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Unlocking the Riddles of Imperial Greek Melodies: The ‘Lydian’ Metamorphosis of the Classical Harmonic System

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Abstract

Building upon Lynch 2022a and 2022b, this article offers the first account of the historical evolution of the Greek harmonic system and notation keys (*tónoi*) that bridges the gap between Classical and Imperial music. This new solution allows us to reconstruct, for the first time, a continuous, if evolving, tradition that stretches from Euripides’ *Orestes* to late antiquity, reconciling key theoretical insights provided by Ptolemy, Porphyry and others with documentary evidence that illustrates the structure of the Imperial harmonic system and its use in the Imperial musical documents (*dDAGM*). This approach also enables us to trace the gradual expansion of the Greek notation system from an initial set of symbols (Α–Ω) to the full array recorded by Aristides and Alypius, mapping its development onto key historical milestones including the revolutionary innovations of the New Musicians and Damon of Oa’s inclusion of the Lydian mode into the Greek modulation system.

Keywords

Harmonics – Imperial musical documents – Canon diagram – Ptolemy – Porphyry – Theon – Damon of Oa – Greek notation

A chronological gap of about two centuries separates the Greek musical documents known to date into two distinct groups: Late Classical/Hellenistic

documents on the one hand, Imperial and Late Antique documents on the other. As we emerge from the darkness that currently enfolds the turn of the era, we are faced with a tectonic shift that transformed the most distinctive features of ancient Greek melodies as well as the notation keys that were employed to write them down, and in so doing fundamentally reshaped the structure and organisation of the Imperial harmonic system.

In contrast with the elaborate chromaticism of late Classical and Hellenistic pieces,¹ Imperial Greek music is in fact largely diatonic with the occasional chromatic twist, and displays a fondness for thirds and fifths that undermined the structural primacy that the fourth enjoyed in the Classical system.²

This major change in style is paralleled by an equally significant shift in the set of notation keys that Imperial musicians used to write their music. As shown in Fig. 1, Imperial musicians abandoned the Classical triad of Dorian-Phrygian-Lydian keys that shaped Classical and Hellenistic music and settled on a different set of keys: Lydian, Iastian and, less frequently, Aeolian (Fig. 1).³

These harmonic and melodic transformations are also accompanied by an important innovation in notation convention. If repeated notes were simply omitted from Classical and Hellenistic songs, they are instead written down in full in Imperial scores, which regularly feature strings of identical notation signs standing over subsequent syllables.⁴ This change in notation convention went hand in hand with an increasing use of complex melismas.⁵

This article offers a new solution to the puzzles raised by these profound shifts, reconciling the practical evidence offered by the Imperial musical documents with key insights preserved by theoretical and technical sources. Whenever appropriate, this discussion will be supported by data concerning the distribution of the notes that are attested in the Imperial musical documents. This dataset⁶ is produced by a new database (*dDAGM*) that collects the musical notes attested in the standard edition of the Greek musical fragments (*DAGM*), for a total of over 3,500 notes.

1 See Lynch 2022b, with further bibliography.

2 Cf. West 1992, 383–90 (“In later Antiquity Greek music became fully diatonic and also less tetrachordal, more addicted to the third as the mid-step of the fifth.”).

3 As shown in Fig. 1, the Iastian triad is attested in full in Imperial Greek music, alongside the Lydian, Hypolydian and Hyperaeolian keys: cf. Pöhlmann and West 2001, 59. For the sake of methodological rigour, Fig. 1 reproduces the key distribution provided by Hagel (2010, 54, Diagram 14), even though the identification of the sub-key employed in a given piece (e.g. Lydian vs Hypolydian) is, at times, a matter of interpretation, and can therefore change in the light of different theoretical frameworks. On the Imperial dating of *DAGM* 17–18, see Lynch (2022a, 400, n. 38) with further bibliography.

4 Cf. the Seikilos song (Fig. 8 below).

5 Cf. West 1992, 202 and 254.

6 Cf. Appendix 3.

Greek Musical Documents

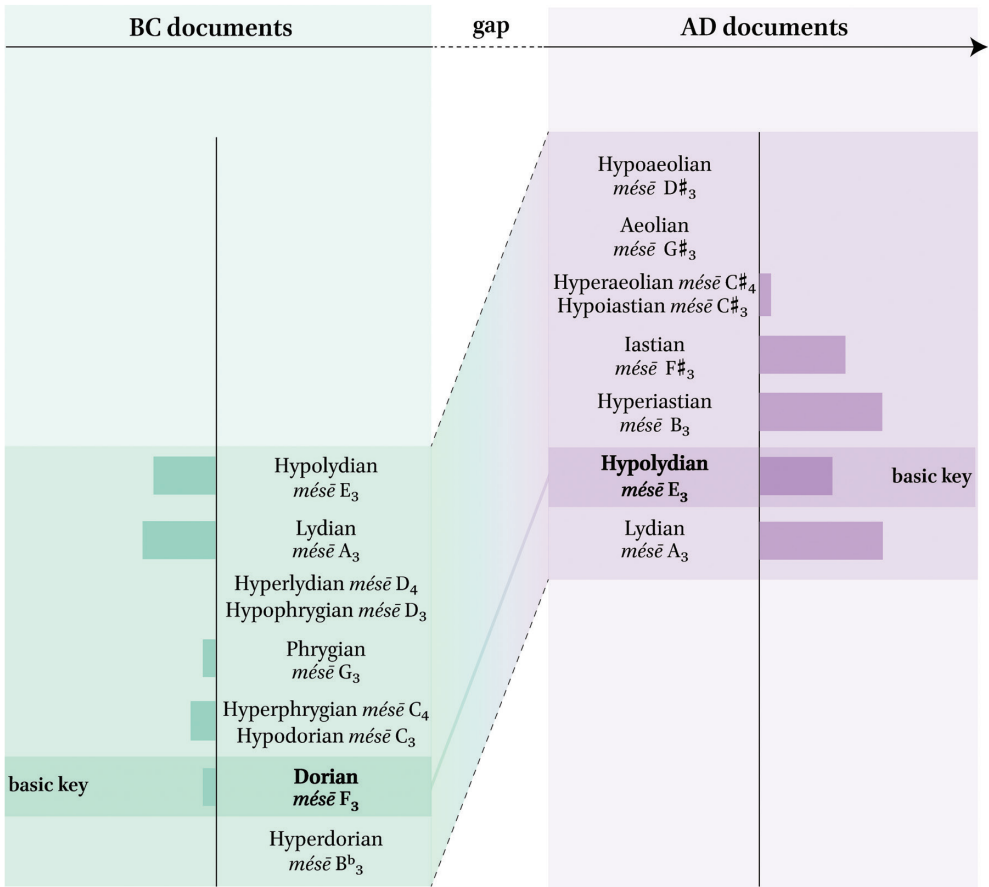


FIGURE 1 The Lydian metamorphosis of the Imperial harmonic system. Imperial musicians abandoned the Classical triad of keys – Dorian, Phrygian and Lydian (Lynch 2022a) – and moved to a new system based on the Lydian, Iastian and Aeolian keys (cf. n. 3)

1 Lydian Keys for Dorian Gates: The Imperial Metamorphosis of the Classical Harmonic System

As discussed in Lynch (2022a), the gradual development of a professional notation system in late Classical times had a problematic knock-on effect, namely the dissolution of a neat and systematic correspondence between the names of the Classical Greek modes (*harmoníai*) and the relative notation keys (*tónoi*). Whilst the Dorian and Phrygian modes intuitively corresponded to the like-named keys, the traditional Lydian mode (*Lydisti*) was surprisingly

assigned to the Hypolydian notation key (*tónos*) and the Tense Lydian mode (*Syntonolydistí*) to the Lydian notation *tónos*.⁷

Lynch 2022a showed how this odd mismatch resulted from a very practical problem faced by late Classical musicians, namely the need to identify a set of terms that would allow them to label consistently all the notation keys that were employed in highly modulating compositions typical of the New Music. These pieces included the traditional modes, and the relative keys, as well as two additional scales that were introduced in the fourth century BC and belonged to the high-pitched, 'hyperbolic' region of the voice set above the Classical central octave (C_3 – C_4).⁸

As shown in §4 below, the highest of these 'new' scales is the Hyperlydian *tónos* – a key that has no parallels in the Classical set of modes and is centred on an unprecedentedly high 'intermediate note' (*mésē* D_4). This note is a fourth higher than the traditional Tense Lydian mode (*mésē* A_3) and sits above the upper limit of the Classical central octave (C_4), making it a 'hyperbolic' tuning.⁹ The Hyperlydian *tónos* is also the highest key that is included in the Greek notation system as a whole. In keeping with its late Classical origins, the Hyperlydian *tónos* featured a number of new musical signs that were expressly devised for this key.¹⁰

It was the need to account for this new 'hyper' scale that produced the anomalous mismatch between the traditional names of the Lydian modes and the relative notation keys. This new hyper-key was a fourth higher than the traditional

7 See Lynch 2022a, esp. 386f., with further bibliography.

8 Cf. *Anon. Bell.* §64: 'The hyperbolic region of the voice is the whole range that stretches beyond the Hypermixolydian', i.e. above Dorian C_4 Γ. Thanks are due to Stefan Hagel for providing the Greek notation fonts employed in this article.

9 Cf. Lynch 2022b.

10 The very shape of these new signs bears witness to their novel character. The vocal sign that represents the new *mésē* of the Hyperlydian key, D_4 U, is clearly an inverted form of Ω – the note that marks the lower boundary the Dorian central octave (C_3). Hyperlydian *mésē* D_4 U is also the first sign that was added above the central set of vocal signs that runs from A to Ω (Fig. 16 below). In similar vein, the instrumental sign that represents Hyperlydian *mésē*, D_4 L, is the first inversion of the sign that marked the top note of the Dorian central octave (*Nétē* $N C_4$; Hagel 2010, 16). In keeping with this, Hyperlydian *mésē* D_4 U L also corresponds to the first string that Philoxenus added to the traditional kithara octave tuning in order to extend its modulating capabilities, inaugurating the 'hyperbolic' tetrachord: cf. Lynch 2018, 319–22, and Lynch 2022b on Athenaeus' *Paeon*. The instrumental triplet of *Nétai* is completed by the sign ΥE_4 , which is set a full tone above the new note D_4 L and therefore identified D_4 L as the *mésē* of the new Hyperlydian key (cf. Cleonid. *Harm.* 201.18–20 Jan, where *mésē* is defined as the note that lies 'below the disjunctive tone' that separates the main tetrachords of the central octave of each *tónos*). These 'new' Hyperlydian signs are employed, for instance, in Limenius' *Paeon* (*DAGM* 21).

Tense Lydian mode (*Syntonolydistí, mēsē A₃*), which was in turn a fourth higher than the traditional Lydian mode (*Lydistí, mēsē E₃*). The new ‘hyperbolic’ scale was therefore identified with the top key of the Lydian set, the Hyperlydian (*mēsē D₄*). As a consequence, the Tense Lydian mode came to correspond to the Lydian notation key (*mēsē A₃*), and the lowest key, the Hypolydian (*mēsē E₃*), ended up representing the traditional Lydian mode (*Lydistí*).

But this mismatch between the denomination of the traditional Lydian modes and the corresponding notation keys did not affect the structure and relative pitch of the modes. As shown in Fig. 2, the defining traits of the traditional Lydian modes were reflected by the distinctive ‘shapes’ (*eídē*), or species, of the octave that were characteristic of the relative notation keys.¹¹

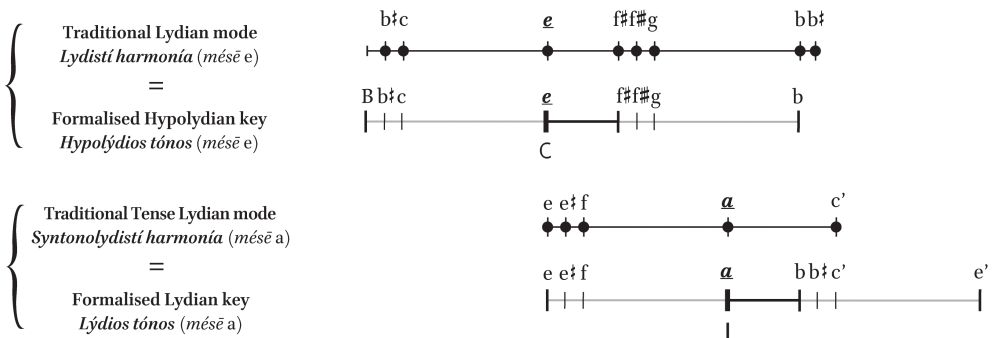


FIGURE 2 Shifts in the designations of the traditional Lydian modes (*harmoníai*) and the relative notation keys (*tónoi*), enharmonic forms *exempli gratia*. Half-sharp signs indicate quarter-tone intervals, sharp-and-a-half signs raise the relative note by three quarter-tones

Broadly speaking, the traditional denominations of the modes (*harmoníai*) that were current in Classical times were gradually replaced in technical literature by the names that professional musicians assigned to the relative notation keys (*tónoi*). But there are a few notable exceptions to this trend,

11 Cf. Lynch 2022a, 389–92. The formal structure of the Tense Lydian mode – one of the scales featured in the earliest modulating system ascribed to Pronomus – implicitly assumes the existence of the Hyperlydian key, which is required in order to identify the top note of this scale c' with Dorian *nētē* N (C_4): see Lynch 2022a, Fig. 5. In similar vein, the structure of the Classical *Lydistí* mode, which includes a note a quartertone higher than the upper boundary of the *Lýdia* octave (B_2 – B_3), implies the existence of the Lydian notation key (Lynch 2022a, Fig. 6). Hence none of the traditional Lydian modes corresponded neatly to a single notation key, and notating these scales in their fully modulating versions required a combination of signs belonging to several Lydian keys. The challenges raised by the integration of the *Lydistí* mode into the Classical harmonic system are summarised in §4 below.

the most important of which is represented by Aristides Quintilianus' discussion and transcription of the *harmoníai* 'employed by the very ancients ... the same modes that the divine Plato too mentions in the *Republic*'.¹² Aristides' account of the Classical modes derives from pre-Aristoxenian sources¹³ and confirms the historical development of the Lydian keys summarised above. In keeping with the traditional identification of the simple Lydian mode *Lydistí* as the 'basic' Lydian tuning, Aristides describes it as '*the Lydian system*'¹⁴ without further qualifications and highlights its theoretical primacy over the other Lydian modes – a primacy that is reflected by its 'perfect', or 'complete', octave structure.¹⁵

An equally exceptional account of the features of the ancient modes is offered in a passage of Ptolemy's *Harmonics* that describes the special patterns of attunement (*harmogaí*) used by Imperial kitharodes. Just as the Classical *harmoníai* preserved by Aristides represent a pre-Aristoxenian account of tunings that were regularly employed by late Classical aulos players, so also Ptolemy's account of the Imperial *harmogaí* is an accurate representation of 'named systems of attunement that were in regular contemporary use' in Alexandria in the second century AD.¹⁶

This fortunate combination of reliable, and complementary, sources of evidence allows us to shed light on the metamorphosis that turned the Classical harmonic and modal systems into their Imperial counterparts, and went hand in hand with the changes in melodic style, notation keys and notation convention summarised at the start of this article.

Ptolemy's account, however, presents us with what seems like a paradox. Without a hint of irony, Ptolemy identifies what he had called Dorian *tónos* all along – i.e. the central point of reference of the Classical harmonic system, which Ptolemy adopted as the basic key of his *Harmonics* – with a tuning that

12 Aristid. Quint. *Mus.* 18.5–19.3, with Fig. 14 below. On the Classical modes and their practical counterparts, see Lynch 2022a and Lynch 2022b.

13 Cf. Barker 2020, 260.

14 *Sýstēma*, lit. 'combination of intervals'. On the relationship between the traditional modes and the technical concept of *sýstēma*, cf. n. 27 below.

15 Aristid. Quint. *Mus.* 18.10–13: '*The Lydian system* (τὸ μὲν οὖν λυδίον σύστημα) consists of a diesis, a ditone, a tone, a diesis, a diesis, a ditone and a diesis; and this was indeed a perfect/complete system (τέλειον σύστημα). On the octave as the most perfect interval, and 'the unit of measurement of melody as a whole', see e.g. [Arist.] *Pr.* 19.35a and 19.39, discussed in Lynch 2020. Aristides highlights the derivative nature of the Tense Lydian mode at the end of the same paragraph: 'the system that is called Tense Lydian (τὸ δὲ λεγόμενον σύντονον λυδίον) consists of a diesis, a diesis, a ditone and three semitones' (Aristid. Quint. *Mus.* 18.23–5).

16 Cf. Barker 2001, 257; see also Barker 1989, 272.

Imperial kitharodes called *Lýdia*, literally ‘the Lydian tetrachords’ (Ptol. *Harm.* 80.14–15). Ptolemy’s good faith is evident, as he elsewhere invites his readers to test the effectiveness and accuracy of his scientific approach by setting up and checking these tunings for themselves. So what is going on here?

