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Abstract

With growing interest in a universal basic income (BI), we provide new results for a majority to benefit from replacing (some) unemployment benefits with BI. Given any income distribution and an extensive margin, such a replacement always benefits those remaining unemployed, raises utilitarian welfare, and benefits a poor - or even a working - majority. Similar results follow with involuntary unemployment, and joint distributions of wages and costs of work. Moreover, using quasi-linear utility with intensive margins, marginal introduction of BI can still benefit a large proportion of the poor whose productivities are below the average, without raising unemployment.

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As income and wealth distribution becomes increasingly skewed even in the most developed economies (Piketty 2014), while job security gives way to precarious employment for many, even better-educated workers (Standing 2011), the idea of a universal basic income (BI), unconditionally available for all citizens (also called a demogrant or citizen's income), is receiving more attention in the media, political discussion, and academia. Such a radical proposition is not a recent innovation. A related experimental system of poor relief emerged in late 18th century England¹, but was not welcomed by the first generation of economists. David Ricardo worried it would reduce the supply of agricultural labor. Thomas Malthus criticized it as likely to encourage population growth, especially of the poor. Jeremy Bentham believed it would discourage work, which is not only economically but also morally valuable. This negative consensus among classical economists was only contested at the time by contemporary political philosopher Thomas Paine (*Agrarian Justice* 1796).

As welfare systems were developed in the postwar decades, a few well-known economists such as James Meade and James Tobin suggested a BI, and Milton Friedman proposed the related negative income tax. In 1968 1,200 economists appealed to the US Congress for income guarantees. But Nixon's "Family Assistance Program" marginally failed to pass the Congress next year and discussion of BI gradually disappeared. Today's welfare systems, with increasing emphasis on wage subsidies for low wage workers but also lower unemployment benefits, are further removed from a BI than welfare in the 1970's (Steensland 2007). Many economists assume a BI, especially in the optimal tax literature, but do not explicitly advocate for its implementation.

¹ Coppola (2014)

The debate re-emerged in the late 90's. Atkinson (1995, 2011) argues in detail for BI together with a categorical transfer, in particular to help the disabled or those with children². Murray (2006) proposes replacing all welfare payments in the U.S. by a \$10,000 p.a. BI combined with a flat tax. Clark and Kavanagh (1996), Fitzpatrick (1999), Offe (2008), Standing (2011), Van Parijs and Vanderborght (2012), Skidelsky and Skidelsky (2012), and Torry (2013) advocate BI as the best response to growing job-insecurity and declining real wages for the less skilled, including many issues beyond the scope of this paper. In Switzerland a planned referendum will decide on BI in 2015, one example of growing political interest in the topic.

A major advantage of BI is to avoid the poverty trap associated with current means-tested welfare systems. These systems generally target the most needy, and are withdrawn or phased out quite rapidly as hours worked increase, or when earnings exceed some threshold. This means that less-qualified workers, particularly under generous European social security systems, may earn little more than they would receive from benefits. Even in the US and UK, the very high effective marginal tax rates faced by unskilled workers entering employment or moving from part time to full time or higher paid work forms a 'poverty trap', which is widely seen as a substantial barrier to leaving dependency (Atkinson, 2015). The new Universal Credit being introduced in the UK does not remove the poverty trap (Hirsch 2015). To overcome this problem Diamond and Saez (2011) argue for a wage subsidy or negative income tax for low earners, to ensure their net income exceeding benefits for the non-employed, and encourage participation. The in-work benefits in the form of tax credits should be rapidly phased out, and the marginal

² He recently proposed (2015) participation-based universal income.

tax should then be low for a broad middle range, but rise substantially for the highest earners, to 70–80 % (Piketty and Saez, 2012). This approach, however does not address the issue of how to help the non-employed. As a radical alternative BI can eliminate the disincentive effect and alleviate poverty for the most needy.

Assuming homogenous labor and random unemployment Van der Linden (2004) and Fabre et al (2014) argue that BI does not raise welfare and would be inferior to existing unemployment insurance. As this paper shows, the benefit of BI to the poor is mainly due to income redistribution from the rich, which would hardly exist if everyone earns the same income. Dolan (2014) points out that Fabre et al (2014) ignore the disincentive effects of means-tested welfare payments; also poverty and unemployment are highly concentrated among the least qualified, not purely random. Hence their model simply removes the main reason for redistributive policies such as BI. Dolan further summarizes evidence that the BI's negative income effect on labor supply would be small compared to the positive incentive effect of mitigating the poverty trap. His own calculations with US data suggest substantial welfare gains from a modest BI to replace most means-tested welfare programs. Gilroy et al (2013) estimate an increased labor supply with BI in the German welfare system.

A crucial decision is the level of BI, and associated taxes. While both right and left wing thinkers see certain attractiveness of BI, the former emphasize its replacement of current welfare and the latter only agree in various degrees and some worry such replacement may hurt the most needy, such as the unemployed. The conventional view holds that either the unemployed will be worse off with a modest BI, or unemployment will rise due to higher taxes and 'subsidized idleness'. Thus, while categorical

unemployment benefits (UB) have existed for a long time, a universal BI still remains an untested and controversial, though increasingly discussed idea, with widely varying views on the ‘optimal’ level. We argue instead that a common ground can be found, and replacing UB with BI need neither raise unemployment nor hurt the unemployed.

