Signatures of the Primordial Universe from Its Emptiness: Measurement of Baryon Acoustic Oscillations from Minima of the Density Field

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Sound waves from the primordial fluctuations of the Universe imprinted in the large-scale structure, called baryon acoustic oscillations (BAOs), can be used as standard rulers to measure the scale of the Universe. These oscillations have already been detected in the distribution of galaxies. Here we propose to measure BAOs from the troughs (minima) of the density field. Based on two sets of accurate mock halo catalogues with and without BAOs in the seed initial conditions, we demonstrate that the BAO signal cannot be obtained from the clustering of classical disjoint voids, but it is clearly detected from overlapping voids. The latter represent an estimate of all troughs of the density field. We compute them from the empty circumsphere centers constrained by tetrahedra of galaxies using Delaunay triangulation. Our theoretical models based on an unprecedented large set of detailed simulated void catalogues are remarkably well confirmed by observational data. We use the largest recently publicly available sample of luminous red galaxies from SDSS-III BOSS DR11 to unveil for the first time a $> 3\sigma$ BAO detection from voids in observations. Since voids are nearly isotropically expanding regions, their centers represent the most quiet places in the Universe, keeping in mind the cosmos origin and providing a new promising window in the analysis of the cosmological large-scale structure from galaxy surveys.

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In the primordial baryon-photon plasma of our Universe, overpressured regions triggered sound waves that stalled at the recombination epoch, imprinting spheres of overdensity fluctuations, measurable in the matter power spectrum as an oscillatory pattern, the so-called baryon acoustic oscillations (BAOs). Any dark matter tracer should encode this signal in its spatial distribution either at early or late cosmic times after cosmic evolution [1–4]. In fact, these oscillations have already been detected in the cosmic microwave background anisotropies [5–8], in the distribution of galaxies [9–14], and more recently in the distribution of the Lyman alpha forest [15–17]. For a review on BAOs and their cosmological implications, see Aubourg *et al.* [18].

Their characteristic scale can be used as a standard ruler to measure the evolving scale of the Universe and to constrain the nature of its driving force, the dark energy component. For this reason a large number of surveys have focused on measuring BAOs or have included them as an integral part of their science, such as the 2dFGRS [19], the SDSS [20], the WiggleZ [21], the BOSS [22], the

SDSS-IV/eBOSS, the DESI/BigBOSS [23], the DES [24], the LSST [25], the J-PAS [26], the 4MOST [27], or the EUCLID survey [28].

Ever since the first detection of the giant Boötes void in 1981 [29] and with the nascent era of galaxy surveys, more evidence for the existence of voids has been found. The presence of voids in the large-scale structure was considered a manifestation of cosmological structure formation transforming the homogeneous Universe into a complex cosmic web structure. This picture was confirmed through numerical simulations (see, e.g., Refs. [30–32]). The classification of voids based on galaxy surveys has turned into a common practice; see, e.g., the CfA [33,34], the IRAS [35], Las Campanas [36], the PSCz [37], the 2dFRGS [38–40], the DEEP2 [41], the 2MRS [42], the SDSS survey [43–48], and the VIMOS survey [49]. Nevertheless, voids are usually considered to be very large rare objects, as compared to galaxies. Their probability distribution function can be used to constrain cosmology in an analogous way to galaxy clusters [50]. The statistics of voids has been studied for a long time (see, e.g., Refs. [51–55]), and an excursion set formalism analogous to the one describing the formation of halos (the compact collapsed dark matter objects hosting galaxies) has been developed [56–59]. Those studies hint towards a hierarchical picture, in which voids can form merger trees through cosmic evolution [60]. Considerable efforts have been made to understand the nature and evolution of voids through theoretical studies with semianalytic studies (see, e.g., Refs. [61,62]) and simulations (see, e.g., Refs. [63–69]).

Nevertheless, there are many different definitions of voids [56,66,67,70–80], which do not necessarily agree with each other (see, e.g., Ref. [81]).

