MODAL EXTENSIONAL MEREOLOGY WITH AN APPLICATION TO SOCIAL GROUPS

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Modal Extensional Mereology with an Application to Social Groups

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Abstract

How does Classical Extensional Mereology (CEM) interact with time and modality? In its first-order logical setting, CEM does not have the formal tools to deal with these dimensions. Nonetheless, it is often charged with endorsing controversial metaphysical theses like mereological essentialism and mereological constantism. Allowing CEM to interact with quantified modal logic (QML) without adding any further mereological commitment clarifies which modal theorems CEM actually implies and eventually undercuts wrong-headed allegations against it. From the logical side, I endorse system *KTB* of modal logic with constant domain semantics, but several theorems are also provable with the adoption of weaker logics (like *KT* and even the simplest *K*). We will also be able to successfully simulate variable domain semantics *via* the procedure of existential relativisation. From the mereological side, I necessitate the ordering axioms and the strong-supplementation axiom, leaving the door open for four different alternatives to deal with the axiom of unrestricted composition. The mereological results emerging from this setting include the necessity of extensionality principles – which does not lead to mereological essentialism – and the existence of possible fusions even when an actualist version of unrestricted composition is adopted.

In the final part of the thesis, I apply the main findings of modal extensional mereology to group membership. After showing why some standard objections are misguided, I argue that the standard account of social groups based on CEM ($\mathrm{CEM}_{\mathrm{SG}}$) fails to be satisfactory because it conceives group membership as *just* parthood. My proposal is to regard group membership as φ-parthood instead. Under this new account (φ-$\mathrm{CEM}_{\mathrm{SG}}$), social groups are not *just* mereological wholes, but mereological wholes with a specified understanding of the parthood relation. Endorsing φ-$\mathrm{CEM}_{\mathrm{SG}}$ allows us to retain an account of social groups based on CEM while avoiding the standard objections $\mathrm{CEM}_{\mathrm{SG}}$ faces.

Contents

Introduction

Although mereology as a discipline is arguably as old as philosophy, it was not until the last century that the term effectively entered the philosophical jargon. The term 'mereology' emerged within the work of Stanisław Leśniewski and its literal meaning is 'the study of parts' (from the Greek $\mu\acute{\epsilon}\rho o\varsigma$ = part and -$\lambda o\gamma\acute{\iota}\alpha$ = study, theory). In short, mereology is the philosophical discipline investigating the relationship between (i) a whole and its parts and (ii) the different parts of a whole. Despite being relatively recent as a term, the part-whole relation has always been regarded as a fundamental ontological relation carving – or amalgamating – nature at its joints. From the ancient times (with philosophers like Parmenides, Democritus, Plato, and Aristotle), to the medieval (Abelard, Thomas Aquinas, Duns Scotus, among the others), until the modern ones (Leibniz and Kant especially), mereology has been characterised mainly by an informal and unsystematic treatment. They had neither received a systematic study, i.e., studied independently of a larger metaphysical framework, nor formally treated until the beginning of the 20^{th} century, when Edmund Husserl and Stanisław Leśniewski shook things up. On the one hand, Husserl considered mereology to be a fundamental piece of the puzzle of formal ontology; on the other, Leśniewski regarded it as an alternative to set theory through which mathematics could be given a more solid foundation. Their starting points, approaches, and aims were undeniably very different, but mereology started to be given a more systematic study thanks to their pioneering work. Despite the undeniable debt that mereology owes to Husserl, it is only with the work of Leśniewski and, subsequently, Alfred Norton Whitehead, Alfred Tarski, Henry Leonard and Nelson Goodman that mereology can be fully considered a formal theory.

This thesis deals with one particular mereological theory known as 'Classical Extensional Mereology' (CEM). CEM is a famous – if not the most famous – mereological theory on the market. Three main principles characterise it:

> EXTENSIONALITY: if two composite things have the same proper parts, i.e., parts that are not identical with the whole they are part of, then they are identical.
>
> UNIVERSALISM: for any things satisfying any condition whatsoever, there is something composed by them.
>
> TRANSITIVITY: given three different things x, y, and z, if x is part of y and y is part of z, then x is part of z.

Although the emergence of non-classical mereologies in recent years has vastly challenged CEM, which used to be seen as 'perfectly understood, unproblematic, and certain' (Lewis, p. 75), it still represents a solid and influential theory of mereology.

The driving force behind this dissertation is twofold. On the one hand, first-order logic is the standard underlying logical apparatus of CEM, which is not, therefore, equipped with the formal tools to deal with the modal and temporal dimensions. On the other, it is often *assumed* in the literature that CEM implies three theses: mereological essentialism, mereological constantism, and composition as identity. Investigating the actual situation, CEM's behaviour, as well as its implications in a modal scenario constitute the substantive aims of this dissertation.

Chapter 1 offers a general introduction to mereology and, more specifically, the mereological theory I endorse (CEM). In the first section, I provide some basic coordinates about mereology as a discipline; in the second, I present and briefly discuss the axiomatisation of CEM in its usual logical setting, namely first-order logic. In **Chapter 2**, I discuss whether it is possible to embed temporal and modal claims within CEM. It is universally acknowledged that first-order logic does not have the formal tools to deal with issues concerning time and modality, so some sort of action is required if we want to make CEM work within a modal or temporal setting. In light of that, I address two distinct yet correlated issues. Firstly, I argue that it is logically wrongheaded to regard CEM as committed to either mereological essentialism, mereological constantism, and composition as identity. Secondly, I follow Cotnoir and Varzi (2021) and consider three possible strategies to implement CEM with temporal and modal claims: four-dimensionalism, increased arity, and quantified modal logic (QML). The rest of the chapter evaluates each strategy by pointing out their respective pros and cons. The aim is to show by the end of the chapter that the third option, QML, is the best among the three.

The heart of this thesis lies in **Chapter 3**. By changing the logical setting of CEM and endorsing QML, we are required to take a stance at least on two main issues: the domain of quantification and the properties of the accessibility relation. For reasons I explain in the first section of this chapter, I endorse system *KTB* of modal logic with constant domain semantics. The subsequent two sections deal with the logical and mereological consequences of this system. As regards the logical consequences, I will discuss the Barcan Formula and its Converse, the issue of mere *possibilia*, the necessity of existence and the necessity of identity. Changing the underlying logical setting of CEM is relatively new to the literature, but the logical consequences of system *KTB* with constant domain semantics are well-known. My original contribution comes in the third and last subsection of the chapter. There I discuss the mereological consequences deriving from the interaction with CEM and the underlying logical system. Here, too, we need to take a decision. In this case, the decision involves which mereological axioms (if any) to necessitate. My standpoint is that in order to obtain some interesting results we need to necessitate the ordering axioms – anti-symmetry included, besides the worries that such a decision might raise – and the strong supplementation axiom. One of the most relevant results that will emerge is that although the necessity of extensionality comes out as a theorem in the system, mereological essentialism is not provable. This formal result confirms the argument I informally put forward in the second chapter. As regards the last axiom of CEM, generally known as 'Unrestricted Composition' or 'Universalism', I will suggest that it does not seem wise to necessitate it. In the last subsection, I will present four alternative ways to deal with it in a modal scenario. I will show that significant results emerge if we endorse an existentially-relativised version of it and decide to necessitate it. One of the most relevant is that there are possible fusions of actual entities. That is, actual entities might compose a non-actual fusion even under a restricted version of the unrestricted composition axiom!

