The Structure of Galaxies
The Division of Stellar Mass by Morphological Type and Structural Component

by

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Submitted for the degree of Doctor of Philosophy in Astrophysics

5th September 2012
Declaration

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For Mum, Dad, Aimz, Soph and Gram.
Oh! I have slipped the surly bonds of Earth
Put out my hand, and touched the face of God.

from *High Flight* by John Gillespie Magee, Jr., 1941
Abstract

The mechanisms which cause galaxies to form and evolve each leave behind distinct structural markers in their wake. Dynamically hot processes (e.g., monolithic collapse, hierarchical merging) give rise to pressure-supported spheroidal structures, including elliptical galaxies and classical bulges. By contrast, dynamically cold processes (e.g., gas accretion, AGN splashback) lead to flattened rotationally-supported disk-like structures, often found on their own or as part of a spiral galaxy. If left in isolation for a sufficient length of time, secular evolutionary processes cause the formation of a bar-like structure within the disk, precipitating the genesis of a rotationally-supported pseudo-bulge. Robustly measuring galaxy structure enables us to ascertain the relative importance of these competing evolutionary mechanisms and, in so doing, help broaden our understanding of how the Universe around us came to be.

This thesis explores the relation between galaxy structure, morphology and stellar mass. In the first part I present single-Sérsic two-dimensional model fits to 167600 galaxies modelled independently in the ugrizYJHK bandpasses using reprocessed Sloan Digital Sky Survey Data Release Seven (SDSS DR7) and UKIRT Infrared Deep Sky Survey Large Area Survey (UKIDSS LAS) imaging data available via the Galaxy and Mass Assembly (GAMA) data base. In order to facilitate this study, we developed Structural Investigation of Galaxies via Model Analysis (SIGMA): an automated wrapper around several contemporary astronomy software packages. We confirm that variations in global structural measurements with wavelength arise due to the effects of dust attenuation and stellar population/metallicity gradients within galaxies.

In the second part of this thesis we establish a volume-limited sample of 3845 galaxies in the local Universe and visually classify these galaxies according to their morphological Hubble type. We find that single-Sérsic photometry accurately reproduces the morphology luminosity functions predicted in the literature. We employ multi-component Sérsic profiling to provide bulge-disk decompositions for this sample, allowing for the luminosity and stellar mass to be divided between the key structural components: spheroids and disks. Grouping the stellar mass in these structures by the evolutionary mechanisms that formed them, we find that hot-mode collapse, merger or otherwise turbulent mechanisms account for \( \sim 46\% \) of the total stellar mass budget, cold-mode gas accretion and splashback mechanisms account for \( \sim 48\% \) of the total stellar mass budget and secular evolutionary processes for \( \sim 6.5\% \) of the total stellar mass budget in the local \((z < 0.06)\) Universe.
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Introduction: Extragalactic Astronomy

1.1 From Cloudy Spots to Island Universes

From antiquity to the turn of the 20th century it was widely believed that humanity’s place in the Universe was somehow special; central to existence. As elucidated by Lynn (1901), the ancient Greek philosopher Democritus (450–370 BC) first proposed that the white band across the night sky which we now refer to as the Milky Way was composed of many stars much like our own. This theory was confirmed in 1610 by Galileo Galilei (1564–1642) who was able to magnify the Milky Way through a telescope and resolve the fainter stars from which it is comprised. The realisation that our own star is part of a much larger network of stars, or a galaxy, was fundamental in reshaping the way in which leading scientists, philosophers and theologians alike came to regard the natural world around us.

Englishman Thomas Wright (1711–1786) of County Durham first postulated that the Milky Way was in fact an extended flattened disk of stars, with our own solar system embedded somewhere within. In this seminal 1750 study, *An original theory or new hypothesis of the Universe*, he also contemplated the possibility of galaxies outside of our own, proposing “the many cloudy spots ... in all likelihood may be external [in] creation”. Around this
time German philosopher Immanuel Kant (1724–1804) and others popularised the notion that these diffuse nebulae may in fact each be an ‘Island Universe’, a galaxy much like our own and yet spatially distant (Kant, 1755). French astronomer Charles Messier (1730–1817) began mapping these nebulae (Messier, 1774) in order to provide an atlas of sources to be avoided in his search for comets. In so doing, he produced one of the first known catalogues of extra-galactic phenomena (albeit, their distant origin unknown to him at the time). Sir Frederick William Herschel (1738–1822) subsequently expanded on Messier’s catalogue, creating an atlas of over 5000 galactic nebulae (Herschel, 1785).

In the mid-19th Century, Yorkshire-born Lord William Henry Parsons, the 3rd Earl of Rosse (1800–1867), oversaw the construction of the ‘Leviathan of Parsonstown’: a 72” telescope situated in central Ireland. The unprecedented scale of this telescope was such that upon re-observing the nebulae previously described by Messier and Herschel he was able to divide them into two distinct types, namely: elliptical nebulae and spiral nebulae. An example of one such spiral nebula is shown in a sketch made by Lord Rosse circa 1850 of M51, now known as the Whirlpool Galaxy (M51), and reproduced in Figure 1.1. Further divisions of these nebulae into elliptical and spiral types followed, e.g., Curtis (1917); Reynolds (1920); Hubble (1926).

By the turn of the 20th Century, detailed analyses of the spectra of nebulae had begun.
1.1. From Cloudy Spots to Island Universes

The exceptionally named American astronomer Vesto Melvin Slipher (1875–1969) found the radial Doppler shift velocity for the Andromeda Nebula had a remarkable velocity of $-300 \text{ km s}^{-1}$ (Slipher, 1913). This velocity measurement put Andromeda into a league of its own when compared with other known Doppler measurements of astronomical phenomena at the time. Slipher hinted at the uniqueness of Andromeda, stating: “The magnitude of this velocity, which is the greatest hitherto observed, raises the question whether the velocity-like displacement might not be due to some other cause, but I believe we have at the present no other interpretation for it.” A broader study made by Slipher shortly thereafter (Slipher, 1915) found that the average velocity for a sample of 15 additional spiral nebulae to be about 25 times the average stellar velocity.

The realisation of the implication of these results, and others (e.g., de Sitter, 1917; Slipher, 1917), led in 1920 to the Great Debate: a pivotal moment in the history of astronomy. The previously favoured notion that the Milky Way and the Universe are one and the same, as championed by Harlow Shapley (1885–1972), was openly challenged by Heber Doust Curtis (1872–1942) during a meeting in Washington, D.C. at the Smithsonian Museum of Natural History. Curtis was a strong advocate that Andromeda and other such nebulae were in fact extra-galactic in origin, the so called Island Universes popularised by Kant 165 years prior. He pointed to the rate of novae in those nebulous systems as anomalously high compared with observations from other parts of the Milky Way, a large flaw in Shapley’s argument if they were indeed a part of our own Galaxy. In any event, the Great Debate did not settle the matter as both parties required additional data in order to back up their claims.

In the early 1920s, Edwin Powell Hubble (1889–1953) working at the Mount Wilson Observatory in the Western United States used the well known period-luminosity relation of Cepheid variable stars to estimate the distances to several nearby nebulae including Barnard’s Galaxy (NGC 6822), Andromeda (M31) and Triangulum (M33). Hubble’s measurements conclusively proved that these systems lie outwith the bounds of the Milky Way and therefore must be extra-galactic, thus settling the topic of the Great Debate. Elliptical and spiral nebulae became known as galaxies in their own right, heralding a new era of extragalactic astronomy. Hubble subsequently tied together distance and radial velocity measurements of these galaxies (Hubble, 1929) to find that the distance to a galaxy correlates closely with its recessional velocity, as shown in Figure 1.2. This relation became known as Hubble’s Law, and is fundamental in our understanding of the expanding Universe (see also Lundmark, 1924 and the...
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Figure 1.2: Hubble’s measurement of the distance and radial velocity component for several nearby galaxies. This linear relation became known as Hubble’s Law, and is fundamental in our understanding of the expanding Universe. This image is taken from Figure 1 of Hubble, 1929.

reviews of Kragh & Smith, 2003; Graham, 2011).

1.2 Galaxy Morphologies

Many different schema for the classification of galaxies into morphological types have appeared in the literature over the last century or so. In this section I review several of the more prominent classification methodologies.

1.2.1 The Hubble Sequence

The first large galaxy catalogues (Herschel, 1864; Dreyer, 1888; Herschel & Dreyer, 1912; Knox-Shaw, 1915) took to describing each system individually, and in painstakingly precise detail. This methodology arose from the need to help describe the exact location of these objects such that it would enable other astronomers to easily re-find them. The desire to classify these nebulae into natural groupings however existed before the extra-galactic nature of these objects was fully understood. As reviewed in Sandage (2005), a purely descriptive classification devised by Wolf (1908) sought to place galaxies into groupings based on their shape and size, however; there was no continuity between these classifications. Furthermore, the Wolf system was extremely verbose. Other systems devised by, e.g., Lundmark; Holmberg; Shapley and others tried to improve on Wolf’s schema, but failed to add the much needed
Perhaps drawing inspiration from Reynolds (1920), and also Jeans (1919), Hubble (1926) introduced what has since become one of the most popular means by which galaxies are classified into distinct groupings; the Hubble sequence. Figure 1.3 shows a graphical representation of the Hubble sequence, named the Hubble tuning fork for its distinctive two tined appearance. Galaxies are arranged into elliptical (left) and spiral (right) groupings. Elliptical galaxies are smooth, red, one-component systems that range from spheroidal (E0) to highly ellipsoidal (E7) in projection. Spiral galaxies are more complex systems containing distinctive, typically blue, spiral arm features within a flat rotating disk and emanating from a central, typically red, spheroidal bulge.

Spiral galaxies are arranged in order of the tightness of the winding in the spiral arm features and the dominance of the central bulge, from tightly wound arms with a large central bulge (Sa) to loosely wound arms with a smaller central bulge (Sc). A further distinction is made based on whether a spiral contains a bar-like feature passing through the central bulge onto which the spiral arms connect. A barred Sa-type galaxy would thus be labelled SBa.

A later introduction of an intermediate class of galaxy, S0 (Hubble, 1936c), contains characteristics of both populations, appearing as smooth systems with no spiral features and yet with an underlying disk like structure. These systems are called lenticulars, owing to their similarity with an optical lens. Elliptical and spiral galaxies are referred to as early-type and late-type, respectively. This naming convention does not imply an evolutionary mechanism, and indeed was never intended to (Baldry, 2008). It draws inspiration from the realm of spectroscopy, whereby simplistic spectra are referred to as ‘early’, and more complex spectra as ‘late’.

### 1.2.2 Revisions to the Hubble Sequence

Many authors have subsequently attempted to introduce revisions to the Hubble tuning fork, highlighting its susceptibility to misclassification owing to the effects of inclination or its over simplicity (Reynolds, 1927). It was Hubble himself who made one of the first attempts to update his classification scheme. As noted in Sandage (2005), Allan Sandage (1926–2010) came across a partially complete manuscript penned by Hubble whilst clearing out Hubble’s office following his death in 1953. The revision sought to fix the discontinuity between the S0 and SBa classes through the introduction of a SB0 class, a barred lenticular. This schema

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1Hubble makes no direct mention of Reynolds’ 1920 paper in his 1926 discussion on galaxy classification schema.
was subsequently published in Sandage (1961) and again in Sandage et al. (1975).

A commonly used extension to the Hubble classification scheme is that advocated by French astronomer Gerard de Vaucouleurs (1918–1995). De Vaucouleurs added an additional axis to Hubble’s tuning fork such that the ring component present in some spiral systems may also be classified (de Vaucouleurs, 1959). This system is summarised in Figure 1.4. Note the distinction between barred galaxies (B), as before, and unbarred galaxies (A), now made explicit in the naming convention. This allows for intermediate structures to be termed AB if the presence of a bar is uncertain or in transition. Similarly, spiral (s) and ring (r) structures may also be co-added to create the rs type. Additionally, the transition lenticular galaxy type S0 is divided into E⁺, S0⁻, S0⁰ and S0⁺, as per Holmberg (1958), allowing for greater resolution between these classes. The Sd type galaxy, a structure with a heavily dominant disk and little to no indication of a bulge, appears as an extension of the Sd class (Shapley & Paraskevopoulos, 1940). Beyond this, the irregular (Sm) and highly irregular (Im) classes are added.

Elliptical classifications (E0–E7) tend to correlate with their inclination relative to the observer in addition to any fundamental properties of the galaxy. Kormendy & Bender (1996) sought to resolve this by reclassifying the early-type arm entirely. Instead of varying as a function of axial ratio, Kormendy and Bender proposed that elliptical galaxies be ordered by isophote shape, as can be seen in Figure 1.5. The ellipticity is still taken into account via the familiar numbering system, with the addition of (boxy) for boxy isophotes, and (disky) for disky isophotes. Often, this is simplified to (b) and (d) for boxy and disky, respectively. Kormendy & Bender (2012) expand further on this system, adopting the increased resolution
1.2. Galaxy Morphologies

Figure 1.4: The de Vaucouleurs extension to the Hubble classification scheme. This widely-used classification system adds an extra dimension for ringed galaxies, and introduces the late type spiral Sd, and irregular classes Sm and Im.
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Figure 1.5: The Kormendy revision of the Hubble tuning fork classification scheme. Elliptical galaxies are now ordered in line with how boxy or disky their isophote is. In addition, greater resolution of the bulge-to-total ratio in lenticular galaxies is added (van den Bergh, 1976). This image has been reproduced from Figure 1 of Kormendy & Bender (2012).

in lenticular bulge-to-total ratios as detailed in van den Bergh (1976). Lenticular galaxies span from bulge dominated (S0a) to disk dominated (S0c) systems, with the logical extension to pure single-component spheroidal (Sph) galaxies appended thereafter.

The previous classification systems discussed all cater to standard ‘giant’ galaxies, with the extrapolation into fainter dwarf systems implied rather than definitively stated. Sandage & Binggeli (1984) introduced one of the first classification schemas that connect dwarf galaxies to their giant counterparts. Figure 1 from their 1984 paper is reproduced here in Figure 1.6, with galaxies arranged from the brightest (top) to the faintest (bottom) in average surface brightness, and hence in mass. The top row represents the classical Hubble sequence, and solid lines indicate a connection to the adjoined system. Dotted lines indicate a potential link between early and late type systems. Dwarf ellipticals (dE) and dwarf lenticulars (dS0) are connected to their giant counterparts as a logical extension into faint surface brightness regimes. Disturbed dwarf elliptical systems (dEpec) lie just below and in-between these two early type dwarfs. From their analysis of 138 dwarf galaxies in the Virgo cluster, Sandage and Binggeli found no dwarf spiral (dSa/dSb/dSc) systems, hence their exclusion from their figure. Moreover, they postulated that these types of systems do not exist in nature (although, see Jerjen et al., 2000b for evidence of a dwarf elliptical galaxy in Virgo that exhibits evidence of spiral structure). Blue Compact Dwarfs (BCDs; Arp, 1965) are connected to the bulge-less late type Sd galaxies via the Sm and Im type systems.
1.2. Galaxy Morphologies

Figure 1.6: The Sandage addition of dwarf systems (lower half) to the classical Hubble sequence (upper row). Dwarf ellipticals (dE) and dwarf lenticulars (dS0) are connected to their giant counterparts as a logical extension into faint surface brightness regimes. From their analysis of 138 dwarf galaxies in the Virgo cluster, Sandage & Binggeli (1984) found no dwarf spiral (dSa/dSb/dSc) systems, hence their exclusion from this figure. Blue Compact Dwarfs (BCDs) are connected to the Sm and Im type galaxies.
1.2.3 Morphology and Magnitude

One of the most fundamental measurements of a galaxy is its total luminosity, or absolute magnitude. Analysis of the luminosity function (LF), the number density of galaxies per unit luminosity interval, allows for the properties of galaxy samples to be more comprehensively studied. Early exploration of the LF led Hubble to (incorrectly) conclude that the global LF is Gaussian in shape (Hubble, 1936c,a,b). This fact was later disputed by Fritz Zwicky (1898–1974) who was able to show that correctly accounting for the effects of selection bias changes the faint-end slope significantly (Zwicky, 1942, 1957, 1964), causing the number density to rise exponentially into the faint dwarf regime.

A distinction is often made between the LF of galaxies in the field and galaxies in over-dense regions, such as a cluster environment (Hubble & Humason, 1931; Morgan, 1961; Abell, 1965; Oemler, 1974). This division is made owing to the different morphological types found within over-dense regions when compared with morphological fractions in field galaxies (see the Morphology-Density Relation; Dressler, 1980). A full description of the LF and its parametrisation via the Schechter luminosity function (Schechter, 1976), is given in Section 5.3.2. Figure 1.7 shows the LF for galaxies in the local field (top) and in the Virgo cluster (bottom), as indicated. This figure is reproduced from Figure 1 of Binggeli et al. (1988). Local field galaxy data has been taken from Kraan-Korteweg & Tammann (1979), whereas Virgo cluster data is from Sandage et al. (1985).

Underneath the total curves, the constituent morphological types are also indicated. These types are: elliptical, E; lenticular, S0; early-type spiral, Sa+Sb; late-type spiral, Sc; disk-dominated/Magellanic spirals, Sd+Sm; dwarf ellipticals, dE; Irregular, Irr; and blue compact dwarfs, BCD. In both the field and cluster environments, the brightest morphological types are giant elliptical galaxies, which dominate at the bright end, and hence, the high mass end. Intermediate brightness galaxies arise in the form of spirals, whereas the secondary upturn in the total LF is caused by Irregular and dwarf systems. In cluster environments, blue compact dwarfs also contribute to this bump at the faint end.

Binggeli further expands on this by plotting the correlation between absolute magnitude and absolute central surface brightness (Binggeli, 1994), reproduced here in Figure 1.8. Here we see the blue cloud (disks, Irregular galaxies) extending from the faint, low luminosity, end of the figure diagonally upwards to intersect perpendicularly with the red sequence (elliptical galaxies, bulges). The red sequence extends from faint, high luminosity systems, to brighter
Figure 1.7: The luminosity functions of galaxies by morphological type in field (top) and cluster (bottom) environments. This figure is reproduced from Figure 1 of Binggeli et al. (1988).
and lower luminosity systems, such as the compact Elliptical M32 (as indicated).

1.2.4 Galaxy Evolution Models

Observations provide key astronomical measurements of observable quantities such as that of the luminosity function (as discussed above) and the stellar mass function (see e.g., Baldry et al., 2012 and Chapter 6) and also several empirical relations such as the Morphology-Density Relation (Dressler, 1980), the Tully-Fisher Relation (Tully & Fisher, 1977) and the Faber-Jackson Relation (Faber & Jackson, 1976) (itself a projection of the Fundamental Plane for elliptical galaxies). However, it is important to put these observational measures in context by contrasting their results with those of advanced galaxy formation and evolution models.

In this section, I discuss the two main means by which galaxy formation and evolution simulations are studied, namely: N-body simulations and semi-analytic modelling.

At the root of both of these methods is a standard ΛCDM cosmology (Peebles, 1982; Blumenthal et al., 1984; Davis et al., 1985). ΛCDM implies that our Universe consists of
1.2. Galaxy Morphologies

a cosmological constant ($\Lambda$), which denotes the energy density of empty space, and cold dark matter (CDM), which is hypothesised to be a form of weakly-interacting non-baryonic matter. A fundamental tenet of this cosmological model is the ‘cosmological principle’; that the Universe is both homogeneous and isotropic, and we as observers on Earth do not occupy a privileged or special vantage point within it. The astrophysical constants that describe the $\Lambda$CDM model are now relatively well established. We opt to use concordance cosmology values of $\Omega_\Lambda = 0.7$, $\Omega_M = 0.3$, $H_0 = 70 \text{km/s/Mpc}$ (see for example the WMAP results of Komatsu et al., 2011) throughout this thesis unless otherwise indicated. Here, $\Omega_\Lambda$ is the ratio between the energy density of the cosmological constant and the critical density of the Universe (that is, the density of a Universe which exhibits a flat Euclidean-like geometry), $\Omega_M$ is the equivalent energy density for matter, and $H_0$ is the Hubble constant.

In a $\Lambda$CDM Universe, galaxies, and the large scale structure that they trace, are believed to have formed from small density perturbations in the very early Universe (Jeans, 1902). These structures continue to grow and evolve via gravitational attraction, giving rise to hierarchical growth via merging and other clustered phenomena (i.e., clusters, groups and filaments, see Efstathiou et al., 1988b; Frenk et al., 1990; Cole et al., 1994). The peak formalism developed by Bardeen et al. (1986) (see also Katz et al., 1993) assumes that the earliest precursor galaxies form only at the highest peaks in the initial density field of the early Universe. The Press-Schechter formalism (Press & Schechter, 1974; Bond et al., 1991; Bower, 1991; Lacey & Cole, 1993) instead assumes that all initial density perturbations form virialised objects (dark matter haloes and galactic structure) when they grow above a given density threshold. Once an initial density distribution has been established, one may evolve their model forwards in time.

A numerical N-body simulation allows for one to calculate the non-linear growth in structure across cosmic time. This is achieved by following the trajectories of multiple dark matter particles of a given mass which are evolving in the most simplistic case; under the effect of gravity (Efstathiou et al., 1985). Typically, either smoothed-particle hydrodynamics (SPH: Gingold & Monaghan, 1977; Lucy, 1977) or adaptive mesh/grid-based methods are employed here. In addition to gravity, more complex models in smaller volumes account for gas physics, e.g.; shock heating, radiative cooling and star formation (Cole et al., 1994), with special attention given to the impacts of mergers and interactions (Barnes & Hernquist, 1992). A particularly useful output of N-body simulations are merger trees (e.g., Kauffmann et al., 1993;
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Cole et al., 1994; Rodrigues & Thomas, 1996; Somerville & Kolatt, 1999; Wechsler et al., 2002; Parkinson et al., 2008). A merger tree shows the merger history of a given structure, allowing for each progenitor mass to be successively identified back to the start of the simulation. An example of such a merger tree is shown in Figure 1.9, reproduced from Figure 2 of Wechsler et al. (2002). Making full use of merger information, and applying gas physics, one is able to produce realistic bulge/disk systems of a known morphology (Negroponte & White, 1983; Governato et al., 2007; Agertz et al., 2011), which is a very useful output for, e.g., the Tully-Fisher relation. Of course, N-body simulations remain extremely computationally demanding, with practical limitations placed on particle densities and masses. Relatively large dark matter particle masses are typically employed (>~10^4 solar masses), which may act to mask small-scale effects and impede on the modelling of core regions.

An additional, or perhaps complimentary approach to the study of galaxy evolution, is that of semi-analytic modelling (SAM; Cook et al., 2009, 2010b,a). SAMs adopt the merger trees mentioned above, and assign properties to each structure based on a physically motivated model. Instead of focusing on the gas dynamics of numerical simulations, SAMs are concerned with the global properties of galaxies, allowing for a level of description unavailable in many contemporary numerical simulations. Assuming some initial cosmology, SAMs must aim to reproduce the low-redshift galaxy properties observed today, such as the Tully-Fisher Relation, the luminosity function and the relationship between luminosity, colour and metallicity. Common SAM components are gas cooling mechanisms; star formation rate history (Lilly et al., 1996; Madau et al., 1996); feedback; metal enrichment; black hole growth; stellar population synthesis, and; mergers via dark matter merging trees (see above, and, e.g., Lacey & Cole, 1993).

By contrasting the final output model Universe parameters with those observed, one is able to comment on the nature of the early Universe, and constrain several key cosmological parameters such as the galaxy luminosity function, galaxy mass function and the effects dark matter. Whilst a more in-depth discussion of both numerical simulations and semi-analytic modelling is beyond the scope of this thesis, it is important to acknowledge that the outputs of these theoretical simulations allow for us to compare our empirical results against them, and consequently draw conclusions on the true nature of galaxy formation and evolution.
1.2. Galaxy Morphologies

Figure 1.9: Structural merger trees for two distinct halos, reproduced from Figure 2 of Wechsler et al. (2002). The merger history of a cluster mass halo ($\mathcal{M} = 2.8 \times 10^{14} \, M_\odot$, left) and a galaxy mass halo ($\mathcal{M} = 2.9 \times 10^{12} \, M_\odot$, right) is shown progressing from the early Universe (top) to the present day (bottom). Lines connect progenitor halos, with the size of the outer circle representing the magnitude of the virial radius of the structure. The numbers along the central spine represent the Universal expansion factor $a = 1/(1 + z)$, as an indication of time.
1.3 The Sérsic Profile

Measuring and quantifying the surface brightness profile of galaxies has been a useful tool for further exploration of intrinsic galactic properties for some time. As was established by Naim et al. (1995), visual galaxy classification remains a subjective measure of the property of a galaxy. Naim found that whilst individual observers agree on the whole, the scatter about each classification can be prohibitively large. Replacing subjective morphological classifications with something more quantitative, such as analysis of the surface brightness profile, is a natural and logical extension of extragalactic analyses.

It is commonplace to approximate the disks of spiral galaxies with an exponential profile (Freeman, 1970; Kormendy, 1977b; Andredakis & Sanders, 1994), as this seems to provide a good indication of the light distribution in those systems (e.g., Bland-Hawthorn et al., 2005). Early-type galaxies are not so standardised. The simplistic model of Plummer (1911), initially developed to model the distribution of light in globular clusters, found some early use in modelling the surface brightness distribution of elliptical galaxies. Similarly, the King profile (King, 1962, 1966; Elson, 1999), again developed to model globular cluster light profiles, is now used by some observers to model the faint nucleated cores of early-type galaxies (e.g., Ferrarese et al., 2006). The Hubble-Reynolds law (Reynolds, 1913; Hubble, 1930; Binney & Tremaine, 2008) was one of the first light profile models developed specifically for elliptical galaxies (or elliptical nebulae as they were known). It is given by

$$I(r) = \frac{I_0}{(1 + r/r_H)^2}$$  \hspace{1cm} (1.1)

where $I$ is the luminosity, $I_0$ is the central luminosity, $r$ is the distance from the centre and $r_H$ is some scaling radius. In many regards, this profile fit has now largely been superseded by de Vaucouleurs’ law: $\log I(r) \propto r^{1/4}$ (de Vaucouleurs, 1948). Whilst there is no theoretical justification for de Vaucouleurs’ law, it has been extremely successful in modelling the surface brightness light profiles of elliptical galaxies and classical bulge components (e.g., Kormendy, 1977a). De Vaucouleurs himself is said to have described it as “a good French curve”.

Initially a generalisation of de Vaucouleurs’ law for early-type galaxies, the Sérsic profile (Sérsic, 1963, 1968; Graham & Driver, 2005) has become a standard measure of the light profile of galaxies across a wide range of morphologies (both early and late type). The Sérsic
equation provides the intensity $I$ at a given radius $r$ as given by:

$$I(r) = I_e \exp \left[ -b_n \left( \frac{r}{r_e} \right)^{1/n} - 1 \right]$$

(1.2)

where $I_e$ is the intensity at the effective radius $r_e$, the radius containing half of the total light, and $n$ is the Sérsic index which determines the shape of the light profile (see Figure 1.10). The value of $b_n$ is a function of Sérsic index and is such that $\Gamma(2n) = 2\gamma(2n, b_n)$, where $\Gamma$ and $\gamma$ represent the complete and incomplete gamma functions respectively (Ciotti, 1991). Varying the Sérsic index parameter $n$ allows one to model a wide range of galaxy profile shapes, with $n = 0.5$ giving a Gaussian profile, $n = 1$ an exponential profile suitable for galactic disks, and $n = 4$ a de Vaucouleurs profile commonly associated with massive spheroidal components such as elliptical galaxies.

### 1.3.1 What Happens Below the Limiting Isophote?

Whilst the surface brightness profile of some galaxies behaves as expected out to very faint magnitudes (e.g., NGC 300: Bland-Hawthorn et al., 2005; Vlajić et al., 2009, NGC 7793: Vlajić et al., 2011), the potential myriad of phenomena present in the outer wings of many systems may cause deviations away from a typical light profile. These include truncated and anti-truncated disks (Erwin et al., 2005; Pohlen & Trujillo, 2006), UV excesses (Bush et al., 2010), tidal debris, halos (Barker et al., 2009; McConnachie et al., 2009) and minor merger fossil records (Martínez-Delgado et al., 2010). In fact, the outer regions of galaxies may defy any systematic profile fitting into a restricted number of structures. The accuracy of any estimation of the background sky and gradients therein will also no doubt affect analyses of these outer structures.

### 1.3.2 Fixed vs Model Aperture Photometry

Since it is not known exactly how the light profile of a galaxy behaves at large radii away from the core regions, several traditional methods of aperture-based flux estimation have been devised. Magnitudes within these isophotal radii, not surprisingly, systematically underestimate the total galaxy light, in particular, relative to the Sérsic magnitude (e.g., Caon et al., 1990, 1993). Graham & Driver (2005) show for example that Kron magnitudes may underestimate the total galaxy flux by as much as $\sim 55\%$ dependent upon choice of the multiple of Kron radii chosen to integrate out to and the profile shape of the galaxy. The comparative value $b_n$ can trivially be calculated within R using the relation $b_n = qgamma(0.5, 2n)$, where $qgamma$ is the quantile function for the Gamma distribution.
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Figure 1.10: The Sérsic profile (Equation 1.2) describes how a galaxy light profile varies as a function of radius, shown here for five distinct values of Sérsic index $n$ (as labelled). (top) Surface brightness at a given radius. (middle) Flux contained within a given radius. (bottom) The magnitude offset between the total magnitude of the galaxy and the Sérsic magnitude at a given radius. Figure reproduced from Kelvin et al. (2012).
for Petrosian magnitudes is considerably worse, underestimating flux by as much as $\sim 95\%$ in the extreme case of a high-Sérsic index object integrating out to thrice the Petrosian radius. In addition to these considerations, below $\mu_B = 27 \text{ mag arcsec}^{-2}$ environmental effects begin to play an increasing role in profile shape determination.

In contrast to traditional aperture methods, studies have repeatedly shown the strength of Sérsic profiling for the majority of elliptical galaxies (e.g., Caon et al. (1993); Graham & Guzmán (2003); Trujillo et al. (2004); Ferrarese et al. (2006)). Tal & van Dokkum (2011) support this viewpoint, showing the light profiles of massive ellipticals are well described by a single Sérsic component out to $\sim 8 r_e$, with evidence for additional flux beyond these radii possibly related to unresolved intra-group light. With regards to disk systems, Bland-Hawthorn et al. (2005) use one of the deepest imaging studies of spiral galaxy NGC 300 to show that an exponential profile ($n = 1$) is a good descriptor of its light profile out to $\sim 17 r_e$. From a sample of 90 face-on late-type galaxies, Pohlen & Trujillo (2006) confirm the accuracy of Sérsic profiling down to $\mu = 27 \text{ mag arcsec}^{-2}$, and suggest up to 10% of their sample show evidence for a deviation from a standard $n = 1$ Sérsic fit (Type I), instead showing a broken exponential profile. These breaks appear in the form of either a downbending (Type II; steeper flux drop-off) or upbending (Type III; shallower flux drop-off) with increasing radii. Importantly, this study also suggests this observed feature is independent of local environment.

1.3.3 Accounting for Uncertainty

It is clear that opinion is divided amongst the community as to how a galaxy behaves below the typical limiting isophote, particularly so in the case of a disk galaxy. Each of these studies does however suggest a more complex structure at large radii than a Sérsic profile extrapolated out to infinity would imply. In order to account for the lack of reliable profile information at large radii, Sérsic magnitudes require some form of profile truncation so as not to extrapolate flux into regimes of which we know little. Two schools of thought exist in terms of appropriate truncation methods, extrapolating flux down to a fixed surface brightness limit or integrating under the profile out to a fixed multiple of the half-light radii. A constant surface brightness limit is more closely related to galaxy gas content, and so has physical meaning. However, this method introduces a redshift dependence on truncated flux, causing different fractions of light to be missed at different redshifts. Truncating at a given multiple of the effective radius assumes that the effective radius is well understood prior to truncation, which owing to the inter-dependency between output Sérsic parameters, is not always evident. It does not
display any redshift dependence however, and is trivial to subsequently recorrect if desired. Corrections are typically minor for most galaxies, becoming most acute in high-index systems (see Figure 1.10).

A sufficiently large truncation radius must be adopted to provide a close estimate of total flux without extrapolating too deep into the region of uncertainty. Sloan Digital Sky Survey (SDSS; Abazajian et al. 2009) model magnitudes (magnitudes based on the best fitting exponential/de Vaucouleurs profile) employ a smooth truncation at $3 \, r_e$ down to zero flux at $4 \, r_e$ for exponential ($n = 1$) profiles and $7 \, r_e$ down to zero flux at $8 \, r_e$ for de Vaucouleurs ($n = 4$) profiles.

We advocate a sharp truncation radius of $10 \, r_e$ for all Sérsic indices (shown by a vertical dashed line in Figure 1.10), which corresponds to an isophotal detection limit of $\mu_r \sim 30$ mag arcsec$^{-2}$, the limit to which galaxy profiles have been studied. Figure 1.11 shows the magnitude offset between the Sérsic profile integrated to infinity and that truncated at $3$, $7$ and $10 \, r_e$ as red, green and blue lines respectively, with shaded areas representing the SDSS tapered limits. A $10 \, r_e$ truncation gives a negligible magnitude offset for $n = 1$, and an offset of $\Delta m \sim -0.04$ for $n = 4$, with larger corrections for higher Sérsic indices. Figure 1.10, middle panel, shows the flux contained within $10 \, r_e$ (dashed vertical line) for various values of $n$. Given a $10 \, r_e$ truncation, $\sim 100\%$ of the pre-truncation flux is retained for $n = 1$, reducing to $96.1\%$ for $n = 4$. Sérsic magnitudes truncated at $10 \, r_e$ are adopted as the standard Sérsic magnitude system throughout this thesis.

### 1.3.4 Magnitude Comparisons

Figure 1.12 shows the offsets between various magnitude systems discussed previously (SDSS Petrosian and Model magnitudes) and Sérsic magnitudes (integrated to infinity and $10 \, r_e$) as a function of Sérsic index (Kelvin et al., 2012). When comparing Sérsic magnitudes integrated to infinity against SDSS Petrosian magnitudes we see the two systems are in good agreement until $n_r \sim 2$, beyond which the magnitude offset relation begins to turn-off from the $\Delta m = 0$ relation. This trend argues that Sérsic magnitudes are recovering an additional $\sim 0.4$ magnitudes for an $n_r = 8$ object which would otherwise have been missed by traditional photometric methods. For the reasons previously discussed however, this value should be taken as a rough ($\sim$upper) estimate of the true amount of flux missed for an object of a given Sérsic index. Truncating the Sérsic index at $10 \, r_e$ reduces the scale of this turn-off, as expected, keeping the two magnitude systems in agreement out to $n_r \sim 3$, however; still
1.3. The Sérsic Profile

Figure 1.11: Magnitude offset between the Sérsic profile integrated out to infinity and that truncated at a given multiple of the effective radius. SDSS model magnitudes (forcing either exponential \((n = 1)\) or de Vaucouleurs \((n = 4)\) profile fits, dotted vertical lines) employ smooth tapering truncation radii, represented by the shaded red and green areas. SIGMA Sérsic magnitudes within GAMA adopt a sharp \(10r_e\) truncation radius, blue line. Figure reproduced from Kelvin et al. (2012).
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providing some measure of turn-off beyond this point. We would expect SDSS Petrosian magnitudes, or indeed any aperture-based photometry, to underestimate total flux for objects with large wings, and so this result is not surprising and a good indication that truncated Sérsic magnitudes are performing as expected.

The final panel in Figure 1.12 compares truncated Sérsic magnitudes against SDSS model magnitudes. The SDSS force fit either an exponential or de Vaucouleurs profile fit (marked on the figure by dashed grey lines) depending upon which profile an individual galaxy most approximates during their calculations. We see clearly the inadequacy of model magnitudes when a more comprehensive Sérsic magnitude is available, with the population of galaxies split into two distinct sub-populations based upon their SDSS forced fit. For a galaxy at $n = 2$ for example, the model magnitude for the galaxy may be offset from its correct magnitude by as much as $\Delta m = \pm 0.3$ magnitudes, with larger offsets observed for high index galaxies. If one constructs a line of best fit for each of these two artificial sub-populations then one finds that the lines pass through $\Delta m = 0$ and $n = 1$ or 4 as appropriate, confirming that SDSS model and truncated Sérsic magnitudes agree for exponential and de Vaucouleurs type galaxies.

1.4 Concluding Remarks

Throughout this thesis I shall show how Sérsic profile measurements of 2D imaging data may be used in the measurement of various intrinsic galaxy properties. These measurements shall include estimates of size, magnitude and concentration, as given by the Sérsic index. I opt to truncate measured Sérsic flux at 10 multiples of the half-light radius, ensuring that flux is not extrapolated below the limiting isophote. Using a combination of Sérsic measurements and visual classifications, I shall reproduce the morphological luminosity functions of Binggeli et al. (1988), verifying the luminosity breakdown by morphology. Furthermore, I shall employ multiple Sérsic profiling to perform bulge-disk decomposition of a large galaxy dataset, enabling the stellar mass breakdown by morphological type, morphological class and galaxy structure to be determined for the first time.
1.4. Concluding Remarks

Figure 1.12: A series of plots displaying offsets between various magnitude systems as a function of $r$-band Sérsic index, with the data points coloured according to their $u - r$ rest colour, as shown. Contours range from the 10th percentile to the 90th percentile in 10% steps. (top) The Sérsic profile integrated out to infinity minus the SDSS Petrosian magnitude and; (middle) the Sérsic profile truncated at 10$r_e$ minus the SDSS Petrosian magnitude. These figures show how Sérsic profiling is able to recover flux in the wings of galaxies that would otherwise be missed by traditional aperture based methods, such as Petrosian apertures. (bottom) The Sérsic profile truncated at 10$r_e$ minus the SDSS model magnitude. SDSS force fit either an exponential ($n = 1$) or de Vaucouleurs ($n = 4$) profile fit to attain their model magnitudes. This figure shows how model magnitudes provide an inaccurate measure of flux for a galaxy whose Sérsic index differs significantly from either $n = 1$ or $n = 4$. Vertical dashed grey lines at exponential ($n = 1$) and de Vaucouleurs ($n = 4$) Sérsic indices are added for reference. Figure reproduced from Kelvin et al. (2012).
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Survey Data

Imaging and Spectroscopic data from multiple surveys has been collected, collated and processed in order to provide a consistent multi-wavelength dataset with which to perform analysis upon. I now discuss the origins of these data.

2.1 The Sloan Digital Sky Survey (SDSS)

The Sloan Digital Sky Survey (SDSS; York et al., 2000; Stoughton et al., 2002) is a spectroscopic and imaging survey covering > 10,000 deg$^2$ of the sky (as of Data Release 7; Abazajian et al., 2009) using a dedicated 2.5 m telescope situated at the Apache Point Observatory in New Mexico, USA. Observations are made in five optical photometric bands: $u'$, $g'$, $r'$, $i'$ and $z'$ (hereafter referred to without their prime nomenclature), whose central wavelengths are approximately 355, 470, 620, 750 and 895 nm respectively with limiting AB magnitudes at 2$\sigma$ of 22.0, 22.2, 22.2, 21.3 and 20.5, respectively. These values are summarised and compared in Table 2.1. The relative transmission throughput for each band can be seen in Figure 2.1. Note the low transmission and narrow filter width of the $u$ and $z$ bands, which go some way in explaining the relatively poorer quality of these bands when contrasted against the other SDSS bands.
Chapter 2. Survey Data

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Table 2.1: Central wavelengths, solar absolute magnitudes and limiting apparent magnitudes for a selection of current- and next-generation wide-area surveys. Solar absolute magnitudes are taken from Table 1 of Hill et al. (2010), and represent the SDSS/UKIDSS bandpasses. Note that all limiting depths are at 5σ (with the exception of SDSS, which are 2σ) and are on the AB magnitude system.

Figure 2.1: Instrument response functions for the current-generation SDSS (Doi et al., 2010) ugriz, UKIDSS (Hewett et al., 2006) YJHK passbands (solid lines) in addition to future-generation and ongoing KIDS ugr and VIKING ZYJHKs passbands (dashed lines). The transmission percentage represents the proportion of incident flux as a function of wavelength which is registered by the CCD detector, assuming typical atmospheric conditions (a 1 mm water vapour column and an airmass of 1.3), with the exception of VIKING filter curves.
The SDSS camera contains 6 columns of CCD chips with 5 chips per column, one for each passband. Each chip is $2048 \times 2048$ pixels in size at a resolution of 0.396 arcseconds pixel$^{-1}$, giving an angular size of $13.51 \times 13.51$ arcminutes per CCD chip. The angular space between columns is slightly smaller than the field of view of a single chip (86%), which is designed as such in order that two successive observations may be interleaved to form a contiguous region. Observations are taken using a drift-scanning technique whereby the pointing of the telescope remains fixed to a celestial great circle, with the CCD columns arranged such that the image of an object moves along a single column, being detected in every filter separately. Each CCD is constantly read out at the same rate as the procession of the night sky, with a total effective exposure time of 53.9 seconds per CCD. A single drift-scan produces an SDSS strip, and two interleaved strips produce a stripe. Stripes are subsequently cut into square fields before being served to the wider astronomy community via the SDSS website\(^1\).

Wide area surveys such as the SDSS are well served by the combination of the drift-scanning technique and continuous CCD readouts as it allows large swaths of the night sky to be observed on a given night without losing time closing the shutter between pointings. However, variation in seeing between the two strips that constitute a stripe has the potential introduce large-scale anomalies in the quality of the imaging data, as can be seen in Figure 2.2. Typically, the FWHM of SDSS data lies in the range 0.9 to 1.6 arcseconds (see Figure 4.3).

### 2.2 The UKIRT Infrared Deep Sky Survey (UKIDSS)

The UKIRT Infrared Deep Sky Survey (UKIDSS; Dye et al., 2006; Lawrence et al., 2007) is a near-infrared (NIR) survey using the UKIRT Wide Field Camera (WFCAM; Casali et al., 2007) instrument on the 3.8 m United Kingdom Infrared Telescope (UKIRT) situated on Mauna Kea, Hawaii. The UKIDSS Large Area Survey (UKIDSS-LAS) is one of five sub-surveys in UKIDSS, providing imaging data across approximately 4000 deg$^2$ in the $Y$, $J$, $H$ and $K$ bands, whose central wavelengths are approximately 880, 1030, 1250, 1630 and 2200 nm respectively and with limiting AB magnitudes at 5$\sigma$ of 21.1, 20.9, 20.2 and 20.3, respectively. These values are summarised and compared in Table 2.1. Note that UKIDSS-LAS magnitudes have been converted from their native Vega calibration onto a standard AB magnitude system in order that they may more easily be compared with other surveys. Conversions are made using the relation $m_{AB} = m_{Vega} + m_{offset}$, where $m_{offset}$ values are 0.634, 0.938, 1.379 and 1.900 for

\(^1\)http://www.sdss.org/
Chapter 2. Survey Data

$YJHK$ respectively (see Hewett et al., 2006). The relative throughput for each band is shown in Figure 2.1.

