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Consumption Inequality and Discount Rate Heterogeneity

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Abstract

Although standard incomplete market models can account for the magnitude of the rise in consumption inequality over the life cycle, they generate unrealistically concave age profiles of consumption inequality and unrealistically less wealth inequality. In this paper, I investigate the role of discount rate heterogeneity on consumption inequality in the context of incomplete market life cycle models. The distribution of discount rates is estimated using moments from the wealth distribution. I find that the model with heterogeneous income profiles (HIP) and discount rate heterogeneity can successfully account for the empirical age profile of consumption inequality, both in its magnitude and in its non-concave shape. Generating realistic wealth inequality, this simulated model also highlights the importance of *ex ante* heterogeneities as main sources of life time inequality.

Keywords: consumption inequality, discount rate heterogeneity, life cycle, risk sharing, incomplete market

JEL Classification: D31 D91 E21

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1   Introduction

A large and growing literature has emphasized the importance of consumption inequality[1]. On the one hand, the usual roles of income and wealth are viewed as only means to consumption, and therefore the consumption inequality is a more direct measure of welfare inequality; on the other hand, consumption inequality, as an inverse measure of consumption insurance, can provide us with valuable information of the market structure and individuals' income processes. In particular, the rise of life cycle consumption dispersion observed in the U.S. data has strong implications for economics theories, as highlighted by Lucas (2003) in his American Economic Association presidential address:

The fanning out over time of the earnings and consumption distributions within a cohort that Angus Deaton and Christina Paxson (1994) document is striking evidence of a sizeable, uninsurable random walk component in earnings.

Lucas' statement is quantitatively verified by Storesletten et al. (2004) in a standard life cycle incomplete market model; since then the life cycle profile of consumption dispersion has become one of the crucial empirical targets in a variety of incomplete market models, including Guvenen (2007) with Bayesian learning, Huggett et al. (2011) with human capital accumulation, Kaplan (2012) with endogenous labour supply and Aguiar and Hurst (2012) with multiple consumption goods. In general, all these incomplete market models have succeeded in accounting for the rise in consumption inequality over the life cycle; and hence these models are employed by their authors to answer, explicitly or implicitly, some important welfare questions.

However, there remain two unsatisfactory discrepancies between the incomplete market models and the U.S. data:

1. These incomplete market models, except for Guvenen (2007)'s model with learning, generate consumption dispersion profiles more concave than the data. To put it bluntly, they have succeeded in matching the *magnitude*, but failed in matching the *shape*. This is a robust feature of the quantitative life cycle incomplete market models, because in the model the old agents' consumption would response less to shocks when approaching retirement age, thanks to the accumulation of savings for retirement.

2. These incomplete market models generate much less wealth inequality than the data. This is a common feature of the standard incomplete market models[2]. Granted, wealth dis-

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[1] Examples include: Deaton and Paxson (1994), Blundell and Preston (1998), Storesletten et al. (2004), Krueger and Perri (2006), Guvenen (2007), Blundell, Preston and Pistaferri (2008).

[2] The incomplete market model can match the empirical wealth inequality by using non-standard income

tribution *per se* is not the focus of these models and all model builders must keep a certain degree of parsimony. Nevertheless, a model's welfare implications regarding life cycle inequality would be seriously doubted, if the wealth inequality of the model agents were unrealistic and systematically downward biased.

The aim of this paper is to fix these two gaps. My conjecture is that these two gaps may be interrelated, and in this paper I explore the possibility of bridging one gap with the other. In particular, I relax the assumption of discount rate homogeneity in the standard incomplete market models. In the context of calibrated quantitative incomplete market models, I estimate the heterogeneous discount factors by matching the empirical moments in wealth distribution, and then I investigate if this estimated model can account for the empirical consumption inequality over the life cycle, in both the magnitude and the shape.

While the preference heterogeneity is well emphasized by applied microeconomists, the macroeconomic model builders are still reluctant to relax the assumption of homogeneous preferences, not because preference heterogeneity is not reasonable, but because it is unobservable. Discount rate is usually elicited from some indirect evidence either by experiments designed to reveal individual's preference (Barsky et al., 1997), or by Euler equation estimation (Lawrance 1991, Alan and Browning 2006, 2010) who measure the discount rate heterogeneity from the unexplained Euler equation residuals. Following Krusell and Smith (1998), it is well-known in the marcoeconomic literature that discount rate heterogeneity, though unobservable, can potentially play an important role in wealth distribution. In the context of life cycle models, discount rate heterogeneity is highlighted by Samwick (1998) for social security reforms, by Hendricks (2007a) for the joint distribution of retirement wealth and earnings and by Hendricks (2007b) for wealth inequality. However, the implication of discount rate heterogeneity on consumption is still neglected. To the author's knowledge, this paper is the first one to investigate the role of discount rate heterogeneity on consumption inequality in an incomplete market model. Perhaps the most related are two recent papers by Badel and Huggett (2012) and Sun (2013) on consumption inequality, both in the context of complete market models.

I begin with the estimation of the distribution of the discount rates in an incomplete market life cycle model. The additional moment I use is the fraction of old agents holding zero or negative wealth, which cannot be explained by a standard homogeneous discount rate life cycle model with realistic social security (Diamond and Housman 1984, Huggett 1996). As consumption inequality is the focus of the paper, I estimate the distribution of discount

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process (Castaneda et al. 2003) or by introducing the entrepreneur behavior and bequest motive (Cagetti and De Nardi 2005).

factors without taking any moments from the consumption data. To evaluate my estimates from another dimension, I also compute the wealth Gini coefficient for the whole population, for the retirement age people and for the coefficient of variance of the retirement wealth / lifetime earnings ratio. Comparing with the data, the difference is small. Interestingly, the estimate in the benchmark model of this paper is 3.5%, which is almost the same as Hendricks (2007a)'s estimate, despite the fact that we use different targets in the wealth distribution.

Model implications with different assumptions of preferences and income processes are compared. After a number of sensitivity analyses, I conclude that the quantitative models with discount rate heterogeneity and heterogeneous income profiles (HIP) can be successful in matching the empirical age profile of consumption inequality and therefore can successfully fix these two above gaps. Without additional information or *ad hoc* model specification, it is impossible for the econometricians to tell apart the predictable income change, such as the heterogeneous income growth, from the unpredictable income change, such as the permanent income shocks. The results of my model implies that the HIP model used by Lillard and Weiss (1979), Hause (1980), Baker (1997), Guvenen (2007, 2009) and Primiceri and Rens (2009) is consistent with the consumption data. Different from Gueven (2007), who resolves the puzzle of the concave age profile of consumption inequality by introducing an unobservable Bayesian learning parameter, I do not use any free parameter to match the consumption data. My model is more parsimonious than Guvenen (2007) and it generates a more realistic wealth inequality.

