Understanding inference

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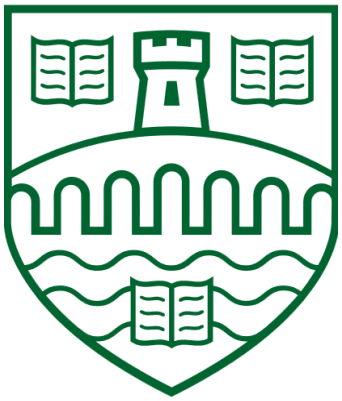

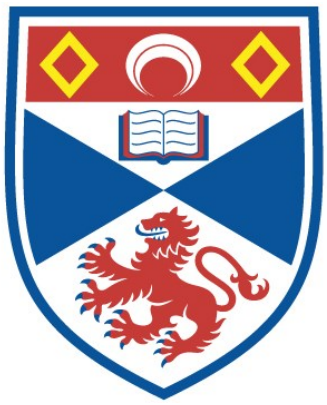

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Thesis: Understanding Inference

Abstract

In this thesis I am investigating the topic of inference. This involves understanding the nature of inferential justification, namely, what grounds the transition from premises to conclusion within an inference. To achieve this, it is necessary to take a view on how logical content features in the discussion. That is, whether the entailment relation can by itself justify the inferential transition. Does the fact that p entails q *justify* the inference from p to q? Since inference is a matter of judgement, contemporary philosophers tend to conclude that logical content cannot perform such justification, but rather, an intermediary act is required. To take the most prominent example, rule following is put forth as such an act which can play this theoretical role.

This thesis rejects the assumption that logical content is prior to judgement. To do this, I first examine the difficulties faced by the contemporary thinkers who accept that there is such a priority. I look at two different accounts, one from Paul Boghossian (2012) and one from Crispin Wright (2014), objecting to both. I then turn to two figures, Frege and Wittgenstein, for a historical discussion of the contrast between possible perspectives. I argue that Frege, in his own way, also accepts the assumption that content is prior to judgement. Wittgenstein's subsequent rejection of Frege's account shines light on the problems with the assumption, still present in the contemporary debate. I defend the Tractarian view of the relation between content and judgement, that content is fundamentally of a kind to be judged. In arguing for such a view of this historical debate, I also argue for a particular way forward with the contemporary debate.

Introductory Chapter

There is an assumption in the contemporary literature on the topic of inference that entailment (an aspect of logical content) is prior to inference (an aspect of judgement). It is therefore assumed based on this that relations of entailment must be transformed into relations of inference. The inferential transition requires justification beyond the fact of entailment.

It follows from the assumption that the logical relation that p entails q does not by itself justify the inference from p to q. One view which shares this assumption is the rule following view of inference, put forth by Paul Boghossian (2012). If p entails q, one is justified in inferring from p to q because they follow a rule of inference which encodes their understanding that the conclusion follows from the premise. Content is understood first, and then the logical relation is turned into a relation of judgement by following the rule of inference.

This assumption leads to theoretical challenges. The rule following view, if one attempts further analysis of the rules involved, leaves us with an explanation that ends up being circular. Following a rule of inference is explained only by the following another rule of inference, which leads to regress. So, a defender of the rule following view must accept that rule following is in fact a primitive action, one which cannot be subjected to analysis. A defender of the view, such as Boghossian, persists even with this counterintuitive defence since they can see no other way that the transition at the level of content can justify a transition at the level of belief.

Sebastian Rödl, on the other hand, argues that understanding Carroll's regress leads directly to an understanding that inference cannot be a further act beyond the judging of the premises. In Carroll's famous example, the tortoise challenges Achill to force him to accept the conclusion Z, if Z follows from A and B. The tortoise argues that it is not sufficient for Achill to merely point out that Z follows from A and B. Further, if the entailment relation is added to the list of premises, then this leads to infinite regress.

The tortoise is never under a greater obligation to accept the conclusion of the inference than he was when the challenge was first given. Rodl's claim, in effect, is that the justification for the inference can never be given by a further act. The rule following explanation is as futile as the tortoise's attempt, the addition of any further act will always lead to the question of subsequent justification for that act.

Yet if we maintain the assumption that content (entailment) is prior to judgement (inference), then a further act is necessitated. The conclusion of chapter 1 is that it is this assumption, then, which must be done away with. What justifies an inference is the fact that the conclusion is entailed by the premise(s). It is not obvious, though, how to go about rejecting this assumption in order to produce a viable account of inference. To motivate a way forward, and to offer a diagnosis of the problems faced by contemporary thinkers in their approach, it is necessary to turn to history. Specifically, to Frege and then Wittgenstein.

In chapter 2, I look to Frege, and I focus primarily on his *Begriffsschrift* (1879), a deeply influential text in the field of logic. Frege presents an inferential calculus, for use in logical deductions. He pairs this with many claims about the nature of logic. The view of inference he outlines as a feature of his system avoids the problems which are the subject of chapter 1. Primarily, The *Begriffsschrift* is able to avoid regress through Frege's distinction between the rules of his system and the axioms underpinning it. Frege is, I argue, not able to defend the idea that rules justify inferential transition. His system of inferential calculus requires rules, and this is clear, but that rules justify inferences in logic does not follow from this.

Fregean rules of inference are licences, they offer permissions or prohibitions against different forms of inference, nothing more. It follows that inference licences are issued when the inference is justified, and a prohibition is issued when it is unjustified. The question of justification, then, is prior to the question of licensing. The argument against Frege in chapter 2 is that we should look for a system of inferential justification without such an intermediary. Wittgenstein's *Tractatus* offers such a system, he sees rules of inference as senseless and superfluous (TLP 5.132).

In chapter 3, I give a Tractarian reading of the relation between content and judgement. This requires an interpretation of what it means for a proposition to be a picture and what it means for these pictures to connect to one another in logical space. Upon understanding these concepts, it is possible to see a system where propositions are such as to be asserted. That is, content is not prior to assertion. Rather, content is fundamentally assertable. In the Tractarian system, content is true because it forms a correct assertion. There is no truth without assertion, there is no content prior to assertion.

I argue that Frege's fundamental flaw is that he, like contemporary thinkers, sees content as prior to judgement. Although he takes the challenge of regress more seriously than defenders of the contemporary rule following view, he runs into even deeper problems.

Developing an objection from Irad Kimhi (2018), I argue that Frege is led by the opposing commitments of his view into contradiction by his own lights. On the one hand, Frege holds that p and not-p, a contradiction, forms perfectly good contents. On the other hand, his 'Intuitive Criterion of Difference' (characterised by Gareth Evans (1982)) states that the thought associated with Sentences S and S' must be different if it is possible to coherently take different attitudes towards the two sentences at the same time. Yet, because 'p and not-p' forms a perfectly good contents, and since Frege defends the possibility of being able to judge the form of contradictions (since it is possible to judge their negation), it follows that 'p and not-p' forms a logically possible judgement. As such, I argue that the judge-ability of 'p and not-p', since p consists in the acceptance of p and not-p consists in the rejection of p, shows that p itself meets the Intuitive Criterion. The singular judgement p has at once two different senses.

In contrast, Wittgenstein would reject 'p and not-p' as contents. On the Tractarian view one cannot accept and reject p simultaneously, it is incoherent. In accepting a proposition, one thereby accepts all the propositions which follow from it and reject all those negated by it. This demonstrates why laws of inference are superfluous. Laws of inference are senseless by the Tractarian perspective since it is not possible to give

propositional form to facts about the proposition (that one proposition follows from another). These facts are internal to propositional form and thereby cannot be explained propositionally, they are shown rather than said (asserted as a proposition).

A possible reaction to this project is that the material in chapter 1 is in fact of a different nature to the material discussed in chapters 2 and 3. The contemporary thinkers are concerned with the process of rational belief formation whereas the historical thinkers are concerned with the foundations of logic. As such, one might think it inappropriate to argue that Wittgenstein is able to offer an answer of the appropriate kind. In response to this I have two thoughts.

First, Wittgenstein offers a diagnosis of the problem with which contemporary thinkers are deeply concerned. Avoiding Carroll's regress is a concern of Wittgenstein's and the modern literature, as such the Tractarian system is built such as to avoid the regress entirely. A Tractarian tortoise could never ask why he ought to accept the conclusion of the inference. If he accepts the premises, he will accept the conclusion. The regress only gets off the ground if one accepts that content is prior to judgement. Propositions, according to Wittgenstein, have it in their nature to relate to other propositions (their place in logical space). Saying anything further about these relations is to say what can only be shown. That both sets of thinkers go to great lengths to try and avoid Carroll's regress show that they are concerned with overlapping subject matter.

Second, even if one is unsympathetic to the system presented by The Tractatus, it is incumbent on such a person to take seriously the challenges Wittgenstein puts forth against, for instance, any kind of rule following view. If one cannot meet this critical challenge, then the philosophical burden is shifted on to the objector against the Tractarian view. The purpose of chapter 1 is to demonstrate the depth of this critical challenge. The purposes of chapters 2 and 3 to motivate, through discussing Frege and Wittgenstein, a way forward grounded in history.

Chapter 1: Contemporary accounts of inference

Introduction

Rational agents infer in a way that consistently accords with logical principles such as Modus Ponens and Modus Tollens. The capacity to systematically infer in this way is foundational to rationality. It is not clear *how* we make inferences. Boghossian (2012) argues for an intentional rule following view of inference. On this view, we *conform* with these logical principles precisely because we are *following* rules which encode their contents. The problem with this is that following rules of inference itself involves inference. The explanation is circular, so we are threatened by regress.[1] Boghossian defends the rule following approach as one with the benefit of meeting the 'Taking Condition' (Boghossian, 2012, 5). The Taking Condition dictates that inference involves *taking* our premises to support their conclusion. Despite its apparent plausibility, much of the growing body of literature has chosen to reject the Taking Condition to avoid the threat of regress.[2] I argue that the Taking Condition can be met without adopting the rule following view.[3] I conclude this chapter by claiming that inferences must be justified by the internal relationship between the propositions involved in the premises and conclusion. Chapters 2 and 3 then investigate two systematic solutions for achieving this goal.

[1] My objection is more precisely that the circularity leads to the explanation being *non-rational* since Boghossian argues that rule following must be primitive (Boghossian, 2012, 17).

[2] See Crispin Wright (2014), Conor McHugh and Jonathan Way (2016), Declan Smithies (2023).

[3] Others including Christopher Blake-Turner (2022), Christian Kietzmann (2018) and Markos Valaris (2020) argue for ways of construing the Taking Condition to retain it and avoid regress. Blake-Turner argues for an account which rejects that the Taking Condition is *prior* to the inference itself (Blake-Turner, 2022, 93), but rather through making an inference one hereby-commits to the premises supporting the conclusion when they draw the conclusion (Blake-Turner, 2022, 96). It is beyond the scope of this paper to address specific issues with these accounts, but rather it is only possible to draw on them when helpful for the argument of this paper.

In section 1, I defend the Taking Condition as an appropriate constraint on inference. In section 2, I outline and argue against the intentional rule following view. In section 3, I develop my positive suggestion that inferring is just a matter of judging the premises appropriately and I show how this vindicates the Taking Condition. In section 4, I examine and resolve an objection against the Taking Condition from Crispin Wright (2014), who argues that inference is a non-rational process. I conclude that my positive suggestion is the only way to meet the demands of the Taking Condition and to thereby affirm inference as a process of reasoning. This conclusion motivates the project of chapters 2 and 3, where I am seeking a system which justifies this positive suggestion.

Section 1: The Taking Condition on inference

The Taking Condition:

> 'Inferring necessarily involves the thinker taking his premises to support his conclusion and drawing his conclusion because of that fact.' (Boghossian, 2012, 5)

Consider how a single premise inference, where one infers A (the conclusion) from the conjunction of A&B (the premise), might meet the Taking Condition. It is necessary that the inferrer take the premise (A&B) of their inference to support the conclusion (A). This means that they must understand the logical connection that A is entailed by A&B. It follows that the inference A is based on the conjunction A&B, since the inferrer takes A&B to support A. The conjunction A&B is a *reason* for inferring A. The inferrer must also draw the conclusion (A) *because* of this understanding, because of the reason (A&B). According to the Taking Condition then, inferring A from A&B = judging A, *caused* by judging A&B and *taking* A&B to support A.

Now consider Michael's succession of beliefs:

> **p**: The car's engine is broken
>
> **q**: If the car's engine is broken, it won't start

r: The car won't start

The premises of the inference (**p**,**q**) function as 'rational justifications' (Blake-Turner, 2022, 86) for the conclusion **r**. Michael's conclusion that the car won't start, is justified by his reasons (The car's engine is broken, If the car's engine is broken, it won't start). Although inferences like this are not 'effortful or demanding' (Boghossian, 2012, 3), like other forms of reasoning, it is still a fundamentally rational process. As with other processes of reasoning, we would expect that inferrers are in *a position to give their reasons*. If I infer **r** from (**p**,**q**), then I should understand (**p**,**q**) to be my reasons for **r**. I can only reason from (**p**,**q**) to **r** if I take (**p**,**q**,) to support **r** and, understanding this, thereby draw the conclusion **r**.

Without the Taking Condition, then, inference would be a non-rational process. If the agent does not take their premises to support the conclusion, then there can be no explanation of their reasoning from the premises to the conclusion.4 Inference would therefore not be something we do (an act in a true sense) but something which happens to us. If Michael reasons from (p,q) to r, it must be that he understands (p,q) to warrant r. The inferrer 'traces a connection of rational support' (Blake-Turner, 2022, 88) which enables them to understand their premises as reasons for the conclusion. This understanding justifies the inference. The Taking Condition captures this rational character of inference. Michael infers r because he takes (p,q) to support r, meaning he reasoned from (p,q) to r and he is in a position to give (p,q) as his reasons for arriving at r.

It is of course possible for Michael to judge **p**, **q** and **r** without inferring **r** from **p** and **q**. In the case where he infers **r** from (**p**,**q**), it is not immediately clear what distinguishes a *mere* set of judgements, from an inference. The Taking Condition tells us that inferring r from (**p**,**q**)= judging **r**, caused by judging (**p**,**q**) and taking (**p**,**q**) to support **r**. This does

[4] In section 4 I reiterate this point and offer a fuller explanation in response to an objection against the Taking Condition from Wright (2014)

not yet answer the question of how the Taking Condition is met by an account of inference, since, as a necessary condition, it only serves to constrain what can count as inference. It places a boundary on inference but does not tell us what kind of process inference is. With single premise inferences such as inferring A from A&B, the conclusion (A) is *part* of the premise. However, with multi-premise inference, although **r** is entailed by **p**,**q**, it looks as though **r** is a *further act,* an extra judgement not contained within the premises (**p**,**q**).[5] So, how might the inferrer make this further judgement?

Section 2i: Intentional rule following

Boghossian argues that inference is a type of rule following (Boghossian, 2008, 492). This is downstream from his idea that rule following explains our ability to form rational beliefs, generally speaking.[6] Inference, a method of rational belief-formation, must also be a type of rule following.

One example of a non-inference rule which allows rational agents to form beliefs is the: '(Email Rule) Answer any email that calls for an answer immediately upon receipt!' (Boghossian, 2012, 11). The agent has a representation of the rule in their mind: If I receive an email that requires a response, I shall reply to it immediately. For it to be the case that the agent *intentionally follows* this rule, it must be on the basis of the (Email Rule) itself that the agent chooses to reply to the email. This representation therefore *encodes* an understanding of rule in the agent's mind, since it expresses the contents of the rule. This allows the agent, whenever they recognise that the 'antecedent of the rule has been satisfied' (ibid.), to follow the (Email Rule). The antecedent in this case, is the

[5] I am outlining a belief of the conventional view, that the conclusion of the inference is not contained within the premises. In section 3 I reject this as the flaw of standard accounts of inference, arguing that the premises already contain the conclusion. I mention it here only to show that I agree that it does *appear* to be the case that the conclusion is a further judgement beyond the premises, especially with multi-premise inferences.

[6] 'Our ability to form rational beliefs is productive: on the basis of finite learning, we are able to form rational beliefs under a potential infinity of novel circumstances. The only plausible explanation for this is that we have, somehow, internalized a rule that tells us, in some general way, what it would be rational to believe under varying epistemic circumstances. (Boghossian, 2008, 483)

receipt of any email that calls for a response. This constitutes the trigger condition of the rule.

So, the agent following the (Email Rule) infers from (representational state + trigger condition). It might be that an agent *conforms* with the (Email Rule) without following it. One might, just out of habit, happen to reliably reply to any email they receive which calls for an answer. The intentional rule follower is different from this sort of conformist in that they reply to the email because of the rule itself, not because of habit. The (Email Rule) therefore *rationalises* the rule followers' choice to reply to emails. That is, 'If I receive an email which warrants a response, I shall reply to it immediately' + 'I received an email which warrants a response' gives a rational explanation for replying to an email. These constitute the rule follower's *reasons* for replying to emails because they caused the rule follower to reply to the email.

