

Reply to comment on 'Perfect imaging without negative refraction'

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Reply to comment on 'Perfect imaging without negative refraction'

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Abstract. I reply to the comment on my paper made by Blaikie 2010 *New J. Phys.* **12** 058001.

Maxwell's fish eye [1] can focus light waves with, in principle, unlimited resolution [2, 3]. Blaikie [4] has performed computer simulations of stationary wave propagation in the two-dimensional (2D) version of Maxwell's fish eye. He numerically reproduces the exact result of [2] where waves are perfectly focused, but he also shows a second simulation where the waves at the image do not converge to a single point. In the first simulation and papers [2, 3] a drain is assumed at the image point, whereas in the second simulation no drain is used. As I argue below, the second solution is in conflict with causality and hence unphysical.

Before explaining the mathematical argument, let me briefly repeat the general reason [2] for the need of a drain in the theory of stationary waves in perfect imaging. Maxwell's fish eye implements the geometry of a finite space, in 2D the surface of a sphere [2] and in its three-dimensional (3D) version the 3D surface of a hypersphere [3]. Imagine light continuously being emitted from a point source in a finite space without absorption. The radiated energy would continuously accumulate; no stationary state is possible without a drain. The radiated wave may be reabsorbed by the source, but in perfect imaging the wave *can* be absorbed at the image where the wave converges into a single point. Maxwell's fish eye facilitates such perfect imaging according to [2]. In practice, when one wishes to capture images in a photoresist or detector, absorption at the image point is the very point of imaging.

Now, here is the mathematical argument. The stationary waves emitted by a point source are known as Green functions, denoted here as $\tilde{G}(\omega)$, where ω is the frequency. There are two principal types of Green functions, retarded and advanced; superpositions of the two represent the most general case. The retarded Green functions are causal, i.e. they vanish before the point source was turned on, whereas the advanced Green functions vanish after the source is

turned on. Clearly, the physically meaningful Green function is the retarded one. We have in the time domain

$$G(t) = \int_{-\infty}^{+\infty} \tilde{G}(\omega) e^{-i\omega t} d\omega. \quad (1)$$

The Green function is retarded if

$$G(t) = 0 \quad \text{for} \quad t < 0. \quad (2)$$

From this requirement it follows that $\tilde{G}(\omega)$ is analytic on the upper half plane of complex frequencies ω and decaying there. Conversely, if $\tilde{G}(\omega)$ is analytic and decaying on the upper half plane, condition (2) is fulfilled: the Green function is retarded and hence causal.

In [2], from a superposition of the two solutions of the wave equation for the Green function in Maxwell's fish-eye mirror, one was selected that decays on the upper half ω plane. Hence this one must be the causal Green function. This Green function has the following asymptotics near the source and image points [2]:

$$\tilde{G} \sim \frac{\ln |z - z_0|}{2\pi}, \quad \tilde{G} \sim e^{i\pi\nu} \frac{\ln |z + z_0|}{2\pi}, \quad (3)$$

where ν depends on frequency and $\pi\nu$ describes the phase delay at the image. The points in the plane of the device are denoted by complex numbers z . The solution has the correct logarithmic asymptotics of a point source in 2D, but also converges at the image point like the field of a point source; the image is a perfect mathematical point: imaging is perfect in Maxwell's fish eye. Blaikie's first simulation agrees with this exact solution, as it should.

It is easy to construct a superposition of the exact retarded and advanced Green functions that describes Blaikie's second simulation. Simply form the expression

$$\tilde{G}' = \frac{\tilde{G}(\omega) - e^{i\pi\nu(\omega) - i\pi\nu(-\omega)} \tilde{G}(-\omega)}{1 - e^{i\pi\nu(\omega) - i\pi\nu(-\omega)}}. \quad (4)$$

As the stationary wave equation depends on ω^2 , the negative-frequency Green function $\tilde{G}(-\omega)$ must be a solution as well, and so is the superposition (4). Near the source, the Green function \tilde{G}' has exactly the same asymptotics (3) as the retarded Green function \tilde{G} , but near the image the phase factors of the two terms cancel: no perfect image is formed. This is what Blaikie observed. His simulation is of course correct, but the stationary wave he simulated is not causal, because it does not decay on the upper half complex plane in frequency ω . Otherwise $\tilde{G}(\omega)$ would also decay on the lower half plane and $G(t)$ would vanish for non-vanishing $\tilde{G}(\omega)$, which is impossible.

The figures in Blaikie's comment create the impression that the two solutions are very similar, apart from the spike at the perfect image. But note that Blaikie's acausal solution (4) is real, because the source is real, whereas the causal solution [2] with asymptotics (3) is complex, unless ν is an integer, because of the phase delay (3) at the image point. Figure 1 shows the full picture of wave propagation along the axis between source and image, including the imaginary parts of the stationary waves (red curves). The causal Green function corresponds to a running wave with complex wave function, whereas the acausal Green function is a standing wave with real wave function, where energy is both supplied and removed by the source. Therefore, to settle the debate about the correct solution for imaging, one could build a microwave fish-eye mirror, let it perform imaging and measure both the real and imaginary parts of the field

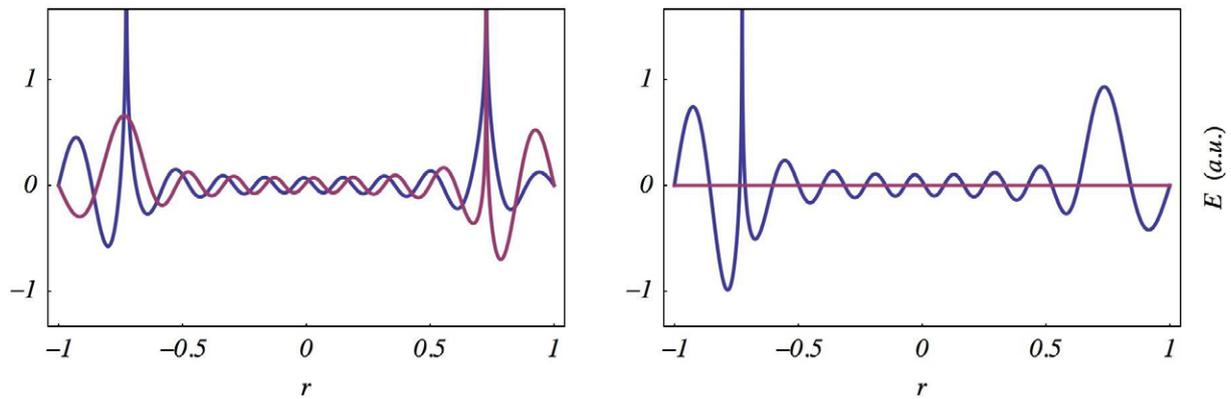


Figure 1. Imaging in fish-eye mirror with (left) and without (right) drain at the image point. The blue curves represent the real part of the electric field (the in-phase component) and the red curves the imaginary part (the out-of-phase component). The figure clearly shows that the wave without drain (right) is real, whereas the wave with drain (left) is complex. This complex wave function (left) describes the Fourier transform of a causal wave, whereas the real wave function (right) is in conflict with causality.

that correspond to the in-phase and out-of-phase components of the microwave radiation. Such an experiment should be able to discriminate between the two mathematically allowed solutions.

References

- [1] Maxwell J C 1854 *Camb. Dublin Math. J.* **8** 188
- [2] Leonhardt U 2009 *New J. Phys.* **11** 093040
- [3] Leonhardt U and Philbin T G 2010 *Phys. Rev. A* **81** 011804
- [4] Blaikie R J 2010 *New J. Phys.* **12** 058001