

# Measuring Dynamic Interaction in Movement Data

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**Running Head:** Measuring Dynamic Interaction

**Keywords:** dynamic interaction, movement data, correlation, GPS, space-time, local statistics, spatial analysis

1 **ABSTRACT:**

2 The emergence of technologies capable of storing detailed records of object locations has  
3 presented scientists and researchers with a wealth of data on object movement. Yet  
4 analytical methods for investigating more advanced research questions from such detailed  
5 movement datasets remain limited in scope and sophistication. Recent advances in the  
6 study of movement data has focused on characterizing types of dynamic interactions,  
7 such as single-file motion, while little progress has been made on quantifying the degree  
8 of such interactions. In this article, we introduce a new method for measuring dynamic  
9 interactions (termed **DI**) between pairs of moving objects. Simulated movement datasets  
10 are used to compare **DI** with an existing correlation statistic. Two applied examples, team  
11 sports and wildlife, are used to further demonstrate the value of the **DI** approach. The **DI**  
12 method is advantageous in that it measures interaction in both movement direction  
13 (termed azimuth) and displacement. As well, the **DI** approach can be applied at local,  
14 interval, episodal, and global levels of analysis. However the **DI** method is limited to  
15 situations where movements of two objects are recorded at simultaneous points in time.  
16 In conclusion, **DI** quantifies the level of dynamic interaction between two moving  
17 objects, allowing for more thorough investigation of processes affecting interactive  
18 moving objects.  
19

## 20 **1 Introduction**

21           The study of individual movement has entered a new era whereby researchers  
22 from various fields can benefit from fine resolution object movement data. Technical  
23 developments associated with location aware technologies, such as GPS, are transforming  
24 representations of movement. Despite improvements in spatially explicit movement  
25 datasets, the scope and sophistication of research questions are limited by a lack of  
26 methods and analysis (Wolfer et al. 2001). Laube et al. (2007) suggest that within  
27 geography, reliance of geographic information systems (GIS) and spatial statistics on 2-  
28 dimensional representations may be limiting the development of more complex analyses  
29 of movement, while disciplines outside of geography may be unaware of the power of  
30 spatial (and space-time) analysis. To optimally utilize new movement datasets, analytical  
31 techniques capable of addressing more advanced research questions are required.

32           Recently, the identification and measurement of *dynamic interactions* between  
33 moving objects has become an active area of research, likely owing to readily available  
34 fine granularity movement data. Dynamic interaction, a term from the wildlife ecology  
35 literature, can be defined as the way the movements of two individuals are related  
36 (Macdonald et al. 1980) or as inter-dependency in the movements of two individuals  
37 (Doncaster 1990). Alternatively, the terms association (Stenhouse et al. 2005), relative  
38 motion (Laube et al. 2005), and correlation (Shirabe 2006) have been used to refer to  
39 dynamic interactions between moving objects in other examples. All of these terms refer  
40 to the same general idea: identifying of how the movements of one individual are related  
41 to another. Recent work on dynamic interactions has focused on methods for identifying  
42 dynamic interaction patterns defined *a priori* (for example single file motion, Buchin et

43 al. 2010; or chasing behavior, de Lucca Siqueira and Bogorny 2011). However limited  
 44 work exists on quantifying the strength of dynamic interactions present in movement  
 45 data. With this in mind we are motivated to investigate methods for measuring the  
 46 strength of dynamic interactions when there is an expectation that such behavior occurs.  
 47 This approach differs from recent developments in movement analysis which focus on  
 48 identifying patterns, defined *a priori*, from large movement databases.

49 The objective of this work is to extend a previously developed statistic (Shirabe  
 50 2006) to a measure capable of quantifying the degree of dynamic interaction between  
 51 moving objects. The new method (termed **DI**) measures dynamic interaction in  
 52 coincidental movement segments, that is, it requires movement data of two individuals  
 53 recorded simultaneously. The **DI** method is separable into components measuring  
 54 dynamic interaction in movement direction (azimuth) and movement distance  
 55 (displacement), termed  $\mathbf{DI}_\theta$  and  $\mathbf{DI}_d$  respectively. Further, **DI** is appropriate with the four  
 56 analysis levels (local, interval, episodal, and global – see Figure 1) identified by Laube et  
 57 al. (2007) with the beneficial property of local values (denoted here using lower-case –  
 58 **di**) that aggregate to the interval, episodal and global values. Lastly, **DI** is derived in a  
 59 way to allow for a time-lagged approach, but also extensions including time- and  
 60 distance-based weighting schemes.

61 < Approximate location Figure 1 >

## 62 **2 Related Work**

63 This research is motivated by an existing technique (Shirabe 2006) for measuring  
 64 the strength of dynamic interactions (termed correlations) present in movement data. The  
 65 use of the term correlation by Shirabe stems from the fact that the statistic takes the form

66 of a Pearson product-moment correlation coefficient. Consider two moving objects  $M^a$   
 67 and  $M^b$ , whose spatial coordinates  $(x, y)$  are recorded coincidentally at discrete times  $t = 1$   
 68  $\dots n$ , termed *fixes*. Now consider for any  $M$  with  $t = 2 \dots n$ ,  $\mathbf{V} = [M_t - M_{t-1}] = [\mathbf{v}_t]$ , is a  
 69 vector time series of  $M$  with  $n-1$  vector segments. A correlation statistic for movement  
 70 data defined this way takes the form (Shirabe 2006):

$$71 \quad r(\mathbf{V}^a, \mathbf{V}^b) = \frac{\sum_{t=1}^{n-1} (\mathbf{v}_t^a - \bar{\mathbf{v}}^a) \cdot (\mathbf{v}_t^b - \bar{\mathbf{v}}^b)}{\sqrt{\sum_{t=1}^{n-1} |\mathbf{v}_t^a - \bar{\mathbf{v}}^a|^2} \sqrt{\sum_{t=1}^{n-1} |\mathbf{v}_t^b - \bar{\mathbf{v}}^b|^2}} \quad (1)$$

72 Where  $\bar{\mathbf{v}} = \frac{1}{n-1} \sum_{t=1}^{n-1} \mathbf{v}_t$  are mean coordinate vectors of  $\mathbf{V}$ . The correlation statistic ( $\mathbf{r}$ )  
 73 is defined over the interval  $[-1, 1]$  with a score of 1 being perfect positive correlation and  
 74 a score of -1 perfect negative correlation, with 0 denoting no correlation.

75 The statistic –  $\mathbf{r}$ , could be advanced in three ways. First, it is dependant on the  
 76 mean vector of each path, and thus measures correlations in movement deviations from  
 77 their respective means. The statistic,  $\mathbf{r}$ , cannot be used for testing direct *interactions*  
 78 between two moving objects unless their corresponding mean vectors are identical or  
 79 near identical. An improved statistic would not rely on this overall mean value. Second,  $\mathbf{r}$   
 80 is unable to disentangle the effects of correlations in movement azimuth and distance,  
 81 while being sensitive to both. Decomposing such a statistic into components based on  
 82 movement direction (termed azimuth) and distance (displacement) would be beneficial,  
 83 as it would allow interactions in these two independent components of movement to be  
 84 analyzed separately. A third improvement would be a statistic that measures the  
 85 interaction of each individual movement segment (i.e., local level - Laube et al. 2007).  
 86 By definition,  $\mathbf{r}$  produces a single resulting value for the entire path (i.e., global level -

87 Laube et al. 2007). When movement patterns are characterized by periods of interactive  
 88 and non-interactive behavior, or varying levels of interactive behavior, a local level  
 89 statistic will allow a finer treatment of dynamic interactions.