Rather than addressing all issues, we focus on the welfare effect of replacing UB with BI and adopt a modest, incremental policy approach. We start from a given level of taxation, UB and (voluntary) unemployment, in a simple model of heterogeneous labor, and then evaluate the impacts of replacement of UB with BI. We first assume labor supply has only an extensive margin. This is motivated by the empirically dominating importance of the participation decision for marginal, low wage workers and inflexibility of working hours for most full-time jobs, which are generally determined by job requirements, not individual preferences. Given this assumption we show that replacing UB with BI can benefit the existing unemployed without raising unemployment. This result is, surprisingly, independent of the income distribution, utility functions and unemployment levels. Moreover, this replacement also benefits a poor majority of the population, and a modified version can even benefit a working majority. This in turn suggests a (redistributive) policy reform which could gain a majority vote.³

Our result also holds in simple cases of involuntary unemployment or joint distributions of wages and cost of work. Moreover, even allowing for intensive margins (with identical elasticity of labor supply for the entire population), we demonstrate that marginal introduction of BI can still benefit most people with lower than the average productivity without raising unemployment. We thus recommend the introduction of BI

³ FitzRoy and Nolan (2015) use numerical simulations to show that a utilitarian optimal BI is preferred by a majority to the corresponding UB with a realistic income distribution, in spite of the higher tax rate.

at roughly the same level to replace the current job seekers' allowance in UK. We further discuss BI's feasibility and other advantages, not included in our model.

We assume a linear tax, not general (non-linear) optimal benefits and tax rates, as reviewed by Diamond and Saez (2011), and neither do we consider 'optimal' levels of employment. Nor do we address issues of growth and dynamic employment, which are obviously beyond the scope of our simple static model.

The paper is organized as follows. We first describe the main model in Section 1. In Section 2 we evaluate the welfare effect of replacing UB with BI given only extensive margins. Sections 3 and 4 demonstrate similar outcomes with involuntary unemployment and joint distributions of wage and cost of work. In Section 5 we show the result is by large still valid even when we allow intensive margins. Section 6 discusses further issues beyond our model, and we conclude with section 7. Proofs are in the Appendix.

1. BASIC MODEL

We first follow Diamond (1980) and others to ignore intensive margins for full time workers. On the other hand, the participation decision for low-wage and part-time workers, many of them female, appears to be highly sensitive to financial incentives (Diamond and Saez, 2011; Colombino et al 2010; Immervoll et al 2007; Saez, 2002). Eissa and Liebman (1996) find that the U.S. Tax Reform Act 1986, which extends tax credit to single mothers, has significant impacts on female labor participation, but no effect on the working hours of single mothers who were already in the labor force. Diamond (1980) and Choné and Laroque (2005 and 2008) developed models of the labor market with only extensive margins, assuming hours and earnings are given as part of the job 'package'. The only choice is thus whether to work, at a wage equal to individual

productivity, or to rely on transfers. When Colombino et al (2010) use numerical simulations to evaluate different welfare systems, they incorporate empirical evidence that the elasticity of labor supply is close to zero for full time workers. This approach represents a useful approximation and simplification for our purposes.

We assume an efficient labor market but with voluntary unemployment. The total population is normalized to one. We assume fixed working times and effort, leaving only the participation decision, given gross earnings and work requirements⁴. Annual earnings reflect workers' productivity, denoted by y , distributed on $[a, b]$, with $0 \leq a < b$, with a distribution function $F(y)$ and density function $f(y)$. Given a flat tax t , a worker's net earnings are $(1 - t)y$. We denote the quantities of BI and UB by B and u . If a person works, his income consists of net earnings plus BI, $y(1-t) + B$; if he does not work his income is just BI plus UB, $B + u$. We allow any mixture of BI and UB systems⁵.

Given only two choices of working or not, utility can be represented by two functions, depending on work status: $W[y(1 - t) + B]$ ⁶ when working and $V(B + u)$ when unemployed. Both functions are continuously increasing and concave.⁷ Since working implies less leisure, $W(m) < V(m)$ for any identical income m . A person with earnings x is indifferent between working or not when the following equality holds:

$$W[x(1 - t) + B] = V(u + B) \tag{1}$$

⁴ Highly paid jobs often require more hours but also offer better job satisfaction (Blundell 1995, Helliwell and Huang 2005). Allowing effort to vary with earnings does not affect our results provided the highly paid jobs are always preferred to the lower ones.

⁵ In the real world, UB may be extended by in-work benefits or tax credits up to some limit on working time or income. For instance job seekers in U.K. can receive allowance up to working 16 hours a week. The decision then is to take a full or part time job up to the limit. While this type of "partial UB" may have less monetary value, the impact is similar, and our model still applies.

⁶ We can obtain our main results allowing different utility functions for different productivity types, assuming monotonicity. For a simple presentation, we only consider identical utility functions.

⁷ With a single consumption good and no saving, this is equivalent to standard utility of consumption.

This equation is similar to the indifference conditions for the marginal worker in Diamond (1980) and Choné and Laroque (2005), with only extensive margins. A person chooses to work if and only if his earnings $y > x$. Given any t , there exists a marginal type x , such that all workers with higher earnings prefer employment, while $F(x)$, or those with lower earnings, choose not to work. The value of x is determined by (1). This assumption will be relaxed in section 4 where costs of work vary for a given productivity and are jointly distributed with income.

We assume that any employed person has a lower marginal utility of income than an unemployed, i.e. $W[y(1 - t) + B] < V(u + B)$ for any $y \geq x$. This is intuitive because the former has more income which reduces her marginal utility, and the latter has more time which allows more efficient consumption. Given x , the economy's total output is equal to $\int_x^b yf(y)dy$, denoted by $Y(x)$. The tax revenue, $tY(x)$, is used to pay u , B and a fixed public expenditure E . So the government budget constraint is:

$$tY(x) = B + uF(x) + E \quad (2)$$

Given t , we may change x , B and u while keeping (1) and (2) equal. If $B + u$ rises, the unemployed will be better off. However, the employed may be worse off. Similarly, when x is fixed, we can choose the value of any one of three variables t , B and u , and the other two will be determined by (1) and (2). We will first show that keeping t constant is unlikely to be socially desirable and then we focus on the welfare effect of changing t given x , so u and B become functions of t (as well as x)⁸.