WFCAM consists of four $2048 \times 2048$ pixel CCDs at an apparent angular resolution of $0.4$ arcseconds pixel$^{-1}$ (equating to $13.65 \times 13.65$ arcminutes per CCD) arranged in a square formation with a gap of $12.83$ arcminutes between CCDs. This formation allows for four unique pointings (pawprints) to be co-added to form a contiguous $0.9 \times 0.9$ degree tile.

Variable UKIDSS coverage across the three GAMA regions (see below) leaves a noticeable imprint in the coverage map shown in Figure 2.2. Several of these frames were manually removed after being checked for imaging accuracy and found to contain corrupt imaging data, such as a frame which has tracked badly or is out of focus.

2.3 The VISTA Kilo-Degree Infrared Galaxy Survey (VIKING)

The VISTA Kilo-Degree Infrared Galaxy Survey (VIKING; Sutherland et al., in prep.) survey is a NIR effort to map $1500$ deg$^2$ of the sky using the VISTA Infrared Camera (VIRCAM; Dalton, 2006) atop the 4 m Visible and Infrared Telescope for Astronomy (VISTA; Emerson et al., 2006; Arnaboldi et al., 2007; Emerson & Sutherland, 2010) situated at ESO’s Cerro Paranal Observatory, Chile. VIKING takes observations in 5 bands, namely, $Z Y J H$ and $Ks$ (note that $K$-short is the same filter used with 2MASS, but not that used for UKIDSS). These bands have central wavelengths of $880$, $1020$, $1250$, $1650$ and $2150$ nm respectively, with limiting AB depths of $23.1$, $22.3$, $22.1$, $21.5$ and $21.1$ magnitudes, respectively. These values are summarised and compared in Table 2.1. The relative throughput, assuming standard atmospheric conditions at the site, is shown in Figure 2.1.

The VIRCAM instrument consists of sixteen $2048 \times 2048$ CCDs at a resolution of $0.339$ arcseconds pixel$^{-1}$ arranged into a $4 \times 4$ square formation (a pawprint). The separation between columns is $90\%$ the width of a single CCD, whereas the separation between rows is $42.5\%$. This formation allows for 6 pawprints to be interleaved in order to form a contiguous $1.5$ deg$^2$ region on the sky.

We note that, as of the time of writing, VISTA VIKING observations are still underway. For this reason, UKIDSS-LAS is still the preferred NIR survey data in use throughout the majority of this thesis, however, owing to the increased depth and spatial resolution of VIKING data, it is used wherever it is feasible and useful to do so.
The GAMA survey is a combined spectroscopic and multi-wavelength imaging programme designed to study spatial structure in the nearby \((z < 0.25)\) Universe on scales of 1kpc to 1Mpc (see Driver et al. 2009 for an overview). The survey, after completion of Phase I, consists of three regions of sky each of 4 deg (Dec) \times 12 deg (RA), close to the equatorial region, at approximately \(9^h\) (G09), \(12^h\) (G12) and \(14.5^h\) (G15) Right Ascension (see Table 2.2 and Figure 2.2 for reference). The three regions were selected to enable accurate characterisation of the large scale structure over a range of redshifts and with regard to practical observing considerations and constraints. They lay within areas of sky scheduled for survey by both SDSS (Abazajian et al. 2009) as part of its Main Survey, and UKIRT as part of the UKIDSS Large Area Survey (UKIDSS-LAS; Lawrence et al., 2007). These data provide moderate depth and resolution imaging data in \(ugrizYJHK\) suitable for analysis of nearby galaxies. The accompanying spectroscopic input catalogue was derived from the SDSS PHOTO parameter (Stoughton et al., 2002) as described in Baldry et al. (2010). The GAMA spectroscopic programme (Robotham et al., 2010) commenced in 2008 using AAOmega on the Anglo-Australian Telescope to obtain distance information (redshifts) for all galaxies brighter than \(r < 19.8\) mag. The survey is \(~99\) percent complete to \(r < 19.4\) mag in G09 and G15 and \(r < 19.8\) mag in G12 (see Table 2.2, column 6), with a median redshift of \(z \sim 0.2\). Full details of the GAMA Phase I spectroscopic programme, key survey diagnostics, and the GAMA public and team databases are given in Driver et al. (2011).

### 2.4.1 Data Processing and Mosaicing

As part of the process of inclusion into the GAMA database (Driver et al., 2011), the survey data discussed above are reprocessed and mosaiced into large per-region per-passband mosaics, as described fully in Hill et al. (2011). These mosaics are commonly referred to as SWARPed images (\textit{swpim}) due to the SWARP software used in their creation (Bertin et al.,

<table>
<thead>
<tr>
<th>Region</th>
<th>RA (J2000)</th>
<th>Dec. (J2000)</th>
<th>(r_{\text{lim}})</th>
<th>(N_{\text{obj}})</th>
<th>(\varepsilon_{\text{comp}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>G09</td>
<td>129°.0 &lt; (\alpha) &lt; 141°.0</td>
<td>(-1°.0 &lt; \delta &lt; 3°.0)</td>
<td>19.4 (19.8)</td>
<td>30,289 (48,548)</td>
<td>99.23% (79.36%)</td>
</tr>
<tr>
<td>G12</td>
<td>174°.0 &lt; (\alpha) &lt; 186°.0</td>
<td>(-2°.0 &lt; \delta &lt; 2°.0)</td>
<td>19.8 (19.4)</td>
<td>50,868 (32,747)</td>
<td>99.12% (99.39%)</td>
</tr>
<tr>
<td>G15</td>
<td>211°.5 &lt; (\alpha) &lt; 223°.5</td>
<td>(-2°.0 &lt; \delta &lt; 2°.0)</td>
<td>19.4 (19.8)</td>
<td>33,205 (51,217)</td>
<td>98.95% (79.27%)</td>
</tr>
</tbody>
</table>

Table 2.2: GAMA region definitions. The GAMA main survey definitions are based on SDSS extinction corrected r-band Petrosian magnitude limits, the depth of which varies between \(r = 19.4\) mag in G09/G15 and \(r = 19.8\) mag in G12. Comparison magnitude limits are shown in brackets for reference. Number counts and redshift completeness are based on objects which passed star-galaxy separation in the GAMA TilingCatv11 (see Baldry et al. (2010) for further details).
Figure 2.2: Coverage in each bandpass across the three GAMA regions is shown, where white space indicates a lack of coverage at that position. Different bandpasses are offset in declination for plotting purposes, with the bottom row (black data points) positioned at the correct GAMA coordinates. The five SDSS bands (ugriz) were taken simultaneously and therefore are represented together, whilst the four UKIDSS bands (YJHK) are shown separately. The data points are coloured according to the local value of the PSF FWHM at that position, giving an indication of the variation in seeing (SDSS data is taken from the r band). See Section 3.5.2 for further details on the derivation of the PSF’s. The bottom row represents a common coverage area, containing galaxies that have been observed across all nine bands (92.8% of the total). A common coverage sample may be derived from this area.
2.4. Galaxy and Mass Assembly (GAMA)

Associated weight map mosaics (\textit{swpwt}) are also constructed. Note that at the time of writing only the $K$ band G09 VISTA data is available, and so the process described below is currently only applicable to the SDSS and UKIDSS data. In constructing these large mosaics, all native reduced frames are downloaded from the respective archives (SDSS DR7 and ROE/WFAU) and scaled to a single uniform zero point (30 mag arcsec$^{-2}$). For SDSS, the input data are the corrected (\textit{fpC}) Data Release 7 (DR7) files, with the data having already been bias subtracted and flat-fielded as part of the SDSS frames pipeline (Stoughton et al., 2002, Section 4.4). The UKIDSS-LAS data has been collected from the UKIDSS Early Data Release (EDR; Dye et al., 2006) and data releases 1 and 2 (DR1; Warren et al., 2007b, DR2; Warren et al., 2007a). The UKIDSS project is defined in Lawrence et al. (2007). UKIDSS uses the United Kingdom Infrared Telescope Wide Field Camera (WFCAM; Casali et al., 2007). The photometric system is described in Hewett et al. (2006), and the calibration is described in Hodgkin et al. (2009). The pipeline processing and science archive are described in Irwin et al (in prep.) and Hambly et al. (2008).

Once these input data have been obtained and calibrated, \texttt{SWARP} is then used to combine them into a single image mosaic at a resolution of 0.339" arcseconds per pixel in the TAN projection system (Calabretta & Greisen, 2002) centred within each GAMA region as appropriate. Note that we are using version 2 \texttt{SWARP} mosaics scaled to a slightly higher resolution (0.339") than the previous version 1 mosaics (0.4") as described in Hill et al. (2011). This increased resolution has been chosen to match that which is expected for future VISTA VIKING data releases, allowing easy cross-wavelength cross-facility comparison of data. Original SDSS and UKIDSS resolutions of 0.396" and 0.4" respectively place a limit on how high one is able to artificially increase the resolution of mosaiced data, requiring increasing amounts of interpolation with increasing artificial resolution. Further details may be found in Liske et al. (in prep.). Version 2 mosaics are a minimum of 193900 $\times$ 79700 pixels each, with each individual FITS file $\sim$ 60GB in size. The process used to create the version 2 mosaics is identical except the regions have been expanded in preparation for GAMA Phase II operations and at higher resolution in preparation for matching to VISTA data in due course$^2$.

As part of the \texttt{SWARP} mosaicing process the background is removed on each individual frame prior to merging using a 256 $\times$ 256 pixel median filtered mesh which itself is median filtered within a 3 $\times$ 3 mesh. The original SDSS and UKIDSS data are typically held in chunks

\footnote{These larger, higher-resolution version 2 mosaics will be released shortly via the GAMA website: \url{http://www.gama-survey.com}.}
of 2048 × 1489 pixels and 2072 × 2072 pixels respectively, at comparable pixel scales (SDSS: 0.396″/pixel, UKIDSS: 0.4″/pixel). At the native pixel resolution the mesh size therefore equates to 101.4″ × 101.4″ and 102.4″ × 102.4″ respectively and so structures with half-light radii less than ~ 17″ should be unaffected by the background smoothing. Note that UKIDSS $J$ band data and selected UKIDSS EDR fields in both $H$ and $K$ bands were microstepped. These data are typically stored in chunks of 4103 × 4103 pixels at a native resolution of 0.2″/pixel, giving a mesh size of 51.2″ × 51.2″.

In addition to the science image frames are the associated weight maps. Because of the zero-point normalisation across all data, and overlapping edge duplication in the SDSS data, the actual weight map values produced by SWARP are an approximation of their correct value. However, the weight maps remain useful as a record of which stars can be associated with which pre-mosaiced frame for the purposes of detailed PSF modelling (see section 3.5.2).
SIGMA: Structural Investigation of Galaxies via Model Analysis

SIGMA (Structural Investigation of Galaxies via Model Analysis) is an automated front-end wrapper introduced in Kelvin et al. (2012) which utilises a wide-range of image analysis software and a series of logical filters and handlers to perform bulk structural analysis on an input catalogue of galaxies. This is primarily achieved through the use of Source Extractor (Bertin & Arnouts, 1996), PSF Extractor (Bertin and Delorme, priv. comm.) and GALFIT 3 (Peng et al., 2010a), with additional packages also created and utilised to aid in the fitting process. Key to this process is the galaxy modelling software GALFIT. GALFIT is able to create a realistic model of each input galaxy by fitting one or more analytical functions (e.g. Sérsic, exponential, Ferrer, Moffat, Gaussian) in multiple combinations. Principle in these functions, and that chosen to perform the bulk of our galaxy modelling, is the Sérsic profile (Sérsic, 1963, 1968; Graham & Driver, 2005), as described in Section 1.3. I now discuss in further detail the operation of SIGMA.
Chapter 3. SIGMA: Structural Investigation of Galaxies via Model Analysis

3.1 SIGMA Setup

The only inputs required by SIGMA are the imaging data itself and the locations of the primary galaxies within them which are to be modelled. All additional parameters and starting values for extra neighbouring objects in the field of view (secondary objects), including PSF evaluation, are determined by SIGMA on the fly on a per-galaxy basis. All scripting and additional programming is written in the open source and freely available R programming language (R Development Core Team, 2010).

Typically, SIGMA is run across multiple-processors, and will output a Comma-Separated Variable (CSV) catalogue, which may subsequently be opened and converted in TOPCAT (Taylor, 2005). The average run-time to profile a galaxy with a single-component (single-Sérsic) fit is 15 seconds per processor\(^1\) sustained over several hundred thousand objects.

3.1.1 Startup

On starting SIGMA, a number of input options can be specified. Some of these are essential in its use, whereas others are designed for testing purposes only. The available input options can be found in the help document, reproduced (from SIGMA version 0.9-0) below.

Listing 3.1: SIGMA help lists the available input options that may be specified when starting SIGMA.

```
$ sigma -h

----- SIGMA Version 0.9-0 ----- 23 Jul 2010

DESCRIPTION
SIGMA (Structural Investigation of Galaxies via Model Analysis) is a 2
dimensional fitting code taking inputs from the GAMA SWarped regions and
producing models using the GALFIT software.

OPTIONS
-a A - append A to output log files
-b # - B-band (default: r)
-c A - input catalogue [.img/csv] (needs at least RA & DEC)
-d - show program defaults
-e # - error generation method (1=GALFIT, 2=BOOTSTRAP)
-h - help (this screen)
-i - interactive mode
-m - make a plot of output .fits files (png format)
-n A - output catalogue name
-o - no headers in output catalogue, only data
-p # - number of sub-processes to spawn
-q # - number of bootstrap runs to generate errors in GALFIT
-s # - subsample, from lower to upper quantile
-t #,# - principle allowed multi-component types (eg: 1,2,5,10)
-v - version number

\(^1\)Using current computer hardware at the University of St Andrews. This consists of a 16 core Intel Xeon E5520 server with 48GB RAM running Ubuntu 10.04.
```
### 3.2 Modules

SIGMA operates in a semi-modular fashion, with an overarching master script calling and linking several key modules within. Each module is specialised in performing and handling a different task. A summary of each module and its purpose is shown in Table 3.1, and a schematic of the SIGMA data-flow process is shown in Figure 3.1.

<table>
<thead>
<tr>
<th>Module</th>
<th>Description</th>
<th>File Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUTTERPIPE</td>
<td>Creates a science and weight map cutout from the master GAMA mosaics using the CFITSIO routine <em>FITSCOPY</em> (Figure 3.2) and performs an additional local background sky subtraction using Source Extractor (Figure 3.3).</td>
<td><code>cutim</code></td>
</tr>
<tr>
<td></td>
<td></td>
<td><code>cutwt</code></td>
</tr>
<tr>
<td>STARPIPE</td>
<td>Determines which frames contributed flux to the primary galaxy, and creates a catalogue of stars that lie within these frames using Source Extractor.</td>
<td><code>psfws</code></td>
</tr>
<tr>
<td></td>
<td></td>
<td><code>psfwt</code></td>
</tr>
<tr>
<td></td>
<td></td>
<td><code>psfct</code></td>
</tr>
<tr>
<td>PSFPIPE</td>
<td>Generates an empirical 2D PSF at the primary object position using PSF Extractor (Figure 3.4).</td>
<td><code>psfss</code></td>
</tr>
<tr>
<td></td>
<td></td>
<td><code>psfim</code></td>
</tr>
<tr>
<td></td>
<td></td>
<td><code>psfsr</code></td>
</tr>
<tr>
<td>OBJECTPIPE</td>
<td>Calculates starting parameters for size, brightness, position angle and ellipticity for the primary galaxy and any secondary neighbours (galaxies and stars) using Source Extractor. A dynamic search algorithm is used to attempt to detect the primary galaxy. Elongated objects (such as satellite trails) are removed from the secondary catalogue and instead added to a bad pixel mask.</td>
<td><code>objct</code></td>
</tr>
<tr>
<td></td>
<td></td>
<td><code>segim</code></td>
</tr>
<tr>
<td>GALFITPIPE</td>
<td>Fits an analytical function in 2D to the science image using GALFIT. Both primary and secondary objects are modelled, with any detected errors/crashes flagged and a fix attempted on a per-galaxy basis (Figures 3.6 and 3.8).</td>
<td><code>segfr</code></td>
</tr>
<tr>
<td></td>
<td></td>
<td><code>extct</code></td>
</tr>
<tr>
<td></td>
<td></td>
<td><code>objim</code></td>
</tr>
</tbody>
</table>

**Table 3.1:** Summary of the modules that comprise SIGMA, a brief description of their purpose, and a list of the file outputs produced by each.

- `x` # - GAMA ID
- `y` # - SIGMA ID
- `z` # - SDSS OBJID

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Chapter 3. SIGMA: Structural Investigation of Galaxies via Model Analysis

Figure 3.1: A flowchart describing SIGMA’s operation. Required inputs are imaging data in the form of GAMA mosaics (including weight maps), and an input catalogue with a list of galaxy coordinates.
3.3 Master Script

When initialising SIGMA a number of options are specified, such as band(s), naming conventions, number of processors and model fit types. SIGMA’s master script handles these requests, and sets up the data and directories for further subsequent processing. To begin, SIGMA’s master script loads into memory the entirety of the input catalogue (GAMA Tiling Catalogue) and defines a naming convention for each primary object based on its own unique identifier (SIGMA_INDEX). A template master CSV catalogue is created into which all of the output data will accumulate as SIGMA loops across each primary galaxy. Once the setup is complete, the master script will loop across every primary object in turn. If for any reason a primary galaxy causes a software crash, with attempted fixes as detailed in subsequent sections unsuccessful, SIGMA will report how far it was able to progress and record a NULL result before proceeding on to the next primary galaxy in the input catalogue. We now discuss each module from Table 3.1 in turn.

3.4 Image Processing

The CUTTERPIPE module creates and prepares the fitting image to be fed into GALFIT. Version 2 mosaics of the three GAMA regions are used as an input to CUTTERPIPE, with a full description of the construction and manipulation of these files given in Hill et al. (2011) and summarised in Section 2.4.1.

CUTTERPIPE’s first task is to create the core cutout of the science image and its associated weight map. Using the WCS information stored in the FITS header of the appropriate mosaiced image, CUTTERPIPE converts the input RA/DEC into an x/y pixel coordinate. The upper and lower limits of a 1201 × 1201 pixel (∼ 400′′ × 400′′) region centred on the primary galaxy are determined. Using the NASA HEASARC package’s CFITSIO subroutine library, namely the routine FITSCOPY, cutouts centred on the primary galaxy on both the mosaiced science image, swpim, and mosaiced weight map, swpwt, are created. These cutouts are named cutim and cutwt respectively. FITSCOPY was found to be the most efficient routine at dealing with the large mosaic files in use, able to quickly analyse the input file and read into memory only the relevant area of interest, thereby reducing file handling time significantly.

The process of creating the GAMA mosaics alters a number of keywords in the FITS header in order to better describe the nature of the mosaiced data. The mosaic headers are copied over to cutim and cutwt during their creation. Several of these keywords are required later
in the fitting process by GALFIT in order to generate a sigma-map (an image showing the 1σ confidence interval at every pixel). CUTTERPIPE reverts these to typical pre-mosaic values which are more appropriate for a smaller single image rather than a larger mosaic. GAIN, RDNOISE, NCOMBINE and EXPTIME are set to values of 0.5, 3, 1 and 1 respectively. These typical values are averages taken from pre-mosaiced data frames.

An estimate of the local background sky is then made with Source Extractor (v2.8.6; Bertin & Arnouts, 1996) using a variable background grid in a $3 \times 3$ mesh configuration. Possible grid sizes are $32 \times 32$, $64 \times 64$ and $128 \times 128$ pixels. The size of the chosen background grid is dependent upon the size of the primary galaxy: larger galaxies will lead to a larger background grid being used so as not to contaminate the sky estimate with galaxy flux. An initial basic estimate of the total size of the primary is given by:

$$r_{\text{tot}} = 2 \times r_{99}$$

(3.1)

where $r_{99}$ is the radius of the primary galaxy which contains 99% of the flux. This is obtained from the Source Extractor parameter FLUX_RADIUS, setting PHOT_FLUXFRAC = 0.99$^2$. Note that a size estimate produced by Source Extractor in this manner is known to be smaller than the true galaxy value, scaling as a function of Sérsic index and thus absolute magnitude. This effect has been accounted for, and does not adversely affect any of the analysis or results here presented. If $r_{\text{tot}} < 128$, CUTTERPIPE rounds up $r_{\text{tot}}$ to the nearest available grid size and performs a background subtraction on the science image as appropriate. If $r_{\text{tot}} \geq 128$, no background subtraction is necessary, as the master GAMA mosaics have a $256 \times 256$ pixel background grid subtraction already applied. Although the value of actual subtracted sky varies with position on the cutout image, the specific value at the position of the primary galaxy, $\rho_{\text{sky}}$ is recorded through the Source Extractor parameter BACKGROUND. The error on the background sky estimate is then given by:

$$\Delta \rho_{\text{sky}} = \frac{\sigma_{\text{sky}}}{\sqrt{0.9 \times n_x \times n_y}}$$

(3.2)

where $\sigma_{\text{sky}}$ is the RMS of background sky counts across the cutout, and $n_x$ and $n_y$ are the dimensions of the cutout in the $x$ and $y$ dimensions respectively. The background sky typically encompasses $\sim 90\%$ of any given cutout, and hence a factor of 0.9 is introduced into the above

---

$^2$Typically, Source Extractor FLUX_RADIUS refers to $r_e$, a radius containing 50% of the flux of the primary galaxy.
3.5 Modelling the PSF

The point spread function (PSF) describes the blurring effect of both the atmosphere and the telescope optics on our imaging data. Observed galaxy images have had their flux redistributed according to this PSF. The galaxy flux most affected by the PSF blurring is that which emanates from the core regions, where the gradient of the light profile is at its steepest. It is therefore crucial to have a good understanding of the PSF when considering fitting smooth analytical galaxy models to imaging data. Furthermore, most current galaxy modelling software weights model fitting towards higher signal-to-noise regions (typically the same core regions), increasing the importance of accurate, reliable PSF estimation.

3.5.1 Finding Stars

STARPIPE uses Source Extractor to create a catalogue of star-like objects with which to create a point-spread function (PSF) in the subsequent PSFPIPE module (section 3.5.2).
The first step is to determine which of the original pre-mosaic frames contain the primary galaxy. This step is non-trivial, as a single cut-out image (cutim) may contain data from several pre-mosaiced frames overlapping at random angles to each other, with only some of the frames contributing flux (and therefore seeing information) to the primary galaxy. Calculating frame ownership is crucial in PSF determination, as using stars from non-contributing frames would skew the PSF estimate away from its true shape at the position of the primary galaxy. A method was devised to determine contributing frames using the information within the GAMA weight maps. Each pre-mosaiced frame is assigned a numerical value based on the global variance of the data for that frame. This value, repeated for each pixel, becomes the weight-map for that individual frame. Weight-map values are essentially unique to several significant figures, and therefore useful in identifying that particular frame. During the SWARP process, overlapping imaging data is median combined (setting the SWARP argument COMBINE_TYPE to MEDIAN) whereas weight-maps are co-added to produce a global weight-map representing the change in the variance across the data. When two or more frames overlap, their individual weight map values are summed. Larger values indicate a greater number of overlapping frames.

The value of the weight map at the primary position is determined, with all pixels of that value clearly contributing flux to the primary galaxy. This defines the initial primary region. However, since this primary region may be an overlap region itself, parent frames must also be determined. The weight-map values of all bordering pixels to the primary region are determined. Higher pixel values indicate a region which contains data from additional frames that did not contribute flux to the primary region, and so these pixels are discarded. Lower values (if any) indicate parent frames of the primary region, and (if they exist), their pixel positions are added to the primary region. This process will continue until all pixels are accounted for across the cutout. As an example of frame determination, contrast Figure 3.2b with the shaded red regions in Figure 3.6.

Pixel determination via this technique is time intensive for the full 1201 × 1201 cutout region, as it requires analysis of 1.4 million pixels for each galaxy. A more efficient method is to reduce the number of pixels that require analysis by simplifying the weight map to its minimal number of pixels which still describe the nature of the data. Duplicate rows and columns in the weight map are removed, producing a simplified weight map, psfws, typically of order ~ 100 × 100 pixels. This thereby reduces the number of pixels needing to be analysed.
3.5. Modelling the PSF

Figure 3.3: A simplified version of the weight map shown in Figure 3.2b. A single cutout from a GAMA mosaic may contain data from several frames observed on different nights in different seeing conditions. The weight map allows us to determine which frames contributed flux to the primary galaxy and which did not, thereby allowing a PSF to be constructed representing the seeing for that galaxy. Simplification of the weight map removes superfluous information contained within the original cutout weight map, reducing the number of pixels to analyse typically by a factor of $\sim 150$, and significantly speeding up STARPIPE.

by a factor of $\sim 150$, significantly speeding up the primary region determination. Figure 3.3 is one such simplification of the cutout weight map shown in Figure 3.2b, in this case reducing the number of pixels from 1.4 million to $\sim 20,000$.

Once a primary region is determined, a local star catalogue must be created. A modified version of cutwt, psfwt, is created, setting all non-primary pixels to a weight value of zero. This will bar Source Extractor detecting any objects in those regions. The central $25 \times 25$ pixel region is also set to a weight value of zero to ensure against the primary galaxy being
falsely classified as a star and used in later PSF analysis.

A Source Extractor parameter file is created to output NUMBER, X_IMAGE, Y_IMAGE, FLUX_RADIUS, FLAGS, FLUX_APER(1), FLUX_MAX, ELONGATION, VIGNET(25,25) and BACKGROUND. The numeric values following VIGNET determine the ultimate size in pixels of the 2D PSF created subsequently in PSFPIPE. A detection threshold of $2\sigma$ above the background is specified, along with a minimum object detection area of 10 pixels, a 25,000 count saturation level and a fixed sky pedestal value of zero counts. The image is filtered through a $5 \times 5$ pixel Gaussian convolution kernel with $\Gamma = 2$ pixels. Source Extractor defaults are used everywhere else. Using these settings, Source Extractor is run across the cutout image cutim using the weight map image psfwt. An output catalogue, psfct, is created in the FITS_LDAC format, a format which saves the image data as well as the catalogue.

3.5.2 PSF Creation

The PSFPIPE module is a wrapper around the PSF extraction software PSF Extractor (PSFEx v3.3.4; Bertin, priv. comm.\(^3\)), and produces a 2D PSF model to be taken into account at the later galaxy modelling stage. PSFEx extracts precise models of the PSF from images pre-processed by Source Extractor, allowing for a wide range of PSF's to be quickly and accurately constructed, including arbitrary non-parametric features present in the PSF.

In brief, the sample of objects from the psfct catalogue created by STARPIPE is initially used for analysis. PSFEx reduces this object list to a star sample based on a set of pre-defined criteria. A signal-to-noise limit of at least 10 is required, and objects with an eccentricity of

$$\left(\frac{a-b}{a+b}\right) > 0.05$$

are removed\(^4\), where $a$ and $b$ refer to the semi-major and semi-minor axes respectively. Each star’s full-width-half-maximum, $\Gamma$, is estimated, with only stars in the pixel range $2 < \Gamma < 10$ accepted. Furthermore, variability in the star sample is limited to the central 50% quantile. After extensive testing on the variation in PSF quality with star sample size, and communication with the authors of PSFEx, we found that a star sample size of at least 10 stars is necessary to ensure that the resultant PSF is not adversely affected by small-number biases. Therefore, if fewer than 10 stars remain in the star sample after selection criteria have been applied, SIGMA will loop back to CUTTERPIPE and expand the cutout region to 1501 $\times$ 1501 pixels ($\sim 500'' \times 500''$). The mean number of stars used for PSF estimation in the $r$ band is 24.4, with 10.2% of cutouts containing fewer than 10 stars after the cutout region has been expanded.

\(^3\)More information on the PSFEx software may be found at http://www.astromatic.net/software/psfex.

\(^4\)PSFEx refers to this quantity as ellipticity rather than eccentricity, however its definition is more akin to that of the latter. We adopt the terminology eccentricity here to avoid confusion with the standard definition of ellipticity used elsewhere, namely, $e = 1 - \frac{b}{a}$. An eccentricity of 0.05 therefore corresponds to an ellipticity of $e \sim 0.095$. 

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3.6 Object Detection

Cutout images of each star are pre-stored in the FITS_LDAC format of psfct, the size of the cutout having been specified at the Source Extraction stage. PSFEx uses the positional information from Source Extractor to mask nearby neighbours to the final star sample, and presents this sample in the output psfss FITS image (Figure 3.4a).

The variation in the shape of the stars in the star sample is then modelled in both $x$ and $y$ as a function of position in the field by a 2D $n^{th}$ order polynomial function. Higher order terms in the fit (i.e. $x$, $x^2$, etc.) describe the variation in the PSF at positions away from the centre of the frame. Within SIGMA, the primary galaxy is always centred in the cutout image, and so a zeroth order polynomial was found to adequately describe the PSF. The best-fit polynomial is sampled at a 1:1 ratio relative to the input data, and an output PSF image psfim is produced of the same size as the input cutout stars, $25 \times 25$ pixels (Figure 3.4b).

As a consistency check, scaled models of psfim are fit to each of the input stars in psfss, and a residual map psfsr produced (Figure 3.4c). Note that some of the PSF residuals still show noticeable structure once the PSF model has been subtracted from the star sample. This is as expected when subtracting a zeroth order PSF model (only accurate at the location of the primary galaxy in the centre) from a star sample taken over a large area on the sky. Those stars with noticeable residuals therefore are typically either significantly spatially separated in the field of view from the primary galaxy or approaching saturation (or both). Both of these factors are accounted for by PSFEx when constructing the model PSF. The stars chosen as part of the star sample are shown in orange circles in Figure 3.6, with each circle numbered according to their position in Figure 3.4a, starting at 1 in the bottom left and increasing horizontally left-to-right and then bottom-to-top.

3.6 Object Detection

A second catalogue of objects optimised for galaxy detection, object, is created in OBJECTPIPE, to be later fed into GALFITPIPE. This catalogue provides the basic starting parameters for the primary galaxy and any secondary galaxies and stars in the frame. OBJECTPIPE also creates a segmentation map of the frame, modified to mask any erroneous regions of flux in the image which may cause fitting problems (e.g., satellite trails). A Source Extractor parameter file is created containing X_IMAGE, Y_IMAGE, MAG_AUTO, FLUX_RADIUS, KRON_RADIUS, A_IMAGE, B_IMAGE, THETA_IMAGE, ELLIPTICITY and CLASS_STAR. These give position ($x/y$), luminosity, size, position angle and ellipticity for the primary and all secondaries in
Figure 3.4: PSFEx generates an empirical PSF (b) from a host sample of representative stars (a). This figure shows 63 sample star cutouts of 25 × 25 pixels each chosen from around GAMA object G00196053, whose real sky positions may be noted in Figure 3.6 (orange circles). Panel (c) represents the residual of each star sample with a scaled form of the PSF subtracted from each. (a) and (c) are scaled logarithmically from $-1 \sigma_{\text{sky}}$ to $40 \sigma_{\text{sky}}$, where $\sigma_{\text{sky}}$ is the typical RMS of the sky in the $r$ band. Source Extractor settings are similar to those used in STARPIPE, excepting a lower detection threshold of $1.8\sigma$ above the background (where $\sigma$ is the RMS as estimated by Source Extractor), and a lower deblending contrast parameter of 0.0001. The Source Extractor Neural-Network Weights V1.3 file is used in the creation of the CLASS_STAR parameter, as well as a standard 5 × 5 pixel Gaussian filter, $\Gamma = 2$ pixels, used during object detection.

OBJECTPIPE calls Source Extractor, and records the results. If initially the primary galaxy is unable to be located within a 5 pixel radius of the input coordinates, OBJECTPIPE will in the first instance decrease the detection threshold in steps of $0.4\sigma$ down to $1\sigma$ above the background until an object is found, re-running Source Extractor as appropriate. This usually occurs with faint objects in the field, or in crowded regions. If the primary object is still unable to be located, the threshold is reset to $1.8\sigma$, and a larger search radius of up to 15 pixels from the input coordinates in 5 pixel steps is tried. This stage accounts for large nearby galaxies whose centroids are not matched to better than 5 pixels, hence requiring a larger detection area. If multiple-matches are found, the largest object will be taken to be the primary galaxy. If at this stage the primary galaxy is still not found, OBJECTPIPE will report a null detection, and move on to the next primary in the input catalogue.

3.6.1 Source Extractor Outputs

Output parameters from Source Extractor are modified by OBJECTPIPE before being fed into GALFITPIPE, with the exception of magnitude which is used unaltered. Position angle is modified to the GALFIT standard (by adding 90 degrees), increasing anti-clockwise from the posi-
3.6. Object Detection

tive \( x \) axis. Ellipticity \( e \) is converted to an axis ratio using the relation:

\[
e = 1 - \frac{b}{a}
\]

with semi-minor axis \( b \) and semi-major axis \( a \).

Half-light radius \( r_e \) is estimated using the relation:

\[
r_e = \sqrt{\left( r_{50}^2 \times \frac{a}{b} \right) - (0.32 \times \Gamma^2)}
\]

where \( r_{50} \) is the (unmodified) Source Extractor half-light radius as given by FLUX_RADIUS (setting PHOT_FLUXFRAC= 0.5) and \( \Gamma \) is the Full-Width Half-Maximum of the PSF of the primary galaxy. The value of 0.32 was derived from simulated test data (see Appendix A of Driver et al., 2005 for further details). A minimum bound on \( r_e \) of 1 pixel is enforced. Modifications of this nature are made in order to account for the difference in radii definitions between Source Extractor and GALFIT. Source Extractor’s FLUX_RADIUS parameter outputs a circularised radius which is based on PSF convolved imaging data. The format of GALFIT’s initial estimate of the half-light radius is that along the semi-major axis which is intrinsic to the object (i.e. - deconvolved from the PSF). Equation 3.4 converts Source Extractor circularised radii into semi-major intrinsic radii more appropriate for GALFIT.

Figure 3.5 displays the before (uncorrected) and after (corrected) Source Extractor half-light radii against output modelled half-light radii from GALFIT for a sample of 167,600 galaxies modelled in the \( r \) band (pixel scale, 0.339 pixels per arcsecond), coloured according to their predicted morphological type as detailed in Section 4.4. Unmodified Source Extractor half-light radii are a poor predictor of final modelled GALFIT half-light radii, as expected. Large galaxies (\( r_e > 4 \) pixels) tend to have their sizes underestimated by Source Extractor by as much as 50%. Following a turn off at \( r_e \sim 4 \) pixels, small galaxies tend to have their sizes overestimated by Source Extractor. Once these data have been corrected, we find a marked increase in the agreement between the two measures, notably so for late-type galaxies (blue data points). Data above \( r_e > 4 \) pixels has been reduced to minimal scatter about a 1:1 correlation. The effect of the turn-off has been significantly mitigated, yet not entirely diminished. This indicates the difficulty in accurate size estimation of galaxies that only occupy a matter of a few pixels. Due to the downhill minimisation employed by GALFIT, it is important to provide input parameters as close as possible to the desired solution. Correcting radii in this manner
Figure 3.5: A comparison between Source Extractor half-light radii (FLUX_RADIUS) and modelled GALFIT half-light radii ($r_e$) in the $r$-band, with data points coloured according to their predicted morphological type as described in Section 4.4. (Left) Uncorrected half-light radii from Source Extractor are a poor initial condition for modelling an object in GALFIT, as Source Extractor radii are circularised and make no attempt to correct for the effect of seeing. (Right) Using Equation 3.4 to correct for ellipticity and the PSF produces a much better starting value for GALFIT, hence reducing the chance of finding local minima during the minimisation phase. The green line represents a 1:1 ratio, for reference.

significantly reduces the chance of GALFIT finding a local-minima during the minimisation phase, and consequently reduces the risk of convergence on a non-physical solution.

3.6.2 Modelling Catalogue

Once starting parameters for the primary galaxy have been determined, a segmentation map of the frame is created to be used as a potential mask for secondary features (e.g., galaxies/stars) should modelling them fail. Secondary objects whose ellipticity is greater than 0.95 are excluded from modelling and will instead be masked, as these are determined to be satellite trails, diffraction spikes or other suitably bad data, and consequently difficult to model. Similarly, secondary objects with a stellaricity index of CLASS_STAR > 0.8 (see Bertin & Arnouts, 1996) are modelled by a PSF within GALFITPIPE, with all others being modelled using a single Sérsic function. A relatively low CLASS_STAR boundary is chosen as tests have shown that a more reasonable fit is produced when fitting a PSF rather than a Sérsic function to ambiguous objects. A graphical representation of detected galaxies, stars, weight-map areas and secondary neighbour determination for G00196053 is shown in Figure 3.6.
Figure 3.6: Final detail analysis plot for G00196053. Green shaded areas represent all detected objects, with stars chosen as part of the PSF star sample circled in orange. The GALFIT fitting area is outlined in yellow, and the weight map frame edges in blue. Secondary objects which will be modelled as nearby neighbours in the fitting process are represented in turquoise. Note how no sample stars are taken from frames which did not contribute directly to the flux of the primary galaxy (areas represented with red shading). Consequently, no stars are taken for PSF analysis from the red shaded areas. The image is scaled logarithmically from $-1 \sigma_{\text{sky}}$ to $40 \sigma_{\text{sky}}$, where $\sigma_{\text{sky}}$ is the typical RMS of the sky in the $r$ band.
3.7 Modelling the Galaxy

The actual modelling is handled by GALFITPIPE, which is a wrapper around the Galfit image analysis software (v3.0.2; Peng et al., 2010a) along with several event handlers and logical filters written in R. Galfit is a 2D parametric galaxy fitting algorithm written in the C language. It allows for multiple parametric functions (such as Sérsic, exponential, Ferrer, Moffat, Gaussian, etc.) to be modelled simultaneously as either multiple components of a single object, multiple objects in a single frame, or combinations thereof.

3.7.1 GALFIT: Galaxy Model Minimisation

GALFIT uses a Levenberg-Marquardt algorithm to fit a 2D function to 2D data, in doing so minimising the global $\chi^2$ until the gradient $\Delta \chi^2$ has become negligible and convergence is reached. When a global minimum is thought to be found, GALFIT introduces a 10 iteration cool-down period, sampling the parameter space around the best-fit parameters in an attempt to overcome the problem of converging on a local rather than global minimum.

Each primary galaxy is fit with a single Sérsic function containing 7 free parameters: object centres $x_0$ and $y_0$; total integrated magnitude $m_{\text{tot}}$; effective radius along the semi-major axis $r_e$; Sérsic index $n$; ellipticity $e$ and position angle $\theta$. Secondaries (galaxies and stars) will also be modelled by either a Sérsic function or a scaled PSF as appropriate. The PSF contains 3 free parameters; $x_0$, $y_0$ and $m_{\text{tot}}$. For additional information on the operation of GALFIT, refer to Peng et al. (2010a).

3.7.2 Modelling the Sérsic Index

3.7.2.1 Sérsic Index Input

All primary inputs to GALFIT are taken from Source Extractor and modified as described in Section 3.6, with the exception of Sérsic index. There is no obvious proxy for Sérsic index in the default Source Extractor parameters file. An approximation was created based on the trend between the output Sérsic index and the ratio between the corrected half-light radius to the Kron radius for a small test sample of trusted galaxies. From this, a derived relation for a predicted variable Sérsic index may be constructed thus:

$$n_{\text{var}} = 10^{-8.6 \left( \frac{r_e}{r_{\text{Kron}}} \right) + 2.8}$$  \hspace{1cm} (3.5)
3.7. Modelling the Galaxy

where $r_e$ is the corrected Source Extractor half-light radius (Equation 3.4) and $r_{\text{Kron}}$ is the Source Extractor Kron radius.

Figure 3.7 shows the density distributions between the variable Sérsic index, $n_{\text{var}}$, and several other initial conditions for a sample of 49,395 galaxies in the $r$ band. The other input parameters (size, position angle, ellipticity, magnitude, position) are not modified. These results show that the final recovered Sérsic index is largely independent of its initial condition, with the notable exception of a bump in the distribution at $n = 0.1$ for $n_{\text{initial}} = 0.1$ and a variable height spike of failed objects at $n \sim 20$. The $n = 0.1$ bump represents galaxies whose initial Sérsic index guess is placed too far away from its true value, and so fails to successfully migrate away from the initial parameter space using the Levenberg-Marquart method employed by GALFIT. It appears that the minor fluctuations found in the main body of the distributions directly correspond to the varying height of the $n \sim 20$ spike. Despite these features, it is clear that the initial Sérsic index is afforded a great deal of variability in order to achieve a successful and consistent fit. The majority of distributions presented in Figure 3.7 show little variation, with similar levels of success and failure. It was therefore felt that a simple $n_{\text{initial}} = 2.5$ would be an appropriate initial condition as it lies in the middle of the expected parameter space, yet not at the value of either of the bimodal peaks.

3.7.2.2 Sérsic Index Output

No explicit constraints were put on the range of acceptable Sérsic indices upon which GALFIT may converge, however, GALFIT has internal limits of $0.05 < n < 20$, where the lower limit is a 'soft' limit (indices scatter around this value) and the upper is a 'hard' limit (indices may not converge above this value). More conservative limits were not enforced on Sérsic index so as not to lead the final results and make presumptions about Sérsic index distributions.

3.7.3 GALFIT Setup

In order for GALFITPIPE to function correctly, it needs the cutout science image from CUTTERPIPE, cutim; the associated segmentation map and object catalogue from OBJECTPIPE, segim and object respectively; and a 2D FITS image PSF representing the PSF at the primary galaxy location from STARPIPE and PSFPIPE, psfim. Note that the weight map (cutwt) is no-longer required at this modelling stage.