90 Measurements of dynamic interaction in movement data have also been  
 91 developed by wildlife researchers interested in a finer understanding of wildlife  
 92 movement processes. The types of interactions studied in wildlife are classified as either  
 93 static or dynamic interactions (Doncaster 1990; Macdonald et al. 1980). Static interaction  
 94 relates to how two individuals use space coincidentally, while dynamic interaction  
 95 reflects how the movements of two individuals are related, for example attraction  
 96 (Macdonald et al. 1980). Typically, measures of dynamic interaction summarize the  
 97 proximity of simultaneous movement points. Doncaster (1990) introduced one such  
 98 measure of dynamic interaction based on the variance/covariance matrix of the spatial  
 99 coordinates of simultaneous wildlife telemetry fixes; others have used Euclidean distance  
 100 as an indicator of interaction (Bandeira de Melo et al. 2007; Stenhouse et al. 2005).  
 101 Stenhouse et al. (2005) further investigated dynamic interaction in grizzly bears (termed  
 102 associations) by measuring dynamic interaction in movement direction (azimuth –  $\theta$ )  
 103 defined as:

$$104 \quad f_t(\theta_t^a, \theta_t^b) = \frac{|-(\theta_t^a - \theta_t^b) - 180|}{180} \quad (2)$$

105 Equation (2) ranges from 0 –1, with values of 1 when direction of movements is identical  
 106 and zero when completely opposite (i.e., at 180°).

107 Measuring dynamic interactions in moving object databases is also directly  
 108 related to a larger body of literature on identifying similar movement trajectories (Sinha  
 109 and Mark 2005; Vlachos et al. 2002; Yanagisawa et al. 2003). Similarity indices are

110 commonly employed as a first-step for identifying broader patterns or for detecting  
111 clusters in larger movement databases (Benkert et al. 2008; Gao et al. 2010). Moving  
112 object pairs that are highly interactive could also be said to follow a similar trajectory in  
113 many of these applications, and the methods for detecting dynamic interactions in  
114 movement data could be useful for detecting similar movement trajectories.

115         Recently, many new techniques have been developed for categorizing various  
116 dynamic interaction patterns commonly found in movement data. Laube et al. (2005)  
117 developed a method for detecting RELative MOTion (REMO) classes based upon  
118 interpreting patterns of movement direction in groups of moving objects. For example,  
119 trend-setting, when one object moves with anticipation of the movement of others, is  
120 identifiable using the REMO approach. Noyon et al. (2007) use changes in inter-object  
121 distance and velocity to identify relative behavior such as collision avoidance. Benkert et  
122 al. (2008) present an algorithm for finding flock patterns in movement databases; which  
123 tests whether a group of moving objects are contained in a circle radius  $r$  over a given  
124 time interval. The study of flocking behavior is useful in the study of wildlife and crowd  
125 dynamics (Batty et al. 2003). Buchin et al. (2010) have developed a method for  
126 identifying single-file motion in groups of moving objects. Single-file motion is detected  
127 using free-space diagrams, derived from the Fréchet distance metric for comparing  
128 polygonal curves (Alt and Godau 1995). Related to single-file motion is the detection of  
129 chasing behavior, identifiable using the algorithm proposed by de Lucca Siqueira and  
130 Bogorny (2011). The methods mentioned above are capable of identifying specific types  
131 of dynamic interactions in movement data as defined *a priori*. However, such methods

132 are unable to quantify the strength of dynamic interactions present, thus motivating the  
 133 development of quantitative measures of dynamic interaction.

### 134 **3 Derivation**

135 In developing a measure of dynamic interaction we consider the rather optimal  
 136 data situation (as in Shirabe 2006) where two moving objects' ( $M^a$  and  $M^b$ ) spatial  
 137 coordinates  $(x, y)$  are recorded coincidentally at discrete times  $t = 1 \dots n$ , termed *fixes*.  
 138 For any  $M$  with  $t = 2 \dots n$ ,  $\mathbf{V} = [M_t - M_{t-1}] = [\mathbf{v}_t]$ , is a vector time series of  $M$  with  $n-1$   
 139 vector segments. For each movement segment define two fundamental properties:  
 140 direction ( $\theta$ ), termed azimuth, and length ( $d$ ), termed displacement. Azimuth ( $\theta$ ) is the  
 141 angle between a movement segment and a constant axis, most commonly the horizontal  
 142 axis (Figure 2a). Displacement ( $d$ ) is the Euclidean distance between two consecutive  
 143 fixes in a movement segment (Figure 2a). We are interested in deriving a measure of  
 144 dynamic interaction that separately quantifies interactions in azimuth and displacement  
 145 (Figure 2b-e).

146 < Approximate location of Figure 2 >

#### 147 *3.1 Azimuth – $\theta$*

148 To investigate the interaction in movement azimuths we take the cosine of the  
 149 angle between them. This is simply calculated as:

$$150 \quad \mathbf{a}_\theta = f_t(\theta_t^a, \theta_t^b) = \cos(\theta_t^a - \theta_t^b) \quad (3)$$

151 where  $\theta_t$  is the angle of movement at time-step  $t$ . Here  $f_t$  has a range of  $[-1, 1]$  as desired.

152 The function  $\cos(\theta_t^a - \theta_t^b)$  is 1 when movement segments have the same orientation, 0

153 when movement segments are perpendicular, and -1 when in complete opposing

154 directions. In practice if either object (or both) do not move (3) is undefined, because  $\theta_t$  is

155 undefined. Thus, we must consider two alternative scenarios; first if one object moves  
 156 and one remains stationary, and second if both objects remain stationary. Here we make  
 157 the assumption that if one moves and the other remains stationary the two objects exhibit  
 158 no directional interaction, and if both are stationary they are positively interactive.  
 159 Considering these two alternative scenarios, a complete definition for (3) is:

$$160 \quad f_t(\theta_t^a, \theta_t^b) = \begin{cases} 0 & , \text{ one of } \theta_t^a \text{ or } \theta_t^b \text{ undefined} \\ 1 & , \text{ both } \theta_t^a \text{ and } \theta_t^b \text{ undefined} \\ \cos(\theta_t^a - \theta_t^b) & , \text{ otherwise} \end{cases} \quad (4)$$

### 161 3.2 Displacement – $d$

162 Interaction in movement displacement could be measured using a variety of  
 163 functions. However, it is desirable to have the function ( $g_t$ ) fall in the range of 0 – 1,  
 164 where a value of 0 represents no interaction and 1 positive interaction. Note there is no  
 165 consideration of negative interaction in displacement. Using this definition  $g_t$  can be  
 166 thought of as a scaling function to  $f_t$ , and maintains the statistic on the range [-1, 1]. We  
 167 propose the following function for  $g_t$ :

$$168 \quad \mathbf{di}_d = g_t(d_t^a, d_t^b) = 1 - \left( \frac{|d_t^a - d_t^b|}{d_t^a + d_t^b} \right)^\alpha \quad (5)$$

