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Competing for Attention: Is the Showiest also the Best?*

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Abstract

We introduce *attention games*. Alternatives ranked by quality (producers, politicians, sexual partners...) desire to be chosen and compete for the imperfect attention of a chooser by investing in their own salience. We prove that if alternatives can control the attention they get, then "the showiest is the best": the equilibrium ordering of salience (weakly) reproduces the quality ranking and the best alternative is the one that gets picked most often. This result also holds under more general conditions. However, if those conditions fail, then even the worst alternative can be picked most often.

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1 Introduction

Choice requires attention, and attention may be imperfect. We study the situation in which alternatives compete to be chosen and can select their own *salience*, namely their effectiveness in being noticed by an imperfectly attentive chooser. Our analysis focuses on general phenomena. The lessons we derive abstract as much as possible from the specific nature of the alternatives, be they producers, politicians, sexual partners, and so on.

When the chooser's attention is imperfect, the most popular choices are not necessarily the best. They may be alternatives that have invested in salience more than their better competitors: they are 'all show and no substance'. Indeed, as we explain later, an inferior alternative may have more incentives than a better one to invest in salience. We explore this issue using (Nash) equilibrium analysis. We show how the technology of salience and attention impacts on the nature of the observed equilibrium and the relationship between quality and salience. We study in particular in what circumstances strategic forces act as a substitute for a chooser's cognitive ability.

We use the concept of a *consideration set*, which originated in marketing science (Wright and Barbour [17]) and has recently become popular in economic models (e.g. Eliaz and Spiegler [5], [6]; Masatlioglu, Nakajima and Ozbay [11]; Manzini and Mariotti [10]) as a tool to model agents with an imperfect ability or willingness to consider all objects of choice that are physically available.[1] In these models, an agent's choices from a menu are guided by a preference relation, but preference is only maximised on the subset of alternatives in the menu that the agent actively considers, or pays attention to. There is evidence of situations in which the consideration set is strictly smaller than the menu. For example, Goeree [7] documents that purchasers of personal computers are typically not aware of all available models when making a choice. [2]

That alternatives can influence the consideration set of the chooser is natural in

[1] See also Roberts and Lattin [14] and Shocker et al. [15].

[2] The failure to seriously consider all alternatives may in fact stem from several sources (it may for example be the outcome of a search process, or of ideological prejudice - see e.g. Wilson [16]). We focus for simplicity on the attention interpretation.

disparate contexts. A minor politician can make an outrageous statement to get noticed by the media and thus enter the voters' consideration set, but he will likely incur a cost in terms of credibility. A single person in search of partners can increase expenditure on hairdressing to get noticed. He may or may not like hairdressing: both situations will be captured in our model. Brand awareness can be increased by increasing the advertising budget. This last example illustrates an important case of our model. If the aim of an advertising campaign is merely to generate *awareness* of the products (rather than to influence tastes) then it is natural to assume that awareness of one brand is largely unaffected by the advertising campaigns for, and the awareness of, other brands. The evidence in van Nierop *et al.* [12] indeed suggests that the probability of being noticed is independent across alternatives.

Assume that while salience is endogenous, the *quality* of an alternative (its position in the preference ranking of the chooser) is fixed. Competition for attention between alternatives gives rise to an interesting strategic situation, which we call an *attention game*. Attention games turn out to have a nice hierarchical structure that ensures the existence of Nash equilibria in *pure* strategies in any finite game in several subclasses (Propositions 1, 2, 3, 4).

We aim to answer the following questions about equilibria of attention games. Suppose that we ignore the effect of technological asymmetries in the production of salience. Then:

- Is there a systematic relationship between quality and salience, and if so, which alternatives choose to be more salient?

- Are alternatives of higher quality chosen more often than those of low quality?

The answer is (Proposition 5) that, in the absence of other asymmetries beside quality, if alternatives can fully control the attention they get (though obviously not the probability with which they are chosen), then the showiest is the best: it can never happen that an alternative a is strictly more salient in equilibrium than an alternative that is better than a. And therefore, because whenever two alternatives are both noticed the inferior one cannot be chosen (recall that the chooser maximises within the consideration set), alternatives of higher quality are chosen more often that those of lower quality. The showiest is also the most chosen.

This result is especially interesting in light of some recent laboratory evidence suggesting that, when strategic aspects are absent, there is a lack of correlation between alternatives' quality and the probability of being noticed (Reutskaya et al. [13], Krajbich and Rangel [8]). In our previous work [10] on individual decision making we also made no assumption of correlation between quality and visibility. Our current result indicates that when strategic factors do operate, they might induce a sharp departure from this baseline. This is particularly relevant in a context such as supermarket choice, which is the environment that Reutskaya et al. [13] replicate experimentally. This study supports the hypothesis of our model that consumers optimise within the consideration set (called there the "seen set"). However it also shows that consumers appear to search randomly with respect to product quality. High quality products are not more likely to be noticed. Because in real supermarkets producers invest heavily to increase the salience of their products, it is hard to assume that strategic forces do not operate, and thus our model suggests the possibility that the lack of correlation observed in the experiment might not continue to hold in the market. But in turn, this assertion depends how robust the the-showiest-is-the best result is, an issue which we now discuss.

In some respects the result holds under quite general conditions. For example, it is not even necessary to assume that increasing salience requires costly effort: if generating salience were a costless or even a rewarding activity, the result would go through. The conclusion can also be generalised to a larger class of games that satisfy a weak supermodularity condition on the technology of salience (Proposition 3), and for which in addition alternatives cannot damage the visibility of other alternatives by increasing own salience. These conditions can be roughly summarised by asserting that the spillovers from competitors' salience are not harmful.

However, the situation could be dramatically different when alternatives lack full control on the attention they get (*relative salience)* and there are harmful spillovers by rivals. In this case salience may confound the quality ranking, even to the point that fully perverse equilibria become possible in which the worst alternative is picked most often (Claim 2). Moreover, our framework assumes that competition for attention is *directional*, in the sense that it is like a shouting competition to get attention in a market. If you shout louder you always increase the chance that somebody will notice your stall - you can never grab more attention by shouting less. When salience is *contextual*, that is, alternatives can choose a location in some space of characteristics (e.g., colour) and they stand out according to how distinct they are from other alternatives, once again salience may confound the quality ranking, and perverse equilibria may occur (Claim 3).

After presenting attention games (section 2) we discuss issues of existence (section 3). The reader who is mainly interested in the core results can skip at no loss to section 4 where the-showiest-is-the-best results are presented. Section 5 describes contexts in which perverse equilibria occur. Section 6 discusses the literature.

2 Attention games

Alternatives in a finite set $A = \{a_1, ..., a_n\}$ wish to be chosen by an agent with imperfect attention. The agent evaluates alternatives by means of a strict preference ordering $\succ$ on A. We refer to the position of an alternative in the ranking as its *quality*, with a lower i indicating a higher quality, that is $a_i \succ a_j$ iff $i < j$.