⁸ The value of u is not indexed to wages or earnings. Otherwise the tax rate would have little impact on the unemployment, as shown by Pissarides (1998).

For simplicity we consider a flat tax only. This assumption is not crucial. In the case of progressive taxes, we can separate the tax structure into two parts, one is a single tax rate t applying to all earnings, and the other includes additional marginal tax rates for high earners. The latter has no effect on the marginal worker since he only faces t , so (1) remains valid. The only change in (2) is additional tax revenue from higher tax rates. If we change t but keep other tax rates fixed, the extra revenue does not change and the impact on (2) is the same as before. Hence this modification will not change our main results obtained below. As we mentioned earlier, our goal is to argue for the superiority of BI over UB in the simplest framework, not to find the optimal tax structure.

2. MAIN RESULTS

We first show that without increasing tax, replacing UB with BI can only benefit the unemployed when it harms everyone else, hence undesirable. The real question then is whether such a replacement can be socially beneficial, when tax can be raised, but without increasing unemployment. In contrast to the conventional wisdom, we find that given x , replacing UB with BI always benefits the unemployed and a poor majority, and is utilitarian welfare improving. Moreover, if we allow x to fall, replacing UB can even benefit a majority of the employed, without hurting those remaining unemployed, thus offering potential political support for BI even when the unemployed have only weak ‘voice’ in the process. We assume that the current equilibrium is characterized by conditions (1) and (2) for given $t > 0$, $u > 0$, $B \geq 0$, $x > a$ and $F(x) > 0$. We first leave t constant and change u , while x and B must be adjusted to keep (1) and (2) valid. If $B + u$ rises, the unemployed can be better off without any tax increase. Differentiating (1) and (2) with respect to u , we obtain the following result (see Appendix A).

Proposition 1: Given any fixed t , $\frac{dx}{du} > 0$ and $\frac{dB}{du} < 0$ always, but $\frac{dB}{du} + 1 > 0$ if and only if $(1 - t)[1 - F(x)] > (u + tx)f(x)$.

Given t , to increase u we have to raise unemployment and lower basic income, but the unemployed may be better off or worse off. If the above inequality does not hold, we have $\frac{dB}{du} + 1 < 0$, so the unemployed are better off if we lower u . This implies a higher B and a lower x , hence more employment. A higher B benefits all employed. So replacing UB with BI would be a Pareto improvement. Unfortunately, this is unlikely as the inequality is usually valid in the real world. For instance, let $t = 0.3$, $F(x) = 0.06$, we get $(1 - t)[1 - F(x)] = 0.66$. UK's job seekers' allowance (£70 per week) is roughly £3640 per year. We take the full-time minimum wage earners' annual income, £11,000 as x , the density $f(x)$ is about 1% per £200⁹. So $(u + tx)f(x) = 0.35 < 0.66$. Hence, we have $\frac{dB}{du} + 1 > 0$. To raise $B + u$, we have to increase u and x but reduce B (infeasible as current $B = 0$). Also this will increase unemployment and hurt all employed. Such a policy to help the unemployed does not seem to be socially desirable. Hence we focus on another option: increase $B + u$ without raising unemployment, i.e. keeping x constant and increase t .

There are additional reasons for fixing x . First, as shown above, it can be viewed as a modified maximin objective subject to no increase of unemployment. Secondly, a given x allows us to evaluate the impact on the fixed group of unemployed, without evaluating the benefit or cost of employment changes. Finally, if a BI is better than pure

⁹ See: <https://www.gov.uk/government/statistics/percentile-points-from-1-to-99-for-total-income-before-and-after-tax>.

UB for a given x , this must hold true when we allow x to change and we will consider this case later to show more people can be better off.

When we raise t without changing x , u and B must adjust to keep (1) and (2) valid. We focus on the effect on $B + u$ first. Conventional wisdom suggests that the UB reduction is unlikely to be fully compensated by the increase in BI because the latter has to be shared by the whole population, and the effect on $B + u$ should depend on the income distribution, utility function and level of unemployment. However, we find a surprising result that the effect is guaranteed positive (see Appendix B).

Proposition 2: Given any x , we have $\frac{dB}{dt} > 0$, $\frac{du}{dt} < 0$ and $\frac{dB}{dt} + \frac{du}{dt} > 0$.

The intuition that a replacement of UB with BI benefits the unemployed goes as follows. When tax rises and UB falls, with fixed x , employment and total output remain unchanged given only extensive margins. Thus replacing UB with BI only leads to an income redistribution. We can prove it must benefit the unemployed by contradiction. Suppose the unemployed are worse off as UB falls, the marginal workers must be worse off as well since they obtain the same utility. More productive workers suffer more from a higher t because they have to pay more tax, but get same BI, so they must be worse off too. Then everyone is worse off. This is impossible, as total output does not change. Hence the unemployed must be better off. In Appendix B we also see the benefit is proportional to $Y(x) - x[1 - F(x)]$. It is high given a large income disparity and becomes zero given homogeneous labor, as assumed in Van der Linden (2004) and Fabre et al (2014). Then, starting with any u and B (including pure UB) the unemployed can be

continuously made better off till u falls to zero, i.e. pure BI. Intensive margins will of course weaken this result and will be discussed in Section 5.

