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# Decision Errors, Legal Uncertainty and Welfare: a General Treatment

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# Decision Errors, Legal Uncertainty and Welfare: a General Treatment

by

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## Abstract

This paper provides a general treatment of the implications for welfare of legal uncertainty. We distinguish legal uncertainty from *decision errors*: though the former can be influenced by the latter, the latter are neither necessary nor sufficient for the existence of legal uncertainty. We show that an increase in *decision errors* will always reduce welfare. However, for any given level of *decision errors*, information structures involving *more* legal uncertainty can improve welfare. This holds always, even when there is *complete* legal uncertainty, when sanctions on socially harmful actions are set at their optimal level. This transforms radically one's perception about the "costs" of legal uncertainty. We also provide general proofs for two results, previously established under restrictive assumptions. The first is that *Effects-Based* enforcement procedures may welfare dominate *Per Se* (or *object-based*) procedures and will *always* do so when sanctions are optimally set. The second is that optimal sanctions may well be *higher* under enforcement procedures involving *more* legal uncertainty.

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<sup>3</sup> We have presented a number of Discussion Papers on legal uncertainty and its implications since 2009. We are grateful for the comments of participants at the IMEDIPA Santorini Workshop, May 28<sup>th</sup> 2009; the CRETE Conference, Tinos, July 9 – 12, 2009; the INTERTIC Conference, Milan, September 15, 2009; the OFT Meeting on the ESRC Project on "Optimal Enforcement Procedures", January 11, 2010; Workshops in the Office of Fair Trading and French Competition Authority in January 2011; the MaCCI Workshop on Competition Policy, Speyer, 13 – 17 June 2011, Seminars at the University of Sussex, September 2011, the Higher School of Economics, Moscow, April 2013, the University of Venice, May 2013 and the IIOC, Boston, 17-19 May 2013. Over this period we have benefited in particular from the comments of Mathew Bennett, Amelia Fletcher, Kai Huschelrath, Kai-Uwe Kuhn, Massimo Motta, Volker Nocke, Martin Peitz, Anne Perrot, Patrick Rey, Anastasia Shchepetova, Yossi Spiegel, Jean Tirole and Thibaud Verge. Of course, all errors and ambiguities remain solely our responsibility. Our research on legal uncertainty has been supported by a research project on optimal enforcement procedures funded by the *Economic and Social Research Council* (ESRC, UK) under grant RES-062-23-2211. Research on the current paper has been supported by the research grant 2591 of the Greek Ministry's of Education Program "Excellence in Research (ARISTEIA)".

## 1. Introduction

Legal Uncertainty, the inability of economic agents to predict with certainty whether a privately beneficial action that they would wish to undertake would be judged by a regulatory authority as socially harmful, be treated as illegal and potentially subject to a penalty, has been the subject of analysis by economists and other social scientists for a long time. The impact of legal uncertainty on the optimal enforcement of economic regulations, the demand for legal advice and the incentives for compliance have been the subject of attention in the Law and Economics literature for at least three decades and, more recently, the literature on the enforcement of Competition Law. A review of contributions to these strands of the literature is also provided in Katsoulacos and Ulph (K&U, 2013a)<sup>4</sup>.

Essentially, traditional literature distinguishes three potential sources of legal uncertainty. The first is uncertainty that agents may face about the *type of their action* - about whether the action is genuinely socially harmful. The second is uncertainty regarding the *liability standard* which we can think of as the threshold level of harm caused by an action such that if the authority perceives the harm caused by a firm's action to be above this threshold it will disallow and penalise the action, while if the perceived level of harm is below this threshold then the authority will allow the firm's action. The third source of legal uncertainty, which has received extensive attention in the literature<sup>5</sup>, arises because authorities are unable to determine the actual harm caused by an action and so have to form some estimate of the harm, and an action is disallowed if the *estimated* value of harm is above the liability standard. Since these estimates contain errors this gives rise to possible Type I and Type II *decision errors* whereby actions that should be allowed are disallowed and actions that should be disallowed are allowed. In their analyses, Craswell and Calfee (1984, 1986) focus on the second and third sources of legal uncertainty and examine their welfare implications considering more specifically how under-compliance and over-compliance are affected. Other very important papers in the Law and Economics tradition that examine the implications of legal uncertainty, arising from the first and third sources above, for the optimal enforcement of economic regulations and the demand for legal advice are those by Kaplow (1990), Kaplow (1995) and Kaplow and Shavell (1992)<sup>6</sup>. The closest in spirit to the present paper, in the sense of examining implications – *albeit* for the demand for legal advice – of *both* the first and the third sources of uncertainty, is the paper of Kaplow and Shavell (1992) to which we return below.

Here, as in K&U (2013a) we propose a formalisation of the concept of legal uncertainty, which can be termed the *information structure approach*. By information structure we mean *what agents know* about the factors that influence the outcome of the enforcement authority's

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<sup>4</sup> “Legal Uncertainty and Competition Law Enforcement Procedures”, Section 2. Paper available in [www.cresse.info](http://www.cresse.info) (Publications).

<sup>5</sup> For example, the papers (mentioned below in the text) in the Law and Economics literature and the more recent contributions in the enforcement of competition law literature including Schinkel, M.P. and J. Tuinstra, (2006), Kwak J (2010) and Lang M (2012),

<sup>6</sup> We neglect here the literature on taxpayer uncertainty in the enforcement of tax laws going back in 1980s, as even less directly related to this paper than the literature we cite above.

decision-making process. It is very important to distinguish what agents know about this process from the *errors* made by the enforcement authority in reaching its decisions. Below, purely on the basis of what agents know, which determines their *perceived probability* of having their actions disallowed, we distinguish and then analyse three different information structures.

To clarify the difference between this approach and just associating legal uncertainty with decision errors, it is important to note that decision errors made by the authority are neither necessary nor sufficient for the existence of legal uncertainty. Thus:

- (i) It is not true that *if* there are no decision errors there will be no legal uncertainty. This would only be true if also agents knew their *type*, that is, if they know the true value of the harm that their action causes to others.
- (ii) It is also not true that when there are decision errors there will be legal uncertainty in the standard sense defined above. After all, it is common to consider as an advantage of *Per Se* legal rules, that they do not involve legal uncertainty but of course *Per Se* legal rules can involve a substantial amount of decision errors.

Firms' perceived probability of having an action that they wish to undertake being disallowed will be influenced by all three sources of legal uncertainty:

- (i) Whether or not they know their true type – the true value of harm on others that their action generates.
- (ii) Whether or not they know the estimate of the harm of their action that the authority will make (which depends on their understanding of exactly how the authority reaches its estimates of harm).
- (iii) Whether or not they know the liability standard that the authority is using.

We assume a zero liability standard that is common knowledge. Then depending on which of the other two sources is relevant in a specific context we distinguish between and analyse three different information structures, those of *No Legal Uncertainty*, of *Partial Legal Uncertainty* and of *Complete Legal Uncertainty*.

We consider Kaplow and Shavell (1992) the seminal paper in the analysis of legal uncertainty – though they themselves did not choose to stress their results with respect to this aspect of their analysis. As we noted above, they use a model closer in spirit than all other models in the literature to the model below in the sense that agents face uncertainty because they may not know their true type *or* because of errors made in determining true harm by the social authority. Using their terminology, agents can be “uninformed” because of either of these types of uncertainty. They can eliminate the uncertainty and become “informed” by getting “legal advice” at a cost. The paper examines whether the demand for legal advice is socially optimal. In terms of the framework used here, we can say that in Section 2 of their paper, agents' information structure is one of *Complete Legal Uncertainty*<sup>7</sup> and by getting

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<sup>7</sup> In this section, agents can be uninformed about whether their action is harmful and if they become informed they learn the true harm that is what the authority will determine if it undertakes an investigation.

legal advice they move to *No Legal Uncertainty*. In Section 3, agents' information structure is one of *Partial Legal Uncertainty* and by getting legal advice they move to *No Legal Uncertainty*. Proposition 3 (in their Section 3) essentially establishes that getting legal advice and moving from *Partial* to *No Legal Uncertainty* is *not* socially optimal – so removing uncertainty reduces welfare. This result is also established below in the different context and objectives of this paper<sup>8</sup>. Thus, one of our main objectives here, unlike in Kaplow and Shavell (1992), is to provide a full and general analysis of the implications for welfare of *changes in the errors* made by the authority *and* of how, for *given* errors, different information structures affect welfare, providing a unified framework that allows an examination and comprehensive comparison of the impact of each one of them. Further, here we are also primarily concerned, unlike Kaplow and Shavell (1992), with a comparison between different enforcement procedures and their implications for optimal penalties<sup>9</sup>. Thus, while, as in Section 2 of Kaplow and Shavell (1992) we find that moving from *Complete* to *No Legal Uncertainty* reduces welfare, we also show the important result that moving from an *Effects-Based* procedure with *Complete Legal Uncertainty* to a *Per Se* procedure can also reduce welfare.

