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Complete Markets Strikes Back: Revisiting Risk Sharing Tests under Discount Rate Heterogeneity

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Abstract

Recent risk sharing tests strongly reject the hypothesis of complete markets, because in the data: (1) the individual consumption comoves with income and (2) the consumption dispersion increases over the life cycle. In this paper, I revisit the implications of these risk sharing tests in the context of a complete market model with discount rate heterogeneity, which is extended to introduce the individual choices of effort in education. I find that a complete market model with discount rate heterogeneity can pass both types of the risk sharing tests. The endogenous positive correlation between income growth rate and patience makes the individual consumption comove with income, even if the markets are complete. I also show that this model is quantitatively admissible to account for both the observed comovement of consumption and income and the increase of consumption dispersion over the life cycle.

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1 Introduction

The hypothesis of complete markets, which is widely used in macroeconomics and finance, has recently been strongly rejected by a number of risk sharing tests which exploit the micro data sets of consumption and income distributions. Some authors (Attanasio and Davis 1996, Blundell, Preston and Pistaferri 2008, BPP hereafter), using constructed panel data of consumption and income, find that the individual or group consumption comoves significantly with income. Others, using consumption data only, find that the cross-sectional dispersion of consumption increases significantly over the life cycle (Deaton and Paxson 1994, Heathcote et al. 2010a). They all argue that if full risk sharing were achievable, the individual consumption would not respond to the individual income change and the consumption dispersion would be constant over the life cycle. Therefore, the hypothesis of complete markets should be rejected. In consequence, this unanimous rejection has motivated more researchers to investigate quantitatively the degree of risk sharing in models with the alternative hypothesis of incomplete markets, using the methodology of the above risk sharing tests (e.g. Storesletten et al. 2004, Heathcote et al. 2010b, Kaplan and Violante 2010, Sun 2010).

This rejection, however, is premature. In this paper, I revisit the implications of these risk sharing tests in the context of a complete market model with discount rate heterogeneity, which is extended to introduce the individual choices of effort in education. I ask: can a complete market model with discount rate heterogeneity pass both types of the risk sharing tests? Drawing quantitative implications of this model, I further ask: with plausible parameters values, is it quantitatively admissible for a complete market model to account for the observed comovement of consumption and income, the increase of consumption dispersion over the life cycle, or even both simultaneously?

The model in this paper begins by relaxing the common assumption of homogeneous discount rates. As the individual discount rate is unobservable, researchers use experimental studies, field studies or structural estimation to elicit the discount rate[1]. Most of the experimental and field studies have reported significant standard deviations in the estimates. Using structural estimation by Euler equation residuals, Lawrance (1991) and Alan and Browning (2006, 2010) strongly reject the hypothesis of homogeneous discount rates. In the macroeconomics literature, the importance of discount rate heterogeneity is also highlighted to account for the life-time wealth inequality of households with similar earnings profiles (Samwick 1998,

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[1] See Frederick et al. (2002) for a survey.

Krusell and Smith 1998, Hendricks 2007).

In addition, the individual choices of effort in education are introduced into the model. Here I focus on the (unobserved) education *quality,* while my model bears the same intuition as the models of the education *quantity* such as the (observed) education levels. Consistent with the model implications, there is strong evidence for the positive correlation between education level and patience, which is verified in the field study from U.S. military downsizing (Warner and Pleeter 2001), in the experiment of Denmark sample with sizable real monetary reward (Harrison et al. 2002), and in the structural estimation of consumption-and-saving models (Cagetti 2003, Alan and Browning 2010). The empirical studies also find that the group with higher education is associated with higher wage growth rate (e.g. Lillard and Weiss 1979, Baker 1997, Guvenen 2009, Low, Meghir and Pistaferri 2010). The above two lines of evidence jointly imply a positive correlation between income growth rate and patience.

The mechanism of my model is as follows. With discount rate heterogeneity, agents with different discount rates have different life cycle consumption paths. Even if markets are complete and consumption paths are deterministic, the consumption dispersion will eventually increase over the life cycle when their consumption paths diverge. Moreover, the more patient agent exerts more effort in education because her discounted future return is higher, which yields higher quality of education and thus higher income growth. As the income growth rate and the consumption growth rate are both increasing in patience, consumption will statistically comove with income. Therefore, with discount rate heterogeneity the risk sharing tests arrive at the conclusion of imperfect risk sharing, even if the markets are actually complete.

This paper contributes to the large literature of the risk-sharing tests since 1990s (e.g. Cochrane 1991, Mace 1991, Altonji et al. 1992, Townsend 1994, Deaton and Paxson 1994, Attanasio and Davis 1996, Blundell and Preston 1998, BPP 2008). In particular, it contributes to the recent researches which cast doubt on the risking sharing tests with the presence of preference heterogeneity (Mazzocco and Saini 2012, Schulhofer-Wohl 2011, Chiappori et al. 2012). Different from this paper, these authors focus on the implications of the heterogeneity in individual risk aversion and find that the hypothesis of full risk sharing cannot be rejected, if there exists risk preference heterogeneity. The time preference in this paper, unlike the risk preference, has no direct relation with risk itself: its comovement of consumption and income simply comes from the intertemporal choices, not from the mutual insurance or from the self-selection into risky income as in the models of heterogeneous risk preferences.

Other papers have addressed the importance of preference on the consumption dispersion in a complete market model. Storesletten et al. (2001) show that the non-separable preferences

between consumption and leisure in a complete market model could possibly generate an increase of consumption dispersion over the life cycle, but it also implies a counter-factual age profile of inequality in hours worked. Badel and Huggett (2012) study in a complete market setup how the preference shocks can account for the life-cycle profile of both consumption and hours dispersion. Their focus is the role of preference shifters, not time preferences.

The income process used in this paper is related to the literature of Heterogeneous Income Profiles (HIP), which is emphasized by Lillard and Weiss (1979), Hause (1980), Baker (1997) and Guvenen (2009). Without additional information or *ad hoc* model specification, it is impossible for the econometricians to tell apart the predictable income change, such as the heterogeneous income growth rates, from the unpredictable income change, such as the permanent income shocks. Although the income processes with homogeneous (unexplained) growth rate are more common in the literature, Primiceri and Rens (2009) estimate a HIP model and find that a significant part of income change over time is predictable. Endogenizing the heterogeneous income growth rates, this paper is also related to the model focusing on ex-ante heterogeneity as in Huggett et al. (2011) and pre-work choice as in Keane and Wolpin (1997).

The rest of the paper is organized as follows. Section 2 presents a simple example to deliver the main message from the empirical risk sharing tests. Section 3 develops a complete market model with discount rate heterogeneity, in which the individual income growth rate is endogenously positively correlated with the individual patience. Section 4 draws the quantitative implications of the model. Section 5 concludes. Proofs are in the Appendix.