Policy makers often consider the impact of higher tax on the working population, not only the unemployed. As we mentioned earlier, since the replacement leads to income redistribution, it will not benefit everyone. As tax t increases, a worker's income, $y(1 - t) + B$ rises if and only if $dB/dt - y > 0$, i.e. when he is sufficiently poor. Hence there is an earning level, y^* , which divide the population into two groups, one better off and the other worse off. This critical y^* is equal to dB/dt . As shown in Appendix B, we denote the ratio of $W[x(1 - t) + B]/V(u + B)$ by ρ . According to our assumption, we have $\rho < 1$.

Proposition 3: Given any x , a worker is better off with a tax increase if and only if his earnings are less than $y^* = \frac{Y(x) + \rho xF(x)}{1 - (1 - \rho)F(x)}$.

It is easy to see that y^* falls with ρ , and has a minimum of $Y(x) + xF(x)$ and a maximum of $Y(x)/[1 - F(x)]$ as ρ approaches to 1 and 0 respectively. So y^* must lie between these two values. Those whose earnings are lower than the former must be better off and those whose earnings are higher than the latter, the average earnings of the employed, must be worse off. $Y(x) + xF(x)$ is certainly higher than $Y(a)$, which is the full employment output and also the average productivity of the population. Empirical data show that average earnings are higher than the median. Hence a majority of the population must be better off. This result provides a political justification for the replacement of UB with BI based on a majority rule.

Also, as UB replacement only leads to income redistribution, the net money transfer must be zero. Every dollar received by a poor person earning less than y^* comes

from a rich person earning more than y^* . Given the same utility function with decreasing marginal utility of income, this type of income transfer hurts a rich person less than it benefits a poor one, and must raise utilitarian welfare. Thus we can conclude:

Proposition 4: Given any x , replacing UB is utilitarian welfare improving.

Proposition 3 implies that the replacement hurts one with the average earnings of the employed, $Y(x)/[1 - F(x)]$. However, the replacement may hurt fewer people if we allow x to fall. By so doing, we need not raise the tax as much as in the previous case. This change will reduce the income transfer to the unemployed, but increase employment and total output, and reduce the loss to the rich. If we replace UB by an equal amount of BI, the unemployed will be indifferent. (1) and (2) imply that t has to rise and x has to fall. As long as we keep $u + B$ constant, we can make more people better off, and maximize political support for this equal replacement of UB with BI (Appendix C).

Proposition 5: An equal replacement of UB with BI can benefit everyone with earnings below $Y(x)/[1 - F(x)]$.

Empirical data indicate that average earnings of the employed are higher than median earnings. Hence the replacement can benefit a majority of the employed, without hurting the unemployed. This ensures political support even if the decision process excludes or gives low weight to the unemployed. The next question is: can such a replacement be a Pareto improvement? If yes, we have the strongest argument to recommend such a reform. Not surprisingly, the answer is generally negative. If one has extremely high earnings, a tiny tax increase will reduce his income so much that cannot be compensated by a (relatively small) BI, and must be worse off. However, if top earnings are not too high, this argument does not apply and replacing UB with BI may

benefit all. To demonstrate such a possibility, we compare two extreme cases: a pure UB system with $u > 0$ and $B = 0$ vs. a pure BI system with $u_1 = 0$ and $B_1 > 0$. Instead of a fixed x , we allow it to fall to x_1 , as in the case of Proposition 5. So $Y(x)$ rises to $Y(x_1)$. The budget constraints are $tY(x) = uF(x) + E$ and $t_1Y(x_1) = B_1 + E$ respectively. To minimize the tax increase, we keep the remaining unemployed indifferent, i.e. $B_1 = u$. The change is a Pareto improvement if the richest are not worse off, i.e. $B_1 \geq b(t_1 - t)$, or

$$\frac{Y(x_1)}{b} + t \frac{Y(x_1) - Y(x)}{u} \geq 1 - F(x) \quad (3)$$

(3) cannot hold if $Y(x)/b$ is close to zero because $t[Y(x_1) - Y(x)] < u[1 - F(x)]$. It does hold if $Y(x)/b$ is close to $1 - F(x)$, though unlikely so as b should be very large.

Our main results do not depend on the utility function, income distribution or initial level of unemployment. The key assumption is the absence of intensive margins and emigration of the rich, which may become relevant if taxes were too high. The assumption of only voluntary unemployment though, is not crucial, and we will extend our model to allow involuntary unemployment and obtain similar results.

3. INVOLUNTARY UNEMPLOYMENT

In the previous section, we only consider voluntary unemployment, or non-employment. In the reality the target of UB is usually the involuntarily unemployed. We now show that our main conclusions still hold with involuntary unemployment as the replacement will benefit all unemployed, either voluntary or involuntary.

We assume that a worker with productivity y has a probability $p(y)$ to find a job, after presumably optimal search, which we do not model explicitly. The involuntary

unemployment is represented by $\Delta F \equiv \int_x^b [1 - p(y)] f(y) dy$, and the loss of the total output due to involuntary unemployment is $\Delta Y \equiv \int_x^b [1 - p(y)] y f(y) dy$. Job search cost amounts to a utility loss of $\sigma(y)$ for worker y . So a worker is indifferent between searching or not when his expected utility of searching, $p(y)W[y(1 - t) + B] + [1 - p(y)]V(u + B) - \sigma(y)$ is equal to the utility of not searching, $V(u + B)$. Simplifying the equality we obtain the condition for the marginal worker x , similar to the previous condition (1):