Apart from the differences in the context and objectives, there are significant differences in the assumptions we utilise relative to Kaplow and Shavell (1992) and the other papers mentioned above. So, in these papers the substantive standard used by the social authority for assessing whether an action is illegal is that of total welfare while we assume that the standard is that of consumer welfare<sup>10</sup>. This is important given that the standard used influences in a critical way the deterrence objectives of the social authority and optimal penalties. Furthermore, the papers mentioned above conflate the probability of being found illegal into a single number while it is important for our analysis to take explicitly into account that this probability is the product of two distinct probabilities, one of which is the probability of been detected to take a potentially harmful action and the other the probability that the authority, making decisions subject to errors, actually decides that the action is harmful. Also, while in the exact context of the issues addressed by all the above papers it may be natural to assume, as the above papers do, that all potential actions undertaken by firms are non-benign<sup>11</sup> (implicitly assuming a probability of allowing benign actions to occur equal to one and thus neglecting Type I errors)<sup>12</sup>, this is certainly not the case for many

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<sup>8</sup> And also, as clarified below, on the basis of a completely different set of assumptions to those utilised in Kaplow and Shavell (1992).

<sup>9</sup> And not just with a comparison across information structures. While comparing different enforcement procedures was the subject also of K&U (2009), as noticed above, in that paper the analysis was restricted to one information structure and only exogenous penalties.

<sup>10</sup> This is a much more natural assumption in the context of many economic regulations (e.g. in the context of Competition Law or the law for sectoral regulation) for which in practice authorities' objective is to maximize consumer welfare

<sup>11</sup> That is, they generate positive or zero (but *not* negative) harm.

<sup>12</sup> An alternative way to put this is to say that Kaplow (1990) and Kaplow and Shavell (1992) just deal with actions that in the terminology below, are *presumptively illegal* while we also have to consider *presumptively legal* actions. This also has serious implications for the results we get on optimal penalties under the different information structures.

business practices<sup>13</sup>, so in our framework we allow for actions that are either socially harmful or socially benign and to allow the authority's decision errors to extend to its assessment of the latter type of actions. An additional difference between the above papers and the present one is that we examine the important phenomenon of *desistance* due to *delays* in the authority's procedures affects the outcomes. Agents' anti-competitive actions will normally take some time before they create benefits and social harm and so the size of these accruing will depend critically on delays in the authority's procedures, which therefore will affect the comparison of procedures and of optimal sanctions.

The present paper considerably extends and generalises our analysis of legal uncertainty in K&U (2013a). Specifically: in K&U (2013a) we do not treat the harm to others caused by the firms' actions as a continuous variable and also we do not explicitly model the influence of estimation errors on welfare. In the present paper harm is treated as a continuous variable and estimation errors are explicitly introduced and modelled under the different information structures. Thus the present paper clearly disentangles the effects of increased estimation and thus of decision errors from that of different information structures facing firms and provides, we believe for the first time, a unified framework that allows an examination and comprehensive comparison of the impact of each one of them.

The analysis leads to a number of contributions. Perhaps the foremost contribution is providing a clear general demonstration of how the "welfare costs" of increased legal uncertainty depends on the interpretation one uses concerning its sources. While these "costs" are certainly positive when increased uncertainty is associated with increased estimation errors on the part of the enforcement authority, the increase in "costs" can actually be negative when, for given estimation errors, firms face information structures with increased levels of uncertainty. We *also* provide general proofs of the results established in K&U (2013a). Thus with the general framework of this paper we establish general proofs of the results that *effects-based* or *rule-of-reason* decision procedures always welfare dominate *Per Se* procedures and that *higher* optimal penalties can be associated with increased legal uncertainty.

The structure of the paper is as follows. In the next Section we set out in detail our model that allows us to disentangle the influence on legal uncertainty of estimation and thus decision errors from that of different information structures facing firms. Then in Section 3 we examine the implications of different information structures for optimal penalties and welfare establishing a number of Lemmas that underline the rest of the analysis. In Section 4 we examine the implications for welfare of increased estimation errors under different information structures and compare the welfare under different information structures for given decision errors with fixed and optimal penalties. In Section 5 we compare *Effects-Based* decision procedures to *Per Se* and in Section 6 we provide a diagrammatic illustration of the different ways of measuring the welfare "costs" of legal uncertainty. Finally, Section 7 concludes.

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<sup>13</sup> Such as, for example, those dealt with by Competition Law.

## 2. The Model

### 2.1 Background

There is a population of firms that might take some action. This may be any action with potentially socially harmful effects condemned by law. This action has some natural life, and all profits, harm to others etc. are measured over this natural life. Any firm will only undertake this action if there is some expected net private benefit/profit from doing so.

However, the action may also have some effect on other agents – consumers, competitors - which may be positive or negative. Let  $b > 0$  denote the private benefit a firm will get over the lifetime of the action in the absence of any intervention from an enforcement authority. Assume that this is distributed across firms in a way that is independent from the harm caused by the action and is given by the density function  $f(b) > 0 \quad \forall b > 0$ .

Let  $h_k$  denote the “harm” caused by firm  $k$  and assume that this is distributed across firms with a density function  $g(h)$  where  $g(h) > 0 \quad \forall h \in (-\infty, +\infty)$ . Let

$$\bar{h} = \int_{-\infty}^{\infty} hg(h)dh$$

be the average harm and say that the action is *Presumptively Illegal* if  $\bar{h} > 0$  and *Presumptively Legal* if  $\bar{h} < 0$ .

**Assumption 1** The density functions  $f$  and  $g$  are common knowledge.

### 2.2 Regulatory / Enforcement Authority

There is a regulatory authority (RA) that investigates a fraction  $\pi$ ,  $0 < \pi < 1$  of actions that are taken, to determine whether or not they are anti-competitive<sup>14</sup>.

We allow for the possibility that this determination takes place before the end of the natural life of the action. If an action is found not to be socially harmful, we say that it has been *approved* by the RA which allows the action continue to end of its natural generating full benefit  $b$  for the firm and full harm,  $h$  for society. On the other hand if an action is found to be socially harmful, we say that it has been *disapproved* by the RA which will therefore stop the action, and, in addition, impose a penalty. So a firm whose action is disapproved suffers both a penalty and a loss of some future profits. Formally we assume that, if an action is disapproved, the firm gets just a fraction  $\Delta$ ,  $0 < \Delta \leq 1$  of the private benefit,  $b$  that it would have got over the natural life of the action and society gets the same fraction of the harm  $h$  it would have suffered over the natural life of the action. The parameter  $\Delta$  is positive and measures the inevitable delay involved in the RA’s learning that an action has occurred, carrying out an investigation and reaching a decision.

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<sup>14</sup> As mentioned explicitly below we take this to mean that the actions cause positive harm to others.

**Assumption 2** The values of both  $\Delta$  and  $\pi$  are common knowledge.

### 2.3 Decision Rule of Regulatory Authority

For any action it investigates the RA has to decide whether to disapprove it – find it to be socially harmful – or approve it – find it was not socially harmful. We assume that:

- the RA operates a welfare standard whereby the only thing it cares about is the harm (to others) caused by the action taken by a firm<sup>15</sup>;
- it does not directly observe the true value of the harm caused by any firm;
- it operates an *Effects-Based* enforcement procedure.