A surprisingly simple solution to this long-standing puzzle¹⁷ is offered by the shift in the denomination of the Classical Lydian modes summarised above, and the consequent identification of the simple tuning *Lydistí* with the Hypolydian notation key. In keeping with the Classical model, the Imperial label *Lýdia* referred to the same octave range covered by the Classical *Lydistí* tuning (B₂–B₃), i.e. the Hypolydian key. As noted by Ptolemy, however, the central tuning of the new Imperial system, *Lýdia*, was shaped in accordance with the Classical Dorian mode. In other words, the Imperial tuning called *Lýdia* was set in the Hypolydian key (B₂–B₃) – a correspondence that is explicitly flagged by Porphyry’s *Commentary* to Ptolemy’s *Harmonics*, as we shall see in §2. Hence the Classical Dorian mode, centred on *mésē* F₃, turned into the Imperial *Lýdia* tuning and was notated by the relative notation key, the Hypolydian *tónos* (*mésē* E₃; Fig. 3).¹⁸

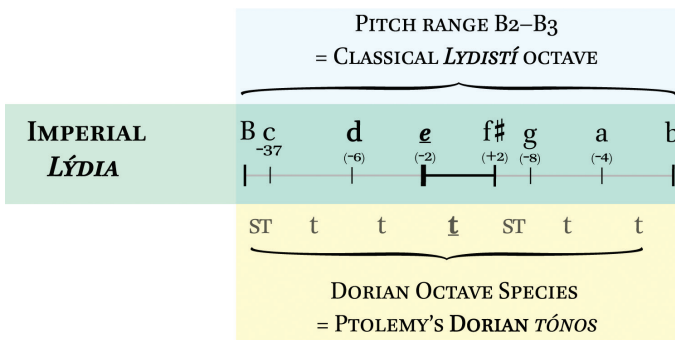


FIGURE 3 The core structure of the Imperial tuning *Lýdia*: a Dorian octave species (St-T-T-T-St-T-T) set in the *Lydistí* octave range B₂–B₃

17 Cf. Barker 1989, 360: “But *Lydia* is in Dorian. There are serious puzzles here”.

18 The identification of Dorian *mésē* with the modern note F₃ and Hypolydian *mésē* with E₃ depends on the structural identification of Lydian *mésē* with modern A₃ first suggested by Hagel 2010 (see esp. 452f.), and confirmed by Lynch 2022a–b. Some evidence about the absolute pitch of the Greek reference note A₃ is now provided by the scale produced by the Hellenistic *Koilē plagíaulos*. Unlike double-pipes, flutes are a fairly reliable source of evidence for the historical reconstruction of absolute pitch standards (Haynes 2002, 7–9). The *Koilē* flute is exceptionally well-preserved and is made of bone, not metal, making the scope for pitch variation fairly limited (Psaroudakēs 2012, esp. 523f). The scale played by the *Koilē* flute can therefore be reconstructed very accurately (Terzēs 2020), and we can take the frequency of its A₄ ~432 Hz as a reliable point of reference. This, of course,

The Hypolydian notation key, therefore, effectively replaced the Classical Dorian as the basic point of reference of the Imperial harmonic system. This change is in many ways akin to a small adjustment in concert pitch which has many parallels in the history of Western music,¹⁹ but had major implications for the wider structure and organisation of the Imperial harmonic system, as we shall see below.

For now, it is important to note that the new foundational role of the Imperial Hypolydian key is fully evidenced by the distribution of the notes attested in the Imperial musical documents. As shown in Fig. 4, 99.1% of the notes attested in Imperial scores do, in fact, fall within the range of the Hypolydian *tónos* E₂–E₄. In keeping with this overarching Hypolydian framework, 84.1% of these notes fall within the Hypolydian central octave B₂–B₃ and include the diatonic notes that form the conjunct as well as the disjunct tetrachords of this key – i.e. the tetrachords that, by definition, shaped the central octave of the Unchanging Perfect System described in theoretical sources.²⁰

does not imply that A₄ ~432 Hz was a standard that was strictly adhered to everywhere and at all times, but it nevertheless provides a historically solid starting point for modern assessments of the scales produced by ancient instruments. Lynch 2023c shows how applying this standard to the Louvre aulos allows us to identify its scales with well-known Imperial aulos tunings: the lower Phrygian and lower Iastian/Hypophrygian.

- 19 On modern pitch standards and their historical evolution, see Ellis 1880 and Haynes 2002. In the modern Western tradition, variations in concert pitch have largely occurred independently from the evolution of the harmonic system. On a few occasions, however, changes in concert pitch caused a re-interpretation of the harmonic value of the scales played by instruments that were designed in accordance with earlier standards: see, for instance, the harmonic ‘re-interpretation’ of Viennese trombones that occurred in the 1780s as a result of the introduction of the new *Wienerton*, ~438 Hz (cf. Haynes 2002, 320f.). In the case of Viennese trombones, the new standard resulted in a simple change of key, because modern Western modality is not directly linked to distinct notation keys. But Aristoxenus and other ancient sources show that the Greek modes were systematically linked to different notation keys – cf. Aristox. *Harm.* 46.17–20 Da Rios: ‘The fifth part is that concerning the *tónoi* in which the *systēmata* are placed when used in melody’, with Barker 2007, 223. Lynch (2022a and 2022b) showed how this link is evidenced by the extant musical documents, and was indeed a defining feature of Classical and Hellenistic Greek music. The significance of this link for the evolution of Greek notation is discussed in §4 below.
- 20 On the Unchanging Perfect System, cf. Fig. 11 and Fig. 12 below. The picture sketched by the Imperial documents is perfectly consistent with the Dorian-based system employed in Classical/Hellenistic music. As shown in Lynch 2022a, 100% of Classical/Hellenistic notes fall within the Dorian *tónos* range (F₂–F₄) and the notes that form the Dorian conjunct tetrachord (ΠΟ Ν/Κ Η) are regularly employed alongside those that belong to the disjunct tetrachord (ΜΛ Κ/Η Γ).

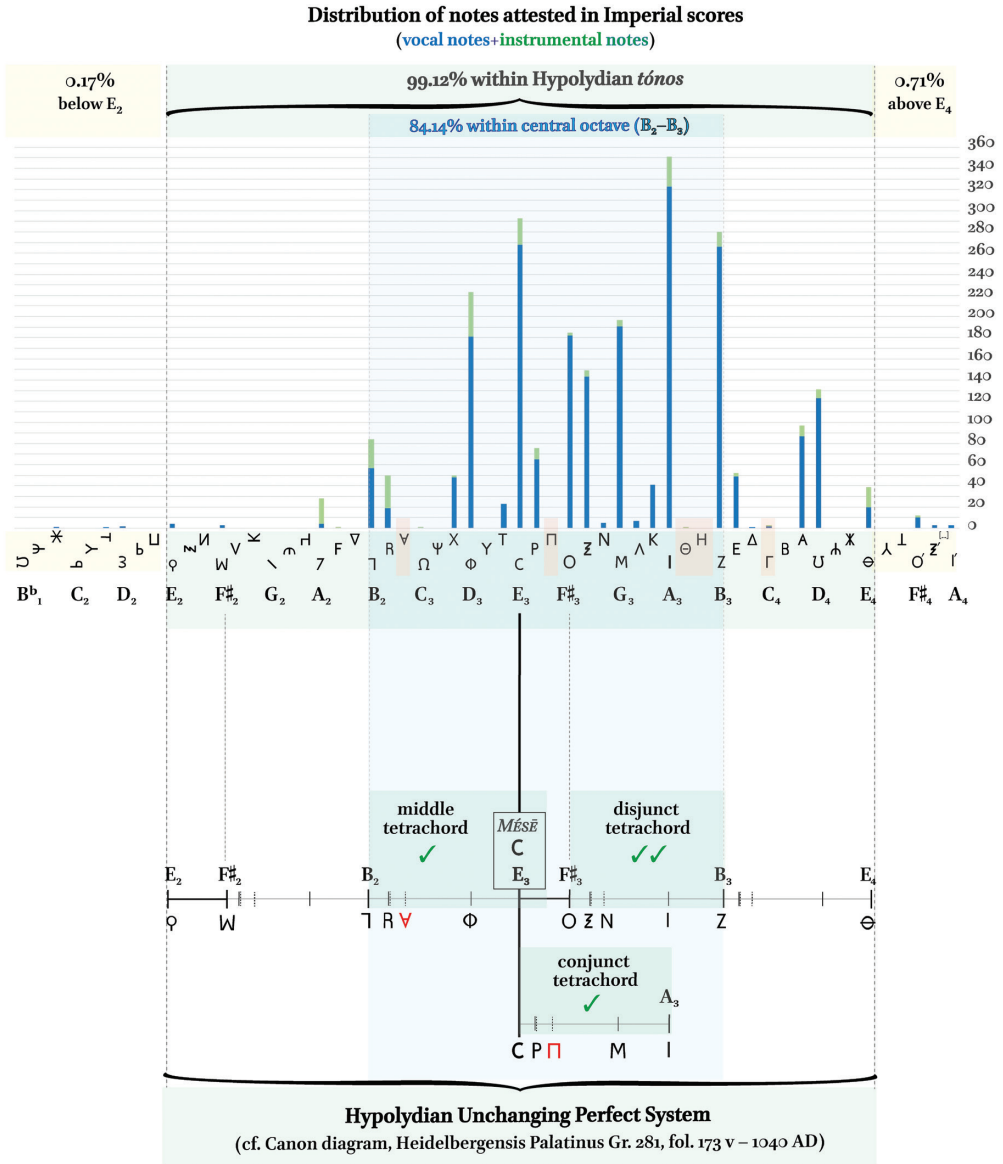


FIGURE 4 The vocal and instrumental notes attested in the Imperial musical documents, and the corresponding structure of the Hypolydian Unchanging Perfect System

The evidence offered by the Imperial musical documents set out in Fig. 4 therefore confirms that Hypolydian *mésē* C E₃ was the central point of reference of the Imperial harmonic system, given that the notes that form the conjunct Hypolydian tetrachord (CPMI) as well as those that form the disjunct

Hypolydian tetrachord (C – OŽIZ) are regularly attested. In keeping with the broadly diatonic character of Imperial music, the diatonic forms of these tetrachords are attested in full, whereas the chromatic variant is included only in the disjunct tetrachord.²¹

In addition to confirming that the Hypolydian *tónos* was the central key of the Imperial harmonic system, the distribution of Imperial notes shows that the Lydian-based hypothesis outlined by Hagel 2010 is structurally untenable. Had the Lydian *tónos* been the basic key of the Imperial system,²² the central note of the Imperial harmonic system should have corresponded to Lydian *mésē* A₃, and this note would have represented the origin of the conjunct and disjunct tetrachords that were by definition included in the central octave of the Unchanging Perfect System. As shown in Fig. 5, however, the notes that correspond to the conjunct tetrachord set above Lydian *mésē* | A₃ are almost entirely absent from the Imperial musical documents, making the Lydian Perfect System structurally incomplete.²³

21 This detail is, again, consistent with the Imperial *harmogai* preserved by Ptolemy, given that only one of these tunings – *Tropiká* – features a chromatic division of its upper tetrachord (Ptol. *Harm.* 80.11–18). Lynch (forthcoming) will show that this chromatic division corresponds to the disjunct Hypolydian *pyknón* OŽN; on the Classical version of this chromatic tuning, see Lynch 2022b, 447–53.

22 Cf. Hagel 2010, 57f.; 95f.

23 Cf. Appendix 3: vocal Θ is not attested (=0%); instrumental V may be attested once (=0.38%), but the reading is doubtful; H is unattested (=0%); the diatonic vocal note Γ may be attested twice in *DAGM* 46 (=0.09%), but one of the two readings is doubtful; the instrumental diatonic note N appears once (=0.38%). The absence of these defining Lydian notes is acknowledged by Hagel 2010 but is explained away on the basis of a questionable historical assumption, namely that “the late treatises, which regularly include the *synēmménon* tetrachord, cannot be taken as evidence of the musical practice of their times, of course” (Hagel 2010, 96). In contrast with the wording of this passage, Hagel’s assumption is far from self-evidently correct, but no justification is provided for this claim and the consequent wholesale dismissal of the evidence provided by Imperial theoretical treatises. The assumption of a *complete* separation between music theory and practice with regard to the most central feature of the Greek harmonic system is not only hard to justify, but would, in fact, make Imperial Greek music a unicum in the history of Western music, setting it apart from its two immediate historical neighbours as well – Classical/Hellenistic Greek music on the one hand, and early Mediaeval music on the other. In both cases, the practical evidence provided by extant scores is clearly linked to contemporary theoretical accounts (see Lynch 2022a and Lynch 2022b on Classical/Hellenistic Greek music; Atkinson 2009 and Christensen 2018 on Mediaeval music; more broadly, Christensen 2006, 6–17). Equally serious methodological issues arise from the fact that Hagel 2010 omits to print the full record of the notes that are attested in the Greek musical documents, i.e. the dataset used in the statistical analyses that inform his reconstruction of the Greek harmonic system, and only includes a number of selections and aggregations of the data that are explicitly designed to support particular

Distribution of notes attested in Imperial scores
(vocal notes+instrumental notes)

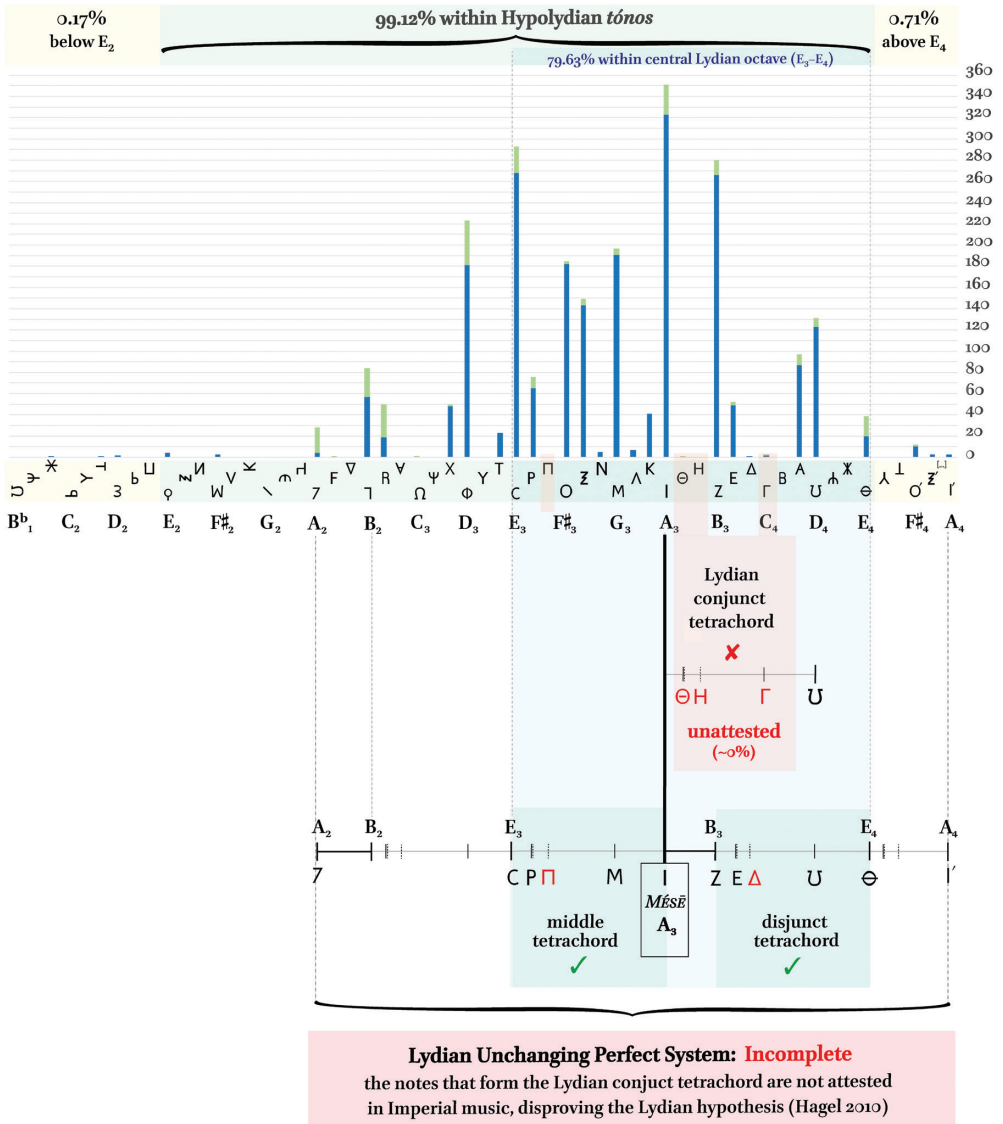


FIGURE 5 The Lydian hypothesis (Hagel 2010) is not consistent with the distribution of notes attested in the Imperial documents

The virtually complete absence of the conjunct Lydian notes marked in red in Fig. 5 shows that the Lydian key was not, and indeed could not, be the central key of the Imperial system, given that the central octave of the Perfect System by definition included both a conjunct and a disjunct tetrachord above its central *mésē* (cf. Fig. 11).²⁴ The notes that form the conjunct and disjunct tetrachords of the Hypolydian *tónos* are conversely attested on a regular basis in Imperial music (Fig. 4).

