A Comment on "Can Relaxation of Beliefs Rationalize the Winner's Curse?: An Experimental Study"

by

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Supplementary Material

In this note, we provide (i) the derivation of the cursed equilibrium and analogy-based expectation equilibrium (which are the same under our specification) and (ii) the derivation of $L1's$ bids with a discrete bid space (which is analogous to ILN's Proposition 3).

A Analogy-Based Expectation Equilibrium and Cursed Equilibrium

The set of signals is $X = \{0, 1, \ldots, 10\}$. Let ω_{x_1,x_2} correspond to the state where player i's signal is x_i ($i = 1, 2$). There are 121 ($= 11 \times 11$) possible states, $\omega_{x_1,x_2} \in \Omega$. The item's value at ω_{x_1,x_2} is $\max\{x_1, x_2\}$. Each state is equally possible. We have

$$\begin{aligned}
\mathcal{P}_1 &= \{\{\omega_{x_1,0}, \omega_{x_1,1}, \omega_{x_1,2}, \omega_{x_1,3}, \omega_{x_1,4}, \omega_{x_1,5}, \omega_{x_1,6}, \omega_{x_1,7}, \omega_{x_1,8}, \omega_{x_1,9}, \omega_{x_1,10}\}_{x_1 \in X}\} \\
\mathcal{P}_2 &= \{\{\omega_{0,x_2}, \omega_{1,x_2}, \omega_{2,x_2}, \omega_{3,x_2}, \omega_{4,x_2}, \omega_{5,x_2}, \omega_{6,x_2}, \omega_{7,x_2}, \omega_{8,x_2}, \omega_{9,x_2}, \omega_{10,x_2}\}_{x_2 \in X}\}.
\end{aligned}$$

where $\mathcal{P}_i$ is the partition of the states from player i's point of view.

Let $\mathcal{A}_i$ be the analogy partitions of the states from player i's point of view ($i = 1, 2$). For the analogy based expectation equilibrium, we assume that $\mathcal{A}_i = \mathcal{P}_i$ for $i = 1, 2$, i.e., the *private information analogy partition* (Jehiel and Koessler (2008, p. 538)). This is visualized in Figure 1. As Jehiel and Koessler (2008, p.539) and Eyster and Rabin (2005, p.1634) note, this specification coincides with the *fully* cursed equilibrium (i.e., $\chi = 1$, in Eyster and Rabin (2005) χ-cursed equilibrium). Thus, the analogy-based expectation equilibrium that we construct is also a fully cursed equilibrium.

Note that $E[X^{max} \mid 10] = 10$. For $x_i \in X \backslash \{10\}$, the expected value of the item is

$$\begin{aligned}
E[X^{max} \mid x_i] &= \left(\frac{1}{11}\right) 10 + \cdots + \left(\frac{1}{11}\right)(x_i + 1) + \left(\frac{x_i + 1}{11}\right) x_i \\
&= \left(\frac{1}{11}\right) \sum_{l=x_i+1}^{10} l + \left(\frac{x_i + 1}{11}\right) x_i
\end{aligned}$$