Once the aforementioned files are in place, an initial fitting region radius on the cutout is defined by:

$$r_x = 2r_{\text{Kron}} (|\cos(\theta)| + (1 - e)|\sin(\theta)|)$$

(3.6)
Figure 3.7: A plot comparing final modelled Sérsic indices for a sample of 49,395 galaxies in the $r$ band given different initial Sérsic indices, as shown. From top to bottom, the initial Sérsic indices fed into Galfit are: variable (see Equation 3.5); 0.1; 0.5; 1.0; 2.5; 4.0; 10.0. Underlying the fixed initial Sérsic index distributions is the distribution for the variable Sérsic index coloured in grey, for reference.
$r_y = 2r_{Kron}(|\sin(\theta)| + (1 - e)|\cos(\theta)|)$ \hspace{1cm} (3.7)

in order to account for the ellipticity $e$ of the object and its position angle $\theta$. Objects within the central $2r_x \times 2r_y$ of the fitting region will be convolved with the supplied PSF at the modelling stage. The segmentation map is modified to unmask all secondary objects in the fitting region, with the resultant map saved into a new segfr file.

A GALFIT feedme file is created containing the starting values for every object being modelled (primary and secondary) as described in Section 3.5.1 and above. A constraints file is used to constrain secondary objects. These objects are constrained in order to reduce the fitting time, and reduce the size of the allowed parameter space. $x_0$ and $y_0$ are constrained to ±3 pixels of their input parameters, ellipticity is constrained to $0 < e < 0.95$ and half-light radius is constrained to $r_{e,initial}/4 < r_{e,final} < 4r_{e,initial}$. A final parameter for sky is added to the bottom of the GALFIT feedme file, fixing the value of the sky to zero counts.

### 3.7.4 Automating GALFIT

GALFIT is initialised, fixing the sky RMS to that measured in OBJECTPIPE. The time taken to converge on a fit scales with the size of the fitting region, and the number of secondaries being fit. Once the GALFIT process has finished, its output (if any) is read and processed. GALFITPIPE scans the primary galaxy for a number of problems in this order:

1. Crash or a segmentation fault
2. Galaxy centre migration of $\sqrt{x^2 + y^2} > r_{e,initial}$
3. An exceptionally large radius of $\log_{10}\left(\frac{r_{e,final}}{r_{e,initial}}\right) > 3$
4. An exceptionally small radius of $\log_{10}\left(\frac{r_{e,final}}{r_{e,initial}}\right) < 3$
5. A high ellipticity of $e > 0.95$

If any of these are detected, a fix will be attempted and GALFIT re-run as appropriate. Fixes attempted vary depending on the problem encountered. If a crash or segmentation fault are detected, GALFIT will be re-run modelling only the primary galaxy, with all secondaries masked. This usually occurs for large nearby objects with a high number of secondary neighbours and foreground stars, providing the Levenberg-Marquart minimisation routine in GALFIT with many local-minima. If the centre migrates away to fit a secondary feature, GALFIT will be re-run with the primary centroids fixed to their starting values. Large or small radii
are initially handled by suggesting a lower starting shape parameter (\(n = 0.5\)). This usually assists GALFIT in finding a way out of any local minima. If this attempt still provides a wildly different size to the input parameter, the size is fixed to the input and GALFIT re-run. Finally, a high ellipticity usually indicates the model has migrated away to fit flocculent secondary features. Re-running GALFIT with a starting ellipticity of \(e = 0.1\), i.e., highly circular, in most cases mitigates this problem. If all fixes have been attempted and problems persist, GALFIT-PIPE will record GALFIT’s best-guess model parameters and move on to the next object in its catalogue, with a flag updated to reflect the fitting history.

### 3.7.5 GALFIT Output

If the fit has been successful, the output multi-HDU FITS file from GALFIT is saved as `objim`, with a catalogue of final modelled secondary objects saved as `extct`. An example model output for GAMA galaxy G00092907 is shown in Figure 3.8. A series of value added measurements are calculated and added to the structural measurements already taken for the primary galaxy. These include \(\mu_0\) (central surface brightness), \(\mu_e\) (surface brightness at the half-light radius), \(\langle \mu_e \rangle\) (average surface brightness within the half-light radius) and \(r_{90}\) (radius along the semi-major axis that contains 90% of the total Sérsic flux), amongst others. These values, along with the output parameters from GALFIT and previous SIGMA modules are added to a comma-separated variable (CSV) catalogue, allowing SIGMA to move to the next primary galaxy in the input catalogue.

### 3.8 Testing SIGMA Through Simulations

A simple test of the accuracy of recovered parameters through a pipeline such as SIGMA lies in the processing of simulated galaxies with known inputs. We created a series of simulated galaxies and placed them into real data frames, upon which SIGMA was then ran across to measure the difference between the input and output quantities. The key simulated parameters of interest are magnitude (\(m\)), half-light radius (\(r_e\)), ellipticity (\(e\)) and Sérsic index (\(n\)). Position angle is notably excluded from this list as one would not expect significant bias in the recovered position angle with varying data quality. To that end, variations in position angle are not measured here.

We define a 4-dimensional grid of starting parameters at which to create a simulated galaxy model. This grid maps entirely our expected parameter space for galaxies within the

\[\text{integrated to infinity}\]
3.8. Testing SIGMA Through Simulations

Figure 3.8: An example model output for GAMA galaxy G00092907. The original SDSS $r$ band image is shown in the top left. The 2D model of this galaxy and its residual (image - model) are also shown as indicated. Inset into the residual is a postage stamp of the PSF constructed for this galaxy. Blue captions within the PSF postage stamp indicate (anti-clockwise from top-left) the full-width half-maximum of the PSF; the number of stars used in creating the PSF; the $\chi^2$ for the PSF model fit and the size of the PSF postage stamp ($8.5'' \times 8.5''$). The 1D profile for this galaxy, calculated by taking the average counts along ellipses centred on the primary galaxy and displayed against the semi-major axis, was created using the IRAF package ELLIPSE. Example ellipses from ELLIPSE have been added to the image in the bottom left to guide the eye, spaced evenly at intervals of 2'' along the semi-major axis. Note that . Inset into the 1D profile are relevant output Sérsic modelling parameters for this galaxy, and the overall $\chi^2$ for the Sérsic model is shown in grey at the bottom of the figure. The 1D profile is a 1D measure of 2D data. Any flux from secondary objects (neighbouring galaxies and stars) lying outside the 1D mask and overlapping with the primary galaxy will be counted as belonging to the primary galaxy by ELLIPSE. The 1D profile therefore should chiefly be used as a guide as to the true light distributions of both the 2D image and 2D model. For plotting, the image data is divided by some scaling constant (150 counts in the $r$ band) and scaled using the arctan function with cut levels at $-\frac{\pi}{4}$ and $\frac{\pi}{2}$.
Table 3.2: The starting values for the creation of a 4-dimensional parameter space, designed to encompass the entire expected range of input GAMA data. Parameters being varied are magnitude \((m)\), half-light radius \((r_e)\), Sérsic index \((n)\) and ellipticity \((e)\).

GAMA dataset. As shown in Table 3.2, we begin with 6 magnitude values, 5 half-light radii, 5 Sérsic indices and 5 ellipticities. This gives 750 unique starting grid positions. In order for any results to be statistically significant, each grid point is simulated 100 times at varying random positions, with median results collected afterwards. This leaves a final simulated catalogue sample of 75,000 galaxies to be modelled by SIGMA.

For each entry in the catalogue, a \(\sim 400 \times 400\) arcsecond patch of sky is cut out centred on the simulated galaxy. Due to computational space limitations, all simulated galaxy positions are located within a 1 square degree region of sky in the \(r\) band G09 field between \(139 < \alpha < 140\) and \(0 < \delta < 1\). The simulated galaxy model is created by the GALFIT software and convolved with the same PSF as measured for its parent cutout frame by the PSFEx software. This convolved model is then linearly co-added with the cutout frame and inserted into the SIGMA pipeline for subsequent detection by Source Extractor and modelling via GALFIT. In reality, the simulated galaxies are constructed on-the-fly within the SIGMA wrapper, raising the total processing time per object per processor to \(20 - 25\) seconds.

Figure 3.9 displays the results of the simulation exercise in an error vector diagram. Arrows represent the difference between the initial (simulated) and final (measured) values of magnitude and half-light radius, where longer arrows signify a greater difference between the two. A dot with no arrowhead shows those grid points where the output measured values of magnitude and half-light radius are in excellent agreement with the input simulated parameters, showing negligible change. The colour of the arrow (or dot) signifies the change in Sérsic index, with red arrows recovering lower Sérsic indices, blue arrows recovering higher Sérsic indices and purple arrows for minimal Sérsic index variation. Green crosses show those grid points where greater than 50% of simulations failed to recover parameters. Variation between input and output ellipticity is not shown here for clarity, however, we note that ellipticity should be calculable if size, total magnitude and Sérsic index are known. The four initial simulated parameters and three final measured parameters shown in this figure combine to create a 7-dimensional space. For reference, representative data from the GAMA Sérsic cat-
3.8. Testing SIGMA Through Simulations

talogue are shown in the background as a 2D density plot, with diagonal lines representing contours of constant average effective surface brightness, namely, $\langle \mu_e \rangle = 22$ and $\langle \mu_e \rangle = 24$ (left to right, respectively).

We find that the largest and faintest galaxies show consistent offsets between their input simulated parameters and their output measured parameters across all values of Sérsic index and ellipticity. These galaxies instead tend to be recovered as smaller and fainter than their input simulated models. For these large low surface brightness galaxies a sizeable fraction of flux resides in the wings of the galaxy within the noise, and so it is not surprising that the recovered model shrinks to a smaller area encompassing relatively less flux. Tempering this trend, we find that the higher the Sérsic index, the greater the offset in recovered size. Higher Sérsic index galaxies have increasing amounts of flux concentrated in the central regions, and owing to the $\chi^2$ minimisation routine employed by GALFIT, it is as expected that these systems will be recovered as smaller than their low-index counterparts. Owing to the inherent coupling between size, magnitude and Sérsic index, we also find that these large faint galaxies are recovered with smaller (redder) Sérsic indices.

What may not be initially apparent is why offsets tend to become smaller with increasing ellipticity. A higher ellipticity finds relatively more flux squashed into the central regions of these galaxy models, increasing the signal-to-noise in these systems and hence improving the chance for a successful recovery of the input simulated parameters.

It is heartening to note that the overwhelming majority of underlying GAMA data lies in regions of healthy parameter space, where the offsets between known simulated inputs and recovered outputs are at a minimum. This situation becomes worse for high index systems, however, as is shown, relatively few data are in the high index ($n \sim 8$) bin. Owing to these simulated tests, it is clear that recovered parameters from the SIGMA pipeline may be taken as credible analogues for the true underlying structural measurements of real galaxies within the GAMA catalogue.
Figure 3.9: Error vector diagram representing the differences between the simulated input parameters of magnitude, half-light radius, Sérsic index and ellipticity and recovered (through SIGMA) values of magnitude, half-light radius and Sérsic index. Longer arrows signify a greater drift in either recovered half-light radius and/or recovered magnitude from their simulated known inputs. The colour of the arrow signifies any additional variation in Sérsic index. Dots represent minimal change between input and output parameters. Green crosses signify parameter space grid points where more than 50% of modelling attempts failed. The two diagonal lines in each sub-plot represent contours of consistent average surface brightness, namely, \( \langle \mu_e \rangle = 22 \) and \( \langle \mu_e \rangle = 24 \) from left to right respectively. The underlying GAMA data as measured by SIGMA in Chapter 4 are shown as density maps, for reference.
Single-Sérsic Models of 167,600 Galaxies Across 9 Wavelengths

Dust and stellar population gradients in both age and metallicity have a huge potential to contribute towards disrupting the apparent shape of a galaxy light profile when measured at distinctly different wavelengths. Quantifying these effects allows one to determine which bands are ‘best’ suited to perform further measurements in, such as structural (bulge-disk) decomposition. Such a band would ideally be least effected by these additional factors and allow the intrinsic structural properties of the underlying galaxy to be more accurately examined. Single-Sérsic measurements such as those performed by SIGMA allow for the dual effects of dust and stellar population gradients to be quantified for both early-type galaxies (ETGs) and late-type galaxies (LTGs).

In this chapter I present SersicCatv07; a catalogue of 167,600 galaxies modelled independently across 9 bandpasses with a 2D single-Sérsic component using the SIGMA pipeline. This catalogue is one of the largest multi-wavelength single-Sérsic catalogues currently available, allowing galaxy structure to be explored from 355 nm (SDSS $u$ band) to 2200 nm (UKIDSS $K$ band), and is currently in use to aid in measurement of the evolution in the size-(stellar
mass) distribution of galaxies (Baldry et al., 2012); explore star formation trends as a function of morphology (Bauer et al., in prep.); to further understand the cosmic SED from 0.1 micron to 1 mm (Driver et al., 2012); to apply dust corrections to galaxies observed at multiple inclinations (Grootes et al., 2012); to explore the dust properties and star-formation histories of local submillimetre selected galaxies Rowlands et al. (2012); better constrain stellar mass measurements by providing total flux corrections (Taylor et al., 2011); comment on the quenching of star formation in the local universe (Taylor et al., in prep.); explore the relation between galaxy environments and their star formation rate variations (Wijesinghe et al., in prep.); provide a new method for automatic morphological classification (Kelvin et al., in prep.); and further explore the relation between environment (i.e., halo mass; Robotham et al., 2011), morphology and structure (Kelvin et al., in prep.).

This chapter is organised as follows. I first define the sample and subsequent processing of this sample through the SIGMA pipeline in Section 4.1 and analyse the outputs in Section 4.2. I discuss the results of this catalogue in Section 4.3 and the utilisation of these results to define two sub-populations (early-type and late-type) in Section 4.4. Finally, I explore the wavelength dependency on recovered structural parameters in Section 4.5 and conclude these results in Section 4.6. A standard cosmology of $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ is assumed throughout.

### 4.1 Single-Sérsic Galaxy Models

#### 4.1.1 Sample Definition

Here I define the sample of galaxies to be processed through SIGMA in single-Sérsic mode. Our initial input is the GAMA-I tiling catalogue, *TilingCatv11*, which contains 169,850 sources. For that subset of sources lying within the deep Stripe 82 region of the SDSS, Baldry et al. (2010) generate a star-galaxy separation parameter based on multi-band (*g*i*J*K*) multi-survey (SDSS and UKIDSS) colour information. Applied globally to the entirety of *TilingCatv11*, 167,600 sources remain classed as galaxy-like. This sample of 167,600 galaxies defines our master input sample.

#### 4.1.2 Processing Through SIGMA

The master input sample is processed through SIGMA independently across nine bands (SDSS: *ugriz* and UKIDSS: *YJHK*), the output of which becomes the SIGMA single-Sérsic master
4.1. Single-Sérsic Galaxy Models

<table>
<thead>
<tr>
<th></th>
<th>Detected</th>
<th>Modelled</th>
<th>Modelled/Detected</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>85,138 (50.8%)</td>
<td>81,120 (48.4%)</td>
<td>95.3%</td>
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<tr>
<td>$g$</td>
<td>165,367 (98.7%)</td>
<td>165,196 (98.6%)</td>
<td>99.9%</td>
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<td>$r$</td>
<td>166,506 (99.3%)</td>
<td>166,384 (99.3%)</td>
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<tr>
<td>$i$</td>
<td>166,675 (99.4%)</td>
<td>166,377 (99.3%)</td>
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<tr>
<td>$z$</td>
<td>163,902 (97.8%)</td>
<td>160,684 (95.9%)</td>
<td>98.0%</td>
</tr>
<tr>
<td>$Y$</td>
<td>156,702 (93.5%)</td>
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<tr>
<td>$J$</td>
<td>152,316 (90.9%)</td>
<td>151,612 (90.5%)</td>
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</tr>
<tr>
<td>$H$</td>
<td>159,464 (95.1%)</td>
<td>158,797 (94.7%)</td>
<td>99.6%</td>
</tr>
<tr>
<td>$K$</td>
<td>157,537 (94.0%)</td>
<td>156,662 (93.5%)</td>
<td>99.4%</td>
</tr>
</tbody>
</table>

Table 4.1: Number counts and percentages of successfully detected and successfully modelled galaxies in SersicCatv07 for each band, in addition to the fraction of those galaxies which have been successfully modelled given that they were initially detected. SersicCatv07 contains 167,600 galaxies in total.

catalogue SersicCatv07, available through the GAMA database\(^1\). Note that at the time of writing, full VISTA coverage across the entirety of the NIR bands is not available, and so its analysis will not be included here. Number counts for the number of successfully detected and successfully modelled galaxies contained within SersicCatv07 are listed in Table 4.1.

The SIGMA master catalogue, SersicCatv07, provides measurements of Sérsic index, half-light radius, position angle, ellipticity and magnitude in addition to extra pre-modelling sky estimation, Source Extraction and PSF Extraction measurements and post-modelling value added measurements as detailed in Section 3.7.5. Magnitudes contained within this catalogue are defined according to the AB magnitude system, and have not been corrected for the effects of foreground Milky Way dust extinction. The catalogue is an output of the GAMA SersicPhotometry data management unit, and contains 527 columns of data; 58 columns per passband, and 5 additional common descriptive columns.

4.1.3 Additional Sub-Samples

From SersicCatv07, additional sub-samples are defined after-the-fact in order to facilitate further analysis of the data, ensuring that selection bias does not adversely affect any conclusions made throughout the remainder of this work. These are defined, in increasing sequential sub-sets, thus:

1. Survey. SersicCatv07 contains sources fainter than the deepest nominal GAMA limit of $r_{\text{petro}} = 19.8$ (corrected for the effects of Milky Way dust attenuation), and so a cut is initially made limiting sources to $r_{\text{petro}} < 19.8$. This reduces the sample size to 150,633 galaxies.

\(^1\)The GAMA database can be found at http://www.gama-survey.org/database.
Chapter 4. Single-Sérsic Models of 167,600 Galaxies Across 9 Wavelengths

<table>
<thead>
<tr>
<th>Name</th>
<th>Number</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>TilingCatv11</td>
<td>169,850 (100%)</td>
<td>Complete GAMA tiling catalogue</td>
</tr>
<tr>
<td>SersicCatv07</td>
<td>167,600 (98.7%)</td>
<td>Removes star-like objects</td>
</tr>
<tr>
<td>Survey</td>
<td>150,633 (88.7%)</td>
<td>Removes r_{petro} &lt; 19.8</td>
</tr>
<tr>
<td>Coverage</td>
<td>138,269 (81.4%)</td>
<td>Requires SIGMA Source Extractor coverage in (ugriz)+Y+J+H+K</td>
</tr>
<tr>
<td>Stellar</td>
<td>116,951 (68.9%)</td>
<td>Requires a match in the StellarMassesv03 catalogue</td>
</tr>
</tbody>
</table>

Table 4.2: Table defining various sample definitions in use throughout this chapter. Cuts are sequential, and include the definitions from previous rows.

2. Coverage. A common coverage sub-sample is then constructed so as not to compare galaxies between bands whose observations are incomplete or have missing data. This is defined using the Source Extractor Auto magnitude SEX_MAG_X (analogous to an elliptical-aperture Kron magnitude) from SersicCatv07, a product of the OBJECTPIPE module, where X=UGRIZJHK. A common region is defined as having a detected Source Extractor magnitude in any of the SDSS bands (ugriz) as well as in each of the UKIDSS bands (YJHK). SDSS data is taken simultaneously at the telescope, and hence the requirement for a detection in only one band. This also has the added benefit of avoiding the poor-quality detection problem evident in the u band (and, to a lesser extent, the z band). The NIR bands are typically of a consistently high quality, and so a detection in each band is specified. However, observational incompleteness can significantly effect the NIR bands, with noticeable UKIDSS footprint gaps visible in the final common coverage area shown in Figure 2.2. The number of detected sources in individual SDSS bands is typically very high, > 97%, with the exception of the u band (see Table 4.1 for reference). The u band data has a detection percentage via this method of 50.8% indicating the poorer quality of the data in that band. For this reason, u band data is excluded from most subsequent analysis throughout the remainder of this chapter, with relations instead extrapolating into the u band wavelength for reference. The common coverage area reduces the sample to 138,269 galaxies.

3. Stellar. Any analysis that makes use of GAMA stellar mass estimates (Taylor et al., 2011) from the StellarMassesv03 catalogue (including rest-frame K-corrected u−r colours) is drawn from a reduced matched-coverage sample of 116,951 galaxies.

These subsets are summarised in Table 4.2.
4.2 Analysis

4.2.1 Background Sky Estimation and Subtraction

As part of the cutout creation process, SIGMA uses a variable background mesh to estimate and subtract the background sky for each galaxy in each band before any other image analysis takes place. Figure Subtracted sky-value distributions are mostly Gaussian in shape, with a small bias to recovering positive sky values most likely owing to background source contamination at the sky estimation stage. The additional correction on top of that already applied at the GAMA mosaicing stage is usually small. In the \( r \) band for example, the sky correction distribution has a 3\( \sigma \)-clipped mean of 0.56 ADU’s (\( \sim 0.01\% \) of the sky pedestal, \( \sim 0.03 \) ADU/RMS) and a standard deviation of \( \sigma = 2.06 \) ADU’s. Longer wavelengths produce larger corrections as expected. The accuracy with which we were able to estimate the sky using this method was found to produce good quality sky estimates in an efficient and relatively fast manner.

An additional spike feature at zero counts present in some bands relates to objects whose determined preferred background mesh size was larger than that already used in the creation of the GAMA mosaics. If this has occurred, SIGMA performs no sky subtraction, and returns zero counts. This feature affects 0.39\% of galaxies in the \( r \) band, and so whilst a larger mosaicing background mesh may be preferred for future surveys, it is not believed to be a major issue affecting these data.

4.2.2 Astrometry

Initial checks were made on the output astrometric accuracy of the SIGMA models. Figure 4.2 shows the computed astrometric offsets between the input SDSS positions and their GALFIT modelled SIGMA positions for all bands, with each sub-plot representing 2 × 2 pixels. Generally speaking, the astrometry is in good agreement with SDSS, with the \( r \) band offset \( c_r = 0.010'' \) (0.029 pixels) and a 1-sigma spread of \( 1\sigma_r = 0.044'' \) (0.130 pixels). The one exception to this is the \( u \) band data, showing a much larger spread in the recovered centroids owing to the poor quality and depth of the data in this band. There are however two interesting features worthy of note in this figure. First, the apparent asymmetry in the SDSS astrometry, particularly noticeable in the higher quality \( r \) and \( i \) bands. Second, the global systematic offsets in the NIR UKIDSS bands (\( Y JHK \)) of approximately 0.07''.

The asymmetry present in the SDSS astrometric data is found to be associated with an
Figure 4.1: Additional subtracted sky in each band represented as fractions of the background sky RMS for that band. Inset into the figure are estimates of the mean RMS to enable conversion back into raw ADU values. Most curves appear Gaussian in shape, with a small (∼2%) offset towards the positive owing to background source contamination during the sky estimation phase. One can see that the additional background sky typically subtracted in the $K$ band is ∼10% of the $K$ band RMS, whereas in the $u$ band this figure rises to ∼30%.
Figure 4.2: Astrometric offsets in RA and Dec between the input SDSS positions and the modelled SIGMA positions for all nine bands. Contours range from the 10th to the 90th percentile in steps of 10% with the peak density of each distribution represented by a yellow cross. Each sub-plot is exactly 2 × 2 pixels in dimension. The global systematic offsets in the NIR UKIDSS data (YJHK), typically ∼ 0.07″ (~ 0.2 pixels), are caused by minor variations in the WCS definitions between SDSS and UKIDSS. SIGMA accounts for this during the modelling phase.
individual SDSS strip\textsuperscript{2} that crosses the G09 field at a large angle of incidence with respect to the equatorial plane. Galaxies whose input imaging data lie in this strip appear to have centroids scattered around $\Delta RA \sim -0.05''$, $\Delta Dec \sim 0.10''$ rather than the origin. This feature is less prominent in the lower quality SDSS bands as it becomes lost in the random scatter, and consequently the effect is most noticeable in the $r$ and $i$ SDSS bands. Since this asymmetry affects only one strip of an SDSS stripe, the error must have been introduced at the splicing stage within the SDSS pipeline. These offsets remain small however, and are not believed to seriously affect this study as they are accounted for during the SIGMA modelling pipeline.

Global systematic offsets in the NIR bands represent minor differences in the WCS calibration between the SDSS and UKIDSS data. Any discrepancy between the imaging data would be carried through to the larger GAMA mosaics. This feature also varies according to GAMA region, with measured offsets of approximately $0.05''$, $0.11''$ and $0.09''$ in G09, G12 and G15 respectively. As with the previous feature, whilst consistent, offsets remain small sub-pixel variations ($\sim 0.2$ pixels) and therefore are not believed to be a major factor affecting cross-band matching between sources within GAMA. These features do not arise at the GALFIT modelling stage, as similar plots comparing input SDSS positions against pre-modelling Source Extractor centroids from SIGMA show similar results albeit with larger spreads. On the contrary, SIGMA should do a better job of recovering true centroids due to GALFIT’s model extrapolation method in estimating centroids. This makes SIGMA robust against astrometric errors such as this by recentring every galaxy at the modelling stage, emphasising the strength of full modelling against basic source extraction.

### 4.2.3 Seeing

An independent measure of the seeing and the form of the PSF for each galaxy in each band is a necessary requirement when considering galaxy modelling. Through PSFEx in PSFPIPE, SIGMA is able to provide robust measurements of the PSF for each galaxy as described in Section 3.5.2 prior to the GALFIT modelling stage. Figure 4.3 shows the recovered PSF full-width half-maxima $\Gamma$ for every galaxy within the SIGMA common sample for all 9 bands. Each density curve has a main peak in the range $0.7'' < \Gamma < 1.4''$, and an additional peak at $\Gamma = 0.4''$, which shall be discussed later.

We note that on average the NIR data is of better seeing than the optical, with the former in the range $0.6'' < \Gamma < 1.3''$ and peaking at around $\Gamma = 0.9''$, and the latter in the range $\Gamma = 0.4''$. In SDSS imaging data, a single run covers a strip. Two strips constitute a stripe, with the second strip offset from the first in order to cover a continuous area.
0.8″ < \Gamma < 1.7″ with variable peaks. These ranges are in good agreement with UKIDSS (K band) and SDSS (r band) seeing targets of \Gamma_{\text{UKIDSS,K}} < 1.2″ and \Gamma_{\text{SDSS,r}} < 1.5″ respectively. Some of the NIR data displays a secondary peak, particularly in the K band, possibly due to the use of microstepping in the taking of some of the UKIDSS data. The worst quality seeing data is in the u band, exhibiting the largest width distribution, and the highest seeing data on average. This distribution of its mean across the GAMA regions is represented in Figure 2.2, with the data points coloured according to the measured seeing at that location. This figure shows significant striping in the SDSS r band due to the drift scan mode of collection, and a measure of consistency coupled with lower average values across each of the UKIDSS bands. This could cause significant problems for image analysis routines, with average seeing doubling on the scale of a few pixels. Modelling the PSF and using that model at the galaxy modelling stage, as in SIGMA, goes some way towards mitigating this issue.

An additional peak at \Gamma = 0.4″ represents those frames where no stars were detected in order to compute the PSF in that region, and so a generic value of \Gamma = 0.4″ is returned. Note that for the majority of bands this problem is minimal, becoming most noticeable in the lower quality u and z band data.

### 4.2.4 Surface Brightness Limits

Consideration of the surface brightness limit beyond which data becomes unreliable at the 1\sigma level is also important. An estimate of the surface brightness limit at any given position may be given by

\[
\mu_{\text{lim}} = ZP - 2.5 \log I_{\text{RMS}}
\]

where ZP is the zero-point of the imaging data, and \( I_{\text{RMS}} \) is the root-mean-square of the background sky per square arcsecond. Note however that this provides a worst-case scenario value to the surface brightness limit, with the real limit likely to be deeper on a per-galaxy basis dependent upon the number of pixels \((n)\) used in constructing the 2D model at large radii from the core region, and scaling as \( \sqrt{n} \). Figure 4.4 shows the global distributions in \( \mu_{\text{lim}} \) for the SIGMA common coverage sample across each bandpass, with the median surface brightness limits in each band given inset into the figure. We note that the shorter wavelengths typically exhibit deeper limits, as expected, with a transition occurring to shallower limits beyond the i-z interface. Figure 4.5 shows the spatial variation of \( \mu_{\text{lim}} \) across the GAMA fields. The deepest \( \mu_{\text{lim}} \) data is represented by blue data points, the shallowest by red. The centroid weighting mechanism employed by GALFIT should minimise the impact of a spatially
Figure 4.3: Recovered full-width half-maximum PSF values from the SIGMA single-Sérsic common coverage sample.
4.3. Results

4.3.1 Case Study Examples

We present example model fits for an individual galaxy across all nine bands and various galaxies separated in magnitude space in Figures 4.6 and 4.7 respectively. These figures each represent the input 2D science image, model and residual along with a 1D profile radiating outwards from the core region of the galaxies along the semi-major axis. The displayed input image is a postage-stamp sub-region of the background corrected cutout from the original

Figure 4.4: Apparent surface brightness limits for all galaxies within the SIGMA single-Sérsic common coverage sample, with median surface brightness values for each band inset. The shorter wavelengths typically exhibit deeper limits (right), as expected, with a transition occurring to shallower limits beyond the $i$-$z$ interface (left). Varying $\mu_{\text{lim}}$, and therefore should not heavily affect the output results from SIGMA.
Figure 4.5: Apparent surface brightness limits for all galaxies within the SIGMA single-Sérsic common coverage sample as a function of their position on the sky in right-ascension and declination. The three GAMA regions are displayed, as indicated, with each band labelled along the right side of the figure. Bands are offset in declination in order to differentiate them from one another. The $K$ band is situated at the correct GAMA coordinates. Surface brightnesses are shown as offsets from the median surface brightness for that band, the values of which are found in Figure 4.4. Blue data points represent the deepest limits and red the shallowest.
4.3. Results

GAMA mosaiced image. The yellow box in each of the input image postage stamps represents the size of the fitting region as determined in GALFITPIPE, the dimensions of which specify the size of the output model FITS image. Recovered Sérsic parameters are listed inset into the 1D profile plot.

Figure 4.6 shows the output SIGMA results for the elliptical galaxy G00032237 across all nine bands. Each image is modelled independently in each band, leading to a variable fitting region size dependent upon local conditions including object density and the physical size of the primary galaxy. The residuals in each band show the high quality of the fit for this particular galaxy, bar some minor core disturbance in the higher quality $r$ and $i$ bands. These bands cover wavelengths that are more sensitive to dust attenuation, in this case potentially highlighting small quantities of dust in the centre of the galaxy possibly related to a recent minor-merger or some form of morphological disturbance. Dust has the effect of perturbing the light profile slightly away from that of a purely single-Sérsic object. Interestingly however, no evidence for dust lanes are evident in the lowest wavelength $u$ band residual. This should not be surprising considering the lower quality data of the $u$ band, hence these small perturbations would be lost in the noise of the image. Barring the $u$ band data, and despite dust attenuation, the recovered Sérsic indices remain relatively stable, ranging from $n = 4.21$ to $n = 4.50$ in $g$ to $K$. Sérsic index peaks in the $J$ band ($n_{\text{max}} = 4.73$) and reaches a minimum in the $r$ band ($n_{\text{min}} = 3.82$, excluding $u$ band data). Modeled ellipticity $e$ and position angle $\theta$ are also in good agreement, with recovered Sérsic magnitude evolving as expected across this wavelength range. Interestingly, the recovered half-light radii show a size-wavelength dependence, ranging from $r_e = 3.74''$ to $r_e = 2.13''$ in $g$ to $K$.

Secondary objects whose object centres lie outside the fitting region but whose flux leaks into it are masked so as not to effect the model fit. One such object can be seen in the upper-right corner of the fitting region in the $r$ band postage stamp in Figure 4.6. The GALFITPIPE module creates a bad-pixel mask using the segmentation map provided by OBJECTPIPE. Should GALFIT reach an error whilst trying to converge on a model, a potential additional fix is to mask all secondary objects in the field of view, and re-run GALFIT. This dynamic masking, whilst not the first choice for producing a model, typically allows high-quality and consistent model data to be extracted from difficult regions where otherwise none would exist.

Figure 4.7 shows nine example galaxies in the $r$ band from the SIGMA common coverage sample, separated in magnitude space in approximately equal SDSS $r$ band magnitude steps.
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Figure 4.6: Model fits for G00032237 across all nine bands. Each column represents (from left to right) the original input image, the model fit to the input image, the residual image (input - model) and the 1D surface brightness profile along the semi-major axis (averaged along the annulus). The fitting region (the region within which 2D modelling takes place) is represented by a yellow box. Recovered 2D intrinsic (i.e., prior to PSF convolution) Sérsic parameters are listed inset into the 1D profile plot. The images are scaled logarithmically from $-\sigma_{\text{sky}}$ to $n\sigma_{\text{sky}}$, where $\sigma_{\text{sky}}$ is the typical RMS of the sky in that band, and $n$ is some scaling constant (generally, $n \sim 40$).
4.3. Results

of $\Delta m_r = 0.5$ from the faintest GAMA limit of $m_r = 19.8$. These galaxies span a wide range of morphologies and environments, exhibiting the large variance in the input data processed by SIGMA. In each case, the residual images show the quality of the fits are relatively high, more so for obvious single-component objects such as the huge elliptical galaxy G00506119 ($m_r = 15.8$) than multi-component objects such as the barred-spiral galaxy G00369161 ($m_r = 16.8$).

In the case of the latter, despite a global single-Sérsic fit to a multi-component object, the resultant model does a good job at describing global parameters such as magnitude. Despite a relatively disturbed fit to the secondary neighbour of G00177815 ($m_r = 18.3$), the quality of the primary galaxy model remains high. This outlines the exclusive use of secondary objects in accounting for additional flux in the wings of primary galaxies. Note that whilst the quality of model fits reflected through the residuals appears to become better at fainter magnitudes, this effect is more likely an example of non-resolved components of a galaxy.

4.3.2 Global Results

Complete distributions for the SIGMA common coverage sample of 138,269 galaxies are shown in Figure 4.8. Here we plot 1D density distributions for recovered model Sérsic indices, half-light radii and ellipticities in each band. Alongside these distributions are displayed the average model galaxies based on median values from the aforementioned parameters.

Recovered Sérsic parameters peak primarily in the range $0.2 < n < 10$, with additional peaks at $n \sim 0.05$ and $n = 20$ arising due to failed fits, discussed in more detail below. The primary range appears bimodal in nature, consisting of two approximately Gaussian-like distributions whose means are $n \sim 1$ and $n \sim 3.5$. These two peaks, for the most massive/brightest systems in GAMA, correspond to the two main galaxy morphologies as originally identified by Hubble, namely, late-type disk-dominated galaxies and early-type spheroid-dominated galaxies for $n = 1$ and $n = 3.5$ respectively. Interestingly, the second of these two peaks does not appear at $n = 4$, which is typically expected for a classic de Vaucouleurs profile. The relative strength of these two peaks shifts with increasing wavelength, with the stronger disk-dominated peak at $n = 1$ giving way to the spheroid-dominated $n = 3.5$ peak at wavelengths longer than the $i$ band. This is believed to be an indicator of the shifting in observed stellar population with wavelength, however, see Section 4.5 for further discussion. In addition, the centroid of the $n = 1$ peak at wavelengths longer than the $i$ band appears to move towards higher Sérsic index values, merging into an elongated shoulder of the relatively stable $n = 3.5$ peak. In the $K$ band, the first peak appears to have a mean centred on $n \sim 1.5$. This should
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Figure 4.7: Model fits for nine galaxies in the $r$ band separated in magnitude space by approximately $\Delta m_r = 0.5$ in the range $15.8 < m_r < 19.8$. Each column represents (from left to right) the original input image, the model fit to the input image, the residual image (input - model) and the 1D surface brightness profile along the semi-major axis (averaged along the annulus). If the fitting region (the region within which 2D modelling takes place) lies within the image thumbnail above, it is represented by a yellow box. If no yellow box is visible, the fitting region is larger than the thumbnail. Recovered 2D intrinsic (i.e., prior to PSF convolution) Sérsic parameters are listed inset into the 1D profile plot. The images are scaled logarithmically from $-1 \sigma_{\text{sky}}$ to $40 \sigma_{\text{sky}}$, where $\sigma_{\text{sky}}$ is the typical RMS of the sky in the $r$ band.

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not be surprising, as optical bands are more likely to probe the young stellar populations in the disks of galaxies whereas the longer wavelengths pick out the older stellar populations within the core regions of a spiral galaxy or in elliptical galaxies. Dust may also be an issue at shorter wavelengths, blocking light from the core regions of galaxies and therefore biasing recovered Sérsic indices towards lower values (for example, see Pastrav et al. (2012)). It is important to note that these data are derived from the same r band selected sample of galaxies observed in different wavelengths, and so these relative shifts in peak positions represent real variances in observed stellar populations, highlighting a wavelength dependence on structural measurements.

The additional peaks at $n \sim 0.05$ and $n = 20$ represent failed fits. For these galaxies, the fitting procedure drifted into an unrealistic parameter space during the downhill minimisation routine employed by GALFIT. Despite attempts to force a better fit from the data within GALFITPIPE, the fits to the images of these objects remain corrupted, and are not appropriate for further analysis. Bad fits occur for many reasons. Typical reasons are over-dense regions introducing too many free-parameters into the minimisation routine, or bad sky subtraction for that region. The corrupt peak values of $n \sim 0.05$ and $n = 20$ arise due to constraints placed by the fitting code GALFIT, and unchanged for the purposes of this study. The upper peak is a hard limit, with galaxies unable to obtain a Sérsic index beyond this value. The lower peak is a result of a consistency check within the GALFIT code that attempts to force a fit at $n > 0.05$, hence leading to a small distribution around this value. These errors that caused these additional peaks are also the cause of those found in the ellipticity distribution, discussed further below. The density of objects within these failed regions scales with wavelength, with the higher-quality bands exhibiting fewer cases than poorer quality bands such as the u band. A conservative estimate ($n < 0.07$ or $n > 19$) places 1.1% (1,456) of r band galaxies within these extremely non-physical regions, rising to 9.1% (12,630) in the worst affected u band.

Distributions of recovered effective radii (along the semi-major axis), converted to kiloparsecs, are also shown. Density profiles at all wavelengths appear relatively smooth, approximating a skewed Gaussian distribution in logarithmic space. Note that these distributions exhibit no additional peaks as observed in the Sérsic index and ellipticity plots. When regarding the median values of these distributions, represented in Figure 4.8 by red dashed lines, we note that the median effective radius of a galaxy ranges from 5.5 kpc in the u band to 3.5
kpc in the $K$ band. This marked decrease in physical size with observed wavelength is again as expected if one expects the longer wavelengths to probe core stellar populations in the bulges of galaxies, whilst shorter wavelengths probe recently formed populations in the disks of galaxies (i.e., inside-out growth, see Trujillo & Pohlen (2005)). The transition wavelength appears to be the $Y$ band, with a median size of $\sim 4.5$ kpc in the $z$ band, and $\sim 3.5$ kpc in the $J$ band, highlighting once more the wavelength dependence on structural measurements and the importance of the right choice of wavelength when comparing galaxy samples. Indeed, there appears to be little size variation at wavelengths longer than the $Y$ band. Clearly, care must be taken when comparing the sizes of galaxies observed at different rest wavelengths.

Ellipticity $(1 - \frac{b}{a})$ measurements remain relatively consistent across all bands, peaking in the range $0.25 < e < 0.35$, and displaying additional peaks at $e = 0$ and $e = 0.95$. Bands $g-K$, excepting the $r$ and $i$ bands, show very similar distributions, with a consistent median value of $e \sim 0.4$ and a modal value of $e \sim 0.35$. The higher quality $r$ and $i$ bands appear to have, on average, more circular recovered ellipticities, with median and modal values of $e \sim 0.35$ and $e \sim 0.25$ respectively, however the shift is minimal. The lower quality $u$ band data leads the ellipticity measurements in that band to be biased towards higher values, caused by the fitting routine being more susceptible to background noise fluctuations and random noise in the frame. Ellipticity measurements presented here are global ellipticities, and there will be variability in ellipticity with increasing radius from the core on a per-galaxy basis caused by additional factors, for example, the effect of seeing or the presence of a bar. The additional peaks directly correspond to those already discussed previously, and represent failed fits. The values of $e = 0$ and $e = 0.95$ correspond to internal GALFIT boundaries.

### 4.3.3 Photometry Comparisons

Figure 4.9 compares SDSS $ugriz$ Petrosian photometry against truncated Sérsic magnitudes as recommended in Section 1.3.3 as a function of SDSS magnitude. Each row represents a different band, with the mode and standard-deviation for varying magnitude subsets inset into the left-hand column sub-plots. Across each band we see a good global agreement between SDSS and recovered Sérsic photometry at all magnitudes, with the variance between the two photometric methods increasing towards fainter magnitudes as expected. The global total spread is larger in the lower quality $u$ band than in the higher quality $r$ band, ranging from $\sigma_u = 0.72$ magnitudes in the former and $\sigma_r = 0.21$ magnitudes in the latter. This trend should not be surprising, as lower quality data presents a unique challenge in recovering 'cor-
4.3. Results

Figure 4.8: Global results from the SIGMA common coverage sample for all nine bands. Each column represents (from left to right) the average model galaxy based on median values for Sérsic index, half-light radius and ellipticity and the distributions for all recovered Sérsic indices, half-light radii (converted into kpc) and ellipticities. The y-axis for each distribution shows the probability density function convolved with a rectangular top-hat kernel with standard deviations of 0.05, 0.05 and 0.02 for index, size and ellipticity respectively. Median values for each distribution are represented by a red dashed line and are used in creating the average model galaxy in the left-hand column. The $r$ band distributions are shown in grey for reference.
rect’ structural parameters, with larger errors expected between different photometric systems for fainter galaxies. In all cases, the peak modal values are typically less than $\Delta m = 0.03$ magnitudes, re-enforcing the notion of good photometric agreement between these two different methods. These measured offsets are also in good agreement with the offsets previously laid out in Hill et al. (2011) using the same input imaging data.

Data points are coloured according to the recovered Sérsic index, highlighting the importance of Sérsic modelling in recovering accurate structural parameters. At all wavelengths, the largest offset between SDSS Petrosian and Sérsic magnitudes is observed in those well-resolved galaxies whose Sérsic indices are large ($n > 4$; red data points). These high-index systems are typified by being highly centrally concentrated with large extended wings, with the flux in the wings of these galaxies most likely to be missed by traditional photometric methods such as Petrosian aperture photometry. By extrapolating the fitted light profile to a given truncation radius, Sérsic photometry is able to recover this missing flux and provide a more accurate measure of ‘total’ magnitude. Unresolved compact high-index galaxies agree well with the Petrosian aperture method.

