TAXATION, UNEMPLOYMENT AND WORKING TIME

IN MODELS OF ECONOMIC GROWTH

by

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Abstract
This paper combines collective bargaining over wages and working time with models of endogenous and neoclassical growth. Public expenditure is funded by taxes on capital and labour supplied by infinitely-lived households in a closed economy. Taxes on labour are generally inefficient in both growth models, there is a "dynamic Laffer Curve", and employment is increased by a reduction of working hours below the collective bargaining level – except in the case of a monopoly union. Although growth is maximised by competitive (efficient) hours, welfare-optimal working time is below the collective bargain when union are 'too weak', and vice-versa.

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Section 1: Introduction

The relationship of the tax system to unemployment has become increasingly important in view of persistently high structural unemployment in Europe. The enduring nature of this problem suggests that unemployment should be considered in the context of models of economic growth and fiscal policy. A notable step in this direction – and an exception to the standard assumption of full employment in growth theories – is the recent paper by Daveri and Tabellini (2000). They develop an overlapping-generations growth model with government and unemployment, and argue both theoretically and empirically that the latter is exacerbated by the taxation of labour.

An equally neglected aspect of economic growth has been the endogenous supply of labour by employed workers. This has obvious implications for the debate over working time and unemployment, and for policy measures such as the 35-hour week introduced in the year 2000 in France. The long-term consequences of such measures cannot be properly evaluated in the usual partial equilibrium context.

In this paper, we extend the approach of Daveri and Tabellini (2000) to incorporate into two simple models of economic growth the endogenous determination of both wages and hours by collective bargaining, while firms choose employment in a right-to-manage (RTM) framework, and government fiscal policy determines tax and distribution parameters. The first model is the basic AK-model of endogenous growth, extended to include bargaining, unemployment and a government sector. The second model is a simple neoclassical model with exogenous technological progress and the same extensions. The use of simple Cobb-Douglas technologies and preferences allows us to derive the balanced growth paths in both models and compare the employment and welfare effects of various policy measures, as well as the conflicts between three classes of infinitely-lived households; capitalists, employed workers, and the unemployed.

In spite of fundamental differences between the two models of growth, we find two remarkable similarities in the welfare and employment effects of policy measures. First, in both models, it turns out that a zero tax on labour is generally optimal, whatever the demand for public goods and government expenditure. Second, and in contrast to widely held views on the likely negative effects of work-sharing or mandatory working time reduction such as the 35-hour week in France, we find positive welfare effects when union bargaining power is 'low enough'. We also find positive employment effects for an hours reduction unless the union is a monopoly union. These are 'second best' results, in the context of balanced growth with equilibrium unemployment, caused by unemployment benefits that are 'too high' for full employment.

As Daveri and Tabellini (2000) have noted, there has been very little discussion of unemployment in the extensive literature on growth and fiscal policy. An exception is Aghion and Howitt (1999), who do consider unemployment caused by technical progress and obsolescence at firm level, with search and deterministic or random matching of unemployed workers with vacancies. However, they do not consider working time, unemployment benefits or fiscal policy. Contensou and Vranceanu (2000) discuss dynamic random matching with variable working time, and dynamic cost minimisation with adjustment costs and labour hoarding, under exogenous demand. These are partial equilibrium models which do not include technical progress, investment, or a government sector. Marimon and Zilibotti (2000) also consider dynamic random matching in the absence of productivity growth or a government sector, but with bargaining over hours and wages. However, they restrict output elasticities of hours and workers to be equal. These models all find a U-shaped relationship between hours and unemployment.

A competitive general equilibrium model of infinitely-lived households and firms with endogenous hours and employment *probabilities* is developed by Fitzgerald (1998). There is no collective bargaining or government sector in this model, which is thus very different from ours, but Fitzgerald's simulations show that reducing hours generate substantial increases in employment probabilities, albeit at the cost of reduced productivity and output. A model of endogenous growth with intermediate products, imperfect competition in product markets, and union bargaining leading to random unemployment of unskilled workers is presented by Palokangas (2000, Chap. 8), but there is no public sector in this model and working time is exogenously given.

Recent empirical studies of working time and unemployment such as Kapteyn, Kalwij and Zaidi (2000), Hunt (1999) and many others surveyed by Koch (2001), find mainly insignificant effects of hours-reduction, though often with a negative sign. They usually find that hourly wages rise after reducing hours in order to maintain weekly earnings, and that this effect is additional to the normal productivity-related secular increase. This finding is somewhat surprising, because it suggests that workers gain extra leisure with no loss of income, and furthermore, that the productivity of a worker-hour increase when working time is reduced.

The latter point has received little attention in the literature, but it has the important implication that other organisational changes occur simultaneously with the reduction of hours, such as faster work (more effort), or less unproductive time on the job. More generally, unions may agree to greater flexibility and other productivity enhancing measures as part of the bargain over work-time reduction, as seems to have happened in France after the recent introduction of a 35-hour week, "La Loi Aubry" (Contensou and Vranceanu, 2000).

Static partial equilibrium models of unemployment and working time have been widely discussed recently, e.g. by Contensou and Vranceanu (2000), Regt (1999) and Houpis (1993), and generally find some scope for increasing employment by reducing hours – in contrast to more sceptical earlier results. A general equilibrium framework introduced by FitzRoy, Funke and Nolan (1999) – and combining unemployment, working time and a government sector with taxation to fund benefits – also finds a U-shaped hours-unemployment relationship; this model has been extended and incorporated into models of growth with unemployment and infinite-horizon households in the present paper.

In Section 2, we continue by laying out the static bargaining solution. This is followed, in Section 3, by discussion of a simple model of endogenous growth with unemployment. Section 4 develops the public sector and the basic inefficiency of labour taxation, while Section 5 considers the welfare and distributional effects of other policies. Section 6 introduces the neoclassical growth model, highlighting differences and similarities with the endogenous growth case. Finally, our conclusions are summarised in Section 7.