This final assumption means that in order to decide what to do in any particular case, it first gathers evidence about the likely consequences of the action taken by any given firm coming under investigation. Based on this, it forms an estimate of the harm caused by the action taken by any firm. Let

$$h_k^e = h_k + \varepsilon_k \quad (1)$$

denote the estimate of the harm caused by firm  $k$  where  $\varepsilon_k$  is the estimation error. Assume that, whatever their true harm, for all firms the estimation error is uniformly distributed on  $[-E, E]$  where  $E \geq 0$ . This allows for the possibility that the authority correctly observes the harm taken by the action – the possibility that  $E = 0$ . Throughout this paper we assume that  $E$  is exogenous.

**Assumption 3** The shape of the distribution of estimation errors, and the exogenous value of  $E$  is common knowledge.

We assume that the decision rule used by the authority is to set a *liability standard*  $\underline{h}$  and then to disapprove the action taken by firm  $k$  if and only if

$$h_k^e > \underline{h}. \quad (2)$$

In principle the liability standard may be positive negative or zero. In this paper we assume that the existence of such a liability standard does not reflect a fundamentally different view between the RA and the firms as to where the boundary between harmful and benign actions lies, but rather it reflects the fact that, recognising that it has only an imperfect measure of harm, the RA may want to be cautious about the decision to approve/disapprove actions and so uses the liability standard as a threshold value that minimises the risk of making wrong decisions. In principle this liability standard could therefore depend on the accuracy of the RA's estimate of harm – i.e. on  $E$  – but in this paper we will treat  $\underline{h}$  as fixed. We also could allow the possibility that firms do not know the fixed liability standard. However, since we are going to allow the possibility that one of the reasons why firms may

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<sup>15</sup> This would certainly be the natural assumption to make in the case of actions potentially prohibited by Competition Law, given that Competition Authorities use consumer surplus as their substantive standard.

not know for sure what decision the authority will make in their case is that they don't know what estimate of harm the authority will make, and since the decision rule depends simply on the difference between the estimate of harm and the liability standard, it will simplify the analysis if we assume that firms know the liability standard. Consequently in what follows we will assume that everyone knows the liability standard, and, furthermore that this is normalised so that  $\underline{h} = 0$

**Assumption 4** It is common knowledge that the RA makes its decision according to (2) and in doing so uses a fixed liability standard  $\underline{h} = 0$

Given our assumptions so far, we can determine the RA's *decision outcome function*,  $\delta(h; E)$ , which defines the fraction of firms with true harm  $h$  whose actions will be disapproved by the RA, when the error bound is  $E \geq 0$ . This is given by:

$$\delta(h; E) = \begin{cases} 0, & h \leq -E \\ \frac{h+E}{2E}, & -E \leq h \leq E \\ 1, & h \geq E \end{cases} \quad (3)$$

Equivalently,  $\delta(h; E)$  is the perceived probability of being disapproved by a firm whose action generates harm  $h$  when the error bound is  $E \geq 0$ . Notice that if there were no estimation errors, so  $E = 0$ , then the decision outcome function is just the first-best one of approving all benign actions and disapproving all harmful ones

Notice also that it follows from Assumptions 3 and 4 that the decision outcome function,  $\delta(h; E)$ , is common knowledge. Since

$$\frac{\partial \delta}{\partial E} = -\frac{h}{2E^2} \quad (4)$$

it follows that:

$$\frac{\partial \delta}{\partial E} > 0 \quad \text{if } h < 0; \quad \frac{\partial \delta}{\partial E} < 0 \quad \text{if } h > 0 \quad (5)$$

So, an increase in the *estimation error* raises the probability ( $\delta$ ) of benign actions being disapproved (Type I *decision errors* or *false convictions*) and increases the probability ( $1-\delta$ ) that harmful actions will be approved (Type II *decision errors* or *false acquittals*).

The costs of Type I and Type II decision error associated with a given rule are, respectively:

$$\begin{aligned} D^I(E) &= -\int_{-E}^0 h \delta(h; E) g(h) dh > 0; \\ D^{II}(E) &= \int_0^E h [1 - \delta(h; E)] g(h) dh > 0 \end{aligned} \quad (6)$$

It is straightforward to show that

$$\frac{dD^I(E)}{dE} = \frac{1}{2E^2} \int_{-E}^0 h^2 g(h) > 0; \quad \frac{dD^{II}(E)}{dE} = \frac{1}{2E^2} \int_0^E h^2 g(h) > 0 \quad (7)$$

so, as expected, we get that:

**Lemma 1:** an increase in the error bound will certainly increase the costs of both Type I and Type II decision errors.

A final property of the RA's decision rule that we wish to discuss is its ability to *Effectively Discriminate* between benign and harmful actions. To understand what this means let

$$\begin{aligned} \bar{h}^D(E) &= \int_{-\infty}^{\infty} \delta(h; E) hg(h) dh \\ &= \int_{-E}^E \delta(h; E) hg(h) dh + \int_E^{\infty} hg(h) dh \end{aligned} \quad (8)$$

be the average harm caused by firms whose actions are *disapproved* under the decision outcome function  $\delta(h; E)$  and

$$\begin{aligned} \bar{h}^A(E) &= \int_{-\infty}^{\infty} [1 - \delta(h; E)] hg(h) dh \\ &= \int_{-\infty}^{-E} hg(h) dh + \int_{-E}^E [1 - \delta(h; E)] hg(h) dh \end{aligned} \quad (9)$$

be the average harm caused by firms whose actions are *approved* by the RA under the decision outcome function  $\delta(h; E)$ .

We say that an *Effects-Based* decision rule can *Effectively Discriminate* iff it has lower decision error costs than the appropriate<sup>16</sup> *Per Se* legal standard and, based on the analysis in Katsoulacos and Ulph (2009), this in turn is true iff

$$\bar{h}^D(E) > 0 > \bar{h}^A(E). \quad (10)$$

Notice that  $\bar{h}^D(0) = \int_0^{\infty} hg(h) dh > 0$ ;  $\bar{h}^A(0) = \int_{-\infty}^0 hg(h) dh < 0$  so (10) is certainly satisfied if  $E = 0$ . Moreover, from (8) and (9) we have:

$$\frac{d\bar{h}^D(E)}{dE} = -\frac{1}{2E^2} \int_{-E}^E h^2 g(h) dh < 0; \quad \frac{d\bar{h}^A(E)}{dE} = \frac{1}{2E^2} \int_{-E}^E h^2 g(h) dh > 0. \quad (11)$$

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<sup>16</sup> A *Per Se Illegal* enforcement procedure is used if all actions of a particular type are disapproved, while a *Per Se Illegal* enforcement procedure is used if all actions of a particular type are approved. The former is appropriate if the type of action is *Presumptively Illegal* and the latter if it is *Presumptively Legal*

Finally, as  $E \rightarrow \infty$ ,  $\bar{h}^D(E) \rightarrow \frac{\bar{h}}{2}$ ;  $\bar{h}^A(E) \rightarrow \frac{\bar{h}}{2}$ .

It follows that there will be some unique critical value of  $E$ , which we denote by  $\tilde{E} > 0$ <sup>17</sup> such that (10) holds iff  $0 \leq E < \tilde{E}$ . If the action is Presumptively Illegal  $\bar{h} > 0$  and  $\tilde{E}$  is defined by  $\bar{h}^A(\tilde{E}) = 0$ . If the action is Presumptively Legal  $\bar{h} < 0$  and  $\tilde{E}$  is defined by  $\bar{h}^D(\tilde{E}) = 0$ .

Now, we are interested in the question whether legal certainty might be a reason to prefer a *Per Se* decision procedure to an *Effects-Based* decision procedure in circumstances where one might otherwise have wanted to use *Effects-Based*. We take this to be circumstances where an *Effects-Based* procedure has lower decision error costs, so in the discussion that follows we will confine discussion to this interval ( $0 \leq E < \tilde{E}$ ). So we make:

**Assumption 6** We assume that the error bound,  $E$ , lies in the interval  $0 \leq E < \tilde{E}$ , and so (10) always holds, so the *Effects-based* procedure can effectively discriminate.

