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4aMU1. Modeling pulse-like lip vibrations in brass instruments

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During the starting transient of a note on a brass instrument it can take several cycles of lip vibration before acoustics reflections from the end of the instrument can influence the lip frequency. Under certain conditions the lip may fail to oscillate at the pitch of the air column resulting in an unwanted pulse-like waveform with relatively low repetition rates (similar to the vocal fry register of phonation in the human voice). This is often observed in the playing of beginners if the lips are insufficiently tense or if the top and bottom lips overlap to a large extent. In this study the reasons for this behavior will be investigated using modeling techniques with the aim of improving the agreement between physical models and measured transients by including the forces responsible for this effect.

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INTRODUCTION

The sound generator responsible for oscillations in brass instruments is usually described as outward striking or sideways striking reeds while some woodwind instruments feature inward striking reeds(Fletcher, 1993). These descriptions may produce reasonable approximations for the behaviour of instruments by approximating the lip or reed by a single mass acting as a forced damped harmonic oscillator, and energy feedback from the passive resonator allows sustain oscillations at frequencies close to the peaks of input impedance of the resonator (Gough, 2007).

Human vocal sound vowel production on the other hand is often described according to a source filter model with the vocal cords able to sustain oscillations in a continuous range of frequencies without the requirement for energy feedback from the acoustic resonance of the vocal tract. The vocal cords are often represented by two masses which open in a convergent shape and close in a divergent shape supplying energy to the upstream mass due to the pressure between the upstream masses being higher during the opening phase than during the closing phase (Ishizaka and Flanagan, 1972).

Single mass and two mass models of pressure driven oscillation often feature nearly sinusoidal oscillations of the masses, with the lip or reed closed for part of each period of vibration in order to produce a discontinuous flow into the air column, producing an excitation containing many harmonics. The vibrations that can be created do not feature the lips/reed closed for a length of time that exceeds the length of time during which the lips/reed are open by a significant factor. While the two mass model does a very good job of representing the usual "modal" register of phonation in the human voice, it is less effective at representing the vibrations where the vocal folds overlap significantly and have low lateral tension, known variously as the vocal fry, pulse register or creak voice.

The vocal fry register has low fundamental frequency of vibration (sub 60Hz) and electroglottographic (EGG) data has shown that the vocal cord opening occurring in singlets, doublets or triplet patterns within each period of vibration rather than always oscillating in a single opening/closing pattern (Blomgren *et al.*, 1998). High speed photography reported by Zemlin has revealed that "the folds are approximated tightly, but at the same time they appear flaccid along their free borders, and subglottal air simply bubbles up between them at about the junction of the anterior two-thirds of the glottis" (Zemlin, 1988) (as also quoted in Blomgren et al. (Blomgren *et al.*, 1998)). It is possible to model a sound similar to the vocal fry by representing the front and back of each vocal fold with multiple masses along the length of the vocal cord and setting the lateral tension to be minimal resulting in an irregular motion of the masses (Kob, 2002), but explanation of the regular periodic vibrations possible for the vocal fry and the underlying mechanism for the repetition rate of those vibrations appears non-obvious.

Sound may also be produced by holding the lips together and passing individual bubbles of air between them to create a sound known as "blowing a raspberry". This method producing sound is clearly similar to the vocal fry in terms of the physics of sound generation, producing low frequency oscillations across a small fraction of the lip width in comparison to standard brass playing and standard one or two mass models (even allowing for two dimensional motion) will not successfully model this sound. Understanding the physics of this method of sound production is clearly beneficial in terms of addressing problems in vocal and brass sound production, understanding the behaviour of the lips/vocal cords during the starting and finishing transients where small fractions of the lips are vibrating and in terms of improving the accuracy of physical modelling.

This paper will focus on physical mechanisms which may be responsible for vibrations where air bubbles between a portion of the lips/vocal cords. We will refer to the vibrating structure as a lip during this paper although the underlying physics is also implicitly applicable to the vocal cords. The description here differs significantly from the description usually used for modelling sound generation in such systems and several order of magnitude estimations are used. No simulations or experiments have been preformed to verify the accuracy of the approximations made thus far and this will follow in later work.

