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# CHOOSING ON INFLUENCE

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**ABSTRACT.** Interaction, the act of mutual influence between two or more individuals, is an essential part of daily life and economic decisions. Yet, micro-foundations of interaction are unexplored. This paper presents a first attempt to this purpose. We study a decision procedure for interacting agents. According to our model, interaction occurs since individuals seek influence for those issues that they cannot solve on their own. Following a choice-theoretic approach, we provide simple properties that aid to detect interacting individuals. In this case, revealed preference analysis not only grants the underlying preferences but also the influence acquired. Our baseline model is based on two interacting individuals, though we extend the analysis to multi-individual environments.

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Individuals sharing the same environment, such as members of the same household, friends from school, colleagues from workplace, influence each others' behaviour on many occasions through different means of interaction such as advice, inspiration, imitation, etc. There is an immense economics literature documenting and analyzing the effect of social interactions in individual decisions, in labor markets (Mas and Moretti, 2009), in education (Zimmerman, 2003; Calvo-Armengol et al., 2009), among teenagers (Evans et al., 1992; Bramoullé et al., 2009), in crime (Glaeser et al., 1996), to name a few. However not enough attention has been paid to the particular decision procedures individuals administer to interact, leaving the microfoundations of social interactions rather unexplored.<sup>1</sup> Appealing to this gap, the current paper presents and studies an individual decision making procedure for interacting agents.

Interaction is defined as the combined act of mutual influence between two or more entities. *Choice on Mutual Influence* considers two individuals in interaction, i.e., two individuals that influence each other in a particular way. In order to describe that 'particular way', we need to answer a simple question: Why and how do people interact? We suggest that people interact because they indeed seek influence from each other for those problems that they cannot solve on their own and through advice or pure imitation, they copy each others' behaviour. Simply put, failing to decide on one's own prompts influence-seeking behaviour. There may be different explanations underlying this behaviour: For instance, informational constraints on a specific problem

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<sup>1</sup>The extensive survey of Blume et al. (2010) on identification of social interactions concludes as follows: 'A final area that warrants far more research is the microfoundations of social interactions. In the econometrics literature, contextual and endogenous social interactions are defined in terms of types of variables rather than via particular mechanisms. This can delimit the utility of the models we have, for example, if the particular mechanisms have different policy implications.'

may be the drive: a person lacking sufficient information to evaluate several options may refer to someone with more expertise or experience on the subject: “These investment decisions are quite difficult for me. Luckily my cousin is a broker. I always ask her when I can’t decide what to invest in.” Or instead, compliance motives may create similar behaviour; a person may especially prefer to comply with another person’s preferences although none of the options is indeed better for herself: “I was not especially in favour of pizza or pasta to be honest, but nobody else on the table was having pasta, so I ordered pizza as well.” Whatever the underlying motivation is, we deduce that individuals influence each other at times of indecisiveness. This observation directly translates into our model in the form of incomplete preferences. We suggest that individuals are susceptible to each other’s influence over those alternatives for which they do not have well-defined preferences.

We present Choice on Mutual Influence as a simple and intuitive decision mechanism for interacting individuals, where interaction simply occurs to enable individuals to refine their choices. We show that this procedure is characterised by three falsifiable behavioural properties on the choice data of individuals. A choice-theoretic approach to interaction becomes appealing especially for this characterization exercise for several reasons: First, the falsifiable properties allows us to detect interacting individuals from their observable choice behaviour. Second, the revealed preference argument gives out the underlying preferences of these individuals, that are generally non-observable. Last but not the least, the representation theorem also grants us the revealed influence among these individuals. In other words, when the choice data of two individuals satisfy certain properties, we can recover the underlying preferences and the influence these individuals acquire from each other, consistent with our

model, which is crucial for policy implications and welfare analysis.<sup>2</sup> Needless to say, neither the rational revealed preference argument nor the revealed preference theories of the recent years that are developed to explain the behaviour of boundedly rational economic agents remain helpful to extract information from the observed choices of interacting individuals, simply because they consider only one individual whose non-standard economic behaviour may only be a result of her own cognitive biases.<sup>3</sup>

Choice on Mutual Influence works as follows: Consider two individuals that are endowed with transitive but not necessarily complete individual preference relations over a set of alternatives. Facing a decision problem, each individual first refers to her own preferences. If individual preferences are complete enough to single out a best preferred alternative for that specific problem, there would be no room for influence. However if maximization of own preferences does not yield a single choice but rather a set of options that are ‘choosable’, then the individuals appeal to each other in a second stage in order to be able to choose from those choosable alternatives. The decision outcome becomes the result of a two-stage maximization process where the second stage involves influence. Let us follow a simple example demonstrating Choice on Mutual Influence.

**An Example: Ian and Jane in the Bakery.** Two close friends, Ian and Jane, go to a bakery that is famous for its Apple Crumble ( $A$ ), Baklava ( $B$ ) and Chocolate Cake ( $C$ ). Let  $\succ_i$  and  $\succ_j$  represent the individual preferences

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<sup>2</sup>For a comprehensive account of welfare analysis under nonstandard choice behaviour see Apestegua and Ballester (2014); Bernheim and Rangel (2007); Cherepanov et al. (2013); Manzini and Mariotti (2014); Masatlioglu et al. (2012); Rubinstein and Salant (2011).

<sup>3</sup>To name a few boundedly rational choice procedures, see Masatlioglu et al. (2012) for limited attention; Masatlioglu and Ok (2005); Apestegua and Ballester (2013) for status quo bias; Rubinstein and Salant (2007) for framing effects.

of Ian and Jane, respectively.<sup>4</sup> Ian likes  $C$  better than  $A$ , but has never tasted that middle eastern desert Baklava, hence does not know how to compare  $B$  to the others. Jane, on the other hand, although she likes  $A$  more than  $B$ , she is indecisive about  $C$ . Hence, preferences are  $C \succ_i A$  and  $A \succ_j B$ , respectively. Unfortunately that day the bakery has run out of Baklava, leaving Ian and Jane with two options to choose from:  $A$  and  $C$ . Maximizing own preferences, Ian chooses  $C$ . However, Jane does not have a preferred option among  $A$  and  $C$ . Thus, she refers to Ian in order to be able to choose. Since  $C \succ_i A$ , Ian's influence leads Jane to choose  $C$ . Now assume that the bakery has all three deserts available. Ian likes  $C$  better than  $A$ . Maximizing his own preferences he will eliminate  $A$ , remaining with two options,  $B$  and  $C$ , that he has no idea how to compare. However, Jane's opinion in this case will not be helpful either to choose either of the options. Jane, on the other hand, eliminates  $B$  since  $A \succ_j B$ , remaining with  $A$  and  $C$ . Influenced by Ian's opinion over  $A$  and  $C$  again, she ends up choosing  $C$ . The following table summarizes Ian's and Jane's choice data for all possible choice problems:<sup>5</sup>

| Menus      | Ian | Jane |
|------------|-----|------|
| <b>ABC</b> | BC  | C    |
| <b>AB</b>  | A   | A    |
| <b>BC</b>  | BC  | BC   |
| <b>AC</b>  | C   | C    |

Choice on Mutual Influence is a natural way of decision making for individuals with incomplete preferences. Notice that unless the other individual has

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<sup>4</sup>Conventionally,  $x \succ_i y$  reads as Ian strictly prefers alternative  $x$  to alternative  $y$ .

<sup>5</sup>For the sake of brevity, we abuse notation and drop set delimiters and commas whenever we refer to menus or choices from menus. For instance, we use  $AB = C_i(ABC)$  to denote Ian's choice of  $A$  and  $B$  from the menu  $\{A, B, C\}$ .

well-defined preferences for all those alternatives that an individual's preferences are incomplete, the choice outcome will not be unique.

Now assume that we observe Ian and Jane's choice behaviour as summarized in the table above, without any information regarding their underlying preferences. Could we infer that they are choosing as if they are getting influenced by each other whenever they cannot decide on their own? Moreover, can we actually recover the hidden preferences and identify the influence they create on each other consistent with this choice behaviour? We answer both of these questions affirmatively relying on our characterization theorem. In a first result, we show that three behavioural properties of the choice data of two individuals are necessary and sufficient to link these individuals' behaviour according to Choice on Mutual Influence. The first property, *Expansion* is an individual rationality property, known to be satisfied by two stage maximization models. Unlike Expansion, the remaining two properties are novel to our model since they are defined over a pair of choice behaviours. *Nullipotency* states that all the influence that could be created among these individuals is already inherent in their behaviour; no further interaction could cause further refinement of their final choices. The key property of our model is the last property, *Consistency of Influence*, which ensures that any violation of consistency, in a standard sense, observed in an individual's choice data indicates the influence acquired. In a way, Consistency of Influence allows us to link individuals to each other by tracing the inconsistencies in their choice behaviour. For two individuals to be interacting according to Choice on Mutual Influence, they have to account for each other's inconsistencies.

