MEASURES OF SOLVENCY IN THE REGULATION OF THE UK LIFE ASSURANCE INDUSTRY

Benjamin R. Gully

A Thesis Submitted for the Degree of PhD at the University of St. Andrews

1999

Full metadata for this item is available in Research@StAndrews:FullText at:
http://research-repository.st-andrews.ac.uk/

Please use this identifier to cite or link to this item: http://hdl.handle.net/10023/2892

This item is protected by original copyright
The problem of designing appropriate solvency regulations is addressed with respect to the U.K. life assurance industry using various theoretical and methodological techniques. These alternative approaches to the measurement of insurer solvency are explored in order to provide a framework for assessing regulations. Reviews of the current insurance regulatory environment as well as an extensive statistical and economic analysis of the life assurance industry provide a practical backdrop to subsequent model building.

Building on these reviews, a ‘Monte-Carlo’ simulation model of an insurer portfolio is constructed to demonstrate additional considerations relevant to solvency regulation. The hypothetical insurance company is assumed to maximise the expected utility of ‘ultimate surplus’, which is taken as an indicator of end-of-period wealth. Five asset classes are used and liabilities are assumed fixed. The simulated run-off performance of the portfolio is evaluated in terms of the probability of insolvency demonstrating a ‘U’ shaped relationship between the risk preference of the insurer and the insolvency probability.

Implications for the design of regulatory constraints are also assessed with respect to the simulations. In particular, the contrast between ex ante and ex post measures of insurer solvency are highlighted with the conclusion taken that current regulations might gain further insight into the underlying solvency performance of insurance companies if they were to use ex ante solvency measures. This subsequent policy prescription is qualified by two factors: first, that the value of simulations and forecasting as an ex ante measure of performance is only as good as the models used to forecast ex ante; and second, that any proposed regulatory shift must be assessed within a cost-benefit analysis. Overall, the simulation analysis suggests that current regulations provide an incomplete picture of the solvency performance of the U.K. life assurance industry.
DECLARATIONS

I, Benjamin R. Gully, hereby certify that this thesis, which is approximately 100,000 words in length, has been written by me, that it is the record of work carried out by me and that it has not been submitted in any previous application for a higher degree.

Date 26/03/00 Signature of candidate

I was admitted as a research student in September 1996 and as a candidate for the degree of Doctor of Philosophy in September 1996; the higher study for which this is a record was carried out in the University of St. Andrews, Scotland (U.K.) between 1996 and 1999.

Date 26/03/00 Signature of candidate

I hereby certify that the candidate has fulfilled the conditions of the Resolution and Regulations appropriate for the degree of Doctor of Philosophy in the University of St. Andrews and that the candidate is qualified to submit this thesis for application for that higher degree.

Date 26/03/00 Signature of supervisor

In submitting this thesis to the University of St. Andrews, I understand that I am giving permission for it to be made available for use in accordance with the regulations of the University Library for the time being in force, subject to any copyright vested in the work not being affected thereby. I also understand that the title and abstract will be published, and that a copy of the work may be made and supplied to any bona fide library or research worker.

Date 26/03/00 Signature of candidate
CONTENTS

LIST OF ILLUSTRATIONS ................................................................. xi
LIST OF TABLES ................................................................................. xiv
ACKNOWLEDGMENTS ........................................................................ xvii

Chapter

1. INTRODUCTION ........................................................................... 19
   Background: The Rationale for Regulation
   Definitions and Objectives
   Structure of Thesis

2. THE LEGAL & ADMINISTRATIVE STRUCTURES
   AFFECTING THE U.K. LIFE ASSURANCE INDUSTRY.................. 32
   Introduction
   Why Regulate?
   The Historical Development of Insurance Legislation
   The European Influence on U.K. Insurance Regulation
   The Current Regulatory Environment
   Insurance Industry Self-Regulation
   Summary
3. EVALUATING INSURANCE COMPANY SOLVENCY: ‘CLASSICAL’ INSURANCE SOLVENCY THEORY

   Introduction
   Quantifying Solvency
   Formal Definitions
   Calculating the Solvency Position of an Insurance Company

   Summary

4. EVALUATING INSURANCE COMPANY SOLVENCY: ALTERNATIVE APPROACHES

   Introduction
   An Alternative Approach to ‘Classical’ Insurer Solvency Theory
   Foundations of Asset/Liability Management (ALM)
   Financial Economics and Portfolio Selection
   Simulation Techniques
   Additional Theoretical Considerations
   Insolvency Prediction: Models of Financial Distress

   Summary

5. MODELLING INSURANCE COMPANY DECISIONS: PORTFOLIO BALANCE, EXPECTED UTILITY AND REGULATION

   Introduction
   The Standard Portfolio Selection Problem
   A General Portfolio Model of a Life Insurer
Acquisitions and Merger Objectives

Summary

8. THE PERFORMANCE OF THE U.K. LIFE ASSURANCE INDUSTRY

Introduction

Life Assurance Performance: a Microeconomic Perspective

Life Assurance Performance: a Macroeconomic Perspective

Summary

9. FIRM-LEVEL STRUCTURE-PERFORMANCE RELATIONSHIPS: EVIDENCE FROM THE U.K. LIFE ASSURANCE INDUSTRY

Introduction

Theoretical and Methodological Background

Brief Literature Review

Research Design

Results and Discussion

Summary

10. OPERATIONAL RESEARCH CONSIDERATIONS IN THE MODELLING OF A LIFE INSURER’S ASSETS AND LIABILITIES

Introduction

Asset Class Selection

The Wilkie Model of Stochastic Investment Returns
The Variance/Covariance Matrix of Returns

Discussion of the Wilkie Model

Liabilities

Summary

11. LIFE INSURER RISK PREFERENCE AND SOLVENCY: A SIMULATION APPROACH ...................... 384

Introduction

The Insurer Optimisation Problem

The Ultimate Surplus Simulation Algorithm

Ultimate Surplus Simulation Output Data

Discussion of Results

Summary

12. CONCLUSIONS .................................................................... 429

Public Policy Perspectives

Regulatory Objectives

The Performance of the Current Regulatory Environment

Considerations in the Measurement of Solvency

A Simulation Perspective

Ex ante Solvency Measurement

Further Remarks
ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1.</td>
<td>The Measurement of Solvency in the EEA</td>
</tr>
<tr>
<td>2.2.</td>
<td>The Regulatory Hierarchy</td>
</tr>
<tr>
<td>3.1.</td>
<td>Solvency: The Regulator’s View</td>
</tr>
<tr>
<td>3.2.</td>
<td>The Insurance Process</td>
</tr>
<tr>
<td>4.1.</td>
<td>The “Cascade” Structure of the 1986 Wilkie Model</td>
</tr>
<tr>
<td>4.2.</td>
<td>The Risk-Based Capital (RBC) System</td>
</tr>
<tr>
<td>5.1.</td>
<td>Mean-Standard Deviation Portfolio Frontier (Risky Assets Only)</td>
</tr>
<tr>
<td>5.2.</td>
<td>Typical Risk Averse Utility Function (A), with Resulting Indifference Map in (B)</td>
</tr>
<tr>
<td>5.3.</td>
<td>Portfolio Selection and Expected Utility</td>
</tr>
<tr>
<td>5.4.</td>
<td>The Probability of Insolvency</td>
</tr>
<tr>
<td>5.5.</td>
<td>Portfolio Selection within a Regulatory Setting</td>
</tr>
<tr>
<td>5.6.</td>
<td>The Envelope Frontier and the Insolvency Constraint</td>
</tr>
<tr>
<td>6.1.</td>
<td>Long-Term and general Insurance Written Premiums</td>
</tr>
<tr>
<td>6.3.</td>
<td>New Yearly Premiums for Individual Long-Term Business in the U.K.</td>
</tr>
<tr>
<td>6.4.</td>
<td>The Proportion of New Yearly Linked-Life Business</td>
</tr>
<tr>
<td>6.5.</td>
<td>Foreign-Controlled Insurers’ Share of the U.K. Life Market</td>
</tr>
<tr>
<td>6.6.</td>
<td>Authorised U.K. and Foreign Insurers</td>
</tr>
<tr>
<td>6.7.</td>
<td>A Lorenz Curve Relationship Between New Business and the Number of Insurers (1992)</td>
</tr>
</tbody>
</table>
7.1. Long-Term Insurance Funds (£m) .................................................... 272
7.2. Breakdown of Long-Term Business Investment Holdings for
    Non-Linked and Linked Funds (%) ........................................... 278
7.3. Long-Term Business Investment Income 1986-1996 .................... 280
8.1. Inter-Firm Profitability of Life Insurers in 1997 ............................ 291
8.2. Breakdown of benefits Paid on U.K. Ordinary Life Assurance
    1986-1996 .............................................................................. 296
8.3. Total Employment in the Insurance Sector ...................................... 304
8.4. Earnings of U.K. Financial Institutions 1996 ............................... 309
10.1. Simulated Inflation Rates ............................................................... 356
10.2. Simulated Dividend Index for Ordinary Shares ............................ 358
10.3. Simulated Dividend Yields ............................................................. 359
10.4. Simulated Equity Price Index ......................................................... 360
10.5. Simulated Yields on (Irredeemable) Fixed Interest Securities ....... 361
10.6. Simulated Short-Term Interest Rates ............................................. 363
10.7. Simulated Yields on a Given Property Portfolio ............................ 365
10.8. Simulated Property Price Income Index ........................................ 366
10.9. Simulated Property Price Index ..................................................... 367
10.10. Simulated Real Returns on Index-Linked Government Securities.... 368
10.11. The Efficient Frontier Using the
       1995 Wilkie Model (Nominal Returns) ................................... 375
10.12. The Efficient Frontier Using the
       1995 Wilkie Model (Real Returns) ........................................... 376
11.1. Optimal Portfolios ..................................................................... 407
11.2. Expected Return and Standard Deviation of Ultimate Surplus...... 414
11.3. The Probability of Insolvency and Regulation .................................. 417
11.4. Margins of Solvency .......................................................................... 425
12.1. Ex ante and Ex Post Regulation of Solvency ..................................... 439
# TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1. Finite-time Ruin Probabilities: Numerical Approximations</td>
<td>115</td>
</tr>
<tr>
<td>4.1. Risk Classes Present in Assets and Liabilities</td>
<td>143</td>
</tr>
<tr>
<td>4.2. Hardy’s Simulation of Insolvency and Relative Insolvency</td>
<td>148</td>
</tr>
<tr>
<td>4.3. Safety-First Rules Used to Define the Size of Downside Risk</td>
<td>153</td>
</tr>
<tr>
<td>4.4. Interested Parties in Solvency Surveillance</td>
<td>159</td>
</tr>
<tr>
<td>4.5. Sample of Risk-Based Capital Charges for Selected Asset Categories</td>
<td>169</td>
</tr>
<tr>
<td>5.1. Variables and Definitions</td>
<td>180</td>
</tr>
<tr>
<td>6.2. Historical U.K. Life Business</td>
<td>216</td>
</tr>
<tr>
<td>6.4. Share of the Global Economy</td>
<td>232</td>
</tr>
<tr>
<td>6.5. Ten Largest Insurance Markets in 1995</td>
<td>233</td>
</tr>
<tr>
<td>6.10. Authorised Insurers</td>
<td>242</td>
</tr>
</tbody>
</table>
6.11. Number of Insurers Authorised for Each Class of Insurance Business
........................................................................................................... 243

6.12. Asset holdings in other Insurance Companies-
Evidence from the Largest Insurers in the UK (1998)....... 245

6.13 Concentration Ratios of the Leading 10 Long-Term
Life Assurance Companies in 1996................................. 249

6.14 Entry to the U.K. Life Assurance Industry............................. 253

7.1. Expectation of Life at Birth According to Death Rates
Assumed for 1994, 2001 and 2031......................................... 259

7.2. Insurance Company Variation in Monthly Premium Rates for
Low Cost With Profits Endowments 1998.......................... 261

7.3. Long Term Expenditure.............................................................. 263

7.4. New Long-Term Business by Distribution Channel..................... 266

7.5. Insurance Companies’ Asset Holdings as a Percentage (%) of
Total Assets.................................................................................. 274


7.7. Takeovers and Mergers in the U.K. Life Assurance
Industry for 1996/97.............................................................. 282

8.1. Inter-Firm Comparison of Selected Proprietary Life Insurers’
Profitability in 1997............................................................... 289

8.2. Inter-Firm Comparison of Selected Proprietary Life Insurers’
Profit Growth Between 1996 and 1997................................. 290

8.3. Inter-Firm Comparison of Selected Mutual Life Insurers’
‘Profitability’ in 1997............................................................ 291

8.4. Top 15 With-Profit Endowment Maturity Values, 1998.......... 293

8.5. 1998 Bonus Declarations for Top 12 Life Companies.............. 294

11.5. Extract from @RISK Convergence Reports................................. 413
11.6. Sensitivity Results for Ultimate Surplus by Asset Class............... 421
11.7. The Probability of Insolvency (%) and Actuarial Portfolio
      Holdings........................................................................... 426
ACKNOWLEDGEMENTS

The author would like to express his thanks to the following, for their support, useful comments and understanding during the drafting of this thesis:

- to Professor Gavin, C. Reid of the Department of Economics, University of St. Andrews, in his capacity as first supervisor;

- to Professor James, A. Brander of the Faculty of Commerce & Business Administration, University of British Columbia (UBC), for the support he has provided in respect of my position and research as visiting scholar at UBC;

- to Dr. M.M.A. La Manna and Dr. G.S. Shea (both University of St. Andrews) for further supervision involvement;

- to the Economic and Social Research Council (E.S.R.C.) who have funded this research full-time;

- to the Scottish Economic Society who have financed the collection of insurance company accounts through the ‘Small Claims Award’;

- and finally to my parents (Richard and Gini), Craig, Mathew, and Ali.
1.0 Background: The Rationale for Regulation

There are two principal motivations for the regulation of life assurance, or more generally financial services, in the U.K. First, there is the problem of information asymmetry, and second there is the problem of systemic risk. These two factors form the foundation of a public interest theory of insurance regulation, where government imposes regulation to correct a perceived market failure for the benefit of the public.\(^1\) Moreover, information asymmetry and risk bearing are also the twin pillars of agency analysis (see Laffont and Tirole (1993)).

1.0.1 Information Asymmetry

Consider in greater detail the nature of the market imperfection that pertains to life assurance. The market for insurance is characterised by asymmetric information, which makes it difficult for policyholders to assess the risks and returns associated with the insurance policy. This problem is especially applicable to life assurance, where the expected returns extend many years into the future.

\(^1\) A comprehensive list of the various sources of market failure and their relationship to regulation can be found in Breyer (1981). Note that the traditional examples of market failure motivating regulation are natural monopoly (Baumol 1977, Waterson 1988), imperfect information (Akerlof 1970, Shapiro 1983, and Milgrom and Roberts 1986), and the provision of public goods (Fisher and Peterson 1976).
There are two types of information asymmetry relevant to this. First, there is the complexity of contract argument where the nature of the contract between the insurer and the policyholder may be difficult to understand, both in respect of the objectives of the plan, and in relation to the charges and investment decisions. Life assurance contracts are long-term in nature with policy terms frequently in excess of 10 years. In theory, in order to compare contracts across insurance companies, it would be necessary for prospective policyholders to compare alternative cashflow profiles associated with different insurers and this, for the most part, is not within the capabilities of policyholders. Thus, regulation of information disclosure is one solution to this information asymmetry, whereby the information disclosed assists the policyholder in forming less biased judgements.

The second information asymmetry relates more specifically to the assessment of insurance company solvency. This problem is compounded for long-term contracts where the expected return is contingent on the solvency (and conduct) of the insurance company long after the contract has been effected. Moreover, the value to the policyholder of that solvency will increase over time, as the policyholder’s financial stake in the insurance company rises. Most policyholders are poorly placed to assess the solvency of an insurance company: most policyholders do not have the expertise to form a judgement on financial solidity, or the resources to acquire the necessary information. Thus, there is some market efficiency in
ensuring that a regulator acts as a collector and monitor of information to ensure adequate safeguards for policyholders.

In the absence of adequate regulation, policyholders may make decisions that are unlikely to be in their best interest, and these decisions may not only have a negative impact individually, but may also have wider implications for the market as a whole. The resulting feedback to insurance companies may encourage and stimulate poor market practice rather than provide competition on the basis of efficiency.

1.0.2 Systemic Risk

The second principal motivation for the regulation of life assurance is systemic risk. The operation of a modern, market economy sees financial institutions make operational decisions, particularly in respect of investments, without direct government control. The systemic risk argument rests on the proposition that a stable financial system provides a favourable environment for efficient resource allocation and therefore promotes economic growth. Moreover, the need to promote stability in the financial system is based on the notion that the failure of a financial institution may, through 'contagion' effects, undermine the stability and public confidence in the financial sector as a whole. This argument is frequently cited as motivation for the regulation of banks because of their payment function. However, the application of this argument to insurance is also relevant because of the central
role that insurance companies play in risk management, which facilitates the smooth
functioning of the economy helping individuals and businesses to plan many years
into the future. The facilitation of inter-generation transfers may also be adversely
affected through the collapse of insurance companies. Thus, the cost of increasing
uncertainty, associated with a lack of confidence in the insurance industry, may
result in a systemic risk.

In order to evaluate the level of systemic risk, there are a number of minimum
standards that are formalised to regulate solvency of institutions selling financial
products or services. These regulations aim to minimise the risk of policyholder
loss and also ensure that in the event of insurance company insolvency, there are a
number of safeguards in place in order to ensure public confidence. Hence,
solvency regulation is motivated here out of a public interest theory of regulation.

1.0.3 Alternative Theories of Regulating Life Assurance

While the public interest or normative theory of regulation is greatly persuasive, it is
important to note that there exist a number of alternative theories of regulating life
assurance. These theories are primarily ‘positive’ in nature and are justified on the
grounds that public interest theories of regulation ignore the fact that regulators have
objectives that may not coincide with those objectives assumed by normative theory
(typically, the maximisation of a welfare index (Baron 1989)). Positive theories
emphasise the roles of different interest groups within the politics of regulation (Olson 1965, Moe 1980, and Becker 1985).

Consider one such group of positive theories that together forms the ‘capture’ theory of regulation. The proposition here is that regulation is primarily motivated by the desire of a grouping, either political or industrial, to attain a position of power or influence. That is, the cartel or capture theory of regulation holds the view that organised interest groups dominate the outcomes of regulatory decisions (see Stigler (1971), Posner (1974), and Peltzman (1976)). Thus, life assurance regulation may in fact be motivated as a direct consequence of lobbying. While the ultimate influence of interest groups on regulation is unclear, there is no doubt that lobbying does occur and that this is significant. For example, there are many debates relating to the legislative changes currently taking place within the regulation of the financial services industry. One such debate relates the extent to which consumers should be held explicitly accountable for effecting a financial contract. The financial services industry is currently lobbying to have an explicit reference in the statute to this consumer responsibility. Although, the outcome of this lobbying is unclear, the significance of this effort is clear and hence the capture theory of regulation is an important contribution. Note that a significant volume of literature has attempted to test interest-group theories of regulation (for examples see Bernstein (1955), Caves (1962), Friedlaender (1969), and Stigler (1971)).
Another theory of regulation that might be relevant to life assurance regulation is the more general economic theory of regulation (Downs 1957). This theory regards regulation as an economic good, subject to the laws of supply and demand. Regulation is supplied by those in a position to do so if the demand for it, in a currency acceptable to the regulator, is sufficient. The 1986 Financial Services Act may be analysed in an economic theory context. Prior to the introduction of the Act, there had been widely publicised cases of investor losses and this resulted in public demand for heightened investor protection to prevent future losses. Thus, the 1986 Financial Services Act can be seen as a response, in part, to the demands of the public. While regulatory theories of demand and supply will be material to an understanding of regulating life assurance, alternative theories of regulation, especially public interest theories, are likely to be more relevant.²

1.1 Definitions and Objectives

"An insurer is solvent if it has sufficient assets to meet its liabilities," (Taylor and Buchanan 1988, 49). However, there is much ambiguity associated with this definition. One of the recurring themes in the measurement and regulation of insurer solvency concerns the complexity associated with a workable definition of solvency. As the eminent actuary, Teivo Pentikäinen, points out:

We should appreciate the fact that the solvency position [of an insurer] is affected by nearly all the economical activities and decision making processes of an insurer... If understood in this broad sense, the solvency issue may have a tendency of spreading out to embrace just about

² The net outcome will probably be a function of all the alternative theories of regulation.
nearly everything. Therefore, it is necessary to limit the considerations in order to confine it reasonably under the title "solvency" and to be content with only making references to many of the relevant factors. (Pentikäinen 1988, 1)

It is a matter of fact that providing a workable definition of solvency is often fraught with complications. However, insurance companies are in the business of assuming risk (and hence variability) and solvency is therefore seen to depend critically on three factors: time, risk, and probability. The relative importance of each of these factors is explored in greater detail in subsequent chapters.

Given the ambiguity surrounding a workable definition of solvency, it is natural to ask whether regulators are asking the 'right' questions in respect of insurance company performance and solvency. Related to this question, is the following: given an appropriate theory of insurer solvency, what types of questions should a regulator be asking? Moreover, are current regulations taking into account the relevant considerations of time, risk and probability in attempting to provide a workable definition of solvency? Here lies the central objective: to expand upon the various approaches to analysing solvency by developing a 'Monte-Carlo' simulation portfolio model of an insurance company to demonstrate some additional considerations that might be deemed relevant to the solvency regulation of life assurance in the U.K.

A hypothetical life assurance company is constructed and assumed to maximise the expected utility of end-of-period wealth, assuming a set of 5 asset classes and a
fixed liability. The run-off performance of the resulting 'optimal portfolios' are then evaluated in terms of their probability of insolvency with associated implications for the design of regulatory constraints being assessed. In particular, the contrast between \textit{ex ante} and \textit{ex post} measures of insurer solvency are highlighted with the conclusion taken that current regulations might gain further insight into the underlying solvency performance of insurance companies if they were to quantify insurer risk preference and use this within an \textit{ex-ante} measure of insurer solvency. Thus, an attempt is made to demonstrate that there are a number of additional factors to consider with respect to the design of solvency regulations. However, any subsequent policy prescriptions must be qualified within a cost-benefit analysis of regulatory shift.

In order to provide a sound institutional background, part of the analysis reviews the current regulatory constraints in the U.K., while another part has focused on providing a full account of the main statistical and economic features of the U.K. life assurance industry. Much of this work is based on personal, professional experience in the life assurance industry, which involved the completion of exams towards the insurance industry's Financial Planning Certificate.

Note here that the term life assurance and the term life insurance are used interchangeably, although there is an historical explanation for these terms. \textit{Assurance} is an earlier term used before the end of the sixteenth century. Its general application is retained in the titles and policy names of many long-established
companies (such as the Legal & General Assurance Society Limited). By contrast, the term *insurance* was first made reference to in relation to fire, but soon became adopted equally with *assurance*. The term *assurance* is primarily related to life business, rather than general insurance such as motor and property. The use of the term *assurance* is retained in the U.K. unlike other countries such as the United States (see Jones-Lee (1976) for discussion), where *insurance* is used exclusively.

1.2 Structure of Thesis

The nature of insurance company solvency is such that this subject matter is interdisciplinary. Insurer solvency has its traditions in the workings of actuarial science. However, more recently, the application of insurance to financial economics, particularly portfolio theory, has provided a fruitful avenue of research and introduced a further dimension to analysis. Moreover, there has been substantial cross-fertilisation between academic disciplines in terms of the use of simulation techniques. As a consequence, the structure and content of this thesis reflects the interdisciplinary nature of insurer solvency. Research is also complimented with an industrial organisation approach to the study of the life assurance industry in the U.K.

In the chapter that follows, a survey of the current legal and administrative environment in which insurance companies operate is presented. The history of insurance legislation dates back to the eighteenth century. The institutional
structure of insurance regulation demonstrates that much of the insurance regulations have been brought about in a reactive, rather than proactive way. The recent changes to financial services regulation are also presented with a discussion of the new financial services regulatory hierarchy. The results show that the regulatory objectives are geared towards minimising insolvency probability subject to a resource constraint.

Chapter 3 provides a survey of the actuarial literature governing insurance solvency. The 'classical' approach to insurer solvency emphasises liabilities and introduces the application of simulation techniques to the subject matter. This risk-theoretic approach to insurer solvency is based around 'ruin theory', a formal framework for evaluating insurer insolvency probability. Moreover, the formal basis for the current solvency regulations in the European Union is presented; this demonstrates that the current regulations have limited theoretical grounding.

Chapter 4 introduces the financial approach to insurer solvency. The developments within financial economics are outlined within an asset-liability management framework. As such, the crossover influences between actuarial science and financial economics are highlighted especially within the use of actuarial portfolio models. The immunization strategy (Redington 1952) is developed formally in conjunction with portfolio theory (Markowitz 1959). Applied research on solvency comes in the guise of 'financial distress' models and a review of this literature is provided. The expressed aim of this applied research is insolvency prediction.
Chapter 5 extends the portfolio analysis of insurer solvency begun in Chapter 4, by illustrating the portfolio approach to the study of life assurance. The results of portfolio balance are established within a risk/return framework and the algebra of efficient portfolios is developed. An operating point on the efficient frontier is selected according to an expected utility maximisation rule; the insurer selects an operating point where its marginal rate of substitution is equal to the objective trade-off between risk and return. In addition, a solvency constraint is constructed formally within the risk/return framework. This constraint is shown to divide the efficient frontier into two operating regions: permissible strategies and non-permissible strategies. Thus, the expected utility maximisation rule is constrained to fall within the permissible set of operating strategies.

Having provided a review of the many alternative approaches to the study of insurer solvency, attention then turns in Chapter 6 to illustrating the structural characteristics of the U.K. life assurance industry. This chapter demonstrates the variety of life assurance markets as well as the variable size and distribution of firms operating in the industry. Chapter 6 represents the first chapter of three chapters, where Chapter 7 details industrial conduct and Chapter 8 presents industrial performance. This work uses the structure, conduct, performance paradigm (SCP) as a format for discussion in order to provide a case study of the life assurance industry. Evidence from Chapter 7 suggests that insurance companies have objectives other than purely maximising rate of return; in particular, the objectives
reflect the ownership structure of the insurer. In terms of industrial performance, there is evidence to suggest that the distribution of profitability is concentrated about the industry average.

Chapter 9 attempts to formalise some of the statistical relationships between firm-level industry structure and performance. The methodological complexities associated with industrial structure-performance econometric models are substantial. However, this chapter attempts to provide some preliminary evidence. A database on 103 insurance companies operating in the U.K. life assurance industry was constructed and results demonstrate that firm size and the intensity of managerial expenses has a significantly negative influence upon profitability and solvency. This chapter brings to a close the industrial organization perspective on the life assurance industry. Such work provides a useful practical grounding upon which to develop subsequent ‘Monte-Carlo’ simulations.

Chapter 10 and 11 develop a portfolio simulation model of an U.K. life insurer. Use is made of the stochastic investment model by Wilkie (1995) to simulate the returns of 5 asset classes: cash, equities, property, fixed-interest government securities and index-linked government securities. The stochastic model of asset returns is used to construct a simulation model of insurer solvency (‘ultimate surplus’). Results demonstrate the central role of risk preference, and specifically a ‘U’ shaped relationship between risk tolerance and insolvency probability. The results also
show that current solvency regulations need to be firm specific as well as needing to take greater account of the dynamic nature of insurer solvency.

Chapter 12 provides some conclusions and follows up on the simulation results of the previous chapter. In particular, Chapter 12 considers the regulatory questions that need to be asked in respect of solvency measurement. The simulations demonstrate that quantifying risk preference is a central regulatory requirement for measuring insurer solvency. Moreover, emphasis is placed upon the distinction between ex post and ex ante measures of insurer solvency, arguing that much of the current regulations use ex post measures that ignore crucial aspects of insurer solvency, namely time and probability. The value of simulations and forecasting as a means of developing ex ante regulatory constraints is discussed and qualified by the fact that ex ante regulations are only as good as the models used to forecast ex ante. Suggestions for future research are also outlined.

Providing a workable definition of solvency, and hence the design of regulatory constraints, is no easy task. While the alternative approaches of actuarial science and financial economics have been reconciled to an extent, much of the current regulations still in force today provide a statistical measurement of solvency, and therefore have a very limited theoretical grounding.
CHAPTER 2

THE LEGAL & ADMINISTRATIVE STRUCTURES AFFECTING THE U.K. LIFE ASSURANCE INDUSTRY

2.0 Introduction

Financial services regulation in general and life assurance regulation in particular have been the focus of regulatory change in recent years. Indeed, the establishment of a single U.K. financial regulator, in the form of the Financial Services Authority (FSA), has transformed the current regulatory framework within which the life assurance industry operates. At this time, regulatory structures and constraints have not yet been finalised. However, the legislative process will culminate in the presenting to parliament of the Financial Services and Markets Bill in 2000, next year. Although much can be said about these legal and administrative changes, many of the underlying principles governing life assurance regulation remain, and as a consequence discussion now turns to addressing these principles.

Any evaluation of regulation must include a detailed description of the current regulatory environment, reflecting the legal and administrative constraints imposed upon the U.K. life assurance industry. This review must form part of the background to any theoretical and empirical modelling. The types of
regulatory constraints imposed on insurers are many and varied, and this contrast exists across all types of insurance markets, both domestic and international. The intensity of these constraints, in terms of the level of intervention, also varies and in many cases is product specific. Finsinger and Pauly (1986), in their cross-national study of insurance regulation, identify six broad constraints that are often imposed upon insurance companies:

1. Regulations which specify the character of the insurance contract [i.e. controls on risk classification];
2. Regulations that place lower limits on the amount or composition of reserves against losses;
3. Regulations which set maximum and/or minimum prices;
4. Regulations covering the treatment of applicants for insurance whom insurers may be unwilling to cover at their current posted prices;
5. Regulations controlling the entry of new firms [other than solvency regulations, such as sales conduct guidelines]; and
6. Regulations associated with the presence of public enterprises in the market (Finsinger and Pauly 1986, 6).

The history of insurance regulation in the U.K. is quite distinct when compared with our European counter-parts, although many of the recent legislative developments have been designed to ‘harmonise’ some of these differences. The
insurance industry has experienced relatively little direct government intervention. Perhaps this result comes as no surprise if we accept the traditional view of the insurance industry as being competitive. However, while competition has been encouraged in order to derive efficiency gains and market discipline, the risk of insolvency has lead to the introduction of a number of regulatory constraints. These constraints consist of restrictions on ‘margins of solvency’ and reserve requirements in the form of a device called the Guarantee Fund. In addition to these explicit constraints, there are a number of further restrictions governing ‘codes of practice’ in respect of marketing practices and more general managerial conduct.

This chapter aims to provide an overview of the main legislative and administrative features governing life assurance in the U.K. Section 2.1 develops the main economic motivations for regulation (as introduced in the previous chapter). Section 2.2 then presents a brief history of the development of regulations in the U.K. and then, in section 2.3, the legislative changes brought about by the European Union are presented. Section 2.4 defines the current regulatory environment, while also emphasising the ongoing legislative changes that are taking place until 2000. By contrast, section 2.5 outlines the industry’s own ‘self-regulation’. A summary is given in section 2.6.
2.1 Why Regulate?

In order to understand the motivation for solvency constraints, reserve requirements, and conduct guidelines, it is useful to analyse the basic life assurance contract. In its simplest form, the life insurer agrees to pay back a certain sum (the sum assured) on the occurrence of the policyholder’s death in return for a policyholder commitment to pay a regular premium to the insurance company. As illustrated in Chapter 1, there are two principal market failures governing the motivation for financial services regulation: systemic risk and asymmetric information. These two ‘failures’ are discussed in the context of the insurance contract.

Solvency and reserve regulation aims to protect the financial solidity of the life-contract, and to reduce the externality that results from the adverse incentives for risk taking. In respect of the financial soundness of the insurer, there is clearly an information asymmetry. The performance of an insurance contract will depend upon the financial strength of the company and the retail customer is unlikely to have the expertise to judge the financial position, or the resources, to obtain the relevant information and advice. Solvency and reserve regulation aims to ensure a minimum standard amongst insurance companies, which in turn keeps the risk of complete market collapse in check.¹

¹Market collapse is defined in the ‘market for lemons’ sense (Akerlof 1971).
Consider next the issue of incentives. The adverse incentives come from the insurer and the policyholder: from the policyholder in the form of the moral hazard problem, and from the insurer in the form of premiums being set too low in the drive to compete, so as to not reflect the expected value of the loss.

A further issue to consider with respect to incentives and agency issues is the effect solvency regulations have on the incentives of management. As there are other regulations to protect the investments of policyholders, the solvency regulations clearly have a further function in addition to just measuring and regulating insurer solvency. The solvency constraints ensure that management and owners are subject to incentives that align their self-interests with the financial well being of the insurance company. How do the regulations do this? Through reputation effects is the answer. Assuming that managers are risk-averse, the damaging reputation effects resonating from the failure of the insurer will be such that there will be a strong incentive for managers to maintain adequate solvency margins. In sum, in the event that an insurer runs into difficulty – as defined by the relevant regulation and solvency measure – it is the reputation ramifications that serve as a constraint on the conduct of management.

Furthermore, the complexity of contracts adds to the uncertainty faced by the policyholder in that there may be difficulty in understanding the details of the contract, for example in respect of policy charges. The adverse incentives,
coupled with this lack of transparency inherent within the insurance contract, demonstrate that the asymmetric information problem is pervasive. As a consequence, this market failure provides an additional motivation for solvency regulation, as well as justification for some conduct guidelines in respect of the marketing of products.

Solvency constraints may come in the form of minimum capital requirements, reserve controls, as well as accounting procedures. The determination of explicit levels of solvency constraints is no easy task and for the most part this has been left to arbitrary rules since the costs and benefits of additional reserves is not easily estimated.\(^2\) The main solvency regulation in the U.K. comes in the form of 'margins of solvency'. These margins are basically a minimum excess of assets over liabilities.

The Guarantee Fund is another form of solvency regulation and forms part of the minimum margin. This fund, "... is an industry-wide arrangement to pay all or almost all of the losses of an insolvent firm," (Finsinger and Pauly 1986, 15). The scheme has the advantage that it provides almost complete protection to consumers. In addition, the fund provides an incentive for the industry to self-regulate as every company has a direct interest in the financial well being of other companies. Any given firm does not want other firms to engage in riskier

\(^2\)There is also the obvious ambiguity that arises in defining accounting conventions, since most of the constraints require an agreed standard for comparison.
projects because it may be called on to finance the guarantee fund if the riskier project of its competitors fails.³

In addition to formal solvency constraints, there are a number of regulatory rules governing the conduct of insurance companies, as set out in the regulator's handbook. These rules aim to ensure a code of practice that is designed to limit misconduct and serve as a basis for the enforcement of consumer protection. Thus, the presence of these regulations, together with the solvency constraints, aims to ensure adequate confidence in the life assurance industry, while also ensuring adequate consumer protection in the event that the solvency regulations fail.

2.2 The Historical Development of Insurance Legislation

Today, legislation relevant to the insurance industry dates back to the eighteenth century. One of the most significant conclusions that may be drawn from the history of insurance legislation in the U.K. is that statutes were frequently motivated as a direct response to some form of financial difficulty experienced in the industry; that is, regulatory change was reactive rather than proactive. This section aims to illustrate these points and to provide a useful historical context in

³ Note that the Guarantee Fund may also establish a moral hazard problem since any losses will be covered in the event of insolvency.
explaining many of the principles underpinning insurance regulation and solvency measurement today.

The historical development of insurance legislation may be divided into insurance specific legislation and more general company legislation. Much of the early regulations governing life assurance were typically company controls covering company reporting and limited liability. However, there were insurance specific cases for regulation; for example, one of the earliest examples of insurance legislation came as a result of insurance gambling. The rapid growth of the insurance industry in the eighteenth century was due, in part, to the rise of insurance gambling (Cockerall and Green 1976). This occurred in spite of some of the legal restrictions imposed on the insurance industry (especially the Bubble Act of 1720, which operated until 1824 and which had encouraged Lloyds of London to grow and dominate the marine insurance market by effectively sheltering Lloyds from competition). Gambling may be contrasted with insurance in the following sense. An insurance contract is a contract:

For the payment of a sum of money, or some corresponding benefit, to become due on the happening of an uncertain event of a character adverse to the likely interest of a person effecting the insurance. (Houseman and Davies 1994, 345)

Therefore, the crucial difference between insurance and gambling is that where a person makes a bet, the hope is that he/she will make a profit; but where a person obtains insurance, he or she seeks to cover a potential loss.
Insurance gambling was allowed to take place since there were no laws at the time establishing the principle of insurable interest. Under the common law of England the courts could enforce a wager on the duration of a human life. Thus, insurance companies did not refuse the issue of policies to individuals who had no interest in the lives being 'assured'. Such a situation generated a spate of speculative activity. As a consequence, the Life Assurance Act of 1774 found that insurance taken out on lives or other events, in which there was no insurable interest, involved 'undesirable' gambling:

Whereas it hath been found by experience that the making of insurances on lives or other events where the assured shall have no interest hath introduced a mischievous kind of gambling. (Life Assurance Act 1774, s.4 [1])

The Life Assurance Act (1774) is still in force today and establishes the nature of insurable interest within an insurance contract. Contrary to the name of the Act, it applies to most other types of insurance.

While the size of the insurance market grew during the eighteenth century, the number of life offices also experienced growth. One of the most noteworthy life offices to come into being was The Equitable Life Assurance Society, which was largely acknowledged for its pioneering use of mortality tables in the calculation of insurance premiums (Cockerall and Green 1976). Many of the other life
offices that emerged in the first half of the nineteenth century specialised in life and fire underwriting; for example, the West Middlesex Assurance Company which was established in 1836. However, many of these newly formed life offices failed after exposure of frauds and profligacy. The highly speculative nature of these organizations made companies very sensitive to changes in premium income and claims. In addition, the problem was made worse since there existed no formal solvency requirements for insurance companies and hence exposure of fraudulent activities was only brought about after a company’s cash flow failed (Cockerall and Green 1976). As a consequence of these problems, a number of select committee reports were initiated with the response of government being to introduce a succession of companies acts; for example, the Select Committee Report of 1841 was followed up with the introduction of the Companies Act 1844. In addition, the 1862 Companies Act introduced limited liability, with the aim of enhancing professional interest in the insurance industry (see Cockerall and Green (1976)).

While the changes to company law provided a useful foundation for insurance regulation, the legislation was still not very effective in stemming the tide of failing life assurance companies. The explanation for this was clear: the introduction of the company legislation was set against a background of a general laissez-faire attitude in the nineteenth century. Thus, while the acts attempted to control for some of the more fraudulent activities there was little in
the way of controls over the solvency of insurance companies: many companies were still heavily undercapitalised. A surge in claims and/or policy surrenders therefore left a company vulnerable and indeed many life offices were unable to meet their current liabilities. This was indeed the case with the Albert Life Assurance Company and the European Assurance Society, both of which failed in 1869 and typified the state of affairs in the insurance industry (Cockerall and Green 1976). Furthermore, the reliance on the market mechanism saw many amalgamations and take-overs as companies attempted to build on their market position. Small, local life offices were frequently the targets for proprietary life companies, in their search for greater premium income and profit. Yet many of the amalgamations took place when the party taken over was technically insolvent and hence the expansion frequently proved highly damaging.

The circumstances that prevailed in the insurance industry during the mid-nineteenth century lead to the enactment of the Life Assurance Companies Act of 1870, which introduced some formal controls on company capital. The Act required that any ‘person’ intending to set up a life assurance company was to deposit £20,000 with the Accountant General for security (Franklin and Woodhead 1980). This was the main constraint imposed on the establishment of a new insurance company. Also stipulated within the Act was a standard layout for revenue accounts and balance sheets, highlighting the need to isolate the funds attributable to life business, and for these life funds to be independently
valued at least once every five years. Furthermore, given the recent surge in amalgamations and take-overs at that time, the Act laid down conditions for future amalgamations. With regard to the threat of insolvency, the courts were empowered with the right to 'wind up' insolvent companies on petition of either a policyholder (in the case of a mutual company or proprietary company) or shareholder (in the case of a proprietary company). Thus, regulatory review was introduced formally in the form of solvency surveillance.

The 1870 Life Assurance Companies Act was not a total success and there were still some insurance company failures that took place towards the end of the nineteenth century and early twentieth century. However, probably the single most important feature of the Act was the introduction of the principle of 'Freedom withDisclosure' which can be seen as one of the major principles upon which regulation in this industry has been based. 'Freedom with disclosure' is the idea that the regulator should require companies to reveal as much information as necessary to enable itself and other interested parties to be able to assess the financial position of a given company. How successful this has been in practice is open to many debates, but it nevertheless introduced an important principle for regulatory control.

In order to appreciate the legislative changes of the early twentieth century, it is worth examining some of the market conditions for insurance in the last half of
the nineteenth century. By the 1850s the market for insurance, especially life assurance, had become fiercely competitive and many additional developments were taking place within the insurance market as a whole. The introduction of new insurance products, not covered in the legislation, reintroduced significant financial distress to the insurance industry. These new forms of insurance business included railway accident, sickness, financial guarantee, burglary, and employers' liability. Also, there was the innovation of industrial assurance, which targeted low-income groups who were attracted to the introduction of new weekly premiums rather than the established annual premium contracts. The forerunner to endowment policies was introduced in the form of bond investment business in 1837, and by the turn of this century a new class of general insurance was introduced in the form of motor insurance. Thus, the number of insurance products available on the insurance market rapidly increased, seeing expansion especially into general lines of insurance.

As already noted, another marked change within the insurance industry concerned the rise in aggressive marketing and the increase in take-overs and amalgamations. The result was the formation of large composite companies. Note that there were also changes in the taxation of insurance: life assurance premiums were deductible against income tax and hence this encouraged sales. The expansion of markets overseas, through the territories, provided another
source of growth and indeed these markets offered little or no competition to the established British companies. Thus, there were many structural changes within the insurance market and these presented new challenges to the established legislation; it became only a matter of time before the regulations governing insurance would need to be changed.

In 1909, the Assurance Companies Act came into force, with the purpose of applying the terms and conditions set out in the 1870 Life Assurance Companies Act to all other classes of insurance (with the exception of marine and motor insurance, where legislation was designed specifically for these lines of insurance). Separate accounts were to be kept for each class of business for which the company was authorised, but specific assets were not yet required to be earmarked (Houseman and Davies 1994). Lloyd’s and other associations of underwriters were excluded from the main provisions of the Act; however, they were required to make deposits with the Board of Trade and to submit periodic returns to the Board.

After the introduction of the 1909 Act, a number of insolvencies continued to occur where the cause of failure lay in other classes of business. The two world wars interrupted some of the insurance legislative changes, although powers were granted to the Board to investigate weak companies and, if necessary, to petition for a company to be closed down. It was not until 1946 when the next
major piece of legislation was introduced in the form of the 1946 Insurance Companies Act. The Act had the expressed aim of extending and strengthening the powers of the Board of Trade and introduced explicitly, for the first time, the so-called ‘margins of solvency’. General and ordinary long-term businesses were differentiated in terms of the rules governing these margins of solvency. For general business, solvency margins were defined as being £50,000 or one-tenth of the general premium income of the preceding year (whichever was the greater). In the case of ordinary long-term business the margin of solvency was set at £50,000 irrespective of the size of premium income. The minimum paid up share capital was set at £50,000.

The margins of solvency, as defined above, were of little significance to insurance companies. As a consequence, they represented very little restriction on the entry of new, inexperienced and poorly managed companies. Insolvencies therefore still occurred. The problems still associated with the insurance industry were illustrated in the motor insurance market where there were nine insurance company failures in the period 1965-67 (for example the failure of the Fire, Auto & Marine in 1965). This again saw an immediate response from government in the form of the 1967 Companies Act, which sought to strengthen the Board of Trade further, providing the Board with all the powers necessary to ‘supervise’ insurance fully (Houseman and Davies 1994). This included the right to refuse authorisation and the power to request information from
companies, including accounts and supporting data in respect of claims settlements.

In spite of these additional changes to the statute book, one of the most spectacular cases of insolvency in recent years occurred in 1971 with the failure of Vehicle and General, a motor insurance company. The Vehicle and General case brought about much of the legislative changes to insurance regulation in the 1970s. For these reasons the history of Vehicle and General will be expanded upon further (see Beale (1972)). In 1961, Vehicle and General started with the minimum capital of £50,000, and enjoyed rapid growth to the point where it accounted for around 10 percent of the private motor insurance market.

While premium income remained high over most of the 1960s this was only explained through the movement from a relatively low-risk portfolio of policies to a relatively high-risk portfolio of policies. Then in 1968 the industry tariff system was abandoned and as a result fierce price competition was generated with the result that Vehicle and General’s expanding premium income ceased and it was no longer able to mask its financial position through expanding premium income with the result that it was declared insolvent in 1971.

The failure of the Board of Trade to stop the insolvency of Vehicle and General was blamed on a number of factors. Aside from the allegations of negligence
and incompetence on the part of the Board, there were two factors considered to be important. First the inadequacy of supervision. The Board relied on the submission of accurate and up-to-date information by insurance companies. This was restricted anyway by the usual lags in information production, but was exacerbated through the introduction of computerised systems, which introduced a number of transitional problems. Second, there was the claims experience of the ‘at-risk’ policies, which was far worse than anticipated. Reserves were simply unable to meet the claims due to poor rating/reserving, and this problem was therefore made worse through the inadequacy of supervision.

The consequences of the Vehicle and General crash were profound and, as noted, this resulted in further legislation. The 1973 Insurance Companies Amendment Act attempted to increase the efficiency in the production of information to the Board of Trade and to address new products, particularly unit-linked policies. Within the Board, the Act provided the Board with more powers of intervention. The need for sound and prudent management was also emphasised with increased staff numbers used to assess information submitted to the Board. Also, it should be noted that the 1975 Policyholder’s Protection Act and the 1977 Insurance Brokers (Registration) Act can be seen as a response, in part, to the Vehicle and General insolvency.
In 1974, there were seven life assurance companies that experienced financial hardship. Five of these were taken over by other insurance companies, the other two, London Indemnity and General and Nation Life, were eventually rescued by a consortium of other life companies. As a result of these changes, London Indemnity and General policyholders had to forgo all surrender values of policies and also accept a 10 percent reduction in maturity values. Nation Life policyholders had to receive nominal payments of 76.5 pence in the pound - this was however spread over many years and therefore the real payments were far less (Houseman and Davies 1994). The failure of these companies was due to the default of its investments, which were concentrated heavily within the property market.

Experience had demonstrated the vulnerability of investors funds to company instability, even with the implementation of a rescue plan. As a result, the 1975 Policyholders Protection Act established the protection of policyholders and others who had been or may have been affected as a consequence of the inability of U.K. authorised insurance companies to meet their liabilities. With this aim the Act set up the 'Policyholders Protection Board' comprising of members from the industry and public, for the purpose of monitoring and enforcing the provisions of the Act. The Policyholders Protection Act ensured that private policyholders of insurance companies in liquidation would normally be granted 90 percent of the benefits promised under their policies (for example in the case
of long-term policies). In the case of compulsory insurance (for example motor 
insurance) policyholders would be guaranteed 100 percent of the guaranteed 
benefit (Policyholder's Protection Act 1975, s.6 and s.7). In the case of a 
company being declared insolvent, the Policyholders Protection Board attempted 
to find a buyer subject to the constraint that policyholders should receive benefits 
of an amount at least as great as those outlined above. If there were no buyers on 
these terms, the Board would then pay out the agreed benefits. Thus, the 
Policyholders Protection Board served as means of indemnifying policyholders 
against insurance company failure.

In order to fulfil its objectives, the Policyholders Protection Board was 
authorised to impose a levy on the insurance industry in order to finance any 
rescue plan. These levies could be imposed on intermediaries (brokers who have 
channelled business to insurance companies), or on insurance companies. 
Insurance companies would be levied on a pro-rata basis. Levies were also 
imposed separately, with respect to general and long-term business 
(Policyholders Protection Act 1975, s.21). Note that just prior to the 
Policyholders Protection Act, there became a formal requirement for each 
insurance company to retain an 'appointed actuary', an individual to monitor the 
solvency of the company at all times.\(^5\)

\(^5\) This enshrined into law what had already been common practice within the life assurance 
industry.
After the introduction of the Policyholders Protection Act, there were to follow almost immediately three more cases of company failure. One company, Lifeguard, stopped taking new business, though after a capital injection continued trading with restrictions imposed on its activities. The other two insurance companies that faced difficulty had their ownership eventually transferred (under the supervision of the Board), and this resulted in benefits being paid to the policyholders that were in excess of those minimum terms stated in the Act. The levy to cover these actions was set at one quarter of one percent of long-term business premium income for the year (Finsinger and Pauly 1986).

The financial distress outlined above was a consequence of the sale of new, unconventional business in the form of guaranteed income bonds. Under guaranteed income bonds, a lump sum was paid by the policyholder and in exchange he or she would get an annual guaranteed return. After the term of the contract expired, the initial sum would be returned. At the time the contracts were established, insurance companies were agreeing to very high surrender values (from 95 percent of the sum invested). Alongside this fact, increasing rates of returns were also being offered in a competitive drive for business. This generated a spate of surrenders in order to effect new policies and thereby gain from these increasing rates of return. As gilts were used to ‘match’ the liability of the bonds, the performance of bonds became a critical determinant of
Moreover, at that time the value of gilts had experienced an extreme fall (because interest rates had risen) and companies faced the disastrous situation that, after commission had been paid, the value of its gilts was actually less than the 95 percent surrender value. The company was therefore running at a loss: a situation that was obviously unsustainable.

While 1975 Policyholders Protection Act was designed to include insurance intermediaries, the role and importance of these intermediaries was addressed in greater detail by the provisions laid out in the 1977 Insurance Brokers (Registration) Act. The Act had the objective of requiring the registration of all persons classed as an insurance ‘broker’ and introduced the regulation of broker’s ‘professional standards’. The Act established the Insurance Brokers’ Registration Council. Insurance brokers were required under the Act to register on the ‘Insurance Brokers’ Register’ which included information such as the broker’s professional qualifications and training. The register had the expressed purpose of promoting professionalism and expertise in the selling of insurance products. Along with controls on conduct there were also professional indemnity requirements. Thus, the Act can be seen as an attempt to raise the professional standing of insurance brokers through the promotion of a system of self-regulation.
While much can be said about the provisions of the 1977 Insurance Brokers (Registration) Act, it has been criticised on the grounds that it only covered those persons who choose to call themselves a broker. As many sellers of insurance did not refer to themselves as a 'broker', the Act left a significant amount of insurance intermediaries unregulated. This state of affairs was to remain unchecked until the implementation of the 1986 Financial Services Act.

In summary, the history of U.K. insurance regulation illustrates that many legislative changes have been reactive rather than proactive, with regulations being frequently introduced in response to a financial upheaval within the insurance industry. More recently, the European influence has introduced legislative change that is more proactive in respect of the single market and this serves as the point of departure for discussion in the next section.

2.3 The European Influence on U.K. Insurance Regulation

One of the motivating factors behind the introduction of insurance legislation over the past 25 years, has been the need to comply with European directives that had the expressed aim of 'harmonising' legislation throughout the member states. After the U.K. joined the European Economic Community (the precursor to the European Union), the first non-life and life directives in 1973 and 1979 respectively, required the U.K. to amend or introduce requirements governing a
whole range of insurance regulations (notably authorisation and reporting standards). These directives and their influence on domestic legislative change are explored here in greater detail.

The influence of the European directives is seen within the solvency regulations as set out in the 1974 Insurance Companies Act. In 1974, the Insurance Companies Act defined very simplistic measures of solvency. The solvency margins for general insurance are given in section 4 of the Act and are also given in Appendix 1. These general insurance solvency margins were a function of the size of the insurance company, as measured by its total premium income. In contrast to the general insurance regulations, the solvency requirement for a company engaged in long-term business was to remain at a margin of £50,000 regardless of premium income.

Solvency regulations for general business were to become more complex with the introduction of the 1977 Solvency Regulations, which introduced a more complicated means of assessing solvency based on past premiums or past claims experience. This resulted in larger margins being required for general insurance business.

It was not until 1981 Insurance Companies Act that more complicated margins of solvency were introduced for long-term business. These changes were a direct

The 1979 Council Directive was one of the first directives to influence government legislation, with a view to co-ordinating EEC laws, regulations and administrative provisions relating to the business of life assurance. The content of this directive focused on the rules of authorisation, the control of solvency margins, and the rules governing the calculations in accounts and statements. Solvency margins were defined using two methods (‘Calculation 1’ and ‘Calculation 2’) both of which took into account of the size of mathematical reserves and the capital at risk, (see Article 19 of the Directive). These calculations formed the basis of subsequent solvency calculations used today.

In addition to the above changes, the Guarantee Fund and the Minimum Guarantee Fund were introduced in Article 20 of the Directive. As noted in the previous section, the Guarantee Fund is an industry wide arrangement to pay all, or almost all of the losses of insolvent insurers; where what can be recovered
from the assets of an insolvent firm is recovered, and any shortfall between recoveries and claims is distributed on a 'pro-rata' basis throughout the rest of the insurance industry. The Minimum Guarantee Fund sets a minimum on the Guarantee Fund contribution made by each company. These contributions from insurance companies are called on if, and when insolvency occurs.

The measurement and regulation of solvency in the European Economic Area is set out in Figure 2.1 below. This illustrates the relative sizes of Guarantee Fund and the Required Margin of Solvency.

Figure 2.1. The Measurement of Solvency in the EEA


Notes.  
(a) GF stands for Guarantee Fund.  
(b) RMS stands for Required Margin of Solvency.  
(c) MS stands for observed Margin of Solvency.
In line with the objective of harmonising laws and regulations and developing the single market, the subsequent directives of 1990 and 1992 respectively, extended the First Life Co-ordination Directive. These directives included the establishment of standardised accounting and actuarial procedures, and the extension of solvency constraints to new financial instruments that have evolved over the last 15 years. Further regulations on insurance conduct (for example marketing) were also introduced, as well as regulations to promote the standardisation of the tax treatment of life assurance across the community.

In line with the directives governing the regulation of life assurance, there have also been similar directives for non-life insurance business. These directives, like their life counterparts, include ‘co-ordination’ objectives aimed at harmonising non-life insurance administration and law. Note that while most general insurance comes under these ‘non-life’ directives, other classes of insurance (for example, motor insurance) have their own set of directives. Also, the Third Life Directive led to a number of further regulatory changes; for example, the 1996 Insurance Companies (Accounts and Statements) Regulations were motivated in part by the directive. Thus, the life directives have had a profound influence upon insurance regulations in the U.K. In particular, these legislative changes have built on the ‘freedom with disclosure’ principle.
2.4 The Current Regulatory Environment

As noted in the introduction, there has been substantial change recently in the structure of U.K. financial services. For this reason, much of the insurance regulations, relating in particular to the regulatory hierarchy, have yet to be finalised. Indeed, the formal legislative development will come next year in the form of the 2000 Financial Services and Markets Bill, which aims to transform the nature of financial services regulation through the use of a newly established single regulator, the Financial Services Authority (FSA). As a result of these changes, a review of the current regulatory framework will be given and this will be supplemented with details on the main regulatory changes, currently being considered.

The prudential regulation of insurance covers three main areas: constraint on actions, requirements for disclosure, and provisions of protection for policyholders in the event of company insolvency. The constraint on action is addressed in the 1982 Insurance Companies Act, while the disclosure legislation is dealt with in the 1996 Insurance Companies (Accounts and Statements) Regulations. As noted in the previous section, policyholder protection is addressed in law by the 1975 Policyholders Protection Act. The 1986 Financial Services Act (soon to be replaced by the 2000 Financial Services and Markets Act) governs the regulation of sales and marketing of financial services.
2.4.1 The 1982 Insurance Companies Act

The persistent inadequacies of previous legislation saw the imposition of greater control over the insurance industry in the form of the 1982 Insurance Companies Act (ICA). This act primarily consolidated previous legislation, in particular the Insurance Companies Acts of 1974 and 1981.\(^6\) The ICA contains a large number of provisions concerning the following areas: the authorisation of insurers, the use of accounting procedures, the definition of solvency margins, the conduct of insurance business, and the role of government and its associated powers of intervention. Following the terms of the Act, any new insurer, or any existing insurer wishing to transact a new class of insurance business, must be authorised by the relevant government department. Up until recently, this department was the Department of Trade and Industry (DTI). However, the FSA has now been given responsibility, with the Insurance Directorate being transferred from the DTI to the new Director of Insurance and Friendly Societies at the FSA.

The details of the ICA are as follows. The ICA sets out the conditions that must be satisfied in order for an insurance company to be authorised to conduct insurance business in the U.K.\(^7\) The authorisation of a new insurance company requires the detailed submission of proposals which must include, "... the manner

---

\(^6\) The 1982 Insurance Companies Act has been amended significantly, for example the recent amendments set out in the Insurance Companies (Amendment) Regulations 1994 (SI1994/3132).

\(^7\) Note that within these conditions is the automatic authorisation of insurance companies previously authorized under the 1974 Insurance Companies Act.
in which a company proposes to carry on business such as financial forecasts and other information as may be required,” (Insurance Companies Act 1982, s.5 (1)). In addition, it must be demonstrated that the proposed insurer exhibits “... integrity and skill, ... fit controllers, ... and sound and prudent management,” (Insurance Companies Act 1982, s.5 (4)). Authorisation, once granted, may then be withdrawn under certain conditions; for example, in the event that a company fails to demonstrate that its conduct has been 'sound and prudent'. While having authorisation withdrawn is theoretically possible, this is very unlikely in practice. Far more likely is that the regulator will require the company to implement a plan to rectify any given problem.

In order to facilitate the process of monitoring and enforcement set out within the ICA, insurance companies are required to submit annual accounts and balance sheets for each class of insurance, together with any additional information requested by the regulator. If the insurer is not trading for profit (as in the case of a ‘mutual’), then it must submit a revenue and expenditure account. The format and contents of these informational requests are laid out in the relevant statutory instrument, (the detail of which will be illustrated in section 2.4.2).

In addition to the annual accounts and balance sheets, the ICA requires companies that are engaged in long-term business to appoint an actuary as

---

8 For the definitions of each class of long-term and general insurance see Appendix 1.
9 That is, the Insurance Companies (Accounts and Statements) Regulations 1996.
‘actuary to the company’. This builds on the previous developments associated with the use of a company actuary. The appointed actuary is required to produce actuarial investigations, and then submit these reports to the regulator for the purpose of allowing the regulator to analyse the financial condition of the company in respect of its long-term business.

A substantial portion of the ICA is set aside to authorise restrictions that are placed on the financial resources of insurance companies. Fundamental to the Act is the distinction between long-term and general insurance business, and in particular the separation of assets and liabilities in relation to the two classes of business (Insurance Companies Act 1982, s.28). Restrictions are also placed on the use of long term business assets, in that these assets must be used only to support long-term business (Insurance Companies Act 1982, s.29). This therefore rules out cross-subsidisation between general and long-term businesses.

Also, the Act stipulates a minimum proportion of the surplus fund (those funds in excess of calculated liabilities) to be allocated to eligible policyholders (Insurance Companies Act 1982, s.30).

There are a number of other constraints specified in the 1982 Insurance Companies Act worth considering. Assets are constrained in ‘form and situation’ (Insurance Companies Act 1982, s.35). That is, assets must be held in denominations of the same currency so that they match the currency in which the
liabilities of the company are denominated, or required to meet.\textsuperscript{10} Also, a company is required to secure its liabilities with assets of an appropriate, "... safety, yield and marketability having regard to different classes of business," (Insurance Companies Act 1982, s.35B). Furthermore, a company must also ensure that the aggregate of premiums paid into a policy and income derived will be sufficient on, "... reasonable actuarial assumptions," to meet all commitments arising in relation to the contract (Insurance Companies Act 1982, s.35B1 (b)).

Central to the Act are the ‘margins of solvency’ and these are set out in the relevant statutory instrument and are based on the E.E.A. standards illustrated in Figure 2.1- see section 2.4.1 (Insurance Companies Act 1982, s.32). U.K. margins of solvency must be applied separately to long-term and general business and, where an insurance company has business overseas, to that portion of the business derived in the U.K. (Insurance Companies Act 1982, s.34). Note the importance of valuation procedures here. The solvency position of a company is highly sensitive to the accounting and actuarial assumptions made. Given this sensitivity, rules governing accounting and actuarial conventions are also provided.

In the event that a company fails to achieve the set margins of solvency, the regulator has the power to impose further conditions. First, a company in

\textsuperscript{10} The advent of the Euro has important implications for asset/liability matching in respect of the number of assets that can be used in ‘matching’ if and when the U.K. joins the currency.
financial difficulty would be required to submit a short-term financial scheme containing modifications designed to rectify the current financial problems. Second, the proposal is then scrutinised by the regulator, which in turn makes its own judgement and recommendations (which may include modifications to the company’s original proposal). Finally, the modifications are agreed upon by the regulator, and then must be implemented in full by the company. If after due course the company still fails to meet the required margins then authorisation may be withdrawn as a last resort.

Another aspect of the 1982 Insurance Companies Act is that it sets out the powers of intervention vested in the regulator and the conditions under which these powers may be exercised. These powers include the right to prevent insurance companies from obtaining new premium income, in order to ensure that companies produce the necessary information for regulatory review. Restrictions may also be imposed on the company’s investments, such as restrictions on the custody of assets and restrictions on the disposal of assets (Insurance Companies Act 1982, s.40 & s.40A). These powers are backed up by a ‘Residual Power’ section that provides the regulator with the power to intervene in almost any way for the purpose of protecting policyholders or potential policyholders.
Procedures are also put in place for the ‘winding-up’ of insurance companies as well as procedures that require any individual who is responsible for the actions of the company (a managing director, chief executive or any other ‘controller’) to be approved by the regulator (Insurance Companies Act 1982, s.59). This provision is used to enforce the regulatory objective that requires that management of insurance companies should be ‘sound and prudent’.

The final main role of the 1982 Insurance Companies Act is in respect of the regulation of insurance company conduct. These provisions include the following: controls on insurance adverts, restrictions on contractual statements, restrictions on the behaviour of insurance intermediaries, and the establishment of a ‘cooling off’ period, during which an individual who has signed a long-term contract can withdraw from the contract and receive a refund in full. Note that it is also a requirement that the regulator prepare a report on the insurance industry in respect of the main provisions of the Act.

2.4.2 The Statutory Instruments

The 1982 Insurance Companies Act sets out the general requirements for regulating authorised insurance companies in law. However, the technical details of these requirements are set out in the relevant statutory instruments. This has the advantage that changes in technical definitions, as a result of say an
E.U. directive, only requires the amendment of the statutory instrument and not the statute itself.

The relevant statutory instruments effected by the 1982 Insurance Companies Act are: first, the 1994 Insurance Companies Regulations, which provides the detailed provisions of the 1982 Insurance Companies Act; and second, the Insurance Companies (Accounts and Statements) Regulations 1996, which specify the format of the data, submitted under the annual reporting to the regulator.

Within the 1994 Insurance Companies Regulations are the calculations for the margins of solvency. The calculations are a function of mathematical reserves and capital at risk (Insurance Company Regulations 1994, Reg. 17). They are approximately the same as those used in the previous 1981 Insurance Companies Regulations, with the exception that the margins are applied to two more categories of long-term business—collective and social insurance. Calculations are also provided for the Guarantee Fund and the Minimum Guarantee Fund. The guarantee fund is set equal to one-third of the relevant margin of solvency (Insurance Company Regulations 1994, Reg. 21), see Figure 12.1.

As mentioned in previous sections, crucial to the calculation of the margins of solvency is the way in which assets and liabilities are treated. The valuation of...
assets (Insurance Company Regulations 1994, Part VIII) and the determination of liabilities (Insurance Company Regulations 1994, Part IX) are therefore set out in greater detail within the 1994 Insurance Companies Regulations.

In order to reduce the risk of insolvency, it is necessary to ensure that the assets of a company are adequately diversified, and thus not overly reliant upon the performance of a few asset classes. To ensure that insurance companies engage in this strategy, there is a specific constraint placed on the valuation of assets. Assets of a certain type are restricted in terms of the extent to which they may count towards total assets. Limits are set in the regulations on the proportion of funds that are allowed to count towards total assets (Insurance Company Regulations 1994, Reg. 57). Any excesses over and above these limits are not counted in the valuation of assets. There is therefore an incentive for a company to spread its investments across a number of asset classes.\(^{11}\)

The long-term business of an insurance company also has restrictions placed on the rate of interest used, when calculating future income streams in solvency assessments. This is designed to remove the incentive to maintain solvency by simply assuming high rates of interest in the future (Insurance Company Regulations 1994, Reg. 23). Interest rates may be constrained in general terms or may be constrained by an explicit rate that has regard to the current interest rate levels and the respective risk involved.

\(^{11}\) Whether this constraint is binding in practice is open to debate.
Further technical information set out in the 1994 Insurance Company Regulations includes restrictions on currency ‘matching’. Where a company’s liabilities are denominated in another currency and this value is in excess of 5 percent of total liabilities there are further constraints to consider. In the event that foreign denominated liabilities account for more than 5 percent of total liabilities, the company is required to hold sufficient assets in the same currency to cover at least 80 percent of this liability (Insurance Company Regulations 1994, Reg. 27). Details of the controls on the conduct of insurance companies and insurance intermediaries are set out as well within this statutory instrument.

While much has been said about the constraints on the actions of insurance companies, the legislation requiring disclosure derive largely from the 1996 Insurance Companies (Accounts and Statements) Regulations. These regulations require the annual reporting of a variety of data covering all aspects of insurance company performance. The form and content of this data reported to the regulator is also given within these regulations. This information may be broken down into a number of different categories, or ‘schedules’ and these are given for completeness in Appendix 1.

In keeping with the provisions of the relevant Acts of Parliament, a distinction is made between long-term and general insurance business. Together with the standard company requirements of providing balance sheets and profit and loss
accounts for each financial year, is the need to demonstrate the actual and required margin of solvency for the purpose of demonstrating the solvency of the insurer. This information must be contrasted against the provisions of the more general Companies Act that attempts to demonstrate the true profitability position of a company so that policyholders and shareholders can judge its worth.

As a supplement to the summary accounts mentioned above, the ‘additional information’ forms also provide a significant source of information. These forms are required for long term and general insurance business. For general insurance a revenue account must be prepared for each accounting class. Also, information regarding premiums, claims and expenses, analysed by accounting class, risk group, and geographical area is also required. With respect to long-term business, the additional information relating to the revenue account includes details of premium income, expenses, assets and new business. These statements must also be calculated for linked and non-linked contracts subtotalled along business lines; for example, life assurance business, general annuity business, or pension business.

From the perspective of an outside observer wanting to evaluate the competitiveness/associated risk of insurance companies and their respective product lines, the extent to which this can be done is limited. Clearly, the
statement of solvency provides an indicator of insolvency risk (though, as will be argued in subsequent chapters, there is some ambiguity associated with the definition of insolvency risk), as one is able to compare 'required margin' with 'actual margin'. The balance sheet breakdown and subsequent 'additional information' also provides an indication of the overall exposure of the company investment portfolio. In addition, the revenue account gives an indicator of the claims portfolio with totals provided on policy claims relating to deaths.

Other information in the 1996 Insurance Company Regulations includes the following. Audited statements must be produced by the director to show that appropriate systems of control are in place and that certain guidelines have been followed in running the business and preparing the disclosure material. Also, as the appointed actuary may be an employee of the company, a statement of his or her remuneration from and, if applicable, his or her shareholdings in, the company is required.

While the accounts and statements provide some insight, the required information is restricted in that the information is aggregated; for example, equity shares are only broken down into 'listed' or 'unlisted'. This aggregation obviously limits the extent to which the regulator can examine individual contracts, and assess the risk associated with a given contract or the competitiveness of the associated premium charged. Such limitations are an
important consideration within any evaluation of regulation and, as subsequent chapters will attempt to demonstrate, efficient insurer solvency regulation might therefore prefer to ask alternative questions to those currently being asked.

Note that as part of the overall prudential regulation framework, there are other areas of additional legislation that is relevant for some lines of insurance business. Pension legislation and the associated tax legislation imposes certain requirements in the form of restrictions on the types of benefit that a policy can provide if it is to benefit from tax exemption. Also, certain annual disclosure requirements to existing policyholders are required (for example, the disclosure of the current value of the pension). Tax legislation also influences product designs in other areas; for example, investment policies qualify for some tax exemption so long as premiums are paid at least annually and the term of the contract is at least 10 years. Finally, it should be noted that the 1975 Policyholders Protection Act provides 'insurance' for policyholders in the event that control and disclosure fail (though a consumer protection review is currently being undertaken by the FSA).

2.4.3 The Regulation of the Sales and Marketing of Financial Services

While the Insurance Companies Act 1982 and the Insurance Brokers (Registration) Act 1977 brought insurance companies and their intermediaries
under much greater supervision, the combined effect of the two Acts still left many providers of financial services products and advice with relatively little supervision. Also, the control of conduct within insurance companies was such that it still left open the potential for significant problems (especially in relation to the poor selling of insurance products). Also there were few regulations relating to dealings with the investing public, and as a result of a number of serious frauds (which involved the loss of significant amounts of investors' funds) there existed significant public awareness in respect of these regulatory shortcomings.

The regulatory environment up until the introduction of the 1986 Financial Services Act, was seen as largely inadequate to deal with the considerable changes that had taken place within the financial services sector. Prior to the Act, the regulatory system had been based around the Prevention of Fraud (Investments) Act of 1939 and 1958. However, this was seen as an inadequate arrangement given the rapid changes that had taken place within the financial services sector since the late 1950s (and indeed the number of financial difficulties experienced during the 1970s and early 1980s). This lead to calls for a regulatory overhaul and greater investor protection. Against this background of concern within the general public, Professor J. Gower was appointed by the government in 1981 to undertake a review of investor protection. His terms of reference included the following: first, to consider the statutory protection
required for private and business investors in securities and other property; second, to consider the need for statutory control of dealers in securities, investment consultants and managers; and, third, to advise on the need for new legislation (The Gower Report 1985).

The interim report by Professor Gower in 1982 included life assurance business within the scope of 'securities' and therefore within its terms of reference. The full report was completed and published in two parts (January 1984 & March 1985). The report proposed a new act, which would provide for basic policy, and have overall surveillance and regulation of investment business undertaken by the government or a new agency. After consultation the report's recommendations were implemented through the passing of the 1986 Financial Services Act. Some of the sections of the Act were effected almost immediately, while the majority of the provisions were brought into force on the 29th April 1988.

The 1986 Financial Services Act introduced a substantial body of law, regulating 'investment business' carried out by firms, companies or individuals in the U.K., or from a 'permanent place of business' in the U.K. (Financial Services Act 1986). Most of the detailed rules governing the sale and marketing of financial services are not specified in the Act, but fall within the rule books of the regulatory bodies established under the Act. It is important to emphasise that the
regulatory framework and associated rulebooks have been the focus of recent regulatory review within the newly established Financial Services Authority.

The 1986 Financial Services Act defined investment business, and through this definition, the range of investments that are to be regulated. The relevancy of the Act for life assurance is that the Act governs those long-term products that have an 'investment element'; for example, endowment policies are regulated under the Act because the premium for the contract is used to build up an investment fund to pay for a mortgage. Products that do not have an investment element, are products such as term assurance contracts.\(^\text{12}\)

While many of the regulations set up under the 1986 Financial Services Act will be replaced by the 2000 Financial Services and Markets Bill, many of the principles introduced in 1986 will remain. For example, requirements are set out for persons involved in investment business to be either ‘authorised’ or ‘exempt’ before they can carry out investment business (Financial Services Act 1986, Ch. III, s.3).\(^\text{13}\) Insurance companies authorised under the Insurance Companies Act 1982 were automatically authorised to conduct long-term business under section 22 of the Act. Also, the aim of providing an all-encompassing rulebook of

\(^{12}\) Note also that general insurance, such as buildings and contents insurance and motor insurance is therefore not covered by the 1986 Financial Services Act.

\(^{13}\) The use of the term ‘person’ is meant to cover any individual, partnership or company conducting regulated investment business.
general principles governing market conduct are to be built upon and standardised across the entire financial services industry in the new legislation.

Consider next the formal regulatory structure governing financial services in the U.K. This too has been the focus of recent change in 1998 and 1999. The 1986 Financial Services Act established a complex regulatory structure with the expressed purpose of supervising the implementation and enforcement of the terms of the Act. Up until recently, the Chancellor of the Exchequer was given many of the powers by the Act, but was allowed to delegate these powers to one or more designated agencies (most of these agencies will remain under the 2000 Financial Services and Markets Bill). Most of the powers were delegated to the Securities and Investment Board (SIB), which has now been replaced by the Financial Services Authority (FSA). This regulator has the power to oversee the 1986 Act and to recognise regulatory institutions covering all aspects of financial service provision. The regulatory hierarchy is given below in Figure 2.2.

The regulatory hierarchy consists of the FSA, which in turn oversees four different types of institutions: 3 Recognised Self-Recognised Organisations (SROs); 9 Recognised Professional Bodies (RPBs); 6 Recognised Investment Exchanges (RIEs); and 2 Recognised Clearing Houses (RCHs). The SROs and the RPBs are the relevant bodies when addressing the insurance sector.
Financial Services Authority Recognises

3 Recognised Self-Regulating Organisations (SROs)
9 Recognised Professional Bodies (RPBs)
6 Recognised Investment Exchanges (RIEs)
2 Recognised Clearing Houses (RCHs)

Note: (a) Example of a SRO is the Personal Investment Authority.
(b) Example of a RPB is the Law Society of Scotland.