Finally, I use the simulated model to revisit a welfare question which was addressed by Keane and Wolpin (1997), Storesletten et al. (2004) and Huggett et al. (2012): which is more important to the lifetime welfare, initial condition or lifetime risks? Storesletten (2004) argue that ex post lifetime shocks are more important than initial conditions, while in the context of a schooling choice model (Keane and Wolpin 1997) and a human captital accumulation model (Huggett et al. 2011), the opposite is claimed to be true. I answer this question again in this calibrated incomplete market model with realistic wealth inequality and consumption dispersion. I find that the life-time inequality in welfare is mostly due to initial conditions. Among those initial conditions, the discount rate heterogeneity is more important than heterogeneous income profiles, which is in turn more important than the fixed effect of income.

The rest of the paper is organized as follows. Section 2 presents the layout of the model and calibrates the model without simulation. Section 3 estimates the distribution of discount rates. Section 4 reports the results of consumption inequality over the life cycle. Section 5 studies the welfare implications of the model through comparing the relative importance of the initial condition and lifetime risks. Section 6 concludes.

2 Model

2.1 The economy

The economy is populated by T overlapping generations, each of which consists of a continuum of agents. Each agent is born (enters the labour market) at age 1 and can live up to a maximum of T periods. Agents face mortality risks. The probability of surviving between age $t-1$ and age t is denoted by ξ_t, with $\xi_1 = 1$ and $\xi_{T+1} = 0$. The unconditional probability of being alive at age t is $s_t \equiv \prod_{\tau=1}^{t} \xi_\tau$. The measure of the new born agents is denoted by μ_1 and the population grows at a constant rate n, implying a stable population structure with $\mu_t = \mu_1 s_t (1+n)^{1-t}$.

Agent i's preference over consumption stream is given by $\sum_{t=1}^{T} \beta_i^{t-1} s_t c_{i,t}^{1-\gamma}/(1-\gamma)$, where β_i is the individual time discount factor, γ is the relative risk aversion and $c_{i,t}$ is the consumption of agent i at age t. When each individual i is born, her discount factor β_i is drawn from a distribution $F(\beta)$ with mean μ_β and variance σ_β^2. In the standard model where the discount rate heterogeneity is absent, $\sigma_\beta^2 = 0$.

Agents enter the labour market at age 1 and the mandatory retirement age is t_R. At working age $t < t_R$, the agents supply inelastically one unit of labour with different wages. The exogenous labour income of agent i is

$$y_{i,t} = (1-\tau)w_{i,t}, \tag{1}$$

where τ is the pension tax, $w_{i,t}$ is the wage income which is assumed to follow

$$\log w_{i,t} = \underbrace{\overline{w} + \kappa_t + \alpha_i + \theta^i t}_{\text{predictable part}} + \underbrace{z_{i,t} + \varepsilon_{i,t}}_{\text{idiosyncratic shocks}}, \tag{2}$$

$$z_{i,t} = \rho z_{i,t-1} + \eta_{i,t},$$

The wage income can be decomposed into a predictable part and an idiosyncratic shocks part. $\overline{w}$ is the average wage and κ_t is the average age profile of income, which is identical to all the agents of the same age. In the stationary equilibrium where there is only a cross-section of overlapping generations, there is no time effect and the concept of cohort and age coincide. The next two terms are heterogeneous in agents: α_i is the fixed effect which is predetermined before the agent enters the labour market, with the variance σ_α^2 ; θ_i is the heterogeneity in the growth rate of individual income, with the variance σ_θ^2. This income process nests both the Heterogeneous Income Profile (HIP) process where $\sigma_\theta^2 > 0$ and the Restricted Income Profile (RIP) process where $\sigma_\theta^2 = 0$. The idiosyncratic shocks part consists of a persistent part $z_{i,t}$ with $z_{i,0} = 0$, $\eta_{i,t} \sim N(0, \sigma_\eta^2)$ and a transitory (i.i.d.) part $\varepsilon_{i,t}$ with $\varepsilon_{i,t} \sim N(0, \sigma_\varepsilon^2)$. The persistent part becomes permanent when $\rho = 1$ or AR(1) when $0 < \rho < 1$.

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After retirement, the agent i receives pension $B_{i,t}$, which is funded by a Pay-As-You-Go system through the pension tax τ. According to the U.S. Old Age pension system, the pension is a concave function of lifetime average income. In computation, I will use the last period non-transitory income as a proxy of the average income to mimic the U.S. pension system.

The market is incomplete in the sense that agents can only have access to a risk-free bond which yields a constant interest rate r. The interest rate is set to be exogenous, but it is straightforward to assume a production function to close the model. In that case, the interest rate is a function of the parameters of aggregate capital and productivity. I use the standard assumption that there exist perfect annuity markets for mortality risks, so that the return of asset is the interest rate plus a survival premium. The agent i's budget constraint is given by

$$c_{i,t} + a_{i,t+1} \leq a_{i,t}(1+r)/\xi_t + y_{i,t}, \tag{3}$$

where $a_{i,t}$ is the asset or financial wealth. The agent cannot leave negative asset at year T and faces a borrowing constraint $a_{i,t+1} \geq \underline{a}_{i,t+1}$, where $\underline{a}_{i,t+1}$ is an ad hoc borrowing constraint which can potentially be a function of current state variables and can be set as low as the natural borrowing constraint. Each individual is endowed with initial wealth $a_{i,0}$.

For CRRA utility function, we can obtain the balanced growth path by dividing all the quantities by the accumulated productivity growth g. Given constant r and $\overline{w}$, each agent i's decision problem can be written recursively as

$$V_i(\alpha, \theta, a, \varepsilon, z; t) = \max_{a'} \left\{ c^{1-\gamma}/(1-\gamma) + \beta_i \xi_{t+1}(1+g)^{1-\gamma} E[V_i(\alpha, \theta, a', \varepsilon', z'; t+1)|z]\right\} \tag{4}$$

subject to

$$c + (1+g)a' \leq a(1+r)/\xi_t + \begin{cases} y_{i,t} & t < t_R \\ B_{i,t} & t \geq t_R \end{cases}$$

$$a' \geq \underline{a}_{i,t+1}$$

$$a_{T+1} \geq 0$$

The terminal period value function is set to $V(:; T+1) = 0$. The equilibrium we study is a stationary recursive competitive equilibrium where the factor prices are constant over time and the age-wealth distribution is stationary. The decision rule is solved by backward induction using Euler equation. To speed up the algorithm, I use the endogenous grid method developed by Carrol (2006). I use linear interpolation with 71 grid points for positive asset and 50 grids for negative asset. Grids on positive asset are formed triple exponentially to make more grids where asset level is lower. I use two discrete states for each of the exogenous state variables $(\alpha, \theta, \varepsilon, \eta)$. 50,000 agents are used in the simulation. In the unit root income process, I use

41 state space for the permanent component. In the AR(1) income process, I discretise the persistent shock by a 41 state Markov chain using the method suggested by Tauchen (1986).