As we shall see in the coming sections, the clear Hypolydian structure evidenced by the musical documents is fully consistent with the theoretical characterisation of the Perfect System that is offered in Imperial technical sources.

2 'Mind the gap, please': Ptolemy, Porphyry, and the *Anonyma Bellermanniana*

As shown above, the Imperial Hypolydian key replaced the Classical Dorian and became the central point of reference of the new harmonic system employed by Imperial musicians. As one would expect, Ptolemy highlights this shift in an attempt to bridge the gap that separates the Classical, Dorian-based theory that shaped his treatise from the Imperial, Hypolydian-based tunings that were familiar to his readers.

To this end, Ptolemy devotes the final section of Book 2 of his *Harmonics* to a detailed analysis of six tunings that were 'played on the *lyra* and the *kithára*'. In this passage, Ptolemy points out that the Classical Dorian *tónos* he had just reconstructed from a purely theoretical point of view corresponded to the tuning that contemporary kitharodes called *Lýdia*, his Phrygian *tónos*

arguments (e.g. Diagrams 18, 71f.). The full dataset has not been released in Hagel's subsequent publications, or in any other format, to date – a lack of transparency that clashes with the ostensibly scientific methodology adopted in Hagel 2010, and the relative professional standards followed in STEM publications to ensure reproducibility. Given that this dataset was not otherwise available to the scholarly community, this omission effectively prevented other scholars from assessing the full set of criteria and interpretative choices that informed Hagel's presentation and reconstruction of the evidence. This dataset was first published in Lynch 2022a (Appendix 3–4, and Figs. 7–10, available in open access; cf. Appendix 3 below) and enabled a new assessment of the evidence at our disposal. The data relative to the Classical and Hellenistic documents is now also available in an interactive digital format (*dDAGM* App).

24 This point is further supported by the selected evidence of Imperial Hypolydian and Lydian scores—see Lynch 2023b.

corresponded to the Imperial tuning called *Hypértropa*, his Hypophrygian *tónos* to the Imperial *Iástia*, and so on.²⁵

In his commentary on Ptolemy's *Harmonics*, Porphyry confirms that Ptolemy's Dorian tuning, *Lýdia*, was in fact notated by means of the foundational key of the Imperial harmonic system, the Hypolydian:²⁶

εἰδέναι δεῖ καὶ τοῦτο, ὅτι οἱ κιθαρῳδοὶ τετράσι τόνοις ὡς ἐπὶ τὸ πλεῖστον ἐχρώντο, τῷ Ὑπολυδίῳ, τῷ Ἰαστίῳ, τῷ Αἰολίῳ καὶ τῷ Ὑπεριαστίῳ.

PORPH. in *Harm.* 156.8–10

It is also necessary to understand this, that kitharodes made use of four keys most of the time: the Hypolydian, the Iastian, the Aeolian and the Hyperastian.

- 25 Ptol. *Harm.* 80.11–18, translated in Barker 1989, 356. The tunings described by Ptolemy, and their relationship to the Imperial musical documents, will be discussed individually in Lynch forthcoming. On the foundational role played by the *Lýdia* tuning, and its equivalence to the central octave of the Dorian *tónos*, see Barker 1989, 360–361, and Fig. 3 above. On *Hypértropa* and *Iástia*, see Lynch 2023c, Figs. 20–21, which show that these tunings correspond to the basic scales played by the Louvre aulos in its Imperial setting. These diagrams also show that the Louvre aulos could produce the modulating tuning *Lýdia* by half-stopping two finger-holes. On the modulating character of *Lýdia* and *Iástia*, see Ptol. *Harm.* 39.6–40.20.
- 26 Hagel (2010, 57f.), in contrast, assumed that the kithara tuning called *Lýdia* corresponded to the Lydian notation *tónos*. On these grounds, Hagel dismissed Porphyry's identification of *Lýdia* with the Hypolydian key as “consistent both with Ptolemy's text and the notational system” but “nevertheless wrong” (2010, 65). Hagel attempts to explain what he sees as “Porphyry's error” by arguing that “towards the end of the second century things might have changed considerably” (2010, 68). In this passage, Hagel assumes that a second structural change took place in Imperial times and made the Late Imperial harmonic system substantially different from the standard Imperial system used by the kitharodes mentioned by Ptolemy. But the second change is nowhere mentioned in the sources, a fact that would be at the very least surprising if another, and seemingly more radical, overturn of well-established musical conventions actually took place in Imperial times. Lynch 2016 showed how similar assumptions concerning allegedly radical, but mysteriously unattested, changes in the meaning of the ancient rhythmical terms *arsis* and *thesis* prevented modern scholars from appreciating the continuity between Classical Greek rhythmical theory and its Imperial Greek and Latin counterparts, as well as the differences between the distinct terminologies developed by ancient rhythmicians and metrists respectively. In similar vein, the alleged inconsistencies identified by Hagel 2010 are a by-product of an unintentional merging of two distinct sets of terms employed in the sources: the traditional terms *Lydistí/Lýdia* that identified different tunings (*harmoníai*), and the new technical labels that professional musicians coined in late Classical times in order to identify the notation keys (*tónoi*) that they employed to notate the traditional tunings as well as their new, ‘hyperbolic’ scales. See §4 below.

In keeping with the centrality of the Classical Dorian *tónos*, Porphyry begins his list with the equivalent central *tónos* of the Imperial system, the Hypolydian (*mésē* E₃). And just as the Classical modulation system was based on the Dorian, Phrygian, and Lydian keys, Porphyry continues by adding the Imperial counterparts of the other core *tónoi*: the Iastian key, which Aristoxenus called ‘lower Phrygian’ because it was a semitone lower than the Classical Phrygian key (*mésē* F#₃, a semitone below Phrygian *mésē* G₃); and the Aeolian key, which Aristoxenus called ‘lower Lydian’ for the same reason (*mésē* G#₃, a semitone lower than Lydian *mésē* A₃).²⁷

Porphyry’s list includes only the most common keys that were employed by Imperial kitharodes, and not all of the options that were available to them.²⁸ In keeping with this, his list is rounded off by the Hyperastian key (*mésē* B₃), which Aristoxenus called Higher Mixolydian.²⁹ Porphyry’s addition of a key related to the Mixolydian mode is not particularly surprising, and follows the historical evolution of the harmonic system outlined by Ptolemy. Ptolemy had, in fact, described the introduction of the Classical Lower Mixolydian *tónos* (*mésē* B^b₃) as the ‘first consonant modulation (*metabolé*)’ of the Classical harmonic system,³⁰ a ground-breaking innovation that was developed by late Classical musicians in order to expand the core modulation system that comprised three basic *tónoi* (Dorian, Phrygian and Lydian).³¹

Unlike the other keys listed by Porphyry, which are a semitone lower than their Classical counterparts, the Hyperastian *tónos* (*mésē* B₃) is a semitone *higher* than the Classical Lower Mixolydian. For this reason, Aristoxenus called the Hyperastian *tónos* ‘higher’ Mixolydian, and contrasted it with a

27 Aristox. *Harm.* 46.17–20 Da Rios, on the *sýstēmata* (cf. n. 19 above). The word *sýstēmata* (literally ‘combinations’ of intervals) is used in other technical works too in order to identify the structure of the traditional modes (*harmoníai*) – see esp. Arist. *Quint.* 15.19f., 18.5f., with Pl. *Phlb.* 17c11–d3. Aristoxenus’ testimony suggests that the full set of *tónoi* was completed in Hellenistic times, but the shift of the central notation *tónos* from the Classical Dorian to Hypolydian occurred much later – sometime around the turn of the era: cf. §4 below. A few hints of this impending transition are, however, provided by Athenaeus’ *Paeon* (128/127 BC): see Lynch 2022b, 453–5.

28 An alternative system is detailed in Fig. 9 below.

29 Aristid. *Quint. Mus.* 20.14–21.1; Cleonid. *Harm.* 203.8–10 Jan (‘there are two Mixolydian keys, a Lower one and a Higher one; the Higher of these keys is also called Hyperastian, whereas the Lower one is also known as Hyperdorian’).

30 Ptol. *Harm.* 62.18–63.1, with Lynch 2018, 317f.

31 Lynch (2022b) discusses the use of the Lower Mixolydian mode in the Dorian setting of the Ashmolean Papyri (DAGM 5–6), and its relationship to the Phrygian tunings employed in Athenaeus’ *Paeon* (DAGM 20). On the Lydian context, see §4 below.

‘lower’ Mixolydian key that was literally a semitone flatter than the ‘higher’ Mixolydian, and was notated by the Hyperdorian key (*mésē* B^b₃).³²

This shift in the relative position of the Mixolydian key had a significant impact on the underlying structure of the Imperial harmonic system. As shown in Fig. 6, the Higher Mixolydian *tónos* is now a *fifth* higher than the basic key of the Imperial system, the Hypolydian (*mésē* E₃), whereas the Classical Lower Mixolydian (*mésē* B^b₃) was a fourth higher than the central key of the Classical system, the Dorian (*mésē* F₃).

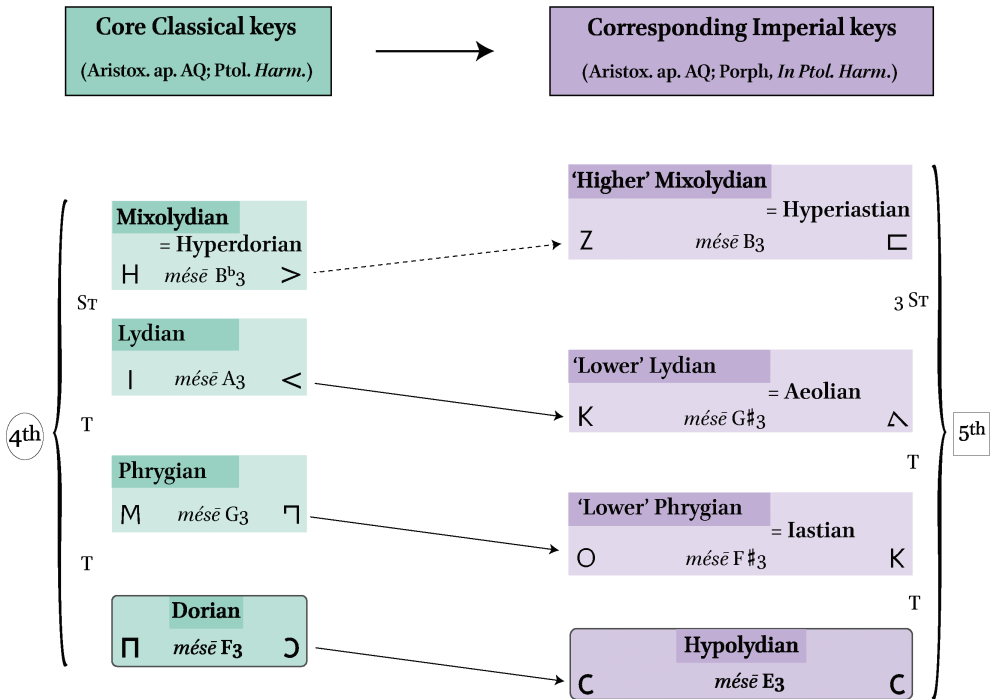


FIGURE 6 Classical keys and their Imperial equivalents (Porph. *in Harm.* 156.8–10). The Classical Dorian, Phrygian and Lydian keys are lowered by a semitone, whereas the key that corresponds to the Classical Mixolydian mode is shifted up by a semitone to the Higher Mixolydian/Hyperastian key

32 Cf. Cleonid. *Harm.* 203.8–10 Jan. The melodic hallmark of the Classical Lower Mixolydian mode – the characteristically undivided tritone F[#]₃–C₄ featured at the top of the scale – was likewise shifted upwards by a semitone (G₃–C[#]₄). This tritone A C[#]₄ – Z G₃ may be attested in a number of Hyperastian musical documents (*DAGM* 42.7 AŽA, 42.8 ŽA; *DAGM* 53, fr. 2.8 AŽ), but these readings are doubtful. On the structure of the Classical Mixolydian mode, see Fig. 7.

This structural change went hand in hand with a tectonic shift in the relationships between the modes of the new Imperial system. As shown in Fig. 7, the range covered by the Classical Lower Mixolydian mode coincided with the central Dorian octave (C_3 – C_4), and Dorian *mésē* ΠF_3 was the common tone that joined the Lower Mixolydian mode with the Dorian framework of the Classical system. The Higher Mixolydian mode was, in contrast, incompatible with the centrality of Dorian *mésē* ΠF_3 , but matched the structure of the new Hypolydian/*Lydistí* mode based on *mésē* $C E_3$.³³ As shown in Fig. 7, this second arrangement had a significant advantage over the Dorian-based one: the *mésē* of the Higher Mixolydian mode (B_3) corresponds to a *pyknón* of the basic Lydian mode, whereas the ‘silent’ *mésē* of the Lower Mixolydian mode (B^b_3)³⁴ had no such equivalents in the Dorian mode.

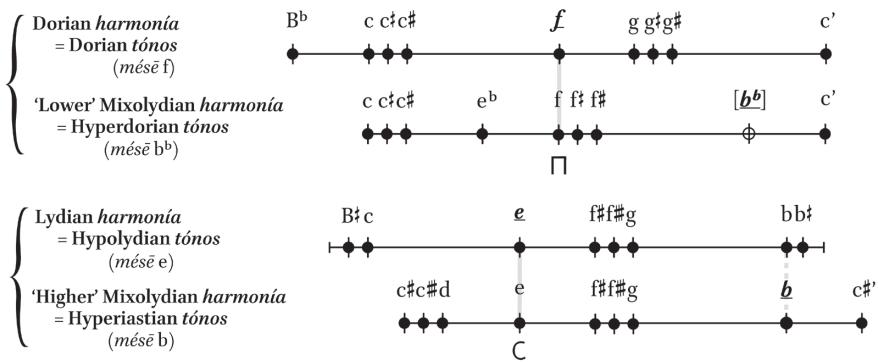


FIGURE 7 The changing alignment of the Mixolydian mode (*mésai* are marked in bold and underlined; half-sharp signs indicate quarter-tone intervals, sharp-and-a-half signs raise the relative note by three quarter-tones)

As discussed in §4 below, the modal ‘tensions’ created by the late-Classical co-existence of the Higher Mixolydian and Hypolydian keys eventually pushed the centre of the tonal system towards Hypolydian *mésē* $C E_3$, establishing the new harmonic setup detailed in Fig. 6. But this new arrangement of keys had

33 The new Higher Mixolydian/Hyperastian *tónos* is included in the 13-*tónoi* system attributed to Aristoxenus, but did not play a central role in the Classical/Hellenistic harmonic system (Lynch 2022b). The Higher Mixolydian variant is likewise included in one of the ‘archaic’ systems of keys detailed by Aristoxenus, but neither of these archaic systems featured the Hypolydian key (*Harm.* 47.1–16, with n. 74 below). The Hypolydian key is, in contrast, included in the ‘Platonic’ system detailed by Aristides (cf. Fig. 14 below). In keeping with this, the integration of the *Lydistí* tuning, and the relative Hypolydian notation key, within the Classical harmonic system is attributed to a legendary fifth-century music theorist, Damon of Oa, and marked a crucial step in the late Classical development of the Greek notation system: cf. §4 below.

34 On the ‘silent’ Mixolydian *mésē* first identified by Lamprocles, see Fig. 14 below.

another significant consequence: it structurally undermined the pivotal role that fourths had in Classical melodies and established the fifth as the underlying building block of the Imperial harmonic system.

The structural centrality of the Imperial fifth $C E_3 - Z B_3$ is reflected by the musical documents, where this interval is often used to define the central range of the tunings employed in a given piece.³⁵ This structural change also lent greater weight to thirds, which are now embedded within the basic framework of the Imperial harmonic system. As shown in Fig. 6, the semitone that separated the Classical Lydian and Lower Mixolydian keys is replaced by a 3-semitone gap in the Imperial system, i.e. the minor third $K G\#_3 - Z B_3$ that is defined by the intermediate notes of the Aeolian and Higher Mixolydian keys. The structural fifth $C E_3 - Z B_3$ is therefore divided into two prominent thirds, $C E_3 - K G\#_3$ and $K G\#_3 - Z B_3$, a feature that is echoed by Imperial melodies including the Seikilos song. As shown in Fig. 8, the basic structural fifth $C E_3 - Z B_3$ that is employed at the start of the Seikilos song is immediately followed by the minor third $Z B_3 - K G\#_3$, and these notes play a prominent role throughout the piece.

$C Z Z K I Z I$
 Ὀ-σον ζῆς φαί-νου,
 $K I Z I K O C O \Phi$
 μη-δὲν ὄ-λως σὺ λυ-ποῦ,
 $C K Z I K I K C O \Phi$
 πρὸς ὀ-λί-γον ἔσ-τι τὸ ζῆν,
 $C K O I Z K C C C X \Gamma$
 τὸ τέ-λος ὁ χρό-νος ἀπ-αι-τεῖ.

FIGURE 8

The Seikilos Song (*DAGM* 23), written in the Iastian *tónos*. This scale corresponds to the core octave of the Louvre aulos (cf. Lynch 2023c, Figs 19–21)

35 See e.g. the beginning of the Seikilos song (Fig. 8) and the Invocation to the Muse (*DAGM* 24), with West 1992, 200–10, Lynch 2023c, Fig. 1. On the structural prominence of the fifth $C E_3 - Z B_3$, cf. Hagel 2010, 287.