Given the choice behaviour of Ian and Jane, the representation theorem ensures that the underlying preferences are uniquely identified, and hence the influence they acquire from each other. Nonetheless, in general the identification does not always sustain uniqueness. In other words, in certain situations,

there are more than one pair of preferences that would yield the same choice data according to Choice on Mutual Influence. In a subsection we explore the extent of this identification problem and find out a regularity condition on the preferences that would guarantee uniqueness.

Although Choice on Mutual Influence is based on only two individuals, the characterization exercise gives us clues about how to uncover the links between individuals in more general settings. By exploiting these clues, it is immediate to extend the analysis to multi-individual settings, where interaction structures take more complicated forms. The second section presents two different extensions for this purpose. In the first one, we focus on a group of interacting individuals that use the unanimity rule in the second stage of decision-making to resolve indecisiveness. The second extension, on the other hand, considers individuals that are influenced by particular individuals on specific problems.

To the best of our knowledge, Choosing on Influence is the first paper to develop a choice-theoretic approach to interaction. The novelty of the paper lies in this approach: We study multiple choice behaviours together and aim to figure out the unobservable elements of interaction, such as the relative identities of the influencer and influencee or the problems for which influence is acquired, out of the observed choice behaviour.

As a two stage maximization process, Choice on Mutual Influence mechanism is clearly related to the other two stage maximization processes studied in the boundedly rational choice literature.<sup>6</sup> The baseline model of two stage maximization would be Rational Shortlist Methods (RSM) proposed by Manzini and Mariotti (2007). RSM essentially refers to a single-valued choice

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<sup>6</sup>In addition to the papers cited, see: Bajraj and Ulku (2015); Garca-Sanz and R. Alcantud (2015); Manzini and Mariotti (2012); Yildiz (2015). For a detailed account of two stage mechanisms, see Horan (2014).

mechanism where a two-stage maximization process yields the chosen alternative uniquely. Two rationality properties, Expansion and a weakening of WARP, Weak WARP are shown to be necessary and sufficient for the choice data to reveal the two binary relations of RSM. A natural and interesting subclass of RSM models, not only for our purposes but also in general, would be RSM with transitive binary relations. Au and Kawai (2011) show that an additional axiom that ensures acyclicity of the revealed preference relation does also ensure transitivity. Horan (2014) proposes a behavioural axiom that does not impose acyclicity directly but does guarantee the existence of transitive rationales. Our model is inherently different from the existing two stage maximization procedures: we consider multiple individuals. The second criteria that the individual uses to choose is not simply another criteria in her mind, but another individual that is also equipped with a choice structure. Hence the behavioural properties we search for are to reveal the mutual relationship between these two individuals, and to identify the specific choice problems that influence is acquired.

The outline of the paper is as follows: The first section is devoted to Choice on Mutual Influence. We describe the model, present the characterizing axioms and the theorem. We also tackle the identification problem in this section. The second section extends the model to multi-individual settings. The third section presents several other uses of our model. The final section concludes. All proofs are left to an appendix.

## 1. CHOICE ON MUTUAL INFLUENCE

1.1. **The Model.** Let  $X$  be a nonempty finite set of alternatives and  $\Omega_X$  be the set of all nonempty subsets of  $X$ . Let 1 and 2 denote two individuals. For any  $i \in \{1, 2\}$ , we define the decision outcomes of  $i$  on  $\Omega_X$  as a choice correspondence  $C_i : \Omega_X \rightrightarrows X$  with  $\emptyset \neq C_i(S) \subseteq S$  for every  $S \in \Omega_X$ .

For  $i \in \{1, 2\}$ , let  $\succ_i$  be the strict preference relation of  $i$  over  $X$ , i.e., an asymmetric and transitive but not necessarily complete binary relation over  $X$ .<sup>7</sup> The set of maximal elements of  $S$  according to  $\succ_i$  will be  $Max(S, \succ_i) = \{x \in S : \nexists y \in S \text{ with } yx \in \succ_i\}$ .<sup>8</sup> If individual preferences are complete enough to single out a best preferred alternative, i.e., if  $Max(S, \succ_i)$  is a singleton, there will be no room for influence. Therefore individual  $i$  would be choosing this maximal element from  $S$ . However, if there are many alternatives that are deemed to be choosable from  $S$ , then  $i$  would be seeking influence over these choosable alternatives. Influenced by  $j$ , the maximal alternatives according to  $j$ 's preferences among those alternatives that are choosable for  $i$  are chosen,  $Max(Max(S, \succ_i), \succ_j)$ .

**Definition 1.** *We say that a pair of choice correspondences  $(C_1, C_2)$  is a Choice on Mutual Influence mechanism, if there exists a pair of asymmetric and transitive binary relations  $(\succ_1, \succ_2)$  such that:*

$$C_1(S) = Max(Max(S, \succ_1), \succ_2) \text{ and}$$

$$C_2(S) = Max(Max(S, \succ_2), \succ_1) \text{ for all } S \in \Omega_X.$$

**1.2. Characterization.** Now suppose we observe individual choice behaviours of two individuals. How could we test whether their behaviour is consistent with Choice on Mutual Influence? Three simple properties are sufficient to infer the interaction between two individuals from  $(C_1, C_2)$ . The first property, a well-known individual rationality property, Expansion (also known as Sen's  $\gamma$ ), states that if an alternative is chosen from two different sets, it has to be

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<sup>7</sup>We stick to strict preferences for simplicity purposes. Our results trivially generalize to the case where indifferences are allowed. Notice that in this case ties will be broken whenever the other individual has a strictly preferred alternative.

<sup>8</sup>Once again we abuse notation and denote an ordered pair  $(x, y) \in X \times X$  simply as  $xy$ .

chosen from the union as well:

**Expansion (EXP).** For any  $x \in X$ ,  $S, T \in \Omega_X$  and  $i \in \{1, 2\}$ , if  $x \in C_i(S) \cap C_i(T)$ , then  $x \in C_i(S \cup T)$ .

Expansion forbids not choosing an alternative from a choice problem, if that alternative is chosen somewhere at the presence of each and all of the alternatives of the problem.

Unlike EXP, the following two properties are not individual rationality properties but are to reveal the mutual relation between 1 and 2. The first of them, Nullipotency is required to link  $C_1$  and  $C_2$  to each other for those problems that they do not yield a single choice. It states that all the influence that could have come from  $j$  is already inherent in  $i$ 's behaviour.

**Nullipotency (NULL).** For any  $S \in \Omega_X$  and  $i, j \in \{1, 2\}$  with  $i \neq j$ , we have  $C_j(C_i(S)) = C_i(S)$ .

The decision outcomes of  $j$  can not be used to further refine  $i$ 's choices, since all possible influence is already exerted.<sup>9</sup>

The last property is the key property of Choice on Mutual Influence in that it allows to identify the influence of individuals on each other. Consistency of Influence states that if an individual influences the other regarding any two alternatives,  $x$  and  $y$ , then she, herself, has to behave consistently with her choice among  $x$  and  $y$  in any decision problem involving those.

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<sup>9</sup>Nullipotency also implies idempotency of the choice correspondences.

**Consistency of Influence (CoI).** For any  $x, y \in X$  and  $i, j \in \{1, 2\}$  with  $i \neq j$ , if there exists  $S \in \Omega_X$  with  $x \in S$  such that  $x = C_i(xy)$  and  $C_i(S) \neq C_i(S \setminus y)$ , then  $C_j(T) = C_j(T \setminus y)$  for all  $T \in \Omega_X$  with  $x \in T$ .

The choice of a single alternative from a binary problem may be the result of two alternative scenarios: Either the individual's preferences dictate this choice or being unable to compare these alternatives, she gets influenced by the other individual. In the former, since individual preferences are transitive, we would never observe an inconsistency involving these two alternatives in choice outcomes.<sup>10</sup> Having the unchosen alternative available in the existence of the chosen alternative would never alter individual's choice. CoI detects the binary problems for which this is indeed not the case. Although  $i$  chooses  $x$  over  $y$  from the binary problem, adding  $y$  to a problem that also includes  $x$  alters her choice behaviour. This is a clear indication of  $i$  being influenced by  $j$  in her choice of  $x$  from the binary problem. But then according to  $j$ 's preferences,  $x$  is clearly a better alternative than  $y$ . Due to transitivity,  $j$  would never choose inconsistently in larger problems: The availability of  $y$  will never change  $j$ 's choice for any problem that also includes  $x$ .