2 The Risk Sharing Tests

The degree of risking sharing can be affected by the nature of market structure and/or the nature of income risks. To test risking sharing, a growing literature uses the information from the micro data sets of consumption and income distributions. In terms of the structure of the data they use, these tests can be classified as two categories: One is to exploit the *joint-distribution* of consumption and income. To do this, one has to construct either a synthetic panel (Attanasio and Davis 1996) or a combined panel data of consumption and income (BPP 2008) and measure the comovement of consumption and income. The other is to use the data sets of consumption and income distributions *separately*, study the change of consumption distribution over the life cycle (Deaton and Paxson 1994) and compare it with the change of income distribution accordingly. These two types of risk sharing tests share the same core methodology which can be illustrated by the following example:

Consider an individual saving problem where she is endowed with a stochastic income process y_{it}, lives for T periods, and only has access to a risk-free bond with net interest rate r. Assume further that the period utility is linear quadratic with the discount factor $\beta = 1/(1+r)$. Assume the borrowing constraints are loose enough to make the first order conditions hold. Solving the model analytically yields that the consumption follows a martingale process, $\Delta c_{i,t} = \pi^{-1} \frac{r}{1+r} \sum_{s=0}^{T-t} \frac{(E_t - E_{t-1})y_{i,t+s}}{(1+r)^s}$, where $\pi \equiv 1 - \frac{1}{(1+r)^{T-t+1}}$, which is a textbook version of the Permanent Income /Life Cycle Hypothesis (PILCH). In the data, we observe the distributions of consumption and income. If we take an agnostic view of either the market structure or the income shocks, can we infer their nature from the data? The logic of the risk sharing tests is: by investigating the comovement of consumption and income and by investigating the change of consumption dispersion, we can identify the nature of the market structure and/or the nature of the income risks. To see this, consider three extreme cases of the composition of the idiosyncratic income y_{it}:

Case A. Pure unit root income shock: $y_{it} = z_{it}$, and $z_{it} = z_{it-1} + \eta_{it}$, where η_{it} is the permanent shock. In this case, $\Delta y_{it} = \eta_{it}$. We can solve analytically $\Delta c_{it} = \eta_{it}$. The comovement of consumption and income is such that the income innovation passes one-to-one to the change of consumption. As the consumption is a unit-root process, the dispersion of consumption increases over the life cycle.

Case B. Pure predictable and heterogeneous income profiles: $y_{it} = \kappa_i t$, where κ_i is the slope of individual income path. Since all the income changes are predictable, the markets are complete. Thus we have $\Delta y_{it} = \kappa_i$ and $\Delta c_{it} = 0$. The comovement of consumption and income is zero, and the dispersion of consumption keeps constant over the life cycle.

Case C. Pure i.i.d. income shock: $y_{it} = \varepsilon_{it}$, where ε_{it} is the transitory shock. If shocks are transitory, we have $\Delta y_{it} = \varepsilon_{it} - \varepsilon_{i,t-1}$ and $\Delta c_{it} = \frac{r}{1+r} \pi^{-1} \varepsilon_{it}$. Notice that $\pi \approx 1$ if $T-t$ is a large number and $\frac{r}{1+r} \approx r$ if r is a small number. Therefore, $\Delta c_{it} \approx r\varepsilon_{it}$ with large $T-t$ and small r. There is a slightly positive comovement of consumption and income, and the consumption dispersion increases slightly over the life cycle.

The risk sharing tests tell us that we can infer the nature of market structure and/or the nature of the shocks from two data sources: 1. the empirical comovement of consumption and income; 2. the empirical dispersion of consumption over the life cycle.

Comovement of consumption and income

To generalize Attanasio and Davis (1996)'s idea, BPP (2008) measure the degree of the transmission of income shocks to consumption growth. They assume that the unexplained log income can be decomposed into a unit root permanent part and an i.i.d. transitory part; they

also assume the log consumption as

$$\Delta \log c_{it} = \phi_{it} \eta_{it} + \psi_{it} \varepsilon_{it} + u_{it}, \tag{1}$$

where η_{it} is the permanent shock and ε_{it} is the transitory shock, u_{it} is the error term. Since ϕ_{it} is the pass-through of permanent income shocks to consumption change, it is a natural measure of (the lack of) consumption insurance.

To create a panel data series of consumption and income, BPP map food data into expenditure data using the estimates of a food demand function, as food data are present in both the Panel Study of Income Dynamics (PSID) and the Consumer Expenditure Survey (CEX). BPP's main finding is that, in the whole sample, the estimate of ϕ and ψ is 0.6423 and 0.0533, respectively. Hence, they infer that markets are incomplete and people may have access to more insurance over and above the self-insurance as implied by the PILCH model. They call it *partial insurance*.

Consumption dispersion over the life cycle

If we do not have BPP's panel data of consumption and income, we have to rely on the distribution of consumption and income separately. It is well known that the income distribution is fanning out over the life cycle. As to the consumption dispersion, Deaton and Paxson (1994) find that the consumption dispersion of each cohort increases over the life cycle by 0.28 log points. Using longer time span from 1980 to 2006, Heathcote et al. (2010a) find that the variance of log consumption increases over the life cycle by 0.057 when controlling for year effects and by 0.13 when controlling for cohort effects. While Heathcote et al. (2010a) do not take a stand on which empirical strategy is better, Heathcote et al. (2005) suggest controlling for year effects[2].

The consensus is that the consumption dispersion increases significantly over the life cycle. Hence, they infer that the markets are incomplete and a large part of the income shocks is very persistent, unpredictable and uninsurable.

The message from all the above risk sharing tests is: either the hypothesis of complete markets must be rejected or, in the context of a PILCH model, the hypothesis of fully predictable heterogeneous income profiles (Case B) must be rejected, or both. Notice the latter is a special case of the former. This rejection, however, is premature. In the rest of the paper, I will revisit

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[2]Recently, several authors argue that the diary survey in CEX is better designed than the interview survey. Attanasio et al. (2007) use the diary survey of the CEX and Attanasio et al. (2010) combine these two surveys. They find that the consumption dispersion from 1980 to 2006 rises twice as much as the result from Heathcote et al. (2010a), who use the interview survey. But they did not report the increase of consumption dispersion over the life cycle.

the implication of these tests in a complete market model with discount rate heterogeneity and endogenous income growth rate.

3   The Model

3.1   The life-cycle problem

Consider an economy populated with a continuum of agents. Agent i is born at age 0 with a discount factor β_i, which is drawn from a distribution F_β with variance $\sigma_\beta^2 > 0$. Each agent's life cycle consists of two stages: education and working. In the education stage, agent i makes a decision related to education, which determines her life-cycle income profile. In the working stage, agent i faces a life-cycle consumption and saving problem, given her income profile predetermined in the education stage.

