes.

The following argument establishes the truth of (every instance of) Higher-Order Necessitism:

Premises of the Sometimes Or Never Argument

(P1-SsNA) Sometimes Or Never.

· $\Box \forall X_{\langle t_1, \dots, t_n \rangle} \Box (\exists \exists x_{t_1}^1 \dots x_{t_n}^n Xx^1 \dots x^n \vee \forall \forall x_{t_1}^1 \dots x_{t_n}^n Xx^1 \dots x^n)$

(P2-SsNrA) Sometimes Quantifier.

· $\Box \forall X_{\langle t_1, \dots, t_n \rangle} \Box (\exists \exists x_{t_1}^1 \dots x_{t_n}^n Xx^1 \dots x^n \rightarrow I_{\langle \langle t_1, \dots, t_n \rangle \rangle} X)$.

(P3-SsNrA) Never Quantifier.

· $\Box \forall X_{\langle t_1, \dots, t_n \rangle} \Box (\forall \forall x_{t_1}^1 \dots x_{t_n}^n Xx^1 \dots x^n \rightarrow N_{\langle \langle t_1, \dots, t_n \rangle \rangle} X)$.

(P4-SsNrA) Thorough Serious Actualism.

³⁰These quantifiers are close to Lewis's *unselective quantifiers* which, according to Lewis (1998, p. 9-10), may 'show up' as the adverbs of quantification 'sometimes', 'never' and 'always'. The main difference is that Lewis's unselective quantifiers simply bind all the variables in a formula φ . For instance, Lewis (1998, p. 10) offers the following semantic account of his unselective quantifier ' \exists ': ' $\exists \Phi$ is true iff Φ is true under some admissible assignment of values to all variables free in Φ '.

Let ' $\hat{x}_{t_1}^1 \dots, x_{t_n}^n (\varphi)$ ' stand for the relation holding between $x_{t_1}^1, \dots, x_{t_n}^n$ such that φ . The following arguments are the analogues of, respectively, the Something Or Everything Argument and the Something Argument:

Premises of the Sometimes Or Always Argument

(P1-SsAA) Sometimes or Always.

$$\cdot \Box \forall X_{\langle t_1, \dots, t_n \rangle} \Box (\exists \exists x_{t_1}^1 \dots x_{t_n}^n X x^1 \dots x^n \vee \forall \forall x_{t_1}^1 \dots x_{t_n}^n \neg X x^1 \dots x^n).$$

(P2-SsAA) Sometimes Quantifier.

(P3-SsAA) Always Quantifier.

$$\cdot \Box \forall X_{\langle t_1, \dots, t_n \rangle} \Box (\forall \forall x_{t_1}^1 \dots x_{t_n}^n \neg X x^1 \dots x^n \rightarrow A_{\langle \langle t_1, \dots, t_n \rangle \rangle} \hat{x}_{t_1}^1 \dots x_{t_n}^n (\neg X x^1 \dots x^n)).$$

(P4-SsAA) Thorough Serious Actualism.

(P5-SsAA) Contradictoriness for Relations.

$$\cdot \Box \forall X_{\langle t_1, \dots, t_n \rangle} \exists Y_{\langle t_1, \dots, t_n \rangle} (\hat{x}_{t_1}^1 \dots x_{t_n}^n (\neg X x^1 \dots x^n) = Y).$$

Premises of the Sometimes Argument

(P1-SsA) Sometimes.

$$\cdot \Box \forall X_{\langle t_1, \dots, t_n \rangle} \Box (\exists \exists x_{t_1}^1 \dots x_{t_n}^n X x^1 \dots x^n \vee \exists \exists x_{t_1}^1 \dots x_{t_n}^n \neg X x^1 \dots x^n).$$

(P2-SsA) Sometimes Quantifier.

(P3-SsA) Sometimes Quantifier 2.

$$\cdot \Box \forall X_{\langle t_1, \dots, t_n \rangle} \Box (\exists \exists x_{t_1}^1 \dots x_{t_n}^n \neg X x^1 \dots x^n \rightarrow I_{\langle \langle t_1, \dots, t_n \rangle \rangle} \hat{x}_{t_1}^1 \dots x_{t_n}^n (\neg X x^1 \dots x^n)).$$

(P4-SsA) Thorough Serious Actualism.

(P5-SsA) Contradictoriness for Relations.

The defence of the cogency of the modal arguments should give an idea of how the cogency of the arguments for Property Necessitism and Higher-Order Necessitism may be defended of criticism. I will now turn to arguments for a related higher-order modal principle.

Schematic versions of the premises of the arguments offered so far in this section enable the formulation of an argument for a principle stronger than Higher-Order Necessitism, namely, the following (quite strong) comprehension principle for higher-order modal logic:

$\hat{\text{Comp}}$. $\Box \exists X_{\langle t_1, \dots, t_n \rangle} (\hat{x}_{t_1}^1, \dots, x_{t_n}^n (\varphi) = X)$

In $\hat{\text{Comp}}$, X is not free in φ and the result of prefixing $\hat{\text{Comp}}$ with any sequence of universal quantifiers of any type (binding parameters in φ) and necessity operators, in any order, is an instance of $\hat{\text{Comp}}$.

Principle $\hat{\text{Comp}}$ has every instance of the following schema as one of its instances:

$$\Box \forall Y_{\langle t_1, \dots, t_n \rangle} \Box \exists X_{\langle t_1, \dots, t_n \rangle} (\hat{x}_{t_1}^1, \dots, x_{t_n}^n (Y x_{t_1}^1 \dots x_{t_n}^n) = X).$$

This schema is equivalent to Higher-Order Necessitism, and so higher-order contingentists must reject $\hat{\text{Comp}}$ on pain of contradiction.

Principle $\hat{\text{Comp}}$ is not only at least as strong as Higher-Order Necessitism but also strictly stronger since Higher-Order Necessitism is consistent with the falsity of some instances of $\hat{\text{Comp}}$. For instance, Higher-Order Necessitism does not imply that for any properties P and Q there is the property of being both a P and a Q , even though this is a consequence of $\hat{\text{Comp}}$.

A principle equivalent to $\hat{\text{Comp}}$ has been recently defended in (Williamson, 2013, §6). Whereas Williamson offers abductive considerations in favour of that principle, schematic versions of the premises of the arguments given above for higher-order necessitism enable the formulation of deductive arguments for $\hat{\text{Comp}}$.

For instance, the following schematic version of the premises of the Sometimes Or Never argument imply $\hat{\text{Comp}}$:

Premises of the Schematic Sometimes Or Never Argument

(P1-SSsNeA) Sometimes Or Never Schema.

$$\cdot \Box(\exists\exists x_{t_1}^1 \dots x_{t_n}^n \varphi \vee \forall\forall x_{t_1}^1 \dots x_{t_n}^n \varphi),$$

where the result of prefixing (P1-SSsNeA) with any sequence of universal quantifiers of any type (binding parameters in φ) and necessity operators, in any order, is an instance of (P1-SSsNeA).

(P2-SSsNeA) Sometimes Quantifier Schema.

$$\cdot \Box(\exists\exists x_{t_1}^1 \dots x_{t_n}^n Xx^1 \dots x^n \rightarrow I_{\langle\langle t_1, \dots, t_n \rangle\rangle} \hat{x}_{t_1}^1 \dots x_{t_n}^n (\varphi)),$$

where the result of prefixing (P2-SSsNeA) with any sequence of universal quantifiers of any type (binding parameters in φ) and necessity operators, in any order, is an instance of (P2-SSsNeA).

(P3-SSsNeA) Never Quantifier Schema.

$$\cdot \Box(\forall\forall x_{t_1}^1 \dots x_{t_n}^n Xx^1 \dots x^n \rightarrow N_{\langle\langle t_1, \dots, t_n \rangle\rangle} \hat{x}_{t_1}^1 \dots x_{t_n}^n (\varphi)),$$

where the result of prefixing (P3-SSsNeA) with any sequence of universal quantifiers of any type (binding parameters in φ) and necessity operators, in any order, is an instance of (P3-SSsNeA).

(P4-SSsNeA) Thorough Serious Actualism.

It should be clear that (P1-SSsNeA) - (P4-SSsNeA) together imply $\hat{\text{Comp}}$, and that this principle is also implied by schematic versions of the Sometimes Or Always Argument and of the Sometimes Argument.

It was shown that the modal arguments for Propositional Necessitism indicate the way to analogous arguments for Higher-Order Necessitism, and indeed for principle $\hat{\text{Comp}}$. My own take is that higher-order contingentists should regard the first premises of these arguments as their weakest premises, just as in the case of the modal arguments.

These premises consist of very weak logical principles. Higher-order contingentists should thus reject the truth of claims such as the claim that necessarily, for every property X , necessarily, something is an X or everything isn't. Yet, these claims are supported by unreflective common sense. It is no wonder that they are principles of very weak logics, since they at least have the appearance of truisms. Arguably, at this point it is best to start focusing on the question what is the correct higher-order necessitist (higher-order) modal theory, leaving Higher-Order Contingentism behind.

3.9 Conclusion

The main aim of the present chapter was that of offering a defence of Propositional Necessitism. Several arguments for this thesis were presented. Since all the arguments presupposed the truth of Thorough Serious Actualism, a preliminary defence of this thesis was mounted. It was shown how Salmon's objection to Thorough Serious Actualism may be rejected. Moreover, a direct argument for Thorough Serious Actualism was offered. Finally, it was seen that Salmon appears to accept all the premises of this argument, and so to hold inconsistent commitments. The discussion revealed the plausibility of the premises of the argument for Thorough Serious Actualism, and the kinds of reasons supporting them. In particular, the argument is based on an understanding of quantification as higher-order predication.

The arguments for Propositional Necessitism here given were based on an understanding of modal expressions according to which these stand for properties of propositions. I showed that an alternative account of modal expressions, based on the notion of truth *at* a world, does not offer the means to reject the cogency of some of the arguments for Propositional Necessitism. On the contrary, the arguments go through unscathed even under this alternative account. It was also shown that a natural way of amending the account of modal expressions in terms of truth at a world, in such a way as to reject the cogency of the arguments for Propositional Necessitism, is ultimately based on an erroneous understanding of modality.

Then, it was shown that the deeper understanding of modality afforded by Kripke semantics presupposes an understanding of modalities as properties of propositions. Thus, from the standpoint of propositional contingentists the problematic assumptions of the arguments for Propositional Necessitism are their first premises, and so the truth of these premises must be rejected by propositional contingentists. Yet, the first premises of these arguments are based on principles of propositional modal logic that are often regarded as constituting very weak commitments. Moreover, eschewing these very weak principles seems to require abandoning expressive resources for talking about what might have been that are ordinarily appealed to.

Afterwards, the Classical Account was revisited, given its *prima facie* incompatibility with Propositional Necessitism. The incompatibility between the Classical Account and Propositional Necessitism arises from the fact that, according to the account, propositions are things that are intrinsically and essentially representational. But this is a problematic commitment of the account, independently of whether Propositional Necessitism is true. The view that things may represent independently of the activities of the subjects doing the representing is surely wrong.

A different problem was also addressed, namely, how may propositions have truth-conditions intrinsically and essentially, and thus independently of the activities of subjects doing the representing, and yet not be representational. The answer, I have suggested, consists in seeing truth, *qua* a property of propositions, as a species of instantiation. Properties are true or false of things and yet are not representational entities. At most, it is the predicates expressing those properties that are

representational entities. Similarly, propositions are true or false and yet are not representational entities. It is sentences that are representational. For a sentence to represent just is for it to have some proposition as its content. And for a sentence to have the first-order property of being true just is for the proposition that is (contingently) its content to have the higher-order property of being true or obtaining. For a sentence to be contingently representational just is for it to contingently express a proposition.

Finally, arguments for Higher-Order Necessitism analogous to the modal arguments for Propositional Necessitism were presented. It was also shown that schematic versions of the arguments for Higher-Order Necessitism support a stronger claim, namely, the comprehension principle $\hat{\text{Comp}}$. The cogency of the arguments for Higher-Order Necessitism and $\hat{\text{Comp}}$ is supported by considerations similar to the ones adduced in favour of the cogency of the modal arguments.

Given the defences of Propositional Necessitism and Higher-Order Necessitism just presented, the question becomes which higher-order necessitist theory is the correct higher-order modal theory. In chapter 5 it is shown that two of the main candidate higher-order necessitists theories turn out to be equivalent, despite their prima facie rivalry. The argument for the equivalence between these two theories appeals to the *Synonymy Account* of theory equivalence. The aim of the next chapter is to develop and defend this account.

4

The Synonymy Account of Theory Equivalence: Noneism and Quineanism

4.1 Introduction

The primary aim of the present chapter is to propose an account of equivalence between theories in metaphysics, the *Synonymy Account*, and to defend its adequacy.¹ The notion of theory equivalence being captured is one concerned with what theories say, i.e., concerning 'the relationship between theory and world'.² A subsidiary aim of the chapter is to apply the account to the debate in metaphysics between *noneists*, proponents of the claim that *some things do not exist*, and *Quineans*, proponents of the thesis that *to exist just is to be some thing*.

The Synonymy Account has two components. The first component consists in an *explication* of theory equivalence as *theory synonymy*. Roughly, two theories are synonymous just in case i) they assert of the same propositions that they stand in the same entailment relations, and ii) are committed to the truth of the same propositions. As shall be seen, the explication to be offered owes much to the formal work developed in (Kuhn, 1977). The second component consists in some criteria for determining when two theories are equivalent. These criteria are heavily influenced by the work developed in (Lewis, 1969, 1974, 1983), as well as in (Hirsch, 2005, 2007, 2008, 2009).

There are at least three reasons why metaphysicians should be interested in theory equivalence and the Synonymy Account. The first reason has to do with recent debates in *metametaphysics* (see, e.g., the papers in (Chalmers et al., 2009)) concerning whether metaphysical disputes are *insubstantial*, and, if so, why. Arguably, theory equivalence offers a sufficient reason for a metaphysical dispute to be insubstantial, at least on one way of understanding *insubstantial*. If two metaphysical theories turn out to be equivalent, then the debate as to which one is true is insubstantial. Thus, if the Synonymy Account of theory equivalence is correct, then it should prove useful to those interested in the debate concerning the insubstantiality of metaphysical debates.

¹The Synonymy Account is expected to be also correct account of equivalence between theories in other areas of inquiry, but this is not the focus of the present chapter.

²See (van Fraassen, 1980, ch. 4, §4).

A different reason why accounts of theory equivalence should be of interest to metaphysicians is that an improved comprehension of theory equivalence promises to afford metaphysicians with a better *understanding* of certain debates, and of what is or should be at stake in those debates. As will be shown, the Synonymy Account delivers the result that it is often more illuminating to understand what is at stake in certain metaphysical debates, such as the debate between noneists and Quineans, as concerning whether certain *expressive resources* are required in order to better describe the world.

Also, the Synonymy Account predicts that certain debates in metaphysics are better construed as concerning whether certain *theories* are true and should be accepted, instead of having to do with the truth of the particular *slogans* used to provide initial characterisations of theories. By 'slogans' what is meant is the initial description of a certain theory as, for instance, Quinean or noneist. Slogans can be misleading. For instance, the theses of Quineanism and noneism are, *prima facie*, contradictory. Yet, theories initially characterised as noneist may turn out to be equivalent to theories initially characterised as Quinean (in which case what proponents of a theory mean with these theses is not what the proponents of the other theory mean with them). Some considerations are offered, in this and in the next chapter, as to why theorists may end up meaning different things with the slogans initially used to characterise their theories.

A third reason why metaphysicians should be interested in an account of theory equivalence concerns *progress* in metaphysics. A direct way of achieving progress concerns ascertaining the truth or falsehood of one or another theory. A more indirect way of achieving progress is by ascertaining the equivalence between certain theories, since the success of a theory typically depends on how well it fares in comparison with its rivals. By appealing to an account of theory equivalence it is possible to avoid *double counting*: in general, the merits and shortcomings of a theory are also merits and shortcomings of the theories that are equivalent to it, since these theories bear the same relationship to the world. To put it differently, since equivalent theories require the same of the world to be true, the choice between equivalent theories is akin to the choice between two sentences requiring the same of the world in order to be true.

The chapter is structured as follows. In §2 the reception of noneism by Quineans is considered with the purpose of extracting some desiderata that should be satisfied by accounts of theory equivalence.

The Synonymy Account is presented in §3. First an explication of theory equivalence as Theory Synonymy is offered, as well as explications of related notions.

Then, in §4, an account of what it takes for a translation scheme to be deeply correct is given, and some principles for determining when this is so are presented. These views are coupled to the explication of theory equivalence as theory synonymy to extract the Synonymy Account.

In §5 the Synonymy Account is applied to the debate between noneists and Quineans. It is first shown that the account satisfies the desiderata laid out in §2. Afterwards, it is shown that the Synonymy Account affords a better understanding of the dialectic between noneists and Quineans and can be expected to shed light on other debates in metaphysics.

In §6 some objections to the adequacy of the Synonymy Account are addressed. Finally, in §7

further applications of the account are pointed out.

Before proceeding I will address a worry as to the relevance of the Synonymy Account. The worry concerns the relationship between theory equivalence and the two main views on the nature of theories, namely, the *syntactic* view and the *semantic* view. According to the syntactic view a theory consists in (or is adequately represented by) a set of sentences of some formal language.³ According to the semantic view a theory consists in nothing but a collection of models, where these are understood as nonlinguistic entities.⁴

Given the availability of the syntactic and the semantic views, it might be wondered if there is any need to provide an account of theory equivalence over and above the relation of being the same theory that arises from these views. The syntactic view gives rise to an account of theory equivalence according to which two theories are equivalent just in case they consist in the same set of sentences of some formal language. The semantic view gives rise to an account of theory equivalence according to which two theories are equivalent just in case they consist in the same set of models.

According to the first account there are no two theories that are both equivalent and (non-trivial) notational variants of one another.⁵ However, this is not right. It is not because ‘ \neg ’ is used for negation instead of ‘ \sim ’ and ‘ \wedge ’ is used for conjunction instead of ‘ $\&$ ’ that we thereby happen to have two non-equivalent theories.

According to the second account there are no two theories that are both equivalent and yet consist in different collections of models. But consider the collection of models consisting in all partially ordered sets such that every pair of elements has both a least upper bound and a greatest lower bound and the collection of models consisting in all algebraic structures that satisfy the commutative, associative and absorption laws. The models in the first class consist in *pairs* of a domain and a relation on that domain. Models in the second class consist of n -tuples with at least a domain and the joint and meet operations on that domain, and so all such models are sequences of three or more elements. Thus, the two collections of models are different. Yet the theories that correspond to the two collections of models are equivalent, corresponding to the theory of lattices.

Thus, theory equivalence consists in something over and above the relation of being the same theory that arises from either the syntactic or the semantic views. Hence, even if one of these views on the nature of theories is correct, an account of theory equivalence is still required.⁶ I here offer the

³The *received view*, put forward by Carnap (1956), Feigl (1970) and Hempel (1965), imposes the stronger constraint according to which theories contain only theoretical terms, which are connected to observational terms via correspondence rules. These contain both theoretical and observational terms. Here, our interest is not in the received view but just in the weaker, syntactic view. For a recent defence of the received view and its history, see (Lutz, 2012).

⁴Different proponents of the semantic view are van Fraassen (1980), Giere (1988), Suppe (1989) and Suppes (2002). Some of these theorists take theories to be set-theoretic predicates, whereas others take theories to be collections of state spaces, and even others allow models to consist of somewhat more concrete entities, such as planets and animals.

⁵Theory T_2 is a notational variant of theory T_1 just in case there is a 1 – 1 function f from the language of T_1 to the language of T_2 such that f maps atomic expressions to like atomic expressions (constants to constants, n -ary predicates to n -ary predicates, connectives to connectives, quantifiers to quantifiers, etc.), f is compositional, and $\{f(\varphi) : \varphi \text{ is a commitment of } T_1\}$ is the set of commitments of T_2 . Theory T_2 is a trivial notational variant of theory T_1 if and only if f is the identity mapping.

⁶In effect, (Halvorson, 2012) surveys three accounts of theory equivalence that would fit naturally with the semantic

Synonymy Account of theory equivalence and argue for its adequacy.

4.2 Noneism, Quineanism and Some Desiderata

Typical examples given by noneists of things that do not exist are *fictional entities*, *possibilia* and *mathematical entities*.⁷ That is, noneists hold that every fictional entity, possible and mathematical entity is something, even though no fictional entity, possible and mathematical entity exists. According to them, Santa Claus, the possible seventh son of Kripke and the number π are all something, and yet none of them exists.

Noneism has been found to be unintelligible by many philosophers (e.g., (Lycan, 1979), (van Inwagen, 1998)). These philosophers, supporters of Quineanism, claim an inability to make sense of the noneist's distinction between existence and being something. According to them, to exist *just* is to be something, and so the claim that some thing does not exist just is the claim that some thing is not some thing. Since the claim that some thing is not something is not only false but also absurd, several Quineans find noneism *unintelligible*.

There are five aspects concerning how Quineans' should understand and engage with noneism that constitute data points for an account of theory equivalence. That is, an account of theory equivalence should be able to accommodate, explain or predict these aspects. The aim of this section is to introduce these data points, which are present in the discussion of the reception of noneism by Quineans present in (Lewis, 1990), (Priest, 2011) and (Woodward, 2013).

The first aspect concerns something that has already been mentioned, namely, the fact that sometimes a theory will be understood as being *absurd* or *unintelligible*, and not just as false, by the proponents of another theory. The second aspect concerns the status of a common social language, such as English, as the means by which proponents of two theories should interpret each other. In order to flesh out what is at stake, consider the question whether the noneist should interpret the allist as meaning with 'some things do not exist' the same as what the Quinean means with 'some things do not exist', namely, the meaning of the English sentence 'some things do not exist'.

As previously mentioned, if Quineans interpret noneists in this way then they will take them to be advocating a view which is absurd or unintelligible. For this reason, Lewis claims that such interpretation of noneists is a *misinterpretation*: 'to impute contradiction gratuitously is to mistranslate' (Lewis, 1990, p. 26).

Call two words *homonymous*, in the context of the present paper, just in case they have the same spelling and pronunciation (thus, according to the way 'homonymous' will be used, homonymous words may have the same meaning). Say that an interpretation is homonymous just in case any word or sentence used by a speaker is interpreted by his interlocutor as having the same meaning as an

view and shows the inadequacy of each one of them. For a recent exchange concerning the adequacy of the semantic view, see also (Glymour, 2013) and (Halvorson, 2013). For a different sort of objection to the adequacy of the semantic view, see (Azzouni, 2014).

⁷Noneist frameworks are developed in, e.g., (Routley, 1980) and (Priest, 2005).

homonymous word or sentence of the interpreter's language. Lewis draws attention to an aspect of theorising which reveals that homonymous interpretation based on the assumption that proponents of different theories share a common language may lead to misinterpretation, even when the two theorists in fact share a language. This aspect concerns the fact that theorists also entertain views on the meaning of the expressions of their language and that these views influence the words they chose to express their commitments.

If proponents of different theories have different views on the meanings of certain expressions of their common language, and one of them chooses to express his position by appealing to some of these expressions, then homonymous interpretation is not guaranteed to lead to correct interpretation. The reason is that the interlocutor will interpret the speaker according to his own views on the meanings of the expressions of their common language, and thus the interlocutor will miss out on what is said by the speaker.

To use one of Lewis's examples, when Berkeley uses the sentence 'the tree in the quad exists' to report one of his commitments, he should not be understood as claiming that the tree in the quad exists, unless we believe, as he does, that 'the tree in the quad' denotes an idea. The problem of interpreting Berkeley homonymously is that by doing so one misunderstands Berkeley's commitments. Since Berkeley holds that everything is mental, if he were to be interpreted homonymously, then he would be understood as contradicting himself, holding at the same time that something non-mental exists (namely, the denotation of 'the tree in the quad') and that everything is mental. Since Berkeley is not contradicting himself he should not be interpreted homonymously, regardless of the fact that he is stating his view in the common language.

The second data point concerning the reception of noneism by Quineans can thus be captured by the slogan that *homonymous interpretation is not sacrosanct*. That is, homonymous interpretation based on the assumption that proponents of two different theories are speaking in a common language sometimes leads to misinterpretation, even when the two theorists are in fact speaking in a common language.

A different reason for thinking that homonymous interpretation is not sacrosanct has to do with the observation that theories come with their own *terms of art*. An interpretation of the term 'fitness', as used in biological theory, as meaning the same as 'fitness' used in the vernacular would lead to misinterpretation.

For simplicity, assume that in such cases there are two different homonymous terms here, rather than one ambiguous term. There are thus two reasons why homonymous interpretation is not sacrosanct, even when theorists are in fact speaking in a common language. The first reason is that theorists may have disagreeing views on the meanings of some terms used. The second reason is that some of the terms employed may be terms of art of the theory. These terms should not be assumed to have the same meaning as homonymous terms of the vernacular.

The third data point can be captured by the slogan that *theories are (sometimes) incommensurable*. Sometimes a theory lacks the conceptual resources to fully interpret a different theory. This point is

made with respect to the relationship between Quineanism and noneism by both Lewis (1990), a Quinean, and Priest (2011), a noneist.

Since homonymous interpretation leads to imputing a commitment to an absurdity, Lewis holds that Quineans should interpret noneists non-homonymously. He suggests that when noneists claim that Santa Claus, the seventh son of Kripke and the number π are all something, Quineans should understand them as claiming that Santa Claus, the seventh son of Kripke and the number π all exist. More generally, Lewis holds that Quineans should understand 'is something', as used by noneists, as having the same meaning as 'exists' as used by Quineans. Thus, according to him, Quineans should understand noneists as advocating *allism*, the position according to which fictional entities, possibilia, mathematical entities and the like all exist.

Importantly, Lewis holds that interpreting noneists as allists *does not suffice* to make noneism (*fully understandable*) to Quineans. He argues that (several) Quineans do not have available the linguistic resources required for understanding the noneist's use of 'exists' since, for instance, Quineans should not understand 'exists' as meaning the same as 'is present', nor as 'is actual'. The reason is that even when the noneist says that it is exactly the present or actual things that, speaking as they do, exist, he still takes this to be a substantive claim.

Thus, according to Lewis, Quineans should understand noneists as being committed to there being a certain distinction between all things, and take them to use 'exists' to mark that distinction. But this does not suffice to make Quineans fully understand the noneist position, since they do not have available the conceptual resources required to understand the noneists' use of 'exists'. That is, they cannot themselves talk about the distinction between things that is picked out by the noneists' use of 'exists'.

Priest explicitly rejects the view that Quineans should interpret the noneists' 'is something' as meaning the same as what they mean with 'exists'. Instead, he holds that Quineans should interpret 'is something' homonymously. Still, the point that the Quinean theory may just lack the resources allowing Quineans to fully understand noneists is also made by Priest. Thus, according to him,

'There is absolutely no reason why, in a dispute between noneists and Quineans, everything said by one side must be translated into terms intelligible to the other. No one ever suggested that the notions of Special Theory of Relativity need to be translated into categories that make sense in Newtonian Dynamics (or vice versa); (...). Though there may be partial overlap, each side may just have to learn a new language game.' (Priest, 2011, p. 251)

That is, according to Priest, it may just happen that the theory held by some philosophers does not afford them the resources required to fully understand a different theory. In other words, the proponents of a theory may lack the resources to fully understand a different theory in terms of the former theory's language.

Thus, Lewis and Priest both hold that Quineans lack the expressive resources to fully understand noneists. One quick remark. It is not meant by this that Lewis and Priest hold that there are expressive

resources such that, if Quineans had them, then they would be able to fully understand noneists. From the fact that Quineans do not possess the expressive resources allowing them to fully understand the noneist theory it does not follow that there are expressive resources such that Quineans do not possess them and that are such that, if Quineans possessed them, they would be able to understand the noneist.

In effect, Lewis and Priest differ in this respect. Priest holds that noneists have available more expressive resources than the ones that are available to the Quinean, whereas Lewis holds that there are no such extra expressive resources to be had. According to Lewis, the sentences of the noneists' language that Quineans cannot interpret are uninterpretable *tout court*. These sentences simply fail to express a proposition.

The fourth and fifth data points are present in Woodward's discussion of the relationship between noneism and allism. Woodward has recently argued that noneism and allism are one and the same view. He argues in the following way:

'Now imagine that we *rewrite* our noneism theory: whereas previously we said that an object exists, we now say that an object is actually concrete, and where we previously said that an object is self-identical, we now say that an object exists. No one seriously thinks that this relabelling exercise has changed anything: all we've done is rewritten the theory in a different way. But our rewritten noneist theory *just is* allism and our new quantifiers are defined in exactly the same way as Quine's!' (Woodward, 2013, p. 191)

What Woodward is alluding to in this passage is the existence of a certain translation from the noneist vocabulary to the allist vocabulary, proposed by him, where 'exists' is translated as 'actually concrete' and 'something' is translated as 'exists'. Woodward claims that this translation 'is guaranteed to always take us from truths to truths and from falsehoods to falsehoods' (Woodward, 2013), and takes this to be evidence for the claim that noneism just is allism.

The present interest is not in Woodward's claim that noneism is allism. Even though, as shall be seen, there is indeed a sense in which noneism just is allism, this claim must be qualified in ways absent in Woodward's discussion. Instead, the present interest is in two observations that fall out of Woodward's discussion. The first is the observation that *theories that would appear to be contradictory if interpreted homonymously are sometimes equivalent*. Woodward's argument, if successful, shows that noneism and allism are one such pair of theories. Furthermore, even if his argument for the equivalence of noneism and allism turns out to be unsuccessful, once it is seen that homonymous interpretation sometimes leads to misinterpretation it can be seen that there can be pairs of equivalent theories that would appear to be contradictory if homonymously interpreted.

The second data point we take from Woodward's discussion is his appeal to translations as a means of showing that 'there is *total* overlap between the conceptual resources of the two theories' (Woodward, 2013, p. 191).

Summing up, the discussion involving the Quinean's reception of noneism reveals that a good account of theory individuation ...

1. ...should predict the conditions under which it is likely for a theory to be received as absurd by the proponents of another theory;
2. ...should not have homonymous interpretation as a mandatory facet of the interpretation of the content of one theory by the proponents of another theory, even when the proponents of the two theories are members of the same linguistic community (broadly speaking);
3. ...should allow for cases in which a theory is intelligible to the proponents of another theory even though the first theory cannot be fully understood in terms of the resources afforded by the second theory;
4. ...should explain how theories that would appear to be contradictory if interpreted homonymously are sometimes equivalent, and offer the means of predicting when this will happen;
5. ...should yield conditions under which translations such as the one proposed by Woodward count in favour of the claim that 'there is total overlap between the conceptual resources of the two theories'.

4.3 The Synonymy Account

4.3.1 Formulations of Theories

The *synonymy relation* is specified in terms of what will be called a *formulation* of a theory. A formulation of a theory T consists in a triple $F_T = \langle L_T, Seq_T, Com_T \rangle$, where L_T is a language – by 'language' is meant, in this context, nothing more than a set of interpreted sentences –, Seq_T is a subset of the set of sequents of L_T , and Com_T is a subset of L_T .

The idea behind a formulation of a theory is that, whatever the ultimate nature of a theory is, a theory is *formulated* in a certain language. The set L_T is a language in which the theory is formulated. This set consists in a set of sentences, understood as meaningful strings, and so decomposable into syntactic strings and their meanings (or so it will be assumed). Moreover, it is a language in the sense that it is the language of a *community*, not just in the sense of being a collection of (interpreted) sentences.

This being said, 'language' and 'sentence' will sometimes be used in other ways. For instance, sometimes 'sentence' will be used to talk solely of the syntactic strings, their meanings being abstracted away. And sometimes 'language' will be used to speak of sets of such sentences, now understood as syntactic strings with their meanings being abstracted away. I will rely on context to disambiguate between these senses of 'sentence' and 'language'.

The set Seq_T consists in the set of sequents of L_T that the theory is committed to being entailments. That is, the set of pairs $\langle \Gamma, \varphi \rangle$, where $\Gamma \subseteq L_T$ and $\varphi \in L_T$, such that, according to the proponents of T , the propositions expressed by Γ entail the proposition expressed by φ . I will be calling Seq_T the entailment relation of formulation F_T of theory T . I will write $\Gamma \models_T \varphi$ whenever $\langle \Gamma, \varphi \rangle \in Seq_T$. Finally, the set Com_T is the set of sentences of L_T to whose truth theory T is committed. That is, Com_T is the set of sentences of L_T that, according to the proponents of T , express true propositions.

I will be calling Com_T the set of commitments of formulation F_T of theory T . In what follows I will for the most part use ‘theory T ’ instead of ‘theory given by formulation F_T ’

Two assumptions will be in place with respect to the commitments and entailment relation of F_T . The first assumption is that Com_T is the same set as the set of sentences φ such that $Com_T \models_T \varphi$. The second assumption is that the entailment relation of F_T is *Tarskian*, i.e., that it is *reflexive*, *transitive* and *monotonic*.

Before proceeding, let me note that, as with ‘sentence’, I will be using the word ‘entailment’ to talk about two different relations, namely, a relation between sentences and a relation between propositions. I will take a set Γ of sentences to entail a sentence φ just in case the propositions expressed by each $\gamma \in \Gamma$ jointly entail the proposition expressed by φ . And, as with ‘sentence’, context will make clear which one of these relations is the one under discussion. Not only will it be assumed that entailment, *qua* relation between sentences, is Tarskian, it will also be assumed that entailment, *qua* relation between propositions, is a Tarskian relation as well.⁸

For illustration, consider a first-order language L_{FL1} without identity and containing as its only non-logical expressions the constant a and the unary predicate P . An example of a formulation of a theory consists in the triple $FL1 = \langle L_{FL1}, Seq_{FL1}, Com_{FL1} \rangle$, where Seq_{FL1} is the set of all multiple premise/single conclusion sequents in language L_{FL1} which are classically valid, and $Com_{FL1} = \{ \varphi : Pa \models_{FL1} \varphi \}$.

Let me make some observations concerning theories and their formulations. The proponents of a theory T may have mistaken views on the meanings of some of the sentences of a language L_T . For instance, there could be proponents of a theory T , formulated in English, that had erroneous views on the semantics of ‘Hesperus’ and ‘Phosphorus’. According to them, ‘Hesperus’ would refer to Venus, whereas ‘Phosphorus’ would refer to Sirius A. On the described scenario, it may be supposed that Com_T contains the sentence ‘Hesperus is a planet’ and also contains the sentence ‘Phosphorus is not a planet’, even though proponents of T are not contradicting themselves in any way. Rather, they just have mistaken views on the semantics of English.

For this reason, even if L_T is an interpreted language, one must be aware of the fact that the sentences in Com_T might not, *according to the proponents of T* , express the propositions that they in fact express. Similarly, proponents of T may take the sentences occurring in the sequents in Seq_T to express different propositions than the ones actually expressed by those sentences. The synonymy relation will be sensitive to the fact that theorists may have erroneous views on the semantics of the language in which their theory is formulated.

A second observation concerns the relationship between formulations of theories and theories themselves. There are two ways in which theories and their formulations come apart. On the one hand, theories may be formulated in different languages, and so they may have different formulations. On the other hand, strictly, speaking, a formulation of a theory may also be a formulation of a *different*

⁸A relation R on $\wp(X) \times X$ is: i) *reflexive* if and only if, if $\gamma \in \Gamma$, then $\langle \Gamma, \varphi \rangle \in R$; ii) *transitive* if and only if, if $\langle \Gamma, \varphi \rangle \in R$ for all $\varphi \in \Gamma'$ and $\langle \Gamma', \psi \rangle \in R$, then $\langle \Gamma, \psi \rangle \in R$; iii) *monotonic* if and only if, if $\langle \Gamma, \varphi \rangle \in R$ and $\Gamma \subseteq \Gamma'$, then $\langle \Gamma', \varphi \rangle \in R$.

theory. For instance, two theories may contain the sentence ‘Phosphorus is a planet’ among their commitments, but because one theory is committed to Venus being a planet, whereas the other theory is committed to Sirius A being a planet.

Finally, and in connection to the discussion in the introduction, concerning the semantic view of theories, note that even formulations of theories appealing to *models* can be seen as having an underlying language, entailment relation and commitment set. Suppose that a theory is presented as a certain subclass X of the class M of models. In such case, a sentence consists in any set belonging to the powerset of M , and the language of the theory consists in the powerset of M (for the present purposes, to count as a sentence it suffices to be a representation of a way things could have been; for instance, the language in which the theory is formulated may not even be compositional). The entailment relation consists in the relation \models_T such that $\Gamma \models_T \varphi$ just in case the intersection of Γ is a subset of φ . Furthermore, the commitment set consists in the class containing all sets of models that are supersets of X .⁹

4.3.2 Theory Synonymy

Roughly, according to the Synonymy Account, two theories are equivalent just in case each has some formulation such that i) these formulations have the same *theoretical structure*, a notion made precise below, and ii) the two theories take the places in this theoretical structure to be occupied by the same propositions.

The following is a preliminary gloss on the notion of sameness of theoretical structure:

Sameness of theoretical structure (Preliminary Gloss). Theories T_1 and T_2 have the same theoretical structure just in case:

1. T_1 and T_2 possess the same entailment structure, and
2. the propositions to whose truth T_1 is committed and the propositions to whose truth T_2 is committed occupy indiscernible places in their common entailment structure.

If besides possessing the same theoretical structure the occupiers of that structure are the same, then T_1 and T_2 are in fact synonymous theories.

4.3.2.1 Entailment Structure

How does one determine the entailment structure of a theory? The following observation will be helpful later on:

⁹Arguably, this proposal can also accommodate van Fraassen’s (1980) view, according to which what is asserted by a theory is that reality can be embedded in some model of a certain set Y of models. Just let Com_T consists in all the sets of models that are supersets of the union of the set Z of sets of models that is such that z belongs to Z if and only if there is some model m in Y such that every model in z can be embedded in m .

Also, a more refined account of entailment can be given provided that a relation \cong between models telling us when two models are representationally the same – e.g., isomorphism – is available. For each sentence φ , let φ_{\cong} be that set which, for each model m in φ , contains the set of all models which bear relation \cong to m . Then, $\Gamma \models_T \varphi$ just in case $\bigcap \Gamma_{\cong}$ is a subset of φ_{\cong} , where $\bigcap \Gamma$ is the intersection of Γ . Yet a different account is possible, provided that a relation \equiv between sets of models U and V telling us when U and V are representationally the same, is available. Assuming such relation is available, then $\Gamma \models \varphi$ just in case $\bigcap \Gamma \equiv \varphi \cap \bigcap \Gamma$.

Commitments of theories. A theory T may claim, roughly, that the proposition expressed by sentence φ entails the proposition expressed by sentence ψ while not claiming that the proposition expressed by φ entails the proposition expressed by sentence χ , even though ψ and χ in fact express the same proposition.

For a rather trivial example, it might be that $a = a \models_T a = a$ and that $a = a \not\models_T a = b$, despite the fact that $a = a$ and $a = b$ express, let us assume, the same proposition. Note that this does not mean that theory T is wrongly committed to the view that $a = a$ and $a = b$ express different propositions. Also, it does not mean that, according to theory T , the proposition expressed by $a = a$ does not entail the proposition expressed by $a = b$. All it shows is that theory T is adopting no commitments with respect to whether the proposition expressed by $a = a$ entails the proposition expressed by $a = b$, nor with respect to whether the propositions expressed by, respectively, $a = a$ and $a = b$ are equivalent.

In general, the fact that $\varphi \models_T \psi$ even though $\varphi \not\models_T \chi$ does not show that T is committed to the view that the proposition expressed by φ does not entail the proposition expressed by ψ , nor to the view that the proposition expressed by ψ is not the same as the proposition expressed by χ . All it shows is that theory T is not committed to the view that the proposition expressed by φ entails the proposition expressed by ψ , and that theory T is not committed to the view that the proposition expressed by ψ is the same as the proposition expressed by χ .

For this reason, the previous gloss on the notion of theory synonymy is not correct. What is asserted by a theory T is not simply that proposition p entails proposition q . Instead, what is asserted is that the proposition that the proponent of T believes is expressed by sentence φ entails the proposition that the proponent of T believes is expressed by sentence ψ .

Theory T asserts that the proposition that is, according to the proponents of T , expressed by $a = a$ entails the proposition that is, according to the proponents of T , expressed by $a = a$. Theory T expresses no commitments as to whether the proposition that the proponents of T believe to be expressed by $a = a$ entails the proposition that the proponents of T believe to be expressed by $a = b$.

I will begin by making precise what is meant with sameness of entailment structure. First, an incorrect precisification is given. By starting this way it is possible to offer extra support for the correct notion of sameness of theoretical structure, and to gain a deeper understanding of that notion.

One initial thought consists in using *sentences* to represent what will here be called *qua propositions*. Thus, the sentence φ can be used to represent the proposition p qua the proposition that is expressed by φ according to proponents of T , and the sentence ψ can be used to represent the proposition q qua the proposition that is expressed by ψ according to (proponents of) T .

Going along for the moment with this option on how to represent qua propositions, a natural gloss on the conditions under which the entailment structure of T_1 is the same as the entailment structure of T_2 is the following:

Sameness of Entailment Structure (Incorrect). T_1 and T_2 have the same entailment structure if and only if there is a bijection f from L_{T_1} to L_{T_2} such that, $f(Seq_{T_1}) = Seq_{T_2}$.

Here, $f(Seq_{T_1}) = \{f(\langle \Gamma, \varphi \rangle) : \langle \Gamma, \varphi \rangle \in Seq_{T_1}\}$, where $f(\langle \Gamma, \varphi \rangle) = \langle f(\Gamma), f(\varphi) \rangle$ and $f(\Gamma) = \{f(\gamma) : \gamma \in \Gamma\}$. Thus, $f(Seq_{T_1})$ is to be thought of as a set of sequents in language L_{T_2} which ‘mirrors’ Seq_{T_1} . The reason why $f(Seq_{T_1})$ may be said to ‘mirror’ Seq_{T_1} is that, for each pair in Seq_{T_1} , there is a ‘mirror pair’ in $f(Seq_{T_1})$. Informally, theories T_1 and T_2 have the same entailment structure, according to the present gloss, just in case Seq_{T_2} ‘mirrors’ Seq_{T_1} .

To see why the above gloss on sameness of entailment structure is incorrect, consider theories T_I , T_{II} and T_{III} . These theories are formulated, respectively in (rudimentary) languages L_{T_I} , $L_{T_{II}}$, $L_{T_{III}}$, where the only sentences of L_{T_I} are \perp , A , B and \top , the only sentences of $L_{T_{II}}$ are \perp , C , D and \top , and the only sentences of $L_{T_{III}}$ are \perp , E , F , G and \top . Consider the following figures:

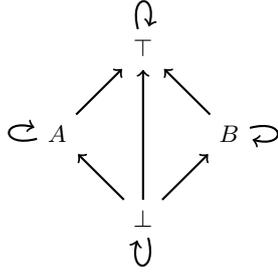


Figure 4.1: \leq_{T_I}

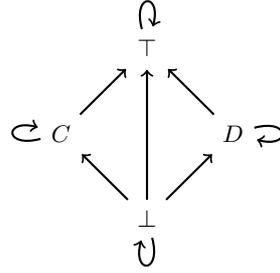


Figure 4.2: $\leq_{T_{II}}$

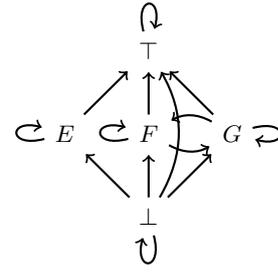


Figure 4.3: $\leq_{T_{III}}$

For every $i \in \{I, II, III\}$, let $\varphi \leq_{T_i} \psi$ if and only if there is an arrow mapping φ to ψ . Also, say that $\psi \leq_{T_i} \Gamma$ if and only if $\varphi \leq_{T_i} \gamma$, for every $\gamma \in \Gamma$. We define Seq_{T_i} as the set of pairs $\langle \Gamma, \varphi \rangle$ such that, for every $\chi \leq_{T_i} \Gamma$: $\chi \leq_{T_i} \varphi$. Thus, for instance $A, B \models_{T_I} \perp, \top$, $C \not\models_{T_{II}} D$, and $E, G, \top \models_{T_{III}} F$.

It should be immediate that there is a bijection f from L_{T_I} to $L_{T_{II}}$ such that $f(Seq_{T_I}) = Seq_{T_{II}}$. Thus, T_I and T_{II} count as having the same entailment structure by the above criterion. However, there is no bijection from $L_{T_I}/L_{T_{II}}$ to $L_{T_{III}}$. The reason is that these languages have different cardinalities to begin with.

There is some reason to think that this is the wrong result, and that, instead Seq_{T_I} , $Seq_{T_{II}}$ and $Seq_{T_{III}}$ all have the same entailment structure. Recall that, so far, the sentences of a theory T are being used to represent the (qua) propositions expressed by them according to the proponents of T . This rules out the possibility of identical qua propositions being represented by different sentences. However, proponents of a theory will want to distinguish between sentences and the propositions that they express, even if these are qua propositions. That is, proponents of a theory T may take the proposition expressed by a sentence φ to be the same as the proposition expressed by sentence ψ (independently of what the identity of this proposition happens to be, and independently of whether φ and ψ in fact express the same proposition). In such case, it is wrong to infer that the entailment structures of T_1 and T_2 are different solely on the basis that, for instance, the languages of T_1 and T_2 have different cardinalities.

The previous observation shows that, in general, there is reason to expect that the gloss on sameness of entailment structure just given will yield the wrong result. Yet, that observation does not, by itself, constitute a positive reason to think that $Seq_{T_{III}}$ has the same entailment structure as $Seq_{T_I}/Seq_{T_{II}}$.

The sameness of theoretical structure between the three theories turns out to be a consequence of the Propositional Identity Presupposition, to be introduced below. Consider first the following hypothesis:

Propositional Identity Hypothesis. Propositions p and q are the same if and only if i) for every set C of propositions, C entails p if and only if C entails q , and ii) for every set C and proposition s , C and p entail s if and only if C and q entail s .

Given the Propositional Identity Hypothesis and the assumption that entailment (qua relation between propositions) is Tarskian, it follows that two propositions are identical just in case each entails the other.

Thus, the Propositional Identity Hypothesis, together with the assumption that Seq_T is Tarskian, allows an to appeal to the following presupposition:

Propositional Identity Presupposition. For each theory T , $\varphi \stackrel{T}{\dashv\vdash} \psi$ only if the proposition expressed by φ according to the proponents of T is the same as the proposition expressed by ψ according to the proponents of T .

By the Propositional Identity Hypothesis, two propositions are the same if and only if they are mutually entailing. According to the Propositional Identity Presupposition, if theorists take sentences S and S' to express mutually entailing propositions, then, a fortiori, S and S' express, according to them, one and the same proposition. They treat the sentences as expressing the same proposition, and so the sentences express, according to them, the same proposition.

Consider the relations Seq_{T_I} ($Seq_{T_{II}}$) and $Seq_{T_{III}}$ once more. Despite the fact that there is no bijection from L_{T_I} ($L_{T_{II}}$) to $L_{T_{III}}$, there is a bijection f from the propositions that are, according to the proponents of T_I/T_{II} , expressed by the sentences of L_{T_I} ($L_{T_{II}}$) to the propositions that are, according to the proponents of T_{III} , expressed by the sentences of $L_{T_{III}}$.

Let f be a function that maps i) the proposition that, according to the proponents of T_I , is expressed by \perp to the proposition that is, according to the proponents of T_{III} , expressed by \perp , ii) the proposition that, according to the proponents of T_I , is expressed by A to the proposition that, according to the proponents of T_{III} , is expressed by E , iii) the proposition that, according to the proponents of T_I , is expressed by B to the proposition that, according to the proponents of T_{III} , is expressed by F (i.e., to the proposition that, according to the proponents of T_{III} , is expressed by G), and iv) the proposition that, according to the proponents of T_I , is expressed by \top to the proposition that, according to the proponents of T_{III} , is expressed by \top . Insofar as it is the case that, for all sentences φ, ψ in the set $\{\perp, E, F, \top\}$, $\varphi \dashv\vdash_{T_{III}} \psi$ only if $\varphi = \psi$, f is a one to one function. And insofar as $F \dashv\vdash_{T_{III}} G$, the function f is also onto, and is thus a bijection.

Furthermore, it should be clear that the propositions that, according to the proponent of T_I , are expressed by the sentences in Γ entail, according to T_I , the proposition that, according to the proponent of T_I , is expressed by the sentence φ , if and only if the propositions that, according to the proponent of T_{III} , are expressed by the sentences in $f(\Gamma)$ entail, according to T_{III} , the proposition

that, according to the proponent of T_{III} , is expressed by the sentence $f(\varphi)$. Thus, it seems reasonable to conclude that T_I has the same entailment structure as T_{III} . It is easy to see that a similar function can be found witnessing the sameness of entailment structure between T_{II} and T_{III} .

This suggests that the qua propositions that a theory T is about are adequately represented not via the sentences of L_T , but instead via sets of sentences of L_T . Let $[\varphi]_{\models_T}$ be the set of sentences ψ of L_T such that $\varphi \models_T \psi$.¹⁰ Then, the qua propositions that T is about can be represented by the set containing $L_T / \models_T = \{[\varphi] : \varphi \in L_T\}$, the quotient set of L_T relative to \models_T . The entailments asserted by theory T to obtain between qua propositions can be captured by the entailment relations asserted by T to obtain between the sets of sentences that, according to the proponent of T , express those propositions. Let $[\Gamma] = \{\{\gamma\} : \gamma \in \Gamma\}$, and $Seq_T^{\models_T} = \{([\Gamma], [\varphi]) : \Gamma \models_T \varphi\}$. The entailment relations asserted by theory T to obtain between qua propositions can thus be captured by the relation $Seq_T^{\models_T}$.

Consider an example. For each $i \in \{I, II, III\}$, let $[\varphi] \leq_{T_i}^{\models_T} [\psi]$ if and only if $\varphi \models_{T_i} \psi$. The following figures provide a representation of $\leq_{T_I}^{\models_T}$, $\leq_{T_{II}}^{\models_T}$ and $\leq_{T_{III}}^{\models_T}$:

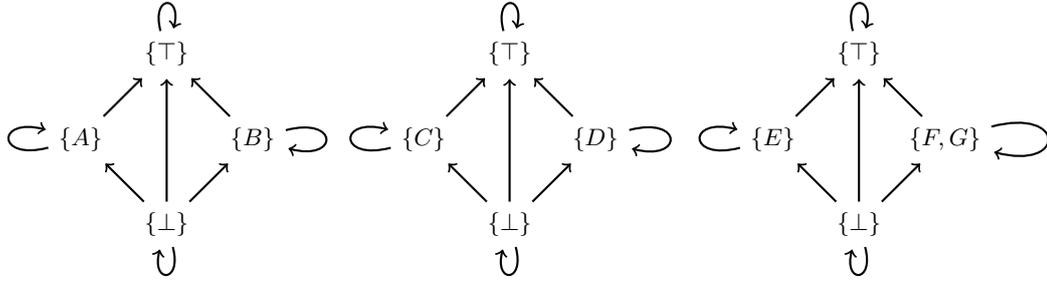


Figure 4.4: $\leq_{T_I}^{\models_T}$

Figure 4.5: $\leq_{T_{II}}^{\models_T}$

Figure 4.6: $\leq_{T_{III}}^{\models_T}$

The diagrams also allow for the representation of the relations $Seq_{T_I}^{\models_T}$, $Seq_{T_{II}}^{\models_T}$ and $Seq_{T_{III}}^{\models_T}$. The reason is that $[\Gamma] \models_{T_i} [\varphi]$ if and only if for every $\psi \leq \Gamma$, $[\psi] \leq_{T_i}^{\models_T} [\varphi]$. Thus, insofar as $Seq_{T_I}^{\models_T}$, $Seq_{T_{II}}^{\models_T}$ and $Seq_{T_{III}}^{\models_T}$ are adequate representations of the entailment relations between qua propositions asserted by, respectively, theories T_I , T_{II} and T_{III} , it should be evident that T_I , T_{II} and T_{III} all have the same entailment structure.

This observation is made more precise by appealing to the notion of similarity:

Definition (Similarity). T_1 and T_2 are similar, $T_1 \sim T_2$, if and only if there is a bijection f from L_{T_1} / \models_T to L_{T_2} / \models_T such that, $f(Seq_{T_1}^{\models_T}) = Seq_{T_2}^{\models_T}$.

The notion of *similarity* is already defined in (Kuhn, 1977). The present proposal, not unrelated to that of Kuhn's, is to precisify sameness of entailment structure in the following way:

Sameness of Entailment Structure (First Version). T_1 and T_2 have the same entailment structure if and only if $T_1 \sim T_2$.

¹⁰We will omit T in \models_T when confusion is unlikely to arise. Similarly, we omit \models_T in $[\cdot]_{\models_T}$ when confusion is unlikely to arise.

Again following Kuhn, let me introduce a notion related to that of similarity, except that this notion appeals directly to mappings between sentences of L_{T_1} and L_{T_2} :

Definition (Similarity via f and g). Let $f : L_{T_1} \rightarrow L_{T_2}$ and $g : L_{T_2} \rightarrow L_{T_1}$. T_1 and T_2 are similar via f and g , $T_1 \overset{f,g}{\sim} T_2$ if and only if:

1. For every $\Gamma \subseteq L_{T_1}$ and every $\varphi \in L_{T_1}$: $\Gamma \models_{T_1} \varphi$ only if $f(\Gamma) \models_{T_2} f(\varphi)$
2. For every $\Gamma \subseteq L_{T_2}$ and every $\varphi \in L_{T_2}$: $\Gamma \models_{T_2} \varphi$ only if $g(\Gamma) \models_{T_1} g(\varphi)$;
3. For every $\varphi \in L_{T_1}$: $\varphi \not\models_{T_1} g(f(\varphi))$;
4. For every $\varphi \in L_{T_2}$: $\varphi \not\models_{T_2} f(g(\varphi))$.

By a (small) generalisation of the result reported in (Kuhn, 1977, p. 69), it can be shown that $T_1 \sim T_2$ if and only if there are functions f and g such that $T_1 \overset{f,g}{\sim} T_2$, assuming that both Seq_{T_1} and Seq_{T_2} are Tarskian.¹¹ This allows us to provide a second, equivalent explication of sameness of entailment structure, namely:

Sameness of Entailment Structure (Second Version). T_1 and T_2 have the same entailment structure if and only if there are functions f and g such that $T_1 \overset{f,g}{\sim} T_2$.

Consider once more theories T_I , T_{II} and T_{III} . We can easily inspect that T_I , T_{II} and T_{III} are all similar to each other. Just let the ‘bottom element’ of each one of $\leq_{T_I}^{\models}$, $\leq_{T_{II}}^{\models}$ and $\leq_{T_{III}}^{\models}$ be mapped to the bottom element of the other structure, the ‘left element’ be mapped to the ‘left element’, the ‘right element’ to the ‘right element’ and the ‘top element’ to the ‘top element’. Thus, the explication of sameness of entailment structure just offered successfully counts T_I , T_{II} and T_{III} as having the same entailment structure.

For examples of theories that are not similar, let $L_{Cl} = L_{Int}$ be a propositional language with logical constants $\neg, \vee, \wedge, \rightarrow$, and whose only non-logical constant is the propositional letter A . Now, let Seq_{Cl} be the set of valid sequents of classical propositional logic in language L_{Cl} and Seq_{Int} be the set of valid sequents of intuitionist propositional logic in language L_{Int} . Even though there are only four elements in L_{Cl}/\models_{Cl} , namely, the elements of the Lindenbaum algebra on one generator for classical logic, there are infinitely many elements in L_{Int}/\models_{Int} , the elements of the Lindenbaum algebra on one generator for intuitionistic logic (i.e., the elements of the Rieger-Nishimura lattice). Hence, Cl and Int are not similar, and thus do not count as having the same entailment structure. This is the intuitively right result.

Kuhn (1977, p. 73-79) also characterises a different relation between theories, the relation of being a fragment:

Definition (Fragment.). Let $f : L_{T_1} \rightarrow L_{T_2}$. T_1 is a fragment of T_2 via f , $T_1 \overset{f}{<} T_2$ if and only if:

¹¹The notion of similarity via f and g is also defined in (Seegerberg, 1982, p. 43), where it is called *syntactic equivalence via f and g* . Pelletier & Urquhart (2003, p. 263) define the notion of translational equivalence. Translational equivalence is quite close to similarity via f and g , except that Pelletier and Urquhart impose the restriction that f and g must be translation schemes. They obtain a notion also defined in (Kuhn, 1977, p. 80), which is called there simply *equivalence via f and g* .

For every $\Gamma \subseteq L_{T_1}$ and every $\varphi \in L_{T_1}$: $\Gamma \models_{T_1} \varphi$ if and only if $f(\Gamma) \models_{T_2} f(\varphi)$.
 T_1 is a fragment of T_2 , $T_1 < T_2$, if and only if there is some f such that $T_1 \overset{f}{<} T_2$.

I will make use of a relation somewhat more stringent than the relation of being a fragment. Let $(Seq_T)^+ = \{\varphi \in L_T : \forall \psi \in L_T (\varphi \models_T \psi \Rightarrow \psi \models_T \varphi)\}$ and $(Seq_T)^- = \{\varphi \in L_T : \forall \psi \in L_T (\psi \models_T \varphi \Rightarrow \varphi \models_T \psi)\}$. The sets $(Seq_T)^+$ and $(Seq_T)^-$ represent, respectively, the set of maximal and the set of minimal propositions according to the entailment ordering.

Then,

Definition (Stringent Fragment.). Let $f : L_{T_1} \rightarrow L_{T_2}$. T_1 is a stringent fragment of T_2 via f , $T_1 \overset{f}{\sqsubset} T_2$, if and only if:

1. $T_1 \overset{f}{<} T_2$,
2. $f((Seq_{T_1})^+) \subseteq (Seq_{T_2})^+$, and
3. $f((Seq_{T_1})^-) \subseteq (Seq_{T_2})^-$.

T_1 is a stringent fragment of T_2 , $T_1 \sqsubset T_2$ if and only if there is a function f such that $T_1 \overset{f}{\sqsubset} T_2$.

