in the pair of modes is conserved.

C *G-S algorithm alternative for the case of the systematic transformation.*

The original G-S algorithm is frequently used in Fourier systems to generate arbitrary distributions of intensities with phase only holographic modulation and while employing fast Fourier transform methods this can be very fast and efficient. If the transformation is neither Fourier nor unitary, the G-S algorithm can still be used in a form known as Yang-Gu algorithm [11]. In this paragraph we introduce this mathematical procedure using our notation. In the initial iteration, we start with the complex superposition of output modes as shown by equation 5. In every following iteration (indexed by t), we change amplitude of the SLM modulation to be uniform:

$$ {}^{t}\bar{M}^{N}_{k,l} = e^{i\cdot\arg\left[{}^{t-1}M^{N}_{k,l}\right]}, \tag{7} $$

calculate complex constitution coefficients of required output modes ${}^{t}a_q$ by a scalar product:

$$ {}^{t}c_q = \sum_{k,l} {}^{t}\bar{M}^{N}_{k,l} \cdot M^{u_q,v_q,w_q\star}_{k,l}, \tag{8} $$

re-balance their amplitudes:

$$ {}^{t}\bar{c}_q = \sqrt{a_q} \cdot e^{i\cdot\arg\left[{}^{t}c_q\right]}, \tag{9} $$

and generate a new SLM modulation:

$$ {}^{t}M^{N}_{k,l} = \sum_{q=1}^{N} {}^{t}\bar{c}_q \cdot M^{u_q,v_q,w_q}_{k,l}. \tag{10} $$

Other algorithms (e.g. weighted G-S [22]) may bring higher efficiency and uniformity for the output fields.

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