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Simultaneous determination of the constituent azimuthal and radial mode indices for light fields possessing orbital angular momentum

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A wide array of diffractive structures such as arrays of pinholes, triangular apertures, slits, and holograms have all recently been used to measure the azimuthal index of individual Laguerre-Gaussian beams. Here, we demonstrate a powerful approach to simultaneously measure both the radial and azimuthal indices of pure Laguerre-Gaussian light fields using the method of principal component analysis. We find that the shape of the diffracting element used to measure the mode indices is in fact of little importance and the crucial step is training any diffracting optical system and transforming the observed pattern into uncorrelated variables. The method is generic and may be extended to other families of light fields such as Bessel or Hermite-Gaussian beams. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4728111]

The direct determination of the complete transversal state of an electromagnetic field and accompanying mode indices is essential for the proper quantification of all light-matter interactions.[1] In particular, light fields with cylindrical symmetry such as Laguerre-Gaussian (LG) beams can possess orbital angular momentum[2] equal to $\ell\hbar$, where ℓ denotes the azimuthal index of the beam. This property is central to a wide range of emergent applications in quantum cryptography,[3] manipulation,[4] astrophysics,[5,6] microscopy,[7] and electron beams.[8] A wide array of diffractive structures such as arrays of pinholes,[5] triangular apertures,[9,10] slits,[11,12] and holograms[13,14] have all recently been used to measure the azimuthal index ℓ of individual LG beams. However, all these experimentally realised approaches measure only one single degree of freedom of LG beams, neglecting the radial component or p index and are thus not applicable for a priori unknown beams. Furthermore, it is unclear which is the optimal aperture and scheme needed to determine simultaneously the azimuthal and radial indices, nor the extent with which such an aperture can tolerate deviations in beam parameters. Here, we demonstrate a powerful approach to simultaneously measure the radial and azimuthal indices (ℓ and p) of LG light fields. We show that the shape of the diffracting element used to measure the mode indices is in fact of little importance and the crucial step is "training" any diffracting optical system and transforming the observed pattern into uncorrelated variables (principal components). Principal component analysis (PCA) refers to an orthogonal transformation that converts multiple measures into a set of linearly independent and uncorrelated variables. The first principal component accounts for the largest variance within the set of measures and each subsequent component reflects the next highest variance whilst ensuring no correlation (orthogonality) to the preceding PCA components. Whilst established for over one hundred years[15] and widely applied in various areas (e.g., data analysis for spectroscopy, face recognition, data compression), this method has never been considered nor used in the case of diffraction theory and the analysis of the transversal state of a light field. Furthermore, by employing the PCA approach, modest fluctuations in beam parameters such as waist size and alignment variations can be tolerated. Our results demonstrate the first complete characterisation of LG beams including both independent degrees of freedom corresponding to the radial and azimuthal indices. The approach is generic and can be expanded to other families of beams such as Bessel or Hermite-Gaussian beams and represents a powerful method for characterising the optical multi-dimensional Hilbert space.[3]

The orthonormal basis set of LG beams is typically characterised by two indices, namely the azimuthal index ℓ which denotes the number of cycles of 2π phase change around the mode circumference and the radial index p, where $p+1$ denotes the number of rings present within the light field. A suite of methods have emerged in the last few years that attempt to measure the azimuthal index of an LG light field. However, it is to be noted, common to all of these schemes is the fact that the incident light field is assumed to possess a radial index $p=0$. In contrast, in virtually all experimental realisations of LG beams, we are presented with light fields that are appropriately described as superpositions of LG modes, each of the same index ℓ but of different p index.[16] Thus, it is crucial to include the influence of both mode indices on the diffraction pattern of any aperture or slit and further ascertain whether using any diffracting aperture we are able to determine both the azimuthal and radial indices simultaneously. The radial index itself adds a major new degree of freedom that may be exploited for quantum communication in its own right. The need for both azimuthal and radial mode analysis is reinforced by recent theoretical work.[17] Furthermore, open questions include the optimal aperture to use and the robustness of the determination of mode indices in the presence of any mis-alignment. The generic PCA approach we implement here addresses all of these issues.