The working stage

After l years of education, each agent starts to work at age l, works for T years and dies at age $T + l$. In the working stage, each agent's income follows a Heterogeneous Income Profiles (HIP) process:

$$
\begin{aligned}
\log y_{it} &= \log w + \sum_{\tau=1}^{t} \theta_{i\tau} + z_{it} + \varepsilon_{it}, \quad (2)\\
z_{it} &= \rho z_{it-1} + \eta_{it} ,
\end{aligned}
$$

where time t denotes the working tenure. w is the average wage. Focusing on the unexplained income change, I assumed away the observed individual fixed effect in this process. The idiosyncratic shocks consist of a permanent (or AR(1) when $\rho < 1$) part z_{it} and a transitory (i.i.d.) part ε_{it}, where $\rho \in (0, 1]$, $\eta_{it} \sim N(0, \sigma_\eta^2)$, $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$, $z_{i0} = 0$. θ_{it} is the individual income growth rate at time t, which is predetermined by the individual education effort choice in the education stage. In the HIP model as estimated by Guvenen (2009), θ_{it} is assumed to be constant over the life cycle and $\sum_{\tau=1}^{t} \theta_{i\tau}$ becomes $\theta_i t$. If we allow the growth rate to vary, we can interpret it as the predictable part of income change (though not predictable by the econometricians), as in Primiceri and Rens (2009) with the restriction of $\rho = 1$. Thus, this HIP model nests both Guvenen (2009) and Primiceri and Rens (2009).

The income shocks at time t are dependent on the stochastic event $s_t \in S$. Denote s^t as $[s_1, s_2..., s_t]$, which is the history of events up to time t. The unconditional probability of history s^t is $\pi_t(s^t)$. The markets are assumed to be complete. Without loss of generality, let us consider the Arrow-Debreu structure in which agents can trade a complete set of contingent claims dated at $t = 1$. By the first and second welfare theorems, the equilibrium allocation is

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Pareto optimal and corresponds to the solution of the social planner's problem with a particular set of Pareto weights. The markets are complete in the sense that for every history s^t, there exist a market of contingent claims with time 1 price $q_t^1(s^t)$ for that history.

Denote $\Theta_i \equiv [\theta_{i1}, \theta_{i2}...\theta_{iT}]$ as the vector of individual income growth rates. The agent i's period utility is assumed to be CRRA with risk-aversion γ and her problem in the working stage is:

$$V(\Theta_i, \beta_i) = \max \sum_{t=1}^{T} \beta_i^{t-1} \{ \sum_{s^t} \pi_t(s^t) \frac{[c_{it}(s^t)]^{1-\gamma}}{1-\gamma} \}$$

$$s.t. \sum_{t=1}^{T} \sum_{s^t} q_t^1(s^t) c_{it}(s^t) \leq \sum_{t=1}^{T} \sum_{s^t} q_t^1(s^t) y_{it}(s^t).$$

The agent i's value function $V(\Theta_i, \beta_i)$ in the working stage is solved in the equilibrium where $q_t^1(s^t)$ clear the markets for all s^t.

The education stage

In the working stage, as individuals have the same (observed) education levels (years) l, they are assumed to have the same base wages w. There are (unobserved) differences in education quality as individuals may differ in their efforts. In specific, at age 0, the agent i chooses an effort level e_i with an individual-specific utility cost function $\omega_i(e_i)$, with $\frac{\partial \omega_i(e_i)}{\partial e_i} > 0$ and $\frac{\partial^2 \omega_i(e_i)}{\partial e_i^2} \geq 0$. This standard cost function captures all the individual-specific cognitive, non-cognitive and financial characteristics in education.

Assume the individual wage growth rate at time t to be increasing and weakly concave in the agent's effort level, i.e. $\frac{\partial \theta_{it}}{\partial e_i} > 0$ and $\frac{\partial^2 \theta_{it}}{\partial e_i^2} \leq 0$, for any t. This assumption can be justified as follows: first, the education quality is an increasing and concave function of effort; second, the learning ability as part of the education quality determines the individual income growth at any time t, which can be viewed as a human capital accumulation model *a la* Hugget et al. (2011). Although this model focuses on the choice of education quality instead of education quantity (years), the main intuitions are the same and it is consistent with the empirical evidence that the group of higher education is associated with higher wage growth rate.

The expected life-time utility after she enters the labor market can be solved in the working stage as $V(\Theta_i, \beta_i)$. Thus the agent i's optimization problem in the education stage is:

$$\max_{e_i} -\omega_i(e_i) + \beta_i^l V(\Theta_i, \beta_i).$$

3.2 Income growth rate and patience

For the whole population, we are interested in the following question: how does the agent's income growth rate correlate with patience? We can derive the following proposition:

Proposition 1 *The invidual income growth rate is positively correlated with the individual patience, i.e.* $\rho_{\theta_t, \beta} > 0$.

Proof. *See Appendix.* ∎

Intuitively, the patient agent would exert more effort because she has higher discounted future benefit than the impatient agent. As higher level of effort generates higher education quality and thus higher income growth rate, the endogenous income growth rate is increasing in patience. It is easy to see that in the special case where we assume constant income growth rate, we have $\rho_{\theta, \beta} > 0$.

For simplicity, this model restricts the effort choice to the education stage only. The intuition still holds if we allow the agents to exert effort in the working stage. The θ_{it}, which may be mis-specified as "shocks" to econometricians, are predictable to the individuals at time t. The more patient agent is more willing to exert effort to increase her human capital and thus income growth θ_{it}, because she always has, *ceteris paribus*, higher discounted benefit of effort than the impatient agent at any working time t.

3.3 Comovement of consumption and income

Let $\mu_i > 0$ be the Lagrange multiplier of agent i's life-time budget constraint, the first-order condition for agent i with history s^t is:

$$\beta_i^{t-1} \pi_t(s^t) [c_{it}(s^t)]^{-\gamma} = \mu_i q_t^1(s^t). \tag{3}$$

Notice that the marginal utility of consumption decreases with the individual discount factor and increases with the Lagrange multiplier. We immediately know that the ratio of consumption of any two individuals does not depend on their own income history; thus each consumer will get full insurance and we can write $c_{it}(s^t) = c_{it}$ for any s^t at t.

Denote $\widetilde{x}$ as the logarithm of any variable x and sum up individual i's consumption across all the states at time t, we get from equation (3) the first difference of log consumption (individual consumption growth rate):

$$\Delta \widetilde{c}_{it} = \frac{1}{\gamma} [\widetilde{\beta}_i - \Delta \log \sum_{s^t} q_t^1(s^t)], \tag{4}$$

where $-\widetilde{\beta} \equiv -\log \beta$ is approximately the discount rate if $E(\beta) \approx 1$. Because the income shocks are fully insured in complete markets, the individual consumption growth rate depends on each agent's discount factor, not the income shocks. Thus, in the OLS estimation by BPP in equation (1), both the permanent and the transitory pass-throughs should be zero.

However, the econometricians do not know each agent's true income process. The best thing they can do is to use the panel data of consumption and income for identification: if they observe one's income and one's consumption are correlated, they conclude that there is "uninsurable" income shock.

