A Comment on “Can Relaxation of Beliefs Rationalize the Winner’s Curse?: An Experimental Study”  
by  
Miguel A. Costa-Gomes and Makoto Shimoji  

Supplementary Material  

In this note, we provide (i) the derivation of the cursed equilibrium and analogy-based expectation equilibrium (which are the same under our specification) and (ii) the derivation of $L_1$’s bids with a discrete bid space (which is analogous to ILN’s Proposition 3).

A Analogy-Based Expectation Equilibrium and Cursed Equilibrium

The set of signals is $X = \{0, 1, \ldots, 10\}$. Let $\omega_{x_1,x_2}$ correspond to the state where player $i$’s signal is $x_i$ ($i = 1, 2$). There are 121 ($= 11 \times 11$) possible states, $\omega_{x_1,x_2} \in \Omega$. The item’s value at $\omega_{x_1,x_2}$ is $\max\{x_1, x_2\}$. Each state is equally possible. We have

$$P_1 = \{\{\omega_{x_1,0}, \omega_{x_1,1}, \omega_{x_1,2}, \omega_{x_1,3}, \omega_{x_1,4}, \omega_{x_1,5}, \omega_{x_1,6}, \omega_{x_1,7}, \omega_{x_1,8}, \omega_{x_1,9}, \omega_{x_1,10}\} | x_1 \in X\}$$

$$P_2 = \{\{\omega_{0,x_2}, \omega_{1,x_2}, \omega_{2,x_2}, \omega_{3,x_2}, \omega_{4,x_2}, \omega_{5,x_2}, \omega_{6,x_2}, \omega_{7,x_2}, \omega_{8,x_2}, \omega_{9,x_2}, \omega_{10,x_2}\} | x_2 \in X\}.$$  

where $P_i$ is the partition of the states from player $i$’s point of view.

Let $A_i$ be the analogy partitions of the states from player $i$’s point of view ($i = 1, 2$). For the analogy based expectation equilibrium, we assume that $A_i = P_i$ for $i = 1, 2$, i.e., the private information analogy partition (Jehiel and Koessler (2008, p. 538)). This is visualized in Figure 1. As Jehiel and Koessler (2008, p.539) and Eyster and Rabin (2005, p.1634) note, this specification coincides with the fully cursed equilibrium (i.e., $\chi = 1$, in Eyster and Rabin (2005) $\chi$-cursed equilibrium). Thus, the analogy-based expectation equilibrium that we construct is also a fully cursed equilibrium.

Note that $E[X^{\max} | 10] = 10$. For $x_i \in X \setminus \{10\}$, the expected value of the item is

$$E[X^{\max} | x_i] = \left(\frac{1}{11}\right)10 + \cdots + \left(\frac{1}{11}\right)(x_i + 1) + \left(\frac{x_i + 1}{11}\right)x_i$$

$$= \left(\frac{1}{11}\right) \sum_{l=x_i+1}^{10} l + \left(\frac{x_i + 1}{11}\right)x_i$$
Figure 1: Partitions of States
<table>
<thead>
<tr>
<th>$x_i$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[X_{\text{max}}</td>
<td>x_i]$</td>
<td>$\frac{36}{11}$</td>
<td>$\frac{58}{11}$</td>
<td>$\frac{61}{11}$</td>
<td>$\frac{60}{11}$</td>
<td>$\frac{70}{11}$</td>
<td>$\frac{76}{11}$</td>
<td>$\frac{83}{11}$</td>
<td>$\frac{91}{11}$</td>
<td>$\frac{100}{11}$</td>
<td>10</td>
</tr>
<tr>
<td>$b^l(x_i)$</td>
<td>5</td>
<td>5.09</td>
<td>5.27</td>
<td>5.54</td>
<td>5.90</td>
<td>6.36</td>
<td>6.90</td>
<td>7.54</td>
<td>8.27</td>
<td>9.09</td>
<td>10</td>
</tr>
<tr>
<td>$b^h(x_i)$</td>
<td>5</td>
<td>5.10</td>
<td>5.28</td>
<td>5.55</td>
<td>5.91</td>
<td>6.37</td>
<td>6.91</td>
<td>7.55</td>
<td>8.28</td>
<td>9.10</td>
<td>10</td>
</tr>
<tr>
<td>$\rho(x_i)$</td>
<td>$\frac{b}{11}$</td>
<td>$\frac{b}{11}$</td>
<td>$\frac{b}{11}$</td>
<td>$\frac{b}{11}$</td>
<td>$\frac{b}{11}$</td>
<td>$\frac{b}{11}$</td>
<td>$\frac{b}{11}$</td>
<td>$\frac{b}{11}$</td>
<td>$\frac{b}{11}$</td>
<td>$\frac{b}{11}$</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1: $x_i$, $E[X_{\text{max}} | x_i]$ and Equilibrium Strategy