As an example of flux recovery, Sérsic photometry for a relatively bright ($r = 18$) high-index ($n \sim 8$) galaxy observed in the $r$ band may recover as much as $\Delta r = 0.5$ magnitudes. Galaxies with indices this high should not be dismissed out of hand as being incorrect. Caon et al. (1993) successfully show that for a deep dataset and employing good sky-subtraction methods it is possible to find galaxies whose central concentrations are of the order $n \sim 15$.

Recovery of missing flux may more clearly be seen in the turn-off shown in Figure 1.12 (middle panel), with the scale of the Petrosian correction a direct function of Sérsic index. In contrast, intermediate and low Sérsic index galaxies ($n < 4$; blue and green data points) agree much more closely with SDSS Petrosian photometry. High-index galaxies are also those whose Sérsic magnitudes have been truncated by the largest amount, and so one must take into consideration the arguments laid out in Section 1.3.3 when analysing these systems.

Figure 4.10 shows a comparison between GAMA AUTO and Sérsic magnitudes as a function of GAMA AUTO magnitude. It should be noted that the GAMA AUTO photometry presented here is version 2 data and different to the version 1 data presented in Hill et al. (2011). Version 2 photometry employs an updated source detection pipeline over a larger area, with a small fraction of version 1 input frames discarded due to erroneous data in that region (e.g., badly focussed frames). The process used in deriving these renewed data is similar in
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Figure 4.9: Comparison between Sérsic magnitudes truncated at 10$r_e$ and SDSS Petrosian magnitudes for the five SDSS bandpasses as a function of SDSS Petrosian magnitude, with the data points coloured according to their Sérsic index in that band (left column). Vertical lines define subsets at magnitudes brighter than those values, with corresponding statistics for mode and standard-deviation inset into the figure. Density plots (right column) show the relative density of objects in $\Delta m$-space for each of the aforementioned subsets.
approach to that previously employed, a full description of which may be found in Liske et al. (2012; in prep.). As in Figure 4.9, there is good agreement between the two photometric systems, with the larger magnitude offsets observed in the resolved high-index systems.

4.4 Two Distinct Galaxy Populations

Figures 4.9 and 4.10 show that Sérsic index plays an important role when considering magnitudes, with higher index galaxies typically recovering more missing flux than their lower index counterparts when compared to traditional photometric methods. In addition, there appears to be a wavelength dependence on the flux difference between high and low index galaxies, which has implications for variations in other structural parameters with wavelength. In order to further analyse this wavelength relationship with structural parameters, we define two galaxy sub-populations based on Sérsic index and $u - r$ rest frame colour (AUTO aperture defined, as found in the GAMA catalogue StellarMassesv03 described in Taylor et al., 2011).

Figure 4.11 shows the relation between $u - r$ rest frame colour and the $K$-band Sérsic index, with the data points coloured according to stellar mass. The bulk of the galaxies appear to lie in two distinct populations, the nature of which have most recently been explored in Baldry et al. (2006); Driver et al. (2006); Allen et al. (2006); Cameron & Driver (2009); Cameron et al. (2009); Mendez et al. (2011), amongst others. These two populations may be typically well described by two overlapping Gaussians. Blue low-index systems correspond to late-type disk-dominated galaxies and red high-index systems to early-type spheroid-dominated galaxies. This is well supported by galaxy stellar mass, with the least massive galaxies appearing disk-dominated, and the most massive appearing spheroid-dominated, as expected. The faintest types of galaxy, namely dwarf systems (dE, dS0), are not represented in our common coverage sample, and so these two peaks do not relate to those morphological classes. We used the positions of peak object density for each sub-population to define a dividing line between them, specifically, the line which lies perpendicular to one connecting the two peaks in object density, bisecting it at the point of lowest object density along the connecting line. The equation of the dividing line is given by:

$$ (u - r)_{rest} = -0.59 \log n_K + 2.07 $$

In order to avoid any potential misclassifications due to the effects of dust attenuation, our longest wavelength $K$-band data was chosen as a measure of central concentration. Sérsic
Figure 4.10: Comparison between Sérsic magnitudes truncated at $10r_e$ and GAMA AUTO magnitudes for all wavelengths as a function of GAMA AUTO magnitude, with the data points coloured according to their Sérsic index in that band (left column). Vertical lines define subsets at magnitudes brighter than those values, with corresponding statistics for mode and standard-deviation inset into the figure. Density plots (right column) show the relative density of objects in $\Delta m$-space for each of the aforementioned subsets.
indices recovered at shorter wavelengths return a steeper dividing line, with the gradient only becoming stable at wavelengths longer than the $z$ band. This effect is characteristic of the effects of dust, and shall be explored in more depth in Section 4.5. Interestingly, Mendez et al. (2011) show that the choice of bands used to quantify colour is less important, and so a standard $u - r$ colour definition is employed in Equation 4.2 for comparison with much of the current literature. Objects bluer than this dividing line will be referred to as disk-dominated late-type galaxies (LTGs), whereas objects redder than this line will be referred to as spheroid-dominated early-type galaxies (ETGs) throughout the remainder of this chapter. It is well known that two 2D Gaussians are able to aptly describe these two populations. It follows therefore that a harsh cut of this nature will no doubt introduce a small fraction of cross-contaminants for galaxies occupying a parameter space in close proximity to this dividing boundary, namely, those galaxies that lie in the wings of the opposing Gaussian function. The amount of contamination will be small however, with the overall trends entirely sufficient for analysing global trends within each sub-population. Improvements to the nature of automatic morphological classification based on global structural measurements exhibited in this chapter will be the focus of future studies presented in later chapters.

4.5 Wavelength Dependency

The observed nature and form of a galaxy varies dependent upon the wavelength at which the observation is taken. These variations reflect physical mechanisms occurring within the galaxy, including, but not limited to; dust attenuation and; intrinsic gradients in stellar population, age and/or metallicity (e.g., Block et al., 1999). In this section I analyse the variance in recovered Sérsic parameters with wavelength, and discuss how this behaviour is characterised.

The key galaxy measurements produced by SIGMA, in addition to improved object centring accuracy are: position angle; ellipticity; Sérsic magnitude; Sérsic index; and half-light radius. Understanding how each of these parameters varies with wavelength is crucial to remove biases when comparing measurements made in different bandpasses. Wavelength bias may also represent real physical bias caused by dust attenuation and stellar population gradients.

4.5.1 Position Angle, Ellipticity and Sérsic Magnitude with Wavelength

Naively, one may expect recovered position angle to show little variance with wavelength, instead varying mainly as a function of the quality of the input data. In line with the cos-
4.5. Wavelength Dependency

Figure 4.11: $K$ band Sérsic index versus $u-r$ rest frame colour, with the data points coloured according to their galaxy stellar mass estimates, as shown. Contours range from the $10^{th}$ to the $90^{th}$ percentile in steps of 10%. The two highest peaks in object density, corresponding to two distinct galaxy populations (late-type and early-type for low-index and high-index respectively), are represented by filled black triangles. The diagonal line lies perpendicular to the line connecting these two peaks, and bisects it at the point of lowest object density along the connecting line, marked on the figure with a plus sign. This dividing line defines two sub-samples, which for the most massive galaxies, relate to disk-dominated systems below the line (LTGs) and spheroid-dominated systems above (ETGs), the equation of which is inset into the Figure.
mological principle, recovered position angle should merely be a random quantity assuming no detector bias, although for small area samples it may be coupled with filamentary structure. On a per-galaxy basis, one might expect minor variations with wavelength to occur in the presence of stellar population gradients in transient local features such as star-forming regions and bars, with different bands being more sensitive to different stellar populations that trace distinct structural components. However, for the SIGMA common coverage sample of 138,269 galaxies one does indeed find no noticeable trend with wavelength for recovered position angle.

Recovered ellipticity remains relatively stable at all wavelengths, instead varying primarily as a function of the quality of the input data, as shown in Figure 4.8. The highest quality $r$ and $i$ bands typically return the most circular galaxy models, whereas the lowest quality $u$ and $z$ bands return more elongated profiles across the same galaxy sample. This is as expected as one reduces the signal-to-noise of the data, with the fitting routine becoming increasingly sensitive to nearby background noise, however, further studies and deeper data are required in order to comment further on this effect.

Finally, recovered Sérsic magnitude is expected to vary as a function of wavelength as per each galaxy’s individual SED, the theory of which is well understood and will not be discussed further. This leaves the apparent central concentration (Sérsic index; $n$) and size (half-light radius; $r_e$) of each galaxy as a function of wavelength to be discussed.

### 4.5.2 Sérsic Index with Wavelength

Figure 4.12 shows the recovered Sérsic indices for each galaxy in the SIGMA matched coverage sample at their rest-frame wavelength, coloured according to their population classification as described in Section 4.4. Considering the population definitions are based on $K$ band Sérsic indices it is reassuring to note that the spheroidal population primarily retain their high Sérsic index values across all wavelengths, and low Sérsic indices similarly for the disk population. This indicates a significant level of consistency in recovered parameters with wavelength, i.e.; a galaxy that appears disk-like in the $g$ band is likely to appear disk-like in the $H$ band, for example. $3\sigma$ clipped mean Sérsic indices are shown for each population in each band, represented by large filled circles coloured as appropriate. Polynomial fits to these mean data points, excluding $u$ band values due to their lower quality imaging data, reveal general trends in Sérsic index with wavelength. The best fit Sérsic index for the disk-dominated
4.5. Wavelength Dependency

The population is given by:

$$\log n_{\text{disk}} = -0.715 \log^2 \lambda_{\text{rest}} + 4.462 \log \lambda_{\text{rest}} - 6.801$$  \hspace{1cm} (4.3)

and similarly for the spheroid-dominated population:

$$\log n_{\text{sph}} = -0.210 \log^2 \lambda_{\text{rest}} + 1.394 \log \lambda_{\text{rest}} - 1.753$$  \hspace{1cm} (4.4)

where $\lambda_{\text{rest}}$ is the rest-frame wavelength of the observation of the galaxy. It is important to remind the reader to be mindful of our sample selection when considering these relations.

Note that we have adopted log-quadratic relations for Equations 4.3 and 4.4. Whilst the spheroid-dominated population may not appear to require a quadratic fit, the disk-dominated population most-likely does. For this reason, the functional form of both equations has been kept the same. The linear relation for the disk-dominated population is given by:

$$\log n_{\text{disk}} = 0.267 \log \lambda_{\text{rest}} - 0.676$$  \hspace{1cm} (4.5)

and the spheroid-dominated population is given by:

$$\log n_{\text{sph}} = 0.170 \log \lambda_{\text{rest}} + 0.024$$  \hspace{1cm} (4.6)

These linear relations are provided for reference only and are not used in any subsequent calculations, with the log-quadratic forms instead being the preferred descriptors of the two populations.

We find that the spheroid population Sérsic indices remain relatively stable at all wavelengths, exhibiting slightly lower Sérsic indices at shorter wavelengths and becoming essentially stable at wavelengths longer than the $z/Y$ interface. Mean ETG Sérsic indices range from $n_g = 2.79$ to $n_K = 3.63$ from $g$ through to $K$, an increase of 0.11 dex, equivalent to 30%. This increase is consistent with the 23% increase reported in La Barbera et al. (2010) over a similar wavelength range. However, whilst the fractional increase is comparable, the absolute values are not; La Barbera et al. find on average Sérsic indices $n \sim 2 – 3$ larger than those reported here. Whilst it is unclear what causes this difference, a potential difference in sample definitions may be important. That study defines ETGs based on a number of SDSS parameters including fracDeV_r; a parameter that describes how well the global light profile
of the galaxy is fit by a de Vaucouleurs profile. A cut of this nature is somewhat analogous to making a Sérsic index cut alone which, as can be seen in Figure 1.12, and again in Figure 4.11, would introduce an element of contamination from the LTG population. If a relatively large number of the ETG sample in La Barbera et al. are in fact bulge-dominated systems with a weak underlying disk then one might expect the Sérsic indices of their bulges to differ somewhat from a traditional de Vaucouleurs profile. A Sérsic index of \( n \sim 6 \) is the value found in Simard et al. (2011) for bulge+disk systems with a well-defined bulge, in good agreement with the offset found here.

The apparent stability found in Sérsic index with wavelength is as expected for relatively dust-free single-component early-type systems, and is interesting to re-confirm empirically. Since the spheroid-dominated population is likely to include a small fraction of misclassified galaxies, as previously discussed, due to the nature of the harsh cut presented in Section 4.4, a small gradient with wavelength should not be surprising. Recent work by Rowlands et al. (2012) suggest that as many as 5.5% of early-type galaxies contain significant fractions of previously unaccounted-for dust, introducing an additional secondary deviation in recovered Sérsic indices with wavelength. Dust in a galaxy is typically centrally concentrated, and so has the effect of masking stellar light emanating from the core regions of a galaxy. Since galaxy fitting algorithms such as GALFIT apply larger weighting to higher signal-to-noise regions, minor deviations at small radii have the potential to drastically affect the recovered structural parameters, including the Sérsic index. Therefore, the addition of dust to the core region of a galaxy would subdue the *cuspiness* of the galaxy and bias the model towards a lower Sérsic index.

The disk population exhibits a larger change in Sérsic index variation with wavelength than that observed for the spheroid population. The recovered mean disk Sérsic indices range from \( n_g = 0.92 \) to \( n_K = 1.40 \) from \( g \) through to \( K \), an increase of 0.18 dex, equivalent to 52%. As with the spheroid population, disk Sérsic indices also become increasingly stable at wavelengths longer than the \( z/Y \) interface. Since we typically expect disk-dominated systems to be dustier than their early-type counterparts, owing to the prevalence of ongoing star-formation in many of these galaxies, then a significant variation in Sérsic index with wavelength should be expected as a consequence of the arguments previously laid out. Since the disk Sérsic index appears stable beyond the \( z/Y \) interface, we can conclude that the effect of dust in these regimes is minimal, and therefore if ‘intrinsic’ disk Sérsic indices are required, one should look
4.5. Wavelength Dependency

to the longest wavelength data available, typically longwards of the $\lambda$ band. In addition to the effects of dust attenuation, we may also consider stellar population gradients. Since this sample is not a pure-disk sample, and instead contains a host of disk-dominated systems, a fraction of galaxies in the disk-dominated population will no doubt contain additional structures such as a bulge and/or a bar. Bulges tend to contain older, redder stars of a higher metallicity than the younger, bluer stars found in the disks of galaxies. Shorter wavelengths are more sensitive to the blue population found in the disk whereas longer wavelengths become increasingly sensitive to the red population. Therefore, any real colour gradients that exist in a galaxy, which are indicative of metallicity and age gradients in the underlying stellar population distribution, would also lead to a change in the measured Sérsic index, dependent upon the wavelength at which that galaxy was observed. A short wavelength is therefore more likely to probe the disk stellar population than a longer wavelength. It is unclear whether the effect of dust attenuation or stellar population gradients are the dominant factor in determining the variation in Sérsic index with wavelength, with a combination of both likely to contribute globally.

We note that the disk-dominated and spheroid-dominated populations, once stabilised, tend towards $n_{\text{disk}} \rightarrow 1.4$ and $n_{\text{sph}} \rightarrow 3.6$ respectively. These values differ from the Sérsic indices typically used to describe late and early-type systems (excluding dwarf galaxies, for which there is a Magnitude-Sérsic index relation), namely $n_{\text{late}} = 1$ and $n_{\text{early}} = 4$ respectively (represented in Figure 4.12 by horizontal grey lines). This may indicate morphological contamination between populations as previously discussed, with some galaxies exhibiting bulge-to-disk ratios away from values of either zero or unity.

4.5.3 Half-Light Radius with Wavelength

Figure 4.13 displays the recovered half-light radii as a function of their rest-frame wavelength. $3\sigma$ clipped means are represented by solid red and blue circles for the disk-dominated and spheroid-dominated systems respectively, with linear fits to these data shown. The best fit linear relation describing the half-light radii in physical units (kpc) for the disk-dominated population is given by:

$$\log r_{e,\text{disk}} = -0.189 \log \lambda_{\text{rest}} + 1.176$$

and for spheroid-dominated systems:

$$\log r_{e,\text{sph}} = -0.304 \log \lambda_{\text{rest}} + 1.506$$
Figure 4.12: Recovered Sérsic index shown as a function of rest-frame wavelength in log-log space, coloured according to the population definitions described in Section 4.4. Blue data-points correspond to disk-dominated galaxies whereas red data-points correspond to spheroid-dominated galaxies. Large red and blue circles show the 3σ-clipped mean Sérsic indices for each respective population in each band, positioned at the median-redshift rest-frame wavelength for that population. Polynomial fits to these mean Sérsic indices are shown for both populations, the equations of which are inset into the figure. Owing to its lower quality imaging data, we exclude the $u$-band data in the calculation of these lines. Vertical lines show the 1σ spread in the data. For reference, grey horizontal lines at $n = 1$ and $n = 4$, equating to exponential and de Vaucouleurs profiles respectively, are added.
where $\lambda_{\text{rest}}$ is the observed rest-frame wavelength for the galaxy. Again, it is important to remind the reader to be mindful of our sample selection when considering these relations.

Using these relations, we observe significant variation in the recovered sizes of galaxies as a function of wavelength. The disk population mean half-light radii range from $r_{e,g} = 4.84$ kpc to $r_{e,K} = 3.62$ kpc from $g$ through to $K$, a decrease in size of 0.13 dex, equivalent to a drop of 25%. The spheroid population exhibits a larger spread from $r_{e,g} = 5.27$ kpc to $r_{e,K} = 3.29$ kpc over the same wavelength range, a decrease in size of 0.20 dex, equivalent to a drop of 38%. This variation in the spheroid population size is in good agreement with studies by La Barbera et al. (2010); Ko & Im (2005), reporting decreases of 29% and 39%$^3$ respectively over a similar wavelength range. Explanations for the variation in recovered size with wavelength include dust attenuation at shorter wavelengths or metallicity gradients within the galaxy. The effects of dust on a galaxy light profile have previously been discussed.

Obscuring the central region of a galaxy would shift the balance of total flux towards larger radii, artificially increasing the half-light radius. This effect is well understood for late-type systems, and indeed has been predicted in the literature notably by Evans (1994); Cunow (2001), and more recently by Möllenhoff et al. (2006); Graham & Worley (2008). Data from these studies are added into Figure 4.13 for reference (as indicated), normalised to the disk half-light radius predicted at $\lambda_{\text{obs}} = 900$ nm. Where possible, we adopt or infer a dust face-on optical depth of $\tau_B^f = 3.8$ (central face-on $B$-band opacity) as recommended in Driver et al. (2007). This value is close to that found in detailed modelling of nearby galaxies by Popescu et al. (2000); Misiriotis et al. (2001); Popescu et al. (2011); Hermelo et al. (in prep.). We also adopt an average inclination galaxy of $\cos i = 0.5$. Evans analyses face-on galaxies alone, whereas Cunow only produces detailed models for $\tau_B^f = 3.0$ systems, and so care should be taken when comparing these data. In all cases we see an excellent agreement between the observed size-wavelength variation and that predicted by these dust model simulations, notably so when compared with the work of Möllenhoff et al., as they employ dust models that can account for both dust attenuation and emission. As a caveat, we note that the population shown in Figures 4.12 and 4.13 does not constitute a volume-limited sample, and so conclusions must remain tentative until further studies can confirm this relation. However, our initial interpretation is that dust models more than adequately account for the apparent size-wavelength relation in late-type disk-dominated galaxies.

$^3$A linear extrapolation of the trends reported in Ko & Im (2005) were applied in order to convert V-K offsets into the g-K wavelength range used here.
It is interesting to note that whilst the spheroid population shows less variation in central concentration, i.e., Sérsic index, than the disk population, it exhibits a larger size variation with wavelength. Dust is not expected to be a dominant factor in the attenuation of light within these systems (although see the earlier discussion regards early-type dust fractions). However, higher optical depth values than that recommended in Driver et al. (2007) would have the effect of skewing the gradient of the dust attenuated size-wavelength relation to match the observed distribution. In addition to the possibility of age/metallicity gradients within spheroids, an alternative explanation for the apparent size variation with wavelength in early-type systems relies upon the interdependency between recovered Sérsic index and half-light radius. A change in the Sérsic index arising due to, e.g. small core dust components, additional unresolved or distorted structure in the core arising due to recent environmental interactions, the influence of an AGN or uncertainty in the PSF may lead to an equivalent corrective change in half-light radius. MacArthur et al. (2003) show that uncertainty on the PSF full-width-half-maximum by as much as $\Gamma = 1.5'' \pm 0.5$, a range encompassing most of the optical data as shown in Figure 4.3, would yield an equivalent measured size variation of 25% for the worst affected compact systems. One concern might be that small numerical uncertainty in Sérsic index would yield artificial changes in recovered size. Using the Sérsic index ranges for both disk and spheroid populations shown in Figure 4.12, and fixing the effective surface brightness and total magnitude in the total luminosity variant of Equation 2 in Graham & Driver (2007), we would expect to see equivalent changes in Sérsic half-light radii of $\Delta r_{e,disk} = 17\%$ and $\Delta r_{e,sph} = 8\%$ for the disk and spheroid populations respectively, i.e., far less than that seen here.

4.5.4 Surface Brightness: Co-variation of Sérsic Index and Half-Light Radius with Wavelength

We have shown how the Sérsic index and half-light radius for the spheroid and disk populations vary with wavelength, however, one must not consider these variations in isolation. All of the output model parameters have a combined effect on the final light profile of a galaxy. Several of these parameters including: Sérsic index; half-light radius; total magnitude; and the background sky display certain levels of inter-dependence (e.g., Caon et al., 1993; Graham et al., 1996). For example, an over-estimation of the sky level would lead to an under-estimation in the total magnitude of that galaxy, and consequently Sérsic index and half-light radius also. In the case of sky however, the signal-to-noise weighting employed by
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Figure 4.13: Recovered half-light radii in kpc as a function of rest-frame wavelength. Red and blue circles show the 3σ-clipped mean half-light radii for spheroid-dominated and disk-dominated galaxies respectively in each band, positioned at the median-redshift rest-frame wavelength for that population. Linear fits to these mean half-light radii are shown for both populations, the equations of which are inset into the figure. Owing to its lower quality imaging data, we exclude the u-band data in the calculation of these lines. Overlaid are data from several authors who predict an increase in the measured half-light radii in late-type systems due to the effects of dust. Further details are available in the text.
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GALFIT somewhat ensures against sky offsets of a few counts from the true value noticeably adversely affecting the fit. Since we would not expect the systematic error in the background sky to show significant trends with wavelength, and the variation in total magnitude with wavelength is well understood, we exclude them from our investigation into the co-variation of structural parameters with wavelength. We now consider the combined effect of varying the Sérsic index and half-light radius in unison, and how this impacts on the overall light profile of a test galaxy from $u \rightarrow K$.

Using Equations 4.3, 4.4, 4.7 and 4.8 we generate estimates of Sérsic indices and half-light radii at equal steps in log-wavelength space for both the spheroid and disk populations. Using the Sérsic relation, and assuming a constant total magnitude for both spheroids and disks of $m_{\text{tot}} = 15$, we create a series of surface brightness light profiles from $u$ through to $K$. Figure 4.14 shows the change in the recovered surface brightness light profiles over the $u \rightarrow K$ wavelength range, with the shaded areas representing the maximal area swept out by these light profiles as they vary in wavelength. This gives us an indication of how changes in recovered structural parameters affect the underlying surface brightness profile. The hatched region represents the worst-case limit at which these light profiles may be accepted as containing significant signal above the background sky level, as given in Section 4.2.4. The vertical dashed line represents a 1 pixel distance from the centre.

Despite the relatively large size variation observed in the spheroid population (a decrease of 38% in $g \rightarrow K$), when considered in conjunction with the Sérsic index variation (an increase of 30% in $g \rightarrow K$) the combined effect amounts to a relatively modest impact on the majority of the recovered light profile. It appears that as the spheroidal size decreases, the Sérsic index increases at a comparative rate. The most noticeable surface brightness variation is found in the central core region, fluctuating by 0.49 magnitudes at the 1 pixel boundary. Since a significant fraction of total flux lies in the core regions of high-index systems, it should not be surprising that a small variation in Sérsic index would produce a relatively large variation in half-light radius. Despite this effect, the majority of surface brightness profile out to large radii remains relatively stable with wavelength, vastly reducing the need for more complex mechanisms as previously discussed.

The variation in size for the disk-dominated population (a decrease of 25% in $g \rightarrow K$) coupled with a relatively large increase in Sérsic index (an increase of 52% in $g \rightarrow K$) yields a similar effect on the surface brightness profile variation as previously described for the spheroid.
population. Surface brightness fluctuates by $\sim 0.86$ magnitudes at the 1 pixel boundary, an increase of 75% on the variation in the spheroid population. Here it appears that the impact of dust attenuation has a particularly distinct effect on the light profile in disk-dominated galaxies, agreeing well with the theoretical predictions for size variation with dust presented in Section 4.5.3.

Whilst no single mechanism can be shown to be entirely responsible for the relations between Sérsic index, half-light radius and wavelength observed across the two populations, it is clear that the large apparent size fluctuations in the spheroid population appear to be initially misleading. Only when considering Sérsic index in conjunction with half-light radius does the true nature of these effects come to the fore. The spheroid population, despite exhibiting large changes in half-light radius with wavelength, maintains a relatively stable surface-brightness profile from $u$ through to $K$. The variation in the disk population with wavelength appears well described by current dust models, however, it is most likely a combination of dust attenuation, stellar population/metallicity gradients, unresolved secondary features in the core region affecting profile fits, and uncertainty on additional parameters such as the PSF that affect the underlying physics in these systems. Future studies aim to further inform this discussion for a limited sub-sample to be presented in Kelvin et al. (2012; in prep.).

\section{4.6 Conclusion}

We have produced high-fidelity automated two-dimensional single-Sérsic model fits to 167,600 galaxies selected from the GAMA input catalogue. These have been modelled independently across $ugrizYJHK$ using reprocessed SDSS and UKIDSS-LAS imaging data. These data have subsequently been delivered to the GAMA database in the form of the catalogue $SersicCatv07$. In order to facilitate the construction of this dataset, SIGMA, an extensive multi-processor enabled galaxy modelling pipeline, was developed. SIGMA is a wrapper and handler of several contemporary astronomy software packages, employing adaptive background subtraction routines and empirical PSF generation on a per-galaxy per-band basis to tailor input data into the galaxy modelling software GALFIT 3. Output results from GALFIT are analysed for predetermined modelling errors such as positional migration, extreme model shape and/or size parameters and adverse nearby neighbour flux. Nearby object masking is employed as a last resort, with secondary neighbours being preferentially modelled simultaneously with the primary galaxy in the first instance.

Using this dataset, we have defined a common coverage area across the three GAMA
Figure 4.14: Surface brightness variation from $u$ through to $K$ for the early-type spheroid-dominated and late-type disk-dominated populations. We generate Sérsic indices and half-light radii in wavelength steps from $u \rightarrow K$ for each population using the trends as described in equations 4.3, 4.4, 4.7 and 4.8. Using these values, surface brightness profiles (without PSF convolution) may be constructed for each wavelength bin. The shaded regions shown here represent the maximal area swept out by these light profiles along the transition from $u$ through to $K$, and represent how much of an effect the reported changes in Sérsic index and half-light radius have on the overall light profiles. The hatched region indicates the brightest limit at which light profiles may be trusted ($\mu_{\text{lim,}K} = 22.07 \text{ mag/arcsec}^{-2}$), and the vertical dashed line represents 1 pixel in distance from the centre. Profiles are produced assuming a constant total magnitude of $m_{\text{tot}} = 15$ for both the spheroid and disk populations.
regions that encompasses 138,269 galaxies, 82.5% of the full sample. This common area contains only those galaxies which have been observed in all nine bands, providing a useful basis upon which to further explore wavelength trends. We define a Sérsic magnitude system that truncates Sérsic magnitudes at 10 \( r_e \). This ensures that flux is not extrapolated below the typical limiting isophote into regions where data quality and quantity is not sufficient to constrain the form of the galaxy light profile. Truncated Sérsic magnitudes appear to be a good descriptor of global galaxy colours and total galaxy flux. For well-resolved disk-like galaxies \((n < 2)\), traditional aperture-based methods are in good agreement with truncated Sérsic magnitudes. For high centrally-concentrated systems however \((n > 4)\), it appears that traditional aperture-based, such as Petrosian magnitudes, may miss as much as \( \Delta m_r = 0.5 \) magnitudes from the total flux budget which is only recovered through Sérsic modelling.

When considering the dataset in \( n \)–colour space we find galaxies appear to exist in two distinct groups. For the most massive systems, we associate these two groups with the spheroid-dominated early-type galaxy (ETG) and disk-dominated late-type galaxy (LTG) populations. Owing to the nature of our input sample selection, these definitions do not extend down to the fainter dwarf population, and so subsequent trends will not represent those systems. We use the longest wavelength \( K \) band Sérsic index measurements in conjunction with rest-frame \( u - r \) colour to define these two populations. Using these definitions, we are able to further probe the variations in recovered structural parameters with wavelength for each population.

We find that the Sérsic indices of ETGs remain reasonably stable at all wavelengths, increasing by 0.11 dex \((+30\%)\) from \( g \) to \( K \) and becoming very stable beyond the \( z/Y \) interface. In contrast to this, we find that LTGs exhibit larger variations in Sérsic index with wavelength, increasing by 0.18 dex \((+52\%)\) across the same wavelength range. Recovered sizes for both the spheroid and disk systems show a significant variation with wavelength, showing a reduction in half-light radii of 0.20 dex \((-38\%)\) in ETGs and 0.13 dex \((-25\%)\) in LTGs from \( g \) to \( K \). Size variation of this scale for disk systems has been well predicted by dust models, highlighting the important role dust attenuation plays when considering structural variations across a broad wavelength range.

We note that spheroidal systems exhibit a larger size variation with wavelength than that found in disk systems. Possible physical explanations for this behaviour include low levels of unresolved dust or the effects of AGN feedback in the core of the galaxy, both of which would affect Sérsic profiling. Significant amounts of dust, such as an increased dust atten-
Chapter 4. Single-Sérsic Models of 167,600 Galaxies Across 9 Wavelengths

The variation optical depth parameter $\tau_{B}$, may allow current dust models to accurately describe the variation in half-light radii we find. It is unlikely however that a significant fraction of our spheroid-dominated population contain sufficient amounts of dust for this to be the case. Large stellar population/metallicity gradients present within individual structures of the galaxy would cause galaxies to look markedly different in different wavelengths, contributing to any concentration-wavelength/size-wavelength variation. In addition to these factors, uncertainties on the measured PSF and background sky must be considered.

However, when considering variations in half-light radius and Sérsic index together with wavelength we find that the large fluctuations in spheroidal parameters amount to a relatively modest impact on the recovered light profile. A comparatively larger effect is noted for the disk systems, particularly in the core region, supporting the presence and effect of dust attenuation in addition to stellar population/metallicity gradients. At a distance of 1 pixel from the central region, spheroid systems display a variation in surface brightness of 0.49 magnitudes from $u$ through to $K$. In disk systems, the comparative figure is 0.86 magnitudes, an increase of 75%. This highlights the importance of not considering recovered parameters in isolation, as the interplay between them has the possibility of masking underlying trends.

The effects of dust attenuation appear to be the dominant factor constraining the variations in structural parameters with wavelength, notably so for the disk-dominated population. In contrast with this, apparent large structural variations in the spheroid-dominated population appear to have a relatively minor effect on the underlying surface-brightness profile than might have been expected. Future studies in Kelvin et al. (2012; in prep.), focussing on a limited sub-sample of this dataset, will provide a deeper understanding of these structural variations with wavelength, enabling us to comment further on the key mechanisms involved in varying structural parameters with wavelength for a host of different morphologies.
In his seminal 1926 paper ‘Extra-galactic Nebulae’, Edwin Hubble established a framework for the morphological classification of galaxies which remains in use essentially unchanged to the present day. From a sample of 400 galaxies, Hubble defined three main sub-groups; Elliptical, Spiral and Lenticular (Hubble, 1926). Elliptical Early-type galaxies show no additional structure beyond a smooth radial light profile. Conversely, late-type spiral galaxies consist of a central spheroidal bulge girt by a flattened extended disk containing spiral arm features, and occasionally with the presence of a bar. Lenticular galaxies fall somewhere in-between, with the traditional late-type bulge and disk features present, potentially with the addition of a bar, and yet the noticeable absence of spiral arm structure.

The rationale for defining these morphological groupings remains a visual one, which has obvious implications for large-scale survey analysis. In this Chapter, we examine potential complimentary methods for automated morphological classification based on global measurements, and the impact these measurements have on the calculation of the total stellar mass within distinct morphology groupings. We begin by defining a volume-limited sample of 4,129 galaxies in Section 5.1. This sample is morphologically classified by eye by three independent
observers in Section 5.2, whereafter the sample is subsequently reduced to 3,845 galaxies. These results are subsequently contrasted against complimentary methods of classification in the absence of visual inspection. In Section 6.1.4 we explore the apparent redshift-bias inherent in the data when using human-eye visual classification, and discuss the best way to resolve this issue. Finally, these results are put into context by calculating the stellar mass within each morphological type and morphological class in Section 6.1. A standard cosmology of \((H_0, \Omega_m, \Omega_\Lambda)=(70 \text{ km s}^{-1} \text{ Mpc}^{-1}, 0.3, 0.7)\) is assumed throughout this chapter.

5.1 Low Redshift Sample

We require the construction of a representative sample of galaxies from the GAMA catalogue. One could use the entire GAMA catalogue, correcting for the effect of Malmquist bias by, for example, applying volume corrections to the data. This would however leave us with a dataset too large to be eyeballed by a small core of experienced observers. The alternative is to construct a volume-limited sample, limited in both redshift and absolute magnitude.

It is important to know how deep in redshift one is able to push and still be able to resolve all the key sub-components that constitute a galaxy (i.e., nucleus, bulge and disk) whilst keeping galaxy number counts high enough so that any final sample remains statistically meaningful. Owing to the small apparent sizes of galaxy nuclei, it would not be possible to create a large enough dataset with potentially resolvable AGN. For this reason, nuclei are excluded from any further galaxy sample considerations.

5.1.1 How Much Structure Can Be Resolved?

Allen et al. (2006), and more recently Simard et al. (2011) have performed bulge-disk decomposition on large galaxy datasets, allowing typical bulge/disk sizes to be estimated. Allen et al. (2006) perform free-Sérsic bulge plus exponential (Sérsic index \(n = 1\)) disk profile fits of 10,095 galaxies in the \(B\) band \((\lambda_{\text{cen}} = 445 \text{ nm})\) using the GIM2D software (Simard et al., 2002). The output of this study includes relevant Sérsic information (magnitude, half-light radius, Sérsic index, position angle, ellipticity) in addition to goodness of fit parameters (reduced \(\chi^2\)) and redshift information from the Millennium Galaxy Catalogue (MGC; Liske et al., 2003). Selecting those galaxies with a bulge-to-total ratio of \(0 < B/T < 1\) (i.e., excluding pure disks and pure elliptical galaxies) with a ‘good’ associated GIM2D fit \((0.5 < \chi^2/\nu < 2)\) and a high-quality redshift \((3 \leq NQ \leq 5)\) leaves a sample of 4,225 galaxies. Of these galaxies, the typical bulge 3-sigma-clipped mean half-light radius is \(1.87 \pm 1.41 \text{ kpc}\) and the mean disk...
5.1. Low Redshift Sample

half-light radius is $7.64 \pm 3.74$ kpc. Note that the errors shown here are the $1\sigma$ distributions of the 3-sigma-clipped data. Disk half-light radii have been calculated by converting disk scalelengths using the relation:

$$r_e = b^n h$$  \hspace{1cm} (5.1)

where $h$ is the disk scalelength, $n$ is the Sérsic index and $b$ is a function of $n$, as described in Section 1.3. For $n = 1$, $b^n = 1.678$.

Simard et al. (2011) perform free-Sérsic bulge plus exponential disk profile fits for a much larger dataset of 1.12 million galaxies in the $r$ band ($\lambda_{\text{cen}} = 622$ nm) using imaging data from the Sloan Digital Sky Survey (SDSS; Abazajian et al., 2009). In a similar fashion to above, the robust mean bulge half-light radius is $3.09 \pm 2.03$ kpc, and the mean disk half-light radius is $7.14 \pm 4.26$ kpc. Whilst the Simard disk sizes are in good agreement with Allen’s, the bulge estimates only agree at the outer edge of their $1\sigma$ errors. Most likely this is owing to the much higher relative resolution of the MGC dataset used by Allen et al. (2006), allowing for much smaller structure to be resolved. In this sense, using the Allen data provides a good lower-boundary on the expected size measurements from our SDSS-based data, whilst Simard’s should act as an expected bulge size measurement.

Previous work (Chapter 4) has shown the impact of dust and stellar population gradients on recovered measured structural parameters (such as half-light radii) with respect to wavelength. For these reasons, the longest wavelength UKIDSS $K$ band has been chosen to perform further structural analyses upon. Mean half-light radii from Allen et al. (2006) and Simard et al. (2011) are converted from their rest-frame observed wavelength values to their predicted $K$ band ($\lambda_{\text{cen}} = 2200$ nm) half-light radii using Equations 4.7 and 4.8 in Section 4.5.3 (Kelvin et al., 2012). These conversions reduce bulge sizes by 38% and 32% (to $1.15 \pm 0.87$ kpc and $2.10 \pm 1.38$ kpc) and disk sizes by 26% and 21% (to $5.65 \pm 2.76$ kpc and $5.62 \pm 3.36$ kpc) for the two studies, respectively.

Figure 5.1 shows the apparent angular size on the sky for a typical bulge and disk observed in the $K$ band, using the values previously calculated, with shaded regions indicating $0.5\sigma$ spread in the data. Regions of overlap (dark shaded regions) indicate those areas where the two aforementioned studies agree to within their $0.5\sigma$ bounds. Typical UKIDSS $K$ band PSF FWHM upper and lower bounds ($0.7''$ to $1.1''$) are represented with dotted horizontal lines. It becomes obvious that resolving the disk is not a problem: the limiting factor is being able to resolve the bulge. Assuming the lower bulge estimate (1.15 kpc) and poor seeing data, this
would limit any volume-limited sample to \( z \sim 0.05 \). Conversely, assuming the upper bulge estimate (2.10 kpc) with favourable seeing data, the upper redshift limit increases to \( z \sim 0.18 \). An upper redshift limit in the range \( 0.05 < z < 0.18 \) therefore, ideally as low as possible to maximise the quality of the input data, is preferred. For this study, we have chosen an upper redshift limit of \( z = 0.06 \).

### 5.1.2 Sample Definition

Using the latest version (version 16) of the GAMA-I tiling catalogue\(^1\) (*TilingCatv16*, see Baldry et al., 2010) we define a volume limited sample of 5,384 galaxy-like (SURVEY\_CLASS > 1) objects whose redshifts \( z \) lie in the range \( 0.013 < z < 0.06 \) with an associated normalised redshift quality \( nQ > 2 \) and an extinction corrected \( r \) band SDSS Petrosian magnitude of \( r < 19.8 \). These redshift limits give our sample a volume of 224555.3 Mpc\(^3\). Note that redshifts have been matched from version 8 of the GAMA spectroscopic catalogue (*SpecAllv08*). The lower redshift limit is chosen so as to remove local objects/stars with large peculiar velocities, whereas the upper redshift limit is within the region where one would expect to still resolve a bulge plus disk system, as shown in Section 5.1.1.

In order to avoid incompleteness contamination from the dwarf population at the low-luminosity end of the sample, we make an additional absolute \( r \) band magnitude cut of \( M_r < -17.4 \). Absolute magnitudes here are based upon SDSS Petrosian \( r \)-band photometry. This reduces our sample to 4,129 galaxies, as shown in Figure 5.2. Note however that a further 19 galaxies are additionally removed after the eyeballing phase (Section 5.2.1) and 265 galaxies are removed at the redshift analysis phase (Section 5.3.1). This sample constitutes our dataset, and shall be used throughout the remainder of this chapter. We note that our sample is more dense than other SDSS-based morphological studies when considering the number of galaxies per unit volume with a given redshift whilst taking into account the respective magnitude ranges of other surveys. For example, our sample is \( \sim 2 \) times more dense than Bamford et al. (2009) and \( \sim 13 \) times more dense than Nair & Abraham (2010), however, we are \( \sim 3 \) times less dense than the INT-based study found in Driver et al. (2006), owing to the increased depth of the MGC survey data relative to the SDSS.

A selection of postage stamp images for 500 randomly selected galaxies from within this volume-limited sample are shown in Figure 5.3.

\(^1\)All GAMA catalogues are available through the GAMA database, available online at http://www.gama-survey.org/database.
5.1. Low Redshift Sample

Figure 5.1: Apparent angular size in arcseconds of both bulges and disks at increasing redshift, with typical bulge/disk half-light radii taken from Allen et al. (2006) and Simard et al. (2011), as indicated. Our preferred wavelength for performing structural decomposition is the $K$ band ($\lambda_{\text{cen}} = 2200$ nm), however, the aforementioned studies provide size estimates in the $B$ ($\lambda_{\text{cen}} = 445$ nm) and $r$ ($\lambda_{\text{cen}} = 622$ nm) bands respectively. To account for any size-wavelength gradient, bulge and disk sizes have been converted into their expected measured $K$ band sizes as per Equations 14 and 15 in Kelvin et al. (2012). This reduces bulge sizes by 38% and 32% and disk sizes by 26% and 21% for the two studies, respectively. Shaded regions denote the 0.5σ variation in the data, with darker overlapping shading indicating those areas where the two studies agree within their own 0.5σ limits. Horizontal dotted lines indicate the typical UKIDSS $K$ band upper and lower seeing limits, and the vertical dashed line indicates the final chosen upper redshift limit of $z = 0.06$. GAMA-I galaxy number counts at increasing redshift are provided along the top axis, having removed known stars, poor redshift data and galaxies fainter than the nominal GAMA limit of $r = 19.8$. 
Figure 5.2: Absolute $r$-band magnitude against redshift for the entire GAMA-I dataset. The blue rectangle outlines the principle redshift and absolute magnitude limits ($0.013 < z < 0.06$ and $M_r < -17.4$ respectively) chosen in defining a complete volume-limited sample of 4,129 galaxies for further morphological analysis. Note that this sample definition already includes additional cuts for normalised redshift quality ($nQ > 2$), a galaxy-like metric (SURVEY_CLASS > 1) and apparent SDSS $r$-band Petrosian magnitude ($r < 19.8$). The smooth curves visible in the data represent the variable $r$-band limits present in the GAMA-I data set, with G09 and G15 complete down to a shallower limit of $r = 19.4$ compared to the $r = 19.8$ limit of G12. This effect should not seriously compromise our dataset over the redshift range of interest.
5.1. Low Redshift Sample

Figure 5.3: Postage stamp examples of 500 galaxies from the volume-limited sample. Galaxies are arranged according to redshift, and within each redshift bin, according to $K$ band Sérsic absolute magnitude (see Section 5.1.3). A representative lookback time in Gyr is provided along the right-hand column. Images are RGB false-colour images using UKIDSS $H$ band and SDSS $i$ and $g$ bands, respectively.
5.1.3 Photometry

5.1.3.1 Absolute (Sérsic) Magnitudes

Although the aforementioned volume-limited sample is defined using SDSS Petrosian photometry, our preferred measure of total magnitude are those derived from single-Sérsic fits to the data. Single-Sérsic model fits have the potential to account for the missing flux in the wings of high central-concentration galaxies, and so side-step the well documented problems with both Petrosian and Kron photometry (see, e.g., Graham & Driver, 2005). Kron photometry is currently favoured as the most robust aperture-based photometric method, more accurately accounting for the fractions of flux missed by the Petrosian apertures. Here, we compare the current standard GAMA photometry, namely that derived from a Kron-like matched-aperture in all nine bands (Auto), against Sérsic photometry derived from a single-Sérsic fit to the galaxy, with the flux truncated at 10 multiples of the effective radius.