Chapter 4 bridges the previous three chapters with the last one, where I investigate a possible application of modal CEM to social groups. There I introduce and defend the notion of φ-parthood, where φ is the predicate modifier that narrows down the original interpretation of 'part' by adding further conditions. In **Chapter 5**, I present a novel account of social groups based on CEM. After introducing the general debate on the ontological status of social groups, I will argue that given what we have proven in the third chapter, $\mathrm{CEM}_{\mathrm{SG}}$ has been unfairly associated with the idea that a fusion (and, therefore, a social group) has 'its parts both essentially and permanently' (Hawley, 2017, p. 399). In other words, $\mathrm{CEM}_{\mathrm{SG}}$ implies neither mereological essentialism, nor mereological constantism, nor the composition as identity thesis. Nonetheless, there are well-founded, serious objections (most famously the transitivity objection and the coextensionality objection) $\mathrm{CEM}_{\mathrm{SG}}$ undeniably faces. I present them in the rest of the section. In the final section, I put forward my proposed account and compare it with the standard one. While $\mathrm{CEM}_{\mathrm{SG}}$ reduces group membership to be *just* parthood, the account I propose (φ-$\mathrm{CEM}_{\mathrm{SG}}$) stems from the idea that the group membership relation is a particular type of φ-parthood. In my view, social groups are *not just* mereological wholes, but mereological wholes with a specified understanding of the parthood relation. I will show that this novel account can turn down the usual objections faced by $\mathrm{CEM}_{\mathrm{SG}}$ and, therefore, that a successful account of social groups based on CEM is attainable.

Chapter 1

Classical Extensional Mereology (CEM)

As A. C. Varzi (1999) nicely put it, the construction of any mereological theory relies on two fundamental steps. The first deals with the minimal features that should characterise every good mereological theory. It has been widely recognised in the literature that parthood is a partial ordering,[1] namely a relation that is reflexive, antisymmetric and transitive, thus our theory should account of them. The second, trickier step requires the identification of more stringent principles that, once added to the most basic ones, will discriminate between the part-whole relation and the other relations, thereby leaving us with a satisfactory formal treatment of the parthood relation. These principles are generally called decomposition and composition principles. Our exposition of CEM will follow this mereological skeleton: basic axioms, supplementation, and composition principles.

1.1 Preliminaries

Consider the following scenarios:

AT THE CHECK-IN

GATE AGENT: I'm pleased to see that you are part of our loyalty program!

PASSENGER: Yes, I joined a couple of years ago and I am very satisfied.

GATE AGENT: Great to hear that. So, I have now attached your part of the luggage tag to your boarding pass. Keep it safe!

DURING THE CRUISE

PASSENGER: Could I have a vegetarian sandwich and a bottle of water, please?

FLIGHT ATTENDANT: Of course. Would you also like a box of crisps? They are part of the meal deal.

[1] These properties hold if parthood is taken as primitive. But if, for instance, proper parthood is chosen as primitive instead, then the relation is irreflexive, asymmetric, and transitive.

PASSENGER: Sure. Could I also ask you to bring a spoon if it's not part of the default tableware for this deal?

FLIGHT ATTENDANT: No problem.

DESCENDING

PILOT: Ladies and gentlemen, we have begun the descent into our destination. The last part of the flight might be shaky due to high winds. Please turn off all portable electronic devices and make sure your seat backs and tray tables are in their full upright position.

PASSENGER A: Flaps will be extended soon then.

PASSENGER B: What are you talking about? What are flaps?

PASSENGER A: Flaps are an important part of an aeroplane wing. They can either increase or decrease the wing's surface area depending on whether an increased lift or drag is needed.

PASSENGER B: I wish I could think about how flaps work instead of the possibility that we could all die soon.

There are three aspects of the use of 'part' in the above examples that is worth highlighting. First, the lexical item employed is the same regardless of the different parthood relations it picks out. For instance, consider the descending scenario. The relevant relationship in 'the last part of the flight' is that of temporal parthood, while in 'flaps are an important part of an aeroplane wing' the part is spatial. Importantly, different classifications of parthood are possible depending on the taxonomy we adopt (e.g., the flaps-wing relationship could also be classified as attached, functional, material, extended, etc., parthood). In the words of McDaniel (2010), 'ordinary language suggests that parthood is topic-neutral or at least highly general' (p.413) so that 'part' can be legitimately used across various contexts of utterance and despite the diversified ontological status of the *relata* involved. This feature of mereology is generally referred to as 'topic neutrality' (as in the quote above) or as 'ontological innocence' (as in the quote below):

> So I claim that mereology is legitimate, unproblematic, fully and precisely understood. All suspicions against it are mistaken. But I claim more still. Mereology is ontologically innocent. (Lewis, 1991, p. 81)

Adopting an Husserlian interpretation, a *formal* theory of parthood should be able to uncover the general principles underlying any parthood relation regardless of our preferred inventory of the world. The second element I want to highlight is that ordinary uses of 'part' (like those in the case scenarios presented above) bring along an implicit meaning that we can disentangle by adopting the mereological notion of 'proper parthood'. In other words, following the arithmetic distinction between less-than ($<$) and less-than-or-equal ($\leq$), mereologists distinguish between a proper sense of parthood, the one that is grasped by everyday usage of 'part', and an improper sense of parthood. We will shortly provide a formal discussion of these fundamental mereological concepts but, informally, we can say that the conceptual difference they bear is intuitively alike to the one between the two following sentences:

(P) The aeroplane is part of itself.

(PP) The wings are part of the aeroplane.

Evidently, (PP) can be unproblematically uttered in the relevant context, whereas (P) might strike many as odd. Despite its prima facie oddness, (P) is philosophically interesting and fundamental - in certain mereological systems, like the one I will present, it is also axiomatically fundamental. On this respect, it is worthwhile to highlight that, in order to put forward an axiom system for mereology, one has to decide which notion to treat as primitive, so that it is then possible to define other notions in terms of it. Taking proper parthood as fundamental is very common in the literature, but it is not mandatory. CEM can be axiomatised by taking different relations (such as parthood, overlap, and disjointness) as primitive without loss of substantive power. Given that mereological notions are inter-definable in CEM,[2] CEM can be axiomatised in different yet mereologically equivalent ways. Evidently, deciding which concept to treat as fundamental is an important choice that is not free of philosophical implications. Different philosophers adopt different strategies depending on their underlying philosophical commitments and this is likely to lead to formulations that are formally, but not philosophically, equivalent. Consider, for example, the different approaches adopted by Leśniewski and Leonard and Goodman in their Calculus of Individuals. The former took proper parthood as primitive – albeit he was aware that alternatives were available – whereas the latter assumed disjointness as fundamental.

The final remark I want to make concerns the fact that, although it might seem intuitive to attribute a temporal dimension to parthood, CEM notoriously regards parthood as atemporal (or timeless). Importantly, this is not the same 'permanent' or 'constant'. In the following chapters, we will realise the importance of keeping 'atemporal' distinct from 'permanent'.

1.2 Axiomatisation

The mereological theory defended by Leśniewski and Leonard and Goodman, viz. CEM, attracted the attention of philosophers for decades and generated a flux of work that now allows us to have a deep understanding of the theory.[3] The pivotal work of those we might dub 'first-wave mereologists' (such as Lesniewski and his disciples, Tarski above all, and Leonard and Goodman) is still very much alive in the minds of present mereologists. Regardless of how significantly things have changed since then, nowadays most mereological theories are still formulated in first-order predicate logic (FOL) and are presented axiomatically. For now, I shall follow suit. At this point, my primary concern is to expose CEM in the most neutral, customary and agreeable way possible. A firm ground will allow me, later on, to 'build upon it'. In outlining CEM, I will follow Cotnoir and Varzi (2021) both notationally[4] and axiomatically.[5] The axiomatisation I am about to present consists of a two-place predicate constant, P, standing for the parthood relation and, as we had already anticipated, a first-order predicate logic with identity, which we can translate into the following preliminary axiom:

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2 For example, it is possible to choose parthood as primitive and define proper parthood in terms of it and yet leave the mereological vigour of the system intact.

3 CEM has been studied in different settings – axiomatic, algebraic, and set-theoretic – giving rise to interesting interpretations of the parthood predicate. For an overview of the different approaches and a sketch of proofs showing their equivalence see Cotnoir and Varzi, 2021, §2.

4 The notational jungle characterising mereology is a well-known pebble in the shoe for mereologists, as it sometimes makes the work of other people less accessible. For a useful list of analogies see Simons (1987), in particular appendix 2.11

5 The only difference is the definition of fusion. Compare mine, D7, with theirs (Cotnoir and Varzi, 2021, p. 29).

A0. Any axiom system adequate for classical first-order logic with identity.