#### 2.4 Penalties

We assume that if the RA disapproves an action then it imposes a penalty that is equal to a fraction  $\phi \geq 0$  of the private benefit that the firm earned until its action was stopped by the RA<sup>18</sup>. So a firm that would have earned a private benefit  $b > 0$  over the natural life of the action will, if its action is disapproved, earn  $\Delta b$  and pay a penalty  $\phi \Delta b$ , generating a net payoff  $\Delta b(1 - \phi)$ . We make the following assumption:

**Assumption 7** The value of  $\phi$  is common knowledge.

#### 2.5 Behaviour of Firms

We assume that when firms decide whether or not to take an action they anticipate that there is a probability,  $\pi$ , of having their action investigated and, if disapproved, having the action stopped and a penalty imposed. If a firm thinks that, if investigated, the probability of having its action disapproved is  $\delta$ ,  $0 \leq \delta \leq 1$ , then the risk of conviction is  $\pi\delta$  and the expected net private benefit from taking the action will be

$$(1 - \pi\delta)b + \pi\delta b\Delta(1 - \phi) = b[1 - \pi\delta((1 - \Delta) + \Delta\phi)]. \quad (12)$$

**Assumption 8** A firm will take the action iff its expected private benefit is strictly positive, i.e. from (12), iff

$$\phi < \frac{1 - \pi\delta(1 - \Delta)}{\pi\delta\Delta}. \quad (13)$$

<sup>17</sup> The precise value of  $\tilde{E}$  will depend on the shape of the density function  $g(h)$ .

<sup>18</sup> In Katsoulacos & Ulph (2013b) we examine optimal penalties when, as is the case for most CAs, penalties are related to revenue rather than profits.

where, remember, that  $\delta$  is a function of  $h$  and  $E$ . The issue is when is this condition satisfied and how does this relate to what firms know about the factors that influence how the RA makes its decisions. This brings us to a discussion of information structures.

## 2.6 Information Structures

We say that firms face *legal uncertainty* if, at the time they make their decision, they do not know for sure what decision the RA will make regarding their action, should it come under investigation.

Given the nature of the RA's decision rule, whether legal uncertainty arises, and, if it does, the precise form it takes could in principle depend on three factors:

- whether a firm knows the true harm,  $h$ , caused by its action, and, if it does not, what precisely it does know;
- whether a firm knows the estimate,  $h^e$ , that the RA will make of the harm caused by its action, and, if it does not, what precisely it does know;
- whether a firm knows the liability standard,  $\underline{h}$  that the RA will use in making its decision and, if it does not, what precisely it does know.

Given our Assumption 4 the third factor is not in play here, so we focus on the first two.

Even with just two potential sources of legal uncertainty there are many different set-ups one could analyse. In what follows we will focus on just three information structures, which capture crucially different configurations. We now define these structures, explain how they could arise in the context of our model, and set out the crucial features they embody.

### Information Structure I: No Legal Uncertainty

This would arise if each firm knew in advance exactly what estimate of harm the RA would make in its situation. This could arise if the RA set out the factors it would measure, the data it would use to measure these, and the calculations it would make, and if firms could costlessly access exactly the data the RA would use in its particular case and perform the calculations before it decided to take the action. Then, since we assume each firm knows the liability standard, each firm would know exactly what decision the RA would reach in its case.

Notice the following:

- (i) While this situation may not often arise in practice, nevertheless it serves to make the point that even though the RA is using an *Effects-Based* procedure there still could be no *legal uncertainty*. Put differently, it is not true that the use of an *Effects-Based* procedure *inevitably* results in the existence of *legal uncertainty*.
- (ii) Because the RA's estimate of harm is imperfect, two firms with the exactly same level of true harm could end up with one knowing for sure that its action will be

approved while the other knows for sure that its action would be disapproved. The fact that there is *variability of decision* across otherwise identical firms is not the same thing as *legal uncertainty* which arises when a given firm is unable to anticipate what decision a RA will make in its particular circumstances.

- (iii) This information structure could arise whether or not firms knew their true level of harm.

A crucial feature of this information structure is that any deterrence effects that arise do not depend on what firms might know about their true harm. Thus:

- as noted, firms with the same level of harm may perceive very different probabilities of disapproval, and so face very different deterrent effects;
- all firms whose actions are disapproved will face exactly the same probability of disapproval (namely 1) irrespective of their true harm;
- all firms whose actions are approved will face exactly the same probability of disapproval (namely 0) irrespective of their true harm;
- nevertheless there is some link between true harm and deterrence in that the greater the true harm the higher will be the fraction of firms who know for sure that their action will be disapproved.

So while there is some statistical sense in which there is a potential *differential deterrence effect*<sup>19</sup> at work – on average actions which are more harmful will be more likely to be deterred than actions that are less harmful, this does not happen at the level of individual firms.

#### Information Structure II: Partial Legal Uncertainty

Here firms are assumed to know their true level of harm,  $h$ , but not the precise estimate  $h^e$  that the RA will make. Instead all they know is that (i) the RA's estimate will lie around their true level of harm with an estimation error; (ii) the RA will disapprove if the estimate of harm is above the liability standard. Given assumptions 3 and 4 **all** firms with harm  $h$  know (a) that estimation errors are uniformly distributed with error bound,  $E$ ; (b) the value of  $E$ ; (c) that the liability standard is  $\underline{h} = 0$ . Consequently, all firms with true harm  $h$  know that the probability of having their action disapproved is  $\delta(h; E)$  as given by (3).

The crucial features of this information structure are that:

- all firms with true harm  $h$  perceive exactly the same probability of disapproval;
- this common probability is an increasing function of true harm  $h$ .

As we will see this sets up a very sharp differential deterrence effect, whereby all firms above a particular level of harm are deterred from taking the action while all those below it

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<sup>19</sup> A differential deterrence effect refers to the effect of firms with more harmful actions perceiving a higher probability that their action will be disapproved if investigated. As shown in Katoulacos and Ulph (2009) this effect is crucial in the comparison between *effects-based* and *Per Se* procedures.

take the action. Thus, by setting the appropriate penalty the authority can ensure that it deters all harmful actions.

### Information Structure III: Complete Legal Uncertainty

Here each firm knows neither its true harm,  $h$ , nor the estimate of harm,  $h^e$ . Indeed it is assumed in this scenario that the degree of uncertainty regarding their true harm is sufficiently great that all that **any** firm knows is that their true harm can take any real number with density  $g(h)$ . As with partial legal uncertainty, the fact that firms do not know the estimate of harm means that firms assume that, if their true harm were  $h$  then the probability of their action being disapproved would be  $\delta(h, E)$ . Taken together these assumptions imply that all firms anticipate exactly the same probability of disapproval:

$$\bar{\delta}(E) = \int_{-\infty}^{\infty} \delta(h, E) g(h) dh, \quad 0 < \bar{\delta}(E) < 1. \quad (14)$$

In this case there is absolutely no *differential deterrence effect* since the probability of disapproval is the same across all firms *irrespective of their true harm*.

## **3. Implications of Different Information Structures for Optimal Penalties and Welfare**

We now consider in turn the implications of each of these information structures for the behaviour of firms, the optimal penalty and consequently for welfare.

### I: No Legal Uncertainty

Under this information structure, all firms know with certainty whether their action will be approved or disapproved if investigated. Specifically, a fraction  $1 - \delta(h, E)$  of firms whose true harm is  $h$  will know for certain that, even if investigated, their action will be approved, and so, at the time they take the action expect to make profits  $b > 0$  and so will take the action. The remaining fraction,  $\delta(h, E)$ , will know that, if investigated by the RA, their action will certainly be disapproved and so, for a given penalty  $\phi > 0$ , will expect to make profits  $b[1 - \pi((1 - \Delta) + \Delta\phi)]$ . Given Assumption 8 and (13) it follows that if  $\phi < \frac{1 - \pi(1 - \Delta)}{\pi\Delta}$  then these firms will take the action.

So if  $\phi < \frac{1 - \pi(1 - \Delta)}{\pi\Delta}$  all the firms will take the action. However, of the firms whose actions would be disapproved if investigated, a fraction  $\pi$  will be investigated and, since their action will be disapproved and stopped, society will lose a fraction  $(1 - \Delta)$  of the harm they would have generated, so the net harm suffered by society in this case is  $\bar{h} - \pi(1 - \Delta)\bar{h}^D(E)$ .