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The experiment (see Figure 1) uses a Helium Neon laser source ($\lambda = 633$ nm, $P_{max} = 5$ mW). The laser beam was additionally sent through a 50 μm pinhole in order to obtain a beam featuring a homogeneous Gaussian intensity profile. The laser beam was then subsequently expanded with a telescope (L_1 and L_2) in order to slightly overfill the chip of a spatial light modulator (SLM, Holoeye LC-R 2500). The SLM operated in the standard first-order diffraction configuration and was used to imprint the vortex phase on the incident beam where the LG beam was created in the far-field of the SLM. In order to filter the first order beam, carrying the vortex, from the unmodulated zero-order beam, a pinhole aperture F was located in the back focal plane of lens L_3. Lenses L_4 and L_5 was then used to image the ℓ and p mode onto a diffracting aperture with feature sizes roughly matching the beam waist. We used two apertures, namely a triple triangular slit aperture and a glass diffuser. Lens L_6 served to create the far-field diffraction pattern on the CCD camera (Basler pi640–210 gm, pixel size: 7.4 μm $\times$ 7.4 μm). We have adjusted the CCD camera exposure time in order to best highlight the pattern morphology for each recorded pattern.

Our approach is to consider the intensity profile of an LG beam after diffraction from a mask or filter. This intensity profile is in general complicated. It is to be noted that all the methods presented here are composed of a training or calibration step in which the response of the optical diffracting system is measured for every single LG beam considered. The second step corresponds to the actual identification or measurement of an unknown LG beam delivering simultaneously its radial and azimuthal indices. The LG beams are created using a SLM that is wavefront corrected using the optical eigenmode (OEi) technique.[18,19]

To illustrate our approach, we begin with a discussion of the far-field diffraction pattern of a general LG beam from

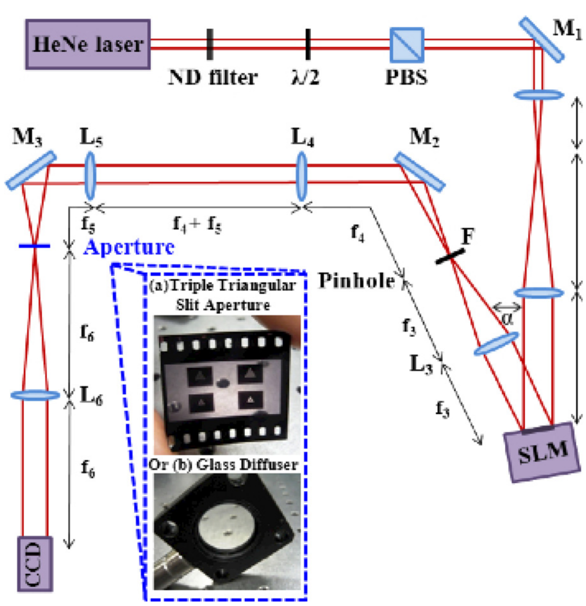

FIG. 1. Schematic of the experimental set-up: L = lens, SLM = spatial light modulator, CCD = charge coupled device camera, and PBS = polarizing beam splitter. Focal widths of lenses: $f_1 = 25$ mm, $f_2 = 100$ cm, $f_3 = 680$ mm, $f_4 = 400$ mm, and $f_5 = 800$ mm. A triple concentric triangular slit aperture or a glass diffuser were used.

an aperture composed of triple concentric triangular slits. This is an extension of the single triangular aperture already considered, which is presently one of the more powerful techniques for determining the azimuthal index[9,10] by simply counting the number of lobes in the far-field diffraction image. Superposing multiple such apertures might appear, at first glance, an appropriate method to determine both the radial and azimuthal indices of an incident field. Indeed, the choice of three concentric apertures aims to probe the radial index of LG beams with a commensurable beam waist. From Figure 2(b), we deduce that the diffraction pattern from the triple triangular slit aperture does not offer any simple way to determine the ℓ and p beam parameters. However, we remark that regardless of the radial index, the pattern orientation does depend on the sign of the azimuthal index. Although, it cannot be excluded that there exists a specifically designed mask that would deliver a simple rule for the detection of both ℓ and p, this is not the case for the triple triangular slit aperture.