For the logical schemas underlying the classical predicate calculus, I refer back to Cotnoir and Varzi (2021), which in turn endorse the axiomatisation proposed by, among others, Mendelson (1964). The admitted rules of inference are *modus ponens* and universal generalisation. I refer back to Cotnoir and Varzi (2021, specifically §1.5) for all the other technical details too. The only element that, for clarity's sake, is worth explicit mention is the iota operator, ι. Given any wff φ and a variable x within the scope of a quantifier, $\iota x \varphi x$ abbreviates a definite description, i.e., a description that picks out one particular (or unique) individual, having linguistic form 'the x such that φx'. The iota operator ι not only allows us to logically represent definite descriptions like 'the present Queen of the United Kingdom' (aka Queen Elizabeth II, as I am writing in January 2022), but also more complex definite descriptions having logical form – following Russell's (1905) characterization – $\exists x((\varphi x \wedge \forall w(\varphi^x_w \to w = x)) \wedge \psi x)$. For instance, consider the sentence 'the present Queen of the United Kingdom loves Pembroke Welsh Corgis'. Assuming that Q stands for the predicate 'is a Queen of the United Kingdom' and L stands for 'loves Pembroke Welsh Corgis', the aforementioned sentence is symbolised as $\exists x((Qx \wedge \forall w(Qw \to w = x)) \wedge Lx)$. Through the iota operator, this can be abbreviated as $L(\iota x Qx)$.

1.2.1 Partial ordering

Bearing in mind that we take the parthood relation, P, as our primitive mereological concept, the following is the most basic set of axioms characterising CEM:

A1. Reflexivity: $\forall x Pxx$

A2. Antisymmetry: $\forall x \forall y((Pxy \wedge Pyx) \to x = y)$

A3. Transitivity: $\forall x \forall y \forall z((Pxy \wedge Pyz) \to Pxz)$

A1 affirms that for any x, x is part of itself (e.g., the carrot cake is part of itself). It forces the parthood relation to be reflexive, qualifying identity as a limit case of parthood. A2 affirms that for any x and y, they cannot be part of each other unless they are identical (e.g., the carrot cake I made and the carrot cake you made cannot be part of each other, unless the cake I made is the same cake you made). Parthood is then antisymmetric, but identity might still be symmetric. A3 affirms that for any x, y, ad z, if x is part of y, and y is part of z, then x is part of z (e.g., if the soft cheese is part of the icing, and the icing is part of the cake, then the soft cheese is part of the cake). Thus, in order to be a parthood relation, the candidate relation must be transitive. Following the terminology adopted in Casati and Varzi (1999), I shall label the mereological theory composed of A1-A3 **M**, or 'Ground Mereology' for long.

A1-A3 are collectively known as 'ordering axioms' and, mathematically, any relation satisfying them is called a *partial order*. Thus, parthood is a partial order.[6] Otherwise put, according to CEM (and many other mereological systems), any relation that fails to satisfy one or more of these properties, automatically fails to be a potential parthood relation. As Simons put it: 'these principles are partly constitutive of the meaning of 'part', which means that everyone who seriously disagrees with them has failed to understand the word' (Simons, 1987, p. 11).

[6] An important remark is that not all partial order relations are parthood relations. In order to be parthood relations, these axioms need to be supplemented.

Although this has been the standard approach for decades, several mereologists have recently questioned that ordering axioms lie at the very core of mereology, posing a number of challenges to CEM and developing 'non-classical' mereologies instead.[7] These are relevant issues that are indeed worth looking into, but which I will not discuss here. Given my present interest and the amount of space that would be needed to engage with those challenges, I will *assume* CEM is a correct mereological theory and, therefore, I will merely endorse its commitment to the ordering axioms. For a detailed and informative discussion of the ordering axioms see the third chapter of Cotnoir and Varzi (2021).

Given A1-A3, we are now in the position to introduce some other useful mereological relations.

D1. Proper Parthood: $PPxy :\equiv Pxy \wedge \neg(x = y)$

D2. Overlap: $Oxy :\equiv \exists z(Pzx \wedge Pzy)$

D3. Underlap: $Uxy :\equiv \exists z(Pxz \wedge Pyz)$

D4. Disjointness: $Dxy :\equiv \neg\exists z(Pzx \wedge Pzy)$

D1 tells us that x is a proper part of y just in case it is a distinct thing from y (e.g., the water is a proper part of the waterfall just in case they are not identical). Thisrelation is a *strict partial order*, namely a relation that is irreflexive, asymmetric, and transitive. D2 tells us that x and y overlap just in case there exists a part z they have in common (e.g., two meetings overlap just in case there is a temporal part[8] that they have in common). This relation is reflexive and symmetric but might fail to be transitive. D3 tells us that x and y underlap just in case there exists a certain z of which they are both parts (e.g., two strings underlap just in case the first string is part of a guitar and the second string is part of it too). As with the overlap relation, the underlap relation is a *tolerance relation*, viz., a relation that is reflexive and symmetric but not necessarily transitive. D4 tells us that x and y are disjoint just in case they do not have any part in common (e.g., the red pastel is disjoint from the blue pastel just in case there is no part they have in common). The disjointedness relation is symmetric and irreflexive. It is well-known that many more mereological concepts (such as proper overlap and proper underlap) could be defined – were someone in need of them – but D1-D4 will suffice for us.

1.2.2 Decomposition

Let's now take a step further in the axiomatisation of CEM. Imagine you are trying to complete a 1,000 pieces puzzle. Piece after piece, you manage to fully assemble it. After some time though, you decide to disassemble it because you want to do it again. In both cases you are, so to say, playing with mereology. By assembling the pieces you adopt an ascending (from parts to whole) mereological procedure, whereas by disassembling them you follow a descending (from whole to

[7] As it has been pointed out by Cotnoir and Varzi (2021) 'non-classic' might be given two different interpretations. A mereological theory might be classified as non-classic because it alters CEM axioms (first way) or because its underlying logic is not first-order predicate logic (second way). In the latter case, a philosopher may decide to go non-classic for logical rather than mereological reasons and, therefore, her mereological intuitions might not differ from CEM's ones. One interesting alternative that might be worth looking at is the 'non-wellfounded mereology' presented in Cotnoir and Bacon (2012). It rejects A2 and it is yet able to include every CEM model of parthood.

[8] To any endurantist that might be reading, sorry!

parts) one. Evidently, they are like two arrows going in opposite directions. The relata they connect are the same and yet they are different mereological operations. In mereology, we talk about 'composition' in the former case and about 'decomposition' in the latter. To have a better grasp of the relationship they bear to one another, consider van Inwagen's Special Composition Question (SCQ):[9]

'When is it true that $\exists y$ the *xs* compose y?' (1995, p.30)

and compare it with the following question, which we might analogously label 'Special Decomposition Question' (SDQ):

'When is it true that there are some *xs* such that y is composed of them?'.

After the ordering axioms, the canonical CEM axiomatizations proceed by fleshing out the decomposition (or supplementation)[10] axioms, which are designed to grasp the intuitive idea that if a certain y has a proper part x, then it must have another one z. In fact, consider the following Hasse diagram:

Figure 1: An unsupplemented model.

It seems counter-intuitive that something has only one proper part. After all, if that was the case, why wouldn't they be the same thing? In the words of Simons,

> How could an individual have a *single* proper part? That goes against what we mean by 'part'. An individual which has a proper part needs other parts in addition to *supplement* this one to obtain the whole. (1987, p.26, author's emphasis)

The following supplementation principle is a standard candidate when outlining CEM:

A4. Strong-Supplementation: $\forall x \forall y(\neg Pxy \rightarrow \exists z(Pzx \land Dzy))$.

A4 affirms that if x is not a part of y, then there exists a further part of x that is disjoint from y (e.g., the wine is a proper part of a bottle of wine because it has a remainder, namely the glass bottle). Intuitively, a remainder z is what is left of x once y is 'removed' and, if we look back at A4, we will notice that the condition of there being a remainder depends on the antecedent $\neg Pxy$.

