

Nonlinear dissipation can combat linear loss

D. Mogilevtsev,^{1,2} A. Mikhalychev,² V. S. Shchesnovich,¹ and N. Korolkova³

¹*Centro de Ciências Naturais e Humanas, Universidade Federal do ABC, Santo André, São Paulo, 09210-170 Brazil*

²*Institute of Physics, Belarus National Academy of Sciences, F. Skarina Ave. 68, Minsk 220072 Belarus*

³*School of Physics and Astronomy, University of St. Andrews, North Haugh, St. Andrews KY16 9SS, United Kingdom*

(Received 5 November 2012; published 28 June 2013)

For nonclassical state generation, we demonstrate the possibility to compensate effects of strong linear loss by engineered nonlinear dissipation. We show that it is always possible to construct such a dissipative gadget that, for a certain class of initial states, the desired nonclassical state can be attained within a particular time interval with an arbitrary precision. Further we demonstrate that an arbitrarily large linear loss can still be compensated by a sufficiently strong coherent or even thermal driving, thus obtaining strongly nonclassical (in particular, sub-Poissonian) stationary mixed states.

DOI: [10.1103/PhysRevA.87.063847](https://doi.org/10.1103/PhysRevA.87.063847)

PACS number(s): 42.50.Nn, 42.50.Dv, 42.50.Ct, 03.65.Yz

I. INTRODUCTION

Nowadays, an engineered dissipation for quantum state manipulation is an intensely developing field. More than a decade ago, it was shown that in systems such as ions in traps it is possible to tailor nonlinear dissipation in a rather wide range [1], and later the concept of the quantum state protection by dissipation was born [2]. Different kinds of nonlinear dissipative apparatuses (aptly nicknamed dissipative gadgets [3,4]) have been shown to be useful for many important tasks, for example, for generating nonclassical and entangled states of few-body and many-body systems [5–12], performing universal quantum computation [4,13], constructing dissipatively protected quantum memory [14], and performing precisely timed sequential operations, conditional measurements, or error correction [13]. The central idea of most dissipative gadgets is to make dissipation to drive the system toward a desired steady state (which is practically independent of the initial state; this interesting feature has been already noticed [15]). Remarkably, in this context, dissipation serves as a helpful quantum resource rather than being a hindrance.

However, in all currently known schemes, the engineered dissipation is far from being a universal efficient tool for combating the usual linear loss, inevitably present in any realistic dissipative gadget. For example, conventional single-photon loss renders the generation of a nonclassical *pure* stationary state impossible for any kind of nonlinear dissipation [3]. Just as it is for the coherent control, it is necessary to minimize linear losses using some extra effort while arranging for the nonlinear terms to produce a desired effect (see, for example, Refs. [9,16]). In particular, it is rather hard to produce a desired state of an electromagnetic field for schemes relying on optical nonlinearities [5–9]. However, in this paper we show that nonlinear dissipation can be made to combat the effects of an arbitrarily strong linear loss (even with the rate far exceeding the rate of nonlinear loss), for both finite-time intervals and for stationary states. This is the main message of our contribution.

To illustrate our argument, we consider a simple Lindblad master equation for a single mode of electromagnetic field undergoing linear and nonlinear loss, quasiclassical and

thermal driving

$$\begin{aligned} \frac{d}{dt}\rho = & -i[H, \rho] + \Gamma(\bar{n} + 1)\mathcal{L}(a)\rho \\ & + \Gamma\bar{n}\mathcal{L}(a^\dagger)\rho + \gamma\mathcal{L}(A)\rho, \end{aligned} \quad (1)$$

where the nonlinear dissipation is described by the Lindblad operator A , the operators a^\dagger, a are the usual bosonic creation and annihilation operators, and the operator H represents the system Hamiltonian. $\Gamma \geq 0$ and $\gamma > 0$ are rates of linear and nonlinear dissipation, and the parameter \bar{n} represents the average number of photons in the thermal pump. The Liouvillian $\mathcal{L}(x)$ acts on the density matrix as $\mathcal{L}(x)\rho = 2x\rho x^\dagger - x^\dagger x\rho - \rho x^\dagger x$. Note that the coherent state is the only possible pure stationary state of Eq. (1) for $\Gamma > 0$ (see [3]), the vacuum state being the stationary state if no driving is present.

The paper consists of two main parts. In the first part (Secs. II and III) we present rather general result for finite-time dynamics described by Eq. (1). Namely, it is always possible, by choice of initial coherent state and nonlinear dissipation, to generate an arbitrary quantum state in a predefined time interval.