Importantly, the deduction of a complicated ℓ and p retrieval rule can be replaced by considering a face-recognition algorithm readily employed in biometric identification.[20] Here, we choose to use the PCA approach to determine the largest variations between the different far-field diffraction patterns from our aperture. The first step of the procedure corresponds to creating a database of all the possible beams that need to be detected. After subtraction of the common mean intensity, we calculate the covariance matrix of these intensity patterns. Its eigenvector with the largest eigenvalue is termed the first "eigenface" corresponding to the largest variability of the LG beam training set. In the same way, one can introduce the second "eigenface" as the image corresponding to the second largest eigenvalue of the covariance matrix. As can be seen from Figure 2(d), projecting the measured beams onto these "eigenfaces" delivers the first and subsequent principal component representation of the measure. We remark that in this representation the LG beams having the same ℓ and p form very tight clusters due to readout noise and small vibrations of the optical system. Finally, representing the diffraction pattern of an unknown beam in the same way, one can use a classification algorithm to determine the membership of beams with unknown mode indices. For simplicity, we chose the nearest neighbour measure for classification[21] but other methods such as the Mahalanobis distance[22] may also be used. Figure 2(f) shows the classification results displaying a 100% efficiency i.e., all unknown beams have been correctly identified.

It is straight forward to deduce the radial index of an LG beam from its intensity profile by simply counting the number of rings in the beam profile. Unfortunately, the complete characterisation of LG beams is more difficult when it has a non zero azimuthal index and thus possesses a topological vortex. Theoretically, the intensity profile of a beam with ℓ azimuthal index is identical to the $-\ell$ case. This makes it impossible to deduce the azimuthal index from the intensity of the beam. The triangular aperture breaks this symmetry and any mask that is not inversion symmetric would be able to distinguish between the two different signs of ℓ. More generally, *any* random mask or aperture can be used to break this symmetry and its diffraction pattern can be used to

