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Signaling through timing of stock splits[☆]

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ABSTRACT

We develop a dynamic structural model of stock splits, in which managers signal their private information through the timing of the split decisions. Our approach is consistent with the empirical evidence which shows that the majority of stock splits have a 2:1 ratio of old-to-new shares, but are announced at various pre-split price levels. Moreover, it explains why split announcement returns are decreasing with the pre-split price. In addition, by matching the model to the data, we estimate the nominal share price preferences of investors and decompose the split announcement return into the value of new information and the signaling cost.

1. Introduction

Stock splits are corporate events that increase or consolidate the number of shares outstanding in a company without any direct effects on capital or cash flows. Existing shares are divided into multiple shares distributed in proportion to the existing shareholders. A striking puzzle associated with stock splits is the existence of the announcement premium — split announcements increase a company's value by 2% to 4% (Grinblatt et al., 1984; Ikenberry et al., 1996). Moreover, companies regularly undertake stock splits despite them being costly.[1] Despite the decrease in frequency after 2008 (Minnick and Raman, 2014; Heater et al., 2023), splits are still a puzzling phenomenon, as the splits by Apple and Tesla in August 2020 illustrate (with 10% and 17.94% announcement returns correspondingly).[2]

The literature is still debating why it could be advantageous to maintain the nominal share price within a certain level: it could improve the liquidity of the stock (Muscarella and Vetsuypens, 1996), increase its appeal to various groups of investors (Lamoureux and Poon, 1987), answer to unsaid social norms (So and Tse, 2000), or address the preferences of irrational investors (Chen et al., 2020). But, irrespective of the actual cause, the premium itself exists only because the split decision provides some new information to investors. If investors could predict the split decision, there would not be any announcement premium, even in the presence of large nominal price preferences. This signaling role of stock splits was first theorized by Brennan and Copeland (1988). In their static model, low-price stocks are costly for shareholders because of the exogenous brokerage fee structure, and managers can signal good private information by choosing a higher split factor in the split decision.

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[1] According to Weld et al. (2009) direct administrative costs are \$250,000-\$800,000 for large firms.

[2] See, Wursthorn (2020) or Cornell (2020) for more details.

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In our paper, we argue that the key piece of information to which investors react is not the chosen split ratio, but rather the timing of the split decision. Towards that goal, we first highlight that relatively few split ratios are used (nearly half of the splits in the US markets happens at a 2:1 ratio of new-to-old shares), while the splits with the same factor can be announced at vastly different price points (from $10 to $100 for 2:1 splits). In addition, we document that the announcement premium for the same split factor is strongly decreasing with the pre-split price.

Then, we develop a realistic dynamic model of signaling in stock splits, in which managers reveal their private information through the timing of the split decision (while using only one split ratio). We consider a company, whose fundamental value is growing (stochastically or deterministically) over time. Investors have nominal price preferences, and, therefore, they require an additional return premium when prices deviate from the optimal region. The company's managers have private information about its prospects (which is a continuous random variable) and, based on this information, they optimally decide at which point in time to undertake the split. The better is their positive information, the less likely is that the future price will become inefficiently low, and, thus, the earlier they will announce the split. We then derive a system of differential equations describing simultaneously the managers' optimal split decision and how the pre-split price level depends on the company's fundamentals. For given values of the model's parameters and given nominal share price preferences, these equations can be solved numerically. Consistently with the data, the model predicts a wide distribution of pre-split prices and a positive announcement premium, which is decreasing with the pre-split price.

Finally, since our model more accurately represents stock split decisions, we can match it to the data and recover the actual nominal price preferences of investors (which prices they prefer and by how much). We use a maximum likelihood estimator on a sample of US 2:1 splits from 1980 to 2013. Our results show that there are significant nominal price preferences and the estimated required return depends on the price level (Fig. 1). The additional premium is relatively small until the price falls to around $50 and then rises sharply. If the stock price is around $20, investors require additional 20 basis points in comparison to more preferable price levels. Moreover, using our estimation, we can decompose the announcement return into the value of the new private information (i.e. the return in case the private information is revealed without the split) and the signaling costs (i.e. the cost of choosing suboptimally low nominal prices). The announcement return, matched to the empirical data, and the value of the revealed private information, calculated from the model, depend on the price level. The difference between the two represents the signaling costs and reaches about 0.5% for a pre-split price of above $50 (Fig. 2).

Our model can be adapted to accommodate different motivations for stock splits. Though, in this paper, we abstract from explaining one rationale for investors' preference for nominal prices, if different theories predict different functional forms of such preferences, our approach allows testing which of them is more consistent with the empirical distribution of pre-split prices and split announcement returns.

The paper proceeds as follows. Section 2 briefly introduces the relevant literature. Section 3 discusses the empirical properties of the stock splits data motivating our model. Section 4 presents a simplified static signaling game that explains how the mechanism of signaling through timing of stock splits works. Section 5 describes the full dynamic model. Section 6 presents the solution of the non-stochastic version of the full model, while Section 7 reports the estimation of the model, and Section 8 concludes.

2. Related literature

In a frictionless market, the nominal share price does not affect the market value of the stocks and there is no optimal price range. Many motivations were proposed to understand why such a range seems to exist in the empirical evidence, so that managers are willing to undertake costly splits, and the market reacts positively to such events.

According to earlier literature, splits could improve the stocks' marketability. For example, a lower share price appeals to individual investors, and can therefore increase the overall demand for the stock. This view is also supported by managers themselves. Dolley (1933) and Baker and Gallagher (1980) report firms' beliefs on managing the nominal price. The most cited purpose for splits is to increase diversity and the number of shareholders.

Many empirical studies test this hypothesis. Some evidence reports a correlation between nominal price and institutional ownership, suggesting an institutional preference for high-price stocks (Gompers and Metrick, 2001; Dyl and Elliott, 2006). Lamoureux and Poon (1987) show an increase in the number of shareholders after a split. However, the evidence is mixed. Some studies show that no significant changes in ownership basis composition happen at the time of a stock split (Mukherji et al., 1997). Chittenden et al. (2010) argue that the constant average nominal price and positive inflation rates are not consistent with the marketability hypothesis.

Another strand of literature motivates the managers' choice to split with improved post-split liquidity. Copeland (1979) develops a theoretical model showing that an optimal price range exists as a result of a trade-off between lower transaction costs and a wider ownership basis post-split. Angel (1997) considers how splits will induce brokers/dealers to provide liquidity through higher market-making profitability due to increased tick size-to-share price ratio. Dennis and Strickland (2003) examines liquidity changes following the 2:1 split of the Nasdaq-100 Index Tracking Stock, concluding that any post-split effects are driven solely by liquidity considerations. However, there is mixed evidence of effects on liquidity; Easley et al. (2001) show that, for example, relative spreads increase after the splits.

More recent contributions introduce behavioral motivations to splits. Firms have a preferable price range that they respect because of social norms and customs. Weld et al. (2009) and So and Tse (2000) test whether firms conform their target prices according to deviations from the median prices of their size-comparable companies or industry peers. This social conformity would explain why the average nominal price has remained broadly constant over the last 90 years. In another study, Birru and Wang

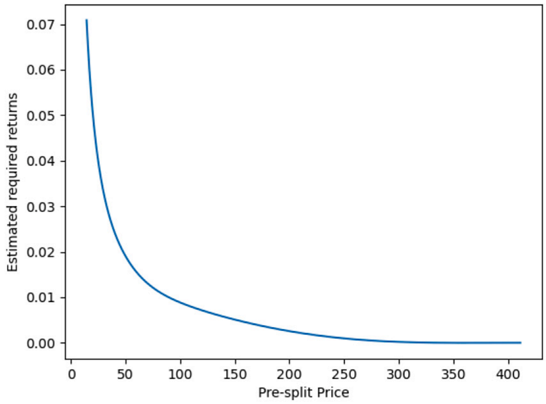

Fig. 1. Nominal share price preferences.
The graph shows how the investors' required return depends on the nominal price level (pre-split Price), as predicted by our model of signaling through the timing of split decisions. We estimate the parameters of the model by matching the empirical distribution of pre-split nominal prices and the relationships between nominal price and announcement abnormal return with the ones predicted by the theory. The model assumes that the required return is a 4th degree polynomial function of the $log(Price)$. The results are presented for the non-stochastic model ($\sigma = 0$) described in the paper.

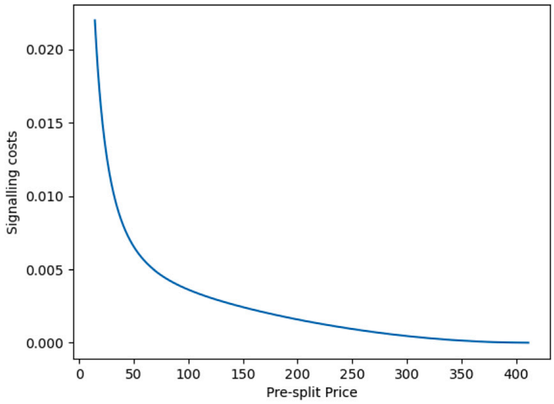

Fig. 2. Signaling costs.
The graph shows how the estimated signaling costs depend on the price level for the non-stochastic model ($\sigma = 0$) described in the paper. This signaling cost is the difference between the value of the managers' private information (the return we would observe if the information was revealed for exogenous reasons) and the actual split announcement premium.

(2016) suggest that investors might overpay to hold low-priced stocks because they systematically overestimate their skewness. Very recently, Chen et al. (2020) show that, when gambling sentiment in the market is high, managers will undertake more splits in order to exploit the spillover on investor demand for lottery-like shares with low nominal prices.

A large literature also discusses the signaling role of splits. The positive market reaction to split announcements could be seen as a disclosure of private positive information to the market (Grinblatt et al., 1984; Chemmanur et al., 2015; Nayak and Prabhala, 2001), such as abnormal increases in future company earnings (Fama et al., 1969; McNichols and Dravid, 1990).[3]

The main theoretical contribution most closely related to our paper is Brennan and Copeland (1988). The authors develop a 2-period signaling model, in which the managers of a company can signal good private information through the split factor. They assume that low-price stocks are costly for shareholders because of the exogenous brokerage fee structure. They found that the managers with good private information split the shares more than it would be efficient ex-post, which allows them to credibly reveal their private information and increase the current stock price.[4]

Our model differentiates itself from Brennan and Copeland (1988) as it is dynamic and it allows the managers to choose the optimal timing of the split. Based on the empirical evidence in Section 3, we argue that this setup matches more closely the actual

[3] However, there is no agreement on the cause–effect relationship between splits and earnings (Huang et al., 2006).

[4] Brennan and Hughes (1991) develop another signaling model in which firms manage their nominal price in order to influence and attract the attention of brokers. Managers with positive signals would find it more convenient to have third independent parties produce positive information about their companies, rather than sharing it directly with the investors. An empirical investigation of their theory is proposed by Chemmanur et al. (2015), looking at institutional investors' role in the signaling of such private information.

split decisions and thus is more suitable for structural estimation. In fact, we observe in the US sample of splits that firms only use a limited number of split factors. Moreover, the announcement premium, proxied by the Cumulative Abnormal Return over a 3-day window (2.36%, on average) decreases as the pre-split price increases. In addition, we do not restrict ourselves to a specific friction driving the nominal price preferences, but focus on estimating such preferences from the data.

Finally, our paper is related to the literature discussing signaling through timing of corporate decisions, in particular to Grenadier and Malenko (2011) (other papers include Morellec and Schürhoff (2011) and Bustamante (2012)). In Grenadier and Malenko (2011) the authors consider a real option problem, in which an agent, undertaking an investment with stochastically changing present value, has superior information about the investment cost. It is optimal to undertake investments with lower costs earlier, and, thus, by choosing the timing of the investment decision the agent can signal their private information. Depending on whether the agent benefits from low or from high beliefs about the true investment cost, the equilibrium investment can happen either earlier or later than in the symmetric information case. In our model, it is optimal to split shares earlier if the firm has better future prospects, and the managers undertaking the split benefit from investors' optimistic beliefs about their private information on the firm's prospects. Thus, similar to Grenadier and Malenko (2011), the splits happen earlier than in the symmetric information case.