2.2 Calibration

Demography The model period is 1 year. Agents begin to work at age 22, which coincides with age 1 in the model. Conditional on surviving, they then work for 45 years, retire at age 66 and die at age 100. Agents are interpreted as households in the data, and hence we chose the conditional surviving rate from the U.S. life table for *females* in 1989-1991. The annual population growth rate is set to $n = 1.0\%$ per year. The interest rate r is set exogenously to be 4%.

Preference The risk aversion is set to $\gamma = 2$. For the estimation of the distribution of β, I will leave it to the next section.

Income process: RIP Vs HIP So far, it is still an open question whether the restricted income profiles (RIP) or the heterogeneous income profiles (HIP) can better represent the household's income process in economic research[3]. I will not take a stand at this moment and use both RIP and HIP as the income process. In the RIP model, I use Storesletten et al. (2004) 's estimation. The coefficient of auto-regression ρ is very close to 1, which is 0.98. I set it to 1 in computation as Storesletten et al. (2004) did, and therefore the persistent shock is actually permanent. The variance of the fixed effect, persistent shock, transitory shocks are $\sigma_\alpha^2 = 0.2105$, $\sigma_\eta^2 = 0.0161$, $\sigma_\varepsilon^2 = 0.0630$, respectively. In the HIP model, I use the income process estimated by Guvenen (2007, 2009). The variance of the wage growth rate is $\sigma_\theta^2 = 0.00038$. The correlation coefficient between the idiosyncratic part of wage growth and the fixed effect is $cov(\theta, \alpha) = -0.002$. The variance of the fixed effect, persistent shock and transitory shock are $\sigma_\alpha^2 = 0.022$, $\sigma_\eta^2 = 0.029$, $\sigma_\varepsilon^2 = 0.047$, respectively. Unlike Guvenen (2007)'s learning story, I assume there is no prior uncertainty about the income process and thus the agent has complete information of her income profile when she enters the labour market.

The secular productivity growth rate is set to $g = 1.5\%$ per year. The average age profile of income κ_t is chosen to match the average income in the U.S. Census 1990.

Pension The pension system in the benchmark model is designed to mimic the U.S. Old Age pension system as follows:

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[3] See Guvenen (2009) for a summary.

$$B_i = \lambda \times \begin{cases} 0.9 y_{i,p,R-1} & \text{for } y_{i,p,R-1} < 0.3\overline{y}_{p,R-1} \\ 0.27\overline{y}_{p,R-1} + 0.32(y_{i,p,R-1} - 0.3\overline{y}_{p,R-1}) & \text{for } \overline{y}_{i,p,R-1} \in (0.3\overline{y}_{p,R-1}, 2\overline{y}_{p,R-1}] \\ 0.81\overline{y}_{p,R-1} + 0.15(y_{i,p,R-1} - 2\overline{y}_{p,R-1}) & \text{for } \overline{y}_{i,p,R-1} \in (2\overline{y}_{p,R-1}, 4.1\overline{y}_{p,R-1}] \\ 1.1\overline{y}_{p,R-1} & \text{for } \overline{y}_{i,p,R-1} > 4.1\overline{y}_{p,R-1} \end{cases}$$

where $y_{i,p,R-1}$ denotes the last working year income excluding the transitory part and $\overline{y}_{p,R-1}$ denotes the average. I use this income instead of the lifetime average income for computational convenience. Guvenen (2007) uses a similar expression where the last period income serves as the proxy for the average lifetime income. I exclude the transitory part because the last period transitory part gives us little information of the average life cycle income and the non-transitory part is still highly correlated with average life cycle income. I rescale the pension system to make the replacement ratio of the model match that of the U.S. data, which is 0.48. It generates 0.92 in the benchmark RIP model and 0.82 in the HIP model. The pension tax τ can be solved directly by the PAYG system, which is 0.1325. I will also discuss the extreme case when there is no pension.

Borrowing constraint In the benchmark setup for both RIP and HIP models, the households are allowed to borrow up to the expected income of next year $\underline{a}_{i,t+1} = -E_t(y_{i,t+1})$, which is the same as used in Storesletten et al. (2004). I also consider two other extreme cases: one is that all households are excluded from any borrowing, i.e. $\underline{a} = 0$; the other is that no ad hoc borrowing constraints is imposed and I only impose the terminal condition that agents cannot die in debt at age $T + 1$. In other words, I set the borrowing constraint as low as the natural borrowing constraint which is not binding in household's optimal solution with CRRA preference.

Initial wealth The initial wealth distribution is calibrated to mimic the wealth distribution of households under age 25 in SCF 1992 (Diaz-Gimenez et. al 1997). I approximate the initial wealth distribution by a log normal distribution whose mean is set to match the initial wealth/income ratio, which is 0.89, and then I calibrate its variance to match the wealth Gini for those young households, which is 0.87.

The above parameters for calibration are partly summarized in Table 1.

Parameter	Value
Relative Risk Aversion	$\gamma = 2$
Mortality Risk	U.S. Female Life table 1991
Interest Rate	$R = 1.04$
Population Growth	$n = 1\%$
Secular Growth	$g = 1.5\%$
Average Wage Growth	U.S. Census 1990
Restricted Income Process	$\rho = 1, \sigma_\alpha^2 = 0.2105, \sigma_\eta^2 = 0.0161, \sigma_\varepsilon^2 = 0.0630$
Heterogeneous Income Process	$\rho = 0.821, \sigma_\alpha^2 = 0.022, \sigma_\eta^2 = 0.029, \sigma_\varepsilon^2 = 0.047$
	$\sigma_\theta^2 = 0.00038, \ cov(\theta, \alpha) = -0.002$
Pension Tax in PAYG	$\tau = 0.1325$
Borrowing Constraints	$\underline{a}_{i,t+1} = -E_t y_{i,t+1}$

3 Estimation of $F(\beta)$

3.1 Methodology

The distribution of β_i is crucial in the model. In a standard model without discount rate heterogeneity, the variance of β_i is restricted to be zero and the conventional procedure of calibrating β is to minimize the distance between the wealth/income ratio in the simulated model and that of the data. In the model with discount rate heterogeneity, however, we need additional moments to identify the distribution of β_i.