Approximately 6,250 firms regulated by the 3 SROs carry out most of the investment business. An SRO is a body which, once recognised by the FSA, has the power to authorise firms to conduct investment business in the U.K. and now, under the Investment Services Directive, throughout the European Economic Area, (FSA 1999). Professional bodies, such as solicitor and accountants, may also be engaged in the sale of financial products such as long-term insurance, though this involvement may be minor. As a consequence these firms may conduct investment business following ‘certification’ by their recognised professional body (RPB), which then regulates both their main activity (e.g. as a solicitor or accountant) and their investment business. Just fewer than 16,000 firms are authorised in this way (FSA 1999). In overseeing
these firms' conduct of investment business, the RPBs have similar responsibilities to that of the SROs.

Note that in addition to the power to supervise all forms of investment business, the FSA is also responsible for authorising insurance companies under the terms of the Insurance Companies Act 1982 (through the Director of Insurance and Friendly Societies). Insurance companies then gain automatic authorisation under the terms of the 1986 Financial Services Act. Most companies have accepted the authority of the Personal Investment Authority (PIA). The PIA is a recently formed SRO recognised by the Securities and Investment Board (the precursor of the FSA) to take over the responsibilities of two older regulatory bodies, the Life Assurance and Unit Trust Regulatory Organisation (LAUTRO) and the Financial Intermediaries, Managers and Brokers Regulatory Association (FIMBRA), together with the retail aspects of the Investment Management Regulatory Organisation (IMRO). This is a direct response to the perceived need for a single regulatory body, responsible for the retail investment sector.

As noted there is an attempt, with the recent changes to financial service regulation, to develop all encompassing principles for financial supervision in the U.K. For this reason, it is worth noting the statutory objectives of the FSA. The draft legislation gives the FSA four objectives: first, to maintain confidence in the financial system; second, to promote public understanding of the financial
system (improving the so-called ‘financial literacy’ of individuals); third, to protect consumers, (while also bearing in mind their own responsibility to inform themselves and to take responsibility for their decisions); and fourth, to reduce the extent to which regulated businesses are used for the purposes of financial crime (FSA 1999). Thus, many of the original aims of the 1986 Financial Services Act are embedded within the new regulatory framework. However, there are notable improvements relating to the streamlining of the regulatory structure and the consistency in rules across the entire financial services industry. Previously, there had been some variation in the regulatory rulebooks, for example, across the different SROs and RPBs. This lead to great ambiguity associated with definitions of financial crime, and also uncertainty in terms of the lines of responsibility within the regulatory hierarchy.

Note also that there has been the suggestion that the FSA’s objectives might be in conflict. Certainly in relation to the degree of consumer protection there is the potential for conflict. The extent to which consumer responsibility is enshrined into law is the source of many debates, with companies wanting explicit reference to consumer responsibility within the draft legislation and consumer groups opposing this move. Note in addition, that there is a review of consumer protection being undertaken, which is also the source of some conflict.
Thus, the regulation of the sales and marketing of investment products has seen substantial regulatory attention over the past 15 years. Indeed, there is a move to include other products within the new financial services regulatory framework, most notably mortgages.

2.5 Insurance Industry Self-Regulation

While formal insurance regulations cover a number of aspects of insurance company operation, it is evident that the extent of intervention in the U.K. insurance industry is still quite limited; for example, there are no explicit controls on premiums or controls on risk classification. Premiums and risk are only indirectly managed through solvency regulation. This regulatory intensity is to be contrasted with other countries, for example France and Germany, where regulations are more interventionist in nature. More emphasis has therefore been placed in the U.K. upon self-regulation as opposed to direct government control of company conduct.

2.5.1 Industry Trade Associations

Besides the formal regulatory structure there is the industry trade association in the shape of the Association of British Insurers (ABI), which provides an
additional source of regulation. The ABI was set up out of the British Insurance Association (formed in 1917 as a trade association open to all U.K. transacting companies which were from British or Commonwealth countries) and various Life Offices' Associations (including the Life Offices' Association (founded 1899) and the Scottish Life Offices' Association (founded 1841)).

The ABI is a trade association for insurance companies that together represent virtually the entire insurance industry. One advantage of the ABI is in its ability to provide self-help to insurers on regulatory matters; for example, the association has a media studio for the provision of training courses designed to meet the requirements of recent legislation on training and competence. The Association also heavily promotes a voluntary code of practice.

Other services provided by the ABI includes protection against fraud through the use of information registers (for example the 'theft register'), which are available to all members in their attempt to minimise illegal claims. There is also a code of practice to assist members on the legal requirements of insurance selling practices. The ABI Statement of Recognised Accounting Practice guides the construction of Companies Act accounts and balance sheets. Thus, the association provides a useful resource to its members. In addition, the ABI

14 The ABI does not have any statutory powers, although it does participate in the annual regulatory review process.
exists to represent the interests of its members to government, civil servants and regulatory bodies such as the PIA.

While the ABI has been useful in providing support for its members, its ability to enforce regulatory agreements has been heavily restricted by the fact that membership is voluntary. The benefit conferred to members is relatively small and therefore there exists very little in the way of effective sanctions that can be used to enforce association rules. A further point to note is that rules imposed upon members frequently yields a competitive disadvantage when compared to insurers who were outside of the given agreement. There is therefore a tendency for members to renge on agreements within a voluntary membership organisation, or else face a potential loss of business.

To illustrate the above point, an example concerns the introduction, by the then Life Offices Association, of controls on commission rates set by its members as part of its ‘Statement of Long Term Insurance Practice’ which aimed to remove commissions as the influencing factor in selling life assurance. These controls were bypassed by non-members and newly formed life offices who specialised in linked life policies (where a benefit is linked to the performance of an investment fund) and who offered better commission rates as part of an ‘aggressive marketing’ campaign. The association members began to renge on the commission agreement in order to match the higher rates of commission
being offered by competitors outside of the agreement. Thus, this eventually lead to the collapse of the agreement (Finsinger and Pauly 1986).

Additional evidence of insurance industry self-regulation comes from a number of other sources. While there is significant self-regulation of insurance companies, the area that has perhaps seen greater self-regulation is that of insurance intermediaries.

The Insurance Brokers (Registration) Act 1977 established a register of brokers with the aim of maintaining professional standards. More recently, the insurance industry in an attempt to meet the need for professionalism in insurance selling advice has introduced industry standard qualifications through the Chartered Insurance Institute.

The Chartered Insurance Institute provides a range of professional qualifications, the main one being that of the Financial Planning Certificate. Once passed, fellows of the Institute are required to adhere to a code of conduct that stands alongside the requirements of the 1986 Financial Services Act. Given the sizeable amount of work that is required to gain the Financial Planning Certificate and the fact that the certificate is an effective entrance requirement to the insurance industry, the Institute has significant powers of influence though the sanction of withdrawing institute fellowship.
2.5.2 The Role of Ombudsmen

Although the FSA is reviewing the complaint procedure, it is worth considering the several voluntary (and some mandatory) Ombudsman schemes used to resolve disputes. The terms of reference and the scope of coverage vary considerably. There are three main schemes worth noting and these are the PIA Ombudsman (PIAO), the Insurance Ombudsman Bureau (IOB) and the Personal Insurance Arbitration Service (PIAS).

The PIAO covers all life assurance with an investment element, but only investigates complaints once all internal company procedures have been exhausted. The PIAO has both voluntary and mandatory jurisdiction. Mandatory jurisdiction relates to businesses regulated by the PIA, while the voluntary jurisdiction applies to products not regulated by the PIA (that is, those products without and investment element such as Personal Health Insurance). The IOB was established in 1982. The IOB deals specifically with complaints about the marketing or administration of an individual policy. Most life cases are, however, dealt with by the PIAO leaving non-life to the IOB. There has been some confusion in the jurisdiction of the Ombudsman schemes, though this too is being addressed in the current regulatory review. Finally, the PIAS was
established by the Chartered Institute of Arbitrators and it has similar scope to the IOB in that it deals only with complaints by policyholders in the U.K., and only after the company in question’s internal complaint procedure has been exhausted without satisfaction. Thus, the Ombudsman schemes offer a degree of consumer redress and hence another channel through which to keep insurance companies in check. There are, however, a number of overlaps across the various schemes that have proven problematic and this has helped motivate the new regulatory review concerning consumer protection.

2.6 Summary

Much of the history of legislative changes to life assurance regulation can be seen to have occurred as a direct response to some form of financial difficulty experienced in the industry. More recently, changes have also been brought about to regulations as a result of European directives with the objective of harmonising laws governing insurance across member states.

The current regulatory environment for insurance companies consists of restrictions on company actions, information disclosure and policyholder protection. These constraints are deemed necessary in order to control for the market failure that is the information asymmetry that exists between the policyholder and the insurance company.
The new legislative proposals attempt to ‘streamline’ the regulatory framework and clarify the overriding principles governing financial service regulation. These changes represent a significant regime shift and the ramifications for the life assurance industry are bound to continue for many years.
CHAPTER 3

EVALUATING INSURANCE COMPANY SOLVENCY:
'CLASSICAL' INSURANCE SOLVENCY THEORY

3.0 Introduction

Assessing insurance company solvency is a complicated matter and as a consequence this subject has been the focus for a variety of academic disciplines. Solvency theory has its foundations in the work of actuaries and risk theory. Traditional research emphasises the liability-side of the balance sheet. This work is established within a framework of minimising the insolvency, or 'ruin' probability. The objective is to provide a workable definition of insurer solvency with a view to writing this definition into legislation. A variety of statistical methods have been adopted in evaluating the insolvency probability for a given insurance company, and in turn defining the appropriate solvency margin that would need to be imposed by a regulator. This work forms the basis of a body of literature that has come to be known as 'classical' insurance solvency theory. This work is to be contrasted against the more general approach of financial economics (and more recent actuarial research) which addresses the role of asset/liability management.

The outline of this chapter is as follows. Sections 3.1 and 3.2 expand upon the definitions and concepts introduced above. Section 3.3 develops some of the 'classical' methods used to evaluate insurer solvency, leading to some general implications for life assurance regulation in Section 3.4.
3.1 Quantifying Solvency

As introduced above, the theory of insurance company solvency is made up of two strands: classical insurance solvency theory, and financial or market theory of insurer solvency. The former strand of literature is concerned with actuarial risk theory. This was developed initially by European actuaries and has been expressed in an extensive and mathematically advanced literature. Solvency is considered within the context of a complex mathematically closed model, which crucially ignores the role of the market process. By contrast, the financial view of insurer solvency was developed primarily in the United States over the last thirty years and rests on the view that economic markets are rational and efficient. While there has been significant overlap between the actuarial and financial approach in recent years, there has been a tradition of limited cross-fertilisation and as a result the discussion proceeds by addressing the classical approach in isolation. Subsequent chapters will develop the financial view and indeed establish some important linkages between these two traditions.

3.1.1 The General Solvency Problem

The 'Classical' research agenda emphasises those variables that should be monitored in order to minimise, or control for the ruin probability. The probability of ruin $\Psi(.)$ is synonymous with the probability of insolvency, where $\Psi(.) = \text{prob}\{Z_t < 0\}$ and $Z_t$ is the amount of free reserves at the end of a given
period. Ruin theory considers in particular the relationship between the premiums charged and the risk insured, and is discussed in greater detail in Section 3.3.

In addition, having established the group of variables to be assessed, the classical research agenda involves specifying the appropriate regulatory response in terms of a required solvency margin. The ‘classical’ literature develops both an empirical and theoretical perspective on the various sources of risk that can jeopardise the solvency of an insurance company, and thereby establishes a quantitative framework within which to build a workable definition of solvency.

The relevant risk categories that have received attention are underwriting risk, inflation risk and investment risk. The latter case of investment risk has received less attention than its counterparts. Moreover, theory has focused on the role of underwriting risk for this has been the traditional work of actuarial science.

One of the fundamental difficulties associated with modelling insurer solvency is that there is no absolute criterion for defining solvency. There are a number of elementary definitions available; for example, the Oxford English Dictionary defines a business as being solvent if it is capable of meeting all pecuniary liabilities. While at first sight this seems a reasonable assumption to make about solvency, closer inspection reveals the fact that it is never possible to guarantee solvency of this kind. Instead, it is necessary to make certain assumptions about

---

1 Most of the work has been centred on the claims process, and much of the earlier work on investment risk has been within financial economics.
liabilities and assets in order to arrive at a workable definition of solvency. Indeed Benjamin (1977) reviews this very issue and distils the problem into the following question: how can an adequate solvency margin be defined so that a workable legal definition can follow? The various considerations affecting this fundamental problem form the basis for discussion.

One of the first points to note is that the concept of solvency applies equally to many forms of business other than insurance. However, unlike many other forms of business, insurance companies are subject to explicit solvency regulations in the form of reserve requirements. Why is this? What makes insurance companies distinct from other companies and hence makes solvency a key regulatory issue is the fact that insurers are by definition in the business of buying risk (variability). At a given time, an insurer’s liabilities are only partially known as they often extend many years into the future and are subject to a great list of uncertainties. These uncertainties have implications for a workable definition of solvency and the appropriate legal specification of the regulations.

First and foremost, given the nature of an insurer’s assets and liabilities, insurer solvency has to be seen as a probabilistic concept. As Kastelijn and Remmerswaal (1986) notes, the probabilistic nature of solvency is a result of two important factors: time and risk.

---

2 There exist bankruptcy laws for business in general, but not specific reserve requirements.
3.1.2 Time

The time consideration depends on how the future is viewed and in particular how to construct a workable definition of solvency while also accounting for future business. The assumption of future business is central to the performance of any solvency definition; future income streams, for example, have the ability to support current performance. This leads to the following questions: to what extent should future business support current operations and in particular, if the future is to be taken into account, what is the relevant time horizon? There are no definitive answers to these questions. However, there have been a number of alternative strategies put forward in order to simplify the problem (Pentikäinen 1988).

The time factor has been addressed by making assumptions specific to the use of future business. First, there are those actuarial methods of solvency that are essentially static and that use balance sheet data. Second, there are those actuarial techniques that attempt to model the future and make solvency a more ‘dynamic’ measure of performance. Related to this assessment is the distinction between methods on a going concern versus methods based on a break-up (or liquidation) basis. Going concern methods take account of future business, while in the case of break-up methods, solvency is considered under the assumption of no new business. Each method has arguments for and against (for discussions see Beard, et al. 1984). The problem faced by both methods is the

---

3 This work involves the calculation and analysis of financial ratios from accounting statements.
choice of the appropriate time horizon; for the break-up case, this will usually be
the expected time that it will take to run off the existing business. By contrast,
the going concern case is less clear. Regulators will usually expect a time
horizon of between one and two years as sufficient (to enable regulators to react),
while management may want a longer term view, especially in the case of life
assurance where its liabilities extend many years into the future. Indeed, these
two groups, regulators and management, may provide a useful perspective on the
problem of defining solvency (Pentikäinen 1988).

While much can be said about the alternative distinctions that can be made
regarding the solvency/time considerations, it is important to emphasise that no
clear method can be deemed as superior. Evidently some account of the future is
important in any final analysis, yet important insights may also be derived from
examining issues within a break-up setting. In reality, the selection usually
comes down to the degree of complexity associated with a given definition.

3.1.3 Risks

In addition to the time consideration, what are the risks that should be considered
when defining solvency? Ultimately, there are many different factors that may
affect the solvency position of insurers and which make up the economic
activities and decision-making of insurance companies. The key is to develop

---

4 The problem is that simply assuming a long enough time horizon may enhance solvency though
greater income streams.
an analysis of those variables that seem reasonable in a discussion of insurer solvency.

In respect of the relevant risks, there are many differences between the various actuarial investigations (see Pentikäinen 1988 for a review). The traditional approach of the actuarial literature has been to consider only risks associated with liabilities, especially the risk of claims fluctuations. This approach has been justified on the grounds that it is the nature of the liabilities (claims) that makes an insurer distinct from other non-insurance business (where there are no explicit solvency constraints). In calculating liabilities, the following aspects have been extensively considered: claims fluctuation; dependency on the composition of the insurance portfolio; inflation; reinsurance; catastrophes; expenses; and, bonuses and profit sharing, particularly relevant to life assurance, (see Beard, et al. (1984)).

The growing importance of investments to insurer solidity in recent years has highlighted the importance of risk factors affecting the insurer’s assets. However, given the predominance of liabilities in the actuarial literature only limited attention has been paid to fluctuation in investment values.

There are further factors that are considered important to the discussion of solvency. Other types of risks might include policy option risks (for example, the risk of policy surrenders) and ‘solidarity’ risks (Beard, Pentikäinen and
A further factor to note is that any prescription vis-à-vis a solvency constraint will be based upon the relevant calculations of assets and liabilities. Therefore, the solvency constraints that are defined are dependent upon the method of valuation for assets and liabilities. An example in life assurance is that of the adequacy of reserves. Here the size of technical reserves can differ significantly depending on the method of reserving employed (Pentikäinen 1988).

Thus, the interplay of many risk factors is a source of complication when attempting to define solvency. The difficulty lies in the fact that many factors are not quantifiable and even when they are quantifiable there may be ambiguity associated with the resulting measure of insurer solvency.

### 3.1.3 Insolvency Probability

Having addressed the roles of time and risk, a further assumption must be made regarding insolvency probability. In particular, what is an acceptable level of insolvency probability? Any answer is fraught with difficulty as an 'acceptable' level of insolvency basically comes down to an arbitrary decision although:

> The choice of the insolvency probability is ... more an illustration of the arbitrariness of the situation than the sole cause of it. (Kastelijn and Remmerswaal 1986, 15)

---

5 Solidarity risks are those risks that bring about a difference between the true actuarial premium and the actual premium charged. The actual premium charged may be calculated without having knowledge of all the relevant risk factors. An efficient insurance operation would therefore try to minimise the discrepancy between the true actuarial premium and the actual premium charged.
The interaction of time and risk with insolvency probability predicted by a given solvency measure is important to stress. If, for example, more risks are considered, or a longer time horizon is used, the probability of insolvency will necessarily increase. These interactions must be carefully evaluated and accounted for in full, before proceeding on towards any acceptance of a workable definition of solvency. Consideration now turns to reviewing some of the classical literature in its attempt to provide a workable definition of solvency.

3.2 Formal Definitions

Having reviewed the background to the insurer solvency problem, what solutions exist to the basic question posed by Benjamin (1977) regarding a practical legal definition of solvency? Before proceeding to some practical examples of legally defined solvency margins, it is useful to consider a theoretical framework from which to view this problem in greater detail. Following Pentikäinen (1988), the development of the economic position of an insurer maybe expressed in terms of an equation or basic algorithm related to year \( t \) as follows:

\[
B_t + I_t = X_t + C_t + D_t + \Delta U_t,
\]

Where \( B \) is the premium income, \( I \) is the net return on investments,\(^6\) \( X \) is the claims paid, \( C \) expenses, \( D \) dividends (and bonuses) and \( \Delta U_t = U_t - U_{t-1} \), the

\(^6\) This is calculated in monetary units and not as a percentage.
change (+/-) of the solvency margin $U$. Equation 3.1 is an algorithm because it is $\Delta U$, that generates a sequence, given an initial condition for $U_0$.

The basic algorithm given in Equation 3.1 is the foundation for model building and simulation work within insurer solvency research. Now, $U$ is the indicator of interest with respect to the insurance company and may be obtained as the difference between assets ($A$) and liabilities ($L$) such that:

\[(3.2) \quad U = A - L\]

While assumptions need to be made regarding the way in which assets and liabilities are valued, this will not be explored in any greater detail at this stage. Instead, it will be assumed that assets are valued according to an acceptable standard (for example market value) and liabilities are estimated using sound actuarial practices (Beard, Pentikäinen and Pesonen 1984).

It is sufficient to say that the claims variable, $X$, has been the focus for research in the actuarial field. Given that a formal expression for an insurance company is presented, it is now possible to ask the question: what is solvency? Pentikäinen (1969) attempts to develop a definition of solvency by examining the perspectives of two groups of interested parties: company management and regulators. The interests of management was argued by Pentikäinen (1969) as being centred on the promotion of company development within a profit maximisation setting, while the regulators were seen as being interested in the promotion of policyholder protection. Moreover this latter requirement, namely
to ensure that claimants and policyholders have secured benefits, was seen by Pentikäinen as a basis of a legal definition of solvency.

Consider first the regulator’s perspective. The regulator in the U.K. is accustomed to screening the solvency position of insurers annually. If a given company is allowed to continue to trade then it may be assumed that the regulator is confident that the insurer will be solvent up until the next review. The regulator’s confidence is still subject to random change and is therefore a matter of probability and statistical significance. The ideas may be expressed in terms of a diagram, see Figure 3.1 below.

Figure 3.1. Solvency: The Regulator’s View


---

7 There are two points to note here. First, the time interval for regulatory review is crucial, since the longer the interval the greater the uncertainty associated with any regulator prediction, and the greater the lag in regulator response to any adverse change. Second, if the company fails the regulatory review, then there will be a number of likely remedial actions before full shutdown is effected.
Following Pentikäinen (1988) and with reference of Figure 3.1, $U$ is typically modelled as a random walk with its path bounded by the $U_{\text{max}}$ and $U_{\text{min}}$ curves, which together delineate the confidence region for prediction. $U'$ stands for the initial solvency margin at time $t=0$, when the last regulatory review occurred.

The progress of the solvency margin until the next review may be seen by the dotted line (as an example) and is subject to fluctuation (and uncertainty about this fluctuation). One of the central problems is to define the confidence bounds $[U_{\text{min}}(T), U_{\text{max}}(T)]$ inside which $U$ can be expected to fluctuate at the next review $t=T$, given a level of statistical probability. The difference between the lower limit $U_{\text{min}}(T)$ and the mean flow, $E(U(T))$ may serve as measure of variation (see bold portion of line).

Solvency constraints are satisfied if the solvency margin, $U$, is at least equal to some legal minimum. This legal minimum is to be determined and is set such that the lower limit of the company solvency projection $U_{\text{min}}(T)$ should not be negative by a given ‘survival probability’ equal to $(1-\varepsilon)$, where $\varepsilon$ is an example of the so-called ‘ruin’ probability $\Psi(.)$. It is the ruin probability that has been paid significant attention within a variety of scenarios in actuarial research.

The issues of time, risk and insolvency probability are evident once more. The choice of the interval $[0,T]$ must be decided upon. As Beard, et al. (1984) note, the fact that regulatory review occurs at discrete intervals allows for violation of the solvency constraint within these intervals. Continuous review, although

---

8 A 95 percent level of probability would therefore mean that 95 times out of 100, the path of the indicator $U$ would be expected to fall within the $U_{\text{max}}$ and $U_{\text{min}}$ limits.
practically near impossible, would seem to be warranted if this risk is to be eliminated completely. However, in practice review frequency will be resource constrained and the European Union convention is one year (Atchinson 1997, 80). Note that the choice of \( T \) will have an effect on the solvency constraint: the greater the interval the greater the solvency margin, in order to maintain confidence over the greater period of time.

The ruin probability \( \Psi(.) \) must be decided upon by the regulator (Pentikäinen 1988, 5). This too will affect the relevant solvency constraint imposed upon insurers. Also, note that the going-concern versus break-up debate may be illustrated in Figure 3.1. The picture set out is that of a going-concern. Business is carried out during the interval \([0,T]\), yet fluctuations in \( U \) are caused by the settlement of claims and other commitments established in a time before this period, as well as due to events occurring within it. In the case of wind-up at \( t=T \) new business is stopped and existing business is wound up, liabilities paid as they fall due using assets as appropriate. The break-up process is, however, subject to ‘run-off’ risks. These risks are associated with deviations in payments to liabilities from that expected (this applies also to the liquidation of assets).\(^9\)

To account for such risks the confidence interval lower bound would need to be extended further.

While much may be said about solvency from the perspective of the regulator, it is important to note that solvency is a vital element of insurance management

\(^9\) A notable deviation may be brought about by, for example, an inflation shock that may make previous assessments of assets and liabilities undervalued.
and may therefore be seen as part of a business strategy (Pentikäinen 1969). There may be many differences between the perspective of the regulator and that of management and this is reflected in, among other things, the time horizon $T$. While the regulator may consider a horizon of 1 to 2 years, management strategy may demand a horizon that stretches many more years into the future. Thus, the role of solvency as part of a business strategy is distinct from the considerations of the regulator: solvency from the management perspective must be balanced along with the need for expansion, and rewards to investors.

It is clear that the considerations of the regulator and management help to clarify some of the issues relating to a workable definition of solvency. Some of the proposed solutions to the practical problem of defining solvency are addressed in the next section and provide a useful overview of the work undertaken in this field. Later, a brief introduction to the formal developments of 'ruin' theory will also be developed.

3.3 Calculating the Solvency Position of an Insurance Company

Having outlined the numerous factors to consider when defining solvency, what practical solutions exist in providing a workable definition of solvency performance? There are a number of alternative solutions proposed in the literature. Most of these proposals are the result of national actuarial working parties. Two such organisations that have played an important role are the Finnish Solvency Working Party (FWP) and the British Solvency Working Party (BWP) (Kastelijn and Remmerswaal 1986). Examples of the FWP work can be
seen in Beard, *et al.* (1984), while examples of both the FWP and BWP maybe seen in Kastelijn and Remmerswaal (1986). These working parties explore solvency issues using the basic simulation algorithm and analysing those risks that are deemed most appropriate to the analysis. Some of results developed are explored further here, and in addition some of the research that helped to form the current regulatory framework within the European Union is illustrated.

The solvency outcome, however that may be defined, is the result of numerous factors. Some of these factors have been mentioned. In order to evaluate the variety of risks that may jeopardise the solvency of insurers, use is made of the so-called ‘insurance process’. The insurance process reflects the variety of observations and experiences associated with solvency management in practice and these are illustrated in Figure 3.2.

Figure 3.2 illustrates the general framework for formal model building, what Pentikäinen (1988) refers to as The Insurance Process (Pentikäinen 1988, 11). The Insurance Process provides a means of identifying the possible sources of uncertainty facing the insurance company, which in turn may affect its solvency position. The flow of insurance business depends on many variables. All known and unknown endogenous and exogenous factors form ‘nature’. Endogenous factors include the size and form of the insurance portfolio, the rating policy, reinsurance, and the business strategy. By contrast, exogenous factors may include the investment market, inflation, and regulatory shift. Nature is not strictly known. However, indicators of the *state of nature* may be disclosed (usually with a time lag) and may therefore be used in subsequent model
construction. Indeed, using this data source together with existing theory on the insurance process and assuming a given set of endogenous factors, it may then be possible to go forward and develop a model of the insurance company and its solvency position.

As Pentikäinen (1988) illustrates, uncertainties are associated with the procedure described above. These uncertainties fall into three main groups: model error, parameter error and stochastic error. Even if the first two error sources are corrected for, it is impossible to eliminate the third error source. The importance
of these error sources will depend on their resulting impact on the solvency position of the insurer. The possible sources of stochastic error include uncertainty from claims, asset values, inflation, and expenses.

Many approaches are adopted in order to evaluate explicitly the solvency position of the insurer. These methods may be divided into three main groups: ratio methods, methods based explicitly on risk theory, and comprehensive model methods. Each method will be addressed respectively in turn.

3.3.1 Ratio Methods

The use of financial ratios to define the solvency position of an insurance company has little theoretical explanation; that is, ratio methods are not based explicitly on risk theory. The ratio methods are used to infer something about the performance of the insurance company. Much of the work in this area has been developed on the basis of a variety of European studies (Kastelijn and Remmerswaal 1986). This research attempts to derive an explicit solvency margin, assuming only a small probability that surplus will be negative (the probability of insolvency/ruin). While some methods are based on the analysis of just one ratio, other methods consider multiple indicators. To illustrate the features of the ratio method, use is made of the research that underpins the current insurance regulations in the U.K.

The basis for the solvency margins implemented within the European Union is taken from the work of Campagne (1961). Campagne’s working party
established a number of empirical results and the work has been one of the most widely cited examples of ratio methodology. Given that this study (and its counterpart for non-life business) has had a profound impact on the nature of solvency regulations in Europe, it is instructive to consider this work in greater detail.

Following Kastelijn and Remmerswaal (1986), Campagne considers three possible ratios for expressing the minimum solvency margin: first, as a percentage of the insured capital; second, as a percentage of the risk capital; and third, as a percentage of the mathematical reserves. After discussion, Campagne opts for the last ratio, a solvency margin as a percentage of mathematical reserves. This is justified on a number of grounds, the most noteworthy being that of the definition problems associated with the first two measures and the fact that mathematical reserves are one of the most important quantities when considering the total investments of life assurance companies.

Having elected the relevant measure, Campagne makes a number of assumptions. First, he expresses a company’s loss, \( L \) per year as a percentage of mathematical reserve, \( V \).\(^{10}\) The ratio \( L/V \) is assumed to be independently and identically distributed for different years and for different companies. On this basis parameters for the \( L/V \) distribution may then be estimated by empirical inspection. Free reserves, \( U \) as a percentage of \( V \) must be such that the

\(^{10}\) Profit is synonymous with a negative loss.
probability of a loss exceeding the reserve in a given year must be less than or equal to a small probability \( \varepsilon \) (that is \( \Psi(\cdot) \), the probability of ruin), that is:

\[
\text{(3.3)} \quad \text{prob}\left\{ \frac{L}{V} > \frac{U}{V} \right\} \leq \varepsilon
\]

Data was collected on 10 Dutch companies for the years 1926-1945. In the original article Campagne (1961) assumes that the ratio \( L/V \) is distributed according to the Pearsonian distribution. The frequency function \( f_{L/V}(L/V) \) was estimated, with the minimum solvency margin being obtained as that fraction \( (L/V^*) \) of the mathematical reserve, such that:

\[
\text{(3.4)} \quad \int_{(L/V^*)}^{\infty} f_{L/V}(L/V) \, d(L/V) = \varepsilon
\]

Campagne estimates the solvency margin for 1 to 5 years and 10 years such that Equation 3.4 is satisfied. The result was intuitive: the longer the time period considered (given \( \varepsilon \)), the greater the required margin. Campagne selected a probability of \( \varepsilon = 5 \) percent which translates to a necessary margin of 4 percent. The significance of this result may be seen in the regulations under the relevant statutory instrument. Under the 1994 Insurance Company Regulations, the definition of UK solvency margins for life and annuity business is that “... there shall be a sum equal to 4 per cent of mathematical reserves,”(Insurance Company Regulations 1994, Reg. 18, s.19).\(^{11}\)

\(^{11}\) A similar treatment is given for non-life insurance solvency margins.
While the ramifications of Campagne’s work are considerable, the methodology employed has come under growing criticism. Indeed, much debate has centred over whether Campagne’s work has actuarial foundations, or whether it is merely the result of an empirical study (for discussion see Byrnes (1986), De Wit (1980) and Miron (1983)). Also, Kastelijn and Remmerswaal (1986) argue that far from resolving the definitional problem associated with the term solvency, the work of Campagne (1961) actually added to the ambiguity. They also criticised the choice of $\varepsilon$, arguing that $\varepsilon$ was arbitrarily selected and was governed by the choice of sample (the choice of the companies). Hardie et al. (1984) also criticised the Campagne study on the grounds that it was too old to be useful and that the independently, identically distributed assumption was not very realistic given the data. They argued that the solvency positions of insurance companies were likely to move in tandem due to underlying factors (for example, the state of the macro-economy).

Also emphasised by Hardie et al. (1984) is the role of ‘implicit’ margins. Implicit margins are common in life assurance and come in the form of conservative estimates over and above the true actuarial estimates of mortality, interest and expenses. It is argued that excessive ‘explicit’ margins may have a detrimental impact on ‘implicit’ margins and given that ‘implicit’ margins are unobservable there may then be unpredictable effects upon the resulting solvency position of the insurance company. Moreover, the fact that implicit margins are
present in life assurance may be an argument to reduce the extent of explicit solvency regulation.\(^\text{12}\)

Aside from the specific criticisms of Campagne’s study there are a number of comments that may be made about the ratio method in particular. This method is basically a straightforward statistical analysis with minimum account of theory, be it risk theory or indeed any other theory. One difficulty is that the use of observed outcomes is the result of the interplay of many factors operating within the insurance process (both known and unknown). It is difficult to separate the total effect according to various risk categories, even if they are known (such as investment risks, underwriting risks, and exposure to inadequate safety loading within the premium). Thus, for the purposes of policy prescription and forecasting the usefulness of such a study is evidently limited.\(^\text{13}\) In order to make greater use of the data and be able to disentangle the roles of the relevant risk factors, any empirical analysis needs to be supplemented by theoretical modelling. The data may then be used to test the usefulness of the model. The initial development of formal models of insurer solvency was based explicitly on risk theory.\(^\text{14}\) This work is illustrated in the next section and is the result, in part, of the research by both the Finnish Working Party and the British Working Party.

\(^\text{12}\) The only problem with this is the fact that it may be very difficult to obtain the necessary data on implicit margins. Precision estimates are necessary in order to ensure adequate regulation.

\(^\text{13}\) Moreover, the account of market processes in determining the market-clearing premium is not present at all within the classical insurance theory.

\(^\text{14}\) It should be noted that analysis of ratios may be extended into a discriminate analysis using a number of different ratios. This has provided a means of classifying companies into those that need investigation and those that do not. This area of work falls under the financial distress literature, developed in recent years within the United States (see Chapter 4).
3.3.2 Methods based on Risk Theory: Claims Fluctuations

This section considers only one type of risk: the risk of fluctuations in aggregate claims. This is the traditional domain of risk theory (see Beard, et al. (1984)). In its simplest form, models are used in order to derive explicit solutions for the solvency margin, given an acceptable level of insolvency probability.

Methods based on claims fluctuations concentrate on the following simplification of the basic algorithm in Equation 3.1:

\[(3.5) \quad \Delta U_i = B_i - X_i\]

Expenses and other variables of the basic algorithm in Equation 3.1 are not normally considered, so that the model usually consists of three main elements: the initial reserve \(U_0\); the aggregate claim amount, \(X_i\) (which will depend on the time interval considered); and the premium income \(B_i\) including the risk loading, where \(B_i\) may be decomposed into a safety loaded risk premium \(P_{s\alpha}\), and an expense loading such that:

\[(3.6) \quad B_i = P_{s\alpha} + c_i B_i\]

Where \(c_i\) is an expense-loading coefficient and \(P_{s\alpha}\) is defined as follows:
(3.7) \[ P_{X} = [1 + \lambda_{r}]E[X_{r}] \]

Where \( \lambda_{r} \) is a safety-loading coefficient.\(^{15}\) The safety loading and expenses may be sources of fluctuation in the premium income \( B_{t} \), since changing actuarial assumptions may affect income. Note that Equation 3.5 will tend towards a converged limit if, on average, premiums are proportional to the expected value of the liability.\(^{16}\)

In order to describe claims fluctuations and underwriting risks, use is made of a Claims Process Model (for full discussion see Beard, et al. (1984)). This model is made up of two components: the claim number process (i.e. the claim frequency) and the individual claim amounts. Following Taylor and Buchanan (1988), these two components may be developed formally as follows. First, assume that coverage of the risk is provided for the time interval \([0, t]\). Further, let \( N_{t} \) be a random variable representing the number of claims occurring in the time interval. Also, let \( X_{1t}, \ldots, X_{N_{t}} \) denote the individual sizes of the \( N_{t} \) claims. Then the aggregate claims amount for the interval \([0, t]\), given by \( X_{t} \), is defined as follows:

(3.8) \[ X_{t} = \sum_{i=1}^{N_{t}} X_{i} \]

\(^{15}\) A safety-loading coefficient is an implicit solvency margin. The greater the coefficient, the larger the assumed margin.

\(^{16}\) Premiums are proportional to, rather than equal to the expected value of the liability because of the existence of implicit margins of solvency (including profit).
To define the claims process in full, it is necessary to specify the distributions for \( N_t \) and \( X_t \). Consider first the variable \( N_t \). There are many alternative distributions that can be considered here. One common distribution assumed is that of the Poisson distribution, where \( N_t \sim Po(\lambda) \), where \( \lambda \) is the parameter of the probability distribution and should not be confused with the safety loading coefficient in Equation 3.7 above. Under the Poisson assumption the probability of a given number of claims is generated as follows:

\[
\text{prob}\{N_t = n\} = \frac{\exp(-\lambda t)(\lambda t)^n}{n!} \quad \text{and}
\]

\[
E[N_t] = \lambda t
\]

The Poisson parameter \( \lambda \) may be interpreted as the expected number of claims per time unit.\(^{17}\) With respect to the individual claim amounts \( X_t \), it is standard to assume that the amounts of the \( N_t \) claims occurring in the interval \([0,t]\) are independently, identically distributed with distribution function \( F(\cdot) \). Let \( G(\cdot,t) \) denote the distribution function of the aggregate claims amount \( X_t \), and this is referred to as the compound Poisson distribution (cPD). Following Kastelijn and Remmerswaal (1986), Equations 3.8 and 3.9 yield the following cPD:

\[
G(x,t) = \sum_{j=0}^{\infty} e^{-\lambda t} \left(\frac{\lambda t}{j!}\right)^j F^{\cdot j}(x)
\]

\(^{17}\) Poisson distributions are considered acceptable for a sufficiently small collection of risks considered over a sufficiently short time interval.
Where $F^{*}(.)$ is the j-fold self-convolution of the distribution function $F(.)$. The choice of $F(.)$ is open to many options. The selection depends on the nature of the risks concerned and the nature of the distribution moments required.

As noted by Kastelijn and Remmerswaal (1986), the usefulness of the cPD can be improved by allowing the Poisson parameter to vary stochastically according to some structure function. This results in the mixed-compound Poisson distribution. It is not necessary to define explicitly the size distribution and/or the structure function, but only to know their lowest three/four moments (in addition to the Poisson parameter, $\lambda$). Thus, returning to Equation 3.5 it is clear that a complex analytical structure can be specified for the claims process $X_t$ and that this can be used to evaluate the solvency performance of the indicator function, $U$, given premium income, $B_t$.

Many additional features have been built into the basic claims process. For example, the claims experience might be subject to considerable variation in the form of a cycle. There are periodic variations in claims frequencies experienced by insurance companies and hence this will be reflected in insurer profit/loss. Even though these periodic variations are irregular, they are typically referred to as “cycles” (Beard, Pentikäinen and Pesonen 1984, 185). Underwriting cycles may occur and manifest themselves as an error in the predicted risk premium $P_t = E[X_t]$, where the error is $\Delta P_t$: 
The error, $\Delta P_i$, yields profit or loss depending on the state of the cycle. The problem is that there is a time lag between the moment at which the error occurs and the moment at which the error is detected and any adjustment can be made.\(^{18}\) Hence, during this period an insurer may accrue profit or loss and this may be revealed as a cycle in the loss ratio $X/B$ and a fluctuation in the solvency margin $U$, but only with a time lag.\(^{19}\)

\subsection*{3.3.3 The Relationship Between Premiums, Claims and Solvency:
Ruin Theory}

While much can be said about the claims process itself, its specific application to classical insurance solvency theory comes in the form of ruin theory (Beard, Pentikäinen and Pesonen 1984, 55). Ruin theory was initiated by Lundberg (1909) and was developed further by Cramér (1930, 1955) and has continued to expand since the time of these works. The theory of ruin considers an insurance fund subject to premium income and claims outflow. The probability that at some stage the insurance fund is exhausted is considered, at which point the insurer may be deemed to be insolvent, or ‘ruined’. The dependency of the solvency position upon risk selection and rating policy is expanded upon within this field of research. A more rigorous description of the ruin problem may be

\(^{18}\) This assumes that it is possible to alter the premiums charged. Many contracts specify fixed premiums for the entire term of the policy.

\(^{19}\) In addition to the long-term variation in exposure levels, there may be other factors influencing the size of premium error including trends, catastrophes, run-off errors, and so forth.
given as follows (see Taylor and Buchanan (1988)). Let the premium, per unit of time, net of expenses be \( c \) such that:

\[
(3.13) \quad c = (1 + \eta) \lambda \alpha_i
\]

Where \( \eta \) is the safety margin (including profit), \( \lambda \) is the Poisson parameter, and \( \alpha_i \) is the average claim size. Consider now the time interval \([0, t]\), where an insurer starts the interval with 'free' reserves, \( w \). The amount of free reserves at the end of the interval will be given by \( Z_t \), where:

\[
(3.14) \quad Z_t = w + ct - X_t
\]

Where \( X_t \) is defined by the aggregate claims process (such as that process given in Equation 3.11). \( Z_t \) is a stochastic process for \( t > 0 \) and is referred to as a surplus process, which tracks the progress of the insurer's free reserves, \( w \).

The case of ruin, or insolvency occurs at time \( t \) if there is shift to the state \( Z_t < 0 \). Conversely, survival over the entire period occurs if \( Z_s > 0 \) for all \( s \in [0, t] \). Let \( \phi(w, t) \) denote the probability of survival to time \( t \), given free reserves, \( w \). Then:

\[
(3.15) \quad \phi(w, t) = \text{prob}\{Z_s > 0 \text{ for } 0 \leq s \leq t : Z_0 = w\}
\]

Now, let \( \Psi(w, t) \) represent the probability of ruin before time \( t \), given that an insurer starts with an initial free reserve of \( w \). Then:
A solution to \( \phi() \) may be explicitly derived using a particular integro-differential equation (see Taylor, *et al.* 1988). There are, however, approximations to the probability of ruin that may be employed. These approximations depend on two specific assumptions: first, the length of the time horizon considered (usually whether it is finite or infinite); and second, the frequency of screening (whether this is continuous or discrete time checking). It is possible under discrete-time checking for an insurer to pass through the state \( Z < 0 \), without being detected so long as \( Z_c > 0 \), where \( Z_c \) is the time of the next inspection and \( c > t \) (Taylor and Buchanan 1988, 63). As a consequence the probability of ruin may need to be altered.

The starting point for a ruin approximation is to assume an infinite time horizon with continuous checking. These assumptions are made because the solutions become analytically tractable. Usually the time-scale considered is chosen in such a way to scale the Poisson parameter, \( \lambda \) to unity. Under these conditions, one of the best known approximations to the infinite-time ruin probability (see Taylor (1976), Taylor and Buchanan (1988), and Teugels (1982)) is given by:

\[
\Psi(w) = 1 - \phi(w, t)
\]

Moreover, the so-called Lundberg inequality states that:
Where $R$ is called the adjustment coefficient and $R>0$. The approximation given above has an intuitive explanation. Recall that $w$ is the initial free reserve. As the ability of the insurer to adjust free reserves increases in response to changes in the insurance process, the probability of ruin falls. $R$ depends on many of the factors mentioned in previous sections, especially the rate at which an insurer can revise its premium in light of its recent claims experience (as specified by the claim size distribution $F(.)$). It might therefore be possible to hypothesise that $R$ may be low for life insurers due to contractual obligations specifying fixed premiums.

The adjustment coefficient, $R$ may itself be approximated. Taylor (1975) provides an approximation based explicitly on the moments of the claim size distribution function. In particular:

\begin{equation}
R = R(\eta, \alpha_2, \alpha_3, \alpha_4), \text{ and } R_1 > 0, R_{\alpha_2} < 0
\end{equation}

Where $\alpha_n$ is the $n$th moment about the origin of the individual claim size distribution; that is:

\begin{equation}
\alpha_n = \int_0^\infty x^n dF(x)
\end{equation}
Taylor (1975) scales $\alpha = 1$ and develops the following results. First, the ruin probability decreases ($R$ increases) as percentage profit margin, $\eta$ increases. The term $\eta$ may also be interpreted as a safety margin. Second, the most important shape parameter of the claim size distribution is the variance and the relationship with the variance is that the longer-tailed distributions generally carry the greater exposure to ruin (a measure of downside risk (see Chapter 4)). The final remark to be made here is that if premiums contain no profit margin, ruin is certain. Note here that the implication for supervision is that the regulator should examine the breakdown of the gross premium. However, this information is not submitted within the annual company returns.\(^{20}\)

The illustration provided by Taylor and Buchanan (1988) demonstrates the techniques applied in the field of classical insurer solvency theory. Note that examples of recent work using a variety of claims distributions include Croux and Veravebeke (1990); Dufresne and Gerber (1993); Gerber (1992); Embrechts and Veravebeke (1982); and, Embrechts and Wouters (1990). From the above analysis of Taylor and Buchanan (1988), it is possible to proceed on and derive explicit expressions for the minimum solvency margin $U_{\text{min}}$ given an assumed level of acceptable ruin probability (Kastelijn and Remmerswaal 1986, 50-69).

While much has been said about the infinite time/continuous checking case, it is important to consider the work that has been developed on the other possible

\(^{20}\) In theory, authorities such as the FSA have the statutory power to obtain any necessary information. In practice this power is rarely exercised due to the complexity of 'process'.

114
variants (that is, finite time/continuous checking, finite time/discrete checking, and infinite time/discrete checking). For examples of each case see Beard et al. (1984). The mathematics becomes increasingly complex as one departs from the infinite time/continuous checking case. However, as a means of circumventing these difficulties, estimates of finite ruin probabilities and their convergence to infinite ruin probabilities have been used. An example is provided by Seal (1972) and is illustrated in Taylor et al. (1988) under the Poisson assumption for the claims distribution function (with a mean of 1 claim per unit time, and $\lambda=0.1$). The results are reproduced in Table 3.1 below:

Table 3.1. Finite-time Ruin Probabilities: Numerical Approximations

<table>
<thead>
<tr>
<th>Time Horizon, $t$</th>
<th>$R_{\text{finite}}$ for initial free reserve=33</th>
<th>$R_{\text{finite}}$ for initial free reserve=44</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>100</td>
<td>0.006</td>
<td>0.001</td>
</tr>
<tr>
<td>200</td>
<td>0.019</td>
<td>0.004</td>
</tr>
<tr>
<td>400</td>
<td>0.033</td>
<td>0.010</td>
</tr>
<tr>
<td>800</td>
<td>0.043</td>
<td>0.015</td>
</tr>
<tr>
<td>1000</td>
<td>0.044</td>
<td>0.016</td>
</tr>
<tr>
<td>2000</td>
<td>0.045</td>
<td>0.017</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.045</td>
<td>0.017</td>
</tr>
</tbody>
</table>


According to Taylor and Buchanan (1988) values of free reserve $w$ were selected in order to produce infinite ruin probabilities that were not too small or too large. They summarise the findings of this work:

First, the amount of the initial free reserves required to reduce infinite time ruin probability to a moderate level is only 30 or 40 times the average size of a single claim; second, with initial free reserves at this sort of level, the finite-time ruin probability attains essentially its infinite-time value over the period required to produce, on average, 1000 claims. (Taylor and Buchanan 1988, 72)
As Taylor and Buchanan (1988) note, the amount of capital required for the first point above would be minute. Hence, if small reserves are consistent with a low ‘classical’ ruin probability, then the fact that insolvencies still occur suggests that insurer insolvency might be due to factors out with the normal consideration of risk theory. This is not that surprising given the variety of other considerations and risks outlined in Section 3.1.

Moreover, solvency research over the last 15 years has developed these other considerations in addition to research in the financial economics arena. The role of these other factors and in particular the operation of the entire basic algorithm in Equation 3.1 is introduced in the next section and falls under the heading of ‘comprehensive models’ of insurer solvency.

Note, as an example of an explicit solvency constraint based on risk theory, the work of Pentikäinen (1988). As far back as 1952 Pentikäinen helped to persuade the Finnish government to accept the risk theoretic approach in designing insurance company regulations. The formula used for the statutory minimum solvency margin was given by:

\[ U_{\min} = aP + b\sqrt{PM} \]

Where the first term covers the random fluctuations in basic probabilities and the second term covers the random fluctuations of claims \( U \) is the required margin,
\( P \) is net premium income and \( M \) is the maximum net retention for a single claim).

### 3.3.4 Methods Based on Comprehensive Models

The previous section considered methods that focused primarily on the effects of claims fluctuations \([X_t]\) upon the solvency position of insurers. Comprehensive models are the more common approach to solvency modelling in recent years and, as mentioned, address alternative sources of risk and their associated interaction. Before proceeding to some examples of this research it is useful to consider the main difficulties associated with comprehensive model construction. Recall the basic algorithm:

\[
(3.20) \quad B_t + I_t = X_t + C_t + D_t + \Delta U_t
\]

Much of the literature has concentrated on just one of the above factors and detailed the effect of this variable on solvency, in isolation from the other variables. As Pentikäinen (1988) notes, a major problem arises in bringing together these sub-models into a fully interacting model, which accounts for all these factors simultaneously (the so-called ‘great convolution’ (Pentikäinen 1988, 25). Thus, the objective of comprehensive models is therefore to develop a model that attempts to address all these difficulties in order to evaluate the overall solvency position of the insurer.
There are a number of ‘comprehensive’ approaches used in the literature. Usually a choice is made between an analytical calculation of ruin probabilities employing risk theory (and estimating the variation ranges, as illustrated in Figure 3.1), or an operational approach using simulation techniques. Due to the complexities involved in deriving a fully analytical solution, simulation is usually relied upon to provide meaningful results. The simulation of claims, inflation, the rate of interest, and so forth may easily be incorporated into the basic algorithm to analyse the effects upon \( U \), the solvency margin. This methodology can be used to provide a framework for setting an explicit constraint.

Whatever the choice of technique, the objective is to construct an overall model of insurer solvency (Pentikäinen 1988, 29). Remembering that insurer solvency is by definition a probabilistic concept the next problem is to determine the joint distributions of the variables contained within the basic algorithm (Kastelijn and Remmerswaal 1986). As discussed, the variables may be affected by nature simultaneously, and are also likely to be mutually dependent. This dependency must be modelled in any estimation procedure otherwise the estimation of the risks will be biased downward (Kastelijn and Remmerswaal 1986). A correlation matrix must therefore be constructed in order to model the ‘great convolution’ and derive meaningful results for the specification of the insurer’s solvency margin. Estimating the full correlation matrix of the model is complicated by the fact that it is difficult to isolate movements between specific factors. There are, however, a number of ways around this and use will now be made of the work by the Finnish Working Party (FWP) to illustrate the
'comprehensive' approach to solvency assessment; for technical detail see Kastelijn Remmerswaal (1986), 85-98.

The Finnish solvency study had two primary objectives (Byrne 1986): first, to safeguard the interests of the insurance consumers; and, second to provide guarantees for the long-term continuation of the operation of an insurance company. In constructing their model, the FWP define a 'standard insurer' that represents an insurer of average size and quality in the sample considered. The FWP conduct sensitivity analysis of the parameters (one by one or in combination with each other), to determine the relative importance of factors; for example, how the size of the portfolio affects the solvency indicators (Beard, Pentikäinen and Pesonon 1984). This analysis was then used to construct an estimate of the correlation matrix and hence calibrate the overall performance of the comprehensive model, evaluating the associated solvency outcome. Subsequent analysis addresses model applicability and also the extent of the three sources of error, mentioned earlier in this section.

A simplified specification for the solvency margin of a going-concern insurance company at time T, arrived at by the FWP, can be given in the following form (Pentikäinen 1988, 42):

\[
(3.21) \quad U_{\text{min}} = aB + b\sigma_x + c + d + eA + f\sigma_A
\]
Where the coefficient, $a$, is a risk loading on premium income $B$ to reflect the risk associated with business cycles or adverse trends. The next term $b\sigma_X$ is a term to account for the fluctuation in claims and is the ordinary mixed compound Poisson fluctuation ($\sigma_X$ is the standard deviation to be calculated for each company). The $c$ term is a margin included for very small companies and the $d$ term is another margin to represent the case where an exceptional risk of catastrophe is imminent. Similar to the representation of the liabilities, the asset side of the balance sheet has two components to reflect its size and variation. Thus, the last two terms represent the asset risks, where $A$ stands for the total amount of assets. Note that the term $\sigma_A$ has a portfolio risk (standard deviation) interpretation and is therefore specified for each insurer (portfolio).

The above solvency measurement in Equation 3.21 represents a ‘short-cut’ method to the great convolution problem. This is therefore an unrefined rule that takes into account the main features of insurance companies in order to provide a means of solvency classification.

3.4 Summary

The need to consider the insurer solvency position is crucial:

If the payments by the insurance companies are not ensured, then there is no real purpose in insuring,” (Kastelijn and Remmerswaal 1986, 117).
The variety of considerations that need to be accounted for are many and varied and as a consequence there is much ambiguity associated with legal definitions and the process of regulating insurance company solvency.

With regard to the three main approaches outlined in section 3.3, each approach has its own merits and drawbacks. Methods based on financial ratios have only a pragmatic justification. The solvency position of an insurer is defined in terms of a straightforward statistical analysis of some observed data on the insurance process; for example claim data. In contrast, methods based on risk theory focus on claims variation $[X_i]$. This theory emphasises the factors that affect liabilities; for example the effect of expenses and inflation. Assets are not explicitly considered and in addition mutual interdependencies are not accounted for.

Methods based on comprehensive models attempt to incorporate both the liabilities and assets, extending the traditional domain of risk theory. Ultimately the objective is to develop a full model that accounts for all the various interconnections between the many factors affecting the solvency position of the insurer. The basic algorithm may be used as a starting point. For each of the variables a sub-model is built, with the interdependencies accounted for in full.

Each of the three approaches to the analysis of insurer solvency varies in terms of the risks that they consider, the time dimension involved, the data required, and the associated complexity. However, there is one very important caveat to all the work outlined above, which serves as the point of departure for discussion in the next chapter. One of the central results to come from classical theory is
that the establishment of suitable premium rates lies at the heart of the solvency of insurers. Yet this conclusion must be checked by the fact that market considerations are not present within this analysis. Moreover, there is little grounding in economic theory in understanding the decision-making process of the company, and the functioning of the industry as a whole. Recent work, both in actuarial science and financial economic analysis, has attempted to build on economic principles while combining this with the simulation methodology outlined above. Attention now turns to addressing these contributions.
CHAPTER 4
EVALUATING INSURANCE COMPANY SOLVENCY:
ALTERNATIVE APPROACHES

4.0 Introduction

Insurance provides a central role in ensuring the smooth functioning of the economy. Insurance companies provide a financial product (a contingent claim on their assets, like a bond) which is set in competition with other financial products. The composition of the insurer's assets and liabilities, rather than being determined exclusively by the actuary, are the outcome of many factors relating to risk and return, which in turn are determined endogenously by the wider economic context.

The previous chapter illustrated the traditional, actuarial view of insurance evaluating the solvency performance of the insurer in terms of a purely risk-theoretic approach, emphasising in particular the role liabilities. Recall that 'classical' models of insurer solvency assume that the level of capital is determined exogenously. Based on this assumption, the analysis then proceeds by attempting to determine the probability that this level of capital will be sufficient to meet the insurer's liabilities. This analysis, while mathematically complex, largely ignores many important features of the insurance process. Insurance is a financial business like any other and as such the industry can be analysed using many of the tools of economic theory. As a consequence, traditional insurance company analysis may be supplemented and enhanced by
various contributions from economics, such as those from the field of financial economics.

This chapter will proceed as follows. Section 4.1 will review briefly the shortcomings of the 'classical' actuarial approach and this will serve as the point of departure for discussion. Section 4.2 will introduce the main themes of the theoretical developments, these fall largely under the inclusive heading of Asset/Liability Management (ALM), see Smink and van der Meer (1997), and discussion commences by reviewing the foundations of ALM. Discussion proceeds in Section 4.3 by demonstrating some of the contributions made by portfolio theory in its application to insurance. Section 4.4 elaborates on simulation techniques as a methodology for evaluating insurer solvency. Section 4.5 reviews some of the other applications of financial economics to the study of insurers, while Section 4.6 addresses the various techniques employed in the early warning/prediction of insurer insolvency emphasising current empirical techniques. Finally, Section 4.7 provides a summary of the main developments within this chapter.

4.1 An Alternative Approach to 'Classical' Insurer Solvency Theory

There are three concerns associated with the traditional risk-theoretic approach, which may be addressed more fully using economic theory. First, although 'classical' solvency theory provides a rigorous account of the underwriting portfolio, its account of the investment portfolio (and its attendant risks) is less than satisfactory. Moreover, the interaction between the underwriting and
investment portfolio, central to understanding the insurer, receives limited assessment. The second problem associated with traditional analysis is that the interaction of the firm with the wider economic environment (within which the insurer must compete and 'survive') is not addressed; that is, the models are closed in nature. The final issue, concerns the way in which solvency is calculated. The traditional literature views solvency in a rather static sense, ignoring the very dynamic nature of solvency.

The three shortcomings outlined above have received differing degrees of attention within the literature (see Laurant (1998)). Recent contributions to the study of insurer solvency have been many and varied. In stark contrast to earlier work, insurer solvency now finds its application in many areas of economics. For the purposes of discussion it is useful to breakdown these developments into theoretical and applied analysis.

Theoretical analysis relates to asset/liability management (ALM) and combines the tools of utility theory, investment analysis, and simulation in order to model the performance of an insurance company as a stochastic process; for example, the an insurer's net cashflow can be analysed by combining a sound theoretical grounding with simulation techniques.\(^1\) Simulation techniques define the insurance company in terms of its output (the underwriting of policyholders). The net cashflow of an insurance company \(S\) is modelled as a stochastic process. The indicator \(S\) is simulated from time \(t=0\) to time \(t=T\), where \(T\) is the time

---

\(^1\) Cash inflows include premiums, investment income, and capital contributions. Cash outflows include loss payments, expenses and taxes.
horizon of the simulation period. Thus, the present market value of $S$ at time $t=0$ yields a measurement of the insurer's solvency position, where $S>0$ ($S<0$) is taken to mean that the insurer is solvent (insolvent). As Kahane, Tapiero, and Jacque (1988) point out, in perfect markets this definition of solvency is the same as the requirement that the market value of assets exceeds the market value of liabilities. Thus, the economic or financial approach places the firm in the context of an economy in general, and financial markets in particular.

Note that in theory, a simulation can be carried out for each insurance company, factoring in firm-specific effects (such as the composition of the underwriting and investment portfolios). The solvency indicator $S_i$ may then be evaluated for insurance company $i$ ($i=1...n$), where $n$ is the number of insurance companies. Any solvency requirements from the perspective of the regulator will be contingent on the value $S_i$ and hence, as a consequence, there will be firm-specific capital constraints.

The applied developments also enrich the analysis of solvency. The applied literature concerns itself with insolvency prediction in the guise of 'financial distress models'. One of the main points to come out of this literature is the fact that standardised capital standards for solvency constraints ignore the firm specific nature of risks. Thus, only a capital standard that takes account of the idiosyncratic nature of insurer will be useful in the prediction/early warning of insolvency. For example, as illustrated shortly, the National Association of Insurance Commissioners (NAIC) has taken up these issues in the United States, with the use of the Risk Based Capital (RBC) model of solvency assessment.
Discussion now turns to addressing these theoretical and applied developments in full. As a starting point for analysis, a review of the foundations of ALM is presented within the next section.

4.2 Foundations of Asset/Liability Management (ALM)

The insurance process (see Figure 3.2) is subject to underwriting risks and investment risks and some of these risks are mutually dependent. Moreover, it is the exploration of the investment risk and its interaction with underwriting risk that remains the point of departure for ALM, since the isolated risks associated with the liabilities of an insurance company (particularly a life insurer) have been explored extensively. Thus, the problem of defining insurer solvency can be viewed in terms of an ALM problem. These techniques build on the analysis of investment risk and are designed to include also the liability structure of a company.

In order to understand the interaction of the underwriting and investment portfolio, a theory is required to model the company as an institution and the investment risk that it faces, given the company’s liability. Both actuaries and financial economists have been involved in the development of asset allocation techniques for many years now. Two of the most frequently used techniques in the analysis of investment risk are immunization theory (Redington 1952) and mean-variance analysis (Markowitz 1952, 1959). Both techniques will be
addressed in turn with the discussion of actuarial techniques being based largely around the work of Ong (1995: Chapter 2).

4.2.1 Actuarial Investment Principles

This analysis of investment risk within actuarial science dates back over a century. As Ong (1995) notes, early investment analysis established a number of actuarial investment principles centred on a minimum risk strategy that, "...considered adverse fluctuations in asset values to be the primary risk to life funds," (Ong 1995, 20).

By contrast, Pegler (1948) argued for a strategy of maximising expected return (Ong 1995). The contrast between this principle and the earlier investment analysis served as a source of debate up until the ALM principle of immunization which stated that assets, "...should be matched to liabilities," (Ong 1995, 21). This argument built on the then recent work of Redington (1952) that suggested the matching of assets and liabilities and presented one of the first formal accounts of ALM through the introduction of the principle of immunization. The theory proposes a method of equating the 'duration' of assets with the 'duration' of liabilities.\(^2\) The theory of immunization has had a major influence on the methods adopted by actuaries in their assessment (valuation) of insurer solvency. Moreover, the influence of immunization can be seen within the current regulations as authorised under the 1982 Insurance

\(^2\) Duration refers to the interest-rate sensitivity of the asset/liability valuations at various points on the yield curve.
Companies Act (Insurance Companies Act 1982, s.18). As a consequence, it is instructive to develop the formal analysis.

Following Ong (1995) let \( L_t \) be the outflow of liabilities (claims plus expenses minus premiums) for year \( t \) and let \( A_t \) be the asset returns (interest plus maturing investments) for year \( t \). \( V_L \) and \( V_A \) are defined as the present value of liability outflows and asset returns respectively at the rate of interest \( \delta \). Assume that \( V_L = V_A \). Furthermore, assume that the interest rate changes by a small quantity, \( \varepsilon \) such that the new rate of interest is equal to \( (\delta + \varepsilon) \). The associated changes in the liability outflows and asset returns mean that their present values are revised to \( V_L' \) and \( V_A' \) respectively. In order to evaluate the net effect upon the valuation (as measured by the surplus) from this small change in the interest rate, an approximation to the amount of surplus may be derived with reference Taylor's expansion, viz.

\[
(4.1) \quad V_A' - V_L' \approx (V_A - V_L) + \varepsilon \frac{d(V_A - V_L)}{d\delta} + \frac{\varepsilon^2}{2!} \frac{d^2(V_A - V_L)}{d\delta^2} + \ldots
\]

The aim of immunization is to ensure that the net change is zero and hence that there is no profit or loss incurred as a result of this interest rate change. With reference to Equation 4.1, it is clear that in order for there to be a zero surplus, all terms composed of \( \varepsilon \) need themselves to be zero. This is achieved when the insurance fund is matched perfectly and every liability outflow is matched by an asset return of equal magnitude and timing (Ong 1995).
If $\varepsilon$ is sufficiently small such that only the first three terms of the Taylor expansion are considered, then immunization is achieved formally so long as:

$$(4.2) \quad \frac{d(V_A - V_L)}{d\delta} = 0, \quad \text{and} \quad \frac{d^2(V_A - V_L)}{d\delta^2} > 0$$

The above equations mean that immunization is satisfied when: (1), the discounted mean term of the asset returns and liability outflow are equal; and (2), the spread of the value of asset proceeds about the mean term is greater than that of the liability outflow (Ong 1995, 22). The insulation of valuations from interest rate movements has obvious benefits in terms of an ALM strategy of eliminating risk. However, as Dardis and Huynh (1996) note, this theory has serious limitations; the most noteworthy limitation being that immunization insulates the insurance fund against profits as well as losses and as a consequence it makes no allowance for anticipated changes (expectations), trading risk and return. Also, they note that in practice immunization would require near continuous re-balancing. The real cost of this in terms of transaction costs makes such a strategy impractical.

While the relative merits and limitations of Redington’s theory are open to many debates, the significance of this contribution to ALM cannot be overstated: immunization provides a useful standard against which to assess alternative investment strategies. In terms of the risk profile of different investment strategies, the immunization strategy therefore constructs a position corresponding to that of minimum risk.
4.3 Financial Economics and Portfolio Selection

In the same year as Redington’s work on immunization, Markowitz (1952, 1959) introduced the idea of asset allocation within a risk-return framework. Markowitz noted that a reduction in portfolio risk, as measured by the standard deviation of return $\sigma$, could only be achieved, without any reduction in return, through diversification into assets whose returns would be uncorrelated (Markowitz 1952). Also introduced by Markowitz, was the set of portfolios referred to as the efficient frontier. This frontier is a curve connecting the risk-return combinations of assets that produce the highest return for each level of risk. The portfolio selected from this set of efficient portfolios is referred to as the optimal portfolio.

The ideas of Markowitz (1952, 1959) can be developed further assuming an economy with uncertainty and a single time period. The utility function of an individual may be expressed as follows:

$$U = U(E(.), \sigma)$$

Where utility, $U$ depends on the expected return $E(.)$ and risk, $\sigma$. Note that restrictions on the preferences of the individual can be expressed to yield risk aversion as follows:
To determine the composition of the efficient portfolios, a problem of quadratic programming must be solved. If \( n \) assets are assumed with respective expected return \( E(R) \), and a variance \( \sigma^2(R) \), the Markowitz problem may be restated as:

\[
\begin{align*}
(4.5) \quad & \min \left\{ \sigma^2(R_p) = \sum_i \sum_j x_i x_j \text{cov}(R_i, R_j) \right\} \\
& \text{s.t.} \ E(R_p) = \sum_i x_i E(R_i) \\
& \text{with} \sum_i x_i = 1, \text{ and } x_i \geq 0 \ \forall \ i
\end{align*}
\]

The advantage of the Markowitz approach is that it has intuitively appealing features; specifically it suggests that diversification pays. It also has the advantage that only the two lowest moments of the return distributions rather than the entire distribution need be calculated. Note that an extensive analytical portfolio analysis is developed in Chapter 5.

Tobin (1958) established another useful property associated with the efficient frontier; namely that of the Separability Theorem. Tobin introduced the existence of a risk-free asset in addition to a set of risky assets. He demonstrated that the optimal portfolio for a mean-variance investor would be a linear function

\[ \text{Note that Outtreville (1986) studies the impact of regulations by quantifying the displacement in the efficient frontiers brought about by a regulatory shift.} \]
of the risk-free asset and an efficient portfolio of risky assets that is the same for all mean-variance investors.

To develop Tobin’s (1958) Separability Theorem, one divides the analysis into two stages: first, the derivation of the mutually optimal portfolio of risky assets is required; and second, the relevant combination of the optimal portfolio of risky assets and the risk-free asset is then to be determined, reflecting the preferences of the investor. The Separability Theorem refines the ideas of Markowitz (1952, 1959) and also underpins the later Capital Asset Pricing Model (CAPM) (Sharpe 1964, and Lintner 1965).

The methods of Tobin (1958) and Markowitz (1952, 1959) required the calculation of all the covariance terms of asset returns; for \( n \) assets, the number of estimates required would be equal to \((n^2 + n)/2\). The computational limitations at the time of Markowitz and Tobin were such that these approaches (even with the contribution made by the Separability Theorem) could only be applied to a small number of asset classes. Such a constraint upon the analysis of investment risk required the portfolio analysis to be simplified further and this was indeed the case with the CAPM, developed simultaneously by Sharpe (1963, 1964), and Treynor (1961), while Mossin (1966), Lintner (1965, 1968), and Black (1972) made further developments.

The basic proposition of the CAPM model is that equilibrium rates of return on all risky assets are a function of their covariance with the market portfolio. This relationship builds on the formal development of Sharpe (1963) that
demonstrated that the returns on securities are related through a common index, $I$. For the case of $n$ securities, the return on security $j$, may be represented through the following relationship:

\[(4.6) \quad R_j = \alpha_j + \beta_j I + \varepsilon_j, \quad for \ j = 1, \ldots, n\]

Here, the parameters of $\alpha_j$ and $\beta_j$ are parameters to be estimated and $\varepsilon_j$ is a random error term with zero mean. Sharpe (1963) argues that the index, $I$ represents a common influence on securities (such as national output). Moreover, the CAPM model assumes that this index is the market portfolio. Sharpe therefore reduces the number of estimates used by Markowitz (1952, 1959) and Tobin (1958) assuming that:

\[(4.7) \quad I = \alpha_{n+1} + \varepsilon_{n+1}\]

Where $\alpha_{n+1}$ is a parameter and $\varepsilon_{n+1}$ is a random variable with mean zero. Thus, this new relationship reduces the estimates of the full covariance matrix to a diagonal matrix, with the number of estimates equal to $(3n+2)$. The performance of this model is similar to that of the Markowitz model (Jarrow and Madan 1997).

Consider further the CAPM, and in particular the market portfolio. Market equilibrium requires that all asset prices must adjust until all assets are held by investors; that is excess demand equals zero. Thus, the market portfolio is
defined as a portfolio that consists of all marketable assets held in proportion to their value weights. The CAPM relates the expected return on security \( j \) to the return on the risk free asset, \( R_f \), and the return on the market portfolio \( R_m \), defined in terms of the security market line, viz.

\[
(4.8) \quad E(R_j) = R_f + \left[ E(R_m) - R_f \right] \frac{\sigma_{jm}}{\sigma_m^2}
\]

Empirically, the return on a security is a linear function of the market return plus a random error term, \( \varepsilon_j \), which is independent, viz.

\[
(4.9) \quad R_j = \alpha_j + \beta_j R_m + \varepsilon_j
\]

The above relationship can also be defined in terms of risk: total risk of any individual asset can be partitioned into two parts—systematic risk, which is a measure of how the asset covaries with the economy (the market), and unsystematic risk, which is independent of the economy (market).

It is worth noting that there are significant problems associated with testing the CAPM including the fact that the market portfolio is not observable and that the market betas are potentially unstable (Huang and Litzenberger 1989). The market portfolio includes assets, such as human capital and other forms of nontraded assets, whose returns are virtually unobservable. As a result,

---

4 The proportion of each asset in the market portfolio is equal to the ratio of the individual asset market value, to the market value of all assets.
empirical work using CAPM employs proxies of the market portfolio. However, these proxies are frequently less than satisfactory as they exclude many assets relevant to the market portfolio (Huang and Litzenberger 1989). In addition, note that Stiglitz (1989) argued that the mutual fund separation theorem has false implications, including that all investors hold widely diversified portfolios and that a company’s debt to equity ratio or dividend policy is inconsequential. Stiglitz (1989) concludes that, “…the major use of the mutual funds theorems has been a cautionary one,” (Stiglitz 1989, 353).

4.3.1 Incorporating Liabilities

The portfolio analysis outlined above emphasises as objects of choice the mean and variance of returns. How does this analysis fit within the analysis of an insurance company and the search for an adequate measure of solvency? The earlier research in mean-variance analysis concentrated almost exclusively on asset funds. This clearly is of limited application to the insurance fund, where the liability structure is of equal importance. However, mean-variance analysis that incorporates institutional liabilities has provided a useful benchmark from which to develop theories of solvency. There are two strands of developments worth emphasising. The first development comes from within financial economics and concerns the portfolio analysis of financial institutions, such as banks and insurance companies. The second development, concerns actuarial portfolio selection models that employ the tools of portfolio theory to analyse various ALM strategies. While the financial literature provides a useful application of mean-variance theory, it is the actuarial literature that provides
some of the more valuable contributions to this analysis. Discussion now proceeds by addressing the financial institution literature, followed by a discussion of the actuarial portfolio models.

One notable aspect of the banking literature that has been applied to mean-variance analysis is in the area of capital adequacy. In particular, the mean-variance approach has been used to analyse the effects of bank capital regulation on the asset and bankruptcy risk of utility maximising banks (for examples of this work see Koehn and Santomero (1980), Kim and Santomero (1988) and Keeley and Furlong (1990)). The literature claims that more stringent capital regulation will increase asset risk (through constraints on portfolio balance) and can therefore increase bankruptcy risk. For example, Kim and Santomero (1988) demonstrate, using a mean-variance analysis, that the use of simple capital ratios in regulating banks is an ineffective way to bound the insolvency risk of banks. The use of ratio constraints was also challenged earlier in the work of Koehn and Santomero (1980). The crucial point here is that the analysis treats the liability simply as a negative asset in order to include it into the portfolio selection problem.

The application of the portfolio selection problem to insurer solvency was established through the work of Hammond, Shapiro and Shilling (1978) who also modelled assets and liabilities within the risk/return framework. Hammond, \textit{et al}. (1978) note that:

\footnote{Keeley and Furlong (1990) have challenged this claim.}
Any attempt to analyze insurer investment and underwriting interaction is greatly complicated by the fact that it is extremely difficult to generate accurate estimates of investment returns associated with each line of insurance. (Hammond, Shapiro and Shilling 1978, 379)

They propose a number of techniques to resolve this problem and indeed the portfolio approach to insurance was developed further through a number of other insurance portfolio optimisation models, see Kahane & Nye (1975), Cummins and Nye (1981), and Outrville (1986). For recent developments in this field see Grossman and Zhou (1996). These models are used to generate mean-variance efficient frontiers. Given these calculated frontiers, it is then possible to determine the operating point on the frontier with reference to a pre-assigned insurer decision rule.

While portfolio theory provides the apparatus to derive the efficient frontier, the selection of an operating point along this frontier requires an additional piece of theory. This additional theory models the operating decisions of an insurance company. Two decision rules frequently used result from ruin theory and expected utility theory. Cummins and Nye (1981) use a portfolio setting to evaluate these decision rules and conclude that these two decision rules generally lead to different operating strategies (Cummins and Nye 1981, 429). For an account of these decision rules and in particular the role of utility theory, see Chapter 5.