SURFACE TENSION

Consider the bottom surface of a top lip displaced by the air pressure to form a half of a spherical cap as shown in figure 1. We will refer to this shape as a bubble in the understanding

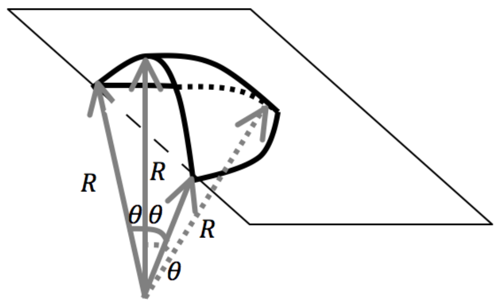

FIGURE 1: Geometry of lip with bubble forming

that it is air filled and forms a continuous air connection to the pressure to the left hand side of the diagram, which is supplied with a higher pressure (from the mouth/lungs) than on the right hand side (outside air). This surface is curved with radius R. If the air pressure difference between the air below the the lip and the pressure above the lip is Δp and the surface tension (force per unit depth) is given by τ the Young-LaPlace equation as given by Batchelor (2000) is:

$$\Delta p = \tau \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \tag{1}$$

where R_1 and R_2 are the radii of curvature along two perpendicular planes and $R_1 = R_2$ implicitly in figure 1. It may be assumed that if there is a strong lateral tension on the lips (as required to sound a note using normal brass instrument playing technique) then the tension will be greater and therefore the radius of curvature will be greater for a given mouth pressure, hence the effect of small bubble formation will be more pronounced when the lateral tension on the lips is low. The curved surface of the lip will be at equilibrium if the surface tension, τ, of the lip (potentially contributed to both by the tension of the lip before displacement and by the lengthening of the lip due to its curvature) satisfies equation 1. There will be an unbalanced force at the corners of the lip leading to bubble growth over time, however. This may be partly mitigated by adhesion from contact with another lip (not shown on the diagram).

It is clear that the lips may close the air inside the mouth during bubble formation such that the pressure below the bubble is equal to that in the mouth and the surface tension, pressure and radii correspond reasonably closely to those given in equation 1. The bubble will grow until it causes a leak to the outside air, allowing air to blow out between the lips. The pressure underneath the lip will then suddenly drop due to the exposure to the outside air (assuming the pressure outside the lips is low) and due to Bernoulli forces. The tension of the curved lip surface is no longer balanced by the mouth pressure and there will be a rapid conversion of the potential energy in the curved lip surface into kinetic energy of the lip causing the lip to slap shut.

WORK DONE IN FORMING BUBBLE

In a one or two mass model will oscillate in such a way that, if the pressure in the mouth is efficient to cause periodic vibration, then each opening follows rapidly after the previous. For the bubble formation model, however, the work done for the bubble growth is worthy of investigation.

For the moment we will assume that there exists a bubble beneath a lip corresponding to half of a spherical cap of radius $R = R_1 = R_2$ with the centre point of the sphere lying on the plane of the surface of the lips inside the mouth as shown in figure 1. Assuming that the radius remains constant while the bubble grows with increasing angle θ, the surface tension in the (growing) curved portion of the lip remains approximately constant during bubble growth. We will assume that the unbalanced component of the force responsible for bubble growth will be much smaller than the overall force produced by the tension (which is then approximately balanced by the pressure), meaning the work done in creating the bubble may be easily calculated as:

$$W = \int \tau dS = \int \left(\frac{\Delta p R}{2}\right) dS = \frac{\Delta p R}{2} \int dS = \frac{\Delta p \pi R^3}{2}(1 - \cos(\theta)) \tag{2}$$

where $S = \pi R^2(1 - \cos(\theta))$ is the surface area of half of a spherical cap. If there are two lips, both displaced symmetrically, then the work done (and therefore the energy stored) to create this shape is double this value.

Estimating the work done on a single lip to form a bubble of radius $R \approx 2$ mm, mouth pressure $\Delta p \approx 1$ kPa and bubble half angle $\theta \approx \pi/6$ radians gives $W \approx 1.6\mu$ J from equation 2.

BUBBLE COLLAPSE

The distance that the bubble extends, perpendicular to the inside mouth surface, in the direction of the outside air will will be given by $R\sin(\theta)$. When this value is equal to the distance between the lips then the air is able to being escaping. The curved surface of the lips will experience a rapidly lowering pressure once the the lips open significantly at their outside surface, and we will approximate the pressure between the lips as zero for the following order of magnitude calculations, while a fuller treatment would involve using the Bernoulli equation and considering the motion of the three dimension shape of the lip surface.