Section 2: The Static Bargaining Solution

We start with the basic bargaining results that will subsequently be embedded in the growth model. Representative competitive firms have a modified Cobb-Douglas production function:

$$Q = A(h-z)^{\alpha} N^{\beta} K^{1-\beta}, \tag{1}$$

where A is a productivity factor, h is average working time per calendar time period, z represents 'non-productive' time for setting up work, maintenance, training etc., N is the number of workers, and K is capital. With single-shift working, hours may represent a utilisation factor, and empirical estimates suggest that α is close to unity[1], while $0 < \beta < 1$. With a unit output price, profit is then given by:

$$\pi = Q - whN - rK \tag{2}$$

where w is the (average) hourly wage and r the capital rental. While fixed non-wage labour costs per worker are not explicitly included, note that the modified Cobb-Douglas function (1) does capture a flexible form non-wage labour cost through the positive factor z – which also allows for a varying output-hours elasticity. Given the wage and hours bargain determined below, firms with the usually observed (Teulings and Hartog, 1998) 'right to manage' (RTM) choose employment according to the first-order condition from (2), giving optimal labour demand:

$$\hat{N} = \left\{\beta A(h-z)^{\alpha} K^{1-\beta} / wh\right\}^{1/(1-\beta)}. \tag{3}$$

[1] See Gianella and Lagarde (2000) for recent estimates of $\alpha \approx 0.9$, and Contensou and Vranceanu (2000) for a review of related work. Multi-shift or continuous production is relatively unusual, but would provide the most favourable situation in which to obtain employment benefits from work-sharing. We do not consider overtime, which is usually a temporary response to random demand shocks of uncertain duration.

To construct the Nash bargaining objective, we assume for simplicity that workers have logarithmic utility of private and public consumption, and wage income is taxed at rate *T*. Public goods are denoted by *P*, so an employed worker has utility:

$$U_{em} = \ln\{wh(1-T)(1-h)P^{\theta}\}, \tag{4}$$

where total time available is normalised at 1, so leisure = 1 – *h*, and $\theta > 0$ is the relative weighting of public goods[2]. Assume that benefits, *B*, are not taxed, so the unemployed have utility:

$$U_{un} = \ln\{BP^{\theta}\}. \tag{5}$$

Even when benefits are taxed, as in the UK, the rate is lower than for most wages, and it is this difference that is crucial for the following. The union objective is then assumed to be the total surplus, $(U_{em} - U_{un})\hat{N}$, and bargaining may be at firm or industry level.

The employer's objective is assumed to be (maximised) cash flow[3] with (expected) optimal employment, or $(1-\beta)A(h-z)^{\alpha}\hat{N}^{\beta}K^{1-\beta}$. Dropping irrelevant public good terms, we then have the bargaining objective, with a weight, $\gamma \geq 0$, reflecting employers' relative bargaining power, as:

$$V = \{\ln[wh(1-T)(1-h)] - \ln B\}\hat{N}\{(1-\beta)A(h-z)^{\alpha}\hat{N}^{\beta}K^{1-\beta}\}^{\gamma}. \tag{6}$$

Optimal wages and hours are then obtained as shown below:

PROPOSITION 1. *The bargaining objective, V, is maximised by* $\hat{w}(\gamma, h, B, T)$ *and* $\hat{h}(\gamma)$, *defined by:*

$$\hat{w}h(1-T)(1-h) = \lambda B, \text{ where } \lambda \equiv \exp\left(\frac{1-\beta}{1+\beta\gamma}\right) > 1 \tag{7}$$

and

$$\hat{h}(\gamma) = \frac{\alpha + z + (\alpha + \beta z)\gamma}{\alpha + 1 + (\alpha + \beta)\gamma}. \tag{8}$$

[2] Different weights for consumption (*wh*), and leisure could be introduced, but would just add notation with little new insight. Note also that individual labour supply is just *h* = ½. This constancy of labour supply reflects the end of the long-term secular decline in hours worked in recent years.

[3] Since we neglect depreciation, this is equivalent to capital income or rental in our constant-returns economy. Employment and capital are chosen after the bargain, which is not affected by *K* in the absence of adjustment costs.

We derive these results by differentiating (6) logarithmically (see Appendix for details of a proof), having dropped irrelevant multiplicative components in β, A and K (including those within $\hat{N}$). The utility mark-up over benefits is analogous to that found by Houpis (1993) in a variety of models, including efficiency wage setting. It is clear that this mark-up decreases with employer power, γ, while optimal hours increase with γ.

When employers have no power ($\gamma = 0$), the resulting *monopoly union* choice of hours is:

$$\hat{h}(0) = \frac{\alpha + z}{\alpha + 1}, \qquad (9)$$

and when unions have no power ($\gamma \to \infty$), the employer's choice of hours tends to the *efficient* or *competitive* market choice, given the alternative utility[4], B. Thus:

$$\lim_{\gamma \to \infty} \hat{h}(\gamma) \equiv \hat{h}(\infty) = \frac{\alpha + \beta z}{\alpha + \beta}. \qquad (10)$$

In this limiting case, of course, there is no mark-up, so $\lambda = 1$, and – as (7) shows – workers just get their alternative utility, which is equal to the benefits available when unemployed. The competitive firm is then a "utility taker", and of course this level of utility or benefits, B, may be higher than the full-employment level.

An interesting question that we can now address is the nature of the general equilibrium relationship between employer (or union) power and employment, with bargaining over both wages and hours. We confirm the partial equilibrium view of wage bargaining with:

PROPOSITION 2. *Employment at the wage-hours bargain is an increasing function of employers' relative power*.

Proof

Some tedious algebra shows that:

$$dN^*\left(\hat{w}\left(\gamma, \hat{h}, B, T\right)\hat{h}(\gamma)\right)/d\gamma > 0, \text{ when } \beta < 1.$$

[4] This can be shown directly, but is quite intuitive. When the union has no bargaining power, the perfectly competitive firm takes worker utility as given, and chooses efficient hours and wage under this constraint. See FitzRoy, Funke and Nolan (2002, forthcoming), and Contensou and Vranceanu (2000).

