smaller than for the case of $l = 500$ nm. This is a direct consequence of Eq. (14). As we increase l, the off-diagonal terms in matrix $\mathbf{R}^{\mathbf{u}}_{12}(l)$ become less important due to the behaviour of the sinc function as l increases. In the limit $l \to \infty$ only the diagonal terms remain. This means that the eigenmodes (eigenvectors of $\mathbf{R}^{\mathbf{u}}_{12}(l)$) in this case correspond to single plane waves as defined in our initial set. The eigenmode (plane wave) with maximum eigenvalue optimises our problem for an infinitely large ROI. The solution found by the FOEi method for $l = 100$ mm

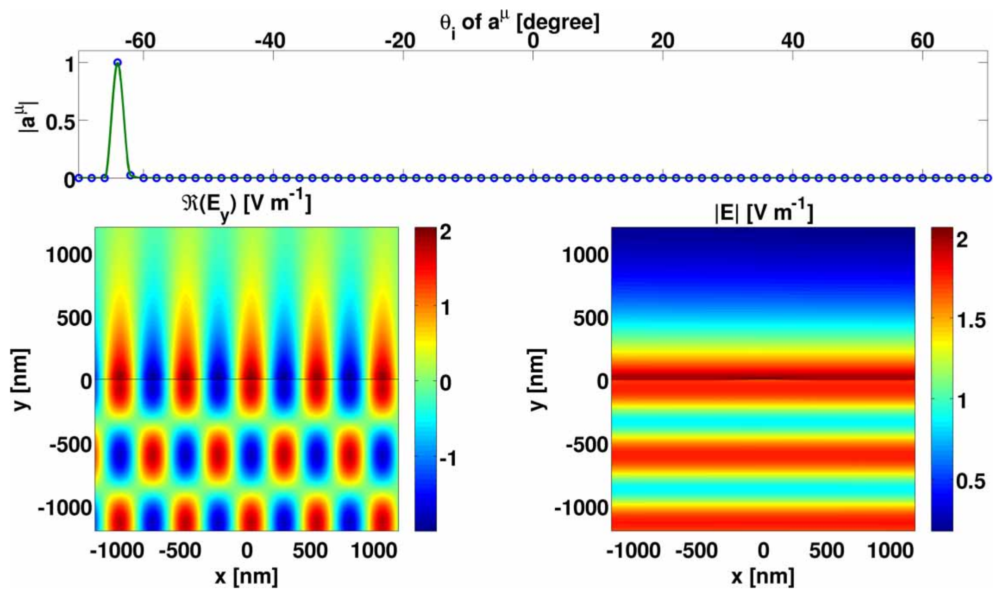

Fig. 6. Field optimising the force difference ΔF for $l \to 100$ mm. As the phase of a^{μ} for zero amplitude $|a^{\mu}|$ is not well defined, it is not displayed in the graph. The optimum angle plane wave is 64°, which is close to critical angle for given interface.

(Fig. 6) is the plane wave near the critical angle. As the phase of a^{μ} for zero amplitude $|a^{\mu}|$ is not well defined, it is not displayed in the graph. The result validates that the FOEi method is working correctly, as the near critical angle plane wave provides the highest intensity and force difference at the interface for infinite system.

The force difference is significantly increased in the cases of small ROI sizes compared with plane wave illuminated system that optimises the sorting for infinite sorting space (Fig. 7(a)). The bulk of this improvement is due to increased intensity of light in ROI, but the periodic pattern of forces indicates a more complex response of the system. This periodic pattern is not related to the discretization of $\mathbf{k}$-space, where we expect periodicity to appear around $14\,\mu$m.

Naturally, we do not wish the sorting ROI to become too small as the experimental realisation would become increasingly complicated. It is interesting to look at the dependence of ΔF on the size of ROI. To show the improvement compared to infinite system, we normalise ΔF by ΔF_{pw}, where ΔF_{pw} is the force difference for optimised infinite system (pw stands for plane wave). The result (Fig. 7(b)) indicates that the gain is significant for a wide range of experimentally interesting ROI sizes. The dip around $l = 14\,\mu$m is present due to discretization of $\mathbf{k}$-space described above. The increase in ratio $\Delta F/\Delta F_{pw}$ for $l > 14\,\mu$m is then equivalent to the formation of second beam focus in ROI due to onset of periodicity.