In what follows I will use ‘sfragment’ instead of ‘stringent fragment’. In order for a theory to count as a sfragment of another theory it is not enough for the first theory to be a fragment of the second theory. It is also required that all the minimal and all the maximal elements in the entailment structure of the first theory be mapped to, respectively, minimal and maximal fragments of the second theory.

This requirement concerns the fact that minimal and maximal elements may be understood as having a special status in a theory, to wit, minimal elements correspond to as propositions which, according to the theorist, are absurd, and maximal elements as propositions which, according to the theorist, are trivial.

The notion of a sfragment affords the resources to explicate a different relationship between the entailment structures of two theories, namely:

Inclusion of Entailment Structure. The entailment structure of T_2 includes the entailment structure of T_1 if and only if $T_1 \sqsubset T_2$.

Consider again theories T_{Cl} and T_{Int} . As noted, it is not the case that these theories are similar. However, $T_{Cl} \sqsubset T_{Int}$. That is, T_{Cl} is a sfragment of T_{Int} . One of the functions witnessing this fact is the function f that, for every $\varphi \in L_{T_{Cl}}$, maps φ to itself. On the other hand, T_{Int} is not a fragment of T_{Cl} , i.e., $T_{Int} \not\sqsubset T_{Cl}$. Again, both of these results are intuitively correct.

4.3.2.2 Theoretical Structure

With the characterisation of sameness of entailment structure in place, the explication of sameness of *theoretical structure* may now be offered. The relevant definition is that of *solid similarity*. Let $Com_T^{\models} = \{[\varphi] : \varphi \in Com_T\}$. Then:

Definition (Solid Similarity.). T_1 and T_2 are solidly similar, $T_1 \approx T_2$, if and only if there is a bijection f from L_{T_1} / \models to L_{T_2} / \models such that, $f(Seq_{T_1}^{\models}) = Seq_{T_2}^{\models}$ and $f(Com_{T_1}^{\models}) = Com_{T_2}^{\models}$.

Our proposal is to explicate sameness of theoretical structure in the following way:

Sameness of Theoretical Structure (First Version). T_1 and T_2 have the same *theoretical structure* if and only if $T_1 \approx T_2$.

If such mapping witnessing the similarity between theories T_1 and T_2 exists, then the (qua) propositions to whose truth T_1 is committed are indistinguishable from the (qua) propositions to whose truth T_2 is committed vis-à-vis T_1 and T_2 's common entailment structure.

For some examples, consider once more the theories T_I , T_{II} and T_{III} . Let $Com_{T_I} = \{A, \top\}$, $Com_{T_{II}} = \{\top\}$ and $Com_{T_{III}} = \{F, G, \top\}$. By appealing to the previous representations of $\leq_{T_I}^{\models}$, $\leq_{T_{II}}^{\models}$ and $\leq_{T_{III}}^{\models}$ we can represent the theoretical structures of T_I , T_{II} and T_{III} , doing so by representing the sets $Com_{T_I}^{\models}$, $Com_{T_{II}}^{\models}$ and $Com_{T_{III}}^{\models}$ with the points in the corresponding structure that are inside the dotted lines:

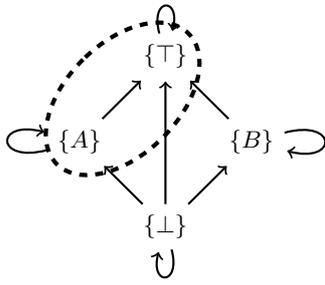


Figure 4.7: Theoretical structure of T_I

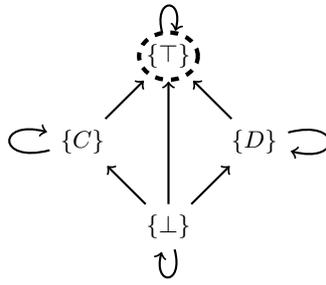


Figure 4.8: Theoretical structure of T_{II}

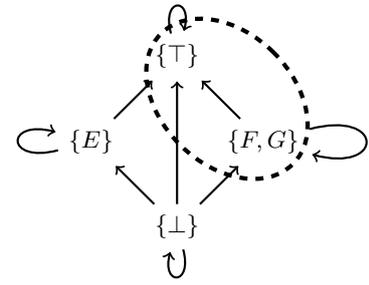


Figure 4.9: Theoretical structure of T_{III}

There are two bijections from L_{T_I}/\models to $L_{T_{II}}/\models$ witnessing the similarity between T_I and T_{II} , namely, the bijection that maps $\{A\}$ to $\{C\}$ and the bijection that maps $\{A\}$ to $\{D\}$. In both of these cases $\{A\}$ is mapped to a set that does not belong to $Com_{T_{II}}^{\models}$, even though $\{A\}$ belongs to $Com_{T_I}^{\models}$. This shows that no proposition asserted to be the case by T_{II} occupies a role in T_{II} 's entailment structure that is indiscernible from the role occupied by the proposition that is expressed by A , according to the proponent of T_I , in T_I 's entailment structure. For instance, the proposition expressed by A entails some other proposition according to T_I even though there is no proposition to whose truth T_{II} is committed which entails, according to T_{II} , some other proposition, since T_{II} is committed only to the truth of one proposition, namely, the proposition that is expressed by \top . Thus, $T_I \not\approx T_{II}$. Hence, T_I and T_{II} do not count as having the same theoretical structure according to the present criterion.

On the other hand, according to the present proposal, T_I and T_{III} do share the same theoretical structure. The bijection f from L_{T_I}/\models to $L_{T_{III}}/\models$ witnessing the similarity between T_I and T_{III} which maps $\{A\}$ to $\{F, G\}$ is such that $f(Com_{T_I}^{\models}) = Com_{T_{III}}^{\models}$. Intuitively, this is the correct result. If anything breaks the equivalence between theories T_I and T_{III} , it must be something having to do with how the proponents of these two theories interpret their respective languages.

Now, where f is any function from L_{T_1} to L_{T_2} , let:

- $T_1 \stackrel{f}{\triangleleft} T_2$ if and only if, $\forall \varphi \in Com_{T_1} \exists \psi \in Com_{T_2}$ such that $f(\varphi) \models_{T_2} \psi$;

One can also define a notion close to that of solid similarity, except that it appeals directly to mappings between the sentences of the languages of T_1 and T_2 :¹²

Definition (Solid Similarity via f and g). Let $f : L_{T_1} \rightarrow L_{T_2}$ and $g : L_{T_2} \rightarrow L_{T_1}$. T_1 and T_2 are solidly similar via f and g , $T_1 \stackrel{f,g}{\approx} T_2$, if and only if:

1. $T_1 \stackrel{f,g}{\approx} T_2$;
2. $T_1 \stackrel{f}{\triangleleft} \stackrel{g}{\triangleright} T_2$.

By appealing to the notion of solid similarity via functions f and g it is possible to explicate sameness of theoretical structure in an alternative albeit equivalent way:

Sameness of Theoretical Structure (Second Version). T_1 and T_2 have the same theoretical structure if and only if there are functions f and g such that: $T_1 \stackrel{f,g}{\approx} T_2$

Let us return to the preliminary gloss on sameness of theoretical structure, which stated that T_1 and T_2 have the same theoretical structure just in case a) T_1 and T_2 possess the same entailment structure, and b) the propositions to whose truth T_1 is committed and the propositions to whose truth T_2 is committed occupy indiscernible places in their common entailment structure. It has been shown how to make this talk of sameness of entailment structure and of occupation of indiscernible places in an entailment structure more precise. Theories T_1 and T_2 have the same entailment structure just in case $T_1 \sim T_2$. The proposition expressed by a sentence φ to whose truth T_1 is committed occupies a place that is indiscernible from the one occupied by the proposition expressed by a sentence ψ to

¹²The following proofs establish that T_1 and T_2 are solidly similar if and only if T_1 and T_2 are solidly similar via functions f and g .

Proof. [$T_1 \stackrel{f,g}{\approx} T_2$ implies $T_1 \sim T_2$] Suppose $T_1 \stackrel{f,g}{\approx} T_2$ and define $h : L_{T_1} / \models \cup L_{T_2} / \models \rightarrow L_{T_1} \cup L_{T_2}$ in such a way that $h([\varphi]) = [f(\varphi)]$. Then, h is a bijection witnessing $T_1 \sim T_2$, by a small generalisation of the result shown in (Kuhn, 1977, pp. 69-70). It will now be shown that i) $h(Com_{T_1}^{\models}) \subseteq Com_{T_2}^{\models}$ and ii) $Com_{T_2}^{\models} \subseteq h(Com_{T_1}^{\models})$.

i) Suppose $x \in h(Com_{T_1}^{\models})$. Then, $x = h([\varphi])$, for some $\varphi \in Com_{T_1}$. So, $x = [f(\varphi)]$. By $T_1 \stackrel{f}{\triangleleft} T_2$, there is a $\psi \in L_{T_2}$ such that $f(\varphi) \models_{T_2} \psi \in Com_{T_2}$. Hence $x = h([\varphi]) = [f(\varphi)] \in Com_{T_2}^{\models}$. So, $h(Com_{T_1}^{\models}) \subseteq Com_{T_2}^{\models}$.

ii) Suppose $x \in Com_{T_2}^{\models}$. Then, $[x] = [\varphi]$, for some $\varphi \in Com_{T_2}$. By $T_2 \stackrel{g}{\triangleleft} T_1$, there is a $\psi \in Com_{T_1}$ such that $g(\varphi) \models_{T_1} \psi$, and thus $g(\varphi) \in Com_{T_1}$. Hence, $f(g(\varphi)) \in f(Com_{T_1})$. So, $\varphi \in f(Com_{T_1})$, by $T_1 \stackrel{f,g}{\approx} T_2$. Therefore, $[\varphi] \in [f(Com_{T_1})] = h(Com_{T_1}^{\models})$. Hence, $Com_{T_2}^{\models} \subseteq h(Com_{T_1}^{\models})$. \square

Proof. [$T_1 \sim T_2$ implies $T_1 \stackrel{f,g}{\approx} T_2$] Suppose $T_1 \sim T_2$. Let h be any bijection witnessing $T_1 \sim T_2$. Let $ch : L_{T_1} / \models \cup L_{T_2} / \models \rightarrow L_{T_1} \cup L_{T_2}$ be any function such that $ch([\varphi]) \in [\varphi]$. Define $f : L_{T_1} \rightarrow L_{T_2}$ and $g : L_{T_2} \rightarrow L_{T_1}$ in such way that $f(\varphi) = ch(h([\varphi]))$ and $g(\varphi) = ch(h^{-1}([\varphi]))$. Then, $T_1 \stackrel{f,g}{\approx} T_2$, by a small generalisation of the result shown in (Kuhn, 1977, pp. 69-70). It will now be shown that i) $T_1 \stackrel{f}{\triangleleft} T_2$ and ii) $T_2 \stackrel{g}{\triangleleft} T_1$.

i) Suppose that $\varphi \in Com_{T_1}$. Then, $[\varphi] \in Com_{T_1}^{\models}$. So, $h([\varphi]) \in h(Com_{T_1}^{\models}) = Com_{T_2}^{\models}$, by $T_1 \sim T_2$, by $T_1 \sim T_2$. Thus, $f(\varphi) = ch(h([\varphi])) \in Com_{T_2}$. Hence, $T_1 \stackrel{f}{\triangleleft} T_2$.

ii) Suppose that $\varphi \in Com_{T_2}$. Then, $[\varphi] \in Com_{T_2}^{\models} = h(Com_{T_1}^{\models})$, by $T_1 \sim T_2$. So, $h^{-1}([\varphi]) \in Com_{T_1}^{\models}$. Thus, $g(\varphi) = ch(h^{-1}([\varphi])) \in Com_{T_1}$. Hence, $T_2 \stackrel{g}{\triangleleft} T_1$. \square

whose truth T_2 is committed just in case $\varphi \in Com_{T_1}$, $\psi \in Com_{T_2}$, $f(\varphi) = \psi$ and $g(\psi) = \varphi$ for some functions f and g witnessing the solid similarity between T_1 and T_2 .

4.3.2.3 Theory Synonymy

Consider the following notions, that of a *correct translation scheme* and a *deeply correct translation scheme*:

Definition (Correct and Deeply Correct Translation Schemes.).

- A function f from L_{T_1} to L_{T_2} is a *correct translation scheme* if and only if, for all $\varphi \in L_{T_1}$, φ and $f(\varphi)$ express the same proposition.
- A function f from L_{T_1} to L_{T_2} is a *deeply correct translation scheme* if and only if, for all $\varphi \in L_{T_1}$, the proposition that, according to the proponents of T_1 , is expressed by φ is the same as the proposition that, according to the proponent of T_2 , is expressed by $f(\varphi)$.

Synonymy between theories is defined as follows:¹³

Definition (Theory Synonymy.). T_1 and T_2 are *synonymous via functions f and g* , $T_1 \stackrel{f,g}{\equiv} T_2$, if and only if $T_1 \stackrel{f,g}{\approx} T_2$ and both f and g are *deeply correct translation schemes*.

T_1 and T_2 are *synonymous* if and only if there are functions f and g such that $T_1 \stackrel{f,g}{\equiv} T_2$.

The main proposal of the present chapter is to explicate theory equivalence via theory synonymy:¹⁴

Theory Equivalence is Theory Synonymy. Theories T_1 and T_2 are equivalent if and only if there are formulations F_{T_1} of T_1 and F_{T_2} of T_2 such that $F_{T_1} \equiv F_{T_2}$.

The reason why theory synonymy is characterised in terms of *deeply correct translation schemes* rather than *correct translation schemes* has to do with Lewis's observations mentioned in §4.2. As was shown there, the interpretation of a theory needs to be sensitive to what proponents of a theory intend to express with the sentences used in their formulation of the theory.¹⁵

Consider again theories T_I and T_{III} . In order to determine whether these theories are synonymous it is not sufficient to consider whether they are solidly similar (and so, whether they have the same theoretical structure). The reason is that proponents of T_I might mean with A something quite different than what proponents of T_{III} mean with F and G . In effect, it might be that what proponents of T_I mean with A is that dinosaurs are extinct, whereas what proponents of T_{III} mean with F and G is that dinosaurs are not extinct. In such case, even though the two theories have the same theoretical structure, they are not synonymous, and thus they are not equivalent. Still, if two theories have a

¹³There is an equivalent formulation of theory synonymy that appeals to bijections witnessing the solid similarity between T_1 and T_2 . However, the present formulation will suffice for our purposes.

¹⁴It will now be relevant to distinguish between theories and their formulations, and so I will do just that.

¹⁵In order to accommodate both semantic indeterminacy and epistemic indeterminacy, the notions of a correct and a deeply correct translation scheme could be generalised, in such a way that sentences express not just one proposition, but instead sets of propositions. In what follows I remain with the simplifying assumption.

different entailment structure this already shows that they are not equivalent. There is no need to consider what their proponents mean with the sentences of their languages.

One of the aims of appealing to solid similarity was that of having a minimally satisfactory necessary condition for theory equivalence which did not require interpretation of the theory's language. Even though there is a sense in which this is indeed the case, note that interpretation is still required at some level. Interpretation plays a crucial role when determining the entailment relation of each theory. It is a tacit assumption of our proposal that when, according to a theory T_1 , Γ entails φ and according to a theory T_2 , Δ entails ψ , the same is meant with the two occurrences of 'entails'.

What is entailment? Entailment is here taken to be a Tarskian relation. It is also assumed that the commitments of a theory are closed under entailment. As a further characterisation, entailment is taken to be necessarily truth-preserving. If Γ entails φ , then necessarily, if all the propositions expressed by all sentences in Γ are true, so is the proposition expressed by φ . In addition, as has been implicit throughout, entailment is assumed to be a relation that holds between classes of propositions and propositions. Finally, entailment is assumed to be such that the hypothesis of propositional identity is true.¹⁶ Besides this, I do not have much more to offer by way of characterisation of the notion. Entailment is a primitive notion of the Synonymy Account of theory equivalence.

Finally, the structural relation of being a stringent fragment via f, \sqsubset^f , in conjunction with the notion of a deeply correct translation, gives rise to the notion of *embeddability*, which will play a role later on:

Definition (Embeddability.). *A theory T_1 is embeddable in theory T_2 just in case there is a deeply correct translation f such that $T_1 \sqsubset^f T_2$.*

This concludes the presentation of the first component of the Synonymy Account of theory equivalence, namely the explication of theory equivalence as Theory Synonymy. The second component of the Synonymy Account consists in some criteria for determining when two translation schemes are deeply correct. Before turning to those criteria, let me briefly offer an account of what it takes for φ to express proposition p according to the proponents of theory T .

4.3.2.4 According To

One approach to explicating what it is for φ to express proposition p in language L according to agent x is, roughly, by appealing to the idea that x believes that he is conforming to a convention of truthfulness and trust in L with respect to φ by treating φ as meaning p . More precisely, consider the following hypotheses:

CF1: Ordinarily, speakers of L believe p when asserting φ ;

CF2: Ordinarily, hearers of L that do not yet believe p come to do so when φ is asserted to them;

CF3: The members of the community of speakers C_L of L believe that CF1 and CF2 hold;

¹⁶This is less helpful than one may think, though. Arguably, 'proposition' can itself be made precise in different ways.

- CF4:** The expectation that CF1 and CF2 will continue to be true gives the members of C_L a good reason to continue to utter φ only if they believe p , and a good reason to expect the same of the other members;
- CF5:** There is among the members of C_L a general preference for them to continue to conform to regularities CF1 and CF2.

Then, According to agent x , φ expresses proposition p in L just in case:

- ACF1:** Ordinarily, x believes p when asserting φ ;
- ACF2:** Ordinarily, if x does not believe p then he comes to do so when φ is asserted to him;
- ACF3:** x believes that CF1 and CF2 hold;
- ACF4:** The expectation that CF1 and CF2 will continue to be true gives x a good reason to continue to utter φ only if he believes p , and a good reason to expect the same of the other members of C_L ;
- ACF5:** x has a general preference for the members of C_L to continue to conform to regularities CF1 and CF2.
- ACF6:** x believes that it is known by the members of C_L that CF1-CF5 obtain, and x believes that the members of C_L all know that it is known that CF1-CF5 obtain, etc.

One important remark before turning to criteria for determining the deep correctness of translation schemes. Even if this is not the correct explication of 'according to agent x , φ expresses proposition p in L ', it should be clear that there is a distinction between the proposition expressed by a sentence in a language L and the proposition that the sentence expresses in L according to an agent x . For the purposes of the Synonymy Account, the latter notion may be left as a primitive of the account.

4.3.3 Deeply Correct Translation Schemes

One difficulty with determining whether theories T_1 and T_2 are synonymous has to do with the fact that the information contained in F_{T_1} and F_{T_2} does not, on its own, suffice to determine whether functions $f : L_{T_1} \rightarrow L_{T_2}$ and $g : L_{T_2} \rightarrow L_{T_1}$ are deeply correct translations. Take any sentence φ of L_{T_1} . The problem is that the proposition which, according to the proponents of T_1 , is the meaning of φ need not be the actual semantic value of φ . Similarly, the proposition that is, according to the proponents of T_2 , the semantic value of $\psi \in L_{T_2}$ need not be the actual semantic value of ψ .

Consider a language L'_{T_i} syntactically just like L_{T_i} and such that the semantic value of each of its sentences φ is the proposition which, according to the proponents of T_i , is expressed by the syntactically identical sentence φ of L_{T_i} (where $i = \{1, 2\}$). The question whether f and g are deeply correct translations can be substituted by the question whether $f' : L'_{T_1} \rightarrow L'_{T_2}$ and $g' : L'_{T_2} \rightarrow L'_{T_1}$ are correct translations.

At this point the problem becomes how to determine whether a translation is correct. One way to do so consists in determining whether the translation is what I have called a *convention-friendly translation*:

Definition (Convention-Friendly Translation.).

Let L_1 be a language of a linguistic community C_1 and L_2 be a language of a linguistic community C_2 . Also, let $L \leq L'$ if and only if L' is a superlanguage of L – i.e., a language which includes all the sentences of L , with the same meanings as the ones those sentences have in L – which is also a language of the community of speakers of L .

A translation f mapping L_1 to L_2 is a convention-friendly translation if and only if there could be a language L such that:

1. $L_2 \leq L$;
2. There is a correct description of the beliefs, desires and intentions of the members of C_1 in L ;
3. This description, in conjunction with the translation of L_1 given by f , yields a description, in L , of the linguistic practices of C_1 as a community of speakers conforming to a convention of truthfulness and trust in the used fragment of L_1 (where the convention of truthfulness and trust is understood as characterised in (Lewis, 1983)).¹⁷

Lewis (1983, p. 167) offers the following characterisation of what it is for a community to be truthful and trusting in a language L :

‘To be truthful in L is to act in a certain way: to try never to utter any sentences of L that are not true in L . Thus, it is to avoid uttering any sentence of L unless one believes it to be true in L . To be trusting in L is to form beliefs in a certain way: to impute truthfulness in L to others, and thus to tend to respond to another’s utterances of any sentence of L by coming to believe that the uttered sentence is true in L .’

Let me illustrate what it is for a translation to be convention-friendly with a simple example. Suppose that we have a true description, in English, of the beliefs, intentions and desires of the community of speakers of French. Suppose also that we have a translation of French into English according to which the sentence ‘le chat est sur le paillason’ is translated as ‘Paris is located in England’. Furthermore, suppose that in the large majority of the occasions in which a speaker of French utters ‘le chat est sur le paillason’, he intends to communicate that the cat is on the mat. In such case the translation is not convention-friendly. The reason is that the description that we obtain in English is not one in which the sentence is commonly uttered by speakers of French when they believe that Paris is located in England. Furthermore, as the example shows, the translation is in fact *incorrect*.

The following principle offers guidance in determining the correctness of a translation in terms convention-friendliness:

Convention-Friendliness Principle. If a plausible candidate for being a correct translation scheme f from L_1 to L_2 is a convention-friendly translation, and all the other translations from L_1 to L_2 that are plausible candidates for being correct translations from L_1 to L_2 are not convention-friendly, then this fact is an excellent reason to believe that f is a correct translation.

¹⁷See also (Lewis, 1969).

Determining whether a translation is convention-friendly is in part a matter of determining the beliefs, intentions and desires of the members of the community of speakers of the source language. Two principles that help in this task are the *rationalisation principle* and the *principle of charity*, proposed in (Lewis, 1974). In a nutshell, according to the *rationalisation principle* the agent should be represented as rational, in such a way that the agent's physical description, as well as the system of beliefs and desires assigned to him, jointly offer explanations of the agent's behaviour that conform to the canons of decision theory. And according to the principle of charity, roughly, we should assign to the agent those beliefs that we would have had if we had been exposed to the same evidence and training of the agent, and the same desires that we would have had if we had the agent's beliefs, training and history. These are principles to which one can appeal in order to evaluate whether a certain translation is convention-friendly.¹⁸

Let me briefly explain why the notion of convention-friendliness requires that the community of speakers conforms to a convention of truthfulness and trust *only* in the used part of L_1 . Suppose that a translation is convention-friendly for all of L_1 . It can nonetheless fail to be a correct translation because a convention-friendly translation may assign the wrong propositions to some of the sentences of the *unused* part of the language. In particular, it will assign the wrong propositions to at least some of those unused, very long and complicated sentences of the language. The problem is, as Lewis notes, that if a speaker were to use such strings, then he would not be *trusted*. Rather, he would be understood as 'trying to win a bet or set a record, or feigning madness or raving for real, or doing it to annoy, or filibustering, or making an experiment to test the limits of what is humanly possible to say and mean' (Lewis, 1992, p. 108). For this reason, there will be no convention of truthfulness and trust with respect to the unused, very long and complicated sentences of the language. So, in general, a convention-friendly translation can be expected to be *incorrect* when defined for the unused and cumbersome sentences of L_1 . Members of the community of speakers of L_1 would think that those sentences would not be used truthfully in L_1 , and so they would not be trusting.¹⁹

Also, the Convention-Friendliness principle appeals to a distinction between the plausible and the implausible convention-friendly translations because there are many different convention-friendly translations from L_1 to L_2 . Where f is a convention-friendly translation from L_1 to L_2 , any mapping g from L_1 to L_2 agreeing with f on the used part of L_1 will be a convention-friendly translation. One way to make precise the notion of a plausible convention-friendly translation would appeal to naturalness, with some account of what makes a translation more natural than another one. This move

¹⁸Lewis (1974) puts these principles at work in a strategy for determining an agent's beliefs, desires and meanings on the basis of our complete knowledge of the agent, *qua* a physical system. No such limited knowledge needs to be assumed for the present purposes. The principles are here given simply as extra resources available to the task of determining whether a certain translation is convention-friendly.

¹⁹The reason why the Synonymy Account is not committed to the stronger principle according to which a translation is convention-friendly if and only if it is correct has to do with the different problems that have been identified in the literature concerning Lewis's account of what it is for a community to speak a language in terms of the members of the community conforming to a convention of truthfulness and trust in the language. These problems have led us to propose a weaker connection between convention-friendliness and correctness. See (Burge, 1975), (Hawthorne, 1990), (O'Leary-Hawthorne, 1993) and (Kölbel, 1998) for some criticisms of Lewis's account.

would be in agreement with what Lewis (1992) says about preferring the straight rather than the bent grammars generating assignments of semantic values for L_1 compatible with there being a convention of truthfulness and trust in the used part of L_1 . But there may be other ways. It is perhaps best to leave the notion of a plausible translation as a primitive, for the present purposes. Inquirers aiming to establish the equivalence between theories will often have already selected the translation schemes which they take to be plausible candidates for being correct translation schemes.

Finally, why is it that the fact that a plausible candidate for being a correct translation scheme f from L_1 to L_2 is a convention-friendly translation, and all the plausible alternative translations from L_1 to L_2 are not convention-friendly, gives only *excellent reason* for believing that f is correct, instead of *implying* that f is correct? The worry here is that there might be no correct translation from L_1 to L_2 whatsoever. The existence of one and only one plausible convention-friendly translation f does not rule out this scenario. Despite this, it is difficult to see what sort of evidence may decide in favour of there being no correct translation from L_1 to L_2 , rather than f being a correct translation from L_1 to L_2 .

One can expect that it will still be difficult to determine whether a translation is convention-friendly. It would be desirable to have a simple procedure for determining whether translation schemes are correct. We propose something close, namely, *Hirsch's rule of thumb*, inspired in Hirsch's (2005; 2007; 2008; 2009) writings on verbal disputes.²⁰ The rule of thumb consists in appealing to judgements concerning the truth of a particular counterfactual statement. For each pair of theories T_1 and T_2 , the antecedent of the counterfactual consists in the description of the following counterfactual scenario:

Disjoint Communities Scenario. There are two different communities, C_{T_1} and C_{T_2} . In C_{T_1} theory T_1 is acknowledged to be the best theory available, and a vast majority of the members of C_{T_1} know all the intricacies of T_1 . In effect, T_1 has become a part of the folk theory of C_{T_1} (what is meant with T being a part of the 'folk theory' of C_T is simply that T is a theory that is implicit in the everyday thought and action of the members of C_{T_1} , just as it is implicit in everyday thought and action that people have beliefs). Furthermore, the meanings that the proponents of T_1 take the sentences of L_{T_1} to have are the meanings that these sentences have in the language of C_{T_1} . In C_{T_2} theory T_2 is acknowledged to be the best theory available, and its intricacies are known by the vast majority of the members of C_{T_2} . In effect, T_2 has become a part of the folk theory of C_{T_2} . Furthermore, the meanings that the proponents of T_2 take the sentences of L_{T_2} to have are the meanings that these sentences have in the language of C_{T_2} . Also, initially, each of these societies was unaware of the existence of the other. Later on, some members mm_{T_2} of C_{T_2} become aware of C_{T_1} , and are given sufficient time to get to know it in detail.

Let f be a translation from L_{T_1} to L_{T_2} . The counterfactual hypothesis is as follows.

²⁰This does not imply that the disputes which Hirsch takes to be verbal turn out to be disputes between equivalent theories. It also does not imply that we agree with Hirsch that what he calls 'common sense ontology' is the correct ontology. In this dissertation I remain neutral on these questions.

Hirschean Counterfactual. If the disjoint communities scenario had obtained, then f would have been a correct translation of the language of C_{T_1} to the language of C_{T_2} by mm_{T_2} .

Hirsch's rule of thumb consists in the following claim:

Hirsch's Rule of Thumb. The Hirschean counterfactual is true if and only if f is a deeply correct translation scheme from L_{T_1} to L_{T_2} .

As previously mentioned, the question whether a translation scheme is *deeply correct* may be substituted by the question whether a related translation scheme is *correct*. The focus on communities C_{T_1} and C_{T_2} and their languages allows us to shift attention from the non-literal use of L_{T_1} and L_{T_2} to the literal use of the languages of the communities C_{T_1} and C_{T_2} . The reason is that the propositions that the proponents of T_1 and T_2 take to be expressed by the sentences of the languages in which their theory is formulated are the propositions that are literally expressed by the sentences of the language of the linguistic communities C_{T_1} and C_{T_2} . Furthermore, the fact that, initially, each one of the communities is unaware of the existence of the other allows for the history of disputes between the proponents of the two theories not to play a role on how the language of each linguistic community is best translated.

To mention the obvious, judgements concerning the truth of the counterfactual require some hold on what would constitute a correct translation. This is a place where the convention-friendliness principle and Lewis's principles of rationalisation and charity come into play. These principles offer some guidance on how to judge the truth of the Hirschean counterfactual. Still, it may turn out to be easier to judge the truth of the Hirschean counterfactual than to use other means to determine whether the translation in question is convention-friendly.

4.4 Applying the Synonymy Account

The present section has two aims. The first aim is to show that the Synonymy Account satisfies the desiderata listed in §4.2. The second aim is to show how the account affords a nuanced understanding of the dialectic of the debate between noneists and Quineans. We can expect the same to be applicable to other debates.

4.4.1 Satisfaction of the Desiderata

According to the first of the desiderata laid out in §4.2, an account of theory equivalence should predict some of the conditions under which it is likely for a theory to be received as *absurd* by the proponents of another theory. The Synonymy Account does yield some predictions concerning when this is likely to happen. Furthermore, these predictions very much agree with the diagnosis as to why some Quineans have understood noneists as being committed to an absurdity.

It is reasonable to suppose that any theory whose entailment structure is such that there is a proposition p which entails every proposition q attributes to p the status of being maximally informative, i.e., of being *absurd*. Suppose that theories T_1 and T_2 appear to be formulated in the same

language (broadly construed), and that T_1 is committed to the truth of a sentence whose homonymous interpretation by the proponents of T_2 is a sentence that according to T_2 , expresses an absurdity. In such case the proponents of T_2 will take T_1 to be absurd.

This prediction of the account can be generalised. The account predicts that a sufficient condition for a theory T_1 to be understood as absurd by the proponents of T_2 is that the homonymous interpretation of some of the sentences to whose truth T_1 is committed be sentences that entail some element in $(Seq_{T_2})^-$, where this is the set of minimal elements according to the ordering of entailment between qua propositions. The reason is that each sentence in $(Seq_{T_2})^-$ is understood by the proponents of T_2 as expressing an absurdity, insofar as these correspond to minimal elements in this ordering.

As we saw in §2, Quineans appear to have understood noneists as speaking gibberish for precisely this reason. Noneists are committed to the truth of ‘some things do not exist’, a sentence which expresses an absurdity according to Quineans. Similar situations may be expected to happen in other debates.

According to the second desideratum, an account of theory equivalence should not have homonymous interpretation as a mandatory facet of the interpretation of the content of one theory by the proponents of another theory, even when the proponents of the two theories are, broadly speaking, members of the same linguistic community. As we have seen in §4.3.3, satisfaction of this desideratum has been written into the Synonymy Account. This requirement underlies the need to appeal to *deeply correct translation schemes*, instead of correct translations, in order to determine whether two theories are synonymous.

The requirement imposed by the third desideratum on an appropriate account of theory equivalence is that any such account should allow for cases in which a theory is intelligible to the proponents of another theory even though the first theory cannot be fully understood in terms of the resources afforded by the second theory.

Notably, when talking about intelligibility and understanding in §4.2, it was not specified what it takes for a theory to be intelligible by the lights of another theory, nor what it takes for a theory T_1 to be fully understandable in terms of the resources of T_2 .

The distinctions introduced in §4.3 allow for an explication of *full understanding*. Full understanding can be cashed out in terms of embedding:

Full Understanding. Theory T_1 is fully understandable in terms of the resources of theory T_2 just in case T_1 is embeddable in T_2 .

According to the notion of intelligibility at play here, intelligibility is easy to get. For T_1 to be intelligible by the lights of T_2 it is enough that T_1 not be understood as an absurd theory by the lights of T_2 . But it should be clear that it is possible for a theory to be neither fully understandable nor an absurd theory from the standpoint of T_2 . Thus, the third desideratum on an adequate account of theory equivalence is satisfied.

According to the fourth desideratum, an account of theory equivalence should explain how theories that would appear to be contradictory if interpreted homonymously are sometimes equivalent, and

offer some means of predicting when this will happen. As has already been remarked, there are cases in which homonymous interpretation is not the correct interpretation of the theories in question. Furthermore, it may happen that two theories that would turn out to be contradictory if homonymously interpreted are such that there are functions f and g establishing the solid similarity between them. In such case the question arises as to whether functions f and g constitute *deeply correct* translation schemes. And we can expect this to be the case sometimes, in which case the theories are in fact synonymous. Thus, the Synonymy Account explains how it is that theories that would appear to be contradictory if interpreted homonymously are sometimes equivalent.

Concerning prediction, by coupling the explication of theory equivalence as theory synonymy with an account of what is required for a translation to be deeply correct, the Synonymy Account has the resources for generating some predictions concerning the equivalence of theories. In §4.3.3 views on how to determine whether a translation is deeply correct were presented. Thus, the Synonymy Account has the resources required for generating predictions concerning the equivalence of theories.

The last of the desiderata previously identified is one according to which any adequate account of theory equivalence should be able to yield conditions under which translations such as the one proposed by Woodward count in favour of the claim that ‘there is total overlap between the conceptual resources of the two theories’. According to the Synonymy Account some translations of the kind discussed by Woodward establish sameness of entailment structure, whereas others go beyond this, establishing sameness of theoretical structure.

Thus, the Synonymy Account takes seriously Woodward’s considerations involving translations. The existence of such mappings is a necessary condition for two theories to be equivalent. Furthermore, if such translations are deeply correct, then the theories turn out to be synonymous, and a fortiori equivalent.

4.4.2 The Synonymy Account and the Noneism vs. Quineanism Dialectic

As shown, the Synonymy Account satisfies the desiderata laid out in §4.2. In the remainder of the section the account is applied to the debate between noneists and Quineans. I begin by spelling out in some detail simple versions of Noneism and Quineanism, respectively, the theories Non_1 , and Qui_1 .

The language of Non_1 , L_{Non_1} , is a first-order modal language with boolean connectives \rightarrow and \neg , modal operator \Box , quantifier \mathfrak{A} (the noneist’s neutral general quantifier), the identity sign, $=$, and as non-logical constants the predicates E (the predicate of existence), F (the predicate that is satisfied by some thing just in case it is fictional), P (the predicate that is satisfied by some thing just in case it could have existed but *actually* does not exist)²¹, and M (the predicate that is satisfied by some thing just in case it is a mathematical entity). The remaining boolean connectives are defined in the usual way, the same applying to \Diamond . The noneist’s neutral particular quantifier, \mathfrak{S} , is defined in the following way: $\mathfrak{S}v(\varphi) =_{df} \neg\mathfrak{A}v(\neg\varphi)$. The *loaded* quantifiers are defined in terms of the neutral quantifiers as follows: $\forall v(\varphi) =_{df} \mathfrak{A}v(Ev \rightarrow \varphi)$ and $\exists v(\varphi) =_{df} \mathfrak{S}v(Ev \wedge \varphi)$. The set L_{Non_1} consists in the set

²¹Note that ‘actually’ is being used with its rigidified reading.

of well-formed formulas of Non_1 .

The language of Qui_1 , L_{Qui_1} , is a first-order modal language with boolean connectives \rightarrow and \neg , modal operator \Box , quantifier \forall (the Quinean's universal quantifier), the identity sign, $=$, and as non-logical constants the predicates E (the predicate of existence), F (the predicate that is satisfied by some thing just in case it is fictional), and P the predicate that is satisfied by some thing just in case it could have existed but *actually* does not exist), and M (the predicate that is satisfied by some thing just in case it is a mathematical entity). The remaining boolean connectives are defined in the usual way, the same applying to \Diamond and \exists . The set L_{Qui_1} consists in the set of well-formed formulas of Qui_1 .

The characterisations of theories Non_1 and Qui_1 to be given make use of the following set of axioms and inference rules:²²

Axioms and Rules of Non_1

- | | |
|--|--|
| (PL) All propositional tautologies. | (E-F) $\mathfrak{A}v(Fv \rightarrow \neg Ev)$. |
| (K) $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$. | (E-P) $\mathfrak{A}v(Pv \rightarrow \neg Ev)$. |
| (T) $\Box\varphi \rightarrow \varphi$. | (E-M) $\mathfrak{A}v(Mv \rightarrow \neg Ev)$. |
| (5) $\Diamond\varphi \rightarrow \Box\Diamond\varphi$. | (MP) $\vdash_{Non_1} \varphi \rightarrow \psi, \vdash_{Non_1} \varphi \Rightarrow \vdash_{Non_1} \psi$. |
| ($\mathfrak{A}1$) $\mathfrak{A}v(\varphi) \rightarrow \varphi[v'/v]$. ²³ | (Nec) $\vdash_{Non_1} \varphi \Rightarrow \vdash_{Non_1} \Box\varphi$. |
| (=1) $v = v$. | ($\mathfrak{A}2$) $\vdash_{Non_1} \varphi \rightarrow \psi \Rightarrow \vdash_{Non_1} \varphi \rightarrow \mathfrak{A}v(\psi)$. ²⁵ |
| (=2) $v = v' \rightarrow (\varphi \rightarrow \psi)$. ²⁴ | |

Axioms and Rules of Qui_1

- | | |
|---|--|
| (PL) All propositional tautologies. | (=2) $v = v' \rightarrow (\varphi \rightarrow \psi)$. ²⁷ |
| (K) $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$. | (EDef) $\exists v'(v = v') \leftrightarrow Ev$. ²⁸ |
| (T) $\Box\varphi \rightarrow \varphi$. | (MP) $\vdash_{Qui_1} \varphi \rightarrow \psi, \vdash_{Qui_1} \varphi \Rightarrow \vdash_{Qui_1} \psi$. |
| (5) $\Diamond\varphi \rightarrow \Box\Diamond\varphi$. | (Nec) $\vdash_{Qui_1} \varphi \Rightarrow \vdash_{Qui_1} \Box\varphi$. |
| ($\forall 1$) $\forall v(\varphi) \rightarrow \varphi[v'/v]$. ²⁶ | ($\forall 2$) $\vdash_{Qui_1} \varphi \rightarrow \psi \Rightarrow \vdash_{Qui_1} \varphi \rightarrow \forall v(\psi)$. ²⁹ |
| (=1) $v = v$. | |

The intended reading of these axioms by, respectively, noneists and Quineans should be clear. Now, let $\Gamma \vdash \varphi$ if and only if there is a finite set Γ' such that $\Gamma' \subseteq \Gamma$ and $\bigwedge \Gamma' \vdash \varphi$, where $\bigwedge \Gamma'$ is any conjunction of all the elements in Γ' . Let $Seq_{Non_1} = \{\langle \Gamma, \varphi \rangle : \Gamma \vdash_{Non_1} \varphi\}$ and $Seq_{Qui_1} = \{\langle \Gamma, \varphi \rangle : \Gamma \vdash_{Qui_1} \varphi\}$. Consider now the following sets of sentences As_{Non_1} and As_{Qui_1} :

²²We could have appealed to a set of models instead. Nothing hangs on this.

²³Provided that v is free for v' , where $\varphi[v'/v]$ results from replacing each free occurrence of v in φ by v' .

²⁴Where ψ differs from φ at most in having v' free at some places where φ has v free.

²⁵Provided that v is not free in φ .

²⁶Provided that v is free for v' , where $\varphi[v'/v]$ results from replacing each free occurrence of v in φ by v' .

²⁷Where ψ differs from φ at most in having v' free at some places where φ has v free.

²⁸Where v' is the first variable of the alphabet if v is not, and v' is the second variable of the alphabet otherwise.

²⁹Provided that v is not free in φ .

The elements of As_{Non_1}

$$(\mathfrak{S}F) \mathfrak{S}v(Fv). \quad (\mathfrak{S}P) \mathfrak{S}v(Pv). \quad (\mathfrak{S}M) \mathfrak{S}v(Mv).$$

The elements of As_{Qui_1}

$$(\exists F) \exists v(Fv). \quad (\exists P) \exists v(Pv). \quad (\exists M) \exists v(Mv).$$

Let $Com_{Non_1} = \{\varphi : As_{Non_1} \models_{Non_1} \varphi\}$ and $Com_{Qui_1} = \{\varphi : As_{Qui_1} \models_{Qui_1} \varphi\}$. The theories Non_1 and Qui_1 are characterised as follows: $Non_1 = \langle L_{Non_1}, Seq_{Non_1}, Com_{Non_1} \rangle$ and $Qui_1 = \langle L_{Qui_1}, Seq_{Qui_1}, Com_{Qui_1} \rangle$.

It is worth pointing out that $\mathfrak{S}x\neg Ex$, the statement of noneism (in the mouths of noneists) is one of the commitments of Non_1 . Also, note that Qui_1 is an allist theory, being committed to the existence of fictional entities, possibilities and mathematical objects – that is, $\exists x(Ex \wedge Fx)$, $\exists x(Ex \wedge Px)$ and $\exists x(Ex \wedge Mx)$ are all commitments of Qui_1 – as well as to the claim that everything exists – $\forall x(Ex)$.

4.4.2.1 Noneism, Allism and Expressive Resources

Lewis notes that homonymous interpretation of noneism by Quineans has the effect that Quineans take noneism to be absurd. In the present case, the kind of interpretation Lewis has in mind is given by the following function h from L_{Non_1} to L_{Qui_1} :

- | | | |
|--|---|------------------------------|
| 1. $h(\neg\varphi)$ is $\neg h(\varphi)$. | 4. $h(\mathfrak{A}v(\varphi))$ is $\forall v(h(\varphi))$. | 7. $h(Mv)$ is Mv . |
| 2. $h(\varphi \rightarrow \psi)$ is $h(\varphi) \rightarrow h(\psi)$. | 5. $h(Fv)$ is Fv . | 8. $h(Ev)$ is Ev . |
| 3. $h(\Box\varphi)$ is $\Box h(\varphi)$. | 6. $h(Pv)$ is Pv . | 9. $h(v = v')$ is $v = v'$. |

According to the interpretation given by h , noneism, captured in the language of Non_1 by the sentence $\mathfrak{S}x(\neg Ex)$, is translated as $\exists x(\neg Ex)$. Whereas $\mathfrak{S}x(\neg Ex)$ is one of the commitments of Non_1 , the sentence $\exists x(\neg Ex)$ of L_{Qui_1} is such that, for every formula φ of L_{Qui_1} , $\exists x(\neg Ex) \vdash_{Qui_1} \varphi$. Thus, it is reasonable to assume that, according to the proponents of Qui_1 , the sentence $\exists x(\neg Ex)$ expresses an absurdity. Hence, the function h offers an uncharitable interpretation of the proponents of Non_1 by the proponents of Qui_1 .

One way of capturing Lewis's suggestion (1990, p. 29) with respect to how Non_1 should be interpreted by the proponents of Qui_1 is as the suggestion that the following function f offers an appropriate interpretation of the sentences of L_{Non_1} in which the noneist's existence predicate does not occur:

- | | | |
|--|---|------------------------------|
| 1. $f(\neg\varphi)$ is $\neg f(\varphi)$. | 3. $f(\Box\varphi)$ is $\Box f(\varphi)$. | 6. $f(Pv)$ is Pv . |
| 2. $f(\varphi \rightarrow \psi)$ is $f(\varphi) \rightarrow f(\psi)$. | 4. $f(\mathfrak{A}v(\varphi))$ is $\forall v(f(\varphi))$. | 7. $f(Mv)$ is Mv . |
| | 5. $f(Fv)$ is Fv . | 8. $f(v = v')$ is $v = v'$. |

Note that, on this interpretation, the sentences $\mathfrak{S}x(Fx)$, $\mathfrak{S}x(Px)$ and $\mathfrak{S}x(Mx)$ are translated as, respectively, $\exists x(Fx)$, $\exists x(Px)$, $\exists x(Mx)$. Furthermore, we have that $\exists x(Fx) \models_{Qui_1} \exists x(Fx \wedge Ex)$, $\exists x(Px) \models_{Qui_1} \exists x(Px \wedge Ex)$ and $\exists x(Mx) \models_{Qui_1} \exists x(Mx \wedge Ex)$. Thus, we get the result that the proponents of Qui_1 would describe the proponents of Non_1 as being committed to the existence of fictional, mathematical and merely possible objects. That is, the function f affords an interpretation of noneism to Quineans whereby noneists are committed to *allism*.

There are different interpretive hypotheses available. Since the present purpose is illustration, it will be assumed that the function f indeed affords a correct interpretation of L_{Non_1} to the proponents of Qui_1 . A challenge remains, namely, how should proponents of Qui_1 interpret $\mathfrak{S}x(\neg Ex)$.

The interpretation of Non_1 that arises from function f does not provide any guidance on how the proponents of Qui_1 should interpret this sentence. Arguably, Lewis's (and Priest's) remarks that Quineans lack the expressive resources allowing them to fully understand noneists are correct as they apply to theories Non_1 and Qui_1 . That is, arguably, theory Qui_1 does not possess the resources required to provide a correct interpretation of $\mathfrak{S}x(\neg Ex)$. A fortiori, proponents of Qui_1 do not have available the expressive resources to fully understand Non_1 . The tools of the Synonymy Account enable us to state this last claim as the claim that Non_1 is not embeddable in Qui_1 .

Lewis's remarks in (1990, p. 29) suggest that the proponent of Non_1 should interpret the proponent of Qui_1 in agreement with the following function g :

- | | | |
|--|---|------------------------------|
| 1. $g(\neg\varphi)$ is $\neg g(\varphi)$. | 4. $g(\forall v(\varphi))$ is $\mathfrak{A}v(g(\varphi))$. | 7. $g(Mv)$ is Mv . |
| 2. $g(\varphi \rightarrow \psi)$ is $g(\varphi) \rightarrow g(\psi)$. | 5. $g(Fv)$ is Fv . | 8. $g(Ev)$ is $v = v$. |
| 3. $g(\Box\varphi)$ is $\Box g(\varphi)$. | 6. $g(Pv)$ is Pv . | 9. $g(v = v')$ is $v = v'$. |

It is easy to see that, under g , $Qui_1 \stackrel{g}{\sqsubset} Non_1$. So, if Lewis's suggestion is right, then g is a deeply correct translation and therefore Qui_1 is embeddable in Non_1 .

At least part of the disagreement between the proponents of Non_1 and Qui_1 becomes clearer after the previous observations. The proponents of Non_1 endorse the view that there are certain expressive resources – corresponding, for instance, to the proposition expressed by $\mathfrak{S}x\neg Ex$ –, which are not available in Qui_1 . The proponents of Qui_1 will disagree insofar as they reject the existence of these extra expressive resources. If, instead, they accept the existence of such expressive resources, then they must acknowledge that their theory is deficient in ways that Non_1 is not, since their own theory is embeddable in Non_1 .

Thus, the Synonymy Account offers the resources to better understand the dialectic between noneists and Quineans. These theorists are fighting about what expressive resources exist and are required to describe the world. The diagnosis of the disagreement between noneists and Quineans as a disagreement concerning the truth of 'some things do not exist' is thus shallow. On the one hand, this diagnosis fails to take into consideration the (real) possibility that noneists and Quineans mean different things by the sentence 'some things do not exist'. On the other hand, the diagnosis neglects the fact that one of the main points of disagreement between noneists and Quineans concerns

the expressive resources required to appropriately describe the world. The Synonymy Account thus provides tools that enable a more nuanced understanding of the debate between noneists and Quineans. This point counts in favour of the Synonymy Account.

4.4.2.2 A Different Quinean Theory

Consider now a different Quinean theory, Qui_2 . This theory is obtained by adding to the language of Qui_1 an extra predicate, C , satisfied by some thing just in case it is *concrete*. Following [Linsky & Zalta \(1996\)](#) and [Williamson \(2013\)](#), the interest is on a notion of concreteness that is *modally flexible*, in the sense that concrete things, such as trees and tables, could have been non-concrete. Thus, this notion is not intended to be synonymous with ‘abstract’, even though part of what it is to be abstract is to be non-concrete. Paradigmatic examples of concrete things are trees, tables, Kripke and the planet Venus. Paradigmatic instances of non-concrete things are Sherlock Holmes, the number 2 and the merely possible seventh son of Kripke.³⁰

The theory Qui_2 is obtained by adding to the axioms of Qui_1 the following:

$$(C-F) \quad \forall v(Fv \rightarrow \neg Cv). \quad (C-P) \quad \forall v(Pv \rightarrow \neg Cv). \quad (C-M) \quad \forall v(Mv \rightarrow \neg Cv).$$

The inference rules of Qui_2 are the same as those of Qui_1 . In addition, $Com_{Qui_1} = Com_{Qui_2}$. The sets Seq_{Qui_2} and Com_{Qui_2} are defined as has been done previously, by appealing, respectively, to the axioms and inference rules of Qui_2 and the set As_{Qui_2} .

Let f' be a mapping from L_{Non_1} to L_{Qui_2} obtained from f by adding the following clause:

$$f'(Ev) \text{ is } Cv.$$

Also, let g' be a mapping from L_{Qui_2} to L_{Non_1} obtained from g by adding the following clause:

$$g'(Cv) \text{ is } Ev.$$

It should be clear that $Non_1 \stackrel{f',g'}{\approx} Qui_2$, even though this does not suffice to establish the synonymy between Non_1 and Qui_2 . Whether the theories are synonymous depends on whether there are pairs of deeply correct translations from the language of one theory to that of the other witnessing their solid similarity. Let me suppose, for the present purposes, that the functions f' and g' are indeed deeply correct translations. In such case $Non_1 \stackrel{f',g'}{\equiv} Qui_2$, and so, according to the Synonymy Account, Non_1 and Qui_2 are equivalent.

First, note that even if the assumption that f' and g' are deeply correct translations is right, from this it *should not be concluded* that noneism just is allism, contra what is suggested in [Woodward \(2013\)](#). The reason is that the focus here is on *particular theories*, Non_1 and Qui_2 . Even though these theories turn out to be equivalent under the assumption that f' and g' are deeply correct, this is not

³⁰A different suggestion, given in [Woodward \(2013\)](#), is to 1) treat ‘concrete’ as synonymous with ‘non-abstract’ and 2) augment the language of the Quinean with predicates intended to stand for concreteness and *being actual*, with the intended reading of *actual* being one according to which the seventh son of Kripke is not actual but could have been.

the case with respect to theories Non_1 and Qui_1 . As was shown, Qui_1 does not even appear to have the expressive resources enabling their proponents to understand what is claimed by the proponents of Non_1 when they advocate the truth of $\exists x \neg Ex$. Yet, Qui_1 and Qui_2 would typically both be counted as allist theories. Hence, the claim that Noneism just is Allism requires qualification because some theories that typically count as allist do not even possess the expressive resources to express the claim of Noneism.

The Synonymy Account reveals that it is often more useful to focus on the truth of *theories* instead of focusing on the truth of *slogans* (such as Noneism, Quineanism and Allism). Suppose that S_1 (e.g., Quineanism) is thought to have the drawback of possessing insufficient expressive resources in comparison to those of theory S_2 (say, Noneism). Suppose that S_1 ers then show how, by appealing to certain extra primitives, they may avoid the objection that S_1 has insufficient expressive resources, and thus show that S_1 is a relevant alternative to S_2 .

The previous discussion of theories Non_1 , Qui_1 and Qui_2 shows that this dialectic is misguided, and that the Synonymy Account affords the resources to see the ways in which this is so. To begin with, when noneists argue that allism is not satisfactory on the basis of insufficient expressive resources, this is best understood as an argument not against allism itself, but instead against a certain theory, or family of theories, that are committed to allism. In addition, by appealing to extra primitives allists in effect express their adherence to theories that are *different* from the ones they started with. Those theories may in fact be better than the ones they started with, and allists may be right in changing their minds. But they are different theories nonetheless.

Finally, if the starting theory under consideration is Qui_1 , the rival noneist theory is Non_1 , and the improved theory is Qui_2 , then the allist will be wrong in claiming that Allism is still a relevant alternative to Noneism on the basis that Qui_2 does not lack expressive resources when compared to Non_1 . Given the assumptions presently in play, Qui_2 and Non_1 are synonymous theories, and so equivalent by the Synonymy Account. Characterising the two theories, Qui_2 and Non_1 , as alternatives insofar as one of them is an allist theory whereas the other is a noneist theory is to mischaracterise the situation. What proponents of Qui_2 mean with ‘some things do not exist’ is different from what proponents of Non_1 mean with ‘some things do not exist’. In general, it may happen that by augmenting the expressive resources of a theory that was proposed as a rival to some other theory with purportedly more expressive resources, the enriched theory turns out to be equivalent to what was previously regarded as an alternative theory.

Are Non_1 and Qui_2 really synonymous theories, and so equivalent? Addressing this question will illustrate the workings of the Synonymy Account. The Synonymy Account recommends the use of Hirsch’s rule of thumb. So, consider two societies Soc_{Non_1} and Soc_{Qui_2} . To make the case rather extreme, imagine that Soc_{Non_1} and Soc_{Qui_2} descend from two different populations of English speakers that were forced to move to two distinct and far away planets, due to some cataclysmic event. The two societies Soc_{Non_1} and Soc_{Qui_2} are constituted by the descendants of these two populations. One of the societies inhabits one of the planets, whereas the other society inhabits the other. Suppose

that:

1. Soc_{Non_1} and Soc_{Qui_2} developed for ages without having any contact with each other;
2. In each of these planets some event took place that led to the destruction of most of the knowledge concerning the origins of the society, in such a way that their current members are all unaware of the fact that they travelled from Earth to their current planet, and that other inhabitants of Earth had to move to a different planet;
3. Theory Non_1 becomes part of the folk theory of Soc_{Non_1} , and that theory Qui_2 becomes part of the folk theory of Soc_{Qui_2} ;
4. At some point in their histories both societies developed the technological means to send tripulated missions to space in search of alien life;
5. Members mm_{Qui_2} of one of these societies, Soc_{Qui_2} , manage to travel to the planet where Soc_{Non_1} is based, and to interact with the inhabitants of Soc_{Non_1} .

The scenario just described is one corresponding to the antecedent of a Hirschian counterfactual. According to Hirsch's rule of thumb, f' is a deeply correct translation from L_{Non_1} to L_{Qui_2} just in case, if the scenario described had obtained, then f' would have been a correct translation of the language of Soc_{Non_1} by mm_{Qui_2} .

To determine whether this is so, the question to be considered is whether f' affords an interpretation of the language of Soc_{Non_1} whereby the members of this society turn out to conform to a convention of truthfulness and trust in their language by the lights of mm_{Qui_2} . In the vast majority of cases in which the inhabitants of Qui_2 would assent to sentences such as the sentence 'some fictional character, α , does not exist and . . .', mm_{Qui_2} would describe them as believing that the content of 'there exists a fictional character, α , that is not concrete and . . .', and as intending to communicate this content to others. Moreover, this generalises to the different sentences for which f' is defined. If this is correct, and there are no other plausible alternative translations, then it should indeed be concluded that the Hirschian counterfactual is true.

To make things more dramatic, we can even conceive mm_{Qui_2} returning to their planet, publishing the translation manual, and this translation manual being used by other members of Soc_{Qui_2} in their visits to Soc_{Non_1} . We can also conceive the possibility of some of these members of Soc_{Qui_2} at some point becoming members of Soc_{Non_1} , quickly becoming speakers of the language of Soc_{Non_1} . Arguably, all this may be conceived as being the case without the members of Soc_{Qui_2} and Soc_{Non_1} ever questioning the adequacy of the translation manual based in f' .

If all this is correct, then the Hirschian counterfactual is indeed true about f' . That is, it is true that if the scenario described had obtained, then f' would have been a correct translation of the language of Soc_{Non_1} by mm_{Qui_2} . Furthermore, a symmetric case may also be considered, with members mm_{Non_1} of Soc_{Non_1} visiting Soc_{Qui_2} . Symmetric considerations would lead to judge as true the claim that if this counterfactual scenario had obtained, then g' would have been a correct translation of the language of Soc_{Qui_2} by mm_{Non_1} . By Hirsch's rule of thumb, f' and g' are deeply correct translations. Given that Non_1 and Qui_2 are strongly similar via f' and g' , Non_1 and Qui_2

are synonymous theories. Finally, given the explication of theory equivalence as theory synonymy, Non_1 and Qui_2 are equivalent theories.

Now, it is important to bear in mind that the theories that have been proposed in connection with the Noneism-Allism debate are more nuanced than Non_1 and Qui_2 . For this reason, I do not want to give much importance to the fact that Non_1 and Qui_2 are, arguably, synonymous theories. The aim of the present discussion has been solely that of offering an example of the application of the Synonymy Account. The aim was not to definitely establish the equivalence between Non_1 and Qui_2 .

To conclude, in this section it was shown that the Synonymy Account satisfies the desiderata laid out in §4.2. It was also shown that the account offers tools enabling a deeper understanding of some debates. On the one hand, the account makes salient the fact that sometimes what is at issue between rival theories is whether to accept the existence of certain distinctions. On the other hand, it rightly changes the focus of debates from slogans to theories.

