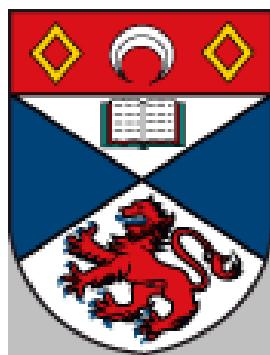


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## **Delay and Haircuts in Sovereign Debt: Recovery and Sustainability**

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# Delay and Haircuts in Sovereign Debt: Recovery and Sustainability\*

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## Abstract

One of the striking aspects of recent sovereign debt restructurings is, conditional on default, delay length is positively correlated with the size of ‘haircut’, which is size of creditor losses. In this paper, we develop an incomplete information model of debt restructuring where the prospect of uncertain economic recovery and the signalling about sustainability concerns together generate multi-period delay. The results from our analysis show that there is a correlation between delay length and size of haircut. Such results are supported by evidence. We show that Pareto ranking of equilibria, conditional on default, can be altered once we take into account the *ex ante* incentive of sovereign debtor. We use our results to evaluate proposals advocated to ensure orderly resolution of sovereign debt crises.

**JEL Classification:** F34, C78

**Key Words:** Debt restructuring, delay, haircuts, growth, sustainability, information

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## 1 Introduction

There is considerable evidence which show that countries have repeatedly defaulted on their external obligations (Reinhart and Rogoff, 2009). What are the common features associated with default on sovereign debts? On several occasions, debt rescheduling or negotiated partial default often involve a reduction of interest rates, if not principal, and typically saddle creditors with illiquid assets that may not pay off for an extended period of time (Reinhart and Rogoff, 2009). Illiquidity and reduction in debt repayment, two striking aspects of recent sovereign debt negotiations, could impose a huge cost to investors.

Both Roubini and Setser (2004) and Sturzenegger and Zettlemeyer (2007) provide evidence that sovereign bond restructurings involve costly delay and argue that delay in sovereign debt restructuring negotiations or lengthy debt renegotiation is widely regarded as inefficient since the sovereign debtor usually suffers from losing access to the international financial market while the creditors cannot realize their investment gains. Table 1 summarizes data from both these papers and provides suggestive evidence that in a number of recent episodes of sovereign debt restructurings, delay length and the size of haircuts are positively correlated.

| Sovereign State | Restructuring negotiations  | Default?       | “Delay” - after default | Face value | Haircut |
|-----------------|-----------------------------|----------------|-------------------------|------------|---------|
| Russia          | 11/1998-7/2000<br>20 months | Yes<br>1/1999  | 18 months               | \$29.1     | 69%     |
| Ukraine         | 1/2000-4/2000<br>3 months   | Yes            | 3 months                | \$2.6      | 40%     |
| Pakistan        | 2/1999-12/1999<br>10 months | No             | —                       | \$0.6      | 30%     |
| Ecuador         | 8/1999-8/2000<br>12 months  | Yes            | 12 months               | \$6.5      | 60%     |
| Argentina       | 9/2003-4/2005<br>19 months  | Yes<br>12/2001 | 40 months               | \$79.7     | 67%     |
| Uruguay         | 4/2003-5/2003<br>1 month    | No             | —                       | \$3.8      | 26%     |

Table 1: Sovereign Debt Restructurings during 1998-2005

Source: Table 14 and 15 in Sturzenegger and Zettlemeyer (2005); Table A.3 in Roubini and Setser (2004)

Why is it so difficult for sovereign debtor and creditors to restructure sovereign debts in an orderly and timely manner? Will faster and more orderly debt restructuring not only help in restoring economic momentum but also reducing the probability of which a crisis occurs in the first place?

From a theoretical perspective, a single creditor, complete information bargaining model of sovereign debt restructuring (for examples, Bulow and Rogoff (1989) and Bhattacharya and Detragiache (1994)) cannot account for delay since, in this class of models, an immediate agreement occurs along the equilibrium path of play. When the size of bargaining surplus is deterministic, the defaulting country and its creditors know exactly the future bargaining surplus; thus, both bargaining parties can reach an agreement immediately after the default.

In this paper, our starting point is the assumption that both the recovery process and the willingness to undertake massive fiscal cuts (by running a primary budgetary surplus, for example) are uncertain. In a single creditor model, we show that multi-period costly delay will exist in a Perfect Bayesian equilibrium driven by the prospect of uncertain economic recovery and the signalling of sustainability concerns.

In our model, the length of delay is positively correlated with the size of creditor losses or haircuts. Although one-period delay occurs to permit for economic recovery, multi-period delay is essential for the debtor to signal sustainability concerns. Relative to the case with one-period delay, with two-period delay, the creditor always receives lower payoffs, which leads to larger haircuts although the debtor could either gain or lose. When the debtor gains from two-period delay, conditional on default, the two scenarios cannot be interim<sup>1</sup> Pareto ranked. However, when the debtor loses from two-period delay, one-period delay interim Pareto dominates two-period delay conditional on default. We use data from Benjamin and Wright (2009) to provide evidence on the positive correlation between delay length and haircuts.

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<sup>1</sup> At the interim stage, payoffs are computed conditional on default but before all uncertainty related to the size of the bargaining surplus and the sustainability concern is resolved.

Finally, we introduce *ex ante* debtor moral hazard so that the probability of default is endogenous. We examine the impact of delay in debt restructuring negotiation on the probability of default. We show that the *ex ante* Pareto ranking of the equilibria with one-period and two-period delay could be different from their interim Pareto ranking of equilibria conditional on default. We then examine some policy interventions that have been advocated to ensure more orderly resolution of sovereign debt crises.

The remainder of the paper is structured as follows. The next subsection presents a review of related literature. Section 2 presents the model and the results. Section 3 is devoted for presenting some supporting evidence. Section 4 is devoted for discussing the issues related to *ex ante* sovereign debtor's incentive. Section 5 presents some policy recommendations, while Section 6 concludes. The proofs and computations underlying some expressions in the main body of the paper are contained in the appendix.

### 1.1 Related Literature

Our result that there could be multi-period delay in a single-creditor model is complementary to a number of other papers that seek to explain delay in debt restructuring. Some scholars highlight the holdout problem or collective action problem among creditors, whereby an individual creditor is better off if he gets paid in full, while other creditors bear the burden of debt restructuring<sup>2</sup>. In Pitchford and Wright (2007), delays in debt restructuring negotiation arise from creditor holdout and free-riding on negotiation effort. In another class of models, costly debt restructuring arises from imperfect creditor coordination with multiple creditors (Kletzer, 2002; Ghosal and Miller, 2003; Haldane et al., 2004; Weinschelbaum and Wynne, 2005; Ghosal and Thampanishvong, 2009), whereby conditional on default, the lack of creditor coordination leads to an inefficient outcome.

Merlo and Wilson (1995, 1998) show that immediate settlement can be socially suboptimal. Sturzenegger (2002) argues that default usually results in large output losses so that delaying settlement could benefit both the

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<sup>2</sup> According to Roubini and Setser (2004), the New York law sovereign bond contracts give each individual bondholder the right to initiate litigation and allow each bondholder to keep for himself anything that he recovers from the sovereign. Roubini and Setser also pointed out that, in case of Argentina, it has to restructure 98 international bonds held by a diverse group of investors, including international institutional investors, domestic pension funds, and hundreds of thousands of retail investors.

debtor and the creditor. Dhillon et al. (2006) examine the Argentine debt swap in 2005 to see whether bargaining theory in the spirit of Merlo and Wilson (1998) can help explain the Argentine final debt settlement as well as the delay in achieving settlement. Based on a dynamic model of sovereign default in which debt renegotiation is modelled as a stochastic bargaining game along the lines of Merlo and Wilson (1995), Bi (2008) finds that delay is beneficial as it permits the debtor country's economy to recover from a crisis. Multi-period delay can arise in Bi (2008) but delay length could be negatively correlated with haircut size.

In a related vein, Benjamin and Wright (2009) analyze the impact of uncertainty on the renegotiation length. Specifically, the debtor and the creditor find it privately optimal to delay restructuring until future default risk (i.e. risk that the debtor will default on the settlement agreement) is low, even though delay means some gains from trade remain unexploited.

Bai and Zhang (2009) study another sovereign debt negotiation game with private information about creditor reservation values. The government uses costly delay as a screening device for the creditors' type so delay arises in the equilibrium. The length of delay positively correlates with the severity of private information and could negatively correlate with haircut size although they show that the presence of a secondary market on sovereign debt could reduce delay length.