Discussion of results: The FOEi method is capable of finding the optimal beam shape for illumination such that the force difference is maximised over the whole sorting region. The computationally intensive calculation of matrices $\mathbf{M}_1$ and $\mathbf{M}_2$ is compensated by the fact that

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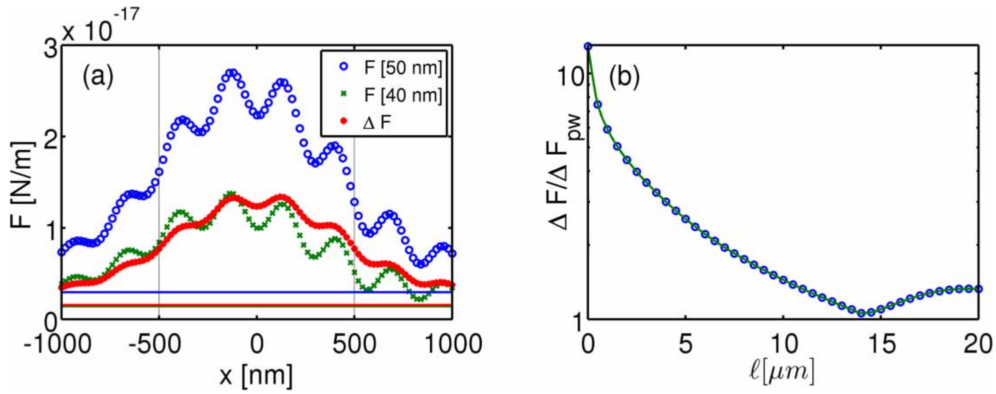

Fig. 7. (a) Blue and green points show forces acting on individual particles in optimised field for $l = 500\,\text{nm}$. Red points show the force difference. The coloured lines show the same but for plane wave illumination optimising the infinite system. (b) The ratio $\Delta F/\Delta F_{pw}$ as a function of ROI size. The dip around $l = 14\,\mu\text{m}$ is due to $\mathbf{k}$-space discretization of angular spectrum representation. The gain in ΔF is significant in the experimentally interesting region.

the same matrices can be used for finding optimal illumination for any size of sorting region. We note that even though the solution does not optimise the vertical force pointing towards the substrate, we found that this is the case for all our solutions. However, the sign of this force is wavelength and particle size dependent and as such the attractive vertical force is not a general feature of the method. Further, the vertical force is not constant in the sorting region, which might introduce some modulation of force difference due to the Faxen correction. To resolve this one may minimise the vertical force and use an auxiliary beam with constant vertical force over the whole sorting region. This would restrict the diffusion of particles in vertical direction in more controlled way. It is possible to modify the method to simultaneously optimise for several parameters of the system. In our case, the full optimised solution for sorting applications of plasmon nanoparticles would involve simultaneous maximalisation of force difference in ROI, minimisation of vertical force, and minimisation of heating. Such a problem reduces to finding matrices (operators) for all parameters of interest and choosing the eigenmodes optimising for such a set of parameters. It is very interesting, for plasmonic sorting in general, to find the beam shape of the field maximising the force difference and minimising the heating. This would also clearly identify the contribution of focusing to the overall improvement of force difference. This is a focus of our ongoing research.

5. Conclusion

We successfully optimised the illumination for sorting gold nanoparticles using our two step approach. Firstly, we found the optimal wavelength maximising the nanoparticle separation and minimising the temperature increase in the system. This is an important consideration in plasmonic systems as the excessive heat increases diffusion and convective effects. Secondly, we found the optimum beam shape of the illumination field for sorting using the method of force optical eigenmodes (FOEi). The applicability of the method was numerically demonstrated for the special case of sorting gold nanoparticles of different size. We plan to extend our approach and perform simultaneous optimisation of several parameters of interest for sorting applications, e.g., minimised heating, maximised force difference along substrate and minimised vertical force. This involves finding operators for all of parameters of interest and choosing the

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eigenmodes optimising them. This will be subject of further work along with the efficient extension of the FOEi method to 3D case.

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