Following again Casati and Varzi (1999), adding A4 to A1-A3 gives us **EM**, or Extensional Mereology for long. The extensional predication derives from the implication that A4 prohibits two distinct things to have the exact same proper parts which, alternatively put, means that they cannot be coextensional. A4 is labelled 'strong supplementation' because among other supplementation principles available, there is one known as 'weak supplementation':[11]

[9] Even though van Inwagen presents the SCQ in relation to material objects, the formulation is general enough to allow for application also outside the domain of material objects and, thus, to be compatible with the alleged topic-neutrality of CEM.

[10] The label 'supplementation principles' was introduced in the literature by Simons (1987).

[11] See Cotnoir and Varzi (2021) for an useful overview of the supplementation principles available.

T1. Weak-Supplementation: $\forall x \forall y(PPxy \rightarrow \exists z(PPxy \wedge Dzx))$.

It is well-known that the mereological theory emerging from the addition of T1 to A1-A3 is much weaker than EM, but there are other ways to obtain CEM without having to renounce to T1. For instance, one can add the following principle to the axiom system:

T2. Proper Parts: $\forall x(\exists z(PPzx) \wedge \forall y(\forall z(PPzx \rightarrow PPzy) \rightarrow Pxy))$.

This move has been famously presented in Simons (1987, p. 28-29), who has also proven that both T1 and T2 are derivable as theorems from A4.[12] However, A4 seems more convenient to adopt since it is capable to do on its own what only the joint work of T1 and T2 can do. For example, A4 autonomously rules out the following model, whereas T1 needs the additional help of T2 to do so:

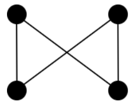

Figure 2: A 'butterfly' model violating T2.

But there's more. There are a bunch of philosophically interesting theorems besides T1 and T2 that can be derived from A4: the no-zero theorem and the so-called 'extensionality principles'.

As Cotnoir and Varzi (2021, p. 26) underline, given $\neg Pxy$ in A4, either (i) y is a proper part of x, or (ii) y is not a proper part of x but the two overlap, or (iii) y and x are completely disjoint.

Mimicking set-theory, we can then provide the following mereological definition:

D5. Difference: $x - y :\equiv \iota z \forall w(Pwz \leftrightarrow (Pwx \wedge Dwy))$.

At this point, one might wonder what the remainder could be if we take our x to be the Universe (U), which is, by definition, the overarching Whole that leaves nothing outside of itself. On this respect, an important remark is needed: the universe of metaphysics should not be confused with the universe of natural sciences. In the words of Aristotle:

> There is a science which studies Being qua Being, and the properties inherent in it in virtue of its own nature. This science is not the same as any of the so-called particular sciences, for none of the others contemplates Being generally qua Being; they divide off some portion of it and study the attribute of this portion, as do for example the mathematical sciences. (*Meta.*VI.1 1003a21-26)

Philosophy is the science of the Being qua Being ((τὸ ὂν ᾗ ὂν), whereas natural sciences study a portion of it. Thus, roughly, the domain of inquiry of the former is larger than that of the latter. Making them coincide is committing oneself to 'physicalism', namely the *metaphysical* thesis according to which everything supervenes on the physical – thereby reducing U to the world of physics. Physics can tell us a lot about the universe insofar as it can be studied through its (powerful) logical-mathematical tools, but it cannot tell us anything that eludes the study of the structure of the physical universe, *pace* Hawking.[13] After this brief yet important digression, we are

[12] There are other possible axiomatisations of CEM adopting T1 as an axiom. Two famous ones are the so-called second and third way in Hovda (2009).

[13] See, for instance, Hawking and Mlodinow(2008), where a full-fledged physicalist view is assumed.

back to the question: does the Universe have a remainder? Given that it is the all-encompassing thing which leaves nothing out of itself, it seems intuitive to say that it has nothing as a remainder. But here, two problems arise. First, if such a remainder exists, what is its ontological status? Second, if such a remainder does not exist, how can we talk about its non-existence? Rather than giving the coordinates of the discussion around the thorny issue of atomism, I shall go straight to the solution that CEM provides. Within our framework, the following theorem can be derived:

T3. No Zero: $\exists x \exists y \neg x = y \rightarrow \neg \exists x \forall y Pxy$.

T3 rules out the existence of a 'bottom' or 'null' mereological element that is part of everything[14] and that we can define as:

D6. Null object: $n :\equiv \iota z \forall y(Pxy \wedge \neg PPyz)$.

In this regard, I shall mention, *en passant*, that the issue of atomism is closely related to the existence or nonexistence of such an object, but I am not going to discuss it. Instead, I would like to point out another crucial feature of CEM: that it is a complete Boolean algebra with zero deleted. And the fact that no null object is admitted by CEM is the most striking difference between CEM and a complete Boolean algebra.[15] As Tarski famously wrote:

> The formal difference between mereology and the extended system of Boolean algebra reduces to one point: the axioms of mereology imply (under the assumption of the existence of at least two different individuals) that there is no individual corresponding to the Boolean-algebraic zero, i.e. an individual which is a part of every other individual. (Tarski, 1935, p. 333, fn. 4)

The second bit of A4-derived theorems are the extensionality principles, namely those principles that, by paralleling the extensionality principle of set theory, rule out the possibility of two distinct coextensional objects, unless they are identical. The following is arguably the most famous extensionality principle:

T4. PP-Extensionality: $\forall x(\exists w PPwx \rightarrow \forall y(\forall z(PPzx \leftrightarrow PPzy) \rightarrow x = y))$.[16]

T4 states that if two composite things x and y have all the same proper parts, then they are identical. It might seem that T4 and T2 bear some similarities, and rightly so. In the presence of A2, T2 is equivalent to T4. T4 expresses a mereological thesis that is as constitutive of CEM

[14] Proof: Suppose there is such a null element, which we shall label n. n is such that for all y, Pny. Now, if y has a remainder, i.e., a certain x disjoint from n, by A4 we know that $\neg Pny$ and n could only be a proper part of y (thereby, $PPny$), as it cannot be disjoint from y. But if n is a proper part of y, this means that y has a remainder, and this cannot be the case as nothing can be disjoint from the bottom element. At this point, we are left with two possibilities: first, ruling out the null element; second, rejecting A4. The latter is not a viable option for those who endorse A4 (and any CEM theory more in general), so we are left with the former and are therefore forced to exclude the existence of a mereological zero.

[15] See Cotnoir and Varzi (2021, §2.2) for a detailed analysis of the interaction between the two.

[16] Proof:

(1)	$\forall x(\exists w PPwx \wedge \forall y(\forall z(PPzx \rightarrow PPzy) \rightarrow Pxy))$	instance of T2
(2)	$\forall x(\exists w PPwx \wedge \forall y(\forall z(PPzy \rightarrow PPzx) \rightarrow Pyx)$	instance of T2
(3)	$\forall x(\exists w PPwx \rightarrow \forall y(\forall z(PPzx \leftrightarrow PPzy) \rightarrow Pxy \wedge Pyx)$	FOL, 1, 2
(4)	$\forall x \forall y((Pxy \wedge Pyx) \rightarrow x = y)$	A2
(5)	$\forall x(\exists w PPwx \rightarrow \forall y(\forall z(PPzx \leftrightarrow PPzy) \rightarrow x = y))$	FOL, 3, 4

as controversial, and we will get a taste of its problematic nature in the following chapters. Other relevant extensionality principles we can derive are:

T5. P-Extensionality: $\forall x \forall y(\forall z(Pzx \leftrightarrow Pzy) \rightarrow x = y)$.[17]

T6. O-Extensionality: $\forall x \forall y(\forall z(Ozx \leftrightarrow Ozy) \rightarrow x = y)$.[18]

T5 states that if two composite things are improper parts of each other, then they are identical, and T6 states that if two composite things overlap all the same things, then they are identical.

1.2.3 Composition

After discussing SDQ, we now move on to SCQ. This means, in practice, that we will be looking at bottom-up mereological operations which will allow us to present a full-fledged axiomatisation of CEM.

The most general intuition underlying composition principles is implicit in SCQ and might be expressed in slogan form as 'composition seems to occur'. Leaving questions of vagueness aside, we can add some milk to our espresso and get a macchiato, or we can add our signature to a document and it becomes a *legally-binding* proof of us getting married or taking out a loan. These seem to be instances of something being created via composition, i.e., not existing before some x and some z were put together. CEM naturally shares this intuition, but goes further. Proponents of CEM also share the intuition that 'the assumption of unique products appears plausible – if two objects overlap, why should there not be a maximal common part?' (Simons, 1987, p.30).