<table>
<thead>
<tr>
<th>$b$</th>
<th>5</th>
<th>5.09</th>
<th>5.10</th>
<th>5.27</th>
<th>5.28</th>
<th>5.54</th>
<th>5.55</th>
<th>5.90</th>
<th>5.91</th>
<th>6.36</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\rho}(b)$</td>
<td>$\frac{1}{11}$</td>
<td>$\frac{10}{121}$</td>
<td>$\frac{1}{121}$</td>
<td>$\frac{8}{121}$</td>
<td>$\frac{3}{121}$</td>
<td>$\frac{4}{121}$</td>
<td>$\frac{6}{121}$</td>
<td>$\frac{1}{121}$</td>
<td>$\frac{10}{121}$</td>
<td>$\frac{7}{121}$</td>
</tr>
<tr>
<td>$b$</td>
<td>6.37</td>
<td>6.90</td>
<td>6.91</td>
<td>7.54</td>
<td>7.55</td>
<td>8.27</td>
<td>8.28</td>
<td>9.09</td>
<td>9.10</td>
<td>10</td>
</tr>
<tr>
<td>$\bar{\rho}(b)$</td>
<td>$\frac{3}{121}$</td>
<td>$\frac{10}{121}$</td>
<td>$\frac{5}{121}$</td>
<td>$\frac{6}{121}$</td>
<td>$\frac{8}{121}$</td>
<td>$\frac{3}{121}$</td>
<td>$\frac{10}{121}$</td>
<td>$\frac{1}{121}$</td>
<td>$\frac{1}{11}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Strategy of player $j$ perceived by player $i$

\[
\begin{align*}
\rho(x_i) &= \left(\frac{1}{11}\right) \left(55 - \frac{x_i(x_i + 1)}{2}\right) + \frac{x_i(x_i + 1)}{11} \\
&= \frac{x_i(x_i + 1) + 110}{22}
\end{align*}
\]

Table 1 shows the value of $E[X_{\text{max}} | x_i]$ for each $x_i \in X$.

Suppose that player $i$ with $x_i$ chooses $b^l(x_i) = \left\lfloor \frac{100E[X_{\text{max}} | x_i]}{100} \right\rfloor$ with probability $\rho(x_i) \in (0, 1)$ and $b^h(x_i) = \left\lceil \frac{100E[X_{\text{max}} | x_i]}{100} \right\rceil$ with $1 - \rho(x_i)$ where

\[
\rho(x_i) = \frac{b^h(x_i) - E[X_{\text{max}} | x_i]}{b^h(x_i) - b^l(x_i)} \in (0, 1)
\]

for each $x_i \in X$. Table 1 shows their values for each $x_i \in X$. It is important to note that for each $x_i$, (i) $b^l(x_i) < E[X_{\text{max}} | x_i] < b^h(x_i)$ and (ii) $\rho(x_i)b^l(x_i) + (1 - \rho(x_i))b^h(x_i) = E[X_{\text{max}} | x_i]$. Let

\[
B^p = \{ b \in B \mid \text{there exists } x_i \text{ such that } b = b^l(x_i) \text{ or } b = b^h(x_i) \}.
\]

Given the strategy specified above, Table 2 shows the strategy for player $j$ perceived by player $i$, $\bar{\rho}$. Note that $\bar{\rho}$ is independent of the state.

Given $\bar{\rho}$, the expected payoff for player $i$ with $x_i$ from $b_i \in B$ is computed as follows: If