CoI ensures that any violation of consistency observed in the choice data is the result of the influence acquired. Therefore the influential individual will never show inconsistencies regarding those alternatives.

Our first theorem shows that these properties are indeed necessary and sufficient for Choice on Mutual Influence.

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<sup>10</sup>Notice that what we refer as an inconsistency is a type of Independence of Irrelevant Alternatives (IIA) violation: Choosing  $x$  uniquely from the menu  $\{x, y\}$  signals that  $y$  is not 'relevant' to the decision maker whenever  $x$  is there, hence it should not affect the choice behaviour in any problem including  $x$ . Thus, finding a set where inclusion of  $y$  alters the choice in  $x$ 's presence is a violation of IIA.

**Theorem 1.** *Let  $C_i : \Omega_X \rightrightarrows X$  for  $i \in \{1, 2\}$ .  $C_1$  and  $C_2$  satisfy EXP, NULL and CoI if and only if  $(C_1, C_2)$  is a Choice on Mutual Influence mechanism.*

The key point of our characterization exercise lies in the following observation:  $i$ 's choice behaviour not only reveals information about  $i$ 's underlying preferences but also  $j$ 's underlying preferences. Any inconsistency in  $i$ 's behaviour corresponds to a pair of alternatives that is uncomparable according to  $i$ 's preferences but ranked by  $j$ 's, as assured by CoI. Any multi-valued choice behaviour, on the other hand, corresponds to alternatives that are neither ranked by  $i$ 's, nor by  $j$ 's preferences, as assured by NULL. All properties are independent as we show in the appendix.

**1.3. Identification.** The characterization theorem ensures that given a particular pair of choice behaviour  $(C_1, C_2)$  satisfying EXP, NULL and CoI, we can recover 'a' pair of revealed preferences  $(\succ_1, \succ_2)$  that would represent  $(C_1, C_2)$  according to Choice on Mutual Influence. But not necessarily this pair is 'the' pair of underlying preferences. In some situations one can actually find other pairs of revealed preferences that would explain the same choice behaviour.<sup>11</sup> But then the following questions arise immediately:

- How accurately can we actually identify the underlying preferences (and hence the influence)?
- Under which conditions the underlying preferences (and hence the influence) can be uniquely identified?

To answer the first question we investigate the part of the preferences that is uniquely identified; i.e., the set of binary pairs possessed by all preference pairs explaining the same choice data. We show that we can indeed recover a major part of underlying preferences. As an answer to the second question,

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<sup>11</sup>We say that  $(\succ_1, \succ_2)$  explains (rationalizes)  $(C_1, C_2)$  if  $C_1(S) = \text{Max}(\text{Max}(S, \succ_1), \succ_2)$  and  $C_2(S) = \text{Max}(\text{Max}(S, \succ_2), \succ_1)$ .

we come up with a regularity condition on the preferences that would allow to restore uniqueness.

According to standard revealed preference argument, choices from binary menus reveal the underlying preferences. Here we cannot apply the standard argument since choices from binary menus may also reflect the influence acquired. On that account we first aim to distinguish the choices from binary menus where an agent influences the other, from the ones that there is no interaction for sure. The latter is indeed simple: for those binary menus where the individuals choose a different alternative uniquely we can be sure that the choices reflect the underlying preferences. In order to detect the former, we look for choice inconsistencies in larger menus. First notice that choices from binary menus define the following mutually exclusive sets of binary comparisons: Disagreements, influences acquired, influences formed, and agreements, where;

- Disagreements of  $i$  from  $j$ :  $P_i = \{xy \in X \times X : x = C_i(xy) \neq C_j(xy)\}$
- Influence of  $i$  over  $j$ :  $Q_i = \{xy \in X \times X : x = C_i(xy) = C_j(xy) \text{ and there exists } S \in \Omega_X \text{ with } x \in S \text{ such that } C_j(S) \neq C_j(S \setminus y)\}$
- Agreements:  $R = \{xy \in X \times X : x = C_i(xy) = C_j(xy) \text{ and } C_i(S) = C_i(S \setminus y) \text{ and } C_j(S) = C_j(S \setminus y) \text{ for all } S \in \Omega_X \text{ with } x \in S\}$ .

In the proof of Theorem 1, we show that  $(\succ_1, \succ_2)$  such that  $\succ_i = (P_i \cup Q_i \cup R) = \{xy \in X \times X : C_i(S) = C_i(S \setminus y) \text{ for all } S \in \Omega_X \text{ with } x \in S\}$  for  $i \in \{1, 2\}$  explains a given pair of choice behaviours  $(C_1, C_2)$ . Indeed this pair of  $(\succ_1, \succ_2)$  is the largest pair that would explain  $(C_1, C_2)$ . However, one can find subsets of  $\succ_1$  and  $\succ_2$  that would explain the same choices. The severity of this overidentification problem is understood by investigating the intersection of any pair of preferences explaining the same choice data.

A first observation is that any pair of preferences explaining a given  $(C_1, C_2)$  has to possess the binary pairs that two individuals disagree on,  $P_1$  and  $P_2$ . These refer to the pairs of alternatives about which the individuals have reverse tastes. Hence,  $(P_1, P_2)$  will be common to any preference pair  $(\succ_1, \succ_2)$  explaining a given  $(C_1, C_2)$ .

Moreover, we are able to detect the pairs of alternatives such that an influence is acquired for sure,  $Q_1$  and  $Q_2$ . Consider a binary problem such that both individuals have chosen  $x$  over  $y$ . If one of the individuals shows inconsistent behaviour in a larger problem, then the choice of  $x$  over  $y$  from the binary problem can only be the result of being influenced and any preference pair resulting in this behaviour will recognize that.  $Q_i$  identifies the pairs  $x, y$  such that individual  $j$  has been influenced by  $i$  to choose  $x$  over  $y$ . Thus,  $(Q_1, Q_2)$  will be common to any preference  $(\succ_1, \succ_2)$  for a given  $(C_1, C_2)$  as well.

Finally, since preferences are defined to be transitive, the ordered pairs that are not necessarily in  $P_i$  or  $Q_i$ , but implied by transitivity of  $\succ_i$  will be common to any  $(\succ_1, \succ_2)$  for the given  $(C_1, C_2)$ . Let us denote the transitive closure of  $P_i \cup Q_i$  as  $tr(P_i \cup Q_i)$ .<sup>12</sup> The following theorem states that the intersection of all pairs of preferences explaining a given choice data is  $(tr(P_1 \cup Q_1), tr(P_2 \cup Q_2))$ :<sup>13</sup>

**Theorem 2.** *Let  $C_i : \Omega_X \rightrightarrows X$  for  $i \in \{1, 2\}$  such that  $(C_1, C_2)$  is a Choice on Mutual Influence mechanism. Then the intersection of all preference pairs  $(\succ_1, \succ_2)$  explaining  $(C_1, C_2)$  is  $(tr(P_1 \cup Q_1), tr(P_2 \cup Q_2))$ .*

Theorem 2 ensures that we can recover a major part of underlying preferences and hence the influence individuals form on each other. The overidentification problem relates to those ordered pairs in  $R$ , the pairs of alternatives

<sup>12</sup>The transitive closure of a binary relation is the smallest transitive relation that contains it.

<sup>13</sup>Notice that this means not only  $tr(P_i \cup Q_i)$  is a common part of all preference pairs explaining a given choice data, but also it is the largest common part.

that both individuals choose the same out of the binary problem, and none of them has shown any inconsistency in a larger menu that would indicate the influence acquired. Hence the model does not help us to associate these ordered pairs uniquely to one of the individuals unless they are a part of the transitive closure of  $(P_i \cup Q_i)$ . Consider the extreme case where two individuals show exactly the same choice behaviour all over. Then since  $P_i = Q_i = \emptyset \neq R$ , we cannot conclude whether two individuals have actually the same sincere preferences or one of them has null preferences and is getting fully influenced by the other. On the other extreme, if we observe a pair of choice behaviours with a null  $R$ , we can completely identify the underlying preferences and the influence. However an empty  $R$  is not a necessary condition for full identification. The identification issue arises due to the fact that although an individual, say  $i$ , is actually indecisive between two alternatives,  $x$  and  $y$ , she does not show any choice inconsistency regarding those, although she is influenced by  $j$  to choose, say  $x$  over  $y$ . But this only happens if  $x$  and  $y$  are too ‘similar’ in terms of their relative comparison to the other alternatives for both of the agents: whatever is better than  $x$  for  $i$  is also not worse than  $y$  and whatever is worse than  $y$  for  $i$  is also not better than  $x$  at least for one of the agents. Moreover no alternative that is better than  $x$  is better than any alternative that is worse than  $y$ , again, at least for one of the agents.<sup>14</sup> This observation translates as a regularity condition on the pair of preferences that allows to recover underlying preferences uniquely.