We now need to take a closer look at Porphyry’s characterisation of the keys represented in Fig. 6. In the passage quoted above, Porphyry describes these four keys as the *tónoi* that kitharodes ‘made use of most of the time’ (ὡς ἐπὶ τὸ πλεῖστον ἐχρῶντο), suggesting that other keys could occasionally be used alongside them. The Imperial documents confirm that this was indeed the case, as the Lydian key features prominently alongside the basic Hypolydian/*Lýdia* tuning. This alternative setup is described in the *Anonyma Bellermanniana*, where we read that ‘kitharodes tune their instruments to four keys: Hyperian, Lydian, Hypolydian, Iastian’.³⁶ The parallel existence of two alternative arrangements of keys is less surprising than it may appear from a modern point of view: as Aristoxenus tells us, two alternative systems of *trópoi* were employed in early Classical times too.³⁷ Fig. 9 represents the two Imperial alternatives side by side.

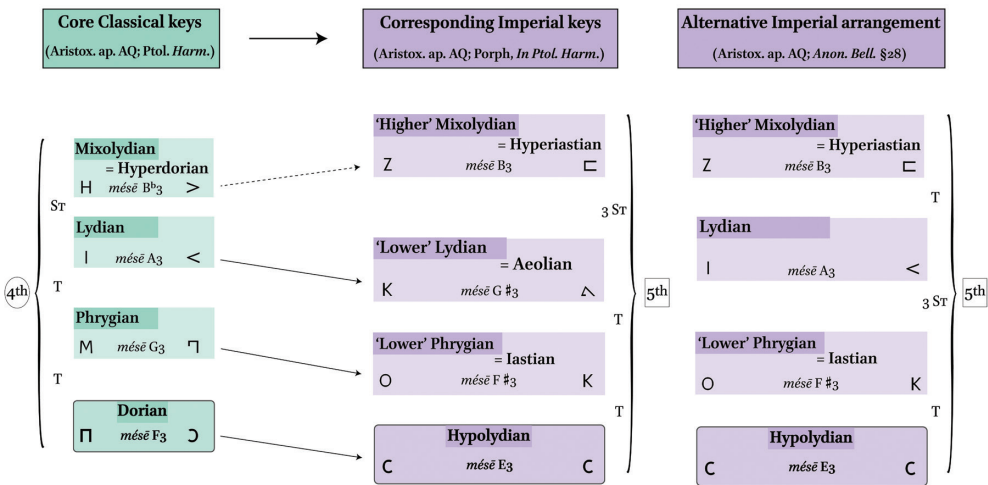


FIGURE 9 Alternative key arrangements employed in Imperial kithara music (Porph. *in Harm.* 156.8–10; *Anon. Bell.* §28.8–10)

The setup described by Porphyry is easier to map onto the Classical system because it features the Lower Lydian=Aeolian key (*mésē* G#₃), which is a semitone lower than the Classical ‘Tense’ Lydian key (*mésē* A₃). As shown

36 *Anon. Bell.* §28.8–10 Najock οἱ δὲ καθαρωδοὶ τέτρασι τούτοις ἀρμόζονται: ὑπεριαστίω, λυδίω, ὑπολυδίω, ἰαστίω.

37 Cf. Aristox. *Harm.* 47.1–15, with Barker 1989, 153f., Barker 2007, 297, and n. 74 below.

in Fig. 9, this downward semitone shift affected the other core keys of the Classical system too: the Classical Dorian key (*mésē* F₃) was replaced by the Hypolydian (*mésē* E₃), and the Classical Phrygian (*mésē* G₃) by the Lower Phrygian, or Iastian, key (*mésē* F#₃). As mentioned above, the upward shift of the Mixolydian to its Higher variant introduced a new element into the picture, producing a fifth instead of a fourth between the basic key of the Imperial system (Hypolydian, *mésē* C E₃) and the top one (Hyperastian, *mésē* Z B₃).

The alternative arrangement described in the *Anonyma Bellermanniana*, in contrast, retains the Classical Lydian key in its original position (*mésē* I A₃), while the other keys are identical to the setting described by Porphyry: Classical Dorian and Phrygian are shifted down by a semitone to Hypolydian and Lower Phrygian respectively, and the Classical Lower Mixolydian key is again shifted up by a semitone to the Higher Mixolydian. Just as in the first setting, this change produced an interval of a fifth between the basic Hypolydian key and the Higher Mixolydian, but this fifth differs from the previous one in its inner arrangement as the minor third is now placed in the middle of the basic fifth C E₃–Z B₃, and not at the top.³⁸

Taken jointly, these structural changes undermined the traditional connotations of the Classical *harmoníai*³⁹ and produced a new, fundamentally diatonic

38 Cf. e.g. *DAGM* 47.11: C O | O C O.

39 The prominence of different Classical modes, and the relative Classical keys, was strictly linked to the relative position of the disjunctive tone *mésē*–*paramésē*. In Classical Greek music, the central note of a given mode (*mésē*) and the tone above it (*paramésē*) identified the ‘tone of disjunction’ that separated the two basic building blocks of Classical scales (i.e. two tetrachords that spanned a fourth each). In keeping with this, the relative *mésē* of a given mode was the most frequent note of Classical pieces set in the relative family of keys (Lynch 2022a). But this conceptualisation became far less relevant in the Imperial system, which is fundamentally diatonic and rooted in fifths and thirds (West 1992, 390). Lynch forthcoming will show how this structural change assigned a greater melodic weight to the note set a tone below *mésē* than to the tone above it; for now, see Lynch 2023b, 3, where Hypolydian *paramésē* O is less frequent than Hypolydian *diátonos* Φ, and Lydian *paramésē* Z is less frequent than Lydian *diátonos* M. In keeping with this, Thrasyllus’ *Division of the Canon* (discussed in §3 below) does not include *paramésē* among the defining fixed notes that are established at the start of his division; see also the unique term *diápemptos* that is used in the *Koinē Hormasia* to indicate the note set a tone below the *mésē* of its Hypolydian scale (cf. Appendix 2B, and n. 51 below). It is also important to note that frequency is not a universal determinant of the structural importance of a note in modal music in general, as was instead the case for Classical Greek modality ([Arist.] *Pr.* 19.20, with Lynch 2022a). In Mediaeval plainchant, for instance, the reference note of a given mode (*finalis*) is not necessarily the most frequent note employed in the relative pieces (cf. Apel 1958, 135f., Wiering 2006, 59, and e.g. the chants transcribed in Atkinson 2009, 166).

system rooted in the structural fifth $C E_3 - Z B_3$, which replaced the Classical fourth $\Pi F_3 - H B^b_3$.

As we shall see in Lynch (forthcoming), the interplay of these harmonic settings defined the essence of Imperial Greek tunings and the character of different groupings. In particular, the first arrangement that features the 'lower Lydian'/Aeolian *tónos* was typical of Imperial pieces set in the Iastian and Aeolian keys – two concepts that are notably linked in the traditional kitharodic term *Iastiaiólía* preserved by Ptolemy.⁴⁰ In keeping with this, Aeolian *mésē* $G\#_3$ plays an important structural role in this first set of keys, as it divides the basic Imperial fifth into two thirds: $C E_3 - K G\#_3$ and $K G\#_3 - Z B_3$.⁴¹ The second setting, in contrast, retains the Lydian key at the same pitch of Classical Tense Lydian (*mésē* A_3), and therefore divides the structural fifth into two interlocking fourths: $C E_3 - I A_3$ and $O F\#_3 - Z B_3$. Lynch forthcoming will show how this second option is accounted for by the 'modulating' kithara tunings described by Ptolemy.⁴²

For the moment, however, it suffices to note that the foundational role of the Hypolydian *tónos* as the Imperial equivalent of the Classical Dorian key – the key that was the fundamental point of reference of the Classical harmonic

40 Cf. Hagel 2010, 59.

41 Cf. West 1992, 383: "Instead of the old tetrachordal structure there sometimes appears a clear sense of the major or minor triad (...) and of the importance of the interval of the third". The transcriptions of Ptolemy's *harmogái* to be offered in Lynch forthcoming show how the string corresponding to $I A_3$ was included in both systems, whereas $K G\#_3$ is not included in Lydian kitharodic tunings. So the difference in the role played by $I A_3$ was a matter of modal prominence: it was a primary degree in the harmonic setting that includes the Classical Lydian *tónos*, but a secondary one in the Iastian-Aeolian system – a modulating *sýstēma* that comprised more than one notation key. On the simple Iastian system, see e.g. the Seikilos Song, Fig. 8. On the concept of *sýstēma*, see n. 27 above.

42 The set of tunings preserved by Ptolemy is almost identical to the list of auletic keys detailed in *Anon. Bell.* §28, which also features the Phrygian/Hypophrygian *tónoi*. Lynch 2023c shows that this addition makes sense not only because of the traditional link between the aulos and the Phrygian mode, but also in the light of the scale produced by the Louvre aulos. In keeping with [Arist.] *Pr.* 19.18, the Louvre aulos scale is an octave higher than its kithara counterpart. This octave shift suggests that the Louvre aulos was a 'kitharistic' aulos (*kitharistérios*), also known as *mágadis* (Ath. 14.634e–f) after the special 'harp-like' effect produced by playing melodies in parallel octaves (*magadízein*). Unlike the note-for-note (*próschorða*) accompaniment that was typical of traditional lyre playing, 'kitharistic' auloi replicated the male register covered by kithara tunings an octave higher (cf. [Plut.] *Mus.* 1141b). This background may also shed light on Aristides' characterization of the Phrygian aulos as a 'feminine' instrument – a characterization that was also typical of harps and many-stringed *kithárai*, as opposed to lower-pitched, and therefore 'male', instruments such as the lyre and simple *kithára* (Aristid. *Quint. Mus.* 85.4–15).

system (Lynch 2022a) – is uncontested in both settings, in full accordance with the practical evidence of the Imperial musical documents.⁴³

3 The Lydian Foundations of the Imperial Harmonic System: The Canon Diagram and Thrasyllus' Division of the Canon

The central role of the Imperial Hypolydian *tónos* is confirmed by a unique diagram that is featured in the earliest codex which preserves ancient Greek music theory, and represents an ancient monochord (*kanón*).⁴⁴ As shown in Fig. 10, this diagram is labelled as 'the Canon' without further qualifications and comes immediately before a passage from Theon of Smyrna's *Mathematics Useful for Reading Plato* that describes 'the division of the Canon' (ἡ δὲ τοῦ κανόνος κατατομή) handed down by Thrasyllus.⁴⁵ This text appears under the heading 'division of the musician's Canon' (μουσικοῦ κανόνος κατατομή) on the facing page of the manuscript, and describes precisely how to slide the bridge of a monochord in order to set up the diatonic and chromatic notes included in the Canon diagram itself (Fig. 10).

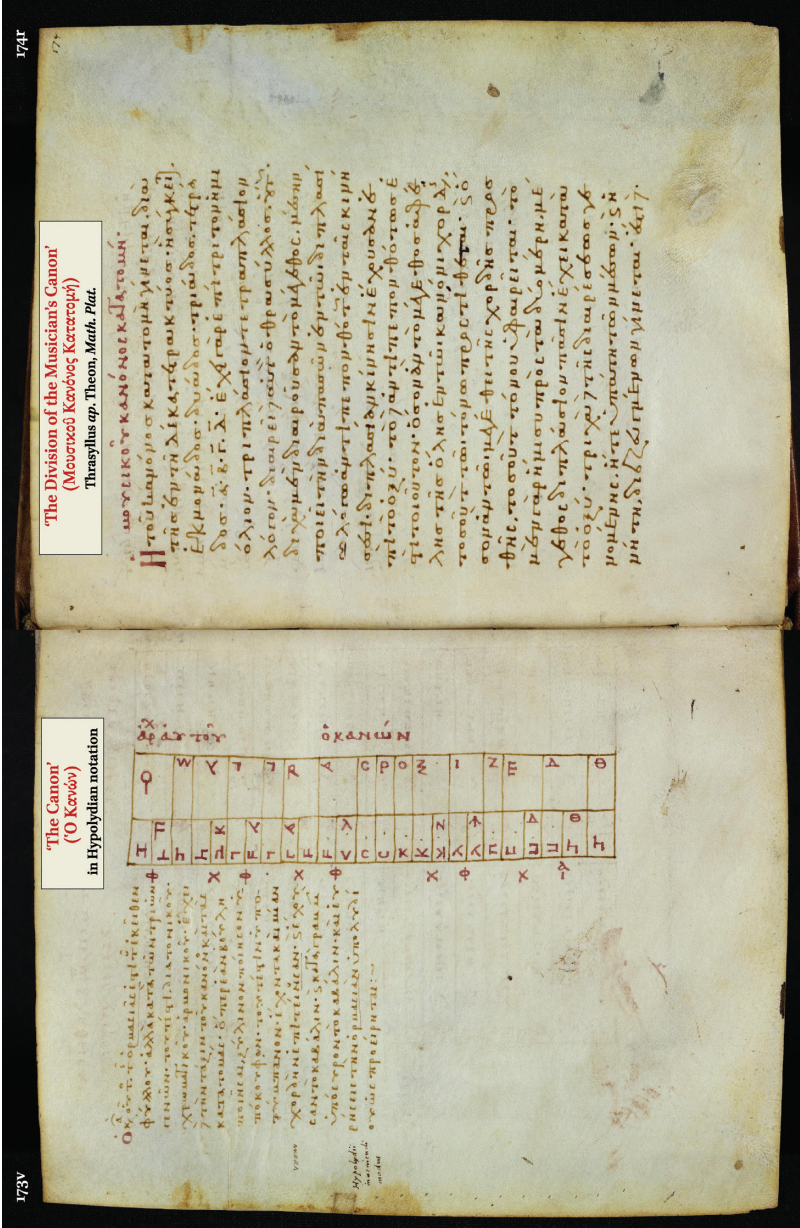
This document provides an effective visual illustration of the fact that the Hypolydian notation *tónos* replaced the Classical Dorian as the underlying point of reference of the Imperial harmonic system. As shown in Fig. 11, the Canon diagram in fact represents the Unmodulating Perfect System by means of the corresponding Hypolydian notes, starting from the lowest note of the system – Hypolydian *proslambanómenos* E₂, aptly labelled as 'the origin' (*arché*) in keeping with Theon's usage.⁴⁶ The diagram then adds the Hypolydian notation

43 Cf. Fig. 4 above.

44 Heidelbergensis Palatinus gr. 281, 173v (1040 AD – cf. Mathiesen 1992, 9–11). The Canon diagram, and the related 'Common Tuning' (*Koinè Hormasia* – Appendix 2A and 2B), appear in a few later codices such as *Monacensis* gr. 104.289r (1557 AD).

45 Theon *Math. Plat.* 87.4–93.9, with Barker 1989, 226–9; Creese 2010, 269–77; Petrucci 2012, 235–7.

46 Theon *Math. Plat.* 89.13–19. In Classical sources, the term *arché* was employed to identify the note *mésē*, and not *proslambanómenos*, because *mésē* was characterised as 'the beginning/origin' of lyre tunings ([Arist.] *Pr.* 19.44 and Arist. *Met.* 5.1018b28–29; cf. Lynch 2020, 130). As highlighted by one of the anonymous readers, this change in the identification of the 'origin' of the Imperial harmonic system reflects a significant step in the historical development of Greek musical theory, and has major implications for the history of Western music more broadly. Lynch forthcoming will show that this change goes hand in hand with the redefinition of the modal connotations of *mésē* in the Imperial system, which is far less dependent on its relationship to *paramésē* and is rather more strictly linked to the placement of the defining 'diatonic' notes of different tunings, especially the diatonic note set a tone below *mésē* itself (cf. n. 39 above).



'The Canon'
(Ο Κανών)
in Hypolydian notation

'The Division of the Musician's Canon'
(Μουσικῶν Κανόνων Κατανομή)
Thrasylus *ap.* Theon. *Math. Plat.*

FIGURE 10 The Canon diagram and the 'Division of the Musician's Canon' handed down by Thrasylus (Theon *Math. Plat.* 87.4–93.9). Heidelbergensis Palatinus gr. 281, foll. 173 v–174r

signs that are needed to define the Greater Perfect System as well as the Lesser Perfect System.⁴⁷ Both systems featured the so-called ‘additional’ tone at the bottom of the scale (*proslambanómenos* $E_2-F\#_2$) and two subsequent tetrachords – the tetrachord of ‘low’ notes (*hypatón*, $F\#_2-B_2$) and an ‘intermediate’ one (*mesón*, B_2-E_3) – but parted ways starting from ‘the middle note’ (*mésē* E_3). The Lesser Perfect System proceeded by joining a tetrachord directly to *mésē* E_3 , forming the so-called ‘conjunct’ tetrachord (*synēmménōn*, E_3-A_3). The Greater Perfect System, in contrast, included a tone of disjunction ($E_3-F\#_3$) and two further tetrachords, respectively known as ‘disjunct’ (*diezeugménōn*, $F\#_3-B_3$) and ‘hyperbolic’ (*hyperbolaión*, B_3-E_4). As shown in Fig. 11, these tetrachords include the relative diatonic and chromatic movable notes, in keeping with the division described in the text that follows the diagram.