\footnote{That is, we choose two numbers in $B$ closest to $E[X_{\text{max}} | x_i]$.}
player $i$ chooses $b_i \in B \setminus B^p$,
\[
\left( \frac{1}{11} \right) \sum_{x_j \in X} \sum_{b < b_i, b \in B^p} \bar{p}(b) \{ \max\{x_i, x_j\} - b \} = \sum_{b < b_i, b \in B^p} \bar{p}(b) \{ E[X_{max} | x_i] - b \}
\]
while if player $i$ chooses $b_i \in B^p$,
\[
\left( \frac{1}{11} \right) \sum_{x_j \in X} \left[ \sum_{b < b_i, b \in B^p} \bar{p}(b) \{ \max\{x_i, x_j\} - b \} + \bar{p}(b_i) \{ \max\{x_i, x_j\} - b_i \} \left( \frac{1}{2} \right) \right]
\]
\[
= \sum_{b < b_i, b \in B^p} \bar{p}(b) \{ E[X_{max} | x_i] - b \} + \bar{p}(b_i) \{ E[X_{max} | x_i] - b_i \} \left( \frac{1}{2} \right).
\]
Note (i) that $b_i = 5$ is a best response for $x_i = 0$, (ii) $b_i = 10$ is a best response for $x_i = 10$, and (iii) that every $b_i < b^l(x_i)$ and $b_i > b^h(x_i)$ cannot be a best response for each $x_i \in X \setminus \{0, 10\}$, meaning that the only remaining bids are $\{b^l(x_i), b^h(x_i)\}$ for each $x_i \in X \setminus \{0, 10\}$.\(^2\) Given $x_i \in \{0, 10\}$, the expected payoff from $b_i = b^l(x_i)$ is
\[
\sum_{b < b^l(x_i), b \in B^p} \bar{p}(b) \{ E[X_{max} | x_i] - b \} + \bar{p}(b^l(x_i)) \{ E[X_{max} | x_i] - b^l(x_i) \} \left( \frac{1}{2} \right)
\]
while the expected payoff from $b_i = b^h(x_i)$ is
\[
\sum_{b < b^h(x_i), b \in B^p} \bar{p}(b) \{ E[X_{max} | x_i] - b \} + \bar{p}(b^h(x_i)) \{ E[X_{max} | x_i] - b^h(x_i) \} \left( \frac{1}{2} \right)
\]
Then, we have
\[
(2) - (1)
\]
\[
= \bar{p}(b^l(x_i)) \{ E[X_{max} | x_i] - b^l(x_i) \} \left( \frac{1}{2} \right) + \bar{p}(b^h(x_i)) \{ E[X_{max} | x_i] - b^h(x_i) \} \left( \frac{1}{2} \right)
\]
\[
= \left( \frac{1}{22} \right) \left[ \rho(x_i) \left\{ E[X_{max} | x_i] - b^l(x_i) \right\} + (1 - \rho(x_i)) \left\{ E[X_{max} | x_i] - b^h(x_i) \right\} \right]
\]
\[
= \left( \frac{1}{22} \right) \left\{ E[X_{max} | x_i] - \left[ \rho(x_i)b^l(x_i) + (1 - \rho(x_i))b^h(x_i) \right] \right\}
\]
\[
= 0
\]
where the last equality comes from the fact that $\rho(x_i)b^l(x_i) + (1 - \rho(x_i))b^h(x_i) = E[X_{max} | x_i]$ for each $x_i \in X$. This shows that both $b^l(x_i)$ and $b^h(x_i)$ are best responses for each $x_i \in X \setminus \{0, 10\}$. The strategy specified above hence constitutes an analogy-based expectation.

\(^2\)More generally, any $b_i < 5.09$ is a best response for $x_i = 0$ and any $b_i > 9.10$ is a best response for $x_i = 10$. 

4
equilibrium with private information analogy partition, and hence is also a fully cursed equilibrium.

B ILN’s Proposition 3 with a Discrete Bid Space

We derive random $L_1$’s best-responses for $BL/SBF$ under the assumption that random $L_0$ randomizes uniformly over $B^0 = \{0, 0.01, \ldots, 9.99, 10\} \subset B$. Given signal $x_i \in X$, the following expression is the expected payoff for player $i$ by bidding $b_i \in B$. Note that we use “cents” instead of “dollars”.

\[
\left( \frac{1}{11} \right) \sum_{x_j \in X} \left[ \left( \frac{1}{|B^0|} \right) \sum_{b < b_i, b \in B^0} \{ \max\{x_i, x_j\} - b \} + \left( \frac{1}{|B^0|} \right) \{ \max\{x_i, x_j\} - b_i \} \left( \frac{1}{2} \right) \right] \\
= \left( \frac{1}{|B^0|} \right) \sum_{b < b_i, b \in B^0} \{ E[X^{\text{max}} | x_i] - b \} + \left( \frac{1}{|B^0|} \right) \{ E[X^{\text{max}} | x_i] - b_i \} \left( \frac{1}{2} \right) \\
= \left( \frac{b_i}{|B^0|} \right) E[X^{\text{max}} | x_i] - \left( \frac{1}{|B^0|} \right) \left( \frac{(b_i - 1)b_i}{2} \right) + \left( \frac{1}{|B^0|} \right) \{ E[X^{\text{max}} | x_i] - b_i \} \left( \frac{1}{2} \right) \\
= \left( \frac{b_i + \frac{1}{2}}{|B^0|} \right) E[X^{\text{max}} | x_i] - \left( \frac{1}{|B^0|} \right) \left[ \frac{(b_i - 1)b_i}{2} + \frac{b_i}{2} \right] \\
= \left( \frac{1}{|B^0|} \right) \left\{ \left( b_i + \frac{1}{2} \right) E[X^{\text{max}} | x_i] - \left( \frac{b_i^2}{2} \right) \right\}.
\]

Given that $b_i \in B^0$ and the expression above is quadratic, the expected payoff is maximized at either $E[X^{\text{max}} | x_i] - 0.01$ or $E[X^{\text{max}} | x_i] + 0.01$ (in dollars). We numerically computed the values of the expected payoff at these points, and selected the bid with the highest expected payoff as the best-response.