Following the (Email Rule) therefore involves inference, but inference, as a process of rational belief-formation in its own right, is itself a form of rule following. Consider:

> (**Modus ponens**):[7] 'If you are rationally permitted to believe both that x and that 'If x then y' then, you are prima facie rationally permitted to believe that y.' (Boghossian, 2008, 472)

Boghossian takes inference to be a case of following such a rule. As we saw with the (Email Rule), the agent follows (**Modus Ponens**) by representing the rule and recognising that the trigger conditions have been met. So, if Michael has a representation of (**Modus Ponens**) then he knows that whenever x and if x then y, then he is rationally permitted to believe that y. When he recognises (**p**,**q**) 'The car's engine is broken, If the car's engine is broken, it won't start' he can thereby follow the rule of inference to conclude that **r**; the car won't start. This rationalises his conclusion **r** since it is *because of the rule* that he concludes **r**.

[7] **Modus Ponens** is repeated in bold in all instances where I am referring to the rule specifically, not the logical principle.

Michael, through intentionally following (**Modus Ponens**), meets the Taking Condition on inference. For Michael to follow the rule of inference, it must be that he understands (**p**,**q**) to support **r** and that he concludes **r** because of this understanding. To meet the criteria of rule following *is* to meet the Taking Condition. This gives us an account of inference which vindicates the Taking Condition.[8]

Section 2ii: The threat of regress and non-rational inference

Following a rule of inference is a process which itself involves inference. Michael infers from **Modus Ponens** + trigger condition to the conclusion that he should follow the rule of inference. This inference, then, is unexplained. The only possible explanation would be to restate the explanation of inference we have just given. So, Michael's action of inferring from **Modus Ponens** + trigger condition is only itself explained by him following a further rule of inference. This further rule following would only lead only to another unexplained inference. So, the intentional rule following view is faced with regress.

Boghossian sees the potential regress problem but offers a two-part defence of the intentional rule following view. First, he claims that his picture (described in Section 2i) of intentional rule following is the only way to meet the Taking Condition (Boghossian, 2012, 16-17). Secondly, he says:

> 'we face a stark choice between attempting to account for our mental lives without something that looks like the traditional notion of person-level reasoning,

[8] I focus only on the *intentional* rule following view in this section, to the exclusion of the *dispositional* rule following view. This is because. I agree here with Boghossian's rather simple critique (see Boghossian (2012,14-15)). Briefly put, if rule following is characterised by our disposition to conform with a rule, then it can no longer be the case that the content of the rule can rationalise our disposition to conform with it. Rather, our disposition must do the rationalising and 'our dispositions cannot explain themselves' (Boghossian, 2012, 14). So, since it is inferring based on the content of the rule which meets the Taking Condition, the dispositional view cannot offer an explanation which competes with the intentional view.

> on the one hand, and being willing to take the notion of following a rule as an unanalyzable primitive, on the other.' (Boghossian, 2012, 17).

For Boghossian, then, the Taking Condition is necessary because it protects our traditional notion of person-level reasoning. By this he means, as we saw in section 1, that inference can only be a rational process which agents perform if they infer the conclusion because they take it to be supported by the premises, which is the Taking Condition.

The conclusion of this two-part defence is that rule following must be the end of our explanation. Any further analysis of rule following leads to regress, and so further analysis must not be possible, rule following must be an unanalyzable primitive explanation. The unexplained inference which rule following contains does not in fact lead to regress. It would only lead to regress, if rule following were capable of non-circular rational explanation. Boghossian is pained by this explanation, admitting that 'it seems we ought to be able to' (Boghossian, 2012, 17) analyse rule following. It is only upon attempting to do so that we are *forced* to conclude that it is impossible. Boghossian prefers the rule following explanation since it defends our traditional notion of person-level reasoning, yet the explanation itself is necessarily non-rational.

Boghossian's choice to persist with primitivist rule following, although it avoids regress, leads to a deep problem for the *kind of explanation* offered by the intentional rule following view. Consider the following explanation:

> 'x looks red to S- (*is explained by)*[9] there is a class of red sense data which belong to x, and are sensed by S' (Sellars, 1997, 32).

The right-hand side of the biconditional offers an explanation for the left-hand side. Red sense data appears to explain 'looking red' in just the same way that molecules bouncing offers a scientific explanation of pressure (ibid.)

Let us assume that red sense data is itself incomprehensible, that it is an unanalyzable primitive. It is the end of our explanation of 'looking red'. If 'red sense data' is

[9] '*Is explained by*' is my addition for clarity

incomprehensible, it therefore lacks the explanatory value of ‘molecules bouncing’.[10] ‘Molecules bouncing’ is a genuine explanation of pressure, since we can offer a further analysis of what ‘molecules bouncing’ means. The capacity for the sentence to be explanatory is underpinned by this further analysis. This same capacity for genuine explanation is not present in our case of red sense data as an explanation of ‘looking red’, if it is true that red sense data is unanalyzable. Likewise, if the goal is to give inference a *rational explanation*, then we cannot meet this goal by using an unanalyzable primitive concept of rule following.

So, allow me to briefly summarise the argument against the intentional rule following explanation. The threat of regress is resolved by accepting rule following to be an unanalyzable primitive. This solution leads to a secondary, insoluble problem. If rule following is basic, although it proposes to rationally explain inference, it does not do so. The picture we are left with is a *non-rational* explanation of inference. Rule following cannot rationalise inference if rule following is primitive. If Michael follows a rule of inference, he meets the Taking Condition, he infers because he takes the premises to support the conclusion via the rule. However, we cannot explain why or how he follows the rule of inference. His choice to follow this rule is basic, non-rational. If we cannot understand how or why Michael follows the rule of inference, then we cannot understand how or why he meets the Taking Condition. If we cannot understand how or why he meets the Taking Condition, which is what makes inference rational, then we cannot understand how or why inference is a rational process. Yet this is the goal of our inquiry.

Section 3: No further act beyond the premises

An assumption made by Boghossian, and other proponents of standard accounts of inference, is that a further act (such as following a rule of inference), is needed to be

[10] For the sake of argument, assume that sense data is incomprehensible. It is clear that rule following, if it is unanalyzable, *is* incomprehensible in the sense that it cannot be rationally explained.

added to these judgements for inference to take place. I do not agree with this assumption. In this section I will develop a positive suggestion; showing how inferential relations could be internal to the judgements which make up inferences.[11] I argue that this allows us to meet the Taking Condition without running into the regress problem and the further issues this causes. It will be the subject of both the following chapters to settle on a theoretical system to underpin this suggestion, yet it is necessary here to characterise my project, since it contrasts sharply with the intentional rule following view just discussed.

Consider the case of *single*-premise inference I described earlier- inferring A from the conjunction A&B. I mentioned in raising this example that, A (the conclusion), is already part of the premise of the inference. It is the first half of the conjunction A&B. When one infers A from A&B, it is clear that nothing needs to be added to the contents of the premise in order to justify the inference, A. Let us then assume a view of judgement, where, in judging A&B, one thereby judges A&B to be *true*.[12] I take it that one cannot judge A&B without judging it to be true and as such, it is a constitutive part of judgement to take oneself to be judging rightly.[13] Both the systems I will discuss in the later

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[11] The idea that inference involves no further act than the judging of the premises is developed from Sebastian Rödl (Rödl, 2018), I differ from Rödl insofar as he argues this emerges directly from *Carroll's* regress (Rödl, 2018, 163), rather than the rule-following regress problem and its implication we looked at in section 2. I am also interested in defending the Taking Condition, a task which Rödl does not take up. It is further work to engage with his argument which would involve a discussion of Carroll's regress and its implications. For helpful discussion on Carroll's regress and its implications on inference see Rödl (2018), Corine Besson (2019) and Barry Stroud (1979).

[12] As Boghossian puts it: 'We needn't always infer from *truths*, if we are to count as inferring. It's enough that we take our premises to be true, that is, judge them to be true' (Boghossian, 2012, 4). The agent who infers judges their premises to be true.

[13]] Kietzmann (2018) also sees the importance of this fact which he argues is one of two 'truisms' about belief (Kietzman, 2018, 299). The other truism about belief, which he calls self-consciousness, is that the

chapters defend such a view, since they each take judgement to involve affirmation, saying “Yes” to the content of the judgement, in different ways.

From this understanding of judgement, we can develop an explanation of single-premise inference. The agent who judges A&B then has committed themselves to the truth of A and the truth of B. The truth of A *is* the conclusion of the inference. So, when the agent judges the premise, they also commit themselves to judging the conclusion. I take this to show, at least in this example of single-premise inference, that judging the premises (which entail the conclusion) *is* to judge the conclusion. We have therefore explained inference without there being any further act beyond the judging of the premises.

Now let's extend this to *multi*-premise inference. When Michael judges (**p**,**q**) (The car’s engine is broken, if the car’s engine is broken, it won’t start), the conclusion r (The car won’t start) is also contained within the premises, if you take the two premises together. The conclusion of the inference is the consequent of **q** and **p** fulfils the antecedent of q. So, r is contained within (**p**,**q**) One might protest though, that this does not help to explain multi-premise inference. It is only upon unifying the two judgements (**p**,**q**) that the conclusion is reached. This is true and the agent is not judging the *conjunction* (**p**&**q**), but rather the agent is judging the two separate judgments (**p**,**q**). So, the objection runs, how can it be that the conclusion is contained within the agent's judgement of the premises, as it is not contained in *either* of the agent’s judgements.

When Michael judges (**p**,**q**), he makes these judgements while recognising the relation between his two judgements. This recognition, that (**p,q**) entails **r**, is internal to the judgement of the premises. If we posit that what justifies the conclusion is internal to the judgement of the premises of an inference, we can derive the following view:

agent who believes that p takes themself to believe that p (ibid.). See Kietzman (2018) for a fuller discussion of these two ‘truisms’.

> ‘‘She who judges C on the basis of A and B understands A and B to provide sufficient grounds for judging C. Yet her recognition that A and B establish the validity of judging C cannot be a judgement added to her judgements A and B.’ (Rödl, 2018, 141)

If this is the case, then Michael’s judging the conclusion is one and the same as his judging the premises, if he judges the premises together. Once again, we can explain inference without needing any further act. This suggestion, therefore, if it can be defended, would remove any need for a *norm* justifying the inferential transition from premises to conclusion.

This position requires a much more substantial theoretical underpinning. This is work I will undertake in sections 2 and 3, when, I will look to Frege and Wittgenstein’s accounts of inference. I mention it here because I want to give the shape of the contrast between this view and the one, I am objecting to. I am also going to give some pre-theoretical motivation for the view.

Is it at all plausible that a judgement of r as true can already contained within the judgements (p,q)? Barry Stroud makes a helpful point about belief:

> ‘‘Believing something involves understanding it, and that in turn appears to involve seeing some of the connections with other things one understands’ (Stroud, 1979, 189).

It is part of believing **p** and believing **q**, that you see some of the relations with other things you understand, in this case, the relation between **p** and **q**: (**r**). To judge **q**, is to judge ‘if the car is broken, it won’t start’. To understand this, surely involves seeing that if the consequent is fulfilled then the consequent will be true, otherwise it would not be possible to make connections from this belief to others. The content of the judgement **q** *is* the conditional judgement, in this case, if **p** then **r**. If the agent does not understand this, they would not be able to understand *any* of the connections with their other judgements, which would give us grounds to say they don’t actually believe it and so cannot judge it to be true. So, part of judging q is to *understand* that if p then r.

With regards to judging p, the most obvious connections are those truths which are necessitated by the truth of p. Once I understand that ‘The car is broken’, my understanding will connect this belief to other beliefs about *what it means for this car to be broken*. If I also believe that **q** ‘If the car’s broken, it won’t start’, it does not take any further work for me to connect (**p**,**q**) which, as I have said, *is* to infer r.

The most obvious objection to this is that, even as rational agents, we are not normatively required to see all connections between all our beliefs at all times. I might also believe ‘If the car is broken, I can’t go to Mexico’ but not draw the inference ‘I can’t go to Mexico’ when I see that ‘The car is broken’. It has already been stipulated though, that I don’t *merely* believe p or q in this sense. Inference is on the basis of judging p and judging q to be true. Part of making a judgement is to hold it in present consciousness and then, holding both judgements in a single present consciousness, is to make the inference. So, you *would* draw the inference ‘I can’t go to Mexico’ if you *chose* to hold the judgments ‘If the car is broken, I can’t go to Mexico, the car is broken’ together in a single consciousness. This explains why we make the inferences we do, and only the ones we do. Being conscious of certain judgements distinguishes the inferences we do make from the inferences we could make.

The Taking Condition is met since the agent takes their conclusion to be supported by their premises and draws the conclusion for this reason. This understanding, that the conclusion is supported by the premises, I have argued, is internal to the very judging of the premises of an inference. Just as there are normative implications for judging **p**, the most obvious being that one should also judge not-**p** to be false, there are normative implications for judging (**p**,**q**). One who judges (**p**,**q**) together, holding both premises in a single consciousness, judges it to be true that **p** and true that **q.** Since **r** is entailed by (**p**,**q**), when a rational agent judges (**p**,**q**) together they form a joint understanding of the implications of both premises, which is to judge **r**.

Because, on such an account, there is no need for rules of inference, it is possible avoid the problems faced by the intentional rule following view. With the rules-based view, we see that rule following cannot be analysed without leading to regress. So, Boghossian chooses to accept rule following as a primitive in his theoretical framework. I argued that

this meant we could not rationally explain inference, since the explanatory concept (rule following) cannot be analysed in the way that we would expect. This section has set out my view (given further substance by the argument of the following chapters), that inference is a rational process, in the sense that it reflects the internal relations between propositions. (**p**,**q**) entail **r**, our judging together of (**p**,**q**) constitutes judging the conclusion **r**.

I am aware, as I mentioned previously, that there is much theoretical work to be done. The hard question of how the relation of *contents* (entailment) can ground a relation of *judgement* (inference) without the need for any norm justifying the transition is answered in chapter 3, where I defend a Tractarian view of inference.

Section 4: An objection: Inference without the Taking Condition

An objection which it is important to address before going further regards the Taking Condition itself. If there is no need for the Taking Condition, then this undermines the motivation I offered for adopting the suggestion I argued for in section 3, and, subsequently, the motivation for the later chapters. This is because the Taking Condition, a rational criterion, requires a similarly rational explanation as to how it is met. An explanation unburdened by the Taking Condition could be *non-rational*. I will outline an objection put forth by Crispin Wright (2014) that the Taking Condition is inappropriate. I will argue that Wright's account fails on its own terms, offering the sort of *non-rational* explanation that he is trying to avoid, with the implication that inference is merely sub-person-level reasoning (contrary to Wright's own view).

Wright agrees with Boghossian that the Taking Condition leads to intentional rule following. Contrary to Boghossian, Wright rejects the rule-following-as-primitive solution to the regress problem (Wright, 2014, 33). For this reason, Wright dismisses both rule following and the Taking Condition. He offers us an alternative account of inference:

> ‘Simple proposal. It says that a thinker infers q from $p_1 \ldots p_n$ when he accepts each of $p_1 \ldots p_n$, moves to accept q, and does so for the reason that he accepts $p_1 \ldots p_n$.’ (ibid.)

Wright does not believe that a thinker needs to take their conclusion to be supported by their premises in order to make an inference. Instead, the act of inference occurs whenever one moves to accept a conclusion for the reason that they accept their premise. Of course, thinkers may meet the Taking Condition (Wright, 2014, 35), but this is not a necessary part of inference-making. Any meeting of the Taking Condition is a further judgement, a judgement about the inference, not a part of the inference itself. One need not understand their premises to support their conclusion via some representational state, it only matters what their reasons for moving to accept the conclusion *actually are* (Wright. 2014, 33).[14] The thinker need not be in a position to know their reason for making their inference.

It is true of course that rational agents are disposed to infer in a rule-like manner. That is, in a way that *conforms* with rules like (**Modus Ponens**) but, on the Simple proposal, inference is not caused by *following* those rules.[15] On the Simple proposal, ‘movement in accordance with basic rules of inference is constitutive of rational thought’ (Wright,

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[14] It is crucial for Wright that representational states play no role in inference. On the rule following view, the agent represents **Modus Ponens** (and infers from it) in order to make the inference, triggering the regress. Wright argues convincingly that the regress will reappear if representational states of any form are involved in inference (Wright, 2014, 31). In fact, he specifies that his rejection of the Taking Condition is contingent on its requiring a representational state (Wright, 2014, 33), and my suggestion in section 3 does not require a representational state. It is still important to engage with his account since he views the Taking Condition as unnecessary, and my suggestion hinges on and argues for its necessity.

[15] In section 3, I argued that the understanding **r** is entailed by (**p**,**q**) is contained in the judging of the premises. So, according to my suggestion the rule-like nature of inference is explained by the understanding of logically valid principles being internal to the content of the premises.

2014, 36). So, rational thought is constrained without being governed by rules. Wright takes this to be possible because the principle of 'Anti-Akrasia (A-A)' (Wright, 2014, 34) constrains rational action without being a type of rule following:

> **The Anti-Akrasia principle (A-A)** 'Provided there is no overriding reason not to, do that which you believe will satisfy your desires' (ibid.)

The A-A principle constrains rational action without it being the case that we follow an A-A rule. Rather, it is constitutive of being rational to conform with A-A. I believe that eating a sandwich will satisfy my desires, so, being rational, I eat one. In doing so, I *conform* with A-A without *following* an A-A rule. Since it is not a rule to be followed, there is no need for a corresponding A-A representational state.

Our process of inference, according to the Simple proposal, works in just this way. Rational agents are disposed to infer in a way that conforms with certain rational principles (Modus Ponens, Modus Tollens). Although the rational disposition to infer is rule-like (just like A-A), in the sense that it is consistent and according to certain rational principles, inferring is not a type of rule following.