In the second part (Secs. IV and V) we prove that, for certain rather simple and practically realizable types of nonlinear loss, it is possible to generate mixed nonclassical stationary states despite an arbitrarily strong linear loss. It can be achieved by driving the mode coherently, or even thermally. The most remarkable feature of this scheme is the possibility to make a stationary state completely independent from the linear loss (despite linear loss being large). We conclude in Sec. VI.

II. GENERATION OF ARBITRARY PURE STATES WITHIN A FINITE-TIME INTERVAL

As we have already pointed out, generation of a pure stationary nonclassical state via the scheme described by the master equation (1) is not possible. However, it is always possible to generate a state approximating the desired pure nonclassical state with any given precision for an arbitrary (but finite) ratio of linear and nonlinear loss rates γ/Γ within a finite time interval.

To demonstrate this, we assume that we have an initial state, $|\alpha\rangle$, and want to produce a target state, $|\phi\rangle$. Then, let us assume that we can construct the following Lindblad operator:

$$A = |\phi\rangle\langle\alpha|a^k, \quad k > 1. \quad (2)$$

Consider now the master equation (1) with the nonlinear loss described by the Lindblad operator (2). In the absence of linear loss, the target state can indeed be readily generated. From Eq. (1), the following equations for the matrix elements can be easily derived:

$$\begin{aligned} \frac{d}{dt}\rho_{\Psi\Psi} &= -2\gamma\mathcal{N}(1 - |\langle\phi|\Psi\rangle|^2)\rho_{\Psi\Psi}, \\ \frac{d}{dt}\rho_{\phi\phi} &= \gamma\mathcal{N}(2\rho_{\Psi\Psi} - \langle\phi|\Psi\rangle\rho_{\phi\Psi} - \langle\Psi|\phi\rangle\rho_{\Psi\phi}), \\ \frac{d}{dt}\rho_{\Psi\phi} &= \gamma\mathcal{N}(\rho_{\Psi\Psi}\langle\Psi|\phi\rangle - \rho_{\Psi\phi}), \end{aligned} \quad (3)$$

where the matrix elements are $\rho_{xy} = \langle x|\rho|y\rangle$, $x, y = \psi, \Psi$; and the vector Ψ is defined as

$$|\Psi\rangle = \frac{(a^\dagger)^k}{\sqrt{\mathcal{N}}}|\alpha\rangle, \quad \mathcal{N} = \langle\alpha|a^k(a^\dagger)^k|\alpha\rangle.$$

From the system (3) the actual matrix elements can be readily obtained

$$\begin{aligned} \rho_{\phi\phi}(t \gg \gamma_{\text{eff}}^{-1}) &= \rho_{\phi\phi}(0) + \rho_{\Psi\Psi}(0) - \langle\phi|\Psi\rangle\rho_{\Psi\phi}(0) \\ &\quad - \langle\Psi|\phi\rangle\rho_{\phi\Psi}(0) + O(e^{-\gamma_{\text{eff}}t}), \end{aligned} \quad (4)$$

where

$$\gamma_{\text{eff}} = \min\{1, 2(1 - |\langle\phi|\Psi\rangle|^2)\}\mathcal{N}\gamma.$$

From the solution (4), we see that if the target state is orthogonal to the initial state, $|\alpha\rangle$, the target state is generated with the fidelity

$$\rho_{\Psi\Psi}(0) = |\langle\alpha|\Psi\rangle|^2 = \frac{1}{\mathcal{N}}|\langle\alpha|(a^\dagger)^k|\alpha\rangle|^2.$$

Now, we intend to find a way of turning readily available classical or semiclassical states into nonclassical states. Hence, a straightforward choice of the initial state is a coherent state. For a coherent initial state $|\alpha\rangle$, the fidelity approaches unity when the amplitude of the initial coherent state is large enough, $|\alpha| \gg 1$. Further, by assuming that the amplitude of the initial coherent state is large enough, we ensure that the initial state is practically orthogonal to the target state. In addition, it is easy to see that the larger $|\alpha|$ is, the quicker the system evolves towards the target [since the decay rate in Eqs. (3) is proportional to \mathcal{N}].

Rapid evolution toward the target state also ensures that the target state can be generated during a time interval when the influence of linear losses is negligible. To see this, we consider the respective time scale of dynamics induced by nonlinear and linear loss. It is possible to do this without solving the master equation (1) by considering the well-known quantum jump approach (see, for example, the classical review [17]). According to this approach, the dynamics of the system is described by the set of stochastic wave functions, or quantum trajectories, say, $|\phi_j(t)\rangle$. As it follows from Eq. (1) and according to quantum jump formalism, in any small

time interval, $[t, t + dt]$, the system's trajectory might change (jump) to

$$|\phi_j(t + dt)\rangle = \frac{A|\phi_j(t)\rangle}{\sqrt{\langle\phi_j(t)|A^\dagger A|\phi_j(t)\rangle}}$$

with probability $p_A = 2\gamma dt \langle\phi_j(t)|A^\dagger A|\phi_j(t)\rangle$; or the trajectory might jump to

$$|\phi_j(t + dt)\rangle = \frac{a|\phi_j(t)\rangle}{\sqrt{\langle\phi_j(t)|a^\dagger a|\phi_j(t)\rangle}}$$

with probability $p_a = 2\Gamma dt \langle\phi_j(t)|a^\dagger a|\phi_j(t)\rangle$.