Consider next the actuarial portfolio selection models. This area has seen a plentiful supply of research, incorporating many different features of the insurance fund. The works of Wise (1984, 1987) and Wilkie (1985) have
provided an influential analytical framework from within which to address issues of asset/liability management (ALM). As Ong (1995) notes, the aim of this work has been to find a more rigorous analytical framework from within which to view ALM strategies, particularly matching strategies.

Wise (1984) and Wilkie (1985) examine the portfolio selection problem with a view to establishing an allocation of assets that best meets specified liabilities. Emphasis is placed upon the performance of the insurer net cashflow (referred to as 'ultimate surplus') at some specified point in time (Sherris 1992, 87). Ultimate surplus is taken as the main indicator function for the performance of the insurance fund. It is defined as the difference between the accumulated asset cash flows and the accumulated liability cash flows at a fixed time horizon, which is taken as the date for the final liability cash flow; that is:

\[(4.11) \quad S = A \left( \sum_{j=1}^{n} w_j R_j \right) - L\]

Where \(A\) stands for the accumulated asset cash inflows, \(w_j\) is the portfolio weight for asset \(j\) with return \(R_j\), and \(L\) represents the accumulated liability outflow. The cash flows are assumed to be stochastic and hence so too are the values of ultimate surplus, \(S\).

Wise (1984) matches the liabilities by minimising the variance of ultimate surplus for a mean ultimate surplus of zero. Wilkie (1985) demonstrated how to incorporate the Wise approach into a mean-variance efficient portfolio
framework. Wise (1987) replies to this development by showing how this more general framework developed by Wilkie related to his matching portfolio.

There are a number of points to note with respect to these developments. First, the portfolio selection problems (similar to the work of Cummins and Nye (1981)) define liabilities in terms of a negative asset. As such they require the means, variances, and covariances of each 'asset' to be estimated. Second, Wilkie (1985) generalises the portfolio selection problem to include the price of the portfolio; thus, Wilkie describes efficient portfolios in the context of a three-dimensional space, mean, variance and price.\(^6\) The price is an attempt to take into account dynamic issues relating to the trade-off between current and future surplus. Finally, Wilkie (1985) attempts to build in a solvency constraint, defining solvency in terms of non-negative ultimate surplus, which ensures that the liabilities are met. Wilkie assumes that the investor wishes to have a particularly small probability of insolvency, viz.

\[
\text{(4.12) } \prob(S < 0) \leq \alpha
\]

Where \(\alpha\) stands for a correspondingly small level of probability and \(S\) is ultimate surplus. Wilkie delineates those portfolios that pass the solvency criteria using numerical techniques.

\(^6\) Wilkie (1985) demonstrates that the matching portfolios established by Wise (1984), were not in general mean-variance efficient.
A further contribution comes from the work of Sherris (1992) who demonstrated how the Wise-Wilkie approach could be fitted into a more general utility maximisation framework. Sherris removes the price dimension, but still treats liabilities as a negative asset. The decision rule adopted by Sherris (1992) was as follows:

\[
(4.13) \quad \text{Max. } E[U(S)]
\]

Where the explicit functional form \( U(\cdot) \) is to be selected depending on the risk characteristics of the investor (Booth and Ong 1994).\(^7\) Sherris (1992) demonstrates that the general utility maximisation framework can be used to address multi-period considerations and can also be used to incorporate risk preference explicitly into the analysis (for example of the multi-period case, see Booth and Ong 1994, and Booth, Chadburn and Ong, 1997).

It is evident that the techniques of portfolio selection have found considerable application in the field of modelling insurer decision-making. This analysis provides an invaluable framework upon which to develop further analysis. Note that this work is explored more fully within the theoretical developments given in subsequent chapters. However, it should be emphasised that this analytical approach has practical limitations, such as in its ability to deal with dynamic issues. Accounting for these complexities requires the use of an additional technique, namely simulation methodology.

\(^7\) Sherris (1992) uses a negative exponential utility function.
4.4 Simulation Techniques

One of the recurring themes throughout this research concerns the difficulty surrounding a workable definition of solvency. It has been shown in previous sections of this chapter that there are many theoretical perspectives from which to view the operations of an insurance company. However, such techniques necessitate restrictive assumptions in order to derive a fully analytical solution. One way to improve on this realism, is to employ a very powerful technique that uses computers to 'simulate' the problem in hand, namely solvency. Simulation techniques define outcomes in terms of probabilistic statements, defining variables in terms of underlying probability distributions. Such an approach has great practical application, although there are still many issues to be resolved. In order to explore this work in greater detail, it is worth reviewing some of the salient features of this methodology within the cash-flow example.

4.4.1 Developing a Stochastic Model of Solvency

Using the 'cash-flow' framework, the financial strength of an insurance company can be simulated in terms of projected cash inflows and outflows. In this case, the occurrence of an insolvency event may be defined as, "...the state which exists when cash inflows are no longer adequate to meet the required outflows for liabilities," (Coutts and Devitt 1988, 1). Since the future stream of inflows

---

8 Note that this definition applies equally well to different lines of insurance, be it general or life business.
and outflows is uncertain, a stochastic model of these cashflows is therefore required.

The variability associated with the cashflow problem lends itself neatly to the use of simulation techniques. Note that Coutts and Devitt (1988) show that the uncertainty associated with assets and liabilities will vary for different lines of insurance. As a consequence, any operational research will need to take account of these insurance-line specific uncertainties, see Table 4.1 below.

Table 4.1. Risk Classes Present in Assets and Liabilities

<table>
<thead>
<tr>
<th>Life assurance</th>
<th>Liabilities</th>
<th>Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non profit business</td>
<td>Relatively predictable in both time and amount.</td>
<td>Mainly predictable in time and amount.</td>
</tr>
<tr>
<td>With profit business</td>
<td>Timing relatively predictable, amount uncertain.</td>
<td>Some uncertainty in respect of time and amount.</td>
</tr>
<tr>
<td>Unit linked business</td>
<td>Timing relatively predictable, amount highly uncertain, but linked to assets.</td>
<td>Predominantly uncertain in respect of time and amount for guarantees.</td>
</tr>
<tr>
<td>Pension business</td>
<td>Timing uncertain, amount highly uncertain.</td>
<td>Significant proportions uncertain in respect of time and amount.</td>
</tr>
<tr>
<td>General insurance</td>
<td>Timing and amount highly uncertain.</td>
<td>Significant proportions uncertain in respect of time and amount.</td>
</tr>
</tbody>
</table>


Table 4.1 illustrates the risk profile present within the insurer’s underwriting portfolio and therefore this will have implications for the investment portfolio. The risk/return characteristics vary across insurance products and this will be reflected in the investment strategy associated with these respective insurance funds.
Since solvency is concerned with the ability of an insurer to meet its liabilities, it must therefore involve an assessment of the cash-inflows and cash-outflows, while also examining the interaction between these two flows. Rather than looking at the company’s financial strength at a point in time, the simulation approach therefore makes projections about the company’s financial condition.

In order to simulate the performance of an insurer, a stochastic model of an insurer’s assets and liabilities needs to be constructed, thereby enabling a probability density function to be built up around the main indicator function, surplus. Such work needs to address a number of considerations. First, the probability distribution of future claims will need to be specified and this will depend on the insurance line; for example, in the case of conventional life assurance business this distribution function will be defined around the mortality table, with the claim quantities being relatively predictable (see Table 4.1).

A second consideration in simulation modelling is with respect to the asset portfolio. The composition of the opening asset portfolio needs to be defined; that is, the relevant proportions of securities that make up the asset portfolio needs to be specified. The main attributes of the asset classes also need to be defined.9

Another consideration with regard to the simulation algorithm concerns the specification of the set of investment decision rules (investment, disinvestment

---

9 For example, in the case of equities, the investment type, market values, valuation date, yields and dividend rate would all need to be specified.
and reinvestment), which in turn will be contingent upon a model for predicting the income and capital values of the assets. These latter considerations with respect to the simulation of the asset portfolio may be addressed with reference to the Wilkie Investment Model (Wilkie 1986). This model has had a profound influence upon the U.K. insurance solvency literature.

The projected rates of inflation, equity prices, dividend indexes and gilt yields are all simulated in Wilkie's stochastic investment model which is considered by Professor Wilkie as, "... the minimum model that might be used to describe the total investments of a life office or pension fund," (Wilkie 1986, 341). Wilkie emphasises the long term, arguing that the actuary's time horizon extends many years into the future:

> It is ... desirable for him [the actuary] to have a stochastic model to describe the way in which appropriate investment variables have moved over the long term, without being too concerned with the very short term fluctuations. (Wilkie, 1986, 341-342)

Given that a life insurer has liabilities that are made up of life contracts operating many years into the future, this would seem an appropriate starting point from which to view investment risk.\(^{10}\)

Wilkie (1986) calibrates a stochastic model of four variables using data from 1919 to 1982: the Retail Prices Index; an index of share dividends; the dividend yield on these same share indices; and the yield on Consols.\(^{11}\) The Wilkie model may be summarised by means of a diagram, see Figure 4.2 (Wilkie 1986, 346).

\(^{10}\) A full account of solvency would also require some account of the short-term risk associated with investments (e.g. the risk (and effect) from a stock market crash).

\(^{11}\) For a formal account of the 'revised' Wilkie model (Wilkie 1995) see Chapter 10.
Figure 4.1 represents a ‘chronological process’ rather than a causal development; the model excludes a full multivariate structure (a possible drawback of the model). The four variables are illustrated and the role of the inflation index can be seen as central to the operation of the Wilkie model. One problem associated with this model concerns the inflation index. Under the assumptions of Wilkie, the probability of negative inflation is non-zero. While in theory negative inflation is possible, the likelihood in the U.K. is such that some papers have censored the lower tail of the inflation index distribution; for example, Hardy (1994) limits negative inflation to no less than −5 percent.

Having defined a model of stochastic investment returns, it is then possible to simulate the effects of different investment strategies. This model, in conjunction with a stochastic model of insurance claims, may be used to build a comprehensive model of solvency, the risk associated with the asset portfolio being therefore generated by the Wilkie model.
As already noted, a significant volume of literature has evolved around simulation techniques. Research into insurer solvency and ALM may be divided into two main strands. The first strand covers work in the U.K. that has tended to concentrate on methods used to test solvency, the so-called ‘dynamic solvency testing procedures’ (Hardy 1993, 1994). This work has been based largely on the stochastic investment model of Wilkie (1986).  

By contrast, the second strand of simulation research has been developed in the U.S. and this approach emphasises the idea that ALM can be seen as a strategy in asset/liability surplus management (ASLM). This work focuses on the U.S. National Association of Insurance Commissioners (NAIC) risk based capital standards, which requires certain minimum surplus amounts to be maintained against specific classes of risk (Hepokoski, 1994). ASLM might, for example, ask the following question: how sustainable are these surplus amounts given different investment strategies undertaken by the insurer? 

In terms of the specific application of simulation techniques to life assurance solvency see the following: Waters (1988), Ross (1989), Brender (1988) Hardy (1993 & 1996), Pentikäinen & Pesonen (1988). For a review of some of the methods employed see Muir & Squires (1996) and Brender & Claire (1994). See also Chadburn (1996) for an evaluation of solvency constraints within the life assurance industry. Throughout this work, simulation analysis is centred on a

---

12 Wilkie has updated and expanded upon his earlier work (Wilkie 1995).
13 The risk-based capital approach is developed in Section 4.6.
basic cash flow model of insurer surplus. One example of the application of simulation methods to life assurance is by examining the effects of different investment and bonus strategies on the solvency position of a with-profit insurer (Ross 1989; Hardy 1993, 1994 and 1996). This work illustrates the realism that can be incorporated into the analysis and the variance in future performance.

Hardy (1996) considers 3 different insolvency measures and analyses different investment strategies (e.g. high and low equity) and different bonus strategies (e.g. high reversionary bonuses) as compared with a 'base' office. For each specified scenario, the effect upon solvency is considered, in particular the insolvency probability, "... that the value of the guaranteed liabilities at the statutory minimum value exceeds the market value of the assets at some year-end," (Hardy 1996, 1018). Some of the results reported by Hardy are given in Table 4.2 below.

<table>
<thead>
<tr>
<th>Office</th>
<th>Relative Insolvency</th>
<th>Relative Insolvency</th>
<th>Low Payout</th>
<th>Relative Insolvency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(1), (2), or (3)</td>
</tr>
<tr>
<td>Base</td>
<td>5.9</td>
<td>3.9</td>
<td>4.6</td>
<td>8.3</td>
</tr>
<tr>
<td>High equity</td>
<td>14.3</td>
<td>9.4</td>
<td>9.4</td>
<td>18.4</td>
</tr>
<tr>
<td>Low equity</td>
<td>0.1</td>
<td>7.2</td>
<td>19.8</td>
<td>20.2</td>
</tr>
<tr>
<td>High Reversionary bonus</td>
<td>24.8</td>
<td>19.1</td>
<td>7.4</td>
<td>30.8</td>
</tr>
</tbody>
</table>


One of the crucial developments to come from this area of research is in relation to defining solvency. The use of simulation techniques has lead to more

---

14 The 'base' office is the yardstick against which to measure alternative scenarios. It usually exhibits those 'expected' characteristics of a life office.
elaborate measures of solvency being constructed. The main approach is to consider the performance of the asset/liability ratio. However, alternative approaches have been developed and indeed the work of Hardy (1996) introduces the notion of 'relative insolvency', where insolvency is said to occur when the performance of a particular life office moves out of line with the performance of the industry as a whole. In relation to Table 4.2, the relative insolvency measure, (1) is concerned with those insurers being declared technically insolvent, while measure (2) is concerned with the case in which the life office has fallen significantly out of line with the industry.

The results demonstrate that, "... the variant [various] offices are vulnerable to different extents to the three relative insolvency tests," (Hardy 1996, 1019). The sensitivity of the solvency position to the proportion invested in equities is understandable, the higher equity offices being more vulnerable to insolvency. This is a very important point since it is in the nature of many U.K. life offices to invest significantly in equities (see Chapter 7).

Furthermore, the second relative insolvency measure is useful for measuring the risk deviation from the industry as a result of different marketing strategies, for example the introduction of a new product. Thus, new and alternative approaches to the recurring problem of defining solvency have been created by the simulation approach.
4.4.2 Stochastic vs. Deterministic

One of the debates within the literature concerns the use of deterministic and stochastic scenarios in the testing of solvency. Hardy (1993) argues that there are three approaches from which to view the solvency of an insurance company. The first approach is the static 'snapshot' approach that uses actuarial valuations of assets and liabilities. The second approach is the deterministic projected model office approach that tests the projected assets and liabilities of an office using a number of deterministic scenarios. The third solvency approach mentioned by Hardy is the stochastic model office approach where the assets and liabilities of an office are projected using stochastic simulation of future performance.

The stochastic and deterministic approaches fall under the heading of 'dynamic scenario solvency testing'. In the deterministic case, the researcher determines the scenarios, while in the stochastic case the scenarios are generated by Monte-Carlo simulation. The main difference between the two approaches is that in the case of stochastic analysis, it is assumed that each stochastically generated scenario is equally likely, while no such assumption is usually made about individual or relative probabilities of the scenarios within deterministic analysis.

Stochastic solvency testing has a simple pass/fail criterion. A life office is referred to as insolvent if its estimated insolvency risk $\alpha$ is greater than some

---

15 This method is adopted in life offices valuations, and is sometimes referred to as 'worst-case scenario testing'.
predetermined value \( \varepsilon \), that is, \(\alpha > \varepsilon\). If a sufficient number of simulations are performed an estimate of the insolvency risk is derived.

By contrast, the deterministic approach is more subjective in nature. If deterministic scenarios are not equally probable then, as a consequence, it is likely that the projected insolvency will be given less ‘weight’ in the researcher’s assessment than projected insolvency under a more probable scenario. Here lies the problem: developing the relative probabilities of different scenarios and implicitly ranking these scenarios is likely to be subjective for the deterministic approach.

In an attempt to gain a greater insight into the relative merits of the two techniques, Hardy (1993, 1994), presents a formal comparison of the stochastic and deterministic approaches. The stated aim was to, “...compare the quality of information available from a set of stochastic simulations with a traditional deterministic sensitivity test approach,” (Hardy 1994, 131). The main conclusion was that a deterministic scenario, “... fails to distinguish relatively safely set-up offices from highly risky offices,” (Hardy 1994, 49).

The use of deterministic scenarios to test solvency is commonly referred to as ‘stress testing’. By contrast, the use of stochastic modelling is an example of ‘value-at-risk’ modelling – a commonly used risk management technique in the analysis of banking institutions (Heffernan 1996). It is important to be clear on these concepts. The main purpose of both stress testing and value-at-risk modelling is to ascertain the effect of adverse events on a financial institution.
However, for a given structure of asset and liability portfolios a value-at-risk model determines a probability of losing a particular amount of capital, without necessarily defining the events that will give rise to the loss (as in the case of the deterministic stress testing). In insurance and pension-fund risk management, the terminology “stochastic investment modelling”, rather than “value-at-risk modelling”, is often used. For a discussion of this approach and the relationship with stress testing see Alexander (1996) and Matten (1996).

4.4.3 Insolvency as an Example of Downside Risk

The discussion of insolvency throughout is to view it in terms of a probabilistic outcome. Assume any one of the basic measures of solvency, such as surplus ($S$), or the asset/liability ($A/L$) ratio. These measures may be defined in terms of a probability, where the probability of insolvency is equal to the probability of negative surplus, or the probability that the $A/L$ ratio is less than unity, respectively. Such measures are examples of downside risk:

The risk measured in relation to the incidence and often the intensity of unfavourable outcomes, the definition of an unfavourable outcome being based on some criteria. (Ong 1995, 32)

Note that where there are no explicit liabilities incorporated into the analysis (as with the original asset fund models), the use of ‘safety-first’ rules can be helpful in determining the size of downside risk. Thus, the analysis would consider the likelihood of an asset fund underperforming a pre-specified, threshold level of performance, $x^*$. Ong (1995) illustrates the use of three safety-first rules, see Table 4.3 below.
Table 4.3. Safety-First Rules Used to Define the Size of Downside Risk

<table>
<thead>
<tr>
<th>NAME</th>
<th>SAFETY-FIRST RULE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roy’s Criterion</td>
<td>Minimise {\text{prob}(x\leq x^*)}</td>
</tr>
<tr>
<td>Katoaka’s Criterion</td>
<td>Maximise {x^<em>}, s.t. \text{prob}(x\leq x^</em>)\leq \alpha</td>
</tr>
<tr>
<td>Telser’s Criterion</td>
<td>Maximise {E(x)}, s.t. \text{prob}(x\leq x^*)\leq \alpha</td>
</tr>
</tbody>
</table>


Table 4.3 illustrates the different max/min problems used to evaluate downside risk. Roy’s criterion minimises the probability that an indicator $x$ falls below the threshold level $x^*$, where $x^*$ is to be determined. By contrast, Katoaka’s criterion maximises the threshold level $x^*$ subject to the probability that $(x \leq x^*)$ is less than or equal to $\alpha$ (where $\alpha$ is to be determined). Telser’s criterion minimises the expected value of the indicator $x$, subject to the same constraint as Katoaka. For a discussion of the relative merits of the measures in Table 4.3 and downside risk in general see Tse, et al. (1993).

4.5 Additional Theoretical Considerations

A number of contributions have been made as a result of applying other aspects of economic theory to insurance. One such case is in the application of the theory of optimal capital structure.

From the works of Modigliani and Miller (1958), it is known that - in the absence of taxes and bankruptcy risk- the value of the firm is invariant to its capital structure, and the leverage level (i.e. gearing) is therefore irrelevant to value. However, in spite of this argument, it is argued by many that there exist
strong reasons to be interested in an optimal capital structure of an insurance company. As Doherty (1988) notes:

The simultaneity of operating and financing decisions for insurance firms raises questions about the relevance of standard capital structure results. (Doherty 1988, 267)

Doherty points out that the application of the Modigliani and Miller (1958) theorem to insurance is restricted by the existence of a number of features that single out insurance in particular and financial intermediaries in general. Doherty (1988) argues that:

Unlike non-financial firms, the insurance firms' "debt" instruments are not traded on the conventional capital markets but on the insurance product market. Consequently, it is necessary to impose the same restrictions on the insurance product market as upon the capital market. To apply M and M to insurance, we must assume perfect product markets as well as perfect capital markets. (Doherty 1988, 267)

Doherty (1988) uses an options pricing model to investigate the impact of the insurance product market on capital structure. He contends that, in a perfect market, the optimal solution is for the insurer to be totally debt financed. However, Doherty also demonstrates that the introduction of market imperfections (such as the risk of adverse selection, Akerlof (1970)) may invalidate the zero capital approach.

The cost of equity capital is another consideration with respect to the analysis of financially distressed proprietary life insurance companies. The cost of capital is the cost to the firm of providing an acceptable return in order to obtain one pound's worth of capital over a given period of time; it is the discount rate used in the net present value (NPV) analysis of new projects.
In the absence of taxes and under the assumption of perfect markets, this cost must be equal to the required rate of return of the market plus the economic rate of depreciation. That is, to satisfy investors the cost of capital must be consistent with the returns investors expect from holding the firm’s securities. Investors value securities on the basis of expected cash flows and risk and demand a higher return on securities that carry higher risks. However, crucially in a distressed situation a company runs out of cash and as a consequence it cannot service its financial obligations.

Note that there are many alternative approaches to the costs of capital – for discussion, see Lusztig, Morck, and Schwab (1997). Lusztig, et al. argue that the cost of capital is a notoriously difficult concept to measure in practice. Factors relevant here include: the difficulty in making forward-looking assessments; and second, the sensitivity of assessments to arbitrary accounting assumptions.

What are the implications of financial distress for an insurance company in financial difficulties? If an insurance company displays financial distress the opportunities for an insurance company to raise additional equity capital is constrained because cash flow ceases. As a result, at the margin of insolvency the cost of capital in such circumstances becomes unbounded to reflect the risk facing potential suppliers of capital.

The risk – generated by the heightened insolvency risk – associated with the supply of additional equity capital is such that the expected return required by
investors is too great to meet by available cash flows. In sum, the cost of capital is so high that the probability of deriving a positive NPV cash flow – and hence benefits from a residual gain that adds to the overall value of the firm – tends towards zero.

In addition to the capital structure perspective, it should be noted that another area of economic theory that has been applied to insurance is that of agency theory. Solvency management highlights the potential for conflicts between different interested parties. These interested parties may include policyholders, insurers, regulators, shareholders, reinsurers, and investors.

The interactions and motivations of the interested parties mentioned above may be evaluated within an agency framework of principal and agent. Agency relationships might include the following: the insurer (agent) and the regulator (principal); and, the insurer (principal) and the policyholder (agent).

The agency theoretical framework highlights the important roles of information asymmetries, adverse selection, moral hazard, signalling, unseen actions, monitoring, and incentives. Note that the agency perspective of solvency is a crucial consideration in the context of designing optimal regulatory structures engaged in solvency surveillance (Baron 1989).

Another important area of consideration that is developed within the agency framework is corporate governance (for discussions see Mayer (1996)). Corporate governance concentrates on the relationship (and the divergence in
incentives) between investors (the principals) and the managers (the agents). In particular, corporate governance concentrates on, "...the ways of bringing the interests of the two parties into line and ensuring that firms are run for the benefit of investors," (Mayer 1996, 4). Mayer (1996) examines the interplay between corporate governance, competition and performance. With respect to investors, such as non-banking financial institutions (for example, pension funds and life assurance firms), Mayer makes the following point:

Despite the high proportion of institution holdings, there is the widely held view that institutions [non-banking institutions] fail to monitor managers. The problem is said to lie in the dispersed nature of the shareholdings. While in aggregate institutions hold a large fraction of corporate equity...this is dispersed amongst a large number of institutions, few of which hold significant fractions of shares in any one firm. There are good reasons why institutions may wish to diversify their holdings across a large number of firms but this has come at the expense of good corporate governance. (Mayer 1996, 11)

Given the significance of institutional investments to life insurer solvency, the importance of effective corporate governance is therefore another factor worthy of consideration. Thus, agency theory provides an invaluable theoretical framework from which to evaluate other issues relevant to an understanding of life insurer solvency.

4.6 Insolvency Prediction: Models of Financial Distress

The problems of insolvency within the insurance industry are well-documented (see Chapter 2). As a result, various approaches to the problem of identifying financial distress have come to be used in empirical work. Financial distress refers to the state in which, "insurers experience liquidation, receivership,
restraining orders, rehabilitation, etc.,” (BarNiv and McDonald 1992, 543). Financial distress models attempt to identify those factors that assist with insolvency prediction. These models vary in terms of their methodology and are often constrained by significant data restrictions. The usefulness of financial distress models is self-evident: the ability to classify and identify those significant factors which serve in the early detection of financially distressed insurers is central to the aim of regulators, legislators, policyholders, insurers, and the public at large.

As in the case of theoretical analysis, it is important to emphasise that insurance is different from other forms of business. As Carson and Hoyt (1996) point out, there exist important differences between non-insurance financial distress and insurer financial distress. These differences stem from the nature of the costs associated with insurer failure, versus the costs associated with the failure of a non-insurer. When a non-insurer is declared insolvent the customer stands to lose no more than the value of the product/service purchased. By contrast, for the policyholder who has bought an insurance contract, he stands to lose not only the premiums invested, “...but may have had losses for which they will not be indemnified- precisely the contingency for which they had sought coverage,” (Carson & Hoyt 1996, 770).

The above point illustrates the fact that there are a number of interested parties involved in solvency surveillance. These groups are summarised with reference to Table 4.4.
Table 4.4. Interested Parties in Solvency Surveillance

<table>
<thead>
<tr>
<th>Interested Party</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regulators</td>
<td>While the exact objective of the regulator may be open to questioning, it is clear that the detection of financial distress is a central consideration.</td>
</tr>
<tr>
<td>Consumers/Policyholders</td>
<td>Consumers and policyholders cannot accurately assess the solidity of insurers due to a variety of information asymmetries.</td>
</tr>
<tr>
<td>Insurers/Managers/Shareholders/Brokers</td>
<td>The self-interest of these individuals would tend to point towards a requirement for financial solidity in the long run. The reputation effects of insolvency mean that the growth of the insurance market may be adversely affected, (Akerlof, 1971).</td>
</tr>
</tbody>
</table>


With reference to Table 4.4, it can be seen that there are three divisions of interested parties. Regulators will be interested in policyholder protection and hence insolvency detection. Consumers have a direct interest in solvency surveillance. Insurers also have self-interest in solvency surveillance from the perspective of market stability.

The interests of those groups given in Table 4.4 have helped to motivate the financial distress research agenda. The research into financial distress uses a variety of techniques. Each technique attempts to model probabilistic statements about insolvency. The multitude of models used in the prediction and early warning of financial distress may be divided into four groups. The first group uses Multiple Discriminant Analysis (MDA), including the linear probability model (LPM). The second group uses conditional probability models (Logit and Probit). The third type of group uses non-parametric techniques, whose assumptions are less restrictive than the LPM, Logit or Probit. The last group
considered deals with a new class of models, the so-called neural network models, which have provided renewed interest in the area of financial distress.

The study of financial distress extends back many years. The earlier studies used accounting data within a straightforward statistical methodology in the classification and prediction of bankrupt firms (for discussions see BarNiv and McDonald 1992). Applied work in this field dates back to the pioneering work of Beaver (1966) who employed a univariate, dichotomous model in the prediction of corporate bankruptcy. By 1968, Altman introduced the idea of using financial ratios within a multivariate framework to predict the failure of a set of manufacturing firms. The model correctly identified 95% of the total sample (Altman 1968).

BarNiv and McDonald (1992) classify corporate bankruptcy detection models according to the following criteria: (1), sample selection procedures; (2), data collection procedures; and (3), classification errors. The majority of these corporate bankruptcy studies use a ‘matched-pair sample’ (one non-failed firm for each failed firm) consisting of the population of failed firms with complete data, assuming that the costs of misclassification (ratio of type I errors to type II errors) are generally equal.16 Note that since the cost of a Type I error is likely to exceed the cost of a Type II error, other studies have used alternative assumptions (BarNiv and McDonald 1992).

16 A type I error is classifying an insolvent insurer as solvent; and, a type II error is classifying a solvent insurer as insolvent.
4.6.1 Multiple Discriminant Analysis

As noted, work on Multiple Discriminant Analysis (MDA) dates back to Altman (1968). The idea is to classify companies in terms of two groups, those companies that require additional regulatory supervision and those that do not. A composite score is calculated for the observations using a procedure which maximises the ratio-score of the 'need attention' group, to the 'not need attention' group.

The MDA approach is illustrated as follows. Assume a set of $n$ ratios, $x_1, x_2, x_3... x_n$, and also a set of coefficients, $c_1, c_2, c_3... c_n$, that are the weights to be used in the calculation of the composite score, $y_j$ for the $j$th company, viz.

\[
(4.14) \quad y_j = \sum_{i=1}^{n} c_i x_i, \forall j
\]

A decision rule can be constructed for the classification of the $j$th company, using a cut-off point, $y^*$ for the composite score. Thus, for example, if the composite score is less than the cut-off $y^*$ a company may therefore be classified as 'distressed' and this may bring about regulatory action. Note that in addition to the MDA, the Linear Probability Model (LPM) was used, though the validity of its underlying assumptions were not addressed in earlier studies (see as an example Eck (1982)).
One of the first studies to apply MDA to solvency surveillance in the insurance (property-liability) industry was the study by Triechmann and Pinches (1973) who used data on 26 companies that experienced financial distress, and 26 companies that did not, for the years 1966-1971. They used six ratios, including for example, the ratio of direct premiums written to surplus. The predictions of the model were close to that of actual experience, with approximately a 5 percent type I error reported. Triechmann and Pinches (1973) also used the same sample to examine the efficiency of the MDA analysis by comparing univariate and multivariate financial ratio models, and concluded that the MDA outperformed the univariate models.

While this method gained popular appeal in the early 1970s, the problems associated with the MDA technique soon became highlighted, with warnings sounded about the misuse and misinterpretation of such models in the classification and prediction of financial distress (Eisenbeis 1977, Altman and Eisenbeis 1978). These concerns related primarily to measurement error and also to the low statistical significance associated with the sample size. More recently the application of MDA has been shown to be dependent on very strong assumptions (BarNiv and Raveh 1989).

Furthermore, although discriminant analysis has the advantage of simplicity, for most countries it is not possible to obtain a large enough sample of distressed firms to be able to apply this method (this applies equally to limited dependent variable models of financial distress). In the U.K., for example, between 1990 and 1996 there were only 2 cases of insurance companies having authorisation
withdrawn due to financial distress (Department of Trade and Industry 1998). It would therefore be impossible to conduct MDA. Also, even if adequate representation can be achieved within the sample, there is then the question of data reliability and the problem of insurance companies manipulating their financial reports to reduce the likelihood of regulatory intervention.

4.6.2 Insurance Regulatory Information System (IRIS)

The operational use of financial ratios and discriminant analysis is illustrated in the Insurance Regulatory Information System (IRIS), developed by the National Association of Insurance Commissioners (NAIC) in the United States.\(^{17}\) The IRIS has served as the baseline solvency screening system for the U.S. IRIS aims to help regulators prioritise insurers that are financially distressed in order that the regulator may allocate their resources more efficiently.

IRIS analysis consists of two stages: a statistical phase and an analytical phase. The statistical phase involves the assessment of a number of financial ratios (11 for property/casualty & 12 for life/health) used to define the performance of each insurer (Lamm-Tennant, Starks and Stokes 1996). In the case of life/health insurance, financial ratios would encompass profitability, investments, and changes in operations as well as any other variable that reflects the overall financial condition of the insurer. The next step, the analytical phase, classifies insurers according to the following decision rule: insurers with four or more

\(^{17}\) The NAIC is an organisation consisting of all the U.S. state insurance regulators from the 50 states, the District of Columbia and the four U.S. territories.
financial ratios outside a pre-specified range, are classified as financially
distressed and as a result come under immediate regulatory scrutiny. In 1992,
under this system, 39.8 percent of life companies were subjected to greater
regulatory investigation (Klein 1995).

With the problems of MDA cited above, it is not surprising that the IRIS system
has been heavily criticised. The system has been criticised for its ‘pass/fail’
nature, its equal weighting of financial ratios, the interdependence between
ratios, the omission of certain ratios or variables that might have significant
predictive value, and its reliance on annual statement data (Thornton and Meader
1977; and, Hershbarger and Miller 1986). The history of IRIS has shown that it
has resulted in a large proportion of Type II errors, mis-classifying solvent
insurers as insolvent (Klein 1995).

A number of studies have attempted to evaluate the IRIS early warning system.
Harrington and Nelson (1986) estimated an OLS equation to predict a firm’s
premium-to-surplus ratio based on a variety of firm-operating characteristics.
Although their model provided promising results, it was still not conclusively
superior to that of IRIS. Hershbarger and Miller (1986) examined the IRIS ratios
and whether exogenous factors should warrant inclusion in financial distress
models. The authors found that the endogenous factors appeared to be more
important than exogenous factors in the prediction of solvency (Hershbarger and
Miller 1986). Thus, the IRIS system provides a useful indicator of financial
distress, although there remain a number of methodological concerns associated
with this approach.
Note that recently, the IRIS system has been updated to include a computerised analytical routine: Financial Analysis and Solvency Tracking System (FAST). The FAST system prioritises companies automatically. It has been introduced by the NAIC to oversee state regulators within a peer review system of insurance regulation.

### 4.6.3 Limited Dependent Variable Models

Having accepted the limitations of MDA analysis and the linear probability model, the use of limited dependent variables may be seen as a step forward in reducing the problems associated with previous work. In limited dependent models, such as logit and probit models, the distributions of the dependent variable or score, $Z$, are conditional on the vector of explanatory variables $x$, and the probability of $Z$, given $x$ is assumed to be distributed according to the logistic distribution (for the Logit model) and the normal distribution (for the probit model). For recent works see BarNiv and Hershbarger (1991), BarNiv (1989, 1990), and Carson and Hoyt (1996).

Martin (1977) uses a logit model to identify bankruptcies among commercial banks. In contrast, BarNiv (1989, 1990) uses a logit model to identify insolvencies in the property-liability insurance industry, and emphasised the respective roles of alternative accounting practices, cashflow, and asset values in explaining insolvency. In terms of comparisons of the relative merits of limited dependent models and discriminant models, there have been a number of studies...
which suggest that the Logit and Probit models are more robust than MDA (Amemiya 1981, BarNiv and McDonald 1992).

In examining the life insurance industry, BarNiv and Hershbarger (1990) applied a number of different methodologies and found that the change in product mix is an important predictor variable of insolvency, and that those insolvent insurers tend to be smaller in size than solvent insurers. Ambrose and Seward (1988) found that financial variables combined with IRIS ratios in a logistic regression model outperformed other methods in distinguishing between solvent and insolvent insurers. Carson and Hoyt (1996) found that surplus and leverage measures are also strong indicators of insurer financial strength.

For a recent review of these techniques and a discussion of economic and market predictors of life assurance insolvency see Chen, et al. (1994). It should be noted that criticisms relating to data reliability are relevant to the discussion of limited dependent variables.

4.6.4 Non-parametric Techniques

Non-parametric techniques have also been used to evaluate the problem of financial distress, see BarNiv and McDonald (1992), and Carson and Hoyt (1996). A valuable contribution came from the introduction of a technique called Recursive Partitioning (Frydman, Altman, and Kao 1985). This technique searches for an optimal linear combination of variables that yield minimum overlap between the ‘distressed’ and ‘not distressed’ groups. Frydman, et al.
(1985) found that Recursive Partitioning compared favourably to MDA and concluded that additional information may be derived from such an approach in the early prediction of bankruptcy; for a more recent comparison of these techniques, see Carson and Hoyt (1996).

Carson and Hoyt (1996) attempt to provide evidence on the relative strength of three types of bankruptcy detection models, the MDA model, the Logit model, and the Recursive Partitioning model. They use data taken from the life insurance industry in the U.S. They conclude that the three models perform reasonable well in classifying financial distress, with the Logit model dominating the MDA and Recursive Partitioning models, in terms of the number of correctly classified solvent insurers. By contrast, the Recursive Partitioning model dominated the Logit and MDA models in terms of the number of correctly identified insolvent insurers. On balance, with the emphasis on insolvency and lower tail of the solvency distribution, the Recursive Partitioning model is therefore to be favoured.

Note that another area of research concerns the application of neural network techniques. This class of model offers great potential for the study of financial distress. Such models attempt to set up the model structure along the lines of the neural networks observed within the human brain. Goss and Ramchandani (1995) compare the insolvency classification accuracy of neural networks with a binary logit regression and a discriminant analysis. The use of a neural network, a non-parametric alternative to past techniques, demonstrates how this
methodology predicts life insurer insolvency more effectively than parametric models.

4.6.5 The American Risk Based Capital Model

Insurance regulations in most western countries (including the U.K.) take the form of fixed or crude proportionate capital and surplus standards. Such regulations are limited in terms of their relationship to risk. However, insurers range widely in terms of their size and the types of risk that they assume, which makes these types of regulations cumbersome at best in attempting to control for insolvency.

The limitations of fixed capital standards in the U.S. resulted in the introduction of risk-based capital standards by the NAIC in 1990. The principle criticisms of the previous system were that the standards were, (1) unrelated to risk, (2) too low for many insurers, and (3) provided an insufficient basis for timely regulatory action (Atchinson 1997). As a consequence, the new RBC standards had the objective of developing a standard of capital adequacy that, (1) was related to risk, (2) raised the safety net for insurers, (3) was uniform among states, and (4) provided appropriate authority for regulatory intervention, (Atchinson 1997).

The risk-based capital system uses a formula that establishes the minimum amount of capital necessary for an insurance company to support its overall business operations, considering its size and risk profile. That amount is then compared to the company's actual statutory capital to determine whether a company is technically solvent. (Atchinson 1997, 60)
The mechanics of the risk-based capital approach are illustrated with reference to the formula for life/health insurance. The formula for life/health insurance is made up of four major categories of risk: asset risk; insurance or pricing risk; interest rate risk; and, business risks (Klein 1995). Asset risk concerns itself with the risk of default and decline of asset value. Insurance, or pricing risk, addresses the risk that premiums (and attendant reserves) will be insufficient to meet liabilities (claims). Interest rate risk then addresses the possibility that an insurer will have liquidity problems from ‘disintermediation’ due to interest rate changes. The final category, business risk, refers to the insurer’s obligation to the guarantee fund and the risk from paying out on this obligation. Consider Table 4.5, which illustrates some of the risk categories used in the assessment of asset risk.

Table 4.5. Sample of Risk-Based Capital Charges for Selected Asset Categories

<table>
<thead>
<tr>
<th>TYPE OF INVESTMENT</th>
<th>Risk-Based Capital (RBC) FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. government bonds</td>
<td>0.000</td>
</tr>
<tr>
<td>Highest quality corporate bonds</td>
<td>0.003</td>
</tr>
<tr>
<td>Cash and short-term investments</td>
<td>0.003</td>
</tr>
<tr>
<td>High quality corporate bonds</td>
<td>0.010 to 0.100</td>
</tr>
<tr>
<td>Bonds in default on principle or interest</td>
<td>0.300</td>
</tr>
<tr>
<td>Mortgages and collateral loans</td>
<td>0.050</td>
</tr>
<tr>
<td>Unaffiliated common stock</td>
<td>0.150</td>
</tr>
<tr>
<td>Real estate</td>
<td>0.100</td>
</tr>
<tr>
<td>Partnerships and joint ventures</td>
<td>0.200</td>
</tr>
<tr>
<td>Reinsurance recoverables</td>
<td>0.100</td>
</tr>
<tr>
<td>Miscellaneous recoverables</td>
<td>0.050</td>
</tr>
</tbody>
</table>


This table shows that the risk-based formulas apply loading factors to various amounts reported in (or related to) the financial statement of an insurance company in order to calculate a company’s required risk-based capital. The application of these loading factors reflects each type of risk mentioned above.
A 'covariance adjustment' is then made to the accumulated risk-based capital charges, to account for the interdependence of that exist between major risk categories. This results in the *adjusted* total risk-based capital amount.

Figure 4.2. The Risk-Based Capital (RBC) System


Notes:
- **A**: Regulator must liquidate or rehabilitate.
- **B**: Regulator may require liquidation or rehabilitation.
- **C**: Regulator may issue a collective order.
- **D**: Insurer or reinsurer must submit a plan for correction to the regulator.

This *adjusted* total risk-based capital is then compared to an insurer’s actual capital (the total adjusted capital (TAC)), to determine a company’s risk-based capital position. Under this assessment, certain company and regulatory actions
are required if a company's TAC falls below its calculated level of risk based capital (Atchinson 1997), see Figure 4.2.\textsuperscript{18}

A voluminous amount of literature has developed around the risk-based capital model. Risk-based capital regulations have been in operation within the banking sector for some time, and research has focused on establishing a theoretical and empirical framework for evaluating such standards (see Grenadier and Hall 1996 and Gjerde and Semmen 1995). A number of perspectives are taken on the merits of such a system within the banking sector, such as the effect of these regulations on credit provision (Engineer 1994; Berger and Udell 1994); for example, Berger and Udell (1994) argue that such standards may have been responsible for a 'credit crunch' in the U.S.

Insurance risk-based capital standards have also been the source of attention within the literature, and have been criticised in terms of the lack of theoretical grounding governing the calculation of the risk-based capital factors (Atchinson 1997). The establishment of a theoretical framework for evaluating these regulations and setting explicit standards is also discussed (Dickinson 1997). The weights assigned to the different categories of risk have, in addition, been the focus for work (Bradley, Wambeke and Whidbee 1991).

Cummins, Harrington & Klein (1995) analyse the risk-based capital formula for property-liability insurance. They use a predictive accuracy test of solvent and

\textsuperscript{18} The TAC is expressed as a percentage of the risk-based capital (the so-called 'authorised control level'). Regulatory action is based on the size of this ratio.
insolvent insurers for the period 1989-1993. They show that the predictive accuracy of the TAC/RBC ratio is poor when this is the only independent variable, but that its accuracy improves significantly when the components of the formula and variables for firm size and organisational form are used as regressors.

Thus, at the applied level, it is evident that a wide variety of methodologies exist in the early warning/prediction of financial distress. These techniques have evolved to produce more accurate predictions, with useful prescriptions for the review and design of insurance regulations. However, data restrictions and methodological concerns have questioned the reliability of such techniques in solvency surveillance.

4.7 Summary

It is possible to view solvency from many different perspectives. As a consequence, it is evident that research into insurance solvency has moved away from the tradition of collective risk theory, into many other areas using alternative theoretical and applied techniques. These alternative approaches are frequently complimentary. Moreover, the interplay of financial economics and empirical research has helped to resolve some of the ambiguity surrounding our understanding of insurer solvency.
CHAPTER 5

MODELLING INSURANCE COMPANY DECISIONS: PORTFOLIO BALANCE, EXPECTED UTILITY AND REGULATION

5.0 Introduction

Previous chapters have served to demonstrate the variety of risks that affect the solvency position of a life assurance company. One particular source of risk, that of investment risk, can be seen as a major contributor to the uncertainty associated with life-insurer solvency measurement. Moreover, the nature of the life assurance contract is such that its future financial position is governed to a large extent by the performance of its investment portfolio. Chapter 4 introduced some of the analytical tools of portfolio analysis. This chapter extends this development by providing an extensive formal account of portfolio theory applied to the issue of insurer solvency regulation.

Life contracts, from basic term assurance to substantive policies such as endowment assurance, are often effected for periods in excess of 10 years, and frequently for periods up to and beyond 25 years. Such contracts are to be contrasted with other forms of insurance, such as general insurance business (e.g. car insurance), where

---

1 A policy is substantive if the premium paid by the policyholder is used to build up a savings fund in addition to providing a financial benefit on the occurrence of death (as in the case of a basic term assurance contract).

2 Substantive life assurance policies are effected for at least 10 years in order that their non-death benefits 'qualify' for tax exemption.
contracts are typically effected for periods of one year. The life contract therefore provides the insurance company with a stream of payments in the form of premium income, which after deductions will be placed into a collective fund (the ‘life fund’) and invested. Here lies the distinction between life assurance and other forms of insurance: the investment opportunity of a life insurer is enhanced by the fact that it is provided with an expected income stream, which extends many years into the future and is reasonably well defined in terms of its magnitude and fluctuation.3 Thus, it is the long-term nature of the life assurance contract that enables an insurance company of this type to engage in an alternative investment strategy to that of any non-life assurance company. The time interval between payment of premium and claim outflow is such that a life insurer has a unique opportunity to profit from employing a long-term investment strategy.

Before proceeding, it is appropriate to consider the issue of insurer liquidity. While a life insurer will have an underwriting portfolio consisting predominantly of long-term contracts, it is important to emphasise that for a given day/month/year, a life insurer, like any other business, will need liquidity in order to service its immediate cash outflows. Cash outflows may come in a variety of forms, such as intermediate policy claims, surrender value entitlements, and the servicing of any other debts. Cash outflows will be governed by the nature of the underwriting portfolio (the composition of policies and their associated risks). For life contracts, the nature of

3 Aside from deaths, fluctuations in premium income might also be brought about by policy surrenders (‘cashing in’).
these liabilities will lend themselves more readily to assessment and prediction (in contrast to general insurance contracts); mortality tables provide valuable information in this regard.\textsuperscript{4} Thus, the relative predictability of claims associated with the underwriting portfolio provides additional motivation for an analytical investigation into the role of the investment portfolio and its influence upon the financial solidity of the insurance company. The success/failure of a given investment strategy (and its attendant risks) can therefore be seen as a central factor in the determination (and understanding) of life-insurer solvency. In order to formalise these ideas, it will be necessary to employ the tools of financial theory and in particular portfolio analysis (Markowitz 1952, 1959). As introduced in Chapter 4, portfolio theory provides an influential analytical framework for modelling insurer decision-making. The theory derives the efficient set of portfolios, from which an insurer (under standard assumptions (see Section 5.3)) selects one portfolio, the 'optimal portfolio'. The selection of an 'optimal portfolio' will depend on the contribution from an additional theoretical development, namely utility theory.

As noted in Chapter 4, portfolio theory has been used in the solvency analysis of a variety of financial intermediaries. For example, portfolio theory has been used as a means of evaluating the regulation of banking institutions, see Morgan (1984), Kim and Santomero (1988), and Keeley and Furlong (1990). The portfolio analysis of life assurance has also been addressed in a number of settings; for example, see

\textsuperscript{4} Prediction is assisted especially in terms of the timing of claims.
Krouse (1970), Krinsky (1985), and Chan and Krinsky (1985) for specific applications to the problem of portfolio optimisation and life insurer decision-making. MacMinn and Witt (1987) use financial theory to model the solvency of an insurance company and examine the insurer’s performance under alternative regulatory standards. Much of the work on the application of portfolio theory to insurance dates back to the work of Kahane (1978).

In line with these developments, discussion will proceed as follows. An outline of the traditional portfolio problem will be given in Section 5.1 and this will be followed by a detailed exposition of a general portfolio model of an insurance company in Section 5.2. The introduction of utility theory to the analysis of insurer portfolio selection will be made in Section 5.3, while in Section 5.4 solvency regulation is illustrated. Analysis is also extended to include ‘insurance leverage’ in Section 5.5. Section 5.6 provides a summary of the main developments.

5.1 The Standard Portfolio Selection Problem

The Markowitz (1952, 1959) portfolio selection problem may be expressed as follows. Initially, consider the case where an investor wants to invest a given sum of wealth into a portfolio made up entirely of \( n \) risky assets (liabilities are excluded for the time being).\(^5\) The portfolio selection problem typically involves investing a

\(^5\)The name ‘investor’ is used as a generic term to cover anything from an individual consumer, to an institutional investor. The important point is that the investor is that unit which has control over the investment decisions.
fixed amount of money, the investor's initial wealth $K_0$, in order to achieve a desired outcome in terms of end-of-period wealth $K$. Since the solution to the portfolio problem does not depend on the size of the initial amount invested, the problem reduces to one of choosing asset proportions (or weights) $w_i$ to be assigned to each of the asset classes, $i=1,...,n$ (Wilkie 1985). Investors make their decisions based upon the first two moments of the return distribution for end-of-period wealth. That is, decisions are based on the expected return $[E(R)]$, which may be taken as a measure of profit, and variance of return $[Var(R)=\sigma_i^2]$, which may be taken as measure of the uncertainty (risk) associated with the expected return. Expected returns for all assets are denoted by the row vector, $e'$ viz.

(5.1) $e'=[E(R_1), E(R_2),..., E(R_n)]$

The covariance of the returns between a pair of assets, $i$ and $j$, is given by $Cov(R_i, R_j)=\sigma_{ij}$. Denote the variance/covariance matrix by $V$, where $V$ is defined as follows:  

(5.2)  $V = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_n^2 \end{pmatrix}$

$^6$Note that $\sigma_j=\sigma_j$, so that the variance-covariance matrix of returns is symmetric. All such matrices are positive semi-definite (Wilkie 1985, 230).
Using the definitions for expected return and variance, the efficient frontier is defined as follows:

The frontier of all feasible portfolios which can be constructed from these $n$ securities is defined as the locus of feasible portfolios that have the smallest variance for a prescribed expected return. (Merton 1972, 1851)

Let $w_i$ be the proportion of initial wealth invested in the $i^{th}$ security, $i = 1 \ldots n$, where by definition $\sum_{i=1}^{n} w_i = 1$. Denote the vector of these proportions, or weights, as follows:

\[
(5.3) \quad \mathbf{w}' = (w_1, w_2, \ldots, w_n)
\]

The efficient frontier is defined by a set of portfolios, which satisfy the following constrained minimisation problem (for a full formal account of this problem see Merton (1972)).

\[
(5.4) \quad \min \left( \frac{1}{2} \sigma_p^2 = \frac{1}{2} \mathbf{w}' \mathbf{V} \mathbf{w} \right)
\]

Subject to the following:

\[
\begin{align*}
\mathbf{e}' \mathbf{w} &= E^* \\
\mathbf{w} &\geq 0 \quad \text{(The 'short sale restriction')} \\
\mathbf{w}' \mathbf{1} &= 1
\end{align*}
\]

\[\text{The inclusion of 1/2 in front of the variance expression is made to simplify the analytical solution.}\]
Where the subscript $p$ refers to the portfolio, $E^*$ is a constant to be specified, and $\mathbf{0}$ is a vector of zeros. The decision variables are the portfolio weights $\mathbf{w}$, and by solving for different values of $E^*$, the efficient frontier can be constructed in expected return/risk space using standard quadratic programming techniques (see Figure 5.1).\(^8\) Markowitz (1959) was the first to define the investor's portfolio selection problem in this way (and also to show that it is equivalent to maximising the investor's expected utility (Levy and Markowitz 1979)).

Figure 5.1. Mean-Standard Deviation Portfolio Frontier (Risky Assets Only)

\[ E(.) \]

\[ E^* \]

\[ V \]


With reference to Figure 5.1, it can be seen that the portfolio frontier is a hyperbola. For a given level of risk there are two alternative portfolios available to an investor.\(^9\)

There are portfolios with returns in excess of $E^*$ and portfolios with returns below $E^*$. $E^*$ represents the return consistent with that of the minimum risk portfolio. Thus, every portfolio with a return in excess of $E^*$ strictly dominates a portfolio of

---

\(^8\)Additional constraints can be imposed on the quadratic program (e.g. solvency constraints).

\(^9\)This is a direct consequence of the quadratic program.
equivalent risk with return below $E^*$. Those portfolios with an expected return greater than, or equal to $E^*$ are defined as the efficient set of portfolios (including the minimum risk portfolio $V$).

5.2 A General Portfolio Model of a Life Insurer

A general portfolio model of a life insurer can be constructed as follows (see Krinsky (1985), Chan and Krinsky (1985), and Cummins and Nye (1981) for this standard treatment of insurance within a portfolio setting). Assume that, for the purposes of exposition, the insurance company is a proprietary life insurer, and that there are $n$ types of possible investments and $(m-n)$ types of life assurance contracts. Table 5.1 provides the names and definitions of the variables to be used within subsequent analysis.

Table 5.1. Variables and Definitions

<table>
<thead>
<tr>
<th>NAME</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_0$</td>
<td>Initial wealth</td>
</tr>
<tr>
<td>$\bar{R}_n$</td>
<td>Rate of return on the $i^{th}$ investment: $i = 1, ..., n$ in period $t$.</td>
</tr>
<tr>
<td>$\bar{R}_m$</td>
<td>Rate of underwriting return on the $i^{th}$ type of insurance contract: $i = (n+1), ..., m$ in period $t$.</td>
</tr>
<tr>
<td>$I_{it}$</td>
<td>Investment in the $i^{th}$ asset, $i = 1, ..., n$ in period $t$.</td>
</tr>
<tr>
<td>$A_i$</td>
<td>Actuarial Reserves for the $i^{th}$ type of insurance contract, $i = (n+1), ..., m$ in period $t$.</td>
</tr>
<tr>
<td>$K_{t0}$</td>
<td>Policyholder’s surplus plus shareholders’ equity in period $t$, where $K_{t0} = S_{t0} + E_{t0}$.</td>
</tr>
<tr>
<td>$S_{Dt}$</td>
<td>Policyholders’ surplus in period $t$.</td>
</tr>
<tr>
<td>$E_{dt}$</td>
<td>Shareholders’ equity at the beginning of period $t$.</td>
</tr>
</tbody>
</table>


Note: (a) The tildes denote the fact that the variable is a random variable.
(b) “Insurance contracts” could be aggregated to distinguish insurance business along insurance product lines (e.g. term assurance, whole life, endowment, etc.).
Following Krinsky (1985), the profit of the insurer in period $t$ is given by $\pi_t$, and is defined as a linear combination of the random rates of return on the $n$ investments and the $m-n$ insurance contracts, viz.

\begin{equation}
(5.5) \quad \pi_t = \sum_{i=1}^{m} I_i R_i
\end{equation}

The rate of return on equity in period $t$, $\tilde{r}_{pt}$, is given by dividing Equation 5.5 by equity $E_{0t}$ in order to obtain the following:

\begin{equation}
(5.6) \quad \frac{\tilde{\pi}_t}{E_{0t}} = \tilde{r}_{pt} = \sum_{i=1}^{m} \left( \frac{I_i}{E_{0t}} \right) \tilde{R}_i = \sum_{i=1}^{m} w_i \tilde{R}_i
\end{equation}

Where, $w_i$ is the $i^{th}$ asset-to-equity ratio for asset $i$ (in period $t$), $i=1,...,n$, and the $i^{th}$ actuarial reserves-to-equity ratio for insurance contract $i$ (in period $t$), $i=(n+1),...,m$.

Expanding 5.6 we have:

\begin{equation}
(5.7) \quad \tilde{r}_{pt} = w_{1t} \tilde{R}_{1t} + w_{2t} \tilde{R}_{2t} + \ldots + w_{nt} \tilde{R}_{nt} + w_{(n+1)t} \tilde{R}_{(n+1)t} + \ldots + w_{mt} \tilde{R}_{mt}
\end{equation}

In order to complete the portfolio model, it is necessary to introduce the balance sheet constraint, which makes certain assumptions regarding the investment funds generated by selling policies (Krinsky 1985, 244) and hence the extent of insurer investment leverage. The investment-funds generating factor needs to be
determined; for example, it could be assumed that one pound of underwriting business (as measured by actuarial reserves) generates one pound of investment funds (Cummins and Nye 1981, 415). Note here that Krinsky (1985) does not explicitly model insurer premiums. Actuarial reserves are taken as a proxy for premium income because of some of the problems, particularly valuation problems, associated with comparing premiums across different contracts. For a discussion of the use of reserves see Krinsky (1985).

Whatever variable is used to define the insurer's activities, the l's (i=1...m) must be determined such as to equate total assets with the sum of liabilities. Hence, the following balance sheet constraint must hold:

\[
\sum_{i=1}^{n} l_{at} = \sum_{i=n+1}^{m} l_{at} + K_{ot} \tag{5.8}
\]

Dividing Equation 5.8 by equity, \(E_{ot}\) results in:

\[
\sum_{i=1}^{n} w_{at} - \sum_{i=n+1}^{m} w_{at} = \frac{K_{ot}}{E_{ot}} = 1 + \frac{S_{ot}}{E_{ot}} \tag{5.9}
\]

The first term on the LHS of Equation 5.9 represents the ratio of the \(i^{th}\) (\(i=1...n\)) asset class investment to equity (the asset weights), and the second term on the LHS represents the ratio of the \(i^{th}\) (\(i=n+1...m\)) contract reserves to equity (the contract
weights). The RHS can be used to simplify Equation 5.9 into a standard weight constraint, assuming that the RHS ratio \((S_{0t}/E_{0t})\) represents another weight \(w_{(m+1),t}\), the surplus-to-equity ratio. Equation 5.9 therefore simplifies to the following:

\[
\begin{align*}
(5.10) & \quad \sum_{i=1}^{n} w_i - \sum_{j=n+1}^{m+1} w_j = 1
\end{align*}
\]

The treatment of liabilities simply as negative assets is clear within the formulation given in Equation 5.10.\(^{10}\) Having established a general portfolio model of an insurance company, it is possible to develop a formal rule for the insurer's portfolio selection problem. The expected value and variance of the rate of return on equity \(\tilde{r}_p\) may be expressed as follows:

\[
\begin{align*}
(5.11) & \quad E(\tilde{r}_p) = \sum_{i=1}^{m+1} w_i E(\tilde{R}_i), \quad \text{and} \\
& \quad Var(\tilde{r}_p) = \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} w_i w_j Cov(\tilde{R}_i, \tilde{R}_j)
\end{align*}
\]

Hence, the expected return \(E\) and variance \(V\) of the insurer's end-of-period wealth, \(\tilde{K}\) is given by:

\(^{10}\) Note that this equation holds for every time period and hence, the use of time subscripts is no longer required.
The Lagrange $L$, defines the insurer’s optimisation problem for the delineation of the efficient set of portfolios:

\[
\begin{align*}
\text{(5.12)} & \quad \text{Minimise} \begin{bmatrix} L = \text{Var}(\tilde{r}_p) - \lambda\left[ E(\tilde{r}_p) - E(\tilde{r}_p)^* \right] \end{bmatrix} \\
\text{Subject to the following constraints:} & \\
\quad & w_i \geq 0 \forall i \\
\quad & \sum_{i=1}^{n} w_i = 1 + \sum_{i=n+1}^{m+1} w_i \\
\end{align*}
\]

Where $E(\tilde{r}_p)^*$ is a constant and this is to be determined by the investor. As in the previous section, the decision variables in the minimisation problem are the $m+1$ portfolio weights. Solving for different values of $E(\tilde{r}_p)^*$ results in the delineation of the efficient frontier.
5.3 Utility Theory and Portfolio Selection

The previous two sections outline the analytical procedure for deriving the efficient frontier. The actual portfolio selected by an insurer from this identified set of efficient portfolios will depend on the use of a pre-specified insurer decision rule. While there are a number of alternative specifications for this decision rule, the use of utility theory and in particular an expected utility maximisation rule is assumed here.\textsuperscript{11}

In order to arrive at an analytical solution for the insurer operating point on the efficient frontier, further assumptions need to be made regarding the investor’s risk preference. A strictly concave objective function $U(.)$ guarantees a unique solution to the insurer’s portfolio selection problem. The solution is determined by equating the insurer’s marginal rate of substitution (MRS), between return and risk, to the marginal rate of transformation (MRT) defined by the slope along the derived efficient frontier. That is, the point of tangency between the efficient frontier and the insurer’s indifference curve provides a solution to the insurer’s portfolio selection problem. Critically, this ‘optimal portfolio’ will depend upon the degree of insurer risk aversion.

\textsuperscript{11} Another decision-rule that can be used to solve for an ‘optimal portfolio’ is derived from ‘ruin’ theory (see Chapter 3, and Cummins and Nye 1981).
Before proceeding, it is worth reviewing the main properties of an insurer's utility function. Let $U(\tilde{K})$ be an insurer utility function, where as before $\tilde{K}$ is end-of-period wealth, such that:\textsuperscript{12}

(5.13) $\tilde{K} = K_0(1 + \tilde{r}_p)$

The utility function $U(\tilde{K})$ is assumed to be well-behaved, continuous, and twice differentiable. In order to specify risk aversion for the insurer, the function must satisfy the following conditions:\textsuperscript{13}

(5.14) $U'(\tilde{K}) > 0$; and $U''(\tilde{K}) < 0$

A utility function that exhibits the above characteristics implies that the insurer prefers more than less; that is, the insurer exhibits monotonicity of wants. Thus, the insurer has a positive marginal utility of wealth. However, Equation 5.14 demonstrates that this marginal utility is increasing at a decreasing rate showing that, "... investors are more concerned with reducing the incidence of adverse outcomes than they are with increasing the incidence of favourable outcomes of

\textsuperscript{12} Implicit within this development is the assumption that the investment decision-makers have the same preferences as those of the owners of capital.
\textsuperscript{13} Examples of explicit functional forms include the quadratic utility function, the logarithmic utility function and the negative exponential utility function (see Chapter 11). All these functions are consistent with the properties of Equation 5.14 but vary according to their relative and absolute risk aversion. For a full account of the various risk preference properties of utility functions see Booth, et al. (1997).
equivalent magnitude,” (Booth, Chadburn and Ong 1997, 3). In other words, an investor exhibiting risk preferences of this type is said to be risk averse, and would therefore reject a ‘fair’ gamble. The utility function and its resulting indifference curves are illustrated in Figure 5.2 below.

![Figure 5.2. Typical Risk Averse Utility Function (A), with Resulting Indifference Map in (B)](image)

Given appropriate specification of the utility function, it is now possible to define the objective of the insurer. Since the insurer faces uncertainty associated with its objective, namely end-of-period wealth, it must therefore maximise expected utility $E(U)$, viz.

\[
\text{(5.15)} \quad \max_{\{w_i : i = 1, \ldots, m+1\}} \{E[U(K)]\} = \mathbb{E}[U(K_0 + \hat{r}_p K_0)]dF(\hat{r}_p) = \mathbb{E}[U(K_0 (1 + \hat{r}_p))dF(\hat{r}_p)]
\]

Where, $F(\hat{r}_p)$ represents the probability distribution function of the random rate of return on the portfolio, $\hat{r}_p$. There is evidence to suggest that an optimisation
A problem that involves maximising expected utility is (approximately) mean-variance efficient. For a detailed discussion on this issue see Levy & Markowitz (1979). For a more general critique of the expected utility paradigm see Ong (1995). Note that the conditions set out in Equation 5.14 establish that the objective function is at least strongly quasi-concave and hence this satisfies the existence and uniqueness of an optimal solution to the expected utility program (Chiang, 1984).

As illustrated in the previous section, the mean \( E \) and variance \( V \) of end-of-period wealth \( \bar{K} \) are respectively:

\[
\begin{align*}
E(\bar{K}) &= K_0[1 + E(\hat{\mu})]; \text{ and} \\
V(\bar{K}) &= K_0^2V(\hat{\mu})
\end{align*}
\] (5.16)

Using the Taylor expansion of the utility function \( U[K_0 + \hat{\mu}K_0] \), around the initial wealth of the insurer \( K_0 \), yields:

\[
(5.17) \quad U(\bar{K}) = \left\{ \frac{U(K_0)}{0!} + \frac{U'(K_0)}{1!}(\bar{K} - K_0) + \frac{U''(K_0)}{2!}(\bar{K} - K_0)^2 + \ldots \right\}
\]

Note that in addition the following simplification can be made:

\[
(5.18) \quad \bar{K} - K_0 = (K_0 + \hat{\mu}K_0) - K_0 = \hat{\mu}K_0
\]
Using this simplification and taking expectations of Equation 5.17, while neglecting higher order terms (Pratt 1964) yields:

\[(5.19) \ E[U(K_0 + \tilde{\sigma}, K_0)] \approx E[U(K_0) + U'(K_0)(\tilde{\sigma}, K_0) + \frac{1}{2} U''(K_0)(\tilde{\sigma}, K_0)^2] \]

\[\approx U(K_0) + U'(K_0)K_0 \left[ E(\tilde{\sigma}) - \frac{1}{2} \Theta \left[ \left( E(\tilde{\sigma}) \right)^2 + \sigma_p^2 \right] \right] \]

\[\approx U[E(\tilde{\sigma}), V(\tilde{\sigma})] \]

Where \( \Theta = -K_0 \left( \frac{U''(K_0)}{U'(K_0)} \right) \) represents the Arrow-Pratt relative risk aversion parameter. Given this objective function, the marginal rate of substitution along a given indifference curve (between expected return and variance) is derived by taking the total differential of Equation 5.19 with respect to \( E(\tilde{\sigma}) \) and \( V(\tilde{\sigma}) = \sigma_p^2 \), viz.

\[(5.20) \ dE(U) = \frac{\partial E(U)}{\partial (E(\tilde{\sigma}))} dE(\tilde{\sigma}) + \frac{\partial E(U)}{\partial \sigma^2} d\sigma^2 \]

\[0 = \left[ U'(K_0) - \Theta U'(K_0)E(\tilde{\sigma}) \right] dE(\tilde{\sigma}) + \left[ -\frac{1}{2} U''(K_0) \Theta \right] d\sigma^2 \]

\[\therefore\]

\[(5.21) \ \frac{d\sigma_p^2}{dE(\tilde{\sigma})} \bigg|_{E(\tilde{\sigma})} = -\left\{ \frac{U'(K_0) - U'(K_0)\Theta E(\tilde{\sigma})}{-(1/2)U''(K_0)\Theta} \right\} = 2 \left\{ \frac{1}{\Theta} - E(\tilde{\sigma}) \right\} \]
The utility maximisation problem can be illustrated with reference to Figure 5.3 below. The diagram illustrates the tangency point between the investor’s indifference curves and the efficient frontier.

![Figure 5.3. Portfolio Selection and Expected Utility](image)

Thus, optimality requires that the MRS be equal to the objective trade-off between risk and return at a given point along the efficient frontier, viz.

\[
(5.22) \quad \frac{d \sigma^2_f}{dE(r_p)} = 2 \left( \frac{1}{\Theta} - E(r_p) \right)
\]

This result may be interpreted as the price of risk; that is the real trade-off between risk and expected return (Ong 1995).
5.4 Incorporating Insurer Solvency Regulation

The previous sections of this chapter have developed the necessary analytical tools for defining the insurer's set of efficient portfolios and selecting an insurer's 'optimal portfolio'. Attention now turns to addressing the role of regulatory supervision of insurer solvency.

As a point of departure for discussion, it is useful to note some of the other developments within the study of insurer solvency regulation. Much of the work on insurance regulation has tended to concentrate on factors that influence capital decisions under default risk. Using the work of Borch (1981, 1982), Munch and Smallwood (1982) and Finsinger and Pauly (1984) consider the possible loss of goodwill in the event of insurer default and show that optimal capital is positively related to the amount of loss that the shareholders would suffer if claim costs were to exceed the firm's financial assets (Finsinger and Pauly 1984, 165). By contrast, applied research has tended to concentrate on quantifying the effects of solvency regulation, using cross-state data in the United States. For example, Munch and Smallwood (1980, 1982) examine why an unregulated market for property-liability insurance would not automatically produce the efficient level of solvency. Munch and Smallwood demonstrate that, "...minimum capital requirements do reduce the number of insolvencies, but that this is achieved solely by deterring the entry of small, relatively risk free firms," (Munch and Smallwood 1980, 262).
While much can be said about the developments within insurance economics, it is useful now to employ the tools of portfolio theory to elaborate further on the role of solvency regulations. Insurer insolvency may be defined as an event where the insurer’s equity capital is completely eliminated, that is:

\[ \text{prob}[\lambda + \bar{\lambda}_p \leq 0] = \text{prob}[\bar{\lambda}_p \leq -1] \]

There are two very important issues to note with respect to this definition of insolvency. First, implicit in a discussion of solvency is the issue of asset (liability) valuation. Valuation and the assessment of an insurer’s solvency position will be an inexact science and a matter of some debate for the regulator. However, following Kahane, et al. (1988), it will be assumed that the market value is equal to the book value.

The second consideration to be made with regard to Equation 5.23 is that the probability rule will be insurer specific, and hence the insolvency definition is specified for a given insurer, insurer \( i \). Given the idiosyncrasies that will exist between insurers within the industry, it is reasonable to expect that the return characteristics and resulting probability distribution will be different for each insurer. These will in turn depend upon the nature of the cashflows (and their respective probability distributions), the composition of the asset portfolio, and other factors that together affect the insurer’s net cash flow position. A regulatory policy
aimed at reducing the probability of insolvency would therefore need to account for these relationships.

Following Kahane, et al. (1988), if the return on equity is normally distributed, the standardised form of equation 5.23 can be written as follows:

\[
\text{prob}[E[R_p] \leq -1] = \text{prob}\left[\frac{\bar{R}_p - E[R_p]}{\sigma} \leq -1 - \frac{E[R_p]}{\sigma}\right] \leq \rho
\]

Where the term \( \rho \) is defined as, the maximum acceptable insolvency probability.

With the normality assumption that \( \bar{R}_p - N(E[R_p], \sigma) \), the standard normal table can be used to find a constant \( \phi(\rho) \), which fulfils the following:

\[
E[R_p] \geq (-1) - \phi(\rho) \sigma
\]

Where \( \phi(.) \) is the inverse of the cumulative standard normal distribution function. The value of \( \phi(.) \) is always negative since the probability of failure addresses only the lower tail of the distribution (that is, the downside risk (see Chapter 4)). A greater absolute value of \( \phi(.) \), \( |\phi(.)| \) corresponds to a lower probability of insolvency.
for a given insurer (portfolio); for example, when the probability of insolvency is 5
percent, \( \phi(p=0.05) \) will equal -1.645, while for a probability of insolvency of 1
percent, the associated value of \( \phi(p=0.01) \) is equal to -3.1. Thus, a simple
functional relationship can be constructed to represent the insolvency constraint in
expected return (\( E[\tilde{r}_p] \)), and risk (\( \sigma \)) space.

The solvency definition in Equation 5.25 defines a half-plane bounded by a straight
line, the insolvency constraint (Kahane 1978). For a given level of insolvency
probability \( p \), the insolvency (or ruin) constraint can be restated as follows:

\[
E[\tilde{r}_p] = (-1) - \phi(p)\sigma
\]

Equation 5.26 provides the means by which to construct the insolvency constraint
graphically in return/risk space (see Kahane, et al. 1988). It represents a constraint
separating those portfolios that pass the insolvency constraint and those portfolios
that fail the constraint. The constraint has a slope equal to \(-\phi(p)\). The
interpretation of this slope coefficient reflects the assumed level of 'acceptable'
insolvency probability (as distinct from the portfolio risk, \( \sigma \)). Note the importance
of the term 'acceptable'. Typically the analysis of solvency assumes exogenously
the regulatory standard governing the insolvency probability. In practice such a
probability would need to be evaluated in terms of its costs and benefits. The
establishment of such a standard would depend upon a number of factors, including
the trade-off that exists between the need for adequate policyholder protection versus the need to ensure commercial efficiency. Therefore, the relationship in Equation 5.26 may be seen as a means of representing the regulator’s preferences, through the construction of ‘degenerate’ indifference curves for a given level of $\rho$.

This relationship measures downside risk in terms of the ‘safety first/minimum $\alpha$’ criteria (Roy 1952) see Chapter 4.

The above points can be illustrated though the use of a diagram, see Figure 5.4. In this diagram the efficient frontier is illustrated in conjunction with two insolvency constraints, $L^*$ and $L^{**}$ reflecting two different levels of ‘acceptable’ insolvency probability, $\alpha^*$ and $\alpha^{**}$ respectively where $\alpha^* < \alpha^{**}$.

Figure 5.4. The Probability of Insolvency

Assume that for whatever reason, the regulator settles for a solvency standard of $\alpha^*$. Thus, portfolios on the frontier to the right of the intersection between the $L^*(\alpha^*)$ ray and the efficient frontier (represented by the point X) are not permissible according to this solvency standard. Conversely, insurance companies represented by portfolios located on the efficient frontier to the left of point X may be classified as being ‘solvent’ for the purposes of regulatory review and intervention assessment.

Having outlined the procedure for deriving the efficient frontier, the means of selecting an operating point on that frontier, and the means of representing ‘degenerate’ regulatory preferences in the form of a solvency constraint, the full model can now be illustrated, see Figure 5.5 below.

Figure 5.5. Portfolio Selection within a Regulatory Setting

In Figure 5.5, the efficient frontier is presented with two alternative operating points at $u^*$ and $u^{**}$, which represent the points of tangency between the insurer’s indifference curves and the efficient frontier. The insolvency constraint intersects the efficient frontier at point $x$. An insurer located at point $u^*$ would be classified as ‘solvent’ while an insurer operating at point $u^{**}$ would be classified as ‘insolvent’. Thus, the efficiency frontier can be differentiated into two sets of strategies: namely, ‘permissible’ and ‘non-permissible’ strategies (Kahane, et al. 1988, 225).

5.5 Solvency Regulation in Terms of Insurance Leverage

Frequently cited as a mechanism for controlling the insolvency risk of insurers, is the regulation of insurance leverage (work in this field dates back to the work of Kahane (1978)). Insurance leverage is defined as the amount of premiums written per pound of equity (Kahane 1978). The portfolio model developed in the previous section can be represented in a slightly different form, modelling the insurer explicitly in terms of its insurance leverage.