**Definition 2.**  $(\succ_1, \succ_2)$  is regular if  $xy, yx \notin \succ_i$  but  $xy \in \succ_j$  imply that at least one of the following holds:

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<sup>14</sup>Notice that if we allow for indifference in addition to indecisiveness, (since  $x$  is indifferent to  $y$  implies that whatever is better (worse) than  $x$  is also better (worse) than  $y$ ) we will never be able to uniquely identify the indifference part of underlying preferences apart from those alternatives that both  $i$  and  $j$  are indifferent on.

- (i) there exists  $a \in X$  with  $ax \in \succ_i$  but  $ay \notin (\succ_1 \cup \succ_2)$
- (ii) there exists  $b \in X$  with  $yb \in \succ_i$  but  $xb \notin (\succ_1 \cup \succ_2)$
- (iii) there exist  $a, b \in X$  with  $ax, yb \in \succ_i$  but  $ab \notin (\succ_1 \cup \succ_2)$ .

With regularity assumption we sustain uniqueness in our characterization result.

**Theorem 3.** *Let  $C_i : \Omega_X \rightrightarrows X$  for  $i \in \{1, 2\}$ .  $C_1$  and  $C_2$  satisfy EXP, NULL and CoI if and only if there exists a unique regular pair of asymmetric and transitive preferences  $(\succ_1, \succ_2)$  that explain  $(C_1, C_2)$ .*

## 2. CHOICE ON SOCIAL INFLUENCE

Choice on Mutual Influence is a simple decision making mechanism for interacting individuals. Despite its simplicity, it is powerful enough to easily extend to more complicated structures and explain more convoluted forms of social interactions. In this section, we present two different extensions to multi-individual settings to demonstrate this.

Through out this section, we consider a group  $N$  of  $n$  individuals,  $N = \{1, 2, \dots, n\}$ , where  $\succ_i$  and  $C_i$  denote the preference relation and choice behaviour of  $i \in N$ , respectively. Facing a decision problem, individuals refer to their own preferences in a first stage, as before. If the first stage maximization results in a unique alternative, there is no room for social influence. If otherwise, they conduct a second stage where social influence is acquired.

In the first model we present, *Choice on Unanimous Influence*, the second stage decision making of an individual involves a consideration of the opinions of all other individuals. If all others agree, then the individual behaves accordingly. In other words, the second stage rationale is the Pareto aggregation of all other individuals' preferences.

The second model, *Choice on Expert Influence*, allows individuals to be influenced by different individuals over different choice problems. We consider individuals with expertise on certain sets of alternatives. If  $i$  is an expert on a set of alternatives, any question regarding those alternatives is only asked to  $i$  by all other individuals of the society.<sup>15</sup>

**2.1. Choice on Unanimous Influence.** Let  $i \in N$ . Facing a decision problem  $S$ ,  $i$  first maximizes  $\succ_i$  over  $S$ . Then, in the second stage,  $i$  maximizes the Pareto binary relation for the rest of the group,  $\succ_{N \setminus i}$ , which is the intersection of all  $\succ_j$ , for all  $j \in N \setminus \{i\}$ :

$$\succ_{N \setminus i} = \{xy \in X \times X : xy \in \succ_j \text{ for all } j \in N \setminus \{i\}\}.$$

**Definition 3.** We say that  $(C_1, C_2, \dots, C_n)$  is a *Choice on Unanimous Influence mechanism*, if there exists  $n$  asymmetric and transitive binary relations  $(\succ_1, \succ_2, \dots, \succ_n)$  such that:

$$C_i(S) = \text{Max}(\text{Max}(S, \succ_i), \succ_{N \setminus i})$$

for all  $S \in \Omega_X$  and for all  $i \in N$ .

The behavioural characterization of Choice on Unanimous Influence closely follows that of Choice on Mutual Influence. EXP stays the same, but we modify NULL and CoI properties to reflect the multi-individual requirements of

<sup>15</sup>Notice that both of the models we present only accommodate direct influence between members of a group, since influence is acquired from preferences. Another interesting setting would be the one where individuals are influenced by another individual's choices, instead of preferences. In those environments, there is also room for indirect influence between individuals. One generalization of our model that would be consistent with this approach would be the following: Individual  $j$  influences  $i$  if  $C_i(S) = C_j(\text{Max}(S, \succ_i))$  for all  $S$ . According to this model,  $i$  first refers to her own preferences and whenever indecisive, directly copies  $j$ 's actions. The behaviour generated by this model is actually only a refinement of the behaviour generated by our model and does not satisfy well-known rationality properties including EXP.

the setting.

**Expansion (EXP).** For any  $x \in X$ ,  $S, T \in \Omega_X$  and  $i \in N$ , if  $x \in C_i(S) \cap C_i(T)$ , then  $x \in C_i(S \cup T)$ .

**Nullipotency' (NULL').** For any  $S \in \Omega_X$  and  $i \in N$ ,  $\bigcup_{j \in N \setminus i} C_j(C_i(S)) = C_i(S)$ .

**Consistency of Influence' (CoI').** For any  $x, y \in X$  and  $i \in N$ , if  $x = C_i(xy)$  and there exists  $S \in \Omega_X$  with  $x \in S$  such that  $C_i(S) \neq C_i(S \setminus y)$ , then  $C_j(T) = C_j(T \setminus y)$  for all  $T \in \Omega_X$  with  $x \in T$  and for all  $j \in N \setminus \{i\}$ .

These three properties are necessary and sufficient to detect a group of individuals that are choosing consistently with Choice on Unanimous Influence model.

**Theorem 4.** *Let  $C_i : \Omega_X \rightrightarrows X$  for  $i \in N$ .  $C_1, C_2, \dots, C_n$  satisfy EXP, NULL' and CoI' if and only if  $(C_1, C_2, \dots, C_n)$  is a Choice on Unanimous Influence mechanism.*

**2.2. Choice on Expert Influence.** Let  $E_i \subset X$  denote expertise of individual  $i$ , i.e., the set of alternatives for which all the members of the group refer to  $i$ 's opinion whenever they are indecisive. For simplicity purposes, we assume that areas of expertise are disjoint;  $E_i \cap E_j = \emptyset$  for  $i \neq j$ . Notice that if an individual  $j$  does not possess any expertise, then  $E_j = \emptyset$ . Moreover, not necessarily all alternatives belong to an area of expertise, i.e., there may exist some alternatives for which no one is an expert.

Choice on Expert Influence works as follows: Facing a decision problem  $S$ ,  $i$  maximizes her own preference  $\succ_i$ . If this maximization does not give out a single alternative, then in a second stage,  $i$  seeks expert influence. If there is  $j \in N$  with expertise on any alternative for which  $i$  is indecisive about,  $i$

acquires influence from  $j$ . Notice that  $i$  can acquire influence from more than one individual in the same choice problem, but that will never be for the same set of alternatives.

Let  $(\succ_j |E_j)$  denote the part of the preference of  $j$  over the alternatives in her expertise, i.e.,  $(\succ_j |E_j) = \{xy \in \succ_j : x, y \in E_j\}$ . Then, the expert rationale of this society will be:  $\succ^E = \bigcup_{j \in N} (\succ_j |E_j)$ .

**Definition 4.** *We say that  $(C_1, C_2, \dots, C_n)$  is a Choice on Expert Influence mechanism, if there exists  $n$  asymmetric and transitive binary relations  $(\succ_1, \succ_2, \dots, \succ_n)$  and  $n$  areas of expertise  $E_1, E_2, \dots, E_n$  such that*

$$C_i(S) = \text{Max}(\text{Max}(S, \succ_i), \succ^E)$$

for all  $S \in \Omega_X$  and for all  $i \in N$ .

Now, consider  $(C_1, C_2, \dots, C_n)$ . What kind of properties on the choice behaviours in this group indicate a Choice on Expert Influence mechanism? Moreover, can we recover the underlying preferences and areas of expertise?