4.5 Objections and Replies

One important charge against the Synonymy Account is that the explication of theory equivalence as Synonymy *overgenerates*, in the sense of counting as synonymous theories that are not equivalent. The objections concern the existence of extra relations between theories that are not taken into consideration by the Synonymy Account, this being the reason why the account overgenerates. Here are three such relations:

1. T_1 **is ideologically more parsimonious than** T_2 . Roughly, theory T_1 is ideologically more parsimonious than theory T_2 just in case T_1 has fewer primitives (or fewer kinds of primitives) than T_2 .³¹
2. T_1 **is more fundamental than** T_2 . T_1 is more fundamental than T_2 just in case the primitive predicates, operators and remaining expressions figuring in the sentences of Com_{T_1} have as their meanings/semantic values entities (e.g., properties, relations, propositional functions) that are more natural than the entities picked out by the primitive predicates, operators and other expressions figuring in the sentences of Com_{T_2} .³²
3. T_1 **has more explanatory power than** T_2 . There are several views on what makes for explanatory power. For instance, perhaps T_1 and T_2 distinguish different sets of sentences, Exp_{T_i} , as being the set of sentences explaining the truth of the remaining sentences to whose truth T_i is committed. Once this is done, some theorists may take explanation to be given by entailment.

³¹See (Cowling, 2013) for the distinction between ideological *quantitative* and ideological *qualitative* parsimony, as well as a defence of the claim that ideological *qualitative* parsimony is an epistemic virtue of theories. Also, see (Quine, 1951) for the distinction between ontology and ideology, and (Sider, 2011, p. 14) for a discussion of ideological parsimony. An appeal to ideological parsimony as an epistemic virtue is present in, e.g., Lewis's (1986) argument for the existence of a plurality of maximal sums of spatio-temporal interrelated objects and in Sider's (2013) argument for mereological nihilism.

³²Perhaps the requirement should be, instead, that the meanings of the primitive predicates, operators and other expressions figuring in the sentences of Com_{T_1}/L_{T_1} be more natural than the meanings of the primitive predicates, operators and other expressions figuring in the sentences of Com_{T_2}/L_{T_2} . The points to be developed later on are independent of which of these glosses is the best way of spelling out when one theory is more fundamental than the other. For a standard defence of the relevance of fundamentality in metaphysical inquiry, see (Sider, 2011).

That is, Exp_{T_i} explains the truth of the sentences in Com_{T_i} insofar as the commitments of T_i are all entailed by Exp_{T_i} . Alternatively, explanation may be understood as 'local'. That is, a theory may distinguish a set of sequents Exp_{T_i} such that $\langle \Gamma, \varphi \rangle$ belongs to Exp_{T_i} just in case the truth of the sentences in Γ explains the truth of φ .

A proponent of ideological parsimony (fundamentality/explanatory power) as a criterion for theory choice advocates that, all things being equal, a more ideologically parsimonious (fundamental/explanatory) theory should be preferred to a less ideologically parsimonious (fundamental/explanatory theory). Thus, Synonymy appears to provide an insufficient criterion for theory equivalence. Even if two theories are synonymous, they should not count as equivalent, because they may still differ in their ideological parsimony (fundamentality/explanatory power).

For each one of these objections there is a straightforward move available. It consists in adding the extra condition in question to the account of equivalence. For instance, the account could be amended in such a way that i) theories T_1 and T_2 are equivalent just in case T_1 is as ideologically parsimonious as T_2 and $T_1 \equiv T_2$; or ii) T_1 and T_2 are equivalent just in case T_1 and T_2 are equally fundamental and $T_1 \equiv T_2$; or iii) T_1 and T_2 are equivalent just in case T_1 and T_2 have equal explanatory power and $T_1 \equiv T_2$. Moreover, the overkill move of adding all the extra conditions to the Synonymy Account is also available.

My view is that there is no need to have these extra constraints figuring in an account of theory equivalence. The general form of the argument is as follows. Either claims concerning *ideological parsimony*, *fundamentality* and *explanation* are not reflected in a theory's commitments, or else they are. If they are not reflected in a theory's commitments, then those claims are not concerned with the relationship between theory and world. If they are reflected in a theory's commitments, then theories that differ in how parsimonious/fundamental/explanatorily powerful they are turn out not to be synonymous.

Considering first the case of ideological parsimony. Either parsimony concerns the way in which a theory says what it says, or it concerns instead what is said by the theory. If the latter, then there is no need to bring in parsimony considerations for judging whether two theories are equivalent. If the former, then it is best to keep those considerations outside of the notion of theory equivalence.

Considerations of ideological parsimony may still have an impact on which one of two equivalent theories are selected. But considerations having to do with the computational complexity of a theory also have an impact on which of two equivalent theories is selected. Yet, even if two theories have different computational complexity, they may still be equivalent. At least it is useful to isolate a sense of equivalence, matching what is said by a theory, whereby a theory's computational complexity is not relevant to the question of whether it is equivalent to some other theory. These considerations apply not only to the case of computational complexity but also to that of parsimony.

Consider now fundamentality. Why should the fact that the meanings of the primitive expressions of a theory are more natural than those of another theory matter for whether the two theories are equivalent? It matters either because i) if a theory is more fundamental than the other, then the

theories say different things, and a fortiori are not equivalent, or ii) if a theory is more fundamental than the other, then the way in which one of the theories says what it says is different from the way the other theory says what it says. If i), then there is no need to appeal to fundamentality. Theory synonymy already distinguishes theories on the basis of what they are committed to. If ii), then some extra work is required to show why the way a theory says what it says matters for whether it is equivalent to another theory.

Arguably, the main motivation for the view that the way a theory says what it says, vis à vis its fundamentality, matters for theory equivalence is as follows. In the case of fundamentality, the way a theory says what it says matters because the way in which a theory says what it says reveals the commitments of the theory with respect to what *the joints of nature are*.³³ There is a different way of spelling out this thought. What is revealed by how fundamental a theory is are the commitments of the proponents of the theory concerning the joints of nature.

However, this motivation does not justify taking fundamentality as a criterion for theory equivalence. The reason is that theorists will often be neutral on what the joints of nature are. This happens in several debates in metaphysics. For instance, theorists interested in the question whether necessarily everything necessarily exists are typically not advocating any views concerning what the joints of nature are. Similarly, several accounts of causation seem to be neutral on this question. Thus, it is unreasonable to take these theories as reflecting commitments concerning fundamentality.

Of course these theories will enjoy some degree of fundamentality. But any theory will enjoy some degree of fundamentality. This is just a consequence of the fact that a theory must be put forward in some language or other, and thus must appeal to expressions which have more or less natural semantic values. If the theorists were told that their theory would be judged on the matter of fundamentality (and they found it fair that they needed to have any commitments on this question), they would reconsider the language that they used to formulate their theory.

Thus, the main motivation for the view that fundamentality, understood as a way a theory says what it says, is required for theory equivalence is unsuccessful. The conclusion is that if fundamentality is not reflected in the commitments of a theory, it does not play a role in determining whether two theories are equivalent.

Note that this is not to say that considerations pertaining to fundamentality are not useful for theory choice. But such considerations are, arguably, best construed as being relevant for deciding between extensions of theories. For any theory, one can consider its extension to a theory just like the original one except that it includes explicit commitments to the joint-carving nature of the semantic values of the expressions occurring in the formulation of the theory. Call this the *joint-carving extension* of a theory. Judgements to the effect that one theory is better than the other insofar as it is more fundamental are best understood as judgements to the effect that the joint-carving extension of the first theory offers a more adequate depiction of the joints of nature when compared to the joint-carving extension of the second theory.

³³Arguably, this is one of the commitments of (Sider, 2011).

Consider now explanatory power. The case for the irrelevance of explanatory power for judging whether two theories are equivalent is the same as the case for the irrelevance of fundamentality. Claims of explanation can be made *part* of a theory explicitly. If a theorist's aim is, at least in part, to put forward explanatory claims, then he can do so directly, adding further sentences to his theory that reflect these claims. It is then possible to assess whether the augmented theory is equivalent to other theories or not. If explanatory claims are not part of the commitments of a theory, e.g., because the theory's proponents wish to remain neutral with respect to this matter, then it is not reasonable to judge the theory according to this criterion. One does better in judging instead the prospects of extensions of the theory obtained by adding to the original theory explanatory claims.

So far, the objections to the Synonymy Account that have been discussed were aimed at showing that the account *overgenerates*, predicting the equivalence of non-equivalent theories. The final objection that I wish to briefly discuss purports to show that the Synonymy Account *undergenerates*, failing to count as equivalent theories that are in fact equivalent. The objection is that the Synonymy Account undergenerates, since it counts as inequivalent theories that are *empirically equivalent*.

I do not have much to say by way of addressing this objection. Methodologically, it is desirable to have a means of classifying theories relative to a relation between theories more stringent than just empirical equivalence since, for instance, theories that differ on their mathematical commitments may still count as empirically equivalent. Furthermore, in this chapter the focus has been on an account of equivalence applicable to *metaphysical* theories. Insofar as, arguably, many metaphysical theories are trivially empirically equivalent, since they are not concerned with empirical matters, all such theories would count as being equivalent *tout court*. But whether metaphysical theories should count as equivalent just because they are not concerned with empirical matters is a highly contentious matter. For instance, mathematical theories should not count as equivalent just because they are not concerned with empirical matters. Thus, with respect to whether metaphysical theories not concerned with empirical matters should count as equivalent, I take the burden of proof to be with the objector.

4.6 Some Further Applications

Sections 4.4 and 4.5 contain the bulk of the defence of the Synonymy Account. Here, some possible applications for the account are considered.

4.6.1 Relationships between Conceptions of Logical Space

Rayo (2013, ch. 2) offers a picture of scientific inquiry whereby inquiry can be seen as divided into three stages. The first stage consists in the choice of a language suited for certain theoretical purposes, whereas the second stage consists in the formulation of a theoretical hypothesis concerning logical space — a *conception of logical space* —, where this is understood as an hypothesis concerning the space of metaphysical possibilities. Say that a 'just is'-statement is a statement of the form 'to be a φ just is to be a ψ '. Rayo holds that a conception of logical space is determined as a function of the set of 'just is'-statements that are held to be true by the theorist. Finally, the last stage of inquiry consists

in narrowing down the possibilities in a conception of logical space that are live possibilities for being the actual world.

This picture of scientific inquiry perforce attaches importance to the relationship between conceptions of logical space. If scientific inquiry requires a conception of logical space, and conceptions of logical space are determined in function of the language of theorists, it is crucial to have the means to say when two theories have 'equivalent', or using Rayo's terminology, *isomorphic* conceptions of logical space. After all, it would make no sense to take two theories to have unrelated conceptions of logical space just because they are formulated in different languages. Besides discussion of some examples, no account of when logical spaces are isomorphic is offered by Rayo.

The Synonymy Account offers the tools required to make better sense of isomorphism between logical spaces. Arguably, conceptions of logical space are adequately equated with pairs $\langle L_T, Seq_T \rangle$. Once this assumption is in place, conceptions of logical space turn out to be isomorphic just in case they are similar via deeply correct translation functions f and g . Rayo discusses one other relation between conceptions of logical space, namely the relation that holds between two conceptions of logical spaces when the first is *more restricted* than the second. This relation can be captured by the relation that holds between two conceptions of logical space just in case the first is *embeddable* in the second.

Thus, theorists sympathetic to Rayo's picture can avail themselves of the resources of the Synonymy Account in order to get a better hold on how to conceive of conceptions of logical space and of the relationships between these.

4.6.2 Metaphysically Necessary Theories

According to a coarse-grained conception of content (Stalnaker, 1984) the content of a proposition consists in nothing but a set of metaphysically possible worlds. Even though the Synonymy Account does not presuppose a coarse-grained account of content, it is compatible with it. One of the difficulties facing the proponents of a coarse-grained conception of content concerns the status of theories which, if true, are necessarily so. All metaphysically necessary theories turn out to have the same content, even though this is implausible.

The Synonymy Account gives proponents of coarse-grained content the tools to distinguish between metaphysically necessary theories (as well as between metaphysically impossible theories). By taking into consideration a theory's entailment structure, metaphysically necessary theories may be distinguished, since two metaphysically necessary theories may have radically different entailment structures. Furthermore, even if the commitments of both theories turn out to be necessarily true, it may still happen that one or both theories, if adequate, would require the existence of propositions or relations between propositions that do not in fact exist or do not in fact obtain. Thus, proponents of coarse-grained content should be sympathetic to the extra theoretical resources offered by the Synonymy Account.

Furthermore, the Synonymy Account offers proponents of the coarse-grained conception with a

picture of the role of metaphysical necessary theories which is, arguably, more attractive than most pictures currently available. The picture that emerges from the Synonymy Account is one in which the main role of metaphysically necessary theories is that of serving as hypothesis concerning the conceptual resources required for describing the world. In effect, one of the main roles of several sentences expressing necessarily true propositions lies in the relationships between contingent propositions that they encapsulate. To give just one example, according to the resulting picture one of the main roles of the commitment to the truth of the sentence ‘necessarily, John is a man only if John is an animal’ is that of encapsulating the fact that, according to the proponents of the theory, the proposition expressed by ‘John is a man’ entails the proposition expressed by ‘John is an animal’. Such commitments impose constraints on the theoretical structure of theories.

Before proceeding, let me remark once more that the Synonymy Account is *not* committed to a coarse-grained conception of content. However, it is hospitable to those who endorse such an account.

4.7 Conclusion

The main aim of this chapter has been that of offering an account of theory equivalence, one applicable to theories in metaphysics. I began by isolating some desiderata that, arguably, any correct account of theory equivalence must satisfy. Afterwards, the Synonymy Account was presented. First, an explication of theory equivalence as Synonymy was offered. Then, some principles for determining whether a given translation scheme is deeply correct were proposed.

In §4.4 it was argued that the account satisfies the desiderata on accounts of equivalence previously laid out. It was also shown that the Synonymy Account has the tools to offer a nuanced understanding of the dialectic between noneists and Quineans (tools that can be expected to apply to other debates in metaphysics). Some objections to the account were considered in §4.5. All of them were found to be unsuccessful.

Finally, two further applications of the account were singled out. It should be clear by now that there are many more. The next chapter is dedicated to one such application, namely, showing that Williamsonian Thorough Necessitism and Plantingan Moderate Contingentism are equivalent.

5

Thorough Necessitism, Moderate Contingentism and Theory Equivalence

5.1 Introduction

Consider a language containing only the propositional connectives, modal and actuality operators, first- and higher-order quantifiers, and identity. What is the true and most comprehensive theory formulated in this language? What is the correct theory of higher-order quantification, modality, identity and their interaction? The thesis of Higher-Order Necessitism was defended in chapter 2 and, more forcefully, in chapter 3. This leaves two main candidate theories, namely, *Williamsonian Thorough Necessitism* (Williamson, 2013, chs. 5-7) and *Plantingan Moderate Contingentism* (Plantinga, 1976).

Williamsonian Thorough Necessitism and Plantingan Moderate Contingentism are, *prima facie*, mutually inconsistent theories. Yet, it is shown in this chapter that their equivalence is a consequence of the Synonymy Account of theory equivalence, developed and defended in chapter 4. It is also shown how to make sense of the equivalence between the two theories, given their apparent mutual inconsistency.

The equivalence between Williamsonian Thorough Necessitism and Plantingan Moderate Contingentism is a significant result in a number of ways. To begin with, the equivalence between the two theories affords a greater understanding of the present state of the debate concerning what is the correct higher-order modal logic, avoiding *double-counting* of theories. This ultimately leads to progress, insofar as it enables theorists to zoom in on the competing candidate higher-order modal theories.

Relatedly, given the defence of Higher-Order Necessitism offered in the previous chapters, Williamsonian Thorough Necessitism and Plantingan Moderate Contingentism are, arguably, the most plausible higher-order modal theories available. If they are indeed equivalent, then we have managed to zoom in on just one theory (up to theory equivalence).

In addition, the equivalence between Williamsonian Thorough Necessitism and Plantingan Moderate Contingentism reveals that debates between proponents of the two theories are *insubstantial*, or

merely verbal. This is so at least insofar as those debates are concerned with the truth of the theories, since one theory is true if and only if the other is. Later in the paper attention will be brought to the mere verballity of the dispute between Thorough Necessitism and Moderate Contingentism, by noting the striking similarity between that dispute and a typical example of a merely verbal dispute.

The results established in this chapter also promise to be useful in two other respects. The synonymy between the two theories is established by appealing to certain mappings between their language. These mappings provide a systematic way to go from entailments in *MC* to entailments in *TN*, and vice-versa.

The mappings also make it possible to map arguments for and against one theory to arguments for and against the other. The merits and shortcomings of the target arguments may reveal the need to reassess the merits and shortcomings of the source arguments. For instance, suppose that an argument A_1 , thought to cause problems to theory T_1 , is mapped to an argument A_2 against T_2 , and that argument A_2 presupposes a certain conception of properties that turns out to be unattractive, given how the notion of *property* is understood by proponents of T_2 . If this is so, then it may turn out that argument A_1 presupposes a certain conception of individuals that turns out to be unattractive, given how the notion of *individual* is understood by the proponents of T_1 . One reason why this unattractive presupposition of A_1 might not have been noticed from the start is that proponents of T_1 and advocates of A_1 might have been conflating different notions of *individual*.

Finally, I want to briefly say how the claims defended in this chapter relate to those defended in Bennett's 'Proxy Actualism' (2006). One of the aims of Bennett's paper is to argue that there are certain structural similarities between Linsky and Zalta's theory and Plantinga's (if Bennett is correct, then those structural similarities are exhibited by Linsky and Zalta's, Williamson's and Plantinga's theories, as remarked in (Nelson & Zalta, 2009)). The other aim is to argue that none of the theories is *actualist*.

The second aim is unrelated to the aims of the present paper. The issue of whether these theories are actualist is not addressed here. As to the structural similarities between Linsky and Zalta's, Williamson's and Plantinga's theories, the present paper advances a claim that is much bolder than Bennett's. It is not just that Williamson's and Plantinga's theories are structurally similar. They are equivalent, and so count for one vis-à-vis the relationship between theories and the world.

The chapter begins with the presentation of Williamsonian Thorough Necessitism and Plantingan Moderate Contingentism. First an overview of the theories is given. Afterwards, detailed formulations of the theories are offered.

The case for the equivalence between the two theories is developed in §5.3. First translations between the languages of the two theories are offered. The solid similarity between the two theories via these translations is established in an appendix to the chapter. The main result of the section is that the translations offered are deeply correct. Since these translations witness the solid similarity between Williamsonian Thorough Necessitism and Plantingan Moderate Contingentism, the two theories are synonymous. Therefore, they are equivalent, on the assumption that synonymy implies equivalence

(an assumption defended in chapter 4). Two other results are established in the section. The first is that the *homonymous* translation between the theories is not deeply correct. The second is that the dispute between proponents of Williamsonian Thorough Necessitism and Plantingan Moderate Contingentism turns out to have the features of typical merely verbal disputes.

In §5.4 three issues related to equivalence between the two theories are tackled. The first consists in making sense of the equivalence between the two theories. How can it be that proponents of, respectively, Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism turn out to differ with respect to the meaning of some of the expressions of their common language, while at the same time taking themselves not to disagree on their meaning? It is suggested that the explanation for this phenomenon is no different from the one to be offered for more mundane cases of merely verbal disputes.

The second issue concerns how to interpret a certain result concerning the relationship between the model-theories offered for Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism. Suppose that the theories are recast in disjoint languages. Then, models for Williamsonian Thorough Necessitism may be extended, thus becoming models of both theories. Similarly, models for Moderate Contingentism may be augmented, thus becoming models of both theories. The result is that such extension does not lead to the same class of models. In light of this result, there is the temptation to think that the theories are not equivalent after all. *Contra* the objection, it is shown that, on the contrary, if the theories are *equivalent*, then this mismatch between the two classes of models is only to be expected.

The third issue concerns translation of reasons. The mappings witnessing the synonymy between Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism enable arguments for one theory to be translated into arguments for the other. They also enable objections to one theory to be translated into objections to the other theory. Such mappings may thus be used either to support both theories, or to reject them. It will be shown how one of the objections to Williamsonian Thorough Necessitism is translated to an objection to Plantingan Moderate Contingentism, and one of the objections to Plantingan Moderate Contingentism is translated to an objection to Williamsonian Thorough Necessitism.

5.2 Moderate Contingentism and Thorough Necessitism

5.2.1 Overview of the Theories

Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism will both be formulated in a higher-order modal language very much like the language $ML_P^{\textcircled{a}}$ defined in §1.3.1.¹ The main difference is that the language considered will have only two constants, namely, '=' and ' $c_{\langle e \rangle}$ '. Given that the higher-order modal theories are intended as general theories concerning the interaction

¹Plantinga does not formulate his theory in terms of higher-order quantification. Yet, clearly, Plantinga's views can be reformulated by appealing to higher-order resources, and so I will be doing just that, since the present focus is on higher-order modal theories.

between quantification and modality, there is no need to consider principles involving constants. The reason why the language will, nonetheless, contain the constant ' $c_{\langle e \rangle}$ ' is that its interpretation, the property of being concrete, turns out to play an important role in the classic necessitist theories, as shall be shown. I will be calling this language $ML_p^{\textcircled{c}}$.

One aspect common to the necessitist theories put forward by LZ (i.e. Linsky and Zalta) and Williamson, and Plantinga's moderately contingentist theory, is that they all have the resources enabling *actualist* accounts of possible worlds' semantics for first-order modal languages (namely, the Literal and the Haecceities Accounts, introduced in §2.2). Briefly, on the face of it, possible worlds' semantics for first-order modal languages implies, in conjunction with the assumption that there *could have been* things that actually are nothing, the thesis of Possibilism (according to which something is actually nothing).

To see why this is so, suppose that the following is true:

- (1) There could have been something such that actually it is nothing.

For instance, there could have been a seventh son of Kripke, despite the fact that actually, nothing is nor could have been a seventh son of Kripke. According to possible worlds' semantics for first-order modal languages, (1) is true if and only if there is a possible world that has in its domain an individual that does not belong to the domain of the actual world. Since the individuals in the domain of the actual world are those that are *actually* something, it follows that there is something that is actually nothing, (e.g., the seventh son of Kripke). That is, it follows that Possibilism is true.

Some theorists find Possibilism problematic, being friendly to Actualism instead. Actualism may be seen as justified by the conjunction of two claims. The first is the claim that actually, p if and only if p is true at a particular world, the *actual world*, i.e., the world that turns out to be realised. The second is the claim that p is true at the actual world if and only if it is the case that p *simpliciter*. From the conjunction of both claims it follows that:

- (2) Actually p if and only if it is the case that p .

So, since everything is such that it is the case that it is something, it follows from (2) that everything is such that actually it is something.²

Linsky and Zalta's and Williamson's theories have the resources required for an actualist account of possible world's semantics for first-order modal languages insofar as their theories are both committed to the truth of *Necessitism*. Since necessarily, everything is necessarily something, it follows that necessarily, everything is actually something. But the claim that necessarily, everything is actually something is equivalent to the claim that it is not the case that there could have been some thing that is actually nothing. In effect, LZ's and Williamson's theories are committed to the falsehood of (1). This

²Note that the view is not that *necessarily, p* if and only if actually, p . It is a notorious feature of the logic of actuality that it is contingent that p if and only if actually p . Supposing that things could have been different in that p is not the case but could have been, it is possible that p , even though, it is not possible that actually p . So, it is possible that p and that it is not the case that actually p . This argument for Actualism was presented in §1.1.

commitment enables them to block the route from possible worlds' semantics for first-order modal languages to Possibilism.

Since Linsky and Zalta's and Williamson's theories are committed to the falsehood of (1), this means that both theories are somewhat opposed common sense. After all, (1) does appear to be true. As I shall show, both theories have the resources to reject the truth of (1) and yet acknowledge that there is some grain of truth in (1), doing so by appealing to the view that some things, such as the possible seventh son of Kripke, are *contingently nonconcrete*.

But let me first turn to the actualist account of possible worlds' semantics afforded by Plantinga's theory. Plantinga's actualist account of possible worlds' semantics turns out to be consistent with the truth of the claim that there could have been something that actually is nothing. Of course, something has got to give. In order to accomplish this the account of possible worlds' semantics offered by Plantinga is nonstandard in that, according to it, the elements of the domains of possible worlds are not *individuals*, but instead *properties*.

According to the nonstandard account of possible worlds' semantics proposed by Plantinga (the Haeceities Account, introduced 2.2.3) instead of *individuals* the domains of possible worlds have as their elements *haeceities*. The elements of the domain of each possible world w are those haeceities such that it is true at w that they have the property of being instantiated. On this understanding of variable-domains possible worlds' semantics, the claim that it is possible that something is the seventh son of Kripke and is actually nothing is true just in case there is a possible world w and haeceity H such that it is true at w that H is coinstantiated with the property of being a seventh son of Kripke, and it is not true at the actual world that H is coinstantiated with the property of being self-identical.³ More generally, (1) is true if and only if the following claim is true:

- (3) There is a possible world w and haeceity H such that it is true at w that H is coinstantiated with the property of being self-identical, and it is not true at the actual world that H is coinstantiated with the property of being self-identical.

Since the truth of (3) is consistent with Actualism, Plantinga's theory has the resources to offer an

³Note that H does not (*actually*) have the property of being coinstantiated with the property of being a seventh son of Kripke. Otherwise, there would be some individual that is a seventh son of Kripke, since if a property is coinstantiated, then there is something that instantiates it.

Equivalently, it is not true at the actual world that H has the property of being coinstantiated with the property of being a seventh son of Kripke. Instead, according to the example in the text what is true of H is that there is some possible world w such that it is true at w that H is coinstantiated with the property being a seventh son of Kripke.

From the fact that it is true at some possible world w that H is coinstantiated with the property of being a seventh son of Kripke it follows that there is some possible world w such that i) it is true at w that H is instantiated and ii) it is true at w that there is some individual x that instantiates H . But it does not follow that (it is true at the actual world that) H is coinstantiated with the property of being a seventh son of Kripke, and so it does not follow that (it is true at the actual world that) there is something that instantiates H . Likewise, for instance, from the fact that there is some possible world w such that it is true at w that Obama is the president of France it does not follow that (it is true at the actual world that) Obama is the president of France.

Also, even though H has the property of being such that it is true at w that it is coinstantiated, it does not follow that there is some individual x that has the property of being such that it is true at w that x instantiates H . Likewise, even though Kripke has the property of being such that it is true at w that he has a seventh son, it does not follow that there is some individual that has the property of being such that it is true at w that it is Kripke's seventh son.

actualist account of possible worlds' semantics for first-order modal languages consistent with the truth of (1).

Plantinga's nonstandard account of possible worlds' semantics requires an abundant conception of haecceities, one on which the following claim is true:

(4) Necessarily, every individual is such that its haecceity is (actually) something.

The reason is that, otherwise, there would not be enough haecceities to populate the domains of all possible worlds. This claim, in conjunction with the thesis that necessarily, every property is necessarily something (a thesis also endorsed by Plantinga) implies that:

(5) Necessarily, every individual is such that its haecceity is *necessarily* something.

Claim (5) is itself a consequence of the conjunction of Higher-Order Necessitism with an *abundantist* conception of higher-order entities. Roughly, according to this conception:

Thorough Abundantism. For any pairing of worlds w with classes of n -tuples of entities, of types t_1, \dots, t_n , that are all something at w , there is a relation whose extension at each world corresponds to the class of n -tuples of entities paired with that world.⁴

Thorough Abundantism will be one of the commitments of Plantingan Moderate Contingentism.

Thorough Abundantism together with Higher-Order Necessitism implies every instance of the following comprehension principle for higher-order modal logic:

Comp. $\exists y_{\langle t_1, \dots, t_n \rangle} \Box \forall x_{t_1}^1 \dots \forall x_{t_n}^n (yx^1 \dots x^n \leftrightarrow \varphi)$

The variables x^1, \dots, x^n may all be free in φ , but the variable y may not. The result of prefixing Comp with any number of universal quantifiers and necessity operators, in any order, is also an instance of Comp.

Briefly, note that Comp has Higher-Order Necessitism as one of its instances. For each sequence t_1, \dots, t_n of types, the following is an instance of Comp:

(6) $\exists y_{\langle t_1, \dots, t_n \rangle} \Box \forall x_{t_1}^1 \dots \forall x_{t_n}^n (yx^1 \dots x^n \leftrightarrow z_{\langle t_1, \dots, t_n \rangle} x^1 \dots x^n)$

So, the result of prefixing (6) with ' $\Box \forall z_{\langle t_1, \dots, t_n \rangle} \Box$ ' is also an instance of Comp, for each sequence t_1, \dots, t_n of types:

(7) $\Box \forall z_{\langle t_1, \dots, t_n \rangle} \Box \exists z_{\langle t_1, \dots, t_n \rangle} \Box \forall x_{t_1}^1 \dots \forall x_{t_n}^n (yx^1 \dots x^n \leftrightarrow z_{\langle t_1, \dots, t_n \rangle} x^1 \dots x^n)$

⁴This formulation of Thorough Abundantism is but a rough sketch of the intended thesis. One reason is that there are some grounds for thinking that if there was such a pairing between worlds and classes of n -tuples of entities, then whatever could have been something would actually be something. Arguably, the claim that whatever could have been something is actually something is a consequence of the not unreasonable claims that i) there is such pairing only if all the things being paired are actually something, and ii) all classes of actual or possible entities are actually something only if all their elements are actually something. Such complications of formulation are unproblematic in the present setting. The model-theoretic formulations of Moderate Contingentism and Thorough Necessitism will ensure that both theories are committed to Thorough Abundantism, in the intended sense, without this commitment implying, on its own, a commitment to the view that necessarily, every thing is actually something.

Since, identity between higher-order entities is here being used as shorthand for necessary coextensiveness, as mentioned in §1.3.2, (7) is just a statement of Higher-Order Necessitism. Thus, Comp implies Higher-Order Necessitism.⁵

Williamson (2013, ch. 6) argues for Necessitism on the basis of Comp, and offers a defence of Comp. Briefly, one of Williamson's arguments from Comp to Necessitism is as follows. Say that a property is the *anti-haecceity of an individual* just in case it is the property of being distinct from that individual, and that it is an *anti-haecceity* just in case it is possible that it is the anti-haecceity of an individual. Comp implies that it is necessary that every haecceity is necessarily something, and that it is necessary that the anti-haecceity of every individual is necessarily something.⁶ Williamson argues that haecceities and anti-haecceities ontologically depend on the individuals that they could have been haecceities and anti-haecceities of.

According to Williamson, the relationship between an individual and the properties that are, respectively, its haecceity and anti-haecceity is particularly intimate. An individual's haecceity could not have been something while the individual that it is possibly an haecceity of was nothing. Similarly, an individual's anti-haecceity could not have been something while the individual of which it is possibly an anti-haecceity was nothing. So, assuming the ontological dependence of haecceities and anti-haecceities on the things that they are haecceities of, if it is necessary that every individual is such that its haecceity, and anti-haecceity, are necessarily something, then it is necessary that every individual is such that it is necessarily something.

Thorough contingentists such as Adams (1981), Fine (1977) and Stalnaker (2012) also endorse

⁵Comp is the comprehension principle equivalent to $\hat{\text{Comp}}$ argued for in §3.8.

⁶Note that the following are instances of Comp:

$$(i) \quad \exists y_{\langle e \rangle} \Box \forall z_e (yz \leftrightarrow (x_e = z)).$$

$$(ii) \quad \exists y_{\langle e \rangle} \Box \forall z_e (yz \leftrightarrow (x_e \neq z)).$$

By prefixing both formulas with $\Box \forall x_e \Box$ one obtains the statements that it is necessary that every haecceity is necessarily something, and that it is necessary that every anti-haecceity is necessarily something:

$$(iii) \quad \Box \forall x_e \Box \exists y_{\langle e \rangle} \Box \forall z_e (yz \leftrightarrow (x_e = z)).$$

$$(iv) \quad \Box \forall x_e \Box \exists y_{\langle e \rangle} \Box \forall z_e (yz \leftrightarrow (x_e \neq z)).$$

Let $Hx_{\langle e \rangle} := \Diamond \exists y_e \Box \forall z_e (xz \leftrightarrow z = y)$ and $Ax_{\langle e \rangle} := \Diamond \exists y_e \Box \forall z_e (xz \leftrightarrow z \neq y)$. The following formulae are formal renderings of the claims that, necessarily, every haecceity is necessarily something, and that necessarily, every anti-haecceity is necessarily something:

$$(v) \quad \Box \forall x_{\langle e \rangle} \Box (Hx \rightarrow \exists u_{\langle e \rangle} (x = u))$$

$$(vi) \quad \Box \forall x_{\langle e \rangle} \Box (Ax \rightarrow \exists u_{\langle e \rangle} (x = u))$$

From (iii) it follows that (v) in the logic neutral **S5** presented in §1.6. Roughly, suppose $M, w, g[x/f] \models_{\mathbf{S5}n} Hx_{\langle e \rangle}$ for some $f \in D_{\langle e \rangle}(w)$. Then, there is $w' \in W$ and $d \in D_e(w')$ such that $M, w', g[x/f, y/d] \models_{\mathbf{S5}n} \Box \forall z_e (xz \leftrightarrow z = y)$. Assume also that $M, w, g \models_{\mathbf{S5}n} (iii)$. Then, $M, w', g[x/f, y/d] \models_{\mathbf{S5}n} \Box \exists r_{\langle e \rangle} \Box \forall z_e (rz \leftrightarrow z = y)$.

In such case, $M, w', g[x/f, y/d] \models_{\mathbf{S5}n} \Box \exists r_{\langle e \rangle} \Box \forall z_e (xz \leftrightarrow rz)$. That is, $M, w', g[x/f, y/d] \models_{\mathbf{S5}n} \Box \exists r_{\langle e \rangle} (x = r)$.

Hence, $M, w, g \models_{\mathbf{S5}n} (v)$. A similar argument establishes that (vi) follows from (iv) in neutral **S5**.

the view that haecceities and anti-haecceities ontologically depend on the individuals that they are haecceities and anti-haecceities of. Whereas they take this fact as evidence for the claim that there could have been higher-order entities — e.g., haecceities and anti-haecceities — that are actually nothing, Williamson takes the ontological dependence of haecceities and anti-haecceities on the individuals that they are haecceities of as evidence for Necessitism. One of Williamson's reasons for such commitment is that he has independent grounds for endorsing the truth of Comp. According to him, principles at least as strong as Comp are required for the general applicability of some logical and mathematical claims formulated in higher-order modal languages.⁷

This suffices to show that Comp plays an important role in Williamson's defence of Necessitism and, *a fortiori*, also in his defence of Thorough Necessitism. As previously mentioned, Thorough Abundantism, together with Higher-Order Necessitism, implies the truth of every instance of Comp. In effect, Williamson adopts a commitment not only to Comp, but also to Thorough Abundantism. Thorough Abundantism will thus be a commitment not only of Plantingan Moderate Contingentism but also of Williamsonian Thorough Necessitism.

Let me now turn to a different commitment of both theories, namely, Thorough Serious Actualism. On Williamson's theory, Thorough Serious Actualism comes out as trivially true. After all, according to it, no possible thing — individual or higher-order entity — could have been nothing. Plantinga's theory is also committed to the truth of Thorough Serious Actualism. Suppose that there could have been something that could have had a property and yet be nothing. According to Plantinga's theory this is equivalent to there being an haecceity that could have been coinstantiated with some property, while not being instantiated. That is, in order for Plantinga's theory to be consistent with the negation of Thorough Serious Actualism, it cannot be that if properties *h* and *g* are coinstantiated, then property *h* is instantiated and property *g* is instantiated, which would border the inconsistent. The commitment to Thorough Serious Actualism is thus an important tenet of Plantinga's theory, and will accordingly also be a commitment of Plantingan Moderate Contingentism.

Finally, as previously mentioned, Necessitism is opposed to common sense. For instance, it does seem that Obama and the Eiffel Tower could both have been nothing. Necessitists such as [Linsky & Zalta \(1994\)](#) and [Williamson \(2013\)](#) accommodate the common sense thought that some individuals could have been nothing by adopting the view that concreteness is not an essential property. Even though it is not true, according to them, that Obama could have been nothing, nor that the Eiffel Tower could have been nothing, it is true, according to them, that Obama and the Eiffel Tower could have been nonconcrete. Obama would have been nonconcrete in those circumstances in which his parents had not met, and the Eiffel Tower would have been nonconcrete in those circumstances in which its actual designers did not design it. In general, whenever it would appear that a thing *x* could have been nothing, what is true according to necessitists is that *x* could have been nonconcrete.

Note that necessitists do not take the claim that *x* is nonconcrete to imply that *x* is abstract. Their view is that there are things that are neither nonconcrete nor abstract. These are what others would

⁷Williamson's argument for Comp is considered in more detail in [§A.5](#).

call mere possibilia – things like the seventh son of Kripke and the being resulting from the union of Sperm and Ovum, where Sperm is a sperm, Ovum is an egg, and Sperm and Ovum have not actually united. Things such as the seventh son of Kripke and the being resulting from the union of Sperm and Ovum are not *abstract*. These are unlike things such as directions and numbers, paradigmatic cases of abstract entities.

Concreteness thus plays an important role in the necessitist theories that have been offered. It enables them to account for the grain of truth in the common sense judgements that things such as Obama and the Eiffel Tower could have been nothing. This allows for the disagreement with common sense to be less radical than it would otherwise be. The mistaken views that Obama and the Eiffel Tower could have been nothing arise from a failure to realise that some things could have been nonconcrete.

So, even though Necessitists like LZ and Williamson are not committed to Contingentism, they are committed to a claim which, according to them, captures the grain of truth in Contingentism, namely:

Accidental Concretism. There could have been some concrete individual that could have been nonconcrete.

Insofar as Accidental Concretism offers necessitists the resources to harmonise their theories with common sense, it will also be a commitment of Williamsonian Thorough Necessitism.

Contingentists, on the other hand, have no reason to endorse Accidental Concretism. They side with common sense in thinking that Obama and the Eiffel Tower could have been nothing, and so have no place for the contingently nonconcrete. According to them, what is true is not that Obama, and the Eiffel Tower, could have been something nonconcrete, but rather that they could have been nothing. Thus, Accidental Concretism will not be a commitment of Plantingian Moderate Contingentism. Instead, Plantingian Moderate Contingentism is committed to *Essential Concretism*, the negation of Accidental Concretism:

Essential Concretism. Necessarily, every concrete individual is necessarily such that if it is something, then it is concrete.

The following list sums up the commitments of Plantingian Moderate Contingentism and Williamsonian Thorough Necessitism identified so far:

	Moderate Contingentism	Thorough Necessitism
Contingentism	✓	×
Necessitism	×	✓
Higher-Order Necessitism	✓	✓
Thorough Actualism	✓	✓
Thorough Abundantism	✓	✓
Thorough Serious Actualism	✓	✓
Accidental Concretism	×	✓
Essential Concretism	✓	×

Detailed formulations of Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism will now be offered.

5.2.2 Formulations of Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism

The formulations of Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism consist in the triples

$$MC = \langle L_{MC}, Seq_{MC}, Com_{MC} \rangle \text{ and } TN = \langle L_{TN}, Seq_{TN}, Com_{TN} \rangle.$$

Both theories are formulated in language $ML_T^{\textcircled{c}}$. That is, $L_{MC} = L_{TN} = ML_T^{\textcircled{c}}$. The characterisation of the remaining elements of MC and TN appeals to the notion of a *generic inhabited model structure*.

Definition (Generic Inhabited Model Structure). *A generic inhabited model structure is any S5-neutral model structure (defined in §1.6) such that D is as follows:*

1. $D_e(w) = d(w)$;
2. $D_{\langle t_1, \dots, t_n \rangle}(w) = \{f \in (\mathcal{P}(\bigcup_{w \in W} D_{t_1}(w) \times \dots \times \bigcup_{w \in W} D_{t_n}(w)))^W : \forall w \in W (f(w) \subseteq D_{t_1}(w) \times \dots \times D_{t_n}(w))\}$.

For simplicity, let me use ' D_t ' as shorthand for $\bigcup_{w \in W} D_t(w)$, for each type t . For each type t , $D_t(w)$ represents the domain of entities of type t that are something at world w . Note that the restriction to functions f such that $\forall w \in W (f(w) \subseteq D_{t_1}(w) \times \dots \times D_{t_n}(w))$ ensures that every instance of Thorough Serious Actualism is satisfied by every model over every generic inhabited model structure. Note also that the definition just given ensures that $D_t(w) = D_t(w')$, for every $w, w' \in W$ and $t \neq e$, and so the satisfaction of Higher-Order Necessitism. The fact that $D_{\langle t_1, \dots, t_n \rangle}(w)$ is not just a proper subset of $\{f \in \mathcal{P}(D_{t_1} \times \dots \times D_{t_n})^W : \forall w \in W (f(w) \subseteq D_{t_1}(w) \times \dots \times D_{t_n}(w))\}$, but instead identical to it, enables both theories to count as thoroughly abundantist theories. The facts that $D_t(w) = D_t(w')$ and $\{f \in \mathcal{P}(D_{t_1} \times \dots \times D_{t_n})^W : \forall w \in W (f(w) \subseteq D_{t_1}(w) \times \dots \times D_{t_n}(w))\}$ together ensure the satisfaction of every instance of Comp.

For a quick example, consider a generic inhabited model structure $\langle W, \odot, R, d, D \rangle$, such that $W = \{1, 2\}$, $\odot = 1$, $d(1) = \{i_1\}$ and $d(2) = \{i_2\}$. That is, according to this generic inhabited

model structure, there are two possible worlds, 1 and 2, 1 is the actual world, and only one individual is something at each world: i_1 is something at world 1 and i_2 is something at world 2.

As to higher-order domains, consider the domain of properties of individuals of each world w , $D_{\langle e \rangle}(w)$. Note that $\mathcal{P}(D_e) = \mathcal{P}(\bigcup_{w \in W} D(w)) = \{\emptyset, \{i_1\}, \{i_2\}, \{i_1, i_2\}\}$. The definition of higher-order domains given above ensures that $D_{\langle e \rangle}(1) = D_{\langle e \rangle}(2) = \{f \in \mathcal{P}(D_e)^W : \forall w \in W (f(w) \subseteq D_e(w))\} = \{f_I, f_{II}, f_{III}, f_{IV}\}$, where $f_I(1) = f_I(2) = \emptyset$, $f_{II}(1) = \{i_1\}$ and $f_{II}(2) = \emptyset$, $f_{III}(1) = \emptyset$ and $f_{III}(2) = \{i_2\}$, and $f_{IV}(1) = \{i_1\}$ and $f_{IV}(2) = \{i_2\}$. That is, the domain of properties of each world has four properties: property f_I , which is instantiated by nothing at both, property f_{II} , which instantiated by i_1 at world 1 and by nothing at world 2, property f_{III} , instantiated by nothing at world 1 and by i_2 at world 2, and property f_{IV} , instantiated by i_1 at world 1 and i_2 at world 2. Note that there is no property that is, for instance, instantiated by both i_1 and i_2 at world 1. This is in full agreement with Thorough Serious Actualism, since i_2 is nothing at world 1, and so it has no properties at such world.

Let me now turn to *generic models*:

Definition (Generic Model). *A generic model is any S5-neutral model based on a generic inhabited model structure whose valuation function is restricted to the language $ML_P^{\textcircled{c}}$.*

The function Val is defined as in §1.6. A formula φ is true in a generic model $M = \langle W, \odot, R, d, D, V \rangle$, $M, w, g \models \varphi$, if and only if $Val_w^g(\varphi) = \{\emptyset\}$. This means that the following holds:

1. $M, w, g \models s_{\langle t_1, \dots, t_n \rangle}^0 s_{t_1}^1 \dots s_{t_n}^n$ iff $\langle Val^g(s^1), \dots, Val^g(s^n) \rangle \in Val^g(s^0)(w)$, where for each i ($1 \leq i \leq n$), s_i^i is either a constant or a variable of type t ;
2. $M, w, g \models \neg\varphi$ iff $M, w, g \not\models \varphi$;
3. $M, w, g \models \varphi \wedge \psi$ iff $M, w, g \models \varphi$ and $M, w, g \models \psi$;
4. $M, w, g \models \Box\varphi$ iff $\forall w' \in W : M, w', g \models \varphi$;
5. $M, w, g \models @\varphi$ iff $M, \odot, g \models \varphi$;
6. $M, w, g \models \forall v_t \varphi$ iff $\forall f \in D_t(w) : M, w, g[v/f] \models \varphi$.

Presently, the interest is in two subclasses of generic models, *MC*-models and *TN*-models. Starting with *MC*-models, these are defined as follows:

Definition (*MC*-Model). *A MC-model is a generic model $\langle W, \odot, R, d, D, V \rangle$ such that:*

1. *There are $w, w' \in W$ such that $d(w) \neq d(w')$;*
2. *$d(w) = V(c_{\langle e \rangle})(w) \cup \{d \in D_e : \forall w' \in W (d \notin V(c_{\langle e \rangle})(w'))\}$.*

The first condition ensures that every *MC*-model satisfies Contingentism. The second condition ensures that every *MC*-model satisfies *Essential Concretism*.

TN-models are defined as follows:

Definition (*TN*-Model). *A TN-model is a generic model $\langle W, \odot, R, d, D, V \rangle$ such that:*

1. *For every $w, w' \in W$: $d(w) = d(w')$;*

2. Some $d \in D_e$ is such that there are $w, w' \in W$ such that $d \in V(c_{(e)})(w)$ and $d \notin V(c_{(e)})(w')$.

When $M = \langle W, \odot, R, d, D, V \rangle$ is a *MC*-model, $w \in W$ and g is a variable-assignment of M , I will use $M, w, g \models_{MC} \varphi$ instead of $M, w, g \models \varphi$, and when $M = \langle W, \odot, R, d, D, V \rangle$ is a *TN*-model, $w \in W$ and g is a variable-assignment of M , I will use $M, w, g \models_{TN} \varphi$ instead of $M, w, g \models \varphi$. Also, I will use $\Gamma \models_{MC} \varphi$ to say that there is no *MC*-model $M = \langle W, \odot, R, d, D, V \rangle, w \in W$ and variable-assignment g such that $M, w, g \models_{MC} \gamma$ for all $\gamma \in \Gamma$ and $M, w, g \not\models_{MC} \varphi$, and I will use $\Gamma \models_{TN} \varphi$ to say that there is no *TN*-model $M = \langle W, \odot, R, d, D, V \rangle, w \in W$ and variable-assignment g of M such that $M, w, g \models_{TN} \gamma$ for all $\gamma \in \Gamma$ and $M, w, g \not\models_{TN} \varphi$. Also, say that $\odot \models_{MC} \varphi$ if and only if $M, \odot, g \models_{MC} \varphi$ for every *MC*-model M and variable-assignment g of M , and that $\odot \models_{TN} \varphi$ if and only if $M, \odot, g \models_{TN} \varphi$ for every *TN*-model M and variable-assignment g of M .

The formulations *MC* and *TN* can now be fully specified. The set Seq_{MC} consists in the set of sequents $\langle \Gamma, \varphi \rangle$ – where Γ is a set of closed formulae of L_{MC} and φ is a closed formula of L_{MC} – such that $\Gamma \models_{MC} \varphi$, and the set Seq_{TN} consists in the set of sequents $\langle \Gamma, \varphi \rangle$ – where Γ is a set of closed formulae of L_{TN} and φ is a closed formula of L_{TN} – such that $\Gamma \models_{TN} \varphi$. Moreover, Com_{MC} consists in the set of closed formulae φ such that $\odot \models_{MC} \varphi$, and Com_{TN} consists in the set of closed formulae φ such that $\odot \models_{TN} \varphi$. I will now turn to the argument for the equivalence between the two theories.

5.3 Equivalence

In what follows the functions $(\cdot)^{TN} : L_{MC} \rightarrow L_{TN}$ and $(\cdot)^{MC} : L_{TN} \rightarrow L_{MC}$ are presented. These mappings will be called, respectively, the *TN*-mapping and the *MC*-mapping. It will be argued that the *TN*- and the *MC*-mappings are deeply correct. The solid similarity via the *TN*- and *MC*-mappings is established in the appendix. Together, these facts establish the synonymy of *MC* and *TN*, and so the equivalence between Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism.

5.3.1 The *TN*- and *MC*-Mappings

Let me begin by defining some predicates that will play a role later on. Say that an individual x_e is *chunky*_(e) just in case x_e is abstract or concrete. That is,

$$Ch_{(e)}(x_e) := cx_e \vee \Box(\neg cx_e).$$

Moreover, say that a higher-order entity $y_{\langle t_1, \dots, t_n \rangle}$ is *chunky*_(t₁, ..., t_n) if and only if, necessarily, if z^1, \dots, z^n fall under it, then z^1 is *chunky*_{t₁}, and \dots , and z^n is *chunky*_{t_n}. That is,

$$Ch_{\langle t_1, \dots, t_n \rangle}(y_{\langle t_1, \dots, t_n \rangle}) := \Box \forall z^1 \dots \forall z^n (yz^1 \dots z^n \rightarrow (Ch_{t_1}(z^1) \wedge \dots \wedge Ch_{t_n}(z^n))).$$

The *TN*-mapping is defined as follows:

TN-Mapping

1. Let

- $(s^i)^{TN} = s^i$, if s^i is a variable or $c_{\langle e \rangle}$;
- $(s^i)^{TN} = v_{\langle e, e \rangle}$, if s^i is $=_{\langle e, e \rangle}$, where $v_{\langle e, e \rangle}$ is the first variable of type $\langle e, e \rangle$ distinct from s^j , for all $j < i$.

Then:

- (a) If there is an i ($1 \leq i \leq n$) s.t. s^i is $=_{\langle e, e \rangle}$, then let $(s^0_{\langle t_1, \dots, t_n \rangle} s^1_{t_1} \dots s^n_{t_n})^{TN} = \exists v_{\langle e, e \rangle} (\Box \forall u_e \forall z_e (vuz \leftrightarrow u = z \wedge (Ch(u) \wedge Ch(v))) \wedge (s^0)^{TN} (s^1)^{TN} \dots (s^n)^{TN})$
 - (b) Otherwise, let $(s^0_{\langle t_1, \dots, t_n \rangle} s^1_{t_1} \dots s^n_{t_n})^{TN} = (s^0)^{TN} (s^1)^{TN} \dots (s^n)^{TN}$;
2. $(\neg \varphi)^{TN} = \neg(\varphi)^{TN}$;
 3. $(\varphi \wedge \psi)^{TN} = (\varphi)^{TN} \wedge (\psi)^{TN}$;
 4. $(\Box \varphi)^{TN} = \Box(\varphi)^{TN}$;
 5. $(@ \varphi)^{TN} = @(\varphi)^{TN}$;
 6. $(\exists v_t \varphi)^{TN} = \exists v_t (Ch(v_t) \wedge (\varphi)^{TN})$.

Let me assume for the present purposes that, according to Plantingans, Noman is a *merely* possible individual. Since Noman is a merely possible individual, and Plantingans are committed to Thorough Serious Actualism, they will endorse the truth of

(8) It is not the case that Noman is self-identical.

If the TN -mapping turns out to be a deeply correct translation, then, according to clause 1., the proposition that is, according to Plantingans, expressed by (8), is the same as the proposition that is, according to Williamsonians, expressed by (9):

(9) It is not the case that Noman is both self-identical and chunky.

Note that Williamsonians accept the truth of the proposition expressed by (9), since Noman is not (actually) concrete, and thus he is not chunky. So, if the TN -mapping turns out to be a deeply correct translation, then the Plantingans' commitment to the truth of the proposition expressed by (8) is not inconsistent with the commitments of Williamsonians.

Similarly, if the TN -mapping turns out to be a deeply correct translation, then the proposition that is, according to Plantingans, expressed by (10)

(10) It is not the case that Noman is something

is the same proposition as the one that is, according to Williamsonians, expressed by (11)

(11) It is not the case that Noman is something chunky.

Plantingans endorse the proposition that, according to them, is expressed by (10), even though they do not endorse the proposition that, according to them, is expressed by (11). And Williamsonians endorse the proposition that, according to them, is expressed by (11), even though they do not endorse the proposition that, according to them, is expressed by (10). So, if the TN -mapping turns out to be

a deeply correct translation, then in endorsing the truth of the proposition that, according to them, is expressed by (10), Plantingans turn out to be in *agreement* with Williamsonians.

This means that, if the *TN*-mapping turns out to be a deeply correct translation, then Williamsonians should understand the moderate contingentists' quantified claims as being restricted to the realm of the chunky. Let me now turn to the *MC*-mapping, starting with a few abbreviations and definitions.

One of the notions required in the formulation of the *MC*-mapping consists in the property that the haecceity of an individual has just in case the individual of which it is an haecceity is *concrete*. To properly formulate an expression standing for such property, one first requirement is a formula stating that a property is the haecceity of an individual. The following formula states that $y_{\langle e \rangle}$ is an haecceity of individual x_e :

$$\Box \forall z_e (yz \leftrightarrow z = x).$$

Let me abbreviate this formula by the expression

$$H(y_{\langle e \rangle}, x_e).$$

The expression

$$c_{\langle \langle e \rangle \rangle} y_{\langle e \rangle}$$

abbreviates the formula

$$\exists z_e (H(yz \wedge c_{\langle e \rangle}(z))).$$

This formula states that y is the haecceity of something concrete.

The mapping also appeals to the relation in which haecceities stand when they are identical. The following formula states that y and z are haecceities and are identical:

$$\Diamond \exists x_e (H(y, x)) \wedge \Diamond \exists x_e (H(z, x)) \wedge y =_{\langle \langle e \rangle, \langle e \rangle \rangle} z$$

This formula is abbreviated as

$$y \simeq z.$$

Besides the above abbreviations, the *MC*-mapping appeals to the following function π having as its domain and range the set of types:

$$\begin{aligned} \pi(e) &= \langle e \rangle \\ \pi(\langle t_1, \dots, t_n \rangle) &= \langle \pi(t_1), \dots, \pi(t_n) \rangle \end{aligned}$$

The function π maps e , the type of individuals, to the type $\langle e \rangle$ of properties of individuals. It maps the type $\langle e, \langle e \rangle \rangle$ of relations between individuals and properties of individuals to the type $\langle \langle e \rangle, \langle \langle e \rangle \rangle \rangle$ of relations between properties of individuals and properties of properties of individuals, etc.

Finally, the *MC*-mapping appeals to what I will call the property of being a *proxy* $_{\langle\pi(t)\rangle}$, for each type t . When $t = e$, the property of being a proxy consists in the property of being an haecceity. As to the other types, a higher-order entity is a proxy just in case it only has proxies in its extension at each world. So,

$$Pr_{\langle\pi(e)\rangle}x_{\pi(e)}$$

abbreviates the formula

$$\diamond\exists y_e(Hx_{\langle e\rangle}, y_e)$$

Moreover,

$$Pr_{\langle\langle\pi(\langle t_1, \dots, t_n \rangle)\rangle\rangle}x_{\pi(\langle t_1, \dots, t_n \rangle)}$$

abbreviates the formula

$$\Box\forall y_{\pi(t_1)}^1 \dots \forall y_{\pi(t_n)}^n xy^1 \dots y^n \rightarrow (Pr_{\langle\pi(t_1)\rangle}(y^1) \wedge \dots \wedge Pr_{\langle\pi(t_n)\rangle}(y^n))$$

The *MC*-mapping is defined as follows:

MC-Mapping

1. Let

- $(s^i)^{MC} = v_{\pi(t)}$, if s^i is the variable v_t ;
- $(s^i)^{MC} = v_{\langle e \rangle}$, if s^i is $c_{\langle e \rangle}$, where $v_{\langle e \rangle}$ is the first variable of type $\langle e \rangle$ distinct from $(s^j)^{MC}$, for all $j < i$.
- $(s^i)^{MC} = v'_{\langle e \rangle, \langle e \rangle}$, if s^i is $=_{\langle e, e \rangle}$, where $v'_{\langle e \rangle, \langle e \rangle}$ is the first variable of type $\langle e \rangle, \langle e \rangle$ distinct from $(s^j)^{MC}$, for all $j < i$.

Then:

- (a) If there is an i ($1 \leq i \leq n$) s.t. s_i is $c_{\langle e \rangle}$ and no j ($1 \leq j \leq n$) such that s^j is $=_{\langle e, e \rangle}$, then let $(s^0_{\langle t_1, \dots, t_n \rangle} s^1_{t_1} \dots s^n_{t_n})^{MC} = \exists v_{\langle e \rangle} (\Box \forall u_e (vu \leftrightarrow c_{\langle e \rangle} u) \wedge (s^0)^{MC} (s^1)^{MC} \dots (s^n)^{MC})$
 - (b) If there is an i ($1 \leq i \leq n$) s.t. s_i is $=_{\langle e, e \rangle}$ and no j ($1 \leq j \leq n$) such that s^j is $c_{\langle e \rangle}$, then let $(s^0_{\langle t_1, \dots, t_n \rangle} s^1_{t_1} \dots s^n_{t_n})^{MC} = \exists v'_{\langle e \rangle, \langle e \rangle} (\Box \forall u_{\langle e \rangle} \forall z_{\langle e \rangle} (v'uz \leftrightarrow u \varrho z) \wedge (s^0)^{MC} (s^1)^{MC} \dots (s^n)^{MC})$
 - (c) If there is an i ($1 \leq i \leq n$) s.t. s_i is $=_{\langle e, e \rangle}$ and a j ($1 \leq j \leq n$) such that s^j is $c_{\langle e \rangle}$, then let $(s^0_{\langle t_1, \dots, t_n \rangle} s^1_{t_1} \dots s^n_{t_n})^{MC} = \exists v_{\langle e \rangle} \exists v'_{\langle e \rangle, \langle e \rangle} (\Box \forall u_{\langle e \rangle} (vu \leftrightarrow c_{\langle e \rangle} u) \wedge \Box \forall u_{\langle e \rangle} \forall z_{\langle e \rangle} (v'uz \leftrightarrow u \varrho z) \wedge (s^0)^{MC} (s^1)^{MC} \dots (s^n)^{MC})$
 - (d) If there are no i ($1 \leq i \leq n$) and j ($1 \leq j \leq n$) such that s_i is $=_{\langle e, e \rangle}$ and s_j is $c_{\langle e \rangle}$, then let $(s^0_{\langle t_1, \dots, t_n \rangle} s^1_{t_1} \dots s^n_{t_n})^{MC} = (s^0)^{MC} (s^1)^{MC} \dots (s^n)^{MC}$
2. $(\neg\varphi)^{MC} = \neg(\varphi)^{MC}$;
 3. $(\varphi \wedge \psi)^{MC} = (\varphi)^{MC} \wedge (\psi)^{MC}$;
 4. $(\Box\varphi)^{MC} = \Box(\varphi)^{MC}$;
 5. $(@ \varphi)^{MC} = @(\varphi)^{MC}$;
 6. $(\exists v_t \varphi)^{MC} = \exists v_{\pi(t)} (\text{Pr}(v_{\pi(t)}) \wedge (\varphi)^{MC})$.

