Accommodating availability bias on line transect surveys using hidden Markov models*

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SUMMARY. Maximum likelihood methods are developed which accommodate intermittent animal availability of animals on line transect surveys. Existing “availability bias” correction methods are shown to be inadequate in general. The new method is applied to an aerial survey of whales, using a hidden Markov model to characterise the availability process.

Keywords: availability bias, surfacing pattern, hidden Markov model, line transect survey

1. Introduction

The methods developed in this paper are motivated by the problem of intermittent availability of cetaceans on line transect surveys. Line transect methods are well developed for situations in which animals are detected with certainty at zero distance. Mark recapture line transect (MRLT) methods using two independent observers are also now well developed, but these estimators may suffer from bias due to neglected heterogeneity in detection probability which affects both observers. The bias arises if both observers tend to see the more detectable animals and this is not taken into account in analysis. MRLT methods are able to deal with such heterogeneity if the variable causing it can be recorded. One source of such

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heterogeneity is animal availability - both observers will tend to see animals which are more frequently available or are available in areas where they are more likely to be detected. This kind of heterogeneity is particularly difficult to deal with because it is cause by animals’ pattern of availability and unlike sources of heterogeneity like animal distance or size, the availability pattern is not observed (although bits of it are).

When animals are continuously available for detection, existing line transect methods are adequate. When they are either continuously available or continuously unavailable for detection for the whole period that they are within detection range, availability bias in existing line transect method estimators can be “corrected” using estimates of the proportion of time animals are available. However, when animals are intermittently available for detection while within range using simple correction factors of this kind can produce very biased estimates (as we show) and there is no generally-applicable method of correction with sound statistical basis in the literature.

We develop methods to address this problem. The method is also readily able to deal with surveys on which the transect line is obscured to the observer (as is the case with many towed hydrophone surveys and aerial visual surveys). The methods we develop are applicable to line transect surveys in which the transect line is searched and surveys in which it is obscured; they allow perpendicular distance detection functions which may or may not be monotonically decreasing with distance; they allow detection probabilities at perpendicular distance zero to be one or to be less than one.

2. Key Notation

We define a Cartesian coordinate system which moves with the observer along a transect line, and has the observer at \((x = 0, y = 0)\). See Figure 1. Forward distance along the transect line is denoted \(y\) and perpendicular distance from the line, \(x\). At time \(t\) animal \(i\) is located at \((x_i, y_{it})\), at an angle \(\theta_{it}\) from the transect line and at radial distance \(r_{it}\) from the observer. Animals are assumed not to move while within detectable range as the observer moves along the transect line.

The observer searches a “pie slice” from angle \(\theta_f\) from the trackline, to angle \(180^\circ - \theta_b\)
degrees from the trackline, out to maximum distance $W$ (the shaded region in Figure 1).

Animals alternate between being available for detection for some random time and being unavailable for some random time. We consider time in discrete units, with equal interval between units. This allows us to model the availability process as a discrete time series with two observable states: $A_t = 1$ if an animal is available at time $t$ and $A_t = 0$ otherwise.

The animal availability process is modelled by allowing animals to be in one of $m$ notional hidden states; the state at time $t$ is denoted $S_t$. Given this state, the probability mass function (pmf) of $A_t$ is $f_{A_t|S_t} = \lambda_{S_t}^{A_t}[1 - \lambda_{S_t}]^{1-A_t}$, where $\lambda_{S_t}$ is the probability of the animal being available, given its hidden state $S_t$. $S_t$ takes on values $1, \ldots, m$ according to a Markov process with $m \times m$ transition matrix

$$
\Gamma = \begin{pmatrix}
\gamma_{1,1} & \cdots & \gamma_{1,m} \\
\vdots & \ddots & \vdots \\
\gamma_{m,1} & \cdots & \gamma_{m,m}
\end{pmatrix}
$$

(1)

where $\gamma_{j,k}$ is the transition probability from state $j$ to state $k$ ($j, k \in \{1, \ldots, m\}$).