From a practical perspective, voids have recently been proposed to give additional cosmological constraints, not only according to their statistics but also according to their shape. The void ellipticity was proposed to probe dark energy [82–85] and to make the Alcock-Paczyński test [86]. In particular, voids can be used to test gravity (see, e.g., Refs. [84,87,88]) dynamical dark energy [84], coupled dark energy [89], and modified gravity [87,90]. They can also be used to measure the Sachs Wolfe effect [91]. However, their sparse population and low signal-to-noise ratio have made them less interesting for clustering analysis. Little work can be found on the measurement of the correlation function of voids; see, however, Refs. [92–94] and, in particular, the recent pioneering study on observations [95].

In this Letter, we propose, for the first time, using the troughs of the density field (from now on called void tracers), meaning the minima in the overdensity field, to obtain additional measurements of the BAOs from the ones corresponding to galaxies. We have developed a Delaunay triangulation void finder based on empty circumspheres constrained by tetrahedra of galaxies Zhao *et al.* [96]. Our voids are close to the classical definition as spherical underdense regions (see, e.g., Refs. [40,51]), including, however, as a crucial difference, overlapping spheres, since we are interested in the distribution of troughs of the density field and account, in this way, for the shape of empty regions.

Our definition crucially increases the statistics of void tracers by about 2 orders of magnitude in contrast to previous studies, in which voids are treated as large connected regions that do not overlap at all or overlap only marginally (see, e.g., Refs. [40,94,95]). The speed of the DIVE void finder has been crucial for this project taking only of the order of minutes to find all the void tracers associated with about half a million objects and with little memory requirements (on a single core: ~18 mins and ~5 Gb, respectively).

In Liang *et al.* [97], we have studied, for the first time, the BAO signal with this void definition on mock catalogues predicting a characteristic correlation function, which includes dips on scales smaller and larger than the BAO peak. These features were exploited to develop a model-independent signal-to-noise estimator, used in turn to determine the radius cuts that provide the optimal signal-to-noise ratio for the BAO signal.

In this Letter we aim to extend the signal-to-noise estimator to detect the BAO signal from voids based on observational data.

To this end, first we define a control sample of accurate mock galaxy catalogues performed with the PATCHY code [98]. In particular, we have produced 100 mocks for each of the following cases: catalogues with and without baryon acoustic oscillations ("wiggle" and "nonwiggle" cases, respectively) in the initial conditions used to simulate structure formation. In particular, we consider complete samples of halos (main and subhalos) in cubic volumes of $(2.5\ h^{-1}\,\mathrm{Gpc})^3$ with number density $3.5\times10^{-4}\ h^3\,\mathrm{Mpc}^{-3}$, similar to the one of the BOSS CMASS galaxy sample at a mean redshift $z = 0.56$. The parameters of the PATCHY code have been calibrated with the large BigMultiDark N-body simulation [99] to accurately match the two- and the three-point statistics (such parameters can be found in Ref. [100]). The cosmological parameters have been consistently chosen to be within Λ cold dark matter Planck cosmology with $\Omega_\mathrm{M} = 0,307115$, $\Omega_\mathrm{b} = 0,048206$, $\sigma_8 = 0,8288$, $n_\mathrm{s} = 0,9611$, and a Hubble constant ($H_0 = 100\ h\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$) given by $h = 0,6777$.

The accuracy of these catalogues has been further demonstrated in several recent papers [101,102].

We have run the DIVE void finder for circumspheres with radii $\geq 16\ h^{-1}\,\mathrm{Mpc}$ on these sets of catalogues in real space and computed the corresponding correlation functions. The results do not show any signal in the nonwiggle case, as expected, while the wiggle case shows a significant BAO signal (see Fig. 1). Hence, both sets of simulations demonstrate that the BAO signal from voids is really