Otherwise, the system might undergo the deterministic change

$$|\phi_j(t + dt)\rangle \approx |\phi_j(t)\rangle - dt \frac{H_{\text{eff}}}{\mathcal{N}_{\text{eff}}}|\phi_j(t)\rangle, \quad (5)$$

where

$$\begin{aligned} H_{\text{eff}} &= -\gamma A^\dagger A - \Gamma a^\dagger a, \\ \mathcal{N}_{\text{eff}} &= \langle\phi_j(t)|(1 - 2H_{\text{eff}}dt)|\phi_j(t)\rangle. \end{aligned}$$

Naturally, for $p_A \gg p_a$, the dynamics dictated by the nonlinear loss will dominate over that dictated by the linear loss [which holds also for the deterministic evolution given by Eq. (5)]. Averaging over trajectories, we arrive at the following condition:

$$\Gamma\langle a^\dagger(t)a(t)\rangle \ll \gamma\langle A^\dagger(t)A(t)\rangle. \quad (6)$$

As long as the condition (6) holds, the influence of linear loss during the time of the target state generation is negligible. However, it is impossible to simultaneously satisfy the condition (6) and to generate exactly the target state. Indeed, to have the unit fidelity of the target state generation, the following equation should hold: $A|\phi\rangle = 0$. Thus, the right-hand side of the expression (6) is strictly zero for the case. In other words, in the close vicinity of the target state the linear loss is always prevailing over the nonlinear loss. However, merely by choice of the initial state $|\alpha\rangle$, it is possible to make this vicinity region arbitrarily small. Indeed, assume that we have generated the state

$$\rho_{\text{app}} = (1 - p)|\phi\rangle\langle\phi| + p\rho_{\text{err}},$$

where $0 < p < 1$ and the density matrix ρ_{err} , orthogonal to $|\phi\rangle\langle\phi|$, denotes the deviation from the generated target state. Then the condition (6) can be rewritten as

$$\Gamma\text{Tr}\{a^\dagger a \rho_{\text{app}}\} \ll \gamma\mathcal{N}p\langle\Psi|\rho_{\text{err}}|\Psi\rangle. \quad (7)$$

It can be seen that for larger \mathcal{N} is, the smaller p will satisfy the condition (7).

Figures 1(a) and 1(b) illustrate the solution of Eq. (1) for the generation of the two-photon Fock state $|2\rangle$ from an initial coherent state $|\alpha\rangle$ with the nonlinear dissipation described by the Lindblad operator $A = |2\rangle\langle\alpha|a^2$. The rate of nonlinear loss, γ , is five times less than the rate of the linear loss, Γ . By increasing the amplitude α the target state $|2\rangle$ is approximated with a larger fidelity and over a shorter time period, despite the presence of rather strong linear loss [Figs. 1(a) and 1(b)].