Unavailable animals are not detected and not all available animals are detected. The farther an available animal is from the observer, the less likely it is that it is detected. To deal with this, we introduce a random variable $\delta_t$, such that $\delta_t = 1$ if the animal is detected at time $t$, $\delta_t = 0$ otherwise and we define the pmf of $\delta_t$ to be

$$
f_{\delta_t|A_t}(\delta_t|x_t, y_t) = \begin{cases} 
  h(x_t, y_t)^{\delta_t}[1 - h(x_t, y_t)]^{1-\delta_t} & \text{if } A_t = 1 \\
  1 - \delta_t & \text{if } A_t = 0
\end{cases}
$$

(2)

We refer to $h(x_t, y_t)$ as the detection hazard; it has a parameter vector $\beta$, which is to be estimated.
3. Likelihood Formulation

We deal initially with situations in which perpendicular distance \(x_i\) of the \(i\)th detected animal is observed, and only the first detection is recorded. Later we consider situations in which forward distance at first detection, \(y_{i,t}\), is also recorded.

For the moment we consider the parameters \(\Gamma\) and \(\lambda = (\lambda_1, \ldots, \lambda_m)\) to be known.

3.1 Detection probability

Given only the state \(S_t\) and position \((x, y_t)\) of an animal at time \(t\), the pmf for \(\delta_t\) is

\[f_{\delta | S}(\delta_t | S_t, x, y_t) = [\lambda_{S_t} h(x, y_t)]^{\delta_t} [1 - \lambda_{S_t} h(x, y_t)]^{1-\delta_t} .\]

We can therefore model the time series of observed \(\delta_s\) as a hidden Markov model (HMM) with states as above, but with \(\delta_t\) considered as arising directly from \(S_t\), according to \(f_{\delta | S}\).

An animal at perpendicular distance \(x\) is within detectable range on occasions \(t = 1, \ldots, T(x)\), where \(T(x)\) is readily calculated from the speed of the observer, the width of time intervals \((\delta_t - \delta_{t-1}; t = 2, \ldots, T(x))\), and the geometry of Figure 1. The animal’s positions \((x, y_t)\) relative to the observer at times \(t = 1, \ldots, T(x)\) is similarly easily calculable.

We use data only up to the time of first detection. The observed \(\delta_s\) for an animal first detected at time \(t\) is therefore a series of \((t-1)\) zeros, followed by a single 1. (It is in principle possible to use data after first detection, although the detection function changes after detection because observers become aware of animals’ presence.) Given the series of states of an animal while in detectable range, \(\underline{S}_{T(x)} = (S_1, \ldots, S_{T(x)})\), and defining \(t\) to be 1 when the animal enters the detectable range, we can write the probability of first observing the animal at time \(t\) as

\[
\Pr(\delta_1, \ldots, \delta_t \mid \underline{S}_{T(x)}, x) = \prod_{u=1}^{t} f_{\delta_u | S}(\delta_u | S_u, x, y_u) \tag{3}
\]

where \(\delta_u = 0\) for \(u < t\) and \(\delta_t = 1\).