In addition, Grenadier and Malenko (2011) also consider a model with a continuum of types and derive the differential equation describing the equilibrium timing strategy. However, with nominal share price preferences, we cannot use exactly the same techniques as they used. When interest rates change with price levels, it is not possible to derive a closed-form solution for the managers' expected payoff at any given point. In addition, in our model, the investors' beliefs about the managers' private signal affect prices even before the split decision and, thus, affect the managers' equilibrium strategy. We were able to derive a system of differential equations describing simultaneously the managers' strategy and how the pre-split price level depends on the company's fundamentals by carefully considering what happens if managers deviate slightly from the equilibrium strategy, while taking into account the above considerations.

3. Empirical properties of stock splits

3.1. Sample

For our empirical analysis, we use a sample of 7643 US non-reverse splits that occurred from 1980 through 2013 in NYSE, NASDAQ or AMEX, announced on the dates reported by CRSP.[5]

We only look at non-reverse splits, the distributional events in which the number of shares outstanding is increased. Only 17.34% of the full sample of splits are consolidating events, and the economics behind reverse and non-reverse splits are quite different.

Moreover, we keep only events that are not announced simultaneously with any other distribution. It is, in fact, common for firms to announce stock splits at the same time as dividend payments, especially firms that manage their nominal price periodically and frequently. In the total sample, about 50% of splits are announced in conjunction with other distributions.

Fig. 3 shows how the frequency of splits changed over time. There are fewer splits in the latest data due to the financial crisis, nevertheless, they are still a significant phenomenon.

3.2. Split factors

To motivate our model, we first look at the distribution of split factors. The split factor is the ratio between the number of new shares issued and the number of existing old shares. Consistently with the literature, we observe that a few round factors tend to be prevailing. As we can see in Table 1, 4 factors represent more than 90% of the sample of non-reverse splits (and still 75% of the total sample including also consolidating splits). These factors are 2:1, 3:2, 5:4, and 3:1.

Past literature on the signaling role of splits focuses on signaling through the split factor. As we see from this evidence, the scope of using the split factor as a signal is quite limited.

3.3. Pre-split price

The second statistic that we look at is the distribution of pre-split prices. We estimate the density function of this distribution from the data. We define the pre-split price as the closing price observed two trading days before the announcement of the event, and the post-event price as the pre-split price plus the abnormal return.

As we can see from Fig. 4, splits with higher factors happen at higher prices, intending to realign the nominal price to a similar average post-split level. More interestingly, the distributions are quite wide and we see that splits with the same factors can happen at completely different price levels.

[5] We correct the announcement dates reported for market closures and weekends, and we keep only those events whose announcement date is plausibly correct.

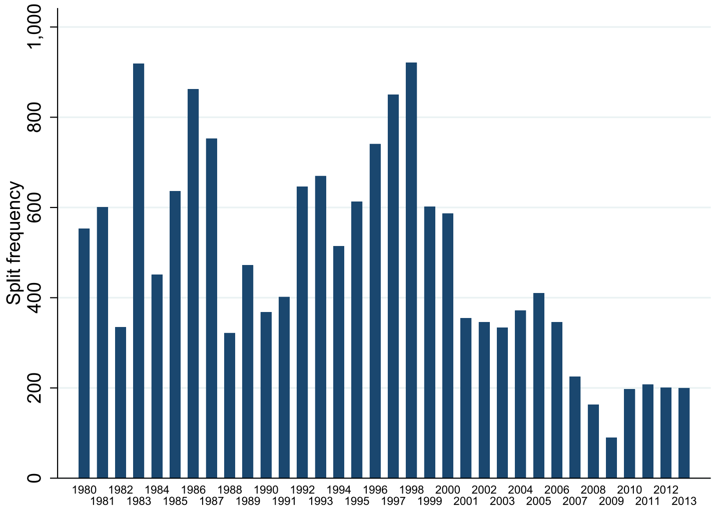

Fig. 3. Split frequencies over time. The graph shows the frequency of all stock splits announced in NYSE, AMEX, and NASDAQ, 1980 through 2013.

Table 1
Split factors.
The table reports the frequencies of split factors for (a) normal and (b) reverse splits in the full sample of splits 1980 to 2013. Normal splits increase the number of shares outstanding, while reverse splits consolidate the capital in a smaller number of shares.

(a) Normal splits			(b) Reverse splits		
Factor	Number	Frequency	Factor	Number	Frequency
2:1	6058	45.1%	1:10	555	19.7%
3:2	4456	33.1%	1:5	523	18.5%
5:4	1118	8.3%	1:4	400	14.2%
3:1	582	4.3%	1:3	272	9.6%
4:3	415	3.1%	1:2	204	7.2%
6:5	210	1.6%	1:20	133	4.7%
4:1	134	1.0%	1:6	130	4.6%

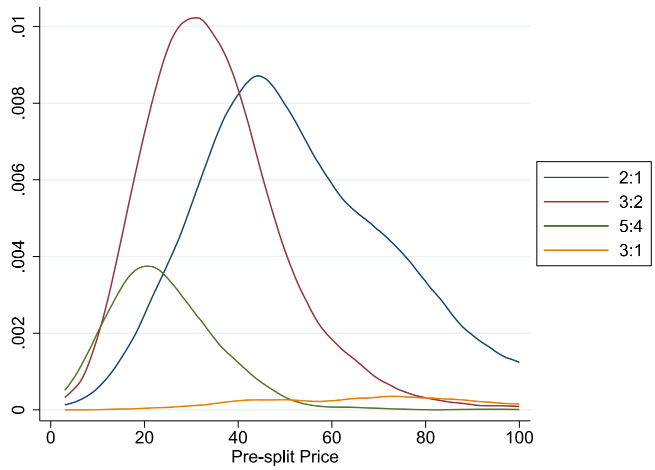

Fig. 4. Split frequencies.
The graph shows the relative frequency of a split with respect to the pre-event price, for different split factors. We restrict to the four most common non-reverse split factors, which together represent 90% of the nonreverse sample. The pre-split price is the closing nominal price 2 days before the announcement of the event. We approximate the density by use of a kernel density estimate (with Epanechnikov kernel).

Table 2
Cumulative abnormal returns at splits announcements.
The table reports the estimated 3-day cumulative abnormal returns around the announcement of the splits. We estimate the abnormal returns at announcements 1980 through 2013 in relation to a market model. Then, we aggregate the abnormal performance for the 3 days around the announcement date ($\tau = -1$ to 1). We report the CARs aggregated over the whole sample, and separately for the five most common split factors, 2:1, 3:2, 5:4, 3:1, and 4:3. We report standard errors and t-statistics (*** 1% confidence level).

Split factor	Average CAR	St.error	t-stat
All splits	0.0236107***	0.000811	29.10
3:1	0.0326***	0.0045	7.13
2:1	0.0256***	0.0013	20.36
3:2	0.0224***	0.0012	18.16
4:3	0.0213***	0.0034	6.15
5:4	0.0202***	0.0023	8.61

3.4. Announcement premium

Finally, we look at the split announcement premiums. We estimate the effect of split announcements on returns, performing a short-term event study around the split announcements. We call the abnormal announcement return of the split, or announcement premium, the Cumulative Abnormal Return predicted in a 3-day window of $\tau = -1$ to 1 days around the announcement of the split. Given event i, announced at date $\tau = 0$:

$$CAR(3)_i = \sum_{\tau=-1}^{1} (r_{i\tau} - r^*_{i\tau}) \tag{3.1}$$

where: $r_{i\tau}$ is the excess simple return observed for the event i at time τ, and $r^*_{i\tau}$ is the normal excess return estimated with the use of a market model in a 110-day window between $t = -120$ *and* -10 trading days before the announcement date:

$$r^*_{it} = \alpha + \beta r_{mt} + \varepsilon_{it} \tag{3.2}$$

and r_{mt} is the CRSP excess market return at time t.[6]

We estimate a highly significant average cumulative abnormal performance of 2.36% on a 3-day window around the event announcements. The results are consistent with the older literature, such as Grinblatt et al. (1984). Table 2 reports the estimated abnormal returns averaged for the overall sample, and for the five more common split factors. The announcement return is clearly increasing as the split factor increases, so higher distributions imply a higher positive response from the market.

We also look at how the announcement return depends on the pre-split prices for different split factors (we consider 2:1, 3:2, 5:4 and 3:1 split factors). Fig. 5 reports a kernel local polynomial regression of the split announcement return with respect to the pre-split price. We can see that the market reaction decreases as the pre-split price increases.

4. A static model

4.1. Setup

In this section we propose a simple static model to illustrate the main intuition. The model is of three periods as follows. The manager of a firm cares about the market value of the firm and holds some private information about the firm in the form of a private signal s distributed according to a cumulative distribution function F over $(0, +\infty)$ with continuous density f.

In the first period, the manager makes a stock split decision $a \in \{0, 1\}$ and the initial number of shares of the firm n is multiplied by a factor λ^a.

In the second period, the private signal is revealed to the public with probability $\pi \in (0, 1)$ for some exogenous reasons, irrespective of the manager's split decision in the first period.

In the final period, a dividend $d \equiv \frac{\theta \cdot s}{n \cdot \lambda^a}$ per share is paid to the investors, where θ denotes a publicly observable attribution of the firm that affects the final dividend.[7] In the meantime, the managers receive their payoff that is assumed proportional to $n\lambda^a p$, where p denotes the share price of the firm.

[6] We also use a Fama–French 3-factor model (Fama and French, 1993) as a robustness check. However, the literature agrees on the fact that, in a short-window event study, a market model would be well-specified and powerful (Brown and Warner, 1985).

[7] In the absence of private information, i.e., $s = 1$, θ simple denotes the final dividend paid by the firm.

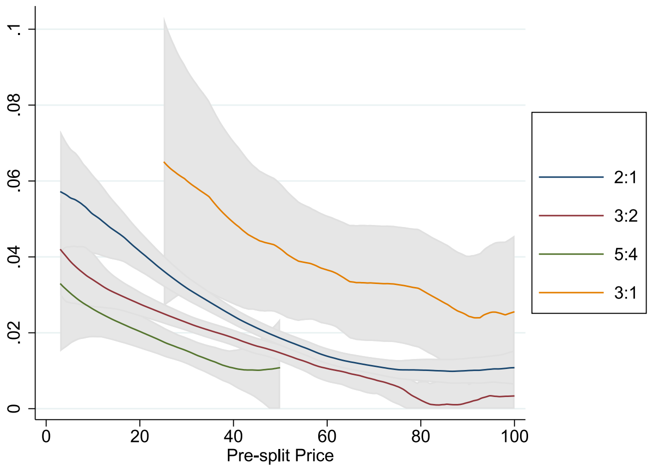

Fig. 5. Announcement abnormal returns.
The graph shows the 3-day Cumulative Abnormal Returns at the announcements of the splits, with respect to the pre-split price. We estimate the abnormal returns at the announcement of the events, 1980 to 2013, in relation to a market model. Then, we aggregate the abnormal performance for the 3 days around the announcement date. We report a kernel-weighted local polynomial (with Epanechnikov kernel) to approximate the distribution of the announcement returns with respect to the pre-split price. The pre-split price is the closing nominal price 2 days before the announcement of the event. We show these announcement returns for the four most common split factors, 2:1, 3:2, 5:4 and 3:1, with 95% confidence intervals.

4.2. Stock split strategy

In the presence of nominal share price preferences by the investors, the share price p is (implicitly) determined by $p = \frac{d}{1+r(p)}$, where $r(p)$ is the nominal interest rate in the form of a loss function, capturing investors' nominal preferences. As a result, we can denote the share price by $p = p(\frac{ys}{\lambda^a})$ where $y \equiv \frac{\theta}{n}$ and the manager's payoff by $u(a; s, y) \propto n\lambda^a p$.