On these grounds, we can begin looking into what CEM's answer to SCQ is. First, we need a mereological definition of composition[19] (generally called 'fusion'):

D7. Fusion: $F\varphi z :\equiv \forall y(Oyz \leftrightarrow \exists x(\varphi x \wedge Oyx))$.

[17] Proof:

(1)	$\forall x \forall y \forall z((Pzy \wedge Pyx) \rightarrow Pzx)$	instance of T3
(2)	$\forall x \forall y \forall z((Pzx \wedge Pxz) \rightarrow x = z)$	instance of T2
(3)	$\forall x \forall y \forall z(((Pzx \wedge Pxy) \wedge Pyz) \rightarrow y = z)$	FOL, 1, 2
(4)	$\forall x \forall y \forall z((Pzx \wedge Pxy) \rightarrow Pzy)$	instance of T3
(5)	$\forall x \forall y \forall z((Pzy \wedge Pyz) \rightarrow y = z)$	instance of T2
(6)	$\forall x \forall y \forall z(((Pzy \wedge Pyx) \wedge Pxz) \rightarrow x = z)$	FOL, 4, 5
(7)	$\forall x \forall y \forall z(((Pzx \wedge Pxy) \rightarrow y = z) \wedge ((Pzy \wedge Pyx) \rightarrow x = z)$	FOL, 3, 6
(8)	$\forall x \forall y \forall z(((Pzx \wedge Pxy) \wedge (Pzy \wedge Pyx)) \rightarrow x = y)$	FOL, 3, 6
(9)	$\forall x \forall y \forall z(((Pzx \rightarrow Pzy) \wedge (Pzy \rightarrow Pzx)) \rightarrow x = y)$	FOL, 1, 4, 8
(10)	$\forall x \forall y \forall z((Pzx \leftrightarrow Pzy) \rightarrow x = y)$	FOL, 9

[18] Proof:

(1)	$\forall x \forall y(\neg \exists (Pzx \wedge Dzy) \rightarrow Pxy)$	contrapositive instance of A4
(2)	$\forall x \forall y(\neg \exists (Pzy \wedge Dzx) \rightarrow Pyx)$	contrapositive instance of A4
(3)	$\forall x \forall y(\neg \exists z((Pzx \wedge Dzy) \wedge (Pzy \wedge Dzx)) \rightarrow Pxy \wedge Pyx)$	FOL, 1, 2
(4)	$\forall x \forall y((Pxy \wedge Pyx) \rightarrow x = y)$	A2
(5)	$\forall x \forall y(\neg \exists z((Pzx \wedge Dzy) \wedge (Pzy \wedge Dzx)) \rightarrow x = y)$	FOL, 3, 4
(6)	$\forall x \forall y(\forall z((\neg Pzx \vee \neg Dzy) \vee (\neg Pzy \vee \neg Dzx)) \rightarrow x = y)$	FOL, 5
(7)	$\forall x \forall y(\forall z((\neg Pzx \vee (Pxz \vee Pxy)) \vee (\neg Pzy \vee (Pyz \vee Pyx))) \rightarrow x = y)$	D4, 6
(8)	$\forall x \forall y(\forall z((Pzx \wedge Pzy) \leftrightarrow (Pzy \wedge Pzx)) \rightarrow x = y)$	FOL, 7
(9)	$\forall x \forall y(\forall z(Ozx \leftrightarrow Ozy) \rightarrow x = y)$	D2, 8

[19] There are several definitions of fusion available in the literature besides the one I am adopting. See Cotnoir and Varzi (2021, §5) for an overview and critical analysis of the alternative definitions.

D7 says that z is the fusion of every x satisfying a certain φ condition ($F\varphi z$) such that those x are overlapped by every y that also overlaps z. In this formulation, φ is any wff with x free and, importantly, φ is left unspecified, viz., no stance is taken on which conditions, if any, the xs must satisfy. We will see in a few lines how CEM implements it but, for now, let's present the consequences of D7. Together with A2 and a theorem of overlap we shall call 'O-Supervenience' ($\forall x \forall y(\forall z(Ozx \to Ozy) \to Pxy)$),[20] D7 validates the following theorem:

T7. Fusion Uniqueness: $\forall z \forall w((F\varphi z \wedge F\varphi w) \to z = w)$.[21]

T7 states that if two composite things are fusions of all the same things satisfying a certain φ, then they are identical.

It rules out, for example, the following model:

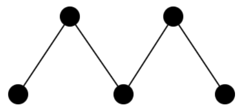

Figure 3: A model violating T7.

Although the uniqueness of fusions is a core characteristic of CEM, there are several ways in which it can be stated depending on the definition of fusion that is chosen. For instance, D7 is entirely defined in terms of overlap, but there are many other formulations available that have a closer affinity with algebraic notions like 'upper bound', 'least upper bound', etc.[22] Leaving this issues aside, we are in a position to provide a definition of unique fusion given D7 and T7:

D8. Unique Fusion $\sigma x \varphi x :\equiv \iota z F\varphi z$.

At this point, we have all the elements to present CEM's answer to SCQ: composition occurs under *any* conditions. In formal terms, the last axiom of CEM can be given the following formulation:

A5. Unrestricted Composition : $\exists x \varphi x \to \exists z F\varphi z$.

This mereological thesis is generally known as *unrestricted composition* or *universalism* and affirms that for *any* φ condition, as long as there is something satisfying it, there exists a fusion of those things satisfying that φ. I would like to pause here to highlight a couple of features of A5. First, A5 is not technically an axiom but rather an axiom *schema*, viz. an infinite set of axioms

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[20] I refer back to Cotnoir and Varzi (2021, p.110) for a proof of this theorem.

[21] Proof:

(1)	$\forall y(Oyz \leftrightarrow \exists x(\varphi x \wedge Oyx))$	D7
(2)	$\forall y(Oyw \leftrightarrow \exists x(\varphi x \wedge Oyx))$	D7
(3)	$\forall z \forall w \forall y(Oyz \leftrightarrow Oyw)$	FOL, 1, 2
(4)	$\forall x \forall y(\forall z(Ozx \to Ozy) \to Pxy)$	O-Supervenience
(5)	$\forall z \forall w(\forall y((Oyz \leftrightarrow Oyw) \to Pwz \wedge Pzw)$	FOL, 3, 4
(6)	$\forall z \forall w((Pwz \wedge Pzw) \to z = w)$	A2
(7)	$\forall z \forall w(\forall y((Oyz \leftrightarrow Oyw) \to z = w)$	FOL, 5, 6
(8)	$\forall z \forall w(\forall y(Oyz \leftrightarrow \exists x(\varphi x \wedge Oyx)) \wedge \forall y(Oyw \leftrightarrow \exists x(\varphi x \wedge Oyx)) \to z = w)$	FOL, 1, 2, 7
(9)	$\forall z \forall w((F\varphi z \wedge F\varphi w) \to z = w)$	D7, 8

[22] I will not dig deeper into the matter, but see Cotnoir and Varzi (2021, §5.1) to have a good grasp of the various alternatives and the relationship between them.

stating that, given a specifiable subset of xs, there is always something overlapping anything they overlap. Second, endorsing the idea that no conditions should be imposed on composition, i.e., that no limitations are imposed on what conditions φ might be filled with, is most often seen as a *very* problematic – if not ravaging – feature of CEM. Following the tradition of examples aiming at accentuate how bizarre fusions could be if A5 is accepted, let φ be the following disjunctive formula: $\sigma x(x = a \vee x = b)$, where a = Donald Trump and b = the four apples I bought this morning.[23] By A5, we are forced to admit that there exist a (unique) fusion composed by Donald Trump and the four apples I bought this morning, and this strikes many as odd, if not simply unacceptable. Accordingly, let φ be the following conjunctive formula: $\sigma x(Pxa \wedge Pxb)$. A5 guarantees that there is a fusion of the product of a = Donald Trump and b = Nancy Pelosi. (Consider, for instance, the product of Donald Trump × Capitol riot and Nancy Pelosi × Capitol riot, namely Capitol riot.)