In practice, BPP estimate the comovement of consumption and income by instruments[3]:

$$\widehat{\phi}_t^{Instr} = \frac{E[\Delta \widetilde{c}_t (\Delta \widetilde{y}_{t-1} + \Delta \widetilde{y}_t + \Delta \widetilde{y}_{t+1})]}{E[\Delta \widetilde{y}_t (\Delta \widetilde{y}_{t-1} + \Delta \widetilde{y}_t + \Delta \widetilde{y}_{t+1})]}, \tag{5}$$

$$\widehat{\psi}_t^{Instr} = -\frac{E(\Delta \widetilde{c}_t \Delta \widetilde{y}_{t+1})}{E(\Delta \widetilde{y}_t \Delta \widetilde{y}_{t+1})}. \tag{6}$$

From equation (4) we know that the individual consumption growth rate is not directly linked with the individual income. However, it is correlated with the individual discount factor, and from Proposition 1 we know that the individual income growth rate and patience are positive correlated. Therefore, BPP's measure by the instrument estimation would no longer be zero. Formally, we have the following proposition:

Proposition 2 *In the complete market model, the BPP's estimators are*

$$\widehat{\phi}_{t,CM} = \frac{cov(\widetilde{\beta}, \theta_{t-1} + \theta_t + \theta_{t+1})}{\gamma \widehat{\sigma}_{\eta, BPP}^2} = \frac{cov(\widetilde{\beta}, \theta_{t-1} + \theta_t + \theta_{t+1})}{\gamma \{ cov(\theta_t, \theta_{t-1} + \theta_t + \theta_{t+1}) + \rho[(\rho^3 - 1)(1 - \rho^{2(t-2)})/(\rho+1) + \rho(\rho-1) + 1] \sigma_\eta^2 \}},$$

$$\widehat{\psi}_{t,CM} = \frac{cov(\widetilde{\beta}, \theta_{t+1})}{\gamma \widehat{\sigma}_{\varepsilon, BPP}^2} = \frac{cov(\widetilde{\beta}, \theta_{t+1})}{\gamma \{ \sigma_\varepsilon^2 - cov(\theta_t, \theta_{t+1}) - (\rho-1)[\rho^3(\rho^{2(t-1)} - 1)/(\rho+1) + 1] \sigma_\eta^2 \}},$$

where $\widehat{\sigma}_{\eta, BPP}^2$ and $\widehat{\sigma}_{\varepsilon, BPP}^2$ are BPP's estimates of the variance of permanent and transitory shocks, respectively. Therefore, the hypothesis of complete markets can NOT be rejected by the observed positive $\widehat{\phi}_{t,CM}$ from BPP's test.

Proof. *See Appendix.* ∎

This proposition simply says that the covariance of income growth rates and patience can be transmitted into the comovement of consumption and income, which may cause the false rejection of the hypothesis of complete maketes by BPP's estimation.

3.4   Consumption dispersion

Let $\lambda_i \equiv 1/\mu_i$ represent the Pareto weight in the corresponding social planner's problem. As we can see for any two agents i and j:

$$\frac{c_{it}(s^t)}{c_{jt}(s^t)} = [(\frac{\beta_i}{\beta_j})^{t-1} \frac{\lambda_i}{\lambda_j}]^{\frac{1}{\gamma}}. \tag{7}$$

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[3]Although the direct OLS and the instrument method are in principle different, Sun (2010) proves that both the instrument and OLS estimators are consistent estimators of ϕ and ψ, if the consumption model (1) is not mis-specified.

If $\beta_i = \beta_j$, the relative consumption ratio is a constant which is determined at the beginning of the life cycle. Testing constant relative marginal utility of consumption is the key logic of the test of complete markets. If $\beta_i \neq \beta_j$, however, the ratio between c_i and c_j will increase exponentially with age. Consider two agents with $\beta_i > \beta_j$. Even if $\lambda_i < \lambda_j$, the impatient agent will consume more in the beginning, but after some time the patient agent will catch up and consume more and more afterwards. In that case the consumption dispersion is expected to decrease at the very beginning, reaches zero after some time and increases convexly since then.

The variance of log consumption can be computed as

$$var(\widetilde{c}_{it}) = \frac{(t-1)^2}{\gamma^2} var(\widetilde{\beta}_i) + \frac{2(t-1)}{\gamma} cov(\widetilde{\beta}_i, \widetilde{\lambda}_i) + var(\widetilde{\lambda}_i). \tag{8}$$

Notice that the last term in the right hand side is constant over the life-cycle, which is the initial level of consumption dispersion. The first term is increasing quadratically with age; the second term is a linear function of age whose slope is dependent on the covariance between patience and corresponding Pareto weight. At the beginning of life, the consumption dispersion may decrease with age if $cov(\widetilde{\beta}_i, \widetilde{\lambda}_i) < 0$; if t is sufficiently large, the consumption dispersion starts to increase convexly with age. We have the following proposition:

Proposition 3 *(i) In the complete market model, the consumption dispersion increases convexly with age after $t \geq t^* = \gamma \rho_{\widetilde{\lambda},\widetilde{\beta}} \frac{\sigma_{\widetilde{\lambda}}}{\sigma_{\widetilde{\beta}}} + 1$. If $\rho_{\widetilde{\lambda},\widetilde{\beta}} > -\frac{T-1}{2\gamma} \frac{\sigma_{\widetilde{\beta}}}{\sigma_{\widetilde{\lambda}}}$, then the consumption dispersion increases over the life cycle. Therefore, the hypothesis of complete markets can NOT be rejected by the observed increasing consumption dispersion over the life cycle.*

(ii) Consumption dispersion over the life cycle increases with $\rho_{\theta_t,\beta}$.

Proof. See Appendix. ∎

3.5 Bond economy with predictable income

Consider a special case of complete markets. The typical life-cycle model of consumption and saving is widely used in the macroeconomics literature. Under this market structure, each agent has only access to a one-period risk-free bond, with the gross interest R which clear the bond markets.

Suppose that the idiosyncratic income change is *fully predictable*, then the individual consumption can be fully smoothed by borrowing and saving through the bond market, i.e., the markets are complete.

The agent i's optimization problem at the working stage is:

$$V(\Theta_i, \beta_i) = \max \sum_{t=1}^{T} \beta_i^{t-1} \frac{c_{it}^{1-\gamma}}{1-\gamma}$$

$$s.t. \sum_{t=1}^{T} \frac{c_{it}}{R^{t-1}} \leq \sum_{t=1}^{T} \frac{y_{it}}{R^{t-1}}.$$

Because the econometricians do not know the agent's true income process, the fully predictable income change might be mis-interpreted as "shocks" to the econometricians. The variance of log consumption in the bond economy can be computed as

$$var(\widetilde{c}_{it}) = \frac{(t-1)^2}{\gamma^2} var(\widetilde{\beta}_i) + \frac{2(t-1)}{\gamma} cov(\widetilde{\beta}_i, \widetilde{\Omega}_i) + var(\widetilde{\Omega}_i), \tag{9}$$

where $\Omega_i \equiv \sum_{t=1}^{T} \frac{y_{it}}{R^{t-1}} / \sum_{t=1}^{T} [\beta_i R^{(1-\gamma)}]^{\frac{t-1}{\gamma}}$. Note that the Pareto weight λ_i in equation (8) now has a concrete interpretation and it is decreasing in the present value of the life-time income. The correlation between Ω_i and β_i might be negative. To have an increasing consumption dispersion, it requires that this correlation to be not too negative, which can be helped by the fact that the more patient agent would have higher life-time wealth. Formally, we can prove in this model that it has the sample properties as shown in the previous case of complete market model with stochastic income process.