## 2 The Model and the Results

### 2.1 The bargaining model environment

In this section, we consider a sovereign debtor who is already in default. Conditional on default, there is a bargaining between the sovereign debtor and the creditor to restructure the defaulted sovereign debts. We refer to the bargaining surplus as the additional tax revenue generated to the sovereign debtor by gaining access to the international capital market once the outstanding debt has been successfully restructured<sup>3</sup>. We suppose that the

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<sup>3</sup>In Eaton and Gersovitz (1981), while in default, the debtor cannot access the international capital market to borrow for consumption smoothing. Bi (2008) assumes that, after a default, the sovereign debtor has no access to outside financing and the length of exclusion from international capital market depends on the renegotiation process. The sovereign debtor needs to repay its defaulted debt in order to regain access to the capital markets.

sovereign debtor uses the tax revenue for one of the two reasons: spending on public good or debt repayment. Thus, debt repayment requires a diversion of funds away from expenditure on public good. We interpret the debtor's offer to the creditor in the debt restructuring negotiation as the amount of tax revenue diverted from provision of public good. The total tax revenue available for bargaining is represented by  $\pi$ , where  $\pi > 0$ .

Since we begin our analysis in this section at a point in which the debtor is in default, we assume that the value of  $\pi$  in the period immediately after default – the initial period – is low and is denoted by  $\pi_L$ . This assumption is along the lines of the argument put forward by Sturzenegger (2002), that is default is associated with large collapse in the economy's output thus, following a default, few resources are available in the debtor country for debt repayment. We, however, allow the future value of  $\pi$  to be stochastic so that, in the subsequent periods,  $\pi$  can continue to be low at  $\pi_L$  with probability  $p$  or grow to a higher level,  $\pi_H$ , with probability  $1-p$ . A growing  $\pi$  can be interpreted as an increase in the tax revenue. It is important to highlight that, if a debt settlement occurs immediately after a default, i.e. in the initial period, in which the bargaining surplus,  $\pi_L$ , needs to be shared between the sovereign debtor and the creditor, the “cake” to be allocated is small and doing so could mean both parties choose to give up on the prospect of economic growth.

As in Dhillon et al. (2006), during the debt restructuring negotiation, the sovereign debtor could have a strategy for reducing the debt vulnerability such as the running of fiscal surpluses to reduce the remaining debt service. Such sustainability constraint essentially restricts the amount available for creditor in the debt negotiation, particularly when the economy of the debtor country is in a recession following a default. In this paper, the debtor's sustainability constraint is represented by  $s$ , and it denotes the minimum fraction of the tax revenue required by the debtor which is consistent with economic and political stability of the debtor country. To simplify our analysis, we assume that there are two types of debtor, Optimistic and Cautious, and the debtor's type is determined by the sustainability constraint. For the Optimistic debtor,  $s$  is close to zero, while the Cautious debtor's sustainability constraint,  $s$ , is close to a level  $\bar{s} > 0$ .<sup>4</sup>

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<sup>4</sup>In the case of Argentina, even after economic recovery, the ratio of the sustainability to the bargaining surplus,  $\bar{s}/\pi$ , is 55 percent (Dhillon et al., 2006).

The Optimistic and the Cautious debtors differ in the utility they obtain from consuming their part of the bargaining surplus. While the utility of the Cautious debtor is linear in her share of the bargaining surplus, for the Cautious debtor, there is a discontinuity in the utility at a threshold settlement level,  $\bar{s}$ , to reflect her sustainability concern on the debt settlement. Formally, let  $u_i$  denotes the utility of type- $i$  debtor, where  $i = \text{Optimistic} (O)$ ,  $\text{Cautious} (C)$ , and let  $\Delta_i$  denotes the type- $i$  debtor's share of the bargaining surplus. It follows that, for the Optimistic debtor,  $u_O = \Delta_O$  for all values of  $\Delta_O$ , while for the Cautious debtor,

$$u_C = \begin{cases} 0 & \text{for } \Delta_C < \bar{s} \\ \Delta_C & \text{for } \Delta_C \geq \bar{s} \end{cases}.$$

In this paper, we capture the debtor's concern on sustainability of debt settlement by assuming the piecewise linearity of preferences. By doing so, it rules out default as a mechanism for risk sharing, which would likely affect welfare computations.

We assume that there is an asymmetric information about the sustainability constraint. The creditor's prior probability over  $\{0, \bar{s}\}$  is  $\{q_0, 1 - q_0\}$ . We assume that the uncertainty with respect to  $\pi$  and with respect to the sustainability constraint are resolved at  $t = 2$ .

In our model, bargaining takes place over a number of time periods,  $t = 1, 2, 3, \dots$ . The bargaining game is specified as follows. We assume that the debtor makes an offer at  $t = 1$ , but, in the subsequent periods, the debtor and the creditor have an equal probability of making an offer. An offer is a number greater than or equal to zero and less than or equal to  $\pi$ . If the offer is accepted, then the sovereign debtor repays the creditor according to the agreement and the game ends; otherwise, both players enter the next period. This process continues until the offer is accepted. In any period in which there is disagreement, the debtor is excluded from having access to the international capital market, which is translated into an assumption that the disagreement payoffs for both players are zero.

The timing of events is summarized in Table 2.

|                | <b>Size of <math>\pi</math></b>           | <b>Debtor's type</b>  | <b>Proposer</b>   | <b>Outcome</b>  |
|----------------|---|---|---|---|
| $t = 1$        | $\pi_L$                                   |   | Debtor makes an offer   | If offer is accepted,<br>game ends.<br>If offer is rejected,<br>game continues. |
| $t = 2$        | Uncertainty<br>over $\pi$<br>is resolved. | Sustainability<br>constraint<br>is revealed to<br>the debtor. | Debtor and creditor<br>make offers<br>with prob. $(\frac{1}{2}, \frac{1}{2})$ | If offer is accepted,<br>game ends.<br>If offer is rejected,<br>game continues. |
| $t = 3, \dots$ |   |   | Debtor and creditor<br>make offers<br>with prob. $(\frac{1}{2}, \frac{1}{2})$ | If offer is accepted,<br>game ends.<br>If offer is rejected,<br>game continues. |

Table 2: Timing of Events

We solve for the Perfect Bayesian equilibrium of the model. We assume that both creditor and sovereign debtor have a common discount factor  $\delta = (1 + r)^{-1}$ , where  $\delta < 1$  and  $r$  is the interest rate charged on sovereign debt.

In order to distinguish between one-period and two-period delay, we consider two cases: (Case 1) stochastic bargaining surplus but there is complete information about sustainability constraint and the debtor's type, and (Case 2) stochastic bargaining surplus and there is asymmetric information about the sustainability constraint and the debtor's type.

### 2.1.1 Case 1: Stochastic bargaining surplus and complete information about the debtor's type

In this case, the debtor's type is known at  $t = 2$  so the creditor knows whether the debtor is Optimistic or Cautious. We begin our analysis at  $t = 2$ . We compute the debtor's and the creditor's second-period payoffs when the debtor is Optimistic and Cautious, respectively. We then calculate the continuation values for each player as we move backward to the first period. The continuation value would limit the offers that can be made by the debtor at  $t = 1$ , when the debtor is a proposer. To capture the point that, at  $t = 2$ , the size of bargaining surplus is stochastic, we suppose that  $E\pi$  denotes the expected size of tax revenue, where  $E\pi = p\pi_L + (1 - p)\pi_H$ .

Delay occurs at  $t = 1$  if the debtor's best offer, which is the excess of the available bargaining surplus at  $t = 1$ ,  $\pi_L$ , over the debtor's own continuation value, falls below the creditor's continuation value. We show that, when the bargaining surplus is stochastic and there is a complete information about the debtor's type, delay occurs whenever the expected growth rate of the economy,  $Eg = \frac{(E\pi - \pi_L)}{\pi_L}$ , exceeds the rate of discount,  $r = \frac{1-\delta}{\delta}$ , regardless of the debtor's type, i.e.  $r < Eg$ . This condition for one-period delay is essentially the same as that in Merlo and Wilson (1998). Our model predicts that, with relatively low growth prospects, there will be an immediate agreement between the debtor and the creditor.

We summarize the above discussion as the following proposition:

**Proposition 1** *When the debtor's type is known but there is an uncertainty with respect to  $\pi$ , there is one-period delay in sovereign debt restructuring whenever the expected growth of the economy exceeds the rate of discount.*

**Proof.** See Appendix A. ■

### 2.1.2 Case 2: Stochastic bargaining surplus and asymmetric information about the debtor's type

In this subsection, we consider the case in which the bargaining surplus can change over time in a stochastic manner and there is asymmetric information about the debtor's type. We solve the model by backward induction, starting from period  $t = 3$ . At  $t = 3$ , the creditor's posterior beliefs over the two types of debtor is denoted by  $q_1$ . Table 3 presents the creditor's offers at extreme values of  $q_1$  and the creditor's payoff<sup>5</sup>.

| Creditor's belief as to debtor's type | Creditor's offer                  | Creditor's Payoff                                   |
|---------------------------------------|-----------------------------------|---|
| $q_1 = 1$<br>(Optimist)               | $\frac{\delta\pi}{2}$             | $\pi - \frac{\delta\pi}{2}$                         |
| $q_1 = 0$<br>(Cautious)               | $\frac{\delta(\pi - \bar{s})}{2}$ | $(\pi - \bar{s}) - \frac{\delta(\pi - \bar{s})}{2}$ |

Table 3: Creditor's offers with extreme beliefs

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<sup>5</sup>The detailed computations of creditor's offers and payoffs are presented in Appendix A.