The idea behind the *MC*-mapping is that when Williamsonians make claims which, according to them, are about what they call individuals, those claims express the same propositions as claims made by Plantingans which, according to them, are about haecceities. Similarly, if the *MC*-mapping is a deeply correct translation then claims which, according to Williamsonians, are about properties of individuals, express the same propositions as claims which, according to Plantingans, are about properties of properties of individuals. And so on.

To give just one example, consider sentence (12):

(12) It is possible that something is neither necessarily nonconcrete nor essentially concrete.

The truth of sentence (12) is a commitment of Thorough Necessitism. In effect, it is a consequence of Accidental Concretism. The negation of sentence (12) is a commitment of Moderate Contingentism, and so it would appear that Plantingans are opposed to the truth of (12). If the *MC*-mapping is a deeply correct translation, then the proposition that, according to Williamsonians, is expressed by (12) is the same as the proposition that, according to Plantingans, is expressed by (13):

(13) It is possible that some haecceity is not instantiated by something concrete, and is not instantiated by something necessarily nonconcrete.

The important observation is that Plantingans happen to be committed to the truth of (13). Since Plantingans accept the truth of Contingentism, while simultaneously endorsing the view that necessarily, every haecceity is necessarily something, they accept the truth of the sentence that it is possible that some haecceity is not instantiated. So, they accept the view that it is possible that some haecceity is not instantiated by something concrete, and is not instantiated by something necessarily nonconcrete. That is, Plantingans accept (13).

5.3.2 Deeply Correct Translation Schemes

The solid similarity between *MC* and *TN* via the *TN*- and *MC*-mappings is established in the appendix. These mappings provide a systematic way to go from entailments in *MC* to entailments in *TN*, and vice-versa. Moreover, the *TN*- and *MC*-mappings turn out to be deeply correct. Thus, *MC* and *TN* are synonymous. A fortiori, Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are equivalent, assuming that synonymy implies equivalence (a thesis defended in chapter 4). The case for the deep correctness of the *TN*-mapping is now presented. It is easy to see how a similar case for the deep correctness of the *MC*-mapping would proceed, and so I will not go through it here.

Let $(\cdot)^{id}$, the *id*-mapping, be the function mapping each formula φ of ML_P^c to itself. Also, let ' S_{MC} ' be shorthand for 'the proposition that is expressed by sentence S according to the proponents of Plantingan Moderate Contingentism', and ' S_{TN} ' be shorthand for 'the proposition that is expressed by S according to the proponents of *TN*'.

First an argument will be formulated addressing those committed to the deep correctness of the

id-mapping. From the standpoint of such theorists, Plantingans and Williamsonians do not differ on what they take the sentences of their common language to mean, and so they are not at the risk of talking past each other. It will be shown that the deep correctness of the *id*-mapping implies, given assumptions that are uncontroversial in the present dialectic, the deep correctness of the *TN*-mapping. So, the *TN*-mapping is deeply correct on the assumption that the *id*-mapping is.

Suppose that the *id*-mapping is deeply correct. Recall the following presupposition of the Synonymy Account, presented in §4.3.2.3:

Propositional Identity Presupposition. For each theory T , $\varphi \stackrel{=}{=} \psi$ only if $\varphi_T = \psi_T$, for every φ, ψ in L_T .

Justification for the Propositional Identity Presupposition comes from the Propositional Identity Hypothesis, mentioned in §4.3.2.3. According to this hypothesis, two propositions are the same if and only if they are mutually entailing. If theorists take sentences S and S' to express mutually entailing propositions, then, a fortiori, S and S' express, according to them, one and the same proposition.

The truth of the Propositional Identity Presupposition is presupposed by the Synonymy Account. It offers the means to give an account of sameness of entailment structure that does not take two theories to have different entailment structures just on the basis of the fact that their languages have different cardinalities. This is precluded because, according to the Presupposition, the mutually entailing sentences of a theory are taken to express the same proposition according to the proponents of the theory. So, the cardinality of a language is not relevant for sameness of entailment structure.

Since the Propositional Identity Presupposition is presupposed by the Synonymy Account, its assumption is, in this context, dialectically unproblematic. After all, the present case for the equivalence between Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism is already premised on the assumption that the Synonymy Account appropriately explicates equivalence between theories.

Consider now the following fact about MC :

Fixidity. $\varphi \stackrel{=}{=}_{MC} (\varphi)^{TN}$, for all $\varphi \in L_{MC}$.

Say that a mapping f from a language L to itself L is *expressively adequate* relative to theory T just in case $f(\varphi)_T = \varphi_T$, for all sentences φ of L . In conjunction with the Propositional Identity Presupposition, Fixidity implies the expressive adequacy of the *TN*-mapping relative to MC :⁸:

Expressive Adequacy of the *TN*-Mapping Relative to MC . $\varphi_{MC} = (\varphi)_{MC}^{TN}$, for every $\varphi \in \text{ML}_p^{\text{@c}}$.

Consider, for instance, the following sentences:

(14) Something could have been nothing.

⁸Here and throughout, ' $(S)_T^f$ ' is shorthand for 'the proposition that is expressed by sentence $(S)^f$ according to the proponents of either Plantingan Moderate Contingentism, if $T = MC$ or Williamsonian Higher-Order Necessitism, if $T = TN$.

- (15) Something chunky could have failed to be chunky.
- (16) There could have been something that could have been nothing.
- (17) There could have been something chunky that could have failed to be chunky.

Since $((14))^{TN} = (15)$ and $((16))^{TN} = (17)$, it follows from Fixidity that $((14))^{TN} \models_{MC} (15)$ and $((16))^{TN} \models_{MC} (17)$. Moreover, it follows from the expressive adequacy of the TN -mapping relative to MC that $(14)_{MC} = (15)_{MC}$ and $(16)_{MC} = (17)_{MC}$.

Let $Ch = \{(\varphi)^{TN} : \varphi \in \text{ML}_P^{c@}\}$ and $(\cdot)^{id}|Ch$, the *chunky restriction* (of the *id*-mapping), be the restriction of the *id*-mapping to the set Ch . Roughly, the domain of the chunky restriction is the domain of sentences about the *chunky*, i.e., the domain of sentences whose quantification is restricted to the realm of the abstract or concrete. Certainly, the *id*-mapping is deeply correct only if the chunky restriction is, that is, only if $(\varphi)_{MC}^{TN} = (\varphi)_{TN}^{TN}$.

The assumption that the chunky restriction is deeply correct, in conjunction with the expressive adequacy of the MC -mapping relative to MC , implies that $\varphi_{MC} = (\varphi)_{TN}^{TN}$, i.e., that the TN -mapping is a deeply correct translation scheme. So, the deep correctness of the *id*-mapping implies, in conjunction with the expressive adequacy of the TN -mapping relative to MC , that the TN -mapping is a deeply correct translation scheme. It is easy to see how a similar argument for the deep correctness of the MC -mapping should proceed. Since both mappings are deeply correct, it follows that MC and TN are synonymous, and thus equivalent.

So, those sympathetic to the view that Plantingans and Williamsonians are not talking past each other should accept the deep correctness of the TN -mapping. The reason is that, as shown, the deep correctness of the *id*-mapping implies the deep correctness of the TN -mapping.

In §5.3.3 it is argued that the *id*-mapping is *not* deeply correct. So, even if the argument just offered is dialectically fruitful, it does not establish the deep correctness of the TN -mapping from the standpoint adopted in the chapter.

Since the deep correctness of the chunky restriction already implies (in conjunction with Fixidity and the Propositional Identity Presupposition) that the TN -mapping is deeply correct, a case for the deep correctness of the TN -mapping need only rely on the assumption that the chunky restriction is deeply correct. In what follows it will be argued that the chunky restriction is indeed deeply correct, and so that the TN -mapping is itself deeply correct.

I think the intuition is already that the chunky restriction is deeply correct. But at this point it is helpful to resort to the procedure for determining whether a translation scheme is deeply correct described in §4.3.3, in particular, whether the relevant Hirschian Counterfactual is true.

Consider a counterfactual scenario CS in which there was a cataclysmic event on Earth forcing humans to abandon the planet and colonise other regions of space. Two communities of English speakers departed in different spaceships to two planets distant from each other, P_{MC} and P_{TN} . These planets are just like Earth, not only in external appearance but also in the chemical compounds

that are present in them, and their appearance. For instance, water is H_2O in both planets, and is the thing that runs in rivers, is drinkable, etc. Both communities turned out to thrive in their new homes. The community in P_{MC} developed into community C_{MC} , whereas the community in P_{TN} developed into the community C_{TN} . The language L_{C_T} of each community C_T is such that the proposition that is, according to the proponents of each theory (formulated via) T , expressed by φ , is the same as the proposition expressed by the sentence φ in the language of C_T . Also, there is in these communities no mismatch between the proposition that typical speakers take φ to express and the proposition that is indeed expressed by φ . To make this reasonable each theory (formulated via) T is assumed to be part of the folk theory of the corresponding linguistic community C_T . One other characteristic of CS is that each community ignores the existence of the other community, and each community ignores its Earthly origin.

Suppose that some members mm_{TN} of C_{TN} eventually discover, in their space explorations, the planet P_{MC} . The members of mm_{TN} are able to observe and interact with C_{MC} during long periods of time, being exposed to a great number of such interactions. I think that the intuition is that if the members of mm_{TN} were to offer a theory accounting for the beliefs, desires, intentions and actions of C_{MC} , alongside with a description of the meanings of $L_{C_{MC}}$, they would not go wrong in taking each sentence $(\varphi)^{TN}$ to have the same meaning in $L_{C_{MC}}$ and in $L_{C_{TN}}$. If mm_{TN} were to offer an account of what is the language in which C_{MC} are conforming to a convention of truthfulness and trust, they would not go wrong in pairing each sentence $(\varphi)^{TN}$ with the meaning of $(\varphi)^{TN}$ in $L_{C_{TN}}$.

For instance, note that there is no divergence on what each community would take to be witnesses for the truth of claims such as (15). Both communities would point to Obama as a witness for the truth of this claim. And none of these communities would take a merely possible physical compound as a witness for the truth of (15), even though both communities would take a merely possible physical compound as a witness for the truth of (17). They would not only agree on what witnesses (15), they would also present similar behaviour, given that they had similar beliefs. Thus, the chunky restriction would be a correct translation if CS had obtained. So, the chunky restriction is a deeply correct translation.

Since the chunky restriction is a deeply correct translation, then the TN -mapping is itself deeply correct, since the TN -mapping is expressively adequate relative to MC . A similar argument would apply to the case of the MC -mapping. Since the TN - and MC -mappings are deeply correct and these mappings witness the solid similarity between MC and TN , it follows that MC and TN are synonymous. Therefore, Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are equivalent.

5.3.3 Deep Incorrectness

Even though the chunky restriction is deeply correct, the *id*-mapping is not. Suppose, absurdly, that this mapping were deeply correct. In such case $\varphi_{MC} = \varphi_{TN}$. Moreover, from the expressive adequacy of the TN -mapping relative to MC it would have followed that $(\varphi)_{MC}^{TN} = \varphi_{MC}$. So,

$(\varphi)_{MC}^{TN} = \varphi_{TN}$. It would have followed again from the assumption that the identity mapping is deeply correct that $(\varphi)_{MC}^{TN} = (\varphi)_{TN}^{TN}$. Hence, it would have followed from the expressive adequacy of the TN -mapping relative to MC and the assumption that the identity mapping is deeply correct that the TN -mapping is expressively adequate relative to TN :

Expressive Adequacy of the TN -mapping relative to TN . $\varphi_{TN} = (\varphi)_{TN}^{TN}$, for every $\varphi \in \text{ML}_P^{\textcircled{c}}$.

This would have been so despite the fact that it is not the case that $\varphi \stackrel{TN}{\models} (\varphi)_{TN}^{TN}$ for every sentence φ of $\text{ML}_P^{\textcircled{c}}$. As it turns out, the TN -mapping is not, after all, expressively adequate relative to TN .

Consider the following sentence:

(18) There could have been something that was nothing.

Note that $((16))^{TN} = (17)$. So, $(16)_{TN} = (17)_{TN}$, by the expressive adequacy of the TN -mapping relative to TN . Moreover, $(16) \stackrel{TN}{\models} (18)$, and so by the Propositional Identity Presupposition it follows that:

Collapse. $(16)_{TN} = (17)_{TN} = (18)_{TN}$

Since $(17) \in \text{Com}_{TN}$, $(17)_{TN}$ is one of the commitments of Williamsonians. Hence, by Collapse, $(18)_{TN}$ is one of the commitments of Williamsonians. But not only is it the case that $(18) \stackrel{TN}{\models} \varphi$, for every $\varphi \in \text{ML}_P^{\textcircled{c}}$, $(18)_{TN}$ expresses an *absurd proposition*, in the sense that it entails every proposition whatsoever, and so a proposition that it is irrational to believe in.

Thus, Collapse implies that Williamsonians are committed to an absurd proposition, and so are, in this sense, irrational. But to so interpret Thorough Necessitists is to interpret them as failing to conform to the Rationalisation Principle mentioned in §4.3.3. Therefore, it is to misinterpret them, in particular because their commitment to $(17)_{TN}$ is one done upon reflection. They do not mean an absurd proposition with (17) .

Also, $(17) \in \text{Com}_{MC}$, and so $(17)_{MC}$ is one of the commitments of Plantingans. From the assumption that the identity mapping is deeply correct it follows that $(17)_{MC} = (18)_{TN}$, and so that Plantingans are also committed to an absurdity. This leads to interpreting Plantingans as failing to conform to the Principle of Rationality, and thus to misinterpretation.

One route for explaining away apparent attributions of irrationality is not available in the present case. Apparent attributions of irrationality are often explained away by distinguishing between the proposition that is the literal meaning of a sentence and the proposition that speakers believe is the meaning of the sentence. For instance, if Tom believes that ‘bought’ means bought and ‘purchased’ means killed, then Tom is not being irrational when he asserts that ‘Dick bought a horse’ and he rejects an assertion of ‘Dick purchased a horse’. But such approach is unavailable, since the present interest is not in what sentences like (17) in fact mean (in English), but rather on what they mean according to Williamsonians and Plantingans.

Collapse and the expressive adequacy of the TN -mapping relative to TN are consequences of i) the Propositional Identity Presupposition, ii) Fixidity, and iii) the assumption that the identity mapping is deeply correct. The Propositional Identity Presupposition is part of the Synonymy Account package, and Fixidity is a theorem about MC . The only assumption left is thus the assumption that the identity mapping is deeply correct. Therefore, this assumption must go.

There are a myriad of related considerations telling against the expressive adequacy of the TN -mapping relative to TN , and so against the deep correctness of the identity mapping. For instance, if CS had obtained, then mm_{TN} would certainly go wrong in taking (17) as meaning in L_{CMC} what (18) means in L_{CTN} . Hence, the TN -mapping is not expressively adequate relative to TN , and the identity mapping is deeply incorrect.

5.3.4 A Typical Case of a Merely Verbal Dispute

Chalmers (2011, p. 515) offers the following example, extracted from William James, of a typical merely verbal dispute:

‘A man walks rapidly around a tree, while a squirrel moves on the tree trunk. Both face the tree at all times, but the tree stays between them. A group of people are arguing over the question: Does the man go round the squirrel or not?’

James (1907, p. 25) solves the problem by distinguishing different senses of ‘going round’, as seems correct:

‘If you mean passing from the north of him to the east, then to the south, then to the west, and then to the north of him again, obviously the man goes round him, for he occupies these successive positions. But if on the contrary you mean being first in front of him, then on the right of him, then behind him, then on his left, and finally in front again, it is quite obvious that the man fails to go round him. . . . Make the distinction and there is no occasion for any further dispute.’

The situation described by James is very similar to the one faced by Plantingans and Williamsonians. The group of people in James’s dispute agree that the sentence ‘the man passes from the north of the squirrel to the east, then to the south, then to the west, and then to the north of him again’ is true, and that the sentence ‘the man is first in front of the squirrel, then on the right of him, then behind him, then on his left, and finally in front again’ is false. Moreover, they agree on the status of these sentences while meaning the same with them. They take themselves to be disagreeing because what one of the parties means with ‘the man goes round the squirrel’ is the same as what *both parties* mean with ‘the man passes from the north of the squirrel to the east, then to the south, then to the west, and then to the north of him again’, whereas what the other party means with ‘the man goes round the squirrel’ is the same as what *both parties* mean with ‘the man is first in front of the squirrel, then on the right of him, then behind him, then on his left, and finally in front again.’

Similarly, Plantingans and Williamsonians agree that the sentence ‘there could have been some chunky things that could have failed to be chunky’ is true, and that the sentence ‘there could have been some haecceities that could have failed to be something’ is false. Moreover, they agree on these sentences while meaning the same with them. They take themselves to be disagreeing because Plantingans take ‘there could have been some thing that could have been nothing’ to express the same proposition as the sentence ‘there could have been some chunky things that could have failed to be chunky’, whereas Williamsonians take ‘there could have been some thing that could have been nothing’ to have the same meaning as the sentence ‘there could have been some haecceity that could have been nothing’.

The situation with Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism thus turns out to be that of a typical merely verbal dispute. As in the case described by James, there is a fragment F of their common language such that: i) proponents of both theories agree on the meaning of each of the sentences in F ; ii) each of the remaining sentences of the language means, according to the proponents of each theory, the same as some sentence in F ; iii) proponents of both theories agree on which sentences in F are true, and which are false. Let $Pr = \{(\varphi)^{MC} : \varphi \in \text{ML}_P^{\text{@c}}\}$. In the case of MC and TN , the fragment F consists in the union of the sets Ch with Pr .

In general, there need not be such a fragment for two theories to be synonymous. Even if two theories are formulated in a common vocabulary, it may be that their proponents agree on the meaning of no sentence. And, of course, equivalent theories may be formulated in different vocabularies.

5.4 Loose Ends

In this section some issues directly connected to the claim that Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are equivalent will be considered. The first of these issues may be seen as a form of incredulous stare. How can it be that the theories are equivalent, if their proponents act as though they are disagreeing, and believe to be doing so? Surely, Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are not equivalent in such case.

I think that the appropriate reply to the incredulous stare consists in offering an explanation of how it can be that the theories are equivalent even if their proponents act as though they are disagreeing, and take themselves to be disagreeing. One such explanation is offered, appealing to the idea that speakers of a language presume to be coordinating on the meanings of its sentences until something crashes, since this presumption secures, for the most part, quick, fruitful and successful communication.

The second issue concerns a certain model-theoretic result that may lead one to the suspicion that Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are not equivalent after all. Contra this suggestion, I argue that the model-theoretic result should be expected if the two theories are equivalent, and so cannot be used to argue against their equivalence.

Finally, I offer an application of the fact that MC and TN are synonymous via the TN - and MC -mappings, namely, the MC -mapping is used to translate an objection to Williamsonian Higher-Order Necessitism into an objection to Plantingan Moderate Contingentism, and the TN -mapping is

used to translate an objection to Plantingan Moderate Contingentism into an objection to Williamsonian Thorough Necessitism.

5.4.1 Making Sense of the Equivalence

Despite the above case for the equivalence between Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism, some will feel unpersuaded. What is still missing is, I think, an explanation of how it can be that these theories, which purport to be rivals, turn out to be equivalent. Such explanation needs to account for how it can be that proponents of Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism turn out to differ with respect to the meaning of some of the expressions of $ML_p^{\textcircled{c}}$, and why it is that they think that they do not disagree on their meaning. In what follows I will propose one explanation for how this may happen.

One misguided objection is that theorists mean the same with the expressions of $ML_p^{\textcircled{c}}$ because they are all competent speakers of English, and in the end the meaning of the expressions of $ML_p^{\textcircled{c}}$ is that of their English analogues.

A problem with this objection is that the logical constants are *technical terms*, even if they are not usually seen as such. Logical constants do not have the same meaning as their natural language analogues. For instance, the natural language ‘if . . . , then . . . ’ does not mean the same as the material conditional, even if typical first year logic exercises require students to translate sentences containing the natural language expression in terms of the material conditional. Those using logical constants should be regarded as already going beyond the resources available in English (when students are told that learning logic is like learning a new language, this is no accident. In part, this is exactly what is going on). The expressions of $ML_p^{\textcircled{c}}$ are thus terms of art of *MC* and *TN*. It is a mistake to think that what proponents of Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism mean with them is the meaning of their English analogues. In particular, note that, in general, English does not possess higher-order resources, contrary to $ML_p^{\textcircled{c}}$.

Even conceding, for the present purposes, that the meaning of the expressions of $ML_p^{\textcircled{c}}$ is that of their English analogues, and that proponents of Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are competent speakers of English, there is another problem with the objection. The objection assumes that competence in English is sufficient for the theorists to mean the same thing. But recall the dispute mentioned by James. Even if the parties in that dispute are all competent speakers of English, they still happen to be involved in a verbal dispute.

Why is this? One explanation as to why this is so is that the meaning of ‘going round’ is *underspecified*. The word can be used to mean what one of the parties mean with it, and it may also be used to mean what the other party means with it. Even if the meaning of the word turns out not to be underspecified, the way its meaning depends on use, and so what it means, may still be (at least currently) inaccessible to competent speakers of English. Speakers use the expression according to what they take it to mean. Since what it means is inaccessible to them, it is natural that their views on what it means will diverge.

The observation that logical constants are technical terms falls short of explaining why proponents of *MC* and *TN* mean different things with them, and why they think that they don't. It is undeniable that proponents of *MC* and *TN* intend to coordinate on the use of the expressions of $ML_p^{@c}$, regardless of whether these are logical terms or not.

The explanation as to why they have differing views on the meanings of some of the expressions of $ML_p^{@c}$ is, I think, similar to the one given for the case of 'going round'. Even if the expressions of $ML_p^{@c}$ are technical terms, it is unreasonable to think that their meanings are both completely specified and fully accessible to their users. Since the meanings of natural language are either incompletely specified or not fully accessible to their users, it is unreasonable to expect anything different to happen with the expressions of $ML_p^{@c}$. That Plantingans and Williamsonians differ slightly on what the meanings of some of the expressions of $ML_p^{@c}$ are, according to them, is only to be expected.

Why should Plantingans and Williamsonians then think that they agree on the meanings of all the expressions of $ML_p^{@c}$? Plantingans and Williamsonians intend to be speaking the same language, and they use the expressions of $ML_p^{@c}$ in mostly the same way. So, it is reasonable to think that they are speaking the same language (which I do think they are). In general, speakers and interlocutors presume that interlocutors and speakers that are members of their linguistic community (of a language *L*) and purport to be speaking in *L* in some communicative exchange mean the same thing with the expressions being used. This presumption secures, quick, fruitful and successful communication, for the most part. Interlocutors do not spend their time interrupting speakers, and speakers do not spend their time asking their interlocutors if they understand what they mean.

It is not that this presumption is completely justified. But, for the most part, what differences there are in what is meant with the expressions used in some linguistic interaction turns out to make little difference for the success of that interaction. Does it matter which of the two alternatives is really meant with 'going round' if the man managed to both 1) pass from the north of the squirrel to the east, then to the south, then to the west, and then to the north of him again, and 2) be first in front of the squirrel, then on the right of him, then behind him, then on his left, and finally in front again? It doesn't. Thus, it is only natural for the presumption that speakers and interlocutors mean the same thing with their sentences and subsentential expressions to be in place in the debate between Plantingans and Williamsonians.

To conclude, Plantingans and Williamsonians do well in presuming that they mean the same thing with the different expressions of their languages, and that their theories are inconsistent. For the most part, the presumption that speakers and interlocutors mean the same thing with the sentences of their common language secures quick, fruitful and successful communication. But this does not mean that they in fact mean the same thing with the different expressions of their language, and it does not mean that their theories are inconsistent. On the contrary, it has been shown that the theories are, after all, equivalent.

5.4.2 Model-Theoretic Mismatch and Quantifier Variance

Take the language $ML_p^{\textcircled{c}}$ and make two copies of it, $MC - ML_p^{\textcircled{c}}$ and $TN - ML_p^{\textcircled{c}}$. The two copies are just like $ML_p^{\textcircled{c}}$ except that each expression of $T - ML_p^{\textcircled{c}}$ is superscripted with T . For instance, \forall^{MC} is the universal quantifier of $MC - ML_p^{\textcircled{c}}$ and \forall^{TN} is the universal quantifier of $TN - ML_p^{\textcircled{c}}$. Let φ^T signal that φ is a formula of $T - ML_p^{\textcircled{c}}$.

Let the TN^* -mapping be a mapping just like the TN -mapping, except that it goes from language $MC - ML_p^{\textcircled{c}}$ to language $TN - ML_p^{\textcircled{c}}$. Similarly, let the MC^* -mapping be a mapping just like the MC -mapping, except that it goes from language $TN - ML_p^{\textcircled{c}}$ to language $MC - ML_p^{\textcircled{c}}$.

For each TN -model M , interpret $TN - ML_p^{\textcircled{c}}$ as the language $ML_p^{\textcircled{c}}$ would be interpreted in M . Let me call each domain of type t of world w the *domain of $\forall^{TN} v_t$ at world w* . Define the domain of $\forall^{MC} v_e$ at a world w as the subset of the domain of $\forall^{TN} v_e$ at world w whose members are the elements of $\forall^{TN} v_e$ that are either concrete or necessarily nonconcrete at w . Let the value of \equiv^{MC} at a world w consist in the set of pairs of elements in the domain of entities of type e of M that are either concrete or necessarily nonconcrete at w . Finally, define the domain of $\forall^{MC} v_t$ at a world w , for all $t \neq e$, as in general models (on the basis of the domain of $\forall^{MC} v_e$).

We have that $M, w \models_{TN} (\varphi)^{TN^*}$ if and only if $M, w \models_{TN} \varphi^{MC}$. The upshot is that each TN -model may thus be seen also as a MC -model. In particular, M satisfies the commitments of both TN (as expected) and MC . I will call any TN -model M expanded in this way an $TN + MC$ -model.

For each MC -model M , interpret $MC - ML_p^{\textcircled{c}}$ as the language $ML_p^{\textcircled{c}}$ would be interpreted in M . Let me call each domain of type t of world w the *domain of $\forall^{MC} v_t$ at world w* . Define the domain of $\forall^{TN} v_e$ at world w as the subset of the domain of $\forall^{MC} v_{\langle e \rangle}$ at world w whose members are possibly haecceities of something. Let the value of \equiv^{TN} at a world w consist in the set of all pairs $\langle o, o \rangle$ of elements o in the domain of $\forall^{TN} v_e$ of M . Define the domain of $\forall^{TN} v_t$ at a world w , for all $t \neq e$, as in general models (on the basis of the domain of $\forall^{TN} v_e$). Finally, define the value of \equiv^{TN} at a world w as the set of haecceities in the domain of $\forall^{MC} v_{\langle e \rangle}$ that are had by some entity of type e that is concrete at world w .

We have that, $M, w \models_{MC} (\varphi)^{MC^*}$ if and only if $M, w \models_{MC} \varphi^{TN}$. The upshot is that each MC -model may thus be seen also as a TN -model. In particular, M satisfies the commitments of both MC (as expected) and TN . I will call any MC -model M expanded in this way an $MC + TN$ -model.

Roughly, each $TN + MC$ -model depicts both one way that modal reality might be according to Williamsonians and how that reality may be redescribed according to how Plantingans interpret $ML_p^{\textcircled{c}}$. Similarly, each $MC + TN$ -model depicts both one way that modal reality might be according to Plantingans and how that reality may be redescribed according to how Williamsonians interpret $ML_p^{\textcircled{c}}$. Since Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are equivalent, the two classes of models should be the same, right?

Well, they aren't. One quick way to see this is by appealing to the following facts:

Mismatch.

1. The domain of $\forall^{TN} v_e$ at world w of any $MC + TN$ -model consists of functions from worlds to sets of things in the domain of $\forall^{MC} v_e$ at world w (roughly, representing haecceities);
2. The domain of $\forall^{TN} v_e$ at world w of $TN + MC$ -models does not consist of functions from worlds to sets of things in the domain of $\forall^{MC} v_e$ at world w .

One may be tempted to see Mismatch as showing that Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are not equivalent. Plantingans must think of the individuals that Williamsonians talk about as *properties* of individuals. But Williamsonians are not talking about properties of individuals. Hence, the two theories are not equivalent. The MC -mapping thus *misrepresents* Williamsonians as speaking about properties of individuals instead of individuals.

Mismatch shows no such thing. The TN - and MC -mappings are deeply correct only if the quantifiers of the two theories have different meanings. That is, the TN - and MC -mappings preserve meaning only if there is quantifier variance, in the sense that the quantifiers of the two theories may have different meanings, even if they are intended to be unrestricted.

The models of each theory reflect how that theory understands 'individual' and the relationship between what they call 'individuals' and 'higher-order entities'. Since Plantingans and Williamsonians take the universal and existential quantifiers to have different meanings, they also take 'individual' and 'higher-order entity' to have different meanings. So, to say that the MC -mapping misrepresents Williamsonians as speaking about properties of individuals instead of individuals is to *equivocate* on 'individual'.

What Williamsonians express in terms of *their* understanding of the first order quantifiers is expressed by Plantingans in terms of how *Plantingans* understand the second-order quantifiers (over haecceities). So, Mismatch does not show that the theories are not equivalent. It shows what was already clear. Plantingans and Williamsonians appeal to expressions with different meanings to describe the same reality. Moreover, Mismatch reminds us that the language of the metatheory is itself not neutral.

Now, a different objection to the claim that TN - and MC - are deeply correct translation schemes is simply that their deep correctness requires quantifier variance. This is thought to be an objection because, according to the objector, quantifier variance is false, the reason being that quantifier expressions pick the joint-carving candidate meanings.

This objection is successful only if there is only one candidate meaning for each quantifier expression. But there does not seem to be convincing justification for the claim that there is only one candidate meaning for each of the quantifier expressions. Hence, the objection is not successful.

5.4.3 Translation of Reasons

Given the synonymy between *MC* and *TN*, arguments for, respectively, Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism can be translated to arguments for the other theory, and arguments against each of these theories can be translated to arguments against the other. Before concluding, *MC*- and *TN*-mappings will be used to translate, respectively, an objection to Williamsonian Thorough Necessitism to an objection to Plantingan Moderate Contingentism and an objection to Plantingan Moderate Contingentism to an objection to Williamsonian Thorough Necessitism.

Since *MC* and *TN* are synonymous via the *TN*- and *MC*-mappings, these translated objections may be used to argue either that the original objections were not compelling after all or that the equivalent theory should be rejected as well, depending on one's persuasion. The question how the translated objections offered should be responded to is, however, outside of the scope of the chapter.

Consider the following claims:

Supervenience. All possible things are such that if they are distinguishable by some general modal property, then they are distinguishable by some general nonmodal property.

- *Example:* If Ganges is distinguishable from Mount Everest because Ganges has the property of being a possible river, whereas Mount Everest does not have that property, then they are also distinguishable with respect to their nonmodal properties, say, because Ganges has the property of being a river, whereas Mount Everest does not have the property of being a river.

General Possibilities. There could have been contingently chunky things *x* and *y* such that: necessarily, *x* has the general modal property of *possibly being an F* if it is something, and it is impossible for *y* to have the *general modal property of possibly being an F*, for some general property *F*.

- *Example:* Necessarily, if Ganges had been something, then it would have had the property of possibly being a river, whereas Mount Everest could not have had the property of possibly being a river.

General Nonmodal Indiscernibility. There could not have been any things *x* and *y* such that *x* and *y* would have been distinguishable by some nonmodal general property in those circumstances in which they fail to be chunky.

- *Example:* No nonmodal general property can distinguish between Ganges and Mount Everest in circumstances in which they are not chunky. Neither Ganges and Mount Everest is a river, nor a mountain, etc. in those circumstances, neither of them is abstract, nor concrete.

The objection to Williamsonian Thorough Necessitism under consideration is that Supervenience, General Possibilities and General Nonmodal Indiscernibility (independently plausible theses, according

to the objectors) together imply the falsehood of Necessitism. I will call this objection to Williamsonian Thorough Necessitism the *supervenience objection*.

To see why these three theses imply the falsehood of Necessitism, assume that the river Ganges, Mount Everest and the general modal property of *possibly being a river* witness the truth of General Possibilities. That is: i) Ganges and Mount Everest are possible things that could have failed to have been chunky, ii) necessarily, Ganges has the property of being possibly a river if it is something, and iii) it is impossible for Mount Everest to have the property of possibly being a river. Let w be a possible world in which Ganges fails to be chunky, and w' be a possible world in which Mount Everest fails to be chunky.

By General Nonmodal Indiscernibility it follows that if w and w' had obtained, then, for every general nonmodal property F , Ganges has F at w if and only if Mount Everest has F at w' . By Supervenience it follows that, for every general modal property F , Ganges has F at w if and only if Mount Everest has F at w' . So, Ganges has the property of possibly being a river at w if and only if Mount Everest has the property of possibly being a river at w' . Since Mount Everest could not have had the property of possibly being a river, it follows that Ganges does not have the property of possibly being a river at w .

Since Ganges does not have the property of possibly being a river at w , it follows that Ganges is nothing at w . But Ganges could have been something. So there could have been something that could have been nothing. That is, Necessitism is false. A fortiori, Williamsonian Thorough Necessitism is false.

The *MC*-translations of the premises of the supervenience objection are the following:

Translation of Supervenience. All possible haecceities are such that if they are distinguishable by some general modal property, then they are distinguishable by some general nonmodal property.

- *Example:* If Ganges's haecceity is distinguishable from Mount Everest because Ganges's haecceity has the property of possibly being instantiated by a river, whereas Mount Everest's haecceity does not have that property, then they are also distinguishable with respect to their nonmodal properties, say, because Ganges's haecceity has the property of being instantiated by a river, whereas Mount Everest does not have the property of being instantiated by a river.

Translation of General Possibilities. There could have been contingently instantiated haecceities x and y such that: necessarily, x has the general modal property of *possibly being an F* if it is something, and it is impossible for y to have the general modal property of *possibly being an F* , for some general property F .

- *Example:* Necessarily, if Ganges's haecceity had been instantiated, then it would have had the property of possibly being instantiated by a river, whereas Mount Everest's haecceity could not have had the property of possibly being instantiated by a river.

Translation of General Nonmodal Indiscernibility. There could not have been any haecceities x

and y such that x and y would have been discernible by some nonmodal general property in those circumstances in which they fail to be instantiated by something.

- *Example:* No nonmodal general property can distinguish between Ganges's haecceity and Mount Everest's haecceity in circumstances in which they are instantiated by nothing. Neither Ganges's haecceity and Mount Everest's haecceity is instantiated by a river, nor by a mountain, etc. in those circumstances.

Assume that Ganges's haecceity, Mount Everest's haecceity and the general modal property of possibly being instantiated by a river witness the truth of the Translation of General Possibilities. That is: i) Ganges's haecceity and Mount Everest's haecceity are possible haecceities that could have failed to have been instantiated, ii) necessarily, Ganges's haecceity has the property of being possibly instantiated by a river if it is something, and iii) it is impossible for the haecceity of Mount Everest to have the property of possibly being instantiated by a river. Let w be a possible world in which Ganges's haecceity fails to be instantiated by something, and w' be a possible world in which Mount Everest's haecceity fails to be instantiated by something.

By Translation of General Nonmodal Indiscernibility it follows that if w and w' had obtained, then, for every general nonmodal property F , Ganges's haecceity has F at w if and only if Mount Everest's haecceity has F at w' . By Translation of Supervenience it follows that, for every general modal property F , Ganges's haecceity has F at w if and only if Mount Everest's haecceity has F at w' . So, Ganges's haecceity has the property of possibly being instantiated by a river at w if and only if Mount Everest's haecceity has the property of possibly being instantiated by a river at w' . Since Mount Everest's haecceity could not have had the property of possibly being instantiated by a river, it follows that Ganges's haecceity does not have the property of possibly being instantiated by a river at w .

Since Ganges's haecceity does not have the property of possibly being instantiated by a river at w , it follows that Ganges's haecceity is nothing at w . But Ganges's haecceity could have been something. So there could have been some haecceity that could have been nothing. That is, Plantingan Moderate Contingentism is false. So, the translation of the supervenience objection constitutes an objection to Plantingan Moderate Contingentism.

Let me now turn to the translation of an objection to Plantingan Moderate Contingentism to an objection to Williamsonian Thorough Necessitism.

Say that a property P is *explanatorily dependent* on xx just in case P 's application conditions are specifiable solely in terms of some of the xx and qualitative properties, and if none of the xx had been something, then P 's application conditions would not have been specifiable solely in terms of individuals and qualitative properties. Also say that xx are contingent just in case it is possible that none of them is something. The *explanatory dependence* objection appeals to the following assumptions:

Fundamentality of Individuals and Qualities. Necessarily, if P is a nonqualitative property, then necessarily, P is something if and only if P 's application conditions are specifiable solely in

terms of individuals and qualitative properties.

Contingency of the Basis. There could have been nonqualitative properties P explanatorily dependent on some contingent xx .

The idea behind the Fundamentality of Individuals and Qualities may be explained by appealing to an example. What guarantees that the haecceity of Tweedledum picks him, rather than Tweedledee, who is qualitatively indiscernible from Tweedledum, in circumstances in which both Tweedledee and Tweedledum are nothing? Nothing seems to guarantee it. So Tweedledum's haecceity is not chunky in circumstances in which both he and Tweedledee are nothing.

Assume that Obama's haecceity and the human beings that are actually something witness the truth of Contingency of the Basis. That is, assume that Obama's haecceity is a nonqualitative property explanatorily dependent on the actual human beings, and that it is possible that none of the actual human beings is something. From Fundamentality of Individuals and Qualities it follows that Obama's haecceity is something, and that it could have been nothing. But then, there could have been some property that could have been nothing, and so Plantingan Moderate Contingentism is false.

To put it differently, according to the Explanatory Dependence objection nonqualitative properties, such as the property of being Obama, ontologically depend on the being of some xx , since their application conditions have to be explained partly in terms of xx . But xx could have been nothing. So, properties such as the property of being Obama could have been nothing. This contradicts Plantingan Moderate Contingentism.

Say that a property P is *chunkily explanatorily dependent* on chunky xx just in case P 's application conditions are specifiable solely in terms of xx and chunky qualitative properties, and if none of the xx had been chunky, then P 's application conditions would not have been specifiable solely in terms of chunky individuals and chunky qualitative properties. Also say that xx are contingently chunky just in case it is possible that none of them is chunky.

The TN -translations of the premises of the supervenience objection are the following:

Translation of Fundamentality of Individuals and Qualities. Necessarily, if P is a nonqualitative chunky property, then necessarily, P is something (and chunky — since every property is necessarily chunky, this turns out to be a redundant predication of chunkyness) if and only if P 's application conditions are specifiable solely in terms of chunky individuals and chunky qualitative properties.

Translation of Contingency of the Basis. There could have been nonqualitative chunky properties P chunkily explanatorily dependent on some contingently chunky xx .

Let the *chunky haecceity of x* be the property of being both x and chunky. The idea behind the Translation of Fundamentality of Individuals and Qualities may be explained by appealing to an example. What guarantees that the chunky haecceity of Tweedledum picks his chunkyness, rather than Tweedledee's, who is qualitatively indiscernible from Tweedledum, in circumstances in which both

Tweedledee and Tweedlsedum are not chunky? Nothing seems to guarantee it. So, Tweedledum's chunky haecceity is not chunky in circumstances in which both he and Tweedledum are not chunky.

Assume that Obama's chunky haecceity and the human beings that are actually chunky witness the truth of Translation of Contingency of the Basis. That is, assume that Obama's chunky haecceity is a chunky nonqualitative property chunkily explanatorily dependent on the chunky actual human beings, and that it is possible that none of the actual human beings is chunky. From Translation of Fundamentality of Individuals and Qualities it follows that Obama's chunky haecceity is something, since it is explanatorily dependent on chunky xx , and that it could have failed to have been something, since xx could have failed to have been chunky. But then, there could have been a chunky property that could have failed to have been something. But this contradicts Williamsonian Thorough Necessitism. According to Williamsonian Thorough Necessitism no property could have failed to have been something.

To put it differently, according to the translation of the Explanatory Dependence objection chunky nonqualitative properties, such as the property of being Obama and chunky, ontologically depend on the chunkyness of xx , since their application conditions have to be explained partly in terms of the chunkyness of xx . But it could have been that none of the xx was chunky. So, properties such as the property of being both Obama and chunky could have been nothing. This contradicts Williamsonian Thorough Necessitism.

5.5 The Correct Higher-Order Modal Theory

What is the correct higher-order modal theory? Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are, at most, sound theories. Even if they are true, they leave questions open, such as the question how many individuals and propositions there are.

Are Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism sound? First, note that two important commitments of these theories have not been defended in the dissertation, namely, the commitment to Thorough Abundantism and the commitment to the propositional modal logic **S5**.

In §3.8 an argument for the comprehension principle $\hat{\text{Comp}}$ was offered. This comprehension principle does not imply Thorough Abundantism (not even in conjunction with Higher-Order Necessitism), even though Higher-Order Necessitism and Thorough Abundantism together imply $\hat{\text{Comp}}$. According to principle $\hat{\text{Comp}}$ for any condition φ there necessarily is a relation that necessarily, obtains between x^1, \dots, x^n if and only if φ . Arguably, the truth of $\hat{\text{Comp}}$ indicates the truth of Thorough Abundantism. It is *because* the necessitation of Thorough Abundantism is true that for any condition φ there necessarily is a relation that necessarily, obtains between x^1, \dots, x^n if and only if φ .

More would have to be said by way of offering a robust defence of Thorough Abundantism. The soundness of the propositional modal logic **S5** will also not be argued for here. As already mentioned in fn. 2 (in ch. 2), arguably, the soundness of **S5** is accepted by most metaphysicians (clearly, those committed to the soundness of **S5** face the challenge of offering a satisfactory reply to objections such

as the one presented in §3.5.3). In this section I want to address a different worry with the claim that Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are sound.

One may think that the theories cannot both be true, since one contains the negation of the other. After all, Contingentism is the negation of Necessitism, independently of whether Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are equivalent. There are two ways of understanding the claim that a theory is true. According to one of these, 'truth' applies to theories themselves, whereas according to the other 'truth' applies to formulations of theories. On the first understanding, two theories may both be true even if they are formulated in the same language and one of them is committed to the truth of sentence φ whereas the other is committed to the truth of $\neg\varphi$. What is required is that the proponents of the theories mean different things with φ . But this is precisely what is going on with Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism.

What if 'true' is understood as applying to the formulations *MC* and *TN* themselves? Which of these formulations is true? There are two salient options in such case. Either the language of the theories is underspecified, or else Plantingans and Williamsonians do not have complete access to the meanings of all the expressions of their common language.

If the language is underspecified, then the notion of truth, for sentences, makes sense only relative to a specification. Arguably, *MC* is true under one specification, and *TN* is true under the other. Compare with the typical case of a verbal dispute presented in §5.3.4. Under one specification, the sentence 'the man is going round the squirrel' is true. Under the other specification, the sentence 'the man is going round the squirrel' is false.

If it is a matter of not having a complete access to the meanings of all the expressions of their common language, then at least one of *MC* and *TN* is false, perhaps both. Compare again with the typical case of a verbal dispute presented in §5.3.4. It might be that none of the options identified by James captures the meaning of 'going round', and that the sentence 'the man is going round the squirrel' is simply false.

Notwithstanding, the issue does not seem terribly important. We understand what the parties involved in the dispute described by James mean, given James's distinction between the different senses of 'going round', and what both parties say is true. That should be enough. Otherwise, the interest shifts from what was going on between the man and squirrel to the semantics of English.

Similarly, the question which of *MC* or *TN* is true, if any, does not seem terribly important. We understand what Plantingans and Williamsonians mean, given that each sentence of their common language means the same, according to them, to a sentence in the set $Ch \cup Pr$ (mentioned in 5.3.4). Moreover, arguably, the restriction of their commitments to the set $Ch \cup Pr$ are all true (assuming the truth of Thorough Abundantism and that **S5** is a sound propositional modal logic).

Why think that the commitments of *MC* and *TN* are all true once restricted to the set $Ch \cup Pr$? Surely, it is true that there could have been something chunky that could have failed to have been chunky. Aren't the Eiffel Tower, Obama, Ganges, Mount Everest, etc. examples of such things? And

certainly it is true, given the truth of Higher-Order Necessitism, that necessarily, every haecceity is necessarily something.

But 'there could have been something chunky that could have failed to have been chunky' just means the same as Contingentism according to Plantingans, and 'necessarily, every haecceity is necessarily something' just means the same as Necessitism according to Williamsonians. Besides Thorough Abundantism and **S5**, these were the remaining problematic commitments of Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism, given the theses defended in the dissertation. So, the restriction of the commitments of each theory to the set $Ch \cup Pr$ is sound. That is all that is worth knowing. Otherwise the interest shifts from metaphysics to semantics.

5.6 Conclusion

The main aim of this chapter was to show that Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are equivalent, on the assumption that the Synonymy Account of theory equivalence is correct. I began by offering an overview of the two theories, highlighting their main commitments. Afterwards, the formulations MC and TN of, respectively, Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism were presented.

In §5.3, the MC - and TN -mappings were specified. It was argued that these mappings are deeply correct. Firstly, it was shown that those committed to the view that Plantingans and Williamsonians mean the same with the sentences of their language are committed to the deep correctness of the MC - and TN -mappings, given the assumption that the Synonymy Account is true. Afterwards, it was shown that even those who reject that Plantingans and Williamsonians mean the same with the sentences of their language are committed to the deep correctness of MC - and TN -mappings. The case for this last claim relied on showing that specific restrictions of their mappings are deeply correct, and that this result suffices for the deep correctness of the unrestricted mappings. In the appendix it is shown that the mappings witness the solid similarity between formulations MC and TN . Thus, MC and TN are synonymous. Therefore, Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are equivalent.

It was also argued in §5.3 that the identity mapping is not deeply correct. If it were deeply correct, then sentences that are clearly used by the proponents of these theories to mean different things would all mean the same according to the proponents of the theories. The section concluded with the presentation of the similarities between the dialectic between Plantingans and Williamsonians and the dialectic of typical verbal disputes. This is no surprise. The dispute between Plantingans and Williamsonians is indeed merely verbal.

Two putative objections to the claim that Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are equivalent were addressed in §5.4. The first objection consisted in a form of incredulous stare. The objection is based on the fact that it seems incredible that two theories are equivalent when their proponents, rational agents and competent speakers of their language, believe otherwise and act as such. In reply to the objection, an explanation was offered of how it can be that

two theories are equivalent when their proponents, rational agents and competent speakers of their language, believe otherwise and act as such. According to the explanation, such situations are to be expected given what it takes to speak a language and to build theories that are likely to go beyond the conventions of that language.

According to the second objection considered in §5.4, a certain model-theoretic result, Mismatch, counts against the equivalence between Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism. It was argued that Mismatch shows no such thing. On the contrary, Mismatch is an expected result under the assumption that the theories are equivalent.

In §5.4, it was shown that, given the synonymy between formulations *MC* and *FN* via the *FN*- and *MC*-mappings, these mappings enable objections to Plantingan Moderate Contingentism to be translated into objections to Williamsonian Thorough Necessitism, and vice-versa, with examples being given. Given that, as mentioned in the beginning of the chapter, Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are, arguably, the best candidate theories available, the hope is that by appealing to the *FN*- and *MC*-mappings it can be shown that at least some of the objections to each theory are not as convincing after all.

Finally, in §5.5 the question what is the correct higher-order modal logic was once more addressed. It was first noted that Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are at most sound. Then, it was pointed out that there are commitments of Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism that have not been defended in the dissertation. Yet, given the theses defended in the dissertation, these commitments do not seem implausible.

Then, one worry was addressed, namely, that the theories cannot both be true (and thus, sound), since one of them is committed to the truth of a sentence φ , whereas the other is committed to the truth of the sentence $\neg\varphi$. It was shown that this does not undermine the truth of both theories. If 'truth' is understood as applying to theories, then what is important is what is meant by φ . Since proponents of these theories mean different things by φ , both theories may still be true.

Moreover, it was shown that if 'truth' is understood as applying to formulations of theories, then the commitments of each theory belonging to the fragment $Ch \cup Pr$ may both be true. Indeed, given the theses defended in the dissertation, they are.

5.7 Appendix

I will begin by presenting functions converting MC -models into TN -models, and vice-versa, and converting variable-assignments of MC -models into variable-assignments of TN -models, and vice-versa. I will then appeal to these functions in order to prove the solid similarity of MC and TN .

Definition (From MC -Models to TN -Models). Let $M = \langle W, \odot, R, d, D, V \rangle$ be any MC -model. The function $(\cdot)^{TN}$ maps M to the following model

$$(M)^{TN} = \langle (W)^{TN}, (\odot)^{TN}, (R)^{TN}, (d)^{TN}(D)^{TN}, (V)^{TN} \rangle, \text{ where:}$$

1. $(W)^{TN} = W$;
2. $(\odot)^{TN} = \odot$;
3. $(R)^{TN} = R$;
4. $\forall w \in (W)^{TN} : (d)^{TN}(w) = \bigcup_{w \in W} d(w)$;
5. $(V)^{TN}(c_{(e)}) = V(c_{(e)})$;
6. $(V)^{TN}(=) = \{ \langle o, o \rangle : o \in \bigcup_{w \in W} d(w) \}$

For each MC -model M , $(Val)^{TN}$ is the valuation function of the model $(M)^{TN}$ assigning values to expressions relative to both variable-assignments and worlds (as defined in 1.6). Moreover, $(Val)^{TN,g}(\varphi)$ is the function mapping each world $w \in (W)^{TN}$ to $(Val)_{w,g}^{TN}(\varphi)$, the value assigned to φ relative to variable-assignment g and world w by the function $(Val)^{TN}$.

Note that $(d)^{TN}(w) = (d)^{TN}(w')$ for every $w, w' \in (W)^{TN}$. Moreover, note that there is a $o \in D_e$, $w, w' \in W$ such that $o \in d(w)$ and $o \notin d(w')$. Thus, there is a $o \in D_e$ such that $o \in V(c)(w)$ and $o \notin V(c)(w')$, for some $w, w' \in W$. Hence, there is a $o \in (d)_e^{TN}$ such that $o \in (V)^{TN}(c)(w)$ and $o \notin (V)^{TN}(c)(w')$, for some $w, w' \in (W)^{TN}$. Thus, for each MC -model M , $(M)^{TN}$ is indeed an TN -model.

The function from TN -models to MC -models is defined as follows:

Definition (From TN -Models to MC -Models). Let $M = \langle W, \odot, R, d, D, V \rangle$ be any TN -model. The function $(\cdot)^{MC}$ maps M to the following model

$$(M)^{MC} = \langle (W)^{MC}, (\odot)^{MC}, (R)^{MC}, (d)^{MC}, (D)^{MC}, (V)^{MC} \rangle, \text{ where:}$$

1. $(W)^{MC} = W$;
2. $(\odot)^{MC} = \odot$;
3. $(R)^{MC} = R$;
4. $\forall w \in W : (d)^{MC}(w) = \{ o \in D_e : d \in V(c)(w) \text{ or } \forall w \in W (o \notin V(c)(w)) \}$;
5. $(V)^{MC}(c) = V(c)$;
6. $(V)^{MC}(=)(w) = \{ \langle o, o \rangle : o \in (d)^{MC}(w) \}$

For each TN -model M , $(Val)^{MC}$ is the valuation function of the model $(M)^{MC}$ assigning values to expressions relative to both variable-assignments and worlds (as defined in 1.6). Moreover, $(Val)^{MC,g}(\varphi)$ is the function mapping each world $w \in (W)^{MC}$ to $(Val)_w^{MC,g}(\varphi)$, the value assigned to φ relative to variable-assignment g and world w by the function $(Val)^{MC}$.

Note that TN -models M are such that there are worlds $w, w' \in W$ and $o \in D_e$ such that $o \in V(c)(w)$ and $o \notin V(c)(w')$. But then, by the definition of $(d)^{MC}$, there will be worlds $w, w' \in (W)^{MC}$ and $o \in (d)^{MC}(w)$ such that $d \notin (d)^{MC}(w')$. So, for each TN -model M , $(M)^{MC}$ is indeed an MC -model.

Besides having MC -models being mapped to TN -models, variable-assignments of MC -models M are also mapped by a function $(\cdot)^{TN}$ to variable-assignments of $(M)^{TN}$, and variable-assignments of TN -models M are mapped by a function $(\cdot)^{MC}$ to variable-assignments of $(M)^{MC}$.

Where g is any variable-assignment of a MC -model M , the definition of the variable-assignment $(g)^{TN}$ of $(M)^{TN}$ is straightforward:

Definition (From MC -variable-assignments to TN -variable-assignments). $(g)^{TN} = g$

The definition of $(g)^{MC}$, requires an appeal to a function, $Proxy(\cdot)$, mapping, for each type t , an element of D_t to its proxy in $(D_{\pi(t)})^{MC}$. The $Proxy(\cdot)$ function works as follows:

Definition (Proxy Function).

1. If $o \in D_e$, then $Proxy(o) = f \in D_{\pi(e)}$ such that, for all $w \in W$:
 - If $o \in V(c)(w)$ or $\forall w' \in W : o \notin V(c)(w')$, then $f(w) = \{o\}$;
 - Otherwise, $f(w) = \emptyset$.
2. If $o \in D_{\langle t_1, \dots, t_n \rangle}$, then $Proxy(o) = f \in D_{\pi(\langle t_1, \dots, t_n \rangle)}$ such that, for all $w \in W$:
 - $f(w) = \{\langle Proxy(o^1), \dots, Proxy(o^n) \rangle : \langle o^1, \dots, o^n \rangle \in o(w)\}$

With the definition of the $Proxy(\cdot)$ function in place, the function mapping each variable-assignment g of a TN -model M , to a variable-assignment $(g)^{MC}$ of $(M)^{MC}$ is defined as follows:

Definition (From TN -variable-assignments to MC -variable-assignments).

1. $(g)^{MC}((v_t)^{MC}) = Proxy(g(v_t))$;
2. Otherwise:
 - $(g)^{MC}(v_t) = g(v_t)$, if $t = e$;
 - $(g)^{MC}(v_t) = f \in D_t$ such that $\forall w \in W : f(w) = \emptyset$, if $t \neq e$ and $v_t \neq (v'_t)^{MC}$, for some variable v'_t .

The following two theorems play an important role in the proof that MC and TN are solidly similar via $(\cdot)^{TN}$ and $(\cdot)^{MC}$:

Theorem 1. For each $\varphi \in L_{MC}$, and each MC -model $M = \langle W, \odot, R, d, D, V \rangle$, $w \in W$ and variable-assignment g of M : $M, w, g \models_{MC} \varphi$ iff $(M)^{TN}, w, (g)^{TN} \models_{TN} (\varphi)^{TN}$.

Theorem 2. For each $\varphi \in L_{TN}$, and each TN -model $M = \langle W, \odot, R, d, D, V \rangle$, $w \in W$ and variable-assignment g of M : $M, w, g \models_{TN} \varphi$ iff $(M)^{MC}, w, (g)^{MC} \models_{MC} (\varphi)^{MC}$.

Lemma 1. For every type t , for every $o \in D_t(w) : (M)^{TN}, w, (g)^{TN}[v_t/o] \models_{TN} Ch(v_t)$.

Proof of Lemma 1.

The proof is by induction on the set of types. For the case where $t = e$, note that, by the definition of a MC -model, every $o \in D_e(w)$ is such that $o \in V(c)(w)$ or $\forall w' \in W : o \notin V(c)(w')$. So, by the definition of $(M)^{TN}$, every $o \in D_e(w)$ is such that $o \in (V)^{TN}(c)(w)$ or $\forall w' \in (W)^{TN} : o \notin (V)^{TN}(c)(w')$. Hence, for every $o \in D_e(w) : (M)^{TN}, w, (g)^{TN}[v_e/o] \models_{TN} Ch(v_e)$.

For the case where $t = \langle t_1, \dots, t_n \rangle$, every $o \in D_t(w)$ is such that, for every $\langle o^1, \dots, o^n \rangle \in o(w)$: $o^1 \in D_{t_1}$ and \dots and $o^n \in D_{t_n}$. So, by the induction hypothesis, $(M)^{TN}, w, (g)^{TN}[v_{t_i}/o^i] \models_{TN} Ch(v_{t_i})$. Hence, $(M)^{TN}, w, (g)^{TN}[v_t/o] \models_{TN} Ch(v_t)$. \square

Lemma 2. For every MC -model M , type t , $w \in W$, variable-assignment g of M and $o \in (D)_t^{TN}(w)$: if $(M)^{TN}, w, (g)^{TN}[v_t/o] \models_{TN} Ch(v_t)$, then $o \in D_t(w)$.

Proof of Lemma 2.

Lemma 2 is established by induction on t . For the case where $t = e$, note that

$$(M)^{TN}, w, (g)^{TN}[v_e/o] \models_{TN} Ch(v_e) \text{ only if } o \in (V)^{TN}(c_{(e)})(w), \text{ or} \\ \forall w' \in (W)^{TN} : o \notin V(c_{(e)})(w).$$

Since $(V)^{TN}(c_{(e)}) = V(c_{(e)})$, it follows that $(M)^{TN}, w, (g)^{TN}[v_e/o] \models_{TN} Ch(v_e)$ only if $o \in V(c_{(e)})(w)$ or $\forall w' \in W : o \notin V(c_{(e)})(w')$. By assumption, $o \in (D)_e^{TN}(w)$. So, by the definition of $(D)_e^{TN}$, $o \in \bigcup_{w \in W} d(w)$. Since $d(w) = V(c_{(e)})(w) \cup \{o \in \bigcup_{w \in W} d(w) : \forall w' \in W (o \notin V(c_{(e)})(w'))\}$, it follows that $o \in d(w) = D_e(w)$.