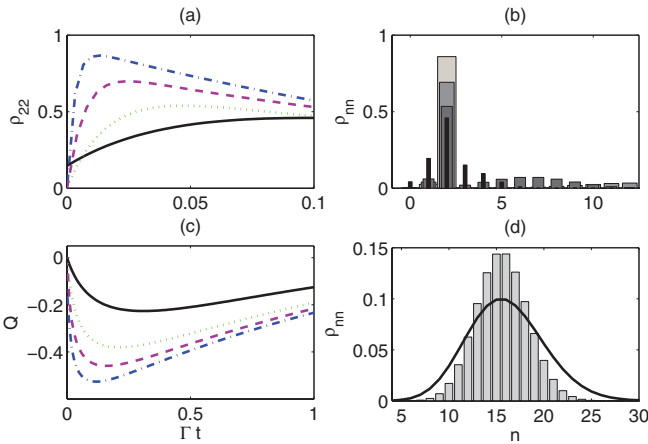


FIG. 1. (Color online) Generation of nonclassical states from a coherent state $|\alpha\rangle$ in the presence of linear loss using designed nonlinear dissipation described by the Lindblad operator A . Thermal driving is absent, $\bar{n} = 0$, and the loss ratio is given by $\gamma = \Gamma/5$. (I) Generation of the Fock state $|2\rangle$ using nonlinear dissipation given by $A = |2\rangle\langle\alpha|a^2$. (a) Fidelity of the target two-photon state as a function of time scaled to linear loss, Γt . Solid, dotted, dashed, and dash-dotted lines correspond to $\alpha = 2, 3, 4, 5$. (b) shows the photon number distribution of the generated state when the fidelity in (a) is maximal; black, dark gray, gray, and light bars correspond to $\alpha = 2, 3, 4, 5$. (II) Generation of sub-Poissonian light and photon-number squeezing using tailored dissipation with $A = a(a^\dagger a - 1)$. (c) Mandel Q parameter vs time scaled to linear loss, Γt ; solid, dotted, dashed, and dash-dotted lines correspond to $\alpha = 2, 4, 6, 8$. (d) Photon number distribution of the generated state for the minimal Q and $\alpha = 8$ (gray bars) in comparison with the Poissonian distribution with the same average number of photons (solid line).

III. FINITE-TIME GENERATION OF NONCLASSICAL STATES FOR REALISTIC NONLINEAR LOSS

We have shown that for an appropriately constructed dissipative gadget it is always possible to combat the linear loss with any predefined rate by choosing the appropriate initial state, in particular, by increasing an average number of photons in the initial state. Of course, there are practical limitations in construction of dissipative gadgets. The consideration above was given for rather artificial nonlinear loss given by the Lindblad operator (2), which might be hard to realize in practice.

However, notably, the condition (6) has a rather general character. If, for a class of states, the jump rate of the nonlinear dissipation far exceeds the jump rate for the linear loss, then an influence of the linear loss for evolution within this class is negligible. Moreover, the dynamics due to the engineered nonlinear dissipation can also be nonexponentially fast. For example, for the realistic two-photon dissipation, quite different initial states can decay toward the same stationary state over the same period of time [18]. So, we can expect that, for feasible nonlinear dissipation schemes, it is possible to generate certain kinds of nonclassical states from quasiclassical states during finite time intervals despite strong linear loss.

Here we demonstrate that it is indeed the case using a class of practically realizable dissipative gadgets. The key element here is the implementation of the particular type of

loss, nonlinear coherent loss (NCL). NCL is described by the Lindblad operators $A = af(a^\dagger a)$, $f(x)$ being a smooth function. This operator is the annihilation operator for the so-called f -deformed quantum harmonic oscillator; eigenstates of this operator are referred to as “nonlinear coherent states” [19]. Any pure state nonorthogonal to an arbitrary Fock state can be exactly represented as a nonlinear coherent state. If it is orthogonal to some Fock states, then one can still devise a nonlinear coherent state closely approximating the state in question [20]. NCL can be realized in practice in a number of schemes (for example, with ions or atoms in traps [1]; Bose-Einstein condensates [21], or even in multicore nonlinear optical fibers [9]). With function $f(x)$ having a finite or countable number of zeros, one can generate Fock states or actually “comb” the initial state by filtering out some predefined set of components in the given basis [22,23]. Using a NCL gadget, it is possible to have significant nonclassicality (in particular, large photon-number squeezing) on the time scales when influence of linear losses is negligible (see, for example, Refs. [9,24,25]). This is illustrated in Figs. 1(c) and 1(d), where the exact solution of Eq. (1) is shown for the nonlinear dissipation described by the Lindblad operator $A = a(a^\dagger a - 1)$ in the presence of linear loss and for an initial coherent state $|\alpha\rangle$. Obviously, this Lindblad operator satisfies condition (6) for a sufficiently large α . At the initial stages of evolution, nonclassical states, namely, photon-number squeezed states, can be generated. The nonclassicality is thereby increased with growing amplitude α in the initial state. Figure 1(c) shows the Mandel Q parameter,

$$Q = [(\langle a^\dagger a \rangle^2) - \langle a^\dagger a \rangle^2] / \langle a^\dagger a \rangle - 1,$$

confirming that the generated light exhibits the sub-Poissonian statistics (for $Q < 0$). Note that the maximum of nonclassicality is reached rather quickly. After that the generated state evolves under the influence of the linear loss towards the classical state. But it happens quite slowly on the time scale of the nonclassical state generation. This is a rather general feature of the dynamics for any dissipative gadget satisfying condition (6) [26].