Let's examine a case of so-called 'basic inference' (Wright, 2014, 36). Basic inference denotes cases of inferential reasoning, according to the Simple proposal, but which appears to not meet the Taking Condition:

> 'Luke and Drew are playing hide-and-seek. Seeing Drew's bicycle leaning against the tree, Luke thinks: 'If he were hiding behind that tree, he would not have left his bicycle there. So, he must be behind the hedge.' (Boghossian, 2012, 6)

Luke, an unsophisticated thinker who is still capable of basic reasoning, moves from his belief about the bike to his belief about Drew's whereabouts. For the intentional rule following view, if we stipulate that Luke is not capable of rule following, since he is not

able to form a representational state encoding the rule, then this cannot be a *true* case of inference.[16] So, it is not clear what *is* happening.

Wright argues that one advantage of the Simple proposal, is that it is well equipped to offer an explanation:

> 'This suggests … that we look at the role of *rules of inference* in basic inference on the model of the role of A-A in rational action. Thus, it is not that some kind of sub-personal acceptance of, say, *modus tollens* underwrites and rationalises- even though sub-personal- the intelligent movements of thought of Boghossian's Hide-and-Seeking children. Rather, *modus tollens* stands to Luke's performance exactly as A-A stands to it.' (Wright, 2014, 36)

So, it is argued, we have a clear case of basic inference where the Taking Condition is not met. And, it is *only* explainable, Wright argues, without acceptance of rules since we have stipulated Luke is too unsophisticated a thinker to follow a rule of inference here that involves an intentional state which registers the content of the rule. Rather, the inference rule plays the role of a constitutive norm (ibid.), just like A-A, which governs Luke's behaviour.

But of course, it must be that Luke *in some sense* takes his premise 'that Drew is not behind the tree' to support 'So, he must be behind the hedge'. Otherwise, Luke would not think '*So*'—'he must be behind the hedge'. There would be no causal relation

[16] I agree with Wright's critique here that Boghossian should not call cases like these instances of 'blind rule-following' (Wright, 2014, 36). On the intentional rule following view, following the rule of inference implies that the Taking Condition has been met. Intentionally following the rule of inference *involves* taking the premises to support the conclusion. If the Taking Condition is not met, then it cannot be intentional rule following. Yet, it looks like Luke performs an inference, which is problematic for Boghossian's view.

between the beliefs at all, at least not in a way that Luke is aware of.[17] The 'So' shows us that Luke believes 'So, he must be behind the hedge' *because* 'If he were hiding behind that tree, he would not have left his bicycle there'. This is to construe the Taking Condition more broadly than Wright and Boghossian since Luke does not represent to himself that his premises support the conclusion. Nonetheless, his understanding that his premises support the conclusion are expressed by his understanding that he believes the conclusion because of the premise.

The first thing to point out is that Wright is correct to take issue with the rule-following-view's reading of the Hide-and-Seek case. Luke makes an inference even though there is no intentional state representing the Taking Condition. Yet, it is still a case where the Taking Condition is met since Luke takes his premises to support his conclusion. The Hide-and-seek case is not a problem for the Taking Condition per se, but rather, it is problematic for the Taking Condition only insofar as it is met via a representational state.

The second consequence of this however is that the Hide-and-Seek case is not a case of so-called basic inference. It does not demonstrate the illegitimacy of the Taking Condition, instead, Luke meets the Taking Condition. For the case to be an example of basic inference, Luke would need to only judge the following 'If he were hiding behind that tree, he would not have left his bicycle there. He must be behind the hedge'. In removing the 'so' we get closer to a basic inference, because we remove the causal connection between the judgements. Only in removing the 'so', would Luke no longer take himself to judge the conclusion because of the premise. At this point, though, we struggle to make any sense of this *being an inference*. If Luke does not know himself to

[17]] I go on to consider inference taking place sub-personally such that the premise(s) causes the conclusion without the agent being consciously aware of this relation.

think the conclusion because of the premise, then how has he *inferred*[18] the conclusion from the premise?

The only remaining possibility for the Simple proposal is sub-personal.[19] Luke's mind has in some way derived the conclusion from premise, without Luke being aware of this causal connection. Inference cannot be a process of reasoning, even sub-personal reasoning, if the agent isn't even in a position to know their reason(s) for believing their conclusion. The Simple proposal claims that inference is still an action (rather than a mere bodily movement) even though it is sub-personal, because it is done for certain reasons. This could still be true even if the agent is not aware of the causal chain. However, if the agent is not aware of the causal chain and they have still made an inference, then this cannot be an action. It is something that happens to the agent. It must be that the agent is *in a position to know their reasons*. If the agent has this understanding, this means they understand the inferential relation between their premise(s) and their conclusion. It follows that they take their premises to support their conclusion, thereby meeting the Taking Condition.

The problem can now be stated clearly. According to the Simple proposal, inference is an act of reasoning *we do* because:

> 'an action proper is distinguished from a 'mere' (as we like to say) bodily movement exactly in that it is done for certain reasons. The notion that inference is something that we do is immediately safeguarded by the Simple proposal, as a

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[18] I should caveat that this framing assumes much of the prior discussion in this paper, some of which from section 1. I have provided support for the claim that inference is a process of reasoning, yet Luke is not even aware of his reason, the inferential relation between premise(s) and conclusion (which he is supposedly reasoning with). I am challenging this idea.

[19] I use sub-personally here in the same sense as Boghossian who defines this type of reasoning as 'sub-conscious, involuntary and automatic' (Boghossian, 2012, 2).

special case of the platitude that *an action* is something that we do' (Wright, 2014, 34).

I infer q from $p_1 \ldots p_n$ if and only if I accept each of $p_1 \ldots p_n$ and move to accept q *because* I accept $p_1 \ldots p_n$. Wright contests that this shows inference to be an action since it is based in reasons. However, we can see now that unless we are conscious of those reasons *as our reasons*, then the process of reasoning is not something *we do* but rather something that *happens to us*. It is not sufficient that it is caused by reasons. We cannot *reason* using reasons that we are not even aware of. 'Reasoning' in this sense would be a mere happening, not a true action.

So, for inference to be a true action, we must be in a position to know our reasons for making inferences, contrary to Wright's account. It is a consequence of being in a position to know one's reason(s) for their inference(s) that the Taking Condition is met. And so, Wright's account fails in his goal to safeguard inference as a rational action since it rejects the Taking Condition. His account therefore fails to achieve its aim.

It follows from my response to Wright's objection that a defender of the Simple proposal is forced to accept that inference is not a process of reasoning *at all*. If it is not a true action (since it is a mere happening), it cannot be an act of reasoning. Inference would just be a *basic mental action*,[20] which is not, strictly speaking, an action. An agent cannot reason from premise(s) to a conclusion if they are not in a position to understand the inferential relation between the premises and the conclusion. It must be something that our minds happen to be disposed towards, a 'mere' basic psychological process. Only on occasions when the Taking Condition happens to be met, which is not a requirement on inference can the agent be said to be engaging in reasoning. The Simple proposal thereby fails to meet its goal to affirm inference as a rational process. In its rejection of the Taking Condition, it is instead forced to view inference as a non-rational mere-psychological disposition of the mind.

[20] This is Wright's language of 'basic mental action' (Wright, 2014, 36). Labelling it an action or not is not important. My contention is that the type of action or happening that inference is on this view cannot be described as a process of reasoning or rational process on the part of the agent.

Conceiving of inference as internal to the premises, as per my argument in section 3, is the only way to make sense of the Hide-and-Seek case. Luke judges the premise 'If he were hiding behind that tree, he would not have left his bicycle there.' and as part of his understanding of that premise he judges that 'He must be behind the hedge'. Of course, it must also be that he judges 'Drew is either behind the tree or the hedge', for him to conclude that he is behind the hedge on the basis of concluding he is not behind the tree. The rule following view cannot make sense of Luke taking his premise(s) to support his conclusion since Luke is not sophisticated enough to represent a rule of inference. But if the Taking Condition is met within the judging of the premises and judging the conclusion is not a further act then there is no representational state. It is contained within Luke's understanding of his premise(s) that he takes the premise(s) to support the conclusion. Thus, the Taking Condition is met, and we can make sense of Luke making an inference.

Conclusion

In section 1, I argue for the Taking Condition. In section 2, I present and then object to the most prominent account of inference which attempts to meet it, the intentional rule following view. In section 3, I develop a positive suggestion; arguing that the way to ameliorate the problems of the rule following view and to meet the Taking Condition is to adopt the view that inference is internal to the judgement of the premises. It cannot be a further act, such as the following of a rule, which justifies the transition from the premises to the conclusion of the inference. In section 4, I look at the Simple proposal which claims to offer an account of inference as a rational process without the need for the Taking Condition. I argue that inference cannot be a rational process on this view, since the inferrer is not required to be self-conscious about their reasons.

To avoid the different problems faced by Boghossian and Wright, it is necessary to turn to historical systematic accounts to continue the project set out by section 3. Both accounts I will examine (*The Begriffsschrift* and *The Tractatus*) view inferential justification as a consequence of the content of propositions, namely, the entailment relations between propositions which is internal to the content. The first of these two

accounts, Frege's *Begriffsschrift*, does employ rules for the purposes of his inferential calculus. However, I will show that they are not the sort of substantial rules put forth by Boghossian, rather the rules serve only to issue inferential licences and prohibitions based on the internal relations between propositions. That is, Frege's account is of the kind I suggest that we need in section 3. While I will eventually reject the need for even licence-rules of this kind, it will be on the basis of a different objection, and we will first need to come to an understanding of Frege's account in order to defend our objection.

Chapter 2: Frege's account of inference

Now that we have considered contemporary accounts of inference, it is time to turn to two competing historical accounts. The first will be Frege's account of inference as presented in *The Begriffsschrift*. Frege puts forth a concept script to be used as an inferential calculus and dedicates most of this text to laying out how this calculus is to be used. What we will be concerned with, though, is his reasoning about the nature of logic. This reasoning, fully understood, will offer a theoretical account of inference, which of course is the aim of our inquiry. I will argue, however (in chapter 3), that Frege's account fails on its own terms because of his separation of content and judgement (content is prior to and therefore external to judgement). Of course, in order to argue this, we will first need to understand Frege's views of content and judgement, and how this relates to inference.

Section 1 examines the nature of assertion and negation in *The Begriffsschrift*. Here, we will see that the Fregean assertion sign serves to transform judgeable contents into judgements. It will also be important to understand that negation occurs at the level of contents, such that a proposition and its negation do not *oppose* each other at the level of judgement. Section 2 examines Frege's chosen rule of inference, Modus Ponens. I will explain how although Frege employs his conditional, that, since it is an axiom within the context of his system alone, he is not vulnerable to Carroll's regress. Section 3 expands upon an objection to Frege put forth by Thomas Ricketts (1985). In effect, Ricketts argues that Frege's rule of inference is redundant because it does not play any role in inferential justification. The conclusion of section 3 is that there is motivation to reach an account which does not seek to justify inferences via any act outside of the judgement of the premises themselves. Chapter 3 is then dedicated to putting forth the account of inference in *The Tractatus* as one of this nature, as well as showing that rules of inference are not just redundant, but that Frege's account fails on his own terms.

Section 1: Assertion and negation

Frege argues that the content of a thought is said to be true through that thought being judged. In Frege's system, judgement is signified through use of his assertion sign. Absent the assertion sign, the thought can be the subject to many other functions which do not assert the truth of the thought. For instance, when I imagine that p, I precisely do not commit myself to the truth of p. The same goes for 'hoping, dreaming, etc.' This distinction is important for Frege. He thinks that 'We cannot correctly express a hypothetical connection between thoughts at all if we cannot express thoughts without affirming them' (BS, S2). Frege chooses here to define content as something which *may* be judged rather than something which is *fundamentally a subject of judgement*, and it is a considered choice. He sees himself as leaving necessary space for hypothetical thinking within his logical system.

Frege's assertion sign serves to mark the set of cases where a thinker believes her thought to be a fact. It prefixes any thought that she judges to be true. Frege argues that without the introduction of this sign, given the other possible attitudes one might take, then the thought is a '*mere combination of ideas*' (BS, S2). Crucially, the assertion sign does not add content to the thought which it expresses as true. It: '*combines the signs that follow it into a totality, and the affirmation expressed by the vertical stroke at the left end of the horizontal one refers to this totality*.' (BS, S2) The assertion sign therefore *only* attaches 'is true' to the end of the thought. It is already in the nature of the thought that it is capable of being judged as true or false. Otherwise, since the assertion sign does not add content to a judgement, it would not be possible for the assertion sign to transform the thought into a judgement. And so, 'Whatever follows the content stroke must have a content that can become a judgement' (BS, S2). A Fregean thought differs only from Fregean judgement insofar as the latter is judged to be true, the content is the same.

This is because Frege's notion of content is his starting point from which he defines judgement. Content is in this sense *prior* to judgement. Frege does not deem all

potential contents as capable of forming judgements. Frege's system begins with a specialised notion of what he calls 'conceptual content' (BS, S3). He defines this as the content of a judgement that from which other judgements can be derived from (BS, S3). So, conceptual content determines the inferential role that a judgement will play with regards to other judgements. He observes that the subject, predicate distinction therefore does not play a role in his system:

> 'The two propositions "The Greeks defeated the Persians at Platea" and "The Persians were defeated by the Greeks at Platea"... Now I call that part of the content that is the *same* in both the *conceptual content*." (BS, S3)

All the judgments which can be derived from the first proposition can also be derived from the second proposition. Therefore, it is the case that both propositions have the same conceptual content, despite their apparent difference in subject and predicate.

Frege further justifies his rejection of the distinction in the following way:

> 'We can imagine a language in which the proposition "Archimedes perished at the capture of Syracuse" would be expressed thus: "The violent death of Archimedes at the capture of Syracuse is a fact". To be sure, one can distinguish between subject and predicate here, too, if one wishes to do so, but the subject contains the whole content, and the predicate serves only to turn the content into a judgement. *Such a language would have only a single predicate for all judgement, namely, "is a fact"*.' (BS, S3)

The same thought is expressed in a language by adding the predicate "is a fact" to all judgements, even though such a language would have "is a fact" as its only predicate. Since the subject predicate distinction plays no role in determining the conceptual content, it serves no purpose for Frege's logical system. The only use for a predicate then, is *to transform a conceptual content into a judgement*, in just the same way that "is

a fact" predicates the truth of a statement. For this reason, Frege says, the assertion sign 'is the common predicate for all judgements' (BS, S3).

Whenever a thinker judges a thought to be true, they may assert it as true, which is signified by their use of the assertion sign. What about judging a claim to be false? Since there is only one predicate for all judgements, which asserts the truth of a thought, it follows that negation operates at the level of content. One asserts (through use of the assertion sign) either a thought or the negation of that thought. 'I therefore regard it as more appropriate to consider negation as an adjunct of a *content that can become a judgement*' (BS, S4). The choice of whether to affirm or deny a thought is a choice of which *content* one is to assert, not what kind of *judgement* one is going to make. Judgement is a choice between the assertion of a thought or its negation.

Frege is explicit about this: 'If a short vertical stroke is attached below the content stroke, this will express the circumstance that *the content does not take place*' (BS, S7). The negation stroke adds nothing to the content beyond the idea that the content does not take place. He follows this with the claim that 'If a negation content stroke is added without an assertion sign, then this 'merely calls upon us to form the idea that A does not take place, without expressing whether this idea is true.' (BS, S7). Once again, Frege is leaving space here for us to conceive of the negation of a thought (like when we imagine something not to be the case) without asserting the negation as true. If a thinker believes that a proposition does not take place, Frege argues, it is sufficient to express this by using the assertion sign to attach "is true" to the negation of a thought.

A consequence of this is that although the truth of p *entails* the falsity of not-p, p does not necessitate by its nature necessitate that not-p is false. This is because p is not fundamentally speaking *an assertion*. P only *becomes* an assertion when it is prefixed by the assertion sign.

A thought and its negation do not oppose each other by their nature. When one asserts p, it is only at this level of judging that p that one takes a normative stand against the assertion of not-p.

> ‘This opposition or conflict is such that we automatically reject one limb as false when we accept the other as true, and conversely.’

(Frege *Posthumous Writings* (PW). Oxford: Blackwell. (Frege, 1979, 8)

The content: not-p, is a separate Fregean thought with a separate conceptual content to p. Yet to *judge* not-p is a choice to reject the content p, which has a different content to ‘p’ (since it is negated). It has an opposite content but, at the level of mere-content, p and not-p do not normatively oppose each other. So, the content ‘p and not-p’ is not incoherent, since, as contents ‘p’ and ‘not-p’ do not reject the other.

Yet Frege says here that the *assertion* p is opposed by the *assertion* not-p. We automatically reject one when we *accept* (assert) the other. This is because negation is an operation at the level of content, rather than an operation at the level of judgement. There is a content ‘p and not-p’, which, at first glance, we could without issue attach the assertion sign too. Of course, doing so would generate a necessary falsehood since only one of these two assertions can be true. However, it would not be a prima facie *incoherent* act to perform. If ‘p and not-p’ is a content which can be ‘hoped, imagined, supposed’ (used in hypothetical thinking), then it appears that it can also be asserted.