When government over-rides the hours bargain $\hat{h}$, say by a mandatory reduction of working time, the wage bargain remains the function of *h* given by $\hat{w}$ in (7). Substituting this equation into the demand for workers, $\hat{N}$, from (3), we obtain labour demand conditional on the optimal wage bargain for any hours as:

$$\hat{N}(\hat{w},h) \equiv N^*(\gamma,h,B,T) = \left\{ \frac{\beta A(1-T)(1-h)(h-z)^{\alpha}}{\lambda B} \right\}^{1/1-\beta} K. \tag{11}$$

The partial equilibrium relationship between employer power, working time, and employment can be summarised in the following:

PROPOSITION 3. *The monopoly union choice of hours,* $\hat{h}(0)$, *maximises employment (minimises unemployment) for any given* γ, *B and T. When relative employer power is positive, (*$\gamma > 0$*), a small reduction of working time below the bargain choice will raise employment. Furthermore, N* is an increasing function of* γ, *and a decreasing function of B and T, for any given h.*

Proof

From (11), we have: $\text{sign}\,\dfrac{\partial N^*}{\partial h} = \text{sign}\left\{\dfrac{\alpha}{h-z} - \dfrac{1}{1-h}\right\}.$

Substituting $\hat{h}(0)$ from (9) shows that *N** is maximised by the monopoly union's optimal hours. Since *N** thus decreases with h for $h > \hat{h}(0)$, and because $\hat{h}(\gamma)$ is an increasing function of employer power, a mandatory reduction of hours below $\hat{h}(\gamma)$ for any positive γ will raise employment *N**. The final claim follows directly from the definition of the mark-up, λ, which decreases with γ, and from labour demand (11).

As in other models of working time, we thus find an inverse U-shaped relationship between employment and hours, with a maximum at $\hat{h}(0)$. For reasonable parameter values, individual labour supply, (½), is less than the monopoly union choice of hours, or $½ < \hat{h}(0)$, but this is not necessarily the case. Related bargaining models were developed by FitzRoy, Funke and Nolan (1999) and Contensou and Vranceanu (2000), but they did not explicitly derive the general effect of a (small) reduction of hours below the bargaining choice, which in our notation is given by $\frac{\partial N^*}{\partial h}\Big|_{h=\hat{h}(\gamma)}$. As Proposition 3 shows, the employment effect is – perhaps surprisingly, always positive in our model when $\gamma > 0$. The numerical simulations in the previous models show that efficiency gains are possible, by correcting for the lack of explicit bargaining over employment in the RTM framework (Contensou and Vranceanu, 2000, p. 169). We shall show analytically that welfare gains are also possible in the two different growth models that follow.

Section 3: Endogenous Growth with Unemployment

In this section, we combine the single period bargaining result from the previous section with a simple model of endogenous growth. Utility in period *t* will be written in logarithmic form as:

$$U_t = \ln\{c_t (1-h) P_t^{\theta}\}, \tag{12}$$

where c_t is private consumption, $\theta > 0$ is the weighting for public consumption, P_t, and total utility with time-preference $\rho > 0$ is then[5]:

$$U = \sum_{t=0}^{\infty} U_t / (1+\rho)^t. \tag{13}$$

We assume 3 classes of individuals. Employed workers have net wage income in period *t* given by $y_t^{em} \equiv w_t h(1-T)$, but no other assets. The unemployed have income from benefits only, $y_t^{un} \equiv B_t$, while capitalists receive interest but obtain no wage income, $y_t^c \equiv (1-\tau) r k_t$, where τ is the capital income tax, *r* is the rate of return which is assumed to be time invariant, and k_t is capital at time t. This sharp division is obviously extreme and dictated by tractability, but does have some advantages

[5] The separable, constant elasticity form is chosen for tractability to yield the bargaining solution of the previous section, with hours that are independent of income and benefits. This seems consistent with the stabilisation of weekly full-time working hours at around 35-40 in most advanced economies, marking the end of the long secular decline in working time (for full time employees) that started around the middle of the 19th Century.

over the usual polar opposite case of representative households and perfect equality, for capturing distributional conflict between classes. We also exclude capital mobility and foreign trade.

The general budget constraint for a household with both wage or transfer income, y_t, and capital income, but no debt, is:

$$c_t = y_t + (1-\tau)rk_t - s_t,\ t = 0,1,..., \tag{14}$$

where k_t is the household's capital and s_t is the household's (non-negative) saving at time t, τ is the capital income tax and r is the interest rate. Starting with some initial capital stock k_0, capital evolves according to:

$$k_{t+1} = k_t + s_t. \tag{15}$$

To obtain the optimal consumption path with exogenous income y_t, consider a small additional saving ds_t, in period t. The proceeds, $\{1+(1-\tau)r\}ds_t$, are consumed in t+1, leaving the rest of the path defined by (14) unchanged. Restricting attention to the additively separable terms ln c_t in utility (12) and (13), we obtain:

$$dU = -\frac{ds_t}{c_t(1+\rho)^t} + \frac{\{1+(1-\tau)r\}ds_t}{c_{t+1}(1+\rho)^{t+1}}. \tag{16}$$

Utility must be stationary on an optimal path so, from $dU = 0$, we obtain the standard discrete-time Euler equation:

$$\frac{c_{t+1}}{c_t} = \frac{1+(1-\tau)r}{1+\rho}. \tag{17}$$

It is convenient to define the corresponding growth rate, g, by:

$$g \equiv \frac{1+(1-\tau)r}{1+\rho} - 1 = \frac{(1-\tau)r-\rho}{1+\rho}. \tag{18}$$

Then $c_t = c_0(1+g)^t$ and, for balanced growth, all other variables must grow at the same rate. From (15) in particular, we have $s_0 = gk_0$, so that initial consumption is defined in terms of the other initial conditions by:

$$c_0 = y_0 + \{(1-\tau)r - g\}k_0. \tag{19}$$

Next, we consider 2 classes of consumers, workers and capitalists, though the former may be unemployed to give 3 different types. Workers own no initial capital, or $k_0 = 0$, so they do not save. If employed, they optimally consume all their wage income, $w_t h(1 - T)$, where $w_t = w_0(1+g)^t$. The bargaining solution in period t is the same as in the static case (7); a constant mark-up of current utility over alternative benefits, which we write as:

$$\hat{w}_t h(1-h)(1-T)=\lambda B_t \text{, where } \lambda=\exp\left(\frac{1-\beta}{1+\beta\gamma}\right) \tag{20}$$

with $B_t = B_0(1+g)^t$, and B_0 chosen by government[6]. In the present case, the Nash bargaining choice of hours is again $\hat{h}$ as in (8). In the following, we continue to assume the constant mark-up, λ, and constant hours, h – which may be set by government at a level different from $\hat{h}$.