$$W[x(1 - t) + B] - \frac{\sigma(x)}{p(x)} = V(u + B) \quad (1')$$

We assume that ratio $p(y)/\sigma(y)$ (weekly) rises with y , because more productive workers have more chances to find jobs per unit of search cost. Then similar to the earlier case, one will search for jobs if and only if his earnings $y > x$. Therefore, given t , u and B there are always a marginal worker x and the associated voluntarily unemployment $F(x)$. The corresponding budget constraint (2) is modified due to ΔF and ΔY as:

$$t[Y(x) - \Delta Y] = B + u[F(x) + \Delta F] + E \quad (2')$$

If ΔF and ΔY are predictable, the government can adjust t , B and u , to keep $F(x)$ fixed as before. Hence, given x , higher t again leads to adjustments in u and B according to (1') and (2'). The only difference between (1') and (1) is the term $\sigma(x)/p(x)$, which is fixed given x . Likewise, (2') only differs from (2) due to ΔF and ΔY , which are also fixed. Thus, similar to Propositions 2 and 3, given x , replacing UB with BI will benefit the unemployed, either voluntary or involuntary, and everyone whose earnings are not higher than $Y(x) - \Delta Y + x[F(x) + \Delta F]$. As we argued earlier, this will benefit a poor

majority. Moreover, we may keep $u + B$ constant so that x will fall and more people can be better off as shown in Proposition 5. By the definition, $Y(x) - \Delta Y$ will rise and $F(x) + \Delta F$ will fall as x decreases. Following the same proof as in Appendix C, we can show that an equal replacement of UB with BI can benefit everyone whose earnings are not higher than $[Y(x) - \Delta Y]/[1 - F(x) - \Delta F]$, which is the average earnings of the employed and must be higher than the median earnings of the employed. Hence we have:

Proposition 6: With involuntary unemployment, replacing UB with BI can benefit a poor majority, and an equal replacement can benefit a poor working majority.

Due to limited space, we do not examine other results in Section 2. Generally speaking replacing UB with BI is still desirable with involuntary unemployment.

4. JONIT DISTRIBUTIONS

So far we implicitly assumed an identical cost of work for all. This may not be true as people have different costs of work, e.g. due to family circumstances. In this section we show that our previous result can be preserved under this situation. Following Diamond (1980) and Choné and Laroque (2005) we consider a simple case where ability and cost of work are jointly distributed. Earnings are distributed with a density function $f(y)$. At each level y , a cost of work c is uniformly distributed in an interval $[0, \bar{c}]$. Everyone has a linear utility, only dependent on income and cost of work. Given t , B , and u , the utility of a type c worker is $y(1 - t) + B - c$, and his utility of not working is $u + B$. So work is preferred if and only if $c < y(1 - t) - u$. Assuming sufficient earning levels and \bar{c} such that $0 < y(1 - t) - u < \bar{c}$ for all y , some choose to work and some do not at any y . There is no “marginal worker” for the whole population, but one for each y .

The share of the workers with productivity y is $[y(1 - t) - u]/\bar{c}$, their number is $[y(1 - t) - u]f(y)/\bar{c}$, and the number of not working is $\{1 - [y(1 - t) - u]/\bar{c}\}f(y)$. The total output is $(1/\bar{c}) \int_a^b yf(y)[(1-t)y - u]dy$; the unemployment $\int_a^b f(y)\{1 - [(1-t)y - u]/\bar{c}\}dy = F(u,t)$. The government budget constraint is

$$\frac{t}{\bar{c}} \int_a^b yf(y)[(1-t)y - u]dy = B + uF(u,t) + E \quad (4)$$

Now we raise tax and replace UB with BI keeping unemployment $F(u,t)$ fixed. A worker is better off if and only if $dB/dt > y$. Similar to our results in section 3, we find that given $t < 0.5$, the replacement can benefit the unemployed and everyone with earnings no more than $\int_a^b yf(y)dy$, which is the average earnings of the population (see Appendix D). It also increases $u + B$, so the unemployed are better off. Since the average earnings are higher than the median earnings of the population, we obtain:

Proposition 7: With joint distributions of productivity and cost of work, replacing UB with BI can benefit a poor majority without changing unemployment.

Hence it seems desirable to replace UB with BI till $t = 1/2$ or $u = 0$. We should notice that this replacement does affect individual employment even though the total unemployment is fixed. It increases employment of less productive workers, while reduces employment of more productive ones. Consequently the total output will fall. But this change can provide extra non-pecuniary benefits. On the one hand unemployment for low earners has strong negative effects on (self-reported) well-being, on the other hand, highly paid employees often suffer from overwork and lack of leisure time. In this simple

case of joint distributions of productivity and cost of work, the total output reduction is compensated by this additional gain in wellbeing/happiness.

5. INTENSIVE MARGINS

So far we have excluded intensive margins, which would generally weaken our conclusions. Nevertheless the key results essentially survive when we allow both intensive and extensive margins. We assume a quasi-linear utility function, hence without income effects on labor supply, which would generally provide more support for income redistribution due to decreasing marginal utility. Instead of a fixed working time, everyone can choose his working hours, h , given his disutility function $h^{1+1/\varepsilon}/(1 + 1/\varepsilon)$, where $0 < \varepsilon < 1$, is the elasticity of labor supply for everyone. Identical elasticity also exaggerates the negative impact of intensive margins as the rich and full time workers usually have low elasticity, even close to zero. Hence our simple model is less favorable to a replacement of UB with BI than a more realistic but complicated approach.

