Collective Suppression of Linewidths in Circuit QED

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We report the experimental observation and a theoretical explanation of collective suppression of linewidths for multiple superconducting qubits coupled to a good cavity. This demonstrates how strong qubit-cavity coupling can significantly modify the dephasing and dissipation processes that might be expected for individual qubits, and can potentially improve coherence times in many-body circuit QED.

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At the root of many of the unexpected effects predicted by quantum mechanics is quantum interference. In the context of quantum optics, one notable consequence of constructive interference is superradiance [1,2], the collective enhancement of radiation from an ensemble of many initially excited atoms. At its heart is the idea that if the atoms remain in a symmetric state, then constructive interference between different final states after emitting a photon can increase the probability of such photon emission events [1]. It is notable that such constructive interference plays a role even when considering the incoherent and irreversible emission of photons into an external environment. This collective effect is distinct from the subnatural linewidth averaging first discussed by [3].

Since the superradiant emission of photons can occur incoherently, it can be expected that similar effects should be visible in the linewidth of a collection of atoms, or artificial atoms, coupled symmetrically to the same solid state environment. Indeed, one may see such an effect by comparing collective and individual decay processes. Consider N two-level systems obeying either individual decay and dephasing: $\dot{\rho} = \dot{\rho}_H + \sum_i (\gamma_\parallel/2)\mathcal{D}[\sigma_i^z] + (\gamma_\perp/2)\mathcal{D}[\sigma_i^-]$ (where $\mathcal{D}[X] = 2X\rho X^\dagger - X^\dagger X\rho - \rho X^\dagger X$, $\{\sigma_i\}$ are Pauli operators describing the two-level systems with associated dephasing ($\gamma_\parallel$) and relaxation ($\gamma_\perp$) rates, and $\dot{\rho}_H = -i[H,\rho]$ describes the Hamiltonian evolution), or collective decay and dephasing: $\dot{\rho} = \dot{\rho}_H + (\gamma_\parallel/2)\mathcal{D}[\sum_i \sigma_i^z] + (\gamma_\perp/2)\mathcal{D}[\sum_i \sigma_i^-]$. If one then calculates the linear absorption spectrum for an environment coupling symmetrically to all two-level systems one finds a total linewidth $1/T_2 = 2\gamma_\parallel + \gamma_\perp/2$ for the case of individual decay, and $1/T_2 = 2\gamma_\parallel + N\gamma_\perp/2$ for collective decay. The linewidth associated with the coupling to a common bath is collectively enhanced, because of the constructive interference of different decay pathways.

A natural context in which such questions arise is solid state realizations of coupled matter-light systems, such as circuit-QED [4,5], where multiple superconducting qubits can be confined in a single microwave cavity [6–8], and so may potentially couple to a common reservoir. In the limit of a good cavity, where a significant part of the vacuum Rabi linewidths is due to non-cavity-mediated decay and dissipation, the distinction between coupling to collective and separate decay channels should be apparent in the dependence of linewidth on the number of qubits present. Even within a single sample, the effective qubit number can be easily varied by detuning the qubits away from resonance with each other [6]. This breaks the symmetry, providing which-path information, and thus destroys the coherence. The question of whether decay and dephasing of multiple qubits is due to separate or collective coupling to the environment may have important consequences for the ability to preserve and manipulate coherence. For example, unexpectedly long coherence in light-harvesting complexes [9] is associated with nontrivial quantum dynamics arising from coupling to common photon modes [10].