The estimation strategy I use is in the spirit of Hendricks (2007a). Besides the normal target of wealth/income, I focus on one particular statistics of the wealth distribution: the fraction of old agents holding zero or negative wealth. Because of the life cycle motive of saving, the standard life cycle models have difficulty in generating significant amount of agents with zero or negative wealth when retirement is approaching, which is at odds with the data (Diamond and Housman 1984)[4]. This model discrepancy with the data implies that there might be a significant amount of sufficiently impatient agent. Hence, it gives us useful information in calibrating the distribution of β_i. Specifically, I choose the fraction of agents holding zero or negative wealth from age 55 to 64 as another target for calibration. In the U.S. data from SCF 1992, this number is 8.9% (Weicher 1997)[5]. My estimation strategy is simpler than that of Hendricks (2007a), who targets the whole age profile of wealth Gini coefficient. I will compare

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[4] Huggett (1996)'s incomplete market model can match this fraction, but his results hinge on the existence of a unrealistically generous lump-sum pension and loose borrowing constraints. Hubbard et al. (1995) argue that this puzzle can be resolved by introducing the asset-based means-tested social insurance system, but it seems not plausible to assume that the main reason why the old households hold non-positive wealth is to get qualified for government transfers.

[5] Diamond and Hausman (1984) calculate the fraction of individuals holding zero or negative wealth in a sample of men aged 45-59. Their result is 7%.

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I approximate the distribution of discount factor $F(\beta)$ by a discrete distribution with two values $\beta_l = \overline{\beta}(1 - \Delta\beta)$ and $\beta_h = \overline{\beta}(1 + \Delta\beta)$. The probability of being an relatively impatient agent is p_l, and the probability of being a relatively patient household is $p_h = 1 - p_l$. The middle value of the discount rate, $\overline{\beta}$, is assumed to be equal to the calibrated value of β in a standard model without discount rate heterogeneity to match the wealth income ratio in the U.S. of 3.1 from SCF 1992 (Diaz-Gimenez et al. 1997). Alternatively, I will also consider the wealth/income ratio of 4.56, which is the average of SCF 1992 and SCF 1998 (Diaz-Gimenez et al. 2002). The two targets for estimating $\Delta\beta$ and p_l are the wealth income ratio again and the average fraction of non-positive wealth household from age 55 to age 64. To save the time for computation, $\Delta\beta$ is assumed to lie on 30 grids, from 0.005 to 0.15, and p_l is assumed to lie on 100 grids, from 1% to 99%. $\Delta\beta$ and p_l are chosen to minimize the loss function which is the sum of the absolute value of the percentage deviation between data and model in these two targets.

3.2 Estimation results

The estimations of β_i are summarized in Table 2. I compute the mean and standard deviation of β_i, which are different, though not far, from $\overline{\beta}$ and $\Delta\beta$. The estimation results will be discussed in the next section.

TABLE 2: ESTIMATION OF DISTRIBUTION OF DISCOUNT RATE

	RIP Model			HIP Model		
	$\overline{\beta}$	$\Delta\beta$	p_l	$\overline{\beta}$	$\Delta\beta$	p_l
Benchmark	0.9870	0.030	64%	0.9972	0.035	66%
No pension	0.9559	0.085	81%	0.9566	0.080	78%
No borrowing	0.9878	0.090	81%	0.9947	0.095	84%
NBC	0.9918	0.020	56%	1.0120	0.015	55%
High Wealth	1.0035	0.045	62%	1.0133	0.050	68%
Alan and Browning (2006)	0.9359	0.087	50%	0.9447	0.087	50%
Hendricks(2007a)	0.9873	0.035	61%	0.9955	0.035	61%

The result shows that $\Delta\beta$ and p_l varies in different model setups. In the benckmark RIP and HIP models, $\Delta\beta$ is 0.03 and 0.035, respectively. In general, the estimates for RIP models is close to those for the HIP models. The middle value of discount factors, $\overline{\beta}$, is higher in HIP models than in RIP models, because a less persistent shock reduces the precautionary motive for saving and thus lowers the wealth income ratio. To match the wealth income ratio, $\overline{\beta}$ has to be increased. When there is no pension or when there is no borrowing, the standard models without discount rate heterogeneity generate too few old agents holding zero

or negative wealth. Increasing both the fraction and the value of relatively impatient agents helps to match this moment. p_l is greater than 50% in all estimation, because discount rate heterogeneity will increase the mean wealth of the economy. To make $\overline{\beta}$ unaltered, the fraction of impatient agents has to be increased.

For comparison, I also report the estimates of mean and variance of β_i from the estimation of Hendricks (2007a) and Alan and Browning (2006)[6]. Simple as it is, my estimation of distribution of β_i in both RIP and HIP benchmark model are very close to the estimation by Hendricks (2007a).

The model-generated fraction of zero or negative wealth households is plotted in Figure 1. We can see that introducing discount factors help the standard HIP or RIP models match this statistics in the data.
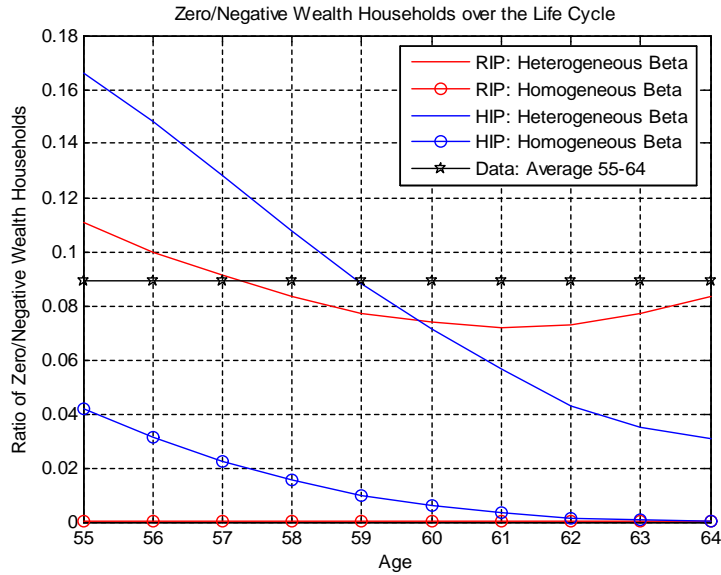

Figure 1: Fraction of Zero/negative Wealth Households (Age 55 – Age 64)

3.3 Other statistics of wealth distribution

Since I only use one particular statistics of wealth distribution, it is informative to see how the model performs in other statistics of wealth distribution. In Table 3, I compute the wealth Gini for the whole population, for the retirement age people and for the coefficient of variance of the retirement wealth / lifetime earnings ratio.