Workers' hourly wage w follows a distribution function $F(w)$ and density function $f(w)$ on $[\alpha, \beta]$. Given tax t , an employed receives after-tax earnings $wh(1 - t)$ and B . So his utility is $wh(1 - t) + B - \frac{h^{1+1/\varepsilon}}{1+1/\varepsilon}$. With optimal labor supply $h = [w(1 - t)]^\varepsilon$, his after-tax earnings are $[w(1 - t)]^{\varepsilon+1}$, where $w^{\varepsilon+1}$ is the earnings without tax, and represents his productivity. His utility becomes $B + \frac{w^{1+\varepsilon}}{1 + \varepsilon} (1 - t)^{1+\varepsilon}$. If unemployed, his income is $B + u$.

So everyone chooses to work if and only if $u < [w(1 - t)]^{\varepsilon+1}/(1 + \varepsilon)$.¹⁰ Denote the marginal worker's wage by s , we have his indifference condition:

$$u = \frac{s^{1+\varepsilon}}{1 + \varepsilon} (1 - t)^{1+\varepsilon} \quad (5)$$

Given s , the unemployment is $F(s)$. Given s and t and the total output is $(1 - t)^\varepsilon$

$$\int_s^\beta w^{1+\varepsilon} f(w)dw = (1 - t)^\varepsilon Y(s), \text{ where } Y(s) \text{ is the output without tax, } \int_s^\beta w^{1+\varepsilon} f(w)dw.$$

Without tax and unemployment, the full employment output, which is also the average

productivity would be $\int_\alpha^\beta w^{1+\varepsilon} f(w)dw \equiv Y(\alpha)$. The government budget constraint is

$$t(1 - t)^\varepsilon Y(s) = B + uF(s) + E \quad (6)$$

We assume the current system only has UB, i.e. $u > 0$, $B = 0$, and $F(s) > 0$.

Obviously, a pure BI is no longer optimal due to intensive margins¹¹. We consider who will be better off from a marginal introduction of BI given $F(s)$. Unlike the previous cases, it will not benefit a person with the average productivity $Y(\alpha)$. Nonetheless we show that it can benefit everyone with earnings not much less than this level.

Proposition 8: Given any unemployment level a marginal introduction of BI can benefit everyone with productivity $w^{1+\varepsilon} \leq (1 - \frac{\varepsilon t}{1-t})Y(s) + s^{1+\varepsilon}F(s)$.

If $\varepsilon = 0$, we obtain the same result as before, the introduction of BI benefits everyone whose productivity is below $Y(s) + s^{1+\varepsilon}F(s)$, which is higher than $Y(\alpha)$. For any

¹⁰ A realistic fixed cost of work can be introduced, so that only those with at least some positive productivity actually work, without changing the basic results.

¹¹ In an earlier version of this paper (FitzRoy and Jin, 2010) we show that pure BI is nonetheless better than pure UB for the unemployed under reasonable conditions.

ε , the threshold is higher than $(1 - \frac{\varepsilon t}{1-t})Y(\alpha)$. We assume the same elasticity for the whole population (though it should be much lower for high income earners). In a plausible case of $\varepsilon = 0.2$ and $t = 0.3$, the threshold is higher than $0.914Y(\alpha)$. Among all OECD countries only Ireland's ratio (0.916) of the median wage to its mean exceed this level and all other country's ratios are lower than 0.9 in 2013¹². If these ratios apply to the whole population, the threshold is almost certainly higher than the median in most countries, and the introduction of BI can benefit a majority¹³.

Moreover, if the public expenditure $E = 0$, then (5) and (6) imply $t/(1-t) = s^{1+\varepsilon}F(s)/(1+\varepsilon)Y(s)$. Substitute this into the threshold in Proposition 8, we obtain:

Corollary: If $E = 0$, given any unemployment level a marginal introduction of BI can benefit everyone whose productivity is below $Y(s) + s^{1+\varepsilon}F(s)/(1+\varepsilon)$.

Regardless of the value of ε , the threshold is higher than $Y(s)$. As $Y(s)/[1-F(s)]$ is the average productivity of the employed, $Y(s)$ is $1-F(s)$ of that average. For instance, with 10% unemployment, the threshold is higher than 90% of the average wage, regardless of how strong the intensive margin is. As we mentioned earlier, the median wage is lower than 90% of average wage in all OECD countries except Ireland. Hence the introduction of BI may benefit a majority of the working population. If the marginal introduction of BI replaces an equal amount of UB allowing unemployment to fall, more people can be better off though the unemployed will be kept indifferent. Such a marginal replacement is very likely to benefit a poor majority or nearly a majority.

¹² The ratio can be calculated from the ratios of minimum wage to the average and median wage, at OECD StatExtracts: <http://stats.oecd.org/INdex.aspx?DataSetCode=MIN2AVE>.

¹³ The distribution of our productivity, $w^{1+\varepsilon}$, is close but not identical to that of w . However, due to its convexity in w , its mean should exceed its median by even a larger margin.

6. FURTHER ISSUES

(i) Feasibility: Since BI would be received by all citizens, it is often argued that its cost would be much higher than existing systems of contingent benefits. Although important transfers such as state pensions (and child benefits until recently) are not means-tested in the UK, there is a widespread misconception that most cash benefits go to the unemployed and lowest earners. However, direct cash transfers to the poorest tenth of households are actually smaller than the transfers to the 2nd, 3rd, 4th and 5th deciles – only the richest receive substantially less.¹⁴ Benefits for the unemployed and lowest earners amount to only 6% of total government spending, providing the least generous support for the poorest among advanced economies except the US. In the UK total cash transfers for welfare (but excluding health-related services such as care for the elderly and disabled) are about £205 billion¹⁵, while £12 billion unclaimed benefits are recorded annually. If this total amount were distributed equally to all citizens, it would provide a BI of about £3600 p.a. Coincidentally, this is approximately equivalent to the job-seeker's allowance (JSA) of £70 per week. Hence we recommend such a BI to replace the current JSA, but with the addition of means-tested housing benefits, due to the catastrophic lack of low-cost housing in parts of Britain. This may benefit a majority of the working population, while keeping the unemployed at least equally well off, as we argued earlier. This BI could provide the average household of 2.2 people with almost £8000. A natural modification would be to pay less for children and raise the rate for pensioners to the current state pension. It would benefit the poorest who do not even

¹⁴ ONS, The effects of Benefits and Taxes on Household Income, 2013/14, 'Table 14, Average incomes, taxes and benefits by decile groups of all households, 2013/14', and Hills (2014).