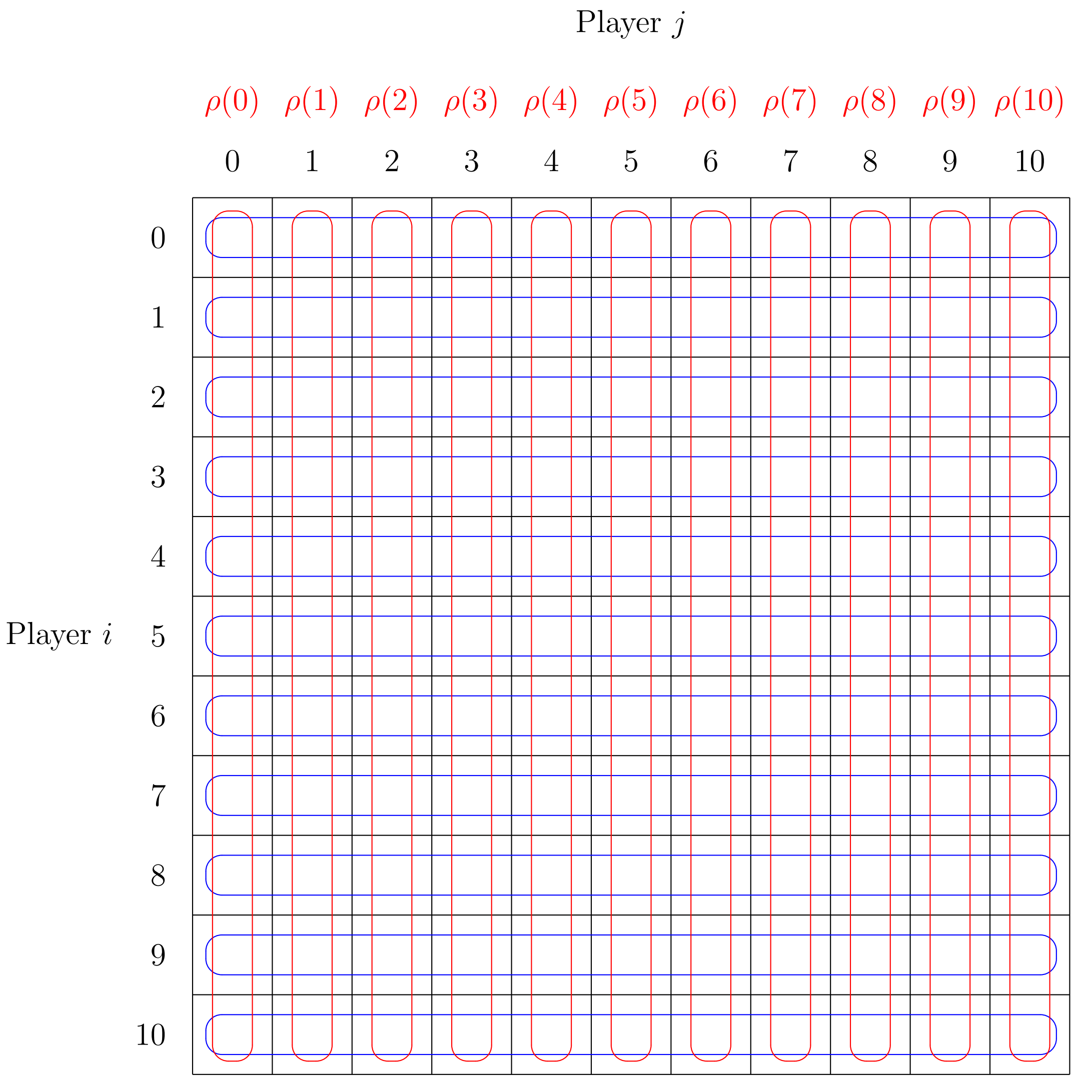

Figure 1: Partitions of States

x_i	0	1	2	3	4	5	6	7	8	9	10
$E[X^{max} \mid x_i]$	5	$\frac{56}{11}$	$\frac{58}{11}$	$\frac{61}{11}$	$\frac{65}{11}$	$\frac{70}{11}$	$\frac{76}{11}$	$\frac{83}{11}$	$\frac{91}{11}$	$\frac{100}{11}$	10
$b^l(x_i)$	5	5.09	5.27	5.54	5.90	6.36	6.90	7.54	8.27	9.09	10
$b^h(x_i)$	5	5.10	5.28	5.55	5.91	6.37	6.91	7.55	8.28	9.10	10
$\rho(x_i)$	-	$\frac{10}{11}$	$\frac{8}{11}$	$\frac{5}{11}$	$\frac{1}{11}$	$\frac{7}{11}$	$\frac{1}{11}$	$\frac{5}{11}$	$\frac{8}{11}$	$\frac{10}{11}$	-

Table 1: x_i, $E[X^{max} \mid x_i]$ and Equilibrium Strategy

b	5	5.09	5.10	5.27	5.28	5.54	5.55	5.90	5.91	6.36
$\bar{\rho}(b)$	$\frac{1}{11}$	$\frac{10}{121}$	$\frac{1}{121}$	$\frac{8}{121}$	$\frac{3}{121}$	$\frac{5}{121}$	$\frac{6}{121}$	$\frac{1}{121}$	$\frac{10}{121}$	$\frac{7}{121}$
b	6.37	6.90	6.91	7.54	7.55	8.27	8.28	9.09	9.10	10
$\bar{\rho}(b)$	$\frac{4}{121}$	$\frac{1}{121}$	$\frac{10}{121}$	$\frac{5}{121}$	$\frac{6}{121}$	$\frac{8}{121}$	$\frac{3}{121}$	$\frac{10}{121}$	$\frac{1}{121}$	$\frac{1}{11}$