For less extreme beliefs,  $0 < q_1 < 1$ , the creditor's expected payoff from a high offer of  $\frac{\delta(\pi-\bar{s})}{2}$ , which is acceptable to both types of debtor, is  $(\frac{2-\delta}{2})(\pi - \bar{s})$ , while the creditor's expected payoff from a low offer of  $\frac{\delta\pi}{2}$ , which is only acceptable to the Optimistic debtor is  $q_1(\pi - \frac{\delta\pi}{2})$ . If  $q_1(\pi - \frac{\delta\pi}{2}) > (\frac{2-\delta}{2})(\pi - \bar{s})$ , the creditor will do better by making a low offer but otherwise for  $q_1(\pi - \frac{\delta\pi}{2}) < (\frac{2-\delta}{2})(\pi - \bar{s})$ . When  $q_1(\pi - \frac{\delta\pi}{2}) = (\frac{2-\delta}{2})(\pi - \bar{s})$ , the two offers give the creditor the same expected payoff. This condition implies that the posterior belief of the creditor is  $q_1 = \frac{\pi-\bar{s}}{\pi}$ . Since  $0 < \bar{s} < \pi$ , it follows that  $q_1 < 1$ . We assume that, if the creditor is indifferent between a low and a high offer, he will choose to make a high offer. Table 4 shows the creditor's offers at  $t = 3$  as a function of his posterior belief.

| Creditor's belief as to debtor's type                     | Creditor's offer                | Expected payoff for creditor          |
|---|---------------------------------|---------------------------------------|
| $q_1 > \frac{\pi-\bar{s}}{\pi}$<br>(Probably an Optimist) | $\frac{\delta\pi}{2}$           | $q_1(\pi - \frac{\delta\pi}{2})$      |
| $q_1 \leq \frac{\pi-\bar{s}}{\pi}$<br>(Probably Cautious) | $\frac{\delta(\pi-\bar{s})}{2}$ | $(\frac{2-\delta}{2})(\pi - \bar{s})$ |

Table 4: Creditor's offers at  $t = 3$  for all values of belief

If, however, the debtor is the proposer, the Optimistic debtor's offer is  $\frac{\delta\pi}{2}$ , while the Cautious debtor's offer is  $\frac{\delta(\pi-\bar{s})}{2}$ , which are the corresponding continuation values of the creditor as computed in Appendix A.

Moving to the second period, the continuation values for the debtor and the creditor, given the common discount factor,  $\delta$ , and the fact that each negotiating party has an equal probability of being a proposer, are presented in Table 5.<sup>6</sup>

| Creditor's belief as to debtor's type                     | Continuation values for debtor  | Continuation values for creditor  |
|---|---------------------------------|---|
| $q_1 > \frac{\pi-\bar{s}}{\pi}$<br>(Probably an Optimist) | $\frac{\delta\pi}{2}$           | $\frac{\delta}{2} [q_1 (\frac{2-\delta}{2}) \pi + \frac{\delta\pi}{2}]$ |
| $q_1 \leq \frac{\pi-\bar{s}}{\pi}$<br>(Probably Cautious) | $\frac{\delta(\pi+\bar{s})}{2}$ | $\frac{\delta(\pi-\bar{s})}{2}$   |

<sup>6</sup>In Table 5, when  $q_1 \rightarrow 1$ , the creditor's continuation value approaches  $\frac{\delta\pi}{2}$ , which is the same as in the case with complete information about the debtor's type.

Table 5: Continuation values for the debtor and the creditor at  $t = 2$

At  $t = 2$ , there is an asymmetric information about the debtor's type. Let us fix the creditor's posterior belief,  $q_1$ , so that the creditor's continuation values are as given in Table 5. We denote such continuation belief by  $q'_1$ , where  $q'_1 \leq \frac{\pi - \bar{s}}{\pi}$ . Corresponding to such continuation belief of the creditor, the debtor's and the creditor's continuation values are given by  $\frac{\delta(\pi + \bar{s})}{2}$  and  $\frac{\delta(\pi - \bar{s})}{2}$ , respectively. In what follows, we show that delay in the second period is necessary for the debtor to signal about her type and the sustainability concerns.

**Lemma 1** *Delay occurs in the second period of the bargaining game whenever the creditor's prior belief that the debtor is Optimistic,  $q_0$ , is sufficiently high, i.e.  $q_0 > \frac{\pi - \bar{s}}{\pi}$ , or when the sustainability concern is sufficiently important, i.e.  $\frac{\bar{s}}{\pi} > (1 - q_0)$ .*

**Proof.** See Appendix A ■

The details of the mixed strategy at  $t = 2$  are as follows: if the debtor is chosen to be a proposer, the Optimistic debtor offers  $(\tilde{x}_2, \pi - \tilde{x}_2)$  with a probability  $(1 - \beta^*)$  and offers  $(x'_2, \pi - x'_2)$  with a probability  $\beta^*$ , where  $\tilde{x}_2 = (\frac{2-\delta}{2})\pi + \frac{\delta\bar{s}}{2}$ ,  $x'_2 > (\frac{2-\delta}{2})\pi + \frac{\delta\bar{s}}{2}$  and  $\beta^* = (\frac{\pi - \bar{s}}{\bar{s}}) \left( \frac{1 - q_0}{q_0} \right)$ , while the Cautious debtor offers  $(x'_2, \pi - x'_2)$  with a probability 1. The creditor rejects the offer  $(\tilde{x}_2, \pi - \tilde{x}_2)$  with a probability  $(1 - \gamma^*)$  and rejects the offer  $(x'_2, \pi - x'_2)$  with a probability 1, where  $\gamma^* = \frac{\delta\bar{s}}{2(\tilde{x}_2 - \frac{\delta\pi}{2})}$ . If, instead, the creditor is chosen to make an offer at  $t = 2$ , the creditor offers  $(\tilde{x}_2, \pi - \tilde{x}_2)$ . The Optimistic debtor accepts such creditor's offer with a probability  $\beta^*$ , while the Cautious debtor rejects the creditor's offer with a probability 1.

Next, at  $t = 3$ , the creditor's belief as to the debtor's type is  $q_1 = \frac{\pi - \bar{s}}{\pi}$ . The creditor's payoff at  $t = 3$  is  $\frac{\delta(\pi - \bar{s})}{2}$ , while the debtor's payoff depends on the debtor's offer at  $t = 2$ . If the debtor's offer is  $(\tilde{x}_2, \pi - \tilde{x}_2)$ , the debtor's payoff at  $t = 3$  is  $\frac{\delta\pi}{2}$ ; however, if the debtor's offer at  $t = 2$  is  $(x'_2, \pi - x'_2)$ , her payoff at  $t = 3$  is  $\frac{\delta(\pi + \bar{s})}{2}$ .

We then move backwards to period  $t = 1$  and calculate the continuation values for the creditor and for the debtor. There are two scenarios to be considered: (1) the debtor knows her own type and (2) the debtor does not know her own type.

If the debtor knows her own type, according to the detailed computations in Appendix B, the continuation value for the Optimistic debtor is  $\delta \left[ \frac{\delta(\pi+\bar{s})}{4} \right]$ , the continuation value for the Cautious debtor is  $\delta \left[ \frac{\delta(\pi+\bar{s})}{2} \right]$  and the continuation value for the creditor is

$$\delta \left[ \frac{\delta(\pi-\bar{s})}{2} \left( 1 - \left( \frac{\pi-\bar{s}}{\bar{s}} \right) (1-q_0) \right) + \left( \frac{\pi-\bar{s}}{\bar{s}} \right) (1-q_0) (\pi-x'_2) \right].$$

On the other hand, if the debtor does not know her own type, again following the detailed computations in Appendix B, the debtor's continuation payoff is  $\frac{\delta^2(\pi+\bar{s})}{2} \left( \frac{2-q_0}{2} \right)$ , while the creditor's continuation payoff is the same as in the first scenario.

From these two scenarios, it follows from the detailed computations in Appendix B that the expected payoff for the creditor from rejecting the debtor's offer at  $t = 1$ ,  $\hat{a}$ , is given by

$$\hat{a} = \delta \left( \frac{\delta(E\pi-\bar{s})}{2} - (1-q_0) \left( \frac{(E\pi-\bar{s})}{\bar{s}} \right) \left[ \frac{\delta(E\pi-\bar{s})}{2} - E\pi + Ex'_2 \right] \right),$$

where  $Ex'_2 > E\pi - \frac{\delta(E\pi-\bar{s})}{2}$  since  $x'_2 > \pi - \frac{\delta(\pi-\bar{s})}{2}$ . An alternative interpretation for  $\hat{a}$  could be the minimum payoff for the creditor to accept the debtor's offer.