Turning to unemployed workers, we assume they are always unemployed and have no initial capital, so that consumption of their benefits, B_t, in each period is optimal. While it is obvious that permanent 'dynastic' unemployment is an extreme case chosen for simplicity, it represents an alternative which is perhaps closer to structural unemployment than the random determination of frictional unemployment in matching models. Empirically, the correlation of poverty and unemployment incidence across generations is quite strong.

Finally, capitalists have no non-capital income, so their $y_0^c=0$. Aggregation across all capitalists yields initial consumption of $c_0^c=(1-\tau)r-g$ if we set total initial capital to unity (K_0 = 1). Then $c_t^c=\{(1-\tau)r-g\}(1+g)^t$ is capitalist consumption at any t, and $K_t = (1 + g)^t$. To determine the interest rate, r, and hence by (18) the growth rate, g, we now consider the production sector. With the constant-returns production function (1), the economy can be represented as a single firm – giving first order conditions for each period in the absence of adjustment costs or depreciation:

$$\beta A_t(h-z)^{\alpha} N^{\beta-1} K_t^{1-\beta}=w_t h \tag{21}$$

[6] In the absence of adjustment costs or uncertainty, the 'realistic' assumption of independent bargaining in each period is consistent with bargaining over the present value of the sequence of all future one-period objective functions (6). Bargaining may be at industry or economy level.

$$(1-\beta)A_t(h-z)^{\alpha}N^{\beta}K_t^{-\beta}=r. \tag{22}$$

The productivity factor, A_t, captures externalities from knowledge production in the private sector through investment, and following the basic endogenous growth model, takes the form $A_t = K_t^{\beta}$. Equilibrium employment, N^*, is constant, and – following (11) – is given by:

$$N^{*1-\beta}=\beta(h-z)^{\alpha}/w_0h=\beta(h-z)^{\alpha}(1-h)(1-T)/\lambda B_0, \tag{23}$$

using (21) and (22), which is the same inverse U-shaped function of hours as the static solution (11)[7]. The equilibrium interest rate or capital rental from (22) is then given by:

$$r^{*1-\beta}=\beta^{\beta}(1-\beta)^{1-\beta}(h-z)^{\alpha}\{(1-h)(1-T)/\lambda B_0\}^{\beta}. \tag{24}$$

Substituting the last equation into (18) we obtain the equilibrium rate of growth as a function of hours and taxes:

$$g^*=\frac{(1-\tau)r^*-\rho}{1+\rho}. \tag{25}$$

Intuitively, a higher rate of interest generates a higher rate of saving by capitalists, which in turn increases the growth rate of capital, and hence also the growth rates of productivity, output and wages. It is then easy to show the following results:

PROPOSITION 4. *The equilibrium rate of growth, g*, and the interest rate, r*, are maximised by efficient* $\hat{h}(\infty)$ *from (10), and are decreasing functions of both capital and labour taxes.*

<u>Proof</u>

From (24), we have: $\text{sign}\,\dfrac{\partial r^*}{\partial h}=\text{sign}\left\{\dfrac{\alpha}{h-z}-\dfrac{\beta}{1-h}\right\}$,

So the maximum is at $\hat{h}(\infty)=\dfrac{\alpha+\beta z}{\alpha+\beta}$, efficient hours from (10). Recall that hours represent a utilisation rate for capital, so the intuition here is that better utilisation of capital generates a higher rate of return.

From (25) and the definition of consumption from (19), we have equilibrium capitalist consumption:

$$c_t^c{*}=\{(1-\tau)r{*}-g{*}\}(1+g{*})^t=\rho\left\{\frac{(1-\tau)r{*}+1}{1+\rho}\right\}^{t+1}. \qquad (26)$$

Not surprisingly, perhaps, capitalist consumption is thus maximised by efficient hours, $\hat{h}(\infty)$. The effects, in general equilibrium, of capital and labour taxes on private and public consumption will be analysed below.

Section 4: The Public Sector

In our model, the government levies taxes on capital and wage income, and spends revenues on transfers to the unemployed, and on public goods that benefit all three classes of households equally according to the utility function (12). To close the general equilibrium model, we need to define public expenditure in terms of tax revenues and the government's budget constraint. For simplicity, we consider only current flows and ignore government borrowing and investment. The public sector budget in each period is thus:

$$Tw_t hN{*}+\tau r{*}K_t{*}=(1-N{*})B_t{*}+P_t{*}. \qquad (27)$$

The labour force is normalised at unity, so $(1 - N{*})$ is unemployment. Given the initial choices of B_0, h, τ and T as policy instruments, and setting $P_t{*} = P_0(1+g{*})^t$ for balanced growth, we see that initial public good expenditure, P_0, is a residual quantity determined by the other, independent policy variables[8]. Cancelling the growth factor, and using $K_0 = 1$ and the wage mark-up (20), we obtain the budget in terms of initial values as:

$$\frac{T\lambda B_0 N{*}}{(1-h)(1-T)}+\tau(1-\beta)(h-z)^{\alpha}N{*}^{\beta}=(1-N{*})B_0+P_0. \qquad (28)$$

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[7] Notice that the growth factors in A_t, $K_t^{1-\beta}$ and w_t have cancelled to leave the constant steady-state employment $N{*}$, when population growth is zero.

[8] Public expenditure is often perceived to be a practical policy choice, but in that case, taxes cannot be chosen independently, and at least one tax will be a function of predetermined P, and other variables. This approach greatly complicates the analysis.

In order to study the efficiency of the tax system, it is convenient to reinterpret (28) for the moment as defining the capital tax, τ, in terms of T, to give a function $\tau(T)$, assuming P_0, B_0 and h are held constant. Clearly, various constraints must be satisfied – thus $N^*(B_0,h,T) \leq 1$, $\tau(T) < 1$ and $T < 1$ must hold, which means intuitively that B_0 must be 'large enough' and P_0 must be 'small enough'. For any admissible parameters, the form of the capital tax as a function of the wage tax, T, is not obvious from the highly non-linear equation (28).