The probability of an animal having the series of states \(\underline{S}_{T(x)}\) while within detectable range is \(\tau_{S_1} \prod_{t=2}^{T(x)} \gamma_{S_{t-1}, S_t}\), where \(\tau_{S_1}\) is the stationary distribution probability for state \(S_1\). Using \(\sum_{\underline{S}_{T(x)}}\) to denote the sum over all possible \(\underline{S}_{T(x)}\), we can write the unconditional
probability that an animal at \( x \) is first detected at time \( t \) as
\[
p(x, y_t) = \sum_{S_{T(x)}} \tau_{S_1} f_{S|S}(\delta|S_t, x, y_t) \prod_{u=2}^{t} \gamma_{S_{t-1}, S} f_{S|S}(\delta_u|S_u, x, y_u) \tag{4}
\]
Following MacDonald and Zucchini (1997), we note that Equation (4) has \( m^{T(x)} \) terms and is computationally very challenging except for small \( T(x) \). However, as they note, it can be rewritten in a computationally more efficient form as follows:
\[
p(x, y_t) = \mathbf{T} \left( \prod_{u=1}^{t} B_u(x, y_u) \right) \mathbf{1}' \tag{5}
\]
where, in our context, \( f(\delta_u \mid x, y_u) = \text{diag}(f_{\delta_u|S}(\delta_u|S_u = 1, x, y_u), \ldots, f_{\delta_u|S}(\delta_u|S_u = m, x, y_u)) \), \( \mathbf{T} = (\tau_1, \ldots, \tau_m) \), \( B_u(x, y_u) = \Gamma f(\delta_u \mid x, y_u) \) and \( \mathbf{1}' \) is a column vector of \( m \) 1s.
It follows that the probability of detecting an animal which is at perpendicular distance \( x \) is the sum of Equation (5) over all possible \( t \)s at which it could be detected:
\[
p(x) = \sum_{t=1}^{T(x)} \mathbf{T} \left( \prod_{u=1}^{t} B_u(x, y_u) \right) \mathbf{1}' \tag{6}
\]
### 3.2 Only perpendicular distance observed
The detection function \( p(x) \) is a function of the availability process parameters \( \Gamma \) and \( \Delta \) and the detection hazard parameters \( \underline{\beta} \). We treat the availability process parameters as known for the moment and write the detection function as \( p(x; \beta) \) to make dependence on \( \underline{\beta} \) explicit. We can then write the likelihood for \( \underline{\beta} \) given that \( n \) animals were detected at distances \( \underline{x} = (x_1, \ldots, x_n) \) as:
\[
L_x(\underline{\beta} \mid \underline{x}) = \prod_{i=1}^{n} \frac{p(x_i; \beta) \pi(x_i)}{\int_{0}^{W} p(x_i; \beta) \pi(x) dx}
\tag{7}
\]
where \( \pi(x) = 1/W \), as is usual with line transect surveys.
3.3 Both perpendicular and forward distances observed

Let \( y_{it} \) be the forward distance at which animal \( i \) is first detected (\( i = 1, \ldots, n \)), let \( y = (y_{1t}, \ldots, y_{nt}) \), and make dependence of \( p(x, y_t) \) on \( \beta \) explicit by writing it as \( p(x, y_t; \beta) \). Then we can write the likelihood for \( \beta \), given \( x \) and \( y \) as

\[
L(\beta | x, y) = L_x(\beta | x) L_{y|x}(\beta | x, y),
\]

where

\[
L_{y|x}(\beta | x, y) = \prod_{i=1}^{n} \frac{p(x_i, y_{it}; \beta)}{p(x_i; \beta)} \tag{8}
\]

4. Correction based on Proportion of Time Available

A common way of correcting for the fact that animals are not continuously available for detection is to multiply the line transect estimate by the inverse of the proportion of time animals are available. This is a valid correction if the survey is instantaneous because this proportion is then a valid estimate of the probability that an animal is available for detection. However, if the survey is not instantaneous correcting line transect estimators in this way can lead to very biased estimation. A commonly-used alternative is to use a method like that of Laake et al. (1997), which uses the estimated probability of an animal being available at least once while in view to correct for availability bias - obtaining the estimate from the expected relative length of periods of availability and of unavailability (i.e. effectively the proportion of time an animal is available).

One can show, however, that any correction factor based on the proportion of time an animal is available or unavailable is not in general sufficient for unbiased correction. To show this, we simulated two availability processes, both with animals being available 20% of the time, but one with frequent short periods of availability, the other with infrequent long periods of availability. Realisations of the availability processes are shown in Figure 3.