To briefly summarise this section, there is a tension here in Frege’s thought. On the one hand, ‘p and not-p’ forms a perfectly good contents, which, it appears, could be asserted. This is because the thought ‘p’ and the thought ‘not-p’ do not oppose each other by their nature. On the other hand, the acceptance (assertion) of p leads us to automatically reject the assertion of not-p, and vice versa. This leaves us with a dilemma regarding ‘p and not-p’. It is undisputedly a coherent thought, so, prima facie, it appears to form a judgeable contents. Yet, as Frege himself says, by accepting p we

automatically reject its negation. The assertions of ‘p’ and ‘not-p’ normatively oppose each other. By Frege’s own lights, the assertion of ‘p and not-p’ would amount to the Fregean thinker taking a stand against themselves, yet this seems to be at least logically possible.

I will return to this tension in Chapter 3 Section 2i, when, in combination with a further principle, I will generate an argument against Frege on his own terms. For now, it is important to understand Frege’s view of the relation between content and judgement. It is because Frege takes judgement to be external to content that it is necessary for him to introduce an inference rule, Modus Ponens, to licence the transition from the premises to conclusion of inferences.

Section 2: The Conditional and Modus Ponens

Frege begins the construction of his inferential calculus from the possibility of affirming or denying the content of judgements. He looks at two judgements and the nexus of possibilities produces by the affirmation and denial of those judgements:

> ‘If A and B stand for contents that can become judgements (S2), there are the following four possibilities:
>
> (1) A is affirmed and B is affirmed;
> (2) A is affirmed and B is denied;
> (3) A is denied and B is affirmed;
> (4) A is denied and B is denied. (BS, S5)

Asserting the conditional then, if B then A, is nothing more than asserting ‘the judgement that *the third of these possibilities does not take place, but one of the three others does*.’ (BS, S5). If B is true, then A must be true. So, if one asserts B + if B then A then one is justified in inferring from B to A (BS, S6). If B then A excludes (3) and B

excludes (2) and (4) (BS, S6). So, our truth tabular explanation shows that from the conjunction of judgements (B & if B then A) the inference to the conclusion that (1) is true can be justified.

Frege employs the inference rule, Modus Ponens, as the only mode of inference needed for his system: 'I employ only this one, at least in all cases in which a new judgement is derived from more than a single one.' (BS, S7). When a thinker makes a conditional assertion (e.g. if B then A), they take a stand on the nexus of possibilities outlined above. Once this normative stance is combined with a normative stance on the propositions themselves, through judgement, a thinker can make inferences according to what is entailed by the logical proposition + their beliefs regarding A or B.

Importantly, Modus Ponens is Frege's chosen inference rule, but his conditional assertions which justify inferences do not seek to express it. They are an axiom *for his system* and only within this necessary context of producing an inferential calculus.

> 'We have already introduced a number of fundamental principles of thought in the first chapter in order to transform them into rules for the use of our signs. These rules and the laws whose transforms they are *cannot be expressed in the ideography because they form its basis*' (my emphasis) (BS, S13)

Modus Ponens is a fundamental principle of thought. Without it, thought itself is not possible. A conditional assertion is not an expression of Modus Ponens, since it cannot be expressed. Rather, the validity of conditional assertions is underpinned by the validity of the underlying principle. Modus Ponens is the basis for the ideography itself, it is not a component within it. Without asserting Modus Ponens then, which is not possible, we are still able to take a stand on the truth table of possibilities. Frege's system thereby demonstrates the validity of the fundamental law of thought through its use of Modus Ponens as an inference rule.

Conditional assertions limit which inferences can be made. They give permissions to infer. When this is combined with judgement(s) about the propositions themselves, further judgements (inferences) can be derived. This is not to say that the relations between propositions are external for Frege. The relations between A and B, if A entails B, are such that it is internal to A that it entails B and internal to B that it is entailed by A. These relations are internal to the propositions. However, this is at the level of content. Inference is an act of assertion, an act of judgement. It concerns the relation between judgements not contents. We have seen that it is not in the fundamental nature of A and B that they are *to be judged* (rather than hoped… etc.). This means that even though A entails B, it does not follow from the judgement that A that one is thereby justified in judging B.

This is why even if A entails B, the assertion of A needs to be combined with the conditional assertion 'if A then B' in order to justify the inference from A to B. A is not fundamentally a subject of judgement. For this reason, it does not, by its nature alone, justify the judgement that B. So, it is in the nature of A that it *entails* B but not that one can *infer* B from A. This shows clearly the need for conditional assertions in Frege's system, which function as axioms. The judgement of A + the conditional (if A then B) can justify the inference from A to B. Without these conditional assertions, which are given their basis by Modus Ponens, no inference could be justified in Frege's view.

Frege clearly distinguishes between the rule of inference which forms the basis for his system and the axioms which operate within it. Peter Sullivan argues that 'no clearer acknowledgement of Carroll's point could be looked for' (Sullivan, 2004, 685). If it were the case that the axioms were an expression of Modus Ponens, there would be a regress problem. Carroll's regress demonstrates that asserting Modus Ponens as a premise in an inference leads directly to regress. Frege's awareness of this allows him to avoid the problem. Reasoning according to Modus Ponens is a fundamental principle of thought, not capable or in need of expression. Its validity as a logical concept is a starting point for reasoning and thereby cannot be articulated within the boundaries of reasoning.

Sullivan's point is that Carroll's regress would only be triggered if Modus Ponens was itself supposed to be Frege's systematic axiom. And so:

> 'It would thus be a mistake to reason that recognition of the validity of a rule cannot constitute, or be grounded in, a judgement- on the ground that an expression of this judgement would then have to be included in a full list of premises on which rests a conclusion drawn in accordance with the rule, so starting the Carroll regress.' (Sullivan, 2004, 685-686)

There is no need here for a further premise expressing Modus Ponens because the recognition of its validity is constituted by the judgement of Frege's conditional assertion. Frege's conditional assertions perform the necessary task of justifying inferences with regards to the context of his logical system. It is not an explanation of the possibility of judgement fundamentally speaking. Justifying its very possibility or foundation is not the task which Frege is undertaking.

As Sullivan puts it, the mistaken reading 'conflates judgement *tout court* with judgement in such and such a particular system' (Sullivan, 2004, 686). Frege's proposed *system* of logic requires the use of his conditional assertions. This is not to say the rule Modus Ponens, the validity of which is shown by the validity of his truth tabular explanation, is itself justified by that rule. This would attribute to Frege a 'meta-perspective' (Sullivan, 2004, 686), such that when he is offering reasoning for his system, he is offering reasoning instead for logic in a general sense. This attribution would only be appropriate if he were arguing that Modus Ponens is *fundamentally* a rule of inference.

There is no reason to think that Frege had this meta-perspective in mind (Sullivan, 2004, 686). Beyond this, part of his note at the beginning of Chapter 2 gives us reason to think precisely the opposite. Frege continues:

> ‘Now it must be admitted, certainly, that the way followed here is not the only one in which the reduction can be done. This is why not all relations between the laws of thought are elucidated by means of the present mode of presentation’ (BS, S13)

Frege takes the laws of thought to exist prior to and outside of his logical system. He is inventing a logic with the goal of reducing the content of all the laws (BS, S13) into an expressible language. He countenances here the idea that there are other, entirely legitimate, ways to attempt to do this. His specific system is a consequence of his attempt at perspicuity (BS, S6). If, on the other hand, he was adopting a meta-perspective on logic, his goal would be quite different. He would be arguing that his logic is the only way to correctly represent the laws of thought. This passage makes it clear that Frege did not see this as his task and that he thought that there were many legitimate ways of representing the laws of thought. This implies that he would have thought that the very idea of taking a meta-perspective on logic would be flawed.

Since Frege does not take on this task, one he would have seen as flawed, he cannot be criticised on its basis. The laws of thought, for Frege, are self-evident. They are not in need of or capable of prior explanation. Their validity can be *shown* by a particular system, such as his, without the law itself being expressed. The validity of Modus Ponens is demonstrated by the possibility of reasoning with conditional assertions, but it is true for Frege that logical reasoning is always going to be from the point of view of a particular system. One cannot reason (as it were ‘from outside of logic’) in order to express the validity of the axioms which underpin logic. As soon as one engages in this sort of reasoning, one uses the very laws of thought that they are trying to prove.

Section 3: Rules and justification

The problem that logic calls for a defence outside of logic, yet any defence outside of logic would be illogical, can be called the ‘logocentric predicament’ (Ricketts, 1985, 3).

Even if it is not a problem that Frege takes himself to be resolving, Thomas Ricketts argues that the tension underlies (and perhaps undermines) Frege's account (Ricketts, 1985, 9).

Ricketts agrees that Frege will not fall into the trap of Carroll's regress, due to his distinction between axioms and rules of inference (Ricketts, 1985,7). Rather,

> 'Frege's axiomatization of the maximally general science takes certain laws as basic. Frege relies on our just coming to recognize the truth of the logical axioms he propounds, assisted perhaps by various elucidatory remarks.' (Ricketts, 1985, 7)

Logical laws, for Frege, cannot be offered prior justification and so must be taken as basic. The truth of these logical laws underpins his system and is shown by the validity of the axioms of his system. It is therefore the case that 'no *statement* can formulate a logical principle' (Ricketts, 1985, 11), since the logical principle is that which underpins the possibility of logical reasoning itself.

We can now clearly define the role of the inference rule in Frege's system. It offers a licence for making inferences without it being the case that the rule plays a premise-role in the inference itself. Frege's systematic axiom, the conditional, is that which plays a premise-role. For example:

P1: p
P2: p implies q (conditional)
C: q

It is true that *because* of Modus Ponens we can infer C from P1&P2, we apply Modus Ponens to the premises. It need not be that we add P3 attempting to express this, for example, p and (p implies q) implies q, as this is just a logical truth for Frege. So,

Carroll's regress cannot get off the ground. The validity of this logical truth underpins the inferential process without itself playing a premise-role in the inferential argument.

Ricketts is cognisant of this, so he offers up a more challenging criticism:

> 'Suppose p and q are propositions of a special science, and suppose the assertion of q provides a basis for the assertion of p, as Frege would say. Then, contrary to Frege, in order to infer p from q, there is never any need first to deduce the conditional 'if q, then p' from "logical laws". The inference does not have to be mediated by the knowledge of the truth of any proposition. Frege urges the replacement in science of enthymatic reasoning with completely explicit, gap-free proofs.' (Ricketts, 1985, 12)

Frege's awareness of Carroll's regress helps him to avoid giving logical truths a premise-role in inference. He recognises that this would lead to regress. So, Modus Ponens is an inference rule only in the sense that it licences inferential transitions. In the example, what justifies conclusion C are for Frege, the premises P1 and P2. This is underpinned by the logical truth but since this is a primitive fact of logical reasoning, it does not form a premise in the argument. Yet, it is possible to deduce p from q, as Ricketts puts it 'in a special science', without need to reproduce Frege's conditional. If p implies q, then it is the singular premise 'p' that provides the basis for the conclusion 'q', nothing more.

It follows that in Frege's own inferential calculus, even though conditional assertions might be necessary for the purposes of performing logical deductions, they do not play a role in terms of *inferential justification*. It is not *because of the conditional* 'if q, then p' that I infer p from q, but because the assertion of q provides a basis for the assertion of p. From 'p' and 'p implies q', one infers q by following the inference rule. There cannot be a further premise 'p and (p implies q) implies q' that one makes the inference. This would amount to an attempt to express Modus Ponens as a premise and would trigger Carroll's regress.

Frege's chosen inference rule of Modus Ponens, then, plays no premise role in inferential arguments. This means that Modus Ponens is a rule which serves only to licence the inferential transition. It is unnecessary to ever consider Modus Ponens when making an inference, since doing so would require an expression of Modus Ponens, which is not possible on pain of regress. Rather, it plays the role of a rule which licences the inferential transition which one understands to be justified based on their grasping of the contents of the premises.

As such, Ricketts charges Frege with not undertaking the 'logicians task' (Ricketts, 1985, 12) which 'is not to state universally applicable principles of inference, but rather to devise a perspicuous notation that renders patent the basis for inferences in the propositions themselves' (Ricketts, 1985, 12).

Ricketts' thought is that taking the conditional to be an axiom of his system is a misstep by Frege. It goes beyond the bounds of what logic can do. If it is the case that Modus Ponens is a valid mode of inference, then this is based on the internal relations between propositions. Since it is these internal relations which justify inferences, for Ricketts, Frege's explanation should be one which seeks to elucidate these. Both the talk of Modus Ponens and the conditional, give external explanations of what Ricketts takes to be a phenomenon internal to the nature of propositions.

Recall here that Frege has a systematic reason for his Modus Ponens rule of inference and his conditional axiom. Frege has a content-first, judgement-second approach to propositions. This, he thinks, leaves space for hypothetical thinking, where the thinker withholds the assertion of a proposition. In this view, it cannot be that inference is a process internal to propositions. What is internal to a proposition is its content, which is not something *fundamentally to be judged*. What this means is that even if the content p entails the content q, it does not follow that one can infer the judgement q from the judgement p. To justify inference from 'p' and 'p implies q' to 'q', the Modus Ponens rule is required to licence the transition.

Frege is therefore subject to the following. On the one hand, I am *justified* in inferring p from q because the content of the assertion that q and the content of the assertion that p is such that the inference is justified (q entails p). On the other hand, the contents of ‘q’ and ‘p’ alone provide no *licence* for inferring from one to the other. Judgement is external to content and so, the thought ‘q’ alone can provide no justification for inferring the thought ‘p’. Relations of content are simply not relations of judgement for Frege.

The problem with this is as follows. Frege is forced to bring in his inference rule, Modus Ponens, to licence inferences from ‘p’ and ‘p implies q’ to ‘q’. Yet, it is also true for Frege that this very inference rule plays no part in the argumentative justification of the inference, since it cannot play a premise-role it merely serves the rule-role of transforming these relations of content into relations of judgement, such that the act of inference can be justified.

The inference rule therefore merely *licences* inferential transitions, through transforming relations of contents into judgements. ‘p’ and ‘p implies q’ does not lead to ‘q’ without the rule licensing the transition. But it is these relations of contents (internal relations between propositions), not the inference rule, which provides the justification for the licencing of inferences. That inferences are licenced in the way is not incidental, or because of Modus Ponens (since it is axiomatic), but because of the way in which propositions relate to each other. For Frege, one is justified in inferring from q to p because q provides a basis for p, they are licenced to do so because of inference rules.

An analogy here will be helpful. One is justified in driving because they are a good (safe) driver. They become licenced based on meeting criteria in a driving test which show them to be a safe driver. Once they become licenced, it is true that their licence is necessary for them to drive on the roads, legally speaking. The licence means they are not prohibited. Yet, they are *justified* in driving on the road not because of the licence *at all*. The justification for their being allowed to drive on the road is grounded in their capacities as a driver.

Ricketts is arguing that the ‘logician’s task’ is one of, in the analogy, uncovering the conditions of what it means to be a good driver, not to devise an efficient system of issuing drivers’ licences. Determining the conditions of justified driving should also settle the question of licencing given the fact that good drivers are justified in driving on the road and so should be licenced. Good (valid) inferences are justified and, based on this justification alone, these are the inferences which should be licenced. In uncovering the relations between propositions (what it means to be a good driver) one uncovers which inferences are in fact licenced based on understanding justification.

Frege, with his mere-licence-providing inference rule, himself accepts that inferences are not *justified* by the inference rule. The rules provide licences without fundamentally impacting on the question of inferential justification.

It is a consequence, though, of Frege’s view that judgement is external to content that he employs inference rules. If it were the case that propositional contents are, in the first instance, assertions, then there would be no need for an intermediary licence. If p entails q, then this very fact is what justifies inferring q from p. The internal relation between the propositional contents would not be such as to only act at the level of contents, but also at the level of judgement.

I take it that, in a perfect world, our account of inferential justification would not include such an intermediary licence. Why? Because *on Frege’s own view*, inference rules which provide the licence are not that which justify the inference. What justifies inferences are the logical laws which underpin an inferential system such as Frege’s. His inference licences are system relative and do not seek to represent these logical laws. Fregean licences are mere biproducts of the necessity of transforming the entailment relation between Fregean contents into the inference relation between Fregean judgements. They play a key role in his logical calculus (which requires rules) but no foundational role of justification. As such, chapter 3 will be spent elucidating the Tractarian view, upon which propositional contents are *fundamentally assertions*.

Conclusion

I am not here, yet, saying that Frege is *wrong*. It is more a question of focus. He is concerned with providing a logical calculus, and, as Sullivan notes, countenances nothing like a meta perspective on logic. His accompanying thoughts are therefore concerned with supporting this system, not with understanding the fundamental justifications of phenomena such as inference. Assuming that such justifications exist, my task is to attempt to uncover what they are. Frege's system cannot be yet discarded since, it does offer *an answer*, it is just one that requires the use of the intermediary mere-licence-providing inference rules. The Tractarian view does not require this, so I will first explain how this is. Then I will be able to argue that Frege is in fact incorrect by his own lights about the nature of content and judgements, which I take to be the key differentiating factor between the two systems.

Chapter 3: Wittgenstein's account of inference

Introduction

Now that we have considered Frege's account of inference in *The Begriffsschrift*, it is possible to contrast it with Wittgenstein's opposing account in *The Tractatus*. Frege's logical system employs rules which govern inferences. This is necessary by his lights because content is prior to judgement. The fact 'that p entails q' is not sufficient by itself to justify the *inference* from p to q because entailment is a relation of content and inference is a relation of judgement. Rules of inference then serve to justify inferences when a content is asserted, since, otherwise, such a justification would be lacking. Wittgenstein on the other hand claims that rules of inference would either be superfluous or senseless (TLP 5.132). This is because he does not share the view that content is theoretically prior to judgement. Rather, as I will show, there is good reason to take the perspective that the fundamental nature of content is that it is *to be judged*. So, it follows that Wittgenstein has no need for rules of inference, since, for one proposition to follow from another *is* for the latter judgement to be justified by the former.