For a given band \(x\), absolute magnitudes \(M_x\) are derived using the standard relation

\[
M_x = m_x - (5 \log_{10} D_L + 25) - k_x - e_x - A_x
\]  

(5.2)

where \(m_x\) denotes the apparent magnitude (in this case, Sérsic or Auto), \(D_L\) is the luminosity distance of the galaxy in megaparsecs, \(k_x\) is the applied \(k\)-correction for band \(x\) (see below), \(e_x\) is the evolutionary correction and \(A_x\) is the Milky Way dust attenuation correction. One would expect minimal evolutionary effects over the narrow redshift range of this sample, and so we do not apply any \(e\)-corrections to these data. We apply the Milky Way dust attenuation correction as given in Table 22 of Stoughton et al. (2002), with UKIDSS values determined by matching UKIDSS database values from the WFCAM Science Archive to the SDSS extinction in the \(r\) band. Further details on this procedure may be found in Liske et al. (2012, in prep.).

5.1.3.2 K-Corrections

The \(k\)-correction (Oke & Sandage, 1968; Hogg, 1999) accounts for the offset between the wavelength range covered by a given passband at redshift \(z\) and the same passband at rest (\(z = 0\)), in addition to accounting for passband stretching. It is essential to include this correction when comparing, e.g., colour information from galaxies at differing redshifts. We derive \(k\)-corrections for each galaxy within our sample using the \textsc{kcorrect} software (v4.2, Blanton & Roweis, 2007), electing to use single-Sérsic truncated photometry in their derivation and correcting to a redshift \(z = 0\) baseline. Figure 5.4 shows the derived \(k\)-corrections across
5.1. Low Redshift Sample

<table>
<thead>
<tr>
<th>Band</th>
<th>Limit</th>
<th>(u)</th>
<th>(g)</th>
<th>(r)</th>
<th>(i)</th>
<th>(z)</th>
<th>(Y)</th>
<th>(J)</th>
<th>(H)</th>
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</tbody>
</table>

Table 5.1: A summary of the best-fit \(k\)-corrections as shown in Figure 5.4. One may use these values as a crude estimate for \(k\)-corrections in band \(x\) in the absence of sufficient photometric information using the relation \(k_x(z) = m_x z\).

all nine passbands for the entire volume-limited sample. One can see that the \(k\)-corrections act to add more flux into the shorter wavelengths (\(ugr\)), have a minimal impact on the flux for passbands spanning intermediate wavelengths (\(izY\)) and act to subtract flux from longer wavelengths (\(JHK\)). The data in Figure 5.4 have been fit with a simple linear model, requiring the line to pass through the \([0, 0]\) origin, and with the gradient \(m\) shown inset into each subpanel within the figure. One can use these values as a crude estimate for \(k\)-corrections in band \(x\) in the absence of sufficient photometric information using the relation

\[
k_x(z) = m_x z
\]

A summary of these \(m_x\) values is given in Table 5.1.

A comparison of Auto and Sérsic absolute magnitudes fully corrected for the effects of dust and bandpass shifting may be found in Figure 5.5. We note a generally excellent level of agreement between the two methods, excepting a slight upturn at the bright end whereby Auto magnitudes begin to recover less flux than their Sérsic equivalents. This is as expected, owing to the arguments laid out in Kelvin et al. (2012). A combination of sky-estimation errors which impact the flux-thresholding methods of Kron apertures and the masking of nearby neighbours (which becomes more of a problem for bright and, hence, larger galaxies) all act to reduce the flux provided by a Kron-like aperture.

5.1.3.3 Absolute Magnitude Limits

The \(r\) band absolute magnitude limit for our volume limited sample (\(M_r = −17.4\)) introduces a colour-dependent limit across the remaining eight passbands in use from the SDSS and UKIDSS. This variable limit has the potential to introduce incompleteness bias when analysing data at other wavelengths, and so we define additional limits down to which the sample remains complete and unbiased as a function of colour for each passband.

The colour-magnitude diagrams in Figure 5.6 show the relation between colour and absolute magnitude, where the long dashed line represents the \(r\) band limit of \(M_r = −17.4\). Data points beyond this line exist owing to the discrepancy between Sérsic magnitudes (shown here) and Petrosian magnitudes (used to construct the volume-limited sample). One can
Figure 5.4: Sérsic $k$-corrections across all nine passbands. $k$-corrections are produced using the KCORRECT software (v4.2, Blanton & Roweis, 2007), electing to use single-Sérsic truncated photometry in their derivation and correcting to a redshift $z = 0$ baseline. The overlaid black lines are linear fits to the data, required to intercept the $z = 0$ axis at $k = 0$. The gradients of these lines are given inset into each sub-panel, and may be used as a crude low-redshift estimate for a $k$-correction using the relation $k_x(z) = m_x z$. 
5.1. Low Redshift Sample

Figure 5.5: A colour comparison of Sérsic photometry derived from single-Sérsic fits to the data against Kron-like (Auto) matched-aperture photometry across all nine bands (ugrizYJHK). Magnitudes have been fully corrected for the effects of Milky Way dust attenuation and bandpass shifting (k-corrections). Density histograms are constructed using a rectangular kernel of width 0.1, with modal values for the distributions inset into the figures, and represented with a dashed line. Note the generally good agreement between the two different photometric methods, except at the bright end where Auto photometry tends towards recovering less flux, as expected (see Kelvin et al., 2012). Single-Sérsic photometry is expected to provide a better measure of flux for bright, highly-centrally concentrated galaxies.
Figure 5.6: Colour-Magnitude diagrams for all nine bands. These values are derived from absolute Sérsic magnitudes truncated at 10 $r_e$ with k-corrections applied. Long-dashed lines represent the volume-limited sample limit of $M_{r,\text{petro}} = -17.4$. Data points beyond this line exist owing to the discrepancy between Sérsic magnitudes and Petrosian magnitudes. Short-dashed lines represent the absolute magnitude at which the main body of data intersects the long-dashed line, and shows down to what magnitude limit this sample is complete down to for that wavelength. These limits are listed in Table 5.2.

We clearly see the two distinct populations (i.e., a bimodal distribution) in the g band data; the blue cloud and red sequence. These two populations are also evident to a lesser extent at all wavelengths.

We define the additional faint-end limits visually as the absolute magnitude in band $X$ (where $X = u g i z Y J H K$) at which the main body of the data intersects the $M_r = -17.4$ line. These passband limits are listed in Table 5.2, and shown as vertical short dashed lines in Figure 5.6.
5.1. Low Redshift Sample

<table>
<thead>
<tr>
<th>Band</th>
<th>u</th>
<th>g</th>
<th>r</th>
<th>i</th>
<th>z</th>
<th>Y</th>
<th>J</th>
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<th>K</th>
</tr>
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<td>−18.3</td>
<td>−18.1</td>
</tr>
</tbody>
</table>

Table 5.2: Absolute Sérsic magnitude limits. These limits denote the faint-end absolute magnitude at which the sample is complete for that band. Limits are defined as the absolute magnitude at which the main body of data in the colour-magnitude diagrams of Figure 5.6 intersect the volume-limited sample faint-end limit of $M_r = −17.4$.

5.1.3.4 Are Sérsic Colours Reliable?

We have previously adopted the $(u−r)$ colour as defined by elliptical matched-aperture Kron photometry (Kron, 1980; Bertin & Arnouts, 1996) as our standard method and measure of colour in order to compare to those colours used in numerous other studies. An alternative option however is to use Sérsic colours as defined by a single-Sérsic fit to the galaxy in both the $u$ and $r$ bands, say. In theory, a Sérsic fit should be a better estimator of ‘total-flux’ present in a galaxy in that band, particularly so for highly-centrally concentrated galaxies (Figure 1.12), and so the colour provided via this method should be a better indication of the actual colour of the galaxy. Moreover, matched aperture photometry makes several key assumptions:

1. The primary passband (i.e., the passband chosen to define the aperture) is of a high enough quality to accurately determine an aperture that encompasses a representative sample of flux from the primary galaxy;

2. The effective size of the galaxy does not sufficiently vary with wavelength;

3. Overlapping flux from secondary neighbours (other galaxies or stars) can either be safely ignored or excluded from the aperture definition.

These assumptions, whilst broadly correct, have the potential to introduce serious errors into the matched-aperture measurements should any one of these criteria not be fully satisfied. Conversely, these requirements are not explicitly imposed on Sérsic colours via single-Sérsic model fitting. Taking each previous point in turn:

1. We have shown in Figure 3.5 that GALFIT size-estimates are more robust than Source Extractor size estimates. In low quality data, a single-Sérsic fit is able to provide a better estimate of total flux than the isophotal thresholding method employed by Source Extractor if the signal flux is still noticeable above background noise fluctuations (however, see below for very low quality data). Additionally, since SIGMA models each band independently of one-another, there is no potential for a poor choice in primary passband affecting the model fit in one of the other bands.
2. It is apparent that size does not remain constant with wavelength, as shown in Figure 4.13. We find half-light radii from $u \rightarrow r$ increasing by as much as $\sim 20\%$ for spheroid-dominated galaxies ($\sim 10\%$ for disk-dominated galaxies). In this scenario, using the $r$ band data has the potential to miss flux present in the wings of the galaxy in the $u$ band should the aperture be too small.

3. The flux-redistribution method employed by GALFIT allows flux (and therefore colour) to be safely accounted for in crowded fields via simultaneous model fitting, removing the need for any form of bad-pixel mask.

However, despite these inherent flaws in the matched-aperture photometric methods, these flaws are well understood within the astronomy community and more often than not can be accounted for should one be familiar with the dataset. In addition, Sérsic modelling is also prone to error, especially so for very low quality data. Should the signal-to-noise ratio for the galaxy be extremely low, GALFIT will have extreme trouble fitting a single-Sérsic component to the image. In these scenarios, matched aperture photometry should always be used. However, on the whole, and for typical data, we would expect Sérsic photometry to be more robust than matched-aperture photometry.

In Figure 5.7 we show global galaxy colour defined using Sérsic magnitudes (truncated at $10r_e$) against colour defined using elliptical Kron-like matched apertures (matched to the $r$ band aperture; standard GAMA photometry) for our volume limited dataset. Both $(u - r)$ and $(g - i)$ colours are shown. For both distributions, we note an excellent level of agreement between the two colour measures within the region where $> 10$ data points exist per bin (i.e., outside of the grey-shaded regions). The scatter is larger in $(u - r)$ as is expected, owing to the poorer quality of the $u$ band data. For this reason, GALFIT will struggle to provide a good fit in the $u$ band, and provide an estimate of the magnitude that may be in error. The bimodal peaks (top-left histogram) are also more washed out when using Sérsic colours than GAMA colours. This is most likely due to the ‘S’ shaped curvature to the relations, with Sérsic measurements finding redder colours for red (E-type) galaxies and bluer colours for blue (Sd-type) galaxies. This has the effect of adding a larger tail to the 1D Sérsic distribution at both the red and blue ends.

The situation for $(g - i)$ is much improved over $(u - r)$ due to the high quality of both $g$ and $i$ band data. The scatter in $(g - i)$ is much tighter about the 1:1 line, and both the Sérsic and GAMA colours produce a bimodal-distribution with short-sharp tails. The blue Sd-type
peak for Sérsic colours exhibits a double prominence, which may come about due to small error offsets, however, we note that this offset is only of the order $\Delta(g - i) \sim 0.1$. Conversely, the red E-type peak appears narrower than its equivalent GAMA peak. On the whole however, the differences between the two systems in $(g - i)$ is small, and further studies are required in order to ascertain which of these methods provides the most reliable means by which the global colour of a galaxy may be estimated.

We note that we expect Sérsic colour estimates to improve once structural decomposition replaces single-Sérsic magnitudes, however, we conclude that a single-Sérsic fit to the data in either the $u$ and $r$ bands or (preferably) the $g$ and $i$ bands provide good analogues for traditional matched-aperture based colour estimation methods, improving on them for highly-centrally concentrated (high Sérsic index) systems.

5.2 Morphological Classification

When presented with a sample of galaxies, there are multiple ways in which the population may be divided into distinct groupings. In increasing order of complexity, typical groupings are:

- **Type** – a broad-brush division between red, old and spheroid-dominated *early-type* galaxies and blue, young and disk-dominated *late-type* galaxies.

- **Class** – a visual identification assigning a morphological class to a galaxy based upon the ellipticity of its outer isophote and the presence or lack of a disk. If a disk is present, additional classifiers exist dependent upon the strength of any spiral arm structure and the indication of a bar.

- **Structure** – a comprehensive measurement of the number, magnitude and astrometric properties of the distinct sub-structures that comprise a galaxy (e.g., spheroid, bulge, thick disk, thin disk, bar, secondary bar, nucleus, pseudo-bulge).

If one is able to ascertain the structure of a galaxy, then its morphological class is known. Similarly, if morphological class is known, the morphological type of the galaxy is also obtained. The inverse is not necessarily true, however.

In this section I discuss various means by which morphological type and class may be obtained, saving a full discussion of structural analysis for Chapter 6.
Figure 5.7: This figure shows galaxy colour defined using Sérsic magnitudes (truncated at 10$r_e$) against galaxy colour defined using elliptical Kron-like matched apertures (matched to the $r$ band aperture; standard GAMA photometry) for our volume limited dataset. The left panel shows $(u - r)$ colour, whereas the right panel shows $(g - i)$, with data points coloured according to their visual morphological classification. Circles represent the median value in a diagonal bin perpendicular to the 1:1 grey line, with the error bars representing the 1σ spread in the data about the median, and the flat error bar heads representing the error on the median ($\sigma/\sqrt{n_{bin}}$). Grey shaded areas show those regions where fewer than 10 objects are present within a bin. The probability density function plots along the outer edge of the figure represent the 1D projection of the data onto each axis, using rectangular bandwidths of 0.1 and 0.05 for the $(u - r)$ and $(g - i)$ panels respectively. For both $(u - r)$ and $(g - i)$, the agreement between matched-aperture GAMA colours and Sérsic colours appears to be extremely high in those regions where > 10 data points exist (non-grey shaded regions), implying that Sérsic colours are indeed a good proxy for the more traditional matched-aperture colour method. As expected, the scatter in $(u - r)$ is larger, owing to the poorer quality of the $u$ band data, with the scatter in $(g - i)$ much tighter. The $(u - r)$ data also appears to exhibit an ‘S’ shaped feature, with red E-type galaxies appearing redder when using Sérsic colours than using GAMA colours, and blue Sd-type galaxies appearing bluer when using Sérsic colours. This feature does not exist for $(g - i)$, suggesting that the poorer quality $u$ band may be responsible for this shape.
5.2. Morphological Classification

5.2.1 Visual Classification (Eyeballing)

The most fundamental method for classification is visual inspection. Given high-quality imaging data and additional information such as colour or velocity dispersion, one is able to trivially determine the structure, class and type of a galaxy. I now discuss our visual classification campaign, from which all subsequent classification methods are judged against.

We created three colour postage stamp images for each galaxy in the volume-limited sample of 4,129 to facilitate eyeball morphological classification. Red, green and blue colours are taken from the UKIDSS $H$ and SDSS $i$ and $g$ bands, respectively. Eyeball classification took place in two phases. Phase 1 postage stamps depict $20'' \times 20''$ with the dynamic range of the images scaled logarithmically and prior decisions made on the lower (black) and upper (white) cut levels. Phase 2 postage stamps depict a larger area of $40'' \times 40''$ and are scaled using the arctan function. We found that the arctan function removes the necessity for a harsh upper or lower cut level. Imposing harsh cuts has the potential to lead to misclassification as it implies a physical boundary in the light profile of a galaxy where none exists. The increased area of the phase 2 postage stamps also allows for the galaxy to be put into context of its local environment, and allows the observer to see more than the core of nearby extended galaxies.

Classification occurs by assigning the postage stamp of a galaxy into a specific directory hierarchy. This hierarchy is shown in Figure 5.8. Note that the ‘Little Blue Spheroid’ (LBS) class was added after the eyeballing phase, and is shown in this figure for reference. The visual classification decisions made eventually filter a galaxy down into its appropriate morphological class, from E to Sd, as indicated. All of the postage stamp images are populated at the top level, and must filter down to the bottom of their appropriate classification tree. The decision tree is essentially binary at each level (apart from the inclusion of the LBS class, which is discussed below). These levels are Early/Late, Single/Multi and Barred/Unbarred.

Early/Late

Galaxies are initially split into early or late-type systems, or more accurately, bulge or disk-dominated, respectively. Colour may be a useful indicator here, however, the apparent gradient and smoothness of the light profile and the central concentration are the main discerning factors.
Figure 5.8: The morphological classification hierarchy used to filter the volume-limited sample of 4,129 galaxies into their appropriate class. Also shown beneath each label are the number of galaxies in the master classification bin for that group, as shown in Table 5.3. The final morphological type at the bottom of this figure depends upon the prior decisions made by the classifier.

Single/Multi

A question of the total number of structural components comprising the galaxy. Early-type single-component galaxies are classical Elliptical systems (or Little Blue Spheroids), whereas early-type multi-component galaxies are lenticular or early-type spiral systems (S0a). Late-type single-component galaxies are bulge-less disk systems (Sd), whereas late-type multi-component galaxies are late-type spiral systems (Sbc).

Barred/Unbarred

The final level of classification determines whether the multi-component system contains a bar structure. If the disk is edge-on, and the presence of a bar cannot be verified, then the galaxy is classified as unbarred.

Little Blue Spheroids (LBS)

After Phase 2 eyeball classification, it became apparent that the inclusion of an additional class comprising ‘Little Blue Spheroids’ was required. These systems are borne out of the early-type single-component systems, and are characterised with a small angular size and typically very blue in colour. More information can be found in Section 5.2.1.1.
Three observers, namely, LSK, SPD and ASGR, independently classified the entire sample of 4,129 galaxies using both the phase 1 and phase 2 postage stamp images. Phase 2 postage stamp images were initially placed into their Phase 1 hierarchy positions as assigned by their classifier in order to speed up and improve the second round of classification.

During the eyeballing phase, 19 postage stamp cutouts were found to be inappropriate due to compromised photometry or not being the main part of the host galaxy. After consensus between all three observers, these galaxies were removed from the sample, further reducing the master catalogue size to 4,110 galaxies. The results of these classifications are summarised in Section 5.2.1.2 below.

5.2.1.1 Little Blue Spheroids

After the eyeballing phase was complete, it became apparent that an additional class describing compact blue galaxies was required. This decision was made after noting the distribution of Elliptical galaxies in $u - r$ colour space, as shown in Figure 5.9. Whilst all other morphologies appear uni-modal in appearance in this space, Elliptical galaxies appear bi-modal, with a dividing line at $(u - r) \sim 1.8$. Eyeballing a sample of galaxies within this secondary blue peak produced many galaxies of a similar type, namely, very small very blue objects. We named galaxies of this new class ‘Little Blue Spheroids’ (LBS). The entire Elliptical sample was re-eyeballed with galaxies divided between Elliptical and LBS classes, as shown in Figure 5.8. Only one observer (LSK) performed the additional round of visual classification for brevity, however, the resultant objects were shared amongst the other eyeballers and agreed upon afterwards as a consistency check. Postage stamp examples of LBS galaxies can be found in Figure A.1.

As shown in Figure 5.9, the new population of LBS galaxies significantly reduces the strength of the secondary peak in the Elliptical galaxy population. Instead, it is replaced with an extended shoulder, hinting at further galaxies that could have potentially been added into this class, however, the quality of the imaging data and the nature of human visual classification precludes removing entirely any possible misclassifications. LBS galaxies number 143 (out of 715 early-type single-component systems), 20% of the previous Elliptical class, with a typical Sérsic index of $n_K \sim 2$ and a physical size of $r_e \sim 0.8$ kpc. The greyscale shading in Figure 5.9 shows the density map for Sd type galaxies. One notes that the new LBS class populates the same parameter space as these pure disk systems. It is apparent that LBS galaxies are misclassified late-type single-component systems and, consequently, will subsequently be
Figure 5.9: The creation and re-designation of the ‘Little Blue Spheroid’ (LBS) morphological class. (Top) $(u - r)$ colour vs. Absolute $K$ band magnitude for the old (left) and new (right) Elliptical class, in addition to the new LBS class. Contours represent 10th to 90th percentiles in 10% steps for all population. (bottom) PDF distributions for these populations. Note that the Sd classes have been scaled down by a factor of 13.83 so as to enable comparison with the LBS class. A rectangular bandwidth of 0.3 is used. Early-type single-component systems, initially all classified as Elliptical, clearly show a non-linear bimodality in the colour-magnitude plane. Visually re-classified LBS galaxies appear to occupy a similar parameter space to late-type single-component systems, implying that they have been misclassified as early-type by virtue of their shape, as per our classification criteria. LBS galaxies will subsequently be added into the Sd class.
5.2. Morphological Classification

Table 5.3: Eyeball results from three independent observers, in addition to the final master classification number counts. The unambiguous column represents the percentage of master galaxies which all three observers agreed upon for that classification. Top to bottom, the table is split into three sections, delineated by a small space. The top section represents absolute classifications, i.e., how often observers agree upon the base classifications regardless of other considerations. The middle section represents the combination of Early/Late and Single/Multi. The bottom section represents the combination of Early/Late and Barred/Unbarred (note that all of these systems are multi-component by definition).

LBS-type galaxies may come about via the intermittent ‘breathing’ star formation predicted in low-mass dwarf galaxies by Stinson et al. (2007), and have been previously isolated observationally by Arp (1965); Sandage & Binggeli (1984) and Brough et al. (2011). The latter study finds that these systems are predominantly low-mass and found in low-density environments, showing similar properties to dwarf irregular galaxies in the Local Volume. We find the Sérsic indices of these systems spanning a wide range of values, with a median of $n_K = 1.62$ and a mean of $n_K = 2.39$. LBS-type galaxies represent an interesting insight into the nature of visual classification, and potential pitfalls therein, however, we shall not pursue this line of investigation further in this body of work.

5.2.1.2 Master Eyeball Classifications

A summary of eyeballing number counts between observers in addition to the final ‘master’ number counts are shown in Table 5.3. Unambiguous denotes the number of galaxies on which all three observers agree on the classification as a percentage of the number of galaxies in the master classification.

The calculation of master classifications is as follows. We require at least two observers to
agree on the classification of a galaxy for that classification to be propagated through to the final master catalogue. Since all decision tree forks are binary in nature, then by definition at least two observers will always be in agreement (except in the case of Barred/Unbarred, whereby not classifying a galaxy as multi-component precludes the option of being able to choose Barred/Unbarred).

Generally, there is mixed-to-good agreement between observers. We first consider how accurately the observer is able to distinguish between Early/Late, Single/Multi and Barred/Unbarred, regardless of other prior classifications. The highest unambiguous fraction is for late-type systems, with 85.3% of galaxies in the late-type Master class designated as such independently by all three observers. Slightly lower, the equivalent ratio for early-type systems is 69.4%. This indicates that the observer typically has the least difficulty in determining whether a galaxy is an early-type bulge-dominated system, or a late-type disk-dominated system. No doubt the colour information included in the three-colour postage stamp images aids classification, with late-type galaxies typically much bluer on average.

There is good agreement for single-component systems also, with the unambiguous population accounting for 79.7% of the master single-component class. However, multi-component systems are more ambiguous, with all three observers agreeing on only 61.7% of galaxies. This ambiguity on the definition of a multi-component system is interesting, in that it leaves a lot of room for error for automated methods. In a similar vein, all three observers find the most disagreement when classifying barred or unbarred systems, with the unambiguous percentages for barred and unbarred systems at 45.0% and 52.9%, respectively.

When considering combined classifications (e.g., is the galaxy early-type and a multi-component system) the level of unambiguous agreement reduces, as expected. Late-type single-component systems retain the highest level of agreement at 72.8%, whereas early-type multi-component systems have the least at 31.6%. The inclusion of a bar further reduces these figures. Late-type multi-component unbarred systems are the most unambiguous at 45.4%, whereas early-type multi-component unbarred systems are the most difficult to agree upon, at only 28.1%.

The level of overlap and agreement between observers is best represented by the Euler diagrams shown in Figure 5.10. Regions where at least two observers overlap constitute the final master classifications, and triple-overlap regions are unambiguous classifications. It is clear that there is excellent agreement between observers for late-type and single-component
5.2. Morphological Classification

The ASGR observer finds substantially fewer multi-component systems compared to LSK and SPD, and of these multi-component systems, ASGR also remains the outlier for the Barred/Unbarred classifications. Interestingly, despite ASGR classifying the fewest multi-component systems (within which a bar may be present), he also classifies the highest number of barred systems. However, this does not stop the barred class from being that with the lowest amount of unambiguous agreement, at only 45.0%. Despite this, Figure 5.10 is an important reminder of the eyeballing process and its use in classification consensus.

Example greyscale postage-stamp images for the various visual morphological classes are shown in Figure 5.11, arranged according to redshift. The LBS class is included here for reference. Further postage-stamp examples for each morphological class (including LBS) may be found in Appendix A.
Figure 5.11: Example postage-stamp cutouts for each morphological class, arranged according to redshift. Inset into each postage-stamp is the GAMA ID of the galaxy, for reference. Images are created from arctan-scaled composite three-colour images (RGB taken from Hig, respectively), with the colours inverted and desaturated. White spaces show regions where no galaxies of that morphology exist.
5.2.1.3 Global Measurements

In Figure 5.12 we show five key global galaxy measurements against each other, coloured according to their morphological classification. The five measurements shown are physical $K$ band half-light radius (kpc), ellipticity, absolute $K$ band Sérsic magnitude, rest-frame $(u - r)_{\text{GAMA}}$ colour and $K$ band Sérsic index. Sérsic measurements come about from a single-Sérsic fit to the $K$ band data. Absolute Sérsic magnitudes are calculated in the standard sense, using Equation 5.2. Note that we have opted to use our longest-wavelength $K$ band measurements in most of these analyses as these data will be least perturbed by the effects of dust attenuation, and so in some sense can be understood to be more robust and analogous to intrinsic properties.

It can clearly be seen that some projections of the data more easily allow distinct morphological groupings to be brought out than others. Absolute magnitude against half-light radius clearly shows a red-sequence of Elliptical-type galaxies progressing from the compact bright end diagonally upwards to the faint extended region of the figure. This should not be surprising for this volume-limited sample as this parameter space is essentially a projection of the fundamental plane of Elliptical galaxies (Djorgovski & Davis, 1987; Dressler et al., 1987), itself an extension of the Faber-Jackson relation (Faber & Jackson, 1976). When plotted against each other, the surface brightness ($\mu$), half-light radius ($r_e$) and velocity dispersion ($\sigma$) are found to lie on a three-dimensional plane, or fundamental plane. The fundamental plane usually takes the form $r_e \propto \sigma^{1.4} \mu^{-0.9}$ (e.g., Jorgensen et al., 1996; Kormendy & Kennicutt, 2004), with the value of the exponents subject to a variation of $\pm 0.1$.

The magnitude-radius sub-plot also picks out a blue-cloud of pure-disk Sd-class galaxies all showing minimal variation in absolute magnitude with radius. When we consider multi-component systems, we find a ‘green-wedge’ of late-type spiral galaxies (Sbc/SBbcd) that fill the triangular gap between the red-sequence and blue-cloud, typically all with high values of half-light radius. These disk-dominated systems have been eyeballed as having multiple components, and yet the global measurements shown here are derived from single-component models. In this case, were structural decomposition invoked in order to separate out the half-light radii and magnitudes for the bulge and disk components, one might expect the disk component to migrate left (fainter but of a comparable size) and the bulge component down and left (fainter and smaller), due to the disk-dominance in these systems.

Finally, and potentially most interestingly, we note that the early-type spiral systems
Figure 5.12: A correlation matrix showing five key global parameters, namely (from left to right): $K$ band half-light radius (in kpc), ellipticity, absolute $K$ band Sérsic magnitude, $(u - r)$ rest frame Kron-like (AUTO) colour and $K$ band Sérsic index. The underlying 1D density plots have associated rectangular bandwidths of 0.1, with the exception of absolute magnitude, which uses a bandwidth of 0.5. Data points are coloured according to their visual morphological classification, as detailed in Section 5.2.1. Distinct groupings of similar colour data points (i.e., same morphology) can be seen, particularly in the case of absolute magnitude against half-light radius where the red-sequence for Elliptical galaxies and the blue cloud for pure-disk galaxies is clearly visible. Inset into the figure is a reproduction of Figure 5.8, for reference.
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(S0a/SB0a) all occupy the same parameter space as the single-component Elliptical galaxies, namely, lying on top of the red-sequence. These results are in excellent agreement with the conclusions of Drory & Fisher (2007), who find that spiral galaxies harbouring classical bulges lie consistently on the red-sequence. Although speculation remains as to how a classical bulge comes into existence, it is widely believed (e.g.: Kormendy et al., 2009) that a classical bulge is analogous to an elliptical galaxy that formed at some early epoch and has subsequently had the opportunity to accrete additional material from the IGM in order to grow a disk (although, see Gadotti, 2009 for a rebuttal of this theory). In contrast to classical bulges, pseudo-bulges are believed to form via secular evolutionary processes present within the disk (Debattista et al., 2006; Gadotti, 2009; Saha et al., 2012). In brief: if left in isolation for a sufficient length of time (i.e., without any major merging events) then a dynamically cold rotating disk system will form a barred structure. The bar acts as a very efficient means by which gas in the disk may be funneled into the core of the galaxy, initiating a new phase of star formation in the central region. A young, blue sub-structure exhibiting a large component of angular velocity and a flattened surface-brightness profile ($n \sim 2$) will form, quite distinct from the underlying classical bulge (if present). This new structure is commonly referred to as a pseudo-bulge. Unlike classical bulges, Drory & Fisher find that galaxies with pseudo-bulges lie in the blue cloud. We find very few multi-component systems overlapping with the main body of the blue cloud, and conclude that structural decomposition is required in order to fully quantify a) which of these galaxies contains a pseudo-bulge and b) where these galaxies lie in relation to the blue cloud.

There are many other useful sub-figures in Figure 5.12 which help to distinguish between the various morphological types. In particular we would like to isolate absolute magnitude vs $(u - r)$ colour, absolute magnitude vs Sérsic index, and $(u - r)$ colour vs Sérsic index as three figures of interest. These all clearly show a bimodality in the data between spheroid-dominated and disk-dominated systems, which are well-fitted by a double-Gaussian profile (Baldry et al., 2004). These figures enable a coarse division of the total population into two groupings. As before, when plotted vs absolute magnitude the late-type spiral (Sbc; green data point) systems sit apart from the centroids of these two groups. However, in the Sérsic index vs $(u - r)$ colour figure, these late-type spirals lie directly between the red and blue populations, exactly where the ‘green-valley’ is expected to lie (Wyder et al., 2007; Salim et al., 2009; Mendez et al., 2011). We make no physical interpretation of this result, merely
draw it out as a point of interest, and note that further structural decomposition may remove this green-valley population, or indeed, may re-enforce it.

In most regimes, the early-type (purple data point) spiral galaxy population very closely matches that of Elliptical galaxies, making them very hard to distinguish on the basis of global measurements alone, with the exception of global ellipticity. Early-type spiral galaxies, ironically, tend to be more elliptical on the whole than their Elliptical counterparts. This is as expected if one imagines these spiral systems are comprised of a circular bulge plus an elongated disk, and so the global measurement of ellipticity will no doubt trace this disk component and skew the final result from a more circular ellipticity. This relation as well as those detailed above are subsequently used in Section 5.2.5 to define hard cuts in an attempt to pick out these morphologies in an automated fashion using only global measurements.

We note that we have not commented on the location and trends of barred structures in Figure 5.12. This is largely because only 109 galaxies within our volume-limited sample (2.7%) have been classified as containing a bar, and so insufficient number counts hinder their further analysis. A larger total sample size (ideally constructed from deeper VST/VISTA data) would increase this number due to increased resolution and volume, opening up further bar investigation.

5.2.2 Morphology by Colour

A common and trivial method of dividing a sample of galaxies is to cut according to some arbitrary colour boundary. Recent studies employing a colour divisions may be found in, e.g., Strateva et al. (2001); Baldry et al. (2006); Driver et al. (2006, 2007); Vika et al. (2009); Mendez et al. (2011); Dariush et al. (2011); Rowlands et al. (2012), to name but a few. The reasoning behind this is clear; galaxies fall chiefly into two main categories: red, old and spheroid-dominated early-type galaxies and blue, young and disk-dominated late-type galaxies, as shown in Baldry et al. (2006); Kelvin et al. (2012) and in Figures 4.11 and 5.12, where a \((u-r) \sim 2\) bisects these two groups. Drory & Fisher (2007) adopt a \((u-r) = 2.2\) colour cut, whereas both Driver et al. (2006) and Cameron et al. (2009) adopt \((u-r) = 2.1\), as seems to be common practice in the literature.

As a means of comparison with the literature, we also adopt a colour cut of \(\text{Auto}(u-r) = 2.1\) to define a red early-type and blue late-type population within our sample. When making this cut, 971 galaxies (23.6%) are defined as early-type, and 3129 (76.1%) as late-type, with 10 galaxies (0.2%) remaining unclassified due to no colour information available in the GAMA
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database because of a null detection in the poorer-quality $u$ band data.

A correlation matrix showing how these data are distributed using colour-defined early-type and late-type populations is shown in Figure 5.13. As expected, morphological resolution is lost when simplifying the data through a binary filter such as this, however, several features and trends are still noticeable. The red-sequence of Elliptical galaxies previously noted in the half-light radius vs absolute magnitude figure is still present. The previous late-type spiral population wedged between this red-sequence and the blue cloud has now been merged into a larger blue cloud. In addition, the bimodal relation in (absolute magnitude)-(Sérsic index) space looks remarkably healthy, with two clearly defined populations emerging as one might expect. This should not be surprising however due to the inherent relationship between Sérsic index and colour, as seen in Figure 4.11.

Figure 5.14a shows how these populations have transitioned from their visual morphological classifications to colour-cut defined early- and late-type populations. On the whole, the majority of visual early-type galaxies remain early-type, as can be seen by the thick green lines connecting the before and after states. Late-type galaxies are even more successfully classified, with $>80\%$ remaining as late-types. The largest source of error is in spheroid-dominated (early-type) galaxies being misclassified as late-type, with 9.8% (404) crossing the early-late boundary. This figure is nearly 50% higher than the number of galaxies transitioning in the inverse direction (235/5.7%). As established by Baldry et al. (2004), one would expect the two dominant populations to be well represented by a double-Gaussian fit to the data, and so misclassification in the wings is a natural consequence of employing a harsh cut such as this. The amount of disparity between the two crosstalk values however indicates that a colour cut of $(u-r) = 2.1$ is potentially too red. We find that a colour cut of $(u-r) \sim 2$ gives a much more equal distribution of misclassifications. Using $(u-r) = 2$ leaves 1135 galaxies defined as early-type (27.6%) and 2965 galaxies as late-type (72.1%), with $\sim 7.7\%$ galaxies alternately-classified in both directions.

5.2.3 Morphology by Sérsic Index

Sérsic index provides a measure of the central concentration in the surface brightness light profile of a galaxy. Typically, one expects high Sérsic indices ($n > 2$) to be associated with spheroidal-dominated systems, and low Sérsic indices ($n < 2$) to be associated with disk-dominated systems (see Figures 4.8, 4.11, 4.12 and 5.12 in addition to, e.g., Kelvin et al., 2012). This useful fact allows a broad-brush division between spheroid-dominated and disk-
Figure 5.13: As for Figure 5.12 but with data points coloured according to a rest-frame \((u - r)_\text{Auto}\) colour cut of \((u - r) = 2.1\).
Figure 5.14: A series of figures representing the transitions from visual morphologies to morphologies as defined by other means, as indicated. In all figures, the thickness and colour of the connecting lines represents the percentage of that population which has transitioned from the previous visual morphology to its new classification. The central crossed arrows represents the ‘crosstalk’, i.e., the percentage of galaxies which have crossed the dashed blue-red boundary and completely changed their type (early $\leftrightarrow$ late).
Allen et al. (2006) divide their population of single-component galaxies into early-type and late-type using a dividing line at \( n = 1.5 \), a conclusion reaffirmed in Cameron et al. (2009). However, for multiple-component systems (i.e., bulge+disk systems) Allen et al. resort to a complex logical filter method, whereas Cameron et al. instead employ a Sérsic index-colour cut (see Section 4.4, also Section 5.2.4 below). Blanton et al. (2003a) also define disk-dominated galaxies as \( n < 1.5 \), yet define spheroid-dominated galaxies as \( n > 3 \), ignoring those potentially problematic overlapping cases in the green-valley region. Guo et al. (2009) note that for their sample of CENs (the central galaxies within groups and clusters) a Sérsic index cut of \( n \sim 3.5 \) is more appropriate to isolate huge Elliptical BCGs. This is no doubt an extension of the apparent linear relation between Sérsic index and magnitude; as has been shown for dwarf Ellipticals which exhibit Sérsic indices of \( n < 2 \) (Caon et al., 1993; Young & Currie, 1994; Jerjen et al., 2000a; Graham et al., 2006). However, we would not expect a large dwarf population within our volume limited sample of 4110 galaxies owing to the faint-end absolute magnitude cut at \( M_r = -17.4 \). Drory & Fisher (2007) find a global Sérsic index division of \( n = 2.5 \) is adequate to divide their population between galaxies which host either a pseudo-bulge (low-index) or a classical-bulge (high-index). A Sérsic index of \( n = 2.5 \) is also used to divide complete populations of galaxies into spheroid-dominated and disk-dominated galaxies in, e.g., Shen et al., 2003; McIntosh et al., 2005; van der Wel, 2008.

It is clear that division by Sérsic index remains a highly debated issue in the literature, however, we choose to divide our population at a Sérsic index of \( n_K = 2.5 \). This value is in agreement with several previous studies and so aids in cross-comparison. \( n_K = 2.5 \) also cuts through the minimum in our own Sérsic index distribution (See Figure 5.12). When making a Sérsic index cut at \( n_K = 2.5 \), we find 1234 galaxies (30.0%) classified as early-type and 2599 galaxies (63.2%) as late-type, with 277 galaxies (6.7%) remaining unclassified due to no \( K \) band Sérsic data. These failed fits arise due to low signal in the \( K \) band data. We note that only 5 galaxies (0.1%) in the \( r \) band failed for the same sample, however, for reasons previously outlined we prefer to use the \( K \) band measurements as they are less affected by dust attenuation.

Figure 5.15 shows how these Sérsic index defined early-type and late-type populations are distributed within a correlation matrix. The bulk of the early-type and late-type eyeball-classified populations are easily identified, with a large contaminant tail visible in both the \( (u-\)
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Figure 5.15: As for Figure 5.12 but with data points coloured according to a $K$ band Sérsic index cut of $n = 2.5$.

$r$) colour and $M_K$ histograms. The red-sequence cloud visible in absolute magnitude against half-light radius is still clearly visible, with a spur of misclassified red objects at high-$r_e$ – even appearing as a secondary peak in the absolute magnitude histogram. Figure 5.14b shows the transition from visual morphologies to Sérsic types. Generally there is good agreement, but it is clear that a Sérsic index cut is not as successful in identifying these two populations as a colour cut. The crosstalk values are relatively high at 8.2% and 11.0%. These contaminants not only come from green-valley populations, but also from Elliptical and (to a lesser extent) Sd-type galaxies also.
5.2.4 Morphology by Sérsic Index and Colour

A combination of \((u - r)\) colour and Sérsic index appears to be a better means by which to divide the population into early and late-type groupings. As can be seen in Figure 4.11, early-type galaxies are typically higher centrally-concentrated and redder, whereas the inverse is true for late-type galaxies. Cameron & Driver (2009) make a cut in index-colour space, finding the two populations are adequately divided along a line given by \((u - r) = 2.325 - 2.074 \log(n)\). Cameron et al. (2009) expand upon this division for multiple-component systems, defining an additional dividing line given by \((u - r) = 3.22 - 2.75 \log(n)\). In Kelvin et al. 2012 we find these two populations are divided along a line given by \((u - r) = 2.07 - 0.59 \log(n)\). We note that the dividing lines in both Cameron & Driver (2009) and Cameron et al. (2009) have a distinctly different gradient to that found in Kelvin et al. (2012). The gradient of the dividing line in Kelvin et al. (2012) is determined as that line which lies perpendicular to a line connecting the two peaks in number density (when plotted in log-linear space). The exact value of the gradient of this dividing line is of less importance however, as very few galaxies are blue and high-index or red and low-index and so very few galaxies will be affected in this regard.

We adopt a dividing line of \((u - r) = 2.07 - 0.59 \log(n)\) as in Kelvin et al. (2012) and as per Section 4.4. 1261 galaxies (30.7%) are defined as early-type, whereas 2564 (62.4%) are defined as late-type. 285 galaxies (6.9%) remain unclassified owing to a lack of either \((u - r)\) colour or UKIDSS \(K\) band Sérsic index information, including two galaxies that have neither (shown in Figure 5.16).
Sérsic index-colour defined populations show a good level of agreement with the visually classified morphologies, as shown in Figure 5.14c. 214 galaxies (5.2%) transition from early-type to late-type, whereas 382 (9.3%) transition from late-type to early-type. The total number of misclassified galaxies found when adopting a combined colour-index cut is reduced compared to either of the colour or Sérsic cuts in isolation.