For delay to occur at  $t = 1$ , when the debtor is a proposer, it should be attractive for the creditor to reject the debtor's best offer at  $t = 1$ . This happens when the best offer of the debtor falls below the creditor's continuation payoff or the expected payoff,  $\hat{a}$ , computed earlier.

**Lemma 2** *The condition for delay at  $t = 1$  is*

$$\pi_L - \delta \left( \frac{\delta(E\pi+\bar{s})}{2} \right) < \hat{a},$$

i.e. the excess of the available bargaining surplus over the Cautious debtor's own continuation value falls below the creditor's expected payoff in the first period.

**Proof.** In what follows, we derive the conditions for delay for each type of debtor at  $t = 1$ . For the Optimistic debtor, the best offer that she can make is the excess of the available bargaining surplus,  $\pi_L$ , over her own continuation value,  $\delta \left( \frac{\delta(E\pi+\bar{s})}{4} \right)$ , i.e.  $\pi_L - \delta \left( \frac{\delta(E\pi+\bar{s})}{4} \right)$ . If such offer falls

below the expected payoff for the creditor,  $\hat{a}$ , the debtor's offer will not be accepted since the creditor does better by waiting until the next period. This results in a delay in the first period. Formally, the condition for first-period delay for the Optimistic debtor is given by

$$\pi_L - \delta \left( \frac{\delta(E\pi + \bar{s})}{4} \right) < \hat{a}. \quad (1)$$

For the Cautious debtor, her best offer is  $\pi_L - \delta \left( \frac{\delta(E\pi + \bar{s})}{2} \right)$ . If such offer falls below  $\hat{a}$ , the offer will not be accepted. Formally, the condition for delay for the Cautious debtor is

$$\pi_L - \delta \left( \frac{\delta(E\pi + \bar{s})}{2} \right) < \hat{a}. \quad (2)$$

Clearly, when condition (2) is satisfied, condition (1) will also be satisfied. Thus, the condition for delay in the first-period is given by condition (2). ■

Therefore, in the case in which the bargaining surplus is stochastic and there is an asymmetric information about the debtor's type and the sustainability concern, there is a semi-separating Perfect Bayesian equilibrium with two-period delay along the equilibrium path of play, initially to permit for economic recovery followed by signalling about the sustainability constraint given that the conditions:

$$\frac{\bar{s}}{\pi} > (1 - q_0) \text{ or } q_0 > \frac{\pi - \bar{s}}{\pi} \text{ and } \pi_L - \delta \left( \frac{\delta(E\pi + \bar{s})}{2} \right) < \hat{a}$$

are satisfied.

Since  $0 < q_0 < 1$ ,  $(\frac{E\pi - \bar{s}}{\bar{s}}) > 0$  and  $Ex'_2 > E\pi - \frac{\delta(E\pi - \bar{s})}{2}$ , by computation, it follows that  $\hat{a} < \frac{\delta E\pi}{2}$  and  $\hat{a} < \frac{\delta(E\pi - \bar{s})}{2}$ . Therefore, the condition for first-period delay both for the Optimistic and Cautious debtor when there is an asymmetric information about the debtor's type, i.e.  $\pi_L - \delta \left( \frac{\delta(E\pi + \bar{s})}{4} \right) < \hat{a}$  and  $\pi_L - \delta \left( \frac{\delta(E\pi + \bar{s})}{2} \right) < \hat{a}$ , respectively, are more stringent than the condition for delay in the full information case, i.e.  $r < Eg$ . The reason is that, under the former, there is a positive risk that the creditor would face a Cautious debtor, which lowers the creditor's continuation payoff.

We summarize the above discussion as the following proposition:

**Proposition 2** *When there is an asymmetric information about the debtor's*

*type and a stochastic bargaining surplus, there exists a Perfect Bayesian equilibrium with two-period delay.*

## 2.2 Comparisons of payoffs with one-period and two-period delay

In what follows, we compare the creditor's payoff in the case with one-period delay and in the case with two-period delay in the mixed strategy equilibrium. This would allow us to establish the correlation between delay and size of haircuts in sovereign debts. Table 6 provides a summary for the creditor's continuation payoffs from rejecting the debtor's offer at  $t = 1$  in the case with one-period and two-period delay.

| Debtor's type     | Creditor's payoffs<br>with one-period delay | Creditor's payoffs<br>with two-period delay |
|-------------------|---|---|
| Optimistic debtor | $\frac{\delta E\pi}{2}$                     | $\hat{a}$                                   |
| Cautious debtor   | $\frac{\delta(E\pi - \bar{s})}{2}$          | $\hat{a}$                                   |

*Table 6: Delay and haircuts in sovereign debt restructuring (in a mixed strategy)*

Basing on Table 6, for the creditor's expected payoff with one-period delay to be greater than the expected payoff with two-period delay, it requires that

$$q_0 \left( \frac{\delta E\pi}{2} \right) + (1 - q_0) \left( \frac{\delta(E\pi - \bar{s})}{2} \right) > \hat{a}.$$

Let  $Ex'_2 = E\pi - \frac{\delta(E\pi - \bar{s})}{2} + \varepsilon$ , where  $\varepsilon$  is a strictly positive number. By computation, the above condition reduces to

$$q_0 \left( \frac{\delta E\pi}{2} \right) + (1 - q_0) \left( \frac{\delta(E\pi - \bar{s})}{2} \right) - \frac{\delta(E\pi - \bar{s})}{2} + (1 - q_0) \left( \frac{E\pi - \bar{s}}{\bar{s}} \right) > 0,$$

which is equivalent to

$$q_0 \left( \frac{\bar{s}}{2} \right) + (1 - q_0) \left( \frac{E\pi - \bar{s}}{\bar{s}} \right) > 0,$$

a condition which always holds.

Thus, conditional on default, relative to one-period delay, the creditor's loss is higher under two-period delay so, in the mixed strategy equilibrium, delay is positively correlated with haircut.

In order to undertake the Pareto ranking of equilibria, we need to compute the debtor's payoffs with one-period and two-period delay. With two-period delay, the debtor's expected payoff is  $\frac{\delta^2(\pi+\bar{s})}{2} \left( \frac{2-q_0}{2} \right)$ , where the detailed computation is contained in Appendix B. By computation, it follows that the debtor's expected payoff with two-period delay is greater than the payoff with one-period delay if and only if:

$$\frac{\bar{s}}{E\pi} > \frac{2(1-\delta) + \delta q_0}{2(1-\delta) + q_0(2-\delta)}. \quad (3)$$

The fraction on the RHS of the above inequality is decreasing in  $q_0$  but increasing in  $\delta$ , while the fraction on the LHS of the inequality is increasing in  $\bar{s}$ . Therefore, the debtor's payoff with two-period delay is likely to be higher than the payoff with one-period delay the higher is the ratio of sustainability constraint to the expected value of the bargaining surplus, the lower is the discount factor and the higher is the creditor's prior that the debtor is Optimistic. A possible intuition behind this condition could be that, both when the discontinuity in the debtor's payoffs is at a higher settlement value and when the creditor has a high prior that the debtor is of an Optimistic type, the value of signalling about the debtor's type and the sustainability concerns is greater for the debtor.

By considering both the debtor's and the creditor's payoff conditional on default with one-period and two-period delay, our results show that, relative to one-period delay, two-period delay is interim Pareto dominated only when the direction of the above inequality (3) is reversed since, in this case, both the debtor's and the creditor's payoffs are lower than with one-period delay. Whenever the above inequality (3) holds, relative to one-period delay, the debtor makes a payoff gain with two-period delay, while the creditor makes a payoff loss. In this case, the two equilibria cannot be interim Pareto ranked.

We summarize the above discussion as the following proposition:

**Proposition 3** *Two-period delay never interim Pareto dominates one-period delay although the reverse could be true.*

### **3 Empirical Evidence on Delay in Sovereign Debt Restructuring Negotiation and Haircuts**

In several studies on sovereign debt restructuring negotiations between a sovereign debtor in default and the international creditors, there have been some documentations that these negotiating parties have encountered some difficulties in reaching mutually advantageous settlements as there is evidence of both delay and haircuts. Roubini and Setser (2004) point out that a number of the sovereign debt restructurings between 1998 and 2005 have been largely characterized by protracted negotiations.

With regards to haircuts, Sturzenegger and Zettelmeyer (2005) find that there are very large variations in the average level of haircuts across the debt restructuring episodes of 1998 to 2005. The present value haircuts<sup>7</sup>, according to their calculations, ranged from around 5 to 20 percent for Uruguay (2003) to over 50 percent for Russia (2000) and over 70 percent for Argentina (2005), with the remaining exchanges falling mostly in the 20 to 40 percent range. Roubini and Setser (2004) and Sturzenegger and Zettelmeyer (2005) have been the two commonly used sources of data for delay length (default dates and date of debt settlement) and estimates of creditor losses or haircuts as cited by several papers including Dhillon et al. (2006), Ghosal and Miller (2005), D'Erasco (2007) and Bi (2008). In fact, there exist small number of estimates of creditor losses produced by different researchers, particularly World Bank (1993), Cline (1995), and Global Committee Argentina Bondholders (2004).