Because of imperfect attention, the agent maximises the preference $\succ$ only on a *consideration set* $C(A) \subseteq A$ of alternatives (the set of alternatives the agent actively considers), which is formed stochastically in the manner explained below. When $C(A)$ is empty, the agent is assumed to pick a default alternative a^* (e.g. walking away from the shop, remaining without a partner, abstaining from voting).

The probability that alternative a_i belongs to $C(A)$ depends on a set of parameters $\sigma_j \in E$, $j = 1, ..., n$, one for each alternative, where E is an interval of the real line. These parameters indicate the ability of each alternative to attract attention (and possibly to dampen or increase the attention paid to the other alternatives). The strategy set for each a_i is a subset $S \subset E$. Unless otherwise specified, we need not assume any particular structure on S. We call $\sigma_i \in S$ the *salience* of a_i, and a list $\sigma = (\sigma_1, ..., \sigma_n) \in S^n$ a *salience profile*. As usual we write σ_{-i} to denote the profile σ with the i^{th} entry omitted, $(\sigma_1, ..., \sigma_{i-1}, \sigma_{i+1}, ..., \sigma_n)$, and (σ_i, σ_{-i}) to denote σ. Given two vectors $v, v' \in \mathcal{R}^n$, we write $v \geq v'$ to signify $v_i \geq v'_i$ for all $i = 1, ..., n$.

The technology of salience is described by functions $p_i : S^n \to (0,1)$, $i = 1, .., n$. Each p_i associates a salience profile with the probability of membership of $C(A)$ for a_i: that is, $p_i(\sigma)$ is the probability that a_i is noticed when the salience profile is σ. We assume that these probabilities are interior, namely there is always an (arbitrarily small) positive probability of being noticed or of not being noticed, independently of salience. The only further assumption we make on the functions p_i is the following:

Own Monotonicity: For all i, for all $\sigma \in S^n$: $\sigma_i > \sigma'_i \Rightarrow p_i(\sigma_i, \sigma_{-i}) > p_i\left(\sigma'_i, \sigma_{-i}\right)$.

Own Monotonicity stipulates that increasing one's own salience strictly increases the probability of being noticed, whatever the salience of the other alternatives. An example of this type of function, which we will consider later in sections 4 and 6, takes the 'Luce form'

$$p_i(\sigma) = \frac{\sigma_i}{\sigma_1 + ... + \sigma_n} \tag{1}$$

with the σ_is chosen on a strictly positive domain.[3] In this example, an increase in salience of the other alternatives is harmful to an alternative. While this is natural in some contexts, our main results do not assume this feature and also allow for the opposite effect. For instance, an alternative a_i which is similar to, or dominated by, another alternative a_j may make the latter more prominent, so that an increase in the salience of a_i is beneficial for the probability of a_j getting noticed. These possibilities are well known and documented in marketing science and psychology as the 'similarity effect' and 'attraction effect'.

The agent picks the preferred alternative among those that he considers. Therefore, the probability $\pi_i(\sigma)$ that alternative a_i is chosen at a salience profile σ is the probability of the compound event that it is considered and that none of the better alternatives

[3]We call this the Luce form in view of the Luce [9] stochastic choice rule, popular in econometrics in the multinomial logit version.

is considered, that is

$$\pi_i(\sigma) = p_i(\sigma) \prod_{k<i} (1 - p_k(\sigma))$$

The payoff to each alternative is the probability of being chosen minus a (possibly negative) cost associated with the salience level that has been selected. An interpretation of this payoff function is that alternatives vie for one single chooser who chooses one alternative, with the chosen alternative getting a unit payoff. Another interpretation is that alternatives care about 'market share' with a continuum of identical choosers each of whom chooses one alternative (the latter interpretation is adopted in Eliaz and Spiegler [5]).

For each $i = 1, ..., n$, let e_i be a function $e_i : S \rightarrow \mathcal{R}$ representing the cost for alternative a_i of selecting a given level of salience. The payoff to alternative i for a pure strategy profile σ is

$$z_i(\sigma) = \pi_i(\sigma) - e_i(\sigma_i)$$

We make no assumption on the functions e_i. In particular, e_i may be convex or concave or neither, and may not even be monotonic increasing. So e_i can represent both costly *effort* (for example an advertising budget) and *elation*, when increasing salience is pleasurable at least on some range (for example, hairdressing to become salient in competition for sexual partners).

An *attention game* is denoted (A, S, z), where $z = (z_1, z_2, ..., z_n)$ with the z_i defined above and satisfying Own Monotonicity.

An attention game has *absolute salience* if, for all i,

$$\sigma_i = \sigma'_i \Rightarrow p_i(\sigma_i, \sigma_{-i}) = p_i(\sigma'_i, \sigma'_{-i})$$

That is, an attention game with absolute salience is one in which an alternative can decide its own probability of being *noticed* independently of the salience choices by the other alternatives. Of course, even in this case the strategic situation is not trivial: the *payoff* of an alternative typically still depends on the salience profile, since it depends on the probability of being chosen which in turns is determined (for all alternatives except the best) by the salience profile. As noted before, the situation captured by absolute salience fits for example the case of when repeated ads in favour of an alternative merely have the function of making the agent actively aware of the alternative ('did you know that people who read book A also read book B?'; 'have you considered cycling to work'?). The hypothesis of absolute salience does not fit any context akin to a shouting competition to get attention in a market, in which how much attention you grab by shouting at a given volume depends on how loud the others shout.

When salience is not absolute it is *relative*.

We study the Nash equilibria of attention games.

3 Existence

While standard existence results apply for many reasonable specifications of the model, in this section we highlight some peculiar features of attention games, which may be useful in applications. Attention games with absolute salience are - thanks to their 'hierarchical' structure - very well behaved in terms of existence properties. A *pure* strategy equilibrium is guaranteed in these games for standard strategy sets (like finite ones) for which in general the extension to mixed strategies or additional assumptions are needed:

Proposition 1 *Let $G = (A, S, z)$ be an attention game with absolute salience. Suppose that S is finite. Then G has an equilibrium in pure strategies.*

And:

Proposition 2 *Let $G = (A, S, z)$ be an attention game with absolute salience. Suppose that S is compact and e_i and p_i are continuous functions of σ_i for all i. Then G has an equilibrium in pure strategies.*

Propositions 1 and 2 are particular cases of more general ones. The existence results below holds for a large class of attention games - including games of absolute salience - namely those satisfying the following condition

Worse Alternative Independence : For all $\sigma, \sigma' \in S^n$, for all i: $\sigma_j = \sigma'_j$ for all $j \leq i \Rightarrow p_i(\sigma) = p_i(\sigma')$.