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[6]I take the estimation of β in the no bequests model of Hendricks(2007a Table 2). Since he use 5 grids for β_i and set β to be 2% or 6% lower or higher than $\overline{\beta}$, I set p_l to the fraction of the agents with 0.94 or 0.98 $\overline{\beta}$ plus half of the fraction of agents with $\overline{\beta}$, which gives $p_l = 61\%$. I set σ to be half of the difference between weighted average of the lowe βs and the high βs, which is 0.035. For Alan and Browning (2006), I assume a symmetric distribution and take its coefficient of variance as σ, which is 0.087. In both cases, I recalibrate the $\overline{\beta}$ to match the wealth income ratio.

	Gini Retirement	Gini All	$\text{CV}_{W/E}$ Retirement
Data	0.62	0.76	1.32
RIP Benchmark	0.65	0.85	1.10
HIP Benchmark	0.52	0.80	1.17

Note: Data is from Hendriks (2007b).

$\text{CV}_{W/E}$ is the coefficient of variance of the retirement wealth/life time earning ratio.

The results show that the RIP model has the Gini coefficient of retirement very close to data. The HIP model has lower Gini coefficients for retirement age people, but it matches the data well in the Gini coefficient for the whole population. The coefficients of variance of both models are lower than the data, but not much. This suggests that my choice of the target statistics of the wealth distribution can generate a realistic wealth inequality.

4 Consumption Inequality

4.1 Empirical evidence

The empirical evidence of the age profile of consumption inequality is drawn from the Consumer Expenditure Survey (CEX) data in the U.S. by most authors. The exact estimates by different authors differ because of the sample selection, the controls for time effect or cohort effect, and the controls for family size. Deaton and Paxson (1994) control for the cohort effect and find that the variance of log consumption increase by 0.25 log points from age 25 to age 65. Their sample periods are before early 1990s. Using slightly different sample, Guvenen (2007) find that the life cycle consumption dispersion increases by 0.21 log points. Heathcote et al. (2005) argue that controlling for time effect and introducing longer sample periods would change the estimation results, and therefore the recent estimation of Heathcote et al. (2010) use CEX waves from 1980 to 2006 and control for time effects. They only calculate the average age group for five years and only report the data until age 60, and the increase of consumption dispersion is 0.06 log points. In another recent study with CEX waves of 1980-2003, Aguiar and Hurst (2012) report an increase of 0.14 log points from age 25 to age 65. They control for the cohort effects and use different controls for family size.

Despite the differences in the exact estimates, the consensus is: the variance of log consumption increases significantly over the life cycle and the shape is roughly non-concave. To joint the debate of HIP vs RIP income models in a similar empirical context, I bring my model to the data of Guvenen (2007), whose shape the standard incomplete market models fail to match and the Bayesian learning model in Guvenen (2007) has succeeded to match. I also plot the age profile from Deaton and Paxson (1994), which is used by Storesletten et al. (2004).

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4.2 Model Vs Data

I consider two benchmark models, one for RIP and one for HIP, where pension is included and borrowing limit is set to the next year's expected income. Age profiles of consumption inequality in both RIP and HIP benchmark models from age 25 to age 65 are shown in Figure 2. Since we are interested in the increase of consumption inequality, not its levels, I normalize the consumption inequality for age 25 to be zero.
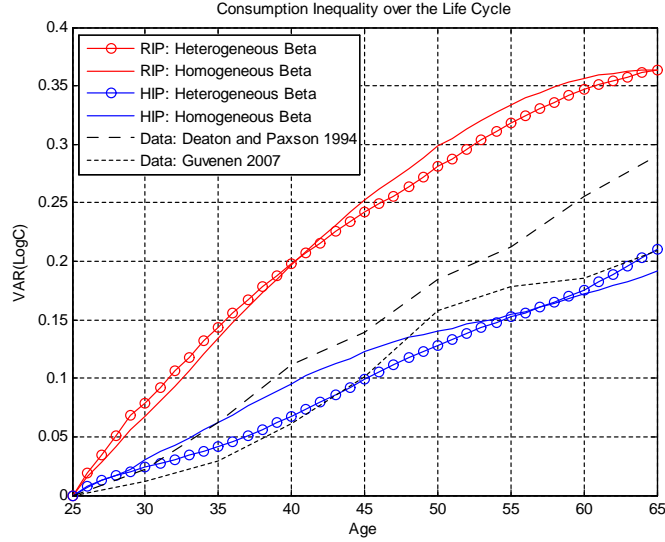

Figure 2: Age Profile of Consumption Inequality: Model Vs Data

Without discount rate heterogeneity, the consumption inequality in both the RIP model and HIP model increase over the life cycle and has a concave age profile. In RIP model, the lifetime increase of consumption inequality is higher than data; in HIP model, the lifetime increase of consumption is less significant and is lower than data. As Lucas (2003) and Storesletten et al.(2004) argued, this evidence of consumption inequality favors the RIP model in which the labour market income process is highly persistent.

With discount rate heterogeneity, the consumption inequality is tilted up, more so from age 50 on. In the RIP model, although the shape is not significantly linear, it does make the profile more convex in the last ten working years. There is no effect of the increase of consumption inequality in the RIP model.

In the HIP model, introducing discount rate heterogeneity makes the consumption inequality increase more. The profile with discount rate heterogeneity becomes approximately linear. HIP model with discount heterogeneity does a good job in accounting for the consumption inequality data, especially for Guvenen's estimation. Unlike Guvenen (2007), the success of

HIP model is not a result of gradual Bayesian learning. Therefore, the key message from this accounting exercise is: without changing the structure of the plain-vanilla life cycle model, adding well-estimated discount rate heterogeneity to a HIP model would not only generate a realistic wealth distribution but also simultaneously account for the age profile of the consumption dispersion, both in magnitude and in shape.