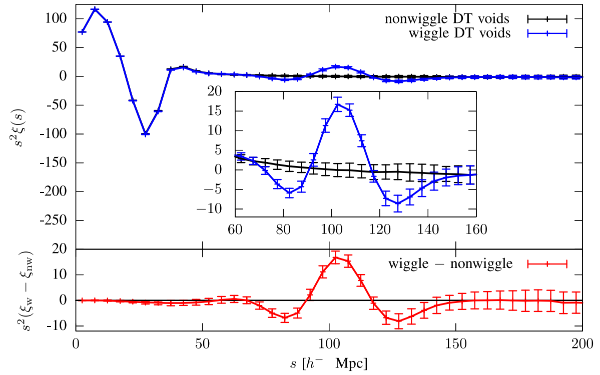

FIG. 1. Correlation functions for the set of 100 PATCHY (full cubic volume at mean redshift 0.56) void tracer mock catalogues (without observational effects) based on seed perturbations with and without BAOs. *Upper panel:* Mean and variance for the following cases: (1) with BAOs, blue solid line and blue error bars, respectively; (2) without BAOs ("nonwiggle"), black solid line and black error bars, respectively. *Lower panel:* Corresponding residual (red solid line and red error bars).

present in our mock catalogues, and we confirm the findings in Liang *et al.* [97]. The two dips around the BAO peak and a singularity around the size (diameter) of the smallest void (~30 h^{-1} Mpc) due to the void exclusion effect can also be clearly seen in Fig. 1. Importantly, the BAO peak is not only seen in the residual after extracting the nonwiggle from the wiggle mock catalogues (see lower panel in Fig. 1) but directly in the correlation function based on the catalogues containing the BAO signal in the seed perturbations (see upper panel in Fig. 1). This is not the case when analyzing disjoint voids (see Fig. 2). The oscillation patterns seen in the correlation functions are not related to the BAOs but are due to hard sphere exclusion effects when the filling factor is high (see [103]), as they can be found both in the wiggle and nonwiggle mock catalogues. There are only tiny differences in the modulation of these oscillations caused by BAOs, which can only be found in the residuals with large error bars (compare upper and lower panels in Fig. 2).

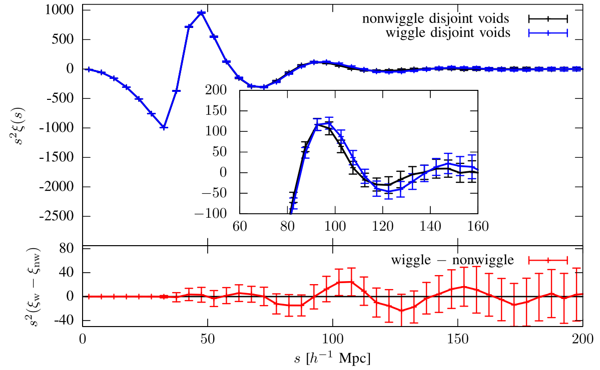

FIG. 2. Same as Fig. 1 for disjoint voids.

We have verified that the majority of the void tracers considered are located in expanding regions and that they are anticorrelated to the halos, hereby demonstrating that our definition of voids yields additional tracers of the large-scale structure (see Ref. [96]).

To detect the void tracer BAO signature in observations, we need to consider mocks resembling the BOSS DR11 CMASS sample in our analysis, including survey geometry, radial selection effects, bias evolution, and redshift space distortions (RSDs).

This work uses data from the Data Release DR11 [104] of the Baryon Oscillation Spectroscopic Survey (BOSS) [105]. The BOSS survey uses the SDSS 2.5 meter telescope at Apache Point Observatory [106], and the spectra are obtained using the double-armed BOSS spectrograph [107]. The data are then reduced using the algorithms described in Ref. [108]. The target selection of the CMASS and LOWZ samples, together with the algorithms used to create large scale structure catalogues (the MKSAMPLE code), are presented in Reid *et al.* [109].

We compute the voids (with radii ≥ 16 h^{-1} Mpc) and the corresponding correlation functions for 1,000 BOSS DR11 CMASS MULTIDARK PATCHY mocks [110]. These galaxy mocks have been calibrated with N-body based reference catalogues from the BIGMULTIDARKsimulation [111] and made publicly available [112]. The radius cut was determined to provide the optimal signal-to-noise ratio for the BAO signal (see Ref. [97]).