In section 1i, I will explain Wittgenstein's view of propositions as logical pictures insofar as it underlies the Tractarian view of contents *as* (fundamentally speaking) contents of judgements. This serves to contrast the opposing account; we saw before that Frege argues that there is a need for an assertion sign to transform contents into judgements; an idea which Wittgenstein argues against. In section 1ii, I will examine the Tractarian notion of logical space. A proposition takes a place in logical space, this place is constituted by the proposition's relation to other propositions, just as a place in physical space consists of the relations to other spatial objects. Logical space lays the foundation for a view of inference as nothing beyond the judgement of the premises. In section 1iii, I will demonstrate that true Fregean propositions lack the aim of correctness in assertion, an aim which is fundamental to the Tractarian notion of truth.

Following this necessary setup, I will argue in section 2i that the notion of sense in *The Begriffsschrift* is undermined by the Intuitive Criterion of Difference.[21] The Criterion states that if different attitudes can be taken towards two sentences, then those sentences must express different thoughts. This criterion leads to contradiction if, as I argue, it is possible for the Fregean agent to judge 'p and not-p', since this would imply that 'p' has two different senses. In section 2ii, I demonstrate that the Tractarian view is not undermined by the Intuitive Criterion, since it is not possible to judge a contradiction.

Having motivated the Tractarian perspective, in section 3i I propose a reading of Wittgenstein's claim that laws of inference are superfluous and senseless, drawing on the distinction between truths which can be said and shown in *The Tractatus*. The truth contained in laws of inference, can only be shown, and, as such, cannot be formulated laws. In section 3ii, I defend the Tractarian view that I have explained with reference to Carroll's regress. I argue that it avoids Carroll's regress through understanding that inference cannot be a further act beyond the judging of the premises of an inference. It follows from the argument of this chapter that Wittgenstein's account of inference should be taken seriously both as an account of inference in its own right, and as a means of diagnosing the problem of inference in the contemporary debate.

Section 1i. Assertion and negation

In *The Tractatus,* Wittgenstein writes the following:

> 'The proposition is a picture of reality for I know the state of affairs presented by it, if I understand the proposition. And I understand the proposition, without its sense having been explained to me.
> The proposition *shows* its sense.

[21] (The criterion is a constraint on the notion of a thought, the substance of which is put forth by Frege himself, but the notion is characterised by Gareth Evans (1982)).

> The proposition *shows* how things stand, *if* it is true. And it *says*, that they do so stand.' (TLP 4.022-4.023)

Wittgenstein's ordering of content and judgement in this passage places him in opposition to Frege. A Tractarian proposition does not *merely* show how things stand; it also asserts that they stand in that way. Recall that for Frege, a thought (an unasserted proposition) is a content. This content then may, or may not be, asserted. It is not the fundamental nature of a thought to be asserted, any more than it is in its nature to be hoped, imagined… etc. I will now give a characterisation of Wittgenstein's pictorial understanding of propositions to shine light on how he diverges from Frege in his view of the relation between propositional content and judgement.

If a proposition is a picture of reality, it follows that one knows the state of affairs presented by a proposition through the very act of understanding the proposition (TLP 4.022). The proposition is a picture, understanding the picture is understanding the proposition. The state of affairs presented by a proposition is just that which is asserted. One understands what state of affairs is presented as soon as (and by the very fact that) they understand the proposition. P, therefore, which is fundamentally a representation of a certain state of affairs, need have nothing added to it in order to be asserted.

This is because a proposition not only shows how things may stand but it asserts that things stand in that way. ('And it *says*, that they do so stand'.) For Wittgenstein, it is not merely the case that the proposition presents a certain state of affairs, as this could be a hypothetical presentation. Rather, the proposition *is* an assertion in the sense that it puts itself forth *as true*. Wittgenstein continues:

> 'The proposition determines reality to this extent, that one only needs to say "Yes" or "No" to make it agree with reality.
> Reality must therefore be completely described by the proposition.' (TLP 4.023)

Affirmation is all that is required for a thinker to assert a proposition. Wittgenstein is clear here that this affirmation does nothing to transform the proposition. The thinker need only say “Yes” or “No” to assert a proposition because the proposition is already (in the form of) an assertion. For this to be the case, the proposition must give a complete description of reality to which nothing need be added. In this way, the proposition is already such as to be asserted, it is an assertion ready to be affirmed or denied.

Returning to the opening passage (TLP 4.022-4.023), we can adopt the Tractarian language to say that a Fregean ‘p’ *shows* a sense without affirming that sense, it *shows* how things stand *if* it is true. This is why mere affirmation (a “Yes”) is not sufficient, rather, an assertion sign is required because ‘p’ is not fundamentally an assertion and so must be *transformed* into an assertion. Wittgenstein deviates in his understanding of ‘p’ in that he thinks that it also *says* that things do so stand (in the sense that ‘p’ shows). This is not a mere presentation since ‘p’ need only be affirmed in order to be asserted. This shows that ‘p’ is already an assertion. When ‘p’ *says* that the picture it presents is true, Wittgenstein is saying that ‘p’ is, by its nature, an assertion.

A Tractarian ‘p’ gives a complete description of reality through being a picture of reality, which represents a certain state of affairs. The propositional content can be called true or false only when it is held up against reality. When the picture ‘p’ is asserted, its truth or falsity is determined as follows:

> ‘The picture agrees with reality or not; it is right or wrong, true or false’ (TLP 2.21)

So, what it means to assert the content of a proposition is to call a certain picture of reality true (to say “Yes” to *that* picture of reality). The picture which one claims to be true is either true or false. The truth or falsity of the picture is determined by comparing it to reality (TLP 2.223). No picture is a priori true (TLP 2.225) (or false).

This means that the correctness conditions of asserting a proposition consists in asserting a proposition which forms an *accurate picture of reality*. The nature of propositional content is to present a picture, with the depiction of reality *as its aim*.

> 'The form of representation is the possibility that the things are combined with one another as are the elements of the picture.
> *Thus,* the picture is linked with reality; it reaches up to it' (TLP 2.151-2.1511)

A picture of reality can be true or false. Asserting a proposition 'p' presents a certain picture and calls it true. Correctness in assertion consists in the picture being true upon comparison with reality. As such, since propositional content is fundamentally assertable, and this content presents a picture of reality, correctness in assertion consists in asserting a proposition which presents a true picture of reality. It is in this way that content, truth and correctness in assertion are internally connected notions for Wittgenstein, none of which could be explained without reference to the others.[22] Propositions are partly constituted by their aim of truth, which is correctness in assertion.

This brief examination of the Tractarian view of propositions as pictures serves to show the stark contrast between Wittgenstein and Frege when it comes to their view of the relation between content and judgement. We can now see why Wittgenstein repudiated the idea of an assertion sign. An assertion sign would only be necessary if a proposition merely hypothetically presented a state of affairs. Rather, as we saw, it shows how things stand *and* asserts that they stand in that way. Assertion is internal to the content of the proposition. Propositional content aims at truth in its representation of reality. Since a proposition is such as to be asserted, nothing need be, logically speaking, added to the proposition in order to transform it into an assertion, beyond the token act of affirming the assertion (saying "Yes" to the proposition).

[22] 'Truth is essentially correctness in assertion.' (Johnston, 2024B, 99)

> ‘(Frege’s assertion sign is logically altogether meaningless; in Frege (and Russell) it only shows that these authors hold as true the propositions marked in this way. The assertion sign belongs therefore to the propositions no more than does the number of the proposition. A proposition cannot possibly assert of itself that it is true.)’ (TLP 4.442)

For Wittgenstein, it follows from the claim that a content’s nature is to be asserted that one should object to an assertion sign as *logically meaningless*. The act of asserting a proposition consists only in saying “Yes” to the proposition, choosing to affirm it. So, it follows that the assertion sign cannot be included in the analysis of propositions. (A meaningful assertion sign would imply that the “Yes” is not sufficient). There can be no role for an assertion sign in logic. It only signifies the contingent fact that Frege, Russell or anyone else holds a particular proposition to be true. It is not needed to explain *how propositions are asserted* (as Frege argues), since analysis of the (Tractarian) proposition deflates any need for an explanation.

The picture theory understanding of the proposition puts Wittgenstein in sharp contrast with Frege, with regards to more than just the assertion sign. When considering Frege’s view, we observed that a proposition and its negation do not oppose each other at the level of propositional content. This is because, for Frege, assertion and negation happen at different levels of content and judgement. Assertion operates at the level of judgement and negation occurs at the level of content. What this implies, is that a content and its negation (p and not-p) do not oppose each other at the level of judgement. One could readily assert ‘p and not-p’, without their judgement lacking a sense. Of course, such a thinker would be incorrect, but this is not the point. They would be asserting a necessary falsehood, yet, in Frege’s system, they could do this without making an incoherent assertion.

If we adopt the Tractarian view of propositions, then it is an incoherent act to assert ‘p and not-p’. There is no such act. This is because both a proposition and its negation occur at the level of propositional content, and both are such as to be asserted. This

means that ‘P and not-P’, because they oppose each other at the level of content, also oppose each other at the level of judgement. The thinker who asserts ‘p and not-p’ *at all*, is only conceivable on the Fregean view, not the Wittgensteinian one. A contradiction at the level of Tractarian content, such as ‘P and not-P’, is a contradiction at the level of Tractarian judgement (it is without sense (TLP 4.461)). This is the sense in which there exists a *strong* opposition between a proposition and its negation for Wittgenstein.

Colin Johnston characterises Wittgenstein’s view of assertion and negation in the following way:

> ‘There are not two distinct acts, assertion and denial. Rather, a thought p opposes its negation not-p, so that denying p is one and the same as asserting not-p.’ (Johnston, 2024A, 1)

This brings into view how a proposition opposes its negation by its very nature. Recall TLP 4.022-4.023, a proposition shows how things stand. It presents a state of affairs. We must understand what this means. Let’s consider another passage for a possible explanation.

> ‘A proposition asserts every proposition which follows from it.
> “*P.q*” is one of the propositions which assert “*p*” and at the same time one of the propositions which assert “*q*”.
> Two propositions are opposed to one another if there is no significant proposition which asserts them both.
> Every proposition which contradicts another, denies it.’ (TLP 5.124-5.1241)

First, a proposition asserts every proposition that follows from it. So, if q follows from p, the assertion of p is the assertion of q. Second, every proposition which contradicts another, denies it. So, when a thinker asserts p, *part of their assertion of p is constituted by their denial of not-p*. It is not a further act to deny not-p. The proposition p is the

denial of not-p. A thinker who attempts to assert ‘p and not-p’ asserts a proposition without sense since they assert a contradiction. A contradiction refers to nothing in the world, so is thereby senseless. It is not a picture of reality. Put differently, one’s understanding of p as true contains the understanding of not-p as false.

This section has served to demonstrate the contrast between the view of content and judgement in *The Tractatus* as opposed to that of *The Begriffsschrift*. This is shown by an examination of Wittgenstein’s divergent view of propositional content, through which we have shown the interrelation of truth and correctness in assertion. It is therefore the case that Tractarian content is of such a nature that it is *not* prior to judgement, but rather that content and judgement are fundamentally connected notions. This puts Wittgenstein in contrast with Frege, whom, I argued (in the previous chapter) does have content as a theoretically prior notion to judgement. The next thing which must be understood is the conception of logic in *The Tractatus* which underpins the view of propositions as logical pictures of reality. This is necessary to explain how it can be that, for example, a proposition asserts all the propositions which follow from it and denies all the propositions which are negated by it.

Section1ii. Logical space

A picture of reality represents a possible situation. Where a physical situation is represented by models of the physical components, a logical situation is represented by the proposition. The space of logical representations is called logical space:

> ‘The picture represents a possible state of affairs in logical space’ (TLP 2.202)

A proposition has sense only insofar as it ‘determines a place in logical space’ (TLP 3.4). This means that the proposition (the picture) must affirm certain possible states of affairs and deny others. The logical picture produced by a proposition consists in precisely the sum of these affirmations and denials. P consists not only in the

affirmation of P, but also the denial of not-p (this is also true in all molecular propositions where the same symbols occur). The logical picture thus represented, the place in logical space, is the totality of the logical inferences which can be drawn from the proposition.

Logical space is a somewhat obscure notion to try and understand, yet it underpins our understanding of *a proposition as a picture of* reality. In physical space, an object's place is constituted by its spatial relationship with other objects. In logical space, a proposition's place is constituted by the proposition's logical relationship with other propositions.

We understand from section 1 that the proposition is a picture in the sense that it calls some propositions true and others false. The proposition gives a logical picture of reality to which one need only say 'Yes' or 'No' to. The picture is of the internal relations between propositions and describes a potential situation, a potential way reality might be. For the purposes of understanding logical space, what is shown by a singular proposition (a singular picture), however, is more than just the *particular place* it determines:

> 'Although a proposition may only determine one place in logical space, the whole logical space must already be given by it.
> (Otherwise denial, the logical sum, the logical product, etc., would always introduce new elements- in co-ordination.)
> (*The logical scaffolding round the picture determines the logical space*. The proposition reaches through logical space.)' (TLP 3.42, my emphasis)

The whole of logical space is given by a singular proposition because the space is constituted by the logical *scaffolding* around the picture of reality. Logical scaffolding, here, consists in the logical relations between propositions (the fact that one proposition follows from another, is negated by another, etc.). This is the way in which a proposition reaches through logical space, it not only asserts that this or that proposition follows

from it, but in its affirmations and denials it shows the very fact that these logical relations hold.

The proposition employs the logical scaffolding, it shows its internal relations to other propositions, which in turn constitute the logical space. A singular proposition determines a particular *place* in logical space; meaning it affirms and denies other propositions. To do this it employs its own logical scaffolding. This logical scaffolding, given by the singular proposition, gives the whole of logical space through the singular proposition. This explains why the denial of a proposition does not introduce new elements. The *possibility* of denial is already given by any singular proposition (which opposes its own negation). So, the proposition employs its own logical scaffolding when it shows its logical relationship with other propositions. Through employing the logical scaffolding, it shows the veracity of the scaffolding, this is the sense in which it *gives* the logical space, since logical space is constituted by the internal relations which form the scaffolding.

That logical relations are internal to propositions is a consequence that Wittgenstein thinks we can observe directly from the nature of thought:

> ‘That logic is a priori consists in the fact that we *cannot* think illogically.’ (TLP 5.4731)

Thought itself, since it requires the use of logic, rests on the validity of logic. So, logic must be valid since it makes thought itself possible. It also follows from the fact that we cannot think illogically that logic can find no justification within thought, and this is why logic must be *a priori*. That the relations between propositions are governed by logic is embedded in the nature of thought. This is why thought itself must simply show the logical relations between propositions to be valid, one cannot seek to justify these relationships through language, since language necessarily operates from within *logic*.

As Wittgenstein puts it:

> ‘Propositions can represent the whole reality, but they cannot represent what they must have in common with reality in order to be able to represent it- the logical form.
> To be able to represent the logical form, we should have to be able to put ourselves with the propositions outside logic, that is outside the world.’ (TLP 4.12)

Logical space can now be characterised more clearly. Propositions are a picture of reality. The picture represents a logical situation, one which consists of affirmations and denials of other propositions. The proposition thereby determines a place in logical space. The proposition cannot represent the logical form, the internal relations which hold between propositions, which underpin the possibility of propositional thought in the first place. To propositionally represent logical form, Wittgenstein observes, one would need to put themselves outside logic, which is to attempt the impossibility of thinking from outside logic[23].

Propositions, then, must simply give, through showing and not saying, logical form. Through showing their own logical form, they show (by using) the logical scaffolding consisting of all logical relations, which gives the whole of logical space. It follows from the nature of logical space that logical form is *such as* to be shown, rather than to be said (asserted) through a proposition. Our understanding also shows us that these relations are intrinsic features of the proposition itself, rather than a product of any external rules/ laws being applied to the proposition. The relations between Tractarian propositions are internal relations between those propositions, which exist as a consequence of the nature of the propositions, rather than anything else.

[23] Wittgenstein here says that one cannot represent the logical form since one would need to put themselves outside the world. Delving into this would require looking at the ‘limits of language mean the limits of the world’ (TLP 5.6) and would distract from the current discussion. This is why I explain the impossibility in terms of the previous point, one cannot think illogically so it follows that one cannot reason from without logic.

We are now able to consider the following passage on inference:

> ‘‘If *p* follows from *q*, I can conclude from *q* to *p*; infer *p* from *q*.
> The method of inference is to be understood from the two propositions alone.
> Only they themselves can justify the inference.
> Laws of inference, which- as in Frege and Russell- are to justify the conclusions, are senseless and would be superfluous.’ (TLP 5.132)

Wittgenstein is defending the claim that if p follows from q, q is sufficient justification to conclude that p. This inference can thereby be understood from (the nature of) the two propositions alone. Consider q once again as representing a possible situation in logical space. This logical picture consists in the affirmations of all propositions which follow from it and denials of all the propositions negated by it. If it is q is *such as* to affirm p, then one need only assert q to be justified in asserting p. The affirmation of p is part of the logical picture given by the assertion of q; it is part of the place in logical space determined by q. On this view, there is no further rule or law needed to justify the inference from one proposition to another.