The characterizing properties of Choice on Expert Influence are also based on the properties of Choice on Mutual Influence. Apart from individual rationality properties, NULL and CoI will be modified to this more sophisticated form of interaction.

Let  $I = \{xy \in X \times X : \text{There exists } i \in N \text{ and } S \in \Omega_X \text{ with } x \in S \text{ such that } x = C_i(xy) \text{ and } C_i(S) \neq C_i(S \setminus y)\}$ . Hence,  $I$  is the set of all binary pairs that are associated with at least one inconsistency in the group. Following the logic of CoI, these are the binary pairs for which an influence has been acquired. Consistency of Expert Influence brings some structure on this set:

**Consistency of Expert Influence (CoEI).** For any  $xy \in I$ , there exist  $j \in N$  such that  $C_j(T) = C_j(T \setminus y)$  for all  $T \in \Omega_X$  with  $x \in T$ . Moreover, for

any  $z \in X$  with  $yz \in I$ , we have  $C_j(T') = C_j(T' \setminus z)$  for all  $T' \in \Omega_X$  with  $y \in T'$ .

CoEI makes sure that for any influence acquired, there is an expert  $j$  behind it. Moreover, if  $j$  is the expert on a pair of alternatives, than any related influence also has to come from  $j$ ; it is not possible to be influenced by individual  $j$  to choose, say,  $x$  over  $y$ , but then to be influenced by another individual  $k$  to choose, say,  $y$  over  $z$ . Notice that this indeed the case since areas of expertise are disjoint.

CoEI accounts for the influence acquired. We now introduce the NULL counterpart of this setting, a property that accounts for the multi-valued decision outcomes. Binding Influence states that if there is an expert on a certain set of issues, it is not possible to be indecisive, yet not to be influenced by this expert:

**Binding Influence (BI).** For any binary chain  $x_1x_2, x_2x_3, \dots, x_{t-1}x_t \in I$ , we have  $x_lx_k \neq C_i(x_lx_k)$  for any  $l, k \in \{1, 2, \dots, t\}$ , for all  $i \in N$ .

The existence of a binary chain in  $I$  indicates the existence of an expert for the alternatives that constitute a part of this chain. BI ensures that the expert is influential. In other words, if an individual is not able to choose a single alternative from a binary set, then there cannot exist an expert with a strict preference over these alternatives.

CoEI and BI are the properties that build the interaction links between individuals. Apart from these, we need two individual rationality properties: EXP and a weakening of WWARP for correspondences, another common property

of two stage maximization procedures (Manzini and Mariotti, 2007).<sup>16</sup>

**Weak WARP\* (WWARP\*).** For any  $x, y \in X$  and  $i \in N$ , if  $x = C_i(xy)$  and  $x \in C_i(S)$  for some  $S \in \Omega_X$  with  $x, y \in S$ , then  $y \notin C_i(S)$ .

WWARP\* prohibits the choice of an alternative  $y$  from a set where another alternative  $x$ , which is uniquely chosen over  $y$  from the binary problem, has been chosen.<sup>17</sup>

**Theorem 5.** *Let  $C_i : \Omega_X \rightrightarrows X$  for  $i \in N$ .  $C_1, C_2, \dots, C_n$  satisfy EXP, WWARP\*, CoEI and BI if and only if  $(C_1, C_2, \dots, C_n)$  is a Choice on Expert Influence mechanism.*

Hence, given a group of individuals whose choice behaviours satisfy the above properties, we can detect the experts, the areas of expertise and the influence that individuals acquire from the experts simply by following choice inconsistencies.

### 3. FURTHER COMMENTS

The previous section aimed to show that Choice on Mutual Influence can easily be extended to more sophisticated interaction environments and the revealed preference approach can be used to detect the particular manner of interaction, underlying preferences and influence acquired in those environments. In this section, we return back to our baseline model and demonstrate

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<sup>16</sup>WWARP for choice functions states that if  $x$  is chosen over  $y$  from a binary menu, and  $x$  is chosen from a set  $S$ , where  $y \in S$ , then  $y$  cannot be chosen from any set  $T$  with  $T \subset S$ .

<sup>17</sup>It is also possible to characterize Choice on Mutual Influence with EXP, WWARP\*, CoI and a weakening of NULL to binary problems. Since weakening NULL comes with the cost of additional WWARP\* axiom, we prefer the previous characterization.

other uses of Choice on Mutual Influence. Let us list several settings where our model and analysis become particularly helpful:

(i) *Influence vs. Homophily*: Homophily refers to the tendency to create social ties with people that are similar to one's self.<sup>18</sup> Both homophily and social influence result in behavioral resemblances between connected people. However it is not trivial to distinguish these two forces from one another: Do socially connected individuals behave similarly because one of them influences the other or do they behave similarly because they indeed share similar tastes that has led them to behave similarly and these similar tastes are the reason they have got socially connected to begin with? This phenomenon is known as the identification problem of homophily and social influence and it is mainly challenged by economists from an econometrical perspective. Many studies document that both effects prevail simultaneously and distinguishing one from the other requires strong parametrical assumptions (Aral et al., 2009; La Fond and Neville, 2010; Manski, 1993; Noel and Nyhan, 2011; Shalizi and Thomas, 2010).<sup>19</sup>

Our model provides a novel approach to this identification problem. Behavior consistent with our testable axioms confirms the existence of social ties between individuals and revealed preference argument allows to recover the underlying preferences and the influence created. Then, identifying homophily

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<sup>18</sup>For an overview of research on homophily in general see McPherson et al. (2001), in couples see Blackwell and Lichter (2004), on economic networks see Currarini et al. (2009).

<sup>19</sup>A major part of the works concentrates around adolescent behavior such as school achievement, use of drugs and recreational activities among high school children (Bramoullé et al., 2009; Calvo-Armengol et al., 2009; Manski, 1993) and innovation diffusion (Aral, 2011; Iyengar et al., 2011). Recently online networks have drawn particular attention since they provide a powerful data source where the network structure is easily observable, hence this structure itself may provide additional information to solve this identification problem (Anagnostopoulos et al., 2008; Aral et al., 2009; Aral and Walker, 2012; Lewis et al., 2012).

becomes an issue of comparing individual preferences. Let us illustrate this point by the example of Ian and Jane. Consider once again their choice behavior from the menus including the alternatives  $A, B$  and  $C$ :

| Menus      | Ian | Jane |
|------------|-----|------|
| <b>ABC</b> | BC  | C    |
| <b>AB</b>  | A   | A    |
| <b>BC</b>  | BC  | BC   |
| <b>AC</b>  | C   | C    |

We observe  $i$  and  $j$  choosing exactly the same out of the menus  $\{A, B\}$ ,  $\{B, C\}$  and  $\{A, C\}$  but showing slightly different behaviours in the larger menu,  $\{A, B, C\}$ . How similar  $i$  and  $j$  are to each other? Do we have any evidence of peer influence in these choices?

As we have discussed in the introduction, choice behaviours of  $i$  and  $j$  do indeed satisfy Expansion, Nullipotency and Consistency of Influence axioms, indicating a Choice on Mutual Influence mechanism. Our identification strategy uniquely reveals the underlying preferences as  $C \succ_i A$  and  $A \succ_j B$ .<sup>20</sup> Hence the influence  $i$  creates on  $j$  is to choose  $A$  over  $B$ , and the influence  $i$  acquires from  $j$  is in her choice of  $C$  over  $A$  both in the binary menu and the larger menu. What about the level of homophily in this pair? Homophily is about the similarity of individuals' tastes, that can be assessed by comparing their preferences. The closer the preferences are to each other, the more similar are individuals' tastes.  $\succ_i$  and  $\succ_j$  do not have any binary comparison in common. However they both fail to compare  $B$  and  $C$ , suggesting rather a

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<sup>20</sup>Since both of them fails to choose uniquely from  $\{B, C\}$ , neither  $\succ_i$  nor  $\succ_j$  ranks  $B$  and  $C$ .  $C_i(ABC) \neq C_i(AC)$ , although  $A = C_i(AB)$  implies that  $\succ_i$  is indecisive over  $A$  and  $B$ , whereas  $A \succ_j B$ . Similarly,  $C_j(ABC) \neq C_j(BC)$ , although  $C = C_j(AC)$  implies that  $\succ_j$  is indecisive over  $A$  and  $C$ , whereas  $C \succ_i A$ .

low level of similarity then no similarity. A proper measure of homophily can be constructed by making use of a similarity function between preferences.<sup>21</sup>

(ii) *Group Influence vs. Group Identification:* Social identity refers to a person's sense of self, based on perceived memberships in social groups.<sup>22</sup> An important insight of social identity theory is about identification; the attainment of the prescribed behaviour of the corresponding social group. Once an individual identifies herself with a group, she begins adopting behaviours consistent with the norms of that group. According to the interpersonal-intergroup continuum argument (Tajfel and Turner, 1986), this behaviour will be somewhere in between two extreme forms of social behaviour. At one extreme, there is the interpersonal behaviour, which refers to the interaction between a group of individuals and is purely determined by individual characteristics. The other extreme, intergroup behaviour, on the other hand, is completely determined by group membership characteristics. We claim that our model accommodates interpersonal-intergroup continuum argument and allows to identify the degree to which group identification has taken place.