¹⁵ Hood and Johnston (2014)

claim their entitled benefits. In contrast to JSA it would not be lost for working more than 16 hours a week, or rejecting unpleasant and low-paid job-offers.

To overcome political opposition from the ‘squeezed middle’¹⁶ and alleviate growing poverty and deprivation, a substantially higher BI would be needed. This would require reform of the tax system to reduce large scale avoidance and evasion by the rich¹⁷, raise effective marginal rates for highest earners, and thus halt the steady rise in the share of GDP appropriated by the top 1%, while real earnings and income for most have been declining since 2008 (Piketty and Saez, 2012; Dorling, 2014). A part of BI funding can also be covered by significant saving from the administrative cost associated with complicated and sometimes inconsistent means-tested welfare programs and various taxation or subsidy schemes.

(ii) Incentives: An unconditional BI would abolish the ‘poverty trap’ faced by low earners under means-tested benefit systems. Modest BI and higher marginal tax rates for the rich are unlikely to reduce their work incentives, as experience in Scandinavia and evidence on labor supply show. There are many benefits associated with reduced inequality and deprivation (Wilkinson and Pickett, 2009). An open question is the response of low earners to BI. Gamel et al (2006) recognize two opposing incentives: the absence of a poverty trap provides a positive incentive, and the income effect of BI might encourage more leisure consumption as a normal good, as low-paid work usually offers little job-satisfaction. But their study does not suggest that BI would substantially reduce

¹⁶ A further complication is that total benefits from various uncoordinated programs vary a lot even among the poorest households, so that some would lose out under a revenue neutral scheme (Torry, 2015)

¹⁷ The ‘tax gap’ due to tax avoidance and evasion is estimated by Tax Research UK to be much higher than official claims, at around £120 billion p.a. <http://www.taxresearch.org.uk/Blog>

labor supply. According to the Dutch CPB report (Ruud de Mooij et al, 2006) based on microsimulation, a BI of 550 euro per month in the Netherlands would only reduce employment by 3.8%. Similarly small income effects are reported by Dolan (2014). Gilroy et al (2013) provide detailed arguments that labor supply would increase with BI in place of current welfare in Germany.

(iii) Freedom: A often neglected aspect is what Van Parijs (1995) called “real freedom for all”, and the scope for substitution between job-satisfaction and productivity. For example, JSA may be withheld if a claimant voluntarily leaves a job or rejects three job offers, say due to poor working conditions, or accepts any work for more than 16 hours per week, no matter how badly paid. So BI would put pressure on employers to provide decent jobs and encourage the poor to leave poverty. It would also eliminate the humiliation and widespread hardship caused by growing sanctions on the poorest welfare recipients who violate any of the numerous conditions for eligibility. Finally, BI would render legal minimum wages, which are often undermined by unpaid overtime or extra ‘effort’, superfluous. Such a fundamental shift of individual bargaining power, going far beyond the best efforts of collective bargainers, would allow the most disadvantaged to search without duress for their optimal combination of wages, working time and conditions. Some workers with a ‘living’ BI might prefer a combination of lower wages and ‘better’ jobs, and employers could benefit from lower labor costs. As Van Parijs and Vanderborgh (2012) and Standing (2011) have emphasized, this shift of bargaining power would generate huge (intangible) welfare benefits for low-wage earners who have the worst jobs and least security. Similarly, Groot (2002) convincingly argues that BI is

essential for compensatory justice. These dimensions are not included in our model and would further strengthen the conclusion obtained here.

7. CONCLUSIONS

In this paper we argue for the superiority of BI over UB. Our model with an extensive margin seems to be a reasonable approximation according to the empirical evidence, as long as tax rates are not excessive, and the main results are confirmed by a simple model with an intensive margin as well. Our findings indicate rather surprising advantages of replacing UB with BI, which mitigates the ‘poverty trap’ effect of UB, raises income for a given number of unemployed, leads to higher utilitarian welfare and benefits a poor majority. Perhaps the most plausible policy is to maintain unemployed utility and benefit a majority of the employed, so we suggest replacing JSA by BI of similar magnitude. The main conclusion is robust to simple examples allowing for involuntary unemployment and a joint distribution of income and cost of work, as well as holding in a simple model combining intensive and extensive margins.

Our simple static model does not consider important issues such as the transition process of the replacement, or responses from the demand side of the labor market. Thus we recommend an incremental approach in introducing and increasing BI, which allows a gradual social and political adjustment. The optimal process and the effects of such a gradual withdrawal of categorical benefits, and various social benefits of BI are interesting topics. Hopefully, our results will encourage further work using more sophisticated models, as a challenge for future research.