Proposition 4 *In the bond economy with predictable income change (a special complete market model),*

(i)the BPP estimators $\widehat{\phi}_{t,CM}$ and $\widehat{\psi}_{t,CM}$ are the same as in the complete market model.

(ii) the consumption dispersion increases convexly with age after $t \geq t^* = \gamma \rho_{\widetilde{\Omega},\widetilde{\beta}} \frac{\sigma_{\widetilde{\Omega}}}{\sigma_{\widetilde{\beta}}} + 1$. If $\rho_{\widetilde{\Omega},\widetilde{\beta}} > -\frac{T-1}{2\gamma} \frac{\sigma_{\widetilde{\beta}}}{\sigma_{\widetilde{\Omega}}}$, then the consumption dispersion increases over the life cycle. The consumption dispersion over the life cycle increases with $\rho_{\theta_t,\beta}$.

Therefore, the hypothesis of bond economy with predictable income change can NOT be rejected by either the positive $\widehat{\phi}_{t,CM}$ or the increasing consumption dispersion over the life cycle.

Proof. *See Appendix.* ∎

In sum, the empirical evidence of the two most popular risk sharing tests is not enough to reveal the true nature of market structure and income shocks. Allowing for heterogeneous discount rates and heterogeneous income growth rates, we can reject neither the hypothesis of complete markets nor the hypothesis of fully predictable income.

4   Quantitative Implications

Theoretically, I have shown that a complete market model can pass the risk sharing tests. To go one step further, I will investigate the *quantitative* significance of the risk sharing tests when markets are complete. I will first discuss the relative insurability between permanent and transitory shocks as implied by my model, and then I ask: with plausible parameters, is it quantitatively admissible for a complete market model to match the observed increase of consumption dispersion, the comovement of consumption and income, or even both simultaneously?

Relative insurability

If the markets are incomplete, the permanent shocks are more difficult to insure against than the transitory shocks. The pass-through of transitory shocks in BPP is 0.0533, which is not significantly from zero and close to the interest rate, consistent with the Permanent Income / Life Cycle Hypothesis. The most informative result of BPP's estimation is the pass-through of permanent shocks $\widehat{\phi}_{BPP} = 0.6422$. BPP call it *partial insurance*. On the one hand, it is less than the one-to-one response of consumption to income in the standard PILCH model as discussed in Section 2, indicating that there is some risk-sharing over and above self-insurance. On the other hand, the magnitude of the pass-through of permanent shocks is more than 10 times larger than that of transitory shocks, which highlights the huge difference in the insurability between permanent and transitory shocks and thus is at odds with the hypothesis of complete markets.

I would argue that the relative magnitude of pass-through between permanent and transitory shocks may not indicate the relative insurability and the huge difference in pass-throughs may not reject the hypothesis of complete markets. Quantitatively, the ratio between the pass-through of permanent shocks and transitory shocks is less informative than we previously thought. To see this, notice that according to Proposition 2, we can write the ratio between the pass-through of permanent shocks to consumption and the pass-through of transitory shocks to consumption as:

$$\frac{\widehat{\phi}_{t,CM}}{\widehat{\psi}_{t,CM}} = \frac{cov(\theta_{it-1} + \theta_{it} + \theta_{it+1}, \widetilde{\beta}_i)}{cov(\theta_{it+1}, \widetilde{\beta}_i)} \frac{\widehat{\sigma}^2_{\varepsilon,BPP}}{\widehat{\sigma}^2_{\eta,BPP}}. \tag{10}$$

This ratio tells us the relative insurability between permanent and transitory shocks. Assume $cov(\theta_{it-1}, \widetilde{\beta}_i,) \approx cov(\theta_{it}, \widetilde{\beta}_i,) \approx cov(\theta_{it+1}, \widetilde{\beta}_i)$, and it becomes

$$\frac{\widehat{\phi}_{t,CM}}{\widehat{\psi}_{t,CM}} \approx \frac{3\widehat{\sigma}^2_{\varepsilon,BPP}}{\widehat{\sigma}^2_{\eta,BPP}}. \tag{11}$$

This is an easily testable implication of the complete market model. If in the data we have the (mis-specified) estimation of the variances of shocks, we can predict this ratio without the knowledge of $cov(\theta_i, \beta_i)$ or γ.

<div align="center">Table 1</div>

Sample	Source	$\widehat{\sigma}^2_{\eta,BPP}$	$\widehat{\sigma}^2_{\varepsilon,BPP}$	$\dfrac{\widehat{\phi}_{t,CM}}{\widehat{\psi}_{t,CM}}$
1979-1981	BPP(2008)	0.0102	0.0368	10.8
1990-1992	BPP(2008)	0.0134	0.0506	11.3
1969-1992	STY(2004)	0.0161	0.063	11.7
1967-1992	Guvenen(2009)	0.015	0.061	12.2
1979-1992	$\widehat{\phi}_{BPP}/\widehat{\psi}_{BPP} = 0.6423/0.0533 = 12.1$			

In Table 1, I list the estimates of permanent and transitory shocks in longer periods by BPP(2008)[4], Storesletten et al. (2004) and Guvenen (2009) with the restriction of $\sigma^2_{\beta} = 0$. All these estimates are from PSID. The ratio between permanent and transitory pass-throughs constructed in the complete market model ranges from 10.8 to 12.2; by BPP's measurement of the comovement of consumption and income, this ratio is 12.1. This is a stunning quantitative result. Without any information of consumption data at all, this rule-of-thumb prediction from this parsimonious complete market model has done a good job in predicting the relative insurability between permanent and transitory shocks, which is quantitatively consistent with the estimates from BPP's panel data of consumption and income. The key message of BPP is that the magnitude of permanent shocks is more than 10 times larger than the magnitude of transitory shocks, and yet the prediction of the complete market model indicates that we should not simply interpret this message as the strong evidence for market incompleteness.

Admissible parameter values

To quantitatively confront the model with the absolute value of the pass-through of shocks, I assume that $\theta_{it} = \theta_t$. The formula for the pass-through of permanent shocks becomes

$$\widehat{\phi}_{t,CM} = \frac{1}{\gamma} \rho_{\theta,\widetilde{\beta}} \frac{3\sigma_\theta \sigma_{\widetilde{\beta}}}{\sigma^2_{\eta,BPP}}. \tag{12}$$

Note that $\widehat{\phi}_{t,CM}$ can also be interpreted as the down-ward bias of the degree of risk-sharing by BPP's method. It depends on the inter-temporal elasticity of substitution and the correlation between income growth rate and patience. $\sigma^2_{\eta,BPP}$ is set to be 0.0118, which is the average of BPP's estimation during 1979-1981 and 1990-1992 period in PSID. θ_i and $\widetilde{\beta}_i$ are assumed to follow joint Normal distribution with mean $\mu_\theta = \mu_{\widetilde{\beta}} = 0$. $\sigma_{\widetilde{\beta}}$ is set to be 0.087, which is

---

[4] In BPP, the variance of permanent and transitory shocks are reported almost yearly and vary immensely over 1979-1992. The only two sample periods with more than one year are 1979-1981 and 1990-1992. I consider the estimates from these two periods to be more robust and comparable with the estimates from other authors.