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<sup>21</sup>One such measure would be Kendall's correlation coefficient  $\tau$ , which measures the correlation between two preferences based on the distance between them:  $\tau(\succ_1, \succ_2) = 1 - 2 \frac{d(\succ_1, \succ_2)}{\max d(\succ_1, \succ_2)}$ , where  $d(\succ_1, \succ_2)$  is the generalization of Kemeny-Snell distance to incomplete preferences and  $\max d(\succ_1, \succ_2)$  denotes the maximum possible distance between two binary relations defined over  $X$  (Bogart, 1973). This distance function simply counts the number of binary pairs on which these preferences do not agree, i.e.,  $d(\succ_i, \succ_j) = |\succ_i \setminus \succ_j| + |\succ_j \setminus \succ_i|$ . Notice that  $\tau$  is equal to 1 when two preferences are identical, and  $-1$  when they are completely reverse. As noted in Bogart (1973),  $\tau$  is the only linear function of  $d$  that takes a value 1 when two preferences are identical, and  $-1$  when they are completely reverse.

<sup>22</sup>Social identity theory is developed by Tajfel and Turner (1986) and introduced to the analysis of economic behaviour by Akerlof and Kranton (2000), leading to a wide empirical, theoretical and experimental literature. For an extensive review of identity economics, see Akerlof and Kranton (2010).

To see this, consider a group  $N$  of individuals and a binary relation that represents the norms of the group (*prescriptions* in Akerlof-Kranton terminology):  $\succ^N$ . Assume that  $\succ^N$  is common knowledge and individuals adopt it in their two-stage decision making procedures. If an individual uses  $\succ^N$  as the first stage relation and her own preference,  $\succ_i$  as the second stage relation, only to refine her choices further, she is said to be identifying herself completely with her group. Hence she will be appealing to the intergroup extreme of the continuum. If, instead, an individual uses  $\succ_i$  in the first stage and refers to  $\succ^N$  in the second stage, then she is only being influenced by the group norms whenever she cannot decide on her own. In this case we can talk about group influence, but to a much lesser degree than the former case. If, on the other hand, an individual uses  $\succ_i$  in the first stage, and gets influenced by some other member(s) of the group at the second stage, then she will be at the other extreme of the continuum, demonstrating interpersonal behaviour. Given the individual decision outcomes, as  $\succ^N$  is known, our identification strategy allows to distinguish these behaviours from each other and detect the degree of group identification.

#### 4. CONCLUDING REMARKS

Interaction is an essential element of daily life and economic decisions and yet microfoundations of interaction have not been explored. This study presents a first attempt in providing a choice-theoretic approach to interaction.

We model interaction as a means to deal with an inherent individual necessity, inability to choose due to incomplete preferences. Our baseline model, Choice on Mutual Influence, despite its simplicity, is flexible enough to grasp at more convoluted forms of interactions. The characterization of Choice on Mutual Influence lays out two important properties of the choice behaviour

consistent with the model: First, choice inconsistencies in one’s behaviour correspond to the influences acquired, and hence are traced back to the preferences of the influential individual. Second, all the influence that can be acquired is already inherent in the choice behaviour. These properties correspond to our CoI and NULL properties respectively. A key observation, which we also explore in the second section of this paper, is that modifications of these two properties indeed aid to characterize various interaction environments.

As the studies of the last decades have shown, expanding the realm of choice data has provided new explanations to the behaviours that were once classified as ‘irrational’. The novelty of this paper lies in its focus on multiple choice behaviours instead of one. We believe there is still a lot to explore once we go beyond more than one individual’s behaviour.

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## 5. APPENDIX

**Proof of Theorem 1.** Necessity is fairly straightforward, thus omitted. We prove the sufficiency part.

For  $i \in \{1, 2\}$ , define  $\succ_i \subseteq X \times X$  as follows:

$$xy \in \succ_i \text{ iff } C_i(S) = C_i(S \setminus y) \text{ for all } S \text{ with } x, y \in S.$$

Notice that  $\succ_i$  is asymmetric by definition since  $xy, yx \in \succ_i$  implies that  $\emptyset = C_i(xy)$ . To see transitivity of  $\succ_i$  take any  $x, y, z \in X$  with  $xy, yz \in \succ_i$ . Take any  $S \in \Omega_X$  with  $x, z \in S$ . If  $y \in S$ , then by  $yz \in \succ_i$ ,  $C_i(S) = C_i(S \setminus z)$ . Let  $y \notin S$  and consider  $S \cup y$ . By  $xy \in \succ_i$ ,  $C_i(S \cup y) = C_i(S)$ . By  $yz \in \succ_i$ ,  $C_i(S \cup y) = C_i(S \cup y \setminus z)$ . By  $xy \in \succ_i$ ,  $C_i(S \setminus z \cup y) = C_i(S \setminus z)$ . But then,  $C_i(S) = C_i(S \setminus z)$ .

Take any  $S \in \Omega_X$ . We now show that  $Max(Max(S, \succ_i), \succ_j) \subseteq C_i(S)$ . Take any  $x \in Max(Max(S, \succ_i), \succ_j)$ . Obviously  $x \in Max(S, \succ_i)$ . Take any  $z \in Max(S, \succ_i)$ . Assume for a contradiction that  $z = C_i(xz)$ . Since  $x \in Max(S, \succ_i)$ , there exists  $T \in \Omega_X$  with  $x, z \in T$  such that  $C_i(T) \neq C_i(T \setminus x)$ . But then, CoI implies that  $zx \in \succ_j$ , contradicting with  $x \in Max(Max(S, \succ_i), \succ_j)$ , as expected. Hence  $x \in C_i(xz)$  for all  $z \in Max(S, \succ_i)$ . But then, EXP implies that  $x \in C_i(Max(S, \succ_i))$ .

For any  $y \in S \setminus Max(S, \succ_i)$ , there exists  $y' \in Max(S, \succ_i)$  such that  $y'y \in \succ_i$  since  $\succ_i$  is transitive. But then  $C_i(Max(S, \succ_i)) = C_i(Max(S, \succ_i) \cup y)$ . Iterative application of the same argument yields that  $C_i(Max(S, \succ_i)) = C_i(S)$ , concluding that  $x \in C_i(S)$ .

We finally show that  $C_i(S) \subseteq \text{Max}(\text{Max}(S, \succ_i), \succ_j)$ . Take any  $x \in C_i(S)$ . By definition of  $\succ_i$ ,  $x \in \text{Max}(S, \succ_i)$ . Assume for a contradiction that there exists  $y \in \text{Max}(S, \succ_i)$  such that  $yx \in \succ_j$ . Notice that this implies  $x \notin C_j(T)$  for any  $T \in \Omega_X$  with  $x, y \in T$ .

- Case 1: Let  $y \in C_i(S)$ . By NULL,  $C_j(C_i(S)) = C_i(S)$ . Since both  $x, y \in C_i(S)$ , we contradict with  $yx \in \succ_j$ .

- Case 2: Let  $y \notin C_i(S)$ : Then, as we have shown earlier,  $y \notin \text{Max}(\text{Max}(S, \succ_i), \succ_j)$ . Since  $y \in \text{Max}(S, \succ_i)$ , there exists  $z \in \text{Max}(\text{Max}(S, \succ_i), \succ_j)$  with  $zy \in \succ_j$  by transitivity of  $\succ_j$ . But then  $z \in C_i(S)$ . By transitivity  $zx \in \succ_j$ , implying that  $x \notin C_j(T)$  for any  $T \in \Omega_X$  with  $x, z \in T$ . But then, by NULL,  $C_j(C_i(S)) = C_i(S)$ , creates the desired contradiction since both  $x, z \in C_i(S)$ .  
□

**Independence of the properties.** Consider  $X = \{x, y, z\}$ ,  $x = C_1(xy)$ ,  $y = C_1(yz)$ ,  $z = C_1(xz)$  and  $x = C_1(xyz)$ . If  $x = C_2(xy)$ ,  $y = C_2(yz)$ ,  $x = C_2(xz)$ ,  $y = C_2(xyz)$ , CoI and NULL are satisfied but EXP is not. If instead,  $y = C_2(xy)$ ,  $z = C_2(yz)$ ,  $x = C_2(xz)$ ,  $x = C_2(xyz)$ , CoI is not satisfied but the others are. Finally, if  $xy = C_2(xy)$ ,  $z = C_2(yz)$ ,  $z = C_2(xz)$ ,  $z = C_2(xyz)$ , NULL is the only one that is not satisfied.