Table 2: Strategy of player j perceived by player i

$$\begin{aligned} &= \left(\frac{1}{11}\right)\left(55 - \frac{x_i(x_i+1)}{2}\right) + \frac{x_i(x_i+1)}{11} \\ &= \frac{x_i(x_i+1)+110}{22}. \end{aligned}$$

Table 1 shows the value of $E[X^{max} \mid x_i]$ for each $x_i \in X$.

Suppose that player i with x_i chooses $b^l(x_i) = \frac{\lfloor 100E[X^{max}|x_i] \rfloor}{100}$ with probability $\rho(x_i) \in (0,1)$ and $b^h(x_i) = \frac{\lceil 100E[X^{max}|x_i] \rceil}{100}$ with $1-\rho(x_i)$ where

$$\rho(x_i) = \frac{b^h(x_i) - E[X^{max} \mid x_i]}{b^h(x_i) - b^l(x_i)} \in (0,1)$$

for each $x_i \in X$.[1] Table 1 shows their values for each $x_i \in X$. It is important to note that for each x_i, (i) $b^l(x_i) < E[X^{max} \mid x_i] < b^h(x_i)$ and (ii) $\rho(x_i)b^l(x_i) + (1-\rho(x_i))b^h(x_i) = E[X^{max} \mid x_i]$. Let

$$B^p = \left\{ b \in B \mid \text{there exists } x_i \text{ such that } b = b^l(x_i) \text{ or } b = b^h(x_i) \right\}.$$

Given the strategy specified above, Table 2 shows *the strategy for player j perceived by player i*, $\bar{\rho}$. Note that $\bar{\rho}$ is independent of the state.

Given $\bar{\rho}$, the expected payoff for player i with x_i from $b_i \in B$ is computed as follows: If

[1] That is, we choose two numbers in B closest to $E[X^{max} \mid x_i]$.

player i chooses $b_i \in B \backslash B^p$,

$$\left(\frac{1}{11}\right) \sum_{x_j \in X} \sum_{b<b_i, b \in B^p} \bar{\rho}(b) \{\max\{x_i, x_j\} - b\} = \sum_{b<b_i, b \in B^p} \bar{\rho}(b) \{E[X^{max} \mid x_i] - b\}$$

while if player i chooses $b_i \in B^p$,

$$\begin{aligned} & \left(\frac{1}{11}\right) \sum_{x_j \in X} \left[\sum_{b<b_i, b \in B^p} \bar{\rho}(b) \{\max\{x_i, x_j\} - b\} + \bar{\rho}(b_i) \{\max\{x_i, x_j\} - b_i\} \left(\frac{1}{2}\right) \right] \\ = & \sum_{b<b_i, b \in B^p} \bar{\rho}(b) \{E[X^{max} \mid x_i] - b\} + \bar{\rho}(b_i) \{E[X^{max} \mid x_i] - b_i\} \left(\frac{1}{2}\right). \end{aligned}$$

Note (i) that $b_i = 5$ is a best response for $x_i = 0$, (ii) $b_i = 10$ is a best response for $x_i = 10$, and (iii) that every $b_i < b^l(x_i)$ and $b_i > b^h(x_i)$ cannot be a best response for each $x_i \in X \backslash \{0, 10\}$, meaning that the only remaining bids are $\{b^l(x_i), b^h(x_i)\}$ for each $x_i \in X \backslash \{0, 10\}$.[2] Given $x_i \in \{0, 10\}$, the expected payoff from $b_i = b^l(x_i)$ is

$$\sum_{b<b^l(x_i), b \in B^p} \bar{\rho}(b) \{E[X^{max} \mid x_i] - b\} + \bar{\rho}(b^l(x_i)) \left\{E[X^{max} \mid x_i] - b^l(x_i)\right\} \left(\frac{1}{2}\right) \tag{1}$$

while the expected payoff from $b_i = b^h(x_i)$ is

$$\sum_{b<b^h(x_i), b \in B^p} \bar{\rho}(b) \{E[X^{max} \mid x_i] - b\} + \bar{\rho}(b^h(x_i)) \left\{E[X^{max} \mid x_i] - b^h(x_i)\right\} \left(\frac{1}{2}\right). \tag{2}$$

