VISUAL FACIAL AGEING USING PLS

Visual ageing of human faces in Three Dimensions using Morphable Models and Projection to Latent Structures

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Abstract: We present an approach to synthesising the effects of ageing on human face images using three-dimensional modelling. We extract a set of three dimensional face models from a set of two-dimensional face images by fitting a Morphable Model. We propose a method to age these face models using Partial Least Squares to extract from the data-set those factors most related to ageing. These ageing related factors are used to train an individually weighted linear model. We show that this is an effective means of producing an aged face image and compare this method to two other linear ageing methods for ageing face models. This is demonstrated both quantitatively and with perceptual evaluation using human raters.

1 INTRODUCTION

Accurate prediction of how a person's appearance will vary with age has a variety of applications, such as aiding in the search for missing persons, planning cosmetic surgery, as well as applications in the movie industry and other visual arts. Most researchers have concentrated on manipulating 2D images, 3D statistical models are a relatively recent innovation. In this paper we develop 3D models of ageing by fitting a Morphable Model to a set of photographs and introduce a new statistical ageing model based on Projection to Latent Structures (PLS) also know as Partial Least Squares.

2 LITERATURE REVIEW

Most previous method for ageing a facial image have concentrated on transforming a 2D image. Cardioidal strain was an early method that relied on the similarity be the mathematical function and facial ageing in children (Pittenger and Shaw, 1975; Pittenger et al., 1975; Mark and Todd, 1983; V. Bruce, 1989). This was later used in a modified form by Ramanathan and Challappa (Ramanathan and Chellappa, 2006). Rowland and Perret used Triangulated Linear warping to create average prototypes of face for two age groups, the difference between the two prototypes was used to define an ageing trajectory (Rowland and Perrett, 1995). Lanits et al. trained a statistical model over a set of face images parametrised by a Principle Components Model (Lanitis et al., 2002). Scandrett et al. also used PCA on a set of 2D images, ageing them using a piecewise linear model, combining the ageing trajectories between age-groups with a historical ageing trajectory from younger images of the individual (Scandrett et al., 2006). Suo et al. explored a different approach by describing the face using a Grammatical Model (Xu et al., 2005) consisting of a hierachical set of face components, (eyes,nose, skin patches etc.). An input face was aged in a probabilistic manor using a dynamic Markov chain to select the most likely set of face components at a target age given the current set (Suo et al., 2007).

The idea of Modelling ageing using 3D models has been around for some time. Mark and Todd applied Cardioidal strain to a 3D model (Mark and Todd, 1983), Hutton and Buxton used Kernel Smoothing to create an ageing model of a set of scanned 3D models (Hutton et al., 2003). More recently Scherbaum el al. (Scherbaum et al., 2007) fitted a Three-dimensional morphable model to database of laser scanned cylindrical depth-maps.

They used these models train a Support Vector Regression model. A new face model could be synthesized from a given set of parameters by 'stepping' through the curved SVR space using a fourth order Runge-Kutta algorithm, using the parameters and an estimated age as the starting point. The curved nature of the SVR model meant that the ageing paths where different depending on the parameters of the input face thus creating a semi-individualised model. However they had only one sample per subject to train the model and so captured population variations and not necessarily the variations due to ageing in a particular individual. Park et al. (Park et al., 2008) fitted a three-dimensional Morphable Model to a set of face images by fitting an Active Appearance Model and extracting a three-dimensional model from the AAM. Ageing was performed by calculating a set of weights between an input face and exemplar faces in the same age group. These weights are then used to build an aged face as a weighted sum of the corresponding faces at the target age.

Many of the statistical methods used lost textural detail such as wrinkles, a few researchers developed methods that attempted to create appropriate textural detail in aged images. Tiddeman et al. used a wavelet transform (Tiddeman et al., 2001) and Markov Models (Tiddeman et al., 2005), Hussein used Bidirectional Reflectance Distribution Functions (Hussein, 2002) and Gandhi used Gaussian filters (Gandhi et al., 2004). These methods work by attempting to replace or adjust the high-frequency components of the image to match the high frequency components of a prototype at the target age.

3 OVERVIEW

Our aim is to be able to take an image of a particular person and to create, and apply, an ageing trajectory specific to that particular individual. Using a set of 3D face models we first separate those factors most related to ageing. Given a training-set of 3D models containing a 'snap-shot' of a number of individuals at various age points, from childhood to early-adult, we then train a set of ageing trajectories for each individual. Finally these trajectories are applied as a weighted sum of trajectories from the train-set.