We follow the methodology presented in Liang *et al.* [97] to deal with the survey geometry and radial selection function. In particular, we use the angular mask from the DR11 galaxy catalogue to filter out the voids identified outside the survey area to construct the observed DR11 void catalogue and the corresponding set of synthetic BOSS DR11 CMASS MULTIDARK PATCHY void light-cone catalogues. To compute the two-point correlation functions, we need to construct a random void catalogue with the same geometry (in both angular and radius directions) as the BOSS DR11 CMASS data. To that purpose we combine 50 BOSS DR11 CMASS MULTIDARK PATCHY void catalogues and reassign the redshift randomly picked from observed data (a.k.a. shuffle method, e.g., see Ref. [14]). This procedure will produce random void catalogues with geometry consistent with the observed data. We avoid using the random galaxy catalogue for the random void catalogue since the distribution of the voids is different, especially at the boundaries of the survey.

Our analysis relies on a factor 2–2.5 more troughs than galaxies (for CMASS North: 1,212,393 troughs—voids with radii ≥ 16 h^{-1} Mpc—vs 566,940 galaxies; and for CMASS South: 472,868 troughs vs 188,582 galaxies). As an example, for the CMASS North we would only have 48,000 disjoint voids.

Finally, we take the BOSS DR11 data and apply the same analysis algorithms, using the same settings. A plot of

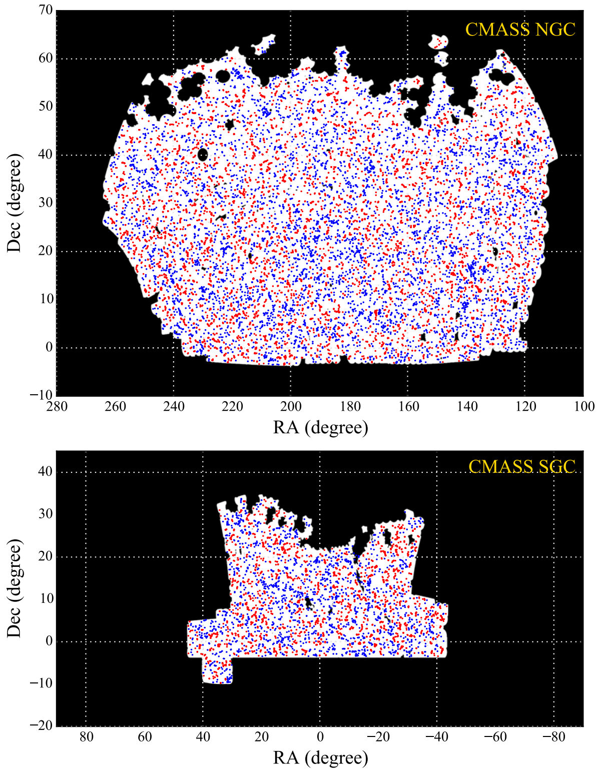

FIG. 3. Sky projection in right ascension (RA) and declination (DEC) of the BOSS DR11 CMASS LRGs (red symbols) and the corresponding void tracer (blue symbols) catalogues. *Upper panel*: Northern galactic cap (NGC). *Lower panel*: Southern galactic cap (SGC). Void tracers obtained in unobserved regions or holes in the mask (caused by, e.g., stars) have accordingly been removed.

the sky projection of the galaxies and their corresponding void tracers clearly illustrates how these tracers trace different regions of the cosmic web (see Fig. 3). The result of these computations shows a remarkable agreement between the theoretical prediction and the observations even towards large scales in contrast to galaxies (see Fig. 4). Here we use the wiggle and nonwiggle simulations to construct the templates of the fitting models to estimate the significance of the BAO detection.