IV. PROTECTION OF NONCLASSICALITY FOR MIXED STATIONARY STATES

Now we demonstrate that for certain classes of nonlinear loss it is possible to obtain mixed nonclassical states in the long-time limit. At the first glance, it seems possible to do this in the way similar to that suggested for generating nonclassicality for finite times. The difference is that instead of choosing the initial state with large number of photons, additional quanta are inserted into the system by external driving. However, driving the system gives rise to quite unexpected and surprising phenomena. More precisely, there is a class of nonlinear losses such that for an arbitrarily large (but finite) ratio $\varepsilon = \Gamma/\gamma$ an influence of losses on the stationary state can be completely eliminated by the sufficiently strong coherent driving and, moreover, even by thermal pumping. Note that the resulting steady state, while being mixed, can still be strongly nonclassical and represent a useful resource [27].

We show now that even a weak NCL can be protected from large linear loss and used for generation of nonclassical

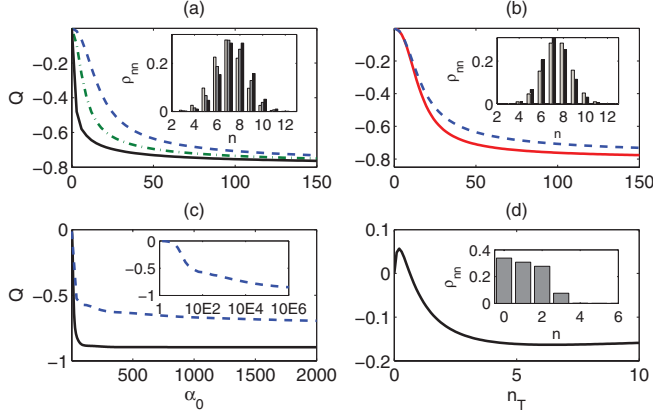


FIG. 2. (Color online) Generation of nonclassical states in the long-time limit using nonlinear coherent loss (NCL) to counteract linear loss. (a) Mandel's Q parameter vs amplitude of the coherent driving $\alpha_0 = \Omega/\gamma$ for the NCL with $A = a(a^\dagger a - 1)$. The Q parameter is derived from the exact solution of Eq. (1); solid, dash-dotted, and dashed curves correspond to the relative loss rate $\Gamma = \gamma, 5\gamma, 10\gamma$. Inset shows photon-number distribution of the stationary state for $\alpha_0 = 150$; light-gray, gray, and black bars correspond to $\Gamma = \gamma, 5\gamma, 10\gamma$. (b) Same for the NCL with $A = a(a^\dagger a - 1)$ using the exact solution of Eq. (1) (dashed line) and using the solution of the approximate equation (9) (solid line). Inset shows photon-number distributions for the solution of the exact (gray bars) and of the approximate (black bars) equations. (c) Exact (dashed) and approximate (solid) solutions for Q parameter for $A = a(a^\dagger a - 1)^2$; the inset shows long-time behavior of Q parameter given by the exact solution. (d) Exact solution for Q factor in dependence of the average number of thermal pumping photons, n_T ; the coherent driving is absent; $A = a(a^\dagger a - 1)^3$.

stationary states. That is, the nonclassicality can be rescued in the long-time limit. We use the classical driving in the form $H = i\Omega(a - a^\dagger)$, where the parameter Ω represents the strength of driving (for simplicity, it is taken to be real). In this section we assume that the thermal pumping is absent, $\bar{n} = 0$. Figure 2 depicts a nonclassicality rescue procedure when generating sub-Poissonian states. Notably, for $A = a(a^\dagger a - 1)$, the Mandel's Q parameter eventually tends to the same limiting value for quite different linear loss rates, $\Gamma = \gamma, 5\gamma$, and 10γ [Fig. 2(a)]. It is remarkable that the generated states are also quite similar [see inset in Fig. 2(a)] for the density matrix].

The nature of this phenomenon can be well illustrated and clarified with the help of the following simple approximation. Eq. (1) can be rewritten as

$$\begin{aligned} \frac{d}{dt}\rho &= \gamma a \rho (B^\dagger - \alpha_0) + \gamma (B - \alpha_0) \rho a^\dagger \\ &\quad - \gamma a^\dagger (B - \alpha_0) \rho - \gamma \rho (B^\dagger - \alpha_0) a \\ &\quad - \gamma a [[\rho, f(a^\dagger a)], f(a^\dagger a)] a^\dagger, \end{aligned} \quad (8)$$

where $B = a([f(a^\dagger a)]^2 + \varepsilon)$ and $\alpha_0 = \Omega/\gamma$. The last term is small in comparison with the others when the inequality

$$\{f(n) - f(m)\}^2 \ll 2f(n)f(m)$$

holds for all n and m , corresponding to essentially nonzero density matrix elements ρ_{nm} . For example, for any power-law nonlinearity $f(x) \sim x^k$, this condition is satisfied when

the driving is sufficiently strong. Neglecting the last term in Eq. (8), we obtain the approximate master equation

$$\begin{aligned} \frac{d}{dt}\rho &= \gamma a \rho (B^\dagger - \alpha_0) + \gamma (B - \alpha_0) \rho a^\dagger \\ &\quad - \gamma a^\dagger (B - \alpha_0) \rho - \gamma \rho (B^\dagger - \alpha_0) a. \end{aligned} \quad (9)$$