We characterize a standard perfect Bayesian equilibrium of this game where the manager's stock split decision is based on a cutoff strategy, i.e., $a^*(s, y) = 1$ iff $s \geq s^*(y)$. In particular, the cutoff point $s^*(y)$ is determined by the indifference condition for the manager with private signal $s = s^*(y)$, i.e., $u(1; s^*, y) = u(0; s^*, y)$. More precisely,

$$\pi n\lambda p(\frac{ys^*}{\lambda}) + (1-\pi)n\lambda p(\frac{y}{\lambda}\mathbb{E}\left[s|s \geq s^*\right]) = \pi np(ys^*) + (1-\pi)np(y\mathbb{E}\left[s|s < s^*\right]) \tag{4.1}$$

$$\frac{\pi}{1-\pi}\left[p(ys^*) - \lambda p(\frac{ys^*}{\lambda})\right] = \lambda p(\frac{y}{\lambda}\mathbb{E}\left[s|s \geq s^*\right]) - p(y\mathbb{E}\left[s|s < s^*\right]).$$

We can also derive the elasticity of the cutoff s^* with respect to the (public) state y from the equilibrium condition:

$$\frac{\partial \ln s^*}{\partial \ln y} = \frac{\frac{\pi}{1-\pi}\left[p'(ys^*) - p'(\frac{ys^*}{\lambda})\right] - p'(\frac{y}{\lambda}\mathbb{E}\left[s|s \geq s^*\right])\mathbb{E}\left[\frac{s}{s^*}|s \geq s^*\right] + p'(y\mathbb{E}\left[s|s < s^*\right])\mathbb{E}\left[\frac{s}{s^*}|s < s^*\right]}{p'(\frac{y}{\lambda}\mathbb{E}\left[s|s \geq s^*\right])\frac{f(s^*)}{1-F(s^*)}(\mathbb{E}\left[s|s \geq s^*\right] - s^*) - p'(y\mathbb{E}\left[s|s < s^*\right])\frac{f(s^*)}{F(s^*)}(s^* - \mathbb{E}\left[s|s < s^*\right]) - \frac{\pi}{1-\pi}\left[p'(ys^*) - p'(\frac{ys^*}{\lambda})\right]}. \tag{4.2}$$

The sign of the elasticity is unfortunately not analytically straightforward to determine due to the complex expressions involved. Nevertheless, we can plot the relationship between s^* and y, given certain specifications of the signal distribution F and the nominal price preference $r(p)$ that are used in the main model in the next Sectins. In the specific, we assume a lognormal distribution of the private signal s, and a quadratic function for the nominal price preferences, and we report a negative relationship between the private signal at the optimal splitting time and the state variable y, which precisely captures the main mechanism. Managers with higher private signals are splitting earlier, when the state variable y is higher (Fig. 6).

5. Full dynamic model

We can now describe the full model, which defines investors' preferences for nominal share prices and derives the signaling equilibrium more specifically in the event of stock splits.

5.1. Modeling the nominal share price preferences

If investors have preferences about the nominal price of a security, they would be willing to pay a premium for the price to be close to the preferred level. Therefore, around the optimal price level, the effective return should be lower than in other price regions. Hence, we can describe the nominal price preferences by describing how the effective return depends on the price.

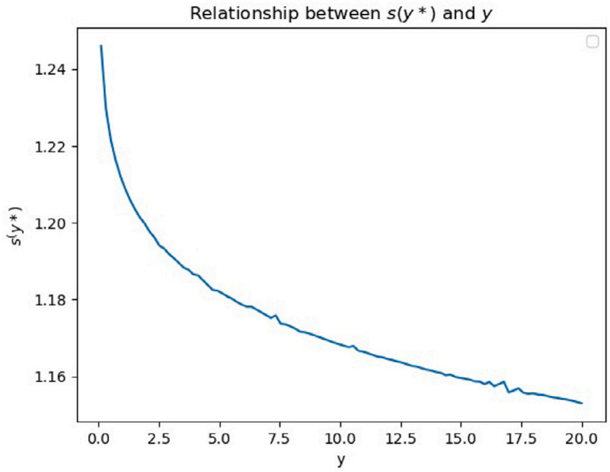

Fig. 6. Relation between s(y) and y in the static model.
The graph shows numerically the negative relationship between the signal at the optimal split point and the state variable y, as in the static model in Section 4. We use the following parameters: $\mu = 0.02,\ \sigma = 0,\ r_0 = 0.05,\ \alpha = 0.0002,\ log(p_0) = 6,\ \gamma = 0.1,\ \pi = 0.66,\ \sigma_s = 0.05$.

For example, consider an asset that pays a final dividend D in one year. The current price p is determined by the effective return $r(p)$:

$$p = \frac{D}{1 + r(p)} \tag{5.1}$$

and we allow the return to depend on the price itself. The price is endogenously determined, therefore the price p will be such that the above equation always holds. If the final dividend changes, then the price would start adjusting to reach a new equality. If investors have nominal price preferences, the asset value can be increased by splitting the number of shares in a way that the resulting share price minimizes $r(p)$.

In a dynamic setting, instead of annual return, we use the instantaneous return, which can also depend on the price level. If p_t is the current price of a security (at time t), we can write the following equations for the evolution of prices:

$$p_t = \frac{E_t\left[p_{t+dt}\right]}{1 + r(p_t)dt} \tag{5.2}$$

or equivalently:

$$\frac{E_t\left[dp_t\right]}{p_t} = r(p_t) \cdot dt \tag{5.3}$$

To determine the current price in a dynamic setting, we need to solve the above dynamic differential equation for all t. Thus, the resulting price depends not only on the current required return determined by the current price level, but also on all future required returns determined by the future price levels.

The above description of the nominal price preferences is consistent with several economic explanations of nominal price preferences proposed by the literature. For example, if the preferences are driven by market illiquidity, this illiquidity creates an additional cost that investors need to be compensated for, which leads to an illiquidity premium in the return. Since the market illiquidity can depend on the price level, the resulting return premium can also depend on the price level. If we believe in the marketability hypothesis, there are some groups of investors restricted to trading within a certain price range. Thus, by choosing the price that appeals to a wider audience, we can achieve better risk sharing and lower overall required return on the asset. Even if we assume conformism to social norms, for such norms to be meaningful, investors should be willing to punish those who deviate. The additional return premium can measure the willingness of investors to enforce the norm. Different economic explanations can potentially lead to different predictions about the $r(p)$. Thus, by estimating the relationship between return and prices, we could test them against each other.

5.2. Company and share price (with symmetric information)

We first introduce the model with symmetric information between managers and investors. Consider a certain company. Every moment with probability $\delta \cdot dt$ the company is liquidated and pays the final dividend equal to θ_t. The final dividend θ_t is publicly observable and is growing stochastically over time according to:

$$\frac{d\theta_t}{\theta_t} = \mu dt + \sigma d B_t \tag{5.4}$$

where B_t is a Brownian motion.

We assume δ to be small,[8] that is, investors expect to receive the dividend sometime far in the future. This assumption is reasonable for our purposes, since investors would expect the company to exist well beyond the next split announcement. In addition, this assumption simplifies the differential equations in the model.

The company is financed by equity and its shares are traded in the financial markets. Suppose that at time t there are n_t shares outstanding, therefore the holder of each share will receive $\frac{\theta_t}{n_t}$ upon liquidation. Denote the price of each share at the time t as p_t.

Investors have nominal price preferences, that is the instantaneous required return $r(p_t)$ depends on the current share price p_t. Because of such preferences, the overall value of the company can be larger if its managers can adjust the number of shares n_t to keep the prices close to the preferred levels.

The main feature of our model is that the timing of the split is endogenously determined. For simplicity, we allow the company to do splits only of a given ratio of new-to-old shares, as $\lambda = 2{:}1$ (i.e. the number of shares outstanding doubles).[9] In addition, we ignore the fixed costs of splits. Since the company is restricted to 2:1 splits, it is not going to undertake them too frequently.[10]

State variable

The current share price depends not on the current dividend θ_t per se, but on the current dividend per share θ_t/n_t. Thus, we can use this ratio as a state variable. Adding the constant δ to simplify further notation, we define:

$$y_t = \delta \frac{\theta_t}{n_t} \tag{5.5}$$

We call y_t the current fundamental value of the company.

When the company is not splitting its shares, the state variable is growing at the same rate as θ_t:

$$\frac{d y_t}{y_t} = \mu dt + \sigma d B_t \tag{5.6}$$

At the point of the split, the ratio is adjusted according to the split factor λ:

$$y_{after} = \frac{\theta_{after}}{n_{after}} = \frac{\theta_{before}}{\lambda \cdot n_{before}} = \frac{y_{before}}{\lambda} \tag{5.7}$$

The share price of the company, at any moment of time, is determined by the current value of state variable y_t :

$$p_t = p(y_t)$$

In addition, the company would always undertake splits when the state variable y_t reaches some optimal y^*.

5.3. Company and share price (with asymmetric information)

In the symmetric information case, the splits will always happen around the same price level $p(y_*)$. In addition, since the time of the split announcement is completely predictable by the investors, there is no split announcement premium. As we have seen in the empirical evidence (Section 3), both these facts are not consistent with the data. Therefore, we can extend the model slightly by adding some asymmetric information between investors and managers, which would make its predictions more empirically relevant.

In particular, we assume that managers possess certain private information s about the firm's prospects, and that the final dividend the company will pay, θ_t, is adjusted by s:

$$\theta_t \cdot s \tag{5.8}$$

Everyone observes θ_t, but s is private information of the managers. Investors have prior beliefs about the signal, believing s to be a random variable with cdf $F(s)$ and pdf $f(s)$. We assume that the distribution of s is continuous within its support. For the numerical solution, we use the log-normal distribution:

$$\log s \sim \mathcal{N}\left(0, \sigma_s^2\right) \tag{5.9}$$

[8] To be specific, we assume $\delta \ll r(p)$ ($r(p)$ is the interest rate) and $\delta \ll \gamma$ (γ is the discount factor for the managers defined in Section 5.4). While δ (probability of receiving the final dividend in the future) is small, θ_t/n_t (final dividend) is instead larger, and, as a result, $\delta\theta_t/n_t$ would have some non-negligible value. Moreover, given that $pt \approx \delta\theta_t/n_t(1/r + \delta - \mu)$ (if the interest rate is constant and equal to r), $\delta p_t \approx \delta^2\theta_t/n_t(1/r + \delta - \mu)$ is not of the same magnitude as $\delta\theta_t/nt$ irrespective of θ_t. Thus, the only specific assumption that we need is $\delta \ll r(p)$, so the term δp to be negligible relative to $r(p)p$ and, thus, can be ignored in the corresponding differential equation.

[9] As we see in the empirical data, this is not a very restrictive assumption, as 45% of normal splits are 2:1. In general, the model could be easily extended to the case in which multiple split ratios are allowed.

[10] This assumption can also be relaxed. For example, we can assume that when the split is undertaken a certain fraction of the company's value is lost. The resulting model will be very similar to the current one.

Investors can infer s from the managers' actions (for example, by observing the split decision), but we assume that they can also learn it over time for exogenous reasons. At any moment in time, with probability $\pi \cdot dt$, the signal is credibly revealed to outside investors.[11] The managers can signal their private information through splits only before it is exogenously revealed. This assumption is crucial for managers to credibly signal their private information. If the information is revealed only with the final dividend, managers' payoff U_t does not directly depend on their private signal. Thus, they would behave in the same way regardless of the signal they have, and their actions would be completely non-informative.

After the signal s is revealed (through the managers' actions or for exogenous reasons), no more additional asymmetric information is introduced. We make this assumption so that the model is tractable. This simplification though should not affect the results significantly. For our model's results, we focus only on the first observed split per each company, before the asymmetric information is revealed. In reality, before future splits, some new asymmetric information will be introduced. Thus, the first split should be representative of splits in general. In our model, we ignore the potential asymmetric information dynamics around future splits assuming they are sufficiently far in the future and should not have any large effect on the current split decisions.