3D data-sets featuring face models from the same individual at various age points are rare and incomplete, however 2D dimensional datasets are more readily available. We therefore opted to use a face-fitting method, to extract a 3D Morphable Model (Blanz and Vetter, 1999) from a two-dimensional image. We obtained a set of photographs by askinh some student volunteers to supply images from a number of key ages. The resulting image set was divided into strata according to the age of the individual when the image was taken see figure 1. The strata show varying numbers of subjects, this is because the dataset does not contain images for every subject in every age-range. Algorithms and methods that require that each individual have an image in each of the included age ranges exclude those individuals for whom data is incomplete. This results in a reduced dataset of 35 individuals.

In this paper, we first briefly outline for process by which we generate the 3D models. We then describe and compare three ageing mechanisms. One, based on average Prototypes is the 3D analog of the 2D method used by Rowland and Perrett (Rowland and Perrett, 1995), the second, an Individualised Linear model, is the 3D analog of work by Lanitis at al (Lanitis et al., 2002) and is identical to the method of Park et al. (Park et al., 2008). And we then introduce a new technique based on Partial Least Squares (Wold, 1966).

4 THREE DIMENSIONAL MORPHABLE MODELS

Three dimensional Morphable models introduced by Blanz and Vetter use Principle Components Analysis to describe the space of human faces as a set of orthogonal basis vectors. Given a set of 3D dimensional face models with a one-to-one correspondence between vertices, we concatenate the vertex positions and colour values as,

$$\mathbf{s} = (x_1, y_1, z_1, x_2, y_2, z_2, \cdots, x_n, y_n, z_n)^T, \tag{1}$$

$$\mathbf{t} = (r_1, g_1, b_1, r_2, b_2, g_2, \cdots, r_m, g_m, b_m)^T \tag{2}$$

Each face is centred by subtracting the mean of all the faces and PCA performed. A reduced set of 40 eigenvectors for each of shape and colour were used to describe the face space. The shape s and colour t of a new face are generated as linear combination of weighted PCA vectors $\mathbf{s}_j$, $\mathbf{t}_j$ and the averages $\hat{\mathbf{s}}$ and $\hat{\mathbf{t}}$.

$$\mathbf{s} = \hat{\mathbf{s}} + \sum_{j=1}^{k} \alpha_j \mathbf{s}_j, \ \mathbf{t} = \hat{\mathbf{t}} + \sum_{j=1}^{k} \beta_j \mathbf{t}_j \tag{3}$$

With the probability distribution over the PCA face-space defined as,

$$p(\mathbf{s}) \approx e^{-\frac{1}{2}\sum_i \frac{\alpha_i^2}{\sigma_{\mathbf{s},i}}} \tag{4}$$

The weights α_j and β_j form the parameter vectors α and β. New faces are created by varying these parameters. In the rest of this paper we will be referring to

Table 1: Ageing dataset stratification

Name	Age Range	Number of subjects	Mean Age	Standard deviation
Mid Child	5-8	50	6.54	0.85
Late Child	8-12	49	10.7	0.94
Student	17-23	47	20.02	1.69

the concatenated shape and colour parameters α, β as the *Face Model* with parameters $\mathbf{p}$. This process is described in more detain in (Blanz and Vetter, 1999).

Fitting a Morphable Model to a face image

Three dimensional scanning equipment are a relatively resent invention, and so databases of three-dimensional models of the same individual taken over a period of many years have yet to be built. However two-dimensional images, in the form of photographs are widely available. In order to build a set of face models we attempt to extract three-dimensional information from these images. Unfortunately for every possible intensity combination in the image an infinite number of shape, colour and lighting combinations exist that could generate it. We therefore limit the space of possible shape combinations to those most likely to be a human face using the PCA shape and colour model, equation (3) from the previous section, which has the probability distribution shown equation (4).

Our fitting method was a simple adaptation of the Lucas Kanade Tomasi algorithm (Baker and Matthews, 2002) from two-dimensional face models to three-dimensional models, this method is similar to that detailed by Blanz and Vetter (Blanz and Vetter, 1999). We use a Taylor series expansion of the l^2-norm of the pixel difference between an input image and the rendered Morphable Model to find the parameters that minimise this difference. To improve the accuracy of the fitting a set of delineated feature points on the two-dimensional image are also matched to their corresponding points on the Morphable Model using the l^2-norm of their separating distance when projected onto the image plane. The result of the fitting operation is a set of vectorised shape and colour parameters $\mathbf{p}$ that describes the face contained in the two-dimensional input image as a three-dimensional Morphable Model.