If τ is a downward-sloping function of T, then the two taxes are substitutes in terms of the funding of a given public expenditure, P_0. Since both taxes reduce growth, this is what might be expected. However, if $d\tau/dT > 0$ for all admissible $T \geq 0$, then any positive wage tax is inefficient (for the given P_0, B_0, h). Raising the capital tax from $\tau(0)$ allows no compensating reduction of the wage tax, but, on the contrary, *both* taxes have to be raised so the growth rate, employment and hence consumption of all classes at any time will fall. To explore this possibility, we differentiate (28) totally with respect to T and, after some lengthy algebra, find that:

$$\text{sign}\frac{d\tau}{dT} = \text{sign}\left\{\tau(T) + \frac{(1-h)(1-T)}{\lambda(1-\beta)} + \frac{T}{1-\beta} - 1\right\}. \tag{29}$$

When $T = 0$, the condition for $\tau'(0) > 0$ reduces to:

$$\tau(0) + \frac{(1-h)}{\lambda(1-\beta)} > 1. \tag{30}$$

For reasonable values of h, β and λ, the second term on the left-hand-side should be approximately equal to 1, so almost any positive $\tau(0)$ should satisfy (30). Indeed, $\tau(0)$ is plausibly greater than ½ as the capital tax rate needed to fund all public expenditure. Now if $\tau(T)$ is an initially increasing function of T, then so is the right-hand-side of (29) because the sum of the other two terms in T also increases. It follows that, if $\tau'(0) > 0$, then $\tau'(T) > 0$ for all $T < 1$. To summarise the quite weak condition for no tax on labour, we conclude with[9]:

[9] It is not clear how this result would be affected by allowing capital mobility. Daveri and Tabellini, who also model a closed economy, argue "that the distorting effects of labour taxes survive in an open economy" (p. 63).

PROPOSITION 5. *Suppose* $\tau(0)>1-\frac{(1-h)}{\lambda(1-\beta)}$; *then, any labour tax is inefficient.*

Since this is a relatively weak condition, as argued above, we shall assume that it holds and adopt the simplification provided by setting T = 0. Further, we now revert to our original interpretation of τ as a policy parameter, so from (28) we can write initial residual public good expenditure in balanced growth as a function of policy instruments:

$$P_0*(\gamma,\tau,h,B_0)=\tau(1-\beta)(h-z)^{\alpha} N*^{\beta} -(1-N*)B_0, \tag{31}$$

where, of course, parameters must be chosen to ensure $P_0* \geq 0$, since there is no borrowing, and we assume that τ satisfies the condition in Proposition 5. Now we have all the components required to study welfare in the balanced growth equilibrium that has been derived so far.

Section 5: Welfare

To start with the simplest case of capitalists, we can write their equilibrium utility from (12), (13) and their consumption (26) as:

$$U^c=\sum_{t=0}^{\infty}\frac{1}{(1+\rho)^t}\left[(t+1)\ln\{(1-\tau)r*+1\}+\theta\ln P_0^*+\theta t\ln\{(1-\tau)r*+1\}\right]$$

$$=(1+\rho+\theta)\ln\{(1-\tau)r*+1\}+\rho\theta\ln P_0^* \tag{32}$$

neglecting constants. Now recall from (24) that $r*$ is independent of the capital tax, τ, while P_0^* increases with τ from (31). Thus the capitalists' optimal tax could in principle be calculated. It can be verified that P_0^* is a decreasing function of benefits, B_0, and so, obviously, is $r*$. Thus, unsurprisingly, capitalists always prefer lower benefits.

This position can be contrasted with that of the unemployed, whose utility turns out to be:

$$U^{un}=(1+\theta)\ln\left\{(1-\tau)r^*+1\right\}+\rho\theta\ln P_0^*+\rho\ln B_0. \tag{33}$$

For employed workers, we find that utility is given by:

$$U^{em} = (1+\theta)\ln\{(1-\tau)r^* + 1\} + \rho\theta\ln P_0^* + \rho\ln B_0 + \rho\ln\lambda. \quad (34)$$

Surprisingly, this differs from unemployed utility (33) only by the constant mark-up terms ρ and λ; all the policy-dependent terms are identical. However, this does not remove all conflict between the employed and unemployed – as we shall demonstrate below.

The first policy conflict to notice is not surprising: capitalists prefer a lower capital tax than workers, because the first term in (32), which is a decreasing function of τ, has a larger coefficient, $(1 + \rho + \theta)$, than the first term in (33) and (34) – which is only $(1 + \theta)$. Of course, as the time preference, ρ, decreases, the difference also declines. The second term, $\rho\theta\ln P_0^*$, is the utility of public goods, which increases with the tax. Thus, the workers' optimal tax, say τ_w^*, follows easily from (31) and either (33) or (34) – and is greater than the capitalists' optimal tax, say τ_c^*. This gives us the following *dynamic Laffer Curve*[10] result:

PROPOSITION 6. *If the capital tax, τ, exceeds τ_w^*, then a tax reduction to τ_w^* is Pareto-improving. However, if $\tau_c^* < \tau < \tau_w^*$, then a reduction to τ_c^* will reduce workers' welfare and increase capitalists' welfare.*

Next, we see that both employed and unemployed utility is maximised by the same value of benefits, say B_0^*, (which will depend on the choice of τ and h)[11]. However, there is still scope for conflict between the employed and unemployed, because there is no reason for optimal B_0^* to also generate full employment. Continuing to hold τ and h fixed, let B_{0f} be full-employment benefits, so that $N^*(\gamma,h,B_{0f},0) = 1$. Then it may be the case that $B_0^* > B_{0f}$. Now it is conceivable that lowering benefits from B_0^* to B_{0f} leaves those workers who were unemployed at B_0^* *worse off* in employment at the lower wage generated by the bargaining process from lower benefits, in spite of

[10] Agell and Persson (2000) present new dynamic Laffer Curves for a representative consumer in an AK-growth model, but without labour supply, and discuss problems with previous definitions such as in Barro and Sala-i-Martin (1995).