5.2.5 Morphology by Global Divisions

As discussed in Section 5.2.1, groupings of similar-type morphologies can easily be seen by eye in Figure 5.12. Here, we discuss using a series of linear quantitative cuts in multiple global measurement planes to construct new estimates of morphological classification. We note that, owing to the small number of barred systems present within the dataset, barred galaxies are not considered here.

Quantitative global divisions are made in several steps. At each stage, only systems which satisfy the following criteria are classified as the given morphology. They are then removed from the catalogue before advancing to the next round of cuts. These morphological divisions, in order, are:

(a) Sbc:
- $M_K < -20$
- $M_K > -5 \log(r_e) - 19$
- $(u - r) < 2.2$
- $n_K < 3$

(b) Sd:
- $M_K > -20$
- $M_K > -5 \log(r_e) - 19$
- $(u - r) < 2$

(c) S0a:
- $e_K > 0.15M_K + 3.6$

(d) E:
- $(u - r) > 2.07 - 0.59 \log(n)$
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Figure 5.17: An attempt to morphologically classify galaxies based on their global photometric and structural measurements. At each stage (a, b, c, d and e) galaxies within the boundaries shown are removed and assigned to the labelled class before proceeding to make the next series of cuts. Statistics for the fraction of correctly identified galaxies within those cuts (‘Correct’) and the fraction of that population which has been found (‘Found’) are also shown. See the text for further details.

Any remaining unclassified objects are assigned to the Sd class. A graphical representation of these stages can be found in Figure 5.17. Statistics for the number of correctly identified galaxies within the cuts (‘Correct’) and the total fraction of that population that have been correctly identified (‘Found’) are inset into the figure.

Figure 5.14d shows the transition statistics. As quantitative divisions such as this allow a greater resolution in our data, a greater number of available morphologies are available. A very large fraction (> 90%) of Sd-class galaxies remain as such, with other morphologies retaining ~ 60% of their visual morphological class. There has also been an improvement...
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on early-type/late-type crosstalk from the previous Sérsic index-colour division. 14.0% of our sample change their type (early/late), with the largest interloping number coming from misclassified Ellipticals.

5.2.6 Morphology by Statistical Analysis

Previous methods for dividing the data using global measurements have relied on relatively simplistic cuts. We know that our populations overlap each other in multi-dimensional space, and so it makes little sense to impose these hard edges in one dimension only in our analyses. A more comprehensive analysis is to look at the data across all 5 global dimensions simultaneously, and use some means by which to assign a probability to each data point that it belongs to a given class. In this section we discuss statistical means by which we are able to perform multi-dimensional analysis on our data.

5.2.6.1 Principal Component Analysis (PCA)

Principal Component Analysis (PCA; Murtagh & Heck, 1987) is a statistical method for decomposing a multi-dimensional dataset into its most significant or ‘principal’ components. This is achieved by applying an orthogonal linear transformation to the data in such a way that principal component 1 (PC1) is a new projection of the data describing the largest variance in the dataset. In order to achieve this, each dimension must be initially scaled such that a unitary step along its axis describes an equivalent unitary step in each of the other dimensions. These data must then be mean subtracted in order that the variance along each dimension describes the direction of variance. PC2 describes the second largest variance in the data under the proviso that it is perpendicular to PC1 (or, more formally, all of the previous principal components), and so on until the \( n \)th principal component is defined, where \( n \) is the dimensionality of the input dataset. Higher-order principal components describe diminishing levels of variance in the data. Typically, there will be a ‘knee’ or turnover in the amount of variance described, enabling some higher-order principal components to be discarded and hence minimising the dimensionality required to accurately describe the dataset. PCA is a useful means by which a dataset containing many different measurements of a set of objects may be broken down into a smaller number of components prior to analysis. We use the R function `princomp` to calculate principle components, which automatically calculates an appropriate scaling for each variable.

We choose to perform PCA on eight key measurements of our dataset, namely: half-light radius, ellipticity, absolute \( K \) band Sérsic magnitude, \( u - r \) colour, \( K \) band Sérsic index, \( K \)
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Figure 5.18: A matrix of scatterplots showing data upon which principal component analysis has been performed. The eight key measurements shown are (in order from left to right): half-light radius, ellipticity, absolute $K$ band Sérsic magnitude, $u - r$ colour, $K$ band Sérsic index, $K$ band position angle, redshift and stellar mass. Data points are coloured according to their eyeball classification morphology, with one-dimensional histograms displayed for each measurement along the primary axis. Simplistic smoothed trend lines are shown by the solid red lines.

Band position angle, redshift and stellar mass. A matrix of scatterplots summarising these data and their correlations may be found in Figure 5.18. The top-left corner of this figure is similar in design and layout to Figure 5.12. Data points are coloured according to their eyeball classifications, with one-dimensional histograms displayed for each measurement along the primary axis. Simplistic smoothed trend lines are shown by the solid red lines. Note the range in correlation between these parameters, from very highly correlated (absolute magnitude against stellar mass) to extremely uncorrelated (position angle against ellipticity).
Figure 5.19 displays the results of our principle component analysis. The left scatter plot is a biplot showing principal component 1 against principal component 2, with axes labelled in both component space (bottom and left axes) and scaled variance space (upper and right axes). Overlaid arrows represent the relative contributions from the eight input variables towards each principal component. One can clearly see that PC1 is primarily defined by stellar mass, Sérsic index, $u - r$ colour and absolute $K$ band magnitude. PC2 is primarily defined by half-light radius and ellipticity. Observe how position angle (and, to a lesser extent redshift) do not feature heavily in either PC1 or PC2. The right bar chart is a screeplot showing the amount of variance in the dataset as described by each principal component. Inset into the figure is a table displaying the scalings applied to each input variable prior to PCA. The first four principal component equations are also reproduced, for reference. Principal components 1 and 2 describe 53.4% of the total variance within the data. In this dataset, the characteristic knee beyond which higher-order principal components may be disregarded as describing diminishingly small fractions of the total variance appears to occur after the first principal component. From this we can deduce that parameter spaces which represent any of stellar mass, Sérsic index, $u - r$ colour or absolute $K$ band magnitude against any of either half-light radius or ellipticity should provide the best discriminator for morphological classification by global means.

5.2.6.2 Linear Discriminant Analysis (LDA)

Linear Discriminant Analysis (LDA) is a method usually employed for pattern recognition of distinct groupings of data within a multi-dimensional dataset; ideally suited for the task of automated morphological classification. LDA is very closely related to PCA in that various projections of the data are analysed in order to ascertain those divisions which most accurately describe assumed populations within the data. Several basic assumptions are made. Firstly, it is assumed that the distribution in each of these assumed populations is well described by a Gaussian function. Secondly, it is assumed that the covariances between variables are identical. We use the R function `LDA` in the MASS package to perform LDA on our dataset. This allows for a posterior probability to be assigned to each data point indicating the likelihood of its morphological type based on the eight input measurements. We assign an LDA morphology to the data by taking the maximum posterior probability for each data point. See Ellis et al. (2005) for a summary of the method by which LDA is performed on a similar dataset for a similar purpose.
Figure 5.19: Results from a principal component analysis of eight key measurements from the volume-limited sample dataset (see above for more details). (left) A biplot showing principal component 1 against principal component 2, with axes labelled in both component space (bottom and left axes) and scaled variance space (upper and right axes). Overlaid arrows represent the relative contributions from the eight input variables towards each principal component. One can see that PC1 (describing the largest variance in the dataset) is primarily defined by stellar mass, Sérsic index, $u - r$ colour and absolute $K$ band magnitude. PC2 (describing the second largest variance in the dataset along a projection perpendicular to PC1) is primarily defined by half-light radius and ellipticity. Observe how position angle (and, to a lesser extent redshift) do not feature heavily in either PC1 or PC2. (right) A screeplot showing the amount of variance in the dataset as described by each principal component. Inset into the figure is a table displaying the scalings applied to each input variable prior to PCA. The first four principal component equations are also reproduced, for reference. Principal components 1 and 2 describe 53.4% of the total variance within the data.
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Figure 5.14e shows the numbers of galaxies within each morphological class as defined by our LDA method, and how the numbers within each morphology have transitioned from their prior eyeball classifications. Each morphology is typically well identified by LDA with the exception of barred galaxies, with a typical success ratio of > 60% identification relative to the eyeball morphology. Owing to the increased complexity of this statistical method, LDA allows for barred galaxies to be identified should their posterior probabilities be large enough. This rarely happens however owing to the fact that barred galaxies never dominate any particular region in any parameter space projection. For this reason, barred galaxies typically are included in their parent unbarred class. The relatively low and balanced crosstalk values show the strength of this method, with less than 10% of the total sample crossing the early-late boundary.

5.2.6.3 Quadratic Discriminant Analysis (QDA)

Quadratic Discriminant Analysis (QDA) follows the same methodology as LDA with the exception that the covariances between variables are allowed to vary. This additional degree of freedom should in theory produce more realistic morphological classifications. Figure 5.14f shows the results of this procedure. Again, large fractions (> 60%) of the data find their way from their assumed eyeball morphologies into their QDA morphologies. Also note the increasing numbers of galaxies identified as containing a bar. The numbers of galaxies transitioning across the early-late boundary is also reduced from the previous LDA, standing at only 8.9% of the total dataset.

5.3 Choosing The Correct Morphology

In the sections above we have described several means by which the sample may be divided into their discrete morphologies. We now seek to determine which of these methods provides the most robust measure of morphology. Whilst absolute cuts in colour, Sérsic index or colour plus Sérsic index allow for the sample to be successfully split into early-type and late-type, they do not provide the resolution we require to pick out more comprehensive morphologies, and so we shall disregard their use throughout the remainder of this section. This leaves us with visual (Eyeball) classifications, global divisions (Cuts), linear discriminant analysis (LDA) and quadratic discriminant analysis (QDA). Since the most intuitive morphological classification scheme to use would be that as defined by eyeball classifications, we shall adopt this as our preferred method, and discuss the merits of other methods in relation to this below.
5.3.1 Variation in Redshift

One would not expect to see large evolutionary variations in morphological fraction over a narrow redshift range such as that used in the creation of our volume limited sample. Figure 5.20 shows the sample data as a function of redshift, with data points coloured according to either their morphological class (left column) or morphological type (right column). From top to bottom, these data are shown relative to their absolute $r$ band Sérsic magnitudes, number density and number fractions. One can clearly see the large-scale structure with redshift appearing as over-dense strips in the scatter plot. Interestingly, the densest regions appear to have the fewest numbers of Sd class galaxies, agreeing well with the classical morphology-density relation of Dressler (1980) which predicts fewer disk-only structures in over-dense regions relative to the field.

Elliptical galaxies (red) show no significant number fraction change with redshift over this redshift range. However, late-type morphologies (Sd and Sbc/SBbcd, blue and green) and early-type spirals (S0a/SB0a, purple/pink) do show some evolution. The number of Sd-class galaxies consistently drop off below a cut-off redshift of $z \sim 0.025$, and are replaced by increasing numbers of early and late-type spiral systems. This trend is also noted with the eyeball morphological type classifications, with a drop off in late-type single-component systems in favour of early-type multi-component systems at lower redshifts. More local systems in our volume-limited catalogue appear by eye to have more dominant spheroidal structures and more components in general than their higher redshift counterparts. This suggests that resolution of these additional components may be causing issues with the visual classifications, however, as previously noted in Section 5.1.1 and in Figure 5.1, previous studies would suggest that both the bulges and the disks should be resolvable in the majority of cases out to a redshift of $z = 0.06$. In addition, if poorly resolved structure is at fault, one might expect a certain cut-off redshift, below which the fractional number of morphologies would plateau.

It appears the problem at low redshift is two-fold. Firstly, only 265 galaxies lie in the redshift range $0.013 < z < 0.025$, constituting 6.4% of our dataset. One might expect small number statistics on galaxies in the local volume would make any fractional measurements meaningless owing to the large uncertainties inherent on them. This certainly appears to be the case here. Secondly, it may be that these low redshift galaxies suffer from too much resolution. When the resolution is below 0.5 kpc even the smoothest classical Elliptical galaxy may look as though it has sub-structure and therefore fool human eye visual classification. If
5.3. Choosing The Correct Morphology

Figure 5.20: Morphology against redshift. Data points are coloured according to their morphological class (left column) and morphological type (right column). (top) Absolute r band Sérsic magnitude as a function of redshift. (middle) Probability density functions for the total (black line) and individual morphologies as indicated, as a function of redshift. The solid grey line shows a representative measure of constant density, for reference. (bottom) The representative fractions of the total number of galaxies for each morphology, as a function of redshift. PDFs have been constructed using a rectangular kernel with a band-width of 0.005.
this were the case, one might expect to find more multi-component systems at low redshift. This trend can clearly be seen in the trade-off between single- and multi-component systems as shown in Figure 5.20.

Bearing these low-redshift issues in mind we opt to remove these 265 low-redshift galaxies by restricting our volume limited sample to the redshift range $0.025 < z < 0.06$, in order that any local-volume effects be removed from our analyses. This reduces our sample size to 3845 galaxies in total, which constitutes a volume of $210120.2 \text{ Mpc}^3$. Owing to the relatively small number of galaxies removed from the sample at this juncture, we do not expect any of our prior analyses to be incorrect in their conclusions. This reduced sample shall be used throughout the remainder of this chapter and thesis.

Do any of our other classification methods provide a viable alternative to Eyeball classification? Figure 5.21 shows the evolution in number fraction with morphology as per the lower-left plot in Figure 5.20, but for the additional three classifications schemes as well, as indicated. Shaded regions indicate $\pm 2\sigma$ Bayesian confidence intervals calculated using the Beta distribution (Cameron, 2011). This plot is now only showing the new redshift limits of $0.025 < z < 0.06$, and has removed the low-redshift anomalies previously encountered. The number fraction of Sd type galaxies for the visual classification schema (Eyeball) is now shown to remain broadly flat within its error boundaries.

Division by quantitative cuts acts to increase the fraction of Sd galaxies at low redshift, and yet with all morphologies still exhibiting a relatively flat redshift distribution. Note that this method does not have the resolution to predict any barred galaxies, hence their absence from this plot. However, the rationale behind choosing appropriate quantitative cut boundaries remain subjective. This, in addition to their inability to resolve barred structure, cause us to discount quantitative global cuts as an appropriate means by which this sample may be automatically classified.

Both LDA and QDA methodologies provide morphological predictions that are largely well fit by a flat horizontal line, indicating minimal evolution in the number fractions with redshift as expected. The only exception is the slight negative gradient in Sd type galaxies, with Sbc type systems showing a slight reduction to compensate. In addition, the Sd type galaxies for QDA show moderately more variation in their number with redshift than those via LDA. However, it appears that both LDA and QDA methods provide a good means by which a sample of potentially biased visually classified morphologies may be ‘cleaned up’.
Figure 5.21: Morphology against redshift by additional methods. This figure shows the evolution with redshift in the number fraction of various morphologies (as indicated) as determined by differing methods. The four comparative methods (see text for full details) are: visual classification (Eyeball); global divisions (Cuts); linear discriminant analysis (LDA) and quadratic discriminant analysis (QDA). Shaded regions indicate $\pm 2\sigma$ binomial confidence intervals. PDFs have been constructed using a rectangular kernel with a band-width of 0.005.
5.3.2 The Luminosity Function

The luminosity function (LF) describes the relative density of galaxies in any given luminosity (or magnitude) bin across a wide range of luminosities. Measurement of the LF allows for constraints to be placed on galaxy formation and evolution models, and as such is a useful and interesting parameter to measure. Understanding the LF allows for the analysis of the number count distribution not only of global galaxy populations, but also sub-populations of galaxies (such as Ellipticals or red galaxies, for example). Here I discuss its measurement and its use in choosing the correct morphological sample.

5.3.2.1 The Schechter Luminosity Function

The Schechter Luminosity Function (Schechter, 1976) is an analytical representation of the luminosity function, describing the effective number of galaxies per unit volume in the luminosity interval $L$ to $L + dL$, where $dL$ is some linear luminosity interval. The number density, $\phi (L) dL = dn$, is given by

$$\phi (L) dL = \phi^* \left(\frac{L}{L^*}\right)^\alpha \exp \left(-\frac{L}{L^*}\right) \frac{1}{L^*} dL \quad (5.4)$$

where $\phi^*$ is some arbitrary normalisation constant, $L^*$ is the characteristic luminosity describing the position of the ‘knee’ in the luminosity function and $\alpha$ gives the slope of the luminosity function at the faint end (where $L \ll L^*$). Note that $\phi^*$, $L^*$ and $\alpha$ are to be determined by minimising a fit to the data. The effect of the Schechter function is to truncate the faint-end power law distribution of galaxies, vastly reducing number counts at luminosities greater than $L^*$.

Logged variant of the Schechter Function: It is usually more convenient to convert the functional form of the Schechter function into other forms to handle, typically, logged data, such as absolute magnitudes in place of linear luminosities. In deriving these equations, a useful precursor is to first convert the Schechter function into a (natural) logged variant. We begin by converting Equation 5.4 into a form that handles logarithmic luminosity intervals. We know that

$$\frac{d}{dL} \ln L = \lim_{dl \to 0} \left[ \frac{\ln(L + dl) - \ln(L)}{dl} \right]$$

$$= \lim_{dl \to 0} \left[ \ln \left(1 + \frac{dl}{L}\right)^{\frac{1}{dl}}\right]$$

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where \( \ln L \) is the natural logarithm of the luminosity \( L \). Using the relation \( u = dL/L \), and therefore \( u \to 0 \) as \( dL \to 0 \), it follows that

\[
\frac{d}{dL} \ln L = \ln \left( \lim_{u \to 0} \left[ (1 + u)^{\frac{1}{u}} \right] \right) = \frac{1}{L} \ln \left( \lim_{u \to 0} \left[ (1 + u)^{\frac{1}{u}} \right] \right) = \frac{1}{L} \ln e
\]

\[
\therefore \frac{d}{dL} \ln L = \frac{1}{L}
\]

or, more usefully,

\[
dL = L \cdot d\ln L \quad (5.5)
\]

On substituting this relation into Equation 5.4, we find

\[
\Phi(L) d\ln L = \phi^* \left( \frac{L}{L^*} \right)^\alpha \exp \left( - \frac{L}{L^*} \right) \frac{L}{L^*} d\ln L
\]

and therefore,

\[
\Phi(L) d\ln L = \phi^* \left( \frac{L}{L^*} \right)^{\alpha + 1} \exp \left( - \frac{L}{L^*} \right) d\ln L \quad (5.6)
\]

where \( \Phi(L) \) is the number density per logged luminosity interval. This form of the equation accepts logarithmic luminosity intervals. Taking exponentials of the luminosity terms in Equation 5.6 gives us

\[
\Phi(L) d\ln L = \phi^* \exp \left[ \alpha + 1 \right] \exp \left[ - \exp \left( \ln \left( \frac{L}{L^*} \right) \right) \right] d\ln L \quad (5.7)
\]

which can be rewritten as

\[
\Phi(L) d\ln L = \phi^* \exp \left[ (\alpha + 1) \ln \left( \frac{L}{L^*} \right) \right] \exp \left[ - \exp \left( \ln \left( \frac{L}{L^*} \right) \right) \right] d\ln L \quad (5.8)
\]

This form of the equation now also handles logged input data. We may now convert this into either its logarithmic base-10 form (for, e.g., log stellar mass input) or its magnitude form.

Log base 10 variant of the Schechter Function: It is typically more natural for astronomers to use \( \log_{10} \) when logging data rather than the less natural natural logarithm, \( \log_e \) or \( \ln \). I shall now derive the log base-10 form of the Schechter function which may be used later to
represent and fit to logged data, such as stellar mass. The logarithm change of base formula,
\[
\log x = \frac{\ln x}{\ln 10}
\] (5.9)
give us the two relations
\[
\ln \frac{L}{L^*} = \ln 10 \cdot \log \left( \frac{L}{L^*} \right) \quad \text{and} \quad d \ln L = \ln 10 \cdot d \log L
\] These relations allow us to rewrite Equation 5.8 thus;
\[
\Phi (L) d \log L = \ln 10 \cdot \phi^* \exp \left[ \ln 10 \cdot (\alpha + 1) \log \left( \frac{L}{L^*} \right) \right] \exp \left[ - \exp \left( \ln 10 \cdot \log \left( \frac{L}{L^*} \right) \right) \right] d \log L
\] (5.10)
which further simplifies to
\[
\Phi (L) d \log L = \ln 10 \cdot \phi^* 10^{(\log L - \log L^*) (\alpha + 1)} \exp \left( -10^{(\log L - \log L^*)} \right) d \log L
\] (5.11)
Finally, substituting log \(L\) and \(\log L^*\) for \(M\) and \(M^*\) respectively the above equation may be rewritten as
\[
\Phi (M) d M = \ln 10 \cdot \phi^* 10^{(\log L - \log L^*) (\alpha + 1)} \exp \left( -10^{(\log L - \log L^*)} \right) d M
\] (5.12)
This is the log base-10 version of the Schechter function I shall adopt throughout the remainder of this thesis.

**Magnitude variant of the Schechter Function:** It is usually more convenient when considering luminosities to re-write the Schechter Function in terms of magnitude. The standard definition of magnitude,
\[
M = 10^{-0.4(M - M^*)}
\]
gives us
\[
\ln L - \ln L^* = -0.4(M - M^*) \ln 10
\] (5.13)
where \(M\) and \(M^*\) are the magnitude and the characteristic magnitude corresponding to \(L^*\), respectively. Differentiating Equation 5.13 with respect to \(dM\) gives us
\[
\frac{d \ln L}{dM} = 0.4 \ln 10
\] (5.14)
Note here the change in sign in order to account for the trend in the opposite sense for magnitude to increase numerically as luminosity increases physically. Substituting Equations...
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5.13 and 5.14 into Equation 5.8 gives us

$$\Phi(M) \, dM = 0.4 \ln 10 \cdot \phi^* \exp \left[ (\alpha + 1)(-0.4(M - M^*) \ln 10) \right] \exp \left[ -\exp(-0.4(M - M^*) \ln 10) \right] \, dM$$

which simplifies to

$$\Phi(M) \, dM = 0.4 \ln 10 \cdot \phi^* 10^{-0.4(M-M^*)(\alpha+1)} \exp \left(-10^{-0.4(M-M^*)}\right) \, dM$$

(5.16)

This is the form of the Schechter luminosity function that I shall adopt throughout the remainder of this thesis.

5.3.2.2 Variation in Luminosity Functions

An analysis of the luminosity functions of each of the morphological classification methods under scrutiny provides a good sanity check that each method is performing as expected. Binggeli et al. (1988) provide an estimate of the total luminosity function for field galaxies in addition to the morphological types that constitute it. I shall use their results as a standard by which to compare my own to.

For each classification method, I fit the total and morphology sub-samples with a single Schechter function (Equation 5.16). As earlier, the barred morphologies are merged with their parent classes owing to the low number of barred galaxies in our volume-limited sample. The total Schechter fit is identical for each method, and is provided for reference only. Schechter fits are performed in the $r$ band, using magnitude bins of 0.25 dex. Any bin containing 3 galaxies or fewer is discarded from the fit, as is any bin fainter than $M_r = -17.4$ (the absolute magnitude limit in the $r$ band for this sample). Errors within each bin measurement are assumed to be Poissonian ($\sqrt{n}$) in nature. Using the remaining data, the Schechter function is fit using the NLMINB routine within the R programming language to perform a $\chi^2$ minimisation to the data.

The results of the Schechter fits are shown in Figure 5.22. Shaded grey areas indicate those regions that lie outwith the limits discussed above, and consequently, data from these regions has not been used for the Schechter fits. Broadly speaking, all classification methods produce large numbers of Sd-type galaxies at the faint end, with spiral galaxies dominating the intermediate magnitude range. However, only the visual classification (Eyeball) method is able to reproduce the number count dominance of the E-type population at the very brightest magnitudes. Following on from Binggeli et al. (1988), one would expect the very brightest
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Figure 5.22: Single-Schechter $r$ band fits to the various morphological classification methods under examination. Each morphology is labelled and coloured according to the inset legend. Barred galaxies have been merged into their non-barred parent class owing to the low number of barred galaxies in the sample. The data is split into bins of 0.25 dex, and errors are Poissonian ($\sqrt{\text{N}}$) in nature. Shaded grey areas ($M_r < -17.4$ and $n < 4$) indicate those regions where data has not been used in the fit. Eyeball classification is the only morphological method under examination which is able to fully reproduce the E-type number-count dominance at the brightest magnitudes, with the LDA classification method underestimating this population at the bright end significantly.

In contrast to the redshift evolution tests performed in Section 5.3.1, the previously promising LDA classification method completely fails to recover bright E-type galaxies, falling to the third brightest unique class behind S0a and Sbc’s, respectively. This drop off in E-type galaxies is surprising when considering the relatively low crosstalk numbers ($\sim 5\%$) between the Eyeball and LDA methods as seen in Figure 5.14, indicating that only the very brightest,
and hence most massive galaxies drop out of the LDA E-type sample. In addition, neither of the remaining two methods (quantitative cuts or QDA) appear to be able to accurately produce the E-type population, although QDA is close. Increasing the depth and quantity of data may push the QDA method into a preferred classification position, however, based on the Schechter fits shown in Figure 5.22 it appears that the Eyeball method is the preferred measure of morphological classification for this volume limited sample.

5.3.3 A Classification Scheme
I have analysed both the redshift distribution and the luminosity functions of each morphological type as given by the four classification methods under scrutiny, namely: visual classification (Eyeball); global divisions (Cuts); linear discriminant analysis (LDA) and quadratic discriminant analysis (QDA).

One would not expect any significant evolution in the morphology fractions with redshift over our initial narrow redshift range ($0.013 < z < 0.06$), however, it was found that the low-redshift end exhibited several anomalies. The number of multi-component systems dramatically increased at the expense of single component systems at redshifts below $z = 0.025$. It appears that the increased physical resolution of these systems causes the human eye to be able to resolve sub-structure on systems where typically one would not expect to find any, such as a classical Elliptical system. Also, only 265 galaxies from our total sample of 4110 lie at a redshift of $z < 0.025$, and so statistical analyses will no doubt suffer from small-number errors. For these reasons, we opt to restrict our sample to the redshift range $0.025 < z < 0.06$. This reduces our sample size to 3845 galaxies across a volume of $210120.2 \text{ Mpc}^3$. Beyond this lower redshift boundary, all eyeball classification morphologies exhibit a relatively flat number fraction with redshift. We rule out the global division method, owing to the subjective nature of its design and its inability to resolve barred structure. Both the LDA and QDA methods provide good analogues for the Eyeball method, however, do not significantly improve on the redshift distribution to become favoured classification schema.

When considering the luminosity function distributions across each morphological type for each method we find that the statistical and quantitative methods prove unable to reproduce those as predicted by Binggeli et al. (1988). Only the Eyeball classification method is able to produce a 'believable' morphology luminosity function, with Elliptical galaxies dominating the bright end. Notably, the LDA method significantly underestimates the E-type galaxies at the brightest end.
Chapter 5. Morphological Analysis

As a further test of the Eyeball classification method, we opt to match our morphological classifications against those of the well-established citizen science project Galaxy Zoo (Lintott et al., 2008). The Galaxy Zoo project brings together morphological classifications made by thousands of volunteer ‘citizen scientists’; typically members of the public rather than professional astronomers. Each classifier is shown a series of false-colour RGB SDSS postage stamp images and asked a series of questions about the visual nature of the galaxy in a similar manner to our method described above in Section 5.2.1. These questions include the smoothness of the galaxy profile, the presence and shape (boxy/disky) of a bar, how tightly wound any spiral arms are and how numerous they are, and if any other unusual features are present. Based upon the answers given, a series of morphological classifications are produced for a large fraction of the SDSS dataset. For comparison with our own data, we employ the Galaxy Zoo 1 data release (GZ1; Lintott et al., 2011) in our analysis below. Specifically, this data is taken from Table 2 of Lintott et al. (2011).

GZ1 contains 667,944 sources down to an SDSS apparent magnitude limit of $r = 17.77$ for all galaxies in the SDSS Data Release 7 which have spectra included. Inclusion of the spectra allows for the bias in the sample to be estimated (see Bamford et al., 2009 for more details), and so is our preferred source of data. Of these 667,944 objects, 1,786 galaxies exhibit a direct match with the galaxies in our volume limited sample of 3,845 (∼ 46%) when matching by SDSS object ID (OBJID). A probability for whether each galaxy is a spiral or an elliptical is determined based upon the number of independent classifications in agreement (or ‘votes’). We use a probability threshold of 80% to isolate a high-fidelity sample of morphological classifications from GZ1 with which to match to our own data. This further reduces our sample to 733 galaxies (19% of our volume limited sample). Of the remaining galaxies, each is assigned to either an elliptical or spiral classification. Our own sample is similarly split into this binary classification system, with ellipticals in the ‘elliptical’ bin, and everything else classified as ‘spiral’.

Figure 5.23 shows the cross-correlation results between our own visual classifications and those provided by Galaxy Zoo. The number of galaxies within each bin are shown as ‘correlation bubbles’, with larger bubbles corresponding to a higher number of objects within that bin. The fraction of galaxies within each classification bin is also quantified as a percentage of galaxies in our own study (left) and of galaxies from Galaxy Zoo (right). As is shown, the entirety of the Galaxy Zoo spiral population are also classified as spirals by our method (i.e.,
5.3. Choosing The Correct Morphology

Figure 5.23: A visual representation of the level of agreement between our eyeball visual classifications and those provided by the Galaxy Zoo project (Lintott et al., 2008, 2011). These figures are constructed using a common dataset of 733 galaxies for which reliable Galaxy Zoo classification data exist. Percentages shown depict the fractional agreement with our own classifications (left) and with the Galaxy Zoo classifications (right), that is; percentages in any given column total 100%. The Galaxy Zoo spiral population is a subset of our own), but not all of our spiral galaxies are found to be spiral in the Galaxy Zoo data. Similarly, the entirety of our elliptical population is classified as elliptical by Galaxy Zoo, but not all Galaxy Zoo ellipticals are elliptical by our method. Using either our own method or Galaxy Zoo as a reference baseline, we find that the primary axis (i.e., the \([E,E] \rightarrow [Sp,Sp]\) axis) remains strong in both cases. The largest level of disagreement arises from those galaxies for which Galaxy Zoo determine their morphological type to be elliptical, with only 77.8% of our sample in agreement. Typically however, the level of agreement between these two projects remains high.

The focus of this section is in weighing up the arguments in favour of and against the various classification methods explored here. Visual (eyeball) classifications provide a good representation of the number fraction variation in morphology with redshift, quite unlike the quantitative cuts method. Both the LDA and QDA statistical methods similarly provide a good number fraction evolution, however, Occam’s razor suggests in this case that visual classifications are the preferred means on which to base a morphological classification schema. Visual classification provides the only realistic morphology luminosity function results, adding further credence to this classification method. In addition, our visual classification results are strongly validated by the strong synergies with the Galaxy Zoo data shown in Figure 5.23. For this reason and the others outlined in this Section, we adopt Eyeball classifications as our
preferred method of morphological classification. Visual classifications are used throughout the remainder of this thesis.
The Mass Function and its Division by Type and Component

The complexity of structure within galaxies ranges from simplistic smooth one-component systems, such as classical elliptical galaxies, to complex multi-component systems, such as barred grand-design spiral galaxies. Each distinct sub-structure exists as a remnant of various competing galaxy formation mechanisms. Developing new methods to automatically and reliably measure galaxy structure with cosmic time will provide constraints on the dominance or existence of the various evolutionary pathways, ultimately helping us answer the question of how some fraction of the baryonic mass of the Universe has evolved.

In this chapter I explore the variation in galaxy space-density distributions in the local Universe through global luminosity and global mass functions. In addition, I provide one of the first measurements of the Morphology Luminosity Function (MLF) and Morphology Mass Function (MMF), showing how the constituent morphologies combine to form the observed global distributions. Finally, I provide structural decompositions of this sample using prior visual classifications as a guide towards choosing the correct structural makeups. With these structural decompositions, I derive the stellar mass function for ellipticals, bulges and disks,
and determine the amount of stellar mass in the local Universe resident in spheroidal (pressure supported systems, formed via dynamically hot processes, e.g. merging and/or collapse) and disk-like (rotationally supported systems, formed via dynamically cold processes, e.g. gas accretion and secular evolution) structures. The data used in this Chapter are taken from the volume limited sample of 3,845 galaxies defined in Chapter 5. A standard cosmology of \((H_0, \Omega_m, \Omega_\Lambda) = (70 \text{ km s}^{-1} \text{ Mpc}^{-1}, 0.3, 0.7)\) is assumed throughout this chapter.

6.1 Number, Luminosity and Mass by Morphology

Here we analyse the variation in global number density, luminosity distributions and discuss the implication this has for the morphological stellar mass breakdown of our sample.

6.1.1 Bivariate Brightness Distribution

The bivariate brightness distribution (BBD; Driver, 1999) relates absolute magnitude to surface brightness, allowing for survey limits to be managed. The BBD also highlights the correlation between luminosity and surface brightness for various populations, and potentially ties key observable quantities to astronomically interesting parameters (i.e., absolute magnitude to stellar mass and effective surface brightness to angular momentum). The BBD is a complimentary orthogonal projection of the Kormendy relation (also known as the HK relation: Hamabe & Kormendy, 1987; Capaccioli et al., 1992), which is a relation between surface brightness against half-light radius for elliptical galaxies.

Figure 6.1 shows the Sérsic \(r\) band absolute magnitude (truncated at 10 \(r_e\)) against the absolute average effective surface brightness (i.e., the average surface brightness within the half-light radius) BBD. The absolute average effective surface brightness, \(\langle \mu^e \rangle\), is given by:

\[
\langle \mu^e \rangle = \langle \mu_e \rangle - 10 \log(1 + z) - k(z)
\]  

(6.1)

The data are represented by a background density grid, constructed from a mesh with intervals of 0.041 in mag and 0.031 in surface brightness (a 256 \(\times\) 256 grid). Darker colours indicate more galaxies lie within that grid element. Solid and dashed black lines show the survey limits that constrict our data frame at the lower and upper redshift bounds respectively. In brief, the minimum size limit is set by the pixel resolution (\(r_e = 0.339''\)), the maximum size limit is set by sky estimation boundaries (\(r_e = 20''\), see below), the faint end limit corresponds to the GAMA \(r\) band Petrosian limit (\(m_r = 19.8\) mag), and the surface brightness limit has been calculated by Jon Loveday (\(\langle \mu^e \rangle = 23.86\) mag/arcsec\(^2\), priv. comm.). The upper size limit
arises from the need for accurate sky estimation within the GALFIT software. An average background sky grid size is $128 \times 128$ pixels. Typically, at least 25% of these pixels must be sky pixels in order for the software to accurately estimate the surface brightness profile of the galaxy out into its wings. For a face-on galaxy, this gives us a maximum radius of $\sim 20''$ before sky estimation becomes an issue.

The limits shown above all vary with redshift even across the modest range covered within our sample. To indicate this, solid straight lines in Figure 6.1 represent our lower redshift boundary of $z = 0.025$ and dashed lines represent our upper redshift boundary of $z = 0.06$. These lines show that as we move from low to high redshift we lose the ability to resolve smaller, fainter, dimmer galaxies, as expected (see Cameron & Driver, 2007). Overlaid on top of the background greyscale map are the contours showing the location of various morphological types (E, red; S0a, purple; Sbc, green; Sd, blue). The barred populations have been merged into their parent classes owing to low number statistics for those two populations. Contours indicate 10 and 100 galaxies, where applicable.

The majority of our sample lies well within the survey boundaries, implying that derived statistics remain robust owing to our high level of completeness. Only the Sd-type population suffers any notable incompleteness, with potential population truncations at the lower size boundary, the faint end boundary and the dim surface brightness boundary. Some extremely faint Ellipticals (or dE) may also be omitted from our sample due to their small sizes. The truncation in the Sd population will have subsequent implications on derived quantities such as the faint-end slope of the luminosity function for that type. In addition, the relatively bright faint-end limits of this volume-limited sample preclude any further exploration of the dwarf and irregular populations of galaxies as may be found in previous studies (e.g., Driver, 1999; Pérez-González et al., 2001; Boselli et al., 2008).

On close examination of the BBD figure we note that elliptical (red) and early-type spiral (purple) populations occupy a similar parameter space, both peaking at $M \sim -21$ and $\langle \mu_e \rangle \sim 20$. E-type galaxies exhibit a uniformly narrow range in surface brightness spanning all magnitudes. Conversely, the late-type spiral Sbc class occupies an intermediate position between the spheroids and the pure-disks. This may arise owing to the complex mix of structure within these systems, whereby a blue dominant disk and a red submissive bulge blend to produce a ‘green valley’ population of galaxies. Structural decomposition may separate out these components and remove the green-valley population, as noted by Chilingarian et al.
Chapter 6. The Mass Function and its Division by Type and Component

Figure 6.1: The $r$ band bivariate brightness distribution for all galaxies within our volume limited sample, displaying absolute Sérsic magnitude against absolute average effective Sérsic surface brightness (i.e., the average surface brightness within the half-light radius). Contours for the morphological types E, S0a, Sbc and Sd are coloured red, purple, green and blue respectively. Numbers inset into the contours represent the number of galaxies to be found at that contour density. The underlying greyscale map shows the number density of all galaxies within this sample, regardless of morphology. Galaxy density number count estimates are produced for each cell with intervals of 0.041 in mag and 0.031 in surface brightness (a $256 \times 256$ grid). Solid and dashed lines represent the survey limits at the lower and upper redshift bounds (see text for more details). Solid lines represent these limits at $z = 0.025$, and dashed at $z = 0.06$, for reference.
In contrast to the E-type galaxies, Sd galaxies rise in number up to the faint end limit and are spread across the full observable range of surface brightnesses. Presumably this trend in Sd-type galaxies would typically extend faintwards towards the dE and Irr classes at the faint and dim end of the figure, were deeper data available.

Based on our BBD it is reasonably safe to conclude that derived luminosity and mass function parameters for the E, S0a and Sbc classes should be robust as the distributions decline prior to the selection limits (i.e., they remain well sampled) whereas the Sd class will suffer a loss of accuracy owing to the lack of completeness at the faint end. However a significant fraction of the Sd sub-population fall well within our survey limits, and so measured parameters may be understood to have been derived via extrapolation.

6.1.2 Morphology Luminosity Function (MLF)

The galaxy luminosity function (GLF) provides a measure of the number density of galaxies for a given volume within discreet luminosity intervals. The GLF is dominated by large numbers of very faint galaxies, whilst the number counts of brighter galaxies drop off sharply beyond some given luminosity ($L^*$). Despite their numerical advantage however, low luminosity systems tend to contribute a relatively small fraction to the total luminosity budget of any given volume (Driver, 1999).

The accurate measurement of the GLF for both field and cluster galaxies (Efstathiou et al., 1988a; Binggeli et al., 1988) has been a hot topic in extragalactic astronomy for some time (Peebles & Hauser, 1974; Felten, 1977). The advent of Schechter’s analytical form of the GLF, the Schechter luminosity function (Schechter, 1976) allowed for the rapid exploration of the GLF at optical wavelengths. Expansion into other wavelength regimes such as the near infrared (e.g., Hill et al., 2010) provide powerful datasets with which to explore new avenues of research including the measurement of the total energy output of the Universe (Driver et al., 2008); constraining models of galaxy formation and evolution (Benson et al., 2003); analysing the implications the GLF may have on the formation of galaxy sub-structure (Cole et al., 2000); and as a complimentary measurement of the mass equivalent, the galaxy stellar mass function (GSMF; Baldry et al. (2008, 2012) and also see Section 6.1.4).

Figure 6.2 shows the GLF across nine GAMA wavelengths ($ugrizYJHK$) for our volume-limited sample, and subdivided by morphological type. Each population (total and morphological type) are binned into absolute Sérsic magnitude bins of 0.25 mag and fit with a single Schechter function. Errors on each bin are taken to be Poissonian in nature. Shaded grey
Chapter 6. The Mass Function and its Division by Type and Component

Figure 6.2: Morphology Luminosity Functions across all nine bands for the various morphological types (coloured points and lines, as indicated) and total populations (black points and lines). Each population has been fit with a single Schechter function. Prior to fitting, the data is split into bins of 0.25 mag, with the error on the measurement per bin taken as Poissonian ($\sqrt{n}$) in nature. Shaded grey areas indicate those regions where data has not been used in the fits. Variable faint-end magnitude limits are given in Table 5.2.

These data provide one of the first measurements of the Morphology Luminosity Function (MLF) using Sérsic photometry, and provide a key insight into the nature of the underlying galaxy populations. The knee in the total Schechter function progresses smoothly towards brighter AB magnitudes as one moves from $u \rightarrow K$, as expected, and is generally well fit by a
single Schechter function until $\sim z$ band. At longer wavelengths, the GLF appears to require a secondary component to aid in fully reproducing the downturn at the bright end and the secondary upturn at the faint end.

When looking at the morphology sub-populations, the faint end is entirely dominated by Sd-class galaxies, with intermediate magnitudes typically containing both the early- and late-type spiral systems. Elliptical galaxies dominate at the brightest magnitude, however, below their $L^*$ knee the number of E-type galaxies remains relatively constant across all wavelengths. The Sd population appears to show the largest variation in MLF with respect to wavelength, with the faint end slope varying wildly from $u \rightarrow K$ as the relative depth of the data in those bands becomes shallower. Owing to our sample selection constraints and the quality of the $r$ band data, one would expect the $M^*$ and $\alpha$ parameters for Sd type galaxies in the $r$ band to be the most well constrained, which is evidenced by the distinctive downturn for Sd type galaxies being the most severe in the $r$ band data. In contrast, the E, S0a and Sbc populations tend to progress steadily towards brighter magnitudes. This is as expected as the majority of these data lies well within our survey selection boundaries (see Figure 6.1).