In the previous section, it was assumed implicitly that the degree of insurance leverage was constant. To illustrate this, discussion will proceed following Kahane, et al. (1988). Assume an insurer is engaged in only one insurance line and has a single investment opportunity. As before, the profit for the period is denoted by $\pi$. A simplified model of insurer profit can be constructed by making profit a linear combination of the profit received from underwriting and investment activities, viz.
Where $\bar{U}$ and $\bar{I}$ represent the total underwriting and investment profit respectively. Assume that there is an indicator of insurer activity $P$, and that assets are defined by $A$. As noted in the previous sections, $P$ can be typically actuarial reserves or premiums. Kahane, et al. (1988) assume that insurance activity is defined in terms of total premiums. Note the contrast with the multi-line insurer model of Section 5.3, which used actuarial reserves as an indicator function of insurer activity. However, as demonstrated shortly, reserves and premiums can be linked through a simple relationship.

Discussion will return to the reserves shortly, but for now assume that the rate of return on underwriting activities $\bar{r}_1$ (in this simplified model) is a function of the premiums written and is given by:

\begin{equation}
(5.28) \quad \bar{r}_1 = \bar{U} / P
\end{equation}

The rate of return on investment activities is given by $\bar{r}_2$ and is defined as follows:

\begin{equation}
(5.29) \quad \bar{r}_2 = \bar{I} / A
\end{equation}
Thus, the rate of return on equity $\tilde{r}_p$, is now defined as:

\begin{equation}
\tilde{r}_p = \frac{P}{E_0} \tilde{r}_1 + \frac{A}{E_0} \tilde{r}_2
\end{equation}

Where, as before, equity capital is given by $E_0$. In this example, the reserves are assumed to be equal to a certain multiple, $f$ of the premiums; this is the reserve 'funds generating factor' where each pound of premiums generates $f$ pounds of reserves. Hence, the balance sheet constraint is now given by:

\begin{equation}
A = E_0 + fP
\end{equation}

The rate of return on equity may now be derived from Equations 5.30 and 5.31 as follows:

\begin{equation}
\tilde{r}_p = k \tilde{r}_1 + (1 +fk) \tilde{r}_2
\end{equation}

Where $k$ denotes the amount of insurer activity, in this case premiums written, per pound of equity ($k=P/E_0$), and this variable is referred to in the literature as "insurance leverage" (as distinct from more general financial leverage). The advantage of the formulation given in Equation 5.32 is that the only decision variable is the insurance leverage, $k$. Thus, the expected return and risk faced by the insurer can be defined exclusively in terms of $k$, viz.
\[ E(\tilde{r}_p) = kE(\tilde{r}_1) + (1 + fk)E(\tilde{r}_2), \text{ and} \]
\[ Var(\tilde{r}_p) = k^2Var(\tilde{r}_1) + (1 + fk)^2Var(\tilde{r}_2) + 2(1 + fk)kCov(\tilde{r}_1, \tilde{r}_2) \]

Equation 5.33 defines the opportunity set for all possible combinations of expected return and risk through varying the degree of insurance leverage, \( k \). In order to extract the influence of \( k \) upon the shape of the frontier it is helpful to re-write Equation 5.33, factoring out \( k \) as follows:

\[ E(\tilde{r}_p) = E(\tilde{r}_2) + k\left[ fE(\tilde{r}_1) + E(\tilde{r}_2) \right] \text{ and,} \]
\[ Var(\tilde{r}_p) = 2fCov(\tilde{r}_1, \tilde{r}_2)k^2 + \left[ 2fVar(\tilde{r}_2) + 2Cov(\tilde{r}_1, \tilde{r}_2) \right]k \]

From Equation 5.34 it can be seen that \( E(\tilde{r}_p) \) increases with insurance leverage \( k \) if \( fE(\tilde{r}_1) \geq -E(\tilde{r}_2) \). Also, the variance is of quadratic form, viz.

\[ Var(\tilde{r}_p) = ak^2 + bk + c \text{, with:} \]
\[ \frac{dVar(\tilde{r}_p)}{dk} = 2ak + b = 0 \Rightarrow k = -\frac{b}{2a} \text{ and} \]
\[ \frac{d^2Var(\tilde{r}_p)}{dk^2} = 2a > 0 \Rightarrow \text{min if } a > 0 \]

As it is reasonable to expect a positive covariance between the rates of return on underwriting and investment activities (Kahane, et al. 1988), one would also expect
that the second derivative of the variance expression to be positive, hence implying that efficient frontier must be concave.\textsuperscript{14}

The concept of insurance leverage can be used to extend the insurance model in previous sections of this chapter. As noted earlier in the multiple contracts/multiple assets portfolio model (Section 5.2 and Section 5.3), use is typically made of actuarial reserves as a measure of underwriting activity (Krinsky 1985). Equation 5.31 provides the means of relating the portfolio model of previous sections with the insurance leverage model in this section. Note that total reserves in this section are defined as $fP$ and this is equivalent to $\sum_{i=n+1}^{m} l_i$ in Section 5.2, for a given fund generating factor $f$. Recall that the delineation of the insurer's efficient frontier, for a multiple contracts/multiple assets portfolio model, may be expressed as follows:

\begin{equation}
(5.36) \quad \text{Minimise } \left\{ L = V(\bar{r}_p) - \lambda \left[ E(\bar{r}_p) - E(\bar{r}_p)^* \right] \right\}
\end{equation}

Subject to

\[ w_i \geq 0, \forall i \]

\[ \sum_{i=1}^{m+1} w_i = 1 \]

\textsuperscript{14}The shape of the efficient frontier will depend upon the exact nature of the relationship between expected return and insurance leverage (Kahane 1978).
Where the expected return on the insurer’s portfolio $E(\tilde{r})^*$, is a constant to be specified and the decision variables in the minimisation problem are the portfolio weights, $w_i$. The solvency regulation of insurance leverage is introduced explicitly into the quadratic program as follows. The optimisation problem in Equation 5.36 is solved subject to a constraint imposed that keeps the insurance leverage $k$ at a constant level, $k_0$. The role of insurance leverage can be incorporated into the analysis by making the following developments. In the multiple contracts/multiple assets case, the sum $\sum_{n+1}^m w_i$ is a proxy for the amount of insurer business (reserves) written for each contract, per-pound of equity. Thus, the insurance leverage constraint is of the following form:

$$(5.37) \sum_{n+1}^m w_i = k_0$$

This new constraint is imposed upon the quadratic program in Equation 5.36 to generate a different efficient frontier for each level of leverage, $k_0$. Repeating the procedure for different values of $k_0$ generates a set of efficient frontiers that together trace out an envelope curve, which in turn is tangential to these respective frontiers (Kahane and Nye 1975). The envelope frontier is illustrated in Figure 5.6, where a movement along the envelope curve is consistent with a change in the level of insurance leverage.
PAGE
MISSING
IN
ORIGINAL
Figure 6.2. New Ordinary Life Business in the U.K., 1986-1996


Figure 6.2 illustrates the rapid growth in single premium business, and also demonstrates that regular premium business has remained relatively constant over the same period. There are a number of points to note with respect to the trends in single and regular premium business. Almost half of new annual business came from mortgage related products (Association of British Insurers 1997b). As a consequence, the depressed state of the housing market in the early 1990s significantly affected the generation of new annual premium business. The role of the housing market is confirmed by the fact that by 1996 there was an upturn in regular premium business, a move consistent with the upturn in the housing market. Thus, the general state of the economy provides a ‘demand’ driven explanation for the relatively poor performance of regular premium contracts.

The only exception to this trend came after the 1987 stock market crash, which adversely affected the sale of single premium policies, (funds generated from single premium plans are typically invested heavily into the equity market).
With respect to the single premium contracts, it should be noted that the growth in single premium contracts is largely a result of the relatively high performance of life savings contracts such as the ‘with-profit’ bond. The performance advantage of these contracts may be explained by the fall in interest rates since 1992, which lead to higher returns being offered by insurers as compared with those returns offered by deposit accounts at banks and building societies (Association of British Insurers 1997b, 13). Thus, there is a performance argument for the success of single premium contracts.

While much can be said about the relatively strong performance of single premium contracts, it is important to note the reason why single premium contracts make up the vast majority of new business income. In terms of the generated funds for a given year, new single premium contracts represent a greater source of new income as they come in the form of a lump sum investment. Such income is therefore comparable to the funds of annual premium policies over the entire term of the contract.

6.4 Basic Life Assurance Products

A review of the various life policies available in the U.K. is now developed. There are six basic types of business that life assurance companies engage in: term assurance, endowment assurance, whole of life assurance, permanent health insurance, and annuities and pensions. The latter category of policies is a relatively new area of development for the life assurance industry, the former five classes making up the traditional business of ‘ordinary’ life assurance.
Products sold by life insurers may be defined in terms of three basic functions: life cover, sickness protection, and a means of savings.\textsuperscript{6} The costs associated with life assurance contracts vary significantly; these costs reflect the extent to which a given contract employs each of the three basic functions. For this reason, it is worth noting the treatment of policy charges within basic life contracts.

Expenses charged on life assurance plans aim to meet three costs: first, the cost of mortality; second, the expenses associated with a contract; and third, investment management costs. Each cost is evaluated in terms of the respective risks facing the policyholder and insurer. Mortality cost is readily calculated from life tables and actuarial assessments. 'Expenses' is a generic term used to cover a variety of insurer costs both fixed and variable. There are also product specific charges that relate to the investment management of life funds.

Frequently misunderstood is the interplay that occurs between the three cost categories. For any given premium, the respective charges will vary during the term. Investment management charges, for example, are typically charged at the beginning of the policy term. Thus, charges of this kind are typically 'front-end loaded' and the percentage of the premium devoted to other functions, such as savings, is correspondingly reduced at the start of the term. As the term of the

\textsuperscript{6} As noted earlier, policies that contain the savings element are referred to as substantive contracts.
plan progresses these ‘front-end loaded’ charges decline whilst the amount of the premium devoted to savings correspondingly increases.\[^7\]

Having explained the charges associated with life contracts, it is now useful to consider the basic product variants. The basis of any life assurance contract is term assurance. Term assurance pays a lump sum only on the death of the life assured within the term of the contract. The plan has no savings element, nor does it provide any protection against illness. The premiums paid go towards providing life cover only and therefore this contract may be expected to be one of the cheapest life contracts available on the market.

Term assurance contracts come in many forms including decreasing term and increasing term contracts. Decreasing term assurance plans are designed for capital repayment mortgages where the amount of protection required reduces over the term (note that the premiums remain constant).\[^8\] By contrast, increasing term assurance plans allow for the sum assured to be increased during the term up to a certain limit without any additional medical questions, (with the premiums revised accordingly).

Endowment policies also pay out a lump sum on the death of the life assured. However, their usefulness lies in their application as a savings device. ‘Endowments’ have become widely used as a means of repayment for

\[^7\] There are many justifications for ‘front-end loading’. The most likely justification is that such a plan will encourage saver loyalty and reduce policy ‘surrenders’ for cash before the end of the term. Charges of this kind make the returns on life assurance particularly small at the start of a policy’s term.

\[^8\] Repayment mortgages, or capital and interest mortgages, calculate a monthly payment that covers the interest of the loan and makes a partial repayment on the capital borrowed.
mortgages. Thus, while life cover is afforded to the life assured, the majority of
the premium paid contributes to the development of the policyholder’s savings
fund.

Another basic life contract is the whole of life policy. Whole of life contracts
have the primary purpose of providing the policyholder with a substantial level
of life coverage, but unlike pure term assurance, premiums do include a savings
element. The ratio of life cover to savings is variable depending on the needs of
the policyholder. Also, a whole of life policy provides a level of protection to
the life assured for the lifetime of the policyholder; there is no policy term.

More recently, many life insurers have sold sickness protection. This contract
offers a benefit (either in the form of a regular payment or lump sum) to the
person assured, dependent on his or her state of health. One of the main forms
of sickness protection is that of Permanent Health Insurance (PHI). PHI is
designed to replace the loss of income for an individual who is unable to work
(for more than a pre-specified period of time) due to illness or accident.
Premium income generated from PHI equalled £301 million in 1996
(Association of British Insurers 1997b, 13). Another main form of sickness
protection is that of Critical Illness Cover (CIC) which pays out a lump sum on
the diagnosis of a particular illness. CIC has been referred to as ‘life assurance
for the living’ (Association of British Insurers 1997b, 16).

Annuities are an additional product offered by life companies. The importance
of annuities comes from the fact that they are typically used as insurance against
'longevity risk'- the risk of extinguishing savings due to living longer than expected. Annuities are a lump sum investment which is used (invested by the insurer on behalf of the policyholder) to produce benefits usually in the form of a regular payment; in effect a premium paid from the insurer to the life assured. Annuities come in two main forms: immediate and deferred. Immediate annuities give the policyholder an immediate income once the premium is paid. By contrast, a deferred annuity may be bought with either a single premium or a regular premium over a number of years in advance of the required annuity income. The ability to sell annuities represents a competitive advantage for life insurers since banks and building societies are not permitted to sale annuities, except through insurance company subsidiaries (Association of British Insurers 1997b).

Related to annuity business is pension business. There are numerous and varied pension contracts available to both private individuals and organisations. Insurance companies are involved in all forms of pension provision. Insurers provide private pensions to employees and the self-employed. They also provide occupational pension schemes where an employer wishes to offer its employees a pension scheme. The employer might also agree to retain an insurance company solely for the purposes of being a fund manager; such a pension scheme is referred to as 'self-administered'. Private pensions and occupational

---

9 Typically, annuities are bought by pension funds.
10 Note that pension contracts sold to institutions by insurance companies are distinct from employer managed occupational pension schemes, which are established and managed by an employer itself for the benefit of some or all of its employees.
11 Depending on the function and size of the employer, it may choose to set up and manage a pension scheme itself.

Pension schemes offered by insurance companies may form the entire provision for retirement or may be used in conjunction with established occupational pension schemes.12 Private pensions allow individuals to retire as early as 50, and take advantage of a tax-free lump sum that can be taken from the proceeds of the pension fund. This arrangement has also lead to the introduction of the so-called 'pension mortgage' whereby the tax-free lump sum is used as a means of repayment for the outstanding balance on a mortgage.

Additional products used for retirement provision are the 'free-standing' additional voluntary contributions. Additional voluntary contributions (AVCs) are a special type of pension arrangement that is available only to employees of organisations who are members of an occupational pension scheme. These plans aim to 'top-up' the retirement funds and are 'free-standing' because the insurance company offers the product, rather than the employer.

Figure 6.3 provides a product breakdown of the new yearly premiums for individual life business in the U.K. over the period 1984-1996. The two most prominent categories of new business are ordinary life assurance contracts and personal pension contracts.

---

12 The selling of private pensions has been the source of regulatory review. Insurance advisors have been criticised for advising that employees 'opt-out' of occupational pensions using private pension plans instead. Such actions resulted in significant losses to consumers and appear to have been justified only on the grounds of greater sales commission.
Figure 6.3. New Yearly Premiums for Individual Long-Term Business in the U.K.

With reference to Figure 6.3, it can be seen that personal pensions have expanded since the mid-1980s with new premiums rising three-fold in the period under discussion. By 1996, total personal pension contributions amounted to £972 million, just short of 40 percent of new yearly premiums. The downturn in new pensions business between 1992 and 1995 may be explained by two factors: first, the economic climate of the time; and second, the controversy and associated negative publicity surrounding the sale of personal pensions in the late 1980s.

There are a number of additional points to note with respect to Figure 6.3. Industrial business has declined, while the introduction of new products, such as
Personal Health Insurance (PHI) and Critical Illness Cover (CIC), has provided the potential for additional growth in new long-term business. PHI business grew by more than 25 percent in the last year of the period under discussion, and the potential for this relatively new line of life business, together with CIC, would seem substantial. Additional Voluntary Contributions have also provided a significant source of new business with income standing at £54 million by 1996.

It should be noted that the contributions of individual product lines to single premium business shows that pension business and ordinary life assurance business are the dominant products. In 1996 pension and ordinary life business together accounted for nearly 90 percent of single premium business (Association of British Insurers 1997b, 13).

The introduction of new products can be seen to play a vital role in maintaining new business income. The motivation for new products can be seen to depend critically on two factors, the degree of competition, and the changes to tax legislation. In particular, tax legislation has been instrumental in the growth of personal pensions and ‘AVCs’ (Association of British Insurers 1997b).

In addition to the basic life products, substantive policies (such as endowment policies) may come in two forms: ‘with-profit’, and ‘unit-linked’. These terms are used to define the investment basis of substantive plans. Traditionally, substantive policies have been invested into the ‘life fund’. If a policy were
with-profit then policyholders would buy a stake in any insurer profit, where the reward for this stake would be in the form of policy bonuses.\textsuperscript{13}

Policy bonuses attached to ‘with-profit’ contracts come in two forms, reversionary (or annual) and terminal. Annual bonuses are attached to the policy each year, based on the performance of the insurer in that year and usually are guaranteed once declared. At the end of the policy term, all the accumulated bonuses are assessed and combined with one final bonus, the ‘terminal’ bonuses. Typically the terminal bonus is much larger than the individual reversionary bonus so that there is an incentive for the policyholder to keep a contract in effect until the end of the agreed term.\textsuperscript{14}

Recently, there has been a growth in the use of unit-linked policies. Under these contracts, premiums are used to purchase directly units in one of the life insurer’s investment funds. These units represent a share in the fund and its price reflects the value of the assets within the fund. Values of the unit-linked funds can go down as well as up, and in order to reduce this risk a variety of unit-linked funds are typically offered with varying degrees of associated risk. The premiums paid to the insurer can go into any number of unit-linked funds and the proportion invested in these funds may be changed on demand (though this may incur a minor ‘switch’ fee).

\textsuperscript{13} Profits relate to the performance of a proprietary life company. A mutual life insurer would have its performance measured in terms of life fund surplus.

\textsuperscript{14} In other words, terminal bonuses aim to keep surrender values down and thereby reduce the incentive to cash-in a policy.
As noted earlier, all unit-linked funds differ in terms of their associated risk profiles. Standard unit-linked funds include the following: a cash fund, where premiums paid go into the short term money markets, (bank deposits and treasury bills); an ‘equity fund’, which specialises in ordinary shares invested principally in the U.K.; and a ‘fixed-interest fund’, where premiums are invested in government stock (including overseas government stocks). In addition, one of the most commonly used unit-linked funds is that of the ‘managed fund’. This fund appeals to those policyholders not wishing to control the day-to-day investment decisions and would rather delegate the responsibility to an insurance company’s investment manager. The managed fund is typically sold as a relatively low-risk fund since it is usually well diversified.

The growth in the popularity of unit-linked policies may be explained in the context of a wider share ownership in the U.K. The investment growth of unit-linked funds is widely acknowledged as being greater than its ‘with-profit’ counterpart (Association of British Insurers 1997b). In addition, unit-linked policies are seen as being far more flexible in order to meet ever-changing policyholder needs. However, there is an important caveat to all of this, namely that of the associated investment risk. While the growth in the ‘with-profit’ funds is guaranteed once declared, the unit-linked plans carry no such assurance. For the risk-averse, the choice would be to invest in the life fund every time, and therefore select a with-profit policy where the variance of return is smaller than that of a unit-linked plan and the accumulation of the investment funds is assured.
The significance of unit-linked policies can be appreciated by inspecting Figure 6.4 below. This outlines the respective proportions of regular premium business invested in unit-linked funds, for ordinary life business, personal pension business, and for ‘free-standing’ AVCs.

Figure 6.4. The Proportion of New Yearly Linked-Life Business


Figure 6.4 shows that there has been a positive trend in the proportion of new annual business being invested into unit-linked plans over the period 1984-1996. This growth was checked by the disturbances to the stock markets in 1987. However in spite of this, the majority of personal pensions and ‘free-standing’ AVCs in 1996 were invested into linked funds. The amount of ordinary life business being invested into unit-linked funds has remained relatively constant, standing at about 40 percent in 1996. The contrast in linked-life proportions, between pensions and ordinary life assurance, may be explained by differences in policyholder risk preference. It is likely, for example, that those individuals purchasing ‘top-ups’ to retirement plans may be willing to take on the greater
risk associated with the stock market than those policyholders who are investing in a mortgage repayment plan for the first time. Thus, risk preference can be seen to play an important role in the explanation of the relative proportions invested into unit-linked funds.

In general, the life assurance industry can be seen to offer products that are flexible and dynamic. Moreover, the changing structure of the market for life assurance can be seen as a response, in part, to the new requirements arising from changing social and economic conditions. Also, such policies have been well adapted to meet changing tax (and social security) regulations (Association of British Insurer 1997b).

6.6 Overseas and Domestic Markets for Life Assurance

In 1995, global gross insurance premiums were £1,360 billion, (Association of British Insurers 1997a, 31). This represented a real growth in worldwide premium income of 4 percent over the previous year. The previous four years had also seen real growth in excess of this figure. While there has been an overall growth in the world insurance market there has also been substantial regional variation in insurance market growth rates (Association of British Insurers 1997b).

Table 6.4 shows the growth experience of the principle regions of the world. This table also presents the different regional shares of the global economy for insurance.
Table 6.4. Share of the Global Economy

<table>
<thead>
<tr>
<th>Region</th>
<th>1993</th>
<th>1995</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asia</td>
<td>35 (6)</td>
<td>35 (8)</td>
</tr>
<tr>
<td>North America</td>
<td>33 (5)</td>
<td>31 (1)</td>
</tr>
<tr>
<td>Europe</td>
<td>27 (8)</td>
<td>30 (3)</td>
</tr>
<tr>
<td>Oceania</td>
<td>1 (6)</td>
<td>2 (-14)</td>
</tr>
<tr>
<td>Latin America</td>
<td>1 (6)</td>
<td>1 (3)</td>
</tr>
<tr>
<td>Africa</td>
<td>1 (1)</td>
<td>1 (17)</td>
</tr>
</tbody>
</table>


Note: Figures in brackets give the real gross-premium annual growth rate (%).

With respect to Table 6.4, there are number of unusual features to note. Africa and Asia have growth rates in excess of the average rate, at 8 percent and 17 percent, respectively. By contrast, the other markets, especially North America and Europe, have reported sluggish growth. This may be explained, in part, by the state of the respective economies. Moreover, recent financial conditions in Asia would probably change the above results quite markedly. In terms of global market share North America, Europe and Asia dominate global premiums, together these regions account for over 90 percent of the global economy.

Out of all of the results, the African result seems a little surprising. Although the absolute size of the African market is small, the growth rate is significant in spite of the many underwriting risks that present themselves and would appear to make prospective policyholders virtually uninsurable. These risks include the uncertainty surrounding the political and economic climate, and health factors such as the spread of HIV and AIDS. However, this rather paradoxical result might be explained in terms of regional growth 'pockets' (such as South Africa) where these risks are reduced. A detailed breakdown of the regional
concentration of insurance business might therefore provide some clarity regarding this issue.

In order to gain an appreciation of the importance of individual country markets to the overall world insurance market, Table 6.5 sets out the ten largest insurance markets in terms of their gross premium income.

Table 6.5. Ten Largest Insurance Markets in 1995

<table>
<thead>
<tr>
<th>Country</th>
<th>Gross Premium Income (£m)</th>
<th>% share of worldwide premium income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan (1)</td>
<td>404</td>
<td>30 1993 30 1995</td>
</tr>
<tr>
<td>USA (2)</td>
<td>395</td>
<td>31 1993 29 1995</td>
</tr>
<tr>
<td>Germany (3)</td>
<td>98</td>
<td>6 1993 7 1995</td>
</tr>
<tr>
<td>France (4)</td>
<td>83</td>
<td>5 1993 6 1995</td>
</tr>
<tr>
<td>UK (5)</td>
<td>81</td>
<td>6 1993 6 1995</td>
</tr>
<tr>
<td>S. Korea (6)</td>
<td>38</td>
<td>2 1993 3 1995</td>
</tr>
<tr>
<td>Italy (7)</td>
<td>24</td>
<td>2 1993 2 1995</td>
</tr>
<tr>
<td>Canada (8)</td>
<td>24</td>
<td>2 1993 2 1995</td>
</tr>
<tr>
<td>Netherlands (9)</td>
<td>22</td>
<td>2 1993 2 1995</td>
</tr>
<tr>
<td>Switzerland (10)</td>
<td>20</td>
<td>1 1993 2 1995</td>
</tr>
</tbody>
</table>

Note: 1995 ranking given in brackets.

The above table illustrates the dominance of the United States and Japan in the worldwide insurance market- these countries account for around 60 percent of worldwide income. The U.K. is ranked fifth (accounting for 6 percent of worldwide premiums) and is of a similar size to the number third and fourth ranked countries, Germany and France respectively. In order to provide an account of the importance of life business to the ten largest countries, Table 6.6 provides details of the level of premiums per person; the so-called ‘insurance

---

15 The above figures are subject to currency conversions and therefore exchange rate performance. This is a critical factor to be mindful of in the light of the Asian crisis in 1997/98 and the recent growth in the strength of sterling.
This provides a means of comparison for insurance expenditures across a number of different countries.


<table>
<thead>
<tr>
<th>Country</th>
<th>1993 (£)</th>
<th>1995 (£)</th>
<th>% Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td>2300</td>
<td>2580</td>
<td>12%</td>
</tr>
<tr>
<td>USA</td>
<td>610</td>
<td>640</td>
<td>5%</td>
</tr>
<tr>
<td>Germany</td>
<td>380</td>
<td>480</td>
<td>56%</td>
</tr>
<tr>
<td>UK</td>
<td>820</td>
<td>680</td>
<td>-17%</td>
</tr>
<tr>
<td>France</td>
<td>660</td>
<td>910</td>
<td>38%</td>
</tr>
<tr>
<td>S. Korea</td>
<td>450</td>
<td>660</td>
<td>47%</td>
</tr>
<tr>
<td>Italy</td>
<td>110</td>
<td>160</td>
<td>45%</td>
</tr>
<tr>
<td>Canada</td>
<td>380</td>
<td>350</td>
<td>-8%</td>
</tr>
<tr>
<td>Netherlands</td>
<td>600</td>
<td>770</td>
<td>28%</td>
</tr>
<tr>
<td>Switzerland</td>
<td>1230</td>
<td>1830</td>
<td>49%</td>
</tr>
</tbody>
</table>


Note: All figures are converted to sterling using the annual average exchange rate.

With reference to Table 6.6, it can be seen that the expenditure on life assurance in 1995 varied significantly across the countries. The largest per-capita spending on life assurance was in Japan (£2,580) as compared with the smallest per capita spending in Italy (£160). In terms of growth, France (38 percent), South Korea (47 percent) and Switzerland (49 percent) posted the largest growth rates, while the U.K. and Canada were alone in having their per-capita spending actually fall by 17 percent and 8 percent, respectively. While the Far East presents a great potential in terms of market growth, the possibilities for expansion have remained heavily constrained by the recent economic difficulties experienced within the region.

---

16 Comparison across countries must be checked by the fact that the public/private provision ratio for benefits, such as pensions, will vary.
The U.K., especially the city of London, has had a long tradition of insurance provision. The development and expansion of the British Empire in the nineteenth century enabled insurers to establish themselves in a number of overseas markets. Today, the U.K. life assurance industry remains strong with the majority of U.K. market share belonging to U.K. controlled insurance companies. Figure 6.5 illustrates the sector shares of foreign controlled companies for new yearly premiums and new single premiums.

Figure 6.5. Foreign-Controlled Insurers' Share of the U.K. Life Market


Another area of interest concerns the contribution of overseas markets to the growth of U.K. insurers. This contribution remains strong with overseas net premium income standing at nearly £14 billion in 1996, approximately 20 percent of long-term business for life insurers in the U.K. (Association of British Insurers 1997b, 8). The significance of the U.K. insurers within overseas markets is difficult to determine since data needs to be collected from those respective overseas markets. However, it is possible to examine the source of

235
U.K. insurer premium income by country of origin. Table 6.7 provides details on this overseas net premium business.


<table>
<thead>
<tr>
<th>Year</th>
<th>EU</th>
<th>Rest of Europe</th>
<th>Canada</th>
<th>USA</th>
<th>Australia &amp; New Zealand</th>
<th>Africa</th>
<th>Other</th>
<th>Overseas Income as % of Total UK Business</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td>(f)</td>
<td>(g)</td>
<td>(h)</td>
</tr>
<tr>
<td>1986</td>
<td>37</td>
<td>2</td>
<td>26</td>
<td>6</td>
<td>21</td>
<td>4</td>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>1987</td>
<td>34</td>
<td>2</td>
<td>17</td>
<td>17</td>
<td>21</td>
<td>1</td>
<td>3</td>
<td>16</td>
</tr>
<tr>
<td>1988</td>
<td>28</td>
<td>4</td>
<td>18</td>
<td>27</td>
<td>20</td>
<td>1</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>1989</td>
<td>28</td>
<td>5</td>
<td>18</td>
<td>25</td>
<td>21</td>
<td>1</td>
<td>2</td>
<td>17</td>
</tr>
<tr>
<td>1990</td>
<td>30</td>
<td>5</td>
<td>18</td>
<td>28</td>
<td>17</td>
<td>1</td>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>1991</td>
<td>30</td>
<td>6</td>
<td>16</td>
<td>29</td>
<td>16</td>
<td>1</td>
<td>2</td>
<td>17</td>
</tr>
<tr>
<td>1992</td>
<td>31</td>
<td>6</td>
<td>13</td>
<td>32</td>
<td>15</td>
<td>1</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>1993</td>
<td>32</td>
<td>6</td>
<td>14</td>
<td>28</td>
<td>15</td>
<td>1</td>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>1994</td>
<td>36</td>
<td>7</td>
<td>8</td>
<td>26</td>
<td>17</td>
<td>1</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>1995</td>
<td>41</td>
<td>6</td>
<td>9</td>
<td>25</td>
<td>12</td>
<td>1</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>1996</td>
<td>42</td>
<td>4</td>
<td>7</td>
<td>28</td>
<td>11</td>
<td>1</td>
<td>7</td>
<td>21</td>
</tr>
</tbody>
</table>


Notes: Columns (a) to (g) give the percentage contribution of the relevant country or region to total overseas business. Column (h) gives the total overseas contribution to life business in the U.K. as a percentage. Percentages are rounded to the nearest percentage point. Also, the figures for Africa prior to 1995 (column (f)) relate to South Africa.

With reference to Table 6.7, there appears to be a number of trends in overseas markets that are noteworthy. Consider first the ranking of the groups. By 1996, the largest contributor to overseas net premium income came from the European Union, which accounted for approximately 42 percent of overseas business.

This is no surprise given the political and economic changes that have occurred in the last decade. The market shares of the remaining groups are (with the relevant percentages being given in parenthesis): USA (28), Australia and New Zealand (11), Canada (7), the rest of Europe (4), and Africa (1). ‘Others’

12 Out of the many changes that have been brought about, the changes that have been particularly instrumental in encouraging growth are the various insurance reforms that have been aimed at harmonising insurance regulations in the E.U.
accounted for about 7 percent. Most markets, other than the USA, have seen a decline in their contributions to U.K. overseas business. For example the decline in business in Australia and New Zealand is significant. For this category, overseas income fell by around 50 percent over the period of concern. In spite of this pattern, total overseas business has continued to rise from 15 percent in 1986 to 21 percent by 1996. This is accounted for largely by the strong overseas performance in the European Union as well as in the USA.

A breakdown of the contributions made by the respective European Union countries to the U.K. long-term premium income is given in Table 6.8. This table details the country share within the E.U. overseas market as a whole. In order to place these proportions within some context the amount of net-premium income generated by the E.U. as a whole is also provided for comparison.

<table>
<thead>
<tr>
<th>Year</th>
<th>Total Net Premium Income from the E.U. (£m)</th>
<th>France</th>
<th>Netherlands</th>
<th>Republic of Ireland</th>
<th>Germany</th>
<th>Denmark</th>
<th>Spain</th>
<th>Italy</th>
<th>Belgium</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>1193</td>
<td>12</td>
<td>35</td>
<td>31</td>
<td>16</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1987</td>
<td>1378</td>
<td>13</td>
<td>32</td>
<td>34</td>
<td>15</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1988</td>
<td>1372</td>
<td>16</td>
<td>34</td>
<td>27</td>
<td>13</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1989</td>
<td>1797</td>
<td>19</td>
<td>34</td>
<td>31</td>
<td>10</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1990</td>
<td>1995</td>
<td>22</td>
<td>35</td>
<td>27</td>
<td>9</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1991</td>
<td>2366</td>
<td>23</td>
<td>34</td>
<td>25</td>
<td>9</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1992</td>
<td>2800</td>
<td>26</td>
<td>23</td>
<td>21</td>
<td>10</td>
<td>7</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>1993</td>
<td>3022</td>
<td>23</td>
<td>30</td>
<td>20</td>
<td>11</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>1994</td>
<td>3881</td>
<td>34</td>
<td>26</td>
<td>15</td>
<td>10</td>
<td>8</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>1995</td>
<td>4654</td>
<td>39</td>
<td>23</td>
<td>12</td>
<td>10</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>1996</td>
<td>5718</td>
<td>48</td>
<td>18</td>
<td>11</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>


Note: Columns (b) to (j) expressed as a percentage of the total European Union business in each year that contributes to the U.K. premium income (see column (a)). Total net premium income from the E.U. countries is given in column (a).
Table 6.8 confirms the significant growth in net premium income from the E.U., 1986-1996. In the last year of the period under discussion income grew by around 22 percent over the previous year. Within the context of a rising premium income from the E.U., France has seen its share increase drastically. In 1986, France accounted for 12 percent of E.U. income, yet by 1996 this amount had increased four-fold to 48 percent. Against this background, there has been a drastic fall in the contributions made by the Netherlands, the Republic of Ireland and Germany. These respective countries have seen decreases in their market shares of between one-third and one-half over the period. By contrast, Spain and Italy have seen moderate increases in their contributions, with both their respective market shares rising to 4 percent of E.U. premium income in 1996.

While much can be said for the significance of the overseas market as a whole, it is important to stress the role of overseas markets to individual U.K. life insurers. What becomes clear is that fact that the life earnings of U.K. life insurers are becoming increasingly dependent upon the performance of overseas markets. In addition, the growth of insurance companies has occurred largely as a result of expanding overseas markets. In 1996, the number one ranked life insurer in terms of world wide life business received overseas premium income of £4.2 billion (see Table 6.9). This amount had grown 59 percent over a previous 4-year period.

Growth in overseas markets has remained strong for many other large insurance companies. Table 6.9 summarises the contribution of overseas earnings within a context of growing world life assurance premium income. This table
demonstrates that for U.K. life insurers, overseas life earnings account for a significant volume of their total life business; for example, four of the insurers have overseas income accounting for more than 40 percent of their total income.


<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Prudential (1)</td>
<td>39</td>
<td>35</td>
<td>43</td>
<td>42</td>
<td>43</td>
</tr>
<tr>
<td>Commercial Union (3)</td>
<td>65</td>
<td>69</td>
<td>76</td>
<td>82</td>
<td>79</td>
</tr>
<tr>
<td>Royal &amp; Sun Alliance (7)</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>38</td>
</tr>
<tr>
<td>Standard Life (2)</td>
<td>21</td>
<td>21</td>
<td>15</td>
<td>22</td>
<td>19</td>
</tr>
<tr>
<td>Norwich Union (4)</td>
<td>32</td>
<td>26</td>
<td>44</td>
<td>30</td>
<td>26</td>
</tr>
<tr>
<td>Legal &amp; General (5)</td>
<td>18</td>
<td>18</td>
<td>23</td>
<td>23</td>
<td>19</td>
</tr>
<tr>
<td>Eagle Star (15)</td>
<td>34</td>
<td>35</td>
<td>41</td>
<td>50</td>
<td>52</td>
</tr>
<tr>
<td>Guardian Royal Exchange (16)</td>
<td>27</td>
<td>29</td>
<td>38</td>
<td>45</td>
<td>44</td>
</tr>
<tr>
<td>Clerical Medical (10)</td>
<td>9</td>
<td>10</td>
<td>12</td>
<td>13</td>
<td>21</td>
</tr>
<tr>
<td>General Accident (8)</td>
<td>27</td>
<td>29</td>
<td>22</td>
<td>19</td>
<td>15</td>
</tr>
</tbody>
</table>


Note: All figures expressed as a percentage. Figure in parenthesis represents the ranking of the UK insurer in terms of its worldwide life business in 1996.

Another pattern to note with respect to Table 6.9 is that the contribution of overseas income varies greatly across the insurers. In 1996, for example, Commercial Union, ranked 3, gained 79 percent of its life business from overseas business, while Standard Life, ranked 2, only saw 19 percent of its life business come from overseas income.

There also appears to be a number of dynamic effects present within the above table. Amongst the smaller insurers, for example Eagle Star and Guardian Royal Exchange, there has been a tendency for the percentage of overseas life business to increase greatly over the period 1992-1996. By contrast, the proportion of
overseas life income received by the larger insurers has remained relatively constant for the greater part of the period under consideration.

6.6 The Number and Size Distribution of Insurers

An examination of industry structure must, by definition, include an account of the number and nature of firms within the life assurance industry. There are many reasons for providing such an account both on theoretical and pragmatic grounds. Typically an industry is defined in terms of its firms, and it is the relative size and distribution of these firms that is usually the focus of theorising in microeconomic theory. Indeed an examination of market shares and firm market concentration will enable conclusions to be drawn on market conditions and the competitive processes that may or may not exist within the life assurance industry. As a consequence of this, a judgement about economic welfare might also be made. Also, information of this kind will be instrumental to an understanding of industry conduct and performance.

A further point to note relates to the public interest and regulation of life assurance. As outlined in previous sections, the importance of various life assurance products to the personal sector is significant and increasing. The growth in substantive contracts, such as mortgage payment plans and personal pension plans, makes the future wealth in the personal sector heavily reliant upon the performance of insurance funds. Moreover, with the growth of insurance funds, the contribution of insurance companies to the economy at large is critical, since insurance funds represent a significant source of capital for trade
and industry. Thus, the regulatory implications of market concentration are clear. Market concentration must therefore be seen as a useful source of information in designing regulatory strategies.

In order to provide an account of industry firms and their respective size distribution, a variety of data will be presented. Discussion proceeds with an examination of firm authorisations, followed by an account of the firm size distribution within the life assurance industry. Use will be made of concentration ratios and a statistical summary produced in the form of a Lorenz Curve will also be presented.

In previous sections, distinctions were made between various insurance markets. As a point of departure for discussion, it is worth noting the breakdown of firms along different market lines. Information on this is derived from the Department of Trade and Industry in its annual report on the insurance industry.\(^{17}\) Insurance Company authorisation is granted along product lines, as defined in the Insurance Companies Act of 1982 (see Chapter 2). There are seven product classes in long-term business and eighteen classes in general insurance. Typically, an insurance company is authorised to conduct general insurance, or long-term insurance, or both forms of insurance. Where an insurer conducts both forms of business it is referred to as a ‘composite’ insurer. Table 6.10 provides details on the total number of insurance company authorisations.

\(^{17}\) The responsibility for producing an annual report on the insurance industry now rests with the Financial Services Authority.
Table 6.10. Authorised Insurers

<table>
<thead>
<tr>
<th>Year</th>
<th>General Only</th>
<th>Life Only</th>
<th>Composite</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>550</td>
<td>215</td>
<td>69</td>
<td>834</td>
</tr>
<tr>
<td>1987</td>
<td>557</td>
<td>213</td>
<td>68</td>
<td>838</td>
</tr>
<tr>
<td>1988</td>
<td>564</td>
<td>209</td>
<td>65</td>
<td>838</td>
</tr>
<tr>
<td>1989</td>
<td>562</td>
<td>206</td>
<td>64</td>
<td>832</td>
</tr>
<tr>
<td>1990</td>
<td>570</td>
<td>203</td>
<td>64</td>
<td>837</td>
</tr>
<tr>
<td>1991</td>
<td>570</td>
<td>202</td>
<td>64</td>
<td>836</td>
</tr>
<tr>
<td>1992</td>
<td>565</td>
<td>196</td>
<td>62</td>
<td>823</td>
</tr>
<tr>
<td>1993</td>
<td>575</td>
<td>194</td>
<td>59</td>
<td>828</td>
</tr>
<tr>
<td>1994</td>
<td>573</td>
<td>191</td>
<td>57</td>
<td>821</td>
</tr>
<tr>
<td>1995</td>
<td>594</td>
<td>174</td>
<td>58</td>
<td>826</td>
</tr>
<tr>
<td>1996</td>
<td>578</td>
<td>177</td>
<td>59</td>
<td>814</td>
</tr>
</tbody>
</table>


The total number of authorisations in the U.K. has remained relatively constant over the period 1986-1996. The number of authorised general insurers has grown from 550 in 1986 to 578 in 1996. The number of long-term life authorisations has fallen in the period by 38 to stand at 177 in 1996. Composite insurers have also seen their numbers decline from 69 to 59. The breakdown of authorisations for U.K. and foreign insurance companies is given in Figure 6.6.

Figure 6.6. Authorised U.K. and Foreign Insurers

Figure 6.6 shows that the number of authorised foreign insurers is comparatively small in all three classes. The total number of U.K. and foreign authorised insurers amounted to 658 and 156 respectively (Association of British Insurers 1997a, 75). The most active market in which foreign insurers operate is in the general insurance market where foreign insurers account for 23 percent. Note that such numbers must be placed in the context of market share. As noted in previous sections, foreign insurers accounted for around one quarter of life business in the U.K. in 1996 (Association of British Insurers 1997a, 29).

As a supplement to the totals presented above, Table 6.11 provides details on life insurer authorisations across product categories for the period 1986-1996.

Table 6.11. Number of Insurers Authorised for Each Class of Insurance Business

<table>
<thead>
<tr>
<th>Year</th>
<th>Life and Annuity</th>
<th>Marriage and Birth</th>
<th>Linked Long Term</th>
<th>Permanent Health</th>
<th>Capital Redemption</th>
<th>Pension Fund Management</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>285</td>
<td>264</td>
<td>280</td>
<td>274</td>
<td>264</td>
<td>272</td>
</tr>
<tr>
<td>1987</td>
<td>283</td>
<td>259</td>
<td>277</td>
<td>272</td>
<td>258</td>
<td>266</td>
</tr>
<tr>
<td>1988</td>
<td>277</td>
<td>251</td>
<td>270</td>
<td>265</td>
<td>250</td>
<td>258</td>
</tr>
<tr>
<td>1989</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>1990</td>
<td>273</td>
<td>239</td>
<td>204</td>
<td>261</td>
<td>238</td>
<td>248</td>
</tr>
<tr>
<td>1991</td>
<td>268</td>
<td>232</td>
<td>199</td>
<td>256</td>
<td>230</td>
<td>240</td>
</tr>
<tr>
<td>1992</td>
<td>262</td>
<td>223</td>
<td>191</td>
<td>250</td>
<td>231</td>
<td>243</td>
</tr>
<tr>
<td>1993</td>
<td>257</td>
<td>217</td>
<td>185</td>
<td>243</td>
<td>215</td>
<td>228</td>
</tr>
<tr>
<td>1994</td>
<td>244</td>
<td>208</td>
<td>234</td>
<td>236</td>
<td>207</td>
<td>217</td>
</tr>
<tr>
<td>1995</td>
<td>240</td>
<td>200</td>
<td>229</td>
<td>230</td>
<td>199</td>
<td>209</td>
</tr>
<tr>
<td>1996</td>
<td>293</td>
<td>192</td>
<td>225</td>
<td>228</td>
<td>192</td>
<td>203</td>
</tr>
</tbody>
</table>


Note: Authorisations quoted as at end of year. Figures for 1989 were not available at time of data collection.

Within the context of a growing insurance market, it can be seen that the number of authorised insurers for each individual class of insurance is falling for every
insurance category except life and annuity. At this stage, without additional information, it is difficult to determine whether the patterns are a result of exit, entry, and/or consolidation of insurance businesses. It should be stressed that counts in the columns of Table 6.11 do not necessarily represent different insurers. In many cases an insurer is authorised to conduct business in all categories (Department of Trade and Industry 1997).

Another distinction to be made is in defining insurance companies in terms of their ownership. In particular, insurers may be defined as mutual or proprietary companies. Mutual insurers are companies that are owned solely by the policyholders and whose objective is to maximise the wealth of these individuals though bonus declarations and life fund surpluses. Proprietary life companies are owned by policyholders and shareholders and issue their own share capital. Mutual companies are in the minority accounting for roughly 1 in 7 insurers in 1996 (Department of Trade and Industry 1997, 14).

While much can be said about the breakdown of mutual and proprietary life insurers, it is also important to consider the extent of interlocking ownership of shares of life insurance companies. If a significant amount of insurance companies’ investments are geared towards other insurance companies, then the role and importance of solvency regulations may be less clear, since insurance companies will have a direct interest in maintaining the financial well being of each other. That is, in effect there would be a mutual insurer function operating across all companies.
Table 6.12 provides some evidence on the size of insurance company share
ownership of other insurance companies. From the table it can be seen that there
is a significant variation in the amount invested by the insurance company into
affiliates; for example, Norwich Union and Standard Life both invest over 2
percent of their total investments in affiliates while Nat-West life and American
Life invest 0 percent and 0.2 percent, respectively. These proportions compare
with a sample average of 0.9 percent. Note that specific information on the
breakdown of share ownership was not available.

<table>
<thead>
<tr>
<th>Name of Insurance Company</th>
<th>Proportion of Total Investments made up be Investments in Other Insurance Companies (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>American Life Insurance Company</td>
<td>0.2</td>
</tr>
<tr>
<td>AXA Sun Life PLC</td>
<td>0.5</td>
</tr>
<tr>
<td>The Equitable Life Assurance Society</td>
<td>0.4</td>
</tr>
<tr>
<td>Legal &amp; General Assurance Society</td>
<td>0.5</td>
</tr>
<tr>
<td>National Westminster Life Assurance Ltd.</td>
<td>0.0</td>
</tr>
<tr>
<td>Norwich Union Life &amp; Pensions</td>
<td>2.5</td>
</tr>
<tr>
<td>The Prudential Assurance Co. Ltd.</td>
<td>0.9</td>
</tr>
<tr>
<td>Scottish Equitable PLC</td>
<td>0.5</td>
</tr>
<tr>
<td>Scottish Widows' Fund &amp; Life Assurance Society</td>
<td>1.2</td>
</tr>
<tr>
<td>The Standard Life Assurance Co. Ltd.</td>
<td>2.1</td>
</tr>
</tbody>
</table>


As Standard & Poor's note, much of the investments in other insurance
companies are in affiliate companies; for example, the Legal & general
Insurance Company is the majority shareholder in Legal & General Fund
Management (Standard & Poor's 1998, 77). That is, the investments in other
insurance companies appears to be the result of 'within group' strategic
management considerations – this is especially the case with the growth in the
number of insurance/financial services groups – rather than any bias in overall
investment allocation decisions. Moreover, anecdotal evidence from the
regulator suggests that whilst the members of a given insurance group may have an incentive to maintain the financial strength of other members within that group (the strategic management considerations), there is little evidence to suggest the operation of wider share ownership amongst other insurance companies.

While much has been said on the number of insurers operating in the life assurance industry, it is important to understand the associated size distribution of companies. Defining insurance company size is a complex task. There is some ambiguity associated with measuring insurer size. Size can be defined in terms of a variety of measures including total premiums written, total sums assured, the life fund size, and the total new business written (Franklin and Woodhead 1980). There is no ‘right’ or ‘wrong’ measure and it is for this reason that caution must be attached to any subsequent conclusion.

Before proceeding, it is worth noting the Lorenz relationship in Figure 6.7 below. This graph determines the extent of equality between two distributions, the number of insurers, and the new business written. Although data was only available up until 1992, it helps provide an indication of market concentration.
With reference to Figure 6.7, there are two relationships present. The forty-five degree line represents the so-called ‘line of equality’. This is a hypothetical relationship that would be present if there were an equal distribution of new business amongst all insurers in the industry. The greater the movement away from this equality line, the greater the inequality between the two distributions. It will be noted that there is a substantial inequality between new business written in 1992 and the distribution of insurers. This gives some indication of what to expect regarding concentration ratios and the distribution of market shares across the industry. In particular, the above pattern is evidence of a few insurers accounting for a disproportionately large amount of new business in the life assurance industry.
Consider next the insurance company concentration ratios. There is a voluminous literature concerning the measurement of industrial concentration. Most of this work concerns the construction of an appropriate index to capture industrial concentration (see Stigler (1968), Adelman (1969), Davies (1979) and Geroski (1983)). The $k$ firm concentration ratio is a frequently adopted indicator of market structure. This measures the ratio of $k$ firm market shares to total market size. The result can give an indication of the extent of market power and dominance of an individual firm or group of dominant firms. For alternative measures of industrial concentration, such as the Herfindahl index, see Kwoka (1985).

It is already known that a large number of insurers operate within the life assurance industry. At first sight this might be taken as a depiction of a relatively competitive market structure, yet it is the relative sizes of the firms across this number of insurers that matters. The relative size measure gives an indicator of market power and, with the use of microeconomic theory, it can then be used to predict associated welfare losses or gains (see Martin (1993)).

Table 6.13 provides details of the market concentration of the top 10 long-term U.K. registered insurance companies for 1996. As already noted, there are a variety of means with which to measure the relative market size. Due to the availability of data, worldwide net premium income will be adopted as a measure of size. Total premiums have been used in previous studies of the life assurance industry (Richards and Colenutt 1975, 155).
Table 6.13. Concentration Ratios of the Leading 10 Long-Term Life Assurance Companies in 1996

<table>
<thead>
<tr>
<th>Rank of Life Companies</th>
<th>Company</th>
<th>1996 Worldwide Net Premium Income £m</th>
<th>% Share of Total Premiums</th>
<th>Cumulative Share of Total Premiums %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Prudential Assurance</td>
<td>9152</td>
<td>13.76%</td>
<td>13.76%</td>
</tr>
<tr>
<td>2</td>
<td>Standard Life Assurance</td>
<td>4253</td>
<td>6.40%</td>
<td>20.16%</td>
</tr>
<tr>
<td>3</td>
<td>Commercial Union Assurance</td>
<td>3669</td>
<td>5.52%</td>
<td>25.68%</td>
</tr>
<tr>
<td>4</td>
<td>Norwich Union Life &amp; Pensions</td>
<td>3517</td>
<td>5.29%</td>
<td>30.97%</td>
</tr>
<tr>
<td>5</td>
<td>Legal &amp; General Assurance Society</td>
<td>2837</td>
<td>4.27%</td>
<td>35.23%</td>
</tr>
<tr>
<td>6</td>
<td>Equitable Life Assurance Society</td>
<td>2830</td>
<td>4.26%</td>
<td>39.49%</td>
</tr>
<tr>
<td>7</td>
<td>Royal &amp; Sun Alliance</td>
<td>2545</td>
<td>3.83%</td>
<td>43.32%</td>
</tr>
<tr>
<td>8</td>
<td>Scottish Equitable plc</td>
<td>1919</td>
<td>2.89%</td>
<td>46.20%</td>
</tr>
<tr>
<td>9</td>
<td>General Accident Fire &amp; Life Assurance Corp</td>
<td>1848</td>
<td>2.78%</td>
<td>48.98%</td>
</tr>
<tr>
<td>10</td>
<td>Sun Life Assurance Society plc</td>
<td>1715</td>
<td>2.58%</td>
<td>51.56%</td>
</tr>
</tbody>
</table>


Note: All company results presented as a percentage of total net premium income in 1996. Cumulative share of the top k firms yields group concentration ratio.

A difficulty that is associated with the above calculations is that the worldwide premium income measure includes overseas business. Thus, insurer size and total market size are an overestimate of the sizes in respect of the U.K. market alone. However, given the fact that the majority of life insurer business comes from the U.K. market, it is believed that in spite of these problems a meaningful result can be still gained by using this data.18

With reference to Table 6.13, it can be seen that the top 10 insurance companies accounted for more than half of premium income in 1996. The largest individual share goes to the Prudential, which accounts for approximately 14 percent of the total. The three-firm concentration ratio accounts for one quarter of net premium income, while the top 5 companies account for over 35 percent. It is also worth

18 U.K. net premium income accounted for just fewer than 80 percent of the worldwide total in 1996 (Association of British Insurers 1997a).
noting the 20 firm concentration ratio equals just fewer than 70 percent of total net premium income (Association of British Insurers 1997, 79).

What is the regulatory implication of the results presented in Table 6.13? In order to assess the regulatory implications, it is necessary to resort to some form of yardstick against which to compare the above results. Einhorn (1964) proposes a yardstick that requires a four-firm concentration ratio to be no greater than 50 percent, assuming that insurance may be classified as a four-digit industry (Einhorn 1964, 43). Based on this yardstick, insurance would therefore pass the regulatory test.

6.7 Entry into the UK Life Assurance Industry

Entry and exit into an industry is another major aspect of industry structure. This aspect of industry structure is another indicator of the extent of competition and the associated economic welfare. Entry and exit provides evidence of the pricing policy and cost structure of incumbents and the consequent 'height' of any potential barriers to entry (see Bain (1956), Stigler (1968), Demsetz (1982) and von Weizsäcker (1980) for discussion of entry barriers). Note that entry and exit conditions may be evaluated in conjunction with the theory of contestable markets in order to provide some guidelines on government intervention (see Baumol (1982)).

Barriers to entry bring about an imbalance in costs between the potential entrant and the incumbent: a potential entrant's unit cost is greater than the cost
associated with that of any incumbent firm. There are a few reasons that may help to explain this phenomenon (Bain 1956). Use will be made of the U.K. life assurance industry to illustrate these possible explanations before providing some empirical evidence on entry.

The theories that have attempted to explain the so-called barriers to entry have done so by making reference to the cost structures of firms. The commonly referred to theories concern absolute cost disadvantages and economies of scale (Bain 1956, 15-16). Absolute cost disadvantages come in a variety of forms. Incumbents will, for example, possess significant know-how and experience in the processing and organisation of life assurance. Experience in underwriting activities will provide invaluable information in making suitable solvency provisions. However, the exact size of the cost disadvantage faced by the potential entrant is difficult to quantify.

Economies of scale might also be expected to exist within the life assurance industry (for recent empirical studies of economies of scale in the U.K. life assurance industry see Hardwick (1994)). Many insurance companies have centralised their operations in underwriting, investment and marketing. Thus, for example, investment management expenses will be expected to be lower per policy unit for larger insurance companies, since investment managers are able to pool investment funds and diversify investment risk. Also, note that the larger insurers will be able to offset a surge in claims more easily than that of a potential entrant.
Regulatory or legal barriers are another barrier to entry. Such constraints are of great importance when attempting to understand entry into the U.K. life assurance industry. The Insurance Companies Act of 1982 sets out the standards by which insurance companies must be regulated. In particular, insurance companies have to gain ‘authorisation’ in order to engage in insurance business. Authorisation is defined in terms of a number of standards in respect of company solvency and management conduct (see Chapter 2). The solvency capital regulations impose a cost upon insurers and this, together with the annual regulatory reviews, imposes important restrictions upon insurers.\textsuperscript{19} Note that Demsetz (1982) is critical of the whole notion of entry barriers and argues that it reflects a failure to define costs correctly (Demsetz 1982, 56).

Having set out some of the potential barriers to entry (especially the legal restrictions), consider Table 6.14, which sets out the changes in authorisations over the past 10 years. This is similar to the information provided in Table 6.10. In Table 6.14, column (a) and (b) represent the total authorisations for domestic and overseas insurers. These totals are the net outcome for the year and as such include authorisation withdrawals and new authorisations. To provide some evidence of new authorisations column (c) is also included. For both foreign and U.K. insurers the total number of authorisations has declined over the period. This might be the result of exit and/or consolidation and take-over. Within this trend there has been a constant influx of new entrants to the industry, although this clearly has not been enough to stop the fall in the headline total of authorisations.

\textsuperscript{19} Other barriers to entry, such as buyer loyalty, will also be important.
Table 6.14. Entry to the UK Life Assurance Industry

<table>
<thead>
<tr>
<th>Year</th>
<th>Total UK Life (a)</th>
<th>Total Foreign Life (b)</th>
<th>New Life Company Authorisation (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>193(n/a)</td>
<td>22(n/a)</td>
<td>3(0)</td>
</tr>
<tr>
<td>1987</td>
<td>191(-2)</td>
<td>22(0)</td>
<td>3(4)</td>
</tr>
<tr>
<td>1988</td>
<td>195(+4)</td>
<td>14(-8)</td>
<td>1(1)</td>
</tr>
<tr>
<td>1989</td>
<td>190(-5)</td>
<td>14(0)</td>
<td>n/a</td>
</tr>
<tr>
<td>1990</td>
<td>190(0)</td>
<td>15(+1)</td>
<td>9(0)</td>
</tr>
<tr>
<td>1991</td>
<td>187(-3)</td>
<td>15(0)</td>
<td>4(0)</td>
</tr>
<tr>
<td>1992</td>
<td>182(-5)</td>
<td>14(-1)</td>
<td>3(2)</td>
</tr>
<tr>
<td>1993</td>
<td>180(-2)</td>
<td>14(0)</td>
<td>4(0)</td>
</tr>
<tr>
<td>1994</td>
<td>173(-7)</td>
<td>18(+4)</td>
<td>4(2)</td>
</tr>
<tr>
<td>1995</td>
<td>160(-13)</td>
<td>14(-4)</td>
<td>6(2)</td>
</tr>
<tr>
<td>1996</td>
<td>163(+3)</td>
<td>14(0)</td>
<td>7(2)</td>
</tr>
</tbody>
</table>


Notes: Columns (a) and (c) show the total number of long-term company authorisations both domestic and foreign. Figures in brackets show the change on the previous year. Column (c) shows the number of new authorisations for the year. The figure in brackets relates to re-issued authorisation where authorisation has lapsed.

In order to provide a full account of entry and hence arrive at an adequate conclusion vis-à-vis government intervention, further details are required in respect of the type of entrant firms and the nature of exit from the industry. These standards are difficult to achieve within the scope of this paper and are restricted by available data. However, the evidence presented above provides an initial indication of contestability.

6.8 Summary

In terms of the structure of the U.K. life assurance industry, there are many indicators by which to measure industrial structure. It is clear that the life
industry has undergone a dramatic change over the last century as a result of many factors not least those of product innovation and premium affordability. With respect to the insurance companies, the firm size distribution provides evidence of modest industrial concentration. The significance of the life assurance industry as a whole cannot be understated and, as Chapter 8 will demonstrate, the implications for the macro-economy are significant.
CHAPTER 7

THE CONDUCT OF THE U.K. LIFE ASSURANCE INDUSTRY

7.0 Introduction

The previous chapter considered some aspects of industry structure relating to the U.K. life assurance industry. Continuing with the structure-conduct-performance (SCP) theme, this chapter considers some of the aspects of industrial conduct. The SCP notion of conduct refers to the operating objectives pursued within the industry and the delivery vehicles that are available for the attainment of these objectives. As such, industrial conduct refers, in part, to the organisational structure of the firm and the relative importance of managerial objectives versus profit maximisation objectives. Given that insurance companies facilitate the smooth functioning of the economy at large, there might also be a public duty function to consider within the assessment of life assurance industrial conduct.

The evaluation of market conduct by empirical means is subject to a variety of constraints. Empirical work is limited by commercial sensitivity, and ambiguity associated with objective definitions. The result is often an incomplete and therefore unsatisfactory evaluation of firm conduct within a given industry. However, it is possible to gauge some aspects of industrial conduct through a number of data sources, provided for the most part by the annual reports of the regulator (see, the Financial Services Authority (1999)).
Having established the background to assessing insurer conduct, it is worth noting the voluminous theoretical literature on market behaviour. Economic theory provides a detailed account of the setting of prices within a max/min framework (for examples of alternative conduct specifications within a SCP setting see Cowling (1976), and Cowling and Waterson (1976)). Work has also focused on a number of accounts of non-price competition (see Beath and Katsoulacos (1991)). Managerial theories of the firm provide an alternative framework from within which to view firm conduct (see Baumol (1958), Marris (1964), and Williamson (1963) for examples).

In line with this theoretical framework a brief overview of the price setting behaviour of life insurers will be provided in Section 7.1, outlining the unique function of the actuary in the price setting behaviour of insurance companies. This is followed by an examination of insurer marketing policy in Section 7.2. While it is not possible to obtain readily information on insurance company objectives (such as through the use of questionnaires), it is useful to review some of the outcomes relating to insurer portfolio holdings; this is another aspect of conduct and is the basis of Section 7.3. Section 7.4 considers the incentives for marketing life assurance and this is followed by a summary in Section 7.5.

Before proceeding, it is important to emphasise the significance of ownership structure to understanding life insurer conduct. As noted in the previous chapter, the ownership of life assurance companies falls into two classifications: mutual life insurers, and proprietary life insurers. Recall that mutual insurers are owned
exclusively by the policyholders while proprietary life insurers are owned by shareholders in addition to policyholders.

While both policyholders and shareholders aim to maximise their respective wealth, policyholders are likely to have slightly different operating requirements to those of shareholders. Policyholders aim to maximise their wealth as measured in terms of life fund surplus and bonus declarations. Thus, in the case of a mutual insurer all excess returns are reinvested for the benefit of policyholders. By contrast, shareholders aim to maximise return on capital and require a certain proportion of this excess return paid out immediately in terms of a dividend. The dividend payment detracts from the size and frequency of any bonus declarations, and thus the accumulation of policyholder wealth. Therefore, it is clear that the conduct of insurance companies will be contingent, at least in part, on the ownership structure of the company.

In addition to the ownership structure there is the issue of control, especially the separation of ownership from management control. A conflict may occur between the objectives of the owners (be they policyholders or shareholders) and the objectives of managers and this is a further factor to consider in relation to any discussion of industrial conduct (see, Barnard (1938), and Drucker (1946)).

7.1 Price-Setting Objectives and the Function of the Actuary

Central to the insurance company’s operations, most notably its price-setting behaviour, is the assessment of the underwriting risk that any given insurance
company faces. This responsibility falls with the Actuary of the company. The Actuary imposes constraints upon the price setting behaviour of insurers and their associated objective function. In agreeing to assume the risk of a given life contract, the insurance company faces the potential of a significant cash outflow in the form of a policy claim. The estimation of this risk (that is, the probability of a claim) is of critical importance to the subsequent performance of the insurance company, as measured by its profitability and solvency. The assessment of this risk, and hence the function of the Actuary, are of direct importance to the policyholders and/or shareholders, as well as the market as a whole since reputation effects might lead, in the limit, to a crisis in consumer confidence (Akerlof 1970).

The likelihood of a claim will affect the price (premium) charged. Thus, while an insurance company may be engaged in a price setting strategy that requires the maximisation of ownership wealth, this objective is subject to a risk constraint. Note that historically, one of the greatest risks to insurer solvency has come from premiums being set too low, thereby making an inadequate provision for claims (Finsinger and Pauly 1986). As a consequence of the significance of the Actuary to insurer conduct, an overview of the actuarial function within the insurer price setting process will be presented.

For all types of annuity and life contracts, the process of actuarial premium determination is basically the same. To illustrate this fact, assume that the contract expenses and profits are normalised to zero. The net premium is then made up of the expected value of the loss and the necessary savings
contribution. Consider first the expected value of the loss. This is assessed from the so-called ‘life tables’. Life tables provide probabilities of surviving a given number of years for a ‘representative’ individual of a given age in the U.K. (Government Actuary’s Department 1994). A life table reveals information on the number of individuals existing at time $t$ surviving to time, $t+n$. The life table shows that the expectation of years to live declines with age. Variations in life expectancy will therefore occur over time, but also over the characteristics of the individual—see Table 7.1 below.

Table 7.1. Expectation of Life at Birth According to Death Rates
Assumed for 1994, 2001 and 2031

<table>
<thead>
<tr>
<th>Region</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>England</td>
<td>74.1</td>
<td>75.4</td>
</tr>
<tr>
<td>Wales</td>
<td>73.9</td>
<td>75.2</td>
</tr>
<tr>
<td>Scotland</td>
<td>72.1</td>
<td>73.5</td>
</tr>
<tr>
<td>Northern Ireland</td>
<td>72.9</td>
<td>74.3</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>73.9</td>
<td>75.2</td>
</tr>
</tbody>
</table>


Note: All figures in years.

Table 7.1 highlights the greater life expectancy of women when compared with men, and also that the life expectancy overall is increasing over time. In addition, there is evidence of regional variation in life expectancy with the English having the greater regional life expectancy within the U.K. While life expectancy may vary across groups, there exists one common pattern: the premium charged for a given level of life cover will necessarily increase with the age of the policyholder in order to cover the fall in life expectancy.

Recall from Chapter 6, a savings element is required for ‘substantive’ life assurance policies.

Life expectancies will be expected to vary according to the background of the individual; for example, life expectancy might be expected to vary across country and region.
Further to the role of life expectancy in calculating a premium, the Actuary must also assess the expected returns on the assets of the investment portfolio. The rate of return that the Actuary expects to receive on the insurer’s assets will directly affect the savings contribution required for substantive policies. Ceteris paribus, the higher the expected return the lower the premium required.

Thus, with respect to the function of the Actuary, it can be seen that there are two informational requirements in order to determine the actuarial provision for premium setting; namely, the life table and the expected return. These two components are used, together with their associated assumptions, to construct the appropriate ‘monetary functions’. These functions exist for each contract type and calculate the net premium per £1 of cover (Franklin and Woodhead 1980).³

In practice, there are a number of additions to the net premium charged. These are centred on the expenses of the insurance contract.⁴ Expenses may come in many forms, they may be specific to the size of the sum assured and premium, as well as varying according to the contract type and investment basis (for example, whether the contract is unit-linked or with-profit). In addition to any implicit profit margins, there will be an explicit profit charge in order to generate an adequate return to the shareholders (in the case of a proprietary life insurer).

³ Monetary functions vary according to the interest rate, the life table, and the contract specifications assumed.
⁴ It is important to stress that within the Actuary’s assessment of the net premium there are likely to be ‘implicit margins’. These implicit margins may be a result of conservative risk assessments.
A further issue to consider in respect of price setting concerns the insurer specific variation in the premiums charged for a given category of risk. While there are standard life tables publicised by the government, an insurer will typically rely on the use of its own mortality estimates (Franklin and Woodhead 1980). These life tables are usually built up as a direct result of the insurer’s own claims experience. This provides some explanation for the variation in premium rates charged within the industry. Table 7.2 provides information on the quoted premiums for a low cost with-profits endowment.

Table 7.2. Insurance Company Variation in Monthly Premium Rates for Low Cost With Profits Endowments 1998

<table>
<thead>
<tr>
<th>Company</th>
<th>Assumed % Growth</th>
<th>ANB 20yrs</th>
<th>ANB 30yrs</th>
<th>ANB 40yrs</th>
<th>ANB 50yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abbey National</td>
<td>7.50%</td>
<td>69.33</td>
<td>69.98</td>
<td>75.18</td>
<td>92.83</td>
</tr>
<tr>
<td>Clerical Medical</td>
<td>7.50%</td>
<td>78.38</td>
<td>80.48</td>
<td>89.57</td>
<td>113.39</td>
</tr>
<tr>
<td>Colonial</td>
<td>7.50%</td>
<td>80.84</td>
<td>80.99</td>
<td>85.99</td>
<td>103.04</td>
</tr>
<tr>
<td>Commercial Union</td>
<td>7.50%</td>
<td>73.5</td>
<td>74.5</td>
<td>81</td>
<td>99.5</td>
</tr>
<tr>
<td>Equitable Life</td>
<td>7.50%</td>
<td>65.38</td>
<td>65.73</td>
<td>70.31</td>
<td>84.74</td>
</tr>
<tr>
<td>Friends Provident</td>
<td>7.50%</td>
<td>71.41</td>
<td>72.41</td>
<td>78.63</td>
<td>96.73</td>
</tr>
<tr>
<td>GA Life</td>
<td>7.50%</td>
<td>73.5</td>
<td>75.41</td>
<td>81.56</td>
<td>102.71</td>
</tr>
<tr>
<td>Legal &amp; General</td>
<td>7.50%</td>
<td>73.63</td>
<td>73.98</td>
<td>79</td>
<td>94.92</td>
</tr>
<tr>
<td>Liverpool Victoria</td>
<td>7.50%</td>
<td>68.7</td>
<td>69.8</td>
<td>75.4</td>
<td>91.9</td>
</tr>
<tr>
<td>Norwich Union</td>
<td>7.50%</td>
<td>73.89</td>
<td>75.35</td>
<td>83.03</td>
<td>105.82</td>
</tr>
<tr>
<td>RNPFN</td>
<td>7.50%</td>
<td>91.58</td>
<td>91.58</td>
<td>100.17</td>
<td>123.33</td>
</tr>
<tr>
<td>Scottish Mutual</td>
<td>7.50%</td>
<td>80.26</td>
<td>81.29</td>
<td>87.06</td>
<td>106.93</td>
</tr>
<tr>
<td>Scottish Provident</td>
<td>7.50%</td>
<td>74.4</td>
<td>74.93</td>
<td>78.73</td>
<td>91.14</td>
</tr>
<tr>
<td>Scottish Widows</td>
<td>7.50%</td>
<td>76.36</td>
<td>76.86</td>
<td>81.39</td>
<td>97.74</td>
</tr>
<tr>
<td>Sun Life</td>
<td>7.50%</td>
<td>76.06</td>
<td>77.08</td>
<td>84.99</td>
<td>110.98</td>
</tr>
</tbody>
</table>


Note: Premiums quoted in pounds for each ‘age-next-birthday’ (ANB). The individual for whom the quote is provided is a male, non-smoker requiring a sum assured of £50,000.

Table 7.2 shows that there is significant variation in the premiums charged for an identical risk class. For all ages, there is a spread of over twenty pounds between the largest and lowest premiums charged, and this amount increases
with the age of the life assured. Given the data in Chapter 6 on the largest life insurers in the U.K., it would appear that the lowest premiums are consistent with the larger life companies, for example Equitable Life posts the lowest rate for all ages of the life assured. The variation in prices posted suggests the existence of non-price competition and product differentiation. As such, the conduct of life assurance companies (as illustrated in Table 7.2) suggests that insurers are seeking to differentiate their product in the search for monopoly rents. This argument is the conclusion of the so-called ‘spatial models’ of imperfect competition (see for example, Hotelling (1929)). More evidence for this form of conduct will be given in the discussion of life assurance marketing in the U.K. (see Section 7.2).

Another pattern to emerge from Table 7.2 is the effect of life expectancy upon the premium charged. This age effect is especially evident when analysing the premium increment for different age groups—such as comparing the premium increment between 40 and 50 years of age with the premium increment between 20 and 30 years of age. Table 7.2 demonstrates that this increment is increasing with age. As a consequence, the standard deviation about the mean premium rates is also increasing with the age group.

The differences in premium rates charged might be explained by the variation in company mortality experiences, and also the company specific experiences associated with the expected return on assets.\textsuperscript{5} These company specific experiences

\textsuperscript{5} Note that for comparison, the insurance companies presented in Table 7.2 assume the same rate of growth for the accumulation of the savings fund. In many cases, insurance companies do not have the same assumed growth rate (Money Facts 1998).
variations may therefore affect the future provisions of the insurance company, which in turn will affect the premium charged.

Before proceeding, it is worth noting the size of some of the different expense categories. Table 7.3 shows the relative size of commission payments in respect of other expenses associated with ordinary life assurance business. In particular, the breakdown between management expenses and taxation is shown.

Table 7.3. Long Term Expenditure

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Expenditure associated with U.K. long-term contracts: Management Expenses</td>
<td>4978</td>
<td>4911</td>
<td>5149</td>
<td>5141</td>
<td>5416</td>
</tr>
<tr>
<td>Commissions</td>
<td>2671</td>
<td>2794</td>
<td>2621</td>
<td>2318</td>
<td>2562</td>
</tr>
<tr>
<td>Taxation</td>
<td>1030</td>
<td>1590</td>
<td>959</td>
<td>2374</td>
<td>2711</td>
</tr>
</tbody>
</table>


With respect to Table 7.3, in 1996 commissions accounted for just over £2.5 billion, an increase of 5.3 percent over the previous year. This amount represents just under 5 percent of U.K. net premium income whilst management expenses accounted for over 10 percent. This demonstrates the significance of these expenses for the insurer, and is also an explanation for the discrepancy between the net and gross premium. Thus, there is additional evidence for the view that the insurance companies are engaged in a strategy that attempts to maximise market share.
7.2 Marketing Objectives

As mentioned, while much can be said about the determination of price within an insurance company, the role of non-price competition is also central to the overall conduct strategy of the life insurer. Product differentiation and market segmentation will allow insurance companies to exploit market opportunities fully. The seminal work of Chamberlin (1933) established product differentiation within a rigorous theoretical framework, emphasising the respective roles of technical and psychological differentiation. The importance of brand loyalty and advertising were also emphasised.

While evidence of technical product differentiation is available, it is more difficult to quantify the significance of psychological product differentiation. Attention will therefore focus on the observable characteristics of the products available in the life assurance industry.

As discussed previously in Chapter 6, products sold in the life assurance market offer many policy options, including policy ‘add-ons’ where a combination of policies may be purchased as a bundle. The packaging of insurance benefits into a single contract is now a popular feature of the insurance product market; for example, Critical Illness Cover (CIC) is offered in conjunction with other contracts such as endowment plans and pension plans and is sold more often in conjunction with other contracts than as a stand-alone contract (Association of British Insurers 1997b). The combination of such plans has lead to a reduction

---

6 A 'stand-alone' contract refers to a policy that is effected for a single benefit, rather than a contract sold in conjunction with a number of other insurance benefits.
in the combined policy charges, as compared with two separate 'stand-alone' contracts offering the same level of benefit.