[11] This is because r^* and P_0^* are both decreasing functions of B_0. Clearly, if initial $B_0 > B_0^*$, then there is again scope for Pareto improvement.

gaining the additional mark-up utility, $\rho \ln \lambda$, in (34). In this case, *all* the gains from increasing employment would accrue to capitalists. More plausible, perhaps, is the case where reducing benefits from B_0* to B_{0f} (or some intermediate value) raises the utility of the newly employed in spite of reducing the utility of those who were already employed at B_0*. In this case, there is a clear conflict of interest between the employed and unemployed at B_0*. Finally, we have a Laffer-type of result for benefits, analogous to Proposition 6: if $B_0 > B_0*$, then reducing benefits to B_0* is Pareto-improving.

If working time is determined by collective bargaining as $\hat{h}(\gamma)$, then the welfare effect of the institutional variable, employer power (γ), is quite complicated. The total effect of γ on employment is positive by Proposition 2. It is straightforward to show – analogously to Proposition 2 – that employer power has a positive total effect on the rate of return for plausible parameter values, or:

$$\frac{dr*\left(\gamma,\tau,\hat{h}(\gamma),B_0\right)}{d\gamma}>0.$$

However, the direct effect of larger γ is to reduce the mark-up term in U_{em}, so we can summarise with:

PROPOSITION 7. *Higher relative employer power raises the welfare of capitalists and the unemployed, but the effect on the welfare of the employed is ambiguous.*

In particular, it cannot be ruled out that U_{em} could be an always increasing, or an always decreasing, function of γ.

Finally, we consider the welfare effects of working time set by government rather than collective bargaining. Going back to equation (31) for initial public expenditure, it is easy to verify that the first term in (31) is maximised by efficient hours, $\hat{h}(\infty)$, while the second term, $(1-N*)B_0$, is obviously maximised by employment-maximising hours $\hat{h}(0)$. The total utility equations (32)-(34) all have a first term which is maximised by the growth-maximising hours $\hat{h}(\infty)$ (see

Proposition 3); while the second term contains public expenditure. However, capitalists put more weight on the first term, so in summary we can conclude this section with:

PROPOSITION 8. *If unemployment is positive and τ and B_0 are given, then the utility of capitalists and workers respectively is maximised by some level of hours* $h_i^* \in (\hat{h}(0), \hat{h}(\infty))$, *where* $h_i^* \equiv \hat{h}(\gamma_i^*)$, *with* $\gamma_i^* \in (0,\infty)$, *and i = c, w, where* $h_c^* > h_w^*$.

Intuitively, the necessity to pay unemployment benefits out of tax revenues reduces residual public good expenditure, and thus causes the socially optimal working time to fall below (statically) efficient – and also growth-maximising – hours $\hat{h}(\infty)$, in order to reduce unemployment. Welfare is an inverse U-shaped function of hours. Clearly, higher initial benefits B_0 imply higher unemployment, and hence a greater weight of this term in utility. Thus, γ_i^* and h_i^* are decreasing functions of B_0. There is a simple corollary to the above:

COROLLARY. *If unions are 'too weak', or employer power, γ, is large enough in an economy with hours set by collective bargaining so that $\gamma > \gamma_c^*$, and hence the bargaining choice of hours exceeds the optimal level,* $\hat{h}(\gamma) > h_c^* = \hat{h}(\gamma_c^*)$, *then a mandatory reduction of working time will raise the welfare of both capitalists and workers. On the other hand, 'excessive' union power, or $\gamma < \gamma_w^*$, means that welfare can be raised by increasing work time above the bargain level* $\hat{h}(\gamma)$.

Finally it should be emphasised that these are 'second-best' results, due to initial benefits being above the full employment level, or $B_0 > B_{0f}$. However, as we have seen that even optimal B_0^* does not necessarily generate full employment, there may still be scope for a welfare-raising reduction of working time, as when actual benefits are optimal, but $B_0^* > B_{0f}$, and hours are excessive because unions are too weak, or $\hat{h}(\gamma) > h^*$.

Section 6: Neoclassical Growth

In contrast to endogenous growth models, the traditional neoclassical approach is to assume an exogenous rate of technological progress that determines the constant rate of growth in equilibrium or balanced growth[12]. This assumption might be most appropriate for a (small) open economy that adopts advancing technology from the rest of the world. However, in contrast to the usual neoclassical approach, we include the working time factor $(h-z)^{\alpha}$ in the Cobb-Douglas production function (1).

The neoclassical model in its simplest form is defined by writing the productivity factor at time t as:

$$A_{tex} \equiv (1+g_{ex})^{\beta t}, \tag{35}$$

where the growth rate, g_{ex}, is now the exogenously given rate of technical progress, rather than being determined by investment as in the previous, endogenous growth, model. The first-order conditions take the same form as before (21, 22), and again we need $A_{tex} = K_{tex}^{\beta}$ for balanced growth, so now:

$$K_{tex} \equiv (1+g_{ex})^{t}, \tag{36}$$

with K_{0ex} = 1, analogously to our previous K_0 = 1. The equilibrium rate of interest, say r_{ex}, then follows *directly* from the Euler condition (18) because the growth rate is now exogenous, as:

$$r_{ex}^{*} = \frac{(1+\rho)g_{ex}+\rho}{1-\tau}, \tag{37}$$

which is just a rearrangement (and different interpretation) of (18).

Writing N_{ex}^{*} for equilibrium employment with exogenous growth, the first-order conditions are the same as (21) and (22) so that the equilibrium rate of interest must now satisfy (37) *as well as* the corresponding first order condition, (22). This constraint then implies that the capital tax, τ, can no longer be chosen as a policy parameter, but is rather determined by (22) and (37) as an equilibrium function, τ_{ex}^{*}, of parameters γ, h, B_0 and T – as follows:

[12] We do not consider transitional paths from one steady-state to another.

$$r_{ex}^{*}=\frac{(1+\rho)g_{ex}+\rho}{1-\tau_{ex}^{*}}=(1-\beta)(h-z)^{\alpha}N_{ex}^{*\beta}, \tag{38}$$

where equilibrium employment is the same function of hours and tax as (23), namely: $N^{*1-\beta}=\beta(h-z)^{\alpha}/w_0h=\beta(h-z)^{\alpha}(1-h)(1-T)/\lambda B_0$.