Consider the element of the curved lip surface of area dA shown in figure 2. We will assume

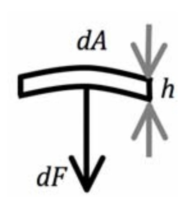

FIGURE 2: Lip element

that the force dF due to the tension of the lip exactly balanced the force due to the pressure in the mouth before closure such that $dF = \Delta p dA$ since bubble growth is mainly caused by the discontinuous force at the corners of the lip curve. When the pressure below the lip vanishes on opening, this becomes a discontinuous force producing acceleration:

$$\ddot{x} = \frac{dF}{dm} = \frac{\Delta p dA}{\rho dA h} = \frac{\Delta p}{\rho h} \tag{3}$$

where dm is the mass of the lip surface element where h is the effective height of the tense part of the lip surface and ρ is the density of the lip material and for $\Delta p \approx 1$ kPa and $\rho \approx 1000$ m/s^2

and $h \approx 1$ mm we have an acceleration of the order of 1000 m/s^2. This approximate acceleration will reduce during closure as the lip gets less tense as the radius of curvature reduces, but ensures rapid closure over a millimetre scale displacement generating a pulse sound.

By considering figure 1, the maximum displacement found on the curved section of lip will be given by $\Delta x = R(1-\cos(\theta))$. If we assume that the force on the surface element in figure 2 reduces linearly with displacement then we approximate the closure by a quarter period of a simple harmonic oscillator. The natural frequency will then be approximately given using the spring constant, k taken from the ratio of the force and displacement for the mass element dm:

$$\omega_0 = \sqrt{\frac{k}{m}} \approx \sqrt{\frac{\left(\frac{\Delta p dA}{\Delta x}\right)}{\rho dAh}} = \sqrt{\frac{\Delta p}{\rho \Delta x h}} \tag{4}$$

which obtains a value of the order of $\omega \approx 310$ Hz to 2 s.f. giving a period time of $T = 2\pi/\omega \approx 3.3$ ms to 2 s.f. indicating that closure would occur in a quarter period of $T/4 \approx 0.8$ ms to 1 s.f. This is consistent with the duration of pulses emitted from the lips when creating a bubble, but the time taken for the next bubble to form may be significantly greater than the closure time, and it is worth investigating why this is so.

APPROXIMATING THE JET

The velocity of a jet across the pressure difference Δp is given by the static Bernoulli equation as:

$$v = \sqrt{\frac{2\Delta p}{\rho_{air}}} \tag{5}$$

The volume of air escaping during a single pulse will depend on the cross-section of the jet opening, which will in turn depend on the time dependent details of the three dimensional motion of the lip surface. In order to assess the order of magnitude we will approximate the average cross-sectional area of the opening as half of the volume of the half spherical cap divided by the lip width. The volume of half of a spherical cap is:

$$V = \frac{\pi R^3(1-\cos(\theta))}{12}\left(3\sin^2(\theta) + (1-\cos(\theta))^2\right). \tag{6}$$

If this cap is to reach the outside air then the distance between the inside of the mouth and the outside air in figure 1 must be $R(\sin(\theta)$ so if the volume of the half spherical cap volume was converted into a constant cross-section opening then this opening would have a cross-section of:

$$S \approx \frac{V}{R\sin(\theta)} \approx \frac{\pi R^2(1-\cos(\theta))}{12\sin(\theta)}\left(3\sin^2(\theta) + (1-\cos(\theta))^2\right). \tag{7}$$

Taking the radius of $R \approx 2$mm and $\theta \approx \pi/6$ radians gives $S \approx 2.8 \times 10^{-7}$ m^2 to 2 s.f.

Using $\Delta p \approx 1$ kPa and $\rho_{air} \approx 1.21$ kg/m^3 in equation 5 gives the jet velocity as approximately $v \approx 40$ m/s and we will approximate a closure time of $T/4$ and an average cross section of $S/2$ giving a lost volume of approximately:

$$dV \approx \frac{S}{2} v \frac{T}{4} \tag{8}$$

which, taking $T/4 \approx 0.8$ ms after equation 4 gives a lost volume of approximately $dV \approx 4.6 \times 10^{-9}$ m^3 or 4.6 mm^3. It is worth noting that the translational kinetic energy in such a jet is then

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}\rho_{air} dV v^2 \tag{9}$$

which is approximately 0.11μ J for this example. It is an order of magnitude smaller than the work done in creating the bubble.