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Appendix A, Proof of Proposition 1:

Given t we differentiate (1) and (2) with respect to u , and obtain:

$$W[x(1-t) + B][(1-t)\frac{dx}{du} + \frac{dB}{du}] = V(u+B)(\frac{dB}{du} + 1) \quad (\text{A.1})$$

$$tY(x)\frac{dx}{du} = \frac{dB}{du} + F(x) + uF'(x)\frac{dx}{du} \quad (\text{A.2})$$

Note $F'(x) = f(x)$, and $Y(x) = -xf(x)$. Dividing both sides of (A.1) by $V(u+B)$, and letting $\rho \equiv W[x(1-t) + B]/V(u+B) < 1$, we get $\rho[(1-t)\frac{dx}{du} + \frac{dB}{du}] = \frac{dB}{du} + 1$.

Combine this with (A.2), we find

$$\frac{dx}{du} = \frac{1 - (1-\rho)F(x)}{(1-t)\rho + (1-\rho)(u+tx)f(x)} > 0 \text{ and } \frac{dB}{du} = -\frac{(1-t)\rho F(x) + (u+tx)f(x)}{(1-t)\rho + (1-\rho)(u+tx)f(x)} < 0.$$

Hence $\frac{dB}{du} + 1 > 0$ if and only if $(1-t)[1 - F(x)] > (u+tx)f(x)$.

Appendix B, Proof of Proposition 2:

Given x we differentiate (1) and (2) with respect to t , and obtain:

$$W[x(1-t) + B](\frac{dB}{dt} - x) = V(u+B)(\frac{dB}{dt} + \frac{du}{dt}) \quad (\text{B.1})$$

$$Y(x) = \frac{dB}{dt} + F(x)\frac{du}{dt} \quad (\text{B.2})$$

Dividing both sides of (B.1) by $V(u+B)$, we get $\rho(\frac{dB}{dt} - x) = \frac{dB}{dt} + \frac{du}{dt}$. Using

this and (B.2) we get $\frac{dB}{dt} = \frac{Y(x) + \rho x F(x)}{1 - (1-\rho)F(x)} > 0$ and $\frac{du}{dt} = -\frac{(1-\rho)Y(x) + \rho x F(x)}{1 - (1-\rho)F(x)} < 0$.

Hence $\frac{dB}{dt} + \frac{du}{dt} = \rho \frac{Y(x) - x[1 - F(x)]}{1 - (1-\rho)F(x)} > 0$ because $Y(x) > x[1 - F(x)]$ always holds.

Appendix C, Proof of Proposition 5:

Now we let x fall to x_1 , while lowering u to u_1 . So new $B_1 > B$, and new $t_1 > t$. To minimize the tax burden for high-income earners, we keep the unemployed indifferent, i.e. $u_1 + B_1 = u + B$. We need to show $B_1 - B > y(t_1 - t)$ for $y \leq Y(x)/[1 - F(x)]$. As (2) implies $B + E = tY(x) - F(x)u$ and $B_1 + E = t_1Y(x_1) - F(x_1)u_1$, we have

$$B_1 - B = t_1Y(x_1) - tY(x) + F(x)u - F(x_1)u_1 \quad (\text{C.1})$$

Since $x_1 < x$, we get $F(x_1) < F(x)$. So $B_1 - B > t_1Y(x_1) - tY(x) + F(x)(u - u_1)$. But $u - u_1 = B_1 - B$, so $B_1 - B > [t_1Y(x_1) - tY(x)]/[1 - F(x)]$. Hence $B_1 - B > y(t_1 - t)$ if

$$\frac{t_1Y(x_1) - tY(x)}{1 - F(x)} > y(t_1 - t) \quad (\text{C.2})$$

When $y < Y(x)/[1 - F(x)]$, (C.2) holds if $Y(x_1) > Y(x)$, which is guaranteed as $x_1 < x$.

Appendix D, Proof of Proposition 7:

Let $Y_1 \equiv \int_a^b yf(y)dy$ and $Y_2 \equiv \int_a^b y^2 f(y)dy$. So $F(u,t) = 1 + [u - (1 - t)Y_1]/\bar{c}$, and the budget constraint is $t[(1 - t)Y_2 - uY_1]/\bar{c} = B + uF(u,t)$. Differentiating them and holding $F(u,t)$ constant, we get $du/dt = -Y_1$, and $[(1 - 2t)Y_2 - uY_1 + tY_1^2]/\bar{c} = dB/dt - Y_1F(u,t)$. Substitute $u = (1 - t)Y_1 + [F(u,t) - 1]\bar{c}$ into the above equality, we find

$$\frac{dB}{dt} = (1 - 2t)\frac{Y_2 - Y_1^2}{\bar{c}} + Y_1 \quad (\text{D})$$

By their definition, we know $Y_2 - Y_1^2 = \int_a^b (y - Y_1)^2 f(y)dy >$, so $dB/dt > Y_1$ if $t < 0.5$.

Moreover, as $du/dt = -Y_1$, we have $du/dt + dB/dt = (1 - 2t)(Y_2 - Y_1^2)/\bar{c} > 0$.

Appendix E, Proof of Proposition 8:

Given s , differentiating (5) and (6) with respect to t , we find that $du/dt = -s^{\varepsilon+1}(1 - t)^\varepsilon$ and $(1 - t)^{\varepsilon-1}[1 - (1 + \varepsilon)t]Y(s) = dB/dt + F(s)du/dt$. So we solve

$$\frac{dB}{dt} = (1 - t)^{\varepsilon-1}[1 - (1 + \varepsilon)t]Y(s) + s^{\varepsilon+1}F(s)(1 - t)^\varepsilon.$$

Since a worker's net utility is $B + \frac{w^{1+\varepsilon}}{1 + \varepsilon}(1 - t)^{1+\varepsilon}$, he is better off if and only if $dB/dt >$

$w^{\varepsilon+1}(1 - t)^\varepsilon$, which gives the result in Proposition 8.