5.4. Manager's objectives

We assume that the company's managers decide when to undertake the split. As a result, we need to specify their objectives. We follow Brennan and Copeland (1988) and assume that managers care not only about current prices, but also about future prices.[12] In particular, we assume that managers maximize the sum of the discounted future overall values of the company[13]:

$$U_t = \int_t^{\infty} e^{-\gamma(\tau - t)} \cdot (n_\tau p_\tau)\, d\tau \to max \tag{5.10}$$

This assumption is consistent with the managers' compensation at time t being proportional to the company value at that time, and, thus, managers maximize the present value of their compensation at their own discount rate γ.[14]

In addition to U_t, we also define the managers' normalized payoff, or their payoff per share:

$$u(y_t) = \frac{U_t}{n_t} \tag{5.11}$$

The benefit of considering the normalized payoff is that, in the case of the linear compensation scheme in Eq. (5.10), it depends only on the state variable y_t. Managers of a larger company expect larger compensation, but their per share compensation $\frac{U_t}{n_t}$ depends only on the current per share dividend $\frac{\theta_t}{n_t}$. Whenever the split takes place, the overall payoff U_t should also be continuous at the split point. Thus, also the normalized payoff changes according to the split factor: $u_{after} = \frac{u_{before} \cdot n_{before}}{n_{after}} = \frac{u_{before}}{\lambda}$.

5.5. Equilibrium structure

Suppose we start with a relatively low fundamental value $y_t = \delta \frac{\theta_t}{n_t}$. This value changes stochastically, but on average it gradually increases over time. At a certain point y, the managers can decide to undertake the split, and this decision depends on their private signal s. By observing or not observing a split at a certain time, investors update their beliefs about the private signal of the managers. In equilibrium, the managers' decision should be optimal given how investors update their beliefs, and the beliefs of the investors should be consistent with the managers' actions.

We focus on finding the separating equilibria in which different manager types, who received different signals s, choose different split points y (assume that the manager undertakes the split when the fundamental value reaches y for the first time). In this case, we can describe the manager's decision by the inverse beliefs function $s(y)$, which shows which signal was received by the managers who announce the split when the fundamental state variable value reaches y for the first time. The managers with more positive asymmetric information will undertake the split sooner, thus $s(y)$ should be monotonically decreasing. We will describe investors' beliefs about the managers' type with the function $\hat{s}(y)$: if investors observe the split at the point y they believe that the managers' private signal is $\hat{s}(y)$. In equilibrium, the managers will choose to split optimally given $\hat{s}(y)$ and the investors' beliefs should be consistent with the managers' choices, that is $\hat{s}(y) = s(y)$.

[11] In a version of the model with continuously paid dividends, the payment itself could provide the information about the signal. The considered setup though is much easier to analyze.

[12] Note that we cannot just assume that the managers always maximize the current value of the company. Then, it is not possible to have splits with an announcement premium. In such case, if the managers can increase the company's value by splitting the shares, they will always proceed (irrespective of any private information they might have). The managers' actions are completely not informative and the investors perfectly predict the split point. Thus, there is no announcement premium. This means that the managers should care not only about the current prices, but also about the future prices. Then, it might be costly to misrepresent the private information in order to increase the current price, as it can have a negative effect on the future price. Thus, split decisions become informative.

[13] Since we assume that $\delta \ll \gamma$, when writing down the managers' objective function we can ignore the possibility that the company is liquidates and that the final dividend $\theta_t \cdot s$ is paid out to shareholders.

[14] In general, managers probably have a concave utility over the received income, but their compensation scheme is normally convex, thus the overall linear compensation is not unreasonable. The model can potentially be extended to other utility functions, but again the linear case is more tractable.

6. Solving the model

It is possible to solve the model from Section 5 and to describe the resulting equilibrium, but it is technically challenging[15] while not being particularly instructive. We will therefore explain how the model can be solved for the special case of $\sigma = 0$, in which the state variable changes according to:

$$\frac{dy_t}{y_t} = \mu dt \tag{6.1}$$

The steps for finding the equilibrium are similar between the full model and this special case. However, the explanation will be clearer in the non-stochastic case, and we will not be distracted by the technical details that arise when dealing with the stochastic state variable.[16]

6.1. Model with symmetric information

We first analyze the model with symmetric information (or equivalently we assume the signal s is equal to 1 for all managers). These results will be useful to describe the prices and managers' expected payoffs after the split in the presence of asymmetric information.

6.1.1. Prices

Suppose the company is not undertaking a split at the moment. Then, the current price is the present value of the payoff at the next moment:

$$p_t = \frac{1}{1 + r(p_t)dt}\left[(\delta dt) \cdot \frac{\theta_t}{n_t} + (1 - \delta dt) \cdot (p_t + dp_t)\right] \tag{6.2}$$

Solving for the price change (while ignoring the higher order terms), we have:

$$\frac{dp_t}{dt} = (r(p_t) + \delta)p_t - \delta\frac{\theta_t}{n_t} \tag{6.3}$$

Given that[17] $\frac{dp_t}{dt} = \frac{dp(y)}{dy} \cdot \frac{dy}{dt} = p_y \cdot \mu y$ and a small δ ($\delta \ll r(p)$), we get the differential equation for the $p(y)$:

$$p_Y \cdot \mu y = r(p) \cdot p - y \tag{6.4}$$

We also need a boundary condition in order to solve for $p(y)$. With symmetric information, the overall company value should not change when the split is undertaken:

$$p\left(y^*\right) = \lambda p\left(\frac{y^*}{\lambda}\right) \tag{6.5}$$

Finally, we need an additional condition that determines the optimal split point y^*.

Lemma 1. *If y^* is the optimal split point, then*

$$r\left(p\left(\frac{y^*}{\lambda}\right)\right) = r\left(p\left(y^*\right)\right) \tag{6.6}$$

The condition (6.6) is quite intuitive — it is optimal to undertake splits to keep the company price in the region with the lowest interest rates.

6.1.2. Managers payoff

If the split is not undertaken at the moment, the current normalized payoff is the discounted future normalized payoff plus the expected payment (we ignore the possibility that the final dividend is paid out here, because δ is assumed to be small, $\delta \ll \gamma$):

$$u_t = \frac{1}{1 + \gamma \cdot dt}\left[p_t \cdot dt + (u_t + du_t)\right] \tag{6.7}$$

Rearranging the terms, while ignoring the higher order terms, and taking into account that $\frac{du_t}{dt} = u_y \mu y$, we get the following differential equation for $u(y)$:

$$u_y \mu y = \gamma u - p(y) \tag{6.8}$$

This differential equation can be solved together with the boundary condition:

$$u\left(y^*\right) = \lambda u\left(\frac{y^*}{\lambda}\right) \tag{6.9}$$

[15] You can refer to the solution reported in the previous version of the paper, as the SSRN working paper series, Iannino and Zhuk (2021).

[16] The split can only happen when we reach some value of y for the first time. Thus, for the stochastic model, since the value of y can also decrease, we need to keep track not only of the current value of y but also of the current maximum of y up to now. In addition, it is much more difficult to derive the condition describing the optimal split choice for managers when y can change stochastically.

[17] Here and in the following sections, t subscript refers to a point in time, such as $p_t = p(t)$, while y and other subscripts refer to a derivative, such as $p_y = \frac{dp(y)}{dy}$.

6.2. Model with asymmetric information

6.2.1. Prices and managers' payoff after the split

In a separating equilibrium, private information is completely revealed after the split. Thus, we can describe the solution after the split using the symmetric information case solution from Section 6.1. We only need to adjust the price because the final dividend is multiplied by the signal s. Thus, the price after the split $p(y,s)$, and the managers' per share expected payoff after the split $u(y,s)$ can be calculated as:

$$p(y,s) = p(y \cdot s)$$
$$u(y,s) = u(y \cdot s)$$

where $p(y)$ and $u(y)$ are, respectively, the price and the managers' per share payoff in the symmetric information case.

In equilibrium, managers will signal truthfully. However, to find the equilibrium we need to know what happens if managers deviate from the equilibrium path. By $u(y,s,\hat{s})$ let us denote the manager's per-share payoff in case the managers' private signal is s, but investors believe it to be $\hat{s}$, and the current fundamental value is y. Note that, after the split, investors learn each moment the true signal with probability $\pi \cdot dt$, and thus, eventually they will learn the true s.

To find the equilibrium we do not need to completely describe the function $u(y,s,\hat{s})$. We only need the derivative of this function with respect to $\hat{s}$ calculated at $\hat{s} = s$:

$$\frac{du(y,s,\hat{s})}{d\hat{s}}|_{\hat{s}=s}$$

The following lemma shows that this derivative is determined by the function $\Pi(y)$, which can be solved in a similar way as $p(y)$ and $u(y)$.

Lemma 2. *The derivative* $\frac{du(y,s,\hat{s})}{d\hat{s}}|_{\hat{s}=s}$ *can be calculated as:*

$$\frac{du(y,s,\hat{s})}{d\hat{s}}|_{\hat{s}=s} = \frac{\Pi(y \cdot s)}{s} \tag{6.10}$$

where $\Pi(y)$ *is the function defined by:*

$$\begin{aligned} &\tfrac{d\Pi}{dy}\mu y = (\gamma+\pi)\Pi - y \cdot \tfrac{dp(y)}{dy} \\ &\Pi\left(y^*\right) = \lambda \Pi\left(\tfrac{y^*}{\lambda}\right) \end{aligned} \tag{6.11}$$

6.2.2. Prices before the split

Denote by $\bar{p}(y)$ the price before the first split, when the current state value is y (this price depends only on y since investors do not know the signal s). We can derive the differential equation for the evolution of the price in a similar way as we did in the case of symmetric information. As before, the current price is the discounted expected payoff in the next moment. However, now several things can happen in the next moment. First, (the first term in Eq. (6.12) below) with probability $\delta \cdot dt$, the final dividend $\theta_t \cdot s$ is paid. Since the company did not implement the split yet, we can eliminate the signal realizations that are above $s(y)$. Second, (and correspondingly, the second term) with probability $\pi \cdot dt$, the private signal s is revealed for exogenous reasons. The share price afterwards is $p(y \cdot s)$, therefore we take the expected value of this expression given current beliefs. Third (and the third term in the equation), with probability $\phi dt = \frac{Prob\{s(y_{t+dt})<s<s(y_t)\}}{Prob\{s<s(y_t)\}}$, managers announce the split. Then, the managers' signal is approximately $s(y)$ and thus the post-split price is $p\left(\frac{s(y)\cdot y}{\lambda}\right)$. Finally (the fourth term), if nothing of the above happens, we get our price in the next moment $\bar{p}_t + d\bar{p}_t$:

$$\begin{aligned} \bar{p}_t \quad = \quad \tfrac{1}{1+r(\bar{p}_t)dt} \Big[&\delta dt \cdot \tfrac{\theta_t}{n_t} E\left[s | s < s(y)\right] + \pi dt \cdot E\left[p(ys) | s < s(y)\right] \\ &+ \phi dt \cdot \lambda p\left(\tfrac{s(y)\cdot y}{\lambda}\right) \\ &+ (1 - \delta dt - \pi dt - \phi dt)(\bar{p}_t + d\bar{p}_t) \Big] \end{aligned} \tag{6.12}$$

After rearranging and simplifying the terms in Eq. (6.12), and taking into account that $\frac{d\bar{p}_t}{dt} = \bar{p}_y \frac{dy}{dt} = \bar{p}_y \cdot \mu y$, we get the differential equation (6.13) for the evolution of the prices before the first split $\bar{p}(y)$ (see the proof of Lemma 3 for details).

Lemma 3. *The price before the first split* $\bar{p}(y)$ *satisfies the following differential equation:*

$$\bar{p}_y \cdot \mu y = r(\bar{p})\bar{p} - y \cdot Q(s(y)) + \pi \cdot [\bar{p} - P(s(y),y)] + \phi \cdot \left[\bar{p} - \lambda p\left(\frac{s(y)\cdot y}{\lambda}\right)\right] \tag{6.13}$$

where $s(y)$ *is the equilibrium investors' beliefs after the split at the point* y*,* f *and* F *are the cdf and pdf of the distribution of signals, and* Q*,* P *and* ϕ *are defined as:*

$$\begin{aligned} &Q(s) = \frac{1}{F(s)}\int_{-\infty}^{s} z \cdot f(z)\,dz \\ &P(s,y) = \frac{1}{F(s)}\int_{-\infty}^{s} f(z)\cdot p(z\cdot y)\,dz \\ &\phi = \frac{f(s(y))}{F(s(y))} \cdot \left|s_y(y)\right| \cdot \mu y \end{aligned} \tag{6.14}$$

6.2.3. Optimal split decision

Managers will choose the split point optimally given the investors' beliefs $\hat{s}(y) = s(y)$, and they are facing the following trade-off. If they undertake the split sooner, then investors will believe that the private signal is higher, which will increase the current value of the company. On the other hand, splitting too soon could be costly if, in the future, private information is revealed, and the resulting price is far below the preferred region. In this subsection, we derive the exact optimality condition.