Recall our discussion of Tractarian negation in section 1i. A proposition opposes its negation (such that the assertion of a proposition denies the assertion of its negation). Understanding p to be true involves understanding not-p to be false. It is in the nature of p that it denies not-p. In just the same way, if p follows from q, then this relation is part of the nature of p and part of the nature of q. So, when q determines a place in logical space, it affirms p. It is internal to q that it affirms p, and it is internal to p that it is such as to be affirmed by p. These logical relations are a priori since they are contained within the nature of thought, they are the logical scaffolding *shown* by the proposition. This is why Wittgenstein then claims, ‘All inference takes place a priori.’ (TLP 5.133). Logic is a priori and so the inferential relations between propositions, which hold based on the internal *logical* relationships of entailment, must also be a priori.

Section 1i demonstrates that content and judgement are interdependent notions for Wittgenstein in a way that contrasts with Frege's view. Section 1ii shows that propositions are pictures of reality in the sense that they determine a place in logical space. Logical space, insofar as a place within it includes the affirmations and denials of other propositional symbols, shows that propositions themselves are sufficient to justify inferences. At this point, then, the divergence Wittgenstein has made from Frege is clear. What is not so clear, is the claim of TLP 5.132 that 'laws of inference are senseless, and would be superfluous'.

We now understand that the logical picture produced by a proposition serves as sufficient justification, by Wittgenstein's lights, to justify its own inferences (the affirmations of which are included in the picture. If p follows from q, the assertion that q justifies the assertion that p. These first two sections show that *The Tractatus* gives us a potential systematic understanding of inferential justification.

In this way, then, we now understand how laws of inference are not necessary on the Tractarian view. They could play no role in the system of *The Tractatus*. So, further discussion of the TLP 5.132 comment may itself seem superfluous. I think, however, that understanding why Wittgenstein claims rules of inference are *senseless*, gets to the heart of the differences between the two accounts. The focus is further warranted by the fact that Wittgenstein has Frege specifically in mind. Investigating this comment gets to Wittgenstein's reasoning *for diverging in the first place*. It is not that laws of inference are merely misconceived by Wittgenstein's lights, but avoiding any need for them is part of the motivation for the kind of account Wittgenstein gives us.

Section 1iii. Truth

Consider that, for Wittgenstein, the molecular sentence (if p then q) has a truth-value which is a function of the atoms it contains (p,q). The logical sign adds nothing substantive, it merely 'stipulates how 'if p then q' is to be compared with reality'

(Ricketts, 1985, 237). It is in this way that the sense of the molecular sentence is a function of the sense of the atomic sentences which constitute it. So, there is nothing added beyond the propositions involved in the inference. The logical sign serves to *project* the truth values of the atomic sentences, showing them in a certain light rather than adding to them. In this light, the truth or falsity of the atoms (p,q) and from this the molecule they make up (if p then q) is then shown. The truth or falsity of the sentence is a product of the truth or falsity of its atoms.

For Frege, 'If p then q' does not project the truth values of (p,q), since it does. Otherwise, the content of a Fregean thought would by its nature provide a judgement, since it would stipulate the conditions under which (p,q) are true/false, but this is not the case. Names of truth-values, Frege says, assert nothing (Ricketts, 1985, 240). The content of a Fregean thought is grasped without grasping the correctness conditions of the thought as a would-be judgement. As Ricketts notes, this would mean straightforwardly that Fregean thoughts (which are merely names of truth values) lack sense, '*as Wittgenstein understands this term*' (Ricketts, 1985, 240, my emphasis). This is because the name of a truth value is not a picture of reality, which, for Wittgenstein, determines the sense of a sentence.

Yet we should be careful here not to incorrectly attribute to Frege a Tractarian view of sense to then argue that Fregean thoughts are fundamentally *senseless*. It is crucially important, in Frege's eyes, that he does not require that sensible content present a certain state of affairs *as true*, to leave space for so-called hypothetical thinking. (This is only to restate his differing view on the relation between content and judgement, and, as of yet, we have no reason to prefer one such view over the other). The content of a thought is not fundamentally a content of judgement for Frege. So, the sense of a proposition must be the truth tabular understanding which is contained within the name of a truth-value such as 'if p then q'. While this is not a picture of reality, since it asserts nothing, it presents conditions of truth which are capable of constituting a picture of reality if asserted. It is a content which is capable of becoming a judgement. A thought

is transformed into a judgement through the use of the assertion sign, yet only a Fregean judgement would contain sense in Wittgenstein's terms.

To see the difference between the two notions of sense, consider the following claim:

> 'Every sentence must *already* have a sense: It cannot be given a sense by affirmation. Indeed its sense is just what is affirmed. And the same applies to negation, etc.' (TLP 4.064)

Consider the goal of the Fregean assertion sign, to assert as true a content and to add nothing to that content. The assertion sign transforms the expression of a content into the expression of a judgement, no more. Yet a content *when asserted* presents a certain state of affairs as true, which will either agree or disagree with what is the case. There is a tension here between the sense of the thought for Frege and the sense of the thought's subsequent assertion. We could say that both the content of the thought and that of the judgement are *about* reality, but only one (the content of the judgement) is *accountable* to reality.

That which is added to the content of the Fregean thought, once asserted, is the accountability of the judgement's content to the actual state of affairs in the world. The thought, once it is judged to be true, is held up against reality as true or false. The Fregean thought is a mere content which is the name of a truth-value, gives the truth conditions of a certain situation without making a claim as to its truth. The thought lacks the *aim,* the aim of truth, which is intrinsic to judgement. Wittgenstein argues that the aim of truth is intrinsic not only to the content of a judgement, but to the sense of a sentence.

Wittgenstein speaks to the relation (as he sees it) of truth and the sense of sentences in the following passage:

> ‘An analogy to explain the concept of truth: a black spot on white paper; you can describe the shape of the spot by saying, for each point on the sheet, whether it is black or white. To the fact that a point is black there corresponds a positive fact, and to the fact that a point is white (not black) a negative fact. If I designate a point on the sheet (a Fregean truth-value), then this corresponds to the assumption that is put forward for judgement, etc. etc.
>
> But in order to be able to say that a point is black or white, I must first know when a point is called black, and when white; in order to be able to say, “p” is true (or false), I must have determined in what circumstances I call “p” true, and in so doing I determine the sense of the sentence.
>
> Now the point where the simile breaks down is this: we can indicate a point on the paper even if we do not know what black and white are, but if a sentence has no sense, nothing corresponds to it, for it does not designate a thing (a truth-value) with properties perhaps dubbed “false” or “true”. The verb of a sentence is not “is true” or “is false”, as Frege thought: rather, that which “is true” must already contain the verb.” (TLP 4.063)

Wittgenstein’s first claim here is that a Fregean truth value equates to the mere designation of a point on the sheet. But in order to *say* whether a point is black or white, he goes on, designating a point is not sufficient. (Say here, is being used in the same sense as we saw in section 1, to mean assert. Recall also that, in Tractarian terms, a Fregean thought *shows* a certain picture of reality without *saying* (asserting) that the picture is true.) The designation must be preceded by an understanding of when a point is called white and an understanding of when a point is called black, the correctness conditions of an assertion. The sense of the sentence is the circumstances in which P is *to be called true*.

Wittgenstein draws an important distinction between his analogy and the concept of truth. One can merely designate a point, in the analogy, without understanding when a

point is to be called white or black. With truth, on the other hand, one refers to *nothing at all* when they make a designation without such an understanding. Their sentence is senseless, which precisely means that the sentence refers to nothing at all. This is why a Fregean truth-value is insufficient since it is *mere* designation. What this shows, Wittgenstein claims, is: that which is true (the truth-value) must already contain the verb (is true or is false). In other words, the content must already be constituted as something which is fundamentally *to be asserted*. The mere designation of a point, the Fregean truth-value in the analogy, lacks sense because it lacks the 'verb' is true or is false. The aim of truth is contained in the aim of assertion itself, and assertion cannot be added to the content, since this is to attempt to add the verb to that which is true, and that which is true must already contain the verb.

This is the consequence of what we already discovered in our analysis of Frege:

> 'Frege held however that prior to having a truth condition,
> a proposition refers to a truth-value, and that it is only through a
> subsequent act of assertion that the proposition comes to have its sense.
> The symbol "p" of the Begriffsschrift formula " ⊢p" refers to a truth
> value, but it does not in itself have a truth condition: for that, we need
> to apply Frege's assertion sign "(Johnston, 2024A, 5)

P' has a content, the understanding contained in its truth table. This means that it expresses a truth or falsity, according to this understanding. Yet, this content is separate from the question of the assertion of this content. Crucially, it is the assertion of the content which makes the content accountable to reality. It follows that Frege's content does not yet contain the understanding (in the analogy) of when a point is to be called black or white. In the analogy, he is merely designating a point. Frege's content is senseless (following Wittgenstein's conception) so, as such, nothing corresponds to it. The content of a Fregean thought, since it does not contain a claim of truth (the verb 'is true', in Wittgenstein's terminology from TLP 4.063) cannot thereby be transformed into a sensible judgement as this is to attempt to add the verb.

The Fregean proposition is senseless in Wittgenstein's view because its content is not accountable to reality. As such it does not call some propositions true, and others false. It shows a certain picture of reality (designates a point, in the analogy), yet it does not assert that picture. It is in this way that the proposition lacks the aspect of truth, which Wittgenstein takes to be crucial, that the goal of a proposition is truth (Dummett, 1959, 143).

Following on from section 1i, it is in this way that we can characterise the Tractarian view of truth as *fundamentally* correctness in assertion. Assertion aims at correctness, and, when it achieves that aim, the proposition can be called true. Truth is not only the goal of assertion, but truth *is* also correct assertion. Wittgenstein's conception of truth as correctness in assertion ties his account of truth to *what is the case*. A proposition is true if it makes a correct assertion about what is the case. Wittgenstein's claim is that a Fregean truth-value, a mere designation of a point on the sheet, misses a crucial aspect of truth by divorcing itself from comparison with reality. The question of truth cannot be separated from the question of (correctness in) assertion. Rather, truth *is* correctness in assertion. The comparison with reality, which is brought by assertion, constitutes part of the sense, of the Tractarian proposition. What is *shown* is part of the sense, but the question is not settled prior to the question of judgement, the question of what is *said* by the proposition.

Section 2i. Frege on the possibility of judging a contradiction

Thus far, I have only demonstrated that Frege's account fails on Wittgenstein's terms, rather than his own. As such, the argument up to this point does not in fact constitute a direct challenge Frege's system. However, there is a principle to which Frege himself is

committed to, which, I will argue, cannot be held in combination with Frege's view of sense. This principle is characterised in the following passage. Gareth Evans writes: 'The sense of a sentence, which is of course a function of the sense of its parts, is (in Frege's terminology) a thought; and the single constraint Frege imposed upon his notion of thought was that it should conform to what we may call "the Intuitive Criterion of Difference," namely that the thought associated with one sentence S as its sense must be different from the thought associated with another sentence S' as its sense, if it is possible for someone to understand both sentences at a given time while coherently taking different attitudes towards them, i.e. accepting (rejecting) one while rejecting (accepting), or being agnostic about, the other.' (Evans, 1982, 18-19)

Irad Kimhi claims, with reference to this passage from Evans, that Frege is forced into contradiction by his own lights via the Intuitive Criterion of Difference (Kimhi, 2018, 54-55).

The Intuitive Criterion states that if one can coherently take different attitudes to two different sentences (S and S') then the two sentences must express different thoughts. This is because if one understands S and S' yet is able to take different attitudes of acceptance/rejection/agnosticism then it must be the case that the sense of S and S' are different. Otherwise, upon understanding S and S', one would realise from their understanding that both sentences express the same thought. This is because understanding a Fregean thought means understanding its truth conditions. If S and S' did express the same thought, the understanding thinker would recognise the identical truth conditions as a part of their act of understanding (grasping). So, one is not able to take different attitudes in a coherent manner towards two sentences unless the thoughts that the sentences express are in fact different.

The prima facie problem for Frege can now be made apparent. Frege's separation of judgement from content makes it such that 'p and not-p' forms a perfectly good content. Since 'p and not-p' forms a perfectly good content, it is also the case that 'p and not-p' forms a possible assertion. Since it is the case that 'p and not-p' forms a coherent

assertion, then it is possible, through asserting this contradiction, to take opposing attitudes towards ‘p’ at the same time. In judging the first half of the conjunction, the thinker accepts p, and, in judging the second half, the thinker rejects p. It follows then from the Intuitive Criterion, that the sentence S (p) and the sentence S’ (p) must express different thoughts, since S is being rejected but S’ is being accepted. So, ‘p’ which of course expresses one thought must in fact express two different thoughts and Frege’s system fails by its own lights. As Kimhi puts it:

> ‘it follows that we can coherently take different attitudes to p at the same time. According to the Intuitive Criterion, then, p has a different sense than p!’ (Kimhi, 2018, 55)

I present this as the prima facie problem because two specific interpretive claims I made about Frege are controversial. As such, it will take time to defend these claims from objection. The first claim I made which is controversial is the following:
Since ‘p and not-p’ forms a perfectly good content, it is also the case that ‘p and not-p’ forms a possible assertion.

A Fregean might object that it does not follow from the fact that ‘p and not-p’ forms a viable logical content to the claim that ‘p and not-p’, therefore, forms a possible assertion. Frege himself says the following about the possibility of judging a contradiction:

> ‘The assertion of a thought which contradicts a logical law can indeed appear, if not nonsensical, then at least absurd; for the truth of a logical law is immediately evident of itself, from the sense of the expression. But a thought which contradicts a logical law may be expressed since it may be negated’

(My emphasis, Frege, “Compound Thoughts,” (CT) in Collected Papers on Mathematics, Logic and Philosophy (Oxford: Blackwell, 1984, 405)

Frege here only countenances the possibility of a contradiction being expressed (forming part of an assertion) insofar as it is being negated. This is because the negation of a contradiction would be a necessarily true judgement, whereas the assertion of a contradiction forms a necessarily false judgement. Furthermore, Frege claims that the truth of a logical law is immediately evident of itself. A thought which contradicts a logical law, then, is absurd since the act goes against this understanding of the logical law as true.

Kimhi draws on this passage to argue that Frege is accepting judging a contradiction as a possible, if absurd, act of judgement. As Kimhi points out, Frege describes 'such assertions as absurd rather than nonsensical' (Kimhi, 2018, 55). The implication being that if the assertion of a contradiction were meaningfully impossible, then the act would be nonsensical. For his purposes, then, he takes the objection I outlined to be dealt with. However, I think this is too quick. It is less than clear here what Frege must mean by absurd.

Let us consider if a Fregean agent can actually judge a contradiction. As Kimhi himself notes with regards to the same passage about contradictions, 'One cannot attach a judgement stroke to it, since by using the judgement stroke one manifests one's capacity to understand logical expressions' (Kimhi, 2018, 55). So, grasping a contradiction (which is a precondition of judgement) will show to the agent that the contradiction is necessarily false. What this means is that it is psychologically impossible to judge a contradiction. The intermediary act of grasping a thought will prevent an agent from ever believing and thereby judging a contradiction.

This clause of the Intuitive Criterion is now brought back into question:

> 'If it is possible for someone to understand both sentences at a given time while coherently taking different attitudes towards them'

The act of understanding (grasping) a contradiction prevents the judgement of a contradiction in the first place. As such, the objection is that the Intuitive Criterion never gets off the ground with regards to ‘p and not-p’. A Fregean agent cannot in fact understand this and judge the contradiction.

What must Kimhi have been thinking then? On Frege’s account, logical content does not ‘exclude psychologically impossible combinations’ (Kimhi, 2018, 55). Otherwise, a contradiction would not have a sense to be expressed, it would just be logical nonsense. Kimhi describes Frege as a psycho/logical dualist because of this. There is not a unity in Frege between the psychologically and logically possible. The role of the contradiction as a psychological impossibility but logical necessity (so that one can judge the logical truth of the negation of a contradiction) demonstrates this. There must be a (logical) thought that is expressed even if that thought cannot be (psychologically) judged.

This contrasts with what Kimhi calls Wittgenstein’s psycho/logical monism (Kimhi, 2018, 50). Wittgenstein’s unification of the psychological and the logical is what allows him to say that one cannot (psychologically) judge a contradiction since a contradiction is (logically) impossible. Since there is no distinction between the constraints on the psychological act of judging and the logical content to be judged (the logical content must itself be a judgement/assertion to be a content) it is therefore both psychologically and logically impossible to judge a contradiction. There is no such content
Frege’s move that one cannot attach the judgement stroke necessary to assert a contradiction is a psychological constraint, it is based on the understanding manifested by the use of the judgement stroke. There is nothing logically wrong with attaching the judgement stroke to a contradiction, there cannot be, a contradiction expresses a thought. For it to be logically impossible, it would need to be the case that a contradiction does not express a coherent thought.

Yet when we are talking about the sense of a sentence, which the Intuitive Criterion is concerned with, we are not speaking psychologically. The possibility of coherently

taking different attitudes towards different sentences (if they have different senses), is a logical notion of possibility. The sense of sentences could not be determined by merely psychological criteria (unless one were a psycho/logical monist like Wittgenstein). The sense of the sentence is its logical content. The possibility of coherently taking different attitudes towards different senses is based on whether or not the senses are in fact logically different. It is incoherent to take different attitudes towards ‘p and p’, not because of some psychological fact, but because ‘p and p’ are one and the same sentence with identical logical content.