169 Where  $|\cdot|$  is the absolute value operator, and  $\alpha$  is a scaling parameter defaulting to 1. The  
 170 function  $g_t(d_t^a, d_t^b)$  approaches zero when  $d_t^a \gg d_t^b$  or vice-versa, and is 1 when  $d_t^a =$   
 171  $d_t^b$ . The effect of the scaling parameter ( $\alpha$ ) on the function  $g_t(d_t^a, d_t^b)$  is demonstrated in  
 172 Figure 3. Parameter  $\alpha$  can be adjusted to place stricter or looser requirements on  
 173 similarity in displacement denoting interaction. As  $\alpha$  is increased larger differences in  
 174 displacement are still considered as positively interactive. A closer examination of (5)

175 reveals that it is undefined when  $d_t^a + d_t^b = 0$ , (i.e., both objects are stationary). If we  
 176 consider both objects remaining stationary as positive interaction, a more robust  
 177 definition of (5) is:

$$178 \quad g_t(d_t^a, d_t^b) = \begin{cases} 1 & , \quad d_t^a + d_t^b = 0 \\ 1 - \left( \frac{|d_t^a - d_t^b|}{d_t^a + d_t^b} \right)^\alpha & , \quad d_t^a + d_t^b > 0 \end{cases} \quad (6)$$

179 < Approximate location of Figure 3 >

180 Thus, for two corresponding movement segments, a measure of dynamic  
 181 interaction is the product between the azimuthal term ( $f_t$ ) and displacement term ( $g_t$ ):

$$182 \quad \mathbf{di}_t(v_t^a, v_t^b) = \mathbf{di}_\theta \times \mathbf{di}_d = f_t(\theta_t^a, \theta_t^b) \times g_t(d_t^a, d_t^b) \quad (7)$$

183 We are motivated to use the functions  $f_t$  and  $g_t$  to provide a statistic that covers the range  
 184  $[-1, 1]$  as was done in Shirabe (2006). Positive values of  $\mathbf{di}_t$  correspond to cohesive or  
 185 positively interactive movements, while negative values can be interpreted as repulsion or  
 186 opposing movements. Values near zero should be interpreted as having no interaction.

187 The  $\mathbf{di}$  statistics measure dynamic interaction based on similarity in azimuth ( $\theta$ )  
 188 and displacement ( $d$ ) of simultaneous movement segments but do not account for the  
 189 proximity of moving objects. Thus,  $\mathbf{di}$  represents a similarity index taken in a normalized  
 190 plane (i.e., the distance between the two objects has no impact on the resulting value).

191 We are motivated to use this type of formulation as the spatial proximity required for  
 192 dynamic interaction to occur is application specific. It is up to the analyst to decide if two  
 193 moving objects maintain a requisite proximity for dynamic interaction to occur, then such  
 194 interaction can be measured using  $\mathbf{di}$ . In cases where actual spatial contact is required, for

195 example when identifying points-of-interest in large movement databases (e.g., Benkert  
196 et al. 2007), the **di** method should not be employed.

197 We have made assumptions in the equations for **di**<sub>θ</sub> and **di**<sub>d</sub> regarding how to  
198 analyze dynamic interactions when objects do not move (i.e., θ is undefined and d = 0).  
199 In certain cases interpretation of these situations will be clear, for example, if one object  
200 stops moving, does the other? However in practice, many applications may not facilitate  
201 such straight-forward interpretation. For example, when studying urban travelers does  
202 stopping at a red-light signify a change to dynamic interaction even if they will  
203 eventually go straight? In light of these concerns, these assumptions can be modified to  
204 accommodate different situations that may arise in various movement scenarios to fit a  
205 given application.

### 206 3.3 Global analysis

207 A global version of the **di** statistic can be used to measure the overall interaction  
208 in a set of movement segments. First, it is useful to recognize that we can identify global  
209 interaction in azimuth or displacement individually by summing the interaction values for  
210 each individual segment and dividing by the number of segments. This form of a global  
211 **DI** gives equal weight to each segment. .

$$212 \quad \mathbf{DI}_\theta(v^a, v^b) = \frac{1}{n-1} \sum_{t=1}^{n-1} \mathbf{di}_\theta \quad (8)$$

$$213 \quad \mathbf{DI}_d(v^a, v^b) = \frac{1}{n-1} \sum_{t=1}^{n-1} \mathbf{di}_d \quad (9)$$

214 A global measure of overall dynamic interaction **DI** can also be derived.

$$215 \quad \mathbf{DI}(v^a, v^b) = \frac{1}{n-1} \sum_{t=1}^{n-1} (\mathbf{di}_\theta \times \mathbf{di}_d) = \frac{1}{n-1} \sum_{t=1}^{n-1} (\mathbf{di}) \quad (10)$$

216 It is important to note that in the local version  $\mathbf{di} = \mathbf{di}_\theta \times \mathbf{di}_d$ , but with the global statistic,  
 217 due to summation rules,  $\mathbf{DI} \neq \mathbf{DI}_\theta \times \mathbf{DI}_d$ . This can make interpretation of global values of  
 218  $\mathbf{DI}$  less straightforward than with local values. However, if we were to alternatively  
 219 define the global version as  $\mathbf{DI} = \mathbf{DI}_\theta \times \mathbf{DI}_d$ , then the equation defined by (10) would no  
 220 longer hold. Thus, interpretation of  $\mathbf{DI}$  values is best done separately for each component  
 221 (i.e.,  $\mathbf{DI}$ ,  $\mathbf{DI}_\theta$ , and  $\mathbf{DI}_d$ ).

222 The global formulation is also appropriate for interval and episodal levels of  
 223 analysis. Here we simply replace  $n$  with some interval or episode length  $n'$ , where  $n' < n$ .  
 224 This type of analysis can be illuminating when analyzing interactions in larger movement  
 225 datasets, where varying levels of dynamic interaction may occur at different points in the  
 226 movement paths.

#### 227 *3.4 Time- and Distance-based Weighting*

228 In instances where the sampling interval of the  $n$  fixes is unequal it is desirable to  
 229 scale the statistic based on the temporal duration of each movement segment. In practice,  
 230 this would give more weight to segments of longer duration and less weight to shorter  
 231 segments. Temporal weighting may also be used to account for missing fixes, common to  
 232 GPS-based tracking data. Let  $\Delta_t$  correspond to the temporal duration of segment  $t$ , where

233  $\sum_{t=1}^{n-1} \Delta_t = T$  is the total duration of the entire movement path. Then a time weighted

234 version of (10) is defined as:

$$235 \quad \mathbf{DI}(v^a, v^b) = \sum_{t=1}^{n-1} \frac{\Delta_t}{T} \mathbf{di}_\theta \times \mathbf{di}_d \quad (11)$$

236 Viewed in light of the uncertainty associated with movement data, this form of temporal  
 237 weighting may be counter-intuitive. That is, it may be logical to assign weights inversely

238 proportional to the duration between fixes; lower weights to segments with higher  
 239 uncertainty (i.e., more time between fixes) and higher weights to segments with higher  
 240 certainty or finer space-time resolution.