We make a cubic spline fit from the wiggle and nonwiggle PATCHY mock correlation function, $\xi_{\rm w}(s)$ and $\xi_{\rm nw}(s)$, respectively, with s being the separation between two void tracers based on the galaxy distribution in redshift space. These two functions are the basis to construct the wiggle model and nonwiggle models for determining the BAO significance. In particular, we apply the following models in the fitting range $60 < r < 160\ h^{-1}$ Mpc. First, we show a wiggle model:

$$\xi_{\rm th}(s) = A[\xi_{\rm w}(s/\alpha) - \xi_{\rm nw}(s/\alpha)] + \xi_{\rm nw}(s/\alpha) + a_0 + a_1/s + a_2/s^2, \qquad (1)$$

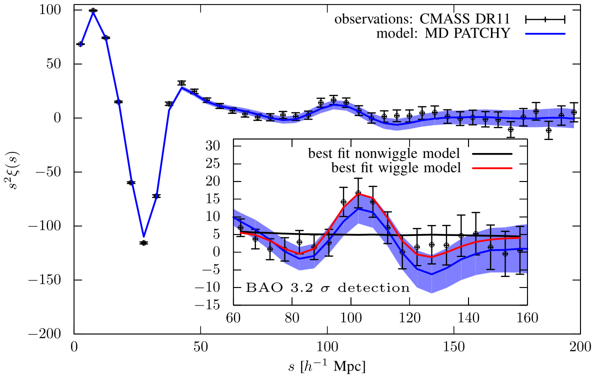

FIG. 4. Correlation functions for the BOSS DR11 CMASS void tracer catalogue (black error bars) and the mean (blue line) and 1σ region (blue shaded) of the corresponding 1,000 light-cone (including evolution from redshift 0.43 to 0.7) MULTIDARK PATCHY DR11 CMASS mock void catalogues (including observational effects: survey geometry, mask, radial selection function, and redshift-space distortions). The wiggle and nonwiggle best fitting models are represented by the red and black solid lines, respectively.

where α is the rescaling factor of BAO, A is the BAO damping factor, and the polynomial models the systematics for the overall shape following Anderson *et al.* [14]. Second, we show a nonwiggle model:

$$\xi_{\rm th}(s) = \xi_{\rm nw}(s/\alpha) + a_0 + a_1/s + a_2/s^2, \qquad (2)$$

which can be obtained from setting $A = 0$ in the wiggle model Eq. (1).

As in Anderson *et al.* [14], we use a template with fixed cosmology. The measurement of alpha can be interpreted as the ratio between the spherically averaged distance scale $D_{\rm V}(z) \equiv [cz(1+z)2D_A(z)2H^{-1}(z)]^{1/3}$ to the pivot redshift ($z = 0.57$) and the sound horizon scale rs at drag epoch with respect to the fiducial model, $\alpha = [D_{\rm V}/r_{\rm s}]/[D_{\rm V}/r_{\rm s}]_{\rm fid}$, where $D_A(z)$ is the angular diameter distance and $H(z)$ is the Hubble parameter. In general, a theoretical correlation function model should be constructed with parameters $\{\Omega_{\rm M}h^2, n_{\rm s}, \Omega_{\rm b}h^2, \alpha\}$, where α absorbs the information of dark energy and curvature. In practice, one might ignore the uncertainties of $n_{\rm s}$ and $\Omega_{\rm b}h^2$ since they are tightly constrained by CMB. While fixing $\Omega_{\rm M}h^2$, we can only measure some quantity which is insensitive to $\Omega_{\rm M}h^2$. Therefore, α should be interpreted as $D_{\rm V}/r_{\rm s}$, which is uncorrelated to $\Omega_{\rm M}h^2$ (e.g., see Table 2 in Ref. [113]).