Alan and Browning (2006)'s structural estimation of the food consumption in PSID. As $\beta \approx 1$, we have $\sigma_{\widetilde{\beta}} \approx \sigma_{\beta}$ when $\sigma_{\widetilde{\beta}}$ is small. For heterogeneous income growth rates, I use Guvenen (2009)'s estimation in PSID: $\sigma_{\theta} = 0.0195$.

The empirical target for the comovement of consumption and income is naturally the pass-through of permanent shocks estimated by BPP: $\widehat{\phi}_{BPP} = 0.642$. If we can match the model with the permanent pass-through, the transitory pass-through will be approximately matched as we have already known that the ratio between these two is consistent with the data.

The other restriction on γ and $\rho_{\theta,\widetilde{\beta}}$ is derived from the life cycle increase of the consumption dispersion in a bond economy with predictable income, a special complete market model.

$$\frac{(T-1)^2}{\gamma^2}\sigma_{\widetilde{\beta}}^2 + \frac{2(T-1)}{\gamma}cov(\widetilde{\beta}_i, \widetilde{\Omega}_i) = \Delta_{1,T}var(\widetilde{c}_{it}). \tag{13}$$

The implicit function of $\sigma_{\widetilde{\beta}}$ and $\rho_{\theta,\widetilde{\beta}}$ can be approximated numerically using simulation of covariance. The gross interest rate R is set to 1.04. It is convenient to do this in the bond economy with predicable income than in an Arrow-Debreu economy and $\widetilde{\Omega}_i$ can be easily simulated. The empirical target for the consumption dispersion is from Heathcote et al. (2010a). I set $T = 31$, as they report the age profile of consumption dispersion for average 5 years group, starting from age 27 (average of 25 to 30) to age 57 (average of age 55 to 60). They find that the variance of log consumption rises by 0.057 over the life cycle, controlling for year effects.

Admissible Parameter Values
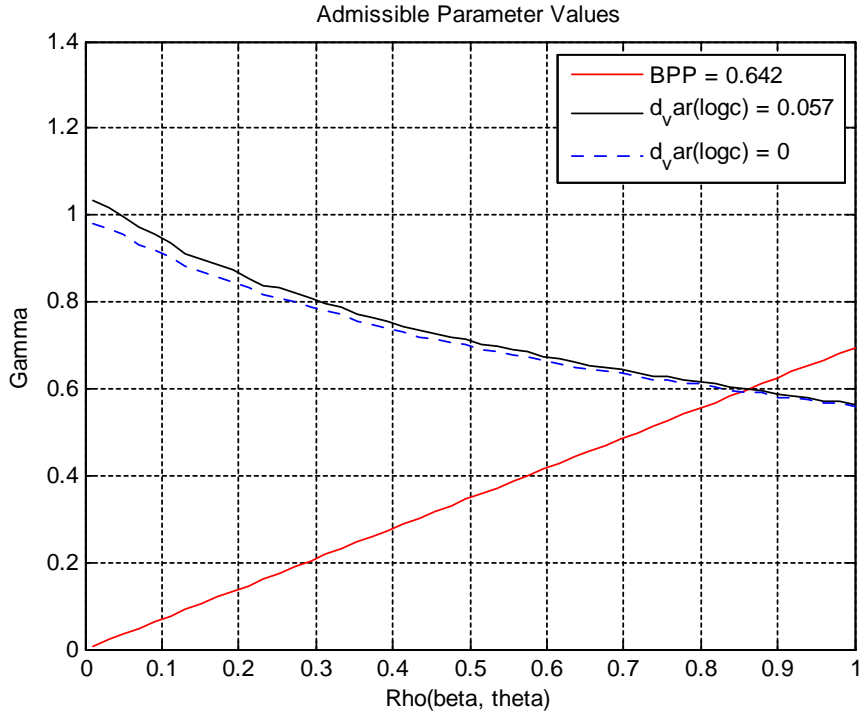
Admissible Parameter Values: γ and $\rho_{\theta, \widetilde{\beta}}$

Figure 1 plots the conditions for admissible $\rho_{\theta, \widetilde{\beta}}$ and γ. The downward sloping black line matches Heathcote et al. (2010a)'s empirical increase of consumption dispersion over the life cycle and the upward sloping red line matches BPP's empirical comovement of consumption and income. These two lines do intersect at a unique pair of $\rho_{\theta, \widetilde{\beta}} = 0.86$ and $\gamma = 0.60$. The relative risk aversion is low, but it is comparable with Gourinchas and Parker (2002)'s estimate of $\gamma = 0.51$ by matching the consumption profile over the life cycle in CEX. It should be noted that: as the preference parameters are unobservable and the income growth rates are unknown to the econometricians, those estimates have already exploited the information of consumption and income. Based on the parameterization which is not model-free, it is too soon to claim the success (or failure) of a complete model by matching (or fail to match) the data. Nevertheless, the key message of this quantitative exercise is: even a parsimonious complete market model, with plausible parameter values, is able to simultaneously account for both the empirical estimates of these two types of risk sharing measures.

According to equation (12), all pairs of positive $\rho_{\theta, \widetilde{\beta}}$ and γ generate positive comovement of consumption and income. I also plot a dashed line to match zero consumption dispersion over the life cycle. Those pairs which lie to the northeast of this line are quantitatively admissible parameters of $\rho_{\theta, \widetilde{\beta}}$ and γ for generating both the positive life-cycle increase of consumption dispersion and the positive comovement of consumption and income; these are the admissible parameter values which would cause the false rejection of the complete market model by the previous risk sharing tests.

The role of $\rho_{\theta, \beta}$ is crucial. To generate a positive comovement of consumption and income, $\rho_{\theta, \beta}$ must be positive. With a high γ, the empirical estimated degree of discount rate heterogeneity is sufficient to generate an increasing consumption dispersion over the life cycle. But with a low γ, the positive correlation between income growth rate and patience is quantitatively necessary.

5 Conclusion

This paper revisits the recent risk sharing tests which use micro data sets of consumption and income distributions. These tests reject the hypothesis of complete markets because in the data: (1) the individual consumption comoves with income and (2) the consumption dispersion increases over the life cycle. These evidence cannot be reconciled with any standard complete market models.

I extend the standard complete market model with the endogenously formed positive correlation between income growth rate and patience, a model feature consistent with the empirical evidence on patience, education and income growth. The main result is: it is not only theoretically possible, but also quantitatively admissible for a parsimonious complete market model to account for the empirical evidence on both the comovement of consumption and income and the increasing consumption dispersion over the life cycle. I conclude that the previous risk sharing tests using micro data sets from consumption and income distributions may underestimate the degree of risk sharing and therefore are not sufficient for rejecting the hypothesis of complete markets.