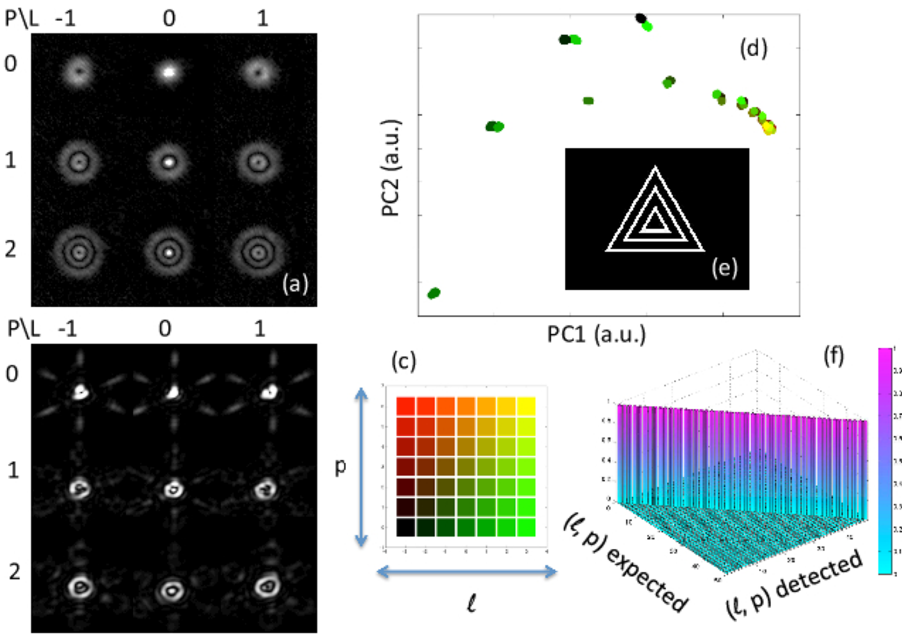

FIG. 2. Triple triangular slit aperture experiment. (a) Table showing the 9 lowest LG modes ($\ell = [-1, 1]$ and $p = [0,2]$) and their (b) far field diffraction pattern of the (e) triple triangular slit aperture. (d) Principal components of the diffraction patterns (c) colour coded for the azimuthal and radial LG index. (f) 3D bar chart of the confusion matrix linking expected and detected beam indices.

detect simultaneously both beam indices. Figure 3(a) shows that the diffraction patterns do not present any prominent features at all and that any distinct LG beam results in a different diffraction pattern of similar overall form. Consequently, the variations, in the plane of the first two principal components, are more evenly distributed not privileging any specific radial index as is the case for the triple triangular slit aperture. This is due to the uniformity of the random mask over which the beam extends for different radial indices. Furthermore, it is to be noted that the use of the random mask presents an experimental benefit as all diffraction patterns have similar peak intensities enabling the intensities acquisition with similar exposure durations. Using the random mask we achieve, as indeed for the triple triangular slit aperture, 100% classification efficiency while eliminating the need to match beam waist and radial index to the size of the triangles used.

The fundamental question of the interdependence between the radial index, beam alignment, and the beam waist is also interesting.[23,24] Importantly, the azimuthal index has only a significance in relationship with the beam axis position and direction while the radial index has significance with regards to the beam waist parameter. Indeed, for centred beams, the waist parameter does not affect the azimuthal index as the total orbital angular momentum is independent of lateral displacement (as long as we have zero transverse momentum).[25] We therefore investigate the influence of the beam waist fluctuation and beam mis-alignment on the classification ability of our scheme. Using the PCA detection method, outlined above, it is possible to study the effect of the variation of these parameters by controllably changing the amplitude profile of the LG beam generating SLM.[26] Figure 4 shows the results when considering the effect of these parameter fluctuations. The first effect is a clear widening of the scattering cluster of each given beam parameter. This is understandable as beam parameter fluctuations naturally induce a certain variability of the intensity profile. Nevertheless, the correct detection can still be achieved provided that either a larger training set is considered and/or a larger dimensionality of the data is allowed by taking more principal components into account. This last point is illustrated in Figure 4(e) where the detection efficiency, defined by the trace of the confusion matrix normalised to the total number of unknown beams considered, is evaluated as a function of the number principal components. We remark here that the beam axis and waist information is

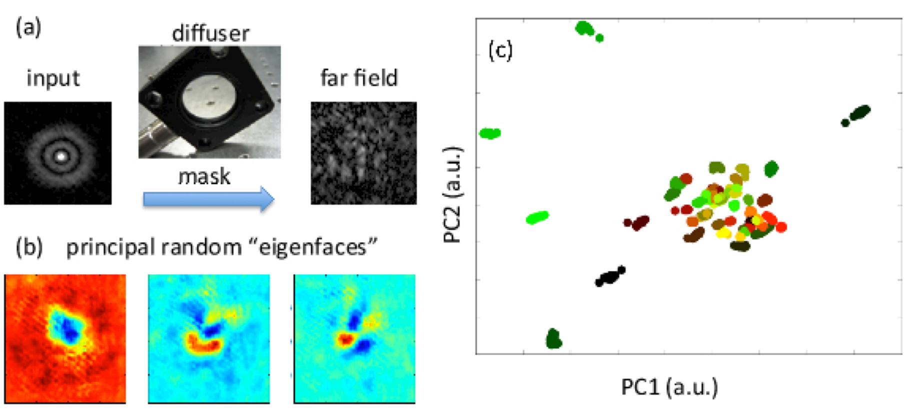

FIG. 3. (a) Experimental implementation of the random mask. (b) First three principal "eigenfaces" based on the far field diffraction patterns ($\ell = [-3, 3]$ and $p = [0,6]$) from the random mask. (c) Principal components using the same colour coding as in Figure 2.