Worse Alternative Independence says that the probability of being noticed for an alternative a_i depends only on the salience of the alternatives which are better than a_i. Note that this is a weaker requirement than absolute salience. To avoid confusion, we stress that in *any* salience game it is true that the probability of being *chosen* depends only on the choice probability (hence on the salience) of better alternatives. What is further asserted by Worse Alternative Independence is that a similar structure holds for the probability of being *noticed* of an alternative in relation to the salience of the other alternatives.

Proposition 3 *Let $G = (A, S, z)$ be an attention game such that Worse Alternative Independence holds. Suppose that S is finite. Then G has an equilibrium in pure strategies.*

Proof: Define $p_1^1 : S \rightarrow (0,1)$ by $p_1^1(\sigma_1) = p_1(\sigma_1, \sigma_{-1})$ for all σ_{-1}. The function p_1^1 is well-defined by Worse Alternative Independence. At a pure strategy equilibrium, alternative a_1 simply solves the one-alternative problem

$$\max_{\sigma_1 \in S} p_1^1(\sigma_1) - e_1(\sigma_1)$$

Given the assumption on S, a solution to this problem exists. Let σ_1^* denote such a solution. We construct an equilibrium recursively. For all $k \leq n$ and $\sigma \in S^n$ denote $\sigma^k = (\sigma_1, ..., \sigma_k) \in S^k$. For $i \leq k$ let

$$p_i^k : S^k \rightarrow (0,1)$$

be given by $p_i^k(\sigma^k) = p_i(\sigma)$ for all $\sigma \in S^n$ (Worse Alternative Independence ensures that p_i^k is well-defined). Suppose that we have defined the components $\left(\sigma_1^*, ..., \sigma_{j-1}^*\right)$ of a pure strategy equilibrium for the first $j-1$ alternatives. Then the j^{th} component σ_j^* is defined by selecting a solution to the problem

$$\max_{\sigma_j \in S} p_j^j\left(\sigma_1^*, ..., \sigma_{j-1}^*, \sigma_j\right) \prod_{k<j}\left(1 - p_k^j(\sigma_k^*)\right) - e_j(\sigma_j) \tag{2}$$

(a solution obviously exists). No alternative a_j can profitably deviate at $\sigma^* = (\sigma_1^*, ..., \sigma_n^*)$ thus constructed. In fact, if it were

$$p_j\left(\sigma_j', \sigma_{-j}^*\right) \prod_{k<j}(1 - p_k(\sigma^*)) - e_j\left(\sigma_j'\right) > p_j(\sigma^*) \prod_{k<j}(1 - p_k(\sigma^*)) - e_j\left(\sigma_j^*\right)$$

for some $\sigma_j^{'} \in S$, then by the definition of the p_i^k also

$$p_j^j\left(\sigma_1^*, ..., \sigma_{j-1}^*, \sigma_j'\right) \prod_{k<j} \left(1 - p_k^j\left(\sigma_k^*\right)\right) - e_j\left(\sigma_j'\right) > p_j^j\left(\sigma_1^*, ..., \sigma_{j-1}^*, \sigma_j^*\right) \prod_{k<j} \left(1 - p_k^j\left(\sigma_k^*\right)\right) - e_j\left(\sigma_j^*\right)$$

so that σ_j^* would not solve problem 2, a contradiction. ∎

By an analogous reasoning and standard facts about the existence of maxima of a function, we also have:

Proposition 4 *Let $G = (A, S, z)$ be an attention game such that Worse Alternative Independence holds. Suppose that S is compact and that e_i and p_i are continuous functions of σ_i for all i. Then G has an equilibrium in pure strategies.*

Compared to standard existence results, note that in the statement of Proposition 4 no convexity assumptions are made - so that Kakutani-type fixed point arguments do not apply - and that the best reply functions could be non-monotonic - so that Tarsky-type fixed point theorems do not apply either. Notice also that p_i is only required to be continuous in σ_i, not necessarily in any σ_j for $j \neq i$.

A non-existence example. The following example of a finite game that only has an equilibrium in mixed strategies shows that Worse Alternative Independence is necessary. There are two alternatives and two levels of salience, high (H) and low (L), $H > L$. Let

$$\begin{aligned} p_1(L, L) &= \frac{1}{2} = p_1(L, H) \\ p_1(H, L) &= \frac{2}{3}, p_1(H, H) = \frac{3}{5} \end{aligned}$$

and

$$\begin{aligned} p_2(L, L) &= \frac{1}{3} = p_2(H, L) \\ p_2(L, H) &= \frac{5}{12}, p_2(H, H) = \frac{2}{3} \end{aligned}$$

The effort functions are given by $e_1(L) = e_2(L) = 0$, $e_1(H) = \varepsilon$ and $e_2(H) = \eta$. Worse Alternative Independence fails since $p_1(H, L) \neq p_1(H, H)$. The matrix below, in which 1 plays rows and 2 plays columns, summarises the payoffs:

$$\begin{array}{ccc} & L & H \\ L & \frac{1}{2}, \frac{1}{2}\frac{1}{3} & \frac{1}{2}, \frac{1}{2}\frac{5}{12} - \eta \\ H & \frac{2}{3} - \varepsilon, \frac{1}{3}\frac{1}{3} & \frac{3}{5} - \varepsilon, \frac{2}{5}\frac{2}{3} - \eta \end{array}$$

It is easy to check that for $\frac{1}{10} < \varepsilon < \frac{1}{6}$ and $\frac{1}{24} < \eta < \frac{7}{45}$ the game has no pure strategy equilibrium.

4 Does salience reveal quality?

We now come to the core question of the paper. We exhibit environments in which, when alternatives are ex-ante symmetric except for the difference in quality (they have access to the same technology of salience), any equilibrium salience order (weakly) correlates with the quality order. The assumption of technological symmetry ensures that we are looking at equilibrium effects that *only* depend on the quality ranking of the alternatives and the directly resulting incentives, and not on any technological advantage that better alternatives may possess.

Our main characterisation results below require no restriction on the structure of the strategy sets S, nor on that of the (common) cost function e.

An attention game is *symmetric* when the following holds:

Symmetry:

(i) For all i, j and all $x, y \in S$:

$$p_i\left(\sigma_1, ..., \sigma_{i-1}, x, \sigma_{i+1}, ..., \sigma_{j-1}, y, \sigma_{j+1}, ..., \sigma_n\right) = p_j\left(\sigma_1, ..., \sigma_{i-1}, y, \sigma_{i+1}, ..., \sigma_{j-1}, x, \sigma_{j+1}, ..., \sigma_n\right)$$

(ii) For all i: $e_i = e$ for some $e : S \to \mathcal{R}$.

The first part of Symmetry says that the effectiveness of salience for getting noticed for a given configuration of the other alternatives' salience is the same for each alternative. More precisely, holding the salience of all alternatives fixed except for a_i and a_j, the attention attracted by a_i with a level of salience x when a_j has salience y is the same as the attention attracted by a_j with salience x when a_i has salience y. The second part of the condition simply says that achieving any level of salience has the same cost for any two alternatives.