The benefit of considering a non-stochastic model (with $\sigma = 0$) is that we can write down an explicit expression for the managers' expected payoff and derive the optimality condition in a very straightforward way. Suppose that at $t = 0$ the current value of the state variable y_0 is below $y(\bar{s})$ (which means that even the managers with the highest type have not yet undertaken the split). If managers with type s decide to announce the split at y, then their expected payoff $w(y, s)$ at $t = 0$ can be written as:

$$w(s,y) = \int_0^{t(y)} e^{-(\pi+\gamma)t} \cdot \left[\bar{p}(y_t) + \pi \cdot u(sy_t)\right] dt + e^{-(\pi+\gamma)t(y)} \lambda u\left(\frac{y}{\lambda}, s, s(y)\right)$$

where $t(y) = \frac{1}{\mu} log(y/y_0)$ represents the amount of time it would take to reach y from the current value of y_0. Before the split announcement $(t < t(y))$ in each point in time, managers are compensated by $\bar{p}(y_t) \cdot dt$, and, in addition, with probability $\pi \cdot dt$ the private signal is revealed, and then managers' expected payoff is $u(sy_t)$. After the split at y, investors believe the signal is $s(y)$ and we can use the function $u(y, s, \hat{s})$ defined in Section 6.2.1 to describe the corresponding expected payoff.

The derivative of the expected payoff with respect to the split point y is as follows:

$$\begin{aligned}\frac{\partial w}{\partial y} &= e^{-(\pi+\gamma)t(y)} \cdot \left[\left(\bar{p}(y_t) + \pi \cdot u(sy_t) - (\pi+\gamma)\lambda u\left(\frac{y_t}{\lambda}, s, s(y)\right)\right)\frac{1}{\mu y} + \right.\\ &\left. + u_y\left(\frac{y_t}{\lambda}, s, s(y)\right) + \lambda u_{\hat{s}}\left(\frac{y_t}{\lambda}, s, s(y)\right) \cdot \frac{ds}{dy}\right]\end{aligned}$$

For a manager of type $s = s(y)$, it would be optimal to undertake the split at y, which gives us the following first order condition:

$$\frac{\partial w}{\partial y}|_{s=s(y)} = 0 \tag{6.15}$$

After rearranging the terms and a few minor adjustments from (6.15), we derive the differential equation (6.16) describing the equilibrium inverse split decision function $s(y)$ (see the proof Lemma 4 for details).

Lemma 4. *The equilibrium inverse split decision function $s(y)$ satisfies the following differential equation:*

$$\frac{ds}{dy} \cdot \frac{y}{s} \cdot \lambda \Pi\left(\frac{ys}{\lambda}\right) \cdot \mu = \pi\left(\lambda u\left(\frac{sy}{\lambda}\right) - u(sy)\right) + \left(\lambda p\left(\frac{sy}{\lambda}\right) - \bar{p}(y)\right) \tag{6.16}$$

We can also derive a second order condition (Lemma 5), which verifies that the first order condition actually corresponds to a maximum. When we solve the model numerically, in addition to finding the optimal $s(y)$ using Eq. (6.16), we also verify that the condition (6.18) also holds.

Lemma 5. *If $\frac{ds}{dy} < 0$, then the second order condition in the manager's maximization problem*

$$\frac{\partial^2 w}{\partial y^2}|_{s=s(y)} = \frac{\partial^2 w}{\partial s \partial y}|_{s=s(y)} \cdot \left(-\frac{ds}{dy}\right) < 0 \tag{6.17}$$

is equivalent to

$$\gamma\left(u(ys) - \gamma\lambda u\left(\frac{ys}{\lambda}\right)\right) - \left(p(ys) - \lambda p\left(\frac{ys}{\lambda}\right)\right) < 0 \tag{6.18}$$

6.2.4. Boundary condition

Eqs. (6.13) and (6.16) form a system of differential equations that can be solved jointly for $\bar{p}(y)$ and $s(y)$. We only need to add the boundary condition.

Suppose first that the prior distribution of signals s is bounded from below and that there exists the lowest signal $\underline{s} > 0$. When we reach the split point $\underline{y}$ corresponding to the lowest type $\underline{s}$, there is no more information asymmetry, as investors can perfectly infer the manager's type from the lack of prior splits. Thus, the split point should be the same as in the symmetric information case:

$$\underline{s} \cdot \underline{y} = y^* \tag{6.19}$$

The price at this point is also equal to the corresponding price in the symmetric information case $p(y^*)$. Thus, we have our boundary condition:

$$\underline{y} = \frac{y^*}{\underline{s}} \tag{6.20}$$

$$s\left(\underline{y}\right) = \underline{s} \qquad \bar{p}\left(\underline{y}\right) = p(y^*) \tag{6.21}$$

Interestingly, the equilibrium can also exist in the unbounded case, when the lowest type does not exist (for example, in the case of log-normal distribution $log(s) \sim N(0, \sigma_s^2)$). Moreover, the unbounded case solution can be approximated by the converging bounded case solutions. Appendix B discusses this in more detail.

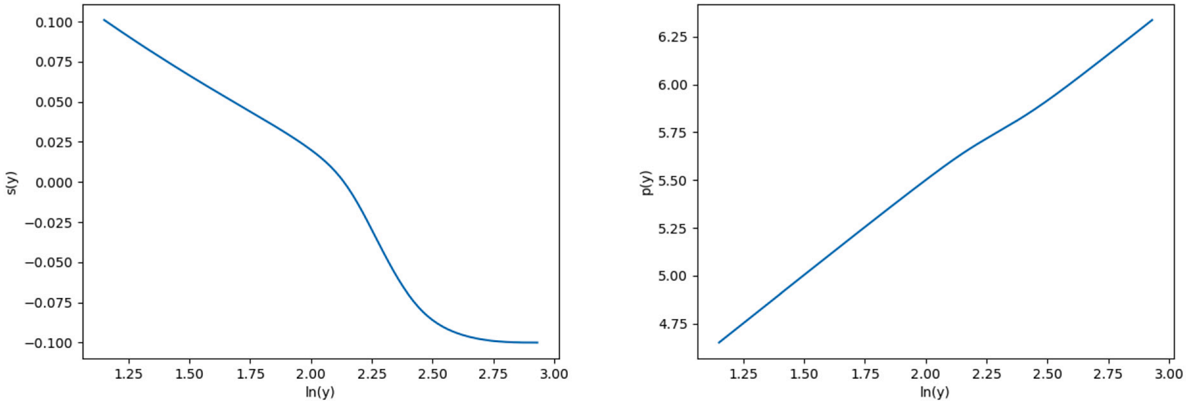

Fig. 7. Solution in the case of the asymmetric information. Price and Signal.
The frame on the left shows how the beliefs about the signal, $s(y)$, should depend on the timing of the split. The frame on the right shows the relationships between the price before the first split, $\bar{p}(y)$, and the fundamental value y. The following parameters are used in the solution: $\mu = 0.02$, $\sigma = 0$, $r_0 = 0.05$, $\alpha = 0.0002$, $log(p_0) = 6$, $\gamma = 0.1$, $\pi = 1$, $\sigma_s = 0.02$.

6.3. Numerical solution

For a given distribution of signals, given values of the parameters and given nominal price costs $r(p)$, the model can be solved numerically. In this subsection, we will discuss how a typical solution looks like and how each parameter affects the results. We will assume the log-normal distribution of private signals,

$$log(s) \sim N(0, \sigma_s^2) \tag{6.22}$$

restricted to $s \geq \underline{s}$, where $log(\underline{s}) = -5 \cdot \sigma_s$. For now, we assume the $r(p)$ is the simplest quadratic function

$$r(p) = r_0 + \alpha \cdot (log(p) - log(p_0))^2 \tag{6.23}$$

The parameter α is the strength of nominal share price preferences, and p_0 is the optimal price point.

Fig. 7 presents a typical solution. The graph on the left shows how the beliefs about the managers' private signal $s(y)$ depend on the split point. The longer the firm waits, the lower is the signal. This happens because it is less costly for managers with high signals to choose to undertake splits early and they choose to do so.

The graph on the right shows how the price before the first split $\bar{p}(y)$ depends on the fundamental parameter y. The price is increasing in y, but the speed of this increase is not uniform. This happens because investors also update their beliefs about the remaining signals when they do not observe splits. The speed of this updating depends on how likely is the split in the next moment and is small for relatively low prices.

From the prior distribution of signals $f(s)$ and the solution functions $s(y)$ and $\bar{p}(y)$, we calculate the predicted distribution of pre-split prices as:

$$f_{\bar{p}}(\bar{p}(y)) = \frac{f(s(y)) \cdot s_y(y)}{\bar{p}_y(y)} \tag{6.24}$$

In addition, we calculate the announcement return premium for any corresponding pre-split price as:

$$r(\bar{p}(y)) = log\left[\lambda \cdot p\left(\frac{s(y) \cdot y}{\lambda}\right)\right] - log\,[\bar{p}(y)] \tag{6.25}$$

Fig. 8 shows the distribution of the pre-split prices and the corresponding announcement return premium for the solution with the above parameters. We see that the splits are possible at different pre-split price levels. In addition, the announcement return premium is always positive and is declining with the pre-split price. When the split is announced, investors eliminate all types lower than $s(y)$, thus the announcement is always good news. Moreover, such premium is higher at lower price levels, when the split is less expected. All of these facts are consistent with the empirical data (Figs. 4 and 5).

We can also look at whether managers are signaling truthfully, looking at the relationship between the signals at the optimal split point, s(y*) and the private signal. From Fig. 9, we evinced that the relations between the true signal of the managers and the market beliefs, i.e. the expected signal inferred from optimal splitting, align on a 45 degree line. In equilibrium, the managers will truthfully reveal their signals, and the signals inferred by the market coincide with the private information of managers. This shows that equilibrium exists at nearly all values of the state variable y.

We also look at how changes in different parameters affect the resulting solution and check whether it is consistent with our intuition. Figs. 10 and 11 show the results of these comparative statics exercises.

The changes in p_0, the optimal price level from Eq. (6.23), just shift the distribution of pre-split prices

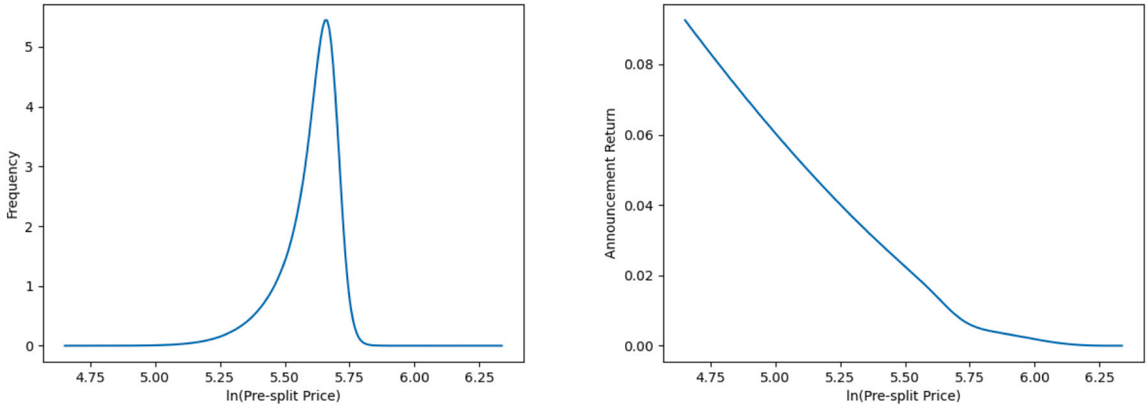

Fig. 8. Solution in the case of the asymmetric information. Return and frequency.
The graph on the left frame shows the distribution of the pre-split prices. The graph on the right frame reports the relationship between the announcement abnormal return and the pre-split price. The following parameters are used in the solution: $\mu = 0.02$, $\sigma = 0$, $r_0 = 0.05$, $\alpha = 0.0002$, $log(p_0) = 6$, $\gamma = 0.1$, $\pi = 1$, $\sigma_s = 0.02$.