5 AGEING METHOD

We applied the face-fitting outlined in the previous section to the photographs in the training set. To produce a set of 3D models of each individual at multiple age-points. We now use this training set to create an ageing model.

Age Prototypes

Prototype face-models were created for each age-stratum by averaging the parameters over all faces in the stratum.

$$\hat{\mathbf{f}}_{\mathbf{s}} = \sum_{i}^{m} \mathbf{p}_i \tag{5}$$

where $\hat{\mathbf{f}}_{\mathbf{s}}$ are the parameters of the averaged face model of all the faces in the stratum s. Here $\mathbf{p}_i$ is the vector of parameters for the i^{th} face model in the stratum. m is the number of faces in the stratum.

A linear transform is created between two strata by creating a vector between the average of each stratum and dividing it by the age difference. Thus, to generate a transform from stratum j to stratum k we take,

$$\mathbf{t} = \frac{\hat{\mathbf{f}}_{\mathbf{k}} - \hat{\mathbf{f}}_{\mathbf{j}}}{\hat{a}_k - \hat{a}_j} \tag{6}$$

where $\hat{a}_j$ and $\hat{a}_k$ are the average ages of the individuals within strata j and k respectively. An input $\mathbf{f}_{input}$ in stratum j is aged towards the age group of stratum k by moving it in the direction of the vector t, multiplying t by the desired number of years.

$$\mathbf{f}' = \mathbf{f}_{in} + (a_t - a_s)\mathbf{t} \tag{7}$$

where $\mathbf{f}'$ is the set of model parameters at the target age, a_s and a_t are ages of the input face and the target age respectively. Clearly this transformis most valid if the target age is within the range of years of the target stratum k.

Individualized Linear Transform

It is well known that faces do not age in an identical manner. In order to generate an ageing trajectory for an unseen individual we exploit the relationship between appearance and ageing trajectory. For each individual in the dataset a linear ageing path is defined as a vector from one sample face in the starting stratum to another in the target stratum containing the end age. If no suitable pair of sample faces can be found the individual is excluded from the dataset. We denote s,e as the start and end ages of the transform respectively, and $\mathbf{p}_i$ and $\mathbf{q}_i$ as the parameters of the face models of the i^{th} individual taken from the start and end strata respectively. We define a single linear ageing function such that the j^{th} parameter of the face model of the individual i at time t is,

$$\mathbf{f}(\mathbf{t})_j = t.\mathbf{a}_{i,j} + \mathbf{b}_{i,j} \tag{8}$$

where $\mathbf{a}$ and $\mathbf{b}$ are sets of weights and $\mathbf{a}_{i,j}$ and $\mathbf{b}_{i,j}$ are the j^{th} weights for the i^{th} individual in the training set. $\mathbf{a}$ defines the gradient of the path in $\Re^n$ and $\mathbf{b}$ the parameters of the face at time $t = 0$. These are defined as,

$$\mathbf{a} = \frac{\mathbf{q} - \mathbf{p}}{e - s} \tag{9}$$

$$\mathbf{b} = \mathbf{p} - s\mathbf{a} \tag{10}$$

These functions can be parametrised using $\mathbf{a}_i$ and $\mathbf{b}_i$ to describe the ageing function f_i for the i^{th} individual. A new ageing path for an unseen individual can be created using a linear weighted sum of the parameters of the ageing functions for each individual in the training set.

$$\mathbf{f}' = \sum_i^n \rho_i \mathbf{f}_i, \qquad \sum_i \rho_i = 1 \tag{11}$$

where ρ_i are a set of weights relating the unseen individual to the ageing path of the i^{th} individual in the dataset. The ρ_i's sum to one, so that that function does not add a scaling factor to the ageing path.