A recent paper by Benjamin and Wright (2009) uncovered different estimates of haircuts in 27 defaults constructed by four different sources mentioned earlier using 5 different methods. Among these four sources, Benjamin and Wright argue that Sturzenegger and Zettelmeyer (2005, 2007) provide the most rigorous measurement for creditor losses, but the estimates are only available for six default episodes. Drawn from a variety of sources, the database on sovereign debt restructuring outcomes – containing data on the occurrence of default and settlement, the outcomes of negotiations as well as measures of economic performance and indebtedness – constructed

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<sup>7</sup>The losses that defaults have inflicted on creditors are largely based on the comparison between the (remaining) payment stream that was originally promised to investors and the payment stream associated with the restructured instrument, both discounted at a common interest rate (Sturzenegger and Zettelmeyer, 2007).

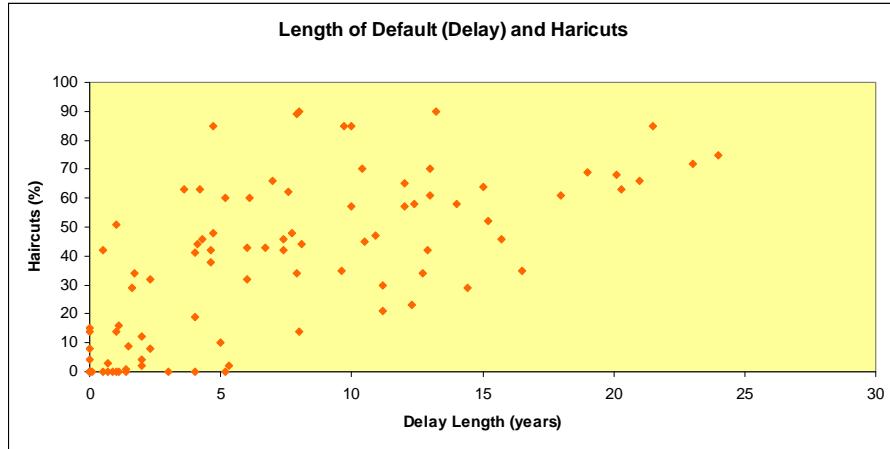


Figure 1: *Delay length and haircuts* (Source: Data on delay length and haircuts are from Benjamin and Wright (2009) Table 15 in Appendix C)

by Benjamin and Wright (2009) covers 90 default episodes by 73 countries during the period of 1989 to 2006.

Basing on the summary statistics they obtained, Benjamin and Wright find that the length of delay falls within the range of 7.4 to 7.6 years. Benjamin and Wright also report that the average creditor experienced a haircut of roughly 40 percent of the value of the debt. Last but not least, they find that there is an evidence that longer delay is associated with larger haircuts, with a correlation between the length of the renegotiation process and the size of haircut of 0.66. By using data on length of renegotiation process and size of haircuts from Benjamin and Wright (2009), we present a scatter plot, which shows a positive correlation between delay length and haircuts, in Figure 1.

### 3.1 A case study of Argentina

Could Argentina be a case in point<sup>8</sup>? In what follows, we provide a discussion on why Argentina could be an example of recent debt restructuring,

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<sup>8</sup>Clearly, a key policy issue in the discussions surrounding the Argentinean debt restructuring is related to issues of debt restructuring. In what follows, we highlight this aspect of the Argentinean experience. This is not because we do not think creditor coordinator is not important in practice (see, for instance, Roubini and Setser (2004, p.298)) but to emphasize issues central to this paper, namely those of recovery and sustainability.

following a default in 2001, which involves a substantial delay and large haircuts. There were different factors that could explain why a restructuring of Argentine sovereign debts involved a substantial delay. We begin by considering the political factors that militated against early restructuring. After a default on sovereign debt was declared at the end of 2001, there was a problem of legitimacy as Argentina was being governed by an interim administration led by President Duhalde. At that time, since the Argentine economy was in a severe recession, the priority of the President was to engineer recovery and “not to pursue outstanding structural reforms, among which debt resolution was the most important” (Bruno, 2004, p.1620). Serious efforts to restructure Argentine debt did not begin until the interim administration was replaced in the elections of 2003, but the striking rate of recovery of GDP during President Duhalde’s administration suggests that “debt restructuring would have been postponed even if there had been no problem of legitimacy” (Dhillon et al., 2006).

It was in September 2003, at the meetings of the IMF and the World Bank in Dubai, that the Argentine government led by President Kirchner finally revealed its negotiating stance. The specific strategy for reducing the debt exposure of the economy involved three principal commitments by the Argentine government, namely to run a primary surplus of 3 percent of GDP, to limit the cost of debt service, and to exempt the preferred creditors from the debt restructuring. The three percent GDP ceiling on debt service restricts the amount that is available for creditors in the debt swap, particularly when GDP is low. Thus, the first two commitments by the government of Argentina effectively determined the overall size of the debt write-down. The third commitment, which requires paying full compensation to the preferred creditors<sup>9</sup> meant there was little left for other private creditors. Without taking account of past-due interest, these constraints left an annual flow of only about a billion dollars on GDP valued at \$137 billion – a ‘Dubai residual’ of less than one percentage point of GDP for private creditors holding debt with a face value of around \$80 billion (Dhillon et al., 2006).

The Dubai proposals articulated by the Argentine government were promptly rejected by creditor groups. However, improvements offered in the course of

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<sup>9</sup>These included both international financial institutions, such as the IMF, the World Bank and the IADB, and domestic bondholders who had lent into arrears.

2004 together with a decline in the global interest rates meant that a debt swap was finally accepted by 76 percent of the creditors in 2005.

To summarize, in the case of Argentina, there were clearly two separate phases leading up to the debt swap in 2005. In the first phase, from end of 2001 to mid-2003, the Argentine economy was recovering strongly from a deep recession and there appeared to be a consensus between both parties to await recovery – a consensus reinforced by the political difficulties faced by the Duhalde regime. As for the second phase, one could interpret the meagre Dubai offer made by Argentina in September 2003 as driven by sustainability concerns. In the context of our analysis, this low offer was designed to be rejected by international creditors, leading to delay and a reappraisal of the type of debtor is Argentina and finally to a settlement that respected these sustainability concerns. It is interesting to take note of the close coincidence of the final debt swap with the sustainability requirement<sup>10</sup> calculated by the Argentine government<sup>11</sup>.

## 4 Debtor's *Ex Ante* Incentives and Efficiency

In Section 2, we have seen that multi-period costly delay arises in a Perfect Bayesian equilibrium driven by prospect of uncertain economic recovery and signalling about the sustainability concerns. What are the implications of delay on the sovereign debtor's *ex ante* incentives? In order to answer this question, we study a very simple model of *ex ante* debtor moral hazard and, by endogenizing the probability of default, examine the impact of delay on the probability of default, i.e. under what conditions would two-period delay, relative to one-period delay, increase the probability of default?

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<sup>10</sup> As already pointed out in Dhillon et al. (2006), the ratio of sustainability constraint to the bargaining surplus,  $\bar{s}/\pi$ , for Argentina has a value of 55 percent, while the debt swap itself is estimated to represent a payoff of about 53 percent.

<sup>11</sup> Anecdotal support for this interpretation is provided in Liashovich (2005, pp. 226-227) in his biography of Mr. Lavagna, who was the Argentine Finance Minister at the time. His writing on the low Dubai offer and the sustainability concerns lying behind it is as follows: “some time before the offer [at Dubai], Lavagna was already preparing the field: he realized that after the offer, ‘there are going to be sad faces everywhere’. And indeed the first reaction of the creditors was of rejection...But the Argentine offensive was not restricted to Dubai. [President] Kirchner in New York, one day after the offer, had an interview with President George W. Bush, who said, ‘Keep on negotiating firmly with the creditors.’ And the Argentine President used the auditorium of UN General Assembly to criticize the international financial organizations for supporting debt reduction and [promoting] growth. ‘It is never been known to recover debts from the dead,’ he said in his speech.”

We assume that the sovereign debtor issues a bond<sup>12</sup> at  $t = 0$ , which promises an interest coupon,  $r$ , in perpetuity. At  $t = 0$ , the sovereign debtor has to choose the level of effort<sup>13</sup>,  $e \in [0, 1]$ , at a cost  $\kappa(e)$ , where  $\kappa'(e) > 0$  and  $\kappa''(e) > 0$ , so that it is more costly for the debtor to exert high effort relative to low effort. At the beginning of  $t = 1$ , the sovereign debtor faces an adverse shock<sup>14</sup> with probability  $h$ . Conditional on the adverse shock, there is a default by sovereign debtor with probability  $1 - e$  due to a lack of available funds to cover outstanding debt payments. This suggests that a low effort implies that the debtor's economy is more likely to be vulnerable to an adverse external shock.