Once again, for games with absolute salience the analysis is very neat:

Proposition 5 *Let G be a symmetric attention game with absolute salience. At any equilibrium of G the salience chosen by an alternative is never lower than that chosen by alternatives of lower quality. That is, let $a_i \succ a_j$ and let σ be a pure strategy equilibrium of G. Then, $\sigma_i \geq \sigma_j$.*

Proposition 5 is implied by a more general result, which holds for a class of games that satisfies the following conditions:

Weak Supermodularity: For all i, all $\sigma_{-i}, \sigma'_{-i} \in S^{n-1}$ with $\sigma'_{-i} \geq \sigma_{-i}$, and all $x, y \in S$ with $x > y$:

$$p_i\left(x, \sigma'_{-i}\right) - p_i\left(y, \sigma'_{-i}\right) \geq p_i\left(x, \sigma_{-i}\right) - p_i\left(y, \sigma_{-i}\right)$$

Cross Monotonicity: For all i, all $\sigma_{-i}, \sigma'_{-i} \in S^{n-1}$ with $\sigma'_{-i} \geq \sigma_{-i}$, $p_i\left(\sigma_i, \sigma'_{-i}\right) \geq p_i\left(\sigma_i, \sigma_{-i}\right)$.

Weak Supermodularity says that the effectiveness for getting noticed of an increase in an alternative's own salience increases with the salience of the other alternatives. Note that this is a condition on the supermodularity of the function p_i only: the whole payoff function need not be supermodular even when Weak Supermodularity is satisfied. We do not defend Weak Supermodularity as a compelling property in all situations;[4] but we note that if salience is absolute then Weak Supermodularity is trivially satisfied.

[4]One example of when Weak Supermodularity might apply is the following. Consider political elections where a candidate's salience is obtained by means of negative campaigning agaist competitors. It is conceivable that the effectiveness of increasing one's own recognition at the expense of little known rivals would be negligible. However, were these other politicians well known (salient), the effect of the same electoral campaign/increase in salience might be effective.

Cross Monotonicity says that an alternative cannot harm the visibility of the rivals by raising own salience: it can only increase it (for example through a "similarity effect") or leave it unchanged (while obviously changing its own visibility), as in games of absolute salience.

Proposition 6 (The showiest is the best). *Let σ be a pure strategy equilibrium of a symmetric attention game satisfying Weak Supermodularity and Cross Monotonicity. Then the salience chosen by an alternative is never lower than that chosen by alternatives of lower quality. That is, $a_i \succ a_j \Rightarrow \sigma_i \geq \sigma_j$.*

Proof: By contradiction, suppose that for some i and j we have $a_i \succ a_j$ but $\sigma_i < \sigma_j$. We use a revealed preference argument. Because σ_i is optimal for alternative a_i, it must provide a weakly higher expected payoff than σ_j, that is

$$\begin{aligned} & p_i(\sigma_i, \sigma_{-i}) \prod_{k<i} (1 - p_k(\sigma)) - e_i(\sigma_i) \geq p_i(\sigma_j, \sigma_{-i}) \prod_{k<i} (1 - p_k(\sigma)) - e_i(\sigma_j) \quad (3) \\ \Leftrightarrow \quad & \left(p_i(\sigma_i, \sigma_{-i}) - p_i(\sigma_j, \sigma_{-i})\right) \prod_{k<i} (1 - p_k(\sigma)) \geq e(\sigma_i) - e(\sigma_j) \end{aligned}$$

using Symmetry (ii) in the second line.

Since $\sigma_i < \sigma_j$, by Own Monotonicity we have $p_i(\sigma_i, \sigma_{-i}) - p_i(\sigma_j, \sigma_{-i}) < 0$. Since $1 - p_k(\sigma) > 0$ for all k by the range assumption on the p_k, it follows that $\prod_{k<i} (1 - p_k(\sigma)) > 0$. Since $i < j$, we have that $\prod_{k<i} (1 - p_k(\sigma)) > \prod_{k<j} (1 - p_k(\sigma))$. Therefore (3) implies

$$0 > \left(p_i(\sigma_i, \sigma_{-i}) - p_i(\sigma_j, \sigma_{-i})\right) \prod_{k<j} (1 - p_k(\sigma)) > e(\sigma_i) - e(\sigma_j) \quad (4)$$

To avoid cumbersome notation, denote, for $x, y \in S$,

$$(y, x, \sigma_{-ij}) = (\sigma_1, ..., \sigma_{i-1}, y, \sigma_{i+1}, ...\sigma_{j-1}, x, \sigma_{j+1}.., \sigma_n)$$

the profile in which a_i plays y, a_j plays x and all other alternatives play as in σ. By Symmetry (i) we have

$$\begin{aligned} & p_i(\sigma_j, \sigma_{-i}) - p_i(\sigma_i, \sigma_{-i}) = \quad (5) \\ & p_j(\sigma_j, \sigma_j, \sigma_{-ij}) - p_j(\sigma_j, \sigma_i, \sigma_{-ij}) \end{aligned}$$

Moreover, by Weak Supermodularity

$$
\begin{aligned}
& p_j\left(\sigma_j, \sigma_j, \sigma_{-ij}\right) - p_j\left(\sigma_j, \sigma_i, \sigma_{-ij}\right) \\
\geq\ & p_j\left(\sigma_i, \sigma_j, \sigma_{-ij}\right) - p_j\left(\sigma_i, \sigma_i, \sigma_{-ij}\right) \\
=\ & p_j\left(\sigma_j, \sigma_{-j}\right) - p_j\left(\sigma_i, \sigma_{-j}\right)
\end{aligned}
\tag{6}
$$

Therefore combining (5) and (6) we have

$$
\begin{aligned}
p_i\left(\sigma_j, \sigma_{-i}\right) - p_i\left(\sigma_i, \sigma_{-i}\right) &\geq p_j\left(\sigma_j, \sigma_{-j}\right) - p_j\left(\sigma_i, \sigma_{-j}\right) \Leftrightarrow \\
p_j\left(\sigma_i, \sigma_{-j}\right) - p_j\left(\sigma_j, \sigma_{-j}\right) &\geq p_i\left(\sigma_i, \sigma_{-i}\right) - p_i\left(\sigma_j, \sigma_{-i}\right) \qquad (7)
\end{aligned}
$$

Using (7) in (4) yields (recall again that $p_j\left(\sigma_i, \sigma_{-j}\right) - p_j\left(\sigma_j, \sigma_{-j}\right) < 0$ by Own Monotonicity)

$$
\begin{aligned}
0 &> \left(p_j\left(\sigma_i, \sigma_{-j}\right) - p_j\left(\sigma_j, \sigma_{-j}\right)\right) \prod_{k<j} \left(1 - p_k\left(\sigma\right)\right) > e\left(\sigma_i\right) - e\left(\sigma_j\right) \\
&\Rightarrow p_j\left(\sigma_i, \sigma_{-j}\right) \prod_{k<j} \left(1 - p_k\left(\sigma\right)\right) - e\left(\sigma_i\right) > p_j\left(\sigma_j, \sigma_{-j}\right) \prod_{k<j} \left(1 - p_k\left(\sigma\right)\right) - e\left(\sigma_j\right)
\end{aligned}
$$