As in (Lanitis et al., 2002) the weighting ρ is defined using the probability distribution of the PCA space of the face model (4). Given two face models, the input face and the face model an individual in the training set, ρ is defined as the probability that the two face models describe the same person. Given that the parameters of the two face models are embedded in a Gaussian PCA space this function is,

$$p(\mathbf{p}_{in}, \mathbf{p}_i) = e^{-\sum_j^n \frac{(p_{in,j} - p_{i,j})^2}{2\sigma_j^2}} \tag{12}$$

where $\mathbf{p}_{in}$ and $\mathbf{p}_i$ are the parameters of the input and i^{th} face model respectively. $p_{in,j}$ is the j^{th} parameter of the input face model. σ_j^2 is the variance of the PCA space in the j^{th} dimension. This function is closely related to the Mahalanobis distance.

This is similar to the method by Park et al. (Park et al., 2008), equation (11) can be combined with equation (3) to derive their method. Our differs in that the weights are based on the PDF (4) of the morphable model rather than linear interpolation.

Partial Least Squares Ageing

The data-set of parameters contains a significant amount of information that is not relevant to ageing. Any statistical analysis needs to separate those factors related to ageing from those that are not related either explicitly or implicitly.

Partial Least Squares (Wold, 1966) also known as a *Projection to Latent Structures* is a statistical distribution similar to Principle Components Analysis that describes mean centred data as weighted linear combination of basis vectors. Unlike PCA which finds directions of maximum variance in the data, PLS attempts to describe a set of dependent variables from a set of predictors. It works by extracting a set of latent vectors that decompose both the dependent and independent matrices in such a manner as to explain as much of their covariance as possible.

If we take the parameters of the face models in the data-set $\mathbf{f}_i$ and use them to build the matrix $X = [\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_n]^T$ such that each row contains the parameters of an individual face model. We define $Y = [age_1, age_2, \ldots, age_n]^T$ where age_i the corresponding ages to the i^{th} face. The rows of both X and Y are then mean centred and scaled by the inverse standard deviation $\frac{1}{\sigma}$

As described by Abdi in (Abdi, 2007), we aim to decompose the independent variables as $X = TP^T$ with $T^T T = I$. T is the *score* matrix and P is the loading matrix. We *estimate* Y as $\hat{Y} = TBC^T$. The diagonal matrix B hold the regression weights, and C is the weight matrix of the dependent variables. See Adbi (Abdi, 2007) for further details on what these mean in practice. The columns of T are the latent vectors that form an exact decomposition of X but only an approximation to Y. The decomposition is found using a iterative algorithm where a latent vector is found that maximizes the covariance between X and Y is found and then subtracted from both. The proportion of variance explained by this vector is found by dividing the sum of squares of the residuals by the the sum of squares of the input matrices X and Y. The algorithm is outlined in 5.1.

PLS, like PCA, can be truncated such that a smaller number of basis vectors are found that approximately span the space of X. We found that the first 6 latent vectors explained 56.3% of the variance and showed little improvement in accuracy thereafter. So we trucated the PLS space to 6 latent vectors.

We separated the parameters into two components; the components most related to ageing and a remainder. As the data used to train the PLS model has been converted to Z-scores by centring the data on the mean and scaling by the standard deviation,

Algorithm 5.1 PLS regression algorithm

Require: Matrix X is the matrix of Z-score model parameters.
Require: Matrix Y is the matrix of Z-score ages.
  SS_x,SS_y are the corresponding total sum of squares for X and Y.
  repeat
    fill $\mathbf{u}$ with random values
    repeat
      $\mathbf{w} = X^T\mathbf{u}$. $\mathbf{w} = \frac{\mathbf{w}}{|\mathbf{w}|}$ Estimate X weights.
      $\mathbf{t} = X\mathbf{w}$. $\mathbf{t} = \frac{\mathbf{t}}{|\mathbf{t}|}$ Estimate X factor scores.
      $\mathbf{c} = F^T\mathbf{c}$. $\mathbf{c} = \frac{\mathbf{c}}{|\mathbf{c}|}$ Estimate Y weights.
      $\mathbf{u} = F\mathbf{c}$. Estimate Y scores.
    until $\Delta\mathbf{u} < \varepsilon$
    $b = \mathbf{t}^T\mathbf{u}$.
    $\mathbf{p} = X^T\mathbf{t}$.
    $X = X - \mathbf{t}\mathbf{p}^T$.
    $Y = Y - b\mathbf{t}\mathbf{c}^T$.
    $\frac{\mathbf{p}\mathbf{t}^T}{SS_x}$ =variance of X explained.
    $\frac{b\mathbf{t}\mathbf{c}^T}{SS_y}$ =variance of Y explained.
    Fill the matrices T,U,W,C,P with the resulting vectors $\mathbf{t},\mathbf{u},\mathbf{w},\mathbf{c},\mathbf{p}$
  until $b < \varepsilon$