The significance of the detection was computed from the difference of the best wiggle and nonwiggle fits yielding a chi-squared per degrees of freedom of $\chi^2/{\rm dof} = 9.9/15$ for the wiggle model and $\chi^2/{\rm dof} = 20.1/16$ for the nonwiggle model. In particular, we measured α by marginalizing over the amplitude A, obtaining $\alpha = 1.000 \pm 0.022$. Converting this finding to an effective distance at $z = 0.57$ would correspond to 2057 ± 45 Mpc, which is compatible with the finding from galaxies alone (see Ref. [14], which found 2056 ± 20 Mpc). One should note that the chi-squared distribution is not very Gaussian for voids. We would therefore take this measurement as a first-order estimate and work on more robust measurements in forthcoming papers.

Relying on these models we find a BAO detection with a significance of 3.2σ (see Fig. 4). We have used the covariance matrices derived from the set of 1,000 mocks to do this analysis analogously to Anderson *et al.* [14]. As a first approximation we assume in the wiggle and nonwiggle models that RSDs can be modeled by a damping term. We plan to investigate RSDs in detail in future work. Incompleteness, veto mask, and the fiber collision are taken into account in the DR11 CMASS mock catalogues and, accordingly, in the void catalogue computations. We do not see in the CMASS void correlation function any strong systematic effects, i.e., strong deviations in the correlation function towards large scales, as it was seen with the CMASS galaxy correlation function [114,115]. The correlation function behaves very much like the theoretical correlation function from the light-cone mocks. With the optimal radius cut used in this study, we found that the number density of voids is insensitive to the number density of galaxies (see Fig. 4 in Ref. [96]). This would explain why a varying number density of galaxies caused by stellar density systematics does not have a significant impact on the void density across the sky.

Questions arise when we measure the clustering of voids: What is the information gain from void tracers directly computed from the distribution of galaxies? And how covariant are these tracers to the galaxies themselves? The construction of void troughs follows the intuitive physical picture of filling the gaps complementary to the high density peaks occupied by the galaxies. Luminous red galaxies (LRGs) are known to reside in high density regions (see, e.g., Ref. [100]). We are thus extending the information on the density fluctuations ($\delta = \rho/\bar{\rho} - 1$) to underdense regions ($\delta < 0$), which based on this galaxy distribution are otherwise set to a constant value ($\delta = -1$). Less massive objects, such as emission line galaxies, could also be used to define underdense regions, but an extended definition with some stellar mass threshold may be required for the estimation of troughs. We note that small voids are equivalent to groups of quartets of galaxies residing in high density regions (see Ref. [96]) and, hence, are expected to deliver redundant information to the galaxies themselves. This is not the case for the large voids considered in this study. In fact, it is clear that the Delaunay voids we construct from tetrahedra of galaxies encode higher order statistics, further constrained by imposing the circumspheres to be empty, which strongly depends on gravitational evolution of the morphology of the cosmic web and hence on all the n-point statistics of the density field (in particular, the three-point statistics; see Ref. [116]). Moreover, our prior knowledge on the radius cut selecting empty circumspheres located in expanding void regions, based on tidal field computations of the underlying dark matter field in simulations (see Ref. [96]), implicitly incorporates knowledge on the void regions beyond the one present in the galaxy distribution. By analyzing the clustering of the troughs (constructed upon the galaxies) we are including higher order information (see Ref. [51]), potentially circumventing a more complicated mathematical formalism needed to extract the full information encoded in the three-dimensional distribution of galaxies. This is supported by recent theoretical work, demonstrating that most of the information gained in BAO reconstruction comes from the three-point statistics with some contributions from the four-point statistics [117], and depends on the environment [118]. In fact, a recent work has presented a 2.8σ detection of BAOs from the three-point correlation function based on BOSS DR12 [119].

The actual information gain we can get from combining void tracers with galaxies in a multitracer analysis remains to be investigated, including whether voids will improve the cosmological constraints from galaxy clustering alone. This analysis may yield little added value in the presence of data covering the underdense cosmic density field, with, e.g., considerably higher number densities than that provided by LRGs. Nevertheless, since void tracers are expected to be less affected by gravitational pull, BAO reconstruction techniques [120] could be less necessary for these tracers, and they may thus yield a less cosmology-dependent estimate of the linear correlation function. We will investigate this in future work.

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