Besides talking of the sum ($\sigma x(x = a \vee x = b)$) and the product ($\sigma x(Pxa \wedge Pxb)$) of a and b, there are two further ramifications of A5 that it is worth pointing out. First, if we take φ to be '$\exists yx = y$', then we are required to recognise that the Universe – the capital U universe we mentioned before – exists and that can be defined as follows:

D9. Universe: $u :\equiv \sigma x \exists yx = y$.

Second, let φ be 'Dxa'. Given A5, we can provide a definition of the fusion of everything that is disjoint from a certain object a:

D10. Complement: $-a :\equiv \sigma x Dxa$.

Importantly, it is not always the case that a complement exists. In particular, the complement of U does not exist – if there were one, the universe would have something outside itself and, therefore, would not be the maximal whole.

Coming back to the general picture, adding A5 to EM gives us a complete axiomatisation of Classical Extensional Mereology (**CEM**). Such axiomatisation – characterised by A1-A5 – corresponds to the so-called 'System 1' presented in Hovda (2009). It is arguably one of the most famous axiomatisations through which CEM is generally presented. However, it has been recently pointed out that this system suffers from redundancy: A1 needs not to be added in the axiom system as it is derivable as a theorem from A3 and A4 (A. C. Varzi, 2019).[24] Thus, the following is our final axiomatisation of CEM:

A2 **Antisymmetry**: $\forall x \forall y((Pxy \wedge Pyx) \rightarrow x = y)$
A3 **Transitivity**: $\forall x \forall y \forall z((Pxy \wedge Pyz) \rightarrow (Pxz))$
A4 **Strong-Supplementation**: $\forall x \forall y(\neg Pxy \rightarrow \exists z(Pzx \wedge Dzy))$
A5 **Unrestricted Composition**: $\exists x \varphi x \rightarrow \exists z F \varphi z$

[23] For the general formula, a and b must be singular terms.

[24] As a matter of fact, Pietruszczak (2018) has been the first to propose such amendment.

Chapter 2

CEM, Time, and Modality

In *Down and Out in Paris and London*, George Orwell's memoir of the time he spent among the penniless during his mid-twenties, he writes:

> It is worth saying something about the social position of beggars, for when one has consorted with them, and found that they are ordinary human beings, one cannot help being struck by the curious attitude that society takes towards them. People seem to feel that there is some essential difference between beggars and ordinary 'working' men. They are a race apart – outcasts, like criminals and prostitutes. Working men 'work', beggars do not 'work'; they are parasites, worthless in their very nature. It is taken for granted that a beggar does not 'earn' his living, as a bricklayer or a literary critic 'earns' his. He is a mere social excrescence, tolerated because we live in a humane age, but essentially despicable. (George Orwell, *Down and Out in Paris and London*, p.)

Take George Orwell. In his life, he has been both a 'beggar' and an 'ordinary working man'. Alternatively put, he has been – is? – a proper part of two distinct social groups: houseless people,[25] on the one hand and ordinary working people on the other. These groups are distinct, but are they entirely disjoint? Well, no. Arguably, these two social groups partially overlap (not every beggar has been or will be an ordinary working man, and vice versa), and George Orwell is only one of the parts they have in common. My language already reveals the direction the discussion is leading to. By A5, we must recognise the existence of a fusion of all x satisfying the φ-condition 'being a beggar and being an ordinary working man'. But, it seems legitimate to ask, what are their temporal and modal features if we want to give them a flesh-and-bone characterisation? In fact, we need to know more about these two fusions: Do they change over time? Could they have been composed by different members? Are they different fusions if membership is altered? These questions refer to more general issues concerning time, modality, and identity. I will start discussing them in this chapter.

[25] Although it is common practice to refer to someone lacking permanent housing as 'homeless', in what follows, I shall refer to them as 'houseless' instead. Very shortly, there are two main interrelated reasons I decided to discard the usual terminology. First, it is a sign of respect towards them not to *assume* that they do not have a home merely because they do not have a house. Second, even from a theoretical point of view, 'home' is a sociological concept that should not be downgraded to 'house'. Although there is no agreement among sociologists about how 'home' is or should be understood (see Mallett (2004) for an overview of the numerous positions on the market), they seem to agree that 'home' is a much richer, socially and existentially loaded concept. Of course, the two concepts might overlap, but someone may have a house without a home.

2.1 How to embed temporal and modal claims?

Let us begin by recollecting CEM's axiomatisation. Would we have a formal apparatus suitable for temporal and modal claims if we wanted to provide an answer to the questions above? Unfortunately, no. As things stand now, we are forced to remain silent about these issues because we do not have the tools to deal with them. Such inadequacy is due to inherent features of CEM stemming from its very first, preliminary axiom that rules out any temporal or modal operators (first-order predicate logic does not come with any) and from mereological considerations concerning the parthood relation, conceived as atemporal and topic-neutral. It is possible to understand why CEM has certain features and not others it if we look back at its historical roots but, however interesting, that would not be relevant for our discussion. What I want to look at here are the strategies we can adopt to overcome this original incompetence of CEM to deal with issues concerning time and modality. Cotnoir and Varzi (2021) identify three possible ways to embed temporal and modal claims into CEM.[26] We can sketch them as follows:[27]

> FOUR-DIMENSIONALISM: ordinary material objects, like tables, trees and human beings, are not only composed by spatial parts, but also by *temporal or modal parts*.
>
> INCREASED ARITY: the logical form of parthood is not two-placed (part, whole), but *three-* (part, whole, time) or *four-placed* (part, whole, time, world).
>
> QUANTIFIED MODAL LOGIC: the formal apparatus through which CEM is given, i.e., the first-order predicate logic enclosed in A0, must be substituted with one allowing for a *quantified modal (and/or temporal) logic*.

Regardless of our chosen strategy, however, there is a price to pay. Mimicking the proverb 'You can't earn stone palaces by honest labour' in Lev Tolstoy's *Too Dear!* short story, we might say that you can't get a satisfying modal or temporal mereology by innocent ontology. In other words, while there is room to defend the ontological innocence of CEM – albeit not without controversy – an upgraded, modality-and-time-friendly version of it would be hopelessly committed in any case. I shall anticipate now that (i) my focus will be on modality and that (ii) the strategy I will be endorsing is the final one. Before getting there, though, a non-negligible evaluation is in place: why do I think replacing first-order predicate logic with a quantified modal one is the best option we have? In the next three sections, I will present and discuss the former two strategies and eventually explain why I endorsed the latter.

2.2 Does CEM imply any particular temporal or modal claims?

Before discussing the strategies on the market, it might be useful to have a preliminary discussion on the temporal and modal commitments displayed by CEM regardless of any approach that might

[26] There are also other strategies available in the literature (see, for instance, Haslanger (2003), Wasserman (2004, 2006), Gallois (2016)). However, those presented by Cotnoir and Varzi (2021) seem to me the most suited in our circumstances.

[27] The exposition of these strategies will be quite coarse-grained and, possibly, unsatisfactorily. For a longer and more in-depth analysis of these two strategies, see Cotnoir and Varzi (2021, §6.2), which has been my main reference source for this chapter.

be adopted. What we will discuss in this section will serve us to have a better grasp of which theses (if any) are *actually* implied by CEM.

When in a temporal or modal framework, CEM is very often associated with one or more of the following theses:[28]

> MEREOLOGICAL ESSENTIALISM (ME): the parts composing a whole compose it necessarily, i.e., if a whole is composed by a certain part, it is composed by it in every possible world.
>
> MEREOLOGICAL CONSTANTISM (MC): the parts composing a whole compose it permanently, i.e., if a whole is composed by a certain part, it is composed by it at every time at which the whole exists.
>
> COMPOSITION AS IDENTITY (CAI): a whole and its parts are identical, so that a whole is nothing over and above the parts that constitute it.