There is, however, a problem with the simple picture of collective enhancement of linewidth when applied to multiple qubits coupled to a microwave cavity. The problem is that the Lindblad terms written in the above are those that would be derived by considering system-reservoir coupling where the system Hamiltonian is that of a single qubit. The importance of using the correct system Hamiltonian in deriving loss terms has long been recognized in the context of ensuring that the correct equilibrium state is reached asymptotically [11,12]. More recently, there has been significant activity on such issues in the context of quantum dots [13–22]. In particular, it has been noted that when the system Hamiltonian is significantly changed, either by external driving [14–22], or (as in the current case) strong qubit-cavity coupling [13,19], it is crucial to recalculate the decay processes in the presence of the *full* system Hamiltonian. This is particularly important when the reservoir has a nonflat frequency dependence. While a sufficiently rapidly varying frequency dependence may prevent a Markovian density matrix being used at all, there is a significant range of parameters where a Born-Markov approach remains valid, but it is necessary to calculate the decay rates using the correct system Hamiltonian, rather than regarding the decay rates as fixed parameters. It is clear that the system of many qubits coupled to a common photon mode is such a case: When the coupling and number of systems becomes large enough, the system has been predicted [23] to undergo a phase transition to a spontaneously polarized state [24]; at this point the frequency of the collective mode should vanish, and care [25] must be taken to avoid unphysical predictions. The importance of using the eigenstates of strongly coupled Hamiltonians has also been pointed out in the context of quantizing superconducting circuits [26].

In this Letter we report measurements and calculations of the linewidth for one, two, and three qubits coupled resonantly to a microwave cavity. In contrast to the well-studied effect of superradiance we observe and explain the narrowing of linewidths in the strong dephasing regime (i.e., dephasing and cavity decay rates are comparable) as the number of qubits is increased. We find the results are compatible with a model of "collective" dephasing processes, where all qubits couple to a single bath, with a spectrum corresponding to $1/f$ noise. We also discuss how varying the cavity-qubit detuning might allow corroboration of this scenario, and a direct measurement of the dephasing bath spectrum.

Figure 1 shows part of the vacuum Rabi transmission spectrum (inset) and the extracted linewidths measured for a microwave cavity with one, two, and three qubits tuned into resonance with the cavity mode. The superconducting microchip sample and setup is similar to the one used in Refs. [6,27]. The transmission spectrum is measured with much less than a single intracavity photon on average and clearly shows the expected $\sqrt{N}$ dependence of the vacuum Rabi splitting with increasing number of qubits (not shown). The single qubit-photon couplings of the three superconducting transmon qubits are $g/(2\pi) = (52.7, 55.4, 55.8)$ MHz. The overcoupled coplanar waveguide resonator has a first harmonic resonance frequency of $\nu_r = 7.0235$ GHz and a quality factor of $Q = 14800$ as measured when the qubits are far detuned (corresponding to a cavity decay rate $\kappa/2\pi = 0.47$ MHz). Time-resolved off-resonant T_1 and T_2 measurements of the three qubits confirm that $T_1 \gg T_2 \approx 150$ ns and that the qubit linewidths have strong dephasing components, i.e., $1/T_2 \approx \kappa$ for each qubit is fulfilled. The dressed linewidths on resonance are therefore expected to be dominated by qubit dephasing as well. While there is a considerable experimental uncertainty to the individual measurement data, see inset in Fig. 1, the extracted linewidth from 6 single qubit, 2 two qubit, and 2 three qubit Rabi peaks (as indicated with error bars), shows a very clear trend of linewidth narrowing as the number of resonant qubits is increased.