Casting doubt on the previous risk sharing tests, this paper highlights the importance of the discount rate heterogeneity in general equilibrium models as emphasized by Browning et al. (1999) and it is the first step towards future research in two directions. One is to design a more proper risk sharing test which does not merely rely on the information of either the consumption dispersion or the comovement of consumption and income. The other is to do the structural estimation of the ex-ante heterogeneity, starting from a complete market framework instead of the more complicated incomplete market models, from which we may learn more about the nature of market structure, the nature of income process and its relationship with discount rate heterogeneity.

6 Appendix

6.1 Proofs

Proof. (Proposition 1)

In the education stage, the optimal effort level is given by

$$\beta_i^l \sum_{t=1}^T [\frac{\partial V(\Theta_{it}, \beta_i)}{\partial \theta_{it}} \frac{\partial \theta_{it}}{\partial e_i}] = \frac{\partial \omega_i(e_i^*)}{\partial e_i}.$$

From the implicit function theorem, we get

$$\frac{de_i^*}{d\beta_i} = -\frac{\beta_i^{l-1} \sum_{t=1}^T \frac{\partial \theta_{it}}{\partial e_i} [\beta \frac{\partial V(\Theta_{it}, \beta_i)}{\partial \theta_{it} \partial \beta_i} + \tau \frac{\partial V(\Theta_{it}, \beta_i)}{\partial \theta_{it}}]}{\beta_i^l \sum_{t=1}^T [\frac{\partial V(\Theta_{it}, \beta_i)}{\partial \theta_{it}} \frac{\partial^2 \theta_{it}}{\partial e_i^2} + (\frac{\partial \theta_{it}}{\partial e_i})^2 \sum_{t=1}^T \frac{\partial^2 V(\Theta_{it}, \beta_i)}{\partial \theta_{it}^2}] - \frac{\partial^2 \omega_i(e_i^*)}{\partial e_i^2}}.$$

In the working stage, the life-time present value of income is increasing in θ_i. We have $\frac{\partial V(\Theta_{it}, \beta_i)}{\partial \theta_{it}} > 0, \frac{\partial V(\Theta_{it}, \beta_i)}{\partial \theta_{it} \partial \beta_i} > 0$ and $\frac{\partial^2 V(\Theta_{it}, \beta_i)}{\partial \theta_{it}^2} < 0$, which come from the standard properties of the consumer's problem. As $\frac{\partial \theta_{it}}{\partial e_i} > 0, \frac{\partial^2 \theta_{it}}{\partial e_i^2} \leq 0, \frac{\partial^2 \omega_i(e_i^*)}{\partial e_i^2} \geq 0$, we can immediately guarantee that $\frac{de_i^*}{d\beta_i} > 0$.

16

Since $\frac{\partial \theta_{it}}{\partial e_i} > 0$, we have $\frac{\partial \theta_{it}}{\partial \beta_i} > 0$. Finally, the property of the covariance of two increasing functions yields $cov(\theta_{it}, \beta_i) > 0$ and $\rho_{\theta_t, \beta} > 0$. ∎

Proof. (Proposition 2)

The "true" pass-through of permanent shocks to consumption growth can be estimated by OLS: since $cov(\Delta \widetilde{c}_t, \eta_t) = cov(\Delta \widetilde{c}_t, \varepsilon_t) = 0$ in complete market model, the pass-through of permanent and transitory shocks would both be zero. From the data, however, the econometricians cannot observe η_t and ε_t. BPP suggest instrument estimation to get the variance and covariance needed. The variance of permanent shocks is identified as:

$$
\begin{aligned}
\widehat{\sigma}_{\eta, BPP}^2 &= E[\Delta \widetilde{y}_t (\Delta \widetilde{y}_{t-1} + \Delta \widetilde{y}_t + \Delta \widetilde{y}_{t+1})] \\
&= E[(\theta_{it} + \rho(\rho-1)z_{t-2} + (\rho-1)\eta_{t-1} + \eta_t + \varepsilon_t - \varepsilon_{t-1}) \\
&\quad (\theta_{it-1} + \theta_{it} + \theta_{it+1} + \eta_{t+1} + \rho\eta_t + \rho^2\eta_{t-1} + (\rho^3-1)z_{t-2} + \varepsilon_{t+1} - \varepsilon_{t-2}) \\
&= cov(\theta_{it}, \theta_{it-1} + \theta_{it} + \theta_{it+1}) + \rho(\rho-1)(\rho^3-1)var(z_{t-2}) + [\rho^2(\rho-1) + \rho]\sigma_\eta^2 \\
&= cov(\theta_{it}, \theta_{it-1} + \theta_{it} + \theta_{it+1}) + \rho[(\rho-1)(\rho^3-1)\sum_{\tau=1}^{t-2} \rho^{2(\tau-1)} + \rho(\rho-1) + 1]\sigma_\eta^2 \\
&= cov(\theta_{it}, \theta_{it-1} + \theta_{it} + \theta_{it+1}) + \rho[(\rho^3-1)(1-\rho^{2(t-2)})/(\rho+1) + \rho(\rho-1) + 1]\sigma_\eta^2.
\end{aligned}
$$

The covariance of consumption growth and permanent shocks is identified as

$$
\begin{aligned}
&E(\Delta \widetilde{c}_t (\Delta \widetilde{y}_{t-1} + \Delta \widetilde{y}_t + \Delta \widetilde{y}_{t+1})) \\
&= E[\frac{\widetilde{\beta}_i - \Delta \log \sum_{s^t} q_t^1(s^t)}{\gamma}(\theta_{it-1} + \theta_t + \theta_{t+1} + \eta_{t+1} + \rho\eta_t + \rho^2\eta_{t-1} + (\rho^3-1)z_{t-2} + \varepsilon_{t+1} - \varepsilon_{t-2})] \\
&= \frac{cov(\theta_{it-1} + \theta_{it} + \theta_{it+1}, \widetilde{\beta}_i)}{\gamma}.
\end{aligned}
$$

The last step uses the fact that $\Delta \log \sum_{s^t} q_t^1(s^t)$ is not individual i specific. Hence,

$$
\begin{aligned}
\widehat{\phi}_{t,CM} &= \frac{E(\Delta \widetilde{c}_t(\Delta \widetilde{y}_{t-1} + \Delta \widetilde{y}_t + \Delta \widetilde{y}_{t+1}))}{E(\Delta \widetilde{y}_t(\Delta \widetilde{y}_{t-1} + \Delta \widetilde{y}_t + \Delta \widetilde{y}_{t+1}))} \\
&= \frac{cov(\widetilde{\beta}_i, \theta_{it-1} + \theta_{it} + \theta_{it+1})}{\gamma \widehat{\sigma}_{\eta, BPP}^2} \\
&= \frac{cov(\theta_{it-1} + \theta_{it} + \theta_{it+1}, \widetilde{\beta}_i)}{\gamma\{cov(\theta_{it}, \theta_{it-1} + \theta_{it} + \theta_{it+1}) + \rho[(\rho^3-1)(1-\rho^{2(t-2)})/(\rho+1) + \rho(\rho-1) + 1]\sigma_\eta^2\}}.
\end{aligned}
$$