By Cross Monotonicity we have

$$
p_k\left(\sigma\right) \geq p_k\left(\sigma_1, \ldots, \sigma_{j-1}, \sigma_i, \sigma_{j+1}, \ldots, \sigma_n\right)
$$

and thus we conclude that

$$
p_j\left(\sigma_i, \sigma_{-j}\right) \prod_{k<j} \left(1 - p_k\left(\sigma_1, \ldots, \sigma_{j-1}, \sigma_i, \sigma_{j+1}, \ldots, \sigma_n\right)\right) - e\left(\sigma_i\right) > p_j\left(\sigma_j, \sigma_{-j}\right) \prod_{k<j} \left(1 - p_k\left(\sigma\right)\right) - e\left(\sigma_j\right)
$$

But this means that alternative a_j would improve by deviating from σ_j to σ_i at profile σ, a contradiction. ■

Why do the conditions on p_i turn out to be important for the result? Let's think of the incentives to invest in salience. On the one hand, this investment attracts attention to oneself by Own Monotonicity. But - depending on the p_is - there may also be a second effect: that of *detracting* attention from other alternatives. This second part of the incentive is *always stronger* for lower quality alternatives. Good alternatives "do not care" whether worse alternatives are noticed or not - their payoff only depends on the probability that even better alternatives are noticed. In the extreme case, the best alternative only cares about its own probability of getting noticed, so that the only incentive it has is of the first type. Thus if the p_is are decreasing in the salience of other alternatives, there is a tendency for the investment in salience to be more profitable the *lower* the quality of an alternative. Cross Monotonicity removes this tendency. Assume now that a worse alternative w finds it profitable to raise salience from l to $h > l$ while a better alternative b chooses l. Can this be an equilibrium? Suppose for simplicity that b is the only other alternative beside w. Note that b's gain in visibility when moving from l to h would be scaled up compared to that of w, by a factor strictly greater than one, given by the (reciprocal of the) probability that b is not noticed: only if b is not noticed, in fact, will w be picked if noticed. So the only reason why b might not want to follow w in raising salience to h is that raising own salience becomes less effective for becoming more noticeable when the rivals raise *their* salience, which w has done. Weak supermodularity eliminates precisely this effect. So if it were profitable for the worse alternative w to raise salience to h, it would be a fortiori profitable for the better alternative b, and therefore the initial configuration could not be an equilibrium.

Note that the the-showiest-is-the-best result is unrelated to any kind of signalling argument. There is no hidden quality to signal to the chooser via costly investment. The reason why lower quality alternatives never produce more salience in equilibrium does not derive either from lower levels of resources or lower unit costs of salience production, both types of asymmetry having been ruled out: every alternative can choose from exactly the same set at exactly the same cost or benefit. The result is purely a function of the cognitive process postulated for the chooser.

At least limited forms of the the-showiest-is-the-best property also hold in other "natural" attention games despite them failing both Weak Supermodularity and Cross Monotonicity. We illustrate the point with an example, in which the probability of being noticed takes the Luce form (1) and alternatives can choose any positive level of salience. Even in the two player case, whether or not the supermodularity condition $p_i\left(x,\sigma_j'\right) - p_i\left(y,\sigma_j'\right) \geq p_i\left(x,\sigma_j\right) - p_i\left(y,\sigma_j\right)$ holds depends on the sign of $\sigma_j'\sigma_j - xy$. Moreover, for any alternative own salience is detrimental for the rivals' chances of being noticed. Nevertheless, we show that the the-showiest-is-the-best result fully applies in the two-alternative case, or to the two best alternatives in any game with $n > 2$.

Claim 1 *Let $G = (A, S, z)$ be a symmetric attention game with $S = \mathcal{R}_{++}$; $p_i(\sigma) = \frac{\sigma_i}{\sigma_1 + ... + \sigma_n}$ for all i, all σ; and with e twice differentiable and weakly convex. Then at any equilibrium σ of G the salience chosen by the best alternative is never lower than that chosen by the second best alternative; that is, $\sigma_1 \geq \sigma_2$.*

Proof: Denote $k = \sigma_3 + ... + \sigma_n \geq 0$. At any interior equilibrium the FOCs for alternatives 1 and 2 must be satisfied

$$\begin{aligned} \frac{\partial \left(\frac{\sigma_1}{\sigma_1 + \sigma_2 + k} \right)}{\partial \sigma_1} &= e'(\sigma_1) \\ \frac{\partial \left(\frac{\sigma_2}{\sigma_1 + \sigma_2 + k} \frac{\sigma_2 + k}{\sigma_1 + \sigma_2 + k} \right)}{\partial \sigma_2} &= e'(\sigma_2) \end{aligned}$$

where e' denotes the first derivative of e. Dividing side by side the two equations and computing we get

$$(k + \sigma_2) \frac{k + \sigma_1 + \sigma_2}{k\sigma_1 + k\sigma_2 + 2\sigma_1\sigma_2 + k^2} = \frac{e'(\sigma_1)}{e'(\sigma_2)} \tag{8}$$

Suppose by contradiction that $\sigma_2 > \sigma_1$. Then, by the weak convexity of e, $\frac{e'(\sigma_1)}{e'(\sigma_2)} \leq 1$. It follows from (8) that

$$\begin{aligned} (k + \sigma_2) \frac{k + \sigma_1 + \sigma_2}{k\sigma_1 + k\sigma_2 + 2\sigma_1\sigma_2 + k^2} &\leq 1 \Leftrightarrow \\ \sigma_1 &\geq \sigma_2 + k \end{aligned}$$

a contradiction in view of $k \geq 0$. ■

The next example shows that Proposition 6 cannot be strengthened to obtain a *strict* correlation between salience and quality.

Example 1 ***(No strict correlation).*** *Assume absolute salience (hence Weak Supermodularity and Cross Monotonicity) and suppose again that there are two alternatives that choose between a high (H) and low (L) level of salience. The probability of being noticed is $\varepsilon < \frac{1}{2}$ for low salience, which has zero cost, while it is $1 - \varepsilon$ for high salience. Low salience is costless while* *high salience costs $\eta > 0$. The matrix below, in which alternative 1 plays rows and alternative 2 plays columns, illustrates.*

$$\begin{array}{ccc} & L & H \\ L & \varepsilon, (1-\varepsilon)\,\varepsilon & \varepsilon, (1-\varepsilon)^2 - \eta \\ H & 1-\varepsilon-\eta, \varepsilon^2 & 1-\varepsilon-\eta, \varepsilon\,(1-\varepsilon) - \eta \end{array}$$

In this attention game there may be two types of equilibria in which salience is only weakly correlated with quality: (H, H) is an equilibrium if $\eta \leq \varepsilon\,(1-2\varepsilon)$, while (L, L) is an equilibrium if $\eta \geq (1-2\varepsilon)\,(1-\varepsilon)$. In addition to these, (H, L) is also an equilibrium if $\eta \in [\varepsilon\,(1-2\varepsilon)\,, 1-2\varepsilon]$, while (L, H) is an equilibrium if $\eta \leq (1-2\varepsilon)\,(1-\varepsilon)$.