Another recent product feature to be introduced to the life assurance market is that of low-cost payment schemes, where a policy is offered on terms that enable the initial premium to start low and increase with each subsequent year to a maximum level (typically after 5 years) for the rest of the term. These payment plans make life contracts more attractive to those individuals at the margin of affordability, thereby supporting the sales growth and market share performance of the insurance companies concerned.

Having outlined briefly the product components of the marketing mix, attention now turns to addressing the various sales and distribution channels through which insurance companies aim to sell and market their products. Insurance companies have five main channels through which to sell their products. These marketing channels are differentiated according to whether the insurer sells to the consumer directly or indirectly through an intermediary. Table 7.4 sets out the new long-term insurance business generated by different distribution channels.

A given insurance company will have a direct sales force, which serves as a representative of the insurer. In addition, the insurance company will rely on

---

7 Direct sales forces have been in operation for some time now. They have their foundations in the nineteenth century when industrial life policies were introduced and insurance contracts were effected through door-to-door selling.
direct selling and promotion through the use of mailings and so-called ‘cold calling’. 8,9

Table 7.4. New Long-Term Business by Distribution Channel

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>New Yearly Premiums—Total individual life and pension business</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IFAs:</td>
<td>29</td>
<td>28</td>
<td>30</td>
<td>35</td>
<td>37</td>
</tr>
<tr>
<td>of which</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Banks</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Building Societies</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Independent Companies/brokers</td>
<td>22</td>
<td>23</td>
<td>25</td>
<td>31</td>
<td>33</td>
</tr>
<tr>
<td>Others</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Company Agents:</td>
<td>68</td>
<td>69</td>
<td>67</td>
<td>62</td>
<td>60</td>
</tr>
<tr>
<td>of which</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Banks</td>
<td>5</td>
<td>6</td>
<td>12</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>Building Societies</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Direct Sales Force</td>
<td>48</td>
<td>50</td>
<td>42</td>
<td>42</td>
<td>40</td>
</tr>
<tr>
<td>Tied Agents</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Direct Selling</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

| **New Single Premiums—Total individual life and pension business** |      |      |      |      |      |
| IFAs:              | 56   | 50   | 53   | 59   | 60   |
| of which           |      |      |      |      |      |
| Banks              | 5    | 3    | 2    | 3    | 3    |
| Building Societies | 3    | 3    | 3    | 3    | 2    |
| Independent Companies/brokers | 41  | 39   | 45   | 49   | 49   |
| Others             | 6    | 5    | 4    | 3    | 6    |
| Company Agents:    | 42   | 48   | 45   | 39   | 39   |
| of which           |      |      |      |      |      |
| Banks              | 3    | 7    | 12   | 11   | 11   |
| Building Societies | 3    | 5    | 6    | 4    | 4    |
| Direct Sales Force | 33   | 34   | 24   | 23   | 23   |
| Tied Agents        | 3    | 2    | 2    | 1    | 1    |
| Direct Selling     | 2    | 1    | 2    | 1    | 1    |


Life assurance companies also have the opportunity to channel business through tied agents and independent financial advisors (IFAs). Tied agents are appointed representatives of the insurance company and are authorised to sell and advise on the products of the associated insurance company. Tied agents may include

8 Details of individuals are readily accessed through consumer databases.
9 Promotional material is regulated by the Financial Services Act of 1986 (see Chapter 2).
banks, building societies and estate agents. By contrast, the IFAs are authorised to advise and sell on the products of any insurance company.

With reference to Table 7.4, the following points are noteworthy. Consider first the new yearly contracts. Here the insurance company is responsible for controlling the majority of new business because at least 60 percent of this new business came from company agents in the period 1992-1996. However, the proportion of this new business has also been declining over the period. In contrast, the proportion of sales accounted for by the IFAs has increased over the period from 29 percent in 1992 to 37 percent in 1996. The growth in independent companies and brokers appears to be a major factor in explaining this trend. While banks and building societies have been able to act as IFAs, the information above implies that most of these institutions favour being company tied agents instead.

When turning attention to the single premium contracts, a slightly different pattern emerges. Here, unlike yearly premium contracts, the majority of new business comes from the IFAs (approximately 60 percent). The independent companies/brokers account for approximately one half of new business. Company agents still account for a significant volume of new business, roughly 40 percent in 1996. The size of the investment associated with single premium contracts may help to explain this trend. The perceived impartiality of IFAs may

10 The growth in the number of IFAs has lead to a significant amount of rationalisation among IFA firms. This has been motivated, at least in part, by the bargaining strength of larger IFAs in respect the levels of commission payments that are agreed upon between the IFAs and the life assurance companies (The Association of British Insurers 1997, 43).
appeal more to those individuals seeking to place a large lump sum investment than those investing in a more modestly sized regular premium contract.

In addition to the mainstream distribution channels, there are two relatively new marketing channels emerging. Recognising the importance of insurance as source of business, banks and building societies have started establishing their own life assurance subsidiaries. This area of the life assurance market has come to be known as bancassurance. In 1996, bancassurance accounted for 14.5 percent of new yearly premiums, and 16.3 percent of new single premiums, (Association of British Insurers 1997a, 29).

Another development in the marketing of life assurance has come in the form of 'execution only' selling. The idea behind 'execution only' contracts is to circumvent the existing regulations on selling regulated insurance products by making the consumer responsible for its decisions within a 'buyer-beware' provision of the insurance contract. The consumer is therefore expected to take full responsibility for what he or she buys.\(^{11}\)

Commission payments help to promote and maintain distribution channels. Table 7.3 illustrated the significance of commission payments to the U.K. life assurance industry. Commission payments have been the focus for competition

\(^{11}\) The responsibility of consumers, in respect of financial contracts, is the focus of recent regulatory review by the Financial Services Authority. In particular, there has been a substantial amount of lobbying occurring which relates to the issue of whether or not the 'buyer-beware' clause should be explicitly stated within the upcoming 2000 Financial Services and Markets Bill. Consumer groups are opposed to such a move, while the financial services industry (especially the insurance industry) claims that consumers must also be held explicitly accountable for the enactment of a contract (FSA 1999).
amongst insurance companies as they compete to gain business referrals. As noted, commission payments amounted to £2,562 million in 1996 (Association of British Insurers 1997a).

The evidence on marketing provides some useful pointers vis-à-vis life assurance company conduct. In particular, it provides some useful insights into the possible incentives that may or may not operate within an insurance company. The use of sales commission payments and an elaborate system of distribution channels provide an indication of the importance of sales growth to insurance companies’ objective functions.

7.3 Portfolio Objectives

The time lag between premium income and claim outflow and the resulting accumulation of funds is specific to life assurance products: life assurance companies are uniquely placed to take advantage of such funds, being able to take advantage of a long-term investment strategy. As a result of this, portfolio management can be seen as an important consideration within a more general asset-liability management (ALM) strategy:

ALM involves the management of risks arising from simultaneous effects of financial market moves on both assets and liabilities. (Smink and van der Meer 1997, 128)

ALM is therefore central to an understanding of insurer conduct. Moreover, ALM strategies provide a useful indicator of the significance of the rate of return as an objective for the life assurance industry.
Smink and van der Meer (1997) conducted an international survey on life assurance ALM management policies. The results cover some 287 insurance companies from France, Germany, Japan, the Netherlands, the U.K., and the United States. The authors show that most insurers have an active ALM policy using a combination of actuarial and financial techniques, although the German and Japanese insurance companies appear to be behind most other countries in implementing an effective ALM policy (Smink and van der Meer 1997, 131).

In terms of individual ALM strategies, there are a number of points to develop. Out of those insurance companies using the ALM techniques, most (over 70 percent) considered asset/liability projections using scenarios important (Smink and van der Meer 1997, 132). Other ALM strategies considered included investment performance measurement, risk-return analysis, interest rate immunization and cashflow matching and these were rated ‘important’ by 70.6 percent, 47.1 percent, 34.3 percent, and 60.8 percent of the sample respectively (Smink and van der Meer 1997, 132). Thus, the emphasis is biased in favour of investment performance analysis and cashflow matching. This is tentative evidence to support the view that it is the investment portfolio and its associated performance that is central to an understanding of life assurance company solvency. Moreover, the emphasis on investment performance would also imply that the rate of return is a very important consideration for insurer conduct (as defined in Chapter 5). For those companies who did not heavily promote ALM, most justified this approach on the grounds that better models were needed, although they also agreed to the need for adequate ALM. In addition, half of

---

12 According to Smink and van der Meer (1997), the countries surveyed account for approximately 70 percent of world premium income.
those who responded claimed that the insurance company regulations were an important restriction to effective ALM. The vast majority of companies also believed that the capital markets provide a sufficient set of instruments for effective ALM.

In terms of the attitudes of life insurers to risk within an ALM strategy, the risks rated important by insurers were credit risks, equity risks and real estate investment risks; currency risks were also considered as undesirable (Smink and van der Meer 1997, 134). Again this result emphasises the role and importance of the investment portfolio, rather than the underwriting portfolio, to life insurer conduct. Also, what becomes clear out of the work of Smink and van der Meer (1997) is the fact that computer scenario testing of financial risks is becoming increasingly important to modern insurer ALM.

The above results provide an indication of the significant considerations that insurance companies need to address in constructing an appropriate ALM strategy. In particular, useful insights into insurer objectives and constraints are made. While rate of return can be seen as a key objective, there is evidence to also suggest that this objective be constrained by specific risk considerations and that these are well understood within insurer ALM.

It is worth making a few additional comments at this stage. Given the variation in industry structure, objectives and constraints faced by asset/liability managers might also be expected to vary; for example, this variation may be cross-sectional or dynamic. As mentioned, cross-sectional effects may be reported
across the different ownership structures of insurance companies, with the objectives of mutual life insurers being different to that of proprietary life insurers. By contrast, judicial review and regulatory shift may bring about a dynamic effect with new regulations imposing additional constraints on any given ALM strategy.

Having reviewed some of the considerations relating to ALM conduct, it is now appropriate to consider the size of insurer portfolio holdings and their significance to the wider economy, see Figure 7.1. Unlike other aspects of market conduct, there exists a substantial amount of information on the composition of long-term insurance investment funds. Long-term funds represent, "...the actuarially calculated amounts held against companies’ liabilities to pay for future claims,” (Association of British Insurers 1997a, 70).

Figure 7.1. Long-Term Insurance Funds (£m)

Figure 7.1 demonstrates the nominal growth in long-term insurance funds over the period 1986-1996. Over this period funds have grown four-fold with the growth in the 1996 being in excess of 10 percent, a significant real increase.\textsuperscript{13}

In order to obtain an idea of the portfolio holdings of U.K. insurance companies, Table 7.5 also provides a breakdown of insurer investments held by asset class, with the value of total assets held by insurers for the period 1986-1995. It is evident that insurance companies engage in a variety of investment activities within the risk/return space; total assets for 1995 accounted for over £500 billion. This confirms the significance of long-term funds as an institutional investor.

In terms of the cross-sectional analysis, insurance companies can be seen to invest significantly overseas in addition to their domestic holdings. The importance of equities to the insurance companies' investment portfolio is clear with 44 percent of asset market value being made up of equities in 1995. This contrasts with public sector securities that account for approximately 20 percent of the total gross asset value.

Table 7.5 also illustrates the significant insurer holdings in property and mortgages that account together for nearly 10 percent of total assets. Note that short-term assets account for approximately 6 percent of asset market value. Note also that total investments account for over 90 percent of total assets and

\textsuperscript{13} The difference between the total value of long-term funds and the market value of total investment funds can be used as a proxy for the reserves held by insurance companies (Franklin and Woodhead 1980).
that this proportion has remained constant over the period. This clearly serves to highlight the importance of investment activities to the solvency position of the insurance company.

Table 7.5. Insurance Companies’ Asset Holdings as a Percentage (%) of Total Assets

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>UK Public Sector Securities</td>
<td>20.39</td>
<td>20.63</td>
<td>17.73</td>
<td>10.83</td>
<td>13.67</td>
<td>13.68</td>
<td>16.18</td>
<td>17.44</td>
<td>17.02</td>
<td>17.02</td>
</tr>
<tr>
<td>Overseas Securities</td>
<td>2.37</td>
<td>2.05</td>
<td>2.12</td>
<td>2.11</td>
<td>2.78</td>
<td>2.96</td>
<td>3.58</td>
<td>2.90</td>
<td>2.59</td>
<td>2.70</td>
</tr>
<tr>
<td>UK Equity</td>
<td>36.12</td>
<td>35.88</td>
<td>35.72</td>
<td>39.25</td>
<td>40.06</td>
<td>41.19</td>
<td>43.71</td>
<td>43.65</td>
<td>44.44</td>
<td></td>
</tr>
<tr>
<td>Unit Trusts</td>
<td>7.18</td>
<td>7.36</td>
<td>7.84</td>
<td>8.98</td>
<td>7.77</td>
<td>8.29</td>
<td>7.47</td>
<td>7.73</td>
<td>6.48</td>
<td>6.81</td>
</tr>
<tr>
<td>Loans and mortgages</td>
<td>2.90</td>
<td>3.19</td>
<td>3.10</td>
<td>2.87</td>
<td>3.34</td>
<td>2.77</td>
<td>2.53</td>
<td>1.86</td>
<td>1.77</td>
<td>1.53</td>
</tr>
<tr>
<td>Investments in Fixed Assets</td>
<td>12.93</td>
<td>14.44</td>
<td>15.94</td>
<td>15.38</td>
<td>14.23</td>
<td>11.27</td>
<td>8.91</td>
<td>7.60</td>
<td>8.45</td>
<td>6.84</td>
</tr>
<tr>
<td>Other Investments</td>
<td>0.12</td>
<td>0.09</td>
<td>0.12</td>
<td>0.25</td>
<td>0.26</td>
<td>0.29</td>
<td>0.52</td>
<td>0.51</td>
<td>0.87</td>
<td>0.31</td>
</tr>
<tr>
<td>Total Investments</td>
<td>92.27</td>
<td>91.27</td>
<td>90.75</td>
<td>90.51</td>
<td>88.10</td>
<td>89.25</td>
<td>89.94</td>
<td>92.65</td>
<td>92.05</td>
<td>91.02</td>
</tr>
</tbody>
</table>

When considering the trends over time, it becomes evident that, on average, asset class proportions remain relatively constant. The holdings in U.K. equities have increased from 36 percent in 1986 to 44 percent in 1995. In addition, the amount held in public sector securities has declined slightly, falling by 3 percent.
over the nine-year period. Short-term assets varied between 5 and 8 percent over the period. It should be stressed that these figures represent assets' market values. As a result, the above trends may not necessarily be taken as an indication of insurers taking a more aggressive equity position in their portfolios, since the changes may merely reflect shifts in underlying asset values; for example, stock market fluctuations might result in these relative holdings being changed significantly.\textsuperscript{14}

What explanations can be given for the above trends? In explaining these trends an account must be made of the requirements imposed by ALM. Table 7.5 demonstrates the significance of three major investment holdings for life assurance companies; namely equities, public sector securities, and property. In terms of the expected return and the variance of return, each asset category represents a distinct asset class.

Consider first the asset class of property. Property is by its very nature a medium to long-term investment vehicle. This therefore provides the insurance company with an instrument to 'match' its liabilities in addition to gilt-edged securities. Moreover, a historical justification for holding property is the belief in relatively high and predictable rates of returns.

Another reason for holding property might be associated with portfolio diversification. Property may provide a return, which is less than perfectly

\textsuperscript{14}In order to appreciate fully the investment activities of insurers, account must be taken of the \textit{net} investment, by class and per-period.
correlated with the return on other investments. One aspect of property that makes it unusual is its disposability. Property, unlike equities or short-term assets, is subject to sizeable transaction costs, both in terms of money and time. Such features make property suitable as an investment for the medium or long-term, but not so suitable as an investment for the short term. Table 7.6 demonstrates the net investment of long-term insurance funds for the period 1994 to 1996 inclusive, where the investments into fixed assets represent about 5 percent of total net investments in 1996.

Table 7.6. Insurance Companies' Net Investment (£m)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>UK Public Sector Securities</td>
<td>8069</td>
<td>6934</td>
<td>10997</td>
</tr>
<tr>
<td>Equities</td>
<td>16803</td>
<td>11165</td>
<td>14414</td>
</tr>
<tr>
<td>Loans &amp; Mortgages</td>
<td>-1052</td>
<td>794</td>
<td>-155</td>
</tr>
<tr>
<td>Fixed Assets</td>
<td>2928</td>
<td>363</td>
<td>1361</td>
</tr>
</tbody>
</table>


Note: Fixed assets include investment in land and existing buildings, property and ground rents in addition to new buildings.

While the above table demonstrates the contribution of property to insurance company net investments, it also serves to reinforce the dominance of equities and public sector securities as first shown in Table 7.5. Table 7.6 confirms that investments by long-term insurance funds are spread across a variety of risk/return classes, thereby ensuring adequate diversification.

15 Evidence on asset class correlation is provided in the simulation studies produced in Chapters 10 and 11.

16 One exception to this rule might be in times of rapidly rising property prices, where the expense of transactions costs can be offset against the profits realised from the increase in property values. When this is the case, the disposability of property increases and it might therefore be more suited to providing a short-term return.
Table 7.6 also illustrates the significance of gilt-edged securities, or gilts. Gilts benefit from the following investment features: a known term, a predictable return and maturity value, and a relatively high degree of marketability. As such, these securities are well placed to serve as a means of ‘matching’ within an immunization ALM strategy (see Chapter 4). Also, they provide an efficient means of portfolio diversification when combined with equities. In terms of risk, public sector securities have returns that are known with certainty. However, the limited capital gains associated with gilts is seen as an important motivation for holding equities, another major destination for long-term investment. Indeed the potential for substantial equity gains is significant. This helps to explain the motivation for holding a large portion of investment holdings in equities (approximately 44% in 1996 (Office for National Statistics 1998b).

Another major explanation of investments into equities comes from the composition of liability portfolio. Unit linked products have grown over the last 25 years. Since most of these funds are equity based, this might explain the growth in equity based investment holdings in recent years. In 1996, unit-linked business accounted for 32 percent of U.K. yearly life business. This contrasts with a figure of 25 percent at the beginning of the decade (Association of British Insurers 1997a, 13). The importance of underwriting portfolio composition upon the investment decisions of life assurance companies is considered below in Figure 7.2.
Figure 7.2 demonstrates the investment holdings of long-term insurance companies and the relationship that this has to the nature of the insurance business written. In particular, a breakdown of the various long-term investment holdings is given for linked long-term business and non-linked long-term business and this demonstrates a number of specific relationships. Recall the basic distinction between linked and non-linked contracts. With linked contracts, the performance of a policy depends upon the performance of the various 'units', which in turn are tied to the performance of individual asset classes. By contrast, non-linked plans aim to tie policy performance to the overall performance of the insurance company.

Figure 7.2. Breakdown of Long-Term Business Investment Holdings for Non-Linked and Linked Funds (%)


Note: IL stands for index-linked and NI stands for non index-linked. Also, the ‘debentures’ category includes loan stock as well as preference and guaranteed stocks and shares.
Considering the results of Figure 7.2, it is clear that differences exist between the investment profiles of linked and non-linked long-term funds. While the largest share of both of these funds is still made up by equities, the total exposure to equity-based investments remains much greater for linked contract funds. The most noteworthy example is the exposure of linked contracts to overseas equities, approximately 15 percent; this contrasts with only 8 percent exposure for non-linked contracts. This might signify that the risk preferences of the unit-linked funds are significantly different from that of the non-linked funds, the latter being more risk averse. This evidence is confirmed with respect to the investment in public sector securities, which attracts more investments from non-linked funds than linked funds. Also important is the exposure to equity unit trusts where the amount being invested by linked funds is over 20 percent, more than ten times the amount invested by non-linked contracts.

In explaining these results, it is important to understand that linked funds offer the policyholder the chance to opt for high risk/high expected return funds. By contrast, non-linked funds leave the investment allocation decision to the investment manager of the life fund. As non-linked funds are considered relatively low risk for the policyholder, one would therefore expect this fact to be reflected in associated investment holdings; for example, non-linked funds would be expected to invest more heavily in land and property and ordinary government securities than linked funds.
Figure 7.2 would seem to support the hypothesis that the composition of insurance company asset holdings is a function of the nature of the business written (and in turn the risk preference of the associated policyholders). In particular, the risk associated with the business written and its respective investment holdings appear to be similar; implying asset/liability matching. This result is not surprising given the requirements of policyholders and the structure of the current U.K. life assurance regulations in respect of immunization (see Chapters 2 & 4). Another important measure of an investment strategy concerns the income derived from the long-term insurance business funds and this is given below in Figure 7.3.

Figure 7.3. Long-Term Business Investment Income 1986-1996
Figure 7.3 provides a breakdown of investment income along linked and non-linked businesses. The illustration shows a rise in total net investment income from £10 billion in 1986 to over £33 billion by 1996. There has been a particularly strong performance in the last year of the period, with income growing at a rate of 10 percent between 1995 and 1996. In addition to this trend, there has been some growth reported in the proportion of income received from linked long-term investment funds. Over the period, investment income associated with linked business has grown from 17 percent in 1986 to 23 percent in 1996. This data is consistent with the explanation associated with Table 7.5; namely, the specific relationship between the insurer’s life funds and the associated investment holdings.

In addition to the above, two further points should be made in respect of insurance company venture capital provision and the use of financial derivatives. In 1996, insurance companies contributed 9 percent to the total venture capital raised in the U.K. (£240 million), (Association of British Insurers 1997b, 6). The growth in such funds is another direct consequence of the unit-linked policies that specialise in these specific investment funds. Insurance company regulations have also reduced the barriers to insurance companies providing venture capital, while also allowing the greater use of financial derivatives such as futures and options in investment activities (Association of British Insurers 1997b, 6).
From the above discussion, it is clear that there are a number of different ALM strategies that are being employed, and that these depend on the ownership structure and the product composition of the insurance company. However, the importance of rate of return remains significant within the objectives of insurance fund managers.

### 7.4 Acquisition and Merger Objectives

The acquisition and merger of insurance companies is another major component of industry conduct. In particular, acquisitions and mergers can be taken as evidence of sales maximisation, subject to a valuation constraint relating to the proposed organisational change and the ability of the insurance company to finance its expansion. The number and size of mergers and acquisitions have been growing in the last five years. Table 6.18 provides evidence on the acquisitions and mergers for the period 1996/97.

#### Table 7.7. Takeovers and Mergers in the U.K. Life Assurance Industry for 1996/97

<table>
<thead>
<tr>
<th>Mergers</th>
<th>Takeovers and Acquisitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>French Insurers AXA and UAP announced plans to merge in November</td>
<td>Acquirer: Halifax</td>
</tr>
<tr>
<td>Royal Insurance and Sun Alliance merged in July</td>
<td>Acquired: Clerical Medical</td>
</tr>
<tr>
<td>United Friendly and Refuge Assurance merged to form United Assurance</td>
<td>Liverpool Victoria</td>
</tr>
<tr>
<td>Lloyds TSB announced plans to merge TSB's insurance business with Lloyds</td>
<td>General Accident: Provident Mutual</td>
</tr>
<tr>
<td>Abbey Life</td>
<td>Life Assurance Holding Corporation: Combined Life</td>
</tr>
</tbody>
</table>

Note that with the introduction of the Euro in 1999, the pace of change in mergers and acquisitions has increased substantially. The development of the Euro-zone sets a challenge to all types of businesses, especially financial businesses. Consolidation is motivated, in part, out of an ambition to establish a pan-European operation and product. To see this change, consider one of the most prevalent insurers, namely the powerhouse of European insurance, the French insurer, AXA Equity Law. In 1998 AXA merged with Sun Life. Then earlier this year, AXA succeeded in buying Guardian Royal Exchange for £3.4 billion (The Economist, February 6th 1999, 83).

In line with the argument outlined above, mergers and acquisitions are typically explained in the insurance industry by one of the following factors: consolidation, globalization, over-capacity in home markets, and access to distribution networks (Association of British Insurers 1997b, 19). The first three factors can be seen as important in the development of insurance company market share in international insurance markets. The latter factor concerns the ability of insurance companies to secure a sales network and establish a dominant market share position.

7.5 Summary

Various aspects of life assurance industry conduct have been illustrated. While there are a number of theoretical accounts relating to firm conduct, there are practical limitations associated with any empirical analysis, not least that of commercial sensitivity. The aim of this chapter has been to provide an overview
of some of the observable aspects of conduct within the U.K. life assurance industry.

There are a number of significant factors to consider in the understanding of insurer conduct. Of great importance are the risk preferences of the insurance company owners, suggesting that there is an insurer objective function that is a constrained utility maximisation rule subject to the articles of association. Thus, insurer conduct is concerned with more than just rate of return maximisation; sales growth is another factor considered important in the conduct of life insurers.
CHAPTER 8

THE PERFORMANCE OF THE
U.K. LIFE ASSURANCE INDUSTRY

8.0 Introduction

Previous chapters have examined the structure and conduct of the U.K. life assurance industry. In line with the structure, conduct, performance (SCP) paradigm, attention now turns to assessing the performance of the life assurance industry. Performance, as defined within the SCP paradigm, can be represented by a myriad of different measures, reflecting aspects of performance such as allocative efficiency, supernormal profits, innovation and equity.

From a regulatory perspective, industrial performance measures attempt to capture the extent to which a given industry departs from a competitive equilibrium. Standards by which to measure industrial performance include the use of rate of return as a performance benchmark, and also the use of the price-cost margin as a performance measure. Consider first, the use of the rate of return, $r$. Performance is measured by the deviation of $r$ from a benchmark $r^*$ that is $r-r^*$ (for examples of this approach within industrial organisation see Bain (1956) and Mann (1971)). Next consider the price-cost margin as another form of performance measure. Using this approach, performance is measured by the deviation of price $P$ from marginal cost $MC$, that is $P-MC$ (for examples of this approach using different conduct assumptions, see Cowling (1976), Cowling and Waterson (1976), Saving (1970), and Geroski (1982)).
While much can be said about the alternative theoretical approaches from which to assess insurance company performance, in practice such terms have no definitive empirical measurement. As a consequence, it is useful to take a ‘basket’ of different empirical performance indicators, both at the microeconomic and macroeconomic level. Although some of these measures may not fall within the traditional SCP notion of performance, they nevertheless provide an indication of industrial performance that is also directly or indirectly related to profitability and efficiency. The indicators that will be discussed here include, firm level profitability, the size of policy benefits, the returns on insurance investment funds, and the solvency assessments by the industry regulators. In addition, the contributions made by insurance companies to the macro-economy can be gauged in terms of employment, overseas invisible trade, and the levels of institutional investment.

Discussion will proceed in Section 8.1 with an evaluation of the microeconomic performance indicators, examining in particular the profitability of insurance companies, the performance of various life funds, and the results of the annual regulatory reports. This discussion will then be followed in Section 8.2 by an examination of the macroeconomic measures of performance, emphasising the role of insurance companies as a source of funds to the capital markets, as well as the contributions made by insurers to invisible earnings in the Balance of Payments. Section 8.3 will bring the SCP discussion to a close, providing a summary of the main developments.
8.1 Life Assurance Performance: a Microeconomic Perspective

This section considers aspects of performance relating to firm-level data. As such, discussion commences by addressing the traditional notion of performance, namely the profitability of firms. As a supplement to this, information is also presented on the performance of different insurance funds, which in turn can provide an indication of the relative sizes of the returns to policyholders and/or shareholders.

8.1.1 Profits in Life Assurance

As reported within previous chapters, life assurance companies are owned by a combination of policyholders and shareholders. As a result, policyholders and shareholders will require a return in terms of profit and life fund performance, respectively.¹

Consider first the significance of profits. The term ‘profit’ has been the subject of historical debate in attempting to find a universally acceptable empirical definition.² In comparing company data from profit and loss accounts, conclusions may only be drawn with respect to the accounting profit. This measure of profit is subject to the accountant’s assumptions underpinning his or her judgements, and as such this restricts the ability to make straightforward

¹ Fund performance for ‘with-profit’ contracts will be measured in terms of the annual bonus declarations.
² Probably one of the most frequently cited explanations of the term profit comes from Frank Knight in *Risk, Uncertainty and Profit* first published in 1921 (Knight 1971).
comparisons of insurer profitability. Add to this the central role of the Actuary, and there are then further assumptions that may vary between insurers.

Recall that the Actuary makes a solvency assessment of an insurance company's liabilities. Only after the full consideration of these liabilities will the Actuary report a surplus on the life fund. On the basis of these results the Actuary then decides how much to allocate to profits and bonuses (for a proprietary life insurer). In the case of a proprietary life assurance company, the allocation between policyholders and shareholders will depend critically upon the assumptions underpinning the calculation of surplus (Franklin and Woodhead 1980, 274). Thus, the fact that so many assumptions need to be imposed by accounting and actuarial assessments necessitates great caution in respect of any inter-firm performance comparison.

Having addressed some of the concerns associated with company data, the measures of performance used here are now defined. As demonstrated, the profitability associated with insurance companies is intimately related with the solvency position and the surplus reported on the life fund. While the surplus allocation decision is relatively straightforward for a mutual insurer, this allocation is less clear for a proprietary insurer. For proprietary insurers, the amount of surplus allocated to shareholders is given in the valuation reports as a transfer to the Profit and Loss Account (Insurance Company Accounts and Statements 1996, Form 60). The allocation of bonuses to the policyholders is given within the Distribution of Surplus Account (Insurance Company Accounts and Statements 1996, Form 60). Transfers to the profit and loss account
therefore provide a measure of the shareholders' share of the insurer's profits and hence the overall performance of the firm. Table 8.1 provides an illustration of the typical returns experienced within the life assurance industry in 1997 using this measure of performance.

Table 8.1. Inter-Firm Comparison of Selected Proprietary Life Insurers’ Profitability in 1997.

<table>
<thead>
<tr>
<th>Name of Firm</th>
<th>Amount Transferred to Profit and Loss Account £000</th>
<th>Worldwide Net Premium Income £000</th>
<th>Long-Term Insurance Fund £000</th>
</tr>
</thead>
<tbody>
<tr>
<td>AXA Equity &amp; Law</td>
<td>63824</td>
<td>4.604906</td>
<td>0.812567</td>
</tr>
<tr>
<td>Eagle Star</td>
<td>106200</td>
<td>8.309859</td>
<td>1.549464</td>
</tr>
<tr>
<td>Legal and General</td>
<td>172800</td>
<td>6.090941</td>
<td>0.922216</td>
</tr>
<tr>
<td>Pearl</td>
<td>100376</td>
<td>7.978203</td>
<td>1.191352</td>
</tr>
<tr>
<td>Prudential</td>
<td>218200</td>
<td>2.384178</td>
<td>0.777196</td>
</tr>
</tbody>
</table>


Table 8.1 provides some examples of the size of surplus transfers to shareholders (column 1). In order to standardise the above results for comparison, account is made of the relative sizes of the insurance companies by calculating the size of surplus transfers as a proportion of the insurer’s worldwide net premium income (column 2), and as a proportion of the insurer’s long-term insurance fund (column 3). It can be seen that, regardless of the relative measure used, there is significant variation in profitability. As will be demonstrated in Chapter 9, the cross-sectional variation in insurer profitability may be explained by a variety of factors. However, it is sufficient to note here that if insurance companies have different liabilities, and actuaries make alternative assumptions on how to treat these liabilities, then inter-firm performance variability is a likely consequence (Franklin and Woodhead 1980, 277).
In addition to cross-sectional variations in profitability, there is also some variability over time. Table 8.2 provides an illustration of performance variation over time.

Table 8.2. Inter-Firm Comparison of Selected Proprietary Life Insurers’ Profit Growth between 1996 and 1997.

<table>
<thead>
<tr>
<th>Name of Firm</th>
<th>1997 Profits as a Percentage of 1996 Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>AXA Equity &amp; Law</td>
<td>126.5</td>
</tr>
<tr>
<td>Eagle Star</td>
<td>129.3</td>
</tr>
<tr>
<td>Legal and General</td>
<td>112.2</td>
</tr>
<tr>
<td>Pearl</td>
<td>31.4</td>
</tr>
<tr>
<td>Prudential</td>
<td>115.2</td>
</tr>
</tbody>
</table>


While the inter-firm variation in profitability is significant over time, it is important to note the relationship between the amount of transferred surplus and the actual amount received by shareholders in the form of dividends. This relationship is relatively straightforward for insurance companies specialising in long-term business only. However, as noted in Chapter 6, many insurance companies are composite insurers. As a consequence, transfers from long-term business to the profit and loss account may be used to subsidise the activities of the general insurance funds and hence the return received by the shareholder will be affected accordingly.

Mutual insurers also make an important contribution to the life assurance industry. A measure of profitability can be gained with reference to the total

---

3 Recall from Chapter 6 that composite insurers are insurance companies that engage in general insurance as well as long-term insurance.
surplus reported in the valuation balance sheet. In contrast to the proprietary life insurers, mutual insurers only allocate their surplus to policyholders.

Table 8.3. Inter-Firm Comparison of Selected Mutual Life Insurers’ ‘Profitability’ in 1997.

<table>
<thead>
<tr>
<th>Name of Mutual</th>
<th>Total Surplus as shown in Valuation</th>
<th>Total Surplus as a Percentage of End of Year Long Term Funds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friends Provident</td>
<td>580836</td>
<td>5.639487</td>
</tr>
<tr>
<td>Scottish Provident</td>
<td>231728</td>
<td>3.90123</td>
</tr>
<tr>
<td>Standard Life</td>
<td>782693</td>
<td>2.521317</td>
</tr>
</tbody>
</table>


Table 8.3 illustrates that the ‘profitability’ associated with mutual insurers is also variable and might also be explained by the function of the Actuary. The distribution of profitability within the life assurance industry is illustrated below in Figure 8.1. The distribution accounts for some 80 percent of U.K. worldwide premium income.

Figure 8.1. Inter-Firm Profitability of Life Insurers in 1997.


Notes: (a) Profit reported as a percentage of total long-term insurance funds.
       (b) Sample size of 103 insurance companies authorised to conduct business in the U.K.
Figure 8.1 depicts the profitability of proprietary life insurers and mutual life insurers using the definitions of profit in Table 8.1 and Table 8.3, respectively.\textsuperscript{4} ‘Profits’ are calculated as a percentage of the total long-term funds. As indicated in the above frequency distribution, the mean profitability for the sample of 103 insurance companies is 4.24 percent. The distribution about this mean is significant with a standard deviation of 2.43 percent. The distribution is relatively symmetrical about the mean with an approximate range in profitability of 14 percent, from −4 percent to +10 percent. However, the rates of return for the majority of insurance companies appear to be concentrated about the mean rate of return (the coefficient of kurtosis was equal to 91.3). This provides evidence to suggest that profit rates are similar across the majority of life insurers in the U.K. life assurance industry.\textsuperscript{5}

8.1.2 Policy performance in Life Assurance

The arguments developed within the previous section served to highlight the ambiguity surrounding much of the reported information within accounting data. The variation in actuarial assumptions complicate assessment and for these reasons the validity and meaningfulness of the profitability results is brought into question. As a consequence, another perspective from which to assess inter-firm performance in the life assurance industry is with respect to the associated performance of the substantive policies. The performance of substantive policies provides an indication of the returns to the policyholders in the form of policy

\textsuperscript{4} Recall that mutual insurers do not make profits per se, but instead have their performance measured in terms of life fund surplus.

\textsuperscript{5} The benchmark for assessing the performance of the life assurance industry is the average rate of return.
bonuses. Thus, emphasis here is placed upon bonus declarations rather than accounting profitability. To provide an account of the variation in bonus declarations across insurance companies, Table 8.4 provides information on the performance of endowment policies as measured by their maturity values listed for a pre-specified individual over a 25-year term.

Table 8.4. Top 15 With-Profit Endowment Maturity Values, 1998

<table>
<thead>
<tr>
<th>Company name</th>
<th>1998 Maturity Value</th>
<th>1998 Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.I.S.</td>
<td>£97,365</td>
<td>13</td>
</tr>
<tr>
<td>Commercial Union</td>
<td>£108,210</td>
<td>4</td>
</tr>
<tr>
<td>Friends Provident</td>
<td>£106,188</td>
<td>9</td>
</tr>
<tr>
<td>GA Life</td>
<td>£120,784</td>
<td>2</td>
</tr>
<tr>
<td>Norwich Union</td>
<td>£100,247</td>
<td>11</td>
</tr>
<tr>
<td>Prudential</td>
<td>£106,278</td>
<td>8</td>
</tr>
<tr>
<td>Rechabite FS</td>
<td>£84,357</td>
<td>15</td>
</tr>
<tr>
<td>Scottish Friendly</td>
<td>£90,301</td>
<td>14</td>
</tr>
<tr>
<td>Scottish Life</td>
<td>£103,325</td>
<td>10</td>
</tr>
<tr>
<td>Scottish Mutual</td>
<td>£108,544</td>
<td>3</td>
</tr>
<tr>
<td>Scottish Provident</td>
<td>£94,820</td>
<td>12</td>
</tr>
<tr>
<td>Scottish Widows</td>
<td>£107,941</td>
<td>5</td>
</tr>
<tr>
<td>Standard Life</td>
<td>£107,379</td>
<td>6</td>
</tr>
<tr>
<td>Tunbridge Wells Equitable FS</td>
<td>£106,573</td>
<td>7</td>
</tr>
<tr>
<td>Wesleyan</td>
<td>£120,959</td>
<td>1</td>
</tr>
</tbody>
</table>


Note: (a) Reported maturity values for a 25-year term endowment.
(b) Results for a male with age-next-birthday equal to 30, contributing £50 per month.
(c) Median maturity value for the top 15 insurers is £106,278.

Table 8.4 demonstrates the significant variation in maturity values of a 'with-profit' endowment policy. The range for the top 15 insurance companies is £36,602. The above results illustrate the variation in policy performance and associated fund performance and bonus declarations. Bonus declarations form the basis of growth for the 'with-profit' endowment plans. Bonuses consist of reversionary and terminal bonuses and the size of these bonuses provides another measure of the returns to the policyholders. An indication of the significance of alternative forms of bonus is given in Table 8.5 below.
Table 8.5. 1998 Bonus Declarations for Top 12 Life Companies

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C.I.S.</td>
<td>3.25% (5)</td>
<td>4% (10)</td>
<td>47%-230%</td>
</tr>
<tr>
<td>Commercial Union</td>
<td>2.5% (8)</td>
<td>4.5% (5)</td>
<td>12%-42.5%</td>
</tr>
<tr>
<td>Clerical Medical</td>
<td>2.5% (8)</td>
<td>4.5% (5)</td>
<td>9.5%-13.6%</td>
</tr>
<tr>
<td>Friends Provident</td>
<td>2.75% (7)</td>
<td>4% (10)</td>
<td>95%-187.5%</td>
</tr>
<tr>
<td>GA Life</td>
<td>3.25% (4)</td>
<td>6.25% (2)</td>
<td>% based on sum assured</td>
</tr>
<tr>
<td>Norwich Union</td>
<td>2.5% (8)</td>
<td>4% (10)</td>
<td>% based on sum assured</td>
</tr>
<tr>
<td>Rechabite FS</td>
<td>10% (1)</td>
<td>N/A</td>
<td>50%</td>
</tr>
<tr>
<td>Scottish Friendly</td>
<td>3.5% (2)</td>
<td>6.5% (1)</td>
<td>4.5% for each full year excluding first six % based on sum assured</td>
</tr>
<tr>
<td>Scottish Provident</td>
<td>3.5% (2)</td>
<td>5.25% (3)</td>
<td>4%-298%</td>
</tr>
<tr>
<td>Standard Life</td>
<td>3% (6)</td>
<td>4.25% (9)</td>
<td>% based on sum assured</td>
</tr>
<tr>
<td>Tunbridge Wells FS</td>
<td>2.5% (8)</td>
<td>5% (4)</td>
<td>special bonus 6.25% of basic sum assured</td>
</tr>
<tr>
<td>Wesleyan</td>
<td>2.5% (8)</td>
<td>4.5% (5)</td>
<td></td>
</tr>
</tbody>
</table>


Notes: (a) Results presented are given with the associated ranking in parentheses.
(b) 'Bonus on Sum Assured' represents reversionary bonuses for the year.

As implied in Table 8.4, there is significant variation in associated insurance company bonus declarations for with-profit policies. This variation is particularly prevalent for terminal bonuses where this bonus can be up to 300 percent of the sum assured. However, the format that terminal bonuses can take may vary between insurers; for example, terminal bonuses may or may not be based on the size of the sum assured. By contrast, the variation in reversionary bonuses is much less significant with a bonus spread of only 7.5 percent of the sum assured. The highest ranked company, in terms of its declared reversionary bonus, is Rechabite FS that declared a 1998 reversionary bonus of 10 percent.

The results depicted in the above table reflect the bonus preferences of the insurer. In particular, it is likely that those insurers that have a relatively poor performance in reversionary bonus declarations are likely to provide greater terminal bonuses. For an example of this contrast between the two types of bonuses see the results for Friends Provident as compared with Rechabite FS; these two insurers demonstrate significant variation in reversionary and terminal bonuses.
bonuses. Regardless of the nature of the policy bonus, it is evident that policyholders receive a relatively modest level of annual (or reversionary) bonuses. Just under half of the total sample of insurance companies interviewed had reversionary bonuses within 2 percent of the average (Moneyfacts 1998, 15). Thus, there is further evidence to support the view that the performances of insurers (as measured in terms of the annual bonus declaration) are centred about the average rate of return.

What explains the reported differences across companies in terms of their bonus declaration strategy? In order to provide an answer to this question, it is necessary to be clear over the differences between reversionary and terminal bonuses. Reversionary bonuses are attached to a policy each year while terminal bonuses can only be attached at the end of the policy term. Bonuses can be used in a strategy that attempts to ensure policyholder loyalty. In a strategy to reduce policy surrenders, insurance companies offer high terminal bonuses that result in the majority of the savings fund being accumulated in the final year of a policy’s term. This therefore encourages policyholder loyalty throughout the full term, in order that a policyholder may derive the maximum benefits from the substantive plan. Note that high reversionary bonuses may represent a significant liability for the insurer at the end of the policy term. Reversionary bonuses, once declared, are guaranteed. These annual bonuses are redeemed at the end of the policy term where the market conditions may be significantly

---

6 Placing greater emphasis upon terminal bonuses means that fund performance will be more heavily dependent upon the market conditions at the end of the policy’s term. In contrast, a bonus strategy that focuses upon reversionary bonuses ensures that the fund performance is more dependent on the market conditions for all the years preceding the end of a policy’s term.

7 In addition to annual and terminal bonuses, there may also be ‘loyalty bonuses’ for policyholders that remain with an insurer over a given period of time.
different to those conditions prevailing at the time of the bonus declaration. Thus, in the event that the market conditions at the end of the policy term are poor, an insurer may have to realise its assets at a greatly depreciated value to that value assumed by the original bonus declaration. By contrast, placing a greater emphasis upon terminal bonuses provides a further area of flexibility for the insurance company in terms of its ability to offset any losses associated with interim bonuses.

The relative sizes of insurance benefits are another measure by which to assess the microeconomic performance of the life assurance industry. Figure 8.2 demonstrates the significance of policy surrendering to the insurance companies’ cash flows.

Figure 8.2. Breakdown of Benefits Paid on U.K. Ordinary Life Insurance 1986-1996


In 1996, surrenders stood at £7085m, some 43 percent of total benefit payments (Association of British Insurers 1997a). This represents a significant drain on
insurer liquidity and also suggests that current policyholder incentive schemes may not be operating as first thought. A further point to note with respect to Figure 8.2 is that the total amount paid out in policy maturities is equal to £6986m in 1996. Death claims represent the smallest proportion of total benefits, accounting for some 14 percent. The relative sizes of death claims, maturities and surrenders have remained similar over the period 1986-1996, although the proportion of benefits paid out in maturities has experienced some growth. In terms of the total benefits, the amount paid was £16352m in 1996, nearly three times the amount paid out in 1986. To provide an account of the product variation in performance, Table 8.6 provides a breakdown of the insurance benefits across 5 different product categories.

Table 8.6. Breakdown of Benefits for Different Product Lines in 1996

<table>
<thead>
<tr>
<th></th>
<th>Ordinary Life</th>
<th>Industrial Life</th>
<th>Annuity</th>
<th>All Pensions</th>
<th>PHI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Death Claims</td>
<td>5.66%</td>
<td>1.13%</td>
<td>0.05%</td>
<td>1.86%</td>
<td>*</td>
</tr>
<tr>
<td>Maturities/Lump sums on</td>
<td>17.32%</td>
<td>3.51%</td>
<td>0.10%</td>
<td>11.52%</td>
<td>*</td>
</tr>
<tr>
<td>Maturity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surrenders/Refunds on Pensions</td>
<td>17.57%</td>
<td>0.74%</td>
<td>0.12%</td>
<td>26.80%</td>
<td>*</td>
</tr>
<tr>
<td>Periodical Payments</td>
<td>n/a</td>
<td>n/a</td>
<td>2.35%</td>
<td>10.39%</td>
<td>*</td>
</tr>
<tr>
<td>Total</td>
<td>40.54%</td>
<td>5.38%</td>
<td>2.63%</td>
<td>50.58%</td>
<td>0.89%</td>
</tr>
</tbody>
</table>


Notes:
(a) All figures expressed as a percentage of total benefits in 1996.
(b) Periodical payments exist only for annuity and pension products.

Table 8.6 illustrates the dominance of ordinary life and pensions benefits within the total benefit payments of insurance companies. Together ordinary life and pension benefits account for over 90 percent of the total insurance benefits. In terms of the breakdown of benefits within each product category, surrenders make up the greatest proportion of these benefits accounting for 44 percent of total benefits. Periodical payments on pensions and annuity business is also
significant accounting for just under 13 percent of the total. The results are
determined largely as a result of product features: pensions and life business are
made up largely of substantive policies that have the option of being surrendered
at any given time for a cash value and hence this explains the sizeable surrender
values reported.

As outlined in Chapter 6, there are many different unit-linked funds available. In
order to provide a brief account of the performance of these funds, Table 8.7
presents details on the performance of ‘Managed’ funds for various life
insurers. Table 8.7 illustrates the variation in fund performance across firm and
time. The largest insurers tend to be located in the mid-point of the distribution
between the highest and lowest return, with insurer performance being relatively
similar overall. As expected, the range between the highest and lowest fund
performances increases with time and associated fund size.

Also note that the spread within the top 10 insurers is much less than the spread
associated with the market as a whole. In particular, the 10-year fund
performance range for the top insurers was equal to £376 in 1998, whereas the
figure for the entire market was £2970 in 1998 (Moneyfacts 1998). In terms of
fund performance volatility, the greatest variation is associated with Sun Life
(2.37 percent), the lowest being reported by Royal & Sun Alliance (1.95
percent). Since the managed fund consists of a mix of investments, it provides a
useful summary indicator of the experience associated with all unit-linked funds.
The composition of managed funds may vary across insurers and thus

8 Recall from Chapter 6 that a managed fund invests into a variety of asset classes.
performance variations might reflect the different mix of assets within these respective managed funds.

Table 8.7. Managed Fund Performance for Largest Insurers

<table>
<thead>
<tr>
<th>Company</th>
<th>Bid/Offer spread %</th>
<th>3 Month (bid-bid)</th>
<th>1 Year (bid-bid)</th>
<th>3 Years (bid-bid)</th>
<th>5 Years (bid-bid)</th>
<th>10 Years (bid-bid)</th>
<th>Volatility (3 years) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highest Performance *</td>
<td></td>
<td>1206</td>
<td>1310</td>
<td>1903</td>
<td>2222</td>
<td>4438</td>
<td>3.55</td>
</tr>
<tr>
<td>Lowest Performance *</td>
<td></td>
<td>998</td>
<td>1058</td>
<td>1214</td>
<td>1386</td>
<td>1468</td>
<td>1.55</td>
</tr>
<tr>
<td>10 Largest Insurers-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prudential</td>
<td>4.01</td>
<td>1113</td>
<td>1222</td>
<td>1573</td>
<td>1774</td>
<td>2697</td>
<td>2.2</td>
</tr>
<tr>
<td>Standard Life</td>
<td>5</td>
<td>1104</td>
<td>1194</td>
<td>1519</td>
<td>1714</td>
<td>2881</td>
<td>2.01</td>
</tr>
<tr>
<td>Norwich Union</td>
<td>5</td>
<td>1101</td>
<td>1184</td>
<td>1509</td>
<td>1762</td>
<td>N/A</td>
<td>2.02</td>
</tr>
<tr>
<td>Legal &amp; General</td>
<td>5</td>
<td>1100</td>
<td>1199</td>
<td>1535</td>
<td>1738</td>
<td>2741</td>
<td>2.23</td>
</tr>
<tr>
<td>Equitable Life</td>
<td>5</td>
<td>1107</td>
<td>1214</td>
<td>1572</td>
<td>1777</td>
<td>2678</td>
<td>2.03</td>
</tr>
<tr>
<td>Royal &amp; Sun Alliance</td>
<td>5</td>
<td>1105</td>
<td>1204</td>
<td>1509</td>
<td>1673</td>
<td>2556</td>
<td>1.95</td>
</tr>
<tr>
<td>Scottish Equitable</td>
<td>5.02</td>
<td>1101</td>
<td>1206</td>
<td>1536</td>
<td>1779</td>
<td>2790</td>
<td>2.23</td>
</tr>
<tr>
<td>General Accident</td>
<td>4.97</td>
<td>1103</td>
<td>1191</td>
<td>1495</td>
<td>1717</td>
<td>2522</td>
<td>2.09</td>
</tr>
<tr>
<td>Sun Life</td>
<td>5</td>
<td>1106</td>
<td>1199</td>
<td>1496</td>
<td>1699</td>
<td>2838</td>
<td>2.37</td>
</tr>
<tr>
<td>Scottish Widows</td>
<td>5.02</td>
<td>1095</td>
<td>1190</td>
<td>1531</td>
<td>1663</td>
<td>2514</td>
<td>2.13</td>
</tr>
</tbody>
</table>


Notes: (a) Figures presented give the fund performance based on a lump sum invested of £1,000.
(b) Results assume bid-to-bid net income reinvested at a given pay date.
(c) Volatility is in performance is captured by the standard deviation of the monthly performance over 3 years.
(d) The size of the insurer is defined in terms of the worldwide net premium income.
(e) The list of the largest insurers excludes Commercial Union Assurance. Figures were not available for this insurance company.
(f) Closing prices as of 31st March 1998.

As noted in Chapter 6, unit-linked plans offer many alternative funds into which premiums can be invested. In contrast to the managed fund, the performance of alternative unit-linked funds varies significantly. Table 8.8 illustrates this variation across the different funds (the managed fund is given for completeness).

Table 8.8 provides evidence on the nature of the risk/return characteristics for the various unit-linked funds. It also illustrates the fund diversity available in
the insurance market. Moreover, the number of new funds available within the unit-linked product space has increased three-fold in the period 1990-1996 and thus this implies a significant rate of innovation (Association of British Insurers 1997b).

Table 8.8. Variation in Life Fund Performance

<table>
<thead>
<tr>
<th>Average Performance over 10 years</th>
<th>Average Volatility over 3 years</th>
<th>Average Fund Size (£m)</th>
<th>Fund Size as a % of Total Average Funds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Managed</td>
<td>2586</td>
<td>2.17</td>
<td>247.2</td>
</tr>
<tr>
<td>Distribution</td>
<td>2900</td>
<td>1.75</td>
<td>180.99</td>
</tr>
<tr>
<td>Friendly Societies</td>
<td>2854</td>
<td>2.16</td>
<td>29.69</td>
</tr>
<tr>
<td>UK Equity General</td>
<td>2994</td>
<td>2.3</td>
<td>119.7</td>
</tr>
<tr>
<td>UK Equity High Yield</td>
<td>3029</td>
<td>2.12</td>
<td>35.33</td>
</tr>
<tr>
<td>UK Smaller Companies</td>
<td>1970</td>
<td>2.88</td>
<td>4</td>
</tr>
<tr>
<td>UK Growth Specialist</td>
<td>2491</td>
<td>2.67</td>
<td>9.43</td>
</tr>
<tr>
<td>International</td>
<td>2631</td>
<td>3.3</td>
<td>31.24</td>
</tr>
<tr>
<td>European</td>
<td>3932</td>
<td>3.23</td>
<td>23.73</td>
</tr>
<tr>
<td>North American</td>
<td>3861</td>
<td>3.69</td>
<td>12.83</td>
</tr>
<tr>
<td>Far East- incl. Japan</td>
<td>1503</td>
<td>4.89</td>
<td>8.15</td>
</tr>
<tr>
<td>Far East- excl. Japan</td>
<td>2953</td>
<td>6.67</td>
<td>6.67</td>
</tr>
<tr>
<td>Japan</td>
<td>812</td>
<td>6.12</td>
<td>8.62</td>
</tr>
<tr>
<td>Commodity &amp; Energy</td>
<td>1434</td>
<td>5.56</td>
<td>0.5</td>
</tr>
<tr>
<td>Money Funds</td>
<td>1775</td>
<td>0.09</td>
<td>14.67</td>
</tr>
<tr>
<td>Currency Funds</td>
<td>1652</td>
<td>1.73</td>
<td>1.4</td>
</tr>
<tr>
<td>Fixed Interest- Sterling</td>
<td>2081</td>
<td>1.37</td>
<td>30.5</td>
</tr>
<tr>
<td>Fixed Interest- International</td>
<td>1805</td>
<td>1.6</td>
<td>3.85</td>
</tr>
<tr>
<td>Index-linked Gilt Funds</td>
<td>2111</td>
<td>1.37</td>
<td>5.27</td>
</tr>
<tr>
<td>Property Funds</td>
<td>1768</td>
<td>1.2</td>
<td>28.54</td>
</tr>
</tbody>
</table>


Notes: (a) Figures give the fund performance based on a lump sum investment of £1,000.
(b) Bid-to-bid net income assumed to be reinvested at pay date.
(c) Volatility is measured by the standard deviation of the monthly performance over 3 years.
(d) Closing prices as at 31st March 1998.

Table 8.8 illustrates that the risk (standard deviation) and return associated with each fund is in line with the predictions of portfolio theory (see Chapter 5). For example, the risk associated with equity based investments is much larger than the risk associated with government securities. Hence, the greater the reported return, the greater the risk (that is, there is a positive risk/return trade-off). However, the Far East fund is the notable exception to this generalisation. This
fund experienced high variation in return, yet at the same time the fund performance was relatively poor. The financial crisis in the Far East explains these unusual patterns, this effect being most prominent in Japan, the major destination for investments of this type (Moneyfacts 1998, 33).

Thus, there exists a substantial amount of available data with respect to the performances of the life policies. Such information is relevant to an assessment of industrial performance because it represents the return to policyholders that are, at least in part, the owners of insurance companies. Moreover, with the ambiguity surrounding profitability assessments, the returns on policies provide useful additional evidence relating to inter-firm performance. In line with the profitability results, the evidence presented on policy performance suggests that substantive policy performance be centred about the industry average.

Note that economies of scale are another microeconomic performance consideration. It is clear that there exist many differences in product performances across insurance companies. While this might be explained by the performance of the investment managers and general investment conditions, it may also be the result of other factors including insurer size and fund size; economies of scale may therefore play an important role. Hardwick (1994) provides a statistical cost analysis of the U.K. life assurance industry analysing the extent of scale and scope economies. Hardwick assumes that a life insurer's outputs can be aggregated into three products: life policies, pensions and health policies:

The results demonstrate that there are positive economies of scope in the production of pensions and permanent health insurance, but
diseconomies of scope otherwise. This evidence suggests that there are potential costs savings achievable from the creation of separate offices for life assurance on the one hand and pensions and permanent health on the other. With regard to economies of scale, the results indicate that there are product-specific economies of scale attributable to all three insurance products. Taking the diseconomies of scope together with the product-specific economies of scale, we find evidence of roughly constant ray average costs for small and medium-sized companies, but significant overall economies of scale for large companies. (Hardwick 1994, 83)

Thus, the findings of Hardwick (1994) confirm the findings in Chapter 7 on mergers and acquisitions; namely, that there is a tendency towards greater industrial concentration within the U.K. life assurance industry. The potential economies of scale will therefore have important implications for the performance of the life assurance industry over the coming years.

8.1.3 Regulatory Perspectives

A further microeconomic indicator of insurer performance comes in the form of the annual regulatory review by the industry regulator. Each year the regulator is required to produce a report on the solvency performance of insurance companies, as authorised in the 1982 Insurance Companies Act. There exists an intimate relationship between solvency and profitability and for this reason it is worth noting the results of the regulator’s report. The report consists of information relating to insurance company authorisations and regulatory interventions. One particular aspect of the report worth noting concerns the powers of intervention.

---

9 Up until the last year, this responsibility fell with the Department of Trade and Industry.
Table 8.9 gives a breakdown of the frequency of regulatory interventions, classified in terms of the motivation for regulatory action. The table demonstrates that the insurance industry is subject to a significant amount of regulatory interventions, although these are not related primarily to the ‘winding up’ of insurers. These interventions are brought about by many concerns relating to the performance of insurer investments, the risk associated with claims, and the regulator’s requirements for additional information on certain insurance companies.

Table 8.9. Number of Insurance Regulatory Interventions 1990-1996

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Power:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Requirements about investments</td>
<td>45</td>
<td>28</td>
<td>28</td>
<td>51</td>
<td>35</td>
<td>27</td>
<td>32</td>
</tr>
<tr>
<td>Maintenance of Assets in EEA</td>
<td>3</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>Custody of Assets</td>
<td>3</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>Freezing of Assets</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>Premium Income Limit</td>
<td>44</td>
<td>22</td>
<td>35</td>
<td>44</td>
<td>29</td>
<td>33</td>
<td>36</td>
</tr>
<tr>
<td>Actuarial investigations</td>
<td>12</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Accelerated accounting information</td>
<td>2</td>
<td>8</td>
<td>9</td>
<td>2</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>General investigations</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>Obtaining information</td>
<td>57</td>
<td>32</td>
<td>37</td>
<td>52</td>
<td>35</td>
<td>40</td>
<td>58</td>
</tr>
<tr>
<td>Obtaining information by production of specific books and papers</td>
<td>2</td>
<td>*</td>
<td>2</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>Residual power to impose require</td>
<td>47</td>
<td>32</td>
<td>42</td>
<td>57</td>
<td>34</td>
<td>31</td>
<td>61</td>
</tr>
<tr>
<td>Successful petitions to wind up an insurance company</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>


Notes:
(a) For the ‘maintenance of assets’ category the figure for 1990 refers to U.K. assets only.
(b) An * signifies that there was no regulatory intervention.

One other important area concerns the use of the regulator’s ‘residual power’. This is imposed if the regulator is concerned about the ability of the insurance company to meet its liabilities (such interventions have risen from 47 in 1990 to 61 in 1996). It should be noted that the above interventions are triggered merely
in response to an early warning signal and as such they aim to act as a preemptive strike on any major risk to insurer solvency. Indeed as the last category in Table 8.8 demonstrates, petitions made by the regulator to the court, requesting authorisation to ‘wind-up’ an insurance company, were minimal; only two cases were reported in the period 1990-1996.

8.2 Life Assurance Performance: a Macroeconomic Perspective

While much can be said on the microeconomic performance indicators of the life insurance industry, there are also performance measures relating to the macro-economy. Life insurers play a critical role as a source of employment, as a source of capital, and also as a source of international trade. These macroeconomic functions will be addressed in turn. Consider insurance employment, where Figure 8.3 provides information on employment over the period 1986-1996.

Figure 8.3. Total Employment in the Insurance Sector


Note: Figures include direct insurance employment and employment auxiliary to insurance.
Total insurance employment amounted to over 200,000 people in 1996. Add to this the auxiliary employment generated by the insurance industry and the total insurance employment was equal to approximately 345,000 in 1996 (Association of British Insurers 1997a, 79). Employment during the period 1986-1996 increased up until 1991, where it reached a peak of 375,000. Since 1991, employment has declined and by 1996 it stood at 345,000. These trends have been matched in male and female employment respectively over the period, and might be explained by the recent trend in insurer mergers and acquisitions (see Chapter 7).

Consider next the role of insurance companies in capital markets. As a point of departure, it is worth considering life offices' own capital issues. Recently, there has been a trend amongst insurance companies towards issuing share capital and therefore becoming proprietary life insurers rather than remaining mutual life insurers. As Table 8.10 illustrates, the rights issues by insurers in the U.K. market is significant with net capital issues growing to one billion pounds in 1996.

<table>
<thead>
<tr>
<th>Year</th>
<th>Net Capital Issues on the UK Market (£m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>606</td>
</tr>
<tr>
<td>1987</td>
<td>-2</td>
</tr>
<tr>
<td>1988</td>
<td>-11</td>
</tr>
<tr>
<td>1989</td>
<td>184</td>
</tr>
<tr>
<td>1990</td>
<td>-29</td>
</tr>
<tr>
<td>1991</td>
<td>63</td>
</tr>
<tr>
<td>1992</td>
<td>394</td>
</tr>
<tr>
<td>1993</td>
<td>1694</td>
</tr>
<tr>
<td>1994</td>
<td>494</td>
</tr>
<tr>
<td>1995</td>
<td>578</td>
</tr>
<tr>
<td>1996</td>
<td>1007</td>
</tr>
</tbody>
</table>

As outlined in Chapter 7, the life assurance industry provides a substantial source of funds to the capital markets with total investment holdings in excess of £500 billion. Holdings of ordinary shares amounted to 44 percent of total holdings in 1995 (Office for National Statistics 1998, Table 1.2). Table 8.11 gives an indication of the relative size of net investments made by different financial institutions.

Table 8.11. Net Investments by Financial Institution as a Percentage of Total Net Investments 1987-1996

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>Self Administered Pension Funds</th>
<th>Long-Term Insurance Funds</th>
<th>Other Than Long-Term Insurance Funds</th>
<th>Investment Trusts</th>
<th>Unit Trusts and Property Unit Trusts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1987</td>
<td>34117</td>
<td>35.24%</td>
<td>33.68%</td>
<td>10.03%</td>
<td>2.58%</td>
<td>18.47%</td>
</tr>
<tr>
<td>1988</td>
<td>24970</td>
<td>46.56%</td>
<td>42.04%</td>
<td>10.66%</td>
<td>-0.50%</td>
<td>1.25%</td>
</tr>
<tr>
<td>1989</td>
<td>37824</td>
<td>38.36%</td>
<td>43.48%</td>
<td>7.39%</td>
<td>1.34%</td>
<td>9.43%</td>
</tr>
<tr>
<td>1990</td>
<td>29764</td>
<td>41.09%</td>
<td>56.30%</td>
<td>1.58%</td>
<td>0.32%</td>
<td>0.71%</td>
</tr>
<tr>
<td>1991</td>
<td>36325</td>
<td>23.46%</td>
<td>67.36%</td>
<td>-0.81%</td>
<td>2.55%</td>
<td>7.43%</td>
</tr>
<tr>
<td>1992</td>
<td>36327</td>
<td>22.45%</td>
<td>64.86%</td>
<td>6.56%</td>
<td>2.19%</td>
<td>3.95%</td>
</tr>
<tr>
<td>1993</td>
<td>51001</td>
<td>8.52%</td>
<td>54.92%</td>
<td>9.00%</td>
<td>5.94%</td>
<td>21.62%</td>
</tr>
<tr>
<td>1994</td>
<td>52531</td>
<td>10.24%</td>
<td>50.41%</td>
<td>10.89%</td>
<td>11.30%</td>
<td>17.17%</td>
</tr>
<tr>
<td>1995</td>
<td>45944</td>
<td>17.39%</td>
<td>57.80%</td>
<td>8.34%</td>
<td>2.61%</td>
<td>13.88%</td>
</tr>
<tr>
<td>1996</td>
<td>64383</td>
<td>17.65%</td>
<td>52.14%</td>
<td>11.57%</td>
<td>-0.23%</td>
<td>18.87%</td>
</tr>
</tbody>
</table>


As the above table demonstrates, long-term insurance funds generate the majority of net investments by financial institutions, accounting for some 52 percent in 1996. This has fallen from a maximum of 67.36 percent in 1991. Thus, the importance of life assurance and pension funds as an institutional investor is clear.

Other significant sources of funds came from self-administered pension funds and unit trusts that in 1996 accounted for 17.65 percent and 18.87 percent, respectively. However, the fact remains that long-term insurance funds, as an
institutional investor, represent a vital source of funds for trade and industry. In particular, when this information is combined with the trends in insurer net investments (see Table 7.6), it also becomes clear that insurers provide an important source of equity finance to industry and trade. Furthermore, since insurance companies purchase significant volumes of government securities—some £10 billion in insurer net investment in 1996—insurance companies may also be seen as an important source of finance for the Government.

Within an analysis of insurance companies and capital markets, it is worth noting the contributions made by insurance companies to overseas trade. The contributions are represented in the form of invisible earnings within the Balance of Payments. The U.K. exports approximately one-third of its GDP each year (Office for National Statistics 1998a). In 1996, invisible earnings (Association of British Insurers 1997b) accounted for £22 billion worth of exports.

Moreover, during the last 15 years it has been the surplus in invisible trade that has helped to limit the effects of a Balance of Trade deficit upon the Balance of Payments. Table 8.12 demonstrates that the growth in the value of goods exported has exceeded the growth in services: the value of goods exported is over three times the value of services exported in 1995. Insurance companies make up a small proportion of the total services exported—approximately 4 percent in 1995. Thus, as an internationally traded product, insurance is still relatively modest in size. Note, however, that there are contributions by insurance companies to interest profit and dividends (IPDs) through insurance

10 Other service exports include tourism, telecommunications, and computer services.
companies making returns on their investments abroad. Between 1993 and 1995 insurance companies' IPD credits grew from £2,914m in 1993 to £3,445m by 1995, an increase of around 18 percent.


<table>
<thead>
<tr>
<th>£ million</th>
<th>1993</th>
<th>1994</th>
<th>1995</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Exports of Goods</td>
<td>121,398</td>
<td>134,666</td>
<td>152,346</td>
</tr>
<tr>
<td>2 Imports of Goods</td>
<td>134,858</td>
<td>145,497</td>
<td>163,974</td>
</tr>
<tr>
<td>1+2=3 Visible Balance</td>
<td>(13,460)</td>
<td>(10,831)</td>
<td>(11,628)</td>
</tr>
<tr>
<td>4 Exports of Services</td>
<td>38,599</td>
<td>41,399</td>
<td>45,254</td>
</tr>
<tr>
<td>5 of which: * insurance companies (a)</td>
<td>419</td>
<td>484</td>
<td>363</td>
</tr>
<tr>
<td>6 Imports of Services</td>
<td>33,083</td>
<td>36,652</td>
<td>39,112</td>
</tr>
<tr>
<td>7 Exports of services</td>
<td>5,516</td>
<td>4,747</td>
<td>6,142</td>
</tr>
<tr>
<td>9 of which: * insurance companies' portfolio investment earnings</td>
<td>1,936</td>
<td>1,711</td>
<td>2,111</td>
</tr>
<tr>
<td>10 Net debits (b)</td>
<td>71,946</td>
<td>69,228</td>
<td>83,567</td>
</tr>
<tr>
<td>12 Transfer credits</td>
<td>5,536</td>
<td>5,616</td>
<td>6,135</td>
</tr>
<tr>
<td>13 Transfer debits</td>
<td>10,543</td>
<td>10,645</td>
<td>13,113</td>
</tr>
<tr>
<td>12-13=14 Net transfers</td>
<td>(5,007)</td>
<td>(5,027)</td>
<td>(6,978)</td>
</tr>
<tr>
<td>20+14=15 Invisible balance</td>
<td>2,707</td>
<td>8,411</td>
<td>8,736</td>
</tr>
<tr>
<td>3+15=16 Current account balance</td>
<td>(10,756)</td>
<td>(2,419)</td>
<td>(2,892)</td>
</tr>
</tbody>
</table>


Notes: (a) Exports of insurance can be shown as negative because the figures are essentially premiums received from abroad less claims paid abroad.
(b) The profits of foreign owned insurers operating in the U.K. (which leave the U.K.) are included in IPD debits.

Additional information on the significance of insurance company overseas returns is derived by comparing the overseas earnings of insurance companies in the U.K. with other financial institutions. Figure 8.4 shows the proportion of overseas earnings made up by the activities of different financial institutions in 1996.
With respect to Figure 8.4, banking and insurance dominate institutional overseas earnings, together accounting for nearly 60 percent in 1996. Total earnings for the year amounted to £22,658m. The activities of pension funds and securities dealers are also significant as together they provide roughly 20 percent of total earnings.

Additional insights may be derived from Table 8.13. This table shows the trends in U.K. overseas earnings for the period 1986-1996 for four types of financial institution. Insurance funds remain a major source of overseas earnings for the period 1986-1996 when compared with the other financial institutions in Table 8.13. Recently, overseas earnings of insurers have tended to increase. By 1996, insurer earnings stood at £6,141m, as compared with £2,157m in 1990. Also noteworthy is the substantial growth in pension fund overseas earnings; these earnings were nearly four times what they had been in 1986. Thus, the overall importance of life assurance to overseas earnings has remained significant.
### Table 8.13. Overseas Earnings of UK Financial Institutions (£m), 1986-1996

<table>
<thead>
<tr>
<th>Year</th>
<th>Banking</th>
<th>Insurance Funds</th>
<th>Pension Funds</th>
<th>Securities Dealers</th>
<th>Other</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>3,934</td>
<td>4,865</td>
<td>615</td>
<td>933</td>
<td>1,925</td>
<td>10,347</td>
</tr>
<tr>
<td>1987</td>
<td>2,740</td>
<td>4,578</td>
<td>695</td>
<td>1,503</td>
<td>1,726</td>
<td>9,516</td>
</tr>
<tr>
<td>1988</td>
<td>2,974</td>
<td>3,344</td>
<td>1,079</td>
<td>1,170</td>
<td>2,117</td>
<td>8,567</td>
</tr>
<tr>
<td>1989</td>
<td>4,067</td>
<td>2,605</td>
<td>1,371</td>
<td>2,048</td>
<td>1,421</td>
<td>10,091</td>
</tr>
<tr>
<td>1990</td>
<td>5,152</td>
<td>2,157</td>
<td>1,356</td>
<td>1,232</td>
<td>1,969</td>
<td>8,997</td>
</tr>
<tr>
<td>1991</td>
<td>5,163</td>
<td>3,723</td>
<td>1,608</td>
<td>480</td>
<td>1,672</td>
<td>10,974</td>
</tr>
<tr>
<td>1992</td>
<td>5,830</td>
<td>3,739</td>
<td>1,486</td>
<td>2,158</td>
<td>2,084</td>
<td>13,213</td>
</tr>
<tr>
<td>1993</td>
<td>1,869</td>
<td>4,960</td>
<td>1,697</td>
<td>2,005</td>
<td>3,694</td>
<td>10,531</td>
</tr>
<tr>
<td>1994</td>
<td>7,046</td>
<td>3,990</td>
<td>1,723</td>
<td>2,185</td>
<td>3,411</td>
<td>14,944</td>
</tr>
<tr>
<td>1995</td>
<td>5,917</td>
<td>6,905</td>
<td>1,845</td>
<td>1,758</td>
<td>4,275</td>
<td>16,425</td>
</tr>
<tr>
<td>1996</td>
<td>7,113</td>
<td>6,141</td>
<td>2,271</td>
<td>2,190</td>
<td>4,943</td>
<td>17,715</td>
</tr>
</tbody>
</table>


### 8.3 Summary

The aim of this paper was to provide an account of the performance of the life assurance industry in the U.K. In line with the results presented in the previous two chapters, use was made of the structure, conduct, and performance (SCP) paradigm as a basis for discussion.

The SCP notion of industrial performance concerns the extent to which an industry departs from a competitive equilibrium. In line with this approach, the first part of this chapter dealt with this concept of performance and it was shown that the profitability of insurance companies is centred about the industry average. Thus, from the perspective of the regulator there is some evidence to suggest that rates of return are approximately equal for the majority of life insurers, and this, according to economic theory, might be the result of competition.
In addition to a discussion of excess returns, evidence was presented on a number of other aspects of insurer performance. While some of this data does not conform to the traditions of SCP, it nevertheless provides a useful context for discussion and also compensates for some of the limitations relating to profitability as a measure of performance in the life assurance industry. Moreover, the importance of life assurance to the economy at large was also highlighted within these developments.

It is important to stress the causality considerations associated with the SCP paradigm. While an attempt has been made to limit any inference of the causality flow between industry structure, conduct and performance, there are, underpinning the analysis, many possible explanations for the flow of causation. Discussion on the flow of causality, as well as the development of some formal statistical relationships within a SCP setting, forms the basis of discussion in the Chapter 9.

In bringing the SCP discussion to a close, it is evident from all of the evidence developed that the life assurance industry is an industry that has undergone dramatic change over the last 100 years. This change has been necessitated out of changing social and economic conditions. The result today is an industry providing insurance coverage across a wide range of risk categories to an ever-increasing consumer base. The challenges of a global insurance market will undoubtedly form the backdrop to a discussion of the U.K. life assurance industry in the twenty-first century.
CHAPTER 9

FIRM-LEVEL STRUCTURE-PERFORMANCE RELATIONSHIPS: EVIDENCE FROM THE U.K. LIFE ASSURANCE INDUSTRY

9.0 Introduction

Previous chapters have served to outline taxonomically the main aspects of the U.K. life assurance industry, referencing the nature of industry structure, conduct and performance. These chapters demonstrated that the life assurance industry has experienced substantial historical change and that it is now made up of a variety of firms, each exhibiting significant variation in terms of their structural characteristics. The aim of this chapter is to take these largely qualitative results and formalise some statistical relationships that may exist between firm-structure and firm-level performance using a cross-section of insurance companies that together makeup the majority of the long-term insurance market in the U.K.