On the other hand, if  $\phi \geq \frac{1-\pi(1-\Delta)}{\pi\Delta}$  all those firms who know for sure that their action will be disapproved will be deterred from taking the action, and so the only firms taking the action are those who know for sure that their action will be approved. Even though some of these will be investigated, by definition their actions will be approved and society will get the full harm to which they give rise. So the harm accruing to society in this case is  $\bar{h}^A(E) = \bar{h} - \bar{h}^D(E)$ .

Social welfare (the negative of harm) under No Legal Uncertainty is therefore:

$$W^{NLU}(\phi; E) = \begin{cases} -\bar{h} + \pi(1-\Delta)\bar{h}^D(E), & \phi < \frac{1-\pi(1-\Delta)}{\pi\Delta} \\ -\bar{h} + \bar{h}^D(E), & \phi \geq \frac{1-\pi(1-\Delta)}{\pi\Delta} \end{cases} \quad (15)$$

Given our Assumption 6 that the error bound,  $E$ , lies in the range  $[0, \tilde{E}]$  over which the RA's decision rule can *Effectively Discriminate*, it follows from (10) that  $\bar{h}^D(E) > 0$  so, from (15), welfare is maximised by setting a penalty that deters all firms who know their action will be disapproved from taking it, and the minimum penalty that does this is given by:

Lemma 2: The *optimal penalty* under No Legal Uncertainty is:

$$\hat{\phi}^{NLU} = \frac{1-\pi(1-\Delta)}{\pi\Delta}. \quad (16)$$

Thus we get:

Lemma 3: The *maximum welfare* under No Legal Uncertainty is:

$$\hat{W}^{NLU}(E) = W^{NLU}(\hat{\phi}^{NLU}, E) = -\bar{h}^A(E) = -\bar{h} + \bar{h}^D(E) > 0. \quad (17)$$

## II: Partial Legal Uncertainty

Under this information structure all firms of type  $h$  think that they face the same probability  $\delta(h, E)$  of having their action disapproved if they are investigated, and so anticipate making expected profits  $b[1 - \pi\delta(h, E)((1-\Delta) + \Delta\phi)]$ .

As with the case of No Legal Uncertainty, if,  $\phi < \frac{1-\pi(1-\Delta)}{\pi\Delta}$  then the penalty is so low that even firms who think that, if investigated, their action will definitely be

disapproved, will find the expected net profit from taking the action is positive, so all firms will take the action, giving rise to total harm  $\bar{h} - \pi(1 - \Delta)\bar{h}^D(E)$ .

However if  $\phi \geq \frac{1 - \pi(1 - \Delta)}{\pi\Delta}$ , then, since  $\delta(h; E)$  is a strictly increasing function of  $h$  for  $h \in [-E, E]$ , we can define  $\tilde{h}(\phi; E)$  as the unique value of  $h \in [-E, E]$  such that

$$\delta(\tilde{h}; E) = \frac{1}{\pi[(1 - \Delta) + \Delta\phi]}, \quad (18)$$

and all firms with  $h \geq \tilde{h}$  will be deterred from taking the action.

So we have that welfare under Partial Legal Uncertainty is given by:

$$W^{PLU}(\phi; E) = \begin{cases} -\bar{h} + \pi(1 - \Delta)\bar{h}^D(E), & \phi < \frac{1 - \pi(1 - \Delta)}{\pi\Delta} \\ -\int_{-\infty}^{\tilde{h}(\phi; E)} [1 - \pi(1 - \Delta)\delta(h; E)]hg(h)dh, & \phi \geq \frac{1 - \pi(1 - \Delta)}{\pi\Delta} \end{cases} \quad (19)$$

Now since there is a one-to-one link between the penalty and  $\tilde{h}$ , it is clear from (19) that the welfare maximising value of  $\tilde{h}$  is  $\tilde{h} = 0$ . That is, the RA wants to set a penalty that deters all harmful actions and ensures that all firms whose actions are benign will take them. Of course some of these will be investigated and, because of decision errors, have their action disapproved and stopped.

Since, from (3),  $\delta(0, E) = \frac{1}{2}$ , it follows that:

Lemma 4: the *optimal penalty* under Partial Legal Uncertainty is

$$\hat{\phi}^{PLU} = \frac{2 - \pi(1 - \Delta)}{\pi\Delta} > \hat{\phi}^{NLU} \quad (20)$$

and

Lemma 5: the *maximum welfare* under Partial Legal Uncertainty is:

$$\hat{W}^{PLU}(E) = -\int_{-\infty}^0 [1 - \pi(1 - \Delta)\delta(h; E)]hg(h)dh > 0. \quad (21)$$

### III. Complete Legal Uncertainty

In this information structure uncertainty is so great that all firms perceive the same probability  $\bar{\delta}(E) = \int_{-\infty}^{\infty} \delta(h, E) g(h) dh$ ,  $0 < \bar{\delta}(E) < 1$  of having their action disapproved.

It follows that if  $\phi < \frac{1 - \pi \bar{\delta}(E)(1 - \Delta)}{\pi \Delta \bar{\delta}(E)}$  then all firms will take the action. As we saw in the previous two information structures this generates welfare  $-\bar{h} + \pi(1 - \Delta)\bar{h}^D(E)$ .

On the other hand if  $\phi \geq \frac{1 - \pi \bar{\delta}(E)(1 - \Delta)}{\pi \Delta \bar{\delta}(E)}$  then no firm takes the action and welfare is zero. So we have that welfare under Complete Legal Uncertainty is given by:

$$W^{CLU}(\phi; E) = \begin{cases} -\bar{h} + \pi(1 - \Delta)\bar{h}^D(E), & \phi < \frac{1 - \pi \bar{\delta}(E)(1 - \Delta)}{\pi \Delta \bar{\delta}(E)} \\ 0, & \phi \geq \frac{1 - \pi \bar{\delta}(E)(1 - \Delta)}{\pi \Delta \bar{\delta}(E)} \end{cases} \quad (22)$$

The implications for the optimum penalty are more subtle than in the previous two Information Structures. If the action is *Presumptively Legal*, so  $\bar{h} < 0$ , then since we have assumed that the error bound lies in the range  $0 \leq E < \tilde{E}$  where the RA can *Effectively Discriminate* and so, from (10)  $\bar{h}^D(E) > 0$  it follows that welfare is always higher if all firms take the action. So in this case the optimal penalty is zero – i.e.  $\hat{\phi}^{CLU} = 0$

If the action is *Presumptively Illegal*, so  $\bar{h} > 0$ , then since our assumption that the RA can *Effectively Discriminate* implies that  $\bar{h}^D(E) > \bar{h} > 0$  it follows that if the fraction of firms that are investigated is so small and the delay in investigating is so large that  $\pi(1 - \Delta) \leq \frac{\bar{h}}{\bar{h}^D(E)}$  then it is best to discourage all firms from taking the action and so the

optimal penalty is  $\hat{\phi}^{CLU} = \frac{1 - \pi(1 - \Delta)}{\pi \Delta \bar{\delta}(E)}$ . However if the fraction of firms investigated is

sufficiently large and the delay in investigating sufficiently small so that  $\pi(1 - \Delta) \geq \frac{\bar{h}}{\bar{h}^D(E)}$

then it is best to allow all firms to take the action and once again the optimal penalty is  $\hat{\phi}^{CLU} = 0$ .

Drawing this all together we have:

Lemma 6: the *optimal penalty* under Complete Legal Uncertainty is given by:

$$\hat{\phi}^{CLU} = \begin{cases} 0, & \bar{h} < 0 \\ 0, & \bar{h} > 0, \quad \pi(1-\Delta) > \frac{\bar{h}}{\bar{h}^D(E)} \\ \frac{1-\pi(1-\Delta)}{\pi\Delta\delta(E)}, & \bar{h} > 0, \quad \pi(1-\Delta) < \frac{\bar{h}}{\bar{h}^D(E)} \end{cases} \quad (23)$$

which implies:

Lemma 7: the *maximum welfare* under Complete Legal Uncertainty is given by:

$$\hat{W}^{CLU}(E) = \begin{cases} -\bar{h} + \pi(1-\Delta)\bar{h}^D(E) > 0, & \bar{h} < 0 \\ -\bar{h} + \pi(1-\Delta)\bar{h}^D(E) > 0, & \bar{h} > 0, \quad \pi(1-\Delta) > \frac{\bar{h}}{\bar{h}^D(E)} \\ 0, & \bar{h} > 0, \quad \pi(1-\Delta) < \frac{\bar{h}}{\bar{h}^D(E)} \end{cases} \quad (24)$$

#### 4. Effects of Increasing Legal Errors and Welfare Comparisons

In the previous two sections we presented a framework in which information structures are separated out from estimation and decision errors. Further in the previous section we derived explicit formulae for welfare under different information structures and shown how this relates to both the error bound and the level of the penalty. We have also derived expressions for maximum welfare under the different information structures as a function of the error bound alone – where we have set the penalty at its welfare-maximising level.