The adverse shock occurs with a positive probability only at the beginning of  $t = 1$ , and the probability of the adverse shock is zero in all subsequent periods. In the absence of the adverse shock, the debtor continues to make any contracted repayments and obtains a continuation, non-contractible payoff<sup>15</sup> (measured in  $t = 0$  payoff units) of  $D > 0$ . The fact that the debtor's payoff,  $D$ , is non-contractible means that  $D$  cannot be attached by the private creditors in the settlement of their claims nor can the sovereign debtor make a credible commitment to transfer such payoff to the private creditors.

If there is a default, the payoff of the debtor is described as in Section 2 and will depend on whether there is one-period or two-period delay in the restructuring of defaulted debts. As a function of the equilibrium prevailing in the post-default debt restructuring game, let  $K$  denote the debtor's expected payoff conditional on default, measured in  $t = 0$  payoff units, where  $K < D$ . The debtor faces the following maximization problem:

$$\max_{e \in [0,1]} h [eD + (1 - e)K] + (1 - h)D - \kappa(e).$$

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<sup>12</sup>It is important to explicitly state that, in this model of *ex ante* debtor moral hazard, since the debtor does not decide how much to borrow, there is no room for default to improve risk sharing, and there is no intensive margin along which borrowing can be adjusted to changes in the cost of borrowing.

<sup>13</sup>In this context, high effort could correspond to a situation where money is borrowed and used to promote R&D in the export sector and low effort could correspond to transferring borrowed money to local elites who are then free to put it in tax havens overseas (see Ghosal and Miller (2003) for more examples of *ex ante* debtor moral hazard and other relevant results).

<sup>14</sup>An example of such adverse shock could be a shock to world oil prices.

<sup>15</sup>Following Eaton and Gersovitz (1981), we interpret this non-contractible payoff as the benefit at  $t = 1$  of a future gain in national output when a debt crisis is prevented at  $t = 1$ .

The first-order condition characterizing an interior solution is:

$$\kappa'(e^*) = h(D - K).$$

Observe that, since  $\kappa''(e) > 0$ , it follows that the optimal level of *ex ante* effort,  $e^*$  (and hence the probability of default,  $(1 - e^*)$ ) is increasing (decreasing) in the probability of adverse shock,  $h$ , and the debtor's non-contractible payoff,  $D$ , but is decreasing (increasing) in the expected payoff, conditional on default, for the debtor,  $K$ .

In the interim debt restructuring negotiation, conditional on default, one has seen that whether or not the length of delay adversely impacts the debtor's payoff depends on whether or not condition (3) holds, i.e. whether or not the debtor's expected payoff with two-period delay is greater than the payoff with one-period delay. When two-period delay gives the debtor higher expected payoff than one-period delay in the interim debt negotiation game, the debtor's payoff conditional on default,  $K$ , with two-period delay increases relative to the one-period delay, while the corresponding *ex ante* choice of effort of the debtor,  $e^*$ , goes down. It follows that, relative to one-period delay, two-period delay increases the *ex ante* probability of default, making the debt crises more likely in the first place.

Sovereign spread,  $S$ , could be calculated by using the formula introduced in Ghosal and Miller (2003) as follows:

$$S = z(1 - R),$$

where  $z$  is the probability of default, and  $R$  is the recovery rate. Applying this formula to the context of our analysis, the probability of default is  $z = (1 - e^*)h$  and the recovery rate,  $R$ , is inversely related to the creditor loss or the size of the haircut. Whenever the two-period delay gives the debtor higher expected payoff than the one-period delay in the interim debt negotiation game (i.e. whenever condition (3) holds), it follows that, relative to one-period delay, sovereign spread with two-period delay is higher since both  $z$  is higher (as  $e^*$  is lower with two-period delay) and  $R$  is lower (as the size of the haircut is higher with two-period delay). Although, when condition (3) holds, the equilibrium payoffs *conditional on default* with one-period delay cannot be efficiency ranked relative to the equilibrium payoffs with two-period delay since the debtor gains while the creditor loses, the *ex*

*ante* payoffs with two-period delay could be lower relative to with one-period delay. Moreover, with an appropriate adjustment of the sovereign spread, the *ex ante* creditor's payoffs will be unaffected by the delay length.

Formally, let  $e_t^*$  (respectively,  $K_t$ ) denote the effort choice of the debtor (respectively, the debtor's expected payoffs conditional on default) with delay length  $t$ , where  $t = 1, 2$ . As long as  $(1 - e_1^*) K_1 \simeq (1 - e_2^*) K_2$  and  $h(e_1^* - e_2^*) D > \kappa(e_1^*) - \kappa(e_2^*)$ , then the sovereign debtor's payoffs are higher with one-period delay relative to two-period delay<sup>16</sup>.

By Proposition 3, relative to one-period delay, two-period delay is inefficient whenever it gives the debtor lower expected payoff at the interim period (conditional on default) than the one-period delay, i.e. when the direction of the inequality in condition (3) is reversed, as in this case, both the debtor's and creditor's payoffs with two-period delay are lower than with one-period delay. However, when the two-period delay gives the debtor lower expected payoff at the interim stage (conditional on default) relative to the one-period delay, with two-period delay, the debtor's *ex ante* level of effort,  $e^*$ , is higher and the probability of default,  $(1 - e^*) h$ , is lower than with one-period delay. Even though, relative to one-period delay, the recovery rate,  $R$ , is lower with two-period delay due to larger size of haircuts, the creditor's *ex ante* payoffs will not be affected as the sovereign spread will be adjusted upwards. However, the debtor's *ex ante* payoffs could actually go up as long as  $(1 - e_1^*) K_1 \simeq (1 - e_2^*) K_2$  and  $h(e_1^* - e_2^*) D > \kappa(e_1^*) - \kappa(e_2^*)$ . Therefore, the *ex ante* Pareto ranking of equilibria with one-period delay and two-period delay could be different from the interim Pareto ranking of equilibria conditional on default.

We summarize the above discussion as the following proposition:

**Proposition 4** *The *ex ante* Pareto ranking of equilibria with one-period delay and two-period delay could be different from the interim Pareto ranking of equilibria.*

## 5 Policy Discussion

Drawing on the results that we have obtained so far, in what follows, we provide a brief evaluation of a number of policy initiatives that are in relation to the sovereign debt restructuring, abstracting away from the issue of

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<sup>16</sup>The latter condition came from  $h e_1^* D - \kappa(e_1^*) > h e_2^* D - \kappa(e_2^*)$ .

creditor coordination but focusing on delay generated by uncertain economic recovery and signalling about the sustainability concerns.

In principle, avoiding efficient delay due to waiting for economic recovery can be achieved by the state-contingent bond contracts<sup>17</sup>. With state-contingent bond contract, since the debtor and the creditor agree *ex ante* how to adjust payments if adverse shocks occur, there is, therefore, no need for both parties to renegotiate the contracts *ex post* should a negative shock takes place. Payments on sovereign debt would then be automatically adjusted upwards in good state (i.e. when output and economic growth are high), and adjusted downwards in bad state (i.e. during an economic recession and low growth). However, there are some broad problems associated with this type of debt contract. First, it is difficult to specify all potential shocks and contingencies in the contract and, second, since this type of contract provides sovereign debtor with insurance against specific contingencies, it could adversely affect the debtor's incentives and results in the problem of debtor moral hazard. These problems could perhaps explain why state-contingent bond contracts could be illiquid.

Argentina did finally issue GDP-linked bonds as a part of the debt swap. These bonds were initially greatly undervalued by the market at the time of issue but have subsequently been priced much more favorably. Could this be a good augury for future debt restructuring<sup>18</sup>?

In the model we propose in this paper, the IMF could help by providing information. When the growth prospects are not common knowledge, for example, the IMF could resolve uncertainty about the future growth. Likewise, when sustainability concerns are in dispute, the IMF could publish its own best estimates, which should remove the need for a Cautious debtor to use delay as a signal for sustainability consideration<sup>19</sup>.

Viewed from the perspective of our model, clearly both of these interventions directly address concerns relating to efficiency of debt restructuring conditional on default. However, their use could be limited by concerns relating to debtor moral hazard especially when the Pareto ranking of equilibria

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<sup>17</sup> See Shiller (2003), Caballero (2003), Borensztein and Mauro (2002) and Roubini and Setser (2004) for further discussion on state-contingent debt contracts.

<sup>18</sup> In this paper, we however, consider a non-contingent debt contract; thus the delay in the initial period is needed to permit the defaulted debtor's economy to recover from the recession and so that more resources could be available for debt settlement.