241 Similarly, we can define a distance-based weighting scheme for (10) where  
 242 movements with larger displacement have increased weight in calculation of the statistic.  
 243 Varying distance-based weights could be used when dynamic interactions of a specific  
 244 movement behavior are of interest. For example in the study of wildlife long directed  
 245 movements are often interspersed with shorter random movements distinguishing  
 246 migratory and foraging behavior (Turchin 1998). Distance weighting could be used to  
 247 tailor the measurement of dynamic interactions to either of migratory or foraging  
 248 behaviors in this case. A possible distance-based weighting scheme would be the average  
 249 displacement of two segments:  $d_t^{avg} = (d_t^a + d_t^b) / 2$ , and  $\sum_{t=1}^{n-1} d_t^{avg} = D$ . Based on the  
 250 average displacement a distance-weighted version of (10) is defined as:

$$251 \quad \mathbf{DI}(\mathbf{V}^a, \mathbf{V}^b) = \sum_{t=1}^{n-1} \frac{d_t^{avg}}{D} \mathbf{d}_t^a \times \mathbf{d}_t^b \quad (12)$$

252 However, the average displacement of two objects movement segments is misleading  
 253 when one object has a large displacement and the other has a small displacement. Thus,  
 254 other distance measures are worth investigating for alternative distance-based weighting  
 255 schemes, keeping in mind that the sum of the weights should equal one. The equations  
 256 (11) and (12) can be combined to provide a time- and distance-based weighting scheme.  
 257 It is important to note that time- and distance-based weighting is really only useful when  
 258 interpreting global results when there is benefit to assigning segments weights based on  
 259 duration or distance.

260 Another interesting extension to studying correlations in movement paths is when  
 261 movements interact with a temporal lag, for example when trend-setting occurs, as  
 262 described by Laube et al. (2005). The **DI** statistic can be modified to evaluate dynamic  
 263 interactions at a temporal lag. To measure dynamic interactions at a temporal lag, select a  
 264 time lag  $-k$ , where  $k$  is generally taken to be a multiple of the fix interval (i.e., if fixes are  
 265 taken at even intervals the time between consecutive fixes). Then we can, alternatively  
 266 define  $\mathbf{di}_\theta$  and  $\mathbf{di}_d$  as:

$$267 \quad \mathbf{di}_\theta = f_t(\theta_t^a, \theta_{t+k}^b) \quad (13)$$

$$268 \quad \mathbf{di}_d = g_t(d_t^a, d_{t+k}^b) \quad (14)$$

269 The global statistics (**DI**, **DI $_\theta$** , **DI $_d$** ) can be computed as before, using the time lagged  
 270 versions of  $\mathbf{di}_\theta$  (13) and  $\mathbf{di}_d$  (14).

## 271 **4 Data**

### 272 *4.1 Simulated Data*

273 Six simulated data sets are used to highlight the utility of the **DI** statistic and the  
 274 benefit of extensions it makes to **r** (Shirabe 2006). A single random walk ( $n = 10$ ) is used  
 275 to generate a movement path that is the bases for the simulation examples. We used  
 276 manual permutations to the spatial coordinates of the original random walk to produce 5  
 277 new movement paths that represent 5 unique dynamic interaction scenarios (Table 1).  
 278 The first scenario simulates two objects moving with strong-positive dynamic interaction.  
 279 The second scenario uses the same two paths as the first scenario, but one is rotated at  
 280  $45^\circ$ , simulating strong interaction in displacement, and low interaction in azimuth. The  
 281 third scenario simulates positive interaction in azimuth and no interaction in  
 282 displacement. The fourth scenario simulates negative interaction in azimuth and no

283 interaction in displacement. The fifth scenario simulates no interaction in azimuth and  
284 strong interaction in displacement. The sixth scenario uses a second independent random  
285 walk to simulate random interactions between two moving objects.

286 < Approximate location of Table 1 >

#### 287 4.2 Athletes – Ultimate Frisbee

288 In team sports players (objects) movements are expected to be highly interactive.  
289 Often a defending player is tasked with “covering” an offensive player, and their  
290 movements are in reaction to that offensive player. In the sport of ultimate frisbee,  
291 offensive players move about the field in an attempt to get open for a pass from their  
292 teammates. Defending players cover them, in an attempt to intercept or dissuade passes  
293 from being completed. As such, in ultimate frisbee the movements of an offensive player  
294 and their defender are highly interactive. We used 5 Hz sports-specific GPS devices  
295 (GPSports, Fyshwick, Australia) to monitor the movements of two ultimate frisbee  
296 players over a one minute segment during a training game. In this example, the two  
297 players cover each other for the entirety of the one minute period. A total of  $n = 276$  GPS  
298 locations (out of a possible 300) were simultaneously recorded. Most of the missing  
299 locations occur when the players are relatively stationary. At 5 GPS locations per second  
300 this represents an extremely detailed movement dataset, appropriate for investigating the  
301 intricate movements of athletes.

#### 302 4.3 Grizzly Bears in Alberta, Canada

303 To further demonstrate **DI**, we investigate a previously published dataset  
304 containing GPS telemetry locations of a number of grizzly bears in Alberta, Canada  
305 (Stenhouse et al. 2005). Stenhouse et al. (2005) revealed that various bear combinations

306 showed evidence of dynamic interaction during different seasons, in particular male-  
 307 female interactions were strongest during spring when mating activity occurs. To  
 308 demonstrate **DI**, we examine one specific male-female bear combination that exhibited a  
 309 relatively strong association during the mating season (male (G006) and female (G010) -  
 310 see Fig. 4 in Stenhouse et al. 2005). Grizzly bear GPS collars were programmed to obtain  
 311 a location fix every four hours, however missing entries are frequent. As a result, only  
 312 112 simultaneous GPS fixes were obtained for the two bears during period from May 28,  
 313 2000 to July 08, 2000. In this example, we incorporate time-based weighting in order to  
 314 account for unevenness in fix intervals (ranging from 4 hours to over 6 days).

## 315 **5 Results**

### 316 *5.1 Simulated Data*

317 Using the six simulated datasets we compared global values for **DI**, **DI<sub>θ</sub>**, and **DI<sub>d</sub>**  
 318 with Shirabe's (2006) **r** statistic (Figure 4) to reveal both the similarities and differences  
 319 between these two methods. In scenario 1, where both movements are highly interactive  
 320 in both displacement and azimuth, **DI** and **r** are very similar. In scenario 2 **DI** and **r** are  
 321 similar, however using the DI method we can identify that interaction is higher in  
 322 displacement (**DI<sub>d</sub>** = 0.977), and lower in azimuth (**DI<sub>θ</sub>** = 0.664). In contrast, scenario 3  
 323 reveals a situation where **DI** and **r** exhibit substantially different results. Using **DI<sub>θ</sub>** and  
 324 **DI<sub>d</sub>** we can further examine the nature of the interaction in both azimuth and  
 325 displacement, in this case **DI<sub>d</sub>** = 0.287 and **DI<sub>θ</sub>** = 0.992. High **DI<sub>θ</sub>** independent of **DI<sub>d</sub>**  
 326 could be useful in measuring interactive movement patterns via different modes of  
 327 transportation (e.g., walking vs. biking), or scale independent movement behavior in  
 328 wildlife. Scenario 4 demonstrates an example where negative dynamic interaction is