Section 9.1 develops the theoretical foundations of this chapter and also reviews the various methodological approaches that have been used to analyse industrial structure-performance relationships, both at the industry-level and at the firm level. Section 9.2 provides a brief literature review. Section 9.3 introduces the components of this empirical analysis, outlining the data, variables and their respective interpretations. Then in Section 9.4, the principle results are presented and discussed. Finally, Section 9.5 brings together a summary of the main developments.
9.1 Theoretical and Methodological Background

Many debates exist within industrial organization concerning the appropriate theoretical and methodological approach to the study of the relationship between industry (and firm) structure and performance. Prior to the 1950s, the dominant approach to the study of industrial organization was that of case studies of individual industries. The seminal work of Bain (1951, 1956) helped to shift research in industrial organization to a higher level though his inter-industry cross-section approach. Since that time, discussion has reigned over the appropriate model specification of structure-performance studies and in particular the violation of three OLS assumptions: first, simultaneous causality; second, measurement error; and third, the omission of relevant explanatory variables. While each assumption is important to ensure unbiased and consistent estimators, the problem of simultaneity has been pervasive amongst scholars of industrial organization in recent years.

To understand these issues further, it is necessary to review briefly the theoretical foundations of empirical studies from the work of Bain (1951, 1956) to the present day. The structure-conduct-performance (SCP) paradigm has proved to be one of the most influential approaches to the study of industrial organization, with its foundations in the work of Mason (1939, 1949) and its advances being made through the work of scholars including Bain (1959) and Markham (1950). The basic proposition was that there existed a unidirectional flow of causality between industry structure, conduct and performance. Thus,
one of the central hypotheses proposed by theory was that profitability varied across industries as the concentration level varied (with no explicit account of conduct). That is, structure helped to explain performance. Explicit formalisation of conduct was introduced to develop formal structure-performance relations; two common examples of this approach are the Cournot-Nash model (Cowling 1976) and the Dominant Firm Model (Geroski 1982).

However, while the SCP approach remains an influential analytical framework, the interpretation of its basic proposition- namely a unidirectional flow of causality- has been concluded to be unsatisfactory in light of causality problems and simultaneity. Thus while structure may affect performance, improved performance may also affect conduct, which in turn may affect structure; that is, there exists the potential for causality feedback between the SCP components. This result led many to question the OLS technique and suggest instead that any structure-performance relation be part of larger system of relations with estimation techniques accounting for this fact (Reid 1989, 48). Note that this approach has itself been the subject of debate in terms of model specification and whether any ‘structure’ variables, in any definitive sense, are truly independent (Schmalensee 1989a). This is a crucial point since independent variables are ‘predetermined’ and may therefore be used as ‘instruments’ in subsequent consistent estimation techniques.

1 Consistent estimation of a ‘recursive’ system may be achieved using OLS. A recursive system exhibits no bias from the application of OLS because such a system exhibits one-way causation and is exactly identified (Reid 1989, 50).
Schmalensee’s point above is emphasised within the more general debate concerning the usefulness of cross-section studies. Cross-sectional studies assume that variations across observations represent variations in long-run equilibrium positions. The task, implied above, is to determine the number of endogenous variables within a given structural equation and to account for this endogeneity within estimation in order to obtain consistent estimators. The difficulty, as Schmalensee (1989a) argues, is that:

In the long-run equilibria ... all variables that have been employed in such studies [cross-section studies] are logically endogenous. (Schmalensee 1989a, 954)

That is, in the long-run market structure is clearly affected by conduct and performance; mergers and acquisitions, for example, will have an impact upon the level of industrial concentration. This poses a significant problem for estimation: the number of logical candidates for use as an instrumental variable in order to obtain consistent estimators is significantly reduced.

Given the theoretical and methodological complexities introduced above, what justification can be made for the presentation of an empirical cross-sectional study of industry structure-performance? Schmalensee (1989a) argues that the aim of cross-sectional studies is really to explain the main patterns of the data set employed:

The appropriate mind-set ... is accordingly that of descriptive statistics, not structural hypothesis testing. (Schmalensee 1989a, 955)

Regression analysis is one such descriptive technique, which may provide an interesting initial insight into the variations across the data. This approach might
also be justified on the grounds that it is most parsimonious. Such work may still be able to demonstrate stable and robust empirical regularities:

It is clearly facile to argue simply that 'everything depends on everything else'. The role of model building is to identify the major causal connections and to establish their quantitative significance. (Reid 1989, 26)

Regardless of the estimation approach assumed, the violation of the other two OLS assumptions (measurement error and omission of relevant variables) must also be brought into consideration when discussing industrial structure-performance relationships.

9.2 Brief Literature Review

As noted in the previous section, empirical research relating to industrial structure-performance relationships is concerned primarily with *inter*-industry variations rather than *intra*-industry variations. However, there exist a number of studies using firm-level data, (see for example, Hall and Weiss (1967), Martin (1983) and Mueller (1986)). More recently, a study of the impact of firm size and age upon firm level performance in India was presented by Majumdar (1997) who showed that older firms were found to be more productive and less profitable, whereas the larger firms were found to be more profitable and less productive. Other recent structure-performance papers include the work of Willis and Rogers (1998), and Bhattacharya & Bloch (1997)- the latter paper provides a useful insight into the methodological difficulties (outlined above) by examining profit-concentration relationships in Australian manufacturing.
While much can be said about the study of industry in general, it is important to re-state that this chapter is concerned with the U.K. life assurance industry. In terms of assessing structure-performance relationships within the life assurance industry little research exists, however notable exceptions include the works of Cummins and Harrington (1987), and the paper by Chidambran et al. (1997) who provided an investigation into the performance of the U.S. property-liability insurance industry. The concentration ratio for each insurance line was found by Chidambran, et al. (1997) to be a significantly negative determinant of performance.

9.3 Research Design

9.3.1 Data

Data was collected on 103 insurance companies for the years 1996 and 1997. These companies all operate in the long-term insurance industry. Together these companies account for approximately 80 percent of U.K. worldwide long-term insurance business.² A comprehensive database of all 103 insurance companies was constructed using accounting data from company accounts and other statutory reporting submitted to the Department of Trade and Industry (the regulator in 1997). In addition to the general company requirements to lodge accounts at Companies House, insurance companies also submit a detailed breakdown of their operations by completing a series of forms as specified in the

² This figure includes both U.K. and overseas business of insurance companies with head offices in the U.K. as well as the U.K. business of branches and subsidiaries of foreign insurers (that is, those insurers with head offices outside the U.K.).
1996 Insurance Companies (Accounts and Statements) Regulations and authorised under the 1982 Insurance Companies Act (section 18). These forms (as discussed in Chapter 2) cover all aspects of insurance company operations including a breakdown of company performance and asset/liability portfolio composition. This data was supplemented by additional information made available in the Post Magazine Insurance Directory 1997 and the DTI's Insurance Annual Report for 1996/1997. All three data sources represent a significant resource upon which to build empirical work on structure-performance relationships at the firm level.

9.3.2 Performance

The performance of insurance companies is captured using two performance indices, profitability and solvency. The former index is commonly used as a measure of economic performance (see Schmalensee (1989b) for a discussion of accounting profitability measures). In particular, profitability is defined as the ratio of total transfers to the profit and loss account (from the insurer surplus on long-term funds), to the total value of long-term funds (see Appendix 2). As noted in Chapter 8, mutual insurers do not make profit per se. However the total surplus reported in the valuation balance sheet is used as a proxy for mutual insurer 'profit'.

By contrast, the second measure of performance is a solvency index (as measured by the solvency margin). This index is less commonly used in studies of structure-performance relationships. However, U.K. regulators use solvency
as one of the main performance signals for insurance companies. Moreover, regulatory intervention is defined through the size of the solvency margin: the ‘available assets for long-term business required minimum margin’ (The Insurance Company (Accounts and Statements) Regulations 1996, Form 9). Insurance company solvency is reported in the ‘statement of solvency’, which is submitted to the regulator and recorded in the statutory annual returns.

It is useful here to stress that while an obvious relationship exists between profitability and solvency as respective performance measures, there are also distinct differences between these two measures. Profitability has strict governance over cash flows that immediately impact upon the net revenue position of an insurance company. By contrast, solvency is a more rigorous measure of performance. Solvency requires the assessment of all the operations of the insurer, not just those impacting upon annual cashflow. It is the role of the Actuary that helps to differentiate the two measures of performance, with the solvency measure requiring an extensive valuation of the risk profile of assets and liabilities that is not accounted for explicitly within a profitability index.

An additional factor to consider in selecting the performance measures is with respect to the ownership of insurance companies. Part of the aim of this research was to try and capture variations in performance across ownership structures. Insurance companies in the U.K. comprise of mutual life insurers and proprietary life insurers. While a measure for profitability can be constructed for mutual insurers (using the reported life fund surplus as a measure of ‘profit’), a more robust measure of performance (in the form of a solvency index) would
strengthen any conclusions because, strictly speaking, mutual insurers do not make profit. It is for this reason and also the regulatory context that the solvency performance index is included within the subsequent analysis.

9.3.3 Structure

In order to explain the firm-level performance across insurance companies in 1997, a wide variety of variables were constructed reflecting the various aspects of insurance company structure. At this stage, it is assumed that the problems associated with simultaneity are not significant, although this issue will be returned to subsequently in section 9.4.3. The variables take the following form:

\[(9.1) \quad PROFIT \ or \ SOLVENCY = a_{1} + b_{1}Z_{1} + b_{2}Z_{2}\]

Where \(b_{1} = (b_{11}, \ldots, b_{1b}, \ldots, b_{113})\) and \(b_{2} = (b_{21}, b_{22}, b_{223})\) and \(Z_{1}\) represents a vector of exogenous intensity variables as well as other indices, and \(Z_{2}\) represents a vector of dummy variables. Each of these variables will be addressed in turn.

In line with other inter-firm studies, AGE and SIZE are included. AGE is seen as important in capturing life cycle effects and indeed the insurance companies in the U.K. have a long history of operation, with an average age of 71 years. As Majumdar (1997) points out, theory is equivocal on the relationship between age and performance: research demonstrates that age may be beneficial in terms of experience, while research also demonstrates that age may bring with it
inflexibility and inertia to change. AGE is defined as the number of years since the insurer was established.

Similar contrasting arguments have been put forward in explaining firm size. The basic proposition is that under competition, profit rates will tend towards equality. However, Baumol (1959) argued that large firms have the resources so that they are able to pursue ventures of such a magnitude that small firms are precluded from taking up the same opportunity. The result is that larger insurers may have the chance to earn higher rates of return than smaller insurers in the long run. Clearly, this assumes some form of capital barrier and the question that is to be discussed in light of the results, is whether or not such a barrier would be expected to exist within the financial sector. Also, Baumol’s proposition assumes that the presence of diseconomies, such as x-inefficiencies (Leibenstein 1976), is outweighed by these economies of scale. For the purposes of this study, SIZE is defined as the natural logarithm of insurer total premium income. ³

Diversification is a means of utilising a firm’s excess resources. Operation of an insurance company across multiple lines of insurance requires a given skill and knowledge base such that the capabilities of the company are enhanced with increasing diversity, with all the attendant positive effects that this may have upon performance. For this reason, DIVERSITY is used to capture this effect and is defined as the total number of insurance lines for which the insurance

³ Size measures are also useful in controlling for heteroskedasticity.
company is authorised to conduct business. This variable also reflects the reality that financial products are significantly differentiated.

The benefits to diversification, as outlined above, will apply to insurance business outside of the long-term market, such as business within the general insurance market. For this reason, the number of authorised insurance lines used in the construction of the DIVERSITY variable includes all insurance markets, both general and long term markets.4

As defined in Chapter 5, an insurance company may be defined in terms of two portfolios, the underwriting portfolio and the investment portfolio. To reflect this theoretical framework, variables were constructed with the aim of representing the main features of these two portfolios. Discussion now turns to addressing these variables.

Consider first the investment portfolio. As noted throughout, the performance of the investment portfolio is seen as crucial to the financial solidity of the insurance company. Recall that the distinguishing feature about life assurance is the long-term nature of the contract, which affords the insurer with a steady stream of income into the future. The investment of the premiums and the attendant rate of return on these insurance funds are seen as critical determinants of insurer performance since they will have a direct effect upon company valuation and an indirect consumer reputation effect, through public reports of poor returns on personal sector savings.

4 This variable will have implications for the performance of so-called 'composite' insurance companies, which operate in both the general and long term insurance markets.
While much can be said about the significance of the investment portfolio, the construction of an adequate financial ratio to measure investment return is often fraught with measurement difficulties; that is, the noise to signal ratio is high. However, the inclusion of the INVESTMENT RETURN variable is still seen as a useful first step in acknowledging the importance of this area to insurance company operation. INVESTMENT RETURN is defined as the ratio of investment income received on long-term business to total assets.

Consider next the underwriting portfolio. The performance of the underwriting portfolio depends on the relative performance of claim outflow and premium income. Clearly the outflow of claims will be central to any insurance company performance evaluation. While the outflow of maturing policies will be planned for in advance, frequently well in advance, the occurrence of policy surrenders are less certain and will clearly have an important impact upon profitability and solvency. Death claims, while being more predictable than policy surrendering, still pose a significant threat to adequate insurer performance. In terms of markets, the role of domestic and overseas markets is also important in respect of claims and hence insurer performance.

For the reasons outlined above, three variables are included: DEATH, SURRENDER, and OVERSEAS CLAIMS. Each variable represents the ratio of the respective claim category to total claims. The impact of these claim categories upon performance would be hypothesised to be negative. However,
the extent and significance of this effect is of interest, especially in respect of overseas markets.

In terms of premium income, the variable SALES GROWTH is included to capture the rate of change in sales between the observation year (1997) and the preceding year, as measured in terms of the total premium income. Such a variable would be expected to capture business-cycle effects and sometimes is also used as a proxy for industry elasticity of demand (Schmalensee 1989a). As with age and size, predictions about the relationship between this variable and performance is ambiguous. On the one hand, greater sales growth may offer an insurance company the potential to make enhanced returns, yet on the other hand, such a structure-performance relationship may serve to act as a signal to new entrants, and hence negatively impact upon long-run returns.5

Two other variables are included to account for the structure of life assurance markets: EXPORTS and SINGLE PREMIUMS. EXPORTS are included to control for the relative income from overseas markets. The operation of this variable will depend upon the relative performance of the domestic and overseas markets. If domestic markets exhibit superior performance over their overseas market counterparts, then insurers with lower export orientation would be expected to have a relatively superior performance than insurers with higher export orientation.

5 This latter argument will depend on the nature of the barriers to entry (Bain 1956).
SINGLE PREMIUMS is included as a variable to control for the product differentiation of insurance products. This variation is defined in terms of premium frequency. Long-term insurance contracts may be subdivided into two types of contracts: single premium contracts and regular premium contracts. While the latter category has been popular traditionally within the life assurance market, the former category has become more common through the growth in single contract bonds and private pension provision (see Chapter 6 on life assurance market structure). To account for this market segmentation the ratio of single premiums to total premiums is measured. This variable may serve as an assessment for an emerging market. Due to the rapid expansion of the single premium contract market, it is hypothesised that the variable has a positive impact upon performance in the long run.

Another important factor to consider in respect of insurance company performance is that of operating expenses and transactions costs (Williamson 1975). AGENCY COST is therefore included as a variable to account for the overall efficiency of an insurance company. AGENCY COST measures the ratio of expenses (including commission) to premium income. Insurance companies often use this ratio to measure internal efficiency in respect of new business (Franklin and Woodhead 1980). Thus, this measure helps to capture firm-specific features, since the ability to reduce the ratio of costs to income depends on skills and abilities present within the firm, which in turn are also likely to generate enhanced returns and therefore have a positive impact upon performance.
Within the managerial view of the firm, the importance of marketing and promotion can be seen as a further factor to control for (Marris 1964, Williamson 1963). The variable MANAGERIAL EXPENSES is defined as the ratio of managerial expenditures to total expenses. The so-called ‘managerial theories of the firm’ provide an influential alternative account to the classical theory of the firm. In particular, the managerial view, which received notable attention during the 1960s, “… is concerned with the consequences of pursuing and attaining certain goals within a given framework,” (Reid 1989, 177). These goals reflect managerial motives and organisational observations. The theories, most notably those of Marris (1964) and Williamson (1963), emphasise the effects of the separation of ownership from control, and the consequent divergence in objectives that can best be represented within a principal agent setting. These theories stress the importance of managerial utility. Williamson (1963) introduces the notion of ‘expense preference’ where, “… managers have a positive preference for expenditures on staff, emoluments and discretionary investment,” (Reid 1989, 182). Thus, it is anticipated that a managerial objective would result in a marked departure from profit maximisation and hence a negative relationship between managerial expenses and performance is hypothesised.

Another aspect of marketing concerns the distribution methods used by insurance companies. DISTRIBUTION CHANNELS is therefore included as a variable to control for the ‘selling costs’ involved in markets for highly differentiated products. DISTRIBUTION CHANNELS is an index that equals the number of distribution channels used by the insurance company in promoting
its products. There are up to three potential means by which insurance companies market their products. Products may be sold through the use of an insurance company’s own sales force, the use of an independent sales force, and also through the use of ‘direct’ selling (see Chapter 7). This variable therefore helps to control for these firm-related factors. It is assumed that a positive correlation exists between the number of channels, and the size of distribution and marketing expenditures. As a consequence, it is hypothesised that an increase in the number of distribution channels will have a positive effect, through product differentiation and monopoly rents, upon performance.

Consider next the vector of dummy variables, $Z_2$. This vector addresses a number of issues relating to the ownership of insurance companies. MUTUAL is a dummy variable used to capture the changing ownership structure across the insurance industry. The variable takes a value of 1 if the insurer is a mutual and 0 if the insurance company is a proprietary life insurer. ‘De-mutualisation’ has been a recent trend amongst building societies and some life insurers and the inclusion of this variable is used to try and detect any performance variation across the ownership structure.

FOREIGN is a variable that attempts to address another aspect of ownership: the impact of foreign ownership. FOREIGN is a dummy representing 1 if there is foreign ownership, and 0 if no foreign ownership is present. This helps to control for performance variation between domestic and foreign companies; for example, foreign firms may display superior performance over their domestic counter-parts due to the additional skills necessary in operating overseas.
GROUP is a dummy variable taking 1 if the insurance company is part of an insurance group and 0 otherwise. This variable controls for the spillover effects that may arise between companies of a similar operation, with a consequential positive impact upon insurance company performance within the insurance group.

9.4 Results and Discussion

9.4.1 Estimation Technique

Two regressions are estimated, one for each of the performance measures PROFIT and SOLVENCY. The results are presented in Table 9.1 and are calculated using the ordinary least squares (OLS) estimator, corrected for heteroskedasticity (White 1980). In the presence of heteroskedasticity the OLS variance-covariance matrix estimator may not be consistent since the variables are correlated with the observation specific variances. White (1980) proposes a consistent estimator of the OLS variance/covariance matrix, using a diagonal matrix with the squared OLS residuals along the diagonal. Leamer refers to this as “White-washing” heteroskedasticity (Leamer 1988).

9.4.2 Discussion

A number of significant statistical relationships are reported in Table 9.1. Some of these results are specific to the performance measure used. Other results are

---

6 See Appendix 2 for a detailed breakdown of this study. The appendix provides details on the definitions of variables, the sample set, and the nature of the residual plots.
robust across the two performance measures. Overall significance and explanatory power associated with the two regressions is also significant.

Table 9.1. Regression Results

<table>
<thead>
<tr>
<th></th>
<th>Profitability</th>
<th></th>
<th>Solvency</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>t-ratio</td>
<td>Coefficient</td>
<td>t-ratio</td>
</tr>
<tr>
<td>SIZE</td>
<td>-0.0432</td>
<td>-2.43*</td>
<td>-0.0239</td>
<td>-3.78**</td>
</tr>
<tr>
<td>AGE</td>
<td>0.0001</td>
<td>0.27</td>
<td>0.0111</td>
<td>1.34</td>
</tr>
<tr>
<td>DIVERSITY</td>
<td>-0.0006</td>
<td>-1.18</td>
<td>0.0008</td>
<td>0.35</td>
</tr>
<tr>
<td>INVESTMENT RETURN</td>
<td>0.3724</td>
<td>3.42**</td>
<td>-0.1046</td>
<td>-0.27</td>
</tr>
<tr>
<td>DEATH</td>
<td>-0.0784</td>
<td>-2.28*</td>
<td>0.3766</td>
<td>2.75**</td>
</tr>
<tr>
<td>SURRENDER</td>
<td>-0.0064</td>
<td>-0.52</td>
<td>-0.225</td>
<td>-4.42**</td>
</tr>
<tr>
<td>OVERSEAS CLAIMS</td>
<td>0.0165</td>
<td>0.08</td>
<td>-2.1778</td>
<td>-2.69**</td>
</tr>
<tr>
<td>SALES GROWTH</td>
<td>-0.0029</td>
<td>-1.19</td>
<td>0.0144</td>
<td>1.54</td>
</tr>
<tr>
<td>EXPORTS</td>
<td>0.2294</td>
<td>1.46</td>
<td>1.5289</td>
<td>2.53*</td>
</tr>
<tr>
<td>SINGLE PREMIUMS</td>
<td>-0.0367</td>
<td>-3.13**</td>
<td>0.0428</td>
<td>0.81</td>
</tr>
<tr>
<td>AGENCY COST</td>
<td>0.0013</td>
<td>1.14</td>
<td>0.0012</td>
<td>0.28</td>
</tr>
<tr>
<td>MANAGERIAL EXPENSES</td>
<td>-0.0478</td>
<td>-3.81**</td>
<td>-0.0395</td>
<td>-2.98**</td>
</tr>
<tr>
<td>DISTRIBUTION CHANNELS</td>
<td>-0.0043</td>
<td>-1.09</td>
<td>0.0059</td>
<td>0.39</td>
</tr>
<tr>
<td>MUTUAL</td>
<td>0.0073</td>
<td>0.78</td>
<td>-0.0652</td>
<td>-1.48</td>
</tr>
<tr>
<td>FOREIGN</td>
<td>-0.0045</td>
<td>-0.44</td>
<td>-0.0969</td>
<td>-2.44*</td>
</tr>
<tr>
<td>GROUP</td>
<td>-0.0021</td>
<td>-0.27</td>
<td>0.0764</td>
<td>1.96</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>0.0651</td>
<td>2.59*</td>
<td>0.3347</td>
<td>3.95**</td>
</tr>
<tr>
<td>R²</td>
<td>0.358</td>
<td></td>
<td>0.583</td>
<td></td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.237</td>
<td></td>
<td>0.505</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>3.99</td>
<td></td>
<td>8.50</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>103</td>
<td></td>
<td>103</td>
<td></td>
</tr>
</tbody>
</table>

Note: (a) **p<0.01; *p<0.05.  
(b) All estimates corrected for Heteroskedasticity (White 1980).

Consider first those results that are significant across the PROFIT and SOLVENCY measures of performance. Table 9.2 provides a summary of these main findings.

Table 9.2. Findings on Size, Death, Managerial Expenses, and Performance in the U.K. Life Assurance Industry

<table>
<thead>
<tr>
<th>Measurement of Performance</th>
<th>Profitability</th>
<th>Solvency</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIZE</td>
<td>(-)</td>
<td>(-)</td>
</tr>
<tr>
<td>MANAGERIAL EXPENSES</td>
<td>(-)</td>
<td>(-)</td>
</tr>
<tr>
<td>DEATH*</td>
<td>(-)</td>
<td>(+)</td>
</tr>
</tbody>
</table>

*The coefficient for this variable reports a significant sign change across performance measures.
The reported coefficients in Table 9.1 demonstrate that SIZE, MANAGERIAL EXPENSES and DEATH are all statistically significant at the 5 percent level for both profitability and solvency (though DEATH reports a sign change across the two performance indices). The coefficients reported for SIZE therefore show that there is initial statistical evidence to support the proposition that larger insurance companies are less profitable and less solvent at the 5 percent level. This would seem to reject Baumol’s (1959) proposition that larger firms may be able to take advantage of performance enhancing business opportunities that are unavailable to the smaller firm. Indeed as noted earlier, the proposition assumes the existence of adequate capital barriers, the form of which might be unclear in the financial sector in general and the insurance industry in particular (aside from the relatively modest U.K. solvency regulations). Moreover, the SIZE result would seem to imply that the presence of diseconomies and so-called x-inefficiencies (Leibenstein 1976) are significant within insurance companies. X-inefficiencies arise from changes in the incentive structure of the firm; x-inefficiencies occur out of reduced effort and productivity, which are a consequence of the changing environmental pressures that occur with increasing firm size.

One further point to note is that there may be an interactive effect with SIZE: the larger insurers may be the older insurers and with this certain life cycle effects could be present. The results for AGE are insignificant at the 5 percent level. However, when an alternative regression run was attempted and the variable SIZEAGE included (defined at the natural logarithm of SIZE*AGE), a negative coefficient was reported for both measures of performance, see Table 9.3 below.
Table 9.3. Results using SIZEAGE

<table>
<thead>
<tr>
<th>Profitability</th>
<th>Coefficient</th>
<th>t-ratio</th>
<th>Solvency</th>
<th>Coefficient</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIZEAGE</td>
<td>-0.0034</td>
<td>-2.761**</td>
<td>-0.0303</td>
<td>-2.052*</td>
<td></td>
</tr>
<tr>
<td>DIVERSITY</td>
<td>-0.0061</td>
<td>-1.059</td>
<td>0.0076</td>
<td>1.497</td>
<td></td>
</tr>
<tr>
<td>INVESTMENT RETURN</td>
<td>0.3765</td>
<td>2.406*</td>
<td>0.0675</td>
<td>0.127</td>
<td></td>
</tr>
<tr>
<td>DEATH</td>
<td>-0.0691</td>
<td>-2.032*</td>
<td>0.1867</td>
<td>1.079</td>
<td></td>
</tr>
<tr>
<td>SURRENDER</td>
<td>-0.5273</td>
<td>-0.419</td>
<td>-0.2922</td>
<td>-4.559**</td>
<td></td>
</tr>
<tr>
<td>OVERSEAS CLAIMS</td>
<td>0.0325</td>
<td>-0.362</td>
<td>-2.0627</td>
<td>-2.693**</td>
<td></td>
</tr>
<tr>
<td>SALES GROWTH</td>
<td>-0.0032</td>
<td>-0.896</td>
<td>0.0104</td>
<td>0.77</td>
<td></td>
</tr>
<tr>
<td>EXPORTS</td>
<td>0.0473</td>
<td>0.574</td>
<td>2.01</td>
<td>2.705**</td>
<td></td>
</tr>
<tr>
<td>SINGLE PREMIUMS</td>
<td>-0.0374</td>
<td>-3.352**</td>
<td>-0.0211</td>
<td>-0.399</td>
<td></td>
</tr>
<tr>
<td>AGENCY COST</td>
<td>0.0009</td>
<td>0.724</td>
<td>0.0013</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>MANAGERIAL EXPENSES</td>
<td>-0.0476</td>
<td>-3.247**</td>
<td>-0.0631</td>
<td>-2.162*</td>
<td></td>
</tr>
<tr>
<td>DISTRIBUTION CHANNELS</td>
<td>-0.0043</td>
<td>-1.088</td>
<td>-0.0009</td>
<td>-0.055</td>
<td></td>
</tr>
<tr>
<td>MUTUAL</td>
<td>0.0063</td>
<td>0.781</td>
<td>0.0214</td>
<td>0.527</td>
<td></td>
</tr>
<tr>
<td>FOREIGN</td>
<td>-0.0054</td>
<td>-0.556</td>
<td>-0.1178</td>
<td>-2.472*</td>
<td></td>
</tr>
<tr>
<td>GROUP</td>
<td>0.0008</td>
<td>0.14</td>
<td>0.0652</td>
<td>1.552</td>
<td></td>
</tr>
<tr>
<td>CONSTANT</td>
<td>0.0624</td>
<td>2.172*</td>
<td>0.6413</td>
<td>2.636**</td>
<td></td>
</tr>
<tr>
<td>Rsq.</td>
<td>0.355</td>
<td>0.456</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AdRsq.</td>
<td>0.244</td>
<td>0.363</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>3.19</td>
<td>4.93</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>103</td>
<td>103</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: (a) **p<0.01; *p<0.05.
(b) All estimates corrected for Heteroskedasticity (White 1980).

With reference to Table 9.3, the coefficient on the variable SIZEAGE was significant at the 5 percent level. As noted in previous chapters, many of the larger insurance companies have a long tradition of operation within the life assurance industry. As a consequence, this interactive result may also help to explain the relatively poor performance of the larger insurance companies.

A further point to note with respect to SIZE is within the context of overall market concentration and the size distribution of the long-term insurance market. Recall that in Chapter 6, results showed that market concentration in the long term insurance market was relatively modest, with the four-firm concentration ratio being approximately 31 percent. This might be additional evidence to

---

7 Most of the other results to be discussed were robust across the change in variables from SIZE and AGE, to SIZEAGE.
support the results in Table 9.1; namely that there exist significant restrictions upon firm size.

Returning to the principal findings in Table 9.2, MANAGERIAL EXPENSES are also a significant variable in explaining the variation in performance across insurance companies. The finding suggests that the greater the proportion of total expenses made up by managerial expenses, the poorer the performance associated with the insurer. This demonstrates that managerial expenses have an important bearing upon firm-level performance outcome in the life assurance industry. Also, this would appear to confirm the view that managers have a positive preference for expenses and that this has a detrimental effect upon overall profitability. As the 'managerial theories' demonstrate, the separation of ownership from control implies that the divergence in incentives between owner and manager may produce a marked departure from the profit predictions associated with the classical theory of the firm. Thus, the results presented in Table 9.1 and 9.2 suggest that MANAGERIAL EXPENSES are an important variable in understanding the performance of insurers and hence the relevance of managerial theories of the firm is signified.

All product variants within the life assurance industry are based around some form of death coverage. As a consequence, a priori, the relationship between DEATH and performance would appear to be important to any understanding of insurance companies operating in the long-term market. The statistical significance of DEATH in explaining performance is confirmed in the results, although there is a notable change in the sign of the coefficient across the
performance measures. The greater the proportion of total claims made up by death claims, the lower the profitability. However, the greater the proportion of death claims, the greater the solvency of an insurance company. This variation in the findings is more difficult to explain. Death claims for a given period will impact negatively on the net cash flow position of an insurance company and hence profitability. However, there is evidently some other mechanism at work in respect of solvency and thus this may reflect the differences between the two measures of performance.

At this point, it may be prudent to recall the distinctions between the two measures of performance. Profit makes up part of the overall solvency position for an insurer. However, solvency is, by definition, an all-encompassing evaluation of an insurance company, taking into account more than just revenue and cost, but rather valuation of all assets and liabilities. For this reason, solvency is arguably a more ‘dynamic’ measure of performance than profitability in terms of the extent to which it makes assumptions about the future. One possible explanation of the above results might be related to the predictability of claims. Death is relatively predictable and for this reason appropriate provisions can be made for such a liability. The greater the proportion of claims made up by these relatively predictable claims, the less likely the opportunity to ‘surprise’ the solvency position of an insurance company. Conversely, the greater the proportion of claims made up by less predictable categories, such as surrendering, the less equipped the insurer may be to meet its liabilities. It is therefore the predictability of claims that may have a crucial impact upon the solvency result reported in Table 9.2.
While much can be said about the significance of variables across performance measures, it is now worth considering the variables significant only with respect to one or other performance index. Evidence relating to the predictability explanation outlined above can also be found using the variable SURRENDER, which is significant at the 1 percent level in explaining solvency variation. Policy surrenders have a negative impact upon the solvency position and the lack of predictability in this claims category may serve as an explanation for this finding. Policy surrenders impose a significant cash outflow that is a lot more difficult to predict than the risk associated with death claims. A similar style of argument might also be found in respect of OVERSEAS CLAIMS: the greater proportion of claims from overseas, the lower the insurer’s solvency performance. Claims from abroad may introduce market specific risk factors; risk factors in the overseas markets that may not be experienced in the domestic market.

The importance of overseas markets is also supported by evidence on EXPORTS, which is statistically significant at the 5 percent level for the solvency performance measure (and the 10 percent level for the profitability measure). The greater the percentage of premium income from overseas, the greater the performance. The ability to appropriate the gains from overseas markets is clearly an important source of increased performance. Indeed, the

---

There are obviously some indicators that can be used for predicting policy surrenders; for example, the overall performance of the economy will have an impact upon the personal sector’s ability to maintain premium payments and hence there is an income effect present within policy surrenders.
skill and knowledge required to operate overseas is such that the expertise
needed may also help to enhance overall performance.

In terms of the solvency index, there is another significant variable, FOREIGN.
This dummy variable shows that there is a negative relationship between
solvency and foreign ownership. On the basis of these results, foreign owned
insurance companies have a negative impact upon solvency performance, when
compared with domestically owned insurers. This might reflect variations in
skill levels inherent within insurers that in turn would be accounted for in
performance.

It is also worth highlighting the variables significant and specific only to the
profitability index. These variables are INVESTMENT RETURN and SINGLE
PREMIUMS. INVESTMENT RETURN is positively related to profitability
such that the greater the investment returns, the greater the reported profitability.
This seems a reasonable result, given the importance of investment activities to
the operation of an insurance company.

By contrast, the greater the proportion of total premiums made up by SINGLE
PREMIUMS, the lower the profitability. This has important implications for the
markets in which insurance companies operate. Note that recent evidence
suggests that single premium contracts make up a growing proportion of total
insurance business (see Chapter 6). The results imply that such business has an
adverse impact upon performance. However, this might not be a long-run result.
A dynamic account might therefore provide additional insights into this result.
It is also worth noting those variables that were insignificant across both performance indices. These variables were DIVERSITY, SALES GROWTH, AGENCY COST, DISTRIBUTION CHANNELS, MUTUAL and GROUP. Evidently there are some important relationships at work and the results presented above are no definitive statement on the relevance or irrelevance of these variables given the methodological concerns mentioned earlier. As noted, the problems associated with measurement error may play a substantial role. The high noise-to-signal ratio present in accounting data makes further comment difficult with respect to intensity variables and other variables constructed out of accounting data.

However, there are a couple of comments worth noting in respect of the dummy variables. The results associated with MUTUAL would suggest that there is no significant difference in insurer performance across ownership structure. Therefore, it might be suggested that the objectives for policyholders in respect of performance are similar to that of shareholders. The GROUP variable is significant at the 10 percent level for the solvency measure and might be explained by the gains to the risk-knowledge base from common ownership.

9.4.3 A Note on Simultaneous Causality

As discussed in section 9.1, estimation problems associated with causality and simultaneity are important considerations within any industrial organization study. While it is not the purpose of this research to explore these issues fully, it
is clearly a very important caveat to the results reported in Table 9.1. For this reason, a brief digression will be made into simultaneity and in particular the relationship between performance and size.

Recall that one of the major issues in evaluating structure-performance relationships, is that of determining the number of endogenous variable. Theory can go some way to answering this question. Consider the simple case of the profit and concentration relationship. Under the original formulation of the SCP paradigm, the relationship and direction of causality between profit and concentration is as follows:

\[(9.2) \quad \text{PROFIT} = f(\text{CONCENTRATION}, Z_0)\]

Where, \(Z_0\) is a vector of other exogenous variables. Ceteris paribus, higher levels of concentration are assumed to lead to greater profitability. However, a regression of this form may lead to spurious results due to problems of causality feedback from \(\text{PROFIT}\) to \(\text{CONCENTRATION}\); that is:

\[(9.3) \quad \text{CONCENTRATION} = f(\text{PROFIT, } Z_1)\]

Where, \(Z_1\) is also a vector of exogenous variables. Thus, increased performance may also bring about greater concentration directly (through competitive advantages from allocative and technical efficiencies) or indirectly via conduct (through aggressive marketing policies). As a consequence, OLS estimation of Equation 9.2 ignores this underlying system within which the structure-
performance relationship is established and hence generates estimates that are biased and inconsistent, since there may exist contemporaneous correlation between the endogenous CONCENTRATION variable and the error term.

Many estimation techniques are available to resolve the problem of simultaneity, assuming adequate knowledge of the underlying system and its 'identification'. To keep matters simple, use of Instrumental Variable (IV) estimation is made here and the only endogenous variable assumed is SIZE through its relation to market share, which in turn forms part of market concentration.9,10

The IV estimation procedure is a single equation technique that uses an 'instrument' for each endogenous variable. Assuming such instruments exist, IV estimators are consistent. Instruments ideally should be strongly correlated with the variable they are serving to replace. Other exogenous variables may be useful candidates because, by assumption, they are exogenous. However, as noted by Schmalensee (1989a), the determination of exogeneity is open to question and as a result in order to maintain a high degree of correlation between instrument and variable, an alternative instrument is proposed here, namely the lagged value of the SIZE variable (SIZE_{t-1}). This is usually correlated with the original independent variable, and although it is correlated with the disturbance vector, because it is lagged it is not contemporaneously correlated with the disturbance.11

---

9 As measured by an appropriate concentration index such as the Herfindahl index of concentration.
10 The problem of determining endogeneity needs to be addressed within any full discussion of simultaneity, with evidence being taken from tests such as the Hausman test (Hausman 1978).
11 This in turn assumes that the disturbance vector is not autocorrelated.
Formally, assume that the matrix of explanatory variables \( \mathbf{X} \) is made up by \( K_1 \) columns of exogenous variables (including the intercept) and \( K_2 \) columns of endogenous variables. The intercept and the exogenous variables serve as their own perfect instruments and new variables must be found to act as instruments for the endogenous variables (at least one for each variable).

Assuming there are \( K_3 \) instruments available, a total of \( K_1+K_3 \) instruments are gathered together to form a matrix \( \mathbf{Z} \). By regressing each column of \( \mathbf{X} \) on \( \mathbf{Z} \) results in \( \mathbf{\hat{X}} \), the desired matrix of final instruments. The \( K_2 \) endogenous variables now have as instruments the best linear combination of all possible instruments. Thus, \( \mathbf{\beta}^IV \) the vector of instrumental variable estimates is produced by regressing \( y \) on \( \mathbf{\hat{X}} = \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{X} \), viz.

\[
(9.4) \quad \mathbf{\beta}^IV = (\mathbf{\hat{X}}'\mathbf{\hat{X}})^{-1}\mathbf{\hat{X}}'y = \left[\mathbf{X}'\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{X}\right]^{-1}\mathbf{X}'\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'y
\]

If \( \mathbf{Z} \) is the same dimension as \( \mathbf{X} \), then Equation 9.4 simplifies to produce \( \mathbf{\beta}^IV = (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'y \). The results associated with this regression are reported in Table 9.4 below. Most of the findings reported in the previous section are also reported after IV correction. SIZE and MANAGERIAL EXPENSES both remain significant at the 5 percent level for both forms of performance measurement. Prior findings, specific to the performance indices, also hold with the only exception being FOREIGN, which is insignificant. Explanatory power is reduced, though the overall significance of the regression is high.
### Table 9.4. Instrumental Variable Regression Results

<table>
<thead>
<tr>
<th></th>
<th>Profitability</th>
<th></th>
<th>Solvency</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>t-ratio</td>
<td>Coefficient</td>
<td>t-ratio</td>
</tr>
<tr>
<td>SIZE</td>
<td>-0.0412</td>
<td>-2.11*</td>
<td>-0.2142</td>
<td>-2.167*</td>
</tr>
<tr>
<td>AGE</td>
<td>0.0001</td>
<td>0.735</td>
<td>0.0075</td>
<td>0.161</td>
</tr>
<tr>
<td>DIVERSITY</td>
<td>-0.0007</td>
<td>-1.289</td>
<td>0.0026</td>
<td>0.851</td>
</tr>
<tr>
<td>INVESTMENT RETURN</td>
<td>0.3699</td>
<td>2.383*</td>
<td>0.0404</td>
<td>0.078</td>
</tr>
<tr>
<td>DEATH</td>
<td>-0.0726</td>
<td>-2.041*</td>
<td>0.3181</td>
<td>2.108*</td>
</tr>
<tr>
<td>SURRENDER</td>
<td>-0.0034</td>
<td>-0.273</td>
<td>-0.2951</td>
<td>-4.518**</td>
</tr>
<tr>
<td>OVERSEAS CLAIMS</td>
<td>-0.0523</td>
<td>-0.584</td>
<td>-1.9745</td>
<td>-2.08*</td>
</tr>
<tr>
<td>SALES GROWTH</td>
<td>-0.0032</td>
<td>-0.884</td>
<td>0.0107</td>
<td>0.686</td>
</tr>
<tr>
<td>EXPORTS</td>
<td>0.0656</td>
<td>0.796</td>
<td>1.7686</td>
<td>1.939</td>
</tr>
<tr>
<td>SINGLE PREMIUMS</td>
<td>-0.0343</td>
<td>-3.308**</td>
<td>-0.0892</td>
<td>-1.193</td>
</tr>
<tr>
<td>AGENCY COST</td>
<td>0.0003</td>
<td>0.279</td>
<td>0.0001</td>
<td>0.02</td>
</tr>
<tr>
<td>MANAGERIAL EXPENSES</td>
<td>-0.0479</td>
<td>-3.266**</td>
<td>-0.0342</td>
<td>-2.47*</td>
</tr>
<tr>
<td>DISTRIBUTION CHANNELS</td>
<td>-0.0037</td>
<td>-1.043</td>
<td>-0.0389</td>
<td>-1.462</td>
</tr>
<tr>
<td>MUTUAL</td>
<td>0.0058</td>
<td>0.635</td>
<td>-0.0734</td>
<td>-1.433</td>
</tr>
<tr>
<td>FOREIGN</td>
<td>-0.0059</td>
<td>-0.682</td>
<td>-0.1087</td>
<td>-2.632**</td>
</tr>
<tr>
<td>GROUP</td>
<td>0.0018</td>
<td>0.301</td>
<td>0.0817</td>
<td>1.847</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>0.0816</td>
<td>3.111**</td>
<td>0.3753</td>
<td>3.565**</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.361</td>
<td>0.422</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
<td>0.242</td>
<td>0.311</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( F )</td>
<td>3.03</td>
<td>4.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N )</td>
<td>103</td>
<td>103</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: (a) **p<0.01; *p<0.05.
(b) \( \text{SIZE}_{t-1} \) is used as an instrumental variable for \( \text{SIZE}_t \).

Although other possible corrections may need to be addressed, the initial findings reported Table 9.4 seem to suggest that the results are robust across different estimation techniques, and [perhaps] this is tentative (stress tentative) evidence to support the view that problems associated with simultaneity are not prevalent here.

Note that while much can be said concerning the theoretical determination of endogeneity, it is important to also note that empirical tests have been devised to test for endogeneity. Such a test was indeed constructed out of the works of Hausman (1978) and Wu (1974). The Hausman test for contemporaneous correlation between the OLS error term and an explanatory variable can be used to test for endogeneity, under the null hypothesis of independence and an
alternative hypothesis of endogeneity. As an illustration, a Hausman test was performed for the variable $SIZE_{1997}$ and the results are reported in Table 9.5 below.

Table 9.5. Results of the Hausman exogeneity test for the variable $SIZE_{1997}$

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Coefficient</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profitability</td>
<td>-0.0231</td>
<td>-1.724</td>
<td>0.0803</td>
</tr>
<tr>
<td>Solvency</td>
<td>-0.0285</td>
<td>-1.436</td>
<td>0.1548</td>
</tr>
</tbody>
</table>

Notes: (a) The omitted variable version of the Hausman test was performed. (b) The t-test was constructed under the null hypothesis of independence and an alternative hypothesis of endogeneity.

The results, presented in Table 9.5, cannot reject the null hypothesis of independence at the 5-percent level (although it can reject the null at the 10-percent level using profitability as the dependent variable). Thus, there is evidence to suggest that the size variable may indeed be independent. However, these results must be treated with great caution; for example, the Hausman test is sensitive to several types of specification error (see Godfrey and Hutton (1994) for discussion). For this reason, the above results may or may not imply endogeneity of the $SIZE$ variable. This section therefore illustrates the complexity associated with simultaneous causality.

9.5 Summary

This paper has provided a cross-sectional study of insurance company performance for the period 1996/1997 using data on 103 insurance companies.

---

12 Rejecting the null hypothesis would therefore imply a need for structural modelling.
13 Note that a similar test was performed for the managerial expense variable. The results demonstrated that there was no statistically significant evidence to suggest that this variable was endogenous.
Findings in respect of SIZE and MANAGERIAL EXPENSES report consistent results with poorer performance associated with larger insurers and greater managerial expenses being associated with poorer performance. In terms of general results specific to each performance measure there are two points to note. With regard to profitability, the variables that perform better are those that reflect the period-by-period net revenue position of the insurance company. By contrast, the solvency performance index appears to reflect underlying factors relating to the risk profile of its liabilities. In addition, the results confirm the importance of overseas markets to the operation of the U.K. life assurance industry.

Methodological issues relating to the fundamental problems associated with empirical work in the field, such as measurement error and simultaneity, provide an important backdrop to the results presented in this chapter. However, the findings presented provide useful preliminary guidance in the explanation and understanding of inter-firm structure performance relationships within the U.K. life assurance industry.

Section 9.6 Implications of SCP study for The Probability of Insolvency

The evidence drawn together in this chapter, built on the developments of the previous 3 chapters, has helped to provide a number of indicators that together could be used to assist in predicting financial distress and in particular the probability of ruin. While the relationship between PROFIT and SOLVENCY has been emphasised, it is worth concentrating on the SOLVENCY variable
since this has been taken from the information submitted to the FSA/DTI and it is this information that assists in the assessment of ruin probability.

Consider the key variables identified in the previous sections that have reported significant results at the 95 per cent level. The variables of interest are SIZE, DEATH, SURRENDER, and OVERSEAS CLAIMS. SIZE, SURRENDER and OVERSEAS CLAIMS have reported a negative influence on SOLVENCY and hence an increase in these indicators would be expected to be matched by an increase in the size of the probability of ruin. Conversely, with respect to DEATH, the results imply that the larger the amounts of death claims the lower the probability of ruin. However, it is important to note that as with all forms of estimation, the results are subject to questioning. This point is illustrated with reference to the negative coefficient on the SIZE variable. Whilst there may be theoretical explanations for this result, it is clear that this result is counter-intuitive and indeed the growth in consolidation is the result of insurance companies aiming to maintain their competitive position and financial strength. As a consequence there is the potential for spurious results to be reported and due caution must therefore be attached to any interpretation.

How can this information be helpful in solvency surveillance? Given that the indicators highlighted above have a robust statistical relationship with SOLVENCY, they could be employed in a qualitative analysis of insurance companies in the first instance. For example, the results could lead to the sub-dividing of the insurance company population into peer groups, where larger insurance companies – assuming a criteria for defining ‘larger’ exists – might be
subjected to a further weighting on their required capital in order to take account of the greater probability of insolvency. A similar treatment might be given with respect to the recent surrender experience.

The preliminary evidence provided within this chapter might be used to construct a measure of insolvency probability using a limited dependent model as a basis for estimation. The effect of a change in the explanatory variable on the probability of ruin could then be evaluated – through an appropriate interpretation of the slope parameters on a limited dependent model – assuming adequate statistical leverage. However, the difficulties in obtaining adequate data are such that an alternative approach is required in order to quantify the probability of insolvency. A simulation model of a life office is one such approach that can be adopted and indeed the approach lends itself quite neatly to the incorporation of additional variables, such as the influence of surrendering. This ‘Monte-Carlo’ simulation approach is the point of departure for the next chapter.
CHAPTER 10

OPERATIONAL RESEARCH CONSIDERATIONS IN THE MODELLING OF A LIFE INSURER'S ASSETS AND LIABILITIES

10.0 Introduction

The previous four chapters have provided a detailed descriptive account of the U.K. life assurance industry. Such information is a prerequisite for subsequent model building. In addition, as emphasised in Chapter 4, the dominant paradigm for the evaluation of insurer solvency is simulation methodology and operational research. This chapter forms the basis of Chapter 11, which uses the 'Monte-Carlo' simulation approach to evaluate a number of regulatory issues. As a forerunner to the principle model and its results in Chapter 11, this chapter attempts to provide a detailed account of the simulation algorithms, while also providing an illustration of the main features of simulation methodology.

The construction of an operational model of a life insurer will, by definition, necessitate the use of certain assumptions regarding the nature of the economic environment in which the insurer operates. Assuming that an 'operational' definition of insurer solvency is a net cash-flow function, further assumptions need to be made regarding the insurer's assets and liabilities (see Chapter 4 for a discussion of solvency operational research). In particular, assumptions regarding the cash inflows from investment returns and the cash outflows from underwriting
claims require detailed specification. Under this formulation, any given solvency outcome will be a consequence of a variety of factors, both internal to the insurer-for example the assessment of underwriting risks policy- and external to the insurer-for example the size of investment returns. These factors will, in turn, affect to differing degrees the respective performances of the investment and underwriting portfolios and hence the overall performance of the insurance companies (see Chapter 5).

In attempting to understand the solvency position of an insurer, it is therefore necessary to specify the main components of the economy in which the 'operational' insurer functions. Moreover, since this is an investigation into the regulation of the U.K. life assurance industry, it would be an advantage for this model to reflect some of the main characteristics of the U.K. economy; this therefore has implications for the estimation or calibration of an insurer solvency simulation model. Having defined the set of algorithms, the cash inflows and cash outflows can be calculated period-by-period so that the insurer's net cash flow position may be evaluated across time, given various hypothesised scenarios.

Discussion will proceed by examining the exact nature of the investment and underwriting portfolios. Emphasis is placed on the operational definition of the investment portfolio, and as a consequence this will therefore form the basis of discussion within this chapter, see respectively Sections 10.1-10.4 inclusive. A review of the main characteristics of the underwriting portfolio is given in Section
10.5. In particular, assumptions concerning the composition of the underwriting portfolio and its associated claims process (since actuarial operational research has already focused heavily on the insurer’s underwriting portfolio (see Chapter 3)). Section 10.6 provides a summary of the main developments and sets the stage for the application of the ‘operational’ insurance company in Chapter 11.

10.1 Asset Class Selection

Providing a full account of the investment activities of a life insurer requires an assumption on the number of asset classes and the nature of their associated return distributions. Such ‘stochastic’ returns are a necessary source of information since they will form the major source of fluctuation within insurer solvency. The relationship between these returns is summarised in the variance-covariance matrix of returns and is discussed in section 10.3. Moreover, this matrix provides the necessary information in order to solve the portfolio balance problem, such as the portfolio selection problems defined in Chapter 5.

In defining the asset mix of a life insurer, it is necessary to ensure that the asset classes used are at least representative (for a detailed discussion of the criteria relating to asset class selection see Booth, et al. (1997)). This provides a useful starting point when considering the selection of alternative asset classes. Chapter 7 illustrated that the investment activities of U.K. insurers are substantial, with a market value of total net assets equal to approximately £566 billion in 1995 (see
Table 7.6 (Chapter 7)). Any well-diversified investor (be it an individual, or institution) will exploit investment activities across the risk/return profile, from the relatively secure assets of government securities to the less secure assets such as equities. As discussed in Chapter 4, greater expected returns are associated with a greater variance about this expected return; that is, according to portfolio theory the risk/return trade-off is positively sloped (Markowitz 1959).

As Chapter 6 highlighted, the composition of the underwriting portfolio is another important factor to consider when establishing the composition of the asset portfolio. The first point to note is that depending on the nature of the policies underwritten, different liquidity requirements may be imposed upon the insurer at different periods of time within the policy term. Thus, there is a need for adequate representation of short-term assets within the asset portfolio of an insurer in order to meet this liquidity requirement.

A further point worth addressing is that policy types themselves vary in terms of risk and return. The majority of long-term funds (aside from the re-investment of positive net returns) will come from substantive policies (see Chapter 6). Within the class of substantive policies there is a wide variation in policy features relating to the investment allocation of premiums. Hence, funds generated by a traditional non-profit life policy would be expected to be invested into government fixed interest securities (low risk and near certain return). By contrast, the growth of unit-linked policies has generated funds which, by definition, are invested into the equities
market with its associated characteristics of greater risk and expected return. Thus, the selection of asset classes must reflect the product specific features of life assurance companies in the U.K.

Having set out some of the practical considerations relating to asset class selection, it is useful to address briefly the theoretical considerations (see Booth, et al. (1997) and Ong (1995)). To start with, it is evident that there needs to be enough asset classes to ensure adequate portfolio diversification. Diversification sets constraints upon the selection of asset classes. In particular, diversification requires at least 2 asset classes. Furthermore, there is the need to avoid sampling error by ensuring that the asset classes are distinct (in terms of their return distribution characteristics).

As Ong (1995) argues:

If two or more asset classes have similar characteristics, the relative preferences for these asset classes are likely to be very sensitive to sampling error and model specification. (Ong 1995, 48)

Thus in relation to the argument made by Ong (1995), asset class ‘distinctness’ may be defined in terms of asset class returns, risks, and especially their associated covariance structure. The probability that a given asset class is not distinct will increase in proportion to the total number of asset classes selected.

The above points serve to highlight some of the considerations, both practical and theoretical, in selecting the number of asset classes. These considerations, together with the availability of suitable model algorithms, have resulted in the selection of 5
asset classes to be used in subsequent operational research. Following the recent literature on modelling stochastic investment returns (see Wilkie (1995), and Ong (1995)), assets are subdivided into the following 5 classes: equities, fixed-interest securities (Consols), index-linked government securities, cash, and property. Such classes are representative and distinct, as defined above. However, it might be that greater asset subdivision could have been achieved without significant loss of distinction. While this may have been the case, the assumption of 5 asset classes is justified on the grounds of parsimony and also because the current literature provides only a limited number of stochastic asset return models.

On the basis of the selected asset categories, there is reasonable representation across the risk/return profile. Consider each of the five asset classes in turn. In the case of index-linked securities, its inclusion seems appropriate given the growth in such investments in recent years and the need for adequate protection against excessive inflation within public sector securities.

Consider next the asset classes of Consols and equities. The fixed interest securities represent the (relatively) safe asset, while equities account for an asset class with greater associated risk. In recent years, equity performance is credited for the growth in substantive policy performance (Moneyfacts 1998). Increasingly nowadays, substantive contracts assume an investment strategy that is centred on the stock market and its associated performance. Thus, it becomes apparent that in
practice equities (and their associated variance of return) will be a central consideration in the measuring and regulation of insurer solvency.

With respect to cash, another asset-subdivision, its inclusion is justified on the grounds that it represents the short-term assets of an insurer's investment portfolio. The inclusion of cash provides the means by which an insurer can meet its liquidity requirements. The final subdivision of assets is property. This asset class has been the subject of debate within the operational research literature (see Ong (1995) for a full account of this debate) and for this reason it is worth expanding on the arguments for and against the inclusion of property within the investment portfolio. The evidence, as exemplified in Table 7.5 (Chapter 7), suggests that property is a significant investment for life insurers. However, it has been argued by some, notably Ong (1995), that property is a unique asset and, as a result, cannot be readily incorporated into a stochastic model of an insurance company. Property is not easily disposed of (or acquired) due to its particularly high transactions costs (included here are the search costs of finding a buyer (seller)). The ability to buy and sell with ease would seem a necessary requirement for continuous portfolio re-balancing and this therefore limits the usefulness of property. However, subsequent simulations consider the run-off performance of an insurer, and is for this reason that property is included.

One final consideration in relation to the number of asset classes to be selected, is that of overseas’ investments. The extent of investments overseas is significant (see
Chapter 7), yet the complexity involved in modelling such activities would seem considerable and it is for this reason that such assets are excluded from the model. However, in spite of this limitation, the vast majority of insurer investment activities are represented within the 5 asset classes assumed.

10.2 The Wilkie Model of Stochastic Investment Returns

The original Wilkie Investment Model (Wilkie 1986) provides a stochastic account of the investment returns for a life office or pension fund (see Chapter 4). The model has been applied to a variety of situations (especially issues pertaining to insurer solidity, see for example Hardy (1993, 1994)). The Wilkie model (Wilkie 1986) provides an estimated econometric model designed to be used in simulating investment returns. In its original formulation, the model comprised of the retail price index (RPI), the yield on Consols (long-term interest rates), and the yield on U.K. equities. While the model received widespread attention and frequent application, it was subject to a variety of criticisms both at the theoretical and econometric level (these concerns are addressed in Section 10.4). To counter some of these concerns, Wilkie revised and updated his original model to include additional asset classes (notably short-term assets, index-linked securities and property), and alternative estimation techniques, such as Autoregressive Conditional Heteroskedasticity (ARCH) models and cointegrated models (Wilkie 1995). The 'revised' Wilkie model has also been the subject of debate and criticism. However, in spite of these criticisms, the Wilkie model has been widely endorsed on the
grounds that it provides a practical means of examining complex issues relating to insurer solvency. As a consequence, use will be made of the ‘revised’ model (Wilkie 1995).  

10.2.1 Simulation Techniques

A full account of the 1995 Wilkie model will be developed here. The model is designed for the expressed purpose of simulation, or ‘Monte-Carlo’ techniques. These techniques use computer models to ‘imitate’ real life situations. Inputs to the system reflect variables that are based on probabilistic statements. It should be noted that the use of simulation techniques has become commonplace because of its ability to handle complex systems (especially with the growth in computer processing power). Note that all simulation results are calculated and presented using the ‘@RISK’ add-in to Microsoft Excel. In particular, ‘summary graphs’ are used to present the results of the output distributions. These graphs are produced by @RISK and summarise the changes in probability distributions across the output variable range. It is assumed that the time horizon for all simulations is 25 years. Within the summary graph, 5 parameters are calculated for the output probability distribution across the time horizon; namely, the mean, two upper band values, and two lower band values. The two upper band values represent respectively the +1

---

1 Implicit here is the relationship between model calibration and model estimation. The Wilkie model (Wilkie 1995) will be used to ‘calibrate’ the insurer model subsequently developed in Chapter 11. This is justified on the grounds that the Wilkie model is based on data that is taken from the U.K. and as such, the model can be seen as representative of the long-term investment climate faced by U.K. life insurers.
standard deviation, and the 95th percentile of each distribution. The two lower band
values represent respectively the -1 standard deviation, and the 5th percentile of
each distribution. Thus, the summary graph charts the performance of these five
parameters through time; the wider the interval between the band values the greater
the associated uncertainty with respect to the output variable.

The number of iterations within a simulation will be 10000 and this number is
justified on the grounds that all the output distributions are ‘converged’.\textsuperscript{2}
Convergence is defined in terms of the respective percentage change in the 5
statistics of the summary graph. @RISK monitors three convergence statistics with
respect to the output distribution: first, the average percentage change in the
percentile values; second, the percentage change in the mean; and third, the
percentage change in the standard deviation. Convergence is defined in terms of the
changes in these statistics over successive iterations. In particular, a distribution is
said to have converged when all its convergence statistics have changed by less than
1 percent for two successive iterations. The number of iterations required for
convergence varies according to a given model’s complexity (see Appendix 3 for all
convergence results).

\textsuperscript{2} Convergence is a central aspect of simulation methodology. Convergence tests the stability of the
output distributions created during the simulation. As more iterations are run, output distributions
typically become more “stable” as the statistics describing each distribution change less and less with
each additional iteration. Thus, it is important to run enough iterations so that the statistics generated
on the outputs are reliable.
10.2.2 The Basic Time Series

Discussion now turns to addressing explicitly the nature of the asset model's computer algorithms (Wilkie 1995). Each asset class is addressed in turn, with the retail price index being defined first because of its central role in generating the sequences of the Wilkie model.3

The Retail Price Index

The value of the Retail Price Index (RPI) is given by the series $Q_t$, where $t$ denotes the time period:

\begin{equation}
Q_t = Q_{t-1} \exp(I_t)
\end{equation}

Where the rate of inflation over the period $(t-1, t)$ is given by $I_t = \ln Q_t - \ln Q_{t-1}$, which in turn is generated by a first order autoregressive series (AR(1)) defined as:

\begin{equation}
I_t = QMU + QA(I_{t-1} - QMU) + QE_t
\end{equation}

$QE_t = QSD.QZ_t$

$QZ_t \sim iid \ N(0,1)$

3 Most of the notation used to define the computer algorithms is taken directly from Wilkie (Wilkie 1995), with time periods denoted in subscript.
The estimates provided by Wilkie for the RPI model are as follows.\textsuperscript{4} The results were $Q_{MU}=0.047$, $QA=0.58$ and $QSD=0.0425$ (Wilkie 1995, 785). Thus, for a given year the rate of inflation is equal to its mean rate (4.7 percent), plus 58 percent of last year’s deviation from the mean, plus a random innovation term which has zero mean and standard deviation of 0.0425. Figure 10.1 shows the results of simulating the rate of inflation ($I_t$) over a 25-year period.

Figure 10.1. Simulated Inflation Rates

![Inflation Rate Chart]

Note: (a) Results given for 10000 iterations. (b) Results over a 25-year period.

With reference to Figure 10.1, it is evident that there exists significant variation in the rate of inflation about its mean rate of 4.7 percent. Note that the model allows for negative inflation. This model feature was criticised especially in the mid-1980s after Wilkie introduced his original model (Wilkie 1986). Recently, the inflation

experience has led most commentators to accept this feature. However, the extent of negative inflation can be constrained (for example Hardy (1993) imposes a minimum inflation rate of (-) 5 percent). As will be demonstrated, the RPI is of central importance to the workings of the Wilkie (1995) model and the generation of its stochastic asset returns.

*Equities: Share Prices and Dividends*

To construct a share price index \( (P_t) \), Wilkie models share dividend yields \( (Y_t) \) and share dividends \( (D_t) \), to give \( P_t = D_t / Y_t \). Consider the index of equity dividends on ordinary shares \( D_t \), first, this is defined as follows:

\[
(10.3) \quad D_t = D_{t-1} \exp\{DW \cdot DM_t + DX \cdot I_t + DMU + DY \cdot YE_{t-1} + DB \cdot DE_{t-1} + DE_t\}
\]

\[
DM_t = DD \cdot I_t + (1 - DD) \cdot DM_{t-1}
\]

\[
DE_t = DSD \cdot DZ_t
\]

\[
DZ_t \sim iid \ N(0,1)
\]

Parameters suggested by Wilkie were \( DW=0.58, \ DD=1.3, \ DMU=0.016, \ DY=(-0.175), \ DB=0.57, \ DSD=0.07 \) (Wilkie 1995, 844). Also, \( DX \) is constrained to equal \((1-DW)\). Thus, for any given year, the change in the logarithm of the dividend index is equal to a function of past and present values of inflation, plus a mean real dividend growth (1.6 percent), plus an influence from last year's dividend yield innovation, plus an influence from last year's dividend innovation, plus a random
error (innovation) term with zero mean and standard deviation equal to 0.075. The series is represented in Figure 10.2.

Figure 10.2. Dividend Index for Ordinary Shares

Notes: (a) Results given for 10000 iterations
(b) Results over a 25-year period.
(c) Base Year =1.

There is positive trend for the dividend index from the base year \( t=1 \) (index =1). In addition, there exists greater uncertainty about the mean as \( t\to25 \). This fluctuation is such that there may be no nominal growth at all in the index over the 25-year period (possibly even a nominal decline). The increasing spread of the probability distributions reflects the accumulation of uncertainty (through time) inherent within the predictions.

Consider the next component of the share price index, namely the dividend yield \( Y_t \) on the same set of ordinary shares. This is defined as:
(10.4) \[ Y_i = \exp\{YW_i + \ln YMU + YN_i\} \]

\[ YN_i = YA \cdot YN_{i-1} + YE_i \]

\[ YE_i = YSD \cdot YZ_i \]

\[ YZ_i \sim iid N(0,1). \]

Wilkie's suggested parameters were \( YW = 1.8, YA = 0.55, YMU = 0.0375, YSD = 0.155 \) (Wilkie 1995, 822). Figure 10.3 demonstrates that dividend yields are relatively constant about the mean.

**Figure 10.3. Simulated Dividend Yields**

![Simulated Dividend Yields](image)

Note: (a) Results given for 10000 iterations.
(b) Results over a 25-year period.

The dividend yield function shows that the logarithm of the dividend yield is equal to its mean value (\( \ln 0.0375 \)), plus 55 percent of its deviation a year ago from the mean, plus an additional influence from inflation (equal to 1.8 times the rate of
inflation), plus a random error term with zero mean and standard deviation 0.155.

The two series, $Y_t$ and $D_t$, can therefore be used to construct the equity price index, $P_t$. $P_t$ is derived by evaluating $(D_t/Y_t)$ using Equation 10.3 and Equation 10.4. The new combined series is illustrated in Figure 10.4 below.

![Figure 10.4. Simulated Equity Price Index](image)

Notes (a) Results given for 10000 iterations.
(b) Results over a 25-year period.
(c) Base year, $t=1$.

**Government Fixed Interest Securities**

The yield on (irredeemable) fixed interest securities $C_t$ is generated by the following series.$^5$

---

$^5$ The Consols model can therefore be used to provide an approximation to long-term interest rates in the U.K.
\( (10.5) \quad C_i = CW.CM_i + CMU.exp\{CN_i\} \)
\[
CM_i = CD.I_i + (1 - CD).CM_{i-1}
\]
\[
CN_i = CAI.CN_{i-1} + CY.YE_i + CE_i
\]
\[
CE_i = CSD.CZ_i
\]
\[
CZ_i \sim iidN(0,1).
\]

Suggested parameters were \( CW=1.0, CD=0.045, CMU=0.0305, CAI=0.9, CY=0.34 \)
and \( CSD=0.185 \) (Wilkie 1995, 862). Thus, at any date the Consols yield is
decomposed into two parts, an allowance for expected future inflation, and a real
yield. The series is presented in Figure 10.5 below.

**Figure 10.5. Simulated Yields on (Irredeemable) Fixed Interest Securities**

Notes: (a) Yield on Consols.
(b) Results given for 10000 iterations.
(c) Results over a 25-year period.
Although the Consols yield is relatively constant, there is a tendency for uncertainty to increase about the mean yield as time increases. This uncertainty may seem an appropriate risk to incorporate into an insurer model of solvency, given the importance of long-term interest rates.

**Cash (Short-term Interest Rates)**

The model presented by Wilkie (1995) for generating the return on cash (and hence short-term interest rates) is given by the autoregressive series, $B_t$:

$$B_t = C_t \exp\{-BD_t\}$$

$$BD_t = BMU + BA.(BD_{t-1} - BMU) + BE_t$$

$$BE_t = BSD.BZ_t$$

$$BZ_t \sim iid \ N(0,1)$$

Parameters suggested for this model were $BMU=0.23$, $BA=0.74$ and $BSD=0.18$ (Wilkie 1995, 871). The series is illustrated below in Figure 10.6. In effect, cash is treated as a one-year (irredeemable) bond.
Figure 10.6. Simulated Short-term Interest Rates

Figure 10.6 demonstrates that the volatility associated with short-term interest rates, about its mean rate of return, is substantial. This will therefore provide another relevant source of uncertainty in modelling insurer solvency.

Property: Yield and Income

A property-price index $A_t$ represents the price and income performance of a given property portfolio and is developed as follows. The index is defined in a similar fashion to that of the equity price index $P_t$ with the relevant components being property yield $Z_t$ and property income $E_t$, with $A_t = E_t/Z_t$. 
Consider first, the property yield $Z_t$, which is given by the following function:

\begin{equation}
Z_t = \exp\{ZN_t\}
\end{equation}

\begin{align*}
ZN_t &= \ln ZMU + ZA.(\ln Z_{t-1} - \ln ZMU) + ZE_t \\
ZE_t &= ZSD.ZZ_t \\
ZZ_t &\sim \text{iid } N(0,1)
\end{align*}

Suggested parameters were $ZMU = 0.074$, $ZA = 0.91$ and $ZSD = 0.12$ (Wilkie 1995, 877). At this point, it is important to emphasise that there are some difficulties associated with these parameter specifications and the ultimate prediction of the property price index $A_t$. The use of these parameters (together with the parameters of the equity share price index) results in a property price index ($A_t$) that consistently out-performs an equity price index ($P_t$). Such a result holds where there is a lower associated risk (variance of return) for property than equity and thus implies a negatively sloped risk-return trade-off between these two asset classes.\(^6\) On the basis of the predictions from portfolio theory, this result seems counter-intuitive. Moreover, the average nominal return for 80 sampled property life funds is roughly 10 percent and this is less than the predicted returns from an equity portfolio of equivalent initial size (Moneyfacts 1998). For this reason the above parameters are amended to produce outcomes which are consistent with the recent evidence.

---

\(^6\) There are many potential explanations for these predictions generated by the Wilkie model. Much of these relate to the uncertainties surrounding the true model and the availability of accurate data (for a discussion of these explanations see Section 10.4).
As a consequence of the above concerns, use is made of the parameters provided by Daykin and Hey (1990) in order to calibrate the Wilkie model and produce outcomes that are more consistent with the risk/return predictions of portfolio theory. The parameters used by Daykin and Hey (1990) were $Z_A=0.6$, $Z_{MU}=0.05$ and $Z_{SD}=0.075$ (Daykin and Hey 1990, 181). As before, this represents a simple autoregressive AR(1) model where the logarithm of property yield is equal to its mean value ($ln0.05$) plus 60 percent of its deviation a year ago from the mean, plus a random error term with zero mean and 0.075 standard deviation, see Figure 10.7.

![Figure 10.7. Simulated Yields on a Given Property Portfolio](image)

Notes: (a) Results given for 10000 iterations.
(b) Results over a 25-year period.

While much is said about the yield on a property portfolio, it is also necessary to consider the property income component of a property-price index. For income on property (especially income in the form of rent), Wilkie attempts to estimate the
income growth over the period 1967 to 1994. The property income index is given by $E_t$, and is defined as follows:

\begin{equation}
E_t = E_{t-1} \exp \{ EW \cdot EM_t + EX \cdot I_t + EMU + EE_t \}
\end{equation}

\begin{align*}
EM_t &= ED \cdot I_t + (1 - ED) \cdot EM_{t-1} \\
EE_t &= ESD \cdot E_t \\
EZ_t &\sim iid\ N(0,1)
\end{align*}

The parameters used by Wilkie were $EW = 1.0$, $ED = 0.13$, $EMU = -0.01$, and $ESD = 0.05$ (Wilkie 1995, 879). See Figure 10.8 for an illustration of simulated property income.

**Figure 10.8. Simulated Property Income Index**

Note: (a) Results given for 10000 iterations. (b) Results over a 25-year period.
Figure 10.8 indicates the upward trend in property income (which includes property rental) together with an expansion in projected uncertainty with time t. The model is centred on the inflation rate \( I_t \), its mean value \( \ln(-0.01) \), plus a random error term with zero mean and 0.05 standard deviation.

Having defined the yield and income components associated with property, a property price index \( A_t \) can be constructed. The resulting property price index is illustrated below in Figure 10.9.

Figure 10.9. Simulated Property Price Index

Notes: (a) Results given for 10000 iterations. (b) Results over a 25-year period.

*Index-Linked Government Securities*

The real yield on index-linked stocks, denoted by \( R_t \) (at time t), is an \( AR(I) \) model and is defined as follows:
\begin{equation}
\ln R_i = \ln RMU + RA(\ln R_{i-1} - \ln RMU) + RBC\cdot CE_i + RE_i
\end{equation}

\[ RE_i = RSD \cdot RZ_i \]

\[ RZ_i \sim iid \ N(0,1) \]

(Note. $CE_i$ is the error term from the model for fixed interest securities)

Suggested parameters for this model were $RMU = 0.04$, $RA = 0.55$, $RBC = 0.22$ and $RSD = 0.05$ (Wilkie 1995, 883). The real yield of these securities fluctuates about the mean return of 4 percent—see Figure 10.10 below (note the modest variations in real yields). Thus the logarithm of the real yield is equal to the logarithm of its mean value ($\ln 0.04$), plus 58 percent of the deviation a year ago from the mean, plus a random innovation from the fixed interest securities, plus its own random innovation with zero mean and standard deviation 0.05.

Figure 10.10. Simulated Real Returns on Index-Linked Government Securities.