The the-showiest-is-the-best property ensures that if there is an alternative that is *uniquely* maximally salient in equilibrium, then that alternative must also be the best one; but it does not exclude that alternatives of differing qualities tie for salience. However, even if we have to settle for a weak correlation, an important and immediate implication of Proposition 6 is:

Corollary 1 (The best gets picked most often) *Let σ be a pure strategy equilibrium of a symmetric attention game satisfying Weak Supermodularity and Cross Monotonicity. Then alternatives of higher quality are chosen with strictly greater probability. That is, $a_i \succ a_j \Rightarrow \pi_i\,(\sigma) > \pi_j\,(\sigma)$.*

Corollary 1 can be read from a revealed preference perspective. Suppose that neither the salience of alternatives nor the agent's preferences are observable to an outside party, but that this party knows the structure of the game. Then, Corollary 1 implies that the observer could still perfectly infer - under the assumption of technological symmetry - the preference ranking of the agent from choice data, simply by checking the choice frequencies.

A second perspective from which Corollary 1 can be read is as showing circumstances in which competitive forces act as a countervailing force to the cognitive limitations of the chooser. Competition pushes the best alternatives to invest sufficiently in salience to overcome the distorsive effects of imperfect attention on the relative popularity of the alternatives.

5 When the ugly duckling is the most popular: Relative and contextual salience

5.1 Relative salience

When salience is relative, and Weak Supermodularity or Cross Monotonicity fail, the neat equilibrium ordering obtained in Proposition 6 may break down. In fact extremely perverse equilibria become possible, in which the worst alternative is selected with the highest probability.

Claim 2 *There are attention games of relative salience with equilibria in which the worst alternative is chosen with the highest probability.*

This claim is shown with a two-alternative example in which $S = \{L, H\}$ and the probability of being noticed has the Luce form. So let

$$p_i(\sigma) = \frac{\sigma_i}{\sigma_i + \sigma_j}$$

for $i \neq j$. We impose the following restrictions on the admissible values of H and L:

$$\begin{aligned} H &> \frac{9}{32} > L > 0 \\ H + L &> \frac{1}{2} \\ H &\in \left[\frac{3}{8} - L - \frac{1}{8}\sqrt{9 - 32L}, \frac{3}{8} - L + \frac{1}{8}\sqrt{9 - 32L}\right] \end{aligned}$$

Let $e_i(x) = x$ for $i = 1, 2$ and $x \in \{H, L\}$, so we drop the subscript. Then the profile $\sigma = (L, H)$ in which the showiest is the worst is a strict Nash equilibrium for all admissible values of L and H. Not all admissible profiles of this type have the property that alternative 2 is picked with higher probability. Such profiles do exist, however (the calculations are in the Appendix).

5.2 Contextual salience

There is a different way in which the the-showiest-is-the-best property may collapse. So far we have assumed (through Own Monotonicity) that salience, whether absolute or relative, is a 'directional' attribute for which 'the more is always the better': the more commercials you produce, the louder you shout, the glitzier your clothes, the more likely - ceteris paribus - you are to get noticed. In some scenarios, however, alternatives can only control variables whose values are not *intrinsically* positive or negative for the aim of attracting attention; whether they are depends, for each alternative, on what the other alternatives do. If everybody else dresses in green you will be salient by dressing in yellow, and viceversa. If all other candidates converge on a given political message, you will stand out by deviating from that message. We call this scenario one of *contextual* salience.

We do not attempt to provide a general model of contextual salience. Rather, we study a simple stylised class of models that generate the perverse result. Suppose that $\sigma_i \in [0,1]$ is now a 'position' selected by alternative a_i in the unit interval. Whether or not the probability that a_i is noticed is increasing in σ_i now depends on the entire profile σ: that is, Own Monotonicity may fail. In particular, we assume that an alternative's probability of being noticed is conferred by its distance from the 'average alternative' (excluding itself):

$$p_i(\sigma) = \alpha_i \left(v_i - \frac{\sum_{j \neq i} v_j}{(n-1)} \right)^2 \in [0,1]$$

where $\alpha_i \in (0,1)$ for all i can be seen for instance as a psychological parameter indicating how naturally inclined the chooser is to notice any given alternative. Finally assume for simplicity a null effort function so that we can take

$$z_i(\sigma) = p_i(\sigma) \prod_{k<i} (1 - p_k(\sigma))$$

Claim 3 *There exist (for some values n and $\alpha_1, ..., \alpha_n$) pure strategy Nash equilibria of the game above in which the worst alternative is chosen with the highest probability.*

The proof consists of fairly tedious calculations and is relegated to an Appendix.

The case of contextual salience we have studied has some superficial similarity with location games à la Hotelling. In that case too alternatives can gain by moving away from the nearest neighbour. However, the payoff structure is in fact very different. This results, unlike a location game, in an existence result with three players, as detailed in the proof of Claim 3.

6 Related literature

Eliaz and Spiegler ([5], Eliaz and Spiegler [6]) study in deep detail strategic aspects of the competition between firms to make their products enter the consideration sets of consumers. Firms deploy marketing strategies devoted to attention grabbing. The choice model at the heart of their work is similar to the one axiomatised in an abstract context by Masatlioglu, Nakajima and Ozbay [11] using revealed preference techniques. However, In these models choice is deterministic. Stochastic choice in a consideration set model has been axiomatised in Manzini and Mariotti [10]: from this perspective, the present work offers a mechanism to endogenise, as equilibrium values, the salience parameters of our previous work.