As pointed out in the text that accompanies the Canon diagram,⁴⁸ a reader who proceeds to reproduce these notes on a monochord ‘shall find the *hormasía* of the Hypolydian, as anticipated’ (εὐρήσεις τὴν ὀρμασίαν ὑπολύδιου ὡς προείρηται). This comment refers the reader back to a table that appears on the previous page of the codex, under the label ‘Common Tuning’ (*Koinē hormasía*).⁴⁹ The interpretation of several details of this table is problematic but one point is beyond doubt – namely the fact that the tuning recorded in the *Hormasía* is rooted in the Hypolydian *tónos* that is reproduced in its right column,⁵⁰ precisely as we are told in the text that accompanies the Canon diagram and in full accordance with the Hypolydian system represented in the Canon diagram itself (Fig. 11).

As shown in Fig. 10, the Canon diagram is followed in the manuscript by Thrasyllus’ division of the Canon. This text, in turn, sheds light on the abbreviations Φ and X that appear at the bottom of the diagram (Fig. 11). Alexandre Vincent (1847, 257) suggested that these abbreviations stand for $\Phi\alpha\upsilon\lambda\omicron\nu$ (‘missing’ or ‘failing’) and $X\rho\omega\mu\alpha\tau\iota\kappa\acute{o}\nu$ (‘chromatic’), indicating respectively semitones that are not included in the Hypolydian system (Φ) and semitones that

47 A handful of irregular notation signs appear in this diagram: some are simply alternative versions of the basic signs, others duplications (see Reinach 1896, 210–13). The systematic displacement of a few signs marked as Φ is discussed at the end of this section (n. 61). These variants do not affect the general interpretation of this document, which is unanimously identified as a ‘Hypolydian Canon’ (Reinach 1896, 209–13; Ruelle 1875, 530–4; Hagel 2010, 98 n. 6). Similar mistakes in the transmission of Greek notation signs occur in manuscripts that reproduce Alypius’ notation tables (368–410 Jan) as well as the notation diagrams preserved in Aristides Quintilianus’ *De musica* (see apparatus *ad* 19–20 and 24–27 W.-I.).

48 See Appendix 1.

49 Heidelbergensis Palatinus gr. 281, 173r – cf. Appendix 2A and 2B.

50 Cf. Vincent 1847, 255–7; Ruelle 1875, 535; Hagel 2010, 128; Najock 2018, 177–80.

produce a chromatic (X) division of the relevant tetrachords.⁵¹ Fig. 12 shows that Vincent's hypothesis is confirmed by the structure of Thrasyllus' division: the semitones that are missing from Thrasyllus' system are in fact marked as Φ in the Canon diagram, whereas the four chromatic notes of the relative tetrachords are marked as X.⁵²

As noted above, Thrasyllus as well as the Canon diagram identify the 'origin' (*arché*) of the Imperial harmonic system with its lowest note, *proslambanómenos* E₂, as opposed to the Classical characterisation of *mésē* as the 'origin' of *harmonía*. In keeping with this significant change,⁵³ the structure of the harmonic system outlined by Thrasyllus differs from the Classical one in two crucial respects.

First, Thrasyllus does not include *paramésē* among the main 'fixed' notes established at the start of his division.⁵⁴ This choice marks a clear shift away

51 The note \circ K F#₃, which is labelled as *chrōmatikḗ* in the *Koinḗ Hormasía*, is interestingly not marked as a 'chromatic' note (X) in the Canon diagram (Fig. 12; Appendix 2A). In the Hypolydian setting of the Canon diagram, \circ K F#₃ represents *paramésē*, marking the beginning of the disjunct tetrachord (F#₃, G₃, A₃, B₃), as well as the chromatic note featured in the modulating, conjunct tetrachord (E₃, F₃, F#₃, A₃). This 'chromatic' conceptualisation originated with Damon's integration of the *Lydistí* mode into the Classical modulating system used on professional *kithárai* (Lynch 2018, 312, and §4 below). As noted by Najock (2018, 178f.), the Hypolydian nature of the notation signs employed in the *Hormasía* is consistent with the first tuning mentioned in the list that appears on the right side of the page ('Hypolydian set in the diatonic genus', Ὑπολυδίου κατὰ τὸ διάττονον – cf. Appendix 2A). This list was written by the same hand that produced the short explanatory text that accompanies the Canon diagram (cf. Appendix 1). A later scribe mistakenly labelled these Hypolydian signs with the corresponding Lydian names (Najock 2018, 178f.), without realizing that this choice was inconsistent with the notation signs employed in this document: if the basic key had indeed been the Lydian, then the note G₃ that is labelled as Lydian *diátonos* in the *Hormasía* should have been notated as M and not Z. The note that is actually employed in the *Hormasía*, Z, is however the appropriate choice for the Hypolydian system recorded in this document. Najock suggests that the mistaken Lydian names were added to the Hypolydian notation signs "sometime between Gaudentius (perhaps 4th century AD) and 1040, the date of the codex" (2018, 179). This hypothesis is compatible with the use of the Lydian *tónos* in diagrams featured in later codices (e.g. Vaticanus gr. 192, 225 r+v, 13th century): at this stage, the mistaken identification of the Lydian notation *tónos* (*mésē* A₃) with the original Lydian modes (Classical *Lydistí* and Imperial *Lýdia*) that were notated by the Hypolydian *tónos* (*mésē* E₃) must have been deeply rooted.

52 These chromatic notes also include *chrōmatikḗ hypatôn* (G#₂), which is strangely missing from Theon's text: cf. Barker 1989, 228 n. 89, and discussion below.

53 Cf. n. 46.

54 Cf. Theon *Math. Plat.* 87.10–89.23. *Paramésē* F#₃ is missing from the core list of fixed notes established at the start of the division and appears at a later stage, when each tetrachord is divided into diatonic and chromatic notes (92.6–9). In this context, F#₃ is labelled as *paramésē* or *chrōmatikḗ synēmménōn*, and corresponds to the string that is called *chrōmatikḗ* in the *Koinḗ Hormasía* (cf. Appendix 2A–B). In contrast, the Euclidean

Thrasyllus' Division of the Canon
(Theon. *Math. Plat.* 89.9–92.27)

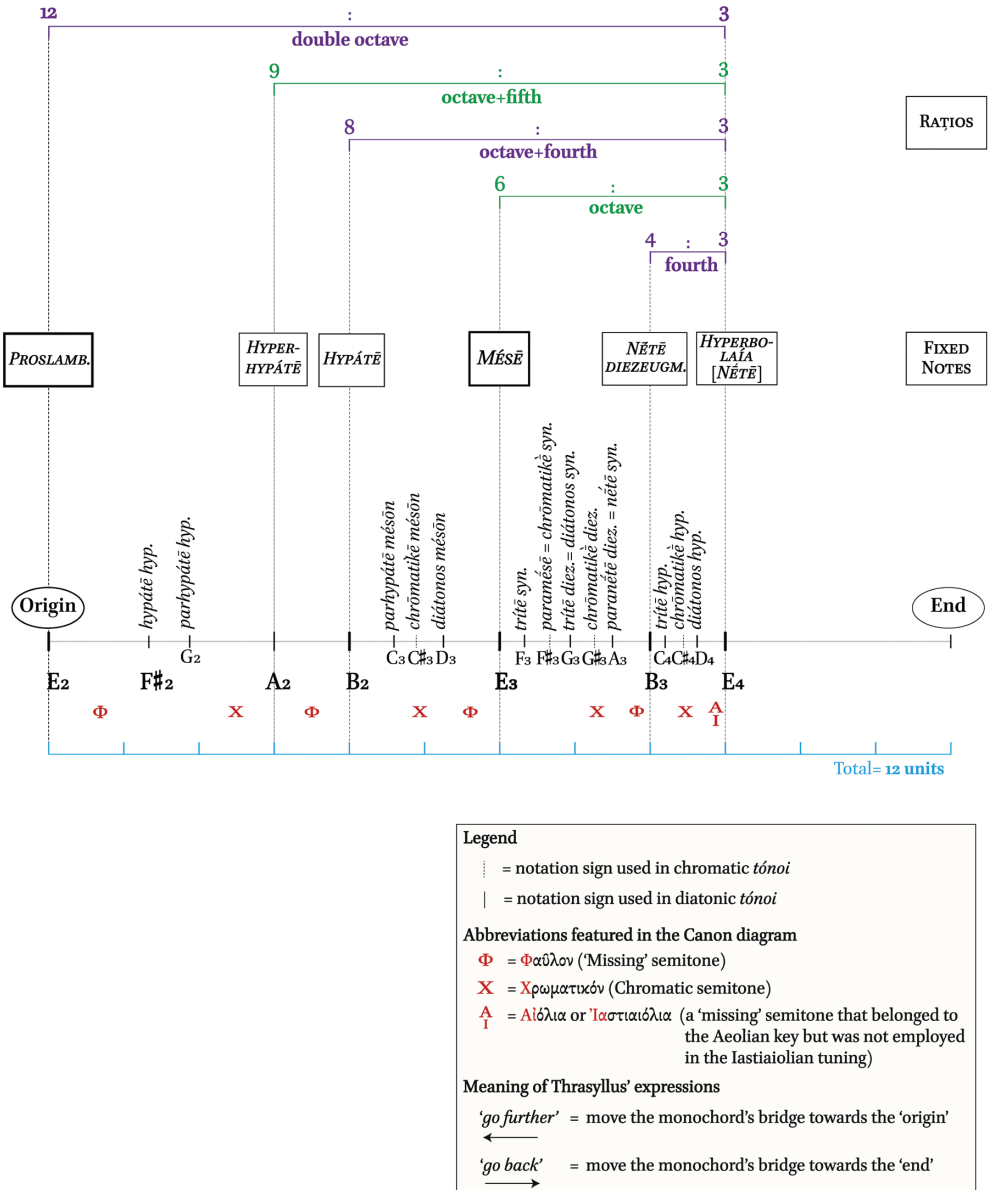


FIGURE 12 Thrasyllus' division of the Canon and the abbreviations featured in the Canon diagram

from the Classical system detailed by Aristoxenus and other sources, a system where *paramésē* played a vital role in defining the central point of reference of each key jointly with the intermediate note *mésē*.⁵⁵ The secondary status of *paramésē* in Thrasyllus' Imperial division, in contrast, reflects a deep theoretical change that is consistent with the reorganization of the Imperial harmonic space discussed in §2 above. The exclusion of *paramésē* from the fixed notes set at the start of the division is compensated by the addition of a new fixed note, *hyperhypátē* A_2 ⁵⁶ – a detail that, again, sets Thrasyllus' division apart from the Classical Perfect System.⁵⁷ An intermediate stage between the Classical system and the Imperial one seems to be reflected by the Euclidean *Sectio Canonis*, a Hellenistic document that includes *hyperhypátē* among the fixed notes established at the start of the division alongside the standard Classical set that includes both *mésē* and *paramésē*. The system detailed in *Sectio* is therefore firmly grounded in the Classical tradition but also reveals an increase in the modal weight of fifths, which eventually replaced the structural fourth that was typical of the Classical harmonic system (Fig. 6 and Fig. 9; cf. Fig. 18 below).⁵⁸

The Canon division provided by Thrasyllus thus reflects the new foundational role played by fifths in the Imperial harmonic system, a setting in which the modal weight of the tone above *mésē* was weakened to the extent that *paramésē* was no longer perceived primarily as a fixed note. As a result, the character of *mésē* as the 'intermediate' note of the harmonic system was defined in a different way too. In contrast with the Classical model, *mésē* was no longer primarily conceived as the 'intermediate' note that marks the disjunctive tone which separates the two main tetrachords of the central octave (e.g.

Sectio canonis includes *paramésē* as well as 'diatonic *lichanòs hypátōn*' (i.e. *hyperhypátē*) among the 'fixed' notes of the immutable system set at the start of the division (*Sectio Can.* 164–165 Jan, with Barker 2007, 205f.). The conceptualisation described in *Sectio* is therefore closer to the Classical system. On the 'theory-ladenness' of different scientific methodologies, the cultural value judgements embedded in them and their epistemological implications, see the classic discussion by Kuhn 1970 and, more recently, Reiss & Sprenger 2020.

55 Cf. Aristox. *Harm.* 28.10–12 and 57.13–59.16 Da Rios; Cleonid. *Harm.* 186.1–7 and 201.18–20 Jan, where *mesē* is defined as the note that sits below the central disjunctive tone (*mésē–paramésē*).

56 Cf. Theon *Math. Plat.* 89.10–21.

57 Cf. n. 55. *Hyperhypátē* is likewise not included in Philolaus' lyre *harmonía* – cf. fr. 6a Huffman, translated and discussed in Lynch 2020, 124f.

58 Cf. Barker 1989, 205 n. 68. Hypolydian/*Lýdia hyperhypátē* A_2 is also the lowest note featured in the instrumental exercises recorded in the *Anonyma Bellermanniana* (DAGM 32–37). Most of these exercises move precisely within the fifth that starts from *hyperhypátē* A_2 and reaches up to *mésē* E_3 .

$B_2 - E_3$ $F\#_3 - B_3$).⁵⁹ In the new Imperial system, *mésē* rather came to be identified as ‘intermediate’ because it stands between two fixed notes that are set a fifth apart from each other (*hyperhypátē* $A_2 - mésē$ $E_3 - nētē$ *diezeugménōn* B_3). Interestingly, this Imperial redefinition of *mésē* intuitively applied to one, and only one, of the Classical modes preserved by Aristides: the Dorian mode, the only Classical *harmonía* that included notes set a fifth above and a fifth below *mésē* F_3 (cf. Fig. 14 below).

The intermediate note of the Classical Dorian mode was therefore the only Classical *mésē* that could be intuitively redefined as ‘intermediate’ in the new Imperial sense of the word – i.e. a note that stands between two fixed fifths. As shown by the *Koinē Hormasía*, the defining notes of the Classical Dorian mode – Dorian *hyperhypátē* B^b_2 , Dorian *hypátē* C_3 , *mésē* F_3 and *nētē* C_4 – were however shifted down by a semitone in the new Imperial system, and came to correspond to Hypolydian/*Lýdia hyperhypátē* A_2 , *hypátē* B_3 , *mésē* E_3 , and *nētē* B_3 .⁶⁰

The interpretative framework provided by Thrasyllus and the Imperial musical documents also explains some strangely regular inconsistencies in the placement of the notation signs that correspond to the ‘missing’ semitones marked as Φ in the Canon diagram (Fig. 11). As long noted by scholars, the notation signs that correspond to the ‘missing’ semitones marked as Φ are misplaced by a fourth.⁶¹ This oddly regular shift affects only these notes and may be explained by assuming that, at some stage in the transmission of this text, a scribe who was no longer acquainted with the keys that were normally used by Imperial Greek musicians misinterpreted the abbreviation Φ as indicating

59 Cf. n. 55.

60 See diagram in Appendix 2B. The unique term *diáemptos* that appears in the *Koinē Hormasía* becomes equally clear on the basis of Thrasyllus’ symmetric construction of fifths centred on *mésē*: *diáemptos* D_3 is a fifth lower than A_3 , the *mésē* of the modulating Lydian tuning that is derived from the basic Hypolydian/*Lýdia* framework of the Canon diagram and the corresponding basic tuning of the *Koinē Hormasía*. As shown in Fig. 11, the fifth above A_3 marks the upper boundary of the Canon diagram as well as the upper boundary of the Hypolydian-centred system employed in the Imperial musical documents (‘hyperbolic’ *nētē* E_4 – Fig. 4).

61 Cf. Ruelle 1875, 530; Reinach 1896, 211f. For example, the notation signs $F \perp$ that appear in correspondence to the first Φ sign from the left side of Fig. 11 represent the note $A\#_2/B^b_2$ and not F_2 – i.e. the note that is actually a semitone higher than *proslambanómenos* $E_2 \varphi H$. The correct placement of the signs $F \perp$ would therefore correspond to the second Φ sign from the left, which is a semitone higher than $A_2 \gamma \vdash$. On other variants of the standard notation signs that appear in this table, see Reinach 1896, 210–13 and n. 47 above.

notes that belong to the Phrygian *tónos*⁶² – a key that was no longer employed in Imperial times (Fig. 1) – instead of the ‘failing’ semitones (Φαῦλοι) that are in fact missing from Thrasyllus’ division and the Imperial musical record (Fig. 4).⁶³ The scribe therefore replaced the Hypolydian signs that originally represented these ‘missing’ notes with signs that belonged to the – now obsolete – Phrygian *tónos*.

In keeping with the original use of the Φ abbreviation (Φαῦλοι), the abbreviation Α̇ that appears at the top of the Canon diagram identifies the highest of the five ‘missing’ semitones marked in this document, which corresponded to the note D#₄ (Fig. 11). This note in principle belonged to the structure of the Aeolian key that is partly included in the Iasti-aeolian tuning described by Ptolemy,⁶⁴ but is in fact missing from Thrasyllus’ division of the Canon. D#₄ is likewise not attested in the Imperial musical scores, just like the other notes marked as Φ in the Canon diagram. In keeping with this, all of the notes that are marked by abbreviations in the Canon diagram are also missing from the diatonic tunings set out in the *Koinḗ hormasía* on the preceding page of the manuscript (Heidelbergensis Palatinus gr. 281, fol. 173r – see Appendix 2A and 2B).