As to the case where $t = \langle t_1, \dots, t_n \rangle$, suppose that $(M)^{TN}, w, (g)^{TN}[v_t/o] \models_{TN} Ch(v_t)$. Suppose also that $\langle o^1, \dots, o^n \rangle \in o(w')$ for an arbitrary $w' \in W$. Then, $(M)^{TN}, w', (g)^{TN}[v_{t_i}/o^i] \models_{TN} Ch(v_{t_i})$, by the definition of $Ch(v_{t_i})$. By the induction hypothesis, $o^i \in D_{t_i}(w')$, for all i such that $1 \leq i \leq n$. But then, $o \in D_t(w)$, by the definition of $D_t(w)$. This proves the theorem. \square

Proof of Theorem 1.

The proof is by induction. The interesting cases are those where (i) φ is atomic and (ii) φ is of the form $\exists v_t(\psi)$, and so these are the ones proved here. Let M be an MC -model. The proofs of these cases go as follows:

(i) $\varphi = s^0 s^1 \dots s^n$.

Since $(V)^{TN}(c) = V(c_{(e)})$, $(g)^{TN} = g$, and $(g)^{TN}[v/V(=)](v) = V(=)$, we have that

$$(Val)^{TN, (g)^{TN}[v/V(=)]}((s^i)^{TN}) = Val^g(s^i) \text{ for all } i, 1 \leq i \leq n.$$

So,

$$\begin{aligned}
(M)^{TN}, w, (g)^{TN} \models_{TN} (s^0 s^1 \dots s^n)^{TN} &\text{ iff } (M)^{TN}, w, (g)^{TN}[v/V(=)] \models_{TN} (s^0)^{TN} (s^1)^{TN} \dots (s^n)^{TN} \\
&\text{ iff } \langle (Val)^{TN, (g)^{TN}}[v/V(=)](s^1), \dots, (Val)^{TN, (g)^{TN}}[v/V(=)](s^n) \rangle \\
&\quad \in (Val)_w^{TN, (g)^{TN}}[v/V(=)](s^0) \\
&\text{ iff } \langle Val^g(s^1), \dots, Val^g(s^n) \rangle \in Val_w^g(s^0) \\
&\text{ iff } M, w, g \models_{MC} s^0 s^1 \dots s^n
\end{aligned}$$

(ii) $\varphi = \exists v_t(\psi)$.

Note that if $o \in D_t$ and g is a variable-assignment of a MC -model M , then $g[v_t/o]$ is also a variable-assignment of M . Moreover, since $(g)^{TN} = g$, for every variable-assignment g of M , the following holds:

$$\mathbf{Fact:} \quad (g)^{TN}[v_t/o] = (g[v_t/o])^{TN}.$$

We thus have that

$$\begin{aligned}
(M)^{TN}, w, (g)^{TN} \models_{TN} (\exists v_t(\psi))^{TN} &\text{ iff } (M)^{TN}, w, (g)^{TN} \models_{TN} \exists v_t(Ch(v_t) \wedge (\psi)^{TN}) \\
&\text{ iff } \exists o \in (D)_t^{TN} \text{ s.t. } (M)^{TN}, w, (g)^{TN}[v_t/o] \models_{TN} Ch(v_t) \wedge (\psi)^{TN} \\
&\text{ iff } \exists o \in (D)_t^{TN}(w) : (M)^{TN}, w, (g)^{TN}[v_t/o] \models_{TN} Ch(v_t) \text{ and} \\
&\quad (M)^{TN}, w, (g)^{TN}[v_t/o] \models_{TN} (\psi)^{TN} \\
&\text{ iff } \exists o \in D_t(w) : (M)^{TN}, w, (g)^{TN}[v_t/o] \models_{TN} (\psi)^{TN} \\
&\quad \text{(by Lemmas 1 and 2)} \\
&\text{ iff } \exists o \in D_t(w) : (M)^{TN}, w, (g[v_t/o])^{TN} \models_{TN} (\psi)^{TN} \\
&\quad \text{(by Fact)} \\
&\text{ iff } \exists o \in D_t(w) : M, w, g[v_t/o] \models_{MC} \psi \\
&\quad \text{(by the induction hypothesis)} \\
&\text{ iff } M, w, g \models_{MC} \exists v_t \psi
\end{aligned}$$

This establishes case (ii): $\varphi = \exists v_t \psi$. As mentioned, the proofs of the remaining cases are straightforward, and are thus omitted. \square

Lemma 3. For all types t , TN -models M and $o \in D_t : Proxy(o) \in (D)_{\pi(t)}^{MC}$.

Proof of Lemma 3.

The proof is by induction on t . For the base case, $Proxy(o) = f$ such that $f(w) = \{o\}$ for all w such that $o \in V(c_{(e)})(w)$ or $o \notin V(c_{(e)})(w')$ for all $w' \in W$, and $f(w) = \emptyset$ otherwise. Given the definition of $(D)^{MC}$, it follows that $Proxy(o) = f$ such that $f(w) = \{o\}$ for all w such that $d \in (D)^{MC}(w)$ and $f(w) = \emptyset$ otherwise.

Suppose that $Proxy(o^i) \in (D)_{\pi(t_i)}^{MC}$, for all i s.t. $1 \leq i \leq n$ and $o^i \in D_{t_i}$. I will show that $Proxy(o) \in (D)_{\pi(t)}^{MC}$, for an arbitrary $o \in D_t$, where $t = \langle t_1, \dots, t_n \rangle$. $Proxy(o) = f$ such that $f(w) = \{ \langle Proxy(o^1), \dots, Proxy(o^n) \rangle : \langle o^1, \dots, o^n \rangle \in o(w) \}$. But then, by the definition of $(D)_{\pi(t)}^{MC}$, $Proxy(o) \in (D)_{\pi(t)}^{MC}$. \square

Lemma 4. $\forall o \in D_t : (M)^{MC}, w, (g)^{MC}[v_{\pi(t)}/Proxy(o)] \models_{MC} Pr(v_{\pi(t)}).$

Proof of Lemma 4.

The proof is again by induction on t . For the case where $t = e$, note that $Proxy(o) = f \in (D)_{\pi(e)}^{MC}$ s.t. $f(w) = \{o\}$ if $o \in (D)_e^{MC}$ and $f(w) = \emptyset$ otherwise. Now,

$$\begin{aligned} & (M)^{MC}, w, (g)^{MC}[v_{\pi(e)}/Proxy(o)] \models_{MC} Pr(v_{\pi(t)}) \text{ iff} \\ & (M)^{MC}, w, (g)^{MC}[v_{\pi(e)}/Proxy(o)] \models \diamond \exists y_e \square \forall z_e (v_{\pi(e)} y_e \leftrightarrow z = y) \text{ iff} \\ & \text{there is some } o' \in (D)_e^{MC} \text{ s.t. for all } w \in W : \\ & Proxy(o)(w) = \{o'\} \text{ if } o' \in (D)_e^{MC}(w) \text{ and } Proxy(o)(w) = \emptyset \text{ otherwise.} \end{aligned}$$

Since o is clearly such a o' , it follows that $(M)^{MC}, w, (g)^{MC}[v_{\pi(e)}/Proxy(o)] \models_{MC} Pr(v_{\pi(t)})$.

For the case where $t = \langle t_1, \dots, t_n \rangle$, note that, by the induction hypothesis:

$$(M)^{MC}, w, (g)^{MC}[v_{\pi(t_i)}/Proxy(o^i)] \models_{MC} Pr(v_{\pi(t_i)}) \text{ for all } o^i \in D_{t_i} \text{ and } i \text{ s. t. } 1 \leq i \leq n.$$

Now, $Proxy(o)(w) = \{\langle Proxy(o^1), \dots, Proxy(o^n) \rangle : \langle o^1, \dots, o^n \rangle \in o(w)\}$. So,

$$\begin{aligned} (M)^{MC}, w, (g)^{MC}[v_{\pi(t)}/Proxy(o)] \models_{MC} \square \forall y_{\pi(t_1)}^1 \dots \forall y_{\pi(t_n)}^n (v_{\pi(t)} y^1 \dots y^n \rightarrow \\ \bigwedge_{1 \leq i \leq n} (Pr_{\langle \pi(t_i) \rangle}(y^i))) \end{aligned}$$

But this means that

$$(M)^{MC}, w, (g)^{MC}[v_{\pi(t)}/Proxy(o)] \models_{MC} Pr(v_{\pi(t)}).$$

□

Lemma 5. For each MC-model M , let f_c^M be a function with domain W and such that:

$$f_c^M(w) = \{h \in D_{\langle e \rangle} : M, w, g[v_{\langle e \rangle}/h] \models_{MC} \exists y_e (\square \forall z_e (v_{\langle e \rangle} z \leftrightarrow z = y) \wedge c_{\langle e \rangle}) y\}$$

Then, for any TN-model M ,

$$Proxy(V(c_{\langle e \rangle})) = f_c^{(M)^{MC}}.$$

Proof of Lemma 5.

$$\begin{aligned} & h \in Proxy(V(c_{\langle e \rangle}))(w) \text{ iff} \\ & \exists o \in D_e(w) \text{ s.t. } o \in V(c)(w) \text{ and } h = Proxy(o) \text{ iff} \\ & \exists o \in (D)_e^{MC}(w) \text{ s.t. } o \in (V)^{MC}(c)(w) \text{ and} \\ & \forall w \in (W)^{MC} \text{ s.t. either } o \in V(c)(w) \text{ or } \forall w' (o \notin V(c)(w')) : h(w) = \{o\}, \text{ and otherwise,} \\ & h(w) = \emptyset \text{ iff} \\ & \exists o \in (D)_e^{MC}(w) \text{ s.t. } o \in (V)^{MC}(c)(w) \text{ and} \\ & \forall w \in (W)^{MC} \text{ s.t. } o \in (D)_e^{MC}(w) : h(w) = \{o\}, \text{ and otherwise } h(w) = \emptyset. \\ & \text{iff } h \in f_c^{(M)^{MC}}(w). \end{aligned}$$

□

Lemma 6. For each MC-model M , let $f_{=}^M$ be a function with domain W and such that:

$$f_{=}^M(w) = \{\langle h, h' \rangle : h, h' \in D_{\langle e \rangle} \text{ and} \\ M, w, g[x_{\langle e \rangle}/h, y_{\langle e \rangle}/h'] \models_{MC} \diamond \exists z_e (\Box \forall u_e (xu \leftrightarrow u = z)) \wedge \diamond \exists z_e (\Box \forall u_e (yu \leftrightarrow u = \\ z)) \wedge \exists z_{\langle e \rangle} \Box \forall u_e (xu \leftrightarrow zu) \wedge \Box \forall u_e (xu \leftrightarrow yu)\}$$

Then, for any TN-model M , $Proxy(V(=_{\langle e, e \rangle})) = f_{=}^{(M)^{MC}}$.

Proof of Lemma 6.

$$\begin{aligned} \langle h, h' \rangle \in Proxy(V(=))(w) \text{ iff } \exists o, o' \in D_e(w) \text{ s.t. } \forall w \in (W)^{MC} : o \in (D)_e^{MC}(w) \text{ only if} \\ h(w) = \{o\} \text{ and} \\ o' \in (D)_e^{MC}(w) \text{ only if } h'(w) = \{o'\}, \text{ and } \forall w \text{ s.t. } o \notin (D)_e^{MC}(w) : \\ h(w) = \emptyset \text{ and } \forall w \text{ s.t. } o' \notin (D)_e^{MC}(w) : h'(w) = \emptyset \text{ and } o = o' \text{ iff} \\ \exists o, o' \in (D)_e^{MC} \text{ s.t. } \forall w \in (W)^{MC} : o \in (D)_e^{MC}(w) \text{ only if } h(w) = \{o\} \text{ and} \\ o' \in (D)_e^{MC}(w) \text{ only if } h'(w) = \{o'\}, \text{ and } \forall w \text{ s.t. } o \notin (D)_e^{MC}(w) : \\ h(w) = \emptyset \text{ and } \forall w \text{ s.t. } o' \notin (D)_e^{MC}(w) : h'(w) = \emptyset \text{ and } h = h' \\ \text{ iff } \langle h, h' \rangle \in f_{=}^{(M)^{MC}}. \end{aligned}$$

□

Lemma 7. For all TN-models M and all $h \in (D)_{\pi(t)}^{MC}$ such that $(M)^{MC}, w, g[v_{\pi(t)}/h] \models_{MC} Pr(v)$ there is an $o \in D_t$ such that $h = Proxy(o)$.

Proof of Lemma 7.

The lemma is established by induction on t . For the case when $t = e$, suppose that

$$(M)^{MC}, w, g[v_{\pi(e)}/h] \models_{MC} Pr(v).$$

Then, there is $o \in (D)_e^{MC}$ such that $h(w) = \{o\}$ for all w such that $o \in (D)_e^{MC}(w)$ and for all w' such that $o \notin (D)_e^{MC}(w') : h(w') = \emptyset$. So, $h = Proxy(o)$. By the definition of $(D)_e^{MC}$, $o \in D_e$.

As to the case when $t = \langle t_1, \dots, t_n \rangle$, suppose that $(M)^{MC}, w, g[v_{\pi(t)}/h] \models_{MC} Pr(v)$. By the definition of $Pr(v_t)$, it follows that, for each $w \in W$ and all sequences $\langle h^1, \dots, h^n \rangle \in h(w)$:

$$(M)^{MC}, w, g[v_{\pi(t_i)}^i/h^i] \models_{MC} Pr(v^i), \text{ for all } i \text{ such that } 1 \leq i \leq n.$$

So, by the induction hypothesis, there is an $o^i \in D_{t_i}$ such that $h^i = Proxy(o^i)$. Now, let o be a function with domain W and such that $o(w) = \{\langle o^1, \dots, o^n \rangle : \langle Proxy(o^1), \dots, Proxy(o^n) \rangle \in h(w)\}$. Clearly, $o \in D_t$. Moreover, $Proxy(o) = h$. So, if $(M)^{MC}, w, g[v_{\pi(t)}/h] \models_{MC} Pr(v)$ then there is an $o \in D_t$ such that $h = Proxy(o)$. This establishes the lemma. □

Proof of Theorem 2.

The proof is by induction. The interesting cases are those where (i) φ is atomic and (ii) φ is of the form $\exists v_t(\psi)$, and so these are the ones proved here. Let M be a TN -model. The proofs of these cases go as follows:

(i) $\varphi = s^0 s^1 \dots s^n$.

Consider the variable-assignment $(g)^{MC}[v_{\langle e \rangle} / f_c^{(M)^{MC}}, v'_{\langle \langle e \rangle \rangle} / f_{=}^{(M)^{MC}}]$. Since

$$f_c^{(M)^{MC}} = Proxy(V(c)) \text{ and } f_{=}^{(M)^{MC}} = Proxy(V(=)) \text{ (by Lemmas 5 and 6),}$$

it follows that $(g)^{MC}[v_{\langle e \rangle} / f_c^{(M)^{MC}}, v'_{\langle \langle e \rangle \rangle} / f_{=}^{(M)^{MC}}] = (g')^{MC}$ for some variable-assignment g' of M just like g except that $g'(v_{\langle e \rangle}) = V(c)$ and $g'(v'_{\langle \langle e \rangle \rangle}) = V(=)$.

Now, note that:

$$(M)^{MC}, w, (g)^{MC} \models_{MC} (s^0 s^1 \dots s^n)^{MC} \text{ iff } (M)^{MC}, w, (g')^{MC} \models_{MC} (s^0)^{MC} (s^1)^{MC} \dots (s^n)^{MC}.$$

Thus, to establish the base case it suffices to show that:

$$(M)^{MC}, w, (g')^{MC} \models_{MC} (s^0)^{MC} (s^1)^{MC} \dots (s^n)^{MC} \text{ if and only if } M, w, g \models_{MC} s^0 s^1 \dots s^n$$

For each i ($0 \leq i \leq n$), let $(s^i)^* = s^i$ if s^i is a variable, $(s^i)^* = v_{\langle e \rangle}$ if s^i is $c_{\langle e \rangle}$ and $(s^i)^* = v_{\langle e, e \rangle}$ if s^i is $=_{\langle e, e \rangle}$:

$$\begin{aligned} & (M)^{MC}, w, (g')^{MC} \models_{MC} (s^0)^{MC} (s^1)^{MC} \dots (s^n)^{MC} \\ & \text{iff } \langle (g')^{MC}((s^1)^{MC}), \dots, (g')^{MC}((s^n)^{MC}) \rangle \in (g')^{MC}((s^0)^{MC})(w) \\ & \text{iff } \langle Proxy(g'((s^1)^*)), \dots, Proxy(g'((s^n)^*)) \rangle \in Proxy(g'((s^0)^*))(w) \\ & \text{iff } \langle g'((s^1)^*), \dots, g'((s^n)^*) \rangle \in g'((s^0)^*)(w) \\ & \text{iff } M, w, g' \models_{TN} (s^0)^* (s^1)^* \dots (s^n)^* \\ & \text{iff } M, w, g \models_{TN} s^0 s^1 \dots s^n \end{aligned}$$

This completes the proof of the base case.

(ii) $\varphi = \exists v_t \psi$. The proof goes as follows:

$$\begin{aligned} & (M)^{MC}, w, (g)^{MC} \models_{MC} (\exists v_t \psi)^{MC} \\ & \text{iff } (M)^{MC}, w, (g)^{MC} \models_{MC} \exists v_{\pi(t)} (Pr(v_{\pi(t)}) \wedge (\psi)^{MC}) \\ & \text{iff there is a } h \in (D)_{\pi(t)}^{MC} \text{ s.t. } (M)^{MC}, w, (g)^{MC}[v_{\pi(t)}/h] \models_{MC} Pr(v_{\pi(t)}) \text{ and} \\ & \quad (M)^{MC}, w, (g)^{MC}[v_{\pi(t)}/h] \models_{MC} (\psi)^{MC} \\ & \text{iff there is } o \in D_t \text{ such that } Proxy(o) = h \text{ and} \\ & \quad (M)^{MC}, w, (g)^{MC}[v_{\pi(t)}/h] \models_{MC} (\psi)^{MC} \\ & \quad \text{(by Lemmas 3, 4 and 7)} \\ & \text{iff there is } o \in D_t \text{ such that } Proxy(o) = h \text{ and} \\ & (M)^{MC}, w, (g')^{MC} \models_{MC} (\psi)^{MC}, \text{ where } g' = g[v_t/o] \\ & \text{iff there is } o \in D_t \text{ such that } M, w, g' \models_{TN} \psi \end{aligned}$$

(by induction hypothesis)

$$\text{iff } M, w, g \models_{TN} \exists v \psi.$$

This concludes the proof of the theorem. \square

Besides Theorems 1 and 2, the proof of the solid similarity via $(\cdot)^{TN}$ and $(\cdot)^{MC}$ of MC and TN to be presented in this appendix appeals to the following lemma:

Lemma 8. For all MC -models M : $M = ((M)^{TN})^{MC}$. For all TN -models M : $M = ((M)^{MC})^{TN}$.

The proof of Lemma 8 is straightforward and thus I shall omit it.

The ingredients required to establish the solid similarity of MC and TN via $(\cdot)^{MC}$ and $(\cdot)^{TN}$ are all in place. Let me now establish the required lemmas:

Lemma 9. $\Gamma \models_{MC} \varphi$ if and only if $(\Gamma)^{TN} \models_{TN} (\varphi)^{TN}$.

Proof of Lemma 9.

(\Rightarrow):

Suppose $\Gamma \models_{MC} \varphi$. Suppose $M, w, g \models_{TN} (\gamma)^{TN}$, for all $\gamma \in \Gamma$. By Lemma 8, it follows that

$$((M)^{MC})^{TN}, w, g \models_{TN} (\gamma)^{TN}, \text{ for all } \gamma \in \Gamma.$$

Moreover, since all $(\gamma)^{TN} \in (\Gamma)^{TN}$ are closed formulae, $((M)^{MC})^{TN}, w, (g')^{TN} \models_{TN} (\gamma)^{TN}$ for some variable-assignment g' of $(M)^{MC}$. But then, by Theorem 1, it follows that $(M)^{MC}, w, g' \models_{MC} \gamma$, for each $\gamma \in \Gamma$.

From the assumption that $\Gamma \models_{MC} \varphi$ it follows that $(M)^{MC}, w, g' \models_{MC} \varphi$. Thus, again by Theorem 1, it follows that $((M)^{MC})^{TN}, w, (g')^{TN} \models_{TN} (\varphi)^{TN}$. And since $(\varphi)^{TN}$ is a closed formula, it follows that $((M)^{MC})^{TN}, w, g \models_{TN} (\varphi)^{TN}$. By lemma 8, it follows that $M, w, g \models_{TN} (\varphi)^{TN}$. Hence, $(\Gamma)^{TN} \models_{TN} (\varphi)^{TN}$.

(\Leftarrow):

Suppose $(\Gamma)^{TN} \models_{TN} (\varphi)^{TN}$. Suppose $M, w, g \models_{MC} \gamma$, for all $\gamma \in \Gamma$. By Theorem 1, it follows that

$$(M)^{TN}, w, (g)^{TN} \models_{TN} (\gamma)^{TN}, \text{ for all } (\gamma)^{TN} \in (\Gamma)^{TN}.$$

From the assumption that $(\Gamma)^{TN} \models_{TN} (\varphi)^{TN}$, it follows that $(M)^{TN}, w, (g)^{TN} \models_{TN} (\varphi)^{TN}$. Finally, again by Theorem 1, it follows that $M, w, g \models_{MC} \varphi$. Hence, $\Gamma \models_{MC} \varphi$. \square

Lemma 10. $\Gamma \models_{TN} \varphi$ if and only if $(\Gamma)^{MC} \models_{MC} (\varphi)^{MC}$.

The proof of Lemma 10 proceeds in a fashion analogous to the proof of Lemma 9, appealing to theorem 2 and lemma 8, and will thus be omitted.

Lemma 11. $\varphi \models_{MC} ((\varphi)^{TN})^{MC}$

Proof of Lemma 11.

$M, w, g \models_{MC} \varphi$ iff $(M)^{TN}, w, (g)^{TN} \models (\varphi)^{TN}$, by Theorem 1 iff $((M)^{TN})^{MC}, w, ((g)^{TN})^{MC} \models ((\varphi)^{TN})^{MC}$, by Theorem 2 iff $M, w, ((g)^{TN})^{MC} \models_{MC} ((\varphi)^{TN})^{MC}$ by Lemma 8 iff $M, w, g \models_{MC} ((\varphi)^{TN})^{MC}$, since φ is a closed formula. \square

Lemma 12. $\varphi \models_{TN} ((\varphi)^{MC})^{TN}$

The proof of Lemma 12 proceeds in a fashion analogous to the proof of Lemma 11, and will thus be omitted.

Corollary 1. MC and TN are similar via $(\cdot)^{MC}$ and $(\cdot)^{TN}$

Corollary 1 is a straightforward consequence of Lemmas 9, 10, 11 and 12.

Lemma 13. $MC \stackrel{(\cdot)^{TN}}{\triangleleft} TN$.

Proof of Lemma 13. Suppose $\varphi \in Com_{MC}$. Let M' be an arbitrary TN -model and g' be an arbitrary variable-assignment of M' . Since $\varphi \in Com_{MC}$, it follows that $(M')^{MC}, \odot, g'' \models \varphi$, for every variable-assignment g'' of $(M')^{MC}$. So, by Theorem 1, it follows that $((M')^{MC})^{TN}, \odot, (g'')^{TN} \models (\varphi)^{TN}$. From Lemma 8 it follows from this that $M', \odot, (g'')^{TN} \models (\varphi)^{TN}$. And since $(\varphi)^{TN}$ is closed, it follows that $M', \odot, g' \models_{TN} (\varphi)^{TN}$. Hence, $(\varphi)^{TN} \in Com_{TN}$. Moreover, $(\varphi)^{TN} \models_{TN} (\varphi)^{TN}$. Hence, there is a $\psi \in Com_{TN}$, namely, $(\varphi)^{TN}$, such that $(\varphi)^{TN} \models_{TN} \psi$, for every $\varphi \in Com_{MC}$. \square

Lemma 14. $TN \stackrel{(\cdot)^{MC}}{\triangleleft} MC$.

The proof of Lemma 14 proceeds as that of 13, appealing to theorem 2 and lemma 8, and is thus omitted.

Finally, we get to the desired result:

Corollary 2. MC and TN are solidly similar via $(\cdot)^{MC}$ and $(\cdot)^{TN}$.

Corollary 2 is a straightforward consequence of Corollary 1 and Lemmas 13 and 14.

6

Conclusion

Mathematics and the natural sciences make extensive use of quantificational resources. Arguably, such use does not stop at the first-order.¹ Also, ordinary thinking is inherently modal, as is thinking in the sciences.

There is some consensus towards classical first-order logic as the correct theory of first-order quantification, and there is some consensus towards **S5** as the correct theory of metaphysical necessity. Matters are murkier with respect to the correct higher-order theory. If there is any consensus, this is where it stops. What is the correct theory of first-order modal logic is up for grabs, and things are even murkier in the territory of higher-order modal logic. This is an unfortunate state of affairs.

The main question addressed in this dissertation was what is the correct higher-order modal logic. Two starting presuppositions were the following:

1. Thorough Actualism is true. Every entity whatsoever, of any type, is actually something.
2. Higher-order quantification is legitimate even if it has no adequate compositional semantic specifiable in English.

In chapter 1 it was shown that Thorough Actualism fits neatly with the Kripke-Stalnaker conception of possible worlds as ways things could have been, and thus as properties or states of the world. No defence of this conception of possible worlds was offered. The main aim was to show the reasonableness of the presupposition. A brief defence of the legitimacy of higher-order quantification was offered also in chapter 1, even though a more robust defence of the legitimacy of such resources would require a lengthier treatment. The main aim of the defence offered there was again to show the reasonableness of the presupposition.

In chapter 2 it was shown that the Propositional Functions Account of the semantics of first-order modal languages is incompatible with typical thoroughly contingentist higher-order modal theories committed to Thorough Serious Actualism. These theories reject the necessary being of haecceities and of attributions of being, whereas the Propositional Functions Account together with Thorough Serious Actualism implies the necessary being of both haecceities and attributions of being.

¹See, e.g., (Shapiro, 1991) for a defence of the indispensability of second-order languages in codifying several mathematical concepts and describing mathematical structures. Mundy (1987) offers a defence of the view that a second-order theory of quantity is superior to first-order theories of quantity .

Imagine a chemical theory C that requires rejection of the different biological theories on offer, without offering the means to see how to get to a substitute theory. For instance, it may be that the different biological theories on offer require that some proteins have shapes that they cannot have according to C . Does this mean that the rival theories are preferable to C ? It all depends on the remaining merits and shortcomings of C in comparison with its rivals. All other things being equal, C 's rivals are preferable.

This is the situation with those thoroughly contingentist theories committed to Thorough Serious Actualism that reject either i) the necessary being of haecceities or ii) the necessary being of attributions of being. These theories are inconsistent not only with the Propositional Functions Account but also with the classic accounts of the semantics of first-order modal languages, namely, the Literal Account and the Haecceities Account. Thus, all things being equal, their rivals are preferable.

Thorough contingentists may retort that not all things are equal. Considerations of a different nature favour the truth of their theories. In chapter 3 the truth of Higher-Order Necessitism was directly argued for. First, a defence of Thorough Serious Actualism was offered. Thorough Serious Actualism is thus a commitment of the correct higher-order modal theory. This already excludes thoroughly contingentist proposals such as the one offered in (Fine, 1977). The bulk of the chapter was dedicated to a defence of Propositional Necessitism. After the presentation of the defence of Propositional Necessitism, arguments for Higher-Order Necessitism analogous to those for Propositional Necessitism were offered. Schematic versions of the premises of the arguments for Higher-Order Necessitism turn out to imply the truth of an even stronger principle, namely, the comprehension principle $\hat{\text{Comp}}$.

The correct higher-order modal theory is thus committed to Thorough Serious Actualism, Higher-Order Necessitism, and the even stronger principle $\hat{\text{Comp}}$. Two main theories have all these principles as their commitments, namely, Williamson's Thorough Necessitism and Plantinga's Moderate Contingentism. Structural similarities between these two theories were already noted in (Bennett, 2006). Given the structural similarities between the theories, could they be equivalent? If so, there is no point in starting the usual comparative evaluation of the theories. The theories require the same of the world in order to be true. One is true if and only if the other is.

To adequately address the question whether the theories are equivalent, some account of what it takes for theories to be equivalent was required. I have found the existing accounts of theory equivalence to be unsatisfactory. At the risk of overgeneralising, the problem is that they are only concerned with the structural features of theories (or, more appropriately, of formulations of theories), not with what theories say about the world.

For instance, judging theories to be similar on the grounds that they consists in the same set of sentences, or the same set of models, or are notational variants, or even on the basis of their similarity or solid similarity, ignores the obvious point that theorists may use syntactically indistinguishable sentences to say radically different things. The structural similarity between theories is a necessary but not a sufficient condition for their equivalence.

Going beyond the structural similarity between theories thus requires doing some philosophy of language, as one may put it. The question that needs to be addressed is what is required from the translations between the languages of the two theories, over and above the constraints stemming from sameness of theoretical structure. Theories are formulated in terms of representational resources, and what is represented via those resources depends on the theorists and what they intend to represent with them.

In chapter 4 an account of theory equivalence was proposed that considered not only what it takes for theories to be structurally similar, but also what is required of translations witnessing their structural similarity, with respect to meaning preservation, for the theories to be equivalent. Moreover, the account also proposed some procedures for determining when those meaning-related desiderata are satisfied.

The thesis that theory equivalence is theory synonymy seems just right. For a notion of equivalence concerned with the relationship between theories and the world, what is required is, from a structural point of view, that the theories have the same entailment structure and that the commitments of each theory occupy the same places in that structure. Besides sameness of theoretical structure, theories must also satisfy a material condition to be equivalent. To wit, there must be mappings between the languages in which the theories are formulated witnessing the sameness of structure of the two theories. This is what it takes for theories to be synonymous.

The defence of the thesis that theory equivalence is theory synonymy pursued a quasi-scientific method. First, desiderata on theory equivalence were extracted from the literature on the dialectic between noneists and Quineans. Surprisingly, there is much convergence in this literature on points concerned with the relationship between theories, even if the theorists involved turn out to disagree on the question whether noneism just is allism.

After the presentation of the Synonymy Account, the thesis that theory equivalence just is theory synonymy was subjected to scrutiny. First, it was shown that the account fits into the data points previously extracted from the debate between noneists and Quineans. That is, it was shown that theory synonymy satisfies the different desiderata previously identified. Afterwards, consequences of the account were extracted, by applying it again to the debate between noneists and Quineans. The account was shown to offer a nuanced understanding of that debate. It reveals that part of the dispute between noneists and allists concerns expressive resources. It also shifts the attention from slogans to the theories falling under those slogans.

Finally, objections were considered to the effect that theory synonymy overgenerates for failing to distinguish between theories differing in i) ideological parsimony; ii) fundamentality; iii) explanatory power. The structure of the reply to these objections consisted in a dilemma. If these features have to do with the relationship between theory and world, then theories differing with respect to these features will also differ in their commitments. If features i)-iii) do not have to do with relationship between theories and the world, having to do instead with properties of the representational devices chosen to formulate the theory, then the fact that the theories differ with respect to them is irrelevant

to the question whether the theories are equivalent.

Equipped with the Synonymy Account of theory equivalence, the question whether Plantingan Moderate Contingentism is equivalent to Williamsonian Thorough Necessitism was addressed in chapter 5. As a preliminary result, it was shown that the synonymy between MC and TN is a consequence of the assumption that proponents of Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism mean the same with the syntactically identical sentences of L_{MC} and L_{TN} . The reason is that the homonymous translation witnesses the sameness of theoretical structure between MC and TN . Therefore, if Plantingans and Williamsonians are not talking past each other, then Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are equivalent, on the assumption that theory equivalence is theory synonymy. Yet, it was argued that the homonymous translation is deeply incorrect. The case for the equivalence between Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism developed in chapter 5 relied on showing that it is very reasonable to think that a certain *restriction* of the homonymous translation is deeply correct, even if the homonymous translation turns out to be deeply incorrect. Since this restriction of the homonymous translation also witnesses the sameness of theoretical structure between MC and TN , it follows that MC and TN are synonymous. Therefore, Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are equivalent, on the assumption that theory equivalence is theory synonymy.

How can it be that the theories are equivalent, given that their proponents believe that they are not? The explanation was seen to lie in the default assumption by speakers and interlocutors that they agree on the meanings of the sentences used in their linguistic interactions, given that those are sentences of a language believed by them to be a common language. This default assumption often leads to quick, fruitful and successful communication, even though it is occasionally false. The illusion of disagreement between Plantingans and Williamsonians is thus explained by their presumption that they mean the same with syntactically identical sentences. For this reason they take the two theories to be contradictory, and so inequivalent, despite the fact that, on this occasion, the presumption of agreement in meaning is false.

Finally, it was argued that Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are both true, given the assumptions that i) **S5** is sound for metaphysical modality (as many theorists take it to be) and ii) Thorough Abundantism is true (an assumption abductively justified by the truth of every instance of $\hat{C}omp$). With the result that Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are equivalent and true theories, progress has been achieved. On the one hand, the fact that the theories are equivalent reveals that there is no need to address the question which one is preferable vis-à-vis its relationship with the world. Since the theories are equivalent, they require the same of the world in order to be true. On the other hand, since Plantingan Moderate Contingentism and Williamsonian Thorough Necessitism are true and substantive theories, they constitute the basis for extensions progressively closer to the complete higher-order modal theory.

Appendices

Appendix A

Strongly Millian First- and Second-Order Modal Logics

A.1 Introduction

Classical first-order logic is widely regarded as being the correct logical system for first-order languages when ‘ \forall ’ is interpreted as having the same meaning as ‘everything,’ and classical second-order logic is regarded as being at least informally sound.¹ Similarly, the modal logic **S5** is widely regarded as being the correct system for the languages of propositional modal logic when ‘ \Box ’ is interpreted as standing for metaphysical necessity.²

However, the system that results from combining classical first-order logic and the propositional modal logic **S5** in the most natural way contains as theorems formulae that, when interpreted, correspond to intuitively false claims. Call this system **LPC=S5**.³ For instance, **LPC=S5** contains as theorems every instance of the following schema, known as the Barcan Formula:⁴

$$(BF) \quad \Diamond \exists x(\varphi) \rightarrow \exists x(\Diamond \varphi).$$

One instance of (BF) is the formula

$$\Diamond \exists x(Skx) \rightarrow \exists x(\Diamond Skx).$$

When interpreted, this formula states that if there could have been something that was a son of Kripke, then, there is something that could have been a son of Kripke. Since there could have been something that was a son of Kripke, it follows that there is something that could have been a son of Kripke. But, arguably, there is nothing that could have been a son of Kripke.⁵

¹We here refer to the deductive systems of second-order logic, not to the class of formulae that are satisfied by every *standard* model.

²Thus, [Williamson \(2013, p. 44\)](#) states that ‘(...) most metaphysicians accept **S5** as the propositional modal logic of metaphysical modality (...): Still, the propositional modal logic **S5** is not universally accepted by metaphysicians. See, e.g., [Salmon, 1989](#)).

³The system **LPC=S5** is described, for instance, in [\(Williamson, 1998\)](#), from where its name has been taken.

⁴See [\(Barcan, 1946\)](#) for a first study of this principle.

⁵For a defence of the view that there is something that could have been a son of Kripke, see [\(Linsky & Zalta, 1994\)](#), [\(Williamson, 2013, ch. 1\)](#).

Other problematic theorems of the system **LPC=S5** are every instance of the following schema, known as the Converse Barcan Formula,

$$(CBF) \quad \exists x(\diamond\varphi) \rightarrow \diamond\exists x(\varphi),$$

as well as the formula

$$(NNE) \quad \Box\forall x(\Box\exists y(x = y)).$$

Formula (NNE) has as its content the implausible thesis that necessarily everything necessarily exists (strictly, speaking, the content of this formula is the thesis that necessarily, everything is necessarily something – in the present paper, being something and existing are treated as equivalent). One of the main problems with (CBF) is that one of its instances, in conjunction with the uncontroversial principle that necessarily everything is something, implies the following unnecessitated version of (NNE),

$$(NE) \quad \forall x(\Box\exists y(x = y)).$$

This formula states that everything necessarily exists. Prima facie, this is false. Kripke would have failed to exist if his parents had never met.

Similarly, the system that results from combining classical second-order logic and the modal logic **S5** contains as theorems formulae that, when interpreted, correspond to theses whose truth is somewhat doubtful. Call this system **SPC=S5**.⁶ For instance, **SPC=S5** contains as theorems every instance of the following schema (a second-order version of (BF)):

$$(BF_M) \quad \diamond\exists X(\varphi) \rightarrow \exists X(\diamond\varphi).$$

One instance of (BF_M) is the formula

$$\diamond\exists X(\exists x(Skx \wedge \Box\forall y(Xy \leftrightarrow x = y))) \rightarrow \exists X(\diamond\exists x(Skx \wedge \Box\forall y(Xy \leftrightarrow x = y))).$$

When interpreted, this formula states (informally) that if there could have been a property X and an individual x such that x was a son of Kripke and X was the property of being x , then there is a property X such that there could have been an x such that x was a son of Kripke and X is the property of being x . But even though it is plausible to think that there could have been such a property X , the (actual) existence of X is controversial. It is often assumed that any property Y which is such that there could have been a y such that Y is the property of being y ontologically depends on the existence of y .⁷ Thus, arguably, property X ontologically depends on the existence of x . Since x does not exist, property X also does not exist. Therefore, (SBF) has at least one false instance.⁸

⁶A description of a higher-order modal system having the system **SPC=S5** as a subsystem can be found in (Gallin, 1975, ch. 3, pp. 71-74). The axioms of the **SPC=S5** are the instances, in a second-order modal language (i.e., a language whose types are restricted to e and, for every n , the n -ary sequence composed of e), of the schemata presented in (Gallin, 1975, pp. 73-74).

⁷Or at least ontologically depends on the existence of everything on which y ontologically depends y . The point made in the main text would still be available with this qualification in place.

⁸Adams (1981), Fine (1985) and Stalnaker (2012) all challenge the existence of property X . In general, these authors reject the claim that every instance of the schema (BF_M) is true.

As in the first-order case, other problematic theorems of **SPC=S5** are every instance of the second-order version of the Converse Barcan Formula, the schema

$$(CBF_M) \quad \exists X(\diamond\varphi) \rightarrow \diamond\exists X(\varphi),$$

as well as the formula

$$(NNE_M) \quad \Box\forall X(\Box\exists Y(\Box\forall x(Xx \leftrightarrow Yx))).$$

If it is assumed that properties are the same if necessarily coextensive, then this formula states that necessarily every property necessarily exists. Even without this assumption, the formula still states the controversial thesis that necessarily, for every property X , necessarily there is some property that is necessarily coextensive with X . One of the problems with (CBF_M) is that it implies, in conjunction with the plausible principle that necessarily, for every property there is some property that is necessarily coextensive with it, the following unnecessitated version of (NNE_M) ,

$$\forall X(\Box\exists Y(\Box\forall x(Xx \leftrightarrow Yx))).$$

But, arguably, it is not the case that necessarily there is some property that is necessarily coextensive with the property of being Kripke.

Call *conservative* any quantified modal logic that does not contain the controversial consequences of the systems **LPC=S5** and **SPC=S5**. Most of the conservative first-order modal logics that have been proposed are either devised solely for languages without individual constants (the paradigmatic example of such a system being the one proposed in (Kripke, 1963)), or else fail to capture the fact that the theorems of classical first-order logic are all actually true (some systems of this kind are the system **G** presented in (Menzel, 1991, pp. 360-363) and the systems put forward in, respectively, (Hughes & Cresswell, 1996, pp. 366-367) and (Stalnaker, 1994)). As to conservative second-order modal logics, to my knowledge these have only been given model-theoretic presentations, and in any case these logics fail to capture the fact that the theorems of classical second-order logical are all true in the actual world.⁹

A common feature of these conservative first-order modal systems is their treatment of individual constants on the model of Millian proper names. Recall the thesis of *Millianism* in the philosophy of language, according to which every proper name of English possesses a referent, and its referent is the proper name's sole contribution to the determination of the truth-conditions of every sentence in

⁹Some conservative systems of first-order modal logic do capture the actual truth of the theorems of classical first-order logic. Some examples are the system **A** described in (Menzel, 1991) and the systems described in (Stephanou, 2005). In general, I am sympathetic to the way individual constants are treated in these logics, taking as values (in a model) only entities in the domain of the actual world. In effect, the logics in question are appropriately called *strongly Millian*, given the terminology used in this chapter. However, the way these systems are presented seems to presuppose a particular take on the question whether logical truths are necessary, requiring a negative answer to the question. In contrast, some of the Millian logics presented here accommodate in a single system the views that i) logical truths are necessary and ii) the truths of quantified modal logic are true in the actual world. Unsurprisingly, this is done by the addition of an actuality operator to the language. In any case, the most important contribution of the present paper lies in the Millian logics it offers for second-order modal languages.

which it occurs. Independently of whether Millianism is true, say that a proper name is *Millian* just in case it possesses a referent and its sole contribution to the determination of the truth-conditions of any sentence in which it occurs consists in its referent. Then, the thesis of *Millianism* may be equivalently formulated as the thesis that every proper name of English is a Millian proper name. The model-theoretic semantics of these alternative systems is such that the value of each individual constant a (in a model M) is some possible individual, and the contribution made by a to the value of any complex expression in which a occurs consists solely in a 's value (in M). Thus, individual constants are treated, in each model, as if they were Millian proper names, and each model provides a representation of that which is, according to the Millian, the semantics of proper names of English. A consequence of such treatment of individual constants is that the logics in question shed some light on the logic of Millian proper names (for instance, on the relationship between Millian proper names and quantification).

Say that a proper name is *strongly Millian* just in case it is a Millian proper name and its referent (actually) exists. *Strong Millianism* is the view that every proper name of English is a strongly Millian proper name. As we shall see in the next section, it is not implausible to think that Millianism implies Strong Millianism.¹⁰ In this paper first-order modal systems treating individual constants on the model of strongly Millian proper names are presented. For this reason, the logical systems characterised here will be called *strongly Millian* systems of first-order modal logic. Strongly Millian first-order modal systems shed some light on the logic of strongly Millian proper names (for instance, on the relationship between strongly Millian proper names and quantification). It will be shown that all the theorems of classical first-order logic (or the result of prefixing them with an actuality operator) are theorems of the strongly Millian first-order modal systems to be proposed. Furthermore, these systems count the schemata (BF) and (CBF) as invalid, and do not have (NNE) as a theorem (nor do they have (NE) as a theorem).

The common conservative alternatives to **SPC=S5** assign to predicates intensions that need not belong to the domain of the actual world. In contrast, the second-order modal systems presented here assign to predicates only intensions in the domain of the actual world. By analogy with the first-order case, these systems may be called *strongly Millian* systems of second-order modal logic. It is shown that all the theorems of classical second-order logic (or their actualisations) are theorems of the strongly Millian systems of second-order modal logic here proposed. Furthermore, these systems count the schemata (BF_M) and (CBF_M) as invalid, and do not have (NNE_M) as a theorem.

The main aim of the present paper is thus to present sound and complete strongly Millian first- and second-order modal logics which enjoy the following attractive features: 1) are conservative; 2) capture a special feature of classical quantified logic, to wit, that its theorems are all true in the actual world.

The reason why several conservative alternatives to **LPC=S5** and **SPC=S5** fail to capture

¹⁰No commitment will be adopted with respect to the truth or falsehood of Millianism (i.e., no commitment will be made with respect to the truth of the claim that every proper name of English is a Millian proper name). A fortiori, no commitment to strong Millianism is adopted.

the actual truth of the theorems of first-order logic is, from the perspective here adopted, that their treatment of individual constants and predicates is too liberal, allowing the denotation of some individual constants and predicates to consist in merely possible entities.

The paper is divided in two parts. In the first part of the paper (§A.2–§A.4) several strongly Millian first- and second-order modal logics are presented. In §A.2 the notions of validity that the strongly Millian logics to be presented aim to capture are introduced, and certain metaphysical and semantic presuppositions of these logics are spelled out. The language, model-theoretic semantics and deductive systems of strongly Millian logics are presented in §A.3. In §4 it is shown that strongly Millian logics have as theorems all the theorems of classical quantified logic, or all their actualisations, and thus that these logics capture the fact that the theorems of classical quantified logic are all actually true. It is also noted that the schemata (BF), (CBF), (BF_M), and (CBF_M) are not valid in these logics, and that they do not have the formulae (NNE) and (NNE_M) as theorems.

In the second part (§A.5–§A.7) a selection of issues concerning the logics presented are discussed. In §A.5, strongly Millian second-order modal logics are applied to an argument in the metaphysics of modality, presented in (Williamson, 2013, ch. 6), purporting to show that the strongest reasonable second-order modal logic has (NNE_M) as one of its theorems. It is shown how the opponent of (NNE_M) may resist the argument by appealing to the strength of strongly Millian second-order modal logics. The similarities between strongly Millian logics and two other proposals in the literature are discussed in some detail in §A.6. Finally, the question whether strongly Millian logics are really second-order is addressed in §A.7. It is shown that this question has different answers depending on one's target conception of properties, and that Millian logics are in fact second-order logics given some popular conceptions of properties.

A.2 Orientation

A.2.1 General Validity and Real-World Validity

General validity and *real-world validity* are two properties of arguments. An argument with premises Γ and conclusion φ is *generally valid* if and only if, for every (admissible) interpretation of the argument's non-logical expressions, it is impossible for the premises of the argument to be true and for the conclusion to be false, independently of how many possibilities there are, and how much variation between possible existents there is. An argument is *real-world valid* if and only if, for every (admissible) interpretation of the argument's non-logical expressions, it is not actually the case that the premises of the argument are true and the conclusion is false, independently of how many possibilities there are and how much variation between possible existents there is.¹¹

¹¹The use of 'actually', in the present formulation of real-world validity, is one where this expression is not context-dependent, referring to the actual world, i.e., the world that obtains. For different conceptions of logical consequence, see (Shapiro, 1998). General validity is akin to Shapiro's preferred conception of logical consequence in that paper. According to that conception, 'Φ is a logical consequence of Γ if Φ holds in all possibilities under every interpretation of the nonlogical terminology in which Γ holds' (Shapiro, 1998, p. 148). The possibility in question here seems to be of a logical nature instead of a metaphysical nature. But see (Shapiro, 1998, p. 147ff.). A conception of logical consequence somewhat akin to

General validity and real-world validity encapsulate a certain neutrality. For instance, on the intended reading of real-world validity and general validity, whether certain arguments have these properties is something which does not depend on whether there is only one or instead many possible worlds. Similarly, whether certain arguments are generally valid or real-world valid is something which depends neither on the number of individuals that exist at each possible world, nor on whether the same or different individuals may be found in different worlds.

Logics which enjoy this kind of neutrality can be theoretically useful. They are useful to theorists who have no commitment to any particular answer to the aforementioned questions. They are also useful to theorists committed to certain answers when these theorists are involved in projects which require them not to beg any of the relevant questions. Importantly, the claim that logics which capture the generally valid or real-world valid arguments are useful should not be confused with the claim that it is only logics fulfilling these desiderata that are correct. Here I remain neutral both on whether the correct notion of validity is captured by general validity or by real-world validity, and on whether there are many equally correct conceptions of validity.¹²

The strongly Millian logics to be presented (as well as their ‘weak’ counterparts) all aim to capture either general-validity or real-world validity. Thus, the logics proposed encapsulate the kind of neutrality present in the notions of general validity and real-world validity. This is not to say that the logics contain no substantial presuppositions. They do, as will now be shown.

A.2.2 Presuppositions

There are certain aspects in which the logics being offered are not neutral. A first aspect concerns a metaphysical doctrine known in the literature as *Serious Actualism* or the *Being Constraint*.¹³ According to this doctrine, it is impossible for there to be individuals such that it is possible for them to stand in some relation without being something. Serious Actualism, as almost every philosophical thesis, has been disputed. Nonetheless, the principle seems quite plausible.

The assumption of Serious Actualism has an interesting consequence. Suppose that Millianism is true. Given the assumption of Serious Actualism (and the assumption that reference is a genuine relation), it follows that the referent of every proper name actually exists. That is, given the plausible assumption that Serious Actualism is true, Millianism implies Strong Millianism, as mentioned in §A.1. According to the intended interpretation of individual constants and n -ary predicate letters (of the target languages), these are all strongly Millian expressions (as this notion is characterised in §A.1).

Concerning the range of the second-order quantifiers, it is assumed that these do not range over

real-world validity, also discussed by Shapiro, is the following: Φ is a logical consequence of Γ if and only if ‘The truth of the members of Γ guarantees the truth of Φ ’ in virtue of the meanings of a special collection of the terms, the ‘logical constants’ (Shapiro, 1998, p. 132).

¹²See (Hanson, 2006) and (Nelson & Zalta, 2010) for a somewhat recent debate over which of general validity and real-world validity is the correct notion of validity. I also remain neutral on what is intended reading of ‘correct’. However, it is shown that the fact that the theorems of classical quantified logic are all true can be successfully captured by appealing to both general validity and real-world validity. Thus, none of these notions is more appropriate than the other when the goal is that of capturing this feature of the theorems of classical quantified logic in the setting of a quantified modal language.

¹³See (Plantinga, 1983, p. 11) and (Williamson, 2013, p. 148-149).

extensional entities such as sets or classes. Instead, they are taken as ranging over entities whose identity criterion is given by necessary coextensiveness. This does not imply a rejection of relations with other identity conditions, but it presupposes that there are entities which are identical if and only if they are necessarily coextensive. More will be said on the intended range of the second-order quantifiers in §A.7.

Given the assumption that the identity criteria for n -ary relations consists in their necessary coextensiveness, identity statements between relations and statements concerning the existence of relations can be formulated without appealing to an extra logical constant having as its intended interpretation the relation of identity between n -ary relations. Let τ^n and τ'^n be n -ary second-order terms, and v_1, \dots, v_n be the first n -ary sequence of individual variables such that there is no $0 \leq i \leq n$ such that v_i occurs in neither τ^n nor τ'^n . The expression $\tau^n \equiv \tau'^n$ abbreviates the formula $\Box \forall v_1 \dots \forall v_n (\tau^n v_1 \dots v_n \leftrightarrow \tau'^n v_1 \dots v_n)$.

The statement that the n -ary relation that is the semantic value of τ^n is something can be captured by the formula $\exists V^n (V^n \equiv \tau^n)$, where V^n is the first n -ary second-order variable distinct from τ^n . Let $E\tau^n$ abbreviate such statement.

Finally, the following formula states that the semantic values of τ^n and τ'^n are the same n -ary relation: $E\tau^n \wedge (\tau^n \equiv \tau'^n)$. We use $\tau^n = \tau'^n$ to abbreviate this formula.

Some of the languages to be adopted contain an operator, λ which, given a formula φ , produces the complex predicate $\lambda v_1, \dots, v_n (\varphi)$. According to the intended interpretation of this expression, it denotes the relation that obtains between v_1, \dots, v_n if and only if φ . For instance, where P stands for the property of being a philosopher and G stands for the property of being Greek, $\lambda x (Px \wedge Gx)$ stands for the property of being a Greek philosopher.

A different presupposition of the strongly Millian logics proposed has to do with the general validity (and, a fortiori, real-world validity) of all 'instances' of a principle concerning the circumstances in which the semantic value of a complex predicate exists. In order to introduce this principle it is helpful to begin with some abbreviations and definitions.

Let Et abbreviate the formula $\exists v (v = t)$, which states the existence of the individual which is the semantic value of t (where t is an individual variable or constant and v is the first individual variable different from t). Also, suppose φ is a formula which contains as individual terms only the terms t_1, \dots, t_j , for some $j \in \mathbb{N}_0$ and as n -ary second-order terms only the terms $\tau_1^n, \dots, \tau_{m_n}^n$ (where m_n is a function of n), for each $n, m_n \in \mathbb{N}_0$. Let $E\varphi$ abbreviate the following conjunction $\bigwedge_{i \leq j} Et_i \wedge \bigwedge_{l \leq n} (\bigwedge_{k \leq m_n} E\tau_k^l)$.¹⁴

Now, consider the following schema:

$$(CComp) \quad E\varphi \rightarrow E\lambda v_1 \dots v_n (\varphi).$$

Intuitively, what (CComp) states is that if the referents of all terms occurring in φ all exist, then there is that n -ary relation that necessarily holds of v_1, \dots, v_n if and only if φ .

¹⁴The empty conjunction is taken to consist in the formula $\forall X^0 (X^0 \rightarrow X^0)$; any other tautologous formula would do the required job.

Let a $@\Box\forall_2$ -closure of a formula φ be any closed formula resulting from prefixing φ with any sequence of $@$, \Box , $\forall v$ (for any individual variable v) and $\forall V^n$ (for any n -ary second-order variable V^n and $n \in \mathbb{N}_0$), in any order. The presupposition is that every $@\Box\forall_2$ -closure of every instance of (CComp) is generally valid. The truth of every $@\Box\forall_2$ -closure of every instance of (CComp) may be seen as a consequence of the following two assumptions:

1. For every ψ , the semantic value of ψ is a function of the semantic values of its component expressions in such a way that necessarily, if the semantic values of all of ψ 's component expressions exist, then the semantic value of ψ itself exists;
2. Necessarily, if the semantic value of ψ exists, then the semantic value of $\lambda v_1 \dots v_n(\psi)$ exists.

From these two assumptions it follows that, for every ψ , necessarily, if the semantic values of all of ψ 's component expressions exist, then the semantic value of ψ itself exists (assuming that the semantic values of the logical constants all necessarily exist). Much more would be required in order to provide an adequate defence of these assumptions, such as a detailed study of the semantics of complex predication. This is outside the scope of the present paper.

Finally, it is worth mentioning the stance on open formulae that will be adopted. The semantics of open formulas is a delicate issue. Our decision has been to adopt a neutral stance on it. For this reason no open formula will occur as an axiom or theorem of the strongly Millian logics offered (nor of their weak counterparts, also presented here).¹⁵

A.3 Strongly Millian Quantified Modal Logics

A.3.1 Languages

The language $\mathcal{P}\mathcal{L}$ contains ' \neg ' and ' \wedge ' as logical constants, and $P_0^0, P_1^0, P_2^0 \dots$ as 0-ary predicate letters. Formulas are constructed in the usual manner. The constants ' \forall ', ' \rightarrow ' and ' \leftrightarrow ' are defined as expected. $\mathcal{P}\mathcal{L}_\Box$ is the language that results from adding the logical constant ' \Box ' to $\mathcal{P}\mathcal{L}$ (with ' \Diamond ' being defined in the usual manner), and $\mathcal{P}\mathcal{L}_{@\Box}$ results from adding '@' to $\mathcal{P}\mathcal{L}_\Box$.

The language $\mathcal{F}\mathcal{L}$ contains the first-order variables x_1, x_2, \dots , the individual constants s_0, s_1, s_2, \dots , and, for each natural number n , the n -ary predicates $P_0^n, P_1^n, P_2^n, \dots$. The logical constants of the language are those of $\mathcal{P}\mathcal{L}$, as well as ' \forall ' (' \exists ' is defined in the usual manner) and '='. By adding ' \Box ' to $\mathcal{F}\mathcal{L}$ one obtains the language $\mathcal{F}\mathcal{L}_\Box$, and by adding '@' to $\mathcal{F}\mathcal{L}$ one obtains the language $\mathcal{F}\mathcal{L}_{@\Box}$.

The language $\mathcal{S}\mathcal{L}$ is obtained from $\mathcal{F}\mathcal{L}$ by adding, for each natural number n , the variables $X_0^n, X_1^n, X_2^n, \dots$. The languages $\mathcal{S}\mathcal{L}_\Box$ and $\mathcal{S}\mathcal{L}_{@\Box}$ are obtained in the expected manner.

Finally, by adding the variable-binding operator λ to $\mathcal{F}\mathcal{L}$ one obtains the language $\mathcal{F}\mathcal{L}_\lambda$, and by adding λ to $\mathcal{F}\mathcal{L}_{@\Box}$ one obtains the language $\mathcal{F}\mathcal{L}_{@\Box\lambda}$. Similarly for language $\mathcal{S}\mathcal{L}$. Given a formula φ , $\lambda v_1 \dots v_n(\varphi)$ is an n -ary complex predicate. When $n = 0$ the result is $\lambda(\varphi)$, a well-formed 0-ary complex predicate.

¹⁵Some discussion on the semantics of open formulae is pursued in section A.6 as a way of contrasting strongly Millian logics with related proposals in the literature.

Given a language L , $Const(L)$ is the set of individual constants of L , $Var(L)$ is the set of individual variables of L , and $Terms(L) = Const(L) \cup Var(L)$. Furthermore, $Pred^n(L)$ is the set of n -ary predicates of L , for each $n \in \mathbb{N}_0$, $Pred(L) = \bigcup_{n \in \mathbb{N}_0} Pred^n(L)$, and $CPred^n(L)$ is the set of n -ary complex predicates of L , for each $n \in \mathbb{N}_0$. The set $SVar^n(L)$ is the set of n -ary second-order variables of L , for each $n \in \mathbb{N}_0$, $SVar(L) = \bigcup_{n \in \mathbb{N}_0} SVar^n(L)$, $STerms^n(L) = SVar^n(L) \cup Pred^n(L) \cup CPred^n(L)$, for each $n \in \mathbb{N}_0$, and $STerms(L) = \bigcup_{n \in \mathbb{N}_0} STerms^n(L)$. Finally, $Form(L)$ is the set of formulas of L and $CForm(L)$ is the set of closed formulas of L .