Frege’s separation of the psychological from the logical implies that he must be concerned with logical possibility in the Intuitive Criterion, since he is concerned with logical content. It follows then, that in the sense of possibility with which Frege is concerned (logical possibility) it is coherent to judge a contradiction. In The Begriffsschrift, Frege gives only two examples of types of contents that are not suited to become judgements. The idea “house”, as opposed to “that there is a house” (the second would be suitable” (BS, S2) and contents containing operations over indeterminate notions (such as removing beans from a heap of beans, as this will not always be true depending on the amount of beans in the heap) (Frege, BS, S27). This is how he bounds the logical possibility of judgements, by excluding certain types of contents as incapable of forming judgements, such that the content stroke of the assertion sign cannot be attached (BS, S2). A contradiction on the other hand does form a content which is suitable for forming a judgement.

My interpretation of Frege I have argued for is as follows: one could (logically speaking) accept ‘p and not-p’. This is a (psychologically) absurd act, such that no rational agent could in fact judge ‘p and not-p’ as their own understanding prevents this. Yet it is not a logically impossible act, since ‘p and not-p’ forms a perfectly good (judgeable) content. Psychological understanding plays no role in this logical possibility, which involves only the type of content. This means that it is possible, in the relevant sense, to judge a contradiction by Frege’s lights. As such, this first objection can be resolved.

The second controversial claim made in my presentation of the prima facie case against Frege must also be defended. Recall:

> ‘Since it is the case that ‘p and not-p’ forms a coherent assertion, then it is possible, through asserting this contradiction, to take opposing attitudes towards ‘p’ at the same time. In judging the first half of the conjunction, the thinker accepts p, and, in judging the second half, the thinker rejects p.’

A Fregean might object that although, following the previous argument, it is logically possible to judge a contradiction, judging the contradiction ‘p and not-p’ does not amount to taking different attitudes towards the thought expressed by ‘p’. In taking an attitude towards p one does not thereby take an attitude towards not-p, and vice versa. Rather, in judging ‘p and not-p’ a Fregean would take attitudes to ‘p’ and ‘not-p’ in isolation. Their acceptance of ‘not-p’ does not involve a rejection of ‘p’ and, as such, in judging a contradiction one would not take opposing attitudes to ‘p’. As such, the Intuitive Criterion would not apply to ‘p’ as I claimed in my presentation of the problem.

To respond to this, let’s look closer at Frege’s view of negation:

> ‘The content of any truth is ‘a content of possible judgment’, but so too is the opposite content. This opposition or conflict is such that we automatically reject one limb as false when we accept the other as true, and conversely. The rejection of the one and the acceptance of the other are one and the same. (Frege, Posthumous Writings, (PW), 1979, 8)

Here, Frege commits himself to an important principle. It is not merely that one commits themself to the acceptance of ‘p’ when they assert ‘p’. But rather, one also commits themself to the rejection of the so-called opposite thought, namely, not-p. Conversely, in accepting not-p one commits themself to the rejection of p. Frege goes so far in this passage as to say that with thoughts of opposite content, the rejection of the one is the acceptance of the other. Call this the opposition principle.

Do not confuse Frege's opposition principle with the stronger form of opposition in The Tractatus. For Wittgenstein, it is in the nature of p to oppose not-p, since contents are fundamentally assertions. For Frege, the assertion of 'p' opposes the assertion of 'not-p', but this is not a consequence of the fundamental nature of the opposite thoughts. Rather, the two thoughts oppose each other because of their opposite contents only once asserted. Although the thoughts themselves have opposite contents they do not normatively oppose each other at the level of thought, only at the level of judgement. This principle must be combined with a second principle. This principle states that when one accepts a conjunction, they necessarily accept both conjuncts. Given that asserting a contradiction involves accepting the truth of the conjunction, and a conjunction is made up of its conjuncts, I will take this principle for granted. Call this the conjunction principle.

As a result of Frege's commitment to the opposition principle and the conjunction principle we are able to refute the Fregean response. The conjunction principle tells us that in accepting the conjunction, one thereby accepts both conjuncts, in this case, 'p' and 'not-p'. The opposition principle tells us that acceptance of one thought involves the rejection of the thought with an opposite content. So, in judging 'p and not-p' one accepts p whilst rejecting p (through their acceptance of 'not-p'). As such, it follows from the Intuitive Criterion that 'p' has a different sense to 'p', as per my initial presentation of the problem.

It is important to note that this argument rests on a Fregeans' acceptance of the Intuitive Criterion. As such, a revisionist Fregean might choose to merely reject the Criterion rather than Frege's entire system. Yet, the opposition principle creates a problem without need for combination with the Intuitive Criterion. Even if judging a contradiction did not trigger the Intuitive Criterion, the act would amount to taking a stand against oneself on Frege's own terms. This might have been part of what Frege was expressing when he called the act absurd. Yet, this absurd act is logically possible in Frege's view

and, I take it, this forms another reason to choose to reject the system rather than to focus on piecemeal solutions.

What I take to be undermined by the above argument is Frege's separation of content from judgement. It should not be logically possible to judge a contradiction, not only because this leads to contradiction when combined with the Intuitive Criterion, but also because it is nonsensical to take a stand against oneself. For a view which rejects this separation, we must turn back to Wittgenstein. This section has served to motivate that choice.

Section 2ii. Wittgenstein and the Intuitive Criterion

It follows that if we accept the Intuitive Criterion, Frege's notion of sense must be misconceived on his own terms. On the other hand, Wittgenstein's notion of sense runs into no such trouble. P and not-p (as we have already seen), relate to each other in a fundamental sense, they oppose each other. One cannot coherently judge p and not-p. Further, as Wittgenstein puts it:

> 'In 'not-p' "p" is not an index but an argument; the sense of "not-p" *cannot* be understood, unless the sense of "p" has previously been understood.' (TLP 5.02)

The sense of not-p, since not-p is the negation of p, is of an opposite nature to p (if one is true, the other must be false). This is a merely factual statement which as yet says nothing about the sense of either proposition, it is a statement which Frege would agree to. According to *The Tractatus* the opposition runs deeper than that. The sense of not-p *consists in its opposition to p*. This is why one cannot understand not-p without *previously* understanding p. Understanding p is a precondition of understanding not-p because asserting not-p consists in the rejection of p.

Wittgenstein uncovers here the same truth about sense which forms the basis of the Intuitive Criterion. We saw with the Intuitive Criterion that the sense of a sentence S is defined in terms of the different attitudes which can be taken towards S, as opposed to some other sentence S'. The sense of S is not an isolated fact, but one relative to other sentences.

> 'The picture represents what it represents, independently of its truth or falsehood, through the form of representation.
> What the picture represents is its sense.
> In the agreement or disagreement of its sense with reality, its truth or falsity consists.' (TLP 2.22-2.222)

Recall that Tractarian propositions are logical pictures of reality. These logical pictures consist in the proposition and the sum of its logical relations (those propositions it affirms and those it denies). This is what the picture represents, and, as Wittgenstein says here, what the picture represents *is* the proposition's sense. P by its nature denies not-p, which is represented in the picture of p, so it is internal to the sense of the proposition. In the same way, all the propositions which are affirmed by p are internal to p's sense.

The Tractarian view here nicely captures the oppositional nature of sense according to the Intuitive Criterion. One can only coherently take opposing attitudes towards two sentences S and S' if the picture of reality presented by the two sentences is different. The falsity of not-p is contained in the truth of p (and vice versa) and thus it *only* makes sense to have opposing accepting/rejecting attitudes of p and not-p. The pictures of the propositions (which are their senses) are necessarily opposed. This constraint (which has it that there is no such assertion 'p and not-p') is necessary to reinforce the Intuitive Criterion.

Frege's contrary view of sense (S4) gets him into trouble because the content of the sentence p is not fundamentally an assertion. One's acceptance of S has no bearing on

one's acceptance of S', since even if the two sentences are opposed, this opposition is not internal to the sense of either sentence. One can coherently judge contradictions because of this fact (even if *psychologically* speaking, one would never in fact judge a contradiction to be true). Yet, if a sense of a sentence S is to be defined in terms of the differences between sentences S and S', then it cannot be that one can coherently judge contradictions to be true, since this is the paradigmatic case of opposing sentences. Being able to coherently judge contradictions means that for any two sentences any attitude set is coherent, undermining the basis for the Intuitive Criterion. It must be incoherent to judge 'p and not-p' if sense is to be defined in terms of the differences between opposing sentences.

I do not think Frege went wrong with the Intuitive Criterion but rather, with his conceptual ordering of judgement and content. If sense is simply a product of the content of a sentence held in isolation, then our *understanding* of opposing sentences should also be held in isolation. We should *disagree* with Wittgenstein's claim that we need to understand p as a necessary precondition of understanding not-p. Yet when, as I argued earlier, the only way to conceive of the assertion not-p is as a rejection of p, I cannot see how one would disagree with such a claim. Gilad Nir (2021) characterises this sort of idea as a view of propositions as one of 'inert atoms; merely understanding each of them does not yet entitle us to take any conclusion to be justified.' (Nir, 2021, 58).

So, if it is not the Intuitive Criterion that explains Frege's misstep it must be his fundamental view of content and judgement. Defining content prior to, and separately from, defining judgement, Frege strips propositional content of its oppositional nature (such that contradictions can form coherent judgements). Frege is forced by his Intuitive Criterion as a constraint on the notion of a thought into contradiction. Yet this contradiction only arises because of his underlying view of the relation between content and judgement.

This objection is not, by this stage in the argument, one which merely assumes a Wittgensteinian perspective. Rather, Frege's account reaches this contradiction *on his own terms*. In view of not wanting to reject Frege's Intuitive Criterion, one is forced to reject the Fregean view of content and judgement. This section has shown that the Tractarian view, that p is fundamentally an assertion, avoids such contradiction. We can therefore derive a positive argument for the Tractarian view:

P1: If one can coherently take different attitudes towards two sentences that they understand then it is the case that the two sentences express two different thoughts (The Intuitive Criterion).
P2: On the Fregean view of judgement, one can coherently judge 'p and not-p'.
P3: Either the senses of 'p' and 'not-p' are not opposed (it follows from P1 that this must be false) or 'p' and not-p' must oppose each other such that the judgement ''p' and 'not-p'' would be incoherent.
P4: For 'p' and 'not-p' to oppose each other at the level of judgement it must be that propositional content is fundamentally the content of a judgement.
C: Propositional content is fundamentally the content of a judgement (The Tractarian view)

Section 3i: Laws of inference

In section 3, we are now well placed to give a plausible Tractarian reading of the claim that laws of inference are senseless and superfluous. I t is now necessary to treat Wittgenstein on his own terms, but I take it that I have sufficiently motivated the choice to do this. Section 2 showed us that we must adopt the Tractarian view of content and judgement to avoid the contradiction Frege runs into. Avoiding this very contradiction is a reason to adopt instead the Tractarian view that contents are fundamentally assertions. Once we have adopted *this* perspective, we are in a position not only to explain what Wittgenstein is saying about laws of inference in TLP 5.132, but to holistically motivate his view.

Allow me first to summarise the argument of this chapter thus far. (S1i) Propositions are a logical picture of reality. This logical picture consists not only of the atomic proposition but also the assertion of all the propositions which follow from it (and the denial of the propositions negated by it). (S1ii) Logical space represents the internal logical relations between propositions such that all propositions are constituted by their relations to each other. We can now see a Tractarian view of inference which requires only the assertion of the premises. A proposition is a picture of reality which, when asserted, affirms the propositions which follow from it. This is an account of inference. It is one which has no need for laws of inference, since propositions are such as to assert all of their inferences. (S1iii) In further contrast with Frege, a Tractarian proposition is fundamentally an assertion and, further, truth is correctness in assertion. This means that the Tractarian conception of truth encompasses the aim of correctness as a part of what constitutes truth.

(S2i) The Intuitive Criterion of Difference forces Frege into contradiction since ‘p and not-p’ forms a possible Fregean judgement. It follows that it is possible to coherently take different attitudes towards p at once, implying that a singular sentence in fact expresses two different thoughts. As such, Frege cannot meet his own criterion. (S2ii) Wittgenstein’s account of sense by contrast, where a proposition's sense consists in the logical picture of reality that represents it, is able to meet the criterion. That the Tractarian view succeeds where *The Begriffsschrift* fails is downstream from the two opposing fundamental views on the relation between content and judgement. It follows that one must adopt the Tractarian view that propositional content is fundamentally a judgement to not be undermined by the Intuitive Criterion of Difference. This, by itself, serves to motivate the Tractarian perspective on content, judgement and sense.

Laws of inference, for Frege, serve as norms justifying the inferential transition from premise(s) to conclusion. It is perhaps prescient to ask why our Tractarian account would repudiate any need for such norms, rejecting them as both superfluous and senseless. Wittgenstein also makes one other claim of superfluity in *The Tractatus*:

> ‘The theory of classes is altogether superfluous in mathematics.’ (6.031)

Russell was shocked by this claim, writing in a letter to Wittgenstein:

> ‘If you said classes were superfluous in *logic* I would imagine that I would imagine that I understood you, by supposing a distinction between logic and mathematics; but when you say they are unnecessary in *mathematics* I am puzzled’ (Monk, 1990, 166)

In my view Russell’s shock here bears significant similarity to shock one is likely to have towards the claim that laws of inference are superfluous and senseless. Russell has no disagreement with the claim that, logically speaking, a theory of classes is unnecessary. Yet, he thinks, surely this does not undermine the *proper role* of a theory of classes, for application in the field of mathematics.

Strictly speaking, following our Tractarian analysis of propositions, laws of inference do not serve to justify the inference from one proposition to another. This, we discovered, is a product of the nature of the propositions themselves. Yet, one might not think it problematic to seek to describe these very relations (in a concept script such as *The Begriffsschrift*) for application in logical notation. I am not arguing here that in such a script which sets out the rules for the conduct of inference, that laws of inference are misplaced. One could not have such a script without rules as a part of it. It is in the context of this kind that laws of inference *do* find their proper role. The role that they carry out, is one of expressing the relations contained within the propositions, the similarity in logical form shared between inferences of the same type (such as Modus Ponens). As such, laws of inference can still serve to represent the inferential relations between propositions.

In light of this, it is incumbent on us to try and reach a reasonable understanding of what Wittgenstein's point is in (TLP 5.132) when he deems laws of inference to be both superfluous and senseless.

> ''If *p* follows from *q*, I can conclude from *q* to *p*; infer *p* from *q*.
> The method of inference is to be understood from the two propositions alone.
> *Only they themselves can justify the inference*.
> Laws of inference, which- as in Frege and Russell- *are to justify the conclusions*, are senseless and would be superfluous.' (TLP 5.132, my emphasis)

Wittgenstein is explicit here that he is only concerned with the question of *inferential justification*. That is, what licences the inferential transition from one proposition to another. In the context of a notation script, laws of inference are unproblematic, yet this is because, in such a context, they are entirely divorced from the question of inferential justification. In such a context, they would not be *laws of inference*, fundamentally speaking, but mere rules demonstrating facts (about the proposition) in the context of the game of generating a notational script. So, it is for the question of inferential justification they are superfluous.

Yet it is still not clear why, in the context of such a game, one would say something *senseless* when they give laws of inference (here, mere rules given in the context of the game). Laws of inference conceived in this way, could serve to express licences and prohibitions dictating the difference between legitimate and illegitimate differences. Although superfluous with regards to inferential justification, it seems as though in the right context laws of inference could carry sense. It is important to note an objection at this stage before progressing further. An objector to the view I'm attributing to Wittgenstein could say the following:

> In the context of a game, there are rules. Rules govern the way in which the game is played, and they are necessary in order to play the game. In chess, rooks can move horizontally and vertically. Is it *true*, fundamentally speaking, that

> rooks move horizontally and vertically? It is not true *outside of the proper context of chess*. So, laws of inference need not seek to express *truths*, rather, they issue licences and prohibitions in their right context, the game of a logical calculus. It follows that, since they seek to express no truth at all, it is not that they *fail* to express a truth or express a falsity. Rather, laws of inference are just rules which issue licences/prohibitions in the context of logical notation, and *this* is their sense.

This objection picks up the thread from chapter 2 section 3. It returns to the question of why it is a problem to have mere-licence providing inference rules. My response to the objector is that the game with which laws of inference are concerned is not self-contained. A notational script seeks to express certain rules which are *valid* and seeks to not express certain rules which are *invalid*. The answer to the question of which rules are which turns on the fundamental question of *which inferences are justified*. This is what dictates which rules are used to play the game of logical notation; the rules of a game are not an open question.

Chess has had rule changes since its initial invention. Upon agreement, it is the sort of game where these rule changes are possible because the game is self-contained. There are not, fundamentally speaking, a right way and a wrong way that chess pieces move.