The theoretical picture sketched by the Canon diagram and Thrasyllus (Fig. 11) is therefore fully consistent with the evidence of the Imperial musical documents. The notes that correspond to the ‘missing’ semitones marked as Φ in the Canon diagram are either unattested or extremely rare in the Imperial

62 Cf. Ruelle 1875, 535; Hagel 2010, 98 n. 6. Given that the Phrygian keys are not employed in the extant Imperial documents (cf. Fig. 4), this interpretation of the abbreviation Φ as indicating the Phrygian *tónos* seems very unlikely. Why should a late source such as the Canon diagram use a rather obscure abbreviation to refer to a key that had been out of use for several centuries? This gap would cover over 900 years of musical practice if we take into account the date of Heidelbergensis Palatinus (1040 AD), but it is of course likely that the source of the diagram was much older than the manuscript itself. Given that the content of this diagram is in full accordance with Porphyry and Theon, as well as Imperial musical practice, it is likely to have originated in the third–fourth century AD at the latest. But even in this case, assuming that the abbreviation Φ referred to the Phrygian or Hypophrygian keys would imply a reference to a key that had not been in use for about 400 years, making it extremely improbable on historical grounds. The interpretation suggested above is, in contrast, fully supported by the evidence of Imperial musical practice and theory.

63 Cf. Vincent 1847, 257.

64 AI for αἰόλιον or IA for ἰασηταιόλια. This tuning, and its relationship to the Imperial musical documents, will be discussed in Lynch forthcoming.

documents.⁶⁵ The same is true for the ‘missing’ Iastian-aiolian semitone marked as \uparrow in the Canon diagram, as neither \flat \uparrow nor \times \searrow $D\#_4$ are attested in the Imperial record. In contrast, the chromatic notes marked as \times in the Canon diagram appear regularly in the Imperial documents, especially in their vocal variety,⁶⁶ with the only exception of the lower chromatic note $G\#_2$ – a note that is interestingly missing from Thrasyllus’ division as well.⁶⁷

As noted in §1, the overall picture outlined by the Imperial musical documents also confirms the foundational role of the Hypolydian key illustrated by the Canon diagram: 99.1% of the notes attested in these documents fall within the range of the Hypolydian *tónos* E_2 – E_4 (Fig. 4) and the handful of notes that fall outside this range are clustered in four documents.⁶⁸ In keeping with this, 84.1% of these notes fall within the Hypolydian central octave B_2 – B_3 .⁶⁹ This figure interestingly rises to 91.7% if we include *hyperhypátē* A_2 , in accordance with Thrasyllus, as well as the note A $C\#_4$ that is required to produce the defining tritone of the Higher Mixolydian tuning included in the Imperial harmonic system (Fig. 7).⁷⁰ Conversely not a single Dorian *mésē* Π \square

65 F_2 is not attested (vocal $\aleph=0.0\%$; instrumental $\mathbb{H}=0.0\%$); $A\#_2$ occurs once in instrumental notation ($\perp=0.38\%$) and never in vocal notation ($F=0.0\%$); $D\#_3$ is not attested (vocal $Y=0.0\%$; instrumental $\perp=0.0\%$); $A\#_3$ does not occur in vocal notation ($\Theta=0.0\%$) and may be attested once in instrumental notation ($V=0.38\%$, but the reading is doubtful). These figures refer to the second notes of the relative chromatic *pykná*, in keeping with the notation of the Canon diagram (Fig. 11). The overall picture is almost identical even if one takes into account the third step of these *pykná*: \mathbb{I} \mathbb{H} are not attested; ∇ \neg are not attested; instrumental \mathbb{T} is not attested, while vocal \mathbb{T} is attested 23 times (1.08%); \mathbb{H} $>$ are not attested; and \times \searrow are not attested. The significant role played by \mathbb{T} in Ptolemy’s *Iastiaíolia* will be discussed in Lynch forthcoming; for the moment, it suffices to say that \mathbb{T} represents a fixed note of this tuning, and not a movable chromatic note.

66 $C\#_3$: vocal $X=48$ occurrences, 2.25%; instrumental $\mathbb{V}=2$ occurrences, 0.75%. $G\#_3$: vocal $K=41$ occurrences, 1.92%; instrumental $\perp=0$ occurrences. $C\#_4$: vocal $A=87$ occurrences, 4.08%; instrumental $\searrow=10$ occurrences, 3.77%.

67 $G\#_2$: both \mathbb{V} \perp and \searrow \mathbb{H} are not attested.

68 0.71% of Imperial notes fall above Hypolydian *néte hyperbolaíōn* E_4 (=17 notes, attested in *DAGM* 44, 46 and 60). 0.17% fall below Hypolydian *proslambanómenos* E_2 (=4 notes, all used in the context of a special mimetic effect in *DAGM* 41 Col I). The octave strokes featured in *DAGM* 17–18 are disregarded: cf. Hagel 2010, 72, n. 45, 277–9 on the late dating of *DAGM* 17–18. The overall picture would not change substantially even if these octave strokes were to be regarded as genuine: in this case, notes that fall above Hypolydian *néte hyperbolaíōn* E_4 would account for approximately 2.5% of the total. As noted in Lynch 2022a, 99% of Imperial notes fall within the Dorian *tónos* F_2 – F_4 vs 100% of Classical/Hellenistic notes, a change that reflects the slight shift in the central point of reference of the two systems.

69 I.e. 74.3% of Imperial instrumental notes and 85.4% of Imperial vocal notes: cf. Fig. 4 and Appendix 3.

70 It is interesting to compare these figures with the Lydian equivalents. 99.5% of the notes attested in the Imperial musical documents fall within the Lydian *tónos* A_2 – A_4 , a figure

F_3 occurs in the Imperial musical documents, showing that this note lost the central role it enjoyed in the Classical system.⁷¹

4 Bridging the Gap: The Hypolydian Logic of the Greek Notation System and Its Historical Development

The foundational character of the Hypolydian key is also reflected by the logic that shapes the organisation of the Greek melodic notation system. Generally speaking, Greek melodic notation is based on groups of three related signs that originally indicated a basic, or ‘natural,’ note and two ‘modifications’ of the basic pitch. This relationship is apparent in instrumental notation, where the shape of the sign that indicates the basic note (e.g. C) is often rotated and inverted to represent the two ‘modified’ notes (e.g. \cup \cup).

As shown in Fig. 13, the set of basic notation signs that fall into the central octave of the Imperial harmonic system (B_2 – B_3) form a Hypolydian octave species (B-c-d-e-f#-g-a-b), in accordance with the theoretical model provided by

that is slightly higher than the Hypolydian one (99.1%). But only 79.6% of these notes fall within the central octave of the Lydian *tónos* E_3 – E_4 , whereas 84.1% fall within the Hypolydian central octave (B_2 – B_3). This comparison shows that these figures cannot be taken in isolation, but must be interpreted in the light of ancient harmonic theory and the relative historical contexts. As shown in §§1–2 above, the Hypolydian *tónos* was the basic point of reference of the Imperial system but the Lydian *tónos* played an important role too, as it represented the ‘modulating’ element of the basic *Lýdia* tuning (Appendix 2B) and defined the minor third characteristic of the second harmonic arrangement employed in Imperial music.

71 On the Classical system, see Lynch 2022a, 398–400. The remarkably consistent picture provided by Imperial sources is further substantiated by Vitruvius’ account of the disposition of special resonating vases (*echea*) that could be used in Roman stone theatres in order to amplify particular notes through sympathetic resonance – cf. Lynch 2023a, esp. Fig. 6, which shows how the distribution of the notes attested in the Imperial scores matches closely the pitch and placement of the resonators described by Vitruvius as well as the abbreviations featured in the Canon diagram. Notes that are marked as X in the Canon diagram correspond to chromatic notes in Vitruvius’ system and are regularly attested in the Imperial record with the exception of $G\#_2$ – a note that, as we have seen above, is omitted from the Canon diagram and Thrasyllus’ division as well. Likewise, the ‘missing’ notes that were marked as Φ the Canon diagram are not included in Vitruvius’ system and are almost entirely missing from the Imperial record too. This also applies to the ‘missing’ Iastian semitone marked as \uparrow , which is not attested in Imperial scores (cf. n. 65). The Lydian hypothesis suggested by Hagel 2010 would, in contrast, align the ‘missing’ semitones marked as Φ with notes that are in fact attested (T and K; the note Δ mistakenly printed in Hagel 2010, 253, Diagram 71, should be A – cf. Hagel 2010, 254, Diagram 72). As discussed in n. 62 above, the related interpretation of Φ as an abbreviation for ‘Phrygian’ (Hagel 2010, 98 n. 6) is problematic, given that the Phrygian key is not attested in the Imperial musical documents (cf. Fig. 1).

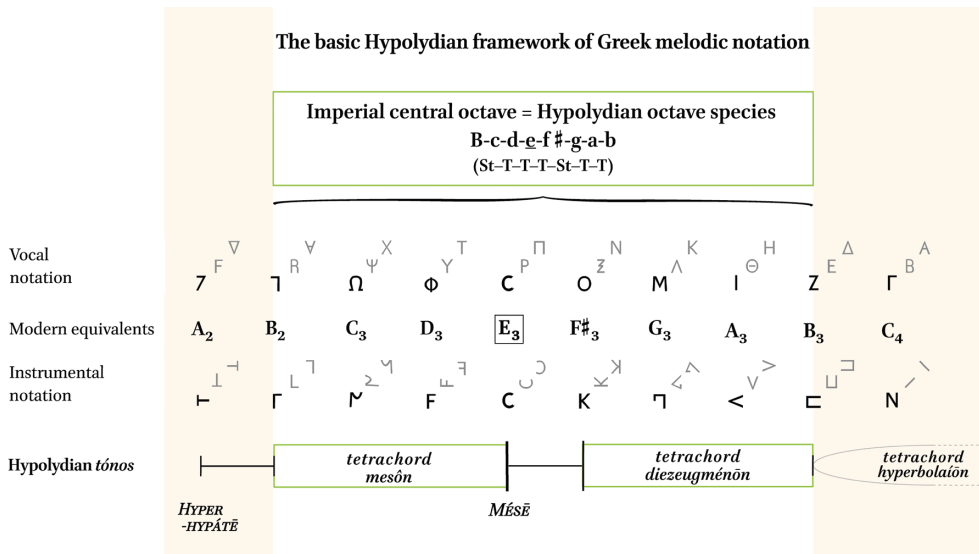


FIGURE 13 The Hypolydian framework of the Greek notation system: the basic notation signs featured in the central octave of the Imperial system (B₂–B₃) form a Hypolydian octave species (B-c-d-e-f#-g-a-b, *mésē* E₃)

the Canon diagram. The Hypolydian key was therefore the basic model for the Greek notation system too.⁷²

But the melodic notation system was not an Imperial invention, of course. In keeping with this, the set of signs reproduced in Fig. 13 covers both the Dorian central octave C₃–C₄ that was typical of Classical Greek music and the

72 Hagel (2010, 28) appreciates that the Hypolydian *tónos* is the natural, or basic, key of the Greek notation system, in contrast with his broader Lydian-based hypothesis. Had the 'natural' key been the Lydian, a basic sign should have been employed to represent the Lydian diatonic note F₃ – a note that is instead represented by an inverted sign (P ⊃) in the Greek notation system; in such a scenario, the typically Hypolydian note F#₃ would, conversely, not have corresponded to the basic signs that actually represent it (O K F#₃). The missing basic sign for F₃ would have been necessary to form the Lydian octave species (e f g a b c' d' e') that would be required by the Lydian hypothesis; the Hypolydian-based structure of the Greek notation system, in contrast, employs basic notation signs to identify the diatonic semitones C₃ ΩΜ and C₄ ΓΝ, producing a Hypolydian octave species (B c d e f# g a b [c'] – Fig. 13). The correspondence of the diatonic semitone G₃ with a basic Lydian sign (M Γ) reflects the Classical origins of the earlier core of the notation system, which was based on the Dorian-Phrygian-Lydian keys (cf. Fig. 14). On the problematic assumptions that shape Hagel's reconstruction of the development of the Greek notation system, and its relationship to ancient harmonic theory more generally, see Laywine 2012 (e.g. 151, 158–68) and n. 23 above.

Hypolydian central octave B_2 – B_3 that is characteristic of Imperial music. These notation signs account for the core set of keys featured in the two Imperial harmonic settings reconstructed above (Fig. 9), and are also the same signs that are needed to notate the Classical *harmoníai* preserved by Aristides Quintilianus (Fig. 14).

This correspondence suggests that the core of this Hypolydian-based notation system must have been perfected in Late Classical times at the latest. But the Classical core of the notation system also bears witness to the original centrality of the Classical Dorian octave C_3 – C_4 . The shape of the instrumental note $N C_4$, in fact, reflects its historical origins as the *Néte* (N) par excellence – the upper Dorian boundary of the Classical central octave. The lower boundary of the Dorian octave ΩC_3 was likewise marked by the last letter of the alphabet series employed in vocal notation ($A - \Omega$).

As shown in Fig. 14, the first triplet that was eventually added below the lower Dorian boundary ΩC_3 features two defining notes of the Classical *Lydistí* tuning – a tuning that, according to Aristoxenus, was first ‘discovered’ by Damon of Oa. As noted elsewhere, this expression cannot mean that Damon literally invented this mode, which was already in use in archaic aulos music.⁷³ Aristoxenus’ testimony rather indicates that Damon discovered how to integrate this traditional aulos mode into the Classical harmonic system – an innovation that is confirmed by the fact that the Hypolydian key is not included in the two ‘archaic’ systems of *tónoi* detailed by Aristoxenus,⁷⁴ but appears in the set of the Classical *harmoníai* preserved by Aristides (Fig. 14).

The first set of *tónoi* mentioned by Aristoxenus also indicates that the Higher Mixolydian mode was already in use in archaic times, but with a significant difference from its standard Classical counterpart, the Lower Mixolydian. In the first ‘archaic’ system detailed by Aristoxenus, the intermediate note of the auletic Mixolydian mode was still identified with the note set a semitone lower than Dorian *mésē*, i.e. $C E_3$, in keeping with the ‘naturalistic’ interpretation of this scale that was common up to the time of Pythocleides.⁷⁵ Its top

73 [Plut.] *Mus.* 136e, with Lynch 2018, 312, and Lynch 2022a, 388.

74 *Harm.* 47.1–16. The first ‘archaic’ system mentioned by Aristoxenus includes the following keys: Hypophrygian, in its auletic form (*mésē* d); ‘Hypo-dorian’ (*mésē* e^b); Mixolydian (‘archaic’ *mésē* e: cf. Fig. 14); Dorian (*mésē* f); Phrygian (*mésē* g); Lydian (*mésē* a). The second archaic system followed ‘the boring of the fingerholes of auloi’ throughout, and comprised the following *tónoi*: Hypophrygian (*mésē* d); ‘Hypo-dorian’ (*mésē* e^{b+}); Dorian (*mésē* f); Phrygian (*mésē* g); Lydian (*mésē* a-); Mixolydian (‘correct’ *mésē* b^b). Cf. Barker 1989, 153f. On the ‘Hypo-Dorian’ key as the *tónos* that is set ‘below the Dorian’, see Lynch 2018, 320 n. 87; on the ‘silent’ *mésē* of the Mixolydian mode, Lynch 2022b, 428–31, and below.

75 Cf. n. 74, with Hagel 2010, 380, Diagram 92. On Pythocleides, see Lynch 2022b, 427f.

But this ‘naturalistic’ interpretation of the archaic Mixolydian *harmonía* did not organise it into well-formed tetrachords,⁷⁷ posing a significant obstacle to its integration into the Dorian-based, Classical modulation system. This problem was eventually solved by a pupil of Pythocleides, an Athenian aulete called Lamprocles, who identified the structurally correct *mésē* of the Mixolydian mode with a silent note (B^b_3) that falls within the undivided tritone that appears at the top of this scale (Lower Mixolydian, Fig. 14). Lamprocles’ re-interpretation of the Mixolydian mode also aligned it with the central Dorian octave, producing the ‘first consonant *metabolé*’ mentioned by Ptolemy.⁷⁸

Lamprocles’ brilliant innovation, in turn, enabled his pupil Damon⁷⁹ to incorporate the *Lydistí* mode into the Classical system by identifying its octave structure as ‘the opposite of the Mixolydian’ species that was discovered by Lamprocles himself.⁸⁰ Lamprocles’ identification of the Lower Mixolydian *mésē* with the note B^b_3 also enabled Damon to employ the note $C E_3$ as the intermediate note of the *Lydistí* mode, given that this note no longer represented the ‘archaic’, but structurally incorrect, *mésē* of the Higher Mixolydian mode (Fig. 14).