329 present (i.e., repulsion). In this case, **DI** is small and negative (**DI** = -0.278) due to low  
 330 interaction in displacement (**DI<sub>d</sub>** = 0.280), while **r<sub>xy</sub>** is large and negative (**r** = -0.805).  
 331 Scenario 5, shows the case where low **DI** is a function of low interaction in azimuth (**DI<sub>θ</sub>**  
 332 = -0.095), despite having a strong level of interaction in movement displacement (**DI<sub>d</sub>** =  
 333 0.979), while **r<sub>xy</sub>** = -0.532. Measurement of high vs. low **DI<sub>d</sub>** independent of **DI<sub>θ</sub>** could be  
 334 used in behavior analysis to identify objects with similar diurnal activity patterns (i.e.,  
 335 temporal patterns of long and short movements). In Scenario 6, both **DI** and **r** show  
 336 values near 0, as would be expected from two independent random motions. It is  
 337 interesting to note that **DI<sub>d</sub>** = 0.649 is relatively high in this example, as the random  
 338 walks used identical parameters for their displacement distributions.

339 < Approximate location of Figure 4 >

#### 340 5.2 Athletes – Ultimate Frisbee

341 In the Ultimate Frisbee example, the two players positively interact in movement  
 342 azimuth (**DI<sub>θ</sub>** = 0.682) and movement displacement (**DI<sub>d</sub>** = 0.730). The global statistic  
 343 shows that a substantial level of interaction exists between the two athletes (**DI** = 0.572).  
 344 Local analysis enables the identification of times/locations where the athletes exhibit  
 345 more or less interactive movements (Figure 5). In the ultimate frisbee example, local  
 346 analysis is more informative than the global measure, as the movement path consists of  
 347 many (shorter) movement segments. Maps of local **di** can be combined with a time-series  
 348 graph of **di**, **di<sub>θ</sub>**, and **di<sub>d</sub>** related to times/locations during the game where the defending  
 349 player did a poor job covering the offensive player. We use episodal level analysis to  
 350 segregate the movement paths into episodes of high vs. low interaction in order to further  
 351 investigate the interactive behavior of these two athletes. For example, from 0 - 20 and 38

352 - 40 seconds (highlighted in blue in Figure 5), high and positive **di** values suggest the  
 353 defending player is providing good defensive coverage (for these two episodes **DI** =  
 354 0.757). While from 20 - 38 seconds (highlighted in red in Figure 5) **di** values are much  
 355 lower, an indication of less interactive movement and poor defensive coverage (for this  
 356 episode **DI** = 0.122).

357 < Approximate location of Figure 5 >

### 358 5.3 Grizzly Bears in Alberta, Canada

359 In the grizzly bear example it was revealed that the male and female bears showed  
 360 substantial interaction (**DI** = 0.578) over the 42 day period from May 28, 2000 to July 8,  
 361 2000, using time-based weighting (see equation (11)) to account for missing fixes.  
 362 Similarly, time weighted results for azimuth (**DI<sub>θ</sub>** = 0.663) and displacement (**DI<sub>d</sub>** =  
 363 0.731) reveal that both azimuth and displacement were strongly related during this  
 364 period. Local analysis revealed that the strong interaction seen with the global results was  
 365 a function of highly cohesive movements during the middle of June, while at the  
 366 beginning of June the two animals show little interaction (see Figure 6). Again we  
 367 perform analysis at the episodal level for separate periods identified visually from the  
 368 local analysis as having low and high dynamic interaction (low interaction: May 28 –  
 369 June 09; high interaction: June 09 – 29). The period of high interactions has a time-  
 370 weighted **DI** = 0.492, while the period of low interaction has a time-weighted **DI** = 0.029.  
 371 Highly interactive behavior by mating grizzly bears is common in this region, as males  
 372 will attempt to confine female movements to a ‘mating area’ (Hamer and Herrero 1990).  
 373 Interpretation of maps and graphs of **di** facilitates the identification of where and when  
 374 such behavior occurs.

< Approximate location of Figure 6 >

## 376 **6 Discussion**

377 **DI** has three fundamental advantages over an existing method (Shirabe 2006) for  
 378 measuring interactions (termed correlations) in movement data. First, the existing method  
 379 follows a traditional correlation coefficient structure and is thus dependent on the mean  
 380 vector of a movement vector time series. In most cases, this mean movement vector will  
 381 have little relevance in the context of the analysis. However, in cases where interactions  
 382 are expected to occur relative to some mean movement trajectory, the method from  
 383 Shirabe (2006) is still advantageous. For instance, two objects moving radially from a  
 384 point (at some acute angle) may exhibit dynamic interaction (e.g., Fig. 4a in Shirabe  
 385 2006). Second, **DI** is explicitly decomposed into components measuring interaction in  
 386 movement azimuth and displacement. This property enables analysts to identify  
 387 situations where movements are related in one component but not the other. For example,  
 388 in scenario 3, **DI<sub>d</sub>** is low, however strong interaction is present in **DI<sub>θ</sub>**, indicating that the  
 389 objects move with similar azimuths but not displacements, a conclusion not discernable  
 390 from the **r<sub>xy</sub>** statistic. Lastly, the **di** statistics we have developed are calculated  
 391 independently for each simultaneous movement segment. The **di** values can be mapped  
 392 and analyzed in a time-series fashion providing a local level analysis. Local analysis  
 393 reveals spatial-temporal information about locations of increased or decreased interaction  
 394 along the movement trajectory. Furthermore, the local level statistics (**di**, **di<sub>θ</sub>**, and **di<sub>d</sub>**) are  
 395 easily aggregated to coarser levels of analysis (*interval*, *episodal*, and *global*).

396 Other research areas where measuring movement interactions could provide new  
 397 and unique insight include transportation, human-activity, and other wildlife and sporting

398 examples. In transportation applications measuring interactions in large movement  
399 databases could be used for generating information on commuter behavior. Examples  
400 from human-activity research where interactions are important include tourist behavior  
401 (e.g., Shoval and Isaacson 2007) or crowd dynamics (Batty et al. 2003). With wildlife  
402 movement data, the detection of interactions is important in the study of resource  
403 selection (Millspaugh et al. 1998) and social behavior (Bandeira de Melo et al. 2007;  
404 Kenward et al. 1993), but also for examining offspring dependency, and inter-/intra-  
405 species behavior. Finally, a number of sporting examples exist where measuring  
406 movement interactions could provide new and unique insight including soccer, American  
407 football, and ice hockey.