We are now able to undertake a number of comparisons to see:

- How, conditioning on the information structure, welfare varies with the error bound on estimation errors;
- How, conditioning on the error bound welfare varies with the information structure.

In the first exercise we can look at how an increase in the error bound on harm estimates affects welfare either holding the penalty constant at some arbitrary level, or assuming that the penalty has been set at the appropriate level by the RA.

In the second exercise, since deterrent effects can vary quite widely across information structures for a given level of penalty, we will confine attention to the case where the penalty has been set at its welfare-maximising level.

Before proceeding, notice that if  $\phi < \frac{1-\pi(1-\Delta)}{\pi\Delta}$  then the penalty is so low that no firm will be deterred from taking the action under any information regime, and information structures play no role in affecting welfare since they matter only because of their impact on deterrence.

So in all the discussion that follows we will assume that the following assumption holds:

**Assumption 9**  $\phi \geq \frac{1-\pi(1-\Delta)}{\pi\Delta}$

#### 4.1 Effects of Increasing Error Bounds on Welfare

Consider in turn the various information structures.

##### I. No Legal Uncertainty

Given Assumption 9 we have

$$W^{NLU}(\phi, E) = \hat{W}^{NLU}(E) = -\bar{h} + \bar{h}^D(E) \quad (25)$$

But then from (11) we have:

$$\frac{\partial W^{NLU}(\phi, E)}{\partial E} = \frac{d\hat{W}^{NLU}(E)}{dE} = \frac{d\bar{h}^D(E)}{dE} < 0. \quad (26)$$

The intuition is straightforward. For the range of penalties under consideration given Assumption 9, with this information structure, all actions that the RA would disapprove are deterred and all actions that it would approve are taken. Given the errors in its decisions this latter category will include actions that are genuinely harmful, while some actions that are genuinely benign will be disapproved and, anticipating this, deterred. The greater the error bounds the larger the number of benign actions deterred (and the greater the benefit of such deterred actions) and the larger the fraction of harmful actions taken (and the greater the harm of such actions). So greater errors cause worse deterrence effects.

So we have:

**Result 1** With No Legal Uncertainty both welfare and maximum welfare are strictly decreasing functions of the error bound.

## II. Partial Legal Uncertainty

Given Assumption 9 it follows from (19) that

$$W^{PLU}(\phi; E) = - \int_{-\infty}^{\tilde{h}(\phi; E)} [1 - \pi(1 - \Delta)\delta(h, E)] hg(h) dh \quad (27)$$

where  $\tilde{h}(\phi, E)$  is the critical value of harm above which all actions are deterred and below which all actions are taken.  $\tilde{h}(\phi, E)$  is defined implicitly by (18). It is straightforward to show that this implies

$$\tilde{h}(\phi, E) = E(2\theta - 1) \quad (28)$$

where  $\theta = \frac{1}{\pi[(1 - \Delta) + \Delta\phi]}$  and so, from Assumption 9,  $0 \leq \theta \leq 1$  which ensures that  $-E \leq \tilde{h}(\phi, E) \leq E$ . Notice that if the penalty happens to have been set at the optimum value as defined by (20), then  $\theta = \frac{1}{2}$ . If the penalty is below the optimum then  $\theta > \frac{1}{2}$  and so  $\tilde{h} > 0$ , whereas if it is above the optimum then  $\theta < \frac{1}{2}$  and so  $\tilde{h} < 0$ . If we differentiate (27) w.r.t  $E$  we get:

$$\frac{\partial W^{PLU}(E)}{\partial E} = - \frac{\pi(1 - \Delta)}{2E^2} \int_{-E}^{\tilde{h}} h^2 g(h) dh - E(2\theta - 1)^2 [1 - \pi(1 - \Delta)\delta(\tilde{h}, E)] g(\tilde{h}) \leq 0 \quad (29)$$

The first term on the right hand side corresponds to the effect of increased errors on *desistance* - the stopping of actions by the RA before the end of their natural life. If decisions are only made when actions have reached their natural life (i.e. if  $\Delta = 1$ ) there is no *desistance* and this effect disappears. Otherwise the first term on RHS of (29) is unambiguously negative and an increase in errors worsens *desistance* because it reduces the quality of decisions: more benign acts are disallowed and fewer harmful actions are disallowed.

The second term on the RHS corresponds to the effect of increased errors on *deterrence*. If the RA has set the optimal penalty (so  $\theta = \frac{1}{2}$ ) then all harmful actions are deterred and all benign actions are taken and an increase in errors has no effect. However, if the penalty is too low then, as we saw,  $\tilde{h} > 0$  and so some harmful actions are taken. But now, from (28), an increase in  $E$  increases  $\tilde{h}$  so even more harmful actions are taken and welfare is reduced. On the other hand, if the penalty is too high then as we saw,  $\tilde{h} < 0$  and so some benign actions are deterred. In this case, from (28), an increase in  $E$  reduces  $\tilde{h}$  so even more benign actions are deterred and again welfare is reduced.

So increased errors can make welfare fall both because they increase decision error costs AND because they generate poorer deterrence effects. From (21), we get:

$$\frac{d\hat{W}^{PLU}(E)}{dE} = -\frac{\pi(1-\Delta)}{2E^2} \int_{-E}^0 h^2 g(h) dh < 0. \quad (30)$$

Since the optimal penalty has been set, this effect works purely through lower quality decisions under desistance, and so, for precisely reasons set out above is zero if  $\Delta = 1$  and unambiguously negative if  $\Delta < 1$ .

We can summarise this discussion in:

**Result 2** If there is Partial Legal Uncertainty then welfare is a strictly decreasing function of the error bound if either the RA makes its decision before the natural life of the action or the RA has set a sub-optimal penalty (or both). Otherwise welfare is unaffected by the error bound.

**Corollary** If the decision is made by the RA only when the natural life of the act has been reached (i.e. if  $\Delta = 1$ ), then *maximum* welfare (i.e. welfare at the optimal penalty given by (30)) is unaffected by the error bound. Otherwise *maximum* welfare is a strictly decreasing function of the error bound.

### III. Complete Legal Uncertainty

Notice that if the RA's decisions are reached only when actions reach their natural life – i.e. if  $\Delta = 1$  – then, from (22) and (24), both welfare and maximum welfare are unaffected by errors under this information structure. This is because, under complete legal uncertainty, depending on the penalty, either everyone takes the action or nobody takes the action. Marginal changes in  $E$  have no effect, so there are no deterrence effects at work. Therefore increased errors can operate only through their effect on the quality of decisions in the presence of desistance and, if  $\Delta = 1$  there is no desistance.

If  $\Delta < 1$  then desistance can only matter in the situations where all firms have taken the action. So when both of these conditions are met an increase in errors unambiguously reduces welfare because it worsens decision error costs: more benign and fewer harmful actions are stopped. So we have:

**Result 3** If there is Complete Legal Uncertainty then:

- (i) If decisions are made by the RA only when the natural life of actions has been reached (i.e. if  $\Delta = 1$ ) then an increase in error bounds has no effect on either welfare or maximum welfare.
- (ii) If decisions are made by the RA before the natural life of actions has been reached (i.e. if  $\Delta < 1$ ) AND if the penalty has deterred all firms from taking the actions

then, once again, an increase in error bounds has no effect on either welfare or maximum welfare.

- (iii) Otherwise an increase in error bounds unambiguously reduces both welfare and maximum welfare.