As shown in Fig. 14, the Classical *Lydistí* mode spanned an octave but was effectively placed in between the Dorian and the Hypolydian notation keys. Damon’s integration of the *Lydistí* mode, and the relative Hypolydian key, into the Classical system therefore required the addition of a new *pyknón* ΓRV immediately below the Dorian boundary ΩC_3 . This new triplet clearly mirrors

18–20). As shown in Fig. 1, the Higher Mixolydian/Hyperastian key is not attested in other Hellenistic musical documents, but is included among the Aristoxenian *tónoi* (Cleonid. *Harm.* 203.8–10 Jan, quoted in n. 29). This key is, in contrast, very common in the Imperial documents and the relative harmonic system (cf. Figs. 1 and 9).

77 Cf. Lynch 2022b, 428, Fig. 4.

78 Cf. n. 30. In this new Dorian-based context, the note that had been mistakenly identified as Mixolydian *mésē* because of its modal prominence, $C E_3$, was shifted down by a semitone and came to correspond to the notes $Y/T E^b_3$ (Lynch 2022b, 428, Fig. 4). In keeping with this, the septimal tone $E^b_3-F_3$ plays a significant role in Classical scores that make use of the Lower Mixolydian mode (Lynch 2022b, 428–37). The historical development of the kithara modulation system seems to have followed a different trajectory: cf. Lynch 2018.

79 *Schol. vet. Plat. Alc.* 1.118c: Πυθοκλείδης μουσικός ἦν, τῆς σεμνῆς μουσικῆς διδάσκαλος, καὶ Πυθαγόρειος, οὗ μαθητῆς Ἀγαθοκλῆς, οὗ Λαμπροκλῆς, οὗ Δάμων.

80 [Plut.] *Mus.* 1136e2–5 ἀλλὰ μὴν καὶ τὴν Ἐπανειμένην Λυδιστί, ἥπερ ἐναντία τῇ Μιξολυδιστί, παραπλησίαν οὖσαν τῇ Ἰάδι, ὑπὸ Δάμωνος εὐρήσθαι φασι τοῦ Ἀθηναίου. The octave species that defined the Mixolydian mode corresponded to ‘the shape (*schēma*) that goes from *paramésē* to *hypaté hypatón*’ ([Plut.] *Mus.* 1136e1), i.e. St–T–T–St–T–T–T. The Hypolydian octave species that corresponds to *Lydistí* is defined by the opposite series of intervals, T–T–T–St–T–T–St.

the top triplet Γ BA that is based on Dorian *Nēte* Γ N C₄, and features the first derivative signs that were produced by modifying the standard letter shapes used in vocal notation. In keeping with this, the vocal note \forall is first attested in a Hypolydian document dated to the third century BC (*P. Vindob. G 13763/1494*), and is employed alongside R in accordance with the structure of the Classical *Lydistí* mode.⁸¹ In similar vein, the instrumental counterpart of vocal \forall , Γ , first appears in the accompaniment to the tragic lament that is preserved in one of the earliest extant musical documents, the *Orestes papyrus* (*P. Vindob. G 2315*, third–second century BC).⁸²

Fig. 14 also shows that Damon's integration of the *Lydistí* mode required the introduction of a new fixed note, Z B₃, and the relative *pyknón* – notes that were not previously featured in the Classical core of the harmonic system (Dorian–Phrygian–Lydian plus Lower Mixolydian). But the integration of the *Lydistí* mode, and the relative Hypolydian octave, within a harmonic system that included the Higher Mixolydian key posed a new, and unprecedented, challenge to the central role of the Classical Dorian mode.

Prior to the integration of the *Lydistí* mode, the Dorian had been the only Classical scale that featured a note set a fifth above its *mésē* F₃. In this context, the Dorian was also the only mode that could modulate seamlessly with the Lower version of the Mixolydian mode via the common tone F₃ and a shared *pyknón* (Fig. 7). The integration of *Lydistí*, however, undermined the previously unique status of the Dorian, given that *Lydistí* also included a note set a fifth above its *mésē* E₃ and could modulate with the Higher Mixolydian mode via two common tones (E₃ and B₃) as well as a shared *pyknón*. As we shall see in a moment, these competing tensions would eventually shift the centre of the harmonic system to Imperial *mésē* E₃.

For now, however, let us keep our focus on the late Classical developments of the Greek harmonic system and the issues raised by the unwieldy structure of the Mixolydian mode. Even though the Mixolydian mode had largely been integrated into the harmonic system by now, a problem still remained unsolved – namely the fact that one of its *pykná* did not correspond to a single notation triplet but featured a mixture of signs taken from neighbouring triplets (Fig. 15).

81 *DAGM* 15.5 RV and 15.7 \forall .

82 Euripides' *Orestes*, 338–44, *DAGM* 3.5 Γ ⊔ and 3.6 Γ ⊔. The other instrumental note attested in the *Orestes Papyrus* is the first 'hyperbolic' note D₄ Γ – a note that was typical of the New Music and its instrumental innovations (cf. n. 10 and Fig. 16 below). Euripides' *Orestes* was first produced in Athens in 408 BC, i.e. around the time of the first performance of Timotheus' masterpiece, the *Persians* (ca 410 BC – LeVen 2014, 95 n. 57). By this time, Damon had already been back in Athens for some twenty years, after returning from the ten-year exile that followed his ostracism (ca 442 BC – cf. Lynch 2013, n. 53).

This Lydian version of the Mixolydian mode is employed in the vocal part of the Euripidean lament preserved in the *Orestes* papyrus (*DAGM* 3), in accordance with the typically tragic character of this mode.⁸⁴ In keeping with the Mixolydian model preserved by Aristides Quintilianus, this lament features the characteristically Mixolydian tritone at the top of the scale ($Z B_3 - \Pi F_3$),⁸⁵ and the same interval is emphasised an octave lower in the instrumental accompaniment ($\forall B_3 - \text{C} F_3$).⁸⁶

But this mode could not properly be called ‘Mixo-lydian’ (*Mixolydistí*) any more, because its core Lydian elements were no longer ‘mixed’ with the Dorian system, as in its previous incarnations.⁸⁷ As shown in Fig. 15, all the notes of the new Lydian ‘Mixolydian’ mode correspond to standard notes of the Lydian *tónos*, producing a conjunct variant of the *Lydistí* octave.

As shown in Fig. 16, the series of inverted signs that was inaugurated by the Lydian *pyknón* TRV was extended in order to notate the extra ‘fixed’ note of the Dorian mode set ‘beyond *hypátē*’ (*hyperhypátē* $B^b_2 \nabla$). In keeping with this, the note $B^b_2 \nabla$ corresponds to the ‘additional tone’ of the Hyperdorian key that was used to notate the Classical Lower Mixolydian mode (*mésē* B^b_3), and is also the note that aulos players would overblow in order to produce the highest note of the Classical harmonic system: the ‘hyperbolic’ *nētē* of the Dorian key ($F_4 \perp$).

The note $B^b_2 \nabla$ also marks the beginning of the second inverted *pyknón* $7F\nabla$, which is included in the lowest mode of the Classical system, the Iastian, and was notated by the Classical Hypophrygian key. In the developed version of the Classical notation system displayed in Fig. 16, the series of inverted vocal signs was therefore extended down to Hypophrygian *proslambanómenos* $3 D_2$, the modified and inverted sigma sign that represents the lowest note of the Hypophrygian key.⁸⁸

84 Cf. Aristox. *ap.* [Plut.] *Mus.* 1136d; [Psell.] *Trag.* 5; Plut. *De recta rat. aud.* 46b2–7, a famous anecdote in which Euripides rebukes one of his choristers for laughing after hearing his demonstration of a Mixolydian song (‘Were you not so insensitive and ignorant, he said, you would not have laughed at me singing in the Mixolydian mode!’; $\epsilon\iota\ \mu\eta\ \tau\iota\varsigma\ \eta\varsigma\ \acute{\alpha}\nu\alpha\iota\sigma\theta\eta\tau\omicron\varsigma,\ \epsilon\iota\ \pi\tau\epsilon,\ \kappa\alpha\iota\ \acute{\alpha}\mu\alpha\theta\acute{\eta}\varsigma,\ \omicron\upsilon\kappa\ \acute{\alpha}\nu\ \epsilon\gamma\acute{\epsilon}\lambda\alpha\sigma\sigma\alpha\varsigma\ \epsilon\mu\omicron\upsilon\ \mu\iota\zeta\omicron\lambda\upsilon\delta\iota\sigma\tau\iota\ \acute{\alpha}\delta\omicron\nu\tau\omicron\varsigma$).

85 Cf. *DAGM* 3.7: $\text{PCPZ}\Pi\Phi$. This tritone $Z B_3 - \Pi F_3$ is, of course, a semitone lower than its Dorian-based, Lower Mixolydian counterpart $\Gamma C_4 - \text{O} F\#_3$, which is attested, for example, in Athenaeus’ *Paeon* (Lynch 2022b, 448–51).

86 *DAGM* 3.5 ($\Phi\Pi\Pi\Pi \succ \text{C}$), 3.6 ($\succ \text{C} \text{ZIZ}$). Similar octave shifts between vocal lines and the relative aulos scales occur in the Lower Mixolydian context of Athenaeus’ *Paeon*: cf. Lynch 2022b, 45of.

87 Cf. Lynch 2022b, 421 n. 12.

88 The fact that the Hypophrygian aulos, and the relative key, was the lowest of the Classical harmonic system is confirmed by Aristox. *Harm.* 47.1–16 (cf. n. 74). In keeping with this, D_3 is also the lowest note that is produced by the Megara auloi reconstructed in Terzēs and Hagel 2022, 57, Fig. 4 (Meg μ , 147.4 Hz $\sim D_{3+7}$), and 60, Fig. 5 (Meg τ , 145.4 Hz $\sim D_{3-17}$).

As mentioned in §1, the Classical harmonic system was further expanded in late Classical times by the introduction of two high-pitched keys set above Dorian *nētē* C_4 , the upper boundary of the central octave. The remaining inverted letters were therefore used to notate these new ‘hyperbolic’ keys:⁸⁹

Phrygian *nētē* $D_4 \cup \perp$ (= Hyperlydian *mésē*)
 with the relative Hyperphrygian *pyknón* $\cup \text{h} \text{X}$
 and
 Lydian *nētē* $E_4 \ominus \text{V}$ (=Hyperlydian *paramésē*)
 with the relative Hyperlydian *pyknón* $\ominus \wedge \perp$

As shown in Fig. 16, the sign that is set immediately above the core Classical system $A - \Omega$ is $D_4 \cup \perp$, a note that completes the central octave of the Phrygian key and, at the same time, made it possible to introduce a new key based on *mésē* C_4 , which was defined by the disjunctive tone $C_4 - D_4$. This notation key was called Hyperphrygian, or Hypermixolydian in Aristoxenian parlance, and corresponded to the Locrian mode that Philoxenus introduced into the late Classical kithara system.⁹⁰

But the note $D_4 \cup \perp$ plays two other important roles. On the one hand, it is two octaves above the note that is represented by the lowest inverted sign of the Greek notation system, Hypophrygian *proslambanómenos* $\exists D_2$. On the other, $D_4 \cup \perp$ corresponds to the intermediate note of the new Hyperlydian key – the highest key included in the Greek notation system as a whole. In order to be defined as a new *mésē* in the Classical sense of the term, $D_4 \cup \perp$ had to be accompanied by a note set a tone above it, $E_4 \ominus \text{V}$ – a note that represents Hyperlydian *paramésē* and, at the same time, completes the central octave of the Lydian key (=Lydian *nētē*). The top note of the Hyperlydian *pyknón*, $\perp F_4$, likewise corresponds to the ‘hyperbolic’ *nētē* of the Dorian key, the very top boundary of the Classical, Dorian-based harmonic system (Fig. 16).

89 The third ‘hyperbolic’ key of the Greek system, the Hyperaeolian (*mésē* $C\#_4$), is a later addition, and represents the Imperial equivalent of the Classical Hyperlydian (*mésē* D_4). In keeping with this, the Hyperlydian key is not attested in the Imperial documents (cf. Fig. 1). Just as the Classical modulation system, and relative notation system, did not feature a key based on *mésē* $D\#_4$, so also the Imperial system does not seem to employ the corresponding key based on *mésē* D_4 .

90 Cf. n. 10 above.

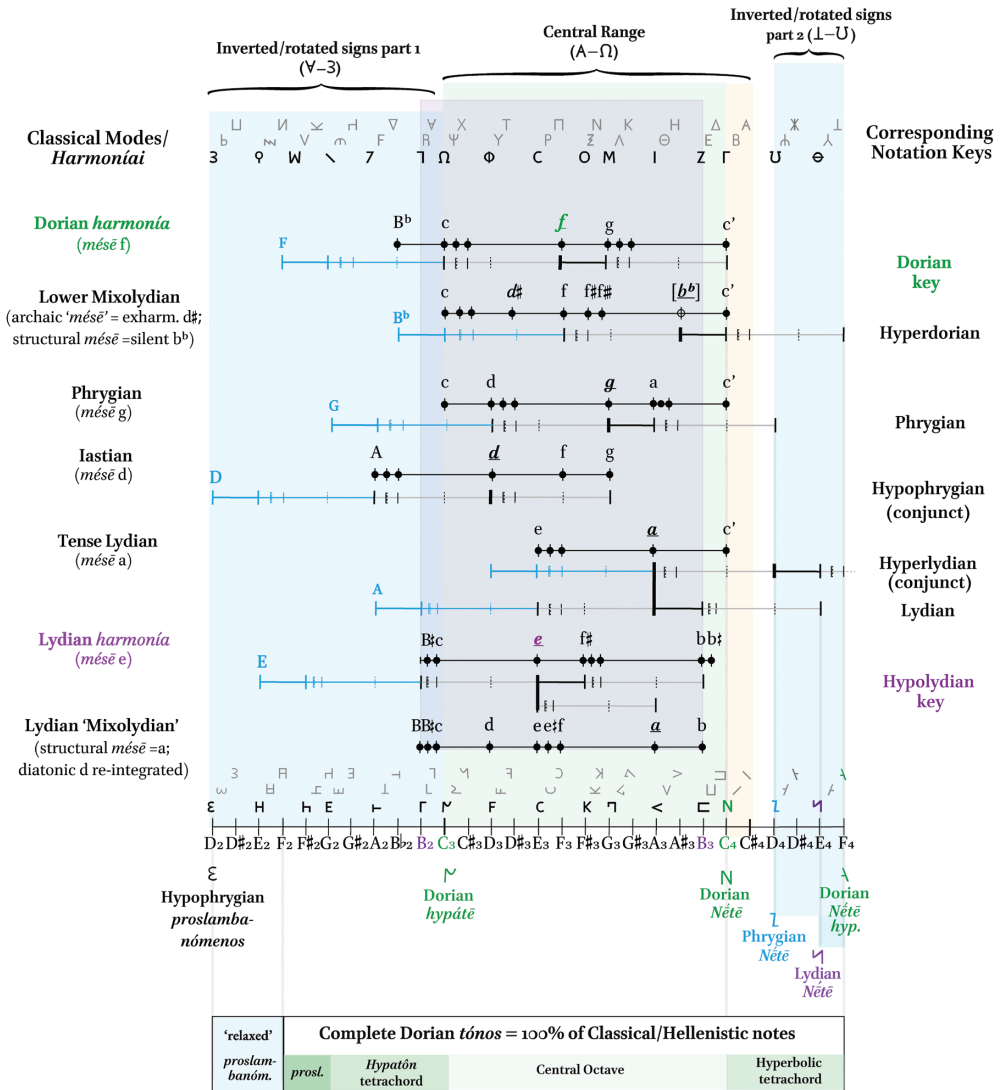


FIGURE 16 The second stage in the development of the Greek notation system is marked by the addition of inverted vocal signs. These inverted signs cover the 'relaxed' register that is set immediately below Dorian C₃ Ω, starting from V and reaching down to 3 (=Hypophrygian proslambanómenos D₂), as well as the new, 'hyperbolic' keys set above A C₄

A new register was eventually added above the upper Dorian boundary F_4 \perp λ , and comprises notes that could be produced as overtones on professional lyres.⁹¹ In keeping with this, the signs featured in this new register are identical to the ones that appear in the central Classical range with the addition of octave strokes (e.g. $O' K' F\#_4$ vs $O K F\#_3$).

The fourth, and final, stage in the development of the Greek notation system entailed the addition of six new letter-based signs at the very bottom of the system. As shown in Fig. 17, the basic note of the higher of these triplets (ΔC_2) defines the 'additional tone' of the formalised Hypodorian key (C_2 – D_2),⁹² and the basic note of the lowest triplet ($\approx B^b_1$) mirrors the inverted omega that sits at the very top of the notation system ($U' D_5$), but was not included in any notation keys. In keeping with this, none of the keys featured in the Greek melodic system is lower than Dorian *hypátē* (ΩC_3), confirming that the Dorian key was the original point of reference for the Classical harmonic system (Fig. 17).

As mentioned above, however, the structural rivalry created by the late Classical introduction of the *Lydistí* mode into the harmonic system undermined the primacy of the Classical Dorian key.⁹³ As shown in Fig. 18, the new *Lydistí* mode could in fact modulate seamlessly with the Higher Mixolydian mode as well as the new Lydian version of the Mixolydian mode, establishing the fifth $C E_3$ – $Z B_3$ as a new point of reference that competed with the Classical fourth defined by the Dorian and Lower Mixolydian mode (ΠF_3 – $H B^b_3$). These competing harmonic tensions eventually led to the birth of the new Hypolydian-centred system that is employed in the Imperial musical documents, where *Lýdia* became the new Dorian (Fig. 18).