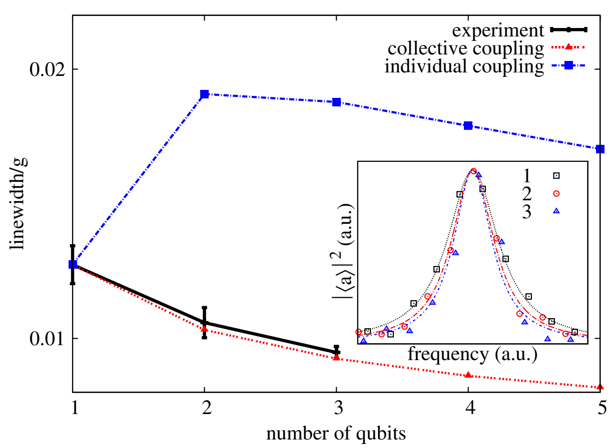

FIG. 1 (color online). Experimental linewidth (black, solid) and theoretical linewidths (collective coupling: red dotted; individual coupling: blue dash-dotted) of an N-qubit-cavity system (error bars indicate the sample standard deviation of the fitted linewidths). Theoretical linewidths are calculated for either collective coupling (lower line) or individual coupling (upper line) to a reservoir with a $1/f$ spectrum [density of states $J(\nu > 0) = 0.0105g/\nu$]. The inset shows the experimental spectrum of the coherently scattered transmission amplitude for 1, 2 and 3 qubits, rescaled and shifted so that peaks are centered at the same frequency for comparison. The lines are the corresponding Lorentzian fits. For every number N of qubits, both Rabi peaks were measured and used to calculate a single linewidth and its uncertainty. The inset shows only one set of Rabi peaks.

As discussed above, if the dephasing could be modeled by Lindblad operators derived for the uncoupled system, then narrowing of the linewidth would be unexpected. For a fixed dephasing rate, one would expect a constant or increasing linewidth for dephasing or decay dominated regimes, respectively. As discussed below, one can, however, directly explain this narrowing as a result of coupling the full system Hamiltonian to a frequency dependent bath; we assume in the following that dephasing is due to a bath with a $1/f$ spectrum [28]. Dephasing arises due to system-bath coupling described by a Hamiltonian $\sum_{i,q} \gamma_q \sigma_i^z (b_{iq}^\dagger + b_{iq})$ coupling the z components of the qubits (represented as Pauli matrices) to bath modes b, $b^\dagger$. Because of the strong qubit-cavity coupling, the system eigenstates are not eigenstates of σ_i^z so that matrix elements of σ_i^z acquire a time dependence. The frequency at which the coupled system samples the bath thus depends on the energy differences between system eigenstates. These depend on the collective Rabi frequency, which scales as $\sqrt{N}$ for N qubits. Thus, the effective decay rate decreases with increasing number of qubits. This simple argument explains the essential origin of the results seen experimentally; however, several complications occur when one actually calculates the effective linewidth using the strongly coupled qubit-resonator Hamiltonian.

As discussed also above, two different scenarios of dephasing can exist, individual coupling to reservoirs and collective coupling. When accounting for the qubit-resonator coupling, the dephasing induced linewidth depends differently on N in these two cases. In the collective coupling case, the linewidth scales as $1/\sqrt{N}$ as the simple argument given above suggests. The dominant effect is sampling the reservoir at the collective Rabi frequency. Note that the dephasing component ($\gamma_{\|}$) is never enhanced collectively, and in the good cavity case the decay component is relatively small. Small corrections to the $1/\sqrt{N}$ behavior occur due to the matrix elements between system eigenstates arising from coupling to the bath and the small but nonzero photon decay rates. In the case of individual coupling, $1/\sqrt{N}$ is still the dominant effect, but as system eigenstates are now delocalized, a competing effect arises. The number of possible decay channels increases with qubit number, as cross-qubit terms induced by the resonator coupling qubits to other baths. This gives rise to an initial increase, followed by a decrease of the linewidth. Figure 1 shows the results of the linewidth vs number of qubits in these two cases of individual and collective decay, assuming a $1/f$ spectrum for the reservoir. Further details of the calculation are given in the following.