The variance of transitory shocks is identified as:

$$
\begin{aligned}
\widehat{\sigma}^2_{\varepsilon,BPP} &= E(\Delta \widetilde{y}_t \Delta \widetilde{y}_{t+1}) \\
&= E[(\theta_{it} + (\rho - 1)z_{t-1} + \eta_t + \varepsilon_t - \varepsilon_{t-1}) \\
&\quad (\theta_{it+1} + \rho(\rho - 1)z_{t-1} + (\rho - 1)\eta_t + \eta_{t+1} + \varepsilon_{t+1} - \varepsilon_t)] \\
&= cov(\theta_{it}, \theta_{it+1}) - \sigma^2_\varepsilon + \rho(\rho - 1)^2 var(z_{t-1}) + (\rho - 1)\sigma^2_\eta \\
&= cov(\theta_{it}, \theta_{it+1}) - \sigma^2_\varepsilon + (\rho - 1)[\rho(\rho - 1)\sum_{\tau=1}^{t-1}\rho^{2\tau} + 1]\sigma^2_\eta \\
&= \sigma^2_\varepsilon - cov(\theta_{it}, \theta_{it+1}) - (\rho - 1)[\rho^3(\rho^{2(t-1)} - 1)/(\rho + 1) + 1]\sigma^2_\eta.
\end{aligned}
$$

The covariance of consumption growth and transitory shocks is identified as

$$
\begin{aligned}
&E(\Delta \widetilde{c}_t \Delta \widetilde{y}_{t+1}) \\
&= E[(\frac{\widetilde{\beta}_i - \Delta \log \sum_{s^t} q^1_t(s^t)}{\gamma})(\theta_{it+1} + (1 - \rho)z_t + \eta_{t+1} + \varepsilon_{t+1} - \varepsilon_t)] \\
&= \frac{1}{\gamma}cov(\theta_{it+1}, \widetilde{\beta}_i).
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\widehat{\psi}_{t,CM} &= -\frac{E(\Delta \widetilde{c}_t \Delta \widetilde{y}_{t+1})}{E(\Delta \widetilde{y}_t \Delta \widetilde{y}_{t+1})} \\
&= \frac{cov(\widetilde{\beta}, \theta_{t+1})}{\gamma \widehat{\sigma}^2_{\varepsilon,BPP}} \\
&= \frac{cov(\theta_{it+1}, \widetilde{\beta}_i)}{\gamma\{\sigma^2_\varepsilon - cov(\theta_{it}, \theta_{it+1}) - (\rho - 1)[\rho^3(\rho^{2(t-1)} - 1)/(\rho + 1) + 1]\sigma^2_\eta\}}.
\end{aligned}
$$

The BPP's test of complete market comes from the fact that if $\sigma_\beta = 0$, we get $cov(\theta_{t+1}, \widetilde{\beta}) = 0$ and thus $\widehat{\phi}_{t,CM} = 0$. As the logarithm function is an increasing function, $\rho_{\theta_t \beta} > 0$ implies $cov(\theta_t, \widetilde{\beta}) > 0$ and thus the empirical finding of $\widehat{\phi}_{t,CM} > 0$ cannot reject the hypothesis of complete markets. ∎

Proof. (Proposition 3)

The variance of log consumption is given by

$$
var(\widetilde{c}_{it}) = \frac{(t - 1)^2}{\gamma^2}var(\widetilde{\beta}_i) + \frac{2(t - 1)}{\gamma}cov(\widetilde{\beta}_i, \widetilde{\lambda}_i) + var(\widetilde{\lambda}_i).
$$

The first statement comes directly from the quadratic function of t. We get t^* by letting the derivative of the variance of log consumption to t be zero. Letting $var(\widetilde{c}_{iT}) > var(\widetilde{c}_{i1})$, we get the condition for the increasing consumption dispersion over the life cycle.

Summing up the first order conditions (3) over all the states at time t, the individual consumption at time t can be derived as

$$c_{it} = \beta_i^{\frac{t-1}{\gamma}} \lambda_i^{\frac{1}{\gamma}} [\sum_{s^t} q_t^1(s^t)]^{\frac{1}{\gamma}}.$$

Substituting c_t^i into the budget constraint, we get

$$\lambda_i^{1/\gamma} = \frac{\sum_t \sum_{s^t} q_t^1(s^t) y_{it}(s^t)}{\sum_t \{\beta_i^{\frac{t-1}{\gamma}} [\sum_{s^t} q_t^1(s^t)]^{\frac{1}{\gamma}+1}\}}.$$

The numerator of the right hand side is the present value of the life-time income, which is increasing in θ_{it}. As $\rho_{\theta_t,\beta}$ increases, $cov(\widetilde{\beta}_i, \widetilde{\lambda}_i)$ would increase and thus the consumption dispersion over the life cycle increases. ∎

Proof. (Proposition 4)

In the working stage, the agent i's log consumption can be solved as

$$\widetilde{c}_{it} = \frac{\log(\beta_i R)}{\gamma}(t-1) - \log \sum_{t=1}^T [\beta_i R^{(1-\gamma)}]^{\frac{t-1}{\gamma}} + \log W_i,$$

where $W_i \equiv \sum_{t=1}^T \frac{y_{it}}{R^{t-1}}$ is the present value of life-time wealth. The increase of log consumption from $t-1$ to t is:

$$\Delta \widetilde{c}_{it} = \frac{\widetilde{\beta}_i}{\gamma}.$$

Compare it with equation (4) in the previous case of Arrow-Debreu economy with stochastic economy. The differences are not individual-specific and thus the covariance between $\Delta \widetilde{c}_t^i$ and any individual income changes would remain the same, which proves the first part of this proposition.

The variance of log consumption in the bond economy can be computed as

$$var(\widetilde{c}_{it}) = \frac{(t-1)^2}{\gamma^2} var(\widetilde{\beta}_i) + \frac{2(t-1)}{\gamma} cov(\widetilde{\beta}_i, \widetilde{\Omega}_i) + var(\widetilde{\Omega}_i),$$

where $\Omega_i \equiv \sum_{t=1}^T \frac{y_{it}}{R^{t-1}} / \sum_{t=1}^T [\beta_i R^{(1-\gamma)}]^{\frac{t-1}{\gamma}}$. The second term is a linear function of t whose slope is determined by $cov(\widetilde{\beta}_i, \widetilde{\Omega}_i)$. We get the t^* by letting the derivative of the variance of log consumption to t be zero. Letting $var(\widetilde{c}_{iT}) > var(\widetilde{c}_{i1})$, we get the condition for the increasing consumption dispersion over the life cycle. Since the life-time wealth W_i is increasing in θ_i, higher $\rho_{\theta_t,\beta}$ would yield larger $cov(\widetilde{\beta}_i, \widetilde{\Omega}_i)$ and thus higher $var(\widetilde{c}_{it})$. ∎

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