**Proof of Theorem 2.** Let  $(C_1, C_2)$  be a Choice on Mutual Influence mechanism. Fix  $i, j \in \{1, 2\}$  with  $i \neq j$ , without loss of generality.

*Necessity.* Consider any  $(\succ_1, \succ_2)$  explaining  $(C_1, C_2)$ . We will show that  $\text{tr}(P_i \cup Q_i) \subset \succ_i$ , hence a subset of the intersection as well. Take any  $xy \in P_i$ . Since  $y = C_j(xy)$ , we have  $xy \notin \succ_j$ . But then  $x = C_i(xy)$  implies that  $xy \in \succ_i$ . Now take any  $xy \in Q_i$ . As there exists  $S \in \Omega_X$  with  $x \in S$  and  $C_j(S) \neq C_j(S \setminus y)$ , we have  $xy \notin \succ_j$ . But then,  $x = C_i(xy)$  implies that  $xy \in \succ_i$ . Thus for  $xy \in (P_i \cup Q_i)$  we also have  $xy \in \succ_i$ . Transitivity of  $\succ_i$  proves the claim.

*Sufficiency.* We now show that for  $\succ_i^* = tr(P_i \cup Q_i)$ , there exists  $\succ_j^*$  such that  $(\succ_1^*, \succ_2^*)$  also explains  $(C_1, C_2)$ . In the proof of Theorem 1 we have shown that  $(\succ_1, \succ_2)$  explains  $(C_1, C_2)$  for  $\succ_1 = \{xy \in X \times X : C_1(S) = C_1(S \setminus y) \text{ for all } S \in \Omega_X \text{ with } x \in S\}$  and  $\succ_2 = \{xy \in X \times X : C_2(S) = C_2(S \setminus y) \text{ for all } S \in \Omega_X \text{ with } x \in S\}$ . Now let  $\succ_j^* = \succ_j$ . We will show that  $Max(Max(S, \succ_i^*), \succ_j) = Max(Max(S, \succ_i), \succ_j)$  and  $Max(Max(S, \succ_j), \succ_i^*) = Max(Max(S, \succ_j), \succ_i)$  for all  $S \in \Omega_X$ .

First notice that  $\succ_i = P_i \cup Q_i \cup R$  and  $\succ_j = P_j \cup Q_j \cup R$ . Now, take any  $S \in \Omega_X$  and  $x \in Max(Max(S, \succ_i^*), \succ_j)$ . Assume for a contradiction that  $x \notin Max(Max(S, \succ_i), \succ_j)$ . There are two possible cases, where both of them result in the desired contradiction as we show below:

-Case 1:  $x$  is eliminated in the first stage: Since  $\succ_i$  is transitive, there exists  $y \in Max(S, \succ_i)$  with  $yx \in (\succ_i \setminus \succ_i^*)$ . But then,  $yx \in R$ , which means  $yx \in \succ_j$ . Since  $\succ_i^* \subset \succ_i$ , we have  $y \in Max(S, \succ_i) \subset Max(S, \succ_i^*)$ . But then,  $yx \in \succ_j$  creates a contradiction with  $x \in Max(Max(S, \succ_i^*), \succ_j)$ .

-Case 2:  $x$  is eliminated in the second stage: Then there exists  $y \in Max(S, \succ_i)$  with  $yx \in \succ_j$ . But since  $y \in Max(S, \succ_i) \subset Max(S, \succ_i^*)$ , this creates a contradiction with  $x \in Max(Max(S, \succ_i^*), \succ_j)$ .

Now take any  $x \in Max(Max(S, \succ_i), \succ_j)$  and assume for a contradiction that  $x \notin Max(Max(S, \succ_i^*), \succ_j)$ . Similarly, we have the following cases, which end with contradictions as desired:

-Case 1:  $x$  is eliminated in the first stage: Since  $Max(S, \succ_i) \subset Max(S, \succ_i^*)$ , we contradict with  $x \in Max(Max(S, \succ_i), \succ_j)$ .

-Case 2:  $x$  is eliminated in the second stage: Then, there exists  $y \in Max(S, \succ_i^*)$  with  $yx \in \succ_j$ . Since  $y \notin Max(S, \succ_i)$ , by transitivity of  $\succ_i$ , there exists  $z \in Max(S, \succ_i)$  with  $zy \in (\succ_i \setminus \succ_i^*)$ . But then  $zy \in R$ , which implies that  $zy \in \succ_j$ . But then, by transitivity  $zx \in \succ_j$ , creating the desired contradiction with  $x \in Max(Max(S, \succ_i), \succ_j)$ .

Finally, we show that  $Max(Max(S, \succ_j), \succ_i^*) = Max(Max(S, \succ_j), \succ_i)$  for all  $S \in \Omega_X$ . Take any  $S \in \Omega_X$  and  $x \in Max(Max(S, \succ_j), \succ_i^*)$  and assume for a contradiction that  $x \notin Max(Max(S, \succ_j), \succ_i)$ . Then, there exists  $y \in Max(S, \succ_j)$  with  $yx \in (\succ_i \setminus \succ_i^*)$ . But then,  $yx \in R$  and hence,  $yx \in \succ_j$ , contradicting with  $x \in Max(S, \succ_j)$ . Now, take any  $x \in Max(Max(S, \succ_j), \succ_i)$ . Since  $\succ_i^* \subset \succ_i$ , we directly have  $x \in Max(Max(S, \succ_j), \succ_i^*)$ , establishing the proof.  $\square$

**Proof of Theorem 3.** We only prove sufficiency. Let  $(C_1, C_2)$  as defined. Consider  $\succ_i = \{xy \in X \times X : C_i(S) = C_i(S \setminus y) \text{ for all } S \in \Omega_X \text{ with } x \in S\}$  for  $i \in \{1, 2\}$ . By the proof of Theorem 1, we know that  $(\succ_1, \succ_2)$  explains  $(C_1, C_2)$ . We only need to show regularity and uniqueness.

To see regularity, take any  $x, y \in X$  with  $xy, yx \notin \succ_i$ , but  $xy \in \succ_j$ , for  $i, j \in \{1, 2\}$  and  $i \neq j$ . Assume for a contradiction that  $(\succ_1, \succ_2)$  is not regular, which means:

- (i) for all  $a \in X$  with  $ax \in \succ_i$ ,  $ay \in (\succ_1 \cup \succ_2)$
- (ii) for all  $b \in X$  with  $yb \in \succ_i$ ,  $xb \in (\succ_1 \cup \succ_2)$
- (iii) for all  $a, b \in X$  with  $ax, yb \in \succ_i$ ,  $ab \in (\succ_1 \cup \succ_2)$ .

By definition of  $\succ_i$ , there exists  $S \in \Omega_X$  with  $x \in S$  such that  $C_i(S) \neq C_i(S \setminus y)$ . There are two possible cases, where both of them result in the desired contradiction as we show below:

-Case 1: There exists  $z \in C_i(S)$ , but  $z \notin C_i(S \setminus y)$ : First, notice that  $z \neq y$ , since  $xy \in \succ_j$  and transitivity of  $\succ_i$  means that there exists  $a \in Max(S, \succ_i)$  such that  $ax \in \succ_i$ , for  $y$  to survive the second stage elimination. But, by (i),  $ay \in (\succ_1 \cup \succ_2)$ , hence  $y \notin C_i(S)$ . Now, let  $z \neq y$ . Then, there exists  $b \in S$  such that  $bz \in \succ_j$ ,  $yb \in \succ_i$ . Moreover there does not exist any  $a \in S$  with  $ab \in \succ_i$ , in particular  $xb \notin \succ_i$  (\*). But then, by (ii),  $xb \in \succ_j$ , which implies  $x \notin Max(S, \succ_i)$  for  $z$  to survive the second stage elimination. By transitivity

of  $\succ_i$ , there exists  $a \in \text{Max}(S, \succ_i)$  with  $ax \in \succ_i$ . By (iii),  $ab \in (\succ_1 \cup \succ_2)$ . By (\*),  $ab \in \succ_j$ , but then  $ab, bz \in \succ_j$  implies that  $z \notin C_i(S)$ , giving the desired contradiction.