In order to disentangle the effects of the reservoir density of states from the effects of the cross-coupling matrix elements, one may instead explore the dependence of linewidth on cavity-qubit detuning. For a single qubit coupled strongly to a cavity, the normal modes are superpositions of photon and qubit states. As the qubit-cavity detuning Δ is varied two effects occur: the nature of the modes changes, and the frequency at which the reservoir is sampled varies as the Rabi frequency $\sqrt{\Delta^2 + g^2}$. The changing nature of the modes means that one mode becomes more photonlike, and the other will have a large qubit weight. The changing Rabi frequency means that the reservoir is sampled at higher frequencies, and so for a $1/f$ spectrum, the linewidth will decrease. The calculated linewidth vs detuning for a single qubit is shown in Fig. 2. Both the changing nature of the modes (causing the linewidths of the two modes to differ) and the reduction of

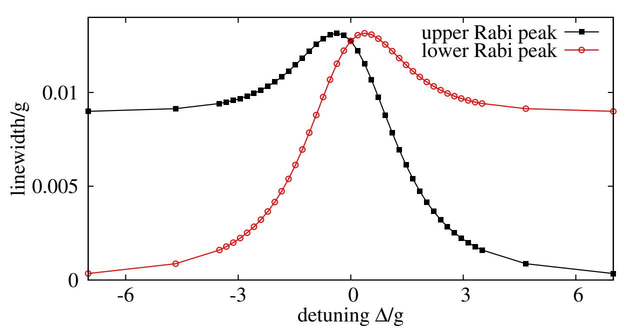

FIG. 2 (color online). Relative theoretical linewidth of a single qubit-cavity system as a function of resonator-qubit detuning. The linewidths of both Rabi peaks (squares: upper peak; circles: lower peak) decrease because the photon part has a small decay rate and the qubit part samples the $1/f$ decay bath at frequencies $\sqrt{\Delta^2 + g^2}$.

the qubit dephasing occur on a similar scale, $\Delta \sim g$, as is clear from the figure. Near $\Delta = 0$ the leading change in linewidth is associated with changing photon and qubit weights. For larger Δ, the main effect is due to the changed dephasing rate sampling at the increasing collective Rabi frequency. For multiple qubits, the detuning dependence of the linewidths still depends on the nature of the coupling (individual vs collective) as well as the density of states of the reservoir. Nonetheless, the detuning dependence of linewidth can corroborate a given model of dephasing, and provide clear information about the real spectrum of the environment which induces dephasing.

We now turn to discuss in more detail how the dephasing rate is calculated using the Born-Markov approximation [29], with the full Hamiltonian. Using Pauli operators $\sigma_i^{x,y,z}$ to represent the qubits and bosonic operators $a, a^\dagger$ for the cavity, we have

$$H_{\rm sys} = \omega a^\dagger a + \sum_{i=1}^{3} \frac{\epsilon_i}{2}\sigma_i^z + \sum_{i=1}^{3} g(\sigma_i^+ a + \text{H.c.}), \tag{1}$$

$$H_{\rm bath} = \sum_{i,q} \gamma_q \sigma_i^z (b_{iq}^\dagger + b_{iq}) + \beta_{iq} b_{iq}^\dagger b_{iq}. \tag{2}$$

b_{iq} are the bosonic modes of the environment with energy β_{iq} whose coupling strength to the system is γ_q. The important quantity for the behavior of the qubits is the combination $J_i(\nu) = \sum_q \gamma_{iq}^2 \delta(\nu - \beta_{iq})$. The total Hamiltonian $H = H_{\rm sys} + H_{\rm bath}$ describes the strongly coupled qubit-resonator system and the dephasing term in the case of coupling to individual reservoirs. For collective coupling one instead has $H_{\rm bath} = (\sum_i \sigma_i^z)\sum_q \gamma_q (b_q^\dagger + b_q) + \sum_q \beta_q b_q^\dagger b_q$. In the interaction picture the equation of motion is

$$\begin{aligned}\dot{\rho} = \sum_i \int_{-\infty}^{\infty} d\nu J(\nu) \int_{-\infty}^{t} dt' \{ &[P_i(t')\rho P_i(t) \\ &- P_i(t)P_i(t')\rho] f_\nu(t - t') + [P_i(t)\rho P_i(t') \\ &- \rho P_i(t^i) P_i(t)] f_\nu(t' - t) \}\end{aligned} \tag{3}$$