The following metalinguistic variables are used: ' a ', ' a' '... range over $Const(L)$; ' ζ^n ', ' ζ'^n ', ... range over $Pred^n(L)$, for each $n \in \mathbb{N}_0$; ' v ', ' v' ', ... range over $Var(L)$; ' V^n ', ' V'^n ', ... range over $SVar^n(L)$, for each $n \in \mathbb{N}_0$; ' t ', ' t' ', ... range over $Terms(L)$; ' τ^n ', ' τ'^n ', ... range over $STerms^n(L)$; ' φ ', ' γ ', ' ψ ' and ' χ ' range over $Form(L)$; ' Γ ', ' Γ' ', ' θ ' range over subsets of $Form(L)$. Sometimes ' φ ', ' γ ', ' ψ ' and ' χ ' are also used as metalinguistic variables ranging over the set comprising all formulas, terms and second-order terms of L .

The formal languages being offered will be used with the presumption that their logical expressions are meaningful, and that their non-logical expressions, if used meaningfully, have certain semantic properties. The intended meaning of \neg and \wedge is their customary, boolean, meaning. The logical constant \forall is intended to express unrestricted universal quantification, $=$ expresses the identity relation (between individuals) and \Box is intended to express metaphysical necessity. As advertised, individual constants are understood as strongly Millian expressions. The same stance is taken towards n -ary predicate letters. Whereas individual constants refer to individuals, n -ary predicate letters refer to n -ary relations.

The connective $@$ is understood as standing for an actuality operator. There are two salient readings of this operator, namely, a context-dependent reading and a context-independent reading. Both readings can be elucidated by appealing to talk of contexts and possible worlds. Briefly, according to the context-dependent reading, a formula of the form $@\varphi$ is true at a context c and possible world u if and only if φ is true at c and w_c , the possible world of context c . According to the context-independent reading of this operator, $@\varphi$ is true at a context c and possible world u if and only if φ is true at c and α , where α is a context-independent expression that refers to this world, i.e. α refers to the world which, according to some possible worlds' theorists, is adequately described as the maximal way things are.

In order to distinguish these two readings, consider the following sentence: 'If Plato had been Aristotle's disciple, then an utterance of the sentence "actually, Plato was Aristotle's disciple," with the sentence being used with its current meaning, would then have been true.'

If 'actually,' as mentioned in the sentence, is understood as a context-dependent expression, then the sentence is true. However, if the relevant reading is the context-independent one, then the sentence is false. The reason is that even if the sentence 'actually, Plato was Aristotle's disciple' had been used in a counterfactual circumstance, it would not have been true, since (in fact) Plato was not Aristotle's disciple.

A.3.2 Model-Theoretic Semantics

According to the stance on the model-theoretic semantics for a language adopted here a language's model-theoretic semantics is a model of the (real) semantics for that language.¹⁶ Thus, the aim of the model-theoretic semantics proposed here is to represent several aspects of the (real) semantics of the target languages. The representational significance of several aspects of the model-theoretic semantics offered are noted during their presentation.

I begin by defining the notions of a inhabited model structure and of a inhabited second-order model structure.

Definition 1 (Inhabited Model Structures).

- An inhabited model structure is a triple $\langle W, d, \alpha \rangle$, where $\alpha \in W$, and d is a function with domain W which assigns to every $w \in W$ a set, and which is such that $\bigcup_{w \in W} d(w) \neq \emptyset$.
- Let $\langle W, d, \alpha \rangle$ be any inhabited model structure. Then, $\langle W, d, D, \alpha \rangle$ is an inhabited second-order model structure, where:
 - D is a function with domain \mathbb{N}_0 and mapping each $n \in \mathbb{N}_0$ to a function $D(n)$ with domain W and such that, for every $w \in W$, $D(n)(w) \subseteq F(n)$, where:
 - * $F(n)$ is the set of all functions f with domain W and such that, for each $w \in W$, $f(w) \subseteq (d(w))^n$.

Unsurprisingly, the set W represents the class of all metaphysically possible worlds, α represents the actual world, d is a function which maps each $w \in W$ to a set that represents the domain of all individuals that exist in the possible world represented by w , and D is a function which maps each natural number n and $w \in W$ to a set that represents the domain of all n -ary relations that exist in w .

For each function $f \in D(n)(w)$ (for each $n \in \mathbb{N}_0$ and $w \in W$), the set $f(w')$ of n -ary sequences of elements in $d(w')$ (for each $w' \in W$) represents the set of sequences of individuals that would have been in the relation represented by f if the possible world represented by w' had obtained. The fact that each function f in $D(n)(w)$ has W as its domain is also representationally significant. Since the present interest is on relations understood to be identical if and only if they are necessarily coextensive, taking the elements of each set $D(n)(w)$ to be functions with W as their domain is the natural option.

Finally, the fact that for each $f \in D(n)(w)$, $f(w') \subseteq (d(w'))^n$ is itself representationally significant. It represents the assumption that necessarily, standing in some relation requires existence. Thus, there is no sequence in $f(w')$ containing elements which are not in $d(w')$, since otherwise the sequence would (wrongly) represent the putative fact that if the world represented by w' had obtained, then the relation represented by f would have obtained between some individuals that would not then exist (i.e., would not exist in the world represented by w').

We are now in a position to define the class of *weakly* Millian models.

¹⁶This picture of model-theoretic semantics is endorsed in, for instance, (Shapiro, 1991, p. 6).

Definition 2 (Weakly Millian Models (W-Models)). Where $\langle W, d, D, \alpha \rangle$ is an inhabited second-order model structure, $M = \langle W, d, D, \alpha, V \rangle$ is a weakly Millian model, a W-model, based on $\langle W, d, D, \alpha \rangle$ for language L , where V is a function with domain $Const(L) \cup Pred(L)$ such that:

1. for every individual constant a of L , $V(a) \in \bigcup_{w \in W} d(w)$;
2. for every n -ary predicate ζ^n of L , $V(\zeta^n) \in \bigcup_{w \in W} D(n)(w)$.¹⁷

The set W is defined as follows: $W = \{M : M \text{ is a W-model}\}$.

The usual notions of a variable-assignment and variable-assignment variant are now defined:

Definition 3 (Variable Assignments of W-models). A variable-assignment g of a W-model M based on an inhabited second-order model structure $\langle W, d, D, \alpha \rangle$ is any function with domain $Var(L) \cup SVar(L)$ and such that:

1. for every individual constant v , $g(v) \in \bigcup_{w \in W} d(w)$;
2. for every n -ary second-order variable V^n , $g(V^n) \in \bigcup_{w \in W} D(n)(w)$;

Definition 4 (Variable-Assignment Variant). Let g be a variable-assignment. The function $g[V/o]$ is a function just like g except that it assigns the object o to the variable $V \in Var(L) \cup SVar(L)$ if the variable V is in the domain of g . Otherwise, $g[V/o] = g$.

The set $As(M)$ is the set of variable-assignments of model M . The value in a W-model M of a term or formula relative to $w \in W$ and $g \in As(M)$ is defined thus:

Definition 5 (Value of a term or formula).

1. If $\varphi = a \in Const(L)$, then $V_{M,w}^g(a) = V(a)$;
2. If $\varphi = \zeta^n \in Pred^n(L)$, then $V_{M,w}^g(\zeta^n) = V(\zeta^n)(w)$;
3. If $\varphi = v \in Var(L)$, then $V_{M,w}^g(v) = g(v)$;
4. If $\varphi = \tau^n t_1 \dots t_n$, then $V_{M,w}^g(\varphi) = \{\langle \rangle : \langle V_{M,w}^g(t_1), \dots, V_{M,w}^g(t_n) \rangle \in V_{M,w}^g(\tau^n)\}$.¹⁸
5. If $\varphi = \neg\psi$, then $V_{M,w}^g(\varphi) = \{\langle \rangle\} - V_{M,w}^g(\psi)$;
6. If $\varphi = \psi \wedge \chi$, then $V_{M,w}^g(\varphi) = V_{M,w}^g(\psi) \cap V_{M,w}^g(\chi)$;
7. If $\varphi = \Box\psi$, then $V_{M,w}^g(\varphi) = \bigcap_{w \in W} V_{M,w}^g(\psi)$;
8. If $\varphi = @v\psi$, then $V_{M,w}^g(\varphi) = V_{M,\alpha}^g(\psi)$;
9. If $\varphi = \forall v\psi$, then $V_{M,w}^g(\varphi) = \bigcap_{o \in d(w)} V_{M,w}^{g[V/o]}(\psi)$;
10. If $\varphi = \lambda v_1 \dots v_n(\psi)$, then $V_{M,w}^g(\varphi) = \{\langle o_1, \dots, o_n \rangle : V_{M,w}^{g[v_1/o_1] \dots [v_n/o_n]}(\psi) = \{\langle \rangle\}\}$;
11. If φ is an n -ary predicate variable V^n , then $V_{M,w}^g(\varphi) = g(V^n)(w)$

¹⁷One could also define the usual notion of a model $\langle W, d, \alpha, V \rangle$ for L based on an inhabited structure $\langle W, d, \alpha \rangle$ by taking V to be a function such that: i) for every individual constant a of L , $V(a) \in \bigcup_{w \in W} d(w)$, and, ii) to every n -ary predicate ζ^n of L , $V(\zeta^n)$ is a function with domain W and such that, for every $w \in W$, $V(\zeta^n)(w) \subseteq (d(w))^n$. It will be simpler to appeal to just one notion of a model in what follows, even though it is easy to see how one can recover models based on inhabited model structures from models based on second-order inhabited model structures.

¹⁸Note that here the convention has been followed of equating $\langle \rangle$, the empty sequence, with the empty set.

$$12. \text{ If } \varphi = \forall V^n \psi, \text{ then } V_{M,w}^g(\varphi) = \bigcap_{f \in D(n)(w)} V_{M,w}^{g[v/f]}(\psi).$$

Given the definition of value of φ in M relative to $g \in As(M)$ and world $w \in W$, $V_M^g(\varphi)$ is that function f with domain W such that, for every $w \in W$ $f(w) = V_{M,w}^g(\varphi)$.

We now define strongly Millian models in terms of weakly Millian models:

Definition 6 (Strongly Millian Models (S-models)). A S-model is any W-model such that:

1. for every individual constant a , $V(a) \in d(\alpha)$.
2. for $n \in \mathbb{N}_0$ and n -ary predicate ζ^n , $V(\zeta^n) \in D(n)(\alpha)$.

The set S is defined as follows: $S = \{M : M \text{ is a S-model}\}$.

The fact that $V(a) \in d(\alpha)$ represents the fact that the individual constant a is being treated as a strongly Millian expression, since the valuation function assigns to a an entity which belongs to the set $d(\alpha)$, a set which represents the domain of individuals that exist in the actual world. Similarly, the fact that $V(\zeta^n) \in D(n)(\alpha)$ represents the fact that the n -ary predicate letter ζ^n has as its denotation an n -ary relation that exists in the actual world, since $D(n)(\alpha)$ represents the domain of n -ary relations that exist in the actual world. Each strongly Millian model is understood as a model of an admissible interpretation of the non-logical expressions of the languages in question.

Now, say that an individual constant is free in φ if and only if it occurs in φ . Similarly for n -ary predicate letters, for any natural number n . Consider the following clause:

13. If $\varphi = \lambda v_1 \dots v_n(\psi)$, then,
 - for every $w \in W$, if for every $m \in \mathbb{N}_0$, individual term t and m -ary term τ^m , if
 - t is a variable or individual constant free in φ only if $V_M^g(t) \in d(w)$, and
 - ζ^m is a m -ary second-order variable or predicate letter free in φ only if $V_M^g(\zeta^m) \in D(m)(w)$,
 then, $V_M^g(\varphi) \in D(n)(w)$.
 - If no individual constants, predicate letters and variables are free in φ , then $V_M^g(\varphi) \in D(n)(w)$.

One can narrow the class of weak and strong Millian models by considering only those that satisfy clause 13:

Definition 7 (W^C - and S^C -models). A W^C -model (S^C -model) is a W-model (S-model) whose valuation function satisfies clause 13. $W^C = \{M : M \text{ is a } W^C\text{-model}\}$, and $S^C = \{M : M \text{ is a } S^C\text{-model}\}$.

The usual model-theoretic notions of truth in a model, validity in a model, satisfiability and validity are now defined.

Definition 8 (Truth in a Model, Validity In a Model, Satisfiability, Validity). Let $\star \in \{W, W^C, S, S^C\}$, $M \in \star$, L be any of the languages previously defined, $\varphi \in Form(L)$ and $\Gamma \subseteq Form(L)$. Then:

- φ is generally true in M iff $\forall w \in W, g \in As(M), V_{M,w}^g(\varphi) = \{\langle \rangle\}$.

- $\langle \Gamma, \varphi \rangle$ is $\star\mathbf{G}$ -valid in M , $\Gamma \models_{\star\mathbf{G}}^M \varphi$, iff $\forall w \in W, g \in As(M): \forall \gamma \in \Gamma (V_{M,w}^g(\gamma) = \{\langle \rangle\})$ only if $V_{M,w}^g(\varphi) = \{\langle \rangle\}$.
- $\langle \Gamma, \varphi \rangle$ is $\star\mathbf{G}$ -valid, $\Gamma \models_{\star\mathbf{G}} \varphi$, iff for all \star -models $M: \Gamma \models_{\star\mathbf{G}}^M \varphi$;
- φ is $\star\mathbf{G}$ -valid, $\models_{\star\mathbf{G}} \varphi$, if and only if $\emptyset \models_{\star\mathbf{G}} \varphi$
- Γ is $\star\mathbf{G}$ -satisfiable iff $\exists \star$ -model $M, w \in W$ and $g \in As(M)$ of M s. t. $\forall \gamma \in \Gamma (V_{M,w}^g(\gamma) = \{\langle \rangle\})$.
- $\langle \Gamma, \varphi \rangle$ is $\star\mathbf{R}$ -valid in M , $\Gamma \models_{\star\mathbf{R}}^M \varphi$, if and only if $\forall g \in As(M): \forall \gamma \in \Gamma (V_{M,\alpha}^g(\gamma) = \{\langle \rangle\})$ only if $V_{M,\alpha}^g(\varphi) = \{\langle \rangle\}$.
- $\langle \Gamma, \varphi \rangle$ is $\star\mathbf{R}$ -valid, $\Gamma \models_{\star\mathbf{R}} \varphi$, iff for all \star -models $M: \Gamma \models_{\star\mathbf{R}}^M \varphi$.
- φ is $\star\mathbf{R}$ -valid, $\models_{\star\mathbf{R}} \varphi$, if and only if $\emptyset \models_{\star\mathbf{R}} \varphi$
- Γ is $\star\mathbf{R}$ -satisfiable iff $\exists \star$ -model M and $g \in As(M)$ s. t. $\forall \gamma \in \Gamma (V_{M,\alpha}^g(\gamma) = \{\langle \rangle\})$.

As is hopefully clear, the notions of $\star\mathbf{G}$ - and $\star\mathbf{R}$ -validity are themselves models of the notions of, respectively, general validity and real-world validity.

A.3.3 Deductive Systems

For each of the systems \star to be discussed, φ is a theorem of the system, $\vdash_{\star} \varphi$, if and only if, for some $n \in \mathbb{N}$, there is an n -sequence of formulas $\langle \varphi_1, \dots, \varphi_n \rangle$ such that $\varphi_n = \varphi$ and either φ_i is an axiom of \star or φ_i follows from previous formulas in the sequence by one of the rules of inference of \star , where $1 \leq i \leq n$, $n \in \mathbb{N}$. An argument having as premises the elements in Γ and conclusion φ is *deductively valid* in \star , $\Gamma \vdash_{\star} \varphi$ if and only if there is a finite $\Gamma' \subseteq \Gamma$ such that $\vdash_{\star} \bigwedge \Gamma' \rightarrow \varphi$, where $\bigwedge \Gamma'$ is any conjunction of the formulae in Γ' .

Now, define a \square -closure of a formula φ as a *closed* formula obtained by prefixing any (finite, perhaps of length 0) string of ' \square ' to φ . The notion of a $\textcircled{\square}$ -closure is defined similarly. Define a \forall -closure of φ as a *closed* formula obtained by prefixing any (finite) string of ' $\forall v$ ' (for any variable $v \in Var(L)$), in any order, to φ , and a \forall_2 -closure of φ as a *closed* formula obtained by prefixing any (finite) string of ' $\forall v$ ' (for any $v \in Var(L)$) and ' $\forall V^n$ ' (for any variable $V^n \in SVar(L)$, $n \in \mathbb{N}_0$), in any order, to φ . Similarly, define a $\textcircled{\square}$ -closure of a formula φ as a *closed* formula obtained by prefixing any (finite) string of ' \square ' and ' $\textcircled{\square}$ '. Besides these notions, we will also make use of the notions of a $\textcircled{\forall}$ -closure, $\textcircled{\forall}_2$ -closure, a $\textcircled{\square}\forall$ -closure, and a $\textcircled{\square}\forall_2$ -closure. These are defined as expected.

I begin by introducing sets of schemata in terms of which the axioms of the different systems will be given:

List of Schemata

- | | |
|--|--|
| | [S5] |
| (PL) All propositional tautologies | (T) $\square\varphi \rightarrow \varphi$ |
| (K) $\square(\varphi \rightarrow \psi) \rightarrow (\square\varphi \rightarrow \square\psi)$ | (5) $\diamond\varphi \rightarrow \square\diamond\varphi$ |

[Act]

$$\begin{array}{ll}
(\textcircled{K}) \ @(\varphi \rightarrow \psi) \rightarrow (@\varphi \rightarrow @\psi) & (\textcircled{\square}1) \ @\varphi \rightarrow \square@\varphi \\
(\textcircled{\neg}) \ @\neg\varphi \leftrightarrow \neg@\varphi & (\textcircled{\square}2) \ \square\varphi \rightarrow @\varphi
\end{array}$$

[FFOL]

$$\begin{array}{ll}
(\forall 1) \ \forall v(\varphi \rightarrow \psi) \rightarrow (\forall v\varphi \rightarrow \forall v\psi) & (\forall =) \ \forall v(v = v) \\
(\forall 2) \ \varphi \rightarrow \forall v\varphi^{19} & (\square =) \ t = t' \rightarrow \square(Et \rightarrow t = t') \\
(\forall E) \ Et \rightarrow (\forall v\varphi \rightarrow \varphi_t^v)^{20} & (\text{Ind}) \ t = t' \rightarrow (\varphi \rightarrow \varphi')^{21} \\
(\diamond E) \ \diamond Et & (\forall @) \ @\forall v\varphi \leftrightarrow @\forall v@\varphi \\
(\text{SA}) \ \tau^n t_1 \dots t_n \rightarrow Et_i
\end{array}$$

[FSOL]

$$\begin{array}{ll}
(\text{SV}1) \ \forall V^n(\varphi \rightarrow \psi) \rightarrow (\forall V^n\varphi \rightarrow \forall V^n\psi) & (\text{SV} =) \ \forall V^n(V^n = V^n) \\
(\text{SV}2) \ \varphi \rightarrow \forall V^n\varphi^{22} & (\text{S}\square =) \ \tau^n = \tau'^n \rightarrow \square(E\tau^n \rightarrow \tau^n = \tau'^n) \\
(\text{S}\forall E) \ E\tau^n \rightarrow (\forall V^n\varphi \rightarrow \varphi_{\tau^n}^{V^n}) & (\text{SInd}) \ \tau^n = \tau'^n \rightarrow (\varphi \rightarrow \varphi')^{23} \\
(\text{S}\diamond E) \ \diamond E\zeta^n & (\text{S}\forall @) \ @\forall V^n\varphi \leftrightarrow @\forall V^n@\varphi \\
(\text{SSA}) \ \tau_1^n = \tau_2^n \rightarrow E\tau_i^n, i \in \{1, 2\}
\end{array}$$

Other Schemata

$$\begin{array}{ll}
(\text{CComp}) \ E\varphi \rightarrow E\lambda v_1 \dots v_n(\varphi). & (\text{R}@) \ @\varphi \rightarrow \varphi \\
(\text{G} =) \ @(t = t). & (\text{R} =) \ t = t \\
(\text{SG} =) \ @(\zeta^n = \zeta'^n) & (\text{SR} =) \ \zeta^n = \zeta'^n \\
(\text{EAb}) \ \lambda v_1 \dots v_n(\varphi)t_1 \dots t_n \leftrightarrow (\varphi_{t_1}^{v_1} \dots \varphi_{t_n}^{v_n} \wedge Et_1 \wedge \dots \wedge Et_n).
\end{array}$$

Inference rule

$$(\text{MP}) \quad \vdash_{\star} \varphi \rightarrow \psi, \vdash_{\star} \varphi \Rightarrow \vdash_{\star} \psi.$$

The rule (MP) is the only inference rule of all the systems to be presented in this section.

The theorems of the system **S5** for $\mathcal{P}\mathcal{L}_{\square}$ consist in the smallest set containing every \square -closure of every instance of every schema of [S5] and closed under (MP).²⁴ The system **S5A** for $\mathcal{P}\mathcal{L}_{\square, @}$, presented in, e.g., (Davies & Humberstone, 1980), is obtained by augmenting the axioms of **S5A** with every \square -closure of every instance of every schema in [Act]. Menzel's (1991, §4) system **G** for $\mathcal{F}\mathcal{L}_{\square}$ is obtained by taking as an axiom any $\square\forall$ -closure of any instance of the schemas in [S5] and [FFML] (except for the schema ($\forall @$), since $\mathcal{F}\mathcal{L}_{\square}$ does not contain an actuality operator).²⁵ By adding any \forall -closure of (R=) to system **G**, one obtains the system **A** for $\mathcal{F}\mathcal{L}_{\square}$, also proposed in

¹⁹Where v is not free in φ .

²⁰Provided that v is free for t in φ .

²¹Where φ' is just like φ except that t' replaces one or more (free) occurrences of t in φ .

²²Where V^n is not free in φ .

²³Where φ' is just like φ except that τ'^n replaces one or more (free) occurrences of τ^n in φ .

²⁴Note that in this presentation of **S5**, the system does not contain the rule of necessitation. Necessitation is an admissible rule of the system. This result is proved by an easy induction on the length of a derivation. The proof relies on the fact that the \square -closure of an axiom is itself an axiom, and that every instance of (K) is an axiom.

²⁵With the difference that no open formulas are theorems of the system just presented. This will be a feature of all the systems to be presented.

(Menzel, 1991, §4).²⁶ The set of theorems of \mathbf{G} is the set of WG-valid formulas of \mathcal{FL}_\square , and the set of theorems of \mathbf{A} is the set of SR-valid formulas of \mathcal{FL}_\square .²⁷

The analogues of \mathbf{G} and \mathbf{A} for the case of second-order languages are now defined. These are, respectively, the systems $\mathbf{WG}_{\mathcal{SL}_\square}$ and $\mathbf{SR}_{\mathcal{SL}_\square}$. The names of these systems indicate their relationship to the different sets of arguments distinguished model-theoretically in A.3.2, as well as the kind of language for which the systems are given. Thus, the system $\mathbf{WG}_{\mathcal{SL}_\square}$ has as theorems precisely the set of WG-valid arguments (composed only of closed formulas) of \mathcal{SL}_\square , and the system $\mathbf{SR}_{\mathcal{SL}_\square}$ has as theorems the set of SR-valid arguments (composed only of closed formulas) of language \mathcal{SL}_\square .²⁸ The names of the remaining deductive systems to be presented in this section follow the same recipe.

Definition 9 (Axioms of $\mathbf{WG}_{\mathcal{SL}_\square}$). Every $\square\forall_2$ -closure of every instance (in \mathcal{SL}_\square) of $[\mathbf{S5}] \cup [\mathbf{FFOL}] \cup [\mathbf{FSOL}]$ (except for the schema $(\forall@)$, since \mathcal{SL}_\square does not contain $@$).²⁹

Definition 10 (Axioms of $\mathbf{SR}_{\mathcal{SL}_\square}$). Every $\square\forall_2$ -closure of every instance of $[\mathbf{S5}] \cup [\mathbf{FFOL}] \cup [\mathbf{FSOL}]$ (except for the schema $(\forall@)$, since \mathcal{SL}_\square does not contain $@$). Every \forall_2 -closure of every instance in \mathcal{SL}_\square of $(R=)$ and $(SR=)$.

Given the system of nomenclatures just used, Menzel's deductive system \mathbf{A} is the same as the system $\mathbf{SR}_{\mathcal{FL}_\square}$, and \mathbf{G} is just the system $\mathbf{WG}_{\mathcal{FL}_\square}$. Furthermore, we have that:

$$\cdot \mathbf{WG}_{\mathcal{FL}_\square} = \mathbf{WR}_{\mathcal{FL}_\square} = \mathbf{SG}_{\mathcal{FL}_\square} (= \mathbf{G}); \quad \cdot \mathbf{WG}_{\mathcal{SL}_\square} = \mathbf{WR}_{\mathcal{SL}_\square} = \mathbf{SG}_{\mathcal{SL}_\square}.$$

In effect, for any $\Gamma \subseteq \text{Form}(\mathcal{FL}_\square)$ and $\varphi \in \text{Form}(\mathcal{FL}_\square)$: $\Gamma \models_{\mathbf{WG}} \varphi \Leftrightarrow \Gamma \models_{\mathbf{WR}} \varphi \Leftrightarrow \Gamma \models_{\mathbf{SG}} \varphi$. That is, the notions of WG-validity, WR-validity and SG-validity all turn out to be extensionally equivalent when the language in question is \mathcal{FL}_\square .

However, by enriching the language with an actuality operator the extensional equivalence between these three model-theoretic notions is broken. Thus,

	$\star = \mathbf{WG}$	$\star = \mathbf{WR}$	$\star = \mathbf{SG}$	$\star = \mathbf{SR}$
$\models_\star a = a$	×	×	×	✓
$\models_\star @ (a = a)$	×	×	✓	✓
$\models_\star @P \rightarrow P$	×	✓	×	✓

In what follows strongly Millian logics capturing the notions of general and real-world validity are presented. Since the extensional equivalence between the notions of WG-validity, WR-validity and SG-validity is broken once an actuality operator is added to quantified modal languages, all the strongly Millian quantified modal logics to be presented in the remaining of this section are formulated for languages containing an actuality operator. The weakly Millian analogues of these logics are presented in §A.9.1 of the appendix.

²⁶Except that the system \mathbf{A} , like \mathbf{G} , also has open formulas as theorems.

²⁷The proof can be found in (Menzel, 1991, §4).

²⁸The completeness proof given in the appendix can easily be adapted to establish this fact.

²⁹Reference to the language in the presentation of the remaining systems is omitted, since the target language can be extracted from the name of the system.

Definition 11 (Axioms of $\mathbf{SG}_{\mathcal{FL}_{@}}^{\square}$). Every $@\square\forall$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}]$. Every closed instance of $(G=)$.

Definition 12 (Axioms of $\mathbf{SR}_{\mathcal{FL}_{@}}^{\square}$). Every $@\square\forall$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}]$. Every $@\forall$ -closure of every instance of both $(R@)$ and $(R=)$.

Definition 13 (Axioms of $\mathbf{SG}_{\mathcal{FL}_{@}\square\lambda}$). Every $@\square\forall$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}]$. Every closed instance of $(G=)$. Every $@\square\forall$ -closure of every instance of (EAb) .

Definition 14 (Axioms of $\mathbf{SR}_{\mathcal{FL}_{@}\square\lambda}$). Every $@\square\forall$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}]$. Every $@\square\forall$ -closure of every instance of (EAb) . Every $@\forall$ -closure of every instance of both $(R@)$ and $(R=)$.

Definition 15 (Axioms of $\mathbf{SG}_{\mathcal{SL}_{@}}^{\square}$). Every $@\forall_2$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}] \cup [\mathbf{FSOL}]$. Every closed instance of $(G=)$ and $(SG=)$.

Definition 16 (Axioms of $\mathbf{SR}_{\mathcal{SL}_{@}}^{\square}$). Every $@\square\forall_2$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}] \cup [\mathbf{FSOL}]$. Every $@\forall_2$ -closure of every instance $(R@)$, $(R=)$ and $(SR=)$.

Definition 17 (Axioms of $\mathbf{SG}_{\mathcal{SL}_{@}\square\lambda}^{\square}$). Every $@\square\forall_2$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}] \cup [\mathbf{FSOL}]$. Every closed instance of $(G=)$ and $(SG=)$. Every $@\square\forall_2$ -closure of every instance of (EAb) .

Definition 18 (Axioms of $\mathbf{SR}_{\mathcal{SL}_{@}\square\lambda}^{\square}$). Every $@\square\forall_2$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}] \cup [\mathbf{FSOL}]$. Every $@\forall_2$ -closure of every instance $(R@)$, $(R=)$ and $(SR=)$. Every $@\square\forall_2$ -closure of every instance of (EAb) .

Definition 19 (Axioms of $\mathbf{SG}_{\mathcal{SL}_{@}\square\lambda}^{\square C}$). Every $@\square\forall_2$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}] \cup [\mathbf{FSOL}]$. Every closed instance of $(G=)$ and $(SG=)$. Every $@\square\forall_2$ -closure of every instance in L of (EAb) . Every $@\square\forall_2$ -closure of every instance of $(CComp)$.

Definition 20 (Axioms of $\mathbf{SR}_{\mathcal{SL}_{@}\square\lambda}^{\square C}$). Every $@\square\forall_2$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}] \cup [\mathbf{FSOL}]$. Every $@\forall_2$ -closure of every instance $(R@)$, $(R=)$ and $(SR=)$. Every $@\square\forall_2$ -closure of every instance of (EAb) . Every $@\square\forall_2$ -closure of every instance of $(CComp)$.

For any $-\mathbf{G}$ -system \star necessitation is an admissible rule.³⁰ That is, for every formula φ , $\vdash_{\star} \varphi \Rightarrow \vdash_{\star} \square\varphi$.³¹ This is the desired result, since $-\mathbf{G}$ -systems purport to capture the notion of general validity, and generally valid formulas are necessary. However, universal generalisation is not, in general, admissible. In \mathbf{SG} -systems \star formulated for languages with an actuality operator there are formulas φ such that, for every individual constant a , $\vdash_{\star} \varphi_a^v$, even though $\not\vdash_{\star} \forall v\varphi$. For instance, for

³⁰By a $-\mathbf{G}$ -system we mean any of the systems \mathbf{WGL} , \mathbf{SGL} , \mathbf{WGC}_L , \mathbf{SGC}_L , with L replaced by the appropriate language.

³¹See footnote 24 for a sketch of the proof.

every individual constant a , $\vdash_{\mathbf{SG}_{\mathcal{FL}_{@}\square}} @Ea$, even though it is also the case that $\not\vdash_{\mathbf{SG}_{\mathcal{FL}_{@}\square}} \forall x @Ex$. Similarly, in \mathbf{SG} -systems for second-order modal languages (with $@$) there are formulas φ such that $\vdash_{\star} \varphi_{\zeta^n}^{V^n}$ for every n -ary predicate ζ^n , even though $\not\vdash_{\star} \forall V^n \varphi_{\zeta^n}^{V^n}$. For instance, for every n -ary predicate letter ζ^n , $\vdash_{\mathcal{SL}_{@}\square} @E\zeta^n$, even though $\not\vdash_{\mathbf{SG}_{\mathcal{SL}_{@}\square}} \forall V^n @EV^n$.

Despite the fact that the rule of universal generalisation is not admissible in all $-\mathbf{G}$ -systems, a related rule is. According to this rule, if φ_a^v is a theorem, then $@\forall v \varphi$ is a theorem. Call this rule ‘actual generalisation’. Similarly, in $-\mathbf{G}$ -systems for second-order modal languages (with $@$), if $\varphi_{\zeta^n}^{V^n}$ is a theorem, then $@\forall V^n \varphi$ is a theorem.³² For instance, not only is it the case that $\vdash_{\star} @Ea$ for every \mathbf{SG} -system \star , it is also the case that $\vdash_{\star} @\forall v @Ev$.

As expected, the rule of necessitation is not admissible in $-\mathbf{R}$ -systems. For instance, for every $-\mathbf{R}$ -system \star , $\vdash_{\star} @P \rightarrow P$, even though $\not\vdash_{\star} \square(@P \rightarrow P)$. Furthermore, the following is also an interesting counterexample to necessitation, namely, $\vdash_{\star} Ea$, even though $\not\vdash_{\star} \square Ea$. Insofar as $-\mathbf{R}$ -systems purport to capture the notion of *real-world validity*, the non-admissibility of the rule of necessitation should be expected: real-world validity does not require necessary truth. However, universal generalisation and actual generalisation are both admissible rules in every one of the $-\mathbf{R}$ -systems presented here.

A.3.4 Soundness and Completeness

We are now in a position to state the relevant result concerning the logics just presented. Let L be any one of \mathcal{FL}_{\square} , $\mathcal{FL}_{@}\square$, $\mathcal{FL}_{@}\square\lambda$, \mathcal{SL}_{\square} , $\mathcal{SL}_{@}\square$, $\mathcal{SL}_{@}\square\lambda$. Also, let $M \in \{W, S\}$, $\Gamma \subseteq \text{Form}(L)$, and $\varphi \in \text{Form}(L)$. Then:³³

Theorem 3 (Soundness and Completeness of Weak and Strong Millian Logics). *For every closed set of formulae Γ of L and closed formula φ of L :*

$$\begin{array}{ll} \Gamma \models_{\mathbf{MG}} \varphi \text{ iff } \Gamma \vdash_{\mathbf{MG}_L} \varphi & \Gamma \models_{\mathbf{MR}} \varphi \text{ iff } \Gamma \vdash_{\mathbf{MR}_L} \varphi \\ \Gamma \models_{\mathbf{MG}^c} \varphi \text{ iff } \Gamma \vdash_{\mathbf{MG}_{\mathcal{SL}_{@}\square\lambda}^c} \varphi & \Gamma \models_{\mathbf{MR}^c} \varphi \text{ iff } \Gamma \vdash_{\mathbf{MR}_{\mathcal{SL}_{@}\square\lambda}^c} \varphi \end{array}$$

In the appendix we prove that $\Gamma \models_{\mathbf{SR}^c} \varphi$ iff $\Gamma \vdash_{\mathbf{SR}_{\mathcal{SL}_{@}\square\lambda}^c} \varphi$. The remaining proofs are quite similar, and simpler.

This concludes the presentation of strongly Millian logics. In the next section it is shown that every theorem of classical quantified logic, or every formula that results from prefixing each theorem of classical quantified logic with $@$, is a theorem of the strongly Millian logics just characterised. It is also shown that strongly Millian logics are *conservative*, in the sense of §A.1.

³²This can be proved by induction on the length of a derivation. The tricky cases are the base cases involving axioms ($@=$) and ($S@=$) (for the case of \mathbf{SG} -systems). But $@\forall v (v = v)$, for each variable v , is an axiom of every $-\mathbf{G}$ -system, from which $@\forall v @ (v = v)$ follows, by an instance of ($\forall @$) and an application of (MP). Similarly, $@\forall V^n (V^n = V^n)$, for each variable V^n , is an axiom of every $-\mathbf{G}$ -system, from which $@\forall V^n @ (V^n = V^n)$ follows, by an instance of ($S\forall @$) and an application of (MP).

³³These results also hold for the weak analogues – presented in the appendix – of the strongly Millian logics characterised in this section.

A.4 Strongly Millian Logics: ‘Classical’ and Conservative

I will begin by defining the notions of a classical inhabited model structure and a classical inhabited second-order model structure.³⁴

Definition 21 (Classical Inhabited Structure).

- A classical inhabited model structure is an inhabited model structure in which $W = \{\alpha\}$;
- A classical inhabited second-order model structure is an inhabited second-order model structure in which $W = \{\alpha\}$;

Definition 22 (Classical Models (CI-models)). A CI-model M is any S-model M based on a classical inhabited second-order model structure.

Later on call classical models will be called *Henkin models*, even though, traditionally, Henkin models do not possess the elements W and α . Finally, CI^{C} -models are defined as follows:

Definition 23 (CI^{C} -models). A CI^{C} -model M is any S^{C} -model M based on a classical inhabited second-order model structure.

Definition 24 (Truth in a Model, Validity In a Model, Satisfiability, Validity). Let $M \in \star$, where $\star \in \{\text{CI}, \text{CI}^{\text{C}}\}$, L be any of the languages $\mathcal{FL}, \mathcal{FL}_{\lambda}, \mathcal{SL}, \mathcal{SL}_{\lambda}$. Also, let $\varphi \in \text{Form}(L)$ and $\Gamma \subseteq \text{Form}(L)$. Then:

- $\langle \Gamma, \varphi \rangle$ is \star -valid, $\Gamma \models_{\star} \varphi$, if and only if for all \star -models M , $g \in \text{As}(M)$: $\forall \gamma \in \Gamma V_{M,\alpha}^g(\gamma) = \{\langle \rangle\}$ only if $V_{M,\alpha}^g(\varphi) = \{\langle \rangle\}$;
- φ is \star -valid, $\models_{\star} \varphi$, if and only if $\emptyset \models_{\star} \varphi$;
- Γ is \star -satisfiable iff $\exists \star$ -model M , and $g \in \text{As}(M)$ of M s. t. $\forall \gamma \in \Gamma V_{M,\alpha}^g(\gamma) = \{\langle \rangle\}$.

We are now in a position to define the deductive systems of classical first- and second-order logic. Consider the following sets of schemata, and inference rules:

	[FOL]
(PL) Every propositional tautology	($\forall 2$) $\varphi \rightarrow \forall v \varphi$ ³⁶
($\forall 0$) $\forall v \varphi \rightarrow \varphi_a^v$ ³⁵	(=I) $a = a$
($\forall 1$) $\forall v(\varphi \rightarrow \psi) \rightarrow (\forall v \varphi \rightarrow \forall v \psi)$	(Ind) $a = a' \rightarrow (\varphi \rightarrow \varphi')$ ³⁷
	[SOL]

³⁴A slightly different presentation of the model-theory is given here, to highlight its continuity with the model-theory of strongly Millian logics.

³⁵Where t is free for v in φ , and φ_a^v results from replacing every free occurrence of v in φ by a .

³⁶Where v is not free in φ .

³⁷Where φ is an atomic formula and φ' is just like φ except that a' replaces one or more occurrences of a in φ .

$$(S\forall 0) \forall V^n \varphi \rightarrow \varphi_{\tau^n}^{V^n} \quad (S\forall 2) \varphi \rightarrow \forall V^n \varphi^{39}$$

$$(S\forall 1) \forall V^n (\varphi \rightarrow \psi) \rightarrow (\forall V^n \varphi \rightarrow \forall V^n \psi)$$

Other Schemata

$$(Ab) \lambda v_1 \dots v_n (\varphi) a_1, \dots, a_n \leftrightarrow \varphi_{a_1}^{v_1} \dots \varphi_{a_n}^{v_n}.$$

Inference Rules

$$(MP) \vdash_{\star} \varphi \rightarrow \psi, \vdash_{\star} \varphi \Rightarrow \vdash_{\star} \psi \quad (UG) \quad \vdash_{\star} \varphi_a^v \Rightarrow \vdash_{\star} \forall v \varphi$$

$$\psi \quad (SUG) \quad \vdash_{\star} \varphi_{\zeta^n}^{V^n} \Rightarrow \vdash_{\star} \forall V^n \varphi$$

The different systems of classical quantified logic are characterised thus:

Definition 25 (Axioms and Inference Rules of $\mathbf{Cl}_{\mathcal{FL}}$). Any closed formula of \mathcal{FL} that is an instance of any schema in $[\mathbf{FOL}]$ is an axiom of $\mathbf{Cl}_{\mathcal{FL}}$. The inference rules of $\mathbf{Cl}_{\mathcal{FL}}$ are (MP) and (UG).

Definition 26 (Axioms and Inference Rules of $\mathbf{Cl}_{\mathcal{FL}\lambda}$). Any closed formula of $\mathcal{FL}\lambda$ that is an instance of any schema in $[\mathbf{FOL}]$, or is an instance of (Ab) is an axiom of $\mathbf{Cl}_{\mathcal{FL}\lambda}$. The inference rules of $\mathbf{Cl}_{\mathcal{FL}\lambda}$ are (MP) and (UG).

Definition 27 (Axioms and Inference Rules of $\mathbf{Cl}_{\mathcal{SL}}$). Any closed formula of \mathcal{SL} that is an instance of any schema in $[\mathbf{FOL}]$ or is an instance of $[\mathbf{SOL}]$ is an axiom of $\mathbf{Cl}_{\mathcal{SL}}$. The inference rules of $\mathbf{Cl}_{\mathcal{SL}}$ are (MP), (UG) and (SUG).

Definition 28 (Axioms and Inference Rules of $\mathbf{Cl}_{\mathcal{SL}\lambda}$). Any closed formula of $\mathcal{SL}\lambda$ that is an instance of any schema in $[\mathbf{FOL}] \cup [\mathbf{SOL}]$ or is an instance of (Ab) is an axiom of $\mathbf{Cl}_{\mathcal{SL}\lambda}$. The inference rules of $\mathbf{Cl}_{\mathcal{SL}\lambda}$ are (MP), (UG) and (SUG).

Where Γ is any subset of closed formulas and φ is any formula (of the relevant language), the following obtains:

$$\Gamma \vdash_{\mathbf{Cl}_{\mathcal{FL}}} \varphi \Leftrightarrow \Gamma \models_{\mathbf{Cl}} \varphi; \quad \Gamma \vdash_{\mathbf{Cl}_{\mathcal{SL}}} \varphi \Leftrightarrow \Gamma \models_{\mathbf{Cl}} \varphi;$$

$$\Gamma \vdash_{\mathbf{Cl}_{\mathcal{FL}\lambda}} \varphi \Leftrightarrow \Gamma \models_{\mathbf{Cl}} \varphi; \quad \Gamma \vdash_{\mathbf{Cl}_{\mathcal{SL}\lambda}} \varphi \Leftrightarrow \Gamma \models_{\mathbf{Cl}^c} \varphi.$$

The relevant results connecting strongly Millian logics and classical quantified logic may now be presented.

Theorem 4. Let $@\Gamma = \{@\varphi : \varphi \in \Gamma\}$. Then:

- (i) $\forall \Gamma \subseteq \text{Form}(\mathcal{FL}), \varphi \in \text{Form}(\mathcal{FL}): \Gamma \vdash_{\mathbf{Cl}_{\mathcal{FL}}} \varphi \Leftrightarrow @\Gamma \vdash_{\mathbf{SG}_{\mathcal{FL}@\square}} @\varphi \Leftrightarrow \Gamma \vdash_{\mathbf{SR}_{\mathcal{FL}@\square}} \varphi;$
- (ii) $\forall \Gamma \subseteq \text{Form}(\mathcal{FL}\lambda), \varphi \in \text{Form}(\mathcal{FL}\lambda): \Gamma \vdash_{\mathbf{Cl}_{\mathcal{FL}\lambda}} \varphi \Leftrightarrow @\Gamma \vdash_{\mathbf{SG}_{\mathcal{FL}@\square\lambda}} @\varphi \Leftrightarrow \Gamma \vdash_{\mathbf{SR}_{\mathcal{FL}@\square\lambda}} \varphi;$
- (iii) $\forall \Gamma \subseteq \text{Form}(\mathcal{SL}), \varphi \in \text{Form}(\mathcal{SL}): \Gamma \vdash_{\mathbf{Cl}_{\mathcal{SL}}} \varphi \Leftrightarrow @\Gamma \vdash_{\mathbf{SG}_{\mathcal{SL}@\square}} @\varphi \Leftrightarrow \Gamma \vdash_{\mathbf{SR}_{\mathcal{SL}@\square}} \varphi;$

³⁸Where τ^n is free for V^n in φ , and $\varphi_{\tau^n}^{V^n}$ results from replacing every free occurrence of V^n in φ by τ^n .

³⁹Where V^n is not free in φ .

(iv) $\forall \Gamma \subseteq \text{Form}(\mathcal{SL}_\lambda), \varphi \in \text{Form}(\mathcal{SL}_\lambda): \Gamma \vdash_{\text{Cl}_{\mathcal{SL}_\lambda}} \varphi \Leftrightarrow @\Gamma \vdash_{\mathbf{SG}_{\mathcal{SL}_{@ \square \lambda}}^{\mathbf{C}}} @\varphi \Leftrightarrow \Gamma \vdash_{\mathbf{SR}_{\mathcal{SL}_{@ \square \lambda}}^{\mathbf{C}}} \varphi$.

In the appendix a proof of item (iv) of theorem 4 is offered.

The strongly Millian logics here proposed are all conservative. This result follows from the fact that $\mathbf{SR}_{\mathcal{SL}_{@ \square \lambda}}^{\mathbf{C}}$ is conservative, since this is the strongest of the strongly Millian logics that have been presented.

The conservativeness of $\mathbf{SR}_{\mathcal{SL}_{@ \square \lambda}}^{\mathbf{C}}$ is established by appealing to theorem 3 and presenting a $\text{S}^{\mathbf{C}}$ -model $M = \langle W, d, D, \alpha, V \rangle$ of which all of the following claims hold:

- Counter (BF)** $\exists g \in \text{As}(M)$ s. t. $V_{M,\alpha}^g(\diamond \exists x(@ \neg Ex)) = \{\langle \rangle\}$ and $V_{M,\alpha}^g(\exists x(\diamond @ \neg Ex)) = \emptyset$;
- Counter (CBF)** $\exists g \in \text{As}(M)$ s. t. $V_{M,\alpha}^g(\exists x(\diamond \neg Ex)) = \{\langle \rangle\}$ and $V_{M,\alpha}^g(\diamond \exists x(\neg Ex)) = \emptyset$;
- Counter (NNE)** $\exists g \in \text{As}(M)$ s. t. $V_{M,\alpha}^g(\square \forall x(\square Ex)) = \emptyset$;
- Counter (BF_M)** $\exists g \in \text{As}(M)$ s. t. $V_{M,\alpha}^g(\diamond \exists X(@ \neg EX)) = \{\langle \rangle\}$ and $V_{M,\alpha}^g(\exists X \diamond (@ \neg EX)) = \emptyset$;
- Counter (CBF_M)** $\exists g \in \text{As}(M)$ s. t. $V_{M,\alpha}^g(\exists X(\diamond \neg EX)) = \{\langle \rangle\}$ and $V_{M,\alpha}^g(\diamond \exists X(\neg EX)) = \emptyset$;
- Counter (NNE_M)** $\exists g \in \text{As}(M)$ s. t. $V_{M,\alpha}^g(\square \forall X(\square EX)) = \emptyset$;

A $\text{S}^{\mathbf{C}}$ -model satisfying all of the above claims is now presented. In the appendix it is shown that the model offered indeed satisfies all of these claims, and that it is indeed a $\text{S}^{\mathbf{C}}$ -model.

Let $W = \{1, 2, 3\}$, d be a function with domain W and such that $\forall w \in W : d(w) = \{0_w\}$. Let Π be the set of all permutations π of $W \cup \bigcup_{w \in W} d(w)$ such that:

- $\pi|_W$ is a permutation of W
- $\pi|_{\bigcup_{w \in W} d(w)}$ is a permutation of $\bigcup_{w \in W} d(w)$;
- $\forall w \in W (d(\pi(w)) = \{\pi(o) : o \in d(w)\})$.

For each $n \in \mathbb{N}_0$, if $d = \langle o_1, \dots, o_n \rangle \in (\bigcup_{w \in W} d(w))^n$ and $\pi \in \Pi$, then, let $\pi(d) = \langle \pi(o_1), \dots, \pi(o_n) \rangle$. Also, for each $n \in \mathbb{N}_0$, if $f \in F(n)$, then let $\pi(f)$ be that function with domain W and such that, for every $w \in W$, $\pi(f)(\pi(w)) = \pi(f(w))$.⁴⁰ Finally, let $\Pi_w = \{\pi \in \Pi : \pi(w) = w \text{ and } \forall o \in d(w)(\pi(o) = o)\}$.

The relevant model $M = \langle W, d, D, \alpha, V \rangle$ is now defined. The sets W and d are the ones previously defined, and $\alpha = 1$. Furthermore:

- $\forall n \in \mathbb{N}_0 : D(n)(w) = \{f \in F(n) : \forall \pi \in \Pi_w(\pi(f) = f)\}$;
- $\forall a \in \text{Const}(\mathcal{SL}_{@ \square \lambda}) : V(a) = 0_1$;
- $\forall \zeta^n \in \text{Pred}^n(\mathcal{SL}_{@ \square \lambda}) \forall w \in W : V(\zeta^n)(w) = (d(w))^n$

Since M satisfies all of Counter (BF), Counter (CBF), Counter (NNE), Counter (BF_M), Counter (CBF_M) and Counter (NNE_M), and M is a $\text{S}^{\mathbf{C}}$ -model, the logic $\mathbf{SR}_{\mathcal{SL}_{@ \square \lambda}}^{\mathbf{C}}$ is conservative, by theorem 3.

⁴⁰Recall that $F(n)$ is the set of all functions f with domain W and such that, for each $w \in W$, $f(w) \subseteq (d(w))^n$.

A fortiori, all the strongly Millian logics presented here are conservative. Combining these two results, it is seen that the strongly Millian logics presented here are conservative and capture the actual truth of the theorems of classical quantified logic.

What should be concluded with respect to conservative systems of classical quantified modal logic which fail to sanction several theorems of classical logic? On the model-theoretic side, these systems do not treat individual constants as strongly Millian expressions. The reason is that the model-theoretic treatment given to individual constants is such that the valuation function of some models assigns to some of these expressions elements that do not belong to the domain of the actual world. However, this fact may not be that significant. If the quantified modal language lacks an actuality operator, and the target notion of validity is that of general validity, then there is no way of capturing the fact that the theorems of classical logic are all true in the actual world, even if individual constants are only assigned to elements in $d(\alpha)$.

In any case, the fact that all theorems of classical logic are true in the actual world is captured by the strongly Millian logics offered. An interesting feature of these logics is that the actual truth of the theorems of classical quantified logic can be captured without appealing to real-world validity (even though, as shown, it can be captured by appealing to real-world validity). Their actual truth can also be captured by appealing to general validity, as long as the expressive resources of the language are augmented (in particular, this can be done once the actuality operator @ is added to the language, as shown here). Thus, the desire to have a conservative and yet 'classical' quantified modal logic should not immediately lead to views such as the view that logical truths are not necessary, or the view that the 'correct' conception of validity is that of real-world validity. Even theorists who hold that every logical truth is necessary have a 'classical' quantified modal logic available to them (in the scope of an actuality operator).

This concludes the first part of the paper. The second part has three aims: i) to show a possible application of strongly Millian logics for second-order modal languages to a current debate in the metaphysics of modality, ii) to compare strongly Millian logics to two other proposals in the literature, and iii) to address the question whether the strongly Millian logics for second-order modal languages that have been proposed are really second-order.

A.5 Comprehension Principles for Second-Order Modal Logic

Necessitism is the thesis that necessarily every individual necessarily exists, a thesis captured by formula (NNE). Contingentism, the contradictory of necessitism, is the thesis that there could have been some individuals that could have failed to exist. Despite the controversial status of necessitism, the thesis has been recently defended by [Linsky & Zalta \(1994, 1996\)](#) and [Williamson \(1998, 2013\)](#). Necessitism and contingentism have higher-order analogues. Higher-order necessitism and higher-order contingentism are, respectively, the thesis that necessarily, every higher-order entity necessarily exists, and the thesis that there could have been some higher-order entity that could have

failed to exist.⁴¹

A recent defence of higher-order necessitism is given in (Williamson, 2013, ch. 6). The defence is based on an argument for the weaker thesis that necessarily every property necessarily exists, regimented by formula (NNE_M). Williamson's argument for (NNE_M) is considered in the present section. The main goal of what follows is that of showing that by appealing to strongly Millian second-order modal logics higher-order contingentists can avail themselves of extra resources for rejecting the cogency of Williamson's argument for (NNE_M).

Let a $\Box\forall_2$ -closure of a formula φ be a closed formula resulting from prefixing φ with any sequence of \Box , $\forall v$ (for any individual variable v) and $\forall V^n$ (for any n -ary second-order variable V^n and $n \in \mathbb{N}_0$), in any order, to φ . Consider the following comprehension principle for second-order modal logic:

$$(\text{Comp}_M) \quad \exists X \Box \forall x (Xx \leftrightarrow \varphi).$$

The variable X is not free in φ , and every $\Box\forall_2$ -closure of any instance of (Comp_M) is itself an instance of (Comp_M).

Note that the formula (NNE_M), repeated below,

$$(\text{NNE}_M) \quad \Box \forall X (\Box \exists Y (\Box \forall x (Xx \leftrightarrow Yx))), \text{ i.e., } \Box \forall X (\Box EX),$$

is an instance of (Comp_M). One of the premises of Williamson's defence of the truth of (NNE_M) is precisely the fact that (NNE_M) is one of the instances of (Comp_M). Williamson argues that the addition, to a modal and second-order deductive system friendly to opponents of (NNE_M), of any set of (natural and sufficiently general) comprehension principles weaker than (Comp_M) results in a system that is 'too weak for reasonable logical and mathematical purposes' (Williamson, 2013, p. 288). He thus takes the strength of (Comp_M) as providing abductive reason to accept the truth of every one of (Comp_M)'s instances. A fortiori, the strength of (Comp_M) gives abductive reason to accept the truth of (NNE_M).

Even though Williamson does not give an explicit characterisation of the deductive system friendly to the opponents of (NNE_M) that he has in mind, it is reasonable to assume that this system is some subsystem of the deductive system $\mathbf{WG}_{\mathcal{FL}\Box}$, if attention is restricted to formulas without free occurrences of variables. The reason is that the class of models singled out in (Williamson, 2013, p. 278) seems to be the class of W -models.⁴² Let me thus focus on the system $\mathbf{WG}_{\mathcal{FL}\Box}$.

The strongest set of comprehension principles friendly to the higher-order contingentist considered by Williamson is the set comprising the following two principles:

$$(\text{Comp}_{MC}) \quad E\varphi \rightarrow \exists X \Box \forall x (Xx \leftrightarrow \varphi).$$

⁴¹In the present context, higher-order entities are n -ary relations, (for each natural number n), relations between n -ary relations, relations between n -ary relations and individuals, relations between individuals and relations that hold between n -ary relations and individuals, and so on.

⁴²Williamson only explicitly defines the function yielding, for each $w \in W$, the 'domain of properties' in w . However, he does say that 'We can ignore higher-order types and polyadic relations since extending the models to them is a routine exercise' (Williamson, 2013, p. 278). The extension of the models to polyadic relations would yield the class of W -models.

Here, X is not free in φ and every $\Box\forall_2$ -closure of any instance of $(\text{Comp}_{\text{MC}})$ is itself an instance of $(\text{Comp}_{\text{MC}})$

$$(\text{Comp}_{\text{M}}^-) \exists X \forall x (Xx \leftrightarrow \varphi).$$

Here, X is not free in φ and every $\Box\forall_2$ -closure of any instance of $(\text{Comp}_{\text{M}}^-)$ is itself an instance of $(\text{Comp}_{\text{M}}^-)$.

Williamson argues that not even the addition of this set of comprehension principles to the deductive system $\mathbf{WG}_{\mathcal{F}\mathcal{L}\Box}$ results in a system sufficiently strong for the purposes of theoretical inquiry. According to him, the resulting deductive system is not sufficiently strong for the application of certain general assumptions made in the context of what he calls ‘second-order modal mathematics’.

Williamson shows this by considering an exemplary assumption made in the context of second-order modal mathematics. Say that y is a *modal upper bound* of property X , under ordering \leq , if and only if necessarily, for every x , if x has X , then it could have been the case that $x \leq y$. That is, y is a ‘modal upper bound’ of property X , under ordering \leq , if and only if $\Box\forall x (Xx \rightarrow \Diamond x \leq y)$.

Also, say that y is a *modal least upper bound* of property X under ordering \leq if and only if i) y is a modal upper bound of X under \leq , and ii) necessarily, for every z , if z is a modal upper bound of X under \leq , then it could have been the case that $y \leq z$. That is, y is a modal least upper bound of property X under ordering \leq if and only if $\Box\forall x (Xx \rightarrow \Diamond x \leq y) \wedge \Box\forall z (\Box\forall x (Xx \rightarrow \Diamond x \leq z) \rightarrow \Diamond y \leq z)$. Consider the assumption that necessarily, for every property X , if X could have had a modal upper bound under \leq , then X could have had a modal least upper bound under \leq , captured by the following formula:⁴³

$$(\text{MCP}) \Box\forall X (\Diamond\exists y \Box\forall x (Xx \rightarrow \Diamond x \leq y) \rightarrow \Diamond\exists y (\Box\forall x (Xx \rightarrow \Diamond x \leq y) \wedge \Box\forall z (\Box\forall x (Xx \rightarrow \Diamond x \leq z) \rightarrow \Diamond y \leq z))).$$

Williamson argues thus:

‘Now the assumption [(MCP)] serves its intended purpose only if it can be properly *applied*. More specifically, from [(MCP)] we must be able to derive any instance of [(MCPi)], by plugging in the formula $[\varphi]$ in place of Xx (where φ may contain x but not y or z free):

$$(\text{MCPi}) \Diamond\exists y \Box\forall x (\varphi \rightarrow \Diamond x \leq y) \rightarrow \Diamond\exists y (\Box\forall x (\varphi \rightarrow \Diamond x \leq y) \wedge \Box\forall z (\Box\forall x (\varphi \rightarrow \Diamond x \leq z) \rightarrow \Diamond y \leq z)), \text{ where } y \text{ and } z \text{ do not occur free in } \varphi.$$

But in general to derive [(MCPi)] from [(MCP)] we need something like (Comp_{M}) , to provide a property over which the second-order quantifier ranges necessarily coextensive with $[\varphi]$. Indeed, we need something like the full modal closure of (Comp_{M}) to derive [(MCPi)] from [(MCP)] in modal contexts for all parameters. We could have reached

⁴³Following Williamson, ‘it does not matter whether $u \leq v$ stands for an atomic formula or a complex one’ (Williamson, 2013, p. 286). It may just abbreviate a formula in which the variables, x , y , and x , z , respectively, occur free.

the same conclusion by considering many other ways of applying second-order modal mathematics. But what guarantee has the contingentist that there even could be a property necessarily coextensive with $[\varphi]$? For example, the parameters in $[\varphi]$ may not be all compossible; informally, it may be impossible for all the relevant objects to be together (Williamson, 2013, p. 287):

Two requirements on the strength of a set of comprehension principles, relative to a deductive system D , are alluded to in this passage. The first is the requirement that any instance of (MCPi) be derivable, in D , from the set of premises containing (MCP) and all instances of all the comprehension principles in S . Call this requirement the *applicability requirement*. The second requirement is connected to Williamson's talk of 'modal contexts'. One way to spell out the requirement is as the requirement that any $\Box\forall_2$ -closure of the following schema, (MCP–MCPi) be derivable, in D from the set of all instances of all the comprehension principles in S :

$$\begin{aligned} \text{(MCP–MCPi)} \quad & \Box\forall X(\Diamond\exists y\Box\forall x(Xx \rightarrow \Diamond x \leq y) \rightarrow \Diamond\exists y(\Box\forall x(Xx \rightarrow \Diamond x \leq y) \wedge \Box\forall z(\Box\forall x(Xx \rightarrow \\ & \Diamond x \leq z) \rightarrow \Diamond y \leq z))) \rightarrow \\ & \rightarrow \Diamond\exists y\Box\forall x(\varphi \rightarrow \Diamond x \leq y) \rightarrow \Diamond\exists y(\Box\forall x(\varphi \rightarrow \Diamond x \leq y) \wedge \Box\forall z(\Box\forall x(\varphi \rightarrow \Diamond x \leq z) \rightarrow \\ & \Diamond y \leq z)), \text{ where } y \text{ and } z \text{ do not occur free in } \varphi. \end{aligned}$$

The appeal to $\Box\forall_2$ -closures accommodates both the 'modal contexts' mentioned by Williamson and his use of free variables, which are banned in the present context. Call this second requirement the *modal applicability requirement*.