On the Tractarian view, if p follows from q, then I can conclude p from q. It is a *true* fact *about the propositions involved* that the inference from one to the other is justified. So, when rules of inference attempt to issue such licences, they attempt to speak about these types of true fact about the propositions. They say, not only, ‘this is how the rook moves in the context of these rules of chess’ but, since the game is not self-contained, they say the equivalent ‘this is how rooks move’. They must make the stronger claim since the inferences are only justified because they are *fundamentally* justified. Laws of inference only take the specific form they do because of the inferences which are in fact justified. Yet, just like outside the context of chess, the stronger claim ‘we should make

inferences in this or that way' is illegitimate outside of the context of the game. Since laws of inference are concerned with the *right* modes of inference, they must make the stronger claim which involves the *true* facts about the internal relations between propositions. In making the stronger claim, they make a senseless claim.

For laws of inference to have sense (so they are not senseless in the first place) they would need to be expressed propositionally, for: 'Only the proposition has sense' (TLP 3.3). Laws of inference, in our present context, would serve to express the relations between propositions in order to licence the inferential transition from one proposition to another. The relations between propositions are internal to the propositions, making them facts *about* the propositions. In dismissing Russell's *Theory of Types*, Wittgenstein says:

> 'Russell's error is shown by the fact that in drawing up his symbolic rules he has to speak about the things his signs mean.
> No proposition can say anything about itself, because the propositional sign cannot be contained in itself' (TLP 3.331-3.332)

A proposition can never speak to its own (logical) form. Since only the proposition has sense, for laws of inference to have sense they must be expressed propositionally. It is a consequence of this those laws of inference (propositions), insofar as they seek to express that which they express (fact(s) about the logical form of propositions), then they seek to express something that cannot be said. (Here, once again, Wittgenstein uses *say* and now *speak* to mean *assert*.) It is important to note here that just as with a correct mathematical application of set theory, one could make correct logical inferences via laws of inference, this does not mean that the laws of inference *even in this context* have sense.

So, my characterisation of the Tractarian view is as follows. With regards to the question of inferential justification, the question with which Wittgenstein is concerned in (TLP 5.132), laws of inference are superfluous. This does not mean that laws of

inference do not find a *proper role* in the context of a logical script in need of rules. In this script, laws of inference have a need, they are not superfluous. Yet, in their proper role, laws of inference are still senseless. Formed into propositions, which they must be to carry sense, laws of inference seek to assert facts about the proposition (facts about themselves). That is, they assert patterns of internal relations between propositions. Wittgenstein, above, argues that these facts about the proposition are *unsayable* by the proposition itself.

To understand *this* claim, we are brought to the Tractarian distinction between showing and saying:

> 'What *can* be shown *cannot* be said' (TLP 4.1212)

That r follows from (p,q), if it is true, can be shown from the nature of the propositions. It will be the case that the logical pictures of the propositions will show that if p and q are true then r must be true. A law of inference like Modus Ponens seeks to express this. Yet, while it is true that an inference of the form of Modus Ponens takes place, this cannot be expressed propositionally in a law of inference. Since the relation *can* be shown from the form of the propositions, it *cannot* be said.

Whatever truth laws of inference seek to express, it is one which Wittgenstein sees as inexpressible. Modus Ponens is not *false but formulated as a law of inference* it speaks to the unspeakable propositional form. Wittgenstein could have equally said the same thing he said about solipsism:

> 'In fact what solipsism *means*, is quite correct, only it cannot be *said*, but it shows itself' (TLP 5.62)

What the law of inference *means*: that there is a commonality in the logical form of propositions which justifies certain patterns of inferences (e.g. Modus Ponens), is correct. This is inarguably true on the Tractarian view, since we can see it from the form

of propositions. Yet, like with solipsism (the meaning of which is also true), the nature of this truth is inexpressible.

Put explicitly:

> ‘If the truth of one proposition follows from the truth of others, this *expresses itself* in relations in which the forms of these propositions stand to one another, and we do not need to put them in these relations first by connecting them with one another in a proposition; *for these relations are internal*, and exist as soon as, and by the very fact that, the propositions exist.’ (TLP 5.131, my emphasis)

Here, Wittgenstein is making the point that one would need no norms to justify inferences since if the truth of one proposition follows from the truth of the other, this expresses itself (as we know, through the logical picture of reality presented by the proposition). So, there is no need for extra propositions, not involved in the relations between the propositions, to express the relation. Further, since the relations are internal to the proposition, the relations cannot be expressed by a proposition (recall that propositions cannot speak of their own form (TLP 3.332). So inferential relations are true facts about propositions, making their truth inexpressible in language. Laws of inference are senseless because they attempt to say a truth which cannot be said.

So, one way to characterise Russell’s point, that the theory of classes is not superfluous *in mathematics*, is to say that the theory of classes is senseless but *not* superfluous. It is senseless when taken as a theory in logic yet necessary for the context of the game of performing mathematical tasks. So, its proper role is as a logically senseless but necessary mathematical tool. This is a plausible interpretation given that Russell is objecting only to the claim of (TLP 6.031) that a theory of classes is superfluous in mathematics, not that it is superfluous in logic. Russell agrees with Wittgenstein that the mathematical truth in the theory of classes is not to be expressed in logic, and so there is no need for a theory of classes for the purposes of achieving a complete logic.

Russell, in accepting the idea that Wittgenstein would be right *if he were talking purely of logic*, thereby accepts the idea that the theory of classes is in fact inexpressible in logic. The same is true, analogously speaking, of laws of inference. The proper role of laws of inference is in the context of the game of logical notation. This does not mean they carry sense, even in the context of this game. The game of logical notation is not self-contained, inferences are made based on true facts about propositions (the internal relations between one another). So, in seeking to represent these true facts about the propositions, for the purposes of the game, laws of inferences render themselves senseless. This does not mean they are superfluous *for the purposes of the game*. Rather, they are superfluous with regards to Wittgenstein's purposes in (TLP 5.132), settling the question of what grounds inferential justification.

Section 3ii. Carroll's regress

The Tractarian account of inference avoids Carroll's regress through its rejection of laws of inference. Its answer to the regress, insofar as it has one, would be that the scenario itself which leads to the regress is incoherent. Since r follows from p and q, the affirmation of r follows from the affirmation of p and q. The (tortoise's) suggestion that one could understand the premises without being under a 'logical necessity' (Carroll, 1895, 279) to accept the conclusion obscures this fact. The tortoise implicitly assumes that the acceptance of the conclusion must be a further act. On the Tractarian view, this is not true. One's acceptance of r is constituted by the acceptance of p and q; since r is a consequent of p and q and propositions assert all the propositions that follow from them. From this perspective, it is incoherent to suggest that one can understand p and q, assert them as true, and yet refuse to conclude r. The affirmation of r is contained within the logical picture that is the assertion of p and q. Achilles is unable to escape the regress, since he is concerned with *logic* 'forcing' (Carroll, 1895, 280) him to accept the conclusion on the basis of the premises, rather than adopting the Tractarian view that the premises themselves already justify the inference.

Requiring laws of inference in order to justify inferences is to require a *norm* of inference. This chapter has argued against specifically the Fregean view found in *The Begriffsschrift* presented in chapter 2. I have, in chapter 1, also looked at contemporary accounts which propose different norms of inference, concluding that all those accounts lead to regress.

Sebastian Rödl has recently argued that an understanding of Carroll's regress leads directly to the conclusion that inference cannot be a further act beyond the premises (Rödl, 163, 2018). I argued in agreement with this claim in chapter 1 and, following a Tractarian view of propositions (a view I am not here attributing to Rödl), we have been able to show this. I also take it to be a positive of the Tractarian view that not only does it avoid the regress, but it also offers a diagnosis of the problem, through distinguishing between those truths which can be said and which can be shown.

As I noted in section 3i, it is true that there are common patterns of inference which are constitutive of thinking. The common assumption, shared by both Frege and contemporary thinkers, is that this must be a truth which we are able to formulate *propositionally*. However, if one accepts that there is a category of truths which can be shown but not said, it does not follow from the truth that there are common patterns of valid inference to the claim that there must be a norm which represents and justifies these patterns. If these patterns are of a kind to be a truth which is shown, it is from our observation that we do indeed infer in certain patterns that we *see* the true facts about how propositions relate to one another. Although these patterns of relations are rule-like, they are facts about the (logical form of) proposition, making us unable to propositionally formulate them.

In Tractarian language, these are *unspeakable* facts which *show* themselves. Philosophy cannot show what propositions have in common with reality, so the truth of the matter must be unspeakable. Philosophy shows the unspeakable by placing limits on the speakable (TLP 4.115-4.12)

Even though I have motivated the Tractarian view, the idea that some truths are inexpressible but *show* themselves will be unconvincing to many. Russell characterised the approach as 'a curious kind of logical mysticism' (Monk, 1990, 165). He derided Wittgenstein's use of the distinction in *The Tractatus* an excuse to avoid difficult thinking (Monk, 1990, 182-183) and refused to accept that facts about the proposition are of an unspeakable kind. Interestingly, as I will now demonstrate, Boghossian himself admits just this kind of truth in order to deal with the problem presented by inference. Although, of course, he uses different words.

Recall in chapter 1, that Boghossian's solution to the regress presented by the intentional rule following view (that rules will require rules to justify themselves, and so on), is to claim that rule following is an unanalyzable primitive. He accepts that the truth of rules is *shown* by their necessity, the need to justify inference in a way which defends rational thought, despite the fact that they are unanalyzable. Boghossian argues that rules offer the correct explanation, yet the nature of their truth cannot be analysed (in the sense that they cannot be given further explanation). It follows from this that the sense in which they are true, can be given no non-circular explanation. We follow rules because we follow rules. Boghossian is conceding that the sense in which we follow rules is inexplicable. This is why I attribute here, to Boghossian's view, the idea that the supposed truth of rules must be 'shown' rather than said.

What is unsatisfying about Boghossian's explanation is that rules of inference, an already mystical concept, is given a mystical explanation. In fact, they can be given no non-circular explanation. We must simply infer from the premise of their supposed necessity to the conclusion that they are a primitive fact. Wittgenstein's explanation on the other hand, has it that *propositions* are the primitive concept. From the analysis of propositions, then, we seek to state only that which can be said, those facts which can be stated by propositions about themselves. It is less mystifying, to discover that propositions can fail to formulate certain truths *about themselves*. Rules, being less fundamental than the proposition (since they are propositions themselves), should be explainable via the language of propositions if they are in any way true.

In summary, it is motivational for the Tractarian view that:

1. From the judgement of Tractarian propositions alone, appropriate inferences are justified (if an entailment holds between propositions, then the inference is justified)
2. The truth, which is supposedly expressed by laws of inference, the validity of certain inferential patterns, is in fact shown by the nature of propositions as pictures of reality (thereby avoiding Carroll's regress)
3. All philosophers who accept the idea of an unanalyzable primitive accept an equivalency to the saying/ showing distinction.
4. Wittgenstein's explanation requires that certain truths are only showable, yet these truths are *about the proposition*.

My goal in this final chapter has not been to show that the Wittgensteinian account of inference is merely an internally consistent view of inference. That is, if one accepts all of *The Tractatus* as true, then they have an unproblematic view of inference *within the context of the system*. Rather, I have been trying to show that at the very least, Wittgenstein is able to avoid real philosophical problems which other philosophers (and their systems) have as yet been unable to. This is the purpose of first (in the previous two chapters) demonstrating the insurmountable task of providing a viable account of inference on other predominant views. It motivates the need to seek to understand *how* Wittgenstein avoids the same issues.

Conclusion

In this final chapter I turn to *The Tractatus*. I present how Wittgenstein, through his idea that contents are fundamentally assertions, avoids the need for laws of inference. In this divergence from Frege, Wittgenstein reaches a different account of sense, where the sense of the proposition includes its correctness conditions *as compared to reality*. Frege's view of sense is undermined by his own Intuitive Criterion of Difference and so his account fails by his own lights. Wittgenstein's view does not face the same problem

because he conceives of judgement as internal to content. We are left with the Tractarian view that the proposition is a picture of reality which asserts all the propositions which follow from it, underpinned by his conception of logical space, forms a coherent account of inference. From this perspective, we reach an understanding of inference which is not undermined by any of the problems discussed in the previous two chapters. *The Tractatus* even offers us a diagnosis of the core of the problem with laws of inference, that is, in attempting to *say* anything about logical laws we *speak* of facts about the proposition and so, we attempt to *say* what can only be *shown*.

Concluding Chapter

This paper defends the Tractarian view of inference as a solution to the problems faced by other contemporary and historical thinkers. In chapter 1 I outlined the intentional rule following the view as it is presented by Boghossian (2012) and the Simple Proposal from Wright (2014). In that chapter, I argued that these views are both unsatisfactory accounts of the nature of inferential justification. The Simple Proposal leads to an account of inference as non-rational and the rule following view leads to regress. Boghossian chooses the rule following view since he sees it as the only way that his Taking Condition could be met. I agree in spirit with the Taking Condition, but not with Boghossian's claim that his rule following view is the only way that the condition can be met. I argue that inference is a rational process where the inferrer must be in a position to know their own reasons, but that we must seek an account which does not run into the regress problem encountered by the rule following view.

Sebastian Rödl argues that the lesson to be learnt from Carroll's regress is that inference cannot consist in a further act beyond the judgement of the premises, such as the following of a rule. In other words, the content of the inference (the relation of entailment between premises and conclusion) is not prior to the act of the judgement of the content. Rather, inference and entailment are two sides of the same coin. Rödl's perspective echoes the Tractarian view.

At first glance, the Fregean view may also appear to assuage Rödl's concern. Frege has the threat of Carroll's regress keenly in mind, distinguishing precisely between the rules of his system and the axioms which underpin his system. These logical axioms manifest themselves without being asserted. The rules on the other hand do have an assertoric role, but it is limited to the issuing of inference-licences in order to transform relations of entailment (content relations for Frege) into relations of inference (judgement relations for Frege). And so, even though inferences are justified by these rules, they merely serve to transform the internal content relations between propositions into relations of judgement.

Since my concern is inferential justification, and I distinguish between mere-licencing and justification, by chapter 3 I am seeking a system which requires no intermediary rule of inference. Either, you have a substantial rule of inference which leads to regress (as in chapter 1), or you reduce the role of the rule to that of mere licencing (as in chapter 2). As such, I can see no proper role for rules of inference for understanding inferential justification.

In chapter 3, then, I begin by outlining the way in which the Tractarian view offers an account of inferential justification while repudiating rules of inference. A proposition is a picture of reality which determines a place in logical space. This place is constituted by the logical relations between those propositions and others, just like your place in physical space is constituted by the spatial relations between you and the rest of physical reality. Wittgenstein's theoretical setup allows him to connect content with judgement. As such, judgement of the premises of an inference is sufficient justification to judge the conclusion.

I then seek to motivate this view of content and judgement, that content is fundamentally assertable content, by returning to Frege. My criticism of Frege is the motivation for taking a Tractarian view over that of The Begriffsschrift. It demonstrates that Frege's deep problem is not with rules specifically, but with his assumption that content is prior to judgement which underpins his need for rules of inference.

I argue, in an objection I develop from Kimhi, that Frege's own Intuitive Criterion of Difference, when combined with his view of the relation between content and judgement, leads to the conclusion that 'p' has two senses at once. The solution to this, I claim, cannot be found within Frege's system. Rather, one must see content and judgement as going hand in hand, with neither notion taking theoretical priority over the other. Only then can one's logical system avoid the theoretical possibility of asserting a contradiction such as 'p and not-p'.

Upon motivating the Tractarian perspective, I adopt it to offer a reading of Wittgenstein's argument against rules of inference. I attempt to understand his claim that they are 'superfluous and senseless' (TLP 5.132) when it comes to understanding inferential justification. The Tractarian diagnosis is that the relations of inference are to be shown, not to be said (asserted). Logical relations between propositions are internal facts about the proposition, which cannot be expressed by a proposition. Insofar as rules of inference seek to say anything about the proposition, they will form a senseless proposition. Insofar as they do not form a proposition (as perhaps, with Frege's mere-licencing rules), they will lack sense, since only propositions have sense.

I conclude chapter 3 by returning to Carroll's regress, and by contrasting the Tractarian resolution with Boghossian's. I argue that Boghossian (unnecessarily) posits rule-following as a theoretical primitive in order to explain inference. This is unnecessary because it adds an additional unanalyzable concept as a basis for explanation. The Tractarian view, on the other hand, comes to an explanation of inference through analysis of the nature of propositions alone. Once this perspective is adopted, and it is a perspective I have attempted to motivate, then an explanation of inference is given without such theoretical baggage.

The assumption made for Carroll's example to work is that, in effect, is that there must be some way to force acceptance of an inferential conclusion through some further act. The example shows us that this cannot be a further piece of language asserting the entailment relation. Wittgenstein makes a compelling case that rules of inference are just as misconceived, not because they fail in their own individual way, but because they fail in just the same way. Both philosophical moves are undermined by their attempt to say what can only be shown. One is forced by their own understanding of the propositions involved to accept the conclusion of an inference; Carroll's example is a simple case of lack of understanding.

I have presented a case that the Tractarian approach, while it may seem unappealing to many, is motivated by real and significant philosophical concerns. It does not merely

form an internally coherent system, but it resolves problems which other systems are ill-equipped to handle. This motivates both a defence of the Tractarian position in its own right and a recognition by contemporary thinkers of the multitude of advantages to Wittgenstein's approach.

It is easy to have Russell's reaction, quoted earlier, and to deride Wittgenstein's integral saying and showing distinction as logical mysticism. Yet no view considered here is innocent of these charges (Boghossian's primitivism is just as mystical to me). The question is not which view is the least mystical, but which view lends us a perspective from which we can have understanding of inference.

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