4.3 Where does the shape come from?

To single out the effect of discount rate heterogeneity, I shut down all the income shocks in the simulated model to study the pure effect of discount rate heterogeneity on the age profile of consumption dispersion. This is very close to the case of complete markets, except for the possible binding borrowing constraints. The residual becomes the consumption dispersion purely due to income shocks. Figure 3 and Figure 4 report the results of this decomposition. The pure effect of discount rate heterogeneity is U-shaped, which is consistent with the finding in a complete market model (Sun 2013). It is clear that most of the increase of the consumption inequality of the last ten working years is attributed to discount rate heterogeneity. Comparing to the homogeneous discount models where the retirement wealth of the old agents is too much to increase the consumption dispersion, it is exactly this property of the discount rate heterogeneity model that resolves the excess concavity puzzle of the standard models.
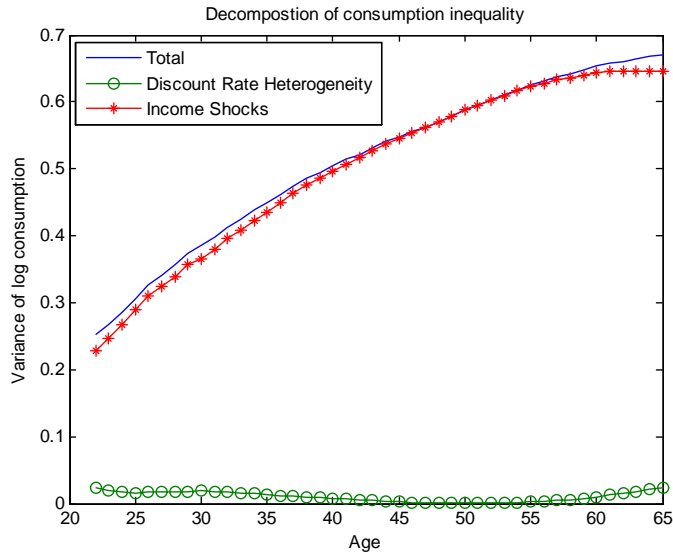

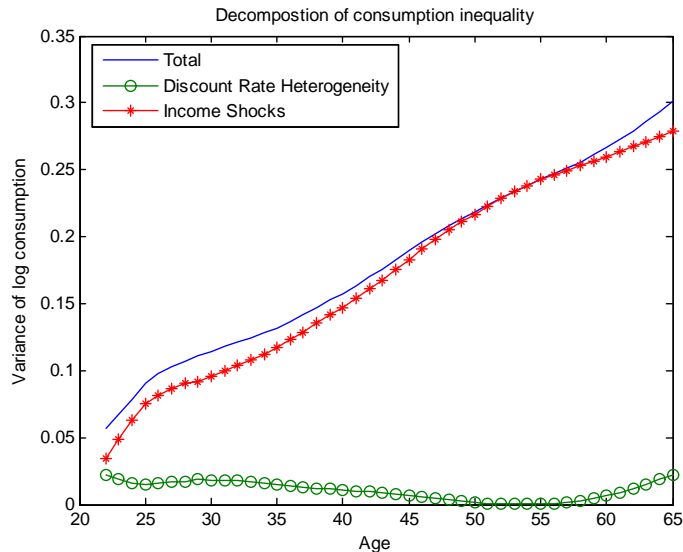Figure 3: Decomposition of Consumption Inequality: RIP Model

Figure 4: Decomposition of Consumption Inequality: HIP Model

4.4 Sensitivity Analysis

I have shown that the benchmark HIP model with discount rate heterogeneity has done a good job in matching the age profile of consumption inequality. Beyond the benchmark models, I will study four different setups: excluding pension, without borrowing constraints, natural borrowing constraints and high wealth/income ratio. For each case, I re-estimate the distribution of β_i, which is reported in Table 1. The age profiles of consumption inequality for each setup in RIP and HIP models are reported in Figure 5 and Figure 6, respectively.

Role of pension Figure 5 and 6 tell us that the shape of consumption inequality would become more convex if pension were excluded. In other words, including pension mitigates the effect of discount rate heterogeneity, especially for the old agents. Without pension, the old households are more willing to save for retirement. To match the model ratio of households holding zero or negative wealth to the data, it requires higher degree of discount rate heterogeneity and higher fraction of relatively impatient households, as we have seen in Table 2. This drives up the consumption inequality between different household groups.
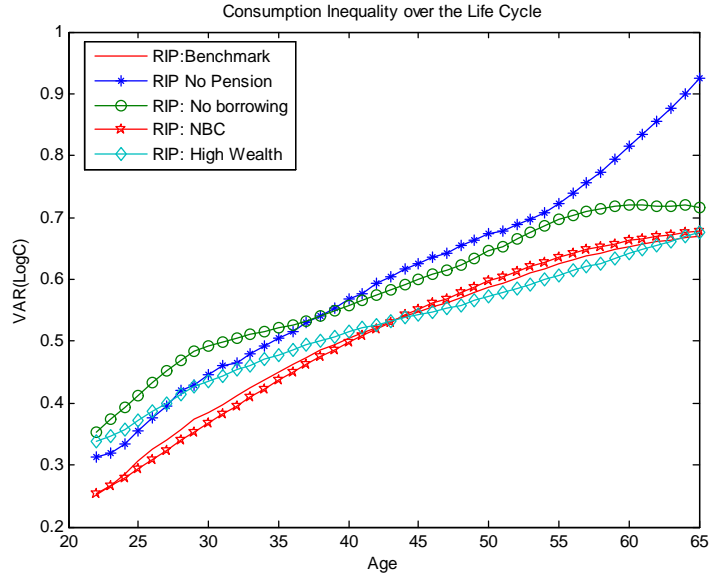
14

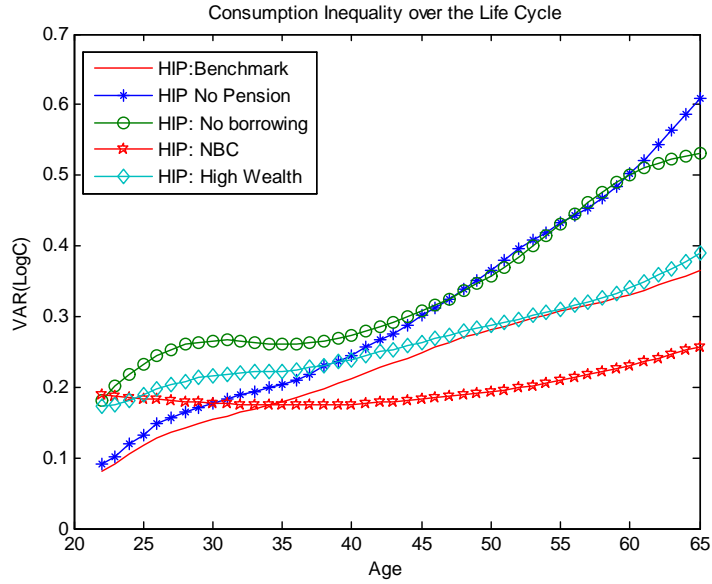Figure 5: Age Profile of Consumption Inequality: RIP Model

Figure 6: Age Profile of Consumption Inequality: HIP Model

Role of wealth-income ratio Although the choice of wealth-income ratio change the distribution of β_i, Figure 5 and 6 show that it gives negligible effect on the age profiles of consumption inequality. This result is different from the standard models with homogeneous discount rates, where the wealth-income ratio is important. In the models with discount rate heterogeneity, higher wealth-income ratio has two effects. Normally it increases $\overline{\beta}$ and thus makes the household holding more wealth for self-insurance, which lowers the consumption

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inequality. On the other hand, when $\overline{\beta}$ is high, $\Delta\beta$ has to be increased to match the fraction of zero or negative wealth agents, which increases the consumption inequality. The results show that these two effects almost cancel each other out, and therefore the consumption inequality is not very sensitive to the choice of wealth-income ratio in the models with discount heterogeneity.