<sup>19</sup> Blustein (2005) provides discussion on the official sustainability assessments of Argentina made by the IMF but kept confidential.

is altered *ex ante* relative to their interim Pareto ranking conditional on default.

Nevertheless, in some situations, namely those where debtor payoffs are lower in the case of one-period delay relative to two-period delay, the information role of the IMF could improve *ex ante* efficiency. A potentially serious challenge to carrying out this informational role is that the IMF faces a conflict of interest: as a senior creditor, it presumably has an incentive to exaggerate sustainability requirements in favour of the debtor to minimize other claims on the debtor's resources. Would such induced compassion for the debtors not be checked by its creditor-dominated Executive Board? If not, this informational task could be delegated elsewhere, perhaps to the Inter-American Development Bank for cases of Latin American debt restructuring, for example. Moreover, conditional on default, the debtor may have an incentive to manipulate the information available to, and subsequently made public by, the IMF.

## 6 Conclusion

One of the striking aspects of the recent sovereign debt restructurings is that, conditional on default, long delay is positively correlated with size of haircut. In this paper, abstracting away from the issues of creditor coordination, we develop a bargaining model to account for this, highlighting the economic recovery and sustainability considerations as complementary reasons for delay.

With stochastic bargaining surplus and asymmetric information about the debtor's sustainability concern, we show that multi-period costly delay arises in a Perfect Bayesian equilibrium, initially reflecting economic recovery followed by signalling about sustainability. A key result of the model shows that prolonged delay is positively correlated with a large haircut. This result is supported by empirical evidence. We, then, examine the *ex ante* incentives of the sovereign debtor and find that the Pareto ranking of equilibria, conditional on default, can be altered once *ex ante* incentives are taken into account. Finally, we use our results for examining a number of policy proposals put forward to ensure an orderly restructuring of sovereign debts. We argue that the IMF could help by providing information on future growth prospect of the debtor country as well as publishing its best

estimates on the sovereign debtor's debt sustainability.

Combining the reasons for delay examined in this paper with those driven by creditor coordination and creditor moral hazard remains a topic for future research.

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## **Appendix A**

*Proof of Proposition 1*

We begin at  $t = 2$ . First, let us consider the case in which the debtor is Optimistic. Let  $x_L$  and  $x_O$  denote the payoffs for the Optimistic debtor when the creditor and the Optimistic debtor are the proposers, respectively and let  $\pi - x_L$  and  $\pi - x_O$  denote the share of the available bargaining surplus for the creditor when the creditor and the Optimistic debtor are the proposer, respectively. Using the standard arguments<sup>20</sup>, to compute the payoffs for the creditor and the Optimistic debtor, the following two equations need to be solved:

$$x_L = \frac{\delta}{2} (x_L + x_O), \quad (\text{A1})$$

$$\pi - x_O = \frac{\delta}{2} (\pi - x_L + \pi - x_O), \quad (\text{A2})$$

where  $\delta$  denotes the common discount factor. Equations (A1) and (A2) have a unique solution, given by  $(x_O^*, x_L^*)$ :

$$x_O^* = \frac{2 - \delta}{2} \pi \text{ and } x_L^* = \frac{\delta \pi}{2}. \quad (\text{A3})$$

According to (A3), at  $t = 2$ , if the Optimistic debtor is a proposer, she offers  $\pi - x_O^* = \frac{\delta \pi}{2}$  to the creditor and if it is the creditor who is the proposer, the creditor's offer is  $x_L^* = \frac{\delta \pi}{2}$ . Given that each negotiating party has an equal probability of being the proposer, it follows that the expected payoffs for the Optimistic debtor and the creditor at  $t = 2$  are given by  $(\frac{\pi}{2}, \frac{\pi}{2})$ , respectively.

Next, we consider the case in which the debtor is Cautious. The Cautious debtor has concern about the sustainability of any debt settlement. The presence of such sustainability constraint reduces the amount of bargaining surplus that is available for debt restructuring negotiation from  $\pi$  to  $\pi - \bar{s}$ . Let  $\tilde{x}_L$  and  $\tilde{x}_C$  denote the payoffs for the Cautious debtor when the creditor and the Cautious debtor are the proposers, respectively, and let  $(\pi - \bar{s}) - \tilde{x}_L$  and  $(\pi - \bar{s}) - \tilde{x}_C$  be the share of the available bargaining surplus for the creditor when the creditor and the Cautious debtor are the proposers, respectively. Using the same logic as above, when the Cautious debtor is the proposer, she always offer  $(\pi - \bar{s}) - \tilde{x}_C^* = \frac{\delta(\pi - \bar{s})}{2}$  to the creditor, while, if the

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<sup>20</sup>Such arguments are as follows. In the equilibrium, either party will agree to a debt restructuring proposal if the proposal offers the party at least as much in discounted present value term as it can expect to attain by waiting until the next period, given the strategies of both parties.

creditor is the proposer, his offer is  $\tilde{x}_L = \frac{\delta(\pi - \bar{s})}{2}$ . With an equal probability that each party is the proposer, the expected payoffs for the Cautious debtor and the creditor at  $t = 2$  are  $((\frac{\pi + \bar{s}}{2}), (\frac{\pi - \bar{s}}{2}))$ , respectively.

Next, we calculate the continuation values for each player as we move to the first period. Let  $E\pi = p\pi_L + (1 - p)\pi_H$  denote the expected size of  $\pi$ . If the debtor is Optimistic, the continuation values for the Optimistic debtor and the creditor are  $(\frac{\delta E\pi}{2}, \frac{\delta E\pi}{2})$ , respectively. However, if the debtor is Cautious, the continuation values for the Cautious debtor and the creditor are  $(\frac{\delta(E\pi + \bar{s})}{2}, \frac{\delta(E\pi - \bar{s})}{2})$ , respectively.

At  $t = 1$ , the sovereign debtor is a proposer. Let  $Eg$  denote the expected growth of the economy, where  $Eg = \frac{(E\pi - \pi_L)}{\pi_L}$  and  $r = \frac{1-\delta}{\delta}$ . In what follows, we present the condition for one-period delay when the debtor is Optimistic and Cautious, respectively.

For the Optimistic debtor, the best offer that she can make is the excess of the available bargaining surplus over his own continuation value given by  $\pi_L - \frac{\delta E\pi}{2}$ . If this offer falls below the creditor's continuation value,  $\frac{\delta E\pi}{2}$ , this offer will not be accepted. Formally, this condition for the first-period delay for the Optimistic debtor is given by

$$\pi_L - \frac{\delta E\pi}{2} < \frac{\delta E\pi}{2} \Leftrightarrow r < Eg.$$

For the Cautious debtor, the best offer that she can make is  $\pi_L - \frac{\delta(E\pi + \bar{s})}{2}$ . If this offer falls below the creditor's continuation value,  $\frac{\delta(E\pi - \bar{s})}{2}$ , this offer will not be accepted. Formally, the condition for the first-period delay for the Cautious debtor is given by

$$\pi_L - \frac{\delta(E\pi + \bar{s})}{2} < \frac{\delta(E\pi - \bar{s})}{2} \Leftrightarrow r < Eg.$$

Therefore, for either type of sovereign debtor, with relatively low prospect of economic growth, there will be no delay in the first period and an immediate agreement occurs. *Q.E.D.*

#### *Proof of Lemma 1*

We begin with the case in which the debtor is chosen to make an offer at  $t = 2$ . Let  $(x_2, \pi - x_2)$  denote the offer made by the debtor, where  $x_2$  and  $\pi - x_2$  are payoff for the debtor and the creditor's share of the bargaining surplus, respectively. Let  $\tilde{x}_2$  be the solution to  $\pi - x_2 = \frac{\delta(\pi - \bar{s})}{2}$ , where

$\frac{\delta(\pi-\bar{s})}{2}$  is the creditor's continuation value. By computation, it follows that  $\tilde{x}_2 = \left(\frac{2-\delta}{2}\right)\pi + \frac{\delta\bar{s}}{2}$ . Let  $x'_2$  be any positive number such that  $\pi - x'_2 < \frac{\delta(\pi-\bar{s})}{2}$ , where  $\tilde{x}_2 < x'_2 < \pi$ . It follows that  $x'_2 > \left(\frac{2-\delta}{2}\right)\pi + \frac{\delta\bar{s}}{2}$ .