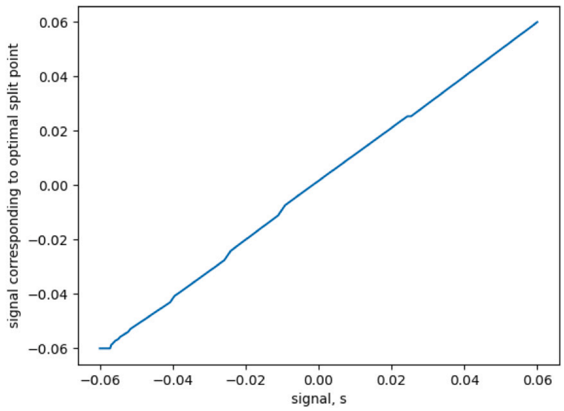

Fig. 9. Optimality.
The graph shows the relation between the true signal of the manager and the expected signal inferred from optimal splitting, which they align on the 45 degree line. In equilibrium, the managers will truthfully reveal their signals, and the signals inferred by the market coincide with the private signals. The following parameters are used in the solution: $\mu = 0.02$, $\sigma = 0$, $r_0 = 0.05$, $\alpha = 0.0002$, $log(p_0) = 6$, $\gamma = 0.1$, $\pi = 1$, $\sigma_s = 0.02$.

The parameter α characterizes the degree of nominal price preferences. When α is increasing, the distribution of splits shifts to the right and the announcement premium becomes larger. When α is high, it is more costly to undertake splits early and it is easier to credibly signal high private information, which leads to later split announcements and higher announcement premium. The announcement premium decreasing with α is consistent with the fact that, when $\alpha = 0$, the signaling is not possible and the announcement premium is 0.

When π, the probability that the signal is revealed to investors for exogenous reasons, is increasing, the distribution of splits shifts to the right and the announcement premium becomes larger. When π is high, it is more costly to deviate from the actual signal, which leads to easier signaling.

Finally, as σ_s, the standard deviation of the ex-ante distribution of private signals, increases, the announcement premium is not much affected, and the distribution of splits shifts to the left. This parameter characterizes how large are the information asymmetries between the managers and the investors. When the information asymmetries are not large, there is no strong need to signal, which means the split prices will be close to the optimal split price in the symmetric information case.

7. Estimation of the model

In this section, we estimate the parameters of our model by matching the empirical distribution of pre-split nominal prices and the relationships between nominal price and announcement abnormal return with the ones predicted by the theory. Since our model predicts a distribution of pre-split prices, we use Maximum Likelihood for the estimation. Using the estimated parameters, we recover the nominal share price preferences of investors and decompose the splits announcement premium into new information and signaling components.

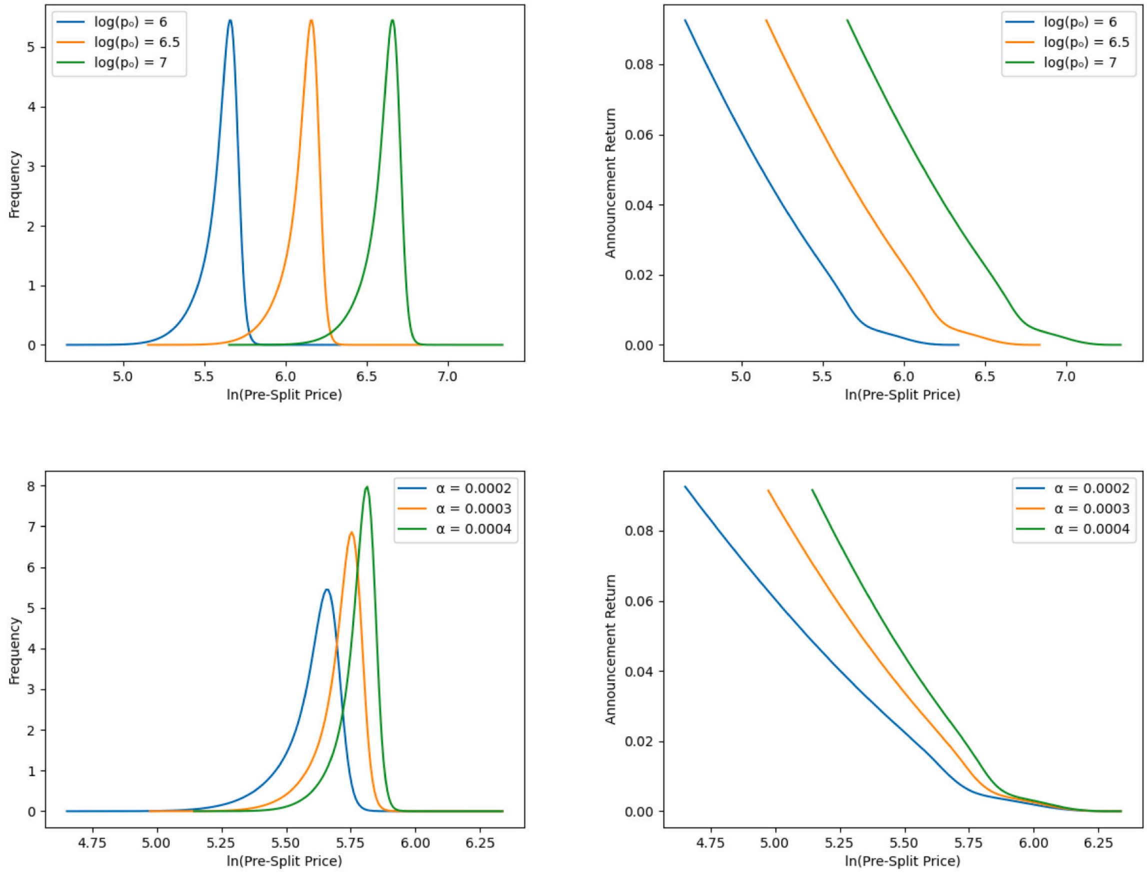

Fig. 10. Comparative statics. p_0 and α.
This figure reports the comparative statics results on the distribution of splits (left frames) and on the announcement returns (right frames). In particular, we report the effects of changes in the optimal price, p_0, in the top frames, and changes in the nominal price preference, α, in the bottom frames. The following parameters are used in the solution (except for the ones shown on the diagrams): $\mu = 0.02$, $\sigma = 0$, $r_0 = 0.05$, $\alpha = 0.0002$, $log(p_0) = 6$, $\gamma = 0.1$, $\pi = 1$, $\sigma_s = 0.02$.

To be consistent with the model, we only select the subset of splits with split factor 2:1. As we have already seen, this type of split represents nearly half of the sample of non-reverse splits. Therefore, we perform our estimation on a total sample of 4396 US split announcements, 1980 to 2013. For each announcement, we calculate the announcement premium r_i and the pre-split price p_i as described in Sections 3.3 and 3.4.

7.1. Parameters and likelihood function

We estimate $r(p)$ as a polynomial function of the $log(p)$ of order n (note that, instead of α_1 we have an equivalent parameter α_k):

$$r(p) = r_0 + \sum_{k=2}^{n} \alpha_k \cdot \left(log\,(p) - log\,(p_0)\right)^k \tag{7.1}$$

For sufficiently large n, this polynomial function can approximate any function $r(p)$. In the estimation, we increase n as long as there is a significant effect on the maximum likelihood.

First, we obtain separately the parameters that do not require structural estimation: $\psi = \{\mu, \sigma, r_0, \gamma\}$. Table 3 contains the estimates that we use and their sources. Then, we estimate endogenously the vector of remaining parameters $\theta = \{\sigma_s^2, \pi, p_0, \alpha_1, \ldots, \alpha_n\}$, where, we recall, σ_s^2 is the variance of the private signal s; π is the probability that the signal is exogenously revealed; γ is the discount rate required by managers; p_0 and $\alpha_2, \ldots, \alpha_n$ are the parameters of function for the nominal price preferences from (7.1). We use Maximum Likelihood estimation, in particular, we calculate

$$\hat{\theta} = arg\max_{\theta} \sum_{i=1}^{N} \log L(r_i, p_i, \theta) \tag{7.2}$$

where N is the number of observations in our sample. The likelihood function for ith observation is defined as

$$L(r_i, p_i, \theta) = f(p_i, \theta) \cdot \frac{1}{\sigma_r(p_i)\sqrt{2\pi}} e^{-\frac{(r_i - r(p_i,\theta))^2}{2\sigma_r(p_i)^2}} \tag{7.3}$$

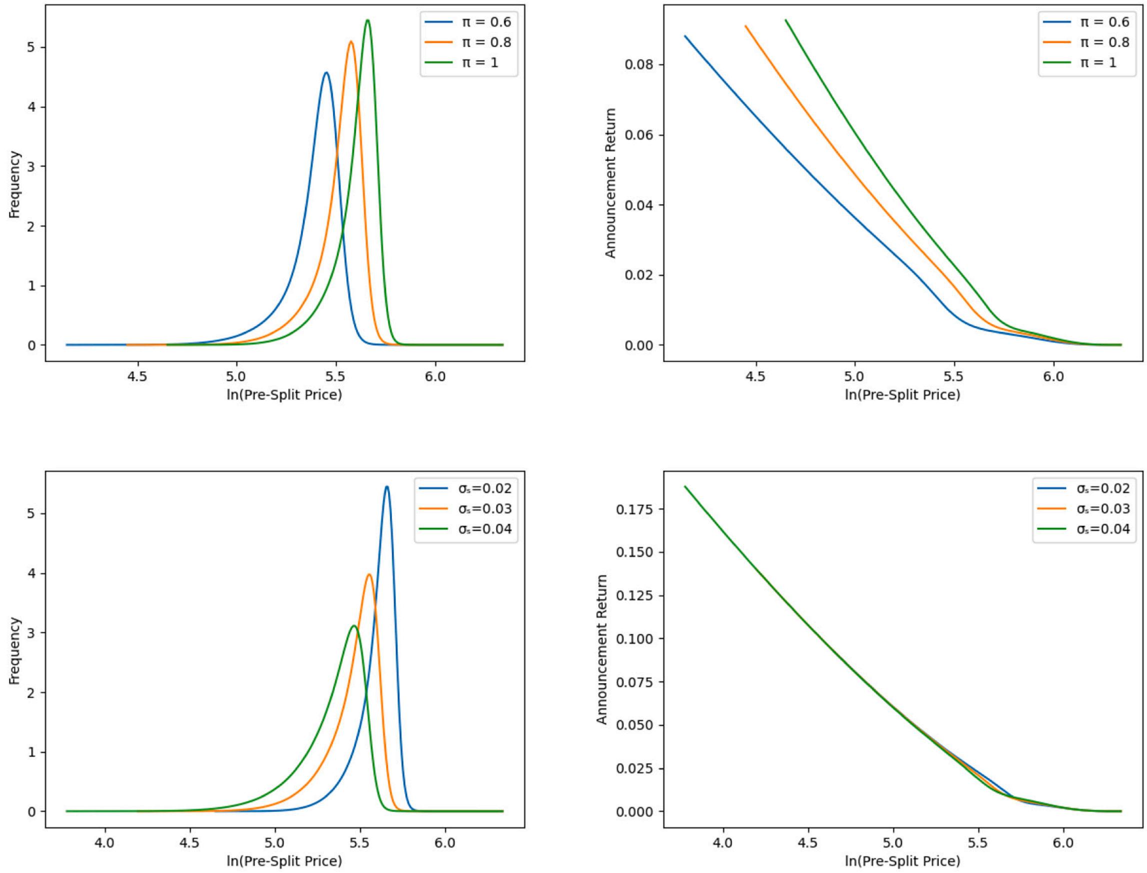

Fig. 11. Comparative statics π and σ_s.
This Figure reports the comparative statics results on the distribution of splits (left frames) and on the announcement returns (right frames). In particular, we report the effects of changes in the probability of private information to be revealed, π, in the top frames, and changes in the variance of the private signal, σ_s, in the bottom frames. The following parameters are used in the solution (except for the ones shown on the diagrams): $\mu = 0.02$, $\sigma = 0$, $r_0 = 0.05$, $\alpha = 0.0002$, $log(p_0) = 6$, $\gamma = 0.1$, $\pi = 1$, $\sigma_s = 0.02$.

where $f(p, \theta)$ and $r(p, \theta)$ are the probability density function of the pre-split prices and the announcement premium that we obtain from the numerical solution of the model, and $\sigma_r^2(p)$ is the non-parametrically estimated variance of returns for a price level p.[18] The first term in the above function is the likelihood of observing price p_i, and the second is the likelihood of observing return r_i for a given p_i. The second term assumes that the actual announcement returns are normally distributed around $r(p_i, \theta)$ with variance $\sigma_r^2(p)$.