ME and MC are 'extremist' mereological theses that have been extensively debated in the literature. In contemporary analytic philosophy, they are generally associated with R.M. Chisholm. CAI is presented here in its most general formulation. However, it is important to stress that there are different versions, depending on the relevant interpretation of the identity relation. Following Cotnoir (2014), we can identify three varieties of CAI: (i) Weak CAI, which argues that the relationship between a whole and its composing parts is like identity; (ii) Moderate CAI, which affirms that the relationship between a whole and its composing parts is non-numerical identity; and (iii) Strong CAI, which says that the relationship between a whole and its composing parts is numerical identity.

As I have already anticipated, my focus will be on modality so that ME will be adequately discussed in the next chapter. Nonetheless, what we will say about ME could be easily transposed in temporal terms so that our findings will virtually apply to MC too. Without anticipating too much, I want to point out the following: *none* of the metaphysical theses above is *implied* by CEM. One might wonder why this is a relevant preliminary point. The answer is that by being almost always combined with some (or all) of them, CEM is often mistakenly taken to imply them. However, that is not the case. The axiomatisation of CEM we presented does not contain any element that might *logically* force one to endorse ME, MC, and CAI – and this is why this chapter deals with *ways* in which we can implement it with temporal and modal notions. Were there to be a modal or temporal thesis implied by CEM, we would not talk of four-dimensionalism, increased arity, and quantified modal logic as possible strategies to embed temporal and modal considerations. Our task would rather be to evaluate which strategy better grasps, or is more suited to, ME, MC, and CAI. That is not to suggest that all possible theses are on par. Some theses might be, in effect, more suited – i.e., more coherent – to CEM's principles, but this depends on philosophical reasons stemming from a *conceptual* rather than logical analysis. There might well be *metaphysical* reasons for doing so, but, strictly speaking, they fall outside the domain of what is *logically* required, which is made clear by the impossibility of stating any of them through the basic vocabulary of CEM. First-order predicate logic is not equipped with formal elements that would allow us to deal with temporal and modal issues. That is something I cannot stress enough. By undergoing a logical analysis of CEM, we would never encounter an element that could force us to reject or accept a particular modal or temporal thesis. We might well be able to formulate

[28] Hovda (2013) recognises that.

one, say MC, in all of them regardless of the different interpretative frameworks, but this by no means implies that it is implicit in CEM's formulation. Once this crucial point is clarified, we can look at the different strategies mentioned above. Let us start with four-dimensionalism.

2.3 Four-dimensionalism

According to four-dimensionalism, spatially located things are four-dimensional entities composed of spatial and temporal parts. This *ontological* thesis is sometimes confused or assumed to be tantamount to the compatible, yet distinct, *explanatory* thesis of perdurantism. The latter, but not the former, affirms that the temporal parts composing a certain material object are what makes it persist. The substantial difference between them is that besides being committed to the existence of temporal parts – which is the ontological claim of four-dimensionalism – perdurantism further argues that things persists *because* they have temporal parts.[29] Historically, perdurantism has been the perfect ally of CEM – endorsed, among the others, by Quine and Lewis – but nowadays four-dimensionalism appears to be preferred, especially given its compatibility with other theories of persistence, e.g., stage theory.[30]

The idea behind this first strategy is straightforward. It stems from the intuition that a temporal part is just like any other part and should, therefore, receive the same treatment.[31] In other words, the intuition is that we can deal with temporal issues by treating temporal parts as time-relativised parts. Consider the following definition given by Sider (2001):

> x is an *instantanous temporal part* of y at instant $t =_{df}$ (1) x is a part of y; (2) x exists at, but only at, t; and (3) x overlaps every part of y that exists at t. (p. 60, author's emphasis)

While it would not be possible to introduce a formal version of this definition within CEM's framework as we presented it, accommodations can be made. By introducing new predicates and axioms regimenting their behaviour, we are able to introduce the notion of temporal part within CEM. For instance, provided that we add a predicate to our language, say T, for 'x exists at time t', we can render the definition above as

D$_*$1. Instantaneous temporal part: $IP_txy := Pxy \wedge (Txt \wedge \forall s(Txs \leftrightarrow s = t)) \wedge \forall z((Pzy \wedge \wedge Tzt) \rightarrow Oxz)$.

For the axioms governing T, I shall not enter the discussion. Albeit interesting, doing so would exceed the aim of this chapter – it is, fundamentally, a different research area. Given my present purposes, it will be sufficient to point out that such an expansion of CEM is theoretically feasible.

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29 See Wasserman (2016) for a careful distinction between the two.

30 See Hovda (2013).

31 We can condense the argument as follows:

1. Parthood is general.
2. Temporal parts exist.
3. If parthood is general, then every part receives the same treatment.
4. Therefore, temporal parts receive the same treatment.

Although a spurious application, the same strategy could be virtually adopted to deal with modality: modal parts are just like any other part and, by making the relevant changes, we could provide a definition of modal part without leaving CEM's paddock. Analogously to the temporal part case, we might introduce a new predicate, say W, for 'x exists at world w' and, get the following definition of modal part by resemblance with D$_*$1:

D$_*$2. Worldbound part: $WBP_w yx := Pxy \wedge (Wxw \wedge \forall u(Wxu \leftrightarrow u = w)) \wedge \forall z((Pzy \wedge Wzw) \rightarrow$ $\rightarrow Oxz)$.

All this might sound promising, but there are at least two problems in place. First, that there are modal parts is far from unobjectionable. Four-dimensionalism about temporal parts has gained considerable support in recent years, but it is still a controversial thesis. So, if discourses of temporal parts do not come without rumbles of discontent, things get more slippery when modal parts are involved. Nevertheless, given that among CEM's meta-theoretical desiderata is topic-neutrality, there is no principled reason why we should not include modal parts in our discourse. If CEM is a mereological theory aiming at describing the fundamental features of the parthood relation regardless of which parts we admit in our ontology, refusing to include modal parts among those allegedly governed by CEM is, strictly speaking, a *metaphysical* rather than a mereological decision. Second, consider what happens if we adopt four-dimensionalism both for temporal and modal parts. Treating modality as another, equally-legitimate dimension along which ordinary material objects extend would commit us to *five*-dimensionalism, which affirms (roughly) that things are composed of spatial, temporal, and modal parts. In the words of a recent supporter,[32]

> Modal parts are four-dimensional worms. Each modal part is world-bound, meaning it exists in a single world. Ordinary objects are *modal worms*, or sums of modal parts. [...]. Just as temporal parts are instantanous, modal parts are world-bound. Just as a temporally extended object is a sum of temporal parts, a modally extended object is a sum of modal parts. Thus any object which is modally extended (or which, in more common terms, *could* have been some way or *must* have been some way) has modal parts. (Graham, 2015, p. 17, author's emphasis)

Five-dimensionalism might sound appealing to those already sharing similar philosophical attitudes, but it results, at best, highly controversial to all the others. Again, that it will not be met with good reception is neither a good nor a sufficient reason to discard it, but it is still worth considering. In this respect, one might point out that five-dimensionalism might be too problematic to defend. However, we could still endorse the less-committing thesis of four-dimensionalism and look for another strategy that works well with four-dimensionalism about time, like counterpart theory.[33] That can be done, but since my focus is on modality – and counterpart theory is a way of interpreting quantified modal logic – the only viable option for me would be to apply four-dimensionalism to modal parts.

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32 See Cotnoir and Varzi (2021, footnote 67) for a list of supporters.

33 Looking at the variant of four-dimensionalism known as stage theory, a particularly good account that is worth mentioning has been defended by Katherine Hawley in *How Things Persist* (2001).