Notes. (a) Results given for 10000 iterations.
(b) Results over a 25-year period.
10.2.3 The ‘Rolled-Up’ Indices

Having defined the basic time series associated with the 5 asset classes, Wilkie (1995) proceeds by constructing a ‘rolled-up’ index for each asset class. These indices represent the total ‘rolled-forward’ nominal returns at time \( t \) for a unit sum invested; such an index therefore gives the compounded returns over a given period of time. The rolled-up series are defined as follows:

\[
PR_t = \frac{PR_{t-1} \{P_t + D_t(1-t_1)\}}{P_{t-1}} \quad \text{(Equities)}
\]

\[
CR_t = CR_{t-1} \left\{ \frac{1}{C_t} \right\} + (1-t_2) \cdot C_{t-1} \quad \text{(Consols)}
\]

\[
BR_t = BR_{t-1} \left\{ 1 + (1-t_3) \cdot B_{t-1} \right\} \quad \text{(Cash)}
\]

\[
RR_t = RR_{t-1} \left\{ \frac{1}{R_t} + (1-t_4) \cdot R_{t-1} \right\} \cdot \frac{Q_t}{Q_{t-1}} \quad \text{(Index-linked)}
\]

\[
AR_t = AR_{t-1} \left\{ A_t + E_t(1-t_5) \right\} \quad \text{(Property)}
\]

Where \( t_i \) (\( i=1 \) to 5) represents the tax rate for the respective asset class (should the tax rate wish to be incorporated into the analysis). However, since the tax calculation is not a simple one with respect to insurance companies and their investment income, gross returns will be assumed.
As an illustration of the application of the Wilkie model, use is made of the generated returns to derive the efficient set of portfolios (see Chapters 4 and 5). In order to calculate the variance/covariance matrix for the standard Markowitz portfolio selection problem, calculations of asset returns are required. To calculate these rates of returns, the following variables derive the nominal and real rates of return for a variable $X_t$, viz.

\[(10.15) \quad FX_t = \frac{X_t}{X_0} \]

\[(10.16) \quad GX_t = 100 \left[ \frac{FX_t}{100} - 1 \right] \]

\[(10.17) \quad HX_t = \frac{FX_t}{FQ_t} \]

\[(10.18) \quad JX_t = 100 \left[ \frac{HX_t}{100} - 1 \right] \]

The variable $FX_t$ is the return over $t$ years from an investment of 1 at time 0, and $GX_t$ is the equivalent compound annual rate of return (expressed as a percentage). $HX_t$ and $JX_t$ are defined in the same way, but they are based on real returns rather than nominal returns and hence are defined relative to the retail price index, $Q_t$.

Table 10.1 and Table 10.2 provide simulation results for nominal and real returns respectively. This information provides a useful illustration of the performance of
the Wilkie model and also demonstrates some other features specific to the model.

Consider first nominal returns—see Table 10.1 below.

Table 10.1. Rates of Nominal Return (%) by Asset Class for Various Terms

<table>
<thead>
<tr>
<th>Asset Class (\textit{GPR}_t)</th>
<th>Mean &amp; Standard Deviation</th>
<th>Term ( t=1 )</th>
<th>Term ( t=5 )</th>
<th>Term ( t=10 )</th>
<th>Term ( t=15 )</th>
<th>Term ( t=20 )</th>
<th>Term ( t=25 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equities ( E(\text{GPR}_t) )</td>
<td>13.42</td>
<td>11.23</td>
<td>10.87</td>
<td>10.79</td>
<td>10.74</td>
<td>10.75</td>
<td></td>
</tr>
<tr>
<td>SD(\text{GPR}_t)</td>
<td>19.65</td>
<td>11.04</td>
<td>6.66</td>
<td>4.62</td>
<td>3.55</td>
<td>2.43</td>
<td></td>
</tr>
<tr>
<td>Consols ( E(\text{GCR}_t) )</td>
<td>8.12</td>
<td>7.94</td>
<td>7.86</td>
<td>7.91</td>
<td>7.83</td>
<td>7.85</td>
<td></td>
</tr>
<tr>
<td>SD(\text{GCR}_t)</td>
<td>7.83</td>
<td>4.97</td>
<td>3.55</td>
<td>2.90</td>
<td>1.96</td>
<td>1.88</td>
<td></td>
</tr>
<tr>
<td>Cash ( E(\text{GBR}_t) )</td>
<td>6.69</td>
<td>6.45</td>
<td>6.28</td>
<td>6.45</td>
<td>6.56</td>
<td>6.51</td>
<td></td>
</tr>
<tr>
<td>SD(\text{GBR}_t)</td>
<td>0.02</td>
<td>1.32</td>
<td>1.36</td>
<td>1.27</td>
<td>1.21</td>
<td>1.18</td>
<td></td>
</tr>
<tr>
<td>Index-linked ( E(\text{GRR}_t) )</td>
<td>9.55</td>
<td>9.37</td>
<td>9.22</td>
<td>8.93</td>
<td>8.96</td>
<td>8.97</td>
<td></td>
</tr>
<tr>
<td>SD(\text{GRR}_t)</td>
<td>12.68</td>
<td>6.83</td>
<td>5.11</td>
<td>3.41</td>
<td>2.73</td>
<td>1.92</td>
<td></td>
</tr>
<tr>
<td>Property ( E(\text{GAR}_t) )</td>
<td>11.76</td>
<td>10.04</td>
<td>9.64</td>
<td>9.87</td>
<td>9.52</td>
<td>9.55</td>
<td></td>
</tr>
<tr>
<td>SD(\text{GAR}_t)</td>
<td>15.33</td>
<td>9.71</td>
<td>5.93</td>
<td>4.05</td>
<td>3.49</td>
<td>2.14</td>
<td></td>
</tr>
</tbody>
</table>

Notes. (a) Results given for 10000 iterations. (b) \( E(\cdot) \) denotes the expected real return. (c) \( SD(\cdot) \) denotes the standard deviation of real return. (d) For evidence of convergence—see Appendix 3.

The rates of nominal returns demonstrate a positive risk/return trade-off. The relative ranking of the asset classes is as follows (from largest expected return to lowest expected return): equities, property, index-linked government securities, fixed-interest government securities, and cash.

Next consider the real returns—see Table 10.2. This table demonstrates a crucial operational distinction between real returns and nominal returns; namely, that the
risk/return trade-off is not the same for nominal and real returns. Note that the summary graphs of the associated real return simulations are given in Appendix 3.

Table 10.2. Rates of Real Return (%) by Asset Class for Various Terms

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Mean &amp; Standard Deviation</th>
<th>Term t=1</th>
<th>Term t=5</th>
<th>Term t=10</th>
<th>Term t=15</th>
<th>Term t=20</th>
<th>Term t=25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equities</td>
<td>E(JPR)</td>
<td>8.45</td>
<td>7.55</td>
<td>5.94</td>
<td>5.93</td>
<td>5.90</td>
<td>5.89</td>
</tr>
<tr>
<td></td>
<td>SD(JPR)</td>
<td>20.67</td>
<td>12.08</td>
<td>7.54</td>
<td>3.52</td>
<td>2.94</td>
<td>2.16</td>
</tr>
<tr>
<td>Consols</td>
<td>E(JCR)</td>
<td>3.57</td>
<td>2.98</td>
<td>2.78</td>
<td>2.96</td>
<td>3.09</td>
<td>3.23</td>
</tr>
<tr>
<td></td>
<td>SD(JCR)</td>
<td>9.35</td>
<td>6.46</td>
<td>5.94</td>
<td>3.10</td>
<td>2.54</td>
<td>1.87</td>
</tr>
<tr>
<td>Cash</td>
<td>E(JBR)</td>
<td>1.24</td>
<td>2.53</td>
<td>2.48</td>
<td>2.37</td>
<td>2.26</td>
<td>2.39</td>
</tr>
<tr>
<td></td>
<td>SD(JBR)</td>
<td>4.30</td>
<td>3.53</td>
<td>2.82</td>
<td>2.38</td>
<td>2.06</td>
<td>1.44</td>
</tr>
<tr>
<td>Index-linked</td>
<td>E(JRR)</td>
<td>3.97</td>
<td>3.96</td>
<td>4.01</td>
<td>4.01</td>
<td>3.99</td>
<td>4.03</td>
</tr>
<tr>
<td></td>
<td>SD(JRR)</td>
<td>3.66</td>
<td>1.47</td>
<td>0.77</td>
<td>0.52</td>
<td>0.37</td>
<td>0.31</td>
</tr>
<tr>
<td>Property</td>
<td>E(JAR)</td>
<td>8.20</td>
<td>6.95</td>
<td>5.89</td>
<td>5.28</td>
<td>4.95</td>
<td>4.78</td>
</tr>
<tr>
<td></td>
<td>SD(JAR)</td>
<td>14.34</td>
<td>5.85</td>
<td>3.68</td>
<td>2.76</td>
<td>2.4</td>
<td>1.96</td>
</tr>
</tbody>
</table>

Notes. (a) Results given for 10000 iterations. (b) $E(.)$ denotes the expected real return. (c) $SD(.)$ denotes the standard deviation of real return. (d) For evidence of convergence- see Appendix 3.

Table 10.2 demonstrates the results of the simulations for 'real' rates of return. With respect to the risk/return trade-off there are a number of points to consider. For equities, property, Consols and cash, the risk/return trade-off is positive and hence in line with the predictions of portfolio theory. However, an anomaly is reported with respect to index-linked government securities; in particular, index-linked securities outperform the real returns of Consols and cash, yet have the lowest overall associated risk (standard deviation). This is a direct consequence of using the real returns as a basis for comparison. From a practical perspective, index-linked securities are designed to provide a return that matches inflation yet
also is as secure as fixed-interest securities. From an operational perspective, the explicit functional forms used to generate the sequence of returns might also be an explanation for the real return results. As a consequence, it is not surprising to see why the real return risk associated with index-linked securities is very small.

10.3 The Variance/Covariance Matrix of Returns

Having defined the nature of nominal and real returns for the five asset categories, the next step towards illustrating the efficient frontier is to construct the associated variance-covariance matrix of returns. The converged estimates of the variance-covariance matrices are given in Table 10.3 and Table 10.4 for nominal and real returns, respectively.

Table 10.3. The Variance/Covariance Matrix for Nominal Returns (25 years)

<table>
<thead>
<tr>
<th></th>
<th>Cash</th>
<th>Consols</th>
<th>Index-linked</th>
<th>Equities</th>
<th>Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>1.39</td>
<td>0.97</td>
<td>1.02</td>
<td>0.94</td>
<td>1.05</td>
</tr>
<tr>
<td>Consols</td>
<td>3.53</td>
<td>0.15</td>
<td>0.83</td>
<td>0.15</td>
<td>2.09</td>
</tr>
<tr>
<td>Index-linked</td>
<td>3.68</td>
<td>1.56</td>
<td>2.05</td>
<td>2.05</td>
<td>4.57</td>
</tr>
<tr>
<td>Equities</td>
<td></td>
<td></td>
<td></td>
<td>5.90</td>
<td>2.05</td>
</tr>
<tr>
<td>Property</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.57</td>
</tr>
</tbody>
</table>

Notes: (a) Results given for 10000 iterations. (b) Variance-covariance estimates based on a 25-year period.

Consider first Table 10.3. The main diagonal of the variance-covariance matrix represents the respective asset variance about return whilst the remaining values off the main diagonal of the matrix represent the covariance \((A, B)\) where, \(A\) and \(B\) are the row and column asset respectively. Only the upper half of the matrix is
completed due to its symmetry. The results reported for the variance/covariance matrix of nominal returns shows that there is only one negative covariance reported, the covariance between Consols and property. Next consider the results reported in Table 10.4.

Table 10.4. The Variance/Covariance Matrix for Real Returns (25 years)

<table>
<thead>
<tr>
<th></th>
<th>Cash</th>
<th>Consols</th>
<th>Index-linked</th>
<th>Equities</th>
<th>Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>2.07</td>
<td>4.62</td>
<td>0.12</td>
<td>3.58</td>
<td>2.46</td>
</tr>
<tr>
<td>Consols</td>
<td>3.49</td>
<td>2.21</td>
<td>0.09</td>
<td>7.57</td>
<td>3.56</td>
</tr>
<tr>
<td>Index-linked</td>
<td>0.09</td>
<td>0.32</td>
<td>0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equities</td>
<td></td>
<td></td>
<td></td>
<td>4.66</td>
<td>3.75</td>
</tr>
<tr>
<td>Property</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.84</td>
</tr>
</tbody>
</table>

Notes: (a) Results given for 10000 iterations. (b) Variance-covariance estimates based on a 25-year period.

It is evident that there exists only positive covariance between the real returns on different asset classes. This might be a feature that is exacerbated when dealing with real returns rather than nominal returns, since the influence of the RPI will be prominent (especially given the structure of the Wilkie model algorithms). Another explanation of the positive covariance terms might be associated with the number of the asset classes; for example, the low number of asset classes may disguise underlying negative covariances that might exist between individual securities (rather than the groups of securities assumed here). However, regardless of the cause, the lack of negative covariance places a limit on the extent to which diversification can take place within this hypothetical insurer asset portfolio.
The Asset-Only Efficient Frontier

The frontier of efficient portfolios with respect to nominal returns and real returns is illustrated in Figure 10.11 and Figure 10.12, respectively. These illustrations show the 'efficient' portion of the hyperbola (for the analytical treatment of efficient frontiers, see Chapter 5).

Figure 10.11. The Efficient Frontier Using the 1995 Wilkie Model (Nominal Returns)

Notes: (a) Asset returns, variances and covariances are based on the results of simulations performed by the Wilkie (1995) model over a 25 year period and presented in Table 10.1 and Table 10.3.
(b) Solutions to the quadratic program provided with the use of 'Solver'- an add-in to Microsoft Excel.
(c) Quadratic program solved by varying risk for a given return, assuming that the portfolio weights sum to unity and that they are all individually non-negative.

The Markowitz (1959) quadratic program (see Equation 5.4 and Figure 5.1 in Chapter 5) was used to calculate the efficient frontiers presented in Figure 10.11 (nominal returns) and 10.12 (real returns). These calculations trace out hypothetical
efficient frontiers (based on the estimates of Section 10.2) and this in turn
demonstrate the operational distinction between nominal and real returns.

Figure 10.12. The Efficient Frontier Using the 1995 Wilkie Model (Real Returns)

Notes: (a) Asset returns, variances and covariances are based on the results of simulations
performed by the Wilkie (1995) model over a 25 year period and presented in Table 10.2
and Table 10.4.
(b) Solutions to the quadratic program provided with the use of ‘Solver’- an add-in to
Microsoft Excel.
(c) Quadratic program solved by varying risk for a given return assuming that the portfolio
weights sum to unity and that they are all individually non-negative.

As discussed in Chapter 5 (see Section 5.3), the problem of selecting an operating
point along the efficient frontier is solved typically with reference to utility theory
(see equations 5.15-5.21 for the derivation of optimality conditions using utility
theory). However, it should be noted that the above illustrations are only with
respect to the five asset classes, with no account of the insurer’s liabilities.
Liabilities are defined in Section 10.5 and are incorporated into utility theory using
the Sherris model (Sherris 1992), see Chapter 11.
10.4 Discussion of the Wilkie Model

Before addressing some of the main points of interest associated with the Wilkie (1995) model, it is important to note the initial conditions for the simulations. The initial conditions need to be defined in order that the simulation may begin at time $t=0$. As Wilkie notes, there are many potential candidates for selecting the initial conditions.

'Neutral' initial conditions are proposed here where, "...the starting values are set at what their long-run means would be if all the standard deviations were zero," (Wilkie 1995, 902). This would seem a reasonable assumption upon which to commence the simulation. The full set of neutral conditions is given below in Table 10.5.

Table 10.5. Initial Conditions.

<table>
<thead>
<tr>
<th>Initial Conditions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I(0) = QMU = 0.047$</td>
</tr>
<tr>
<td>$Y(0)% = 4.0811%$</td>
</tr>
<tr>
<td>$C(0)% = 7.75%$</td>
</tr>
<tr>
<td>$B(0)% = 6.1576%$</td>
</tr>
<tr>
<td>$R(0)% = RMU% = 4.0%$</td>
</tr>
<tr>
<td>$Z(0)% = ZMU% = 5.0%$</td>
</tr>
<tr>
<td>$DM(0) = CM(0) = EM(0) = QMU = 0.047$</td>
</tr>
<tr>
<td>$YE(0) = DE(0) = 0$</td>
</tr>
<tr>
<td>$Q(0) = D(0) = P(0) = E(0) = A(0) = PR(0) = CR(0) = BR(0) = RR(0) = AR(0) = 1$</td>
</tr>
</tbody>
</table>

(for the construction of indices)

The previous sections have been used to develop the series of stochastic asset returns and demonstrate that the Wilkie model’s strength is in terms of its practical usage and ease of interpretation. Most of the series are built around an
autoregressive structure. Moreover, considering the structure of the Wilkie model, for example the structure of the retail price index (RPI), it is clear that there is an *adaptive* expectation mechanism underpinning much of the model.

As noted earlier in this chapter, some of the predictions relating to the property price index are inconsistent with the predictions of portfolio theory (Markowitz 1959). One of the difficulties in constructing a statistical model is that it is constrained by data availability and estimation techniques. While many alternative estimation techniques were explored, the available data with respect to property income and yield has been questioned (Ong 1995, 49). As a result, it is not surprising to find unusual predictions. However, the inclusion of property is still seen as a valuable addition to the analysis and for this reason this asset class remains within the analysis.

In terms of the robustness of the security returns it is useful to make reference to a recent paper on the sensitivity of life office simulation outcomes to differences in asset model structure (see Chadburn and Wright (1999)). Chadburn and Wright examine whether the choice of stochastic asset model structure is a material factor in determining the output and hence interpretation of stochastic actuarial asset-liability modelling experiments, or whether such differences can effectively be removed by appropriate parameterisation and hence remain under simple effective control by the

---

378

7 The generation of real yields for index-linked securities also provides a result that might be model specific.
used. In accordance with this objective they use various simulation output measures related to both solvency and policy payout levels to test their sensitivity to asset model structure. Three different stochastic asset models were tested: the Wilkie (1995) model, a Vector auto-regressive (VAR) model, and a so-called first order auto-regression (AR(1)) model. The paper makes the following observations:

Simulated probabilities of ruin appear to be reasonable robust to model structure. Other output measures, particularly the simulated bonus rates and policy maturity values, are distinctly sensitive to model structure, reflecting residual differences which exist in the simulated asset returns from the different models over shorter durations. (Chadburn and Wright 1999, 27)

Thus, a discussion of model structure is highlighted as a basic consideration and the output of a simulation system will be a function of the asset and liability model. Hence, the results of any given solvency assessment need to be interpreted in light of the characteristics of the underlying stochastic model.

The significance of the Wilkie (1995) model in its application to life office analysis is seen as enough justification for its use within the simulation work performed here and for this reason discussion proceeds to preparing for the simulation work in Chapter 11. Note that model assumption will also be a consideration in the implementation of any policy change vis-à-vis solvency measurement (see Chapter 12).
Related to the above point regarding the use of statistical models, Huber & Verrall (1997) critically assess the Wilkie model within a methodological context. In particular, they debate the relative merits of an essentially 'data-based' methodology with that of a theoretical methodology where:

Data based methods assume that the main features of the financial system can be adequately approximated by directly modelling [historical] data... [and] ... theoretical methods assume that the economy is too complex to learn about it directly and that a substantial part of the historical data is time specific. (Huber and Verrall 1997, 1)

Huber and Verrall highlight the fact that the Wilkie model is largely 'data-based' and is therefore likely to fall foul of the theorists. While this might be the case, on balance it is argued here that the practical advantages of using the Wilkie model outweigh these concerns. Moreover, since the objectives here are to examine insurer solvency issues relating to the U.K. life assurance industry, the use of 'historical' experience as a starting point for discussion would seem a reasonable assumption.

10.5 Liabilities

An extensive account of the life insurer's assets has been presented. In order to complete the preparations for subsequent simulations, the liabilities of the life insurer need to be defined. The operations of insurance companies may be broken down into annual cohorts of policies (Waters 1988). The overall performance of the insurance company will depend on the performance of these cohorts. For ease of
illustration and subsequent analysis, discussion will focus on just one of these
cohorts. As a consequence, liabilities are assumed to be fixed and are defined by a
single cohort of policies made up of non-smoking males, with an age next birthday
equal to 35, and a policy sum assured equal to £50,000. At time $t=0$, 1000 identical
policies are issued by the insurer simultaneously and these policies will therefore
define the underwriting composition of the life insurer.

The policy type assumed is the (full) endowment assurance policy. The endowment
assurance plan provides a guaranteed benefit on the occurrence of death and also
provides this benefit, if death has not already occurred, at the end of the policy term.
Such contracts are used as a means of repayment on mortgages and will have
premiums far greater than those quoted for regular low-cost endowment plans where
the maturity value of the substantive plan is not guaranteed. The premium for
endowment assurance is an approximation of the net premium, calculated on the
basis of the assumed mortality rate and the associated ‘monetary’ function (Faculty
of Actuaries 1989) assuming 5 percent interest. The calculated premium is £1075
per year.

The claim outflows are defined as follows. If the policyholder dies before the end of
the policy term, payment of the sum assured will occur before the policy matures. If
the policyholder survives the term of the contract, the payment will be made and be
equal to the sum assured. Withdrawals from policy contracts are normalised to zero.
While the size of the payment outflow is defined above (equal to £50,000), the timing of this payment will depend upon mortality rates. Therefore, it is assumed that mortality is deterministic and follows the projections of the Government Actuaries Department (Government Actuaries Department 1994).

### 10.6 Summary

The assets and liabilities of a hypothetical life assurance company have been defined. These definitions will form the basis of the simulation model used in Chapter 11 in order to evaluate life insurer solvency within a regulatory setting. Five asset classes were selected: equities, Consols, cash, index-linked securities and property. Use is made of the 1995 Wilkie model of stochastic investment returns. On the liability-side, a single cohort of endowment assurance contracts was assumed.

From the simulation results of real returns, index-linked securities seem the most stable both over the short-term and longer-term. This is a direct consequence of the way in which the asset class is ‘operationalised' and also serves to illustrate the sensitivity of outputs to the way in which a simulation model is constructed. One of the problems associated with simulation methodology is that scenarios are frequently generated that might not be easily explained in practice. However, in terms of nominal returns the results (that are the returns) are as expected and hence in line with the predictions of portfolio theory.
As a point of departure for discussion in Chapter 11, returns generated from the Wilkie model (Wilkie 1995) are assumed to be representative of the insurer’s financial environment, and as a result these returns are used as a proxy for insurer expectations regarding the future performance of its associated investments. Thus, the returns generated by the Wilkie model will be employed in an insurer optimising routine for the purposes of solving an insurance company’s portfolio selection problem at the start of the solvency simulation, at time $t=0$. 
CHAPTER 11

LIFE INSURER RISK PREFERENCE AND SOLVENCY: A SIMULATION APPROACH

11.0 Introduction

This chapter uses a ‘Monte-Carlo’ simulation approach to address solvency considerations in the regulation of U.K. life assurance industry. As such, this analysis builds on the formal developments of the previous chapter, Chapter 10. The previous chapter constructed an ‘operational’ model of a life insurer’s assets and liabilities employing simulation techniques to reproduce investment returns.

Having defined the assets and liability in Chapter 10, a simulation model of insurer solvency is constructed in this chapter. The aim of the ‘operational’ insurance company is defined as follows: the insurer must decide how to allocate its premium fund amongst five asset classes in order to meet a fixed liability (see Chapter 10). A hypothetical insurance company is constructed and its decision-making modelled in terms of an expected utility maximisation rule. The insurance company decisions are evaluated in terms of their solvency outcome at the end of the simulation period. The regulatory implications are discussed with respect to the use of solvency margins as a means of regulation, and also with respect to the more general considerations in the measurement (and regulation) of insurer solvency. While a number of different aspects of insurer solvency will be discussed in this chapter, the
simulations have been designed to focus on answering to the following two questions:

1) What is the probability that the premium fund together with any accrued investment-income will be insufficient to meet the cost of the claims?

2) What 'margin of solvency' is required in order that the operational insurer will pass a predetermined regulatory standard?

This chapter builds on the basic framework developed by Waters (1988), extending his essentially statistical approach to modelling solvency to include a sound theoretical basis for portfolio selection. As already noted, an expected utility maximisation rule defines the objective for the insurance company. Such a programming problem will be a function of a number of factors; for example, the characteristics of the return distributions associated with each asset class, the risk preference of an insurer, and also the characteristics of the liability portfolio. This latter factor marks an important point of departure from the traditional asset portfolio selection problem (see Chapter 4). Indeed, the significance of liabilities to the portfolio selection problem has been the focus of recent work in this area- see for examples Wilkie (1985), Wise (1987), Sherris (1992) and Booth and Ong (1994). In line with this work, an insurer portfolio model is defined and asset weights are derived for various levels of risk aversion, given the appropriate specification of the insurer's utility function. The solvency performance associated with each level of risk aversion (that is, each 'optimal portfolio') is then evaluated.
The structure of this chapter is as follows. Section 11.1 develops the theoretical basis for the insurer's investment strategy. Section 11.2 presents the solvency simulation algorithm with the simulation results being given in Section 11.3. The explicit role of risk preference together with its associated solvency implications is discussed in Section 11.4. Note that the reader is referred to Chapter 10 for a detailed analysis of the simulation algorithms used to model the stochastic asset returns. Section 11.5 provides a summary of the main findings.

11.1 The Insurer Optimisation Problem

11.1.1 The Asset-Only Portfolio Selection Problem

Recall from Chapter 5 that the portfolio selection problem is defined in terms of maximising the expected utility of terminal wealth and that a solution to this problem occurs where the insurer's indifference curves are tangential to the efficient frontier. The efficient frontier is therefore derived by solving the following minimisation problem for different values of portfolio expected return \( E^* \), viz.

\[
(11.1) \quad \min_w \left\{ \frac{1}{2} w'Vw \right\}
\]

Subject to:  
\[ e'w = E^* \]
\[ w \geq 0 \]
\[ w'1 = 1 \]

\(^1\) That is, the insurer's 'optimal portfolio' is where the Marginal rate of Substitution (MRS) is equal to the Marginal Rate of Transformation (MRT) in risk/return space.
Where $V$ is the variance-covariance matrix and $w$ is a vector of weights. The characteristics of the asset-only efficient frontier are illustrated in Figure 10.11 and Figure 10.12. As explained in Chapter 5, utility theory provides the means of selecting an 'optimal portfolio' from the set of efficient portfolios. Moreover, the respective risk/return characteristics of the asset classes are incorporated into the minimisation problem in Equation 11.1 and asset classes are selected assuming a given level of insurer risk aversion.

The hypothetical life assurance company constructed in this chapter consists of five asset classes $i=1,2,3,4,5$, respectively cash, Consols, index-linked government securities, equities and property. Each asset class is subject to uncertainty concerning its performance over the simulation period (the 1995 Wilkie model (see Chapter 10) defines this uncertainty). The asset mix selected by the insurer will be a row vector of portfolio weights $w' = (w_1, w_2, w_3, w_4, w_5)$, where the $w_i$'s ($i=1...5$) represent the portfolio weights assigned to each of the asset classes per unit sum invested. In the absence of any explicitly defined liabilities, the variance-covariance matrix, denoted by the matrix $V$, is a $5*5$ symmetric matrix.

Having defined the required information for portfolio balance, the quadratic minimisation problem can be solved for different values of expected return, $E^*$. An expected utility objective function is used to select the 'optimal portfolio' amongst all efficient portfolios. Note that this framework does not take into account the
risk/return profile of the liabilities and it is for this reason that additional assumptions need to be incorporated into the asset-only approach.

11.1.2 Insurer Optimisation in the Presence of Liabilities

Consider the explicit development of a portfolio model in the presence of fixed liabilities. In order to incorporate these liabilities into the quadratic program, use is made of the model developed by Sherris (1992). The Sherris model provides an analytical framework for evaluating the portfolio selection problem of a life fund, selecting assets to meet a fixed liability. As Sherris notes, liabilities for life funds are fixed in terms of their marketability and disposability (Sherris 1992, 88). It is for this reason that the simplification is made and liabilities are therefore assumed fixed.\footnote{Recall that under the general portfolio model of a life assurance company (see Chapter 5), different insurance lines were treated as negative assets and assumed to be as marketable as the asset classes.} Sherris (1992) demonstrates that the portfolio selection problem for a life fund may be expressed in terms of the mean and variance of \textit{ultimate surplus}. Ultimate surplus is defined in terms of a net cash flow at a fixed time horizon; it is the difference between the accumulated asset funds and the accumulated liability. Thus, ultimate surplus provides an 'operational' definition of insurer solvency.

Following Sherris (1992), it will be assumed that the initial value of assets held to meet the liability is denoted by $A$, and that this value consists of two components, $C$ and $K$. $C$ is a fixed component and belongs to the fund claimants, and is represented by the premium funds (the expected value of the liability). By contrast, $K$ is a
variable component and is the provision for adverse conditions, that is \( K \) is the insurer's equity. The appropriate size of \( K \), in order to pass a predetermined regulatory standard, will be investigated in Section 11.4. For now, the initial amount of assets is given by the following expression:

\[
(11.2) \quad A = (1 + p)C
\]

Where \( p = K/C \) and this represents the size of the insurer's solvency margin.

Furthermore, there are assumed to be five asset classes \((i=1,2,3,4,5)\) in addition to one liability, \( L \). With respect to the asset classes, the cash flows at the end of the simulation period, assuming that all intermediate cash flows are reinvested until the end of period, are random variables and may be denoted by \( R_i \) \((i=1\ldots5)\). These amounts are all per unit of security purchased and therefore the cash flow of security \( i \) is given by:

\[
(11.3) \quad R_i = (1 + \mu_i)
\]

Where \( \mu_i \) is the rate of return on asset \( i \) over the period with an expected value of \( E_i \), a variance of \( V_i = \sigma_i^2 \), and a covariance with asset \( j \) of \( C_{ij} = \sigma_{ij} \). Note, since it is only the mean, variance and covariance of asset returns that determine the asset allocation, it is not necessary to specify the total cash flows for these assets in the maximisation problem.
Consider next the liability $L$. The amount of the ‘rolled-forward’ liability will be a random variable with expected value $E_L$ and variance $V_L = \sigma_L^2$. These amounts are in pounds and square pounds, rather than per unit (Sherris 1992, 90). The variability comes from the fluctuations in claim rates over the interval (if claims were assumed to be stochastic) and in the rates used to compound the funds to their ‘ultimate’ liability. The liability cash flow may be correlated with the asset returns and hence the covariance with asset $i$ is given by $C_{Li} = \sigma_{Li}$. Thus, the variance-covariance matrix used in the insurer’s portfolio selection problem is defined as follows (only the top-half of the matrix is specified due to the symmetry of the variance-covariance matrix):

\[
V = \begin{pmatrix}
\sigma_1^2 & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} & \sigma_{1L} \\
\sigma_{21} & \sigma_2^2 & \sigma_{23} & \sigma_{24} & \sigma_{25} & \sigma_{2L} \\
\sigma_{31} & \sigma_{32} & \sigma_3^2 & \sigma_{34} & \sigma_{35} & \sigma_{3L} \\
\sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_4^2 & \sigma_{45} & \sigma_{4L} \\
\sigma_{51} & \sigma_{52} & \sigma_{53} & \sigma_{54} & \sigma_5^2 & \sigma_{5L} \\
\sigma_{L1} & \sigma_{L2} & \sigma_{L3} & \sigma_{L4} & \sigma_{L5} & \sigma_L^2
\end{pmatrix}
\]

(11.4) \[V = \begin{pmatrix}
\sigma_1^2 & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} & \sigma_{1L} \\
\sigma_{21} & \sigma_2^2 & \sigma_{23} & \sigma_{24} & \sigma_{25} & \sigma_{2L} \\
\sigma_{31} & \sigma_{32} & \sigma_3^2 & \sigma_{34} & \sigma_{35} & \sigma_{3L} \\
\sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_4^2 & \sigma_{45} & \sigma_{4L} \\
\sigma_{51} & \sigma_{52} & \sigma_{53} & \sigma_{54} & \sigma_5^2 & \sigma_{5L} \\
\sigma_{L1} & \sigma_{L2} & \sigma_{L3} & \sigma_{L4} & \sigma_{L5} & \sigma_L^2
\end{pmatrix}\]

In addition, the vector of expected returns is given by $e'$ as follows:

\[
e' = (E_1, E_2, E_3, E_4, E_5, E_L)
\]

(11.5) \[e' = (E_1, E_2, E_3, E_4, E_5, E_L)\]

The ultimate surplus function $S$ may now be defined. Ultimate surplus is a random variable because it is a function of the random amount of the ‘ultimate’ liability cash flow and the random amount of the ‘ultimate’ cash flows associated with the 5 asset
classes. Surplus is also a function of the vector of asset weights $w'$; these weights
are the decision variables for the insurance company and are defined within ultimate
surplus as follows:

\[
S = A(w_1 R_1 + w_2 R_2 + w_3 R_3 + w_4 R_4 + w_5 R_5) - L
\]

Note that Sherris uses the result in Equation 11.2 to show that ultimate surplus is
equivalently stated as (Sherris 1992, 91):

\[
S = C(1 + p) \left[ w_1 R_1 + w_2 R_2 + w_3 R_3 + w_4 R_4 + w_5 R_5 \right] - \left( \frac{1}{1 + p} \right) R_L
\]

The above formulation makes explicit the role of the rate of return on liabilities
$R_L = L/C$ and demonstrates the effect of the solvency margin, $p$. This formulation is
not used here and is presented merely to demonstrate the explicit role of the
solvency margin. For this reason, discussion proceeds with reference to Equation
11.6 above.

In addition to the five asset weights ($i = 1 \ldots 5$), the weight assigned to the insurer’s
liability is constrained to equal ($-1$) to reflect the fact that the liability is fixed, and
thus the vector of weights $w'$ is now given by:

\[
w' = (w_1, w_2, w_3, w_4, w_5, -1)
\]
The expected value and variance of ultimate surplus will therefore be given respectively by:

\begin{align}
E_S &= w'e \\
V_S &= x'Vx
\end{align}

Having set out the information required for solving the portfolio selection problem, the probability of insolvency can be defined by two states of ultimate surplus. If the fund is solvent, then ultimate surplus satisfies the following inequality, \( S \geq 0 \). In the event of solvency, the providers of equity \( K \) will be entitled to the full surplus \( S \) and the claimants (the policyholders) will be paid the full liability, equal to the amount \( L \). By contrast, where the fund is insolvent, \( S < 0 \), the fund claimants will receive \( L+S < L \ (S<0) \), and the providers of equity will be liable for the full loss in the event of unlimited liability.

The solution to the insurer's portfolio selection problem is now made with reference to the following expected utility maximisation rule:

\begin{align}
\text{Maximise} \{E[U(S)]\}, \text{ subject to :} \\
&\quad w'1 = 1 \text{ and } w \geq 0
\end{align}

Sherris (1992) notes that a solvency constraint can be incorporated into the maximisation problem in Equation 11.11. The constraint specifies some pre-
determined level of insolvency probability ($\alpha$).\(^3\) If the cumulative distribution function of ultimate surplus is given by $F_S(.)$ then the solvency constraint may be given by:

\[(11.12)\] $F_S(0) \leq \alpha$

In order to arrive at explicit solutions to the above maximisation problem, assumptions need to be made regarding the specification of the insurer’s utility function. Discussion turns to addressing this issue in the next sub-section.

11.1.3 Insurer Optimisation and the Specification of the Utility Function

It is evident that numerical solutions to the general portfolio problem in Equation 11.1, and the specific portfolio problem in Equation 11.11, will require explicit specification of the insurer’s utility function. As illustrated in Chapter 5, there are many factors that need to be taken into account when considering the selection of an explicit functional form for a utility function. These factors relate primarily to the risk preference properties of the utility functions concerned.

Before proceeding further, there are a number of points to note. Chapter 5 assumed that an insurer would maximise the expected utility of terminal wealth; a function of

\(^3\) Alternatively, a constraint could be placed on the margin of solvency such that assets are required to exceed the liabilities by some minimum amount, $p$.\(^3\)
expected return and variance, based on a second order Taylor series approximation.\textsuperscript{4}

Tobin (1958, 1963) showed that an investor who maximises expected utility can be analysed in terms of a mean and variance approximation if at least one of the following conditions hold: first, that the investor has a quadratic utility; and second, that the returns are normally distributed.

Subsequent work attempted to analyse the applicability of the mean-variance approximation to the case where neither of Tobin’s two conditions held. The work of Tsiang (1972) demonstrated that the mean-variance approximation is still applicable in the absence of Tobin’s conditions. Furthermore, Levy and Markowitz (1979) tested the conclusion Tsiang assuming a variety of utility functions and return distributions. They demonstrated that, for an investor who maximises expected utility, the mean/variance approximation to expected utility is still applicable in the absence of quadratic utility and normally distributed returns (Levy and Markowitz 1979, 316).

Having mentioned one of the possible functional forms for the insurer’s utility function, it is instructive to examine this function in greater detail together with two other types of functional form, the logarithmic utility function and the negative exponential utility function. Consider first the logarithmic utility function, which is defined as follows:

\textsuperscript{4} This approach imposes explicit restrictions upon the functional form used to specify the insurer’s utility function (see Chapter 5).
(11.13) \( U\{\text{Wealth (}= W)\} = \ln(W) \)

Where \( U'(W) = \frac{1}{W} > 0 \) and \( U''(W) = -\left\{ \frac{1}{W^2} \right\} < 0 \), and \( W \) is wealth. The above functional form yields an absolute risk aversion measure \( \alpha(W) \) defined as:

\[
(11.14) \alpha(W) = \left\{ \frac{U''(W)}{U'(W)} \right\} = \frac{1}{W}
\]

The degree of absolute risk aversion is therefore decreasing in wealth and hence different investors will require a different risk premium for a given risky investment depending on the value of their initial wealth. By contrast, the relative risk aversion parameter, \( \sigma(W) \) is defined for the logarithmic utility function as follows:

\[
(11.15) \sigma(W) = -W \left\{ \frac{U''(W)}{U'(W)} \right\} = W \alpha(W) = 1
\]

Thus, there is constant relative risk aversion exhibited by the logarithmic utility function. That is, an insurer will have constant risk aversion to a proportional loss of wealth even though the absolute loss increases with wealth.

Consider next the quadratic utility function. This functional form is defined below in Equation 11.16.
(11.16) \[ U(W) = aW - bW^2 \]

In the case of quadratic utility, the absolute and relative risk aversion parameters are respectively:

(11.17) \[
\alpha(W) = \frac{-2b}{a - 2bW}; \quad \frac{d\alpha(W)}{dW} > 0 \quad \text{and} \quad \\
\omega(W) = \frac{-2b}{(a/W) - 2b}; \quad \frac{d\omega(W)}{dW} > 0
\]

Thus, the quadratic utility function exhibits increasing absolute and relative risk aversion. Such a result is counter-intuitive: an investor defined by quadratic utility would be more averse to a given percentage loss in wealth as wealth increases.

The final utility function to be considered is the negative exponential function where utility, absolute risk aversion, and relative risk aversion are respectively defined as follows:

(11.18) \[ U(W) = -\exp\{-aW\} \]

Where \( \alpha(W) = a \) and, \( \omega(W) = aW \). For the negative exponential utility function, the parameter \( a \), is the only parameter considered and this represents the degree of absolute risk aversion, where greater values of \( a \) represent increasing absolute risk aversion. This utility function exhibits constant absolute risk aversion: all insurers
with this utility function, regardless of the size of their initial wealth, would require the same risk premium for a risky project of a given amount. Thus, all insurers with a given risk parameter \( a \), would have the same utility maximising portfolio for different starting values of initial wealth. Thus, it is for this very reason (and the unusual risk properties associated with alternative functional forms), that the negative exponential utility function is used in subsequent discussion. One further point should also be noted with regard to the selection of the negative exponential utility function. This point relates to the issue of local versus global risk aversion (Pratt 1964). Pratt (1964) demonstrated that the use of a constant risk aversion parameter (such as \( a \) in Equation 11.18) avoided the complication of distinguishing between ‘local’ or ‘global’ risk aversion (Pratt 1964, 122).

Given an explicit functional form in order to define insurer preferences, utility can now be expressed in terms of ultimate surplus, \( S \). In addition, to be consistent with previous work in this area (see Sherris (1992), Ong (1995), Booth, et al. (1997)), the absolute risk aversion parameter \( a \), will be represented in terms of a risk tolerance parameter \( r \), viz.

\[
(11.19) \quad U(S) = -\exp\left(-\frac{1}{10^6} S \right)
\]

---

5 The absolute risk aversion measure \(-u''(W)/u'(W)\) is a ‘local’ measure; risk aversion in terms of small risks. An investor A is globally more risk averse than another investor B, only when it can be shown that for every risk, investor A’s cash equivalent of the risk is smaller than that of investor B (Pratt 1964, 124).
The absolute risk aversion parameter \( a \), is set equal to \( \{1/10^6r\} \), where \( r \) is the measure of risk tolerance. Hence, increasing values of \( r \) represent increasing risk tolerance of the insurer.\(^6\) The scaling of \( 10^6 \) was found to be necessary in order that a reasonable variation in portfolio composition would be produced using values of \( r \) between 1 and 34. It is important to note that the magnitude of this scaling is a direct consequence of the size of the liability. Recall from Chapter 10 that the insurer’s liability consists of 1000 identical full endowment policies covering a £50,000 sum assured for an annual premium of £1075.

\[ \text{11.1.4 The Life Insurer’s Portfolio Selection Problem} \]

Analysis proceeds by evaluating the asset allocation for different levels of risk tolerance \( (r=1, \ldots, 34) \), assuming a given level of initial wealth.\(^7\) In order to derive analytical solutions to the expected utility maximisation problem in Equation 11.11, it will be assumed that the underlying probability distribution of ultimate surplus is normal. While alternative distribution assumptions could be made, the normality assumption is adopted here in order to simplify the analytical solution to the portfolio selection problem.

---

\(^6\) In other words, the risk tolerance parameter \( (r) \) is equal to the inverse of the absolute risk aversion parameter \( (a) \).

\(^7\) Experimental evidence on fund manager risk tolerance suggests that, after appropriate scaling corrections, the size of \( r \) is 25, and hence \( a =0.04 \) (Sharpe and Tint 1990).
$E_S$ and $V_S$ are respectively the mean and variance of ultimate surplus, $S$. The portfolio selection problem may now be defined in terms of an explicit function relating utility to ultimate surplus $S$, and risk tolerance $r$, viz.

\[(11.20) \quad \text{Maximise}\{E[U(S)]\} = \text{Maximise} \left\{ E \left[ -\exp \left( -\frac{1}{r} S \right) \right] \right\} \]

Subject to:

$\mathbf{w}'\mathbf{1}=1$

Non-negativity restrictions may also be imposed on the portfolio weights and $S$ is defined in Equation 11.6. There are two methods by which a solution to the above problem can be developed analytically: first, by using moment generating functions; and second, by using Taylor’s expansion. The latter approach was addressed in Chapter 5.\(^8\) The method of moment generating functions is a method that has been developed extensively in the actuarial literature as a means of solving the maximisation problem in Equation 11.20. Moment generating functions are a powerful analytical tool in statistics, and as the name suggests these functions are used to generate moments for a random variable $X$ with cumulative density function $F_X$ (Casella and Berger 1991). The $n^{th}$ moment of a given probability distribution is equal to the $n^{th}$ derivative of the moment generating function (MGF), $M_X(t)$ evaluated at $t=0$, where:

\(^8\) This is only an approximation where the distribution is not normal (and the utility function is not quadratic).
(11.21) \( M_X(t) = \mathbb{E}[\exp(tX)] = \int_{-\infty}^{\infty} e^{tx} f_X(x) \, dx \)

For the normal distribution with mean \( E_S \) and variance \( V_S \), the MGF is given by:

(11.22) \( M_X(t) = \exp\left( tE_S + t^2 \frac{V_S}{2} \right) \)

Where \( t = -1/r \) and \( r \) is the risk tolerance parameter. In order to maximise the expected utility expression in Equation 11.20, use is made of the following result:

(11.23) \( \text{Maximise}\{ \mathbb{E}[-\exp(tS)] \} = \text{Minimise}\{ \mathbb{E}[\exp(tS)] \} \)

Given the normality assumption, the above problem can be set up using the normal moment generating function for ultimate surplus, \( S \). Equation 11.23 may then be restated as follows:

(11.24) \( \text{Minimise}\left[ \exp\left( E_S t + \frac{V_S t^2}{2} \right) \right] \)

For the two-asset case, the variance \( V_S \) and expectation \( E_S \) of ultimate surplus \( S \) is given by:
\[ E_S = A w_1 E_1 + A w_2 E_2 - E_L \]
\[ V_S = A^2 w_1^2 V_1 + A^2 w_2^2 V_2 + V_L + 2A^2 w_1 w_2 C_{12} - 2A w_1 C_{1L} - 2A w_2 C_{2L} \]

Hence in this case, the optimal asset allocation is derived by differentiating 11.24 with respect to the portfolio weights, \( w_i \), (see Appendix 4 for a guide to the portfolio solutions in the Sherris (1992) model). In order to solve for the vector of portfolio weights \( w' \), given each level of risk tolerance \( r \) (\( r=1...34 \)), the expected return vector \( e' \) needs to be defined, together with its associated variance-covariance matrix, \( V \). The expected returns and variances are defined for each of the five ‘rolled-up’ asset classes in addition to the ‘rolled-up’ total cash outflow associated with the liability (assuming tax rates are normalised to zero). The expected returns and variances are defined below in Table 11.1 and are set at their ‘converged’ values.

**Table 11.1. Expected Returns and Variance-Covariance matrix (25 years)**

<table>
<thead>
<tr>
<th>Assets:</th>
<th>Var-Cov Matrix</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Expected 'Rolled-up'</td>
<td>Cash</td>
</tr>
<tr>
<td>Cash</td>
<td>6.02</td>
<td>6.5025</td>
</tr>
<tr>
<td>Consols</td>
<td>8.24</td>
<td>6.76</td>
</tr>
<tr>
<td>Index-linked</td>
<td>9.66</td>
<td>25.81</td>
</tr>
<tr>
<td>Equities</td>
<td>17.4</td>
<td></td>
</tr>
<tr>
<td>Property</td>
<td>11.36</td>
<td></td>
</tr>
<tr>
<td>Liability:</td>
<td>54284270</td>
<td></td>
</tr>
</tbody>
</table>

Note: Only top half of matrix given due to symmetry of variance covariance matrix.

---

9 This is equivalent to minimising the logarithm of the moment generating function.

10 Convergence is defined as the case in which the mean, percentiles and standard deviation of the output distributions have respectively changed by less than 1 percent for two consecutive iterations (see Appendix 4 for details on output distribution convergence).
Due to nature of the maximisation problem, the nominal accumulations of assets are reported in Table 11.1 for the initial sum invested over a 25-year period. The set-up of the problem requires that the liability be defined in terms of the total cash outflow (Sherris 1992, 90). Note that the cashflows are an alternative formulation to the nominal and real returns reported in Table 10.1 and Table 10.2, respectively. Following Wilkie (1985), intermediate claims are paid for by borrowing cash at the stochastic rate of (short-term) interest (see Figure 10.6). This provides the source of variability in the expected value of the liability. This assumption, namely the financing of liabilities through the borrowing of cash, is justified on the grounds that such financing represents the lowest opportunity cost to the insurer. Alternative means of finance- such as depleting asset funds- would impose a greater real cost.

A couple of additional remarks need to be made with regard to the construction of the liability cash-outflow. Recall from Chapter 10, that mortality is assumed to be deterministic. Based on this assumption, the liability cash outflow at year \( t \) \((t=1...25)\) will be defined by the number of people dying at year \( t \) (as specified by the mortality rates) multiplied by the sum assured (£50,000). Thus, the timing and size of each cash outflow is known for each year. However, the valuation of each annual cash outflow (intermediate claims) is evaluated at time \( t=25 \) in order to define the ‘ultimate liability’ (intermediate claims plus policy maturities) and hence construct the measure of solvency, ultimate surplus. As stated above, the interest rate used to compound the intermediate claims to time \( t=25 \) is the short-term
interest rate and this is therefore the source of the random fluctuation in the ‘ultimate liability’.

Having outlined some of the general considerations with respect to the insurer’s optimisation problem, attention now returns to providing an explicit solution to the ‘optimal portfolio’ using the information in Table 11.1. As noted, the moment generating function in Equation 11.24 can be used to solve the expected utility maximisation problem. The definition of ultimate surplus $S$ (see Equation 11.6) requires the value of initial wealth ($A$) to be specified in order to solve for the ‘optimal portfolios’. The calculation of initial wealth requires a number of further simplifying assumptions.

The initial funds available for investment at time $t=0$ are assumed to be equal to the discounted stream of premium payments, a total of £11,900,098. Consider how this sum was calculated in further detail. One of the considerations in designing the simulation model was the way in which to treat the stream of premium payments. The difficulty in investing premiums year after year is that the problem is essentially a dynamic program. Investment strategy and risk-aversion are therefore a function of time and an optimal path must be derived for these variables in order to solve for the year-by-year asset-allocation problem. Since a dynamic program requires a significant extension to the analysis, it is assumed that the problem should remain a single-period problem. Moreover, the single period framework can still provide some useful insights and the important features of a life insurer; namely, that the
determination of insurer asset allocation involves explicit consideration of the liabilities. As a result, at time $t=0$ the insurer is assumed to have the present value of all future premiums associated with the cohort of policies. This discounted stream accounts for deterministic mortality and the resultant reduction in the number of total polices (and premium income) throughout the term.$^{11}$

11.1.5 The ‘Optimal Portfolios’

Given that the aim of this paper is to address some basic issues in solvency regulation, particularly in respect of the risk aversion and asset allocation, it is believed that the single-period framework can provide some useful insights. Table 11.2 presents the ‘optimal portfolios’, the numerical solutions to the insurer’s portfolio selection problem in Equation 11.24 assuming the variance-covariance matrix in Table 11.1.

Table 11.2 reports the solutions to the expected utility maximisation rule using the numerical techniques available on Microsoft Excel’s add-in, ‘Solver’. The associated values of surplus, variance and expected utility are given in addition to the portfolio weights. The table reports details of the optimal asset mix for a life fund comprising 1000 full endowment policies effected over a 25-year period (see Chapter 10). The results demonstrate a number of trends across the risk tolerance scale.

$^{11}$ Note that the premium steam is discounted to time $t=0$ using the mean long-term interest rate (see Figure 10.5).
Table 11.2. Optimal Portfolios

<table>
<thead>
<tr>
<th>Risk tolerance, ( \tau )</th>
<th>Cash</th>
<th>Consols</th>
<th>Index-linked</th>
<th>Equities</th>
<th>Property</th>
<th>Expected Utility</th>
<th>Mean Surplus</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.7</td>
<td>0</td>
<td>0.3</td>
<td>0</td>
<td>0</td>
<td>-0.00869</td>
<td>30223390</td>
<td>27293510</td>
</tr>
<tr>
<td>2</td>
<td>0.352539</td>
<td>0</td>
<td>0.41618</td>
<td>0.014807</td>
<td>0.216474</td>
<td>-0.01264</td>
<td>73294439.6</td>
<td>48793080</td>
</tr>
<tr>
<td>3</td>
<td>0.032922</td>
<td>0</td>
<td>0.47405</td>
<td>0.056705</td>
<td>0.436323</td>
<td>-0.02445</td>
<td>78328251.6</td>
<td>51094589</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0.479674</td>
<td>0.061753</td>
<td>0.458573</td>
<td>-0.05278</td>
<td>57637675.2</td>
<td>49733271</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0.453343</td>
<td>0.092564</td>
<td>0.45092</td>
<td>-0.13157</td>
<td>81128027.7</td>
<td>52941660</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0.42726</td>
<td>0.123416</td>
<td>0.449324</td>
<td>-0.18685</td>
<td>83873203.1</td>
<td>55225765</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0.401177</td>
<td>0.154268</td>
<td>0.444555</td>
<td>-0.23607</td>
<td>86618378.1</td>
<td>57895201</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0.375094</td>
<td>0.185119</td>
<td>0.439786</td>
<td>-0.27898</td>
<td>89363556.1</td>
<td>60899321</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0.349011</td>
<td>0.215971</td>
<td>0.435018</td>
<td>-0.31621</td>
<td>92108730.6</td>
<td>64191148</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0.322928</td>
<td>0.246823</td>
<td>0.430249</td>
<td>-0.34856</td>
<td>94853905.7</td>
<td>67728746</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0</td>
<td>0.296845</td>
<td>0.277674</td>
<td>0.425481</td>
<td>-0.37682</td>
<td>97599090.3</td>
<td>71475647</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0</td>
<td>0.270762</td>
<td>0.308526</td>
<td>0.420712</td>
<td>-0.39039</td>
<td>100344264</td>
<td>75400627</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0</td>
<td>0.244679</td>
<td>0.339738</td>
<td>0.415943</td>
<td>-0.40163</td>
<td>103089440</td>
<td>79477324</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>0</td>
<td>0.218596</td>
<td>0.370229</td>
<td>0.411775</td>
<td>-0.42355</td>
<td>105834617</td>
<td>83683565</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>0</td>
<td>0.192513</td>
<td>0.401081</td>
<td>0.406406</td>
<td>-0.44303</td>
<td>108579796</td>
<td>88000781</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>0</td>
<td>0.16643</td>
<td>0.431933</td>
<td>0.401637</td>
<td>-0.45064</td>
<td>111324970</td>
<td>92413407</td>
</tr>
<tr>
<td>17</td>
<td>0</td>
<td>0</td>
<td>0.140437</td>
<td>0.462784</td>
<td>0.396869</td>
<td>-0.47608</td>
<td>114070150</td>
<td>96908428</td>
</tr>
<tr>
<td>18</td>
<td>0</td>
<td>0</td>
<td>0.114264</td>
<td>0.493636</td>
<td>0.3921</td>
<td>-0.4902</td>
<td>116815328</td>
<td>1.01E+08</td>
</tr>
<tr>
<td>19</td>
<td>0</td>
<td>0</td>
<td>0.088181</td>
<td>0.524488</td>
<td>0.387331</td>
<td>-0.50301</td>
<td>119560502</td>
<td>1.06E+08</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>0</td>
<td>0.062098</td>
<td>0.555339</td>
<td>0.382563</td>
<td>-0.51467</td>
<td>122305680</td>
<td>1.11E+08</td>
</tr>
<tr>
<td>21</td>
<td>0</td>
<td>0</td>
<td>0.036015</td>
<td>0.586191</td>
<td>0.377794</td>
<td>-0.52534</td>
<td>125050863</td>
<td>1.16E+08</td>
</tr>
<tr>
<td>22</td>
<td>0</td>
<td>0</td>
<td>0.009932</td>
<td>0.617043</td>
<td>0.370325</td>
<td>-0.53513</td>
<td>127569956</td>
<td>1.2E+08</td>
</tr>
<tr>
<td>23</td>
<td>0</td>
<td>0</td>
<td>0.649295</td>
<td>0.350705</td>
<td>0.50477</td>
<td>-0.54472</td>
<td>129950018</td>
<td>1.24E+08</td>
</tr>
<tr>
<td>24</td>
<td>0</td>
<td>0</td>
<td>0.682408</td>
<td>0.317592</td>
<td>0.53595</td>
<td>-0.56251</td>
<td>132330801</td>
<td>1.28E+08</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
<td>0</td>
<td>0.715521</td>
<td>0.284479</td>
<td>0.5621</td>
<td>-0.57799</td>
<td>134710142</td>
<td>1.33E+08</td>
</tr>
<tr>
<td>26</td>
<td>0</td>
<td>0</td>
<td>0.748635</td>
<td>0.251365</td>
<td>0.57047</td>
<td>-0.57799</td>
<td>137090206</td>
<td>1.37E+08</td>
</tr>
<tr>
<td>27</td>
<td>0</td>
<td>0</td>
<td>0.781748</td>
<td>0.218252</td>
<td>0.57799</td>
<td>-0.57799</td>
<td>139470269</td>
<td>1.41E+08</td>
</tr>
<tr>
<td>28</td>
<td>0</td>
<td>0</td>
<td>0.814861</td>
<td>0.185139</td>
<td>0.58484</td>
<td>-0.59134</td>
<td>141850332</td>
<td>1.46E+08</td>
</tr>
<tr>
<td>29</td>
<td>0</td>
<td>0</td>
<td>0.847974</td>
<td>0.152026</td>
<td>0.59134</td>
<td>-0.59744</td>
<td>144230394</td>
<td>1.5E+08</td>
</tr>
<tr>
<td>30</td>
<td>0</td>
<td>0</td>
<td>0.881087</td>
<td>0.118913</td>
<td>0.60317</td>
<td>-0.60317</td>
<td>146610457</td>
<td>1.55E+08</td>
</tr>
<tr>
<td>31</td>
<td>0</td>
<td>0</td>
<td>0.9142</td>
<td>0.0858</td>
<td>0.60858</td>
<td>-0.60858</td>
<td>148990519</td>
<td>1.59E+08</td>
</tr>
<tr>
<td>32</td>
<td>0</td>
<td>0</td>
<td>0.947314</td>
<td>0.052686</td>
<td>0.61367</td>
<td>-0.61367</td>
<td>151370582</td>
<td>1.64E+08</td>
</tr>
<tr>
<td>33</td>
<td>0</td>
<td>0</td>
<td>0.980427</td>
<td>0.019573</td>
<td>0.62038</td>
<td>-0.62038</td>
<td>152777435</td>
<td>1.66E+08</td>
</tr>
</tbody>
</table>

With reference to Table 11.2, for the least risk tolerant insurer, the optimal portfolio consists of 70 percent cash and 30 percent index-linked government securities. By contrast, for the greatest insurer risk tolerance, an optimal portfolio consisting of

\[ \text{Note that numerical trials demonstrated that this optimal portfolio was the limit case for the least risk-tolerant insurer.} \]
100 percent equities was reported. Between these two limit cases, the optimal portfolios vary in terms of their respective composition.

Note that the results in Table 11.2 show that the non-negativity constraint on the portfolio weights is binding with respect to Consols. This result holds for all levels of risk tolerance. Chapter 10 demonstrated that index-linked government securities dominated Consols in terms of expected real return and risk and this operational distinction might be reflected in the optimal portfolios. In addition, as Ong (1995) notes, a combination of securities (e.g. cash and index-linked securities) may strictly dominate another security (e.g. Consols). Changes in the portfolio composition illustrate a linear trade-off across the risk tolerance scale, although there are some distinct breaks in this trade-off (see Figure 11.1).

11.2 The Ultimate Surplus Simulation Algorithm

Having established the criteria for asset selection and presented the ‘optimal portfolios’, it is now necessary to turn to the simulation model of ultimate surplus. Assuming that expenses are normalised to zero, the ultimate surplus of the fund is equal to the net-cashflow at the end of the simulation period (t=25). The ultimate surplus is a combined profit/performance measure of the insurer’s underwriting and investment portfolios.
With reference to Chapter 10, the simulation model provides details on the respective performances of five asset classes, cash \((B_i)\), Consols \((C_i)\), index-linked government securities \((R_i)\), equities \((P_i)\), and property \((A_i)\) (Wilkie 1995). The U.K. mortality estimates define the liability outflow, which in turn are assumed fixed. Initial conditions are assumed neutral\(^{13}\). This can be justified on the grounds that such an assumption rules out any carried forward effects from before the stage at which the cohort of policies was issued.

For a given period of time, the ultimate surplus simulation generates the returns associated with the 5 asset classes. The neutral initial conditions enable the simulation to proceed from, \(t=0\) to \(t=25\). Having 'rolled forward' the accumulated nominal funds for assets and liabilities, the performance of 'ultimate' surplus can be assessed at time \(t=25\). In order to appreciate the mechanics of simulating ultimate surplus, Table 11.3 illustrates the main assumptions and the steps involved in the simulation process.

From the layout in Table 11.3, it is important to stress the relationship between the insurance company and its policyholders. The insurance company is involved in a contract with 1000 policyholders that commits the insurer to make future payments to the policyholders under various conditions as defined in the contract\(^{14}\). The

\(^{13}\) Starting values are 'neutral' in that they are set equal to their long-run means assuming standard deviations are equal to zero.

\(^{14}\) In this simplified insurer model, the payment of an insurance benefit occurs either when the policyholder dies, or when the term of the contract expires at time \(t=25\). Policy surrenders are normalised to zero.
insurer uses the stream of premium payments to invest in assets that provide the insurer with a return that can be used to meet its liability.\textsuperscript{15}

Table 11.3. Generating a Simulation of Ultimate Surplus

To simulate the present value of ultimate surplus associated with 1000 policies of a given type (which may in turn be used as a measure of profit), the following assumptions are made:

A.1 Claims are deterministic and follow the national population projections for 1994 (Government Actuary’s Department 1994). Borrowing funds at the short-term rate of interest pays for interim claims.

A.2 At \( t=0 \) the asset portfolio is constructed according to the expected utility maximisation rule in Equation 11.24.

A.3 All nominal returns are reinvested in the class from which they came from, assuming zero transactions costs.

A.4 The investment experience associated with all 5 asset classes is defined according to the Wilkie (1995) stochastic investment model, with the modifications for property (see Chapter 10).

With the assumptions specified, the algorithm for the generation of ultimate surplus is defined in terms of a number of stages:

S.1 At \( t=0 \), generate the total premium fund accounting for interim claims using A.1.

S.2 From A.2 generate the returns for the 5 asset classes given A.4 and accumulate the premium fund for successive years according to A.3.

S.3 Repeat S.2 until all liabilities are 'run-off' at the end of the time interval, \( t=25 \).

S.4 Calculate ultimate surplus, \( S \) then:

1. If at the end of the term all the claims (maturity) payments are so large that they cannot be met by cash inflows plus the realisation of all investments, then ultimate surplus is negative and the insurer is strictly insolvent (i.e. \( S<0 \)).

2. If ultimate surplus is greater than zero after all claims and maturity payments are made then the value of remaining assets is a measure of the company's financial strength (i.e. \( S>0 \)).

S.5 Repeat the whole process 10,000 times, building up a probability distribution of ultimate surplus and its 'convergence' according to predefined criteria (see Chapter 10). The proportion of occasions when the assets prove insufficient to meet liabilities may be used as a proxy for the probability of insolvency.

\textsuperscript{15} As mentioned in the previous chapter, an insurance company can be viewed in terms of a collection of policy cohorts. The simulation model therefore analyses the performance of just one of these cohorts.
A further point to note with respect to the simulation algorithm is the way in which the insolvent insurers have been included within the simulations. The main point to note is that the solvency assessment is performed at $t=25$ and hence it is possible that an insurer could be technically insolvent before the end of the simulation; for example, an insurer could be insolvent at $t=10$. In such an event the simulation allows the insurer to proceed up until the next screening at $t=25$ and hence insolvency during the 25 year interval is not accounted for within this analysis.

While in practice regular annual screening would remove such an insurer before $t=25$, the approach is justified on the grounds that the problem of discrete screening exists whatever the time interval assumed (in practice a year). Moreover, since continuous screening is not feasible from the point of view of a resource constraint, changes to the time interval does not necessarily add anything further to the analysis or change any of the basic propositions put forward here. Indeed, the problem of allowing insolvent insurers to proceed until the next screening remains.
11.3 Ultimate Surplus Simulation Output Data

The basic hypothesis is that there exists a positive relationship between risk tolerance and insolvency probability. The central role of risk preference in the measurement and regulation of insurer solvency is therefore emphasised. Thus, the importance of the optimal portfolios that are calculated in Section 11.1 is clear: optimal portfolios are the analytical link between the risk preference of the insurance company and the solvency outcome of the simulation.

Note that while in practice the exact nature of this relationship between risk tolerance and solvency outcome is subject to empirical research and many debates, the ultimate surplus simulation model will define the explicit linkage. The results of the simulations are reported in Table 11.4.

Table 11.4 reports, in terms of probabilistic statements, the values of ultimate surplus and the nature of the associated probability distribution function. The reported surpluses are in present value terms in order that a discussion can be made regarding the use of reserves at time $t=0$. Note that for a graphical representation of the ultimate surplus probability distributions see Appendix 4.
Table 11.4. Simulation Results for Ultimate Surplus & Insolvency Prediction (25 years)

<table>
<thead>
<tr>
<th>Risk Tolerance</th>
<th>Expected Surplus (£)</th>
<th>Standard Deviation (£)</th>
<th>Expected Surplus (Standardised)</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Insolvency Probability (Surplus&lt;0) (%)</th>
<th>Reserves (£)</th>
<th>Margin of Solvency (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3279783</td>
<td>2593015</td>
<td>1.264853</td>
<td>1.29</td>
<td>8.71</td>
<td>3.74</td>
<td>1144285</td>
<td>15.49807</td>
</tr>
<tr>
<td>2</td>
<td>6101077</td>
<td>3581152</td>
<td>1.703663</td>
<td>0.69</td>
<td>3.99</td>
<td>1.692</td>
<td>710246</td>
<td>5.968405</td>
</tr>
<tr>
<td>3</td>
<td>8621236</td>
<td>5520325</td>
<td>1.561726</td>
<td>1.19</td>
<td>5.95</td>
<td>1.427</td>
<td>418703</td>
<td>3.518484</td>
</tr>
<tr>
<td>4</td>
<td>9370603</td>
<td>5956693</td>
<td>1.573122</td>
<td>1.21</td>
<td>6.1</td>
<td>0.92</td>
<td>-520788</td>
<td>-4.77363</td>
</tr>
<tr>
<td>5</td>
<td>9811736</td>
<td>6218954</td>
<td>1.577715</td>
<td>1.13</td>
<td>5.27</td>
<td>1.35</td>
<td>370754</td>
<td>3.115554</td>
</tr>
<tr>
<td>6</td>
<td>10165460</td>
<td>6513164</td>
<td>1.560756</td>
<td>1.33</td>
<td>7.15</td>
<td>1.28</td>
<td>12975</td>
<td>0.109033</td>
</tr>
<tr>
<td>7</td>
<td>1049784</td>
<td>7225964</td>
<td>1.452371</td>
<td>1.94</td>
<td>11.45</td>
<td>0.74</td>
<td>-232228</td>
<td>-1.95148</td>
</tr>
<tr>
<td>8</td>
<td>10905810</td>
<td>7482450</td>
<td>1.457519</td>
<td>1.77</td>
<td>9.65</td>
<td>1.31</td>
<td>223893</td>
<td>1.881438</td>
</tr>
<tr>
<td>9</td>
<td>11628850</td>
<td>800423</td>
<td>1.453529</td>
<td>1.68</td>
<td>8.59</td>
<td>1.06</td>
<td>84725</td>
<td>0.711969</td>
</tr>
<tr>
<td>10</td>
<td>11753600</td>
<td>8725850</td>
<td>1.346986</td>
<td>2.21</td>
<td>13.81</td>
<td>1.17</td>
<td>171930</td>
<td>1.444778</td>
</tr>
<tr>
<td>11</td>
<td>12209480</td>
<td>8601134</td>
<td>1.41952</td>
<td>1.39</td>
<td>6.25</td>
<td>1.27</td>
<td>134529</td>
<td>1.130486</td>
</tr>
<tr>
<td>12</td>
<td>11756680</td>
<td>8593904</td>
<td>1.368026</td>
<td>1.58</td>
<td>7.33</td>
<td>1.35</td>
<td>338378</td>
<td>2.843489</td>
</tr>
<tr>
<td>13</td>
<td>12300220</td>
<td>7975299</td>
<td>1.326127</td>
<td>1.84</td>
<td>9.41</td>
<td>1.38</td>
<td>195775</td>
<td>1.645155</td>
</tr>
<tr>
<td>14</td>
<td>13099050</td>
<td>10533250</td>
<td>1.243591</td>
<td>1.89</td>
<td>9.1</td>
<td>1.64</td>
<td>869320</td>
<td>3.703515</td>
</tr>
<tr>
<td>15</td>
<td>13528270</td>
<td>11109840</td>
<td>1.217684</td>
<td>2.27</td>
<td>12.8</td>
<td>1.52</td>
<td>456759</td>
<td>3.838279</td>
</tr>
<tr>
<td>16</td>
<td>14398170</td>
<td>11416760</td>
<td>1.261143</td>
<td>1.64</td>
<td>7</td>
<td>1.72</td>
<td>661777</td>
<td>5.661105</td>
</tr>
<tr>
<td>17</td>
<td>14673960</td>
<td>12560380</td>
<td>1.168274</td>
<td>2.18</td>
<td>10.8</td>
<td>2</td>
<td>464346</td>
<td>3.902035</td>
</tr>
<tr>
<td>18</td>
<td>14942600</td>
<td>12714990</td>
<td>1.175196</td>
<td>2.15</td>
<td>10.5</td>
<td>1.84</td>
<td>835781</td>
<td>7.023312</td>
</tr>
<tr>
<td>19</td>
<td>15623430</td>
<td>13647240</td>
<td>1.144805</td>
<td>2.48</td>
<td>13.35</td>
<td>2.24</td>
<td>1076972</td>
<td>9.050132</td>
</tr>
<tr>
<td>20</td>
<td>15494480</td>
<td>14484850</td>
<td>1.069702</td>
<td>2.87</td>
<td>16.38</td>
<td>2.29</td>
<td>1035894</td>
<td>8.70492</td>
</tr>
<tr>
<td>21</td>
<td>15675400</td>
<td>13830960</td>
<td>1.133356</td>
<td>1.95</td>
<td>9.46</td>
<td>3.15</td>
<td>1053544</td>
<td>8.853238</td>
</tr>
<tr>
<td>22</td>
<td>16980350</td>
<td>16590290</td>
<td>1.023511</td>
<td>2.63</td>
<td>16.86</td>
<td>3.44</td>
<td>169229</td>
<td>1.422081</td>
</tr>
<tr>
<td>23</td>
<td>16559770</td>
<td>18548730</td>
<td>0.892674</td>
<td>6.99</td>
<td>11.34</td>
<td>2.7</td>
<td>1503447</td>
<td>12.6359</td>
</tr>
<tr>
<td>24</td>
<td>17464200</td>
<td>17461580</td>
<td>1.00015</td>
<td>2.33</td>
<td>12.45</td>
<td>3.53</td>
<td>1613737</td>
<td>13.5607</td>
</tr>
<tr>
<td>25</td>
<td>17426100</td>
<td>15825620</td>
<td>1.101132</td>
<td>2.06</td>
<td>9.91</td>
<td>2.72</td>
<td>1455641</td>
<td>12.23218</td>
</tr>
<tr>
<td>26</td>
<td>17581090</td>
<td>18421040</td>
<td>0.954403</td>
<td>2.91</td>
<td>17.15</td>
<td>3.03</td>
<td>1828282</td>
<td>15.36359</td>
</tr>
<tr>
<td>27</td>
<td>17580480</td>
<td>17540180</td>
<td>1.002298</td>
<td>2.38</td>
<td>12.98</td>
<td>4.49</td>
<td>2217335</td>
<td>18.63291</td>
</tr>
<tr>
<td>28</td>
<td>17746970</td>
<td>18668440</td>
<td>0.95064</td>
<td>2.76</td>
<td>15.22</td>
<td>5.07</td>
<td>1960416</td>
<td>16.47395</td>
</tr>
<tr>
<td>29</td>
<td>18201070</td>
<td>20546450</td>
<td>0.88585</td>
<td>3.06</td>
<td>18.67</td>
<td>4.5</td>
<td>2816817</td>
<td>23.67054</td>
</tr>
<tr>
<td>30</td>
<td>17707610</td>
<td>20443740</td>
<td>0.918159</td>
<td>3.13</td>
<td>25.78</td>
<td>5.42</td>
<td>2531492</td>
<td>21.27287</td>
</tr>
<tr>
<td>31</td>
<td>17807440</td>
<td>19030370</td>
<td>0.935738</td>
<td>3.22</td>
<td>26.33</td>
<td>5.8</td>
<td>659207</td>
<td>55.95926</td>
</tr>
<tr>
<td>32</td>
<td>19391840</td>
<td>21736210</td>
<td>0.89214</td>
<td>3.06</td>
<td>18.22</td>
<td>6.72</td>
<td>5335910</td>
<td>44.83921</td>
</tr>
<tr>
<td>33</td>
<td>19882020</td>
<td>22448980</td>
<td>0.885654</td>
<td>3.51</td>
<td>29.42</td>
<td>7.13</td>
<td>7556363</td>
<td>63.49833</td>
</tr>
<tr>
<td>34</td>
<td>20114706</td>
<td>21007510</td>
<td>0.957501</td>
<td>3.69</td>
<td>31.6</td>
<td>7.8</td>
<td>7946512</td>
<td>66.77686</td>
</tr>
</tbody>
</table>

Notes. (a) Simulation results of (discounted) ultimate surplus reported for 10,000 iterations for each level of risk tolerance.
(b) Standardised expected surplus is given by dividing the expected value by its standard deviation.
(c) The coefficient of skewness measures the departure from symmetry in the probability distribution by expressing the difference between the mean and the median relative to the standard deviation. For a normal distribution a 0 coefficient would be expected. Thus, for a positively (negatively) skewed distribution, the coefficient is positive (negative).
Table 11.4 — Continued

(d) The coefficient of kurtosis measures the distribution of the observations across the range, that is whether the distribution is platykurtic (flat), leptokurtic (peaked), or mesokurtic (neither flat nor peaked). A value of 3 is reported for the normal distribution. A value greater (smaller) than 3 implies a greater (smaller) tendency for leptokurtosis in the probability distribution.
(e) The probability of insolvency measures the probability that the reported ultimate surplus will be negative.
(f) The 1st reserve column reports the amount of reserves (capital) that the insurer would need to have so that the probability of not requiring any further capital is 99 percent. This value is calculated using the lower one-percent point of the distribution.
(g) Using the reserves calculated in (f), the margin of solvency is calculated by evaluating the required reserves as a percentage of the present value of the premium income.

As noted in Chapter 10, the stability of the output distributions is necessary in order to demonstrate stable relationships between inputs and outputs. For this reason, evidence of convergence is given in Table 11.5.