Single Schechter fit parameters are shown for all populations in Tables 6.1 (All); 6.2 (E); 6.3 (S0a); 6.4 (Sbc) and 6.5 (Sd). In addition to the Schechter fit parameters, we also calculate the luminosity density for each population at each wavelength. The luminosity density, $j$, is the integral under the Schechter function curve and is given by

$$j = \int_{0}^{\infty} L\phi(L)dL = \phi^*L^*\Gamma(\alpha + 2)$$  \hspace{1cm} (6.2)

Alongside the characteristic knee in the Schechter luminosity function, $L^*$ (or $M^*$ in terms of magnitude), the two other fitted parameters are the slope of the faint end of the LF, $\alpha$, and the normalisation constant, $\phi^*$. Whilst the error on the latter parameter may be estimated via some simplistic method such as jackknife resampling of the data set, the well-known covariance between $\alpha$ and $M^*$ would result in their jackknife errors being systematically underestimated. An alternative approach is to produce ‘error ellipses’ which map out the $\chi^2$ parameter space around the best fit values. This technique involves re-fitting the data set fitting for $\phi^*$ alone whilst adopting a fixed pair of input $\alpha$ and $M^*$ parameters as defined by a regularly spaced grid about the best fit values. The resultant $\chi^2$ surface then allows for 1, 2 and 3$\sigma$ errors to be estimated as the contours which lie at $\Delta \chi^2 = 2.30, 6.17$ and 11.8, respectively."
Table 6.1: Single Schechter Luminosity Function Fit Parameters for the total GLF as shown in Figure 6.2. From left to right, columns are: GAMA passband; the knee in the Schechter function ($M^*$); the slope of the faint end of the Schechter function ($\alpha$); the normalisation constant for the Schechter function fit ($\phi^*$); the $\chi^2$ goodness of fit parameter ($\chi^2/\nu$); the luminosity density ($j$). Errors on $M^*$ and $\alpha$ are taken from the 1σ error ellipses shown in Figure 6.3. All others errors are estimated using the relation $\sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2$, where $\bar{x}$ is the best fit parameter, $x_i$ is the best fit parameter as given from a jackknife resampled variant of the data set and $N$ is represents the number of jackknife volumes. We adopt $N = 10$.

We applied this error-ellipse technique to the nine bands of photometry from our dataset, the results of which are shown in Figure 6.3. The total and morphology error ellipses are shown, as indicated. Successive contours relate to 1, 2 and 3σ errors on each parameter. The strong diagonal covariance between these two parameters has a strong impact on all of the measurements shown, notably so for the Sd population. The faint end slope of the Sd class in the $u$ band, $\alpha_u = -0.69 \pm 0.03$, is particularly poorly constrained. This is as expected owing to the poor quality and relatively shallow depth of the $u$ band in conjunction with the completeness issues for the Sd population as shown in Figure 6.1. Only in the $r$ band is the error on the value of $\alpha$ for the Sd class lower than any of the other morphological types. Conversely, whilst $\alpha$ is typically well constrained for the Elliptical populations, the value of the knee in the Schechter function is not. In the $z$ band for example, the turnover is found at $M^*_z = -19.66 \pm 0.27$; a relatively large uncertainty. Also note the relative stability in recovered $\alpha$ values for all populations over all wavelengths, excepting the $u$ band as discussed above.

Each of the morphology sub-populations appear to be well fit by a single Schechter function, with reduced $\chi^2$ parameters typically lying within the range $0.5 < \chi^2/\nu < 3$. The only notable exception to this is the Sd population. The knee of the Sd population may in some
Figure 6.3: Error ellipses for each Schechter function fit shown in Figure 6.2. These ellipses are generated by constructing a regularly spaced grid of input $M^*$ and $\alpha$ values in steps of 0.01 each and fitting for the normalisation constant $\phi^*$ in the Schechter function, producing a $\chi^2$ map about the coordinates of the best fit. Successive contours represent the 1$\sigma$, 2$\sigma$ and 3$\sigma$ error boundaries ($\Delta \chi^2 = 2.30, 6.17$ and 11.8 respectively). Note the significant diagonal elongation between these parameters, particularly for the Sd class population. This highlights the covariant relationship between $M^*$ and $\alpha$. 

6.1. Number, Luminosity and Mass by Morphology
## Chapter 6. The Mass Function and its Division by Type and Component

### Table 6.2: As Table 6.1 but for Elliptical galaxies.

<table>
<thead>
<tr>
<th>Band</th>
<th>( M^* ) (mag)</th>
<th>( \alpha )</th>
<th>( \phi^*/10^{-3} ) (dex(^{-1})Mpc(^{-3}))</th>
<th>( \chi^2/\nu )</th>
<th>( j/10^8 ) (L(_\odot)Mpc(^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u )</td>
<td>(-19.79^{+0.45}_{-0.68} )</td>
<td>(-1.03^{+0.23}_{-0.22} )</td>
<td>0.88 ( \pm ) 0.50</td>
<td>0.68</td>
<td>0.26 ( \pm ) 0.17</td>
</tr>
<tr>
<td>( g )</td>
<td>(-21.12^{+0.30}_{-0.34} )</td>
<td>(-0.82^{+0.12}_{-0.12} )</td>
<td>1.03 ( \pm ) 0.19</td>
<td>1.11</td>
<td>0.31 ( \pm ) 0.04</td>
</tr>
<tr>
<td>( r )</td>
<td>(-21.82^{+0.26}_{-0.28} )</td>
<td>(-0.76^{+0.09}_{-0.08} )</td>
<td>1.08 ( \pm ) 0.13</td>
<td>1.31</td>
<td>0.40 ( \pm ) 0.03</td>
</tr>
<tr>
<td>( i )</td>
<td>(-22.33^{+0.32}_{-0.37} )</td>
<td>(-0.87^{+0.09}_{-0.09} )</td>
<td>0.89 ( \pm ) 0.03</td>
<td>1.09</td>
<td>0.47 ( \pm ) 0.06</td>
</tr>
<tr>
<td>( z )</td>
<td>(-23.10^{+0.46}_{-0.65} )</td>
<td>(-1.00^{+0.09}_{-0.09} )</td>
<td>0.60 ( \pm ) 0.02</td>
<td>1.95</td>
<td>0.68 ( \pm ) 0.03</td>
</tr>
<tr>
<td>( Y )</td>
<td>(-22.96^{+0.39}_{-0.52} )</td>
<td>(-0.93^{+0.09}_{-0.09} )</td>
<td>0.71 ( \pm ) 0.03</td>
<td>0.74</td>
<td>0.67 ( \pm ) 0.03</td>
</tr>
<tr>
<td>( J )</td>
<td>(-22.96^{+0.33}_{-0.40} )</td>
<td>(-0.92^{+0.09}_{-0.08} )</td>
<td>0.72 ( \pm ) 0.03</td>
<td>0.95</td>
<td>0.70 ( \pm ) 0.03</td>
</tr>
<tr>
<td>( H )</td>
<td>(-23.15^{+0.33}_{-0.38} )</td>
<td>(-0.89^{+0.08}_{-0.08} )</td>
<td>0.78 ( \pm ) 0.03</td>
<td>1.52</td>
<td>1.03 ( \pm ) 0.04</td>
</tr>
<tr>
<td>( K )</td>
<td>(-22.83^{+0.35}_{-0.43} )</td>
<td>(-0.89^{+0.09}_{-0.09} )</td>
<td>0.80 ( \pm ) 0.05</td>
<td>1.26</td>
<td>1.22 ( \pm ) 0.07</td>
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</tbody>
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### Table 6.3: As Table 6.1 but for S0a galaxies.

<table>
<thead>
<tr>
<th>Band</th>
<th>( M^* ) (mag)</th>
<th>( \alpha )</th>
<th>( \phi^*/10^{-3} ) (dex(^{-1})Mpc(^{-3}))</th>
<th>( \chi^2/\nu )</th>
<th>( j/10^8 ) (L(_\odot)Mpc(^{-3}))</th>
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</thead>
<tbody>
<tr>
<td>( u )</td>
<td>(-18.25^{+0.20}_{-0.21} )</td>
<td>(0.61^{+0.31}_{-0.28} )</td>
<td>2.69 ( \pm ) 0.12</td>
<td>1.20</td>
<td>0.28 ( \pm ) 0.01</td>
</tr>
<tr>
<td>( g )</td>
<td>(-19.49^{+0.17}_{-0.17} )</td>
<td>(1.16^{+0.31}_{-0.27} )</td>
<td>2.10 ( \pm ) 0.24</td>
<td>3.11</td>
<td>0.35 ( \pm ) 0.02</td>
</tr>
<tr>
<td>( r )</td>
<td>(-20.27^{+0.17}_{-0.17} )</td>
<td>(0.95^{+0.29}_{-0.25} )</td>
<td>2.40 ( \pm ) 0.24</td>
<td>1.82</td>
<td>0.45 ( \pm ) 0.03</td>
</tr>
<tr>
<td>( i )</td>
<td>(-20.68^{+0.17}_{-0.16} )</td>
<td>(0.89^{+0.27}_{-0.23} )</td>
<td>2.46 ( \pm ) 0.21</td>
<td>1.50</td>
<td>0.56 ( \pm ) 0.03</td>
</tr>
<tr>
<td>( z )</td>
<td>(-21.00^{+0.17}_{-0.17} )</td>
<td>(0.78^{+0.26}_{-0.23} )</td>
<td>2.55 ( \pm ) 0.16</td>
<td>1.29</td>
<td>0.70 ( \pm ) 0.04</td>
</tr>
<tr>
<td>( Y )</td>
<td>(-21.16^{+0.16}_{-0.16} )</td>
<td>(0.69^{+0.24}_{-0.21} )</td>
<td>2.61 ( \pm ) 0.17</td>
<td>0.33</td>
<td>0.75 ( \pm ) 0.05</td>
</tr>
<tr>
<td>( J )</td>
<td>(-21.26^{+0.16}_{-0.15} )</td>
<td>(0.78^{+0.24}_{-0.21} )</td>
<td>2.47 ( \pm ) 0.22</td>
<td>1.03</td>
<td>0.88 ( \pm ) 0.06</td>
</tr>
<tr>
<td>( H )</td>
<td>(-21.63^{+0.16}_{-0.16} )</td>
<td>(0.67^{+0.23}_{-0.20} )</td>
<td>2.59 ( \pm ) 0.21</td>
<td>1.43</td>
<td>1.34 ( \pm ) 0.12</td>
</tr>
<tr>
<td>( K )</td>
<td>(-21.30^{+0.16}_{-0.16} )</td>
<td>(0.69^{+0.23}_{-0.21} )</td>
<td>2.59 ( \pm ) 0.19</td>
<td>1.14</td>
<td>1.58 ( \pm ) 0.13</td>
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</tbody>
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### Table 6.4

<table>
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<th>Band</th>
<th>( M^* )</th>
<th>( \alpha )</th>
<th>( \phi^*/10^{-3} )</th>
<th>( \chi^2/\nu )</th>
<th>( j/10^8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(mag)</td>
<td>(dex(^{-1})Mpc(^{-3}))</td>
<td>(L(_\odot)Mpc(^{-3}))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( u )</td>
<td>(-18.21^{+0.20}_{-0.21} )</td>
<td>(0.28^{+0.28}_{-0.25})</td>
<td>(4.11 \pm 0.15)</td>
<td>(1.82)</td>
<td>(0.33 \pm 0.03)</td>
</tr>
<tr>
<td>( g )</td>
<td>(-19.35^{+0.18}_{-0.16} )</td>
<td>(0.59^{+0.21}_{-0.18})</td>
<td>(3.90 \pm 0.32)</td>
<td>(3.88)</td>
<td>(0.35 \pm 0.04)</td>
</tr>
<tr>
<td>( r )</td>
<td>(-19.79^{+0.21}_{-0.21} )</td>
<td>(0.73^{+0.26}_{-0.23})</td>
<td>(3.72 \pm 0.53)</td>
<td>(4.25)</td>
<td>(0.37 \pm 0.05)</td>
</tr>
<tr>
<td>( i )</td>
<td>(-20.30^{+0.19}_{-0.19} )</td>
<td>(0.45^{+0.20}_{-0.18})</td>
<td>(3.85 \pm 0.24)</td>
<td>(4.47)</td>
<td>(0.43 \pm 0.06)</td>
</tr>
<tr>
<td>( z )</td>
<td>(-20.58^{+0.18}_{-0.18} )</td>
<td>(0.39^{+0.19}_{-0.16})</td>
<td>(3.83 \pm 0.12)</td>
<td>(3.76)</td>
<td>(0.53 \pm 0.07)</td>
</tr>
<tr>
<td>( Y )</td>
<td>(-20.51^{+0.19}_{-0.19} )</td>
<td>(0.43^{+0.19}_{-0.16})</td>
<td>(3.63 \pm 0.18)</td>
<td>(3.61)</td>
<td>(0.47 \pm 0.05)</td>
</tr>
<tr>
<td>( J )</td>
<td>(-20.96^{+0.18}_{-0.18} )</td>
<td>(0.10^{+0.15}_{-0.13})</td>
<td>(3.48 \pm 0.21)</td>
<td>(2.83)</td>
<td>(0.59 \pm 0.06)</td>
</tr>
<tr>
<td>( H )</td>
<td>(-21.28^{+0.18}_{-0.18} )</td>
<td>(0.04^{+0.14}_{-0.12})</td>
<td>(3.57 \pm 0.24)</td>
<td>(2.77)</td>
<td>(0.91 \pm 0.11)</td>
</tr>
<tr>
<td>( K )</td>
<td>(-21.02^{+0.17}_{-0.16} )</td>
<td>(0.06^{+0.13}_{-0.11})</td>
<td>(3.53 \pm 0.15)</td>
<td>(3.29)</td>
<td>(1.10 \pm 0.11)</td>
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</tbody>
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Table 6.4: As Table 6.1 but for Sbc galaxies.

### Table 6.5

<table>
<thead>
<tr>
<th>Band</th>
<th>( M^* )</th>
<th>( \alpha )</th>
<th>( \phi^*/10^{-3} )</th>
<th>( \chi^2/\nu )</th>
<th>( j/10^8 )</th>
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<td>(mag)</td>
<td>(dex(^{-1})Mpc(^{-3}))</td>
<td>(L(_\odot)Mpc(^{-3}))</td>
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<td></td>
</tr>
<tr>
<td>( u )</td>
<td>(-17.21^{+0.27}_{-0.30} )</td>
<td>(-0.66^{+0.51}_{-0.47})</td>
<td>(12.64 \pm 1.52)</td>
<td>(1.11)</td>
<td>(0.31 \pm 0.04)</td>
</tr>
<tr>
<td>( g )</td>
<td>(-18.34^{+0.22}_{-0.25} )</td>
<td>(-0.92^{+0.27}_{-0.26})</td>
<td>(11.35 \pm 2.00)</td>
<td>(1.16)</td>
<td>(0.27 \pm 0.02)</td>
</tr>
<tr>
<td>( r )</td>
<td>(-18.66^{+0.16}_{-0.16} )</td>
<td>(-0.61^{+0.18}_{-0.17})</td>
<td>(12.24 \pm 1.00)</td>
<td>(2.04)</td>
<td>(0.24 \pm 0.01)</td>
</tr>
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<td>( i )</td>
<td>(-19.14^{+0.22}_{-0.23} )</td>
<td>(-1.02^{+0.23}_{-0.21})</td>
<td>(8.94 \pm 1.50)</td>
<td>(1.17)</td>
<td>(0.27 \pm 0.02)</td>
</tr>
<tr>
<td>( z )</td>
<td>(-19.63^{+0.28}_{-0.33} )</td>
<td>(-1.31^{+0.24}_{-0.23})</td>
<td>(5.75 \pm 1.82)</td>
<td>(1.45)</td>
<td>(0.35 \pm 0.04)</td>
</tr>
<tr>
<td>( Y )</td>
<td>(-19.70^{+0.30}_{-0.33} )</td>
<td>(-1.41^{+0.24}_{-0.22})</td>
<td>(4.61 \pm 1.58)</td>
<td>(1.11)</td>
<td>(0.34 \pm 0.06)</td>
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<tr>
<td>( J )</td>
<td>(-19.70^{+0.31}_{-0.36} )</td>
<td>(-1.33^{+0.26}_{-0.24})</td>
<td>(4.55 \pm 1.44)</td>
<td>(1.47)</td>
<td>(0.31 \pm 0.04)</td>
</tr>
<tr>
<td>( H )</td>
<td>(-20.20^{+0.33}_{-0.39} )</td>
<td>(-1.54^{+0.23}_{-0.21})</td>
<td>(3.17 \pm 1.15)</td>
<td>(0.90)</td>
<td>(0.56 \pm 0.09)</td>
</tr>
<tr>
<td>( K )</td>
<td>(-19.80^{+0.41}_{-0.51} )</td>
<td>(-1.51^{+0.31}_{-0.28})</td>
<td>(3.49 \pm 1.32)</td>
<td>(1.23)</td>
<td>(0.63 \pm 0.12)</td>
</tr>
</tbody>
</table>

Table 6.5: As Table 6.1 but for Sd galaxies.
cases lie outwith the fitting limits, and so the overall fit may instead be better suited to a single exponential function. An alternative (or possibly additional) explanation is that we are beginning to see the impact of dE, Sm and Irregular type galaxies creeping into the Sd class. Since the classification method employed in Section 5.2.1 is insensitive to these classes, it is relatively likely that they would appear as late-type (blue) single-component systems, i.e., Sd. If this were true then a second Schechter parameter fit would be required to accurately bring out this population.

It is evident however that at wavelengths longer than the $z$ band a single Schechter fit to the total GLF is a poor fit, reaching a maximum goodness of fit value of $\chi^2/\nu = 3.77$ in the $Y$ band. This is as expected if one considers that the GLF is comprised of an initial red spheroidal 'bump' at bright magnitudes and then a subsequent blue disk 'bump' at fainter magnitudes, as can clearly be seen in Figures 6.2 and 6.4, and noted in, e.g., Loveday et al. (2012). A single Schechter function is unable to account for the intricacy in this distribution.

We elect to fit the total GLF with a double Schechter function with a shared knee, whilst maintaining single Schechter fits to the morphology sub-populations. The free parameters for the double Schechter fit are $M^*$, $\alpha_1$, $\phi^*_1$, $\alpha_2$ and $\phi^*_2$. The results of this fit are shown in Figure 6.4 for all nine bands, and the fit parameters given in Table 6.6. It is instantly apparent that the GLF is more naturally fit with a double Schechter function of the form applied than a single Schechter function, particularly so for the longer NIR passbands. All $\chi^2$ parameters beyond the $z$ band show a significant improvement in the quality of the fit. However, the shortest wavelengths show little need for the extra parameter, with the goodness of fit showing a mild worsening in the $u$ band, again most likely owing to the poorer quality of the $u$ band data. Nevertheless, the overall fits appear robust, and so we advocate a double Schechter form for the total GLF but single Schechter function forms for the morphology sub-population MLFs.

A summary of both the single and double Schechter fits to the GLF in addition to the adopted single Schechter fits to the MLFs in the $r$ band are shown in Figure 6.5. Also shown are several other contemporary single Schechter fits to similar $r$ band data, scaled to our preferred cosmology of $(H_0, \Omega_m, \Omega_\Lambda) = (70, 0.3, 0.7)$ and $k$-corrected where necessary from $r_{0.1}$ to $r$. There is generally good agreement between our global luminosity function fits and those of other studies. The variable faint end limit between surveys makes a comparison of the faint end slope problematic, however, the $M^*$ and $\phi^*$ parameters agree well to within their errors. The need for a second Schechter component in the $r$ band is less evident than
6.1. Number, Luminosity and Mass by Morphology

Figure 6.4: Morphology luminosity functions across all nine bands for the various morphological types (coloured points and lines, as indicated) and total populations (black points and lines). Each morphological population has been fit with a single Schechter function and is identical to those as shown in Figure 6.2. Total populations have been fit with a double Schechter function. Prior to fitting, the data is split into bins of 0.25 mag, with the error on the measurement per bin taken as Poissonian ($\sqrt{n}$) in nature. Shaded grey areas indicate those regions where data has not been used in the fits. Variable faint-end magnitude limits are given in Table 5.2. The additional Schechter function for the total population allows for the notable upturn at faint magnitudes to be properly accounted for, especially at longer wavelengths.
Chapter 6. The Mass Function and its Division by Type and Component

\[
\begin{align*}
M^\ast \alpha_1 \phi^*_{\alpha_1} / 10^{-3} \alpha_2 \phi^*_{\alpha_2} / 10^{-3} \chi^2 / \nu_j / 10^{-8} (\text{mag}) (\text{dex}^{-1} \text{Mpc}^{-3}) (\text{dex}^{-1} \text{Mpc}^{-3})(\text{L}_\odot \text{Mpc}^{-3}) \\
\mu \pm 0.76 \pm 0.30 \pm 0.06 \\
\gamma \pm 0.28 \pm 0.18 \pm 0.06 \\
\lambda \pm 0.18 \pm 0.08 \pm 0.06 \\
\pi \pm 0.69 \pm 0.22 \pm 0.06 \\
\nu \pm 0.52 \pm 0.32 \pm 0.06 \\
\text{Table 6.6: Double Schechter luminosity function fit parameters for the total GLF as shown in Figure 6.4. From left to right, columns are: GAMA passband; the shared knee in the Schechter function (M_\ast); the primary slope of the faint end of the Schechter function (\alpha_1); the primary normalisation constant for the Schechter function fit (\phi^*_{\alpha_1}); the secondary slope of the faint end of the Schechter function (\alpha_2); the secondary normalisation constant for the Schechter function (\phi^*_{\alpha_2}); the \chi^2 goodness of fit parameter for the second order term of the Schechter function (\chi^2 / \nu); the luminosity density (j). Errors are estimated from jackknifed resampling using the relation }
\sigma^2 = \frac{N - 1}{N} \sum_{N_i = 1}^N \left( \frac{x - \bar{x}}{\bar{x}} \right)^2, \text{ where } x \text{ is the best fit parameter, } \bar{x} \text{ is the best fit parameter estimated from jackknife resampling using the relation for the shared knee in the Schechter function (M_\ast), the luminosity density (j), the primary and secondary normalisation constants for the Schechter function (\phi^*_{\alpha_1}, \phi^*_{\alpha_2}), the \chi^2 goodness of fit parameter (\chi^2 / \nu); the sample mean of the parameter fit to the data is shown in Figure 6.4. From left to right, columns are: GAMA passband;}
\end{align*}
\]
6.1. Number, Luminosity and Mass by Morphology

Figure 6.5: Morphology luminosity functions in the r band as fit by a single-Schechter function in addition to both a single and double-Schechter total luminosity function shown in grey and black, respectively. Each morphology is labelled and coloured according to the inset legend. Prior to fitting, the data is split into bins of 0.25 mag, with the error per bin assumed as Poissonian ($\sqrt{n}$) in nature. Shaded grey areas ($M > -17.4$ and $n \leq 3$) indicate those regions where data has not been used in the fits. Schechter fit parameters from the global fits (inset, top left) in addition to single-Schechter fits from other studies are also shown for reference. Where appropriate, Schechter fit data from other studies has been k-corrected back to a $z = 0$ rest frame using the relation as shown in Equation 5.3. Blanton et al., 2003b; Montero-Dorta & Prada, 2009; Loveday et al., 2012 have been corrected in this fashion, whereas Benson et al. (2007); Hill et al. (2010) and Driver et al. (in prep.) have not. Note that the Benson et al. (2007) values have been scaled up by a factor of 10.

at longer wavelengths, however, its effects in causing a steeper drop off at the bright end can clearly be seen in improving the fit to the data.
6.1.3 Stellar Mass-to-Light Ratio

The stellar mass-to-light ratio, $\Upsilon$, is the ratio of the total stellar mass contained within a galaxy to its luminosity in a given band. Finding correlations between this ratio and observable quantities such as magnitude or colour (e.g., Baldry et al., 2006) allows for 1st order stellar mass estimates to be calculated for systems where no stellar mass information is initially available. Note that every galaxy in our volume limited sample has a stellar mass estimate, and so this method has not been used to generate mass estimates here.

We adopt stellar masses as given by the StellarMassesv06 catalogue (v6.7) available through the GAMA database. The construction of this catalogue is detailed extensively in Taylor et al. (2011). To summarise this study in brief, a series of Bruzual & Charlot (2003) ‘composite stellar population’ spectral models adopting a Chabrier (2003) Initial Mass Function and using a Calzetti et al. (2000) dust attenuation law are created. These spectra are subsequently rescaled by some normalisation factor in order that the synthetic spectral flux matches that as defined by a Kron-like aperture from Hill et al. (2011), whereby the value of the normalisation factor determines the stellar mass for that system.

Stellar mass-to-light ratios $M_\star/L_X$ are determined for each galaxy in each band (where $X = ugrizYJHK$), using Solar absolute magnitudes as given in Table 2.1 to calculate $L_X/L_\odot$. Luminosities are derived from single Sérsic absolute magnitudes, corrected for the effects of Milky Way dust absorption and extinction.

Figure 6.6 shows $M_\star/L_X$ as a function of absolute magnitude, $M_X$, for all galaxies within our volume limited sample. Data points are coloured according to their morphological type, as indicated. Moving from the shortest u band to the longest K band, note how the early-type populations (E, S0a, SB0a) transition downwards and flatten into a single flat plane beyond the z band. In addition, the spread of the late type (Sbc, SBcd, Sd) populations flatten out and begin to merge into the plane established by the early-type galaxies. The solid red and blue lines show a linear fit to the early- and late-type galaxy populations, the equations of which are inset into each panel. The H band early- and late-type best fit lines map out a very similar trend in both populations. As such, we conclude that the H band Sérsic photometry provides the best proxy for stellar mass in the absence of extensive stellar mass modelling.

6.1.4 Morphology Mass Function (MMF)

Perhaps a more natural arena within which to analyse the galaxy population is in relation to stellar mass rather than luminosity. Several other studies have previously measured the galaxy
6.1. Number, Luminosity and Mass by Morphology

Figure 6.6: The stellar mass-to-light ratios for all galaxies within our volume limited sample across all nine bands. Data points are coloured according to their morphological type (as indicated), and are plotted as absolute Sérsic magnitude in band $X$ as a function of mass-to-light ratio, $\Upsilon = M_\star/L_X$. Linear best fits to the early-type (E, S0a, SB0a) and late-type (Sbc, SBbcd, Sd) populations are shown as red and blue solid lines, respectively, the equations of which are given inset into the figure.
Chapter 6. The Mass Function and its Division by Type and Component

Table 6.7: Double Schechter stellar mass function fit parameters for the total GSMF as shown in Figure 6.7. From left to right, columns are: the shared knee in the Schechter function ($M^*$); the primary slope of the faint end of the Schechter function ($\alpha_1$); the primary normalisation constant for the Schechter function fit ($\phi_1^*$); the secondary slope of the faint end of the Schechter function ($\alpha_2$); the secondary normalisation constant for the Schechter function fit ($\phi_2^*$); the $\chi^2$ goodness of fit parameter ($\chi^2/\nu$).

<table>
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<tr>
<th>$M^*$ (mag)</th>
<th>$\alpha_1$</th>
<th>$\phi_1^*/10^{-3}$ (dex$^{-1}$Mpc$^{-3}$)</th>
<th>$\alpha_2$</th>
<th>$\phi_2^*/10^{-3}$ (dex$^{-1}$Mpc$^{-3}$)</th>
<th>$\chi^2/\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.55 ± 0.09</td>
<td>−0.23 ± 0.33</td>
<td>4.88 ± 0.67</td>
<td>−1.43 ± 0.16</td>
<td>1.07 ± 0.68</td>
<td>0.84</td>
</tr>
</tbody>
</table>

stellar mass function (GSMF; Baldry et al. (2008); Peng et al. (2010b); Baldry et al. (2012), and advocate the double Schechter form of the GSMF We also adopt to fit the stellar mass function with a double Schechter model, and a single Schechter model for the morphology mass function (MMFs) that constitute it. The results of these fits are shown in Figure 6.7, and the Schechter fit parameters in Tables 6.7 and 6.8.

The double Schechter form of the GSMF fits the bimodal form of the total population very well, with a goodness of fit parameter of $\chi^2/\nu = 1.21$. As can be seen, the initial high-mass peak is primarily made out of spiral galaxies with some small contribution from Elliptical type systems. As was seen for the MLF in Section 6.1.2, Elliptical galaxies appear to exist uniformly across a wide range of masses. Conversely, the low-mass end is dominated by Sd-type systems.

6.1.5 Stellar Mass Breakdown by Morphology

Figure 6.8 shows the stellar mass breakdown for the entirety of our volume limited sample of 3,845 galaxies. Within each classification bubble, values for the median logged stellar mass (left) and the fraction by stellar mass (right) are shown. We note that for our local-Universe sample approximately 2/3 of the stellar mass is locked up within early-type (Elliptical and S0a/SB0a) systems. This is in stark contrast to the results table as presented in Figure 5.8, whereby ~ 69% of our sample by number were disk-dominated late-type galaxies. The ratio of mass division between Elliptical and S0a type galaxies is split evenly at ~ 1/3 each of the total stellar mass budget. These two populations alone account for ~ 65% (bound) of the total stellar mass within the complete sample, and yet only account for ~ 27% (unbound) by number.

Sd type galaxies contribute only ~ 8% of the mass whilst accounting for ~ 53% of the population (to $M_r < -17.4$ mag) by number. This statistic should not be surprising, as the
Figure 6.7: Morphology Mass Functions in the $r$ band as fit by a single-Schechter function in addition to a double-Schechter total mass function shown in black. Each morphology is labelled and coloured according to the inset legend. The data is split into mass bins of 0.1 dex, with the error per bin assumed to be Poissonian ($\sqrt{n}$) in nature. Shaded grey areas ($\log M/M_\odot < 9$ and $n \leq 10$) indicate those regions where data has not been used in the fits. Schechter fit parameters from the global fit (top right) in addition to fits from other studies are also shown for reference.
Morphology log $\mathcal{M}^*/M_\odot$ $\alpha$ $\phi^*/10^{-3}$ $\chi^2/\nu$

<table>
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<th>Morphology</th>
<th>$\log\mathcal{M}^*/M_\odot$</th>
<th>$\alpha$</th>
<th>$\phi^*/10^{-3}$</th>
<th>$\chi^2/\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>10.92 ± 0.46</td>
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<td>0.94 ± 0.50</td>
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<tr>
<td>S0a</td>
<td>10.32 ± 0.06</td>
<td>0.54 ± 0.22</td>
<td>2.80 ± 0.14</td>
<td>0.89</td>
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<tr>
<td>Sbc</td>
<td>10.23 ± 0.08</td>
<td>−0.32 ± 0.11</td>
<td>3.19 ± 0.29</td>
<td>1.27</td>
</tr>
<tr>
<td>Sd</td>
<td>9.75 ± 0.15</td>
<td>−1.51 ± 0.18</td>
<td>2.44 ± 1.13</td>
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</table>

Table 6.8: Single Schechter stellar mass function fit parameters for the Morphology Mass Functions (MMFs) as shown in Figure 6.7. From left to right, columns are: the morphological class; the knee in the Schechter function ($\mathcal{M}^*$); the slope of the faint end of the Schechter function ($\alpha$); the normalisation constant for the Schechter function fit ($\phi^*$); the $\chi^2$ goodness of fit parameter ($\chi^2/\nu$). Errors are estimated from jackknifed resampling using the relation 

$$\sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_j - \bar{x})^2,$$

where $x$ is the best fit parameter, $x_j$ is the best fit parameter as given from a jackknife resampled variant of the data set and $N$ is the number of jackknife volumes. We adopt $N = 10$.

Figure 6.8: The breakdown of stellar mass into morphological type. Within each classification bubble, values for the median logged stellar mass (left) and the fraction by stellar mass (right) are shown. We find that $\sim 2/3$ of our sample are early-type spheroid-dominated galaxies, with the remainder in late-type typically blue and star-forming systems. Despite accounting for $> 50\%$ of the number density, Sd-type galaxies only account for $\sim 8\%$ of the stellar mass. The majority of mass lies within the older redder spheroid dominated E and S0a-type galaxies. Barred galaxies also notably contribute to the stellar mass budget, in both cases providing a significantly higher mass fraction than their equivalent number fractions.
### Fraction by number

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<tr>
<td>Index/Colour</td>
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<tr>
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<td>Eyeball</td>
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### Fraction by mass

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<td>QDA</td>
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<td>27</td>
</tr>
<tr>
<td>Eyeball</td>
<td>68</td>
<td>32</td>
</tr>
</tbody>
</table>

**Figure 6.9:** The number fraction (top) and mass fraction (bottom) of all galaxies within our volume limited sample for a series of complimentary classification methods are shown. Our preferred method of morphological classification (Eyeball) is highlighted at the bottom of each table.

Bluer diskier galaxies are more likely to be younger star-forming systems which are typically lower mass and pre-merger in their nature. Since hierarchical merging events are known through simulations to aid in the production of more massive, red, dead spheroidal systems, then some evolutionary hierarchy in mass is to be expected. The Sbc class accounts for \( \sim \frac{1}{5} \) of both the total number (18%) and stellar mass (20%) budgets.

A summary of both the number and stellar mass breakdown via all the various types of classification thus far examined is provided in Table 6.9. The methods shown, in increasing order of complexity are: by colour; by Sérsic index; by a combination of Sérsic index and colour; by quantitative global parameter cuts; by LDA statistical analysis and by QDA statistical analysis. Drawn out from the main list at the bottom of both tables are the number and stellar mass breakdowns as determined via visual classification. Visual classification remains our preferred means by which to divide our sample.

## 6.2 Bulge-Disk Decomposition

We now split the galaxies within our volume limited sample into their constituent bulges and disks using the SIGMA software developed in Chapter 3. We use the morphological classifications as given in Chapter 5 and in Section 6.1 above as inputs to provide prior information
Chapter 6. The Mass Function and its Division by Type and Component

and help choose the best model fit.

6.2.1 Method

The SIGMA software (Chapter 3) is designed to accept complicated inputs and produce more than just single-Sérsic model fits to galaxies. Whilst this should not affect single-component systems, such as Elliptical or disk-only Sd-type galaxies, both the bulge and disk components of a spiral galaxy typically require a Sérsic component fit each. On top of these, additional Sérsic components may also be used to model, e.g., a bar, secondary disk, pseudo bulge or an AGN. On occasion it may be more appropriate to use functions other than the Sérsic equation to fit some structures. For example, a bar structure is sometimes fit using the Ferrer profile (Binney & Tremaine, 1987) and a nucleated AGN is typically better fit using a PSF derived empirically from point-sources in the surrounding field (or using an analytical PSF if its form is well known). Exploration of these additional structures via structural decomposition is exceedingly complex, and beyond the scope of this Chapter and this thesis. In addition, one and two component systems comprise \(\sim 97.5\%\) of our sample by number, and so modelling up to two components should not adversely affect our global conclusions. For these reasons, in this Section I shall be modelling up to two components only (a bulge and a disk), where appropriate.

6.2.1.1 SIGMA Setup and Application

Using our volume limited sample of 3845 galaxies as an input, SIGMA was used to fit four distinct model types to each galaxy in each of the nine bands under consideration (ugrizYJHK). This is so that the ‘best’ model may be chosen after-the-fact by analysis of various goodness of fit parameters, rather than relying too heavily on prior information. These four model types are shown in Table 6.9 in increasing order of complexity. This is indicated in the number of free parameters available for each model fit. Certain constraints are placed on each model type in order to keep fit parameters in healthy parameter spaces. Several of these constraints are detailed in Table 6.9. Additional constraints are placed on all models regardless of model type. We require the best-fit model component number 1 (i.e., bulge) to be within 5 pixels of its input coordinate (in both x and y) and for the disk component to be within \(\pm 1\) pixels of the bulge component, where applicable. Model half-light radii must be within the range \(1/1000 < r_{e,\text{out}} / r_{e,\text{in}} < 1000\). In addition to constraints, model fit conditions are also placed on the Sérsic fit parameters, as shown in Table 6.9. These are to ensure that the bulge component does not ‘flip’ with the disk, thereby simplifying any analysis which follows. Bulge and disk
6.2. Bulge-Disk Decomposition

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>(k)</th>
<th>Constraints</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>M01</td>
<td>Single Sérsic</td>
<td>7</td>
<td>(0 &lt; e &lt; 0.95, 0.3 &lt; n &lt; 15)</td>
<td></td>
</tr>
<tr>
<td>M02</td>
<td>De Vaucouleurs Bulge</td>
<td>12</td>
<td>(0 &lt; e_b &lt; 0.3), (e_b &lt; e_d), (2 &lt; \frac{r_{ed}}{r_{eb}} &lt; 1000)</td>
<td></td>
</tr>
<tr>
<td>+ Exponential Disk</td>
<td></td>
<td></td>
<td>(0 &lt; e_d &lt; 0.9)</td>
<td></td>
</tr>
<tr>
<td>M03</td>
<td>Sérsic Bulge</td>
<td>13</td>
<td>(0 &lt; e_b &lt; 0.3, 1 &lt; n_b &lt; 15), (e_b &lt; e_d), (2 &lt; \frac{r_{ed}}{r_{eb}} &lt; 1000)</td>
<td></td>
</tr>
<tr>
<td>+ Exponential Disk</td>
<td></td>
<td></td>
<td>(0 &lt; e_d &lt; 0.9)</td>
<td></td>
</tr>
<tr>
<td>M04</td>
<td>Sérsic Bulge</td>
<td>14</td>
<td>(0 &lt; e_b &lt; 0.3, 0.3 &lt; n_b &lt; 15), (e_b &lt; e_d), (2 &lt; \frac{r_{ed}}{r_{eb}} &lt; 1000)</td>
<td>(n_b &gt; n_d)</td>
</tr>
<tr>
<td>+ Sérsic Disk</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.9: Bulge-Disk model types and descriptions. From left to right, columns show (1) model name, (2) basic model description, (3) the number of free parameters, \(k\), available for fitting within the model, (4) constraints applied to model parameters during the SIGMA/GALFIT3 fitting process, (5) conditions placed on the relations between bulge and disk model parameters. Additional constraints are applied to all model types, and therefore are not detailed in this table. See the text for further details.

constraints and conditions are indicated by a subscript \(b\) and \(d\), respectively.

Single-Sérsic model fits are started at a Sérsic index of \(n = 2.5\), as per Chapter 4. Bulge Sérsic components are started at \(n = 4\) and disk Sérsic components at \(n = 1\). For De Vaucouleurs and Exponential fits, the input Sérsic indices are fixed at these starting values. To estimate a starting magnitude for multi-component fits, the flux from a Kron-like AUTO aperture as given by Source Extractor is split evenly between the bulge and the disk. The bulge starting half-light radius is taken to be \(0.25 \times r_{e, in}\), where \(r_{e, in}\) is the half-light radius estimated by Source Extractor. The disk starting half-light radius is taken to be \(2 \times r_{e, in}\). In an attempt to help distinguish the bulge from the disk, the input position angle for the disk is offset from the bulge by \(\theta = +90^\circ\). Other SIGMA inputs such as the PSF, background sky estimation and definition of the fitting region (hence the determination of which secondary objects are to be modelled and which masked) are calculated in an identical manner to that as previously employed in Chapters 3 and 4.

6.2.1.2 Choosing a realistic model

Applying four unique model-type fits to each galaxy in our sample of 3845 galaxies takes an average of \(\sim 3\) minutes per galaxy per band, or \(\sim 25\) minutes per galaxy across all 9 bands. Each model output contains a goodness of fit parameter, \(\chi^2\), as well as its reduced statistic, \(\chi^2/\nu\), where \(\nu\) is the number of degrees of freedom in the model fit. The number of degrees of freedom, \(\nu\), is typically given by \(\nu = n - k\), where \(n\) is the number of data points (pixels) used in the fit minimisation and \(k\) is the number of free parameters. Choosing the lowest
\( \chi^2/\nu \) value for each galaxy should give the model fit which most accurately models all of the flux in the original image. However, model types with a greater number of free parameters are more likely to produce a fit of this type, as one would expect. Therefore, assuming the lowest \( \chi^2/\nu \) statistic will preferentially choose multi-component systems with more degrees of freedom over more simplistic model types. Often this result is not the desired result, as the simpler system contains the more ‘realistic’ fit parameters. One way around this is to choose the \( \chi^2/\nu \) which is closest to unity. This should classify those very low \( \chi^2/\nu \) parameters (\(< 1\)) as having overfit the data, and hence discard those models. This method however relies upon the uncertainties in the model fit to be very well understood, and hence may not be the most robust method for model choice. For this reason, using the \( \chi^2/\nu \) statistic in isolation is not convenient for choosing the most ‘realistic’ model type.

Another means by which the correct model type may be chosen is to employ a robust logical filter such as that utilised in Allen et al. (2006). This involves creating a large catalogue of input model types, such as our own, and then filtering these model types down to their final ‘best’ fit using a series of criteria. This criteria may be how many times the surface brightness profiles of the bulge and disk cross over, or the relative dominance of the bulge through analysis of the bulge-to-total ratio. This method has been used in Allen et al. (2006) with great success, however; it does require a good working knowledge of your input data set in advance in order to determine the logical filter decision nodes. Indeed, there may be unforeseen galaxy types that will not easily fit into any given logical filter, regardless of its complexity. For these reasons, we opt not to employ a logical filter in this study at this time, however; we encourage their use for future structural decomposition analyses.

We adopt the Bayesian Information Criterion (BIC) as our model selector. BIC accounts for the overfitting typical of more complex models by weighting model fits according to the number of free parameters within that model. Specifically, the BIC is given by

\[
\text{BIC} = \chi^2 + k \cdot \ln(n) \tag{6.3}
\]

where \( \chi^2 \) is the (complete) goodness of fit parameter, \( k \) is the number of free parameters and \( n \) is the area (number of pixels) used to minimise the fit. The fitting area \( n \) is estimated using the relation \( n = \pi r_e^2 (1 - e) \). We use the single-Sérsic fit results in the \( r \) band as given in Chapter 4 as our input \( r_e \) and \( e \) values. The \( r \) band has been chosen for its quality and relative depth compared to the other bands. Using this relation, the most ‘realistic’ model fit is simply
6.2. Bulge-Disk Decomposition

Given by that model that has the lowest BIC.

Whilst this provides a best fit model, it does not enable us to morphologically classify that model. For example, a single-component fit may be preferred, but it remains unclear whether this single-component represents an Elliptical or an Sd galaxy. As mentioned above, a logical filter could also be applied here after the BIC phase has been complete, or a more simplistic cut such as Sérsic index or colour could be applied in order to separate out the galaxies. We opt instead to use our prior morphological eyeball classifications to help guide our analyses. For each galaxy, if the visual classification results show the galaxy as being either an Elliptical or Sd-type galaxy, the single-Sérsic fit is used regardless of the relation between BIC values. If however the classification is anything other than E or Sd, the lowest BIC value from models M02, M03 and M04 is taken as the bulge-disk decomposition for that galaxy. In these cases, even if the single-component fit has the lowest BIC value, a multi-component solution will be adopted. This combination of visual classifications with the Bayesian Information Criterion, or VBIC, constitutes our preferred method by which we decompose galaxies into bulges and disks, and shall be used to provide the bulge+disk parameters throughout the remainder of this chapter.