Intuitively, having lost a degree of freedom through the exogenous growth factor g_{ex}, the capital tax cannot be chosen independently in the neoclassical model, when we maintain the labour tax, T, and benefits, B_0, as policy variables[13]. Rearranging (38) gives:

$$\tau_{ex}^{*}(\gamma,h,B_0,T)=1-\frac{(1+\rho)g_{ex}+\rho}{(1-\beta)(h-z)^{\alpha}N_{ex}^{*\beta}}. \tag{39}$$

Clearly, τ_{ex}^{*} is a decreasing function of B_0 and T, and an increasing function of γ. Residual initial public expenditure is, thus, a function of the labour tax, T, and – following (28) – is given by:

$$P_{0ex}^{*}=\frac{\lambda B_0 N_{ex}^{*}T}{(1-h)(1-T)}+\tau_{ex}^{*}(1-\beta)(h-z)^{\alpha}N_{ex}^{*\beta}-(1-N_{ex}^{*})B_0. \tag{28$'$}$$

It is easy to see that P_{0ex}^{*} is a decreasing function of B_0. Corresponding to Proposition 5, we can now again show the inefficiency of labour taxation in a different setting:

PROPOSITION 9. *Residual initial public expenditure, P_{0ex}^{*}, is a decreasing function of the labour tax, T.*

The proof of Proposition 9 can be found in an Appendix.

Turning to questions of welfare, we will assume henceforth that T = 0, and note that expressions for equilibrium total utility take the same forms as before [(32)-(34)], but now with r_{ex}^{*} and P_{0ex}^{*} reflecting exogenous technical progress. There is also a remarkable similarity in the role of working time between the endogenous growth and neoclassical models. The growth rate is

[13] An alternative approach would be to retain τ as a policy instrument, and then use (24′) to define equilibrium benefits as a function of taxes and hours.

unaffected by changing hours in the latter, because the expression $(1-\tau_{ex}^{*})r_{ex}^{*}=(1+\rho)g_{ex}+\rho$ only contains exogenous parameters, and hence the first terms in the utility equations corresponding to (32)-(34) are constants, which are obviously not influenced by government policy. Residual initial equilibrium public expenditure with $T = 0$ is now given by:

$$P_{0ex}^{*}(h,B_0,0)=\tau_{ex}^{*}(1-\beta)(h-z)^{\alpha} N_{ex}^{*\beta}-(1-N_{ex}^{*})B_0. \tag{31′}$$

Since the tax $\tau_{ex}^{*}(\gamma,h,B_0,0)$ is a decreasing function of B_0, and of course employment declines with the level of benefits, it follows that public expenditure P_{0ex}^{*} is a declining function of benefits, B_0. From (33) and (34) in terms of exogenous growth variables, there must again be a benefit level, say B_{0ex}^{*}, that is optimal for the employed and unemployed, and a Laffer-type Pareto improvement when 'excessive' benefits ($B_0 > B_{0ex}^{*}$) are reduced. Since the growth rate is now constant, the (respective) first terms in workers' utility (33) and (34) are unaffected by benefits, and so we can conclude that $B_{0ex}^{*}>B_0^{*}$. From (23′), N_{ex}^{*} is obviously maximised by $\hat{h}(0)$ as in Proposition 4, and similarly from (24′), it is easy to see that $\tau_{ex}^{*}(\gamma,h,B_0,0)$ is maximised by efficient hours $\hat{h}(\infty)$. Thus, in (31′) we have the sum of terms maximised by $\hat{h}(\infty)$ and $\hat{h}(0)$ respectively, but in the neoclassical utilities corresponding to (32)-(34), there is no first term maximised by $\hat{h}(\infty)$. Thus, we have a result related to Proposition 8 as follows:

PROPOSITION 10. *If unemployment is positive then the utility of capitalists and workers respectively is maximised by hours* $h_{exi}^{*}\in\left(\hat{h}(0),\hat{h}(\infty)\right)$*, with i = c, w,* $h_{exi}^{*}<h_i^{*}$.

Due to the reduced 'weight' attached to $\hat{h}(\infty)$, which maximises growth in the previous model, optimal hours in the neoclassical case are lower than in the endogenous growth model. Remarkably perhaps, the possibility of welfare-enhancing reduction of working time remains if the initial collective bargain sets longer hours than optimal due to too little union power. Finally, note that increasing employer power, γ, lowers the mark-up and raises employment as before, but now also increases the equilibrium tax, τ^*_{ex}. Thus employer power again has two conflicting effects on U_{em} in (34), and no simple conclusion is possible.

Section 7: Conclusions

This paper has combined a number of aspects of real-world economies that have been developed in disparate strands of the literature. Thus, collective bargaining over working time as well as wages does not seem to have been incorporated into standard models of economic growth. We have extended our previous static model of bargaining, employment and taxation to fund unemployment benefits, to include other public goods, taxes on labour and capital, and three classes of infinitely-lived households in two different growth models. In spite of the fundamentally different nature of the basic endogenous and neoclassical growth models, we find that labour taxation is inefficient in both, thus extending the results of Daveri and Tabellini (2000) for an overlapping generations economy with a monopoly union and wage bargaining. We also find a "dynamic Laffer Curve" – thus, there is a Pareto-optimal capital tax rate in the endogenous growth model; and there is a Pareto-optimal level of benefits in both growth models.

Surprisingly, we also find that a (small) reduction of collectively-bargained working time raises employment whenever unions have less than monopoly power. Our Cobb-Douglas assumptions allow for explicit solutions which show that bargainers will choose longer hours than the welfare maximum when union are 'too weak', and fewer than the optimum number of hours when unions are 'too strong'. Our finding that union power has ambiguous effects on the welfare of the employed is also surprising.

Various distributional conflicts, including the conflicts between employed and unemployed, are made precise. Most of our results depend on unemployment benefits, and hence the wage mark-up, being set at a level which is too high for full employment – presumably for distributional reasons. Among the many limitations of the models dictated by tractability are the assumptions that public goods have only direct consumption value, rather than also enhancing productivity; that capital is immobile; and that debt is excluded.