Thermal energy and repressurisation

The thermal kinetic energy contained within the lost volume of air is approximately $E_T = \gamma P dV$ which equates to approximately 1100 μ J for the calculations above (given the air pressure is $P \approx 1 \times 10^5$ Pa and dimensionless specific heat capacity at constant volume of $\gamma = 5/2$), but this energy is not converted to acoustic energy of course and is in proportion to the mass lost so the pressure inside the mouth/lung system will go down proportionally with the reduced mass of air inside according to the ratio:

$$\sigma = \frac{dV}{V} = \frac{dp}{P} \tag{10}$$

where dV is the volume of air lost to the jet from a mouth/lung system of V, leading to a pressure drop of dp from the starting pressure of P. This pressure loss can only be recovered by performing work by compressing the gas in adiabatic pressure change (say by pumping air using muscles such as the diaphragm). The pressure and volumes after adiabatic repressurisation will be:

$$P_2 = \beta P_1, \quad V_2 = \beta^{1/\gamma} V_1 \tag{11}$$

where β is the ratio of the pressure after decompression and the pressure before decompression and $\beta = 1 + \sigma$ if P_2 returns us to the initial pressure before the jet was emitted. This leads to a change in thermal energy of:

$$\Delta E_T = \gamma(P_2 V_2 - P_1 V_1) = \gamma P_1 V_1 \left(\beta^{\left(1-\frac{1}{\gamma}\right)} - 1\right) \tag{12}$$

If we assume that the volume lost will be much smaller than the volume in the mouth/lung system ($dV << V$ and therefore $\sigma << 1$) then the binomial theorem gives:

$$\Delta E_T = \gamma P_1 V_1 \left((1+\sigma)^{\left(1-\frac{1}{\gamma}\right)} - 1\right) \approx \gamma P_1 V_1 \left(1 - \frac{1}{\gamma}\right)\sigma \approx (\gamma - 1) P_1 dV \tag{13}$$

which is independent of the mouth/lung system volume and gives us an estimate of the work done to repressurise of $\Delta E_T \approx 680 \mu$ J which is much larger than the work done in order to create a bubble for this example.

Consequences for playing

Clearly bubbles are likely to form when the lateral tension of the lips/cords is small, leading the possibility of small radius bubbles in the Young-LaPlace equation (1) creating a weak, rasping sound. If the lateral tension is large then the radius implied by the Young-LaPlace equation will be large and the standard model of sound production applies.

Bubble formation requires that the lips have some depth front to back in order for lip bubbles to develop to an appreciable size. If the lips overlap to a great degree in the vertical direction, even when sufficient lateral tension applies, then conventional sound production is not possible as the pressure required for opening the lips goes up and the radius of bubble formation reduces. Future work will include the displacement of the supporting structures above the lip surface as this will play a role in controlling the threshold pressure required before bubble generation. Bubble generation occurring at a particular point requires that the lips/cords are not perfectly uniform along their length and this will have to be included in a model which reproduces such behaviour.

The physics behind the stages necessary to avoid trumpet playing producing undesirable weak pulse-like waveform are clear, namely that the lateral tension of the lips should be large and the lips should not overlap too much. If a problem note seems to requires a very large pressure then blowing harder will probably make the problem worse. When bubble generation does occur the repetition rate will depend on the rate of energy input of the lungs/mouth system and the volume of air ejected in each bubble and will not necessarily tune to the resonances of attached air columns.

Conclusions

A description has been given for bubble formation, collapse and repressurisation within an air driven lip or vocal cord type oscillator. Order of magnitude estimates of parameters have been calculated for the example of bubbles emitted from the lips under minimal transverse tension, when the lips are overlapping to a sufficient extent that that a non-trivial back pressure is required to release a bubble of air. Future work should include time domain simulation and experiments to validate the application of the laws of physics described here.

If the lips are brought together in such a way that they overlap significantly and lateral tension is minimal for the given back pressure then small bubbles (across a small fraction of a lip/cord) may be formed such that pulses are emitted. For the example taken, the dominant form of energy input necessary to determine the rate at which bubbles are created was that required to repressurise the system after mass is ejected in a jet during bubble opening. The bubble radius will be large if the lateral lip tension is large, and if the lips do not overlap too much this will lead to the bulk of the lip/cord vibrating in the manner usually described in the literature. Including the effect of bubble formation in time domain simulations may improve the agreement between models and experiment, particularly during starting transients where the lips may begin in an overlapping position before the back pressure is increased leading to them parting.

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