408         We use simulated movement data to highlight the advantages of **DI** over an  
409 existing method in a small set of specific scenarios designed to show the range of  
410 dynamic interactions present in movement data. When two movements are highly  
411 interactive (e.g., scenario 1) both methods successfully identify the high level of dynamic  
412 interaction. Also, when two movements show opposing or repulsive movements (e.g.,  
413 scenario 4) both methods are able to identify this behavior. The value of the **DI** method is  
414 demonstrated in scenarios 3, 4, and 5, where interactions in either azimuth or  
415 displacement are coupled with no interaction in the other component. This type of  
416 analysis may be useful, for example, when object movement is dependent on a temporal  
417 factor. For instance, many wildlife species are active only at specific times of the day and  
418 remain dormant during other periods. Measuring positive dynamic interactions in  
419 displacement, irrespective of azimuth, may be useful in identifying whether or not

420 different species or individuals operate with similar circadian cycles (Merrill and Mech  
421 2003).

422         The example from athletes playing ultimate frisbee demonstrates the value of  
423 measuring dynamic interactions at the local and episodal levels of analysis. Local and  
424 episodal analysis revealed periods of varying degrees of dynamic interaction, which can  
425 be related to player performance (i.e., how well the defensive player was able to cover the  
426 offensive player). In many team sports, player evaluation has traditionally been  
427 conducted by human observers. More recently, data driven analyses have become  
428 common in the evaluation of players in team sports (e.g., Fearnhead and Taylor 2011).  
429 When a player's movement can be directly related to specific abilities, for instance the  
430 soccer example in Laube et al. (2005), the measurement of dynamic interactions, using  
431 the **DI** method can enhance player evaluation using novel sport-specific movement  
432 datasets.

433         The **DI** method we have developed requires that movement locations be recorded  
434 simultaneously. Such a tidy form of movement data (i.e., where objects locations are  
435 recorded simultaneously) may not always be available, limiting the ability to implement  
436 this method. In such cases, path interpolation methods (e.g., Tremblay et al. 2006) could  
437 be used to estimate the locations of one object at coinciding times. Similarly, in many  
438 applications the assumption that movement data are collected at a regular interval is not  
439 satisfied (e.g., with movement data collected using cell-phone records). This is also the  
440 case in many wildlife telemetry studies where missing fixes are common. In the grizzly  
441 bear example, we demonstrate the value of temporal weighting the **DI** statistic to account  
442 for uneven sampling intervals. Further, we highlighted how local and episodal analyses

443 can provide unique and valuable insights into the nature of dynamic interactions present  
444 in movement datasets. Local analysis reveals the times and locations of dynamic  
445 interactions not discernable from global level statistics. When comparing male and  
446 female grizzly bears, the dynamic interactions were likely due to mating behavior. This  
447 example demonstrates the value of quantifying dynamic interactions in wildlife  
448 movement datasets, as they can be related directly to specific social activities.

449         When movement data are collected at too fine a granularity, the movement  
450 process (e.g., dynamic interaction) can be masked by data noise (termed over-sampling,  
451 Turchin 1998). In these cases, down-sampling can be used to reduce data redundancy in  
452 the movement path and improve the process signal to noise ratio. The **DI** statistics can  
453 then be computed on the re-sampled movement dataset, as another form of interval and/or  
454 episodal analysis (e.g., Laube et al. 2007). Variations of this procedure at different  
455 interval and episodal scales can lead to increasingly complex and cross-scale  
456 investigations of dynamic interactions in moving object datasets. Recently, Laube and  
457 Purves (2011) have discussed the impact that movement data granularity (i.e., sampling  
458 resolution) has on metrics used to quantify and describe movement trajectories (e.g.,  
459 mean speed). The **DI** method is similarly impacted by the granularity at which  
460 movement data are represented. For example, at a coarse granularity objects may exhibit  
461 positive dynamic interactions, while at a fine granularity their movements may show  
462 negative dynamic interaction (see Figure 7). Both the granularity at which the data are  
463 represented and analysis level selected will impact the results and subsequent  
464 interpretation of **DI**. One of Laube and Purves (2011) main recommendations is that

465 movement data analysis be conducted across a range of scales (granularities and analysis  
466 levels) to correctly understand observed patterns.

467 < Approximate location of Figure 7 >

## 468 **6 Conclusions**

469 Movement data are being collected for a variety of research agendas involving the  
470 study of humans, their vehicles, and wildlife. Central to analyzing movement data is the  
471 measurement of dynamic interactions between pairs of moving objects. We have  
472 developed a new statistic (**DI**) for measuring dynamic interactions in discrete movement  
473 data (e.g., with a GPS). The basic properties of movement segments – azimuth and  
474 displacement, are used to detect dynamic interactions in azimuth, displacement, and  
475 overall movement. The **DI** method can be applied at four analysis levels (local, interval,  
476 episodal, and global - Laube et al. 2007) associated with movement data, and results can  
477 be aggregated across analysis levels. We introduce both time- and distance-based  
478 weighting schemes that can be useful in specific situations. The measurement of dynamic  
479 interactions at a temporal-lag, an example of trend-setting (Laube et al. 2005), can be  
480 easily incorporated. Like many spatial analysis techniques the **DI** method is impacted by  
481 the granularity at which movement data is represented. A detailed investigation of cross-  
482 scale effects is warranted to provide a better understanding of how the measurement of  
483 dynamic interaction is impacted by changing data granularities.

484 In some situations the nature of movement interactions will not simply involve  
485 two moving objects, but rather involve two moving objects impacted by a third. Consider  
486 the grizzly bear example; the bears exhibit varying levels of dynamic interaction over the  
487 course of the time period. The level of interaction is likely affected by their position

488 relative to the location of other objects, including other bears, roads, or sources of  
489 attraction or repulsion (i.e., food or danger). Future research will develop approaches for  
490 measuring third-party interactions, whereby pairs of moving objects interact with respect  
491 to a third stationary or moving object.

492 To those wishing to measure dynamic interactions with their own applications we  
493 have developed code for implementing **DI** in the statistical software package R (R  
494 Development Core Team 2011), for more information please visit:

495 <insert link to website here>

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505

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Table 1: Simulated movement scenarios, depicting different types of dynamic interactions, used to examine the differences between the new interaction statistic (DI) and an existing method ( $\mathbf{r}$ ).

<b>Scenario</b>	<b>Azimuth (<math>\theta</math>)</b>	<b>Displacement (<math>d</math>)</b>
<b>1</b>	Positive interaction	Interaction
<b>2</b>	Positive interaction (rotated by $45^\circ$ )	Interaction
<b>3</b>	Positive interaction	No interaction
<b>4</b>	Negative interaction	No interaction
<b>5</b>	No interaction	Interaction
<b>6</b>	Random	Random