As shown in the appendix of (Williamson, 2013, ch. 6), the applicability and modal applicability requirements are not satisfied, relative to the deductive system $\mathbf{WG}_{\mathcal{FL}\Box}$, by the set comprising the comprehension principles (Comp_{MC}) and (Comp_{M}^-). Williamson concludes from this fact that the opponent of (NNE_{M}) does not have available a set of natural and sufficiently general comprehension principles yielding a reasonable deductive system strong enough for the practice of second-order modal mathematics. As he puts it, replacing (Comp_{M}) with (Comp_{MC}) and (Comp_{M}^-) 'prevents second-order logic from adequately serving the logical and mathematical purposes for which we need it' (Williamson, 2013, p. 288).

Williamson takes this fact as providing abductive reason to accept the truth of every instance of (Comp_{M}) and, a fortiori, of (NNE_{M}). Since similar arguments can be run for analogues of (Comp_{M}) of every type, the weakness of the comprehension principles available to the higher-order contingentist constitutes abductive reason to accept the truth of higher-order necessitism.

Strongly Millian second-order modal logics offer the higher-order contingentist extra resources for resisting Williamson's abductive argument. One strategy available to the higher-order contingentist involves establishing the following two claims:

- (A) There are natural and reasonable deductive systems stronger than $\mathbf{WG}_{\mathcal{FL}\Box}$ which are friendly to the higher-order contingentist and furthermore satisfy the applicability requirement (or an actualised version of it).

(B) We have no good reason to accept the truth of every $\Box\forall_2$ -closure of every instance of (MCP–MCPi). If true, (A) shows that the fact that the applicability requirement is not satisfied by the system $\mathbf{WG}_{\mathcal{FL}\Box}$ does not provide evidence for the view that higher-order contingentists do not have available second-order modal logics that are both strong and reasonable. However, by itself, (A) does not suffice to block Williamson’s argument. If one grants the truth of every $\Box\forall_2$ -closure of (MCP–MCPi), then the higher-order contingentist incurs the burden of finding a reasonable deductive system of second-order modal logic friendly to higher-order contingentists and which has every $\Box\forall_2$ -closure of every instance of (MCP–MCPi) as a theorem. Thus, either higher-order contingentists offer such a deductive system, or else they must reject the truth of every $\Box\forall_2$ -closure of every instance of (MCP–MCPi).

There are strongly Millian logics which satisfy the applicability requirement (or a requirement quite close to it) and yet fail to satisfy the modal applicability requirement. Given the availability of these logics, the option of rejecting the truth of every $\Box\forall_2$ -closure of (MCP–MCPi) seems promising. We thus focus on the strategy for replying to Williamson which involves establishing claims (A) and (B).

Consider first the following lemma:

Lemma 15. *For any formula φ whose only free variable is, at most, V^n , and any formula ψ whose only free variables are, at most, v_1, \dots, v_n :⁴⁴*

- (i) $(\text{Comp}_{\text{MC}}), \Box\forall V^n\varphi \vdash_{\mathbf{SG}_{\mathcal{SL}_{\Box\lambda}}} @\varphi_{\lambda v_1 \dots v_n}^{V^n}(\psi)$;
- (ii) $(\text{Comp}_{\text{MC}}), \Box\forall V^n\varphi \vdash_{\mathbf{SR}_{\mathcal{SL}_{\Box\lambda}}} \varphi_{\lambda v_1 \dots v_n}^{V^n}(\psi)$.

The following corollary shows that once the assumption that individual variables and n -ary predicate letters are Millian expressions is taken seriously a result close to the applicability requirement is available to the opponent of NNE_M .

Corollary 3. *For any formula φ whose free variables are at most x :*

- (i) $(\text{Comp}_{\text{MC}}), (\text{MCP}) \vdash_{\mathbf{SG}_{\mathcal{SL}_{\Box\lambda}}} @(\Diamond\exists y\Box\forall x(\lambda x(\varphi)x \rightarrow \Diamond x \leq y) \rightarrow \Diamond\exists y(\Box\forall x(\lambda x(\varphi)x \rightarrow \Diamond x \leq y) \wedge \Box\forall z(\Box\forall x(\lambda x(\varphi)x \rightarrow \Diamond x \leq y) \rightarrow \Diamond y \leq z)))$;
- (ii) $(\text{Comp}_{\text{MC}}), (\text{MCP}) \vdash_{\mathbf{SR}_{\mathcal{SL}_{\Box\lambda}}} \Diamond\exists y\Box\forall x(\lambda x(\varphi)x \rightarrow \Diamond x \leq y) \rightarrow \Diamond\exists y(\Box\forall x(\lambda x(\varphi)x \rightarrow \Diamond x \leq y) \wedge \Box\forall z(\Box\forall x(\lambda x(\varphi)x \rightarrow \Diamond x \leq y) \rightarrow \Diamond y \leq z))$.

Corollary 3 is an instance of lemma 15. From corollary 3 (by appealing to schema (EAb)) it follows that:

Corollary 4. *For any formula φ whose free variables are at most x :*

- (i) $(\text{Comp}_{\text{MC}}), (\text{MCP}) \vdash_{\mathbf{SG}_{\mathcal{SL}_{\Box\lambda}}} @(\Diamond\exists y\Box\forall x(\varphi \rightarrow \Diamond x \leq y) \rightarrow \Diamond\exists y(\Box\forall x(\varphi \rightarrow \Diamond x \leq y) \wedge \Box\forall z(\Box\forall x(\varphi \rightarrow \Diamond x \leq y) \rightarrow \Diamond y \leq z)))$;

⁴⁴Recall that the systems $\mathbf{SG}_{\mathcal{SL}_{\Box\lambda}}$ and $\mathbf{SR}_{\mathcal{SL}_{\Box\lambda}}$ contain no formulae with free variables as theorems. Item (i) of Lemma 15 is proved by appealing to axioms $(\Box\Box 2)$, $(@K)$, $(S\forall E)$, (Eab) , $(A=)$ and $(SA=)$. Item (ii) of Lemma 15 is proved by appealing to axioms $(\Box\Box 2)$, $(@K)$, $(S\forall E)$, (Eab) , $(R=)$, $(SR=)$ and $(R@)$.

- (ii) $(\text{Comp}_{\text{MC}}, (\text{MCP}) \vdash_{\mathbf{SR}_{\mathcal{SL}_{@ \square \lambda}}} \diamond \exists y \square \forall x (\varphi \rightarrow \diamond x \leq y) \rightarrow \diamond \exists y (\square \forall x (\varphi \rightarrow \diamond x \leq y) \wedge \square \forall z (\square \forall x (\varphi \rightarrow \diamond x \leq y) \rightarrow \diamond y \leq z))$.

Arguably, corollaries 3 and 4 show that the applicability requirement is met by the comprehension principle $(\text{Comp}_{\text{MC}})$ with respect to the logics $\mathbf{SG}_{\mathcal{SL}_{@ \square \lambda}}$ and $\mathbf{SR}_{\mathcal{SL}_{@ \square \lambda}}$. These results establish claim (A).

The modal applicability requirement is not satisfied by the set whose comprehension principles are $(\text{Comp}_{\text{MC}})$ and $(\text{Comp}_{\text{M}}^-)$. That is, it is not the case that all $\square \forall_2$ -closures of $(\text{MCP} - \text{MCPi})$ are derivable in $\mathbf{SG}_{\mathcal{SL}_{@ \square \lambda}}$ or $\mathbf{SR}_{\mathcal{SL}_{@ \square \lambda}}$ from the set containing all instances of comprehension principles $(\text{Comp}_{\text{MC}})$ and $(\text{Comp}_{\text{M}}^-)$.⁴⁵ Thus, an appropriate reply to Williamson's argument for (Comp_{M}) requires more than just the appeal to the deductive strength of the strongly Millian logics $\mathbf{SG}_{\mathcal{SL}_{@ \square \lambda}}$ and $\mathbf{SR}_{\mathcal{SL}_{@ \square \lambda}}$. That is, a successful reply to Williamson's abductive argument for (NNE_{M}) based on the strength of the strongly Millian logics $\mathbf{SG}_{\mathcal{SL}_{@ \square \lambda}}$ and $\mathbf{SR}_{\mathcal{SL}_{@ \square \lambda}}$ requires establishing claim (B).

Williamson offers no consideration in favour of the truth of every $\square \forall_2$ -closure of every instance of $(\text{MCP} - \text{MCPi})$. Arguably, this makes Williamson's argument less than satisfactory. The reason is that the most natural defence of the claim that every $\square \forall_2$ -closure of every instance of $(\text{MCP} - \text{MCPi})$ is true is unavailable to him.

To see this, note that the natural reason for supporting the truth of every $\square \forall_2$ -closure of every instance of $(\text{MCP} - \text{MCPi})$ consists in pointing out that all of these formulae are also $\square \forall_2$ -closures of instances of a more general schema, namely, the following:

$$(\square \text{S}\forall 0) \quad \square \forall X \varphi \rightarrow \varphi$$

The problem, in the present context, with such a defence of the claim that every $\square \forall_2$ -closure of every instance of $(\text{MCP} - \text{MCPi})$ is true is that higher-order contingentists simply reject the truth of every $\square \forall_2$ -closure of every instance of $(\square \text{S}\forall 0)$.

For instance, higher-order contingentists reject the truth of the following formula:

$$\square \forall Y \square (\square \forall X (\exists Z (\square \forall x (Xx \leftrightarrow Zx)) \rightarrow (\exists Z (\square \forall x (Yx \leftrightarrow Zx)))).$$

To see why, note that a straightforward consequence of this formula in the minimal higher-order contingentist deductive system $\mathbf{WG}_{\mathcal{SL}_{\square}}$ consists in the formula

$$\square \forall Y \square \exists Z (\square \forall x (Yx \leftrightarrow Zx)).$$

This formula is just (NNE_{M}) (up to substitution of bound variables). Thus, higher-order contingentists simply reject the claim that every $\square \forall_2$ -closure of every instance of $(\square \text{S}\forall 0)$ is true. A fortiori, a defence of the truth of every $\square \forall_2$ -closure of every instance of $(\text{MCP} - \text{MCPi})$ that appeals to $(\square \text{S}\forall 0)$ is unavailable to Williamson, for such defence would be question-begging.

Note also that corollary 4 has as a consequence that every (closed) instance of $(\text{MCP} - \text{MCPi})$ is a theorem of the deductive system $\mathbf{SR}_{\mathcal{SL}_{@ \square \lambda}}$, and that every instance of the following schema is a theorem of both $\mathbf{SG}_{\mathcal{SL}_{@ \square \lambda}}$ and $\mathbf{SR}_{\mathcal{SL}_{@ \square \lambda}}$:

⁴⁵This can be shown by slightly adapting the countermodel provided in the appendix of (Williamson, 2013, ch. 6).

(MCP–@MCPi) $\Box\forall X(\Diamond\exists y\Box\forall x(Xx \rightarrow \Diamond x \leq y) \rightarrow \Diamond\exists y(\Box\forall x(Xx \rightarrow \Diamond x \leq y) \wedge \Box\forall z(\Box\forall x(Xx \rightarrow \Diamond x \leq z) \rightarrow \Diamond y \leq z))) \rightarrow$
 $\Diamond\exists y(\Box\forall x(\varphi \rightarrow \Diamond x \leq y) \rightarrow \Diamond\exists y(\Box\forall x(\varphi \rightarrow \Diamond x \leq y) \wedge \Box\forall z(\Box\forall x(\varphi \rightarrow \Diamond x \leq z) \rightarrow \Diamond y \leq z)))$, where y and z do not occur free in φ .

Thus, the higher-order contingentist may adopt the option of taking any intuition that apparently supports the truth of every $\Box\forall_2$ -closure of every instance of (MCP–MCPi) as an intuition supporting, at most, the actual truth of every instance of (MCP–MCPi).⁴⁶

Despite these considerations in favour of the higher-order contingentist, it must be noted that the absence of any example of a $\Box\forall_2$ -closure of an instance of (MCP–MCPi) which should be regarded as false by the lights of the higher-order contingentist might be regarded as providing sufficient abductive support for the truth of every $\Box\forall_2$ -closure of an instance of (MCP–MCPi) – and, a fortiori, of (NNE_M).

We will not go further into the matter here. Hopefully, the present discussion suffices to show how higher-order contingentists may avail themselves of strongly Millian logics in order to offer a reply to Williamson’s abductive argument for the truth of (NNE_M). The upshot is that by appealing to strongly Millian second-order modal logics higher-order contingentists avail themselves of extra resources for rejecting the cogency of Williamson’s argument for (NNE_M). If the line of reply to Williamson’s argument suggested in this section turns out to be successful, then there is reason to think that some strongly Millian modal logics available to higher-order contingentists are not only reasonable but also sufficiently strong second-order modal logics.

A.6 Other Proposals

There are at least two proposals in the literature that share some similarities with strongly Millian logics. One of these is Kaplan’s proposal on how classical first-order logic can be recovered once a ‘context-sensitive’ interpretation is given to free variables (Kaplan, 1989). The other proposal is Menzel’s (1991) logic **A**, a logic reflecting certain of Prior’s intuitions motivating his logic **Q**, albeit with a different notion of possibility. I will briefly compare strongly Millian logics with these two proposals.

The logic **A** has already been presented in §A.3.3. The main motivation behind this system is also one of the main motivations of the strongly Millian logics proposed, namely, to capture the fact that the theorems of classical quantified logic are all true in the actual world. In effect, we noted in §A.3.3 that **A** is appropriately regarded as a strongly Millian logic, namely, the logic **SR** _{$\mathcal{FL}\Box$} .

The logic **A** purports to capture the notion of real-world validity. One of the things I have tried to do here has been to show that the special feature of classical quantified logic captured by **A** can also be captured by appealing to a property of arguments other than real-world validity. As shown in

⁴⁶This strategy is not unlike the strategy appealed to by necessitists when faced with the claim that our intuitions support the view that individuals like Michael Jordan exist contingently. Necessitists point out that those intuitions may be seen as supporting instead the weaker thesis that individuals like Michael Jordan are contingently concrete.

§A.4, the (arguably) more common notion of general validity is also able to capture the actual truth of every theorem of classical quantified logic, as long as a (context-insensitive) actuality operator is present in the language. Besides this point, the notorious difference between the strongly Millian logics offered here and Menzel's logic **A** concerns the logics' underlying languages, with languages with an actuality operator and second-order modal languages (with and without a λ operator) also being considered here.

Thus, the present paper can be seen as extending Menzel's insight of treating individual constants as strongly Millian expressions to the case of n -ary predicates, and showing that the fact that the theorems of classical quantified logic are all true in the actual world can be captured by appealing not only to the notion of real-world validity, but also to the notion of general validity. The relationship between strongly Millian logics and Kaplan's proposal concerning the logic of free variables will now be considered.

Briefly, Kaplan's account of the semantics of context-sensitive expressions requires a notion of truth relativised both to contexts of use and circumstances of evaluation. Formally, the role of contexts of use in Kaplan's account of the semantics of context-sensitive expressions is that of providing parameters required for the determination of the content of these expressions and the sentences containing them. Those contents are then true relative to some circumstances of evaluation, and false relative to others. Kaplan remarks that, given this formal understanding of a context of use, variable-assignments may be understood as parameters provided by context. They are required in order for the content of formulas containing occurrences of *free variables* to be determined. On this way of understanding the semantics of free-variables, the variable-assignment of a context assigns to each variable an individual in the domain of the possible world of the context.⁴⁷

For each context of use there is a circumstance of evaluation that is the circumstance of evaluation of that context of use. For the present purposes, let a circumstance of evaluation consist just in a possible world. Intuitively, the possible world of a context is the possible world in which the sentence would be used if used in that context. Given this feature of contexts of use, from the doubly-relativised conception of truth it is possible to extract a conception of truth relativised solely to contexts of use. A formula is true relative to a context of use c if and only if it is true relative to c and possible world w_c , the possible world of context c . Equivalently, a formula is true relative to a context of use c if and only if the content it expresses relative to c is true relative to possible world w_c .

Kaplan offers a conception of validity which appeals to truth in a context of use. According to this conception, an argument with premises Γ and conclusion φ is *Kaplan-valid* if and only if there is no context of use such that all premises in Γ are true in that context of use and φ is false in that context of use. Call this conception 'Kaplan-validity'. Kaplan-validity is intended to capture a special feature of arguments. The Kaplan-valid arguments are those arguments which cannot be used in a context c without it being the case that the conclusion is true (relative to c and w_c) if all of the premises are

⁴⁷As pointed out in (Kaplan, 1989, p. 592), for bound occurrences of variables the role of variable-assignments is not that of providing a parameter required for the determination of the content of sub-formulas in which the variables occur.

true (relative to c and w_c).

The context-sensitive understanding of the semantics of free variables previously sketched has as a consequence that any instance of the schema $\forall v\varphi \rightarrow \varphi_v^v$ is Kaplan-valid, even though it is not the case that every instance of the schema $\Box(\forall v\varphi \rightarrow \varphi_v^v)$ is Kaplan-valid.⁴⁸ In effect, once Kaplan-validity is assumed, the quantified logic for free variables is *classical*, even though, in the scope of a necessity operator, the quantified logic is free.⁴⁹

This reveals a structural similarity between strongly Millian logics and the logic resulting from the adoption of Kaplan-validity and of the context-sensitive understanding of the semantics of free variables proposed by Kaplan. The logic of individual constants is classical in any **SR** system. More generally, the logic of individual constants is classical, in any strongly Millian logic, when the formulas of the language are in the scope of the actuality operator, @. However, as in the case of free variables, in the scope of a necessity operator the quantified logic is free also in the case of strongly Millian logics.

Consider another ‘actuality’ operator, A , understood according to its *context-dependent* reading. That is, it is assumed that the meaning of the operator A is such that a formula of the form $A\varphi$ is true at a context c and possible world w if and only if φ is true at c and w_c , the possible world of context c . Note that a formula φ is Kaplan-valid if and only if $A\varphi$ is Kaplan-valid, with the quantified logic of free variables being free for any formula φ of \mathcal{FL} and \mathcal{SL} in the scope of the operator A .

Despite the structural similarities between strongly Millian logics and the logic of free variables resulting from the adoption of Kaplan’s semantic proposal, the logic for a language whose semantics for free variables is the one proposed by Kaplan and which takes individual constants to be strongly Millian expressions is not perforce one in which classical quantified logic is preserved in the scope of @. And similarly for the operator A .

For an example, consider the notion of Kaplan-validity. Let a be any individual constant, understood as a strongly Millian expression, and v any individual variable. Suppose that it is possible that no actually existing thing exists (with ‘actually’ being understood here in its context-independent sense), even though some other thing does, and let w_{c^*} be a counterfactual possible world witnessing this possibility statement, for some context c^* . Then, even though AEv is Kaplan-valid, for any individual variable v , AEa is not. To see that AEa is not Kaplan-valid, note that AEa is true at context of use c^* if and only if Ea is true at c^* and w_{c^*} , if and only if the referent of a exists in the world w_{c^*} . Since a is a strongly Millian expression, the referent of a is some actually existing individual (with ‘actually’ being understood here in its context-independent sense). But then, the referent of a does not exist in w_{c^*} , and thus Ea is false with respect to c^* and w_{c^*} . Similarly, $@Ev$ is not Kaplan-valid. To see this, note that $@Ev$ is true at c^* if and only if $@Ev$ is true at c^* and w_{c^*} , if and only if Ev is true at c^* and the actual possible world, α , if and only if the variable-assignment of context c^* assigns to v an

⁴⁸Assuming that Kaplan-validity is neutral with respect to whether the same or different individuals may be found in different possible worlds. As Kaplan notes, this neutrality is absent in Kaplan’s Logic of Demonstratives. The formula $\Box\forall x\Box Ex$ is logically valid in the Logic of Demonstratives.

⁴⁹Kaplan (1989, p. 594) reports that this fact has been pointed out to him by Harry Deutsch.

individual that exists in α . But the variable assignment of c^* assigns to v an individual that exists in w_{c^*} and, by hypothesis, no actual individual exists in w_{c^*} . Hence, the formula $@Ev$ is not true at c^* . Thus, the schemas AEt and $@Et$ both have some instances which are not Kaplan-valid, where t may be replaced by any individual constant or variable.

A similar point can be made by appealing to a different notion of validity, *independent validity*, where a formula φ is independently valid if and only if φ is true relative to every context of use and every circumstance of evaluation. Even though, for every individual constant a , the formula $@Ea$ is independently valid and, for any individual variable v , the formula AEv is independently valid, we also have that the formula $A Ea$ is not independently valid, and the formula $@Ev$ is not independently valid.⁵⁰ Therefore, the logic of individual terms is classical neither under the scope of $@$ nor under the scope of A when the notion of validity in question is independent validity.

The upshot is that, despite the structural similarities between Kaplan's proposal and strongly Millian logics, these proposals are in fact different, and may lead, depending on one's target conception of validity, to classical quantified logic to be recovered neither under the scope of $@$ nor under the scope of A .

A.7 Second-Order?

Before concluding, I want to address a possible worry concerning the strongly Millian logics for second-order languages that have been proposed. In a nutshell, the worry is that these are not *really* second-order.

Some philosophers hold that there are arguments formulated in \mathcal{SL} which are really valid, even though they are not valid in every Henkin model. Equivalently, some philosophers hold that there are arguments formulated in \mathcal{SL} which are really valid, even though they are not valid in every CI-model. A simple example is the argument whose premise is the statement that there are at least two things, and whose conclusion is the statement that there are at least four non-coextensive properties.

For the present purposes, I will focus on the conception of validity as *general validity*, spelled out in subsection A.2.1. Let an *absolutist* be a philosopher who holds that there are arguments formulated in second-order languages that are generally valid, despite the fact that they are not valid in every CI-model, and a *relativist* be a philosopher who holds that every generally valid argument is valid in every CI-model for that language.

The reason why absolutists reject the claim that all generally valid arguments are valid in every

⁵⁰Take any context of use c and possible world w . The formula AEv is true at c and w just in case the formula Ev is true at c and c_w . But the variable-assignment of context c assigns to the variable v an individual that exists at w_c (i.e., at the possible world of c). Hence, Ev is true relative c and w_c , and thus AEv is true relative to c and w , for any c and w . That is, AEv is independently valid.

Now, take any context of use c and world w . $@Ea$ is true relative to c and w if and only if Ea is true relative to c and α , if and only if the referent of a exists in α . But, for every a , the referent of a exists in α , given the assumption that a is a strongly Millian expression. Hence, $@Ea$ is independently valid.

Finally, the context c^* previously mentioned can also be used to show that some instances of $A Ea$ and $@Ev$ are not independently valid

Henkin model has to do with the fact that, according to them, certain Henkin models fail to adequately represent the relationship between the range of the first- and second-order quantifiers. In particular, absolutists hold that the relationship between the set $d(\alpha)$ and the set $D(n)(\alpha)$ (for each $n \in \mathbb{N}_0$) does not always adequately represent the relationship between the class of all individuals and the class of all n -ary relations. For the present purposes, let me focus on the sets $d(\alpha)$ and $D(1)(\alpha)$. Absolutists hold the following thesis about the relationship between the range of first- and second-order quantifiers:

Abundantism For every subclass of the class of all individuals there is a property that is instantiated by all and only the elements in that subclass.

Models $M \in \text{Cl}$ in which the definition of value relative to a variable-assignment g is such that $V_{M,\alpha}^g(\forall V\varphi) = \{\langle \rangle\}$ if and only if, for every element f in $(\mathcal{P}(d(\alpha)))^{\{\alpha\}}$, $V_{M,\alpha}^{g[V/f]}(\varphi) = \{\langle \rangle\}$ are in agreement with the thesis of Abundantism, whereas the remaining models in Cl are not. Thus, absolutists take any Cl -model in which $D(1)(\alpha) \neq (\mathcal{P}(d(\alpha)))^{\{\alpha\}}$ to be a model which inadequately represents the relationship between the range of the first- and second-order quantifiers.

One way to put the matter is that, from the standpoint of absolutists, Cl -models in which it is the case that $D(1)(\alpha) \neq (\mathcal{P}(d(\alpha)))^{\{\alpha\}}$ fail to capture the fact that $\forall V$ is intended to express (unrestricted) universal quantification over properties, even though models with this feature would be appropriate if, instead of unrestricted universal quantification, $\forall V$ was intended to express *restricted* universal quantification over properties. In the present context, this means that absolutists hold that certain Henkin models invalidating certain arguments do not represent possibilities in which the premises of the argument are true and the conclusion is false, since they fail to depict the correct relationship between the ranges of the first- and second-order quantifiers.

Following Shapiro (1991), say that a Cl -model is *full* just in case, for every $n \in \mathbb{N}_0$, $D(\alpha)(n) = (\mathcal{P}(d(\alpha))^n)^{\{\alpha\}}$. The considerations presented above lead absolutists to hold that only *full* Henkin models adequately represent the relationship between the ranges of the first- and second-order quantifiers. Since there are some arguments valid in every *full* Henkin model that are invalid in some Henkin models, absolutists thereby hold that some generally valid arguments are invalid in some Henkin models.

Abundantism also favours the view that the strongly Millian logics for second-order modal languages proposed here fail to capture the class of generally valid arguments formulated in those languages. Let us focus on S -models. For every $w \in W$, let

$$D^*(n)(w) = \{g : g(w) \subseteq (d(w))^n \ \& \ \forall w' \text{ s. t. } w' \neq w \ \& \ w' \in W(g(w') = \emptyset)\}.$$

A minimal requirement for a S -model to appropriately represent the truth of Abundantism seems to be that $D^*(n)(\alpha) \subseteq D(n)(\alpha)$. Furthermore, it is plausible to think that proponents of Abundantism also adhere to its necessitation. Call this thesis *Necessitated Abundantism*. A minimal requirement for a S -model to appropriately represent the truth of Necessitated Abundantism seems to be that, for every $w \in W$, $D^*(n)(w) \subseteq D(n)(w)$.

Now, there are S-models which do not even satisfy the constraint that $D^*(n)(\alpha) \subseteq D(n)(\alpha)$. Theorists committed to Abundantism will hold that these S-models fail to appropriately represent the relationship between the ranges of the first- and second-order quantifiers.⁵¹ A fortiori, theorists committed to Necessitated Abundantism will hold also hold that S-models fail to appropriately represent the relationship between the ranges of the first- and second-order quantifiers.

Let a *weakly full S-model* be a S-model such that $D^*(n)(\alpha) \subseteq D(n)(\alpha)$, and a *full S-model* be a S-model such that, for each $w \in W$, $D^*(n)(w) \subseteq D(n)(w)$. Some arguments $\langle \Gamma, \varphi \rangle$ are SG-valid in every weakly full S-model even though there are S-models in which $\langle \Gamma, \varphi \rangle$ is SG-invalid, and some arguments $\langle \Gamma, \varphi \rangle$ are SG-valid in every full S-model even though there are S-models in which $\langle \Gamma, \varphi \rangle$ is SG-invalid. Hence, proponents of Abundantism should hold that there are generally valid arguments which are SG-invalid. Similarly, proponents of Necessitated Abundantism should hold that there are generally valid arguments that are SG-invalid. Given that one of the aims of strongly Millian logics is that of capturing the notions of general validity and real-world validity, these seem bad news, requiring a reappraisal of Abundantism and Necessitated Abundantism. Since Necessitated Abundantism implies Abundantism, in what follows I will focus solely on the thesis of Abundantism. By showing that there are good reasons for rejecting Abundantism (given certain conceptions of properties) it is shown, a fortiori, that there are good reasons to reject Necessitated Abundantism (given those conceptions of properties).

Shapiro, the main advocate of the legitimacy of the notion of validity extensionally captured by validity in every full Henkin model, commits himself to the truth of Abundantism only given an extensional understanding of 'property' as what he calls a 'logical set'. He takes the notion of a logical set to be akin to an indexical notion: given a universe of discourse, a logical set is any subclass of this universe.⁵² Clearly, Abundantism is true if properties are understood as logical sets. However, in the present paper the focus is on properties understood as entities which, in general, could have been instantiated by individuals other than the ones actually instantiating them. Hence, the fact that Abundantism is true if properties are understood as logical sets does not show that Millian logics do not capture the notion of general validity. Even though Absolutists typically intended the second-order quantifiers of second-order logic to range over logical sets, the second-order quantifiers of strongly Millian logics are not intended to range over logical sets.

This shows that the typical reason for supporting Abundantism put forward by Absolutists does not constitute a reason for rejecting the claim that every generally valid argument is SG-valid, since the second-order quantifiers of Millian logics are not intended to range over entities whose criterion of individuation is extensional. But it does not show that other conceptions of properties, ones where properties are not extensionally conceived, aren't themselves committed to the truth of Abundantism.

⁵¹To give a simple example, let $M = \langle W, d, D, \alpha, V \rangle$, where $W = \{1\}$, $\alpha = 1$, $d(\alpha) = \{a\}$. For each n , let f_n be a function with domain W and such that $f^n(\alpha) = (d(\alpha))^n$. Let $D(n)(\alpha) = \{f^n\}$. Also, let f_\emptyset be a function with domain W and mapping α to the empty set. We have that $f_\emptyset \in D^*(1)(\alpha)$, even though $f_\emptyset \notin D(1)(\alpha)$. But then, $D^*(1)(\alpha) \not\subseteq D(1)(\alpha)$.

⁵²See (Shapiro, 1991, pp. vii, 18-22 and 63-64).

in which case there are generally valid arguments which are not SG-valid.

But there are several sparse conceptions of properties which, arguably, are not committed to the truth of Abundantism. For instance, a popular conception of properties takes these to be individuated by their nomological roles. Call this conception of properties the *nomological conception* of properties. Arguably, there are subclasses of individuals for which there is no nomological property instantiated by all and only the elements of the class, where a nomological property is a property whose individuation criterion is given by its nomological role. This means that Abundantism is false if understood as concerning properties individuated according to the nomological conception.

Recall that one of the presuppositions of the present paper is that the second-order quantifiers range over properties whose criterion of individuation is given by necessary coextensiveness. It is unclear whether there could have been two properties with different nomological roles which were nevertheless necessarily coextensive (arguably, there could not have been two properties which were not necessarily coextensive but that nevertheless had the same modal profile). If there could have been two such properties, then the fact that Abundantism is false if understood as concerning properties individuated according to their nomological role might seem not to be of importance to the present paper.

However, even if it is conceded that there could have been two properties with different nomological roles which were nevertheless necessarily coextensive, the fact that Abundantism is false according to the nomological conception is still revealing. The reason is that even on the nomological conception it is still the case that properties have *modal profiles*. One of the options for the range of the second-order quantifiers is to take them as ranging over modal profiles of nomological properties. This option is pursued in what follows.

It is reasonable to think that modal profiles of nomological properties are ontologically dependent on nomological properties, in the sense that necessarily, a modal profile of a nomological property exists if and only if there is a nomological property with that modal profile. It has been shown that there is at least a subclass X of individuals for which there is no nomological property instantiated by all and only the elements of the class. Therefore, given the ontological dependence of modal profiles of nomological properties on nomological properties, there is no modal profile of a nomological property which has X as its extension in the actual world. Hence, Abundantism is false given the assumption that the range of second-order quantifiers consist in modal profiles of nomological properties.

It also seems plausible to assume that there is no necessary connection between the range of the first-order quantifiers and domain of nomological properties. Hence, arguably, if the range of the second-order quantifiers is assumed to consist in modal profiles of nomological relations, then general validity and SG-validity will extensionally coincide.

This point generalises. So long as the second-order quantifiers are understood as ranging over modal profiles of properties conceived in such a way that there is no necessary connection between the range of the first-order quantifiers and the domain of properties, it is reasonable to assume that general validity and SG-validity will extensionally coincide. This is yet another instance of the neutrality of

strongly Millian logics. These logics may be used to reason about modal profiles of properties under different conceptions of properties, as long as those conceptions do not imply a necessary connection between the range of the first-order quantifiers and the domain of properties.

Finally, it is relevant to point out that there is no effective deductive system whose theorems are all and only those formulas that are SG-valid in every weakly full S-model (for the same reason that there is no effective sound and complete deductive system for second-order logic with full Henkin models).⁵³ Thus, just as absolutists should be interested in the deductive system $\mathbf{C1}_{\mathcal{SL}}$ – since $\mathbf{C1}_{\mathcal{SL}}$ enables them to reason about logical sets with the guarantee that they will not be inferring falsehoods from truths –, even proponents of Abundantism should be interested in strongly Millian logics, despite the fact that they are not really second-order according to them. Strongly Millian logics still afford Abundantists with deductive systems which can be used for reasoning about modal profiles with the guarantee that no falsehoods will be inferred from true premises.

A.8 Conclusion

In this paper complete strongly Millian first- and second-order modal logics have been presented. Some of their presuppositions were made salient, and it was shown that they capture a special feature of classical first- and second-order logic, to wit, that the result of prefixing any classical theorem with an actuality operator is a theorem of these logics. Insofar as the Millian logics proposed capture the notions of real-world validity and general validity for their underlying languages, the result of prefixing any theorem of classical quantified logic with an actuality operator yields a generally valid and real-world valid formula. This result holds even if a neutral stance with respect to questions such as whether necessarily everything necessarily exists is maintained, since as shown, the strongly Millian logics proposed are all conservative, in the sense discussed in §A.1.

In the second part of the paper a possible application of strongly Millian second-order modal logics to the debate between higher-order contingentists and higher-order necessitists was presented. It was shown that strongly Millian logics promise to provide higher-order contingentists with the resources required to reject an argument for higher-order necessitism recently put forward by Williamson.

The logics were also compared to other proposals in the literature. It was seen that even though Kaplan's understanding of the behaviour of free variables is structurally similar to the strongly Millian stance adopted here, the logic of a language containing both strongly Millian expressions and a context-sensitive treatment of free variables is not guaranteed to capture the specialness of classical quantified logic. In particular, it was shown that the result of prefixing some theorems of classical quantified logic with A does not yield a Kaplan-valid formula, nor an independently valid formula, and similarly for the result of prefixing some theorems of classical quantified logic with $@$.

Finally, a worry to the effect that strongly Millian logics are not really second-order was addressed. The crux of the worry was identified as having to do with the question whether strongly Millian

⁵³Similarly, and for the same reason, there is no effective deductive system whose theorems are all and only those formulas that are SG-valid in every full S-model.

second-order logics capture the notion of general validity. It was shown that whether this is so depends on the conception of properties in which one is interested, and that for some conceptions of properties there is good reason to think that strongly Millian logics do capture the notion of general validity.

A.9 Appendix

A.9.1 Weakly Millian Logics

The ‘weakly correlates’ of the strongly Millian deductive systems characterised in §A.3.3 are now presented. The only rule of inference of all of these systems is (MP):

Definition 29 (Axioms of $\mathbf{WG}_{\mathcal{FL}_{@}\square}$). Every $@\square\forall$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}]$.

Definition 30 (Axioms of $\mathbf{WR}_{\mathcal{FL}_{@}\square}$). Every $@\square\forall$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}]$. Every $@\forall$ -closure of every instance of (R@).

Definition 31 (Axioms of $\mathbf{WG}_{\mathcal{FL}_{@}\square\lambda}$). Every $@\square\forall$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}]$. Every $@\square\forall$ -closure of every instance of (EAb).

Definition 32 (Axioms of $\mathbf{WR}_{\mathcal{FL}_{@}\square\lambda}$). Every $@\square\forall$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}]$. Every $@\square\forall$ -closure of every instance of (EAb). Every $@\forall$ -closure of every instance of (R@).

Definition 33 (Axioms of $\mathbf{WG}_{\mathcal{SL}_{@}\square}$). Every $@\forall_2$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}] \cup [\mathbf{FSOL}]$.

Definition 34 (Axioms of $\mathbf{WR}_{\mathcal{SL}_{@}\square}$). Every $@\square\forall_2$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}] \cup [\mathbf{FSOL}]$. Every $@\forall_2$ -closure of every instance of (R@).

Definition 35 (Axioms of $\mathbf{WG}_{\mathcal{SL}_{@}\square\lambda}$). Every $@\square\forall_2$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}] \cup [\mathbf{FSOL}]$. Every $@\square\forall_2$ -closure of every instance in L of (EAb).

Definition 36 (Axioms of $\mathbf{WR}_{\mathcal{SL}_{@}\square\lambda}$). Every $@\square\forall_2$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}] \cup [\mathbf{FSOL}]$. Every $@\forall_2$ -closure of every instance of (R@). Every $@\square\forall_2$ -closure of every instance of (EAb).

Definition 37 (Axioms of $\mathbf{WG}_{\mathcal{SL}_{@}\square\lambda}^C$). Every $@\square\forall_2$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}] \cup [\mathbf{FSOL}]$. Every $@\square\forall_2$ -closure of every instance of (EAb) and of (CComp).

Definition 38 (Axioms of $\mathbf{WR}_{\mathcal{SL}_{@}\square\lambda}^C$). Every $@\square\forall_2$ -closure of every instance of every schema in $[\mathbf{S5}] \cup [\mathbf{Act}] \cup [\mathbf{FFOL}] \cup [\mathbf{FSOL}]$. Every $@\forall_2$ -closure of every instance of (R@). Every $@\square\forall_2$ -closure of every instance of (EAb) and of (CComp).

A.9.2 Strongly Millian logics: ‘Classical’

I now turn to the proof of item (iv) of theorem 4, i.e., that $\Gamma \vdash_{\mathbf{Cl}_{\mathcal{SL}_\lambda}} \varphi \Leftrightarrow @\Gamma \vdash_{\mathbf{SG}_{\mathcal{SL}_{@ \square \lambda}}^{\mathbf{C}}} @\varphi \Leftrightarrow \Gamma \vdash_{\mathbf{SR}_{\mathcal{SL}_{@ \square \lambda}}^{\mathbf{C}}} \varphi$. It is shown that $\Gamma \vdash_{\mathbf{Cl}_{\mathcal{SL}_\lambda}} \varphi \Leftrightarrow @\Gamma \vdash_{\mathbf{SG}_{\mathcal{SL}_{@ \square \lambda}}^{\mathbf{C}}} @\varphi$ and $@\Gamma \vdash_{\mathbf{SG}_{\mathcal{SL}_{@ \square \lambda}}^{\mathbf{C}}} @\varphi \Leftrightarrow \Gamma \vdash_{\mathbf{SR}_{\mathcal{SL}_{@ \square \lambda}}^{\mathbf{C}}} \varphi$ is now offered. Item (iv) of theorem 4 follows straightforwardly from these two proofs.

Proof. ($\Gamma \vdash_{\mathbf{Cl}_{\mathcal{SL}_\lambda}} \varphi \Leftrightarrow @\Gamma \vdash_{\mathbf{SG}_{\mathcal{SL}_{@ \square \lambda}}^{\mathbf{C}}} @\varphi$). First, it is shown that i) $\Gamma \vdash_{\mathbf{Cl}_{\mathcal{SL}_\lambda}} \varphi \Rightarrow @\Gamma \vdash_{\mathbf{SG}_{\mathcal{SL}_{@ \square \lambda}}^{\mathbf{C}}} @\varphi$.

Afterwards, it is shown that ii) $@\Gamma \vdash_{\mathbf{SG}_{\mathcal{SL}_{@ \square \lambda}}^{\mathbf{C}}} @\varphi \Rightarrow \Gamma \vdash_{\mathbf{Cl}_{\mathcal{SL}_\lambda}} \varphi$.

Proof of i):

Suppose $@\Gamma \not\vdash_{\mathbf{SG}_{\mathcal{SL}_{@ \square \lambda}}^{\mathbf{C}}} @\varphi$. By theorem 3, we have that $@\Gamma \not\vdash_{\mathbf{SG}^{\mathbf{C}}} @\varphi$. Hence, there is a $\mathbf{S}^{\mathbf{C}}$ -model $M = \langle W, d, D, \alpha, V \rangle$ and $g \in \text{As}(M)$ s. t. $\forall \gamma \in \Gamma (V_{M,w}^g(@\gamma) = \{\langle \rangle\})$ and $V_{M,w}^g(@\varphi) = \emptyset$. Thus, $\forall \gamma \in \Gamma (V_{M,\alpha}^g(\gamma) = \{\langle \rangle\})$ and $V_{M,\alpha}^g(\varphi) = \emptyset$. Let

$$M|_{\{\alpha\}} = \langle \{\alpha\}, d|_{\{\alpha\}}, D|_{\{\alpha\}}, \alpha, V|_{\{\alpha\}} \rangle$$

where, $\forall n \in \mathbb{N}_0$, $D|_{\{\alpha\}}(n)(\alpha) = \{f|_{\{\alpha\}} : f \in D(n)(\alpha)\}$, $\forall a \in \text{Const}(\mathcal{SL}_\lambda)$, $V|_{\{\alpha\}}(a) = V(a)$, and $\forall \zeta \in \text{Pred}^n(\mathcal{SL}_\lambda)$, $V|_{\{\alpha\}}(\zeta) = V(\zeta)|_{\{\alpha\}}$.

Clearly, $\forall g' \in \text{As}(M|_{\{\alpha\}})$, $\psi \in \text{Form}(\mathcal{SL}_\lambda)$: $V_{M,\alpha}^{g'}(\psi) = V_{M|_{\{\alpha\}},\alpha}^{g'}(\psi)$. Furthermore, $\forall g' \in \text{As}(M|_{\{\alpha\}})$, $\forall \gamma \in \Gamma$: $V_{M,\alpha}^g(\gamma) = V_{M,\alpha}^{g'}(\gamma)$, and $V_{M,\alpha}^g(\varphi) = V_{M,\alpha}^{g'}(\varphi)$, since no free variables occur in φ , nor in any $\gamma \in \Gamma$.

Thus, $\exists g' \in \text{As}(M|_{\{\alpha\}})$ s. t. $\forall \gamma \in \Gamma (V_{M,\alpha}^{g'}(\gamma) = \{\langle \rangle\})$, and $V_{M|_{\{\alpha\}},\alpha}^{g'}(\varphi) = \emptyset$.

It remains to show that $M|_{\{\alpha\}}$ is a $\mathbf{Cl}^{\mathbf{C}}$ -model. It suffices to show that $\forall g \in \text{As}(M|_{\{\alpha\}})$, $\forall \psi \in \text{Form}(\mathcal{SL}_\lambda)$: $V_{M|_{\{\alpha\}}}^g(\lambda v_1 \dots v_n(\psi)) \in D|_{\{\alpha\}}(n)(\alpha)$.

Note that $\forall g \in \text{As}(M|_{\{\alpha\}})$: $\forall t \in \text{Terms}(\mathcal{SL}_\lambda) (V_M^g(t) \in d(\alpha))$ and $\forall \zeta \in \text{Pred}^n(\mathcal{SL}_\lambda) \cup \text{SVar}^n(\mathcal{SL}_\lambda) (V_M^g(\tau^n) \in D(n)(\alpha))$. We have that $\forall g \in \text{As}(M|_{\{\alpha\}})$: $\forall t \in \text{Terms}(\mathcal{SL}_\lambda) (V_M^g(t) \in d(\alpha))$ and $\forall \zeta \in \text{Pred}^n(\mathcal{SL}_\lambda) \cup \text{SVar}^n(\mathcal{SL}_\lambda) (V_M^g(\zeta) \in D(n)(\alpha))$. Therefore, $\forall g \in \text{As}(M|_{\{\alpha\}})$, $\forall \psi \in \text{Form}(\mathcal{SL}_\lambda)$: $V_M^g(\lambda v_1 \dots v_n(\psi)) \in D(n)(\alpha)$, since M is a $\mathbf{S}^{\mathbf{C}}$ -model. But then, by the definition of $D|_{\{\alpha\}}$, and the fact that

$$\forall g \in \text{As}(M|_{\{\alpha\}}) \forall \psi \in \text{Form}(\mathcal{SL}_\lambda) (V_{M,\alpha}^g(\lambda v_1 \dots v_n(\psi)) = V_{M|_{\{\alpha\}},\alpha}^g(\lambda v_1 \dots v_n(\psi))),$$

it follows that:

$$\forall g \in \text{As}(M|_{\{\alpha\}}), \forall \psi \in \text{Form}(\mathcal{SL}_\lambda) : V_M^g(\lambda v_1 \dots v_n(\psi))|_{\{\alpha\}} = V_{M|_{\{\alpha\}},\alpha}^g(\lambda v_1 \dots v_n(\psi)) \in D|_{\{\alpha\}}(n)(\alpha).$$

Therefore, $M|_{\{\alpha\}}$ is in fact a $\mathbf{Cl}^{\mathbf{C}}$ -model. A fortiori, $\Gamma \not\vdash_{\mathbf{Cl}^{\mathbf{C}}} \varphi$. But then, by the completeness of $\mathbf{Cl}_{\mathcal{SL}_\lambda}$, we have that $\Gamma \not\vdash_{\mathbf{Cl}_{\mathcal{SL}_\lambda}} \varphi$.

Hence, by contraposition, we get that $\Gamma \vdash_{\mathbf{Cl}_{\mathcal{SL}_\lambda}} \varphi \Rightarrow @\Gamma \vdash_{\mathbf{SG}_{\mathcal{SL}_{@ \square \lambda}}^{\mathbf{C}}} @\varphi$.

Proof of i):

$$\begin{aligned}
\Gamma \not\vdash_{\mathbf{Cl}_{S\mathcal{L}_\lambda}} \varphi &\Leftrightarrow \Gamma \not\vdash_{\mathbf{Cl}^C} \varphi \text{ (By the completeness of system } \mathbf{Cl}_{S\mathcal{L}_\lambda}\text{)} \\
&\Leftrightarrow \exists M = \langle W, d, D, \alpha, V \rangle \in \mathbf{Cl}^C, \exists g \in \text{As}(M) (\forall \gamma \in \Gamma (V_{M,\alpha}^g(\gamma) = \{\langle \rangle\}) \text{ and } V_{M,\alpha}^g(\varphi) = \emptyset) \\
&\Rightarrow \exists M \in \mathbf{S}^C, \exists g \in \text{As}(M) (\forall \gamma \in \Gamma (V_{M,\alpha}^g(@\gamma) = \{\langle \rangle\}) \text{ and } V_{M,\alpha}^g(@\varphi) = \emptyset) \\
&\Rightarrow \exists M \in \mathbf{S}^C, \exists g \in \text{As}(M) \exists w \in W (\forall \gamma \in \Gamma (V_{M,w}^g(@\gamma) = \{\langle \rangle\}) \text{ and } V_{M,w}^g(@\varphi) = \emptyset) \\
&\Rightarrow @\Gamma \not\vdash_{\mathbf{SG}^C} @\varphi \\
&\Rightarrow @\Gamma \not\vdash_{\mathbf{SG}_{S\mathcal{L}_{@□\lambda}}^C} @\varphi \text{ (Theorem 3)}
\end{aligned}$$

Therefore, $@\Gamma \vdash_{\mathbf{SG}_{S\mathcal{L}_{@□\lambda}}^C} @\varphi \Rightarrow \Gamma \vdash_{\mathbf{Cl}_{S\mathcal{L}_\lambda}} \varphi$. \square

Proof. $(@\Gamma \vdash_{\mathbf{SG}_{S\mathcal{L}_{@□\lambda}}^C} @\varphi \Leftrightarrow \Gamma \vdash_{\mathbf{SR}_{S\mathcal{L}_{@□\lambda}}^C} \varphi)$

$$\begin{aligned}
@\Gamma \not\vdash_{\mathbf{SG}_{S\mathcal{L}_{@□\lambda}}^C} @\varphi &\Leftrightarrow \Gamma \not\vdash_{\mathbf{SG}^C} \varphi \text{ (Theorem 3)} \\
&\Leftrightarrow \exists M \in \mathbf{Cl}^C, \exists w \in W (\forall \gamma \in \Gamma (V_{M,w}^g(@\gamma) = \{\langle \rangle\}) \text{ and } V_{M,w}^g(@\varphi) = \emptyset) \\
&\Leftrightarrow \exists M \in \mathbf{Cl}^C, \exists w \in W (\forall \gamma \in \Gamma (V_{M,\alpha}^g(\gamma) = \{\langle \rangle\}) \text{ and } V_{M,\alpha}^g(\varphi) = \emptyset) \\
&\Leftrightarrow \Gamma \not\vdash_{\mathbf{SR}^C} \varphi \\
&\Leftrightarrow \Gamma \not\vdash_{\mathbf{SR}_{S\mathcal{L}_{@□\lambda}}^C} \varphi \text{ (Theorem 3)}
\end{aligned}$$

Thus, $@\Gamma \vdash_{\mathbf{SG}_{S\mathcal{L}_{@□\lambda}}^C} @\varphi \Leftrightarrow \Gamma \vdash_{\mathbf{SR}_{S\mathcal{L}_{@□\lambda}}^C} \varphi$ \square

A.9.3 Strongly Millian Logics: Conservative

Let me now turn to the results concerning the model M characterised in page 210.

Clearly, all of Counter (BF), Counter (CBF), Counter (NNE) are satisfied by M . Thus, it remains to show that Counter (BF_M), Counter (CBF_M), Counter (NNE_M) are all satisfied by M , and that M is indeed a \mathbf{S}^C -model.

I will begin by showing that M satisfies Counter (BF_M), Counter (CBF_M) and Counter (NNE_M).

Consider the functions $f^{0_1}, f^{0_2} \in F(1)$, defined as follows:

- $f^{0_1}(1) = \{0_1\}, f^{0_1}(2) = f^{0_1}(3) = \emptyset$;
- $f^{0_2}(1) = f^{0_2}(3) = \emptyset, f^{0_2}(2) = \{0_2\}$;

Note that $\forall \pi \in \Pi_1 : \pi(f^{0_1}) = f^{0_1}$, and $\forall \pi \in \Pi_2 : \pi(f^{0_2}) = f^{0_2}$. Thus, $f^{0_1} \in D(1)(1)$, and $f^{0_2} \in D(1)(2)$.

It is now shown that $\exists \pi \in \Pi_2$ such that $\pi(f^{0_1}) \notin D(1)(2)$, and thus M satisfies Counter (CBF_M), and that $\exists \pi \in \Pi_1$ such that $\pi(f^{0_2}) \notin D(1)(1)$, and thus M satisfies Counter (BF_M).

Let $\pi_1 = (1)(2, 3)(0_1)(0_2, 0_3)$, and $\pi_2 = (1, 3)(2)(0_1, 0_3)(0_2)$. Clearly, $\pi_1 \in \Pi_1$, and $\pi_2 \in \Pi_2$. However,

$$\pi_1(f^{0_2})(w_2) = \pi_1(f^{0_2})(\pi_1)(3) = \pi_1(f^{0_2}(3)) = \pi_1(\emptyset) = \emptyset \neq f^{0_2}(w_2) = \{0_2\}.$$

Thus, $\pi(f^{0_2}) \neq f^{0_2}$, and therefore $f^{0_2} \notin D(1)(1)$.

Similarly,

$$\pi_2(f^{0_1})(w_1) = \pi_2(f^{0_1})(\pi_2)(3) = \pi_2(f^{0_1}(3)) = \pi_2(\emptyset) = \emptyset \neq f^{0_1}(w_1) = \{0_1\}.$$

Hence, $\pi(f^{0_1}) \neq f^{0_1}$, and therefore $f^{0_1} \notin D(1)(2)$. This result is not only sufficient to show that M satisfies Counter (BF_M) and Counter (CBF_M), it also shows that M satisfies Counter (NNE_M).

It remains to show that M is in fact a S^C -model. Clearly, M is a S -model. To see this, note that i) $o_1 \in d(\alpha)$, and thus, for every $a \in \text{Const}(L)$, $V(a) \in d(\alpha)$; and ii) for every $w \in W$, $\pi \in \Pi_w$, n -ary predicate letter ζ^n , $\pi(V(\zeta^n)) = V(\zeta^n)$, and thus $V(\zeta^n) \in D(n)(\alpha)$.

Hence, to prove that M is a S^C -model it suffices to show that M obeys condition 13. spelled out in page 202. For every $w \in W$, $\pi \in \Pi_w$, let $\pi(g)(x) = \pi(g(x))$, for every variable-assignment g . Our proof that M obeys condition 13. spelled out in page 202 relies on the following lemma:

Lemma 16. For every $w \in W$, $\pi \in \Pi_w$, $\varphi \in \text{Terms}(\mathcal{SL}_{@ \square \lambda}) \cup \text{STerms}(\mathcal{SL}_{@ \square \lambda}) \cup \text{Form}(\mathcal{SL}_{@ \square \lambda})$, $n \in \mathbb{N}_0$, $a \in \text{Const}(\mathcal{SL}_{@ \square \lambda})$ and $\zeta^n \cup \text{Pred}^n(\mathcal{SL}_{@ \square \lambda})$:

if a occurs in φ , then $V(a) \in d(w)$, and if ζ^n occurs in φ , then $V(\zeta^n) \in D(n)(w)$ only if:

$$\pi(V_M^g(\varphi)) = V_M^{\pi(g)}(\varphi).$$

Let us suppose that Lemma 16 has been established. Assume that for every $t \in \text{Terms}(\mathcal{SL}_{@ \square \lambda})$, $\tau^n \in \text{Pred}^n(\mathcal{SL}_{@ \square \lambda}) \cup \text{SVar}^n(\mathcal{SL}_{@ \square \lambda})$: if a occurs in φ , then $V_M^g(t) \in d(w)$, and if τ^n occurs in φ , then $V_M^g(\tau^n) \in D(n)(w)$. We have that

$$\pi(V_M^g(\lambda v_1 \dots v_n(\varphi))) = V_M^{\pi(g)}(\lambda v_1 \dots v_n(\varphi)).$$

But now, let v be any individual variable free in $\lambda v_1 \dots v_n(\psi)$. Then, $\pi(g)(v) = \pi(V_M^g(v)) = V_M^g(v)$ (since $V_M^g(v) \in d(w)$). Furthermore, $V_M^g(v) = g(v)$. That is, $\pi(g)$ and g agree in all of the individual variables free in $\lambda v_1 \dots v_n(\psi)$. By similar reasoning the same conclusion is reached for any n -ary second-order variable free in $\lambda v_1 \dots v_n(\psi)$. But then $\pi(g)$ and g agree in their assignments to all the free variables in $\lambda v_1 \dots v_n(\psi)$. Therefore,

$$V_M^{\pi(g)}(\lambda v_1 \dots v_n(\varphi)) = V_M^g(\lambda v_1 \dots v_n(\varphi)).$$

Hence,

$$\pi(V_M^g(\lambda v_1 \dots v_n(\varphi))) = V_M^g(\lambda v_1 \dots v_n(\varphi)).$$

Thus, if Lemma 16 holds, then condition 13. is satisfied, and a fortiori M is a SR^C -model.

I will thus proceed to prove Lemma 16. The proof is by induction on the complexity of φ . Suppose that for every $n \in \mathbb{N}_0$: for every $a \in \text{Const}(\mathcal{SL}_{@ \square \lambda})$, $\zeta^n \cup \text{Pred}^n(\mathcal{SL}_{@ \square \lambda})$: if a occurs in φ , then $V(a) \in d(w)$, and if ζ^n occurs in φ , then $V(\zeta^n) \in D(n)(w)$.

Proof.

1. If φ is an individual constant a , then $V(a) \in d(w)$. Therefore, $\pi(V(a)) = V(a)$. Thus, $V_M^{\pi(g)}(a) = V(a) = V_M^g(a) = \pi(V_M^g(a))$. Similarly for every n -ary predicate letter;
2. If φ is an individual variable v , then $V_M^{\pi(g)}(v) = \pi(g)(v) = \pi(g(v)) = \pi(V_M^g(v))$.
3. If φ is of the form $\tau^n t_1 \dots t_n$, then

$$\begin{aligned}
\langle \rangle \in V_M^{\pi(g)}(\varphi)(\pi(w)) &\Leftrightarrow \langle V_M^{\pi(g)}(t_1), \dots, V_M^{\pi(g)}(t_n) \rangle \in V_M^{\pi(g)}(\tau^n)(\pi(w)) \\
&\Leftrightarrow \langle \pi(V_M^g(t_1)), \dots, \pi(V_M^g(t_n)) \rangle \in \pi(V_M^g(\tau^n))(\pi(w)) \text{ (I.H.)} \\
&\Leftrightarrow \pi(\langle V_M^g(t_1), \dots, V_M^g(t_n) \rangle) \in \pi(V_M^g(\tau^n))(\pi(w)) \\
&\Leftrightarrow \pi(\langle V_M^g(t_1), \dots, V_M^g(t_n) \rangle) \in \pi(V_M^g(\tau^n)(w)) \\
&\Leftrightarrow \langle V_M^g(t_1), \dots, V_M^g(t_n) \rangle \in V_M^g(\tau^n)(w) \\
&\Leftrightarrow \langle \rangle \in V_M^g(\varphi)(w) \\
&\Leftrightarrow \langle \rangle \in \pi(V_M^g(\varphi))(\pi(w))
\end{aligned}$$

4. If φ is of the form $\neg\psi$, then:

$$\begin{aligned}
\langle \rangle \in V_M^{\pi(g)}(\varphi)(\pi(w)) &\Leftrightarrow \langle \rangle \notin V_M^{\pi(g)}(\psi)(\pi(w)) \\
&\Leftrightarrow \langle \rangle \notin \pi(V_M^g(\psi))(\pi(w)) \text{ (I.H.)} \\
&\Leftrightarrow \langle \rangle \notin \pi(V_M^g(\psi)(w)) \\
&\Leftrightarrow \langle \rangle \notin V_M^g(\psi)(w) \\
&\Leftrightarrow \langle \rangle \in V_M^g(\neg\psi)(w) \\
&\Leftrightarrow \langle \rangle \in \pi(V_M^g(\varphi))(\pi(w))
\end{aligned}$$

The case where φ is of the form $\psi \wedge \chi$ proceeds similarly.