we must convert the parameters of the input face $\mathbf{f}$ to Z-scores also. We denote the Z-score converted face as $\bar{\mathbf{f}}$. The parameters of a face model in PCA space are related to the parameters of the face in PLS space as $\dot{\mathbf{f}} \approx \mathbf{g}P$. Since the loading matrix P is not generally orthogonal in PLS regression $\mathbf{g}$ is approximated using least squares regression,

$$\mathbf{g} = (P^T P)^{-1} P\bar{\mathbf{f}} \tag{13}$$

The PCA face model parameters can be recovered from the PLS space as $\bar{\mathbf{f}}' = \mathbf{g}P$ and converted from Z-scores to the original PCA parameter space using $\mathbf{f}' = \bar{\mathbf{f}}'\sigma + \hat{\mathbf{f}}$.

In general the recovered $\bar{\mathbf{f}}' \neq \bar{\mathbf{f}}$, so we compute the residual $\mathbf{r}$ as $\bar{\mathbf{f}} = \mathbf{g}P + \mathbf{r}$. Ageing is performed using the Individualized Linear ageing Transform described earlier on the PLS model parameters ($\mathbf{g}$) instead of the PCA model parameters ($\mathbf{p}$). After the face is aged the residuals $\mathbf{r}$ are added back in. The overall algorithm is outlined in 5.2.

6 Results

The results of ageing a face model using these methods is shown in figure 1.

Algorithm 5.2 PLS ageing algorithm

  Train PLS model.
  for Face model i in the training set **do**
    Convert parameters of model i to Z-scores. $\bar{\mathbf{f}} = \frac{\mathbf{f}-\hat{\mathbf{f}}}{\sigma}$.
    Calculate parameters in PLS space using $\mathbf{g}_i = (P^T P)^{-1} P\bar{\mathbf{f}}_{\mathbf{i}}$, equation (13).
  end for
  Train Individualised Linear ageing model on PLS face models $\mathbf{g}$.

Require: Input face model with parameters $\mathbf{p}$
  Convert parameters $\mathbf{p}$ to Z-scores. $\bar{\mathbf{f}} = \frac{\mathbf{f}-\hat{\mathbf{f}}}{\sigma}$.
  Convert $\mathbf{p}$ from PCA model space to PLS space. $\mathbf{g} = (P^T P)^{-1} P\bar{\mathbf{f}}$, equation (13).
  Calculate residuals, $\mathbf{r} = \bar{\mathbf{f}} - \mathbf{g}P$.
  Age $\mathbf{g}$ using Individual Linear ageing model in PLS space.
  Recover PCA parameters from PLS parameters. $\bar{\mathbf{f}}' = \mathbf{g}'P$.
  Add residuals $\mathbf{r}$ to $\bar{\mathbf{f}}'$.
  Convert from Z-scores to original model-space. $\mathbf{f}' = \bar{\mathbf{f}}'\sigma + \hat{\mathbf{f}}$.

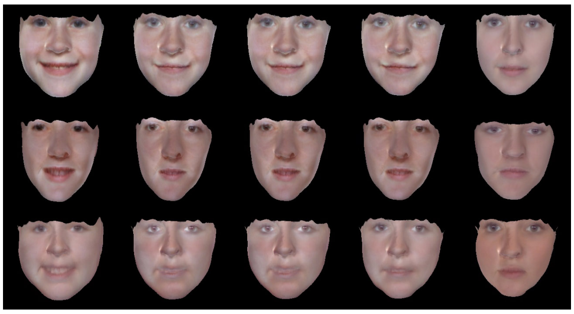

Figure 1: Examples of aged face images. Each row contains a different individual. The columns show, from left to right, the original mid-child face model for each individual, the mid-child face model aged to student age using the Prototyping method, the Individual Linear method and the PLS method. The right-most column shows the original face-model for the individual at student age.