Notice that errors affect welfare:

- only through deterrence effects if there is No Legal Uncertainty;
- through both lower quality decisions in the presence of desistance and deterrence effects if there is Partial Legal Uncertainty;
- only through lower quality decisions in the presence of desistance if there is Complete Legal Uncertainty

#### 4.2 Effects of Different Information Structures on Welfare (for given Error Bound)

In this section we take the error bound,  $E$ , as given and examine the effects of different information structures on welfare. For the reasons given above we will assume that in each case the RA has chosen the appropriate penalty and so we compare maximum welfare under the three information structures. We start by considering the case where the RA makes no errors ( $E = 0$ ).

*Case 1. No Errors:  $E = 0$*

If there are no errors then the RA will approve all benign actions and disapprove all harmful ones. So under both Partial Legal Uncertainty and No Legal Uncertainty, if the appropriate penalty is set all benign actions will be taken and all harmful actions deterred. Even though some benign actions will be investigated they will be approved and so none will be stopped. It follows that under both Partial Legal Uncertainty and No Legal Uncertainty welfare is just equal to that which would prevail in the First-Best – which we denote by  $\widehat{W}^{FB}$  - whereby only benign actions are taken and society gets the full benefit of these. Welfare is strictly positive.

So formally we have:

**Result 4** If there are no errors,  $E = 0$ , then welfare under both Partial Legal Uncertainty and No Legal Uncertainty is the same and equals that under the First Best. Formally

$$\widehat{W}^{NLU}(0) = \widehat{W}^{PLU}(0) = \widehat{W}^{FB} = -\int_{-\infty}^0 hg(h)dh > 0 \quad (31)$$

Under Complete Legal Uncertainty two possible situations can arise. The first is that no action is taken. This will be the case if the action is Presumptively Illegal and the fraction of firms investigated is sufficiently low and the delay in reaching a decision sufficiently high that the optimal penalty to set is one that deters all actions. The second is that all firms take the action. Even if benign actions are investigated, if there are no errors they will be

approved. So as with both Partial Legal Uncertainty and No Legal Uncertainty society will get the full benefit from all benign actions. However all harmful will be taken. A fraction of these will be investigated and if investigated, certainly disapproved and stopped – but with a delay. So there will be a cost of all the harmful actions that society has to suffer either in full or in part. So in either of these two cases welfare is unambiguously lower than with either Partial Legal Uncertainty or No Legal Uncertainty.

So welfare under Complete Legal Uncertainty is

$$\widehat{W}^{CLU}(0) = \text{MAX} \left[ 0, \left( -\int_{-\infty}^0 hg(h) dh \right) - (1 - \pi(1 - \Delta)) \int_0^{\infty} hg(h) dh \right] \quad (32)$$

Comparing (31) and (32), we find:

**Result 5** If there are no errors,  $E = 0$ , then under Complete Legal Uncertainty welfare is lower than under both Partial and No Legal Uncertainty. Formally:

$$0 \leq \widehat{W}^{CLU}(0) < \widehat{W}^{NLU}(0) = \widehat{W}^{PLU}(0) = \widehat{W}^{FB} \quad (33)$$

*Case 2 Positive Error Bound:  $E > 0$*

Notice that, from (21) we have:

$$\widehat{W}^{PLU}(E) = -\int_{-\infty}^{-E} hg(h) dh - \int_{-E}^0 [1 - \pi(1 - \Delta)\delta(h, E)] hg(h) dh \quad (34)$$

Also, from (17) we have:

$$\widehat{W}^{NLU}(E) = -\int_{-\infty}^{-E} hg(h) dh - \int_{-E}^0 [1 - \delta(h, E)] hg(h) dh - \int_0^E [1 - \delta(h, E)] hg(h) dh \quad (35)$$

Subtracting (35) from (34) we get:

$$\widehat{W}^{PLU}(E) - \widehat{W}^{NLU}(E) = -[1 - \pi(1 - \Delta)] \int_{-E}^0 \delta(h, E) hg(h) dh + \int_0^E [1 - \delta(h, E)] hg(h) dh > 0 \quad (36)$$

The first term on the RHS of (36) is positive and relates to the benign actions that, because of errors, will be disapproved by the RA. Under Partial Legal Uncertainty, with the appropriate penalty, they will all be taken, but there will be a loss arising because some of these will be investigated and stopped thereby losing some of the remaining benefit they would have conferred. However with No Legal Uncertainty all of these actions will be deterred and so society gets none of their benefit. Since it is better to lose just some of the benefit than all of it welfare is higher under Partial Legal Uncertainty. The second term on the RHS of (36) relates to the harmful actions that, because of errors, will be approved. These will all be deterred under Partial Legal Uncertainty (with appropriate penalty) but will be taken under conditions of No Legal Uncertainty. So this factor too implies that welfare is higher with Partial Legal Uncertainty than with No Legal Uncertainty.

So we have proved:

**Result 6** If there are errors,  $E > 0$ , then welfare is higher under Partial Legal Uncertainty than under no Legal Uncertainty. Formally we have proved that for all  $E > 0$

$$\widehat{W}^{PLU}(E) > \widehat{W}^{NLU}(E) > 0 \quad (37)$$

Now consider what happens under Complete Legal Uncertainty. As explained above, under Complete Legal Uncertainty two possible situations can arise.

The first is that no action is taken. This will be the case if the action is Presumptively Illegal and the fraction of firms investigated is sufficiently low and the delay in reaching a decision sufficiently high that the optimal penalty to set is one that deters all actions. In this case social welfare is zero and so unambiguously worse than under No Legal Uncertainty which, from (17) is positive.

The second is that all firms take the action. In this case, from (24):

$$\begin{aligned} \widehat{W}^{CLU}(E) &= -\bar{h} + \pi(1-\Delta)\bar{h}^D(E) = -\int_{\infty}^{-E} hg(h)dh - \int_{-E}^{\infty} [1-\pi(1-\Delta)\delta(h,E)]hg(h)dh \\ &= -\int_{\infty}^{-E} hg(h)dh - \int_{-E}^{\infty} [1-\delta(h,E)]hg(h)dh - [1-\pi(1-\Delta)]\bar{h}^D(E) \\ &= -\int_{\infty}^{-E} hg(h)dh - \int_{-E}^E [1-\delta(h,E)]hg(h)dh - [1-\pi(1-\Delta)]\bar{h}^D(E) \end{aligned} \quad (38)$$

If we subtract (38) from (35) then we get:

$$\widehat{W}^{NLU}(E) - \widehat{W}^{CLU}(E) = [1-\pi(1-\Delta)]\bar{h}^D(E) > 0 \quad (39)$$

Given our Assumption 6, that the RA can *Effectively Discriminate*, it follows from (10) that  $\bar{h}^D(E) > 0$  so the expression on the RHS is positive<sup>20</sup>. This captures the fact that whereas under No Legal Uncertainty all actions that will be deemed to be harmful will be deterred, under Complete Legal Uncertainty these will be taken and will be stopped only to the extent that they are investigated and, even then, with a delay.

So we have proved:

**Result 7** If there are errors,  $E > 0$ , then welfare is higher under No Legal Uncertainty than under Complete Legal Uncertainty. Formally we have proved that for all  $E > 0$

$$0 \leq \widehat{W}^{CLU}(E) < \widehat{W}^{NLU}(E) \quad (40)$$

**Corollary 1** If the RA uses an *Effects-Based* decision procedure, and if it makes errors but these are not large enough so it can *Effectively Discriminate* then there is a clear welfare

<sup>20</sup> Except of course in the case – which we have ruled out by assumption – where all actions are investigated with no delay – so  $\pi = 1$ ,  $\Delta = 0$  and this term is zero.

ranking of welfare across information structures: Partial Legal Uncertainty welfare dominates No Legal Uncertainty which welfare dominates Complete Legal Uncertainty. Formally

$$0 \leq \widehat{W}^{CLU}(E) < \widehat{W}^{NLU}(E) < \widehat{W}^{PLU}(E). \quad (41)$$

Notice also that if, under Complete Legal Uncertainty, it is optimal to deter all actions and so  $\widehat{W}^{CLU}(E) = 0$ , then, from Result 1, the gap between welfare under No Legal Uncertainty and welfare under Complete Legal Uncertainty decreases as the error bound grows. If, under Complete Legal Uncertainty, it is optimal to set a zero penalty and encourage all firms to take the action then, from (39) and (11) it also follows that the gap between welfare under No Legal Uncertainty and welfare under Complete Legal Uncertainty decreases as the error bound grows. So we have proved:

**Corollary 2** The gap between welfare under No Legal Uncertainty and welfare under Complete Legal Uncertainty decreases as the error bound grows. Formally

$$\frac{d \left[ \widehat{W}^{NLU}(E) - \widehat{W}^{CLU}(E) \right]}{dE} < 0 \quad (42)$$

We can summarise the results of this Section in Figure 1. Rather than proliferate cases, this is drawn on the assumption that  $\Delta < 1$  and that under Complete Legal Uncertainty welfare is positive. If instead we had assumed that  $\Delta = 1$  then welfare under both Partial Legal Uncertainty and Complete Legal Uncertainty would be flat rather than decreasing. The other alternative is that welfare under Complete Legal Uncertainty is zero. But the essential features of the relative ranking of welfare under the various information structures is still captured in Figure 1.