For Eq. (9), the stationary state ρ^s is the eigenstate of the operator B . It satisfies

$$B\rho^s = \alpha_0\rho^s, \quad \rho^s B^\dagger = \alpha_0\rho^s.$$

This leads to the following recurrence relation for the diagonal (in the Fock-state basis) elements of the steady state:

$$\rho_{n,n}^s = \rho_{n-1,n-1}^s \frac{\alpha_0^2}{n(f(n)^2 + \varepsilon)^2}. \quad (10)$$

It should be noted that for $f(x)$ growing faster than x , the relation (10) means that the density matrix elements, ρ_{nn} , decrease with growth of the photon number n faster than for any coherent state, as for the coherent state

$$\frac{\rho_{nn}^{\text{coh}}}{\rho_{n-1,n-1}^{\text{coh}}} = O(1/n). \quad (11)$$

Thus, the state described by the density matrix ρ^s is nonclassical. Namely, its Glauber P function cannot be non-negative everywhere on the phase plane [28].

Assuming that the steady-state photon number distribution is peaked at n_0 and using Eq. (10), we get a simple condition for determining n_0

$$\rho_{n_0,n_0}^s \approx \rho_{n_0-1,n_0-1}^s \Rightarrow n_0\{[f(n_0)^2 + \varepsilon]^2\} \approx \alpha_0^2.$$

This condition indicates that, for sufficiently strong classical driving and for function $f(n)$ increasing monotonically for sufficiently large $n > 0$, relation $f(n_0)^2 \gg \varepsilon$ holds always. That is, the influence of linear losses on the generated steady state will always be negligible. The recurrence relation (10) also allows us to estimate the width of the photon-number distribution of the stationary state. Assuming that the value of the diagonal element $\rho_{n,n}^s$ changes only weakly for small changes δn of the photon number n around the value n_0 , from Eq. (10) we obtain the following expression:

$$\begin{aligned} \rho_{n_0+\delta n,n_0+\delta n}^s \\ \approx \rho_{n_0,n_0}^s \exp \left\{ -\frac{|\delta n|(|\delta n| + 1)}{2n_0} \left(1 + \frac{4n_0 f'(n_0) f(n_0)}{f(n_0)^2 + \varepsilon} \right) \right\}, \end{aligned} \quad (12)$$

where $f'(n) = df(n)/dn$. From Eq. (12) we can estimate the variance $\Delta^2 n$ of the steady-state photon number distribution

$$\Delta^2 n \approx n_0 \left(1 + \frac{4n_0 f'(n_0) f(n_0)}{f(n_0)^2 + \varepsilon} \right)^{-1}. \quad (13)$$

Equation (13) has important implications. That is, our approximation shows that in the limit of strong driving the steady state produced by the NCL will always be a photon-number squeezed state, provided that $f'(n_0) > 0$. The squeezing can be quite high. For example, consider $f(n) = n - 1$, which has been proven feasible using three-well potential in Bose-Einstein condensates [21] or three-core nonlinear fibers [9]. For the simple NCL with $f(n) = n - 1$, the value

$\Delta^2 n \rightarrow n_0/5$ is asymptotically reached for $n_0 \approx (\alpha_0)^{2/5} \rightarrow +\infty$. This value corresponds to the Mandel parameter equal to -0.8 . Figure 2(b) shows that the approximation to Eq. (8) indeed gives a rather good estimate for both the Mandel parameter and the generated state.

It should be noted though, that for rapidly increasing $f(n)$ the approximation works somewhat worse. It can be easily seen from Eq. (13), since it predicts Q parameter close to -1 for rapidly increasing $f(n)$, e.g., $Q \sim -4k/(4k+1)$ for $f(n) = n^k$. $Q = -1$ corresponds to the Fock state. But when the photon number distribution becomes very narrow, the assumption of slowly changing ρ_{nn} near maximum of the photon number distribution can hardly be applied. Nevertheless, the approximation still provides a qualitatively correct description as illustrated in Fig. 2(c) for $f = (x-1)^2$, where exact and approximate solutions for the Q parameter are compared. Hence even for $f(x)$ increasing rather rapidly one can indeed have highly pronounced nonclassicality unaffected by any linear loss. However, it happens at the expense of quite strong coherent driving required [see Fig. 2(c)].