Differential attention to alternatives is not necessarily tied to a consideration set interpretation. One interesting example is the recent work by Echenique, Saito and Tserenjigmid [4], who study a Luce-type (Luce [9]) model of stochastic choice in which the *perception* of alternatives by the decision maker is hierarchical. An alternative can only be chosen if the alternatives that precede it in a perception priority ranking is (randomly) not chosen. Given that in our setting the set of alternatives is fixed, it turns out that our model is also consistent with the Echenique *et al*. interpretation. More precisely, in a slightly simplified version of their choice model, the primitives are a utility function $u : A \to \mathcal{R}$ and a perception ordering $\succ_p$.[5] The probability that $a \in A$ is chosen is

$$\frac{u(a)}{\sum_{b\in A} u(b) + u(a^*)} \left(\prod_{c\in A: c\succ_p a} \left(1 - \frac{u(c)}{\sum_{b\in A} u(b) + u(a^*)}\right)\right)$$

Defining, for all $a \in A$

$$\mu(a, A) = \frac{u(a)}{\sum_{b\in A} u(b) + u(a^*)}$$

[5]In the general version the perception ordering is a weak order, rather than a strict order, thus allowing for “perception ties".

this choice model falls within the class of choice models we are considering. To see this, set $\succ = \succ_p$and set $\sigma_i = u(a_i)$, so that $p_i(\sigma) = \mu(a_i, A)$. In this interpretation, we assume that alternatives cannot change the perception ranking, just as in the consideration set interpretation we assumed that they cannot change the quality ranking. However, they can compete by striving to raise their quality (utility) and thus increase the probability of being selected in the event that no alternative that is higher ranked in the perception ordering is itself selected. Observe that $\mu(a_i, A)$ is increasing in the utility of a_i and decreasing in the utility of the other alternatives, so that Own Monotonicity for an attention game is satisfied. The model is not one of absolute salience, and not even one satisfying Worse Alternative Independence, since $p_i(\sigma)$ depends on the utility of all other alternatives including inferior ones. Moreover it fails Weak Supermodularity. However, as shown by Claim 1 in the two alternative version the the-showiest-is-the-best feature holds under mild assumptions on the cost function. Finally, note that as Echenique *et al.* observe, their model has in general (i.e. with a variable A) very different observable implications from that of Manzini and Mariotti [10].

7 Concluding remarks

Wrapping up, in this paper we have highlighted several features of mechanisms for attracting attention. These features can have profoundly different effects on the equilibrium outcomes of a competition to be chosen by a chooser with imperfect attention. We have endeavoured to study these effects abstracting as much as possible from the specific context in which competition takes place. In one mechanism, attracting more attention is a directional phenomenon, a matter of increasing a quantity (advertising, shouting, expenditure on hairdressing): the more, the better. Such increases may or may not have spillovers on the attention grabbed by rivals. And such spillovers when they exist may or may not be benign, that is, increasing own salience may (i) increase or decrease the attention devoted to others, and (ii) it may increase or decrease the gains others derive from raising their own salience. Under quite general conditions it is only the situation with non-benign spillovers (in either of the two senses (i) and (ii)) in which it may happen that the worst alternatives grab the most attention and are the most chosen. Otherwise, equilibrium forces push better alternatives to invest weakly more in salience; observed frequencies of choice fully reveal any hidden quality ordering to an external observer; and strategic competition replaces an agent's cognitive ability. Yet another kind of mechanism operates when salience is non-directional, that is when grabbing attention is a matter of positioning oneself appropriately in a space of characteristics vis a vis the competitors. In this case again salience may confound quality; the choices of cognitively limited choosers may be completely unrevealing of preferences; and competitive forces are no substitute for individual cognitive power.

In the present work we have considered quality a fixed characteristic of the alternatives. In future research, we plan to study the situation in which both salience and quality are a variable of choice.

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8 Appendix: Calculation for Claim 2

For (L, H) to be an equilibrium we need

$$\begin{aligned} z_1(L,H) &= \frac{L}{L+H} - e(L) > \frac{H}{2H} - e(H) = z_1(H,H) \\ z_2(L,H) &= \frac{H}{L+H}\left(1 - \frac{L}{L+H}\right) - e(H) > \frac{L}{2L}\left(1 - \frac{L}{2L}\right) - e(L) = z_2(L,L) \end{aligned}$$

that is

$$\begin{aligned} e(H) - e(L) &> \frac{1}{2} - \frac{L}{L+H} \\ e(H) - e(L) &< \left(\frac{H}{L+H}\right)^2 - \frac{1}{4} \end{aligned}$$

With our choice of cost function the equilibrium condition for alternative 1 becomes

$$\begin{aligned} e(H) - e(L) &= H - L > \frac{1}{2} - \frac{L}{L+H} \Leftrightarrow \\ (H-L)(2H+2L-1) &> 0 \end{aligned}$$

holds given our restrictions on H and L. Turning to the equilibrium condition for 2:

$$\begin{aligned} e(H) - e(L) &= H - L < \left(\frac{H}{L+H}\right)^2 - \frac{1}{4} \Leftrightarrow \\ \frac{1}{4}(H-L)\frac{\left(8HL + 4H^2 + 4L^2 - 3H - L\right)}{(H+L)^2} &< 0 \end{aligned}$$

The inequality is verified since the quadratic at the numerator has roots $H_{1,2} = \frac{3}{8} - L \pm \frac{1}{8}\sqrt{9 - 32L}$ in view of our restrictions on L and H.

So far we have shown that the worse alternative has strictly higher salience than the better alternative in all admissible profiles (L, H). But not all admissible profiles have the property that alternative 2 is picked with higher probability than alternative 1. Such profiles do exist, however. For example, let $H = \frac{2}{5}$and $L = \frac{4}{25}$ (it is easy to check that this profile is admissible), so that

$$\pi_2(L,H) = \frac{\frac{2}{5}}{\frac{2}{5}+\frac{4}{25}}\left(1-\frac{\frac{4}{25}}{\frac{2}{5}+\frac{4}{25}}\right) = \frac{25}{49} > \frac{2}{7} = \frac{\frac{4}{25}}{\frac{2}{5}+\frac{4}{25}} = \pi_1(L,H)$$

■

9 Appendix: Proof of Claim 3

We consider the case of three alternatives and show that the position profile $\sigma^* = (0,0,1)$ is a Nash Equilibrium with the desired property. For a generic profile $\sigma = (\sigma_1, \sigma_2, \sigma_3)$, the choice probabilities are given by

$$z_1(\sigma) = \left(\sigma_1 - \tfrac{\sigma_2+\sigma_3}{2}\right)^2 \alpha_1$$
$$z_2(\sigma) = \left(\sigma_2 - \tfrac{\sigma_1+\sigma_3}{2}\right)^2 \alpha_2 \left(1 - \left(\sigma_1 - \tfrac{\sigma_2+\sigma_3}{2}\right)^2 \alpha_1\right)$$
$$z_3(\sigma) = \left(\sigma_3 - \tfrac{\sigma_1+\sigma_2}{2}\right)^2 \alpha_3 \left(1 - \left(\sigma_1 - \tfrac{\sigma_2+\sigma_3}{2}\right)^2 \alpha_1\right)\left(1 - \left(\sigma_2 - \tfrac{\sigma_1+\sigma_3}{2}\right)^2 \alpha_2\right)$$

so that

$$z_1(0,0,1) = \tfrac{1}{4}\alpha_1$$
$$z_2(0,0,1) = \tfrac{1}{4}\alpha_2\left(1-\tfrac{1}{4}\alpha_1\right)$$
$$z_3(0,0,1) = \alpha_3\left(1-\tfrac{1}{4}\alpha_1\right)\left(1-\tfrac{1}{4}\alpha_2\right)$$