## 5. *Effects-Based* vs *Per Se* Decision Procedures

So far we have focussed on an *Effects-Based* decision procedure that can *Effectively Discriminate* and examined how it performs under various information structures. We have shown that, under Complete Legal Uncertainty it is optimal to have either all firms take the action or none. It is natural to ask how this compares with a *Per Se* decision procedure which either disapproves all actions – if the type of action is *Presumptively Illegal* – or else approves all actions – if the type of action is *Presumptively Legal*.

It is clear that welfare under *Per Se* is therefore:

$$W^{PS} = \begin{cases} -\bar{h} & \bar{h} < 0 \\ 0, & \bar{h} > 0 \end{cases} \quad (43)$$

If we compare this with (24) then we see that, if the action is *Presumptively Legal*, then welfare is higher under an *Effects-Based* procedure with Complete Legal Uncertainty because even though, in both cases, there is no deterrence and all actions are taken, at least under an *Effects-Based* procedure some actions are investigated and stopped – albeit with a delay. If the *Effects-Based* procedures can *Effectively Discriminate* then all actions that are disapproved are, on average, harmful, so society gains from having investigations undertaken

On the other hand if the type of action is *Presumptively Illegal* then under *Per Se* all firms’ actions will be stopped under all circumstances and welfare will be zero. However, as we have seen above, if the fraction of firms investigated is sufficiently high and the delay is not too great, then, under Complete Legal Uncertainty it would be better to set a zero penalty, have all actions taken, and pursue harmful actions through desistance.

So we have shown:

**Result 8** If the RA uses an *Effects-Based* decision procedure that can *Effectively Discriminate* then even under Complete Legal Uncertainty welfare is higher than under *Per Se*. *A fortiori*, an effectively discriminating *effects-based* procedure *always* welfare dominates *Per Se*.

## 6. Measuring the Welfare Costs of Legal Uncertainty

The key message of this paper has been that in discussing the impact of Legal Uncertainty on welfare and the optimal choice of decision procedures it is very important to disentangle the effects of increased estimation and thus decision errors and of different information structures. The implications of the former have been examined in the previous literature. However, this literature, failed to consider how different information structures facing firms affect welfare for given estimation and decision errors of a decision procedure used by an enforcement authority – e.g. Caswell and Caffee (1984, 1986). The contribution of this paper has been to provide, we believe for the first time, a framework that clearly disentangles these two effects whilst allowing an examination of the impact of each one of them.

The central idea is captured in Figure 2, which shows clearly the significance of disentangling these two effects for measuring the welfare costs of legal uncertainty. Suppose it were assumed that firms know the true value of the harm that their actions create and also the liability standard and there is *Partial Legal Uncertainty*. Then in the absence of errors, it could be argued, correctly, that there is no legal uncertainty since each firm can accurately predict what decision the RA will make in their case. However, if the RA cannot accurately estimate harm and has, say, an error bound  $E_0$  then, firms can no longer predict with certainty what decision the RA will make – there is now legal uncertainty - and, with Partial Legal Uncertainty, welfare is lower. One might therefore think of  $\widehat{W}^{PLU}(0) - \widehat{W}^{PLU}(E_0)$  in Figure 2 as a measure of the cost of legal uncertainty.

But this ignores two crucial issues:

- even if the RA operates an *Effects-Based* decision procedure, and even though this is subject to error, it still could be the case that there is No Legal Uncertainty;
- if we maintain an information structure under which there is No Legal Uncertainty then welfare falls even faster as the error bound increases.

So if we used as an alternative measure of the cost of legal uncertainty one which focused purely on differences in information structure and held estimation errors constant then we would use  $\widehat{W}^{NLU}(E_0) - \widehat{W}^{PLU}(E_0)$  in Figure 2 and find that this “cost” was actually negative. This radically transforms ones perception about the welfare “costs” of legal uncertainty.

## 7. Conclusions

This paper offers a unified framework that allows an examination and comprehensive comparison of the impact of on the one hand increased legal uncertainty associated with increased estimation and thus decision errors by enforcement authorities and, on the other, of increased legal uncertainty associated with different information structures that firms face when considering to undertake privately beneficial but potentially socially harmful actions. These information structures relate to *what firms know* about the factors that influence the outcome of the enforcement authority’s decision-making process. We show that it is very important to distinguish what agents know about this process from the errors made by the enforcement authority in reaching decisions.

While welfare “costs” are certainly positive when increased legal uncertainty is associated with increased estimation errors on the part of the enforcement authority, the increase in “costs” can actually be negative when, for given estimation errors, firms face information structures with increased levels of uncertainty. This is not to say that we think legal uncertainty is irrelevant – we have shown that Complete Legal Uncertainty is definitely harmful compared to other information structures with an *effects-based* procedure, though it is better to use *effects-based* even with Complete Legal Uncertainty than *Per Se*. Our conclusion is that one can only have a meaningful assessment of the effects of legal uncertainty on welfare and on the choice of enforcement procedures by being very clear about the information structures that firms face and by disentangling these from the decision errors made by enforcement authorities. We also provide general proofs of the results established in K&U (2013a). Thus with the general framework of this paper we establish that *effects-based* or rule-of-reason decision procedures always welfare dominate *Per Se* procedures and that higher optimal penalties can be associated with increased legal uncertainty.

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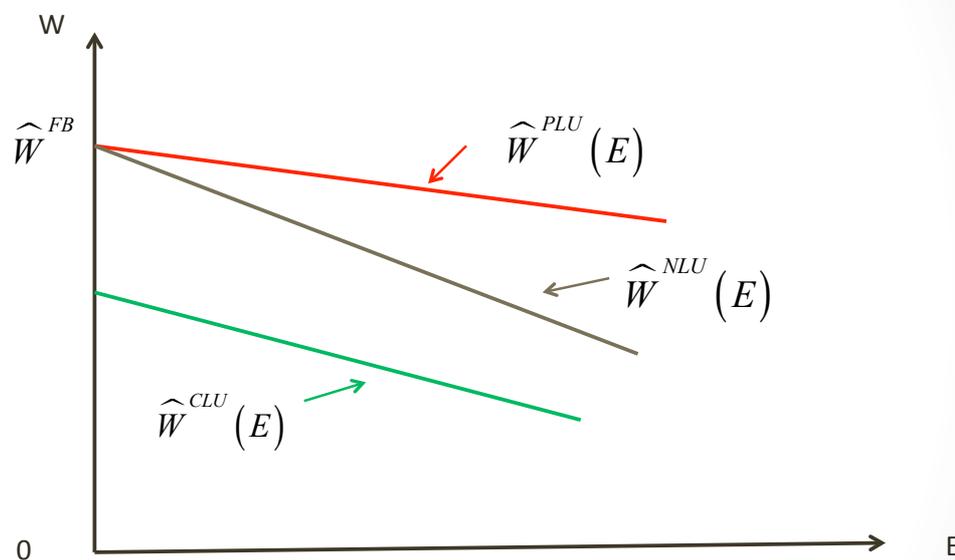
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Figure 1



# Figure 2