In keeping with its new Hypolydian framework, only three families of notation keys are employed in the Imperial musical documents: Lydian, Iastian and (very occasionally) Aeolian.⁹⁴ As shown in Figs. 9 and 19, these are precisely the Imperial counterparts of the Dorian, Phrygian and Lydian keys that formed the core of the Classical modulation system.⁹⁵

91 Kithara players could easily produce harmonic overtones an octave higher than the basic pitch of a given string by touching it lightly at the halfway point while plucking or striking it with the plectrum (cf. [Arist.] *Pr.* 19.23). This octave shift is reflected by the addition of octave strokes to the notes featured in this new register. In contrast, overblown harmonics produced on the aulos are a twelfth higher than the basic note (e.g. overblowing F_2 produces C_4 , and not F_3).

92 Cf. Lynch 2022a, 394, nn. 25f.

93 An intermediate phase seems to be reflected by the inclusion of *hyperhypátē* among the fixed notes of the Hellenistic *Sectio Canonis* (cf. §3, with nn. 56f. above).

94 Cf. Fig. 1.

95 On the primacy of the Dorian key in the Classical harmonic system, see Lynch 2022a and 2022b. A detailed discussion of the relationship between the Imperial notation keys,

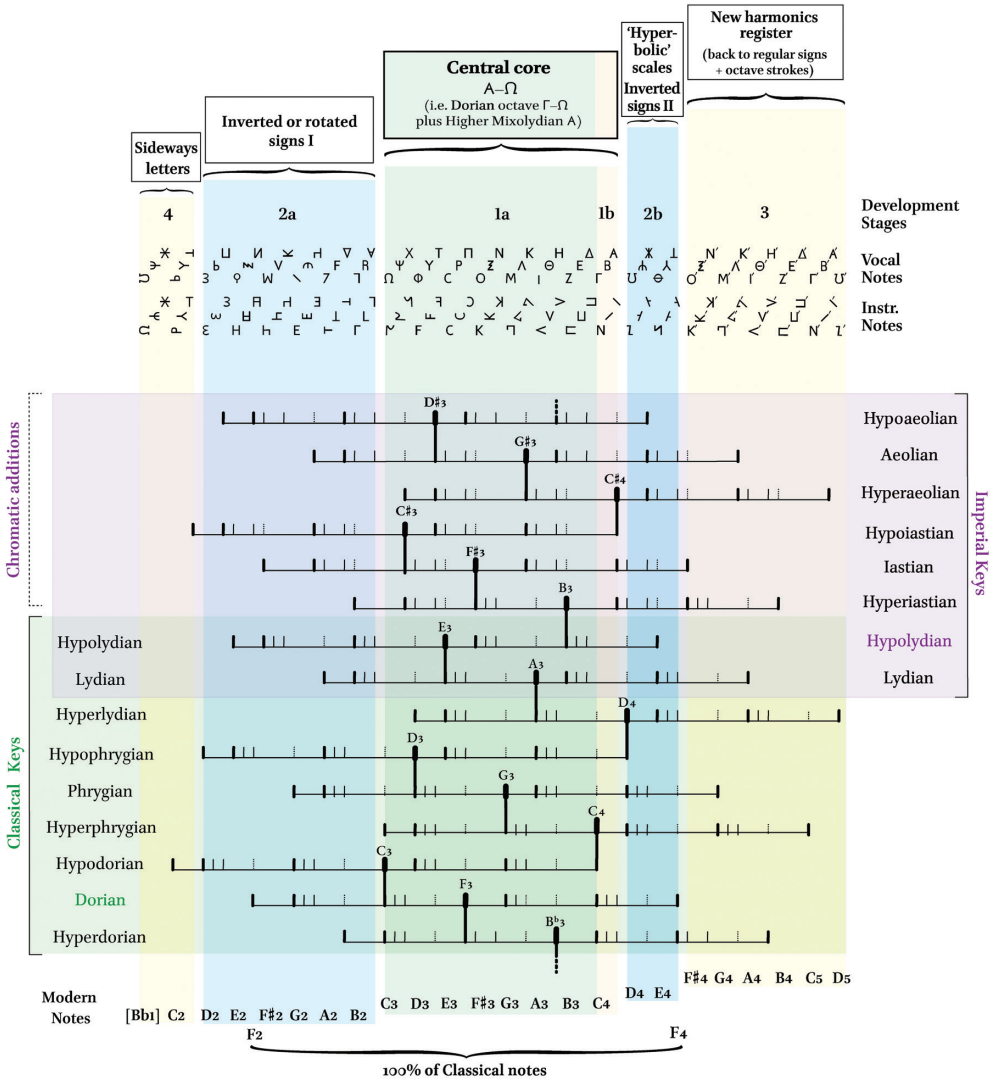


FIGURE 17 The final stages of the development of the Greek notation system: 3. addition of the overtone register set above the hyperbolic Dorian tetrachord; 4. addition of six sideways letters that represent the 'additional' tone of the Hypodorian key

Classical system

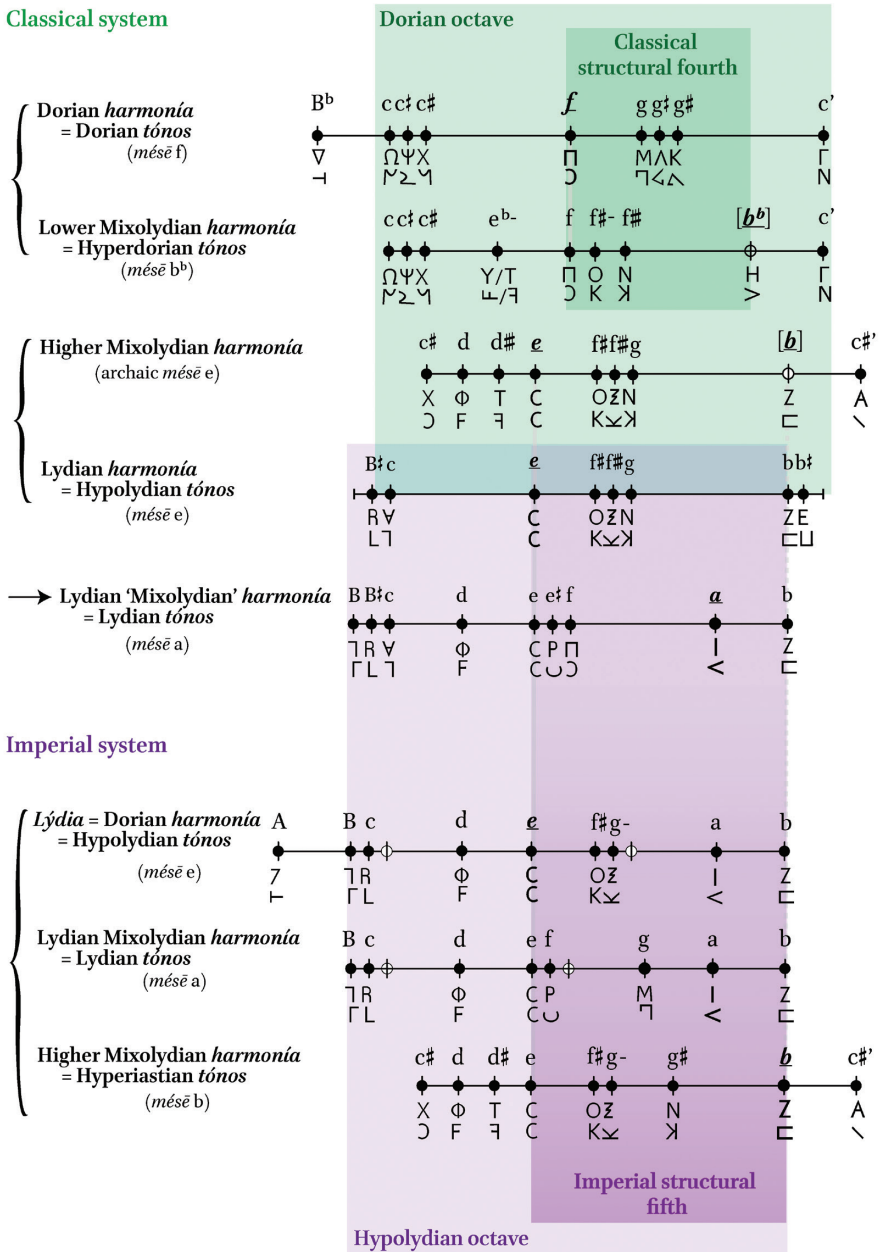


FIGURE 18 Bridging the gap: the structural rivalry between the Classical Dorian mode and *Lydistí* eventually led to the birth of the new Imperial system, where *Lýdia* became the new Dorian and the structural fifth C E₃-Z B₃ replaced the Classical fourth Π F₃-Η B₃

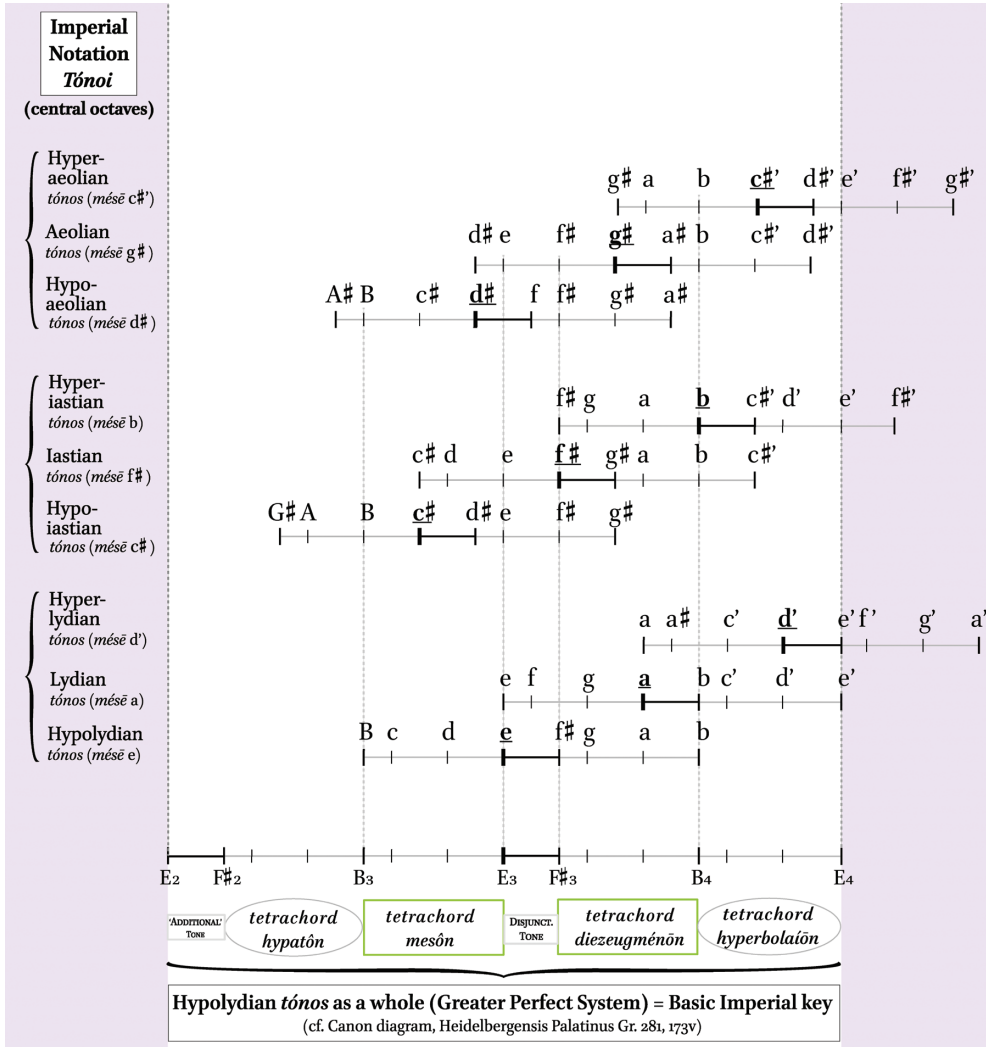


FIGURE 19 The notation keys employed in the Imperial musical documents and the underlying Hypolydian tónos

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the kithara tunings preserved by Ptolemy, and the corresponding scales played by other Imperial instruments, goes well beyond the scope of this article, and will be offered in Lynch forthcoming. A preview of the Louvre aulos scales, and the relative tunings, is provided in Lynch 2023c.

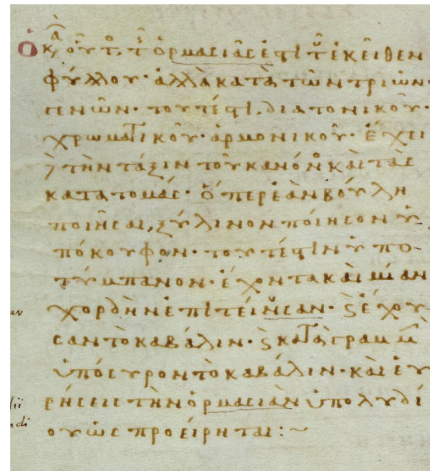
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Appendix 1 Explanatory text that accompanies the Canon diagram (Palatinus gr. 281 Fol 173v)

ὁ κανὼν, οὗτος, τῆς ὀρμασίας ἐστὶ τοῦ ἐκείθεν
 φύλλου· ἀλλὰ κατὰ τῶν τριῶν
 γενῶν· τουτέστι διατονικοῦ·
 χρωματικοῦ· ἁρμονικοῦ· ἔχει
 δὲ τὴν τάξιν τοῦ κανόνος καὶ τὰς
 κατατομάς· ὅ περ ἐὰν βούλη
 ποιῆσαι, ξύλινον ποιήσον ὑ-
 πόκουρον· τουτέστιν ὑπο-
 τύμπανον· ἔχοντα καὶ μίαν
 χορδὴν ἐπιτείνουσαν· καὶ ἔχου-
 σαν τὸ καβάλιν· καὶ κατὰ γραμμὴν
 ὑπόσυρον τὸ καβάλιν· καὶ εὐ-
 ρήσεις τὴν ὀρμασίαν ὑπολυδί-
 ου ὡς προεῖρηται· ~



This is the Canon of the tuning (*hormasías*) that is on the previous leaf; but it is set down according to three genera: that is to say, diatonic; chromatic; [en]harmonic. It displays the organisation of the Canon and its divisions. If you ever wished to build it, build a wooden box as a foundation, that is to say a ‘drum’ [i.e. a sound box]; it must also have a single string, stretched along [the box]; and the string must have a bridge (*kabálin*). Follow the diagram as you slide the bridge, and you shall find the tuning (*hormasían*) of the hypolydian, as mentioned earlier.

Appendix 2A The 'Common Tuning' (Koinē Hormasia) – Heidelbergensis Palatinus gr. 281 Fol 173r

173

ἡ κοινὴ ὁρμασίαν ἀπὸ τῆς ἑξῆς ἔβληθη εἰς αὐτὴν
ἀριστερῆς χειρὸς Δεξιᾶς χειρὸς

λυδί ουκ τοδ τον	προλαμβανόμε κ	Ζ Τ	Διάπεμπο α	Φ Ε	ὑπολύδι το διάτον:
	μέση κ	Ι Χ	ὑπάτη α	Σ C	ὑπερύδι το διάτονον:
	νήτη α	Θ Υ	χρωματικὴ α	Ο Κ	ὑπερολίον κ' το χ' τὸ νήτην:
	συνημένη α	Ψ Ζ	διάτονη α	Ξ Υ	ὑπολατὶ καὶ το χρωματικ:
	συνημένη α	Υ Ζ	μέση α	Ι Χ	ὑπερολίον κ' το διάτονον:
	διάτονος ο	Ξ Κ	παράμεσος α	Ζ Ε	κ' το διάτονον λυδίου κ' τ
	διάτονος ο	Ξ Κ	τρίτη α	Ε Η	τριώνητην ὑπερφρεγίον
	παράμεσος ο	Κ Ζ	συνημένη α	Ψ Ζ	το ἑναρμόνι: ὑπεριατὶκ
	τρίτη α	Ε Η	νήτη α	Θ Υ	το διάτονον ὑπεριατὶκ
	διάπεμπο α	Φ Ε	ὀξεῖα χρωματικὴ ο	Ο Κ	ὑπεριατὶκ το ἑναρμόνι:
	ὑπάτη α	Σ C	ὀξεῖα χ' τὸ νοε ο	Ξ Υ	
	παρυπάτη α	Ρ Υ	ὀξεῖα μέση ο	Ι Χ	
	χρωματικὴ ο	Ο Κ	ὀξεῖα παράμεσος ο	Ζ Ε	
	μέση ο	Ι Χ	ὀξεῖα τρίτη ο	Ε Η	
	παράμεσος ο	Ζ Ε	ὀξεῖα λυνημένη ο	Ψ Ζ	
	τρίτη α	Ε Η	ὀξεῖα νήτη ο	Θ Υ	