-Case 2: There exists  $z \in C_i(S \setminus y)$ , but  $z \notin C_i(S)$ : First assume that  $z$  survives the first stage elimination in  $S$  but there exists  $a \in (\text{Max}(S, \succ_i) \setminus \text{Max}(S \setminus y, \succ_i))$  with  $az \in \succ_j$ . Since  $\text{Max}(S, \succ_i) \subset \text{Max}(S \setminus y, \succ_i) \cup \{y\}$ , we have  $a = y$ . But then, by (ii),  $xz \in \succ_2$ , which implies  $x \notin \text{Max}(S \setminus y, \succ_i)$  for  $z$  to survive the second stage elimination. By transitivity of  $\succ_i$ , there exists  $a \in \text{Max}(S \setminus y, \succ_i)$  with  $ax \in \succ_i$ . By (iii),  $az \in \succ_j$ , but then  $z \notin C_i(S \setminus y)$ . Hence  $z$  cannot survive the first stage elimination in  $S$ , although it survives the first stage elimination in  $(S \setminus y)$ , which implies  $yz \in \succ_i$  and for all  $a \in S$  with  $a \neq y$ , we have  $az \notin \succ_i$ , in particular  $xz \notin \succ_i$  (\*). But then, by (ii),  $xz \in \succ_j$ , which implies  $x \notin \text{Max}(S \setminus y, \succ_i)$  for  $z$  to survive the second stage elimination. By transitivity of  $\succ_i$ , there exists  $a \in \text{Max}(S \setminus y, \succ_i)$  with  $ax \in \succ_i$ . By (iii),  $az \in (\succ_1 \cup \succ_2)$ . By (\*),  $az \in \succ_j$ , but then  $z \notin C_i(S \setminus y)$ , giving the desired contradiction in order to establish that  $(\succ_i, \succ_j)$  is regular.

Now we only need to show uniqueness. Assume for a contradiction that there exists another regular pair  $(\succ'_1, \succ'_2) \neq (\succ_1, \succ_2)$  that explains  $(C_1, C_2)$ . Take any  $\succ'_i \neq \succ_i$  for  $i \in \{1, 2\}$ . There are two possible cases, where both of them result in the desired contradiction as we show below:

-Case 1: There exists  $x, y \in X$  with  $xy \in (\succ'_i \setminus \succ_i)$ : Then, there exist  $S \in \Omega_X$  with  $x, y \in S$  such that  $C_i(S) \neq C_i(S \setminus y)$ . But since  $\succ'_i$  is transitive  $\text{Max}(S, \succ'_i) = \text{Max}(S \setminus y, \succ'_i)$  and hence  $C_i(S) = C_i(S \setminus y)$ , absurd.

-Case 2: There exists  $x, y \in X$  with  $xy \in (\succ_i \setminus \succ'_i)$ : Since for all  $S \in \Omega_X$  with  $x, y \in S$ ,  $C_i(S) = C_i(S \setminus y)$ ,  $yx \notin \succ'_i$  and  $xy \in \succ'_j$ . Now notice that if  $C_i(S) = C_i(S \setminus y)$  for all  $S$  with  $x \in S$ , then the following holds:

(i)  $yb \in \succ_i$  implies  $xb \in (\succ_1 \cup \succ_1)$  for any  $b$  in  $X$ : Assume for a contradiction that there exists  $b \in X$  with  $yb \in \succ_i$  but  $xb \notin (\succ_1 \cup \succ_2)$ . Then,  $C_i(bxy) \neq xb = C_i(xb)$ , absurd.

(ii)  $ax, yb \in \succ_i$  implies  $ay, ab \in (\succ_1 \cup \succ_2)$  for any  $a, b \in X$ : Assume for a contradiction that there exists  $a, b \in X$  with  $ax, yb \in \succ_i$  but  $ab \notin (\succ_1 \cup \succ_2)$ . But then  $C_i(abxy) \neq ab = C_i(abx)$ , absurd. Now assume for a contradiction that there exists  $a, b \in X$  with  $ax, yb \in \succ_i$  but  $ay \notin (\succ_1 \cup \succ_2)$ . But then  $C_i(abxy) = ay \neq C_i(abx)$ , which is absurd.

But then, since  $xy, yx \notin \succ'_i$  and  $xy \in \succ'_j$ , (i) and (ii) contradicts with regularity, establishing the proof.  $\square$

**Proof of Theorem 4.** For  $i \in N$ , define  $\succ_i \subseteq X \times X$  as follows:

$$xy \in \succ_i \text{ iff } C_i(S) = C_i(S \setminus y) \text{ for all } S \text{ with } x, y \in S.$$

The proof replicates the proof of Theorem 1 by using  $\succ_{N \setminus i}$  instead of  $\succ_j$  and using the quantifier ‘for all  $j$ ’, instead of ‘ $j$ ’, wherever necessary. The properties are used exactly the same way with their two individual counterparts.  $\square$

**Proof of Theorem 5.** We only prove sufficiency. For  $i \in N$ , define  $\succ_i \subseteq X \times X$  as follows:

$$xy \in \succ_i \text{ iff } C_i(S) = C_i(S \setminus y) \text{ for all } S \text{ with } x, y \in S.$$

As shown in the proof of Theorem 2,  $\succ_i$  is asymmetric, transitive and  $C_i(S) = C_i(\text{Max}(S, \succ_i))$  for any  $S \in \Omega_X$ . Now, for  $j \in N$ , define  $E_j$  as follows:

$$x, y \in E_j \text{ iff } xy \in \text{tr}(\succ_j | E'_j)$$

where,

$$x, y \in E'_j \text{ iff } xy \in (I \cap \succ_j) \text{ and for all } yz, tx \in I, \text{ we have } yz, tx \in \succ_j.$$

Notice that by CoEI, for any  $xy \in I$ , there exists  $j \in N$  such that  $x, y \in E'_j \subseteq E_j$ . If  $E_j \cap E_k \neq \emptyset$  for some  $j, k \in N$ , then discard  $E_j \cap E_k$  either from  $E_j$

or  $E_k$ , randomly. Notice that this does not break transitivity of  $(\succ_j | E_j)$  or  $(\succ_k | E_k)$  thanks to CoEI.

Finally, let  $\succ^E = \bigcup_{j \in N} (\succ_j | E_j)$ .  $\succ^E$  is asymmetric and transitive, since each  $(\succ_i | E_i)$  is asymmetric and transitive and areas of expertise are disjoint.

Now take  $i \in N$  and  $S \in \Omega_X$ . First, take  $x \in \text{Max}(\text{Max}(S, \succ_i), \succ^E)$ . For any  $z \in \text{Max}(S, \succ_i)$ , we have  $x \in C_i(xz)$ . This holds since  $z = C_i(xz)$  and  $x \in \text{Max}(S, \text{succ}_i)$  imply that  $zx \in I$ . But then  $zx \in \succ^E$ , contradicting with  $x \in \text{Max}(\text{Max}(S, \succ_i), \succ^E)$ . Hence,  $x \in C_i(xz)$  for all  $z \in \text{Max}(S, \succ_i)$ . But then, by EXP,  $x \in C_i(\text{Max}(S, \succ_i) = C_i(S, \succ_i))$ .

Now take  $x \in C_i(S)$ . By definition of  $\succ_i$ ,  $x \in \text{Max}(S, \succ_i)$ . Assume for a contradiction that  $x \notin \text{Max}(\text{Max}(S, \succ_i), \succ^E)$ . But then, since  $\succ^E$  is transitive, there exists  $y \in \text{Max}(\text{Max}(S, \succ_i), \succ^E)$  such that  $yx \in \succ^E$ . By the previous part,  $y \in C_i(S)$ . If  $x = C_i(xy)$ , since  $x \in C_i(S)$ , by WWARP\*, we have  $y \notin C_i(S)$ , contradiction. If  $y = C_i(xy)$ , since  $y \in C_i(S)$ , by WWARP\*, we have  $x \notin C_i(S)$ , contradiction. Finally, if  $xy = C_i(xy)$ , then by BI, there does not exist a binary chain  $yz_1, z_1z_2, \dots, z_tx \in I$ , which contradicts with  $yx \in \succ^E$ , establishing the proof.