As a first attempt to combine collective bargaining over wages and working time with fiscal policy in growth models, our results clearly need to be treated with caution. However, the basic qualitative agreement between results from the two fundamentally different models of endogenous and neoclassical growth does suggest an element of robustness. In contrast to widely held views, we have confirmed not only the possibility that working time reduction can increase employment, but we have also shown that such intervention is more likely to be beneficial when union power is weak.

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Appendix

Proof of Proposition 1.

Beginning with the bargaining objective:

$$V = \{\ln[wh(1-T)(1-h)] - \ln B\}\hat{N}\left\{(1-\beta)A(h-z)^{\alpha}\hat{N}^{\beta}K^{1-\beta}\right\}^{\gamma}, \tag{6}$$

when we differentiate logarithmically, V's multiplicative components in β, A and K (including those within $\hat{N}$), will disappear. Effectively, ln V takes the form:

$$\ln V = \ln\{\ln[wh(1-T)(1-h)] - \ln B\} + \frac{\alpha(1+\gamma)}{1-\beta}\ln(h-z) - \frac{1+\beta\gamma}{1-\beta}\ln(wh).$$

We can find the first-order condition for the optimal wage – proceeding term by term, as follows:

$$\frac{\partial \ln V}{\partial w} = \frac{(h(1-T)(1-h))/(wh(1-T)(1-h))}{\ln[wh(1-T)(1-h)] - \ln B} + 0 - \frac{1+\beta\gamma}{1-\beta}\frac{h}{wh}$$

$$\Rightarrow \qquad \frac{1}{\{\ln[\hat{w}h(1-T)(1-h)] - \ln B\}}\frac{1}{\hat{w}} = \frac{1+\beta\gamma}{1-\beta}\frac{1}{\hat{w}}$$

$$\Rightarrow \qquad \ln[\hat{w}h(1-T)(1-h)] - \ln B = \ln[\hat{w}h(1-T)(1-h)/B] = \frac{1-\beta}{1+\beta\gamma}$$

$$\Rightarrow \qquad \hat{w}h(1-T)(1-h) = \lambda B, \text{ where } \lambda = \exp\left(\frac{1-\beta}{1+\beta\gamma}\right)$$

The first-order condition for optimal hours can be obtained as follows:

$$\frac{\partial \ln V}{\partial h} = \frac{(w(1-T)(1-h) - wh(1-T))/(wh(1-T)(1-h))}{\ln[wh(1-T)(1-h)] - \ln B} + \frac{\alpha(1+\gamma)}{1-\beta}\frac{1}{h-z} - \frac{1+\beta\gamma}{1-\beta}\frac{w}{wh}.$$

Setting this derivative to zero, and also using the optimal wage:

$$\frac{(1-2\hat{h})/\hat{h}(1-\hat{h})}{(1-\beta)/(1+\beta\gamma)} + \frac{\alpha(1+\gamma)}{1-\beta}\frac{1}{\hat{h}-z} - \frac{1+\beta\gamma}{1-\beta}\frac{1}{\hat{h}} = 0$$

$$\Rightarrow \qquad \frac{(1-2\hat{h})(1+\beta\gamma) - (1-\hat{h})(1+\beta\gamma)}{\hat{h}(1-\hat{h})} + \frac{\alpha(1+\gamma)}{\hat{h}-z} = 0$$

$$\Rightarrow \qquad (\alpha(1+\gamma))/(\hat{h}-z) = (1+\beta\gamma)/(1-\hat{h}) \qquad \Rightarrow \qquad \hat{h}(\gamma) = \frac{\alpha + z + (\alpha+\beta z)\gamma}{\alpha + 1 + (\alpha+\beta)\gamma}. \tag{8}$$

Proof of Proposition 9.

Let us consider P_{0ex}^{*}, N_{ex}^{*} and τ_{ex}^{*} as functions of the labour tax. We can use logarithmic differentiation to obtain:

$$\frac{dN_{ex}^{*}}{dT}=-\frac{N_{ex}^{*}}{(1-\beta)(1-T)}, \qquad \frac{d\tau_{ex}^{*}}{dT}=-\frac{\beta(1-\tau_{ex}^{*})}{(1-\beta)(1-T)}.$$

We now split P_{0ex}^{*} into 3 parts, as in (28′) and differentiate with respect to T:

$$\frac{d}{dT}\left(\frac{\lambda B_0 N_{ex}^{*} T}{(1-h)(1-T)}\right)=\frac{\lambda B_0 N_{ex}^{*}(1-\beta-T)}{(1-\beta)(1-h)(1-T)^2},$$

$$\frac{d}{dT}\left(\tau_{ex}^{*}(1-\beta)(h-z)^{\alpha} N_{ex}^{*\beta}\right)=-\frac{\beta(1-\beta)(h-z)^{\alpha} N_{ex}^{*\beta}}{(1-\beta)(1-T)}=-\frac{\beta(1-\beta)(h-z)^{\alpha}(1-h)(1-T)N_{ex}^{*\beta}}{(1-\beta)(1-h)(1-T)^2},$$

$$\frac{d}{dT}\left(-(1-N_{ex}^{*})B_0\right)=-\frac{B_0 N_{ex}^{*}}{(1-\beta)(1-T)}.$$

Thus, using (23′) alongside the fact that $N_{ex}^{*}=N_{ex}^{*\beta}N_{ex}^{*1-\beta}$, we obtain:

$$\frac{dP_{0ex}^{*}}{dT}=\frac{N_{ex}^{*\beta}}{(1-\beta)(1-h)(1-T)^2}\left\{-\beta T(h-z)^{\alpha}(1-h)(1-T)-\frac{B_0(1-h)(1-T)\beta(h-z)^{\alpha}(1-h)(1-T)}{\lambda B_0}\right\}$$

$$=\frac{-\beta(h-z)^{\alpha} N_{ex}^{*\beta}}{\lambda(1-\beta)(1-T)}\{\lambda T+(1-h)(1-T)\},<0. \qquad \text{[Q.E.D.]}$$