Then, we have

$$\begin{aligned} & (2) - (1) \\ = & \bar{\rho}(b^l(x_i)) \left\{E[X^{max} \mid x_i] - b^l(x_i)\right\} \left(\frac{1}{2}\right) + \bar{\rho}(b^h(x_i)) \left\{E[X^{max} \mid x_i] - b^h(x_i)\right\} \left(\frac{1}{2}\right) \\ = & \left(\frac{1}{22}\right) \left[\rho(x_i) \left\{E[X^{max} \mid x_i] - b^l(x_i)\right\} + (1 - \rho(x_i)) \left\{E[X^{max} \mid x_i] - b^h(x_i)\right\}\right] \\ = & \left(\frac{1}{22}\right) \left\{E[X^{max} \mid x_i] - [\rho(x_i) b^l(x_i) + (1 - \rho(x_i)) b^h(x_i)]\right\} \\ = & 0 \end{aligned}$$

where the last equality comes from the fact that $\rho(x_i) b^l(x_i) + (1 - \rho(x_i)) b^h(x_i) = E[X^{max} \mid x_i]$ for each $x_i \in X$. This shows that both $b^l(x_i)$ and $b^h(x_i)$ are best responses for each $x_i \in X \backslash \{0, 10\}$. The strategy specified above hence constitutes an analogy-based expectation

[2] More generally, any $b_i < 5.09$ is a best response for $x_i = 0$ and any $b_i > 9.10$ is a best response for $x_i = 10$.

equilibrium with private information analogy partition, and hence is also a fully cursed equilibrium.

B ILN's Proposition 3 with a Discrete Bid Space

We derive *random* $L1$'s best-responses for BL/SBF under the assumption that *random* $L0$ randomizes uniformly over $B^0 = \{0, 0.01, \ldots, 9.99, 10\} \subset B$. Given signal $x_i \in X$, the following expression is the expected payoff for player i by bidding $b_i \in B$. Note that we use "cents" instead of "dollars".

$$
\begin{aligned}
& \left(\frac{1}{11}\right) \sum_{x_j \in X} \left[\left(\frac{1}{|B^0|}\right) \sum_{b<b_i, b \in B^0} \{\max\{x_i, x_j\} - b\} + \left(\frac{1}{|B^0|}\right) \{\max\{x_i, x_j\} - b_i\} \left(\frac{1}{2}\right) \right] \\
= & \left(\frac{1}{|B^0|}\right) \sum_{b<b_i, b \in B^0} \{E[X^{max} \mid x_i] - b\} + \left(\frac{1}{|B^0|}\right) \{E[X^{max} \mid x_i] - b_i\} \left(\frac{1}{2}\right) \\
= & \left(\frac{b_i}{|B^0|}\right) E[X^{max} \mid x_i] - \left(\frac{1}{|B^0|}\right) \left(\frac{(b_i - 1) b_i}{2}\right) + \left(\frac{1}{|B^0|}\right) \{E[X^{max} \mid x_i] - b_i\} \left(\frac{1}{2}\right) \\
= & \left(\frac{b_i + \frac{1}{2}}{|B^0|}\right) E[X^{max} \mid x_i] - \left(\frac{1}{|B^0|}\right) \left[\frac{(b_i - 1) b_i}{2} + \frac{b_i}{2}\right] \\
= & \left(\frac{1}{|B^0|}\right) \left\{ \left(b_i + \frac{1}{2}\right) E[X^{max} \mid x_i] - \left(\frac{b_i^2}{2}\right) \right\}.
\end{aligned}
$$

Given that $b_i \in B^0$ and the expression above is quadratic, the expected payoff is maximized at either $E[X^{\max} \mid x_i] - 0.01$ or $E[X^{\max} \mid x_i] + 0.01$ (in dollars). We numerically computed the values of the expected payoff at these points, and selected the bid with the highest expected payoff as the best-response.