Role of borrowing constraints As to the sensitivity of borrowing constraints, I consider two extreme cases: no borrowing and natural borrowing constraints (NBC). In Figure 5 and 6, we can see the effect of excluding borrowing is to tilt up the consumption inequality profile and is very close to the profile of no-pension economy for most part of the life cycle. Intuitively, excluding borrowing has a similar effect as excluding pension, which makes the households save more and thus requires higher degree of discount rate heterogeneity to match the data. The former is the precautionary saving motive and the latter is the life cycle saving motive.

Interestingly, the shape of profiles in the no borrowing case becomes concave in the last ten working years. It is because of the with-in group inequality of the low β households decreases when approaching retirement. For some of the low β households, they are too impatient to save, even when retirement is near. They are willing to borrow in the no borrowing case, which causes the borrowing constraints of those agents to be binding. Therefore, the consumption for the borrowing constrained households is lower than that derived from the optimal consumption rule when the borrowing constraint is not binding. When approaching retirement, some of the previously borrowing constrained agents become unconstrained. Since this increase of consumption happens to the households with relatively lower consumption level, the with-in group consumption inequality decreases.

When there are only natural borrowing constraints, in the RIP model the result does not differ much from the benchmark case, because the borrowing constraints of expected next year's income would be hardly binding for any households. In the HIP model, some agents with lower wage growth rate are more likely to hold zero or negative wealth and the borrowing constraints in the benchmark model are more often binding. Hence, the consumption inequality profile goes in the opposite direction as the no borrowing case. It is U-shaped, which is similar to the previous decomposition with no idiosyncratic shocks.

The choice of borrowing constraints would not be a problem for the standard model without discount rate heterogeneity, since the fact that in the model no one will be borrowing constrained near retirement makes the borrowing constraints quantitatively less important for consumption inequality. If there does exist discount rate heterogeneity, however, the assumption about borrowing constraints is not innocuous and thus the borrowing constraint should

be chosen carefully.

5 Initial condition Vs Life-time risk

Difference quantitative models deliver different sources of life-time inequality. In models with discount rate heterogeneity and heterogeneous income profiles, there are possibly four sources of inequality: fixed effect, individual income growth rate, individual discount rate and realization of income shocks. These four different dimensions of inequality contribute in different amount to the lifetime inequality of consumption, wealth, and welfare.

The lifetime money equivalent welfare can be defined as

$$M = u^{-1}\left[\frac{(1-\beta_i)E\sum_{t=1}^{T}\beta_i^{t-1}u(c_{i,t})}{1-\beta_i^T}\right] \tag{5}$$

Note that in the standard lifetime welfare calculation, the welfare is higher for the household with higher β, even if the consumption streams are identical. To remove this direct effect, I discount the life-time welfare by $\frac{1-\beta_i}{1-\beta_i^T}$. The total variance of welfare can be decomposed into with-group and between-group variance as follows

$$VarM = E(Var(M|x)) + Var(E(M|x)) \tag{6}$$

where $x = \alpha, \beta, \theta$ or all of these three variables. The results are summarized in Table 4.

TABLE 4: DECOMPOSITION OF LIFE TIME WELFARE: INITIAL CONDITIONS VS SHOCKS					
	Initial Conditions				Income Shocks
	Only a	Only θ	Only β	All	
HIP heterogeneous β	1%	62%	80%	96%	4%
HIP homogeneous β	14%	54%	-	86%	14%
RIP heterogeneous β	16%	-	48%	96%	4%
RIP homogeneous β	40%	-	-	40%	60%

The contribution of fixed effect to the RIP model with homogeneous discount rate is 40%, which is close to the estimate of 0.47 in Storesletten et al. (2004). When discount rate heterogeneity is present, the fixed effect can only account for 16% percent of total inequality and the total variance accounted for by initial condition is 96%, which is close to the estimate of 0.9 in Keane and Wolpin (1997) and higher than 64% in Huggett et al. (2011). In the HIP model, the heterogeneous income profile alone can account for 62% for the lifetime inequality and the heterogeneous discount factor alone can account for 80% of the total inequality. In short, the initial condition becomes more important when there is *ex ante* heterogeneity in either

discount rates or income profiles. Among the initial conditions, the discount rate heterogeneity is the most important, and the fixed effect is the least important. Generating realistic wealth inequality, this simulated model highlights the importance of *ex ante* heterogeneities as main sources of life time inequality.

6 Concluding Remarks

In this paper, I investigate the role of discount rate heterogeneity on consumption inequality in the context of incomplete market life cycle models. The major effect of the discount rate heterogeneity is to make the consumption inequality profile more convex, especially for the household when approaching retirement. Since in the U.S. data the age profile of consumption inequality is non-concave and in the standard model it increases concavely, the presence of discount rate heterogeneity brings the model closer to the data.

To generate a more realistic wealth inequality than the standard incomplete market models, I estimate the distribution of discount rates using moments from the wealth distribution. I find that the model with heterogeneous income profiles (HIP) and discount rate heterogeneity can successfully account for the empirical age profile of consumption inequality, both in its magnitude and in its non-concave shape. Different from Guvenen (2007), who resolves the puzzle of the concave age profile of consumption inequality by introducing an unobserbable Bayesian learning parameter, I do not use any free parameter to match the consumption data and my model also generates a more realistic wealth inequality.

When discount rate heterogeneity is present, the borrowing constraints and pension become more important for consumption inequality. These caveats must be noticed when studying consumption inequality in standard models with homogeneous preferences. As to the normative question asked by Keane and Wolpin (1997) , Storesletten et al. (2004) and Huggett et al. (2011), the simulated model with realistic wealth inequality also highlights the importance of *ex ante* heterogeneities as main sources of life time inequality.

As Browning et al. (1999) warned us, using micro data to calibrate the preference parameters in quantitative models may have potential flaws and has to be taken carefully, especially when there is preference heterogeneity. This paper, among a few others, takes this view seriously. Without changing the structure of the plain-vanilla life cycle model, adding well-estimated discount rate heterogeneity gives us important implications for the consumption inequality, for the nature of income processes and for the lifetime welfare.

References

[1] Alan, Sule and Martin Browning, 2006. Estimating Intertemporal Allocation Parameters using Simulated Expectation Errors, University of Oxford, Department of Economics Discussion Paper Series 284.