[Figure 3 about here.]

Animals were detected using a two-dimensional detection hazard function like that used by Skaug and Schweder (1999): \( h(x, y) = \exp\left( -(x^{\gamma_1} + y^{\gamma_2}) / \sigma \right) \), where \( x \) is perpendicular
distance, \( y \) is forward distance, and \( \gamma_1, \gamma_2 \) and \( \sigma \) are parameters. An example of the perpendicular distance detection functions resulting from use of identical detection hazard functions with each of the availability processes illustrated in Figure 3 is shown in Figure 4. Since the proportion of time animals are available is the same in both cases, and the detection hazard functions are the same in both cases, it is apparent that a correction factor based only on the proportion of time animals are available for detection is inadequate (even if that proportion were known).

[Figure 4 about here.]

5. Application

The methods developed above were applied to an aerial survey of whales on which only perpendicular distances were recorded. The availability process was estimated from tags placed on 12 separate animals. A model like that described above for perpendicular distance data only was fitted to the data by maximum likelihood, treating \( \theta_f \) as a parameter to be estimated and setting \( \theta_b = 90^\circ \). The fit is shown in Figure 5.

The model was also fitted by resampling segments of the observed availability process instead of fitting a HMM to it. Estimates differed by less than 5% from those obtained using the HMM.

[Figure 5 about here.]

6. Discussion

Conventional methods for correcting for availability bias on line transect surveys of animals with intermittent availability use only the proportion of time animals are available (and the proportion of time they are unavailable) for detection. While this provides a useful correction when the surveys are effectively instantaneous (i.e. the time animals are within detectable range is really very small compared to the length of periods of availability), it is in general insufficient for correction of availability bias.

The method developed here provides a general-purpose method for dealing with availability bias on line transect surveys. It requires that the availability process be characterised by
more than just the mean length or proportion of time animals are available. This characterisation can be via modelling availability data (from tags, for example) using hidden Markov models, or by resampling from a sample of availability process realizations. The method deals both with intermittent availability and with an area about the transect line being obscured.

**References**


Figure 1. Notation for field of view and location. The vertical arrow is the transect line, with the arrow indicating direction of the observer's movement. The observer is located at the circle on the transect line. The black whale shape shows the location of target animal $i$ at time $t$. The perpendicular, forward and radial distances from observer to animal are $x_i$, $y_{it}$, and $r_{it}$. Observers search the shaded “pie slice” from angles $\theta_f$ to $180^\circ - \theta_b$ from the the direction of movement of the observer along the trackline, out to a radial distance $W$. 
Figure 2. Schematic representation of the hidden Markov model (HMM) for a single animal. For times $t^* = (t - 1), t, (t + 1)$: $S_{t^*}$ are the hidden states, $A_{t^*}$ are the availability indicators, $\delta_{t^*}$ are the detection indicators, $\gamma_{t^*,t^*+1}$ are the transition probabilities, and $x_{t^*}$ and $y_{t^*}$ are the Cartesian coordinates of the animal relative to the observer. $f_{A|S}(\cdot)$ is the pmf of the availability indicator given state, and $f_{\delta|A}(\cdot)$ is the pmf of the detection indicator given availability and position.
Figure 3. Realisations of two availability processes, both with animals being available 20% of the time.
Figure 4. Perpendicular distance detection functions resulting from applying the detection hazard function \( h(x, y) = \exp(-x^{1.25} + y^{1.25}/1500) \) to each of the availability processes illustrated in Figure 3.
Figure 5. Maximum likelihood fit to whale perpendicular distance data. The model estimates that $\theta_f = 25^\circ$ (i.e. that an area 25$^\circ$ either side of the trackline is obscured); $\theta_b$ was set to 90$^\circ$ because observers searched only forward of abeam. For the availability process, a 2-state hidden Markov model was fitted to the tag data; from this model, animals are estimated to be available 19.8 of the time.