Table 11.5. Extract from @RISK Convergence Reports

<table>
<thead>
<tr>
<th>Name of Output</th>
<th>Percentage Change in Percentiles</th>
<th>Percentage Change in Mean</th>
<th>Percentage Change in Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discounted surplus (r=5)</td>
<td>0.08</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>Discounted surplus (r=10)</td>
<td>0.09</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>Discounted surplus (r=15)</td>
<td>0.10</td>
<td>0.06</td>
<td>0.01</td>
</tr>
<tr>
<td>Discounted surplus (r=20)</td>
<td>0.05</td>
<td>0.06</td>
<td>0.01</td>
</tr>
<tr>
<td>Discounted surplus (r=25)</td>
<td>0.08</td>
<td>0.03</td>
<td>0.20</td>
</tr>
<tr>
<td>Discounted surplus (r=30)</td>
<td>0.07</td>
<td>0.05</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Note. Convergence is defined where the percentage change in all three-convergence statistics is less than 1 percent for at least 2 successive iterations.

Table 11.5 illustrates the convergence in the ultimate surplus distribution associated with 10000 iterations. The results demonstrate that there is significant stability associated with the output distributions (the ultimate surplus probability distribution function, for a given risk tolerance).
11.4 Discussion of Results

11.4.1 Ultimate Surplus

With reference to the Table 11.4, there are important points to note, particularly with respect to the form of the probability distribution for ultimate surplus, and the central role of risk preference. Consider first, the relationship between expected surplus and surplus standard deviation for given levels of risk tolerance—see Figure 11.2. Ceteris paribus, the greater the risk tolerance, the greater the expected surplus and the greater the standard deviation of ultimate surplus—see Figure 11.2.

![Figure 11.2. Expected Return and Standard Deviation of Ultimate Surplus](image)

With reference to Figure 11.2, expected surplus increases from approximately £3.3 million for the least risk tolerant insurer, to roughly £20 million for the most risk tolerant insurer. For these two limiting cases, the standard deviation rises
respectively from £2.6 million to £21 million. Thus, the variation about the ultimate surplus expected value increases with risk tolerance. Note that for the most risk tolerant insurer the standardised expected ultimate surplus is less than unity.\textsuperscript{16} This will have clear implications for the spread of the surplus distribution and hence the probability of insurer insolvency.

Consider next the results of two higher moments of the ultimate surplus distribution function; namely, skewness and kurtosis. The simulations demonstrate that the distribution of surplus becomes more positively skewed with increasing risk tolerance. In addition, evidence on kurtosis implies that the observations themselves are becoming more concentrated at the peak with increasing risk tolerance; the distribution is becoming increasingly leptokurtic. This evidence therefore suggests that as risk tolerance increases, the ultimate surplus probability distribution tends toward a lognormal distribution function. Note that skewness and kurtosis have important implications for the measurement of insolvency probability and the design of appropriate solvency constraints.

\textbf{11.4.2 The Central Role of Risk Preference in Measuring Solvency}

While much can be said about the moments of the surplus distribution function, discussion now turns to considering insolvency prediction and the relationship between investment strategies and ultimate surplus. An examination of the financial

\textsuperscript{16} Recall that the standardised expected value is equal to the expected value divided by the standard deviation.
strength of the insurance company requires probabilistic statements about the ability of the insurer to meet its liabilities when they fall due. That is, the investment of premium income into various asset classes must be such that their combined return is expected to cover the liability with only a degree of probability.

As emphasised throughout, it is the probability associated with insurer insolvency that is the crucial point for regulators. This probability sets the position of the boundary dividing 'permissible' optimal portfolios from 'non-permissible' optimal portfolios (see Chapter 5). In the absence of any regulations regarding the size of this downside risk, there is the incentive for highly risk averse insurers to assume excessive portfolio risk by selecting an otherwise 'non-permissible' optimal portfolio, and therefore be more susceptible to insolvency.

In order to define the regulations relating to the size of insurer insolvency probability, there are a number of questions that need to be addressed by the regulators. First, what is an acceptable probability of insolvency? Second, related to the first question, what is the probability of failing the imposed regulatory standard? As a consequence of these two questions, the regulator may then ask the following question: what kinds of measures should be adopted in order that the regulator can effectively monitor insurance company performance?

---

17 If the value of liabilities were exactly known, then it would be feasible to design an investment strategy that would in turn provide a payoff that matched the liability exactly (i.e. the fund would be immunized (see Chapter 4)). However, since the value of the liabilities is not known exactly, because of the random fluctuations inherent in inflation and interest rates, the insurer's information set is therefore incomplete. Hence, it is only possible to make probabilistic statements about the solvency performance of the insurance company.
In order to answer the three questions posed above, it is necessary to understand the central role of risk preference; it is the risk preference of the insurer that provides the variation in the composition of the investment portfolio and consequently the variation in the performance of ultimate surplus (and hence the insolvency probability). The relationship between insolvency probability and risk tolerance is given below in Figure 11.3.

Figure 11.3. The Probability of Insolvency and Regulation

The above graph illustrates the probability that ultimate surplus is negative (the probability of insolvency), conditional on the level of risk tolerance. The probability of insolvency is presented in conjunction with an exogenously assumed regulatory standard, viz.

\[(11.26) \text{Probability } \{\text{Ultimate Surplus } \leq 0\} = 0.01\]
Note that this constraint is derived purely for illustrative purposes; in practice this constraint would also be the subject of economic analysis within a cost-benefit setting. Figure 11.3 demonstrates that the probability of insolvency increases to a maximum of 7.8 percent where $r=3.4$. The composition of this portfolio is entirely made up of equities. Thus, for 100 insurance companies with such a portfolio composition, roughly 8 insurers would have insufficient funds to pay for claims.

The illustration in Figure 11.3 demonstrates the variation in insolvency probability across the risk tolerance scale. The figure also demonstrates that most portfolios fail the assumed regulatory standard. In particular, out of the thirty-four portfolios simulated, only two portfolios passed the regulatory standard, at $r=4$ and $r=7$. Across the full range of risk preference, insolvency likelihood varies such that there is a 'U' shaped relationship between the probability of insolvency and the risk tolerance of the hypothetical life assurance company; the central role of risk preferences is therefore highlighted.

The 'U' shaped relationship between the probability of insolvency and the risk tolerance of the insurer can be expounded as follows. Consider the solvency performance associated with risk tolerance $r=1$. Here the probability of insolvency is 3.74 percent; this insolvency probability is nearly three times the size of the next highest risk category $r=2$. Moreover, for the four lowest risk categories $r=1,...,4$, and also $r=6,7$ there is a negative relationship between insolvency probability and risk tolerance. Thereafter, there exists a positive relationship between insolvency
probability and insurer risk tolerance. Thus, overall there is a ‘U’ shaped relationship between risk tolerance and insolvency probability over the entire risk tolerance scale. For a rational investor, portfolios to the left of the optimal portfolio with minimum insolvency-probability \((r=7)\) are strictly dominated by other optimal portfolios generated by greater risk tolerance \((r>7)\), which offer a greater expected surplus for a given level of insolvency probability. This result can be demonstrated with respect to the lowest risk tolerance category \(r=1\), which has an insolvency probability consistent with \(r=25\), yet offers a lower expected surplus.

The importance of the ‘U’ shaped relationship between risk tolerance and insolvency prediction needs explaining further, since there are clear implications for the design of regulatory standards. There are two explanations developed here: first, there is the interaction between changes in expected returns (and portfolio risk) as risk tolerance changes; and second, there is the role of model specification.

Consider the interaction between risk tolerance and expected returns/standard deviation. At first sight, it appears that the more spread investments are across the different asset classes, the lower the insolvency probability; that is, diversification reduces insolvency probability (see Table 11.2 and Table 11.4). While this seems an intuitive explanation, the evidence provided in Table 11.2 and Figure 11.2 cast doubt on this view. The major difficulty with this explanation is that the benefits of diversification are not reported in the standard deviation results. If there are benefits to solvency from diversification, then there should be benefits reported in terms of
portfolio risk (standard deviation); that is, if the ‘U’ shaped pattern in insolvency probabilities resulted from diversification, then there should be a ‘U’ shaped pattern reported in the portfolio standard deviation (this is evidently not illustrated in Figure 11.2). More likely, is the fact that the position and size of the probability distribution is changing with risk tolerance – through the changing preference set of the insurer and the consequential effect that this has on portfolio return and risk – and that it is the interaction of expected return and standard deviation with risk tolerance that is leading to the results presented in Figure 11.3.

A further explanation of the ‘U’ shaped relationship between insolvency probability and risk tolerance might come from the assumed stochastic model. In particular, the functional forms of the simulation algorithms will be important and in particular the role of inflation is crucial. As Chapter 10 demonstrated, inflation (or specifically the Retail Price Index (RPI)) is central to the evolution of the 1995 Wilkie model. Since the respective influence of the RPI upon the five asset classes will vary, a change in the optimal portfolio’s composition might introduce model specific effects; for example it might introduce a variation in the role of inflation and hence have implications for insolvency prediction. Thus, the model specific considerations are important in the measurement of future insurer solvency. Evidence for the ‘model argument’ comes from ‘sensitivity’ analysis and the results that are reported in Table 11.6 below. Sensitivity analysis demonstrates the sensitivity of the ultimate surplus distribution function to its input distributions.
Table 11.6. Sensitivity Results for Ultimate Surplus by Asset Class

<table>
<thead>
<tr>
<th>Risk Tolerance</th>
<th>R²</th>
<th>Cash</th>
<th>Consols</th>
<th>IL Gilts</th>
<th>Equities</th>
<th>Property</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.77</td>
<td>0.17</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>5</td>
<td>0.67</td>
<td>0.12</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.16</td>
</tr>
<tr>
<td>10</td>
<td>0.53</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.14</td>
<td>0.09</td>
<td>0.17</td>
</tr>
<tr>
<td>15</td>
<td>0.47</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.11</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>20</td>
<td>0.44</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.13</td>
<td>0.04</td>
<td>0.15</td>
</tr>
<tr>
<td>25</td>
<td>0.46</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.14</td>
<td>0</td>
<td>0.12</td>
</tr>
<tr>
<td>30</td>
<td>0.51</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.17</td>
<td>0</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Note. Average standardised coefficients reported across all input distributions for given asset class.

The sensitivity analysis performed by @RISK involves the use of multivariate stepwise regression. Stepwise regression is used in sensitivity analysis because it removes all variables that provide an insignificant contribution to the simulation model (at the 5 percent level of statistical significance). The results that are reported in Table 11.6 are normalised regression coefficients for each input variable. Thus, for example, a regression value of 0 indicates that there is no significant relationship between the input and the output, while a regression coefficient of +1 or -1 indicates a +1 or -1 standard deviation change in the output for a 1 standard deviation change in the input. $R^2$ values are reported for completeness.

The results in Table 11.6 demonstrate the model specific variations as risk tolerance changes. In particular, the explanatory power of the input distributions also has a 'U' shaped relationship with risk tolerance. This is a direct result of the respective changes in the composition of optimal portfolios. Consider the role of inflation within the surplus simulation. The evidence confirms that for all classes of risk
tolerance, the importance of fluctuations in inflation is seen as a significant explanation of fluctuations in ultimate surplus. This confirms the model specific effects present within the structure of the 1995 Wilkie model (see Chapter 10).

With regard to the other results reported in Table 11.6, the signs of the reported coefficients are as expected, and the respective role of equities is confirmed as statistically significant in generating some of the variation associated with ultimate surplus. In line with the changes in portfolio composition, the relative significance of the different asset classes also changes. Thus, for example, where \( r \to 0 \), short-term interest rates become increasingly significant in explaining fluctuations in ultimate surplus, while by contrast, as \( r \to 34 \) the equity performance becomes increasingly significant.

11.4.3 Regulatory Considerations

As an illustration of the role of solvency constraints, consider the capital that an insurance company would require in order that the probability of needing any further capital to finance its activities is less than 1 percent (the assumed regulatory standard in Equation 11.26). The final two columns in Table 11.3 provide some guidance to the capital adequacy problem outlined above. Total reserves and the margin of solvency are reported in Table 11.3.\(^\text{18}\) Recall that Equation 11.2 sets total assets equal to two components- a variable (capital) component \( K \), and a fixed

\(^{18}\) Note that the calculated margin of solvency used in this study is a theoretical measure and must be contrasted with the regulations imposed on insurers in practice (see Chapter 2).
component $C$ (that belongs to the fund claimants and is the expected value of the liability). At the start of the simulation the value of $K$ has been assumed to be equal to zero. An attempt is made to calculate retrospectively the value of $K$ that would have ensured that the regulatory constraint in Equation 11.26 was non-binding.

Before proceeding to the reserve results, it is important to understand the basis of the reserve calculations. Waters (1988) highlights two points in this regard. First, required reserves are evaluated with respect to ultimate surplus and therefore at the end of the policy term. That is, with a 0.99-degree of probability, the calculated reserves ensure that the insurance company would have enough funds to meet its liability. However, it might be more reasonable to assume that the reserve funds are made available from the beginning of the policy term, since at time $t=0$ the liability was assumed (Waters 1988, 60). The second point developed by Waters (1988), concerns the source of capital and how this in turn is serviced. On the basis of the reserve calculation in Table 11.4, it is assumed that reserves would be invested on the same basis as the premium funds; for example, for an optimal portfolio made up entirely of equities, the reserves are calculated assuming that they too are invested entirely into equities. This seems an unreasonable assumption, particularly where preferences are polarised at the two ends of the risk tolerance scale and where the risk of insolvency is correspondingly increased.

The latter point outlined above needs greater elaboration and is the point of departure for further research. In particular, a richer theory of solvency regulation
would need to incorporate constraints on the composition of reserves, which would itself be a function of portfolio composition. While the above points will be material to the discussion, it is asserted that the results in Table 11.3 provide a useful introduction to some of the issues relating to insurer solvency regulation. As a consequence, consider the results relating to reserves and margins of solvency in Table 11.4. Although there are some significant variations about trend, the trend is positive as the required reserves rises with the risk tolerance of the insurer. This is an intuitive result since the variation in ultimate surplus is increasing with risk tolerance. The possibility of insurer insolvency necessitates the requirement for some form of solvency constraint. Thus, Table 11.4 shows that the required 'margin of solvency' for a given premium fund size varies significantly with risk tolerance. Margins rise to a maximum of 67 percent of the premium fund size- a significant provision for adverse outcomes in order that the solvency regulatory standard is adhered to. Figure 11.4 illustrates the variation in the solvency margin along the risk tolerance scale.
Figure 11.4 reflects the ‘U’ shaped relationship that exists between insolvency probability and insurer risk tolerance. In addition, the required margin of solvency is insurer-specific; depending on the risk preference of the insurer, the required solvency margin will vary in order to ensure that the solvency constraint in Equation 11.26 is passed. Note also that there exists a substantial jump in the margin of solvency between $r=30$ and $r=31$. This may, for example, indicate that there is a critical point beyond which it is not possible to exploit the gains from diversification and, as a result, a more than proportionate response in terms of an increase in the required solvency margin is required.

Now it is useful to compare the insolvency probabilities associated with the optimal portfolios with those outcomes from actual portfolios operated in practice. As a basis for comparison, the portfolio allocations for the insurance industry as a whole
will be used in the simulation. Data on the industry investments are taken from Table 7.5 (page 274) and are simplified below in Table 11.7 for the purposes of using the 5-asset class model assumed here. In addition, the probability of insolvency is also calculated for 10 of the larger insurers using their asset allocations for 1998.

Table 11.7. The Probability of Insolvency (%) and Actual Portfolios Holdings (%)

<table>
<thead>
<tr>
<th>Name of Insurance Company</th>
<th>Cash</th>
<th>Consols</th>
<th>Index-linked</th>
<th>Equities</th>
<th>Property</th>
<th>Probability of Insolvency</th>
</tr>
</thead>
<tbody>
<tr>
<td>American Life Insurance Co.</td>
<td>52.4</td>
<td>31.2</td>
<td>15.6</td>
<td>0</td>
<td>0.8</td>
<td>1.56</td>
</tr>
<tr>
<td>AXA Sun Life PLC</td>
<td>3.3</td>
<td>25.1</td>
<td>9.5</td>
<td>54.5</td>
<td>7.6</td>
<td>0.58</td>
</tr>
<tr>
<td>The Equitable Life Assurance Society</td>
<td>6.5</td>
<td>25.5</td>
<td>10</td>
<td>51.6</td>
<td>6.4</td>
<td>0.63</td>
</tr>
<tr>
<td>Legal &amp; General</td>
<td>1.1</td>
<td>27.3</td>
<td>11.7</td>
<td>49.4</td>
<td>10.5</td>
<td>0.44</td>
</tr>
<tr>
<td>Nat West Life</td>
<td>0.7</td>
<td>88.1</td>
<td>11.2</td>
<td>0</td>
<td>0</td>
<td>1.46</td>
</tr>
<tr>
<td>Norwich Union Life &amp; Pensions Ltd</td>
<td>1.3</td>
<td>25.4</td>
<td>3.4</td>
<td>59.2</td>
<td>10.7</td>
<td>0.62</td>
</tr>
<tr>
<td>Prudential Assurance</td>
<td>2.3</td>
<td>10.3</td>
<td>7.4</td>
<td>69.8</td>
<td>10.2</td>
<td>0.51</td>
</tr>
<tr>
<td>Scottish Equitable</td>
<td>0.9</td>
<td>50.9</td>
<td>19.4</td>
<td>28.8</td>
<td>0</td>
<td>0.58</td>
</tr>
<tr>
<td>Scottish Widows</td>
<td>4.1</td>
<td>25.6</td>
<td>13.2</td>
<td>51.1</td>
<td>6</td>
<td>0.76</td>
</tr>
<tr>
<td>Standard Life</td>
<td>2.8</td>
<td>30.6</td>
<td>6.3</td>
<td>51.3</td>
<td>9</td>
<td>0.53</td>
</tr>
<tr>
<td><strong>Industry Average</strong></td>
<td><strong>8.74</strong></td>
<td><strong>13.3</strong></td>
<td><strong>5.99</strong></td>
<td><strong>62.6</strong></td>
<td><strong>9.37</strong></td>
<td><strong>0.48</strong></td>
</tr>
</tbody>
</table>

     (b) The number of iterations during the simulation was 10,000.
     (c) All convergence criteria successfully passed.

With respect to Table 11.6 there are a number of points to note. With respect to the portfolio holdings themselves, it is evident that most portfolios are diversified across all asset classes – notable exceptions to this pattern are the American Life Insurance Company and Nat West Life. Moreover, all insurance companies – except the two aforementioned insurers – have insolvency probabilities less than 1 percent using this simulation model. Correspondingly, the level of margin required would be negative under the current regulatory assumptions. However, the illustrative regulatory standard may in fact be quite high, as one failure in every one hundred –
given that there are approximately 160 authorised life insurers operating in the UK—would see an insurance company insolvency every year on average. As noted previously, the determination of what is the acceptable level of insolvency is a matter for cost/benefit analysis.

With respect to the optimal portfolios, the actual portfolios perform rather better with most of the insolvency probabilities reported in Table 11.6 reporting insolvency probabilities significantly less than the optimal portfolio with the lowest insolvency probability (optimal portfolio, \( r=7 \) (probability 0.74 percent)). The outcome is in line with what might be expected.

### 11.5 Summary

The use of simulation methodology has provided a means of illustrating the central role of risk preferences in measuring and regulating the solvency of life assurance companies. The financial strength of a hypothetical insurance company was assessed in this chapter by making probabilistic statements about the accumulated cash inflows and the accumulated cash outflows, as measured by the ultimate surplus at a fixed time horizon, \( t=25 \). The results and methodology demonstrate the complexities involved in quantifying insurer solvency. In particular, the models used to simulate are a critical consideration. Sensitivity analysis can provide an indication of the significance of these model specific factors.
The above point serves to illustrate the point that, as with any form of model building, results and conclusions are frequently assumption specific. One of the most important assumptions concerns the use of portfolio run-off performance as opposed to the use of portfolio going-concern performance. The absence of new business in the measurement of the insurer's solvency position is unrealistic at best. The inflow of new cash from new policy cohorts will introduce important crossover effects (such as cross-subsidy effects between different policy cohorts). However, the run-off case is justified on the grounds that it provides a relatively simple framework from which to demonstrate a number of central issues relating to insurer solvency measurement. Also, the assumption of new business introduces additional complications, such as the complication associated with determining the new business time horizon.

Despite the methodological concerns, the central role of risk preference in the solvency assessment of life assurance companies is clear. The implications for the design of solvency regulation are discussed in Chapter 12. Note that experimental economics can provide an indication of the actual level of risk tolerance; for example there is evidence to suggest that an appropriately scaled risk tolerance parameter \( r \) would equal 25 for a fund manager (see Sherris (1992)).\(^{19}\) On the basis of this experimental evidence and the simulations used in this chapter, the resulting insolvency rate is significant, suggesting that a cost-benefit analysis of regulatory shift might be worthwhile.

\(^{19}\) There are a number of caveats to the experimental evidence including the sensitivity of results to the size of initial wealth, and the utility function assumed.
12.0 Public Policy Perspectives

A number of alternative approaches to the study of solvency in the U.K. life assurance industry have been explored and used to demonstrate factors that are worthy of further consideration in respect of the measurement of insurer solvency. The implications for public policy are explored in greater detail within this chapter, with particular emphasis on the possible directions for future research. Before proceeding, it is worth stating some of the policy questions that have been considered in respect of the current solvency regulations governing the U.K. life assurance industry. These questions are as follows:

1. What are the objectives of the regulator?

2. What has economic theory got to say about the current regulatory environment?

3. What has the simulation analysis got to say about the value of the current information submitted to the regulator for the purposes of solvency screening?
   
   3(a) What measures of solvency are current regulations actually providing?
   
   3(b) Are these the types of measures that are consistent with the conclusions of the simulation analysis?
   
   3(c) If not, what types of measures should the regulator be seeking to identify?
The above questions serve to act as a focus for discussion within this chapter. Discussion proceeds in Section 12.1 with an analysis of the regulatory objectives associated with U.K. life assurance solvency regulation. Section 12.2 provides a performance appraisal of the current regulatory environment pertaining to life assurance. Section 12.3 develops some general considerations in the measurement of solvency, while in Section 12.4 the implications from a simulation approach to solvency are developed. Section 12.5 outlines considerations specific to *ex ante* solvency measurement, while closing remarks are presented in Section 12.6.

### 12.1 Regulatory Objectives

In order to understand the motivation for regulation, it is worth reviewing some of the conclusions that can be drawn from current regulations. There have been a variety of regulatory actions that have been in the public interest, aimed at correcting a perceived market failure, such as information asymmetry. The public interest argument can be applied to most of the regulatory changes, particularly those relating to the authorisation of insurance companies; for example, the 1982 Insurance Company Act (see Chapter 2). These regulatory changes emphasise quality control, using insurance company accounts and statements (submitted to the regulator) as a signal for insurer solvency.

The historical evidence would tend to suggest a degree of regulatory capture. One notable example is within the Insurance Ombudsman scheme, which has terms of reference that limits either the scope or the basis of judgement of the ombudsman.
Also, there is evidence of regulatory responses to consumer demands, such as the 1975 Policyholders Protection Act, which followed widespread public concern over the Vehicle and General collapse in the early 1970s.

A further point to note with respect to the underlying theories of regulation concerns the lack of flexibility in legislation. With the pace of innovation in financial services there is clearly a need for insurance legislation to be flexible, this was clearly not the case with the 1870 and 1909 Life Assurance Acts which were inadequate to deal with new types of insurance products. Moreover, the pace of financial innovation has highlighted the inadequacies of the 1986 Financial Services Act, especially with respect to the regulation of financial crimes. The development of a new unified financial regulator to oversee all aspects of financial supervision is an attempt to rectify these inadequacies. This regulatory shift also demonstrates a pattern that is consistent throughout history; namely that regulatory shift is primarily reactive in nature, rather than proactive.

While much has been said about the evidence to support alternative theories of regulation in the U.K. life assurance industry, what can be concluded about the regulator’s objectives (where the regulator is defined as the Insurance Director at the Financial Services Authority (formerly the Insurance Directorate at the DTI))? U.K. insurance regulations, in particular solvency margins, aim to provide an ‘early-warning’ mechanism of financial distress within the life assurance industry. Do
these regulations aim to minimise the risk of insolvency? The answer to this question is not that clear.

The accounts and statements submitted under the 1982 Insurance Companies Act can be seen as a signalling device to pre-empt financial distress and the declaration of insolvency. Changes to the regulations governing the informational requirements may therefore be seen as a means of improving the quality of the insurer solvency signal, rather than necessarily minimising the risk of insolvency, although this too is an important consideration.

While there is clearly a need to restrict the probability of insolvency, and as a result ensure the quality of any solvency signal specified in the regulations, the regulator's objective still does not fit into an unconstrained max/min problem. This is because of the fact that it is highly inefficient to have an objective that aims to eliminate the probability of insolvency completely; the resources that would be required in order to eliminate the probability of insolvency would be exorbitant. Thus, the regulatory objective is focused on limiting the frequency of insolvency, while setting this objective within a cost/benefit framework; that is, the regulator minimises insolvency probability subject to a resource constraint.
12.2 The Performance of the Current Regulatory Environment

What can be said about the performance of the current prudential regulatory structure? In terms of financial crises within the life assurance industry, the current regulatory system has performed relatively well over the last twenty years, with only one 'life' case referred to the Policyholders Protection Fund (Department of Trade and Industry 1998). In addition, the Appointed Actuary scheme has proved a valuable means of insolvency control in ensuring that insurance companies do not assume excessive risks. However, there are a number of weaknesses and strains that are associated with the current system. One of the major problems concerns the frequency of regulatory review and the time lags involved in detecting financial distress. Forms are submitted to the regulator six months after the end of the year to which they relate. As a consequence, a possible 18 might pass before the regulator would detect financial distress. The speed of 'risk-related supervision' (see Chapter 5) needs to be improved so that public information disclosure is more timely. The potential for improving the speed of information disclosure is significant given the advances in information technology since the establishment of the 1982 Insurance Companies Act. Thus, the time lag in solvency-signal transmission results in a solvency measure that is outdated and unlikely to provide an accurate measurement of the current solvency performance exhibited within the U.K. life assurance industry.
The need for a more responsive, dynamic measure of solvency is highlighted when considering the pace of product innovation within the life assurance industry. As demonstrated in Chapters 6-9, the continued development of the life assurance industry has been supported through numerous product innovations (see Chapter 6), which supports new business, provides evidence for non-price competition (see Chapter 7), and yields the potential to earn monopoly rents (see Chapters 8 and 9). Furthermore, Chapter 6 highlighted more general financial innovation, such as the use of financial derivatives in asset/liability management. The importance of equities to the long-term performance of life insurers is clear: life insurers are heavily reliant upon the performance of the stock market (see Chapter 7). In sum, dynamic solvency testing, such as assessing an insurance company's ability to withstand a stock market crash of variable size, becomes another crucial consideration (see Chapter 4). Current insurer solvency measures must therefore be more dynamic in design.

Another argument in favour of making the current regulatory assessments more up-to-date comes from organisational shifts, such as mergers and demutualisations (see Chapter 7), which represents significant adjustments that may adversely affect the performance of the company (see Chapter 9). All these changes mean that timely and flexible regulatory reviews are critical for the effective measurement of solvency in the U.K. life assurance industry (see Chapters 3 and 5).
12.3 Considerations in the Measurement of Solvency

The actuarial and financial approaches to insurer solvency (respectively Chapters 3 and 4-5) have highlighted the numerous methods by which to assess and measure solvency. Although these two approaches have a tradition of being developed in isolation from each other, they both highlight three attributes of solvency that are central considerations: the considerations of time, risk, and probability. The use of portfolio theory has become the dominant paradigm for analysing issues pertaining to insurance solvency in both actuarial science and economics (see Chapters 4 and 5). This theoretical framework is complimented by stochastic simulation methodology that attempts to incorporate further the underlying probabilistic nature of insurer solvency (see Chapters 10 and 11).

Paulson and Dixit (1988) argue that the main requirements for any sound investigation of solvency are as follows: first, to keep at a minimum the number and impact of assumptions made; second, to use actual company, industry-wide data as much as possible; third, to identify information that should be incorporated in prospective studies if it is not currently available; and fourth, to integrate the salient features and timing aspects of solvency into a cash flow model, which faithfully captures the essential character of the insurance firm subject to market and regulatory behaviour (Paulson and Dixit 1988, 41).
While it is difficult to ensure that all of the above considerations are satisfied fully, the points of Paulson and Dixit (1988) provide a valuable guide for a discussion of solvency measurement. Throughout this thesis, it is evident that there is no simple answer regarding the 'right' measure of insurer solvency. Any measure of solvency, by definition, necessitates the use of assumptions that contain subjective elements, which are a direct consequence of the time, risk and probabilistic attributes of solvency. Moreover, as Chapter 9 demonstrated, much of the current available data contains a high noise-to-signal ratio and this limits its usefulness. Also the static, \textit{ex post} measures of solvency used in the U.K. regulations can never incorporate fully the dynamic factors that affect solvency. This fact is acknowledged implicitly within the U.K. insurance regulations. Regulatory actions are defined in terms of the 'required margin of solvency'. As noted earlier, these solvency measures are used as an early warning signal, rather than a definitive criterion for the declaration of insolvency. Hence, the inadequacies associated with \textit{ex post} solvency measurement means that definitive regulatory action (i.e. the ability to declare an insurer solvency or insolvency) is constrained heavily.

The limitations associated with the current solvency measures yields a number of opportunities for the improvement of solvency measurement. However, these improvements must be checked by three qualifications. The first qualification is that any regulatory change must be assessed in terms of its cost and benefit. The second qualification (related to the first qualification) is that a full account of market processes must be made in the design of regulations. This qualification
acknowledges the fact that increasing the intensity of a solvency constraint imposes restrictions upon the ability of insurers to allocate their resources effectively and in addition may impose entry barriers, which might also serve to reduce productive efficiency (see Chapter 6). The third qualification to consider in respect of regulatory change is that the theoretical analysis in Chapter 5 demonstrated the potential for regulatory constraints to be in conflict and hence a trade-off may exist between alternative measures of solvency. Thus, the implementation of a set of regulatory constraints in order to achieve a desired outcome (insurer solvency) needs to be fully integrated rather than implemented in a piecemeal approach.

12.4 A Simulation Perspective

Having acknowledged some of the more general considerations in the measurement and regulation of solvency, it is now worth considering the detailed implications from the simulations performed in Chapters 10 and 11. While the mechanics of simulations are relevant to discussion, it is the application of this abstract discussion to the practical problem of solvency measurement that yields some of the more interesting insights.

There is a critical distinction between the simulation approach to measuring solvency and the measurement approach assumed by the insurance regulators in the U.K. The simulations provide an *ex ante* measure of solvency, 'ultimate surplus' (see Chapter 11), in contrast to the *ex post* measures constructed out of the accounts
and statements currently submitted to the regulator. The distinction between *ex ante* and *ex post* is important, since solvency measurement is, by definition, involved in making predictions that are necessarily subject to future random fluctuations. For this reason, it is believed that in order to improve insurer solvency measurement the use of an *ex ante* measure of insurer solvency would be beneficial. In particular, an *ex ante* solvency measure incorporates the respective considerations of time, risk and probability far more than the current regulations.

In order to illustrate these ideas in greater detail, a graphical illustration of the *ex ante* solvency measure is given below in Figure 12.1. The diagram illustrates the respective roles of *ex ante* and *ex post* regulation assuming the following: first, that there are two measures of solvency, *ex post* and *ex ante*; second, that solvency $S$ is defined in terms of a solvency margin (recall that the 'ultimate surplus' measure used in Chapter 11 is also a solvency margin measure); third, that regulatory review operates discrete rather than continuous monitoring; fourth, that insurer solvency performance, prior to the period under consideration, has been passed as satisfactory; and fifth, that the time horizon for *ex ante* solvency measurement is $t+25$. In addition, it is assumed that an appropriate measure of *ex ante* solvency can be constructed (this point will be revisited in the next section).

Consider now the basic flow of argument with reference to Figure 12.1 below. Analysis commences assuming that the regulator only uses an *ex post* solvency measure.
Figure 12.1. *Ex ante* and *Ex Post* Regulation of Solvency

Notes: (a) XP denotes an *ex post* measurement of insurer solvency. 
(b) XA denotes an *ex ante* measurement of insurer solvency.

At time $t=0$ the regulator measures the *ex post* solvency performance of a given life assurance company. This measure of solvency is similar to the current regulations, and the *ex post* required solvency margin is given by the bold portion of the line $XP(0)$. The observed margin is equal to $PS(0)$ and on this basis the insurer is signalled as solvent. Under the current arrangements this measurement of solvency allows the insurer to proceed to time $t=1$ (the actual path is illustrated by the dotted line), where again *ex post* solvency measurement $XP(1)$ reports an observed margin of $PS(1)$, which passes the required margin (the bold portion of $XP(1)$). It might be the case that the observed solvency margin is less than the required margin. Under
this scenario, a regulatory response would then be initiated (for a discussion of the current regulatory corrective action see Chapter 2).

Now consider the inclusion of ex ante solvency measurement and its regulation. As noted in the previous sections, an ex post measure of solvency might ignore crucial factors that are relevant to solvency measurement (e.g. time, risk, and probability). Thus, an ex ante measurement of solvency may help to clarify the respective roles of these factors and thereby strengthen the current ex post regulations.

To illustrate the advantage of an ex ante solvency measure, consider the following example. The form of ex ante solvency measurement consists of predictions about the future, and as a result only confidence statements can be made about the associated solvency measurements. At time $t=0$ ($t=1$) the regulator measures ex ante the projected solvency of the insurer at time $t=25$ ($t=26$). These predictions are defined in terms of confidence intervals, respectively $A$ to $B$, and $C$ to $D$. It might be the case that ex ante solvency constraints impose a limit on this confidence interval (as shown by the bold section of the $XA(25)$ and $XA(26)$ lines).

Figure 12.1 shows that at time $t=0$ the insurer passes both the ex post and the ex ante measures of solvency. However, this is not the case at time $t=1$ where the insurer passes the ex post measure, but fails the ex ante measure; the lower confidence bound of the ex ante measure is below the regulatory minimum at $D$. This might, for example, reflect changing market conditions and more pessimistic
scenario predictions used within simulations. The *ex ante* solvency measurement may indicate insolvency risks that are not identified by *ex post* measures of solvency and thus there is a greater likelihood that the regulator can initiate a pre-emptive strike against pending insurer insolvency. Note that the regulatory constraints are dynamic, in that the solvency margins and confidence bounds can be altered to account for changing conditions (such as short-term volatile movements in the stock market).

### 12.5 *Ex ante* Solvency Measurement

It is important to stress a number of additional points that relate to the design and implementation of *ex ante* solvency measurement. Consider first, the design of *ex ante* solvency measures. Guidance to this can come from a variety of considerations, such as those factors highlighted by Paulson and Dixit (1988). However, discussion here focuses on the simulation model used in the preceding chapter, Chapter 11.

Central to the simulation analysis and the insolvency probability calculation, is the risk preference of the insurer. Thus on the evidence of the simulations, an *ex ante* measurement of solvency requires the measurement of insurer risk preference. Risk preference might be inferred from a variety of sources.\(^1\) Chapter 2 illustrated that

---

\(^1\) The separation of ownership from control and the predictions of agency theory (see Chapter 4) suggest that there may be a divergence between the risk preferences of the owners and the risk preferences of the investment managers. This point is therefore an important consideration in the assessment of insurer risk preference.
there is substantial data available in the accounts and statements submitted to the regulator regarding the portfolio composition of assets and this information might therefore be used to infer something about the insurer's risk preference. However, much of the submitted data are aggregate in nature and a detailed breakdown of the composition of assets and claims is not available (although the regulator could request this data).

Questionnaires might be another useful resource from which to gauge insurer risk preference (although these are often fraught with design complications). For example, these questions might be used to construct an insurer risk preference index, though the weights assigned to each of the questions would need to be evaluated.\(^2\) In sum, based on the theoretical and methodological approaches used in Chapters 10 and 11, the measurement of \textit{ex ante} insurer solvency requires the adequate assessment of insurer risk preference.

Another issue to arise with respect to the design of \textit{ex ante} solvency regulation is that \textit{ex ante} measurement (and regulation) is only as good as the models used to forecast \textit{ex ante}. This issue introduces a plethora of additional considerations, from specific problems of model specification and estimation, to more general methodological debates concerning the predictive and explanatory value of simulation models. With respect to model specification and estimation, this clearly

\(^2\) The problem of insurers manipulating answers is a concern with questionnaires. If, for example, questions are not weighted equally then insurers may have an incentive to concentrate only on those questions weighted heavily and manipulate their answers accordingly in order to ensure that they pass the \textit{ex ante} solvency constraint.
is an area for future research. In Chapter 10, use was made of the Wilkie model (Wilkie 1995) to calibrate a simplified insurer solvency model in Chapter 11. However, there are a number of alternative approaches that could have been adopted with respect to model specification and estimation (indeed Wilkie (1995) reviews some of these alternative model specifications).

Also important to any discussion of \textit{ex ante} solvency measurement are the methodological concerns associated with simulation methodology. These concerns relate to the problems of the ‘black-box’ approach, and in particular the relative importance of model explanation and model prediction in defining a criterion for \textit{ex ante} model ‘validation’. A full account of the relative methodological merits of simulations is given in Balci (1994), Pidd (1996), and Robinson (1997). Many considerations relate to the views espoused by Friedman’s instrumentalism, the influential assertion that economic theories should be judged by the accuracy of their predictions rather than by the realism of their assumptions (Friedman 1953). Evidently, prediction is a central objective for solvency regulations, however questions persist regarding explanation; the black box remains. While it is possible to test the correlation between input and output distributions, it might not be possible to reach a sensible, plausible explanation of what the results mean. Moreover, an attempt has been made throughout this thesis to demonstrate how understanding a concept (solvency) provides invaluable guidance to its ultimate measurement and prediction.\footnote{The simulations used in the previous chapters are justified in terms of their function as an ‘exploratory simulation’, rather than as a ‘confirmatory simulation’ (Robinson 1997).}
Hardy (1994) summarises some of the main concerns associated with the use of stochastic simulation techniques. The first concern is that the level of uncertainty is such that not enough is known about parameter variability in order to assign probability distributions. The second concern is that the understanding of stochastic methods is at best incomplete. The third concern is that the results cannot be adequately explained. The final concern highlighted by Hardy is that stochastic simulation will inevitably throw up situations which could not happen in practice and hence challenge model validity. The concerns highlighted by Hardy are by no means conclusive and are open to debate. However, it is useful to bear these concerns in mind when designing an ex ante measure of insurer solvency.

Of course the concerns about model validity in the ex ante measurement of insurer solvency has repercussions for the effectiveness and credibility of regulation. The introduction of new regulations will be subject to lobbying and the use of a given model needs to be justified; for example, the use of the Wilkie (1995) model as a basis for assessment would require some explanation for the selection of the model. There are two considerations in this regard: first, model sensitivity tests could be performed, (such as a convergence test of the stability of the predicted ruin probabilities across alternative model specifications) allowing the insurer to adopt more than just one model; and second, related to the first point, insurers are governed by the valuation guidelines put down by the Faculty of Actuaries and therefore guidelines could be extended to include the use of stochastic modelling techniques, allowing insurers to report their own solvency assessments with the
regulator performing model sensitivity tests. Whatever the assumption made, ultimately the regulator, through its sanction of withdrawing insurer authorisation can enforce its view on solvency measurement.

A caveat to all of the discussion on *ex ante* regulation is that regulatory changes must be clearly evaluated in terms of their costs and benefits. Resource constraints mean that eliminating downside risk (see Chapter 4) is unrealistic and hence regulatory shift must therefore take account of any resulting reallocation of resources.

12.6 Further Remarks

Any ultimate theory of solvency regulation in the U.K. life assurance industry needs to take account of many features relating to solvency measurement, not least the consideration of time. A full 'dynamic program' of insurer solvency regulation is therefore required. The basis for solvency simulation in Chapter 11 was essentially a single-period, insurer optimisation portfolio model. While this has provided a useful analytical tool for discussing various issues relating to solvency measurement in the life assurance industry, the analysis nevertheless remains incomplete. A full dynamic model of insurer decision-making might therefore be a useful point of departure for future research. The use of dynamic optimisation has already been established (see Booth, *et al.* (1997)), however this analysis is only partially
dynamic and key variables, such as the insurer’s risk preference, is assumed constant throughout the period of analysis. Risk preference has been shown to play a central role in solvency measurement. A dynamic theory, where risk tolerance $r$ is a function of time $r(t)$ and the movement of this variable through time is given by $\dot{r} = \frac{dr}{dt}$, will be of critical importance in measuring insurer solvency and hence designing appropriate regulations. Thus, a dynamic theory of insurer decision-making may shed more light on the problem of solvency measurement.

Further considerations with respect to insurer solvency measurement are many and varied. Most considerations relate to an assessment of the impact from new solvency measures; for example, measuring the impact from insurers reallocating their resources in order to meet the solvency constraints. The opportunity cost associated with alternative means of financing the solvency constraint is another important issue. The role of ‘implicit’ margins in solvency measurement (see Chapter 3) is a further consideration. Note that the role of reinsurance has not been addressed and this is an additional area for future research.

In conclusion, the challenges to the newly established FSA regulator and the current insurance legislation are significant. The need for more accurate measurements of solvency is central to promoting public confidence within the U.K. life assurance industry. The greater understanding from recent theoretical and methodological advances suggests that current regulations are providing an incomplete solvency measure and that ‘a unified theory of insurer solvency regulation’ might suggest a
more focused solvency signal, reflecting the considerations of solvency measurement that are probability, time, and risk. Thus, the potential to reduce the level of 'noise' associated with current solvency measurements is significant, although due consideration must be given to the forecasting limitations of ex ante solvency measurement and the resource constraints imposed upon regulators.
APPENDIX 1: LEGAL MEASUREMENTS OF INSURER SOLVENCY

A.1.1. Solvency Margins for General Insurance in 1974

<table>
<thead>
<tr>
<th>Case</th>
<th>Relevant Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>The general premium income of the body in its last financial year did not exceed £250,000.</td>
<td>£50,000</td>
</tr>
<tr>
<td>The income in that year was greater than £250,000 but did not exceed £2,500,000.</td>
<td>One-fifth of the income in that year</td>
</tr>
<tr>
<td>The income in that year exceeded £2,500,000.</td>
<td>The aggregate of £500,000 and one-tenth of the amount by which the income in that year exceeded £2,500,000.</td>
</tr>
</tbody>
</table>

Source: The Insurance Company Act 1974, s.4.

A.1.2 Statutory Definitions of Long Term Insurance Business

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Life and Annuity</td>
</tr>
<tr>
<td>II</td>
<td>Marriage and Birth</td>
</tr>
<tr>
<td>III</td>
<td>Linked Long Term</td>
</tr>
<tr>
<td>IV</td>
<td>Permanent Health</td>
</tr>
<tr>
<td>V</td>
<td>Tontines</td>
</tr>
<tr>
<td>VI</td>
<td>Capital Redemption</td>
</tr>
<tr>
<td>VII</td>
<td>Pension Fund Management</td>
</tr>
<tr>
<td>[VIII]</td>
<td>Collective Insurance</td>
</tr>
<tr>
<td>[IX]</td>
<td>Social Insurance</td>
</tr>
</tbody>
</table>

Note: Classes VIII and IX have been inserted as a result of European Directives.

A.1.3. Statutory Definitions of General Insurance Business

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Accident</td>
</tr>
<tr>
<td>2</td>
<td>Sickness</td>
</tr>
<tr>
<td>3</td>
<td>Land Vehicles</td>
</tr>
<tr>
<td>4</td>
<td>Railway Rolling Stock</td>
</tr>
<tr>
<td>5</td>
<td>Aircraft</td>
</tr>
<tr>
<td>6</td>
<td>Ships</td>
</tr>
<tr>
<td>7</td>
<td>Goods in Transit</td>
</tr>
<tr>
<td>8</td>
<td>Fire and Natural Forces</td>
</tr>
<tr>
<td>9</td>
<td>Damage to Property</td>
</tr>
<tr>
<td>10</td>
<td>Motor Vehicle Liability</td>
</tr>
<tr>
<td>11</td>
<td>Aircraft Liability</td>
</tr>
<tr>
<td>12</td>
<td>Liability for Ships</td>
</tr>
</tbody>
</table>

A.1.4. Information ‘Schedules’ submitted to the Regulator

<table>
<thead>
<tr>
<th>Schedule Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Balance Sheet and profit and loss account (forms 9-17).</td>
</tr>
<tr>
<td>2</td>
<td>General Business: revenue account and additional information (forms 20-39).</td>
</tr>
<tr>
<td>3</td>
<td>Long term business: revenue account and additional information (forms 40-45).</td>
</tr>
<tr>
<td>4</td>
<td>Abstract of valuation report prepared by the appointed actuary (forms 46-61).</td>
</tr>
<tr>
<td>5</td>
<td>General business: additional information on business ceded.</td>
</tr>
<tr>
<td>6</td>
<td>Certificates by directors and actuary and report of auditors.</td>
</tr>
<tr>
<td>7</td>
<td>Regulations revoked.</td>
</tr>
</tbody>
</table>

## APPENDIX 2: DATA ON THE U.K. LIFE ASSURANCE INDUSTRY

### A.2.1. Sources and Definitions of Variables

<table>
<thead>
<tr>
<th>Variables</th>
<th>Definitions</th>
<th>Data Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.04, 0.02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solvency</td>
<td>Solvency margin (1997 assets for long term business required minimum margin) divided by total assets 1996.</td>
<td>1</td>
</tr>
<tr>
<td>(0.18, 0.25)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size</td>
<td>Natural logarithm of total long-term premium income.</td>
<td>1</td>
</tr>
<tr>
<td>(11.23, 2.84)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>Number of years since the insurer was established.</td>
<td>2</td>
</tr>
<tr>
<td>(71.05, 61.16)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>'SizeAge'</td>
<td>Natural logarithm of: age multiplied by total long-term premium income.</td>
<td>1 &amp; 2</td>
</tr>
<tr>
<td>(17.27, 3.12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diversity</td>
<td>Total number of insurance lines for which the insurer is authorised to conduct business under the terms of the Insurance Companies Act 1982.</td>
<td>3</td>
</tr>
<tr>
<td>(9.38, 6.98)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment Return</td>
<td>Investment income receivable on long term business before deduction of tax, divided by total assets.</td>
<td>1</td>
</tr>
<tr>
<td>(0.06, 0.04)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Death</td>
<td>Total death claims incurred divided by total claims.</td>
<td>1</td>
</tr>
<tr>
<td>(0.15, 0.19)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surrender</td>
<td>Total surrender of long term policies, divided by total claims.</td>
<td>1</td>
</tr>
<tr>
<td>(0.39, 0.28)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overseas Claims</td>
<td>Total overseas claims divided by total claims.</td>
<td>1</td>
</tr>
<tr>
<td>(0.02, 0.11)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales Growth</td>
<td>Total long-term premium Income in 1997 minus the 1996 total divided by the 1996 total.</td>
<td>1</td>
</tr>
<tr>
<td>(0.37, 0.78)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exports</td>
<td>Long term premium Income from overseas markets divided by total long-term premium income.</td>
<td>1</td>
</tr>
<tr>
<td>(0.03, 0.11)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single Premiums</td>
<td>Total premium income from single premium contracts, divided by total long-term premium income.</td>
<td>1</td>
</tr>
<tr>
<td>(0.46, 0.32)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agency Cost</td>
<td>Total expenses (including commission) divided by total long-term premium income.</td>
<td>1</td>
</tr>
<tr>
<td>(0.79, 3.72)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Managerial Expenses</td>
<td>Total management expenses in connection with acquisition and maintenance of business, divided by total expenses.</td>
<td>1</td>
</tr>
<tr>
<td>(0.77, 0.50)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distribution Channels</td>
<td>Number of distribution channels used for marketing of insurance products.</td>
<td>3</td>
</tr>
<tr>
<td>(1.7, 1.02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mutual</td>
<td>= 0 for Proprietary Life Insurer, = 1 for Mutual Insurer.</td>
<td>1,2&amp;3</td>
</tr>
<tr>
<td>Foreign</td>
<td>= 0 for domestic ownership, = 1 for foreign ownership.</td>
<td>1,2&amp;3</td>
</tr>
<tr>
<td>Group</td>
<td>= 0 if insurer does not belong to an insurance group, = 1 if insurer does belong to an insurance group.</td>
<td>1,2&amp;3</td>
</tr>
</tbody>
</table>

* Mean and standard deviation shown in parenthesis.


<table>
<thead>
<tr>
<th>INSURANCE COMPANY NAME</th>
<th>COMPANY NUMBER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abbey Life Assurance Company Limited</td>
<td>ac000846</td>
</tr>
<tr>
<td>Aegon Life Assurance Company (UK) Limited</td>
<td>ac001338</td>
</tr>
<tr>
<td>Albany Life Assurance Company Limited</td>
<td>ac001368</td>
</tr>
<tr>
<td>Allchurches Life Assurance Limited</td>
<td>ac000957</td>
</tr>
<tr>
<td>Allied Dunbar Assurance PLC</td>
<td>ac000960</td>
</tr>
<tr>
<td>Ambassador Life Assurance Company Limited</td>
<td>ac000963</td>
</tr>
<tr>
<td>America Life Insurance Company Limited</td>
<td>ac000744</td>
</tr>
<tr>
<td>AXA Equity and Law Life Assurance Society</td>
<td>ac000004</td>
</tr>
<tr>
<td>Barclays Life Assurance Company Limited</td>
<td>ac000943</td>
</tr>
<tr>
<td>Black Sea and Baltic General Insurance Company</td>
<td>ac000525</td>
</tr>
<tr>
<td>Bradford Insurance Company Limited</td>
<td>ac000816</td>
</tr>
<tr>
<td>Britannia Life Holdings Limited</td>
<td>sa004348</td>
</tr>
<tr>
<td>Britannic Assurance PLC</td>
<td>ac000205</td>
</tr>
<tr>
<td>Britannic Unit Linked Assurance Limited</td>
<td>ac001516</td>
</tr>
<tr>
<td>British Equitable Assurance Company Limited</td>
<td>ac000089</td>
</tr>
<tr>
<td>British Life Office Limited</td>
<td>sa000179</td>
</tr>
<tr>
<td>Caledonian Insurance Company</td>
<td>ac000055</td>
</tr>
<tr>
<td>Canada Life Assurance Company</td>
<td>ac000197</td>
</tr>
<tr>
<td>The Canada Life Assurance Company of Great Britain Limited</td>
<td>ac001204</td>
</tr>
<tr>
<td>Canterbury Life Assurance Company Limited</td>
<td>ac001328</td>
</tr>
<tr>
<td>Carlyle Life Assurance Company Limited</td>
<td>ac000918</td>
</tr>
<tr>
<td>Century Life PLC</td>
<td>ac000255</td>
</tr>
<tr>
<td>City of Westminster Assurance Company Limited (The)</td>
<td>ac000975</td>
</tr>
<tr>
<td>Colonial Mutual Life Assurance Society Limited</td>
<td>ac000164</td>
</tr>
<tr>
<td>Commercial Union Life Assurance Company Limited</td>
<td>ac001286</td>
</tr>
<tr>
<td>Consolidated Life assurance Company Limited</td>
<td>ac001544</td>
</tr>
<tr>
<td>Co-operative Insurance Society Limited</td>
<td>ac000158</td>
</tr>
<tr>
<td>Cornhill Insurance PLC</td>
<td>ac000281</td>
</tr>
<tr>
<td>Criterion Life Assurance Limited</td>
<td>ac000914</td>
</tr>
<tr>
<td>Cuna Mutual Insurance Society</td>
<td>ac001737</td>
</tr>
<tr>
<td>Customs Annuity &amp; benevolent fund</td>
<td>ac000130</td>
</tr>
<tr>
<td>Domestic and General Life Assurance Company Limited</td>
<td>ac001707</td>
</tr>
<tr>
<td>Eagle Star Life Assurance Company Limited</td>
<td>ac001654</td>
</tr>
<tr>
<td>Ecclesiastical Insurance Office PLC</td>
<td>ac000214</td>
</tr>
<tr>
<td>The Equitable Life Assurance Society</td>
<td>ac000063</td>
</tr>
<tr>
<td>Eurolife Assurance Company Limited</td>
<td>ac001217</td>
</tr>
<tr>
<td>Fidelity Life Assurance Limited</td>
<td>ac000953</td>
</tr>
<tr>
<td>FP Life Assurance Limited</td>
<td>ac000917</td>
</tr>
<tr>
<td>Forester Life Limited</td>
<td>ac000953</td>
</tr>
<tr>
<td>Friends' Provident Life Office</td>
<td>ac000417</td>
</tr>
<tr>
<td>General Accident Fire and Life Assurance Corporation PLC</td>
<td>sa000256</td>
</tr>
<tr>
<td>General Accident Life Assurance Limited</td>
<td>ac000074</td>
</tr>
<tr>
<td>Gisborne Life Assurance Company Limited</td>
<td>ac000862</td>
</tr>
<tr>
<td>Guardian Assurance PLC</td>
<td>ac000057</td>
</tr>
<tr>
<td>Halifax Life</td>
<td>ac001725</td>
</tr>
<tr>
<td>Hambro Assured PLC</td>
<td>ac001633</td>
</tr>
<tr>
<td>Hamilton Life Assurance Company Limited</td>
<td>ac001547</td>
</tr>
<tr>
<td>Hill Samuel Life Assurance Limited</td>
<td>ac001196</td>
</tr>
<tr>
<td>Legal &amp; General Assurance Society Limited</td>
<td>ac000013</td>
</tr>
<tr>
<td>Legal &amp; General Assurance (Pensions Management) Ltd.</td>
<td>ac000830</td>
</tr>
<tr>
<td>London Aberdeen &amp; Northern Mutual Assurance Society Limited</td>
<td>ac001386</td>
</tr>
<tr>
<td>INSURANCE COMPANY NAME</td>
<td>COMPANY NUMBER</td>
</tr>
<tr>
<td>-------------------------------------------------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>London and Manchester Assurance Company Limited</td>
<td>ac000111</td>
</tr>
<tr>
<td>London and Manchester (Managed Funds) Limited</td>
<td>ac001537</td>
</tr>
<tr>
<td>London Life Linked Assurances Limited</td>
<td>ac001477</td>
</tr>
<tr>
<td>London Life Managed Funds</td>
<td>ac001352</td>
</tr>
<tr>
<td>M&amp;G Life Assurance Company Limited</td>
<td>ac000830</td>
</tr>
<tr>
<td>Medical Sickness Annuity and Life Assurance Society Limited</td>
<td>ac000494</td>
</tr>
<tr>
<td>Merchant Investors Assurance Company Limited</td>
<td>ac001211</td>
</tr>
<tr>
<td>Midland Life Limited</td>
<td>ac000964</td>
</tr>
<tr>
<td>National Farmers Union Mutual Insurance Society Limited</td>
<td>ac000426</td>
</tr>
<tr>
<td>National Provident Institution Limited</td>
<td>ac001303</td>
</tr>
<tr>
<td>National Mutual Life Assurance Society (The)</td>
<td>ac000177</td>
</tr>
<tr>
<td>The Northern Assurance Company Limited</td>
<td>ac000233</td>
</tr>
<tr>
<td>Norwich Union Life Insurance Society</td>
<td>ac000088</td>
</tr>
<tr>
<td>Old Mutual Life Assurance Company Limited</td>
<td>ac001159</td>
</tr>
<tr>
<td>Pearl Assurance PLC</td>
<td>ac000105</td>
</tr>
<tr>
<td>Permanent Insurance Company Limited</td>
<td>ac000792</td>
</tr>
<tr>
<td>Pinnacle Insurance Company PLC</td>
<td>ac001264</td>
</tr>
<tr>
<td>Pheonix Assurance PLC</td>
<td>ac001119</td>
</tr>
<tr>
<td>Premium Life Assurance Company Limited</td>
<td>ac000112</td>
</tr>
<tr>
<td>The Prudential Assurance Company Limited</td>
<td>ac000016</td>
</tr>
<tr>
<td>Reliance Mutual Insurance Society Limited</td>
<td>ac000696</td>
</tr>
<tr>
<td>J. Rothschild Assurance PLC</td>
<td>ac001669</td>
</tr>
<tr>
<td>Royal Heritage Life Assurance Limited</td>
<td>ac001114</td>
</tr>
<tr>
<td>Royal Life Insurance Limited</td>
<td>ac001508</td>
</tr>
<tr>
<td>Royal Life (unit linked Pension funds) limited</td>
<td>ac001307</td>
</tr>
<tr>
<td>Royal London Mutual Insurance Society</td>
<td>ac000246</td>
</tr>
<tr>
<td>Royal National Pension Fund for Nurses</td>
<td>ac000165</td>
</tr>
<tr>
<td>Scottish Amicable Life Assurance Society</td>
<td>sa047842</td>
</tr>
<tr>
<td>Scottish Life Assurance Company Limited</td>
<td>sa000151</td>
</tr>
<tr>
<td>Scottish Equitable PLC</td>
<td>sa144517</td>
</tr>
<tr>
<td>The Scottish Provident Institution</td>
<td>sa000045</td>
</tr>
<tr>
<td>Scottish Widows Fund and Life Assurance Society</td>
<td>sa000594</td>
</tr>
<tr>
<td>Scottish Widows Unit Funds Limited</td>
<td>sa074809</td>
</tr>
<tr>
<td>Skandia Life Assurance Company Limited</td>
<td>ac001439</td>
</tr>
<tr>
<td>Stalwart Assurance Company Limited</td>
<td>ac000900</td>
</tr>
<tr>
<td>Standard Life Assurance Company</td>
<td>sa000038</td>
</tr>
<tr>
<td>Sterling Life Limited</td>
<td>ac000966</td>
</tr>
<tr>
<td>Sun Alliance &amp; London Assurance Company Limited</td>
<td>ac001587</td>
</tr>
<tr>
<td>Sun Life Assurance Company of Canada (UK) Limited</td>
<td>ac001187</td>
</tr>
<tr>
<td>Sun Life Assurance of Canada</td>
<td>ac000174</td>
</tr>
<tr>
<td>Sun Life Assurance Society PLC</td>
<td>ac000049</td>
</tr>
<tr>
<td>Swiss Life (UK) PLC</td>
<td>ac001661</td>
</tr>
<tr>
<td>Teachers Assurance Company Limited</td>
<td>ac001772</td>
</tr>
<tr>
<td>Transatlantic Life Assurance Company Limited</td>
<td>ac001336</td>
</tr>
<tr>
<td>TSB Life Limited</td>
<td>ac000971</td>
</tr>
<tr>
<td>United Friendly Insurance PLC</td>
<td>ac000133</td>
</tr>
<tr>
<td>Unum Limited</td>
<td>ac001231</td>
</tr>
<tr>
<td>Virgin Direct Life Limited</td>
<td>ac002452</td>
</tr>
<tr>
<td>Wesleyan Assurance Society</td>
<td>ac000062</td>
</tr>
<tr>
<td>Windsor Life Assurance Company Limited</td>
<td>ac00125</td>
</tr>
<tr>
<td>Winterthur Life UK Limited</td>
<td>ac000285</td>
</tr>
<tr>
<td>Woolwich Life Assurance Company Limited</td>
<td>ac000465</td>
</tr>
<tr>
<td>Yorkshire Insurance Company Limited</td>
<td>ac000245</td>
</tr>
<tr>
<td>Zurich Life Assurance Company Limited</td>
<td>ac000840</td>
</tr>
</tbody>
</table>
A.2.3 Estimation Residuals: Residual Plot for Profitability (Table 9.1)

A.2.4. Estimation Residuals: Residual Plot for Solvency (Table 9.1)
APPENDIX 3: SIMULATION OUTPUT FOR STOCHASTIC ASSET RETURNS

A.3.1. Convergence in Wilkie Model Asset Output Distributions

<table>
<thead>
<tr>
<th>Name of Output Distribution (Wilkie Abbreviation)</th>
<th>% Change in Percentile</th>
<th>% Change in Mean</th>
<th>% Change in Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of Inflation (series I)</td>
<td>0.09</td>
<td>0.06</td>
<td>0.08</td>
</tr>
<tr>
<td>Dividend Yields (Series Y)</td>
<td>0.05</td>
<td>0.06</td>
<td>0.11</td>
</tr>
<tr>
<td>Yields on Consols (Series C)</td>
<td>0.12</td>
<td>0.32</td>
<td>0.27</td>
</tr>
<tr>
<td>Yield on Cash (Series B)</td>
<td>0.08</td>
<td>0.13</td>
<td>0.06</td>
</tr>
<tr>
<td>Yield on Property (Series Z)</td>
<td>0.06</td>
<td>0.10</td>
<td>0.09</td>
</tr>
<tr>
<td>Yield on Index-Linked Securities (Series R)</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Note. Results report the percentage change in the summary statistics of the output distribution for each simulation using 10000 iterations.

A.3.2. Convergence in Nominal Returns (25 years only)

<table>
<thead>
<tr>
<th>Name of Output Distribution (Wilkie Abbreviation)</th>
<th>% Change in Percentile</th>
<th>% Change in Mean</th>
<th>% Change in Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equities (GPR)</td>
<td>0.41</td>
<td>0.35</td>
<td>0.41</td>
</tr>
<tr>
<td>Consols (GCR)</td>
<td>0.35</td>
<td>0.37</td>
<td>0.29</td>
</tr>
<tr>
<td>Cash (GBR)</td>
<td>0.61</td>
<td>0.71</td>
<td>0.68</td>
</tr>
<tr>
<td>Index-Linked Securities (GRR)</td>
<td>0.33</td>
<td>0.44</td>
<td>0.59</td>
</tr>
<tr>
<td>Property (GAR)</td>
<td>0.37</td>
<td>0.53</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Note. Results report the percentage change in the summary statistics of the output distribution for each simulation using 10000 iterations.
### A.3.3. Convergence in Real Returns (25 years only)

<table>
<thead>
<tr>
<th>Name of Output Distribution (Wilkie Abbreviation)</th>
<th>% Change in Percentile</th>
<th>% Change in Mean</th>
<th>% Change in Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equities (JPR)</td>
<td>0.34</td>
<td>0.38</td>
<td>0.32</td>
</tr>
<tr>
<td>Consols (JCR)</td>
<td>0.41</td>
<td>0.33</td>
<td>0.28</td>
</tr>
<tr>
<td>Cash (JBR)</td>
<td>0.56</td>
<td>0.64</td>
<td>0.68</td>
</tr>
<tr>
<td>Index-Linked (JRR)</td>
<td>0.02</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>Property (JAR)</td>
<td>0.18</td>
<td>0.16</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Note. Results report the percentage change in the summary statistics of the output distribution for each simulation using 10000 iterations.

### A.3.4. Convergence in Variance/Covariances (Nominal Returns)

<table>
<thead>
<tr>
<th>Name of Output Distribution (Wilkie Abbreviation)</th>
<th>% Change in Percentile</th>
<th>% Change in Mean</th>
<th>% Change in Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance (Cash)</td>
<td>0.11</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>Variance (Consols)</td>
<td>0.06</td>
<td>0.03</td>
<td>0.12</td>
</tr>
<tr>
<td>Variance (Index-linked)</td>
<td>0.09</td>
<td>0.11</td>
<td>0.13</td>
</tr>
<tr>
<td>Variance (Equities)</td>
<td>0.08</td>
<td>0.12</td>
<td>0.09</td>
</tr>
<tr>
<td>Variance (Property)</td>
<td>0.07</td>
<td>0.10</td>
<td>0.08</td>
</tr>
<tr>
<td>Covariance (Cash, Consols)</td>
<td>0.04</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>Covariance (Cash, Index-linked)</td>
<td>0.06</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>Covariance (Cash, Equities)</td>
<td>0.08</td>
<td>0.10</td>
<td>0.07</td>
</tr>
<tr>
<td>Covariance (Cash, Property)</td>
<td>0.06</td>
<td>0.09</td>
<td>0.07</td>
</tr>
<tr>
<td>Covariance (Consols, Index-linked)</td>
<td>0.11</td>
<td>0.07</td>
<td>0.12</td>
</tr>
<tr>
<td>Covariance (Consols, Equities)</td>
<td>0.05</td>
<td>0.08</td>
<td>0.11</td>
</tr>
<tr>
<td>Covariance (Consols, Property)</td>
<td>0.04</td>
<td>0.08</td>
<td>0.06</td>
</tr>
<tr>
<td>Covariance (Index-linked, Equities)</td>
<td>0.09</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>Covariance (Index-linked, Property)</td>
<td>0.11</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Covariance (Equities, Property)</td>
<td>0.07</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Note. Results report the percentage change in the summary statistics of the output distribution for each simulation using 10000 iterations.
A.3.5. Convergence in Covariances (Real Returns)

<table>
<thead>
<tr>
<th>Name of Output Distribution</th>
<th>% Change in Percentile</th>
<th>% Change in Mean</th>
<th>% Change in Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance (Cash)</td>
<td>0.13</td>
<td>0.08</td>
<td>0.11</td>
</tr>
<tr>
<td>Variance (Consols)</td>
<td>0.11</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>Variance (Index-linked)</td>
<td>0.01</td>
<td>0.00</td>
<td>0.03</td>
</tr>
<tr>
<td>Variance (Equities)</td>
<td>0.09</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>Variance (Property)</td>
<td>0.12</td>
<td>0.11</td>
<td>0.08</td>
</tr>
<tr>
<td>Covariance (Cash, Consols)</td>
<td>0.08</td>
<td>0.09</td>
<td>0.10</td>
</tr>
<tr>
<td>Covariance (Cash, Index-linked)</td>
<td>0.07</td>
<td>0.09</td>
<td>0.10</td>
</tr>
<tr>
<td>Covariance (Cash, Equities)</td>
<td>0.08</td>
<td>0.11</td>
<td>0.13</td>
</tr>
<tr>
<td>Covariance (Cash, Property)</td>
<td>0.12</td>
<td>0.07</td>
<td>0.09</td>
</tr>
<tr>
<td>Covariance (Consols, Index-linked)</td>
<td>0.11</td>
<td>0.09</td>
<td>0.14</td>
</tr>
<tr>
<td>Covariance (Consols, Equities)</td>
<td>0.17</td>
<td>0.09</td>
<td>0.13</td>
</tr>
<tr>
<td>Covariance (Consols, Property)</td>
<td>0.08</td>
<td>0.12</td>
<td>0.08</td>
</tr>
<tr>
<td>Covariance (Index-linked, Equities)</td>
<td>0.11</td>
<td>0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>Covariance (Index-linked, Property)</td>
<td>0.12</td>
<td>0.15</td>
<td>0.09</td>
</tr>
<tr>
<td>Covariance (Equities, Property)</td>
<td>0.10</td>
<td>0.09</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Note. Results report the percentage change in the summary statistics of the output distribution for each simulation using 10000 iterations.

A.3.6. Illustration of Real Returns for Equities

![Graph showing real returns over years](image)
A.3.7. Illustration of Real Returns for Consols

![Graph of Real Returns for Consols]

A.3.8. Illustration of Real Returns for Cash

![Graph of Real Returns for Cash]
A.3.9. Illustration of Real Returns for Index-Linked Government Securities

A.3.10. Illustration of Real Returns for Property
A.4.1. Guide to Optimal Portfolios (2 Asset Case)

Ultimate Surplus $S$ (with initial asset funds equal to $A$):

$$S = A(w_1 R_1 + w_2 R_2) - L$$

$S$ depends on the random rates of return on the two assets $R_1$ and $R_2$, and on the liability $L$. The portfolio problem for the insurer is given by:

$$\max E - \exp \left( -\frac{S}{r} \right) \quad \text{subject to } w_1 + w_2 = 1$$

The mean $E$ and variance $V$ of ultimate surplus is given by:

$$E = A w_1 E_1 + A w_2 E_2 - E_L$$
$$V = A^2 w_1^2 V_1 + A^2 w_2^2 V_2 + V_L + 2A^2 w_1 w_2 C_{12} - 2A w_1 C_{1L} - 2A w_2 C_{2L}$$

Where $E$, $V$, and $C$ denote, the respective expected value, variance and covariance. Assuming that $S$ is normally distributed, then using the normal MGF, the problem can be seen as maximising the following function:

$$E \left[ \exp \left\{ tS \right\} \right] = \exp \left\{ Et + \frac{Vt^2}{2} \right\}, \quad \text{where } t = -(1/r)$$

(Subject to $w_1 + w_2 = 1$)

This is equivalent to minimising the logarithm of the above function handling the constraint by substituting, $w_2 = 1 - w_1$, directly into the objective function.

Differentiating the objective function with respect to $w_i$ and setting the derivative equal to zero, yields the optimal asset weights as a function of initial assets.

$$w_1 = \frac{(C_{1L} - C_{2L}) + A(V_2 - C_{12}) + r(E_2 - E_1)}{A(V_1 - 2C_{12} + V_2)}$$

$$w_2 = \frac{(C_{2L} - C_{1L}) + A(V_1 - C_{12}) + r(E_1 - E_2)}{A(V_1 + V_2 - 2C_{12})} = 1 - w_1$$
## A.4.2. Convergence in Expected Returns and Variances (25 years)

<table>
<thead>
<tr>
<th>Name of Output Distribution</th>
<th>% Change in Percentile</th>
<th>% Change in Mean</th>
<th>% Change in Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Return (Cash)</td>
<td>0.06</td>
<td>0.08</td>
<td>0.05</td>
</tr>
<tr>
<td>Expected Return (Consols)</td>
<td>0.08</td>
<td>0.07</td>
<td>0.09</td>
</tr>
<tr>
<td>Expected Return (Index-linked)</td>
<td>0.07</td>
<td>0.08</td>
<td>0.10</td>
</tr>
<tr>
<td>Expected Return (Equities)</td>
<td>0.11</td>
<td>0.09</td>
<td>0.12</td>
</tr>
<tr>
<td>Expected Return (Property)</td>
<td>0.07</td>
<td>0.09</td>
<td>0.08</td>
</tr>
<tr>
<td>Expected Return (Liability)</td>
<td>0.11</td>
<td>0.16</td>
<td>0.14</td>
</tr>
<tr>
<td>Variance (Cash)</td>
<td>0.07</td>
<td>0.06</td>
<td>0.09</td>
</tr>
<tr>
<td>Variance (Consols)</td>
<td>0.09</td>
<td>0.13</td>
<td>0.11</td>
</tr>
<tr>
<td>Variance (Index-linked)</td>
<td>0.06</td>
<td>0.09</td>
<td>0.10</td>
</tr>
<tr>
<td>Variance (Equities)</td>
<td>0.13</td>
<td>0.11</td>
<td>0.16</td>
</tr>
<tr>
<td>Variance (Property)</td>
<td>0.12</td>
<td>0.09</td>
<td>0.13</td>
</tr>
<tr>
<td>Variance (Liability)</td>
<td>0.16</td>
<td>0.13</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Note. Results report the percentage change in the summary statistics of the output distribution for each simulation using 10000 iterations.
A.4.3. Convergence in Covariances (25 years)

<table>
<thead>
<tr>
<th>Name of Output Distribution</th>
<th>% Change in Percentile</th>
<th>% Change in Mean</th>
<th>% Change in Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariance (Cash, Consols)</td>
<td>0.05</td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>Covariance (Cash, Index-linked)</td>
<td>0.03</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>Covariance (Cash, Equities)</td>
<td>0.04</td>
<td>0.06</td>
<td>0.04</td>
</tr>
<tr>
<td>Covariance (Cash, Property)</td>
<td>0.06</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>Covariance (Cash, Liability)</td>
<td>0.03</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>Covariance (Consols, Index-linked)</td>
<td>0.13</td>
<td>0.06</td>
<td>0.03</td>
</tr>
<tr>
<td>Covariance (Consols, Equities)</td>
<td>0.03</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>Covariance (Consols, Property)</td>
<td>0.07</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>Covariance (Consols, Liability)</td>
<td>0.09</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>Covariance (Index-linked, Equities)</td>
<td>0.04</td>
<td>0.09</td>
<td>0.07</td>
</tr>
<tr>
<td>Covariance (Index-linked, Property)</td>
<td>0.13</td>
<td>0.11</td>
<td>0.04</td>
</tr>
<tr>
<td>Covariance (Index-linked, Liability)</td>
<td>0.09</td>
<td>0.04</td>
<td>0.12</td>
</tr>
<tr>
<td>Covariance (Equities, Property)</td>
<td>0.08</td>
<td>0.09</td>
<td>0.08</td>
</tr>
<tr>
<td>Covariance (Equities, Liability)</td>
<td>0.06</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>Covariance (Property, Liability)</td>
<td>0.04</td>
<td>0.08</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Note. Results report the percentage change in the summary statistics of the output distribution for each simulation using 10000 iterations.
A.4.4. Distribution of Ultimate Surplus \((r=5)\)

![Probability distribution for Ultimate Surplus with \(r=5\)].

A.4.5. Distribution of Ultimate Surplus \((r=10)\)

![Probability distribution for Ultimate Surplus with \(r=10\)].
A.4.6. Distribution of Ultimate Surplus ($r=15$)

VALUES IN TEN MILLIONS

A.4.7. Distribution of Ultimate Surplus ($r=20$)

VALUES IN TEN MILLIONS
A.4.8. Distribution of Ultimate Surplus ($r=25$)

A.4.9. Distribution of Ultimate Surplus ($r=30$)
REFERENCES


Chadburn, R.G. and Wright, I.D. 1999. The Sensitivity of Life Offices Simulation Outcomes to Differences in Asset Model Structure. *Actuarial Research Paper* (City University): No. 120.