V. THERMAL DRIVING

The generality of the condition (6) suggests that the coherent driving is not the only option for preserving nonclassicality of the stationary state. Indeed, here we demonstrate that incoherent (thermal) driving can produce a nonclassical sub-Poissonian stationary state (which is really rather counterintuitive). However, thermal driving leads also to the appearance of some quite specific phenomena.

Indeed, in the absence of the coherent driving ($\Omega = 0$), from exact Eq. (8) the following recurrence relation is obtained:

$$\rho_n = \rho_{n-1} \frac{\bar{n}}{(\bar{n}+1) + (\gamma/\Gamma)f^2(n)}. \quad (14)$$

The denominator in Eq. (14) is always greater than the numerator. Therefore, the photon number distribution is a monotonically decreasing function. For

$$f^2(n) \ll \varepsilon(\bar{n}+1), \quad (15)$$

the relation (14) between the density matrix elements corresponds to ordinary thermal state with the mean photon number \bar{n} . In the opposite case,

$$f^2(n) \gg \varepsilon(\bar{n}+1), \quad (16)$$

the density matrix elements ρ_{nn} decrease with the growth of n faster than for thermal states. Equation (14) implies that for $f(x)$ growing faster than x , the matrix elements decrease faster than matrix elements of the coherent state [the relation (11) does not hold in this case]. So, the stationary state is nonclassical. Figure 2(d) shows an example of such a thermal rescue for the nonlinear loss with $f(x) = (x-1)^3$. Thus, remarkably, the thermal excitation is able to produce photon antibunching. However, it is easy to see that the minimal Q value always corresponds to some finite \bar{n} . When the intensity of the thermal driving increases, all the nonclassicality will be eventually washed out.

One can also encounter a special situation when both of the conditions (15), (16) are satisfied, each in a different region of the photon number distribution. In this case the photon

number distribution for the stationary state can resemble the one for thermal state when n is small enough, $n < n_0$. It will then rapidly decrease for large n ($n > n_0$), being effectively truncated at certain point $n = n_0$. This truncation point can be roughly estimated from the equation

$$f^2(n_0) = \varepsilon(\bar{n}+1). \quad (17)$$

This situation is illustrated in the inset in Fig. 2(d).

Concluding this section, we present a curious example of nonclassical state generation in the limit of very strong thermal pumping ($\bar{n} \rightarrow \infty$) and, simultaneously, very strong nonlinear driving

$$f(n)/(\varepsilon\bar{n}) \rightarrow \infty$$

for the number of photons exceeding some threshold value, n_0 . We assume that below this value the nonlinear driving is not acting; i.e., $f(n) = 0$ for $n \leq n_0$. From Eq. (14), the resulting stationary state is an equally weighted mixture of all the Fock-state projectors $|0\rangle, \dots, |n_0\rangle$

$$\rho = I_{n_0} = \frac{1}{n_0+1} \sum_{n=0}^{n_0} |n\rangle\langle n|.$$

Thus, it is possible to generate truncated unit operator in this scheme.

VI. CONCLUSION

We have shown that dissipative gadgets can be made extremely robust with respect to linear losses unavoidably affecting any realistic engineered dissipation scheme. First, we have shown that it is always possible to generate any desired pure target state during the time interval when the influence of linear loss is negligible. We provided an example illustrating such scenario using coherent input states. The larger the difference in average number of photons between the initial state and the target state, the more robust the scheme can be made. We have also shown that realistic, already available nonlinear loss schemes can also generate strongly nonclassical states during finite time intervals.

Next, remarkably, there exists a class of dissipative gadgets based on the nonlinear coherent loss (NCL) for which the influence of linear loss can be completely annihilated by a sufficiently strong coherent driving. The resulting state is mixed. However, this stationary mixed state can be close to the Fock state and thus exhibits strong photon-number squeezing. Furthermore, there exists a class of dissipative gadgets, for which nonclassicality of the stationary state can even be rescued by using usual incoherent thermal driving.

Note that the possibility to annihilate completely an influence of an arbitrarily strong linear loss has deep implications for devising on-demand generators of nonclassical states far beyond the simplest single-mode case considered here. To name a few possibilities for further developments, it is straightforward to generalize the scheme to multimode case designing, for example, a robust on-demand generator of different kinds of entangled states impervious to independent linear losses in each mode. Moreover, using a combination of different kinds of nonlinear losses (which is usually the case

for nonlinear losses appearing in practice [9,21,24]), a wide variety of nonclassical states can be generated.