and thus

$$z_3(\sigma^*) > z_2(\sigma^*) > z_1(\sigma^*) \tag{9}$$

for suitable values of α_i, e.g. provided that $\alpha_3 > \min\left\{\frac{\alpha_2}{4-\alpha_2}, \frac{4\alpha_1}{(4-\alpha_1)(4-\alpha_2)}\right\} \in (0,1)$. To check that the above is an equilibrium, observe that first derivatives of the payoff functions with respect to own salience are:

$$\frac{\partial(z_1(\sigma))}{\partial\sigma_1} = (2\sigma_1 - (\sigma_2+\sigma_3))\,\alpha_1$$

$$\frac{\partial(z_2(\sigma))}{\partial\sigma_2} = (2\sigma_2 - (\sigma_1+\sigma_3))\,\alpha_2\left(1-\left(\sigma_1-\tfrac{\sigma_2+\sigma_3}{2}\right)^2\alpha_1\right) + \alpha_1\alpha_2\left(\sigma_2-\tfrac{\sigma_1+\sigma_3}{2}\right)^2\left(\sigma_1-\tfrac{\sigma_2+\sigma_3}{2}\right)$$

$$\begin{aligned}\frac{\partial(z_3(\sigma))}{\partial\sigma_3} &= \alpha_3(2\sigma_3-(\sigma_1+\sigma_2))\left(1-\left(\sigma_1-\tfrac{\sigma_2+\sigma_3}{2}\right)^2\alpha_1\right)\left(1-\left(\sigma_2-\tfrac{\sigma_1+\sigma_3}{2}\right)^2\alpha_2\right)\\ &+\alpha_1\alpha_3\left(\sigma_3-\tfrac{\sigma_1+\sigma_2}{2}\right)^2\left(\sigma_1-\tfrac{\sigma_2+\sigma_3}{2}\right)\left(1-\left(\sigma_2-\tfrac{\sigma_1+\sigma_3}{2}\right)^2\alpha_2\right)\\ &+\alpha_2\alpha_3\left(\sigma_3-\tfrac{\sigma_1+\sigma_2}{2}\right)^2\left(1-\left(\sigma_1-\tfrac{\sigma_2+\sigma_3}{2}\right)^2\alpha_1\right)\left(\sigma_2-\tfrac{\sigma_1+\sigma_3}{2}\right)\end{aligned}$$

It is seen immediately that, with $\frac{\sigma_2+\sigma_3}{2} \in [0,1]$ being a minimum for $z_1(\sigma)$, alternative 1's best reply is a corner solution, i.e. either $\sigma_1 = 1$ (if $\frac{\sigma_2+\sigma_3}{2} \leq \frac{1}{2}$) or $\sigma_1 = 0$ (if $\frac{\sigma_2+\sigma_3}{2} \geq \frac{1}{2}$). Turning now to alternative 2, we see that

$$\left.\frac{\partial(z_2(\sigma))}{\partial\sigma_2}\right|_{\substack{\sigma_1=0\\ \sigma_3=1}} = \frac{1}{8}\alpha_2(2\sigma_2-1)\left(8-4\alpha_1\sigma_2^2-\alpha_1-5\alpha_1\sigma_2\right)$$

with three roots,[6]

$$\begin{aligned} r_1 &= \frac{1}{2} \\ r_2 &= \frac{1}{8\alpha_1}\left(-5\alpha_1+\sqrt{128\alpha_1+9\alpha_1^2}\right) \geq r_1 \\ r_3 &= -\frac{1}{8\alpha_1}\left(5\alpha_1+\sqrt{128\alpha_1+9\alpha_1^2}\right) < 0 \end{aligned}$$

There are two candidate best replies at $\sigma_2 = 0$ and $\sigma_2 = \min\{1, r_2\}$. Letting $\alpha_1 \in \left(0, \frac{4}{5}\right)$ ensures that $\sigma_2 = r_2$ is not a best reply.[7] The choice probabilities corresponding to the two remaining candidate best replies are:

$$\begin{aligned} z_2(0,0,1) &= \frac{1}{4}\alpha_2\left(1-\frac{1}{4}\alpha_1\right) \\ z_2(0,1,1) &= \frac{1}{4}\alpha_2(1-\alpha_1) < \frac{1}{4}\alpha_2\left(1-\frac{1}{4}\alpha_1\right) \end{aligned}$$

so that, regardless of the size of α_1 and α_2, alternative 2 cannot profitably deviate from σ^*.

Finally consider alternative 3:

$$\left.\frac{\partial z_3(\sigma)}{\partial\sigma_3}\right|_{\substack{\sigma_1=0\\ \sigma_2=0}} = \frac{1}{8}\alpha_3\sigma_3\left(-8\alpha_1\sigma_3^2-8\alpha_2\sigma_3^2+3\alpha_1\alpha_2\sigma_3^4+16\right)$$

Of the three distinct roots of the polynomial,[8] one is negative, one is larger than unity and one is $\sigma_3 = 0$, with $\left.\frac{\partial z_3(\sigma)}{\partial\sigma_3}\right|_{\substack{\sigma_1=0\\ \sigma_2=0}} > 0$ for $\sigma_3 \in (0,1)$. It follows that $z_3(0,0,\sigma_3)$ is

[6] To check the inequality in the second line below observe that

$$\frac{1}{8\alpha_1}\left(-5\alpha_1+\sqrt{128\alpha_1+9\alpha_1^2}\right) \geq \frac{1}{2} \Leftrightarrow$$
$$\frac{128}{\alpha_1 72} \geq 1$$

which holds true always.

[7] Observe that $r_2 > 1$ if and only if $\alpha_1 \in \left(0, \frac{4}{5}\right)$.

[8] The roots are 0 and $\pm 1.1547\sqrt{\frac{1}{\alpha_1\alpha_2}\left(\alpha_1+\alpha_2-\sqrt{\alpha_1^2-\alpha_1\alpha_2+\alpha_2^2}\right)}$, where the non zero roots are double roots.

maximised for $\sigma_3 = 1$, with corresponding choice probability

$$z_3(\sigma^*) = \alpha_3 \left(1 - \frac{1}{4}\alpha_1\right)\left(1 - \frac{1}{4}\alpha_2\right) > 0 = z_3(0,0,0)$$

Then $\sigma^* = (0,0,1)$ is a Nash equilibrium in which, provided that $\alpha_1 \in \left(0, \frac{4}{5}\right)$ and $\alpha_3 > \min\left\{\frac{\alpha_2}{4-\alpha_2}, \frac{4\alpha_1}{(4-\alpha_1)(4-\alpha_2)}\right\}$, condition (9) holds, so that the worst alternative has the highest probability of being chosen ■