Quantitative evaluation

The Mahalanobis distance between two sets of the face parameters has been used by (Lanitis et al., 2002; Lanitis et al., 2004; Scandrett et al., 2006) to measure the similarity between two faces in terms of the probability that they are the same face.

$$d(\mathbf{a},\mathbf{b}) = \sqrt{\sum_i \frac{(\mathbf{a}_i - \mathbf{b}_i)^2}{\sigma_i^2}} \tag{14}$$

where $\mathbf{a}$ and $\mathbf{b}$ are two face models each composed of a vector of shape and colour parameters. σ_i is the standard deviation of the i^{th} component of the PCA model.

In our case we can use this measure to determine the dissimilarity between an aged face and a known ground truth at the target ages. In order to determine the comparative effectiveness between different methods of ageing we used a leave-one-out method. Figure 2 shows the results of ageing using the prototyping, individualized linear and PLS methods. We can clearly see that the 'Individual Linear' method gives an improvement in accuracy over the 'Prototyping' method with a lower average error and the PLS method shows a marked improvement over both.

Table 2: Standard deviation weighted RMSE between shape and colour parameters of aged face model and a known ground-truth model for each individual in the data-set. With 93 subjects for each method.

Ageing Method	RMSE	Standard Deviation
Prototyping	8.86	1.84
Individual Linear	8.69	1.92
PLS	7.4	1.4

Perceptual evaluation

The Mahalanobis distance between two face models provides a quantitative description of the error between the aged model and the known ground truth at the target age. However this measure may miss ageing cues that human raters would be able to detect. We performed a series of tests with human raters to evaluate the ability of the methods to produce images of the required age.

Each user was shown a single image of a rendered face model at a time and asked to estimate the age of the face shown. The age is selected from a range between 5 and 30 to the nearest year. The stimuli are a selection mid-child faces aged to student age by the three-methods, prototyping, individualized linear and PLS, together with the rendered face-models of the individuals at the source and target age. Thus we have five groups for each individual in the face model data-set. The images are presented with the faces in a uniform pose, with uniform lighting conditions on a black background. Only the face is shown with no peripheral details such as hair on display, as such ageing cues are limited to those in the face area. The images where presented on public website with users asked to estimate the age in years of the face shown. The website generated a significant amount of traffic, with an average of 105 age estimations per image, and just under 5000 for each ageing method being trialled.

Table 3 shows the mean perceived age in years for the face models aged by the different methods as well as the mean ages of the rendered models of the original face models. Table 4 shows the mean age difference in years between the perceived age of the individual after the ageing method is applied and the target age the algorithm was attempting to recreate. We can see that all the methods succeed in ageing the faces towards the target age, but vary in how much they age the face model. The worst method is the Individual Linear method ageing the faces to a mean of 16.801 and a mean difference of -3.8874, there is a statistically significant difference between the Individual Linear and the Prototyping methods (p=0.026152, t=1.9408, dp=10180) using a one-tailed independent t-test and between the Individual Linear and the PLS methods (p=0.017934, t=-2.3674, dp=9195). The PLS method also shows a statistically significant improvement over the Prototyping method (p=0.22495, t=-0.75562, dp=9145).

Table 3: Mean (μ) and standard deviation (σ) of the human rated ages for faces ages by each method

Ageing Method	μ	σ	Count
Prototyping	17.048	6.7605	5090
Individual Linear	16.801	6.8449	5092
PLS	17.115	6.6780	4987
Student	17.026	5.9044	6205
Mid Child	12.762	6.0626	4678

Table 4: Mean (μ) and standard deviation (σ) of the error in years in human rated ages for faces ages by each method

Ageing Method	μ	σ	Count
Prototyping	-3.6614	6.1888	4596
Individual Linear	-3.8674	6.1688	4646
PLS	-3.5643	6.1098	4551
Student	-3.3737	5.4273	5855
Mid Child	6.2135	6.0815	4678

7 CONCLUSIONS

We have described a method of ageing 3D morphable models by a method based on *Projection to Latent Structures* or *Partial Least Squares*. This method shows an improvement over the others tested both in quantitative measures, in terms of similarity to a known ground-truth, and in perceptual evaluation by human raters. Due to its reliance on face-fitting methods the success of this method depends on the quality of the face model produced in the fitting stage.

Improved fitting techniques or a database of three-dimensional scans of the same person over several year, would improve the accuracy of these ageing methods. Other authors have used Quadratic and Cubic functions (Lanitis et al., 2002) in two-dimensions or non-linear Kernel methods such as Support Vector Regression (Scherbaum et al., 2007) in three-dimensions, so and obvious extension is examine non-linear individualised ageing paths.

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