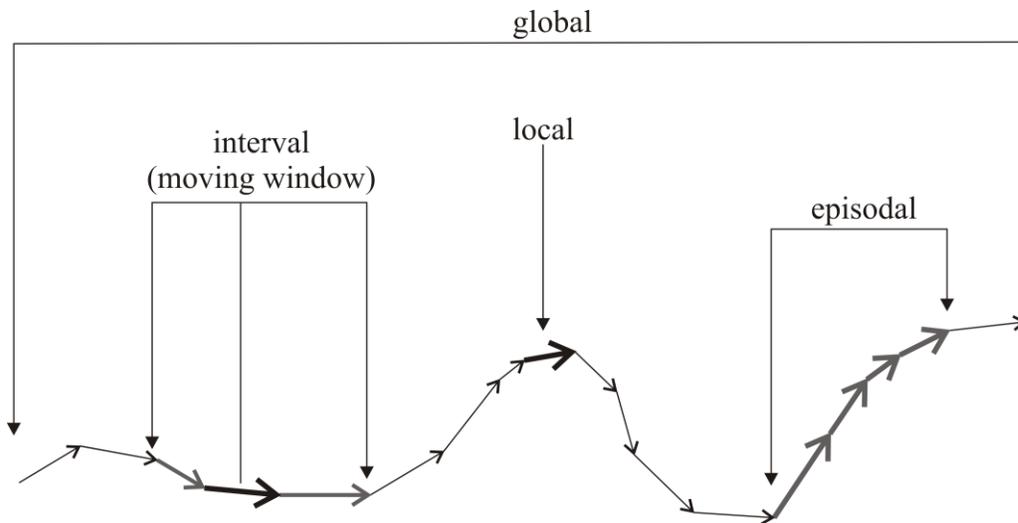
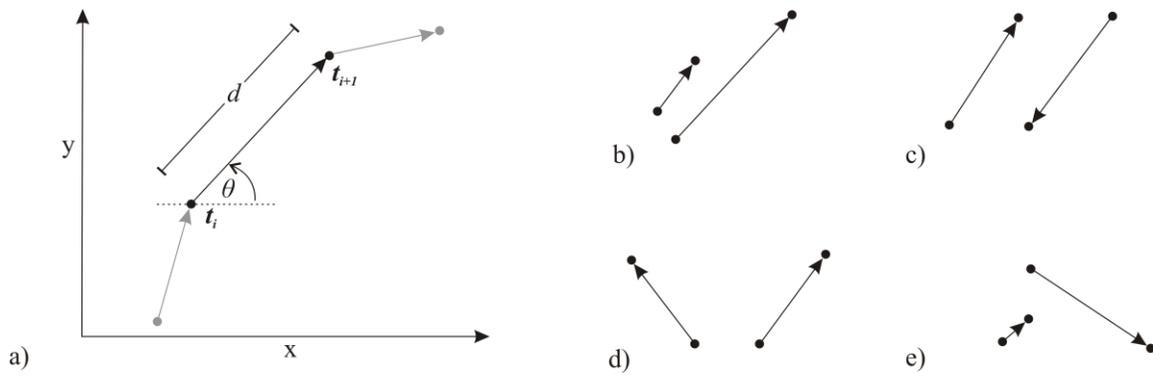


Figure 1: Diagram of four analysis levels used in movement data analysis (after Figure 2 in Laube et al. 2007). Local level statistics are calculated for each individual movement segment. Interval level analysis computes a running average statistic using a moving window. Episodal level analysis computes the statistic over a selected ‘episode’ or period of the dataset. Global level analysis computes the statistic over the entire movement path.



a) Diagram of movement properties azimuth ( $\theta$ ) and displacement ( $d$ ).  
 Examples of movement segments that exhibit: b) positive interaction in  $\theta$  and low interaction in  $d$ ; c) negative interaction in  $\theta$  and high interaction in  $d$ ; d) no interaction in  $\theta$  and high interaction in  $d$ ; and e) no interaction in  $\theta$  or  $d$ .

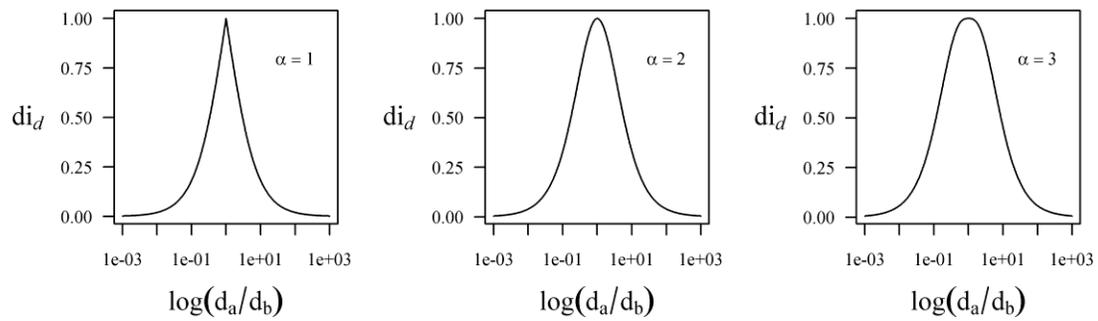


Figure 3: Relationship between  $\log(d_a/d_b)$  and  $di_d$ , for values of  $\alpha = 1, 2, 3$ .

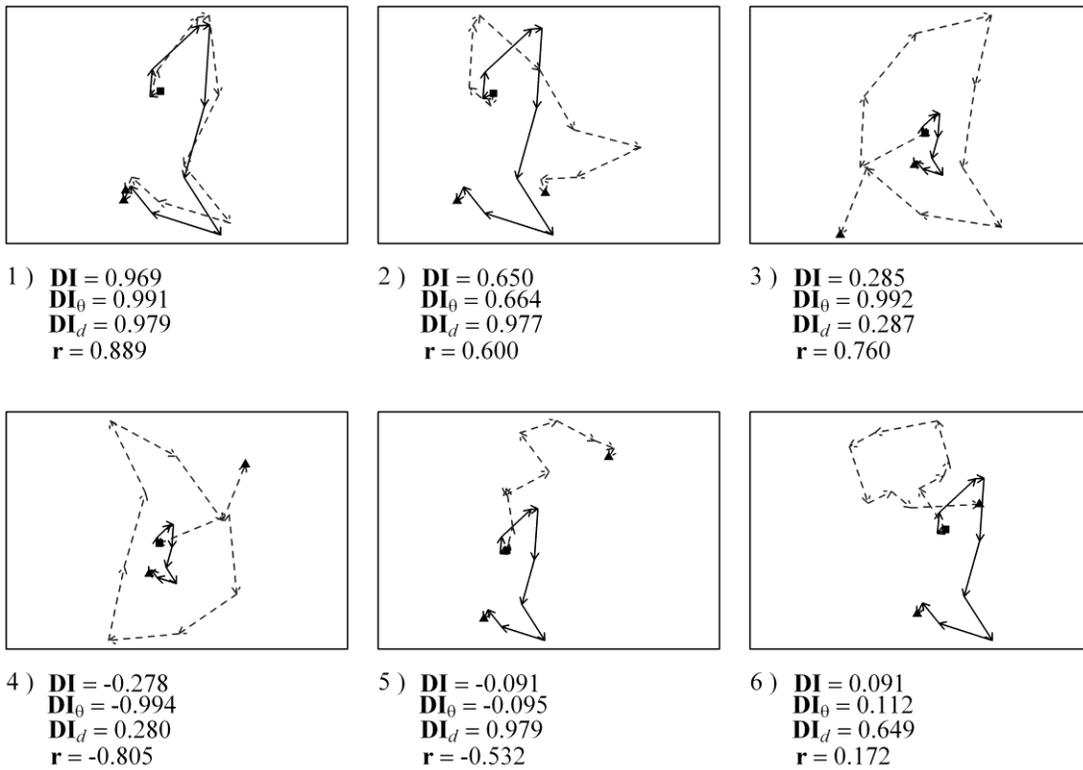


Figure 4: Results from global analysis of 6 simulated example scenarios, comparing the new **DI** method with the Shirabe (2006) correlation statistic – **r**. Original path is solid and black, while the path in dashed grey portrays variations based on six simulated scenarios (see Table 1).