In a mixed strategy<sup>21</sup>, the Optimistic debtor may randomize between a high offer  $\pi - \tilde{x}_2$  and a low offer  $\pi - x'_2$ . Suppose the Optimistic debtor offers  $(\tilde{x}_2, \pi - \tilde{x}_2)$  with a probability  $(1 - \beta)$  and offers  $(x'_2, \pi - x'_2)$  with a probability  $\beta$ , while the Cautious debtor offers  $(x'_2, \pi - x'_2)$  with a probability 1. Recall that the creditor's posterior belief at  $t = 3$ , is given by  $q_1 = \frac{\pi - \bar{s}}{\pi}$ . Updating the belief according to the Bayesian updating rule implies that, in the equilibrium, the Optimistic debtor offers  $(x'_2, \pi - x'_2)$  with a probability  $\beta$  such that  $q_1 = \frac{\beta q_0}{\beta q_0 + (1 - q_0)}$ , where  $q_0$  is the creditor's prior belief. It follows that

$$\frac{\beta q_0}{\beta q_0 + (1 - q_0)} = \frac{\pi - \bar{s}}{\pi}. \quad (\text{A4})$$

Solving equation (A4) for  $\beta$  yields:

$$\beta^* = \left(\frac{\pi - \bar{s}}{\bar{s}}\right) \left(\frac{1 - q_0}{q_0}\right).$$

To ensure that  $\beta^* < 1$ , it requires that

$$\left(\frac{\pi - \bar{s}}{\bar{s}}\right) \left(\frac{1 - q_0}{q_0}\right) < 1 \Leftrightarrow (1 - q_0) < \frac{\bar{s}}{\pi} \Leftrightarrow q_0 > \frac{\pi - \bar{s}}{\pi}. \quad (\text{A5})$$

Condition (A5) implies that, for a delay in the second period to exist, the sustainability concern needs to be sufficiently important or the creditor's prior that the debtor is Optimistic is sufficiently high since, in these two cases, the value of signalling is greater. As long as  $q_0 > \frac{\pi - \bar{s}}{\pi}$ , there exists  $\beta \in (0, 1)$  which solves equation (A4). For the creditor, by observing  $x'_2$ , it follows that  $q_1 = \frac{\pi - \bar{s}}{\pi}$ . Given  $q_1 = \frac{\pi - \bar{s}}{\pi}$ , the creditor is indifferent between accepting  $(\tilde{x}_2, \pi - \tilde{x}_2)$  and rejecting it (and obtains his continuation value).

From condition (A5), we have  $q_0 > \frac{\pi - \bar{s}}{\pi}$ . Let us denote the lower bound for  $q_0$  by  $\underline{q} = \frac{\pi - \bar{s}}{\pi}$ . It is obvious that  $q'_1 < \underline{q}$ . The creditor accepts  $(\tilde{x}_2, \pi - \tilde{x}_2)$  with a probability  $\gamma$  and rejects it with a probability  $(1 - \gamma)$ . After observing the offer  $(x'_2, \pi - x'_2)$ , the creditor rejects the debtor's offer with a probability 1. The debtor's expected payoff from an offer  $(\tilde{x}_2, \pi - \tilde{x}_2)$  is given by  $\gamma\tilde{x}_2 + (1 - \gamma)\left(\frac{\delta\pi}{2}\right)$ , where  $\frac{\delta\pi}{2}$  is the continuation value for the Optimistic debtor,

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<sup>21</sup>See Spence (1974) and Fudenberg and Tirole (1991).

while the debtor's expected payoff from an offer  $(x'_2, \pi - x'_2)$  is  $\frac{\delta(\pi + \bar{s})}{2}$  (with a probability 1 since the creditor chooses to reject this offer with certainty), which is the continuation value for the Cautious debtor. It follows that there exists  $\gamma \in (0, 1)$ , which is a solution to the following equation:

$$\gamma \tilde{x}_2 + (1 - \gamma) \left( \frac{\delta\pi}{2} \right) = \frac{\delta(\pi + \bar{s})}{2}. \quad (\text{A6})$$

Solving equation (A6) for  $\gamma$  yields:

$$\gamma^* = \frac{\delta\bar{s}}{2(\tilde{x}_2 - \frac{\delta\pi}{2})}.$$

Next, we consider the situation in which the creditor is chosen to be a proposer at  $t = 2$ . Suppose that  $q_0 > \frac{\pi - \bar{s}}{\pi}$ . The creditor offers  $(\tilde{x}_2, \pi - \tilde{x}_2)$  and the Optimistic debtor accepts the offer with a probability  $\beta^*$ , while the Cautious debtor rejects the creditor's offer with a probability 1. Given the preceding computations, it is clear that both types of debtor choose their best-response and, given the debtor's strategy, and given that the creditor's prior satisfies the condition  $q_0 > \frac{\pi - \bar{s}}{\pi}$ , the creditor cannot do better either: if the creditor makes an offer which gives him a payoff greater than  $\pi - \tilde{x}_2$ , such offer would be rejected with probability 1 by both types of debtor.

*Q.E.D.*

## Appendix B

*Scenario 1: The debtor knows her own type*

The continuation value for the Optimistic debtor is given by:

$$\delta \left[ \frac{1}{2} (\beta^* x'_2 + (1 - \beta^*) \tilde{x}_2) + \frac{1}{2} \left( \frac{\delta\pi}{2} \right) \right],$$

where the first term refers to the debtor's payoff when she is the proposer at  $t = 2$ , while the second term refers to the debtor's payoff when the creditor is a proposer at  $t = 2$ . Recall that an offer of  $(x'_2, \pi - x'_2)$  from the debtor is rejected by the creditor with a probability 1; therefore, the debtor obtains a payoff of  $\frac{\delta(\pi + \bar{s})}{2}$ . However, if the debtor makes an offer  $(\tilde{x}_2, \pi - \tilde{x}_2)$ , the creditor accepts the debtor's offer with a probability  $\gamma^*$  and rejects the debtor's offer with a probability  $(1 - \gamma^*)$ . It follows that the continuation

value for the Optimistic debtor is

$$\frac{\delta}{2} \left[ \beta^* \left( \frac{\delta(\pi + \bar{s})}{2} \right) + (1 - \beta^*) \left( \gamma^* \tilde{x}_2 + (1 - \gamma^*) \left( \frac{\delta\pi}{2} \right) \right) + \frac{\delta\pi}{2} \right].$$

After a simplification, we find that the Optimistic debtor's continuation value is given by  $\delta \left( \frac{\delta(\pi + \bar{s})}{4} \right)$ .

The continuation value for the Cautious debtor is  $\delta \left( \frac{\delta(\pi + \bar{s})}{2} \right)$  since, at  $t = 2$ , the Cautious debtor offers  $(x'_2, \pi - x'_2)$  with a probability 1 and the creditor rejects the debtor's offer of  $(x'_2, \pi - x'_2)$  with a probability 1.

Finally, the continuation value for the creditor is given by

$$\delta \left[ q_0 \left( \beta^* (\pi - x'_2) + (1 - \beta^*) (\pi - \tilde{x}_2) \right) + (1 - q_0) \left( \frac{\delta(\pi - \bar{s})}{2} \right) \right],$$

where the first term refers to the creditor's payoff if the debtor is Optimistic and the second term refers to the creditor's payoff if the debtor is Cautious. After a simplification and a substitution of the term  $\tilde{x}_2$ , we obtain:

$$\delta \left[ \frac{\delta(\pi - \bar{s})}{2} \left( 1 - \left( \frac{\pi - \bar{s}}{\bar{s}} \right) (1 - q_0) \right) + \left( \frac{\pi - \bar{s}}{\bar{s}} \right) (1 - q_0) (\pi - x'_2) \right].$$

*Scenario 2: The debtor does not know her own type*

Under this scenario, the creditor's continuation value is the same as in Scenario 1, but the debtor's continuation payoff is

$$q_0 \frac{\delta}{2} \left( \beta^* x'_2 + (1 - \beta^*) \tilde{x}_2 + \frac{\delta\pi}{2} \right) + (1 - q_0) \delta \left( \frac{\delta(\pi + \bar{s})}{2} \right),$$

which after substitution and simplification becomes

$$\frac{\delta^2(\pi + \bar{s})}{2} \left( \frac{2 - q_0}{2} \right).$$

In the first period, the sovereign debtor is a proposer. It follows from the two scenarios we considered above that the expected payoff for the creditor from rejecting the debtor's offer at  $t = 1$ ,  $\hat{a}$ , is given by

$$\hat{a} = \delta \left( \begin{array}{l} p \left[ q_0 (\beta^* (\pi_L - x'_2) + (1 - \beta^*) (\pi_L - \tilde{x}_2)) + (1 - q_0) \left( \frac{\delta(\pi_L - \bar{s})}{2} \right) \right] \\ + (1 - p) \left[ q_0 (\beta^* (\pi_H - x'_2) + (1 - \beta^*) (\pi_H - \tilde{x}_2)) + (1 - q_0) \left( \frac{\delta(\pi_H - \bar{s})}{2} \right) \right] \end{array} \right).$$

After a simplification, we obtain:

$$\hat{a} = \delta \left( \frac{\delta(E\pi - \bar{s})}{2} - (1 - q_0) \left( \frac{(E\pi - \bar{s})}{\bar{s}} \right) \left[ \frac{\delta(E\pi - \bar{s})}{2} - E\pi + Ex'_2 \right] \right),$$

where  $Ex'_2 > E\pi - \frac{\delta(E\pi - \bar{s})}{2}$ .