with $P_i(t) = e^{iH_{\rm sys}t}\sigma_i^z e^{-iH_{\rm sys}t}$ for individual reservoirs and $P(t) = e^{iH_{\rm sys}t}(\sum_i \sigma_i^z)e^{-iH_{\rm sys}t}$ for collective dephasing. We assume the reservoir density of states to be $J(\nu > 0) \propto 1/\nu$ and $f_\nu(\tau) = f_\nu^*(-\tau) = (n_\nu + 1)e^{-i\nu\tau} + n_\nu e^{i\nu\tau}$ depends on the Bose-Einstein occupation n_ν which we may assume to be zero, since $k_B T \ll g$.

By assuming $\rho = \rho(t)$ on the right-hand side of Eq. (3) we make the standard Markov approximation [29], but retain the crucial dependence on the collective Rabi frequency by evaluating the remaining integrals with the full system Hamiltonian [17]. For a general system Hamiltonian, Eq. (3) is not in Lindblad form and so does not necessarily preserve positivity of the density matrix equation [29]. For short times, this is not an issue [17], but for long times (as matters for steady states), positivity violation becomes a problem [21,30]. One may, however, obtain a Lindblad form by dropping any terms which are time dependent in the interaction picture [31]. This is analogous to the rotating wave approximation where non-energy-conserving transitions are perturbatively suppressed. The resulting time evolution is equivalent to Eq. (3) over short times, but avoids unphysical positivity violations as discussed extensively in [31]. The resulting Lindblad terms have the form

$$\mathcal{L}_d = \sum_{i\alpha\beta\gamma\delta} (2r^i_{\gamma\delta}\rho r^{i\dagger}_{\beta\alpha} - r^{i\dagger}_{\beta\alpha} r^i_{\gamma\delta}\rho - \rho r^{i\dagger}_{\beta\alpha} r^i_{\gamma\delta}), \quad (4)$$

with $r^i_{\alpha\beta} = \sqrt{A_{\alpha\beta}}|\alpha\rangle\langle\alpha|\sigma^z_i|\beta\rangle\langle\beta|$ where α, β are eigenstates of the system Hamiltonian and $A_{\alpha\beta} = \Gamma(\epsilon_\alpha - \epsilon_\beta)$ is a transition-energy dependent decay rate, $\Gamma(\delta) = \pi[J(\delta)(n(\delta)+1) + J(-\delta)n(-\delta)]$. The summation over states is restricted to energy-conserving transitions $E_\alpha - E_\beta = E_\delta - E_\gamma$. For collective decay, the sum over i disappears, and $r_{\alpha\beta}$ is defined in terms of matrix elements of the operator $\sum_i \sigma^z_i$ instead.

This dephasing term is of Lindblad form as required to preserve positivity of the density matrix. However, it is not simply a collection of qubit dephasing terms added *ad hoc* to the density matrix equation of motion. Instead, we naturally obtain dephasing in terms of system eigenstates, sampling the reservoir density of states at the system's natural transition frequencies. To calculate the coherent photon scattering spectrum we numerically solve the steady state of this density matrix equation in the presence of a weak drive. (Since the drive is weak, it does not itself modify the decay rates.) The spectrum is then found by calculating $|\langle a\rangle|$ vs detuning, and the linewidth plotted in Figs. 1 and 2 is extracted by fitting this spectrum to a Lorentzian.

In conclusion, we have shown how strong matter-light coupling and a nontrivial reservoir spectrum can produce a nontrivial *suppression* of linewidth, which can, nonetheless, be explained within a Markovian approximation. Our calculations explain why linewidths in an N-qubit-cavity system can decrease with the number of qubits. We further make testable predictions for an off-resonant qubit-cavity system and offer a way to probe the reservoir density of states.

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