7.2. Results

Table 4 reports the estimated parameters $\hat{\theta}$. We report the estimates for the non-stochastic model ($\sigma = 0$) described in the previous sections.[19] The standard deviation of the distribution of managers' private signals σ_s is around 3% (which means that revealing managers' private information changes a company's value by 3% on average) The probability per unit of time (in years) that the private information is revealed by exogenous factors π is 0.33 for the non-stochastic model. (Thus, in 1 month the information is revealed with a probability of approximately $0.33 \times \frac{1}{12} \approx 2.75\%$).

[18] We calculate $\sigma_r^2(p)$ as

$$\sigma_r^2(p) = \sum_i K\left(\frac{log(p_i) - log(p)}{h}\right) \cdot (r_i - \mu_r(p))^2$$

$$\mu_r(p) = \sum_i K\left(\frac{log(p_i) - log(p)}{h}\right) \cdot r_i$$

where $K(.)$ is Epanechnikov kernel and h is a bandwidth, which was equal to 0.5 in the calculation.

[19] The estimates for the stochastic model ($\sigma = 0.1$) can be found in Iannino and Zhuk (2021).

Table 3
Exogenous parameters.
The table reports the values of the exogenous parameters used for the Maximum Likelihood estimation of our model and their sources.

Parameter	Explanation	Proxy	Value
μ	Mean growth rate of the final dividend θ_t	Mean growth rate of dividends from Strebulaev and Whited (2011)	0.02
σ	Standard deviation of the growth rate of the final dividend θ_t	Standard deviation of the growth rate of dividends from Strebulaev and Whited (2011)	0.00/0.10
r_0	Minimum discount rate required by the market	One-year average market return in our sample (own calculation). The average return is a reasonable proxy because the results are not very sensitive to r_0 and the nominal share price effects are small.	0.05
γ	Discount factor of the managers	Discount factor of the managers from Taylor (2010)	0.10

Table 4
Estimated parameters.
The table reports the Maximum Likelihood estimated parameters from the model; $\hat{\sigma_s}$ is the standard deviation of the managers' private signal; $\hat{\pi}$ is the probability that this signal is revealed for exogenous reasons; $\widehat{log(p_0)}$ and $\hat{\alpha_2}, \hat{\alpha_3}, \hat{\alpha_4}$ are the parameters from the equation $r(p) = r_0 + \sum_{k=2}^{4} \alpha_k \cdot \left(log\,(p) - log\,(p_0)\right)^k$ describing the relationship between the required return and the price level, or the nominal share price preference. We report standard errors and t-statistics (*** 1% confidence level). The results are presented for the non-stochastic model ($\sigma = 0$) described in the paper.

	Parameter	St.Error	T-statistic
$\hat{\pi}$	0.3314***	0.02068	16.031
$\hat{\sigma_s}$	0.03037***	0.00095	32.0284
$\widehat{log(p_0)}$	5.7113 ***	0.00540	1057.89
$\hat{\alpha_2}$	0.0001728 ***	0.0000065	26.5117
$\hat{\alpha_3}$	0.0000830 ***	0.0000011	75.8638
$\hat{\alpha_4}$	0.00002184 ***	0.0000000037	5978.06

The most interesting estimates are $\hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4$, which are the parameters of the polynomial function from Eq. (7.1). They represent the relation between the nominal price and the return required by the market, the so-called nominal share price preferences. The coefficients are small, but highly significant, confirming the existence of non-trivial nominal price effects in the market. It is therefore costly for the managers to falsely signal to the market. The optimal price level is represented by $\widehat{log(p_0)}$, and it is also highly significant. Fig. 1 shows the resulting nominal share price preferences.

To check how closely our model can match the data, we compare the distributions of pre-split prices and the announcement premiums for each price level resulting from both the model estimation and the data. Fig. 12 reports the distributions of the pre-split prices in the top frame, and the relationships between announcement return and pre-split price in the bottom frame. We see how the estimates from the model and from the data fit very closely when looking at the distribution of the pre-split prices. In this case, the likelihood of splitting shares at different prices from the sample is very closely matches by the model one both in mean and volatility. This result is consistent with our mechanism of signaling through the timing of splits. The relationships between the abnormal announcement return and the pre-split price level is negative and nonlinear in both the model and the data, though the curvature is not as perfectly matched. The curve for the model is steeper for smaller prices, but this due to the functional form we assume for r(p) in the numerical solution.

7.3. Announcement return decomposition

When managers announce a stock split at a relatively low price level, this allows them to reveal positive private information. However, this comes at a cost, because prices are now at an inefficiently low level. Using the estimated parameters of our model we can calculate what is the value of the private information and how large is the signaling cost for any given price level.

When we estimate the model we match the split announcement return:

$$r = log\left[\lambda \cdot p\left(\frac{s(y)\cdot y}{\lambda}\right)\right] - log\,[\bar{p}(y)] \tag{7.4}$$

with the empirically observed one. We then can calculate the value of the unobserved asymmetric information as a return on the stock price if private information of the managers was revealed without a stock split:

$$\{Value\,of\ Information\} = log\,[p\,(s(y)\cdot y)] - log\,[\bar{p}(y)] \tag{7.5}$$

The difference between the two returns is the cost incurred due to signaling.

$$\{Cost\,of\ signaling\} = log\,[p\,(s(y)\cdot y)] - log\left[\lambda \cdot p\left(\frac{s(y)\cdot y}{\lambda}\right)\right] \tag{7.6}$$

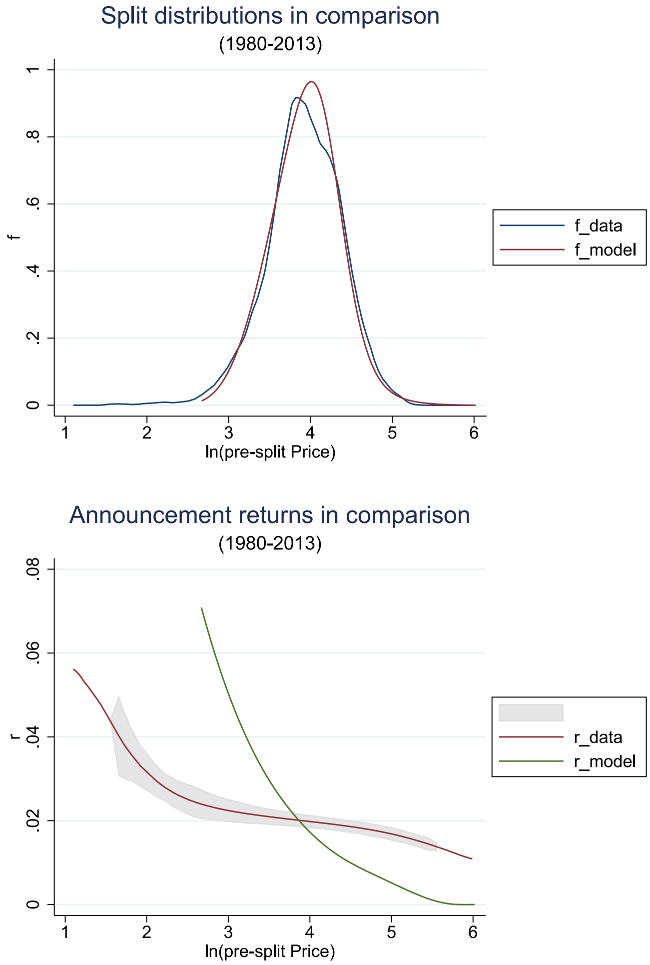

Fig. 12. Comparing data with the model.
On the top frame, we report the distribution of pre-split price as estimated in the data and in the model. The bottom frame reports the relationships between announcement return and pre-split price as estimated in the data and in the model, including 95% confidence intervals.

Fig. 2 shows that the estimated signaling costs depend on the pre-split price. In particular, it shows how the announcement return, matched to the empirical data, and the value of the revealed information, calculated from the model, depend on the price level. The difference between the two is the signaling costs and it is small for large prices, but reaches 0.5% of a company's value for prices around $50.

8. Conclusion

In the paper, we develop a dynamic structural model of stock splits, in which managers signal their private information through the timing of the stock split decision. The model is a more realistic representation of the signaling mechanism of the splits, and, as far as we know, it is the first model of stock splits that is suitable for direct empirical estimation. In the model, investors have preferences about the nominal price levels, measured by the additional return premium they require to hold stocks with nominal prices far away from the desirable level. Managers have private information about their company's prospects, that they can convey with a split announcement. We derive a set of differential equations that describe how price levels and optimal split decisions of the managers depend on the company's fundamentals. These equations can be solved numerically for any given values of the parameters. Consistently with the empirical data, the model predicts a positive split announcement premium which declines with the pre-split nominal price.

We estimate the parameters of the model empirically with a maximum likelihood. The estimation allows us to recover the investors' actual nominal price preferences, and to describe which are the preferable prices and to which extent. We reject the hypothesis that investors do not have nominal price preferences. In addition, we decompose the split announcement premium into unobservable private information and the cost of signaling.

Our model can be used to test different theories of stock splits, as long as the nominal share price preferences they generate are not identical.

Data availability

The authors do not have permission to share data.

Appendix A

Proof of Lemma 1

1. If $r\left(p\left(y^*\right)\right) < r\left(p\left(\frac{y^*}{\lambda}\right)\right)$, then it would be optimal to delay the split from $y = y^*$ by small Δy (one shot deviation), which would increase the price at $y = y^*$ while not affecting the prices beyond $y + \Delta y$, and, thus, would be preferred by both managers and investors.
2. If $r\left(p\left(y^*\right)\right) > r\left(p\left(\frac{y^*}{\lambda}\right)\right)$, then it would be optimal to undertake the split already at $y^* - \Delta y$ (for small Δy). Then, the prices beyond y^* remain unaffected, but the company value at $y^* - \Delta y$ would increase, and, thus, again this benefits both investors and managers.