[2] Alan, Sule and Martin Browning, 2010. Estimating Inter-temporal Allocation Parameters using Synthetic Residual Estimation, *Review of Economic Studies* **77**, 1231-1261.

[3] Aguiar, Mark. and Hurst, Erik, 2012. Deconstructing Lifecycle Expenditure. Mimeo.

[4] Badel, Alejandro and Mark Huggett, 2012. Interpreting life cycle Inequality Patterns as an Efficient Allocation: Mission Impossible?, Federal Reserve Bank of St. Louis Work Papers.

[5] Baker, Michael, 1997. Growth-Rate Heterogeneity and the Covariance Structure of Life Cycle Earnings, *Journal of Labor Economics* **15**, 338-375.

[6] Barsky, Robert, Thomas Juster, Miles Kimball and Matthew Shapiro, 1997. Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study, *Quarterly Journal of Economics* **112**, 537-579.

[7] Blundell, Richard, Luigi Pistaferri, and Ian Preston, 2008. Consumption dispersion and Partial Insurance, *American Economic Review* **98**, 1887-1921.

[8] Blundell, Richard and Ian Preston, 1998. Consumption Inequality and Income Uncertainty, *Quarterly Journal of Economics* **113**, 603-640.

[9] Browning, Martin, Lars Peter Hansen and James J. Heckman, 1999. Micro Data and General Equilibrium Models, Handbook of Macroeconomics.

[10] Cagetti, Marco and Mariacristina De Nardi, 2005. Entrepreneurship, Frictions and Wealth, *Journal of Political Economy* **114**, 835-870.

[11] Carrol, Christopher, 2006. The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems. *Economics Letters* **91**, 312–320.

[12] Castaneda, Ana, Javier Diaz-Gimenez and Jose-Victor Rios-Rull, 2003. Accounting for Earnings and Wealth Inequality, *Journal of Political Economy* **111**, 818-857.

[13] Deaton, Angus and Christina Paxson, 1994. Intertemporal Choice and Inequality, *Journal of Political Economy* **102**, 384-394.

[14] Diaz-Gimenez, Javier, Vincenzo Quadrini, and Jose-Victor Rios-Rull, 1997. Dimensions of Inequality: Facts on the U.S. Distribution of Earnings, Income and Wealth, *Federal Reserve Bank of Minneapolis Quarterly Review*, 3–21.

[15] Diaz-Gimenez, Javier, Vincenzo Quadrini, and Jose-Victor Rios-Rull, 2002. Updated Facts on the U.S. Distribution of Earnings, Income and Wealth, *Federal Reserve Bank of Minneapolis Quarterly Review*, 2–35.

[16] Diamond, P. A. & Hausman, J. A., 1984. Individual Retirement and Savings Behavior, *Journal of Public Economics* **23**, 81-114.

[17] Guvenen, Fatih, 2007. Learning your Earning: Are Labor Income Shocks Really Very Persistent?, *American Economic Review* **97**, 687-712.

[18] Guvenen, Fatih, 2009. An Empirical Investigation of Labor Income Processes, *Review of Economic Dynamics* **12**, 58-79.

[19] Hause, John, 1980. The Fine Structure of Earnings and the On-the-Job Training Hypothesis, *Econometrica* **48**, 1013-1029.

[20] Heathcote, Jonathan, Fabrizio Perri and Giovanni L. Violante, 2010. Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States, 1967-2006, *Review of Economic Dynamics* **13**, 15-51.

[21] Heathcote, Jonathan, Kjetil Storesletten and Giovanni L. Violante, 2005. Two Views of Inequality Over the life cycle, *Journal of the European Economic Association* **3**, 765-775.

[22] Hendricks, Lutz, 2007a. How Important is Discount Rate Heterogeneity for Wealth Inequality?, *Journal of Economic Dynamics & Control* **31**, 3042-3068.

[23] Hendricks, Lutz, 2007b, Retirement Wealth and Lifetime Earnings, *International Economic Review*, **48**, 421-456.

[24] Hubbard, Glenn, Jonathan Skinner and Stephen Zeldes, 1995. Precautionary Saving and Social Insurance **103**, 360-399.

[25] Huggett, Mark, 1996. Wealth Distribution in Life Cycle Economies, *Journal of Monetary Economics* **38**, 469-94.

[26] Huggett, Mark, Gustavo Ventura and Amir Yaron, 2011. Sources of Lifetime Inequality, *American Economic Review* **101**, 2923-54.

[27] Kaplan, Greg, 2012, Inequality and the Lifecycle. *Quantitative Economics* **3**, 471-525.

[28] Keane, Michael. and Kenneth Wolpin, 1997. The Career Decisions of Young Men, *Journal of Political Economy* **105**, 473-522.

[29] Krueger, Dirk and Fabrizio Perri, 2006. Does Income Inequality Lead to Consumption Inequality? Evidenceand Theory, *Review of Economic Studies* **73**, 163-193.

[30] Krusell, Per and Anthony A. Smith Jr., 1998. Income and Wealth Heterogeneity in the Macroeconomy, *Journal of Political Economy* **106**, 867-896.

[31] Lawrance, Emily, 1991. Poverty and the Rate of Time Preference: Evidence from Panel Data, *Journal of Political Economy* **99**, 54–77.

[32] Lillard, Lee A. and Yoram A.Weiss, 1979. Components of Variation in Panel Earnings Data: American Scientists, 1960-70, *Econometrica* **47**, 437-54.

[33] Lucas, Robert E., Jr., 2003. Macroeconomic Priorities, *American Economic Review* **93**, 1-14.

[34] Primiceri, Giorgio and Thijs van Rens, 2009. Heterogeneous life cycle Profiles, Income Risk and consumption dispersion, *Journal of Monetary Economics* **56**, 20-39.

[35] Samwick, Andrew, 1998. Discount Rate Heterogeneity and Social Security Reform, *Journal of Development Economics* **57**, 117–146.

[36] Storesletten, Kjetil, Christopher Telmer and Amir Yaron, 2004. Consumption and Risk Sharing Over the Life Cycle, *Journal of Monetary Economics* **51**, 609-633.

[37] Sun, Gang, 2013. Complete Markets Strikes Back: Revisiting Risk Sharing Tests under Discount Rate Heterogeneity, Working Paper.

[38] Tauchen, George, 1986. Finite State Markov-chain Approximations to Univariate and Vector Autoregressions, *Economics Letters* **20**, 177–181.

[39] Weicher, John, 1997. The Rich and the Poor: Demographics of the U.S. Wealth Distribution, *Federal Reserve Bank of St. Louis Review*, 25-37.