5. If φ is of the form $@\psi$, then:

$$\begin{aligned}
\langle \rangle \in V_M^{\pi(g)}(\varphi)(\pi(w)) &\Leftrightarrow \langle \rangle \in V_M^{\pi(g)}(\psi)(\alpha) \\
&\Leftrightarrow \langle \rangle \in \pi(V_M^g(\psi))(\pi(\alpha)) \text{ (I.H.)} \\
&\Leftrightarrow \langle \rangle \in \pi(V_M^g(\psi)(\alpha)) \\
&\Leftrightarrow \langle \rangle \in V_M^g(\psi)(\alpha) \\
&\Leftrightarrow \langle \rangle \in V_M^g(@\psi)(w) \\
&\Leftrightarrow \langle \rangle \in \pi(V_M^g(\varphi))(\pi(w))
\end{aligned}$$

6. If φ is of the form $\Box\psi$, then:

$$\begin{aligned}
\langle \rangle \in V_M^{\pi(g)}(\varphi)(\pi(w)) &\Leftrightarrow \forall w' \in W : \langle \rangle \in V_M^{\pi(g)}(\psi)(w') \\
&\Leftrightarrow \forall w' \in W : \langle \rangle \in \pi(V_M^g(\psi))(w') \text{ (I.H.)} \\
&\Leftrightarrow \forall w' \in W : \langle \rangle \in \pi(V_M^g(\psi)(w')) \\
&\Leftrightarrow \forall w' \in W : \langle \rangle \in V_M^g(\psi)(w') \\
&\Leftrightarrow \langle \rangle \in V_M^g(\Box\psi)(w) \\
&\Leftrightarrow \langle \rangle \in \pi(V_M^g(\varphi))(\pi(w))
\end{aligned}$$

7. If φ is of the form $\forall v\psi$, then:

$$\begin{aligned}
\langle \rangle \in V_M^{\pi(g)}(\varphi)(\pi(w)) &\Leftrightarrow \forall o \in d(\pi(w)) : \langle \rangle \in V_M^{\pi(g)[v/o]}(\psi)(\pi(w)) \\
&\Leftrightarrow \forall o \in d(w) : \langle \rangle \in V_M^{\pi(g)[v/\pi(o)]}(\psi)(\pi(w)) \\
&\Leftrightarrow \forall o \in d(w) : \langle \rangle \in V_M^{\pi(g[v/o])}(\psi)(\pi(w)) \\
&\Leftrightarrow \forall o \in d(w) : \langle \rangle \in \pi(V_M^{g[v/o]}(\psi))(\pi(w)) \text{ (I.H.)} \\
&\Leftrightarrow \forall o \in d(w) : \langle \rangle \in \pi(V_M^{g[v/o]}(\psi)(w)) \\
&\Leftrightarrow \forall o \in d(w) : \langle \rangle \in V_M^{g[v/o]}(\psi)(w) \\
&\Leftrightarrow \langle \rangle \in V_M^g(\forall v\psi)(w) \\
&\Leftrightarrow \langle \rangle \in \pi(V_M^g(\varphi))(\pi(w))
\end{aligned}$$

The case where φ is of the form $\forall V^n\psi$ proceeds similarly.

8. If φ is of the form $\lambda v_1 \dots v_n(\psi)$, then:

$$\begin{aligned}
\langle o_1, \dots, o_n \rangle \in V_M^{\pi(g)}(\varphi)(\pi(w)) &\Leftrightarrow \langle o_1, \dots, o_n \rangle \in (d(\pi(w)))^n \\
&\text{and } \langle \rangle \in V_M^{\pi(g)[v_1/o_1 \dots v_n/o_n]}(\psi)(\pi(w))
\end{aligned}$$

Let $\pi(o'_i) = o_i$, for each $1 \leq i \leq n$. We have that:

$$\begin{aligned}
\langle o_1, \dots, o_n \rangle \in V_M^{\pi(g)}(\varphi)(\pi(w)) &\Leftrightarrow \langle o'_1, \dots, o'_n \rangle \in (d(w))^n \\
&\text{and } \langle \rangle \in V_M^{\pi(g)[v_1/o'_1 \dots v_n/o'_n]}(\psi)(\pi(w)) \\
&\Leftrightarrow \langle o'_1, \dots, o'_n \rangle \in (d(w))^n \\
&\text{and } \langle \rangle \in \pi(V_M^{g[v_1/o'_1 \dots v_n/o'_n]}(\psi))(\pi(w)) \text{ (I.H.)} \\
&\Leftrightarrow \langle o'_1, \dots, o'_n \rangle \in (d(w))^n \\
&\text{and } \langle \rangle \in \pi(V_M^{g[v_1/o'_1 \dots v_n/o'_n]}(\psi)(w))
\end{aligned}$$

$$\begin{aligned}
&\Leftrightarrow \langle o'_1, \dots, o'_n \rangle \in (d(w))^n \\
&\text{and } \langle \rangle \in V_M^{g[v_1/o'_1 \dots v_n/o'_n]}(\psi)(w) \\
&\Leftrightarrow \langle o'_1, \dots, o'_n \rangle \in V_M^g(\lambda v_1 \dots v_n(\psi))(w) \\
&\Leftrightarrow \pi(\langle o'_1, \dots, o'_n \rangle) \in \pi(V_M^g(\lambda v_1 \dots v_n(\psi))(w)) \\
&\Leftrightarrow \langle \pi(o'_1), \dots, \pi(o'_n) \rangle \in \pi(V_M^g(\lambda v_1 \dots v_n(\psi))(\pi(w))) \\
&\Leftrightarrow \langle o_1, \dots, o_n \rangle \in \pi(V_M^g(\varphi))(\pi(w))
\end{aligned}$$

This concludes the proof. □

A.9.4 Completeness

In what follows completeness is proved for the more general cases discussed, namely, those involving the deductive systems $\mathbf{SG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C$ and $\mathbf{SR}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C$. I begin by stating several lemmas about the deductive system $\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C$. These lemmas will be used in the proofs to be given later on.

Lemma 17.

$$\begin{array}{ll}
\text{(i)} \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} \varphi \Rightarrow \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} \square\varphi \text{ } (\square\text{I}) & \text{(iii)} \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} \varphi_a^v \Rightarrow \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} \forall v\varphi \text{ } (\forall\text{I}) \\
\text{(ii)} \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} \varphi \Rightarrow \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} @\varphi \text{ } (@\text{I}) & \text{(iv)} \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} \varphi_{\tau^n}^{V^n} \Rightarrow \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} \forall V^n\varphi \text{ } (\forall\text{VI})
\end{array}$$

Lemma 18.

$$\begin{array}{ll}
\text{(i)} \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} (@\varphi \wedge @\psi) \rightarrow @(\varphi \wedge \psi) & \text{(iii)} \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} @(@\varphi \rightarrow \varphi) \\
\text{(ii)} \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} @@\varphi \leftrightarrow @\varphi
\end{array}$$

Lemma 19.

$$\begin{array}{ll}
\text{(i)} \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} \forall vEv & \text{(iii)} \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} \forall V^nEV^n \\
\text{(ii)} \Gamma \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} \varphi_t^v \Rightarrow \Gamma \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} \forall v\varphi^{54} & \text{(iv)} \Gamma \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} \varphi_{\tau^n}^{V^n} \Rightarrow \Gamma \vdash_{\mathbf{WG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C} \forall V^n\varphi^{55}
\end{array}$$

The proofs of lemmas 17, 18 and 19 are trivial, and thus omitted.

The deductive systems $\mathbf{SG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C$ and $\mathbf{SR}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C$ are now proved to be complete with respect to, respectively, \mathbf{SG}^C -validity and \mathbf{SR}^C -validity, for arguments composed of *closed* formulae. The proof of soundness is established by an induction on the length of a derivation, as usual. The proof of soundness is omitted: the present focus will be on establishing its converse.

Appealing to the usual Henkin method, it is shown that every $\mathbf{SG}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C$ -consistent set of formulas is \mathbf{SG}^C -satisfiable, and that every $\mathbf{SR}_{\mathcal{S}\mathcal{L}_{@}\square\lambda}^C$ -consistent set of formulas is \mathbf{SR}^C -satisfiable. Let $\star \in$

⁵⁴Where v does not occur in Γ

⁵⁵Where τ^n does not occur in Γ .

$\{\mathbf{SG}_{\mathcal{S}\mathcal{L}_{@□\lambda}}^C, \mathbf{SR}_{\mathcal{S}\mathcal{L}_{@□\lambda}}^C\}$, unless noted otherwise. Also, let $\overline{\star} = \mathbf{SG}^C$, if $\star = \mathbf{SG}_{\mathcal{S}\mathcal{L}_{@□\lambda}}^C$, and $\overline{\star} = \mathbf{SR}^C$, if $\star = \mathbf{SR}_{\mathcal{S}\mathcal{L}_{@□\lambda}}^C$. It will be shown how, given any \star -consistent set Γ of sentences of $\mathcal{S}\mathcal{L}_{@□\lambda}$, a sequence U of sets of sentences U_j of $\mathcal{S}\mathcal{L}_{@□\lambda}$ can be constructed containing information directly relevant for the construction of an \mathbf{M}^C -model M $\overline{\star}$ -satisfying Γ .

As expected, to each U_j in sequence U will correspond an element $w_j \in W$ in the model. Each set U_j will contain information determining the ‘individuals and n -ary relations that exist in w_j ’ as well as which formulas of L are true at w_j . Roughly, a formula belongs to U_j if and only if it is true at w_j . The set Γ is guaranteed to be $\overline{\star}$ -satisfied by some world of W owing to the fact that the sequence U is constructed in such a way that one of the sets of formulae in the sequence, the set U_k , is a superset of Γ . Thus, every formula in Γ is true in w_k , the element of W corresponding to U_k .

The presence of axioms with a distinctively ‘actualistic’ flavour (namely, the instances of the schemas (G=) and (SG=) of axiom system $\mathbf{SG}_{\mathcal{S}\mathcal{L}_{@□\lambda}}^C$, and the instances of the schemas (R=), (SR=) and (R@) of axiom system $\mathbf{SR}_{\mathcal{S}\mathcal{L}_{@□\lambda}}^C$) requires that an element of U be selected for containing information that is distinctively concerned with the actual world. In the construction to be provided, this element will consist in the set U_0 . Thus, the set U_0 will be a superset of the set containing every instance of (R=), (SR=), and (R@).

The distinguished world of the model M that will be extracted from the sequence U will satisfy, relative to M , every instance of (R=), (SR=) and (R@). The difference between \mathbf{SG}^C -satisfaction and \mathbf{SR}^C -satisfaction is reflected on the relationship between the sets U_k and U_0 . Consideration of the notion of \mathbf{SR}^C -satisfaction requires that $k = 0$, since the elements in Γ must all be true in the designated world of the model. On the other hand, \mathbf{SG}^C -satisfaction requires that $k \neq 0$, since some sets of formulas are \mathbf{SG}^C -satisfiable, even though they are not true in any designated world of any \mathbf{S}^C -model (an example is given by the set $\{@P, \neg P\}$).

The methods used in the proof are similar to those in (Hodes, 1984) and (Menzel, 1991), which are themselves similar to those used in (Gallin, 1975) and (Fine, 1980).⁵⁶ In particular the present proof will follow closely the proof in (Menzel, 1991, pp. 364-370). The reader will be directed to aspects of that proof at several stages.

I begin by laying out some useful definitions. Let L be any language, $\Gamma \subseteq \text{Form}(L)$ and \star be any deductive system.

Definition 39. Γ is \star -consistent if and only if there is no formula φ such that $\Gamma \vdash_{\star} \varphi$ and $\Gamma \vdash_{\star} \neg\varphi$.

Definition 40. Γ is maximal if and only if, for every formula φ of L , either $\varphi \in \Gamma$ or $\neg\varphi \in \Gamma$.

Definition 41. Γ is \exists_1 -complete if and only if, for every formula φ of L , $\exists v\varphi \in \Gamma$ if and only if Ea , $\varphi_a^v \in \Gamma$, for some individual constant a .

Definition 42. Γ is \exists_2 -complete if and only if, for every formula φ of L , $\exists V^n\varphi \in \Gamma$ if and only if $E\zeta^n$, $\varphi_{\zeta^n}^{V^n} \in \Gamma$, for some n -ary predicate letter ζ^n .

⁵⁶A different completeness proof, for a different axiomatisation of a system similar to $\mathbf{SR}_{\mathcal{F}\mathcal{L}_{@□}}$, can be found in (Stephanou, 2005).

Definition 43. An ω -sequence $S = \langle S_0, S_1, \dots \rangle$ of sets of formulas of language L is \star -consistent just in case the set $\{\diamond \wedge \Gamma_i : i < \omega \text{ and } \Gamma_i \text{ is a finite subset of } S_i\}$ is \star -consistent.

Let $S[i, \{\varphi_1, \dots, \varphi_n\}]$ be the result of replacing S_i in S with $S_i \cup \{\varphi_1, \dots, \varphi_n\}$. Where $n = 1$, $S[i, \varphi]$ is written instead of $S[i, \{\varphi\}]$.

Definition 44. Let $j < \omega$ and $\varphi \in Form(L)$. φ is \star -consistent _{j} with S if and only if $S[j, \varphi]$ is \star -consistent.

Definition 45. An ω -sequence S is \diamond -complete if and only if, for any $j < \omega$, $\diamond\varphi \in U_j$ iff $\exists k < \omega$ such that $\varphi \in U_k$.

Definition 46. An ω -sequence S is $@$ -complete if and only if, for any $j < \omega$, $@\varphi \in U_j$ iff $\varphi \in U_0$.

Let 'G' and 'R' abbreviate, respectively, ' $\mathbf{SG}^C_{\mathcal{SL}_{@ \square \lambda}}$ ' and ' $\mathbf{SR}^C_{\mathcal{SL}_{@ \square \lambda}}$ '. Let $\Gamma^\star \subseteq Form(\mathcal{SL}_{@ \square \lambda})$ be any \star -consistent set, $Const'$ be a countable set of new individual constants, $Pred^n$ be a countable set of new n -ary predicate letters, for every $n \in \mathbb{N}_0$, and L' be $\mathcal{SL}_{@ \square \lambda} + Const' + \bigcup_{n \in \mathbb{N}} (Pred^n)$. Let $\xi = \{\langle j, \varphi \rangle_i\}_{i < \omega}$ be an enumeration of all pairs $\langle j, \varphi \rangle$, where $j < \omega$ and $\varphi \in Form(L')$, ord_i be the first element in the i th pair in ξ , and φ_i be the second element in the i th pair in ξ . Also, let $ESet = \{Ea : a \in Const(L)\}$ and $@Set = \{@\varphi \rightarrow \varphi : \varphi \text{ is a closed formula } \in L'\}$. Now, for each $n \in \mathbb{N}$, let $SESet^n = \{E\zeta^n : \zeta^n \in Pred^n(L)\}$, and $SESet = \bigcup_{n \in \mathbb{N}} SESet^n$. For each $i < \omega$, two ω -sequences of sets $U_j^{G,i}$ and $U_j^{R,i}$ of formulae of L' are defined, where $j < \omega$. $U_0^{G,0} = ESet \cup SESet \cup @Set$, $U_1^{G,0} = \Gamma^G$, and $U_j^{G,0} = \emptyset$, for $j > 1$. $U_0^{R,0} = ESet \cup SESet \cup @Set \cup \Gamma^R$, and $U_j^{R,0} = \emptyset$, for $j > 0$. $U^{\star,i+1}$ is now defined. If φ_i is not $\mathbf{WG}^C_{\mathcal{SL}_{@ \square \lambda}}$ -consistent _{ord_i} with $U^{\star,i}$, then $U^{\star,i+1} = U^{\star,i}$ and if φ_i is $\mathbf{WG}^C_{\mathcal{SL}_{@ \square \lambda}}$ -consistent _{ord_i} with $U^{\star,i}$, then:

1. If φ_i is of the form $\exists v\psi$, then $U^{\star,i+1} = U^{\star,i}[ord_i, \{\varphi_i, \psi_a^v, Ea\}]$, where a is a variable from $Const'$ present at most in $@Set$;
2. If φ_i is of the form $\exists V^n\psi$, then $U^{\star,i+1} = U^{\star,i}[ord_i, \{\varphi_i, \psi_{\zeta^n}^{V^n}, E\zeta^n\}]$, where ζ^n is a n -ary predicate letter from $Pred^n$ present at most in $@Set$;
3. If φ is of the form $\diamond\psi$, then $U^{\star,i+1} = U^i[ord_i, \varphi_i][n, \psi]$, where n is the least ordinal > 0 such that $U_n^{\star,i} = \emptyset$.
4. If φ_i is of neither of the above forms, then $U^{\star,i+1} = U^{\star,i}[ord_i, \varphi_i]$.

For each $j < \omega$, let $U_j^\star = \bigcup_{i < \omega} U_j^{\star,i}$ and $U^\star = \langle U_0^\star, U_1^\star, \dots \rangle$. The following lemmas will be relevant later on:

Lemma 20. If $\theta = \{\diamond \wedge \Gamma_k : k < \omega\}$, where each Γ_k is a finite subset of $Form(L')$, then, if $\theta \cup \{\varphi\}$ is $\mathbf{WG}^C_{\mathcal{SL}_{@ \square \lambda}}$ -inconsistent, so is $\theta \cup \{\diamond\varphi\}$.

Proof. See (Menzel, 1991, p. 365). The proof relies on item (i) of Lemma 17 and on the axiom-schemata (PL), (K) and (5). \square

Lemma 21. For every $i < \omega$, $U^{\star,i}$ is $\mathbf{WG}^C_{\mathcal{SL}_{@ \square \lambda}}$ -consistent.

Proof. The proof is by induction on i . For the base case, consider first the case where $\star = G$. Suppose, for reductio, that $U^{G,0}$ is $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -inconsistent. This means that the set

$$\{\diamond \bigwedge \theta : \theta \text{ is a finite subset of } ESet \cup SESet \cup @Set\} \cup \{\diamond \bigwedge \theta : \theta \text{ is a finite subset of } \Gamma^G\}$$

is $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -inconsistent. From this it follows that there are finite sets $\theta_0 \subseteq U_0^{G,0}$ and $\theta_1 \subseteq U_1^{G,0}$ such that $\{\diamond \bigwedge \theta_0, \diamond \bigwedge \theta_1\}$ is $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -inconsistent. Let $Z = \{@\varphi : \varphi \in U_0^{G,0}\}$. We have that $Z \cup \Gamma^G$ is G -consistent, since all elements of Z are theorems of G .

To see this, take any $\varphi \in Z$. We have that $\varphi = @Ea$ for some individual constant a , or $\varphi = @E\zeta^n$ for an n -ary predicate letter ζ^n , or $\varphi = @(@\varphi \rightarrow \varphi)$. If $\varphi = @Ea$, then φ follows from (G=), (SA) and (@K). If $\varphi = @E\zeta^n$, then φ follows from (SG=), (SSA) and (@K). If $\varphi = @(@\varphi \rightarrow \varphi)$, then φ follows by item (iii) of Lemma 18. Since $Z \cup \Gamma^G$ is $\mathbf{SG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -consistent, it is also $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -consistent, since $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ is a subsystem of $\mathbf{SG}^C_{\mathcal{SL}_{\@ \square \lambda}}$.

Now, $Z \cup \Gamma^G \vdash_{\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}} @\varphi$, for every $\varphi \in \theta_0$. Thus, $Z \cup \Gamma^G \vdash_{\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}} \bigwedge \{@\varphi : \varphi \in \theta_0\}$. Furthermore, since $\vdash_{\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}} (@\psi \wedge @\chi) \rightarrow @(\psi \wedge \chi)$ for every φ and ψ (lemma 18), it follows that $Z \cup \Gamma^G \vdash_{\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}} @ \bigwedge \theta_0$. In addition, $\vdash_{\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}} @\varphi \rightarrow \diamond \varphi$, for every φ (by axiom ($\square @2$)). Thus, $Z \cup \Gamma^G \vdash_{\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}} \diamond \bigwedge \theta_0$. Note also that $Z \cup \Gamma^G \vdash_{\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}} \bigwedge \theta_1$, since $\theta_1 \subseteq \Gamma^G$. Furthermore, $\vdash_{\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}} \varphi \rightarrow \diamond \varphi$, for every φ (by (PL) and ($\square @2$)). Hence, $Z \cup \Gamma^G \vdash_{\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}} \diamond \bigwedge \theta_1$. But this means that $Z \cup \Gamma^G \vdash_{\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}} \diamond \bigwedge \theta_0 \wedge \diamond \bigwedge \theta_1$. Since $Z \cup \Gamma^G$ is $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -consistent, $\{\diamond \bigwedge \theta_0, \diamond \bigwedge \theta_1\}$ must be $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -consistent as well ζ .

Thus, $U^{G,0}$ is $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -consistent.

Consider now the case where $\star = R$. Suppose, for reductio, that $U^{R,0}$ is $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -inconsistent. It follows from this that there is a finite set $\theta \subseteq U_0^{R,0}$ such that $\{\diamond \bigwedge \theta\}$ is $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -inconsistent. Since Γ^R is $\mathbf{SR}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -consistent, $\Gamma^R \cup ESet \cup SESet \cup @Set$ is $\mathbf{SR}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -consistent, since every element of $ESet \cup SESet \cup @Set$ is an axiom of $\mathbf{SR}^C_{\mathcal{SL}_{\@ \square \lambda}}$. Thus, $\Gamma^R \cup ESet \cup SESet \cup @Set$ is also $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -consistent, since $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ is a subsystem of $\mathbf{SR}^C_{\mathcal{SL}_{\@ \square \lambda}}$. Now, $\Gamma^R \cup ESet \cup SESet \cup @Set \vdash_{\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}} \bigwedge \theta$, since $\theta \subseteq \Gamma^R \cup ESet \cup SESet \cup @Set$. Thus, $\Gamma^R \cup ESet \cup SESet \cup @Set \vdash_{\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}} \diamond \bigwedge \theta$. But this means that $\{\diamond \bigwedge \theta\}$ is $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -consistent ζ .

Hence, $U^{R,0}$ is $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -consistent.

The proof of the induction cases goes exactly as in (Menzel, 1991, pp. 366-367), except for the induction cases where φ is $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -consistent $_j$ with $U^{\star,i}$ and $\varphi = \exists v\psi$, and where φ is $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -consistent $_j$ with $U^{\star,i}$ and $\varphi = \exists V^n\psi$. Here only the first case is considered. The proof of the remaining case is exactly the same, except that it appeals to the second-order variants of the theorems appealed in the case where $\varphi = \exists v\psi$.

So, let $\varphi = \exists v\psi$ and assume φ is $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -consistent $_j$ with $U^{\star,i}$. Suppose, for reductio, that $U^{\star,i+1}$ is not $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -consistent. This means that there are finite $\Gamma_k \subseteq U_k^{\star,i+1}$ such that $\{\diamond \bigwedge \Gamma_k : k < \omega\}$ is not $\mathbf{WG}^C_{\mathcal{SL}_{\@ \square \lambda}}$ -consistent.

Consider first the case where $0 \neq j$. Let $\Delta = \{\diamond \bigwedge \Gamma_k : 0 < k < \omega, k \neq j\}$, $\Gamma'_j = \Gamma_j - \{\exists v\psi, \psi_a^v, Ea\}$, $\Gamma'_0 = \Gamma_0 - \{@\varphi \rightarrow \varphi : a \text{ occurs in } \varphi\}$. We have that the set $\Delta \cup \{\diamond \bigwedge \Gamma'_j \wedge$

$\exists v\psi \wedge \psi_a^v \wedge Ea\}) \cup \{\diamond \wedge \Gamma_0\}$ is $\mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C$ -inconsistent. So,

$$\begin{aligned} \Delta \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \neg(\diamond \wedge \Gamma_0 \wedge \diamond(\wedge \Gamma'_j \wedge \exists v\psi \wedge \psi_a^v \wedge Ea)). \\ \text{So } \Delta \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \diamond \wedge \Gamma_0 \rightarrow \neg \diamond(\wedge \Gamma'_j \wedge \exists v\psi \wedge \psi_a^v \wedge Ea). \\ \text{So } \Delta \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \diamond \wedge \Gamma_0 \rightarrow \square \neg(\wedge \Gamma'_j \wedge \exists v\psi \wedge \psi_a^v \wedge Ea). \\ \text{So } \Delta \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \diamond \wedge \Gamma_0 \rightarrow \neg(\wedge \Gamma'_j \wedge \exists v\psi \wedge \psi_a^v \wedge Ea). \\ \text{So } \Delta \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \diamond \wedge \Gamma_0 \rightarrow ((\wedge \Gamma'_j \wedge \exists v\psi) \rightarrow (Ea \rightarrow \neg \psi_a^v)). \end{aligned}$$

Let $\Gamma''_0 = \Gamma'_0 \cup \{\@ \neg \wedge \Gamma'_0 \rightarrow \neg \wedge \Gamma'_0\}$. We have that:

$$\begin{aligned} \wedge \Gamma''_0 \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \@ \neg \wedge \Gamma'_0 \rightarrow \neg \wedge \Gamma'_0. \\ \text{So } \wedge \Gamma''_0 \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \wedge \Gamma'_0 \rightarrow \neg \@ \neg \wedge \Gamma'_0. \\ \text{So } \wedge \Gamma''_0 \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \wedge \Gamma'_0 \rightarrow \@ \wedge \Gamma'_0. \\ \text{So } \wedge \Gamma''_0 \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \wedge \Gamma'_0 \wedge \Gamma''_0 \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C \@ \wedge \Gamma'_0. \end{aligned}$$

We have that $\vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C \@(\@ \varphi \rightarrow \varphi)$, by Lemma 18. Hence, for every χ in which a occurs, $\wedge \Gamma''_0 \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C \@(\@ \chi \rightarrow \chi)$. Now, for any set θ , let $\@ \theta = \{\@ \varphi : \varphi \in \theta\}$. Thus, for every finite subset θ of $\{\@ \varphi \rightarrow \varphi : a \text{ occurs in } \varphi\}$:

$$\begin{aligned} \wedge \Gamma''_0 \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \wedge \@ \theta. \\ \text{So } \wedge \Gamma''_0 \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \@ \wedge \theta \text{ (Lemma 18)}. \\ \text{So } \wedge \Gamma''_0 \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \@(\wedge \Gamma'_0 \wedge \theta) \text{ (Lemma 18)}. \\ \text{So } \wedge \Gamma''_0 \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \diamond(\wedge \Gamma'_0 \wedge \theta) \text{ (axiom } (\square \@ 2)). \\ \text{So } \diamond \wedge \Gamma''_0 \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \diamond(\wedge \Gamma'_0 \wedge \theta) \text{ (axioms (PL), (K) and Lemma 17)}. \end{aligned}$$

Hence, $\diamond \wedge \Gamma''_0 \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C \diamond \wedge \Gamma_0$. Therefore:

$$\begin{aligned} \Delta, \diamond \wedge \Gamma''_0 \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & (\wedge \Gamma'_j \wedge \exists v\psi) \rightarrow (Ea \rightarrow \neg \psi_a^v). \\ \text{So } \Delta, \diamond \wedge \Gamma''_0, \wedge \Gamma'_j \wedge \exists v\psi \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & Ea \rightarrow \neg \psi_a^v. \\ \text{So } \Delta, \diamond \wedge \Gamma''_0, \wedge \Gamma'_j \wedge \exists v\psi \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \forall v(Ev \rightarrow \neg \psi) \text{ (Lemma 19 (ii) - } a \text{ occurs in no} \\ \text{premise)}. \end{aligned}$$

$$\begin{aligned} \text{So } \Delta, \diamond \wedge \Gamma''_0, \wedge \Gamma'_j \wedge \exists v\psi \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \forall v Ev \rightarrow \forall v \neg \psi \text{ (axiom } (\forall 1)). \\ \text{So } \Delta, \diamond \wedge \Gamma''_0, \wedge \Gamma'_j \wedge \exists v\psi \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \forall v \neg \psi. \\ \text{So } \Delta, \diamond \wedge \Gamma''_0, \wedge \Gamma'_j \wedge \exists v\psi \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \neg \exists v\psi, \\ \text{And } \Delta, \diamond \wedge \Gamma''_0, \wedge \Gamma'_j \wedge \exists v\psi \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C & \exists v\psi. \end{aligned}$$

Thus, $\Delta \cup \{\diamond \wedge \Gamma''_0\} \cup \{\wedge \Gamma'_j \wedge \exists v\psi\}$ is $\mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C$ -inconsistent. By Lemma 20, $\Delta \cup \{\diamond \wedge \Gamma''_0\} \cup \{\diamond(\wedge \Gamma'_j \wedge \exists v\psi)\}$ is $\mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C$ -inconsistent. But this contradicts the $\mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C$ -consistency _{j} of $\exists v\psi$ with $U^{\star, i}$ (note that $\Gamma''_0 \subseteq U^{\star, i}$) ∇ .

Consider now the case where $j = 0$. Let $\Gamma_0^a = \{\@ \varphi \rightarrow \varphi : \@ \varphi \rightarrow \varphi \in \Gamma_0 \text{ and } a \text{ occurs in } \varphi\}$, and $\Gamma'_0 = \Gamma_0 - \{\exists v\psi, \psi_a^v, Ea\} \cup \Gamma_0^a$. Then,

$$\Delta \vdash \mathbf{WG}_{S\mathcal{L}@ \square \lambda}^C \neg \diamond(\wedge \Gamma'_0 \wedge \wedge \Gamma_0^a \wedge \exists v\psi \wedge \psi_a^v \wedge Ea).$$

$$\begin{aligned}
& \text{So } \Delta \vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} \square \neg (\bigwedge \Gamma'_0 \wedge \bigwedge \Gamma_0^a \wedge \exists v\psi \wedge \psi_a^v \wedge Ea). \\
& \text{So } \Delta \vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} \neg (\bigwedge \Gamma'_0 \wedge \bigwedge \Gamma_0^a \wedge \exists v\psi \wedge \psi_a^v \wedge Ea). \\
& \text{So } \Delta \vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} (\bigwedge \Gamma'_0 \wedge \bigwedge \Gamma_0^a \wedge \exists v\psi) \rightarrow (Ea \rightarrow \neg \psi_a^v).
\end{aligned}$$

Where $@\Delta = \{@\delta : \delta \in \Delta\}$, we also have that

$$@\Delta \vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} @\bigwedge \Gamma'_0 \rightarrow (@\bigwedge \Gamma_0^a \rightarrow @(\exists v\psi \rightarrow (@Ea \rightarrow @\neg \psi_a^v))).$$

Furthermore, for every element $\gamma \in \Gamma_0^a$, $\vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} @\gamma$. Thus, $\vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} @\bigwedge \Gamma_0^a$. This means that

$$@\Delta, @\bigwedge \Gamma'_0, @\exists v\psi \vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} (@Ea \rightarrow @\neg \psi_a^v).$$

But then,

$$@\Delta, @\bigwedge \Gamma'_0, @\exists v\psi \vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} @\forall v(@Ev \rightarrow @\neg \psi_a^v).$$

Therefore,

$$@\Delta, @\bigwedge \Gamma'_0, @\exists v\psi \vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} @\forall v @Ev \rightarrow @\forall v @\neg \psi.$$

Since $\vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} @\forall v @Ev$, we get the result that:

$$\begin{aligned}
& @\Delta, @\bigwedge \Gamma'_0, @\exists v\psi \vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} @\forall v @\neg \psi. \\
& \text{So } @\Delta, @\bigwedge \Gamma'_0, @\exists v\psi \vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} @\forall v \neg \psi. \\
& \text{So } @\Delta, @\bigwedge \Gamma'_0, @\exists v\psi \vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} \neg @\exists v\psi. \\
& \text{And } @\Delta, @\bigwedge \Gamma'_0, @\exists v\psi \vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} @\exists v\psi.
\end{aligned}$$

Let $\Gamma''_0 = \{\bigwedge \Gamma'_0, \bigwedge \Gamma'_0 \rightarrow @\bigwedge \Gamma'_0\} \cup \{\exists v\psi \rightarrow @\exists v\psi\}$. Note that:

$$\Delta \vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} @\Delta.$$

Moreover,

$$\begin{aligned}
& \Gamma''_0 \vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} @\bigwedge \Gamma'_0. \\
& \text{and so } \Gamma''_0, \exists v\psi \vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} @\bigwedge \Gamma'_0
\end{aligned}$$

Then,

$$\begin{aligned}
& \Delta, \Gamma''_0, \exists v\psi \vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} @\exists v\psi, \\
& \text{and } \Delta, \Gamma''_0, \exists v\psi \vdash_{\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C} \neg @\exists v\psi
\end{aligned}$$

Thus, $\Delta, \Gamma''_0, \exists v\psi$ is $\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C$ -inconsistent. By lemma 6, $\Delta, \diamond(\Gamma''_0 \wedge \exists v\psi)$ is $\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C$ -inconsistent. This contradicts the $\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C$ -inconsistent $_j$ of $\exists v\psi$ with $U^{\star,i}$ (note that $\Gamma''_0 \subseteq U_0^{\star,i}$). ζ

Hence, $U^{\star,i}[\text{ord}_i, \{\varphi_i, \psi_v^v, Ev'\}]$ is $\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C$ -consistent. This concludes the proof. \square

Lemma 22. U^{\star} is $\mathbf{WG}_{S\mathcal{L}@\square\lambda}^C$ -consistent.

Proof. Follows from the fact that proofs are finite and Lemma 21. \square

Lemma 23. If φ is $\mathbf{WG}_{S\mathcal{L}@□\lambda}^C$ -consistent _{j} with U , then $\varphi \in U_j$.

Proof. See (Menzel, 1991, p. 367). □

Lemma 24. For every $j < \omega$, U_j^\star is maximal.

Proof. See (Menzel, 1991, p. 368). □

Lemma 25. For every $j < \omega$, U_j^\star is \exists_1 -complete and \exists_2 -complete.

Proof. I here focus only on the case of \exists_2 -completeness. The left-to-right direction (if $\exists V^n \varphi \in U_j^\star$, then $E\zeta^n, \varphi_{\zeta^n}^{V^n} \in U_j^\star$) follows directly from the definition of U_j^\star . For the right-to-left direction, suppose that $E\zeta^n, \varphi_{\zeta^n}^{V^n} \in U_j^\star$. Suppose also, for reductio, that $\exists V^n \varphi \notin U_j^\star$. By Lemma 24, it follows that $\neg \exists V^n \varphi \in U_j^\star$. Consider the set $\theta = \{\neg \exists V^n \varphi, E\zeta^n, \varphi_{\zeta^n}^{V^n}\}$. Since U_j^\star is $\mathbf{WG}_{S\mathcal{L}@□\lambda}^C$ -consistent, $\{\diamond \wedge \theta\}$ is $\mathbf{WG}_{S\mathcal{L}@□\lambda}^C$ -consistent, the reason being that $\theta \subseteq U_j^\star$. But we have that:

$$\begin{aligned} & \vdash_{\mathbf{WG}_{S\mathcal{L}@□\lambda}^C} \neg \exists V^n \varphi \rightarrow \forall V^n \neg \varphi, \text{ and thus} \\ & \wedge \theta \vdash_{\mathbf{WG}_{S\mathcal{L}@□\lambda}^C} \forall V^n \neg \varphi. \text{ Furthermore,} \\ & \wedge \theta \vdash_{\mathbf{WG}_{S\mathcal{L}@□\lambda}^C} E\zeta^n. \\ & \text{So } \wedge \theta \vdash_{\mathbf{WG}_{S\mathcal{L}@□\lambda}^C} \neg \varphi_{\zeta^n}^{V^n}. \\ & \text{So } \wedge \theta \vdash_{\mathbf{WG}_{S\mathcal{L}@□\lambda}^C} \varphi_{\zeta^n}^{V^n}. \\ & \text{So } \diamond \wedge \theta \vdash_{\mathbf{WG}_{S\mathcal{L}@□\lambda}^C} \diamond (\varphi_{\zeta^n}^{V^n} \wedge \neg \varphi_{\zeta^n}^{V^n}) \text{ (axiom (K)).} \end{aligned}$$

But, $\vdash_{\mathbf{WG}_{S\mathcal{L}@□\lambda}^C} \neg \diamond (\varphi_{\zeta^n}^{V^n} \wedge \neg \varphi_{\zeta^n}^{V^n})$. This means that $\{\diamond \wedge \theta\}$ is $\mathbf{WG}_{S\mathcal{L}@□\lambda}^C$ -inconsistent \nexists .

Hence, $\exists V^n \varphi \in U_j^\star$. □

Lemma 26. U^\star is \diamond -complete.

Proof. See (Menzel, 1991, p. 368). □

Lemma 27. U^\star is $@$ -complete.

Proof. Suppose $\varphi \in U_0^\star$, and, for reductio, that there is $j < \omega$, $@\varphi \notin U_j^\star$. Consider the sets $\Gamma_0 = \{\varphi, @\neg\varphi \rightarrow \neg\varphi\}$ and $\Gamma_j = \{\neg @\varphi\}$. Clearly, $\Gamma_0 \subseteq U_0^\star$. Furthermore, $\Gamma_j \subseteq U_j^\star$, since U_j^\star is maximal (by Lemma 24), and thus either $@\varphi \in U_j^\star$ or $\neg @\varphi \in U_j^\star$. But:

$$\begin{aligned} & \Gamma_0 \vdash_{\mathbf{WG}_{S\mathcal{L}@□\lambda}^C} @\neg\varphi \rightarrow \neg\varphi. \\ & \text{So } \Gamma_0 \vdash_{\mathbf{WG}_{S\mathcal{L}@□\lambda}^C} \varphi \rightarrow @\varphi \text{ (axiom (@}\neg\text{))}. \\ & \Gamma_0 \vdash_{\mathbf{WG}_{S\mathcal{L}@□\lambda}^C} \varphi. \\ & \text{So } \Gamma_0 \vdash_{\mathbf{WG}_{S\mathcal{L}@□\lambda}^C} @\varphi. \\ & \text{So } \Gamma_0 \vdash_{\mathbf{WG}_{S\mathcal{L}@□\lambda}^C} \square @\varphi \text{ (axiom (@}\square\text{1))}. \\ & \text{So } \diamond \wedge \Gamma_0 \vdash_{\mathbf{WG}_{S\mathcal{L}@□\lambda}^C} \diamond \square @\varphi \text{ (axiom (K))}. \\ & \text{So } \diamond \wedge \Gamma_0 \vdash_{\mathbf{WG}_{S\mathcal{L}@□\lambda}^C} \square @\varphi \text{ (Axiom (5))}. \end{aligned}$$

Furthermore,

$$\begin{aligned}
& \Gamma_j \vdash \mathbf{WG}_{S\mathcal{L}_{@ \square \lambda}}^C \neg @\varphi. \\
& \text{So } \Gamma_j \vdash \mathbf{WG}_{S\mathcal{L}_{@ \square \lambda}}^C \neg \square @\varphi \text{ (axiom (T))}. \\
& \text{So } \diamond \wedge \Gamma_j \vdash \mathbf{WG}_{S\mathcal{L}_{@ \square \lambda}}^C \diamond \neg \square @\varphi \text{ (axiom (K))}. \\
& \text{So } \diamond \wedge \Gamma_j \vdash \mathbf{WG}_{S\mathcal{L}_{@ \square \lambda}}^C \neg \square @\varphi \text{ (Axioms (K), (T) and (5))}.
\end{aligned}$$

But, this contradicts the $\mathbf{WG}_{S\mathcal{L}_{@ \square \lambda}}^C$ -consistency of U , contrary to lemma 22 ζ .

Thus, $\varphi \in U_0^\star \Rightarrow @\varphi \in U_j^\star$, for every $j < \omega$.

For the left-to-right direction, suppose that there is $j < \omega$ such that $@\varphi \in U_j^\star$, even though $\varphi \notin U_0^\star$. Let $\Gamma_0 = \{@\varphi \rightarrow \varphi, \neg\varphi\}$ and $\Gamma_j = \{@\varphi\}$. Then:

$$\begin{aligned}
& \Gamma_0 \vdash \mathbf{WG}_{S\mathcal{L}_{@ \square \lambda}}^C \neg @\varphi. \\
& \text{So } \diamond \wedge \Gamma_0 \vdash \mathbf{WG}_{S\mathcal{L}_{@ \square \lambda}}^C \diamond \neg @\varphi \text{ (axiom (K))}. \\
& \text{So } \diamond \wedge \Gamma_0 \vdash \mathbf{WG}_{S\mathcal{L}_{@ \square \lambda}}^C \neg \square @\varphi. \\
& \text{So } \Gamma_j \vdash \mathbf{WG}_{S\mathcal{L}_{@ \square \lambda}}^C @\varphi. \\
& \text{So } \Gamma_j \vdash \mathbf{WG}_{S\mathcal{L}_{@ \square \lambda}}^C \square @\varphi \text{ (axiom } (\square @1)). \\
& \text{So } \diamond \wedge \Gamma_j \vdash \mathbf{WG}_{S\mathcal{L}_{@ \square \lambda}}^C \diamond \square @\varphi \text{ (axiom (K))}. \\
& \text{So } \diamond \wedge \Gamma_j \vdash \mathbf{WG}_{S\mathcal{L}_{@ \square \lambda}}^C \square @\varphi \text{ (axiom (5))}. \\
& \text{So } \diamond \wedge \Gamma_0, \diamond \wedge \Gamma_j \vdash \mathbf{WG}_{S\mathcal{L}_{@ \square \lambda}}^C \square @\varphi \wedge \neg \square @\varphi.
\end{aligned}$$

But this contradicts the $\mathbf{WG}_{S\mathcal{L}_{@ \square \lambda}}^C$ -consistency of U^\star , since Γ_0 is a finite subset of U_0^\star (again, $\neg\varphi \in U_0^\star$ by lemma 24) and Γ_j is a finite subset of U_0^\star . Thus, $@\varphi \in U_j^\star \Rightarrow \varphi \in U_j^\star$. \square

For each individual constant a of L' , let $[a]^\star = \{a' : \text{for some } j < \omega, a = a' \in U_j^\star\}$. Also, for each n -ary predicate letter ζ^n of L' , let $[\zeta^n]^\star$ be a function with domain $\{U_j^\star : j < \omega\}$ and such that, for each $j < \omega$, $[\zeta^n]^\star(U_j^\star) = \{[a_1]_1, \dots, [a_n]_n : \zeta^n a_1 \dots a_n \in U_j^\star\}$. The canonical W^C -model for $\mathbf{WG}_{S\mathcal{L}_{@ \square \lambda}}^C$, $M^\star = \langle W^\star, d^\star, D^\star, \alpha^\star, V^\star \rangle$, for Γ^\star is now defined.

Let $W^\star = \{U_j^\star : j < \omega\}$, $d^\star(U_j^\star) = \{[a]^\star : Ea \in U_j^\star\}$, $D^\star(n)(U_j^\star) = \{[\zeta^n] : E\zeta^n \in U_j^\star\}$, $\alpha^\star = U_0^\star$. For every individual constant a of L' , $V^\star(a) = [a]^\star$, and for every n -ary predicate ζ^n of L' , $V^\star(\zeta^n) = [\zeta^n]^\star$. The function D^\star is that function mapping each $n \in \mathbb{N}_0$ and $U_j \in W^\star$ to $D^\star(n)(U_j^\star) = \{[\zeta^n]^\star : E\zeta^n \in U_j^\star\}$.

Lemma 28. M^\star is a W^C -model for L' . Furthermore, for every closed formula φ of L' , $g \in As(M)$:

$$V_{M^\star, U_j^\star}^{\star, g}(\varphi) = \{\langle \rangle\} \Leftrightarrow \varphi \in U_j^\star.$$

Proof. Clearly, $\alpha^\star \in W^\star$. Consider now the valuation function V^\star . By axiom ($\diamond E$), for every individual constant a of L' there is a $j < \omega$ such that $Ea \in U_j^\star$. Thus,

$$V^\star(a) = [a]^\star \in \{[a']^\star : Ea' \in U_j^\star\} \subseteq \bigcup_{j < \omega} \{[t]^\star : Et \in U_j^\star\} = \bigcup_{w \in W^\star} d^\star(w).$$

Similarly for any n -ary predicate letter ζ^n of L' , $V^\star(\zeta^n) \in D^\star(n)(U_j^\star)$, for some $j < \omega$, by axiom (S \diamond E). The fact that $D^\star(n)(U_j^\star) \subseteq F(n)$, for each $j < \omega$ and $n \in \mathbb{N}_0$, follows from axiom (SA).

Consider the following function σ , whose domain consists in the set of pairs $\langle U_j, \varphi \rangle$ such that $U_j \in W^\star$ and φ is a closed term or a closed formula of L' :

1. If $\varphi \in \text{Const}(L')$, then $\sigma_{U_j^\star}(\varphi) = [\varphi]$
2. If φ is a closed n -ary (simple or complex) predicate of L' , then $\sigma_{U_j^\star}(\varphi) = \{ \langle [a_1]_1, \dots, [a_n]_n \rangle : \varphi a_1 \dots a_n \in U_j^\star \}$
3. If φ is a closed formula of L' , then $\sigma_{U_j^\star}(\varphi) = \{ \langle \rangle \}$ if and only if $\varphi \in U_j^\star$.

For each variable $v \in \text{Var}(L')$, each $g \in \text{As}(M^\star)$ is such that $g(v) = [a]^\star$, for some $a \in \text{Const}(L')$. Similarly, $g(V^n) = [\zeta^n]^\star$, for some n -ary predicate letter $\zeta^n \in \text{Pred}^n(L')$. So, for each formula or term φ of L' having as free individual variables exactly the variables v_1, \dots, v_n and as free n -ary second-order variables exactly the variables $V_1^n, \dots, V_{m_n}^n$, where $m_n \in \mathbb{N}_0$, let $(\varphi)^g$ be the closed formula or term that results from substituting each v_i for a chosen individual constant a for which $g(v_i) = [a]^\star$, and each V_i^n for a chosen n -ary predicate letter ζ^n for which $g(V_i^n) = [\zeta^n]^\star$. Call the sequence of chosen individual constants and n -ary predicate letters a *representing sequence* for φ and g .⁵⁷

This is well defined, in the sense that the value of $\sigma_{U_j^\star}((\varphi)^g)$ does not depend on the chosen representing sequence. That is, let $\varphi[\alpha]$ be the result of uniformly replacing the variables in φ by the constants and predicate letters in the representing sequence α for φ and g . Let β be any other representing sequence for φ and g . We have that $\sigma_w(\varphi[\alpha]) = \sigma_w(\varphi[\beta])$ for every closed formula or closed term φ . The proof of this fact appeals to (Ind) and (SInd).

It is now shown that, for every $w \in W^\star$, variable-assignment $g \in \text{As}(M^\star)$ and every term or formula φ of L' :

$$V_{M^\star, U_j^\star}^g(\varphi) = \sigma_{U_j^\star}((\varphi)^g),$$

where $V_{M^\star, U_j^\star}^g(\varphi)$ consists in the value of φ in M relative to U_j^\star and g (as defined in Definition 5).

It is clear that $V_{M^\star, U_j^\star}^g(\varphi) = \sigma_{U_j^\star}((\varphi)^g)$ when φ is an individual constant, an n -ary predicate letter, an individual variable, an n -ary predicate variable or of the form $\tau^n t_1 \dots t_n$. It is proven that $V_{M^\star, U_j^\star}^g(\varphi) = \sigma_{U_j^\star}((\varphi)^g)$ when φ is of the forms $@\psi$, $\forall v\psi$ and $\lambda v_1 \dots v_n(\psi)$. The proofs of the remaining cases are analogous.

⁵⁷The expression 'representing sequence' is taken from (Gallin, 1975, p. 35).

• $\varphi = @\psi$.

$$\begin{aligned}
V_{M^\star, U_j^\star}^g(\varphi) = \{\langle \rangle\} &\Leftrightarrow V_{M^\star, U_j^\star}^g(@\psi) = \{\langle \rangle\} \\
&\Leftrightarrow V_{M^\star, U_0^\star}^g(\psi) = \{\langle \rangle\} \\
&\Leftrightarrow \sigma_{U_0^\star}((\psi)^g) = \{\langle \rangle\} \text{ (I.H.)} \\
&\Leftrightarrow (\psi)^g \in U_0^\star \\
&\Leftrightarrow @(\psi)^g \in U_j^\star \text{ (Lemma 27)} \\
&\Leftrightarrow (@\psi)^g \in U_j^\star \\
&\Leftrightarrow \sigma_{U_j^\star}((\psi)^g) = \{\langle \rangle\}
\end{aligned}$$

• $\varphi = \forall v\psi$.

$$\begin{aligned}
V_{M^\star, U_j^\star}^g(\varphi) = \{\langle \rangle\} &\Leftrightarrow V_{M^\star, U_j^\star}^g(\forall v\psi) = \{\langle \rangle\} \\
&\Leftrightarrow \forall o \in d(U_j^\star) : V_{M^\star, U_j^\star}^{g[v/o]}(\psi) = \{\langle \rangle\} \\
&\Leftrightarrow \forall o \in d(U_j^\star) : \sigma_{U_j^\star}((\psi)^{g[v/o]}) = \{\langle \rangle\} \text{ (I.H.)} \\
&\Leftrightarrow \forall o \in d(U_j^\star) : (\psi)^{g[v/o]} \in U_j^\star \\
&\Leftrightarrow \forall a \in \text{Const}(L') \text{ s.t. } Ea \in U_j^\star : (\psi)^{g[v/[a]^\star]} \in U_j^\star \\
&\Leftrightarrow (\forall v\psi)^g \in U_j^\star \text{ (Lemma 25)} \\
&\Leftrightarrow \sigma_{U_j^\star}((\forall v\psi)^g) = \{\langle \rangle\}
\end{aligned}$$

• $\varphi = \lambda v_1, \dots, v_n(\psi)$.

$$\begin{aligned}
V_{M^\star, U_j^\star}^g(\varphi) &= V_{M^\star, U_j^\star}^g(\lambda v_1, \dots, v_n(\psi)) \\
&= \{\langle o_1, \dots, o_n \rangle \in (d(U_j^\star))^n : V_{M^\star, U_j^\star}^{g[v_1/o_1 \dots v_n/o_n]}(\psi) = \{\langle \rangle\}\} \\
&= \{\langle o_1, \dots, o_n \rangle \in (d(U_j^\star))^n : \sigma_{U_j^\star}(\psi)^{g[v_1/o_1 \dots v_n/o_n]} = \{\langle \rangle\}\} \text{ (I.H.)} \\
&= \{\langle o_1, \dots, o_n \rangle \in (d(U_j^\star))^n : (\psi)^{g[v_1/o_1 \dots v_n/o_n]} \in U_j^\star\} \\
&= \{\langle [a_1]_1^\star, \dots, [a_n]_n^\star \rangle : Ea_1 \in U_j^\star \ \& \ \dots \ \& \ Ea_n \in U_j^\star \ \& \\
&\quad (\psi)^{g[v_1/[a_1]^\star \dots v_n/[a_n]^\star]} \in U_j^\star\} \\
&= \{\langle [a_1]_1^\star, \dots, [a_n]_n^\star \rangle : \lambda v_1 \dots v_n(\psi)^g a_1 \dots a_n \in U_j^\star\} \text{ (axiom (EAb))} \\
&= \sigma_{U_j^\star}((\lambda v_1 \dots v_n(\psi))^g)
\end{aligned}$$

This establishes that $V_{M^\star, U_j^\star}^{g^\star}(\varphi) = \sigma_{U_j^\star}((\varphi)^g)$. Since $(\varphi)^g = \varphi$ when φ is closed, we have that, for every closed formula φ of L' , $g \in \text{As}(M)$: $V_{M^\star, U_j^\star}^{g^\star}(\varphi) = \{\langle \rangle\} \Leftrightarrow \varphi \in U_j^\star$.

In order to show that M^\star is a WC -model it remains to prove that condition 13. stated on page 202 is satisfied. The proof is as follows:

Suppose $\varphi = \lambda v_1 \dots v_n(\psi)$, χ_1, \dots, χ_n are all the parameters free in φ , and that, for every i such that $1 \leq i \leq n$ and arbitrary $j < \omega$, $V_{M^\star}^g(\chi_i) \in d^\star(U_j^\star)$, if χ_i is an individual constant or variable, and that $V_{M^\star}^g(\chi_i) \in D^\star(m)(U_j^\star)$, if χ_i is an m -ary predicate letter or m -ary second-order variable. To prove: $V_{M^\star}^g(\varphi) \in D^\star(n)(U_j^\star)$. We have that $V_{M^\star}^g(\chi_i) = \sigma_{U_k^\star}((\chi_i)^g)$, for any $k < \omega$, and thus that $E(\chi_i)^g \in U_j^\star$ if χ_i is an individual constant or variable, and that $E(\chi_i)^g \in U_j^\star$ if χ_i is a m -ary predicate letter or predicate variable. Thus, by (CComp), $E(\lambda v_1 \dots v_n(\psi))^g \in U_j^\star$. Furthermore, by \exists_2 -completeness, we have that there is a n -ary predicate letter ζ^n such that $E\zeta^n \in U_j^\star$ and $\Box \forall v'_1 \dots v'_n (\zeta^n v'_1 \dots v'_n \leftrightarrow (\lambda v_1 \dots v_n(\psi))^g v'_1 \dots v'_n) \in U_j^\star$. But this implies that $\sigma_{U_j^\star}(\zeta^n) = \sigma_{U_j^\star}((\lambda v_1 \dots v_n(\psi))^g)$, and that $\sigma_{U_j^\star}(\zeta^n) \in D^\star(n)(U_j^\star)$. But $\sigma_{U_j^\star}((\lambda v_1 \dots v_n(\psi))^g) = V_{M^\star}^g(\lambda v_1 \dots v_n(\psi))$, as previously shown. Thus, $V_{M^\star}^g(\varphi) \in D^\star(n)(U_j^\star)$. The case where φ has no parameters is similar, and is thus omitted.

Thus, M^\star is a W^C -model. This concludes the proof. \square

Now, let $\mathcal{M}^\star = \langle W^\star, d^\star, D^\star, \alpha^\star, \mathcal{V}^\star \rangle$, where \mathcal{V}^\star is the result of restricting the valuation function V^\star of M^\star to the individual constants and n -ary predicate letters of $\mathcal{SL}_{\@ \square \lambda}$.

Lemma 29. \mathcal{M}^\star is an S^C -model for $\mathcal{SL}_{\@ \square \lambda}$.

Proof. Since for every individual constant a of $\mathcal{SL}_{\@ \square \lambda}$, $Ea \in U_0^\star$, we have that $\mathcal{V}^\star(a) \in d(\alpha^\star)$. Similarly, $\mathcal{V}^\star(\zeta^n) \in D(n)(\alpha^\star)$, for every n -ary predicate letter $\zeta^n \in \text{Pred}^n(\mathcal{SL}_{\@ \square \lambda})$. Furthermore, the result of restricting the function $V_{M^\star, U_j^\star}^{\star, g}$ to language $\mathcal{SL}_{\@ \square \lambda}$ is a valuation function that assigns to each formula or term φ of $\mathcal{SL}_{\@ \square \lambda}$ a value $V_{M^\star, U_j^\star}^{\star, g}(\varphi)$ in such a way as to satisfy conditions 1. to 13. This concludes the proof of the lemma. \square

Lemma 30.

There is a variable assignment g such that, for every $\varphi \in \Gamma^G$, $\mathcal{V}_{\mathcal{M}^G, U_1^G}^{G, g}(\varphi) = \{\langle \rangle\}$.

There is a variable-assignment g such that, for every $\varphi \in \Gamma^R$, $\mathcal{V}_{\mathcal{M}^R, U_0^R}^{R, g}(\varphi) = \{\langle \rangle\}$.

Proof. This is a straightforward consequence of Lemma 28, since every element of Γ^G belongs to U_1^G , by construction of U^G , and every element of Γ^R belongs to U_0^R , by construction of U^R . \square

Since Γ^\star is an arbitrary \star -consistent set of closed formulae of $\mathcal{SL}_{\@ \square \lambda}$, it follows that:

Lemma 31. Every \star -consistent set of closed formulae Γ of $\mathcal{SL}_{\@ \square \lambda}$ is $\overline{\star}$ -satisfiable.

The completeness of the systems $\mathbf{SG}_{\mathcal{SL}_{\@ \square \lambda}}^C$ and $\mathbf{SR}_{\mathcal{SL}_{\@ \square \lambda}}^C$ follows from the previous lemma:

Theorem 5 (Completeness of $\mathbf{SG}_{\mathcal{SL}_{\@ \square \lambda}}^C$ and $\mathbf{SR}_{\mathcal{SL}_{\@ \square \lambda}}^C$). For every $\Gamma \subseteq \text{Form}(\mathcal{SL}_{\@ \square \lambda})$ such that every $\gamma \in \Gamma$ is a closed formula, for every closed formula $\varphi \in \text{Form}(\mathcal{SL}_{\@ \square \lambda})$: $\Gamma \models_\star \varphi \Rightarrow \Gamma \vdash_\star \varphi$.

The proofs of the completeness of the other strongly Millian logics presented here are similar to the proof just given, and simpler.

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