ACKNOWLEDGMENTS

This work was supported by Belarusian Republican Foundation for Fundamental Research (BRFFI) of Belarus, by

National Academy of Science of Belarus through the Program “Convergence”, by the External Fellowship Program of the Russian Quantum Center at Skolkovo, by the Brazilian Agency FAPESP (Project No. 2011/19696-0) (D.M.) and has received funding from the European Community’s Seventh Framework Programme (FP7/2007-2013) under Grant Agreement No. 270843 (iQIT).

-
- [1] J. F. Poyatos, J. I. Cirac, and P. Zoller, *Phys. Rev. Lett.* **77**, 4728 (1996).
- [2] A. R. R. Carvalho, P. Milman, R. L. de Matos Filho, and L. Davidovich, *Phys. Rev. Lett.* **86**, 4988 (2001).
- [3] B. Kraus, H. P. Buchler, S. Diehl, A. Kantian, A. Micheli, and P. Zoller, *Phys. Rev. A* **78**, 042307 (2008).
- [4] F. Verstraete, M. M. Wolf, and J. Ignacio Cirac, *Nature Phys.* **5**, 633 (2009).
- [5] H. Ezaki, E. Hanamura, and Y. Yamamoto, *Phys. Rev. Lett.* **83**, 3558 (1999).
- [6] M. Alexanian and S. K. Bose, *Phys. Rev. Lett.* **85**, 1136 (2000).
- [7] H. Ezaki, E. Hanamura, and Y. Yamamoto, *Phys. Rev. Lett.* **85**, 1137 (2000).
- [8] T. Hong, M. W. Jack, and M. Yamashita, *Phys. Rev. A* **70**, 013814 (2004).
- [9] D. Mogilevtsev and V. S. Shchesnovich, *Opt. Lett.* **35**, 3375 (2010).
- [10] M. B. Plenio, S. F. Huelga, A. Beige, and P. L. Knight, *Phys. Rev. A* **59**, 2468 (1999).
- [11] A. S. Parkins and H. J. Kimble, *Phys. Rev. A* **61**, 052104 (2000).
- [12] S. Clark, A. Peng, M. Gu, and S. Parkins, *Phys. Rev. Lett.* **91**, 177901 (2003).
- [13] M. J. Kastoryano, M. M. Wolf, and J. Eisert, *Phys. Rev. Lett.* **110**, 110501 (2013).
- [14] F. Pastawski, L. Clemente, and J. I. Cirac, *Phys. Rev. A* **83**, 012304 (2011).
- [15] It is interesting to note that for finite times and two-photon loss a possibility to generate nonclassical states were predicted in very early work of A. Bandilla and H.-H. Ritze, *Phys. Lett. A* **55**, 285 (1975); H.-H. Ritze, *Annalen der Phys.* **33**, 207 (1976).
- [16] K. Stannigel, P. Rabl, and P. Zoller, *New J. Phys.* **14**, 063014 (2012).
- [17] M. B. Plenio and P. L. Knight, *Rev. Mod. Phys.* **70**, 101 (1998).
- [18] V. V. Dodonov and S. S. Mizrahi, *Phys. Lett. A* **223**, 404 (1996).
- [19] V. I. Manko, G. Marmo, E. C. G. Sudarshan, and F. Zaccaria, *Phys. Scr.* **55**, 528 (1997); V. I. Manko and R. Vilela Mendes, *J. Phys. A* **31**, 6037 (1998).
- [20] Z. Kis, W. Vogel, and L. Davidovich, *Phys. Rev. A* **64**, 033401 (2001).
- [21] V. S. Shchesnovich and D. S. Mogilevtsev, *Phys. Rev. A* **82**, 043621 (2010).
- [22] A. Mikhalychev, D. Mogilevtsev, and S. Kilin, *J. Phys. A: Math. Theor.* **44**, 325307 (2011).
- [23] A. B. Mikhalychev, D. S. Mogilevtsev, and S. Ya. Kilin, *Nonlinear Phenom. Complex Syst.* **14**, 241 (2011).
- [24] V. S. Shchesnovich and D. Mogilevtsev, *Phys. Rev. A* **84**, 013805 (2011).
- [25] A. Bandilla, *Opt. Commun.* **23**, 299 (1977).
- [26] A. Mikhalychev, D. Mogilevtsev, A. M. Ishkhanyan, and S. Kilin [Nonlinear Phenom. Complex Syst. (to be published)] (2013).
- [27] Note that a robust on-demand generator of strongly non-Gaussian states is very much needed for many tasks in quantum information science and quantum-enabled technology. To see how entangled states can be produced from non-Gaussian states, see, for example, Ulf Leonhardt, *Essential Quantum Optics* (Cambridge University Press, Cambridge, 2010).
- [28] J. Peřina, *Quantum Statistics of Linear and Nonlinear Optical Phenomena*, 2nd ed. (Springer, Berlin, 1991).