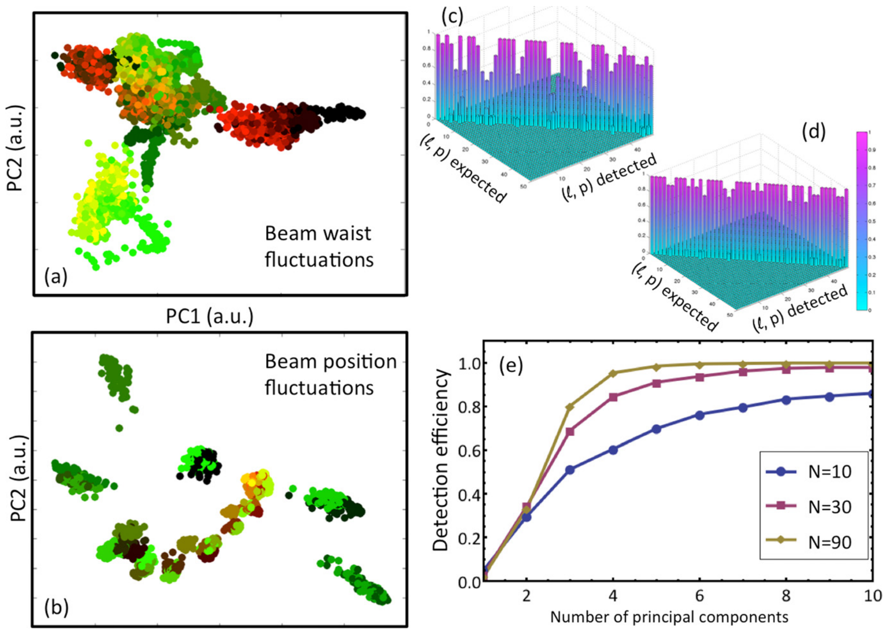

FIG. 4. Experimental study of position and beam waist fluctuations. (a) and (b) Principle components using the same colour coding as in Figure 2 for (a) the beam waist fluctuations of 10% relative to waist and (b) beam position fluctuations of 10% relative to the waist size. (c) and (d) Respective confusion matrices for the same cases. (e) Detection efficiency as a function of the number of principal components used for the beam parameter identification where N is the number of far field diffraction images used as a training set for the PCA.

actually included in the "training set" and as such its explicit knowledge is not necessary to correctly identify the azimuthal and radial indices.

In this letter, we have presented a straight forward approach to simultaneously determine both the radial and azimuthal indices of LG beams by projecting the far-field diffraction pattern onto a set of uncorrelated variables. This method is robust and can even tolerate a certain degree of beam mis-alignment and beam waist variations. Whilst we concentrate here on the LG family of optical beams, the approach we present is generic and can be readily applied to the detection of other families of beams such as Hermite-Gaussian or Bessel beams. The method presented here is limited by the number of detectable optical degrees of freedom of the far-field diffraction pattern.[27] Indeed, low resolution cameras would greatly decrease the distinguishability between the different LG beams and decrease the number of modes that can be detected. Finally, with suitable training, one could envisage the use of this approach to detect low order aberrations that can be described by the family of Zernike polynomials. The mode determination method may be applied beyond the field of electromagnetic waves to sound and matter waves, for example, and electron beams. In future work, we will expand the method to explore complex superpositions of LG beams, enabling the encoding, decoding, and manipulation of the radial and azimuthal degrees of freedom.

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