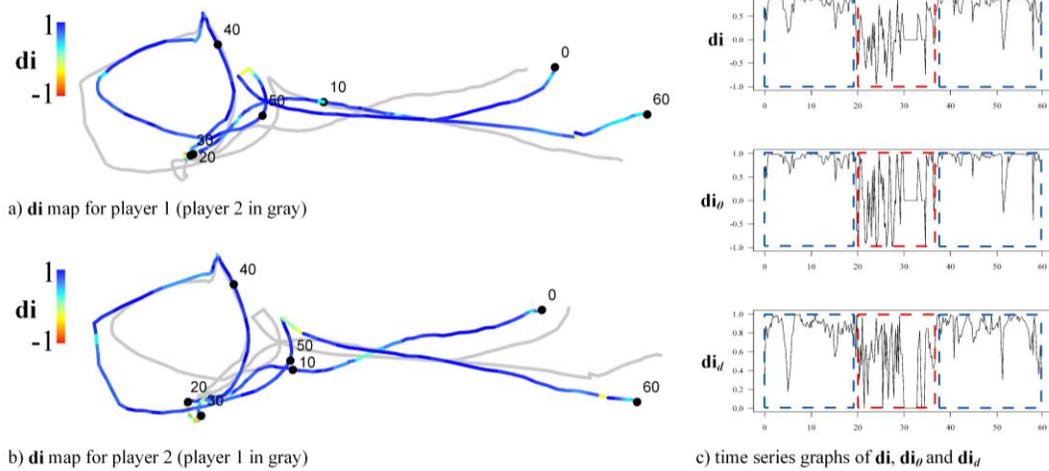


Figure 5: Local analysis showing maps of  $\mathbf{di}$  values for a) player 1, and b) player 2, from the ultimate frisbee example. c) time series graphs of  $\mathbf{di}$ ,  $\mathbf{di}_\theta$ , and  $\mathbf{di}_d$  can be used to identify periods of high and low dynamic interaction. Highlighted in blue in the time series graphs (c) are periods where player 1 does a good job covering player 2 ( $\mathbf{DI} = 0.757$ ). Highlighted in red is a period where the player 1 does a poorer job covering player 2 ( $\mathbf{DI} = 0.122$ ).

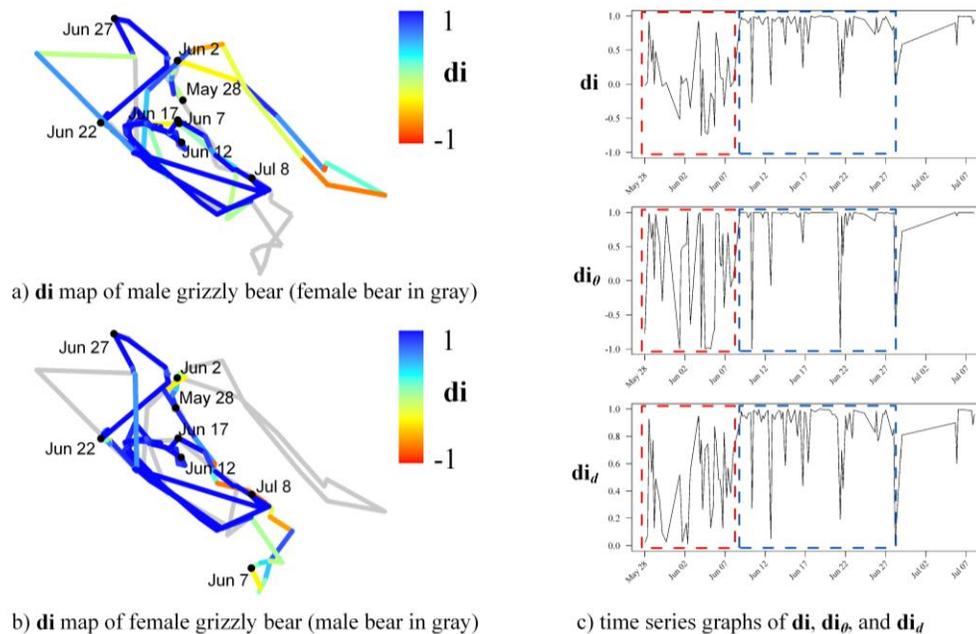


Figure 6: Local analysis showing maps of  $DI$  values for a) the male grizzly bear (G006), and b) the female grizzly bear (G010), from the grizzly bear example. c) time series graphs of  $DI$ ,  $DI_\theta$ , and  $DI_d$  can be used to identify periods of high and low dynamic interaction. Highlighted in red in the time series graphs (c) is a period where the bears exhibit low dynamic interaction ( $DI = 0.029$ ). Highlighted in blue is period where the bears exhibit strong dynamic interaction ( $DI = 0.492$ ), in this example indicative of mating behavior.

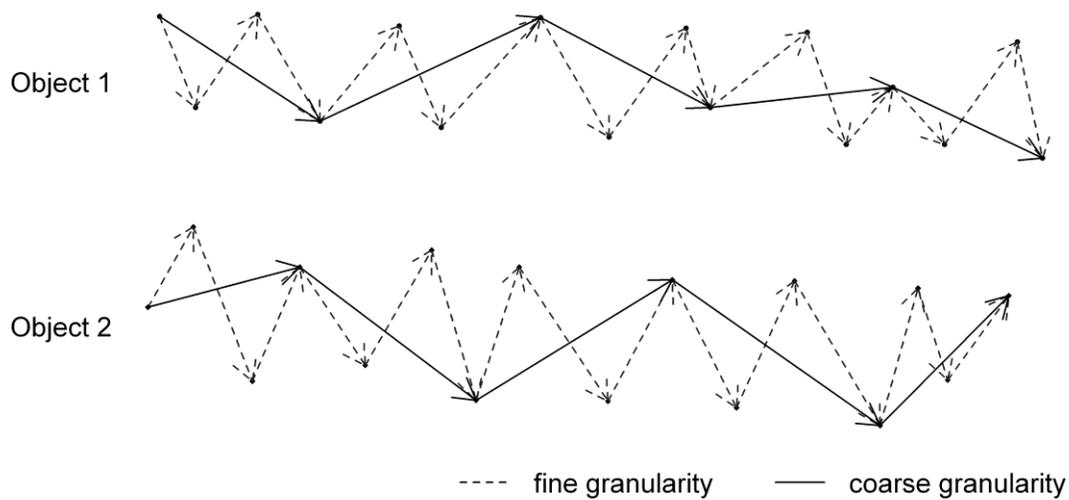


Figure 7: A pair of moving objects that exhibit negative dynamic interaction when analyzed at a fine granularity (dashed line,  $\mathbf{DI} = -0.47$ ) but positive dynamic interaction when analyzed at a coarser granularity (solid line,  $\mathbf{DI} = 0.49$ ). This example illustrates how changes in data granularity can impact results and interpretation of  $\mathbf{DI}$ .