Proof of Lemma 2

1. Every moment the signal is revealed with probability πdt. The probability that the signal is not yet revealed by the time t is $e^{-\pi t}$. After the signal is revealed, the manager's utility is $u(s \cdot y)$; before the signal is revealed the company will behave as if the information is actually $\hat{s}$.
2. Before the private signal is revealed, the company will undertake splits at the following points (denote also T_0):

$$\begin{array}{llll} \text{1st split} & \hat{s} \cdot y_{T_1} = y^* & \Rightarrow & T_1 = \frac{1}{\mu} ln\left(\frac{y^*}{\hat{s} y_t}\right) \\ \text{2nd split} & \frac{\hat{s} \cdot y_{T_2}}{\lambda} = y^* & \Rightarrow & T_2 = \frac{1}{\mu} ln\left(\frac{y^*}{\hat{s} y_t}\right) + \frac{1}{\mu} ln(\lambda) \\ \dots & \dots & \dots & \dots \\ \text{k-th split} & \frac{\hat{s} \cdot y_{T_k}}{\lambda^{k-1}} = y^* & \Rightarrow & T_k = \frac{1}{\mu} ln\left(\frac{y^*}{\hat{s} y_t}\right) + \frac{k-1}{\mu} ln(\lambda) \\ \dots & \dots & \dots & \dots \end{array}$$

3. Then, we rewrite the managers' overall utility as:

$$u(y, s, \hat{s}) = \sum_{i=0}^{N} \int_{t=T_i}^{T_{i+1}} e^{-\gamma t} e^{-\pi t} \left(\lambda^i p\left(\frac{\hat{s} \cdot y_t}{\lambda^i}\right) + \pi \cdot \lambda^i u\left(\frac{s \cdot y_t}{\lambda^i}\right)\right) dt$$

4. The derivative of this expression with respect to $\hat{s}$ is:

$$\begin{aligned} u_{\hat{s}}(y, s, \hat{s}) &= \sum_{i=0}^{\infty} \int_{T_i}^{T_{i+1}} e^{-(\gamma+\pi)t} \cdot \left(y_t p_y\left(\frac{\hat{s} \cdot y_t}{\lambda_i}\right)\right) dt \\ &+ \sum_{i=1}^{\infty} e^{-(\gamma+\pi)T_i} \left(\pi \cdot \lambda^{i-1} u\left(\frac{s \cdot y_{T_i}}{\lambda^{i-1}}\right) - \pi \cdot \lambda^i u\left(\frac{s \cdot y_{T_i}}{\lambda^i}\right)\right) \frac{dT_i}{d\hat{s}} \end{aligned} \tag{A.1}$$

5. At $\hat{s} = s$ the second term disappears (since u is continuous at the split points):

$$u_{\hat{s}}(y, s, \hat{s} = s) = \sum_{i=0}^{\infty} \int_{T_i}^{T_{i+1}} e^{-(\gamma+\pi)t} \cdot \left(y_t p_y\left(\frac{s \cdot y_t}{\lambda^i}\right)\right) dt = \frac{1}{s} \Pi(ys)$$

   where

$$\Pi(y) = \sum_{i=0}^{\infty} \int_{T_i}^{T_{i+1}} e^{-(\gamma+\pi)t} \cdot \left(y_t p_y\left(\frac{y_t}{\lambda^i}\right)\right) dt$$

6. $\Pi(y)$ solves the following differential equation:

$$\frac{d\Pi}{dt} = \Pi_y \cdot \mu y = (\gamma + \pi)\Pi - p_y \cdot y$$

7. By comparing the values of $\Pi\left(y^*\right)$ and $\Pi\left(\frac{y^*}{\lambda}\right)$ we also get the following boundary condition:

$$\Pi\left(y^*\right) = \lambda \Pi\left(\frac{y^*}{\lambda}\right).$$

Proof of Lemma 3

1. We can calculate the probability of the split in the next moment in the following way. The split will happen if the signal realization is between $s(y_t)$ and $s(y_{t+dt})$:
$$\frac{Prob\{s(Y_{t+dt}) < s < s(Y_t)\}}{Prob\{\infty < s < s(Y_t)\}} \approx \frac{f(s(y_t)) \cdot |ds(y_t)|}{F(s(y_t))} = \frac{f(s(y_t))}{F(s(y_t))} \left|s_y(y_t)\right| \cdot dy_t$$
Here $f(s)$ and $F(s)$ are the pdf and cdf for of the prior distribution of s. Since $\frac{dy}{dt} = \mu y$, we get that:
$$\phi(y) = \frac{f(s(y))}{F(s(y))} \cdot \left|s_y\right| \cdot \mu y$$
2. In Eq. (6.12) we substitute:
$$E[s \cdot | s < s(y)] = \frac{1}{F(s(y))} \int_{-\infty}^{s(y)} z \cdot f(z)\, dz = Q(s(y))$$
$$E[p(ys) | s < s(y)] = \frac{1}{F(s(y))} \int_{-\infty}^{s(y)} p(y \cdot z) \cdot f(z)\, dz = P(s, y)$$
After rearranging the terms we get the desired result.

Proof of Lemma 4

1. Since $u(y,s) = u(ys)$ then $u_y(y,s) = s \cdot u_y(ys)$, which in turn implies that
$$u_y\left(\frac{y}{\lambda}, s, s\right) = u_y\left(\frac{y}{\lambda}, s\right) = s \cdot u_y\left(\frac{sy}{\lambda}\right)$$
Now using the differential equation (6.8) we get
$$u_y\left(\frac{y}{\lambda}, s, s\right) \cdot \mu y = \lambda \cdot u_y\left(\frac{sy}{\lambda}\right) \cdot \frac{\mu ys}{\lambda} = \gamma \cdot \lambda u\left(\frac{sy}{\lambda}\right) - \lambda p\left(\frac{sy}{\lambda}\right)$$
2. Finally we also take into account that
$$\lambda u_{\hat{s}}\left(\frac{y_t}{\lambda}, s, s\right) = \frac{\lambda}{s} \cdot \Pi\left(\frac{ys}{\lambda}\right)$$

Proof of Lemma 5

1. The second derivative $\frac{\partial^2 w}{\partial y^2}|_{s=s(y)}$ can be calculate as (since $\frac{\partial w}{\partial y}|_{s=s(y)} = 0$)
$$\frac{\partial^2 w}{\partial y^2}|_{s=s(y)} = \frac{\partial^2 w}{\partial y^2}|_{s=s(y)} - \frac{d}{dy}\left(\frac{\partial w}{\partial y}|_{s=s(y)}\right) = \frac{\partial^2 w}{\partial s \partial y} \cdot \left(-\frac{ds}{dy}\right)$$
2. The cross derivative $\frac{\partial^2 w}{\partial s \partial y}|_{s=s(y)}$ is equal to
$$\frac{\partial^2 w}{\partial s \partial y}|_{s=s(y)} = \frac{e^{-(\pi+\gamma)t(y)}}{\mu y} \cdot \left[\pi y \cdot u_y(ys) - (\pi+\gamma)\lambda u_s\left(\frac{y_t}{\lambda}, s, s\right) + u_{ys}\left(\frac{y_t}{\lambda}, s, s\right)\mu y\right] \tag{A.2}$$
3. From Eq. (A.1)
$$u_s(y,s,s) = \pi \sum_{i=0}^{N} \int_{t=T_i}^{T_{i+1}} e^{-(\gamma+\pi)t} y_t u\left(\frac{s \cdot y_t}{\lambda^i}\right) dt = \frac{\pi}{s} \Psi(ys)$$
where
$$\Psi(y) = \sum_{i=0}^{\infty} \int_{T_i}^{T_{i+1}} e^{-(\gamma+\pi)t} \cdot y_t u_y\left(\frac{y_t}{\lambda^i}\right) dt$$
and solves $\Psi_y \cdot \mu y = (\gamma+\pi)\Psi - y \cdot u_y$ with boundary condition $\Psi(y^*) = \lambda \Psi\left(\frac{y^*}{\lambda}\right)$.
4. As a result
$$u_s\left(\frac{y}{\lambda}, s, s\right) = \frac{\pi}{s}\Psi\left(\frac{ys}{\lambda}\right)$$
$$u_{ys}\left(\frac{y}{\lambda}, s, s\right) = \pi \Psi_y\left(\frac{ys}{\lambda}\right)$$
which implies that
$$\begin{aligned}(\pi+\gamma)\lambda u_s\left(\frac{y}{\lambda}, s, s\right) - u_{ys}\left(\frac{y}{\lambda}, s, s\right)\mu y &= \frac{\pi\lambda}{s}\left[(\pi+\gamma)\Psi\left(\frac{ys}{\lambda}\right) - \Psi\left(\frac{ys}{\lambda}\right) \cdot \mu \frac{ys}{\lambda}\right] \\ &= \frac{\pi\lambda}{s} \cdot \frac{ys}{\lambda} \cdot u_y\left(\frac{ys}{\lambda}\right)\end{aligned}$$

5. Finally Eq. (6.8) implies that

$$ys \cdot u_y(ys) = \gamma u(ys) - p(ys)$$
$$ys \cdot u_y\left(\frac{ys}{\lambda}\right) = \gamma u\left(\frac{ys}{\lambda}\right) - p\left(\frac{ys}{\lambda}\right)$$

and if we substitute these expressions into (A.2) we get that

$$\frac{\partial^2 W}{\partial y^2}\Big|_{s=s(y)} = \frac{\pi e^{-(\pi+\gamma)t(y)}}{\mu^2 ys} \cdot \left[\gamma\left(u(ys) - \gamma\lambda u\left(\frac{ys}{\lambda}\right)\right) - \left(p(ys) - \lambda p\left(\frac{ys}{\lambda}\right)\right)\right] \cdot \left(-\frac{ds}{dy}\right)$$

and the required condition directly follows.

Appendix B

In this appendix, we show that even for unbounded distributions of signals there could exist a equilibrium and that this equilibrium could approximated by the corresponding bounded solutions equilibria.

Suppose, we have an unbounded distribution of signals. Denote by $s(y, \underline{s})$ and $\bar{p}(y, \underline{s})$ the beliefs and price functions describing the separating equilibrium for the case when we restrict it to $s \geq \underline{s}$.

We will assume that the prior distribution of signals satisfies (which is true for lognormal distribution)

$$\frac{s \cdot f(s)}{F(s)} \to +\infty \qquad as \qquad s \to 0$$

The probability of the split happening in the next moment is:

$$\frac{Prob\{s(s_{t+dt}) < s < s(y_t)\}}{Prob\{s < s(y_t)\}} \approx \frac{sf(s(y))}{F(s(y))}\left|\frac{d\log s}{d\log y}\right| \cdot \mu dt$$

As we show below $\frac{d\log s}{d\log y} \approx -1$ for small s. Given the condition on the distribution function, we get:

$$\phi \to \infty \qquad as \qquad s \to 0$$

This means that when signals are very small at any point of time, investors believe that the split will almost surely happen in the next moment. Then the price that they are paying for the stock should be equal to the current after-split price:

$$\bar{p}(y) = \lambda p\left(\frac{s(y)\cdot y}{\lambda}\right) \tag{B.1}$$

If the we substitute (B.1) into (6.16), the last term drops out and we get:

$$\frac{ds}{dy} \cdot \frac{y}{s} \cdot \lambda\Pi\left(\frac{ys}{\lambda}\right) \cdot \mu = \pi\left(\lambda u\left(\frac{sy}{\lambda}\right) - u(sy)\right)$$

Denote $z = sy$. Then taking into account that $\frac{s_y}{s} \cdot y = \frac{d\log z}{d\log y} - 1$ we can rewrite the above equation as:

$$\frac{d\log z}{d\log y} = \frac{\pi\left(\lambda u\left(\frac{z}{\lambda}\right) - u(z)\right)}{\lambda\Pi\left(\frac{z}{\lambda}\right)\cdot\mu} + 1$$

Let us describe the solution $z(y, \underline{s})$ for the distribution restricted to $s \geq \underline{s}$. At the boundary condition (the upper bound), we have:

$$z(\underline{y}, \underline{s}) = \underline{s} \cdot \underline{y} = y^*$$

since $\lambda u\left(\frac{y^*}{\lambda}\right) - u(y^*) = 0$, the corresponding derivative is:

$$\frac{d\log z}{d\log y}\Big|_{y=\underline{y}} = +1$$

Thus, $z(y, \underline{s})$ is increasing in y, and the further we deviate from the upper bound $\underline{y}$ the smaller is the function. However, we can never go beyond the point z^* at which the right hand side of the derivative is 0:

$$\frac{\pi\left(\lambda u\left(\frac{z^*}{\lambda}\right) - u(z^*)\right)}{\lambda\Pi\left(\frac{z^*}{\lambda}\right)\cdot\mu} - 1 = 0$$

The larger is the distance between the current point $log(y)$ and the upper bound, $log(\underline{y}) = log(y^*) - log(\underline{s})$, the smaller is $z(y, \underline{s})$ and the closer we are to the fixed point z^*. Which means that, in the limit, when $\underline{s}$ becomes very small:

$$z(y, \underline{s}) \to z(y) = z^* \qquad as \qquad \underline{s} \to 0$$

The above convergence was derived assuming that investors always believe that split will happen immediately. This is not exactly precise in the actual equilibrium:

$$z(y, \underline{s}) \to z(y) \approx z^* \qquad as \qquad \underline{s} \to 0$$

but the smaller are the signals we are considering (or equivalently the larger is y), the more precise it would be. As a result, in the limit:

$$z(y) \rightarrow z^* \qquad as \qquad y \rightarrow \infty$$

or equivalently:

$$s(y, \underline{s}) \cdot y \rightarrow z^* \qquad as \qquad \underline{s} \rightarrow 0.$$

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