6.2.2 Case Studies

Below I present case study examples of the structural decomposition process for six galaxies, one per morphological type. Figures 6.10 to 6.15 are all arranged in a similar fashion. From top to bottom, the model types shown are (1) Single Sérsic, (2) De Vaucouleurs bulge plus exponential disk, (3) Sérsic bulge plus exponential disk, and (4) a Sérsic bulge plus Sérsic disk (i.e., a double free Sérsic model). For each model type, a series of four images (left) and two line profile plots (right) are shown. The four images are (clockwise from top left) (a) the original SDSS $r$ band image, centred on the galaxy of interest, (b) the best fit model for this galaxy as output by SIGMA, (c) the residual image (data - model), and (d) a detail image, showing additional information. Within the detail image, thin light-blue ellipses represent the single-Sérsic ellipse contours, separated at intervals of 5'' along the semi-major axis for reference. The shaded purple region represents those pixels determined to belong to the main (primary) galaxy. Pixels from this region are used to generate subsequent goodness of fit statistics. Shaded orange and light green regions represent secondary modelled and secondary masked neighbours, respectively. The solid green ellipse shows the single-Sérsic half-light radius ellipse, and is reproduced on each model plot for reference. Solid blue and
red ellipses represent the equivalent bulge and disk half-light radii respectively for each model type.

The 1D line profiles to the right of the figure show a 1D measure of the 2D models represented. These 1D measures are constructed using the ELLIPSE package in the IRAF software, fixing the ellipticity and position angles to the single-Sérsic measure. The top sub-plot shows the 1D surface brightness light profile, whereas the bottom sub-plot shows the residual between the data and model fit. Black points show measurements from the original data. Green, red and blue solid lines show model fit values to the global, bulge and disk parameters, respectively. The horizontal grey dashed line at the bottom of the image represents the value of one standard deviation above the background sky value. Inset into the 1D figure are the five key Sérsic measurements for the modelled components (truncated magnitude $m$, half-light radius $r_e$, Sérsic index $n$, ellipticity $e$ and position angle $\theta$) and the total fraction of flux held within that component ($f$). The total combined non-truncated magnitude of all components ($m+$) is also shown. Below these values in the bottom-left of the 1D light profile are the number of free parameters ($k$), the primary $\chi^2/\nu$ statistic ($P$) and the global GALFIT-derived $\chi^2/\nu$ statistic ($G$). The primary $\chi^2/\nu$ statistic is derived in the usual sense, but only using the pixels within the shaded purple regions for analysis.

I hazard caution when interpreting 1D measurements as the model fits are not minimised in one-dimensional space. Often, the reduction in dimensionality which accompanies 1D radial measurement has a tendency to mislead the eye regards the goodness of fit, especially for edge-on ($e > 0.8$) systems. Therefore, the 1D measurements should be used as a guide to the eye only.

Figure 6.10 shows a case study example elliptical galaxy, G346888. This galaxy is chosen to be elliptical by virtue of its prior morphological classification, however; as can clearly be seen, some of the other model fits produce a significantly worse fit to the data. M02 and M03 leave a large residual imprint, which is not highlighted in the 1D profile plots. This is because the centroids have been offset from their ‘best’ value, and so an averaged radial annulus will act to mask this problem. This highlights one of the dangers of 1D galaxy analyses. The poor quality centroid offsets are brought out in the goodness of fit parameters, with M02 and M03 significantly worse than M01. M04 produces a good fit to the data, with $P$ and $G$ parameters closer to unity than M01. However, BIC would penalise the M04 model significantly, and so the M01 would be favoured as the most ‘realistic’ fit to the data even without prior visual
Figure 6.11 shows the model fit combinations to G417433, an S0a type galaxy. This galaxy is clearly a large edge-on bulge plus disk system. A single Sérsic fit fails in producing both the circular central component and the extended highly-elliptical surrounding disk, falling somewhere in between with a compromise ellipticity. Note the distinctive hourglass impression left in the single-Sérsic residual image. Also note the lack of any indication of a problematic fit in the 1D image. A surface brightness slice along the semi-major axis rather than azimuthally averaged would most likely solve this problem however. The introduction of a secondary component instantly improves the goodness of fit, with only the dust lanes present in the disk evident in the residual images. Increasing the number of free parameters continues to increase the goodness of fit, with the lowest BIC value for this system being the M04 model. Using this double-free Sérsic model, the B/T ratio is 0.46.

Figure 6.12 shows an SB0a type galaxy, G505952. These types of systems contain at least three components by eye, and so one would not expect a one or two component fit to accurately represent all of the flux from this galaxy. This is evidenced by the large residuals present for all model types. A single-Sérsic fit does a good job at modelling the lions-share of the flux present within the galaxy, and as such, gives a good estimate of the total Sérsic magnitude. Additional components tend to reduce the quality of the fit. The bulge component is elongated in all three cases, hitting the upper limit of $e = 0.3$ in an attempt to model both the bulge and the bar. In a similar sense, the disk has adopted the position angle of the bar rather than the required position angle of the disk approximately $90^\circ$ offset. The lowest BIC value is for M01, however, since this galaxy has been confirmed as a multi-component system by visual classification, M03 is adopted in its place. One would expect that the addition of a third component would allow for the bar to be accurately modelled, and hence producing a lower BIC value than M01.

Figure 6.13 shows a late-type spiral galaxy, G609912. This galaxy has a small apparent bulge and a disk containing distinctive spiral arm features, easily seen in several of the residual images. A prominent secondary neighbour is also modelled close in to this galaxy. M01 produces a good overall fit to the data, but M04 is clearly preferred and has the lowest BIC value. Both the M02 and M03 models produce an inferior fit to the data. A common trend amongst Sbc-type galaxies is the need for an intermediate Sérsic index bulge ($n \sim 2$) usually accompanied by a low index disk ($n < 1$, further details available in Section 6.2.3). The
Figure 6.10: Bulge-Disk decomposition case study for an Elliptical galaxy. From top to bottom, the model types shown are (1) Single Sérsic, (2) De Vaucouleurs bulge plus exponential disk, (3) Sérsic bulge plus exponential disk, and (4) a Sérsic bulge plus Sérsic disk (i.e., a double free Sérsic model). See the text for a full description of this Figure.
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Figure 6.11: As per Figure 6.10, but for an S0a type galaxy.
Figure 6.12: As per Figure 6.10, but for an SB0a type galaxy.
constraints imposed on the M02 and M03 models do not allow for these combinations of bulge and disk Sérsic indices to occur naturally, and so prohibit their fits from being preferred. This late-type spiral galaxy has a B/T ratio of 0.26.

Figure 6.14 shows SBbcd type galaxy G517070. This system contains a small bulge, bar and a dominant disk. A single-Sérsic fit to the data vastly fails to reproduce the flux in the core regions, underestimating by as much as 1 magnitude. In a similar sense to Figure 6.13 above, M02 and M03 do not allow for the Sérsic indices of the bulge and disk components to be low enough to produce a ‘good’ fit to the data, and so leave large flux residuals in both the core and intermediate regions. The M04 model contains enough freedom to allow for this, and so is preferred, with a B/T ratio of 0.07. The small bulge-to-total ratio only allows for the bulge to dominate in the very core regions.

Figure 6.15 shows the Sd classified galaxy G99078. As with the elliptical galaxy above, a single-Sérsic model fit has been selected by default owing to the prior morphological classifications adopted. Perhaps unexpectedly, the addition of extra components reduces the quality of the fit despite the extra number of free fit parameters in these models. This is because the constraints and conditions imposed on the multi-component model types act to force the most realistic multi-component fit to the data. Since these data are not multi-component, the model fits get progressively worse.

6.2.3 Structural Measurements

In this section I discuss the results of the structural decomposition performed on our volume limited dataset. In total, 33% of our sample of 3845 galaxies (1269) require a secondary component as determined by visual classification. Of these multi-component galaxies, ∼14% (181) are best fit using a De Vaucouleurs bulge plus exponential disk model, ∼20% (259) by a free Sérsic bulge plus exponential disk model and ∼65% (829) by a double free Sérsic model.

Figure 6.16 shows the Sérsic index distributions of all galaxies, single and multi-component. Shaded grey bars show the single-Sérsic fits presented in Chapter 4, whereas red and blue bars show the newer M01 to M04 model fits described in this chapter. The small offset between the distributions for E and Sd-type galaxies arises due to the different constraints placed on these two runs (such as limiting the allowed range of Sérsic indices), however; as can be seen, this does not severely impact on the distributions. The median Sérsic index for elliptical galaxies is $n \sim 4$, as expected. Sd-type galaxies also have a modal $n \sim 1$. 
Figure 6.13: As per Figure 6.10, but for an Sbc type galaxy.
6.2. Bulge-Disk Decomposition

Figure 6.14: As per Figure 6.10, but for an SBBcd type galaxy.
Figure 6.15: As per Figure 6.10, but for an Sd type galaxy.
Figure 6.16: Bulge-Disk decomposition results showing \( r \) band Sérsic index distributions for single and multi-component systems. Histogram bin widths are in steps of 0.05 dex. The underlying grey population are the Sérsic index distributions for the same galaxies using the single-Sérsic fits as presented in Chapter 4, for reference. Red and blue bars denote spheroid and disk populations, where appropriate. Note the small offset between the E and Sd type grey populations and their new single-Sérsic distributions. This arises owing to the different constraints applied to the two runs of SIGMA.
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The S0a-type single-Sérsic models have a median Sérsic index of \( n \sim 3 \), a compromise Sérsic index between the bulge and disk component. Structural decomposition allows for the bulge components at \( n \sim 4 \) and the disk component at \( n \sim 1 \) to be separated, and their flux apportioned appropriately. This trend is also replicated for SB0a-type galaxies, however; the effects of small number statistics and the need for an additional third component throw relatively larger errors on these results. The sharp peaks at \( n = 1 \) and \( n = 4 \) arise because of the fixed exponential and de Vaucouleurs models adopted. Overall, S0a galaxies tend to show a classical bulge + disk like structure. Sbc type galaxies however do not show this trend. The Sérsic indices of Sbc bulges are typically lower than their S0a counterparts, with a median value of \( n \sim 2 \). Accompanying this is typically a low Sérsic index disk, at \( n \sim 0.6 \). A possible explanation for this discrepancy could lie in the formation and evolutionary histories of S0a and Sbc-type galaxies. If early-type spirals are spheroidal structures that have subsequently grown a disk, one would expect this classical bulge to have retained many of its original spheroidal characteristics, such as a Sérsic index of \( n = 4 \). In contrast to this, Sbc-type galaxies may have formed as a disk-dominated structure and subsequently grown a central bulge. This process can occur following the formation of a bar, which acts as an efficient means by which gas and stellar material may be funnelled into the core regions of the galaxy. The resultant ‘pseudo-bulge’ tends to be flatter (\( n \sim 2 \)) whilst having a higher rate of angular momentum and a younger stellar population than their classical counterparts (see e.g., Gadotti, 2009). Note however that a full distinction between classical bulges and pseudo-bulges remains unclear, and so we shall be using these basic distinctions that S0a galaxies harbour a classical bulge and Sbc galaxies a pseudo-bulge as a simplistic means of further analysis of the data.

Bulge and disk half-light radii in physical units are shown in Figure 6.17. Elliptical and Sd galaxies show a skewed log-normal distribution, with E-type galaxies skewed towards larger \( r_e \) and Sd-type galaxies skewed towards smaller \( r_e \). Bulge and disk half-light radii are well fit with a standard log-normal distribution. Regardless of morphology, disk sizes in multi-component systems are relatively consistent, peaking at \( r_e \sim 5 \) kpc. The bulge sizes of S0a-type galaxies are typically smaller than their Sbc counterparts by as much as a factor of 2. Also note the increase in recorded bulge sizes from Sbc to SBbcd-type galaxies. This is most likely an artefact of the ‘bulge’ component compromising its fit between the actual bulge and the additional bar.
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Finally, Figure 6.18 shows the recovered absolute magnitudes arising from our bulge-disk decompositions. Elliptical galaxies exist almost uniformly at all magnitude intervals, whereas Sd type galaxies peak at a relatively faint magnitude. Interestingly, the bulge magnitudes never dominate the flux in the S0a galaxies, and are $\sim 2$ magnitudes fainter than the disk component for Sbc galaxies.

6.2.3.1 Division of Global Measurements by Component

Several global measured parameters need to be applied to the measured sub-components in order to produce additional complimentary measurements. Here I detail how our $k$-corrections and total stellar masses have been divided between our bulge and disk populations.

The $k$-corrections measured in Section 5.1.3.2 have been derived using global single-Sérsic measurements. In order to calculate sub-component $k$ corrections, we first calculate the total component flux by summing the flux in the two sub-components (converting to maggies first). The global $k$-correction is then applied to this value, and the flux re-divided between the two components using the previously calculated component-to-total ratios.

We derive stellar masses for sub components by use of their component luminosity fraction to total ratios. For bulges, the bulge stellar mass equates to the SIGMA-derived $r$ band B/T
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Figure 6.18: Bulge-Disk decomposition results showing $r$ band absolute magnitude distributions for single and multi-component systems. Histogram bin widths are in steps of 0.2 mag.

ratio multiplied by the (linear) total stellar mass of the system, and similarly for the disks, substituting the $r$ band $D/T$ ratio for $B/T$. This method should provide reasonable estimates of component stellar masses, as the $M/L$ ratio’s for each morphology follow a relatively linear relation as shown in Figure 6.6.

6.3 Luminosity and Mass Functions of Spheroids and Disks

In this section I analyse the Structural Luminosity Functions (SLFs) and Structural Mass Functions (SMFs) for both spheroids and disks. These structural decompositions are described in Section 6.2 above.

Figures 6.19 and 6.7 show the $r$ band luminosity and mass functions of all spheroid (red line) and disk (blue line) components for all galaxies. Each structural type is fit with a single Schechter function, whilst the global population (black line) is fit by a double Schechter function, as per Figures 6.4 and 6.7. The Schechter fit parameters are inset into each figure, for reference. The disk population is well described by a single Schechter function, accurately describing the faint low-mass end slope well, and mapping the number density turnover beyond $L^*$. The Schechter fits imply that the number density of disks are dominant at all luminosities and masses. This arises in part because a single Schechter fit to the spheroid population does not have sufficient flexibility to be able to model the large number of bright high-mass
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Figure 6.19: Structural luminosity functions for spheroid and disk components. The data are binned into r band magnitude bins of 0.25 mag prior to fitting. Spheroid (red) and disk (blue) components are fit with a single Schechter function, whereas the global GLF (black) is fit using a double Schechter function. Schechter fit parameters are inset into the top-left of the figure, for reference.

spheroids, typically elliptical galaxies. It appears that the spheroid population is better fit by two Schechter functions with alternate \( L^* / M^* \) turnover points. Despite this bright-end discrepancy, disk galaxies dominate the number density at all luminosities fainter than \( M \sim -22 \) mag or at masses less than \( \log M \sim 10.5 \).

Figures 6.21 and 6.22 show the SLF and SMF for ellipticals, bulges and disks. The spheroid populations has been split into those spheroids that don’t host a disk (ellipticals) and those which do (the bulges of S0a/Sbc type galaxies) in an attempt to better model the spheroid population with a series of Schechter functions. The disk SLF/SMF Schechter fit has not
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Figure 6.20: Structural Stellar Mass Functions for spheroid and disk components. The data are binned into stellar mass bins of 0.1 dex prior to fitting. Spheroid (red) and disk (blue) components are fit with a single Schechter function, whereas the global GLF (black) is fit using a double Schechter function. Schechter fit parameters are inset into the top-right of the figure, for reference.

\[
\begin{align*}
\log (M^*, \alpha, \phi^*/10^{-3}, \alpha_2, \phi_2^*/10^{-3}) \\
&= (10.55, -0.23, 4.88, -1.43, 1.07) \text{ All} \\
&= (10.32, -0.67, 4.42) \text{ Spheroid} \\
&= (10.70, -1.22, 2.03) \text{ Disk}
\end{align*}
\]
changed. Splitting spheroidal structures into these two distinct sub-structures and modelling each with a single Schechter function instantly improves the quality of the fit to the data. Elliptical galaxies have the freedom to adopt a brighter higher-mass turnover point, whereas bulge galaxies have a much lower turnover point at $\Delta L^* \sim 2$ mag. The bright high-mass population of spheroidal/elliptical galaxies is now modelled by one of these contributing functions.

Overall, splitting the galaxies within our volume limited sample into disks, bulges and ellipticals and fitting each with a single Schechter function provides a good fit to our data, and is our preferred means for describing this population.

A further division of the bulge population into classical bulges and pseudo-bulges is explored in Figures 6.23 (SLF) and 6.24 (SMF). Noting the central concentration and half-light

\[ \phi(M, \alpha, \phi^*/10^{-3}) \]

\(-21.93, -0.82, 0.97) \text{ Elliptical}

\(-19.51, -0.12, 6.07) \text{ Bulge}

\(-20.67, -1.05, 5.69) \text{ Disk} \]
Figure 6.22: Structural stellar mass functions as per Figure 6.20 but for elliptical, bulge and disk components.
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radius differences between the bulge populations of S0a and Sbc galaxies as shown in Section 6.2.3, we advocate the classification of S0a bulges as ‘classical’ and Sbc bulges as ‘pseudo’. This analysis is useful for an exploration of the data, but a more robust definition should preferably be applied once additional information can be obtained, such as velocity dispersion measurements of the bulge stellar populations.

We find that pseudo-bulges are less massive than their classical counterparts, dominating the spheroidal number densities only at the lowest masses and faintest magnitudes. Whilst both the classical and pseudo-bulges are well described by individual Schechter function fits, their combined populations are also well described by a single Schechter function. Classical bulges dominate at intermediate masses, and provide much of the number density required to produce the double Schechter bump that appears in the global fit at log $M \sim 10.5$.

It may be helpful at this point to contrast our results against those found via numerical simulations. The Millennium Simulation (Springel et al., 2005), so named because of its size, is a high-resolution N-body simulation performed by the Virgo Consortium with the aim of studying the formation and evolution of galaxies brighter than the Small Magellanic Cloud. The simulation follows the evolution of $\sim 10^{10}$ particles in the redshift range $0 < z < 127$ in a cube of size $500 \, h^{-1} \, \text{Mpc}$ on a side, assuming a standard $\Lambda$CDM cosmogony. The Millennium Simulation is notable as it marked a significant increase in particle number density and spatial and temporal resolution from previous studies (e.g., Colberg et al., 2000 [10$^9$ particles, 3000$h^{-1}$ Mpc]; Evrard et al., 2002 [10$^9$ particles, 3000$h^{-1}$ Mpc]; Wambsganss et al., 2004 [10$^9$ particles, 320$h^{-1}$ Mpc]). This simulation has been exceptionally successful at reproducing some key observables, such as confirmation that black holes are able to form early on in the history of the Universe. A follow up simulation by the same team, dubbed Millennium-II (Boylan-Kolchin et al., 2009), used the same number of particles in a volume $1/5$ of the linear size of its parent in order to follow the particle evolution at smaller scales (Guo et al., 2010).

Guo et al. (2011) apply a complex galaxy formation model to both the Millennium Simulation (MS) and Millennium-II (MS-II) N-body simulations in order to test for the recoverability of global galaxy properties at low and high particle resolution. Using data from Baldry et al. (2008) and Li & White (2009), Guo et al. (2011) show that the increased particle density of MS-II is able to much more accurately predict the low-mass end of the galaxy stellar mass function. The MS typically underestimates the number of low-mass gravitationally bound
Figure 6.23: Structural luminosity functions as per Figure 6.19 but for elliptical, classical bulge, pseudo bulge and disk components.
Figure 6.24: Structural stellar mass functions as per Figure 6.20 but for elliptical, classical bulge, pseudo bulge and disk components.
structures below $\log M \sim 9.5$ stellar masses, the so called ‘missing-satellites’ problem, owing to resolution issues. This is particularly important here, as our data shown in Figure 6.24 displays an apparent upturn in the global galaxy stellar mass function at approximately this stellar mass interval. The MS-II simulations however are able to faithfully recover this supposed low-mass dwarf population of galaxies.

Dependent upon the type of merger, Guo et al. (2011) allow for structures to assemble into either spheroidal/bulge structures (via major mergers) or disks (via minor mergers). This allows for a simplistic morphological class to be attributed to each system according to its bulge-to-total ratio ($B/T$). They define elliptical galaxies as those that have a $B/T > 0.7$, pure disks with a $B/T < 0.03$, and the remainder classified as spiral systems. This allows for a direct comparison with our own data presented here. Based on their simulations, Guo et al. (2011) find that elliptical galaxies should dominate the stellar mass budget at stellar masses larger than $\log M \sim 11$ in the local Universe, which is in excellent agreement with our own results. Pure disk-only structures dominate at all other masses, excepting the range $8.5 < \log M < 9.5$, where spiral systems dominate the stellar mass budget. It is highly likely that if the stellar mass from the separate components were split between their bulge and disk sub-structures, as we have shown above, that the stellar mass residing in disks would continue to dominate at all stellar masses. This strong agreement between the empirical data shown in this study and the analytic models produced by Guo et al. (2011) provide strong evidence that the collapse, satellite accretion, disk growth via gas accretion, hierarchical merging, feedback and eventual disruption via harassment mechanisms remain the preferred means by which galaxies and galaxy structures form and evolve.

6.4 Relations

In this section I analyse the Sérsic index against colour relation and the stellar mass against half-light radius relation for the components described above.

6.4.1 Sérsic Index - Colour

Figure 6.25 shows the Sérsic index-colour relation for the two types of bulge (classical, purple; pseudo, green) only, with histograms projecting both axes into their frequency distributions along that axis. The solid line features at $n = 1$ and $n = 4$ arise owing to the choice of models as described in Section 6.2. There is a tendency for pseudo-bulges to be bluer ($u - r < 2$) and have a lower Sérsic index ($n < 3$) than their classical counterparts, as has been previously
6.4. Relations

Figure 6.25: The $r$ band Sérsic index against $u - r$ colour for both bulge types. Classical bulges are coloured purple, and pseudo-bulges coloured green. The Sérsic index histogram calculates population frequencies in bins on 0.065 dex, whilst the colour histogram calculates population frequencies in bins of 0.2.

noted by, e.g.; Kormendy & Kennicutt (2004); Drory & Fisher (2007); Cameron et al. (2009). However, we note that the scatter between these two populations remains prohibitively high, particularly so for the case of Sérsic index, such that any division along either axis in defining a classical/pseudo-bulge boundary would lead to large contamination fractions.

6.4.2 Mass - Radius

The relation between stellar mass and half light radius is shown in Figure 6.26 for elliptical (red), classical bulge (purple), pseudo-bulge (green) and disk (blue) populations. Solid lines represent contours within which 50% of that population lie. The most massive structures
are giant ellipticals, whilst the physically smallest structures are classical bulges. Elliptical galaxies and classical bulges occupy a similar parameter space, with a significant level of overlap in this mass-size plane. This overlap re-enforces the assertion made in Kormendy & Kennicutt (2004) that Elliptical galaxies and classical bulges are essentially the same object; the latter happens to have grown a disk around it. Disks are the physically largest structures, and occupy a wide range of stellar masses. Lying between disks and elliptical galaxies are the pseudo-bulge structures. Since pseudo-bulges are formed via secular evolutionary processes from gas and stellar material in the disk, it is not surprising that pseudo-bulges share much in common with the disk population, as seen here.
6.5 Stellar Mass Budget of Spheroids and Disks

The structural analyses performed allow for the complete stellar mass budget by morphological type, morphological class and structure to be calculated for the first time in our local volume. Figure 6.27 shows the stellar mass present within increasingly specific sub-populations. These populations are, from top to bottom, (1) morphological type, (2) morphological class, (3) structure, and (4) the evolutionary mechanism which formed this stellar mass. Within each ellipse is the name of the sub-population (top), the percentage of total stellar mass that sub-population accounts for (middle) and the upper and lower error estimates on the percentages (bottom). Errors are estimated using the q-beta function (Cameron, 2011), taking the number fractions for each population to generate expected $2\sigma$ upper and lower bounds. The number fraction error offset is then multiplied by the mean population mass to estimate the errors in stellar mass. Grey lines indicate some or all of the mass from that sub-population contributes towards the mass in the connecting sub-population.

We find that 32% of the stellar mass within our sample lies in elliptical galaxies, 14% in classical bulges, 6.5% in pseudo-bulges and 48% in disks. The majority of this disk stellar mass is derived from disks in multi-component spiral systems, and not from pure disk-only systems. The stellar mass dominance of elliptical galaxies is as expected following on from the Schechter function fits to the data in Figure 6.22 where a small number of giant elliptical galaxies contribute a large amounts of mass to the population mass budget.

The effect of barred structures on these stellar mass estimates has not been accounted for at this stage, and provides an intriguing avenue for future research. Naively we may expect a proportion of the bulge and disk stellar mass to be syphoned off into a new bar class, reducing the amount of overall stellar mass in spheroids and disks. Typically, we find during the galaxy modelling phase that the bulge component will attempt to form a compromise profile between the bulge and the bar. If this is the case, one may expect the stellar mass impact to more adversely affect the bulge populations than the disk population. However, only $\sim 2.5\%$ of our sample of galaxies contains a bar structure, and so any modification of these final stellar mass breakdowns owing to the impact of bars should be minimal.

Grouping these structures by the evolutionary mechanisms that formed them, we find that hot-mode collapse, merger or otherwise turbulent mechanisms account for $\sim 46\%$ of the total stellar mass budget, cold-mode gas accretion and splashback mechanisms account for $\sim 48\%$ of the total stellar mass budget and secular evolutionary processes for $\sim 6.5\%$ of the total
Figure 6.27: The complete stellar mass breakdown of all galaxies within our local volume limited sample. Top to bottom, stellar mass is divided by (1) morphological type, (2) morphological class, (3) structure, and (4) the evolutionary mechanism which formed this stellar mass. Within each ellipse is the name of the sub-population (top), the percentage of total stellar mass that sub-population accounts for (middle) and the upper and lower error estimates (bottom). Grey lines indicate some or all of the mass from that sub-population contributes towards the mass in the connecting sub-population.
stellar mass budget in the local ($z < 0.06$) Universe. These results provide one of the first measurements of the stellar mass breakdown by structure and evolutionary processes. The progression and division of stellar mass by various indicators using visual-BIC Sérsic structural decomposition constitutes our final result.

To put this result into context, Figure 6.28 traces the mass-energy density breakdown for the local Universe. All energy derived from the Big Bang is divided into its three main constituent parts (excluding thermal radiation, e.g., the cosmic microwave background), namely: dark energy, dark matter and baryonic (normal) matter. Normal matter is further divided amongst the main observationally verified constituents of the observable Universe: gas, stars, black holes and dust. Finally, the stars in galaxies are divided by structure into the amount of stellar mass present in disks, ellipticals, classical bulges and pseudo-bulges.
Figure 6.28: The mass-energy density breakdown for the local Universe; from cosmology to matter to structure.
In this chapter I provide an overview of the key results presented throughout this thesis (Section 7.1), and explore possible future avenues of research (Section 7.2).

7.1 Results

In this thesis I have outlined the development of the SIGMA galaxy profile fitting software, which acts as a wrapper around several contemporary astronomy software packages (Chapter 3). SIGMA provides estimates of galaxy size, magnitude, concentration and several additional structural parameters which may be used in further analysis of galaxy properties. Using data from both the SDSS and UKIDSS (Chapter 2), I have applied SIGMA to 167,600 galaxies independently across 9 wavelengths ($ugrizYJHK$) in single-Sérsic mode (Chapter 4), and to a volume-limited sample of 4,110 galaxies across 9 wavelengths in multi-Sérsic mode (Chapters 5 and 6). Using these structural measurements, I have explored the wavelength dependency on recovered structural parameters (Chapter 4), performed morphological classification and morphological analysis of a volume-limited sample (Chapter 5) and calculated the luminosity function and mass function for galaxies in the local Universe, divided by morphological type, morphological class, and galaxy structure (Chapter 6).
Below I detail the key results of this thesis in bullet point format.

- It is not clear exactly how the surface brightness profile of a galaxy behaves below a given limiting isophote. In order to account for this uncertainty when extrapolating Sérsic profile flux, we opt to truncate Sérsic model fits at 10 multiples of the effective radius (Section 1.3.3). This provides a good estimate of ‘total’ flux, showing good agreement with both traditional fixed aperture (Petrosian, Kron) and model (cf. SDSS) magnitudes for low Sérsic index systems \((n < 4)\), and much improved photometry for high Sérsic index systems \((n > 4)\), recovering as much as \(\Delta m = 0.5\) mag in the \(r\) band.

- From an analysis of the background sky at the position of each galaxy in each band, I find that the SDSS \(ugri\) bands exhibit significantly deeper surface brightness limits than their longer wavelength counterparts (Section 4.2.4). However, the effects of seeing and dust attenuation are more acute at these short wavelengths. For statistical analyses where depth is not a primary issue, the use of the UKIDSS \(K\) band is strongly advised. For structural analyses where depth is crucial in the resolution of galaxy sub-components, the use the the SDSS \(r\) band is advised.

- The distribution of global Sérsic index measurements for the entirety of the the GAMA-I sample is bi-modal in nature; peaking at \(n \sim 1\) and \(n \sim 3.5\) for the disk-dominated and spheroid-dominated populations, respectively (Section 4.3.2). The relative strength of these two peaks shifts with increasing wavelength, with the stronger disk-dominated peak at \(n = 1\) giving way to the spheroid-dominated \(n = 3.5\) peak at wavelengths longer than the \(i\) band (Figure 4.8). This shift in recovered global Sérsic index is caused by a combination of stellar population gradients within each galaxy (old, red, spheroid and young, blue disk components), and the effects of dust attenuation at shorter wavelengths impinging on the emitted light from the core (typically spheroidal) regions of the galaxy.

- A similar trend is noted in global half-light radii, with shorter wavelengths recovering typically larger radii (from the disk stellar population) and longer wavelengths recovering smaller radii (from the unmasked bulge/spheroid stellar population).

- Recovered global ellipticities remain relatively consistent at all wavelengths, indicating that passband choice should not adversely affect estimation of the ellipticity of the system, and hence; inclination.
7.1. Results

- Using both the global Sérsic index ($K$ band) and rest-frame $u - r$ colour, I divide our galaxy sample into two distinct populations using the relation $(u-r)_{\text{rest}} = -0.59 \log n_K + 2.07$ (Equation 4.2 in Section 4.4, and Figure 4.11). Galaxies below this line (typically blue, low Sérsic index) are defined as disk-dominated late-type galaxies. Galaxies above this line (typically red, high Sérsic index) are defined as spheroid-dominated early-type galaxies.

- The variation in recovered global Sérsic index with wavelength for disk-dominated late-type galaxies is given by $\log n_{\text{disk}} = -0.715 \log^2 \lambda_{\text{rest}} + 4.462 \log \lambda_{\text{rest}} - 6.801$ (Equation 4.3) and for spheroid-dominated early-type galaxies is given by $\log n_{\text{disk}} = -0.715 \log^2 \lambda_{\text{rest}} + 4.462 \log \lambda_{\text{rest}} - 6.801$ (Equation 4.4).

- The variation in recovered global half-light radii with wavelength for disk-dominated late-type galaxies is given by $\log r_{e,\text{disk}} = -0.189 \log \lambda_{\text{rest}} + 1.176$ (Equation 4.7) and for spheroid-dominated early-type galaxies is given by $\log r_{e,\text{sph}} = -0.304 \log \lambda_{\text{rest}} + 1.506$ (Equation 4.8).

- The mean Sérsic index of early-type galaxies shows a smooth variation with wavelength, increasing by 30 per cent from $g$ through $K$. Late-type galaxies exhibit a more extreme change in Sérsic index, increasing by 52 per cent across the same range (Section 4.5.2). Early-type and late-type galaxies exhibit a 38 and 25 per cent decrease, respectively, in half-light radius from $g$ through $K$.

- Considering the co-variation of half-light radius and Sérsic index together with wavelength we find that the large fluctuations in spheroidal parameters amount to a relatively modest impact on the recovered light profile. A comparatively larger effect is noted for the disk systems, particularly in the core region, supporting the presence and effect of dust attenuation in addition to stellar population/metallicity gradients. At a distance of 1 pixel from the central region, spheroid systems display a variation in surface brightness of 0.49 magnitudes from $u$ through to $K$. In disk systems, the comparative figure is 0.86 magnitudes, an increase of 75%; a warning for the consideration of recovered model parameters in isolation (Section 4.5.4).

- A single-Sérsic fit to a galaxy in either the $u$ and $r$ bands or (preferably) the $g$ and $i$ bands provides a good analogue for traditional matched-aperture based colour estima-
tion methods, improving on them for highly-centrally concentrated (high Sérsic index) systems (Section 5.1.3.4).

- We constructed a volume limited sample of 3,845 galaxies (0.025 < z < 0.06 and $M_r < -17.4$) and morphologically classified these galaxies via visual classification into: elliptical, 14%; S0a, 13%; SB0a, 0.71%; Sbc, 19%; SBbcd, 1.9%; and Sd, 52% (Section 5.2). We identify an additional class of ‘Little Blue Spheroids’ within the Sd population with median a Sérsic index of $n_K = 1.62$ and a mean of $n_K = 2.39$.

- Statistical methods for automated morphological classification fail to reproduce the dominance of elliptical galaxies at the bright end of the luminosity function or, conversely; the high-mass end of the stellar mass function. Using 5 key global structural measurements ($K$ band half-light radius, ellipticity, absolute magnitude, Sérsic index and $u - r$ colour), we trialled the linear discriminant analysis and quadratic discriminant analysis routines. (Sections 5.2.6.2 and 5.2.6.3). For the reasons outlined therein, we adopt visual classifications as our preferred method of morphological classification.

- The majority of our sample lies well within the survey boundaries (Figure 6.1), implying that derived statistics remain robust owing to our high level of completeness. Only the Sd-type population suffers any notable incompleteness, with potential population truncations at the lower size boundary, the faint end boundary and the dim surface brightness boundary. The truncation in the Sd population has implications on derived Schechter fit parameters at the faint-end slope of the luminosity function (Section 6.1.1).

- Both the global luminosity function (at all wavelengths) and the global mass function require a double Schechter component fit with a joint ‘knee’ in order to accurately fit the faint/low-mass upturn (Sections 6.1.2 and 6.1.4). Individual morphologies (E, S0a, Sbc, Sd) are well described by a single Schechter function fit.

- In our field sample of galaxies from the local Universe we find approximately 2/3 of the stellar mass is locked up within early-type (Elliptical and S0a/SB0a) systems. The remainder is in late-type (Sbc/SBbcd and Sd) galaxies. The ratio of mass division between Elliptical and S0a type galaxies is split evenly at ~ 1/3 each of the total stellar mass budget. Sd type galaxies contribute only ~ 8% of the mass whilst accounting for ~ 53% of the population (to $M_r < -17.4$ mag) by number (Section 6.1.5).
7.1. Results

• We employed the SIGMA software to fit each galaxy in our local sample with four different model types: single Sérsic; de Vaucouleurs bulge plus exponential disk; Sérsic bulge plus exponential disk, and; a double free Sérsic (Section 6.2). Using a combination of prior visual classifications and the Bayesian Information Criterion, I establish a means by which each galaxy in our sample is divided into its main constituent parts, namely: single component ellipticals, bulges and disks.

• In total, 33% of our local sample of 3845 galaxies (1269) require a secondary component as determined by visual classification. Of these multi-component galaxies, \( \sim 14\% \) (181) are best fit using a De Vaucouleurs bulge plus exponential disk model, \( \sim 20\% \) (259) by a free Sérsic bulge plus exponential disk model and \( \sim 65\% \) (829) by a double free Sérsic model. (Section 6.2.3).

• Noting the central concentration and half-light radius differences between the bulge populations of S0a and Sbc galaxies (Section 6.2.3), we advocate the classification of S0a bulges as 'classical' and Sbc bulges as 'pseudo'. Our classical bulges lie on the Kormendy relation faintwards of elliptical galaxies, whereas our pseudo-bulges appear as a spur to this relation, highlighting the differences between these two populations (See also Figure 6.26).

• The luminosity and stellar mass functions of galaxies in the local volume are composed of four principle components, namely: elliptical galaxies; classical bulges; pseudo-bulges, and; disks. Each component is typically well described using a single Schechter function fit. Elliptical galaxies contribute the majority of mass at the high-mass end of the stellar mass function. Disks dominate elsewhere. Classical bulges are typically more massive than their pseudo-bulge counterparts (Section 6.3).

• Using simplistic colour or Sérsic index cuts (or a combination of the two) is not sufficient in order to delineate the classical and pseudo-bulge populations (Section 6.4.1).

• Figures 6.27 and 6.28 constitute our main and final results. Using a combination of robust automated Sérsic profiling, visual eyeball classification and logical bulge-disk decomposition, we find that 32% of the stellar mass within our sample lies in elliptical galaxies, 14% in classical bulges, 6.5% in pseudo-bulges and 48% in disks.

• Grouping these structures by the evolutionary mechanisms that formed them, we find
that hot-mode collapse, merger or otherwise turbulent mechanisms account for $\sim 46\%$ of the total stellar mass budget, cold-mode gas accretion and splashback mechanisms account for $\sim 48\%$ of the total stellar mass budget and secular evolutionary processes for $\sim 6.5\%$ of the total stellar mass budget in the local ($z < 0.06$) Universe.

### 7.2 Future Work

I am extremely interested in extending the area, depth and, importantly, redshift baseline of my future work. We live in an exciting era, with a host of surveys on new instruments emerging (VST KIDS, VISTA VIKING, SkyMapper Southern Sky Survey) and a range of existing high-quality surveys yet to be fully exploited (SDSS Stripe 82, GOODS HST/ACS). I hope to use my software and these data in the following ways:

**Structural Decomposition**

The most complex galactic systems are comprised of spheroidal components, bulges (classical and pseudo), disks (thick and thin), bars, nucleated bars, rings, dust lanes and dust attenuation, spiral arms, AGN, lenses and a veritable myriad of additional exotic phenomena. As shown in Figure 5.1, an insurmountable problem blighting current and previous-generation wide-area surveys is the inability to resolve many of these components in sufficient numbers for their subsequent statistical analyses to be meaningful. The increased resolution and depth afforded by KIDS and VIKING should allow for these comprehensive and robust large-scale studies of galactic structure to be undertaken for the first time. Providing full structural decomposition models of these systems will allow for the complete stellar mass breakdown into finer structural divisions as well as to explore any dependence on environment and in particular halo mass.

**Three Dimensional Structural Analyses**

A more natural arena within which to analyse the structure of galaxies may be in their native three-dimensions, rather than in their two-dimensional projection we observe here on Earth. The advent of cheap and powerful mass computing makes the application of complex three-dimensional modelling relatively attainable, and; if performed correctly, should allow for the construction of a complimentary data-set to traditional two-dimensional structural analysis catalogues. The increased resolution afforded via this technique should significantly improve galaxy evolutionary analyses.
7.2. Future Work

Non-Parametric Galaxy Fitting

Some galactic structures defy parametrisation via any given function, such as that given by the Sérsic equation. By taking advantage of advances in computing power and capabilities, it is becoming increasingly possible to employ novel methods by which galactic structure is analysed. One such method is non-parametric fitting, whereby the residual analysis of a parametric fit to the galaxy (or some form of unsharped mask) enables the flux in structures such as disturbed spiral arms or star forming knots to be fully accounted for.

Automated Morphological Classification

We know that morphology appears to be linked to the local environment. Current methods for morphological classification rely on the human eye, making the process increasingly time-consuming with growing quantities of data. Global and structural measurements of colour, size and concentration allow for automatic classification. This enables further exploration of the morphology-density relation; fully understanding why the colour and star-formation rate of galaxies appears linked to the local density.

Observational Tests of Simulated Galaxies

The realms of theoretical and observational astronomy hold important pieces of the galaxy evolutionary puzzle. Applying our observational structural measurement techniques to simulated data will allow for unprecedented access to the internal dynamics of a galaxy population. Knowing what’s going on inside the galaxy, and what’s being observed, should allow for current structural catalogues to be put into a new perspective. This joining of theory and observation may have huge implications on our understanding of galaxy structure, for example, the distinction between a classical and a pseudo-bulge, or the importance of a bar in the creation of a lens.

Structural Tests of Dynamical Observations

In a similar vein, uniting two-dimensional structural decomposition techniques to galaxies with known IFU measurements will provide direct confirmation that structure does indeed trace distinct dynamical components. Combining simulation, structural decomposition and IFU measurements of galaxies from surveys such as CALIFA and SAMI will provide an unheralded wealth of information with which to tackle the question of exactly how classical and pseudo-bulges differ, in addition to opening up exploration of bars, multiple disks and spiral arm features.
Chapter 7. Summary

Tracking Galaxy Evolution

Different evolutionary mechanisms (hot mode collapse and merger, cold mode gas accretion and more cosmologically recent secular evolution) each leave behind distinct structural tracers in their wake. Mapping the relative abundance of structure across cosmic time allows for a measure of the relative importance of each evolutionary process to be calculated. This can be achieved by applying our low-redshift techniques to high-redshift data from, e.g., the Hubble Space Telescope, and upgrading our low-redshift data to forthcoming ground and space-based facilities, e.g., VST KIDS, VISTA VIKING, Euclid. This blueprint of how mass evolves through the emergence of structure shall provide a key insight into our knowledge of galaxy formation and evolution scenarios.
Below I provide postage stamp examples of each morphological type as defined in Figure 5.8. These types are: Little Blue Spheroids (LBS), Figure A.1; ellipticals, Figure A.2; lenticular/early-type spirals, Figure A.3; barred lenticular/early-type spirals, Figure A.4; late-type spirals, Figure A.5; barred late-type spirals, Figure A.6; and; disk-dominated spirals, Figure A.7. Each figure is arranged according to its global $K$ band Sérsic index and rest-frame $u - r$ colour. Postage stamp images are created from false-colour $H i g$ input data.
Figure A.1: Little Blue Spheroids
Figure A.2: Ellipticals
Figure A.3: S0a
Figure A.4: SB0a
Appendix A. Morphology Postage Stamps

Figure A.5: Sbc
Figure A.6: SBbcd
Appendix A. Morphology Postage Stamps

Figure A.7: Sd
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