2.4 Increased arity

Let's now discuss the second strategy. It stems from the idea that we need to reconsider the logical form of the parthood relation altogether: rather than being a binary relation, parthood is a ternary relation holding between parts, wholes, and times. As Judith Thomson, one of the fiercest proponents of this thesis, highlights, this is a commonsensical view of parthood (thereby its attractiveness):

> It is really the most obvious common sense that a physical object can acquire and lose parts. Parthood surely is a three-place relation, among a pair of objects and a time. (Thomson, 1983, p. 213)

Alternatively, one could follow Hudson (2001) and defend the idea that the third slot is occupied by space-time. Regardless of the inevitable, and sometimes substantial, differences between these two positions, there is a fundamental trait they have in common: that the parthood relation is substantially altered. Increasing the arity of the parthood relation leads to an ascription of time, or space-time, or (as we will soon see) modality, to the DNA of the parthood relation. Admittedly, this is not an easy pill to swallow. It would be possible to 'quantify out' time (or space-time, or modality) easily enough,[34], but the parthood relation would be substantially altered, thereby leading to a relativisation-adapted axiomatisation of CEM. This is a crucial point. How would the ordering axioms, the decomposition and composition principle change? Cotnoir and Varzi (2021, §6.2.2) provide a useful overview in carefully evaluating the consequences of increasing the arity of the parthood relation. From their work, it is clear that relativised versions of the relevant axioms (and therefore derived theorems) are inevitably weaker than their atemporal counterparts. Consider, for instance, the puzzle about Tibbles, the Cat.[35] Does Tibbles coincide with his proper part Tib after losing his tail? The time-relativised version of an extensionality principle like T6 would reply affirmatively to this question, but that would go against CEM. A whole can never be identical to one of his proper parts, no matter how big that part is. A4 rules this possibility out. Moreover, T6 is a stronger principle than its relativised version because it states that two things are identical if they *atemporally* overlap all the same parts. And being atemporally identical is much more demanding than being temporarily so. This might look like a drawback in the eyes of a CEM supporter, but it might be seen as a good reason to change the arity of the parthood relation. It seems that having a relativised version of extensionality principles would grant the thought that Tibbles coincides with his proper part Tib after losing his tail – an intuition many might have regarding material objects. However, the crucial point here is that if we want to maintain the mereological import of CEM after a change in the arity of parthood, adjustments in the system (addition of predicates, strengthening of axioms, etc.) will be needed. But, after all, why couldn't the parthood relation be fundamentally temporal? It would be dogmatic to rule this possibility out only based on how CEM was originally conceived. The *metaphysical* claim that parthood is a two-placed relation is by no means more valuable than the claim that it is three- or four-placed. Moreover, for those who are happy to endorse mereological pluralism[36] — the view according to

[34] I am grateful to my primary supervisor, Dr. Aaron J. Cotnoir, for stressing this point in an early draft.

[35] See Wiggins (1968).

[36] The opposite view is known as 'mereological monism', which affirms that there is only one *fundamental* parthood relation.

which there is more than one *fundamental* parthood relation – atemporal parthood could be said to hold for *abstracta*, while temporal parthood for *concreta*. Such a position would contrast with CEM's original desideratum of generality, but it is not incompatible with it. Now, what about modality?

We have already anticipated that, as for four-dimensionalism, it is legitimate, at least in principle, to give modality the same treatment reserved for time. It would be interesting to reflect on whether this strategy maintains the same appeal once applied to modality. It seems intuitive to think that there is something *intrinsically* temporal about Tibbles having a tail and not having it (anymore). But would people find as much intuitive the thought that parthood is *intrinsically* modal? Rather than having modality as a parameter, codifying parthood as 'x is part of y at world w' implies that every evaluation of a mereological claim is world-bound. Thus, there is no 'absolute' notion of parthood in this framework. Given that this is an under-explored area of research, assessing the implications of this strategy is not straightforward. Nonetheless, besides adapting what we have said about time to modality, we can sketch some of the issues that are likely to arise once parthood-at-a-world is taken as primitive. First, it seems that a notion of unbound parthood (and of all the other mereological concepts) could be defined by adopting a strategy we mentioned above: quantifying out modality. This means that we could arguably get unbounded mereological formulas when needed, e.g., for talking about mathematical entities, while maintaining the commitment to a fundamental three-placed parthood relation. However, it seems methodologically less complex to have a two-placed relation and *then* explore how it behaves in a modal dimension. Stipulating that parthood-at-a-world is fundamental implies granting logical and metaphysical priority to it. That a dish might or might not contain nuts depending on the world under consideration is prior to the state of affairs in the actual world, and that is highly controversial. A different, but related point involves the primitive notion of parthood-at-a-world itself. In the formulation of CEM we saw in 1.2, the primitive P is said to be a 'predicate constant'. Contrarily to the logical constants, its extension is fixed in the actual world but might change across possible worlds. Now, consider what the extension of the primitive predicate constant 'parthood-at-a-world' would be in our actual world. Arguably, it would have the same extension of the unbound parthood predicate – parthood-at-$w_@$ is the standard parthood predicate. But what happens when we move to a different world? In the standard case, the parthood predicate has different extensions across different worlds unless the axiomatisation is modified to make room for ME. The parthood-at-a-world predicate should behave similarly insofar as it is a predicate constant rather than a logical one. Thus, we have two alternatives here. Either we fix the extension of the parthood-at-a-world across possible worlds, or we must admit that its extension might change from $w_@$ to w_1, w_2, etc., unless otherwise stipulated. If we go the latter way, it seems inevitable to ask ourselves, why should we want to make the 'parthood-at-a-world' predicate our primitive if it behaves the same way as the one we already have? Evidently, as for four-dimensionalism, one of the benefits is that we could decide to jointly treat time and modality by assuming that parthood is a four-placed relation,[37] but this will inevitably require a more significant deal of good arguments in its defence, and, at least in the case of modality, the actual benefits of this change are dubious. If we go the former, we fall into meta-logical considerations like 'what is a logical constant?', 'can there be predicates behaving like logical constants? And if yes, what is the difference between them?'.

[37] See Plantinga (1975).

2.5 Quantified Modal Logic

So far, we have encountered two different ways of dealing with time and modality. On the one hand, four-dimensionalism approaches them from an onto-mereological perspective: temporal issues concern, strictly speaking, temporal parts, while modal issues concern modal parts. On the other, increasing the arity conceives time and modality as fundamental components of the parthood relation. These two strategies have in common the shared approach of changing the language of mereology to make room for temporal and modal discourse. The third strategy we will consider, quantified modal logic, distinguishes itself by approaching modality (and, potentially, time) from a different angle. Rather than undergoing a mereological revision, it proposes to revise the logical setting of mereology. Given mereology's dependence on its logical substratum, the underlying idea is that we could change that without touching the mereological axiomatisation. Switching from a first-order logic to a quantified modal one provides us with formal tools independently suited (that is, independently of mereology) for temporal and modal purposes. This section will be devoted to presenting the formal apparatus with which I will work for the rest of the dissertation; thus, it will be longer than the previous two. In the last section of the chapter, I will provide an evaluation of the three strategies. I will argue that regardless of the particular theory of quantified modal logic one might endorse, changing the underlying logical setting of mereology is the best option we have to supply CEM with modal and temporal tools.

2.5.1 Syntax

The vocabulary of QML comprises:

- Connectives: $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction), $\rightarrow$ (material implication), $\leftrightarrow$ (material equivalence). $\neg$ and $\rightarrow$ are taken as primitives and the other connectives are defined as usual:

$$\varphi \wedge \psi :\equiv \neg(\varphi \rightarrow \neg\psi)$$
$$\varphi \vee \psi :\equiv \neg\varphi \rightarrow \psi$$
$$\varphi \leftrightarrow \psi :\equiv (\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$$

- Punctuation marks: (,).

- Variables: v_0, v_1, v_2, ... and $x, y, z, ...$ for arbitrary variables.

- Constants: $k_0, k_1, k_2, ...$ and $a, b, c, ...$ for arbitrary constants.

- n-place predicate symbols, for any $n > 0$: $P^0_n, P^1_n, P^2_n, ...$

  In particular, the binary predicate of identity (symbolised as $=$) and parthood (symbolised as P) are assumed as primitives. Other relevant predicates will be defined as we proceed in our discussion.

- Arbitrary wffs: $\varphi, \psi, \chi, ...$

- Quantifiers: $\forall$ (universal quantifier), $\exists$ (existential quantifier). $\forall$ is taken as primitive and $\exists$ is defined as:

$$\exists x\varphi :\equiv \neg\forall x\neg\varphi$$

- Modal operators: $\Box$ (metaphysical necessity), $\Diamond$ (metaphysical possibility). $\Box$ is taken as primitive and $\Diamond$ is defined as:

  $$\Diamond\varphi :\equiv \neg\Box\neg\varphi$$

As for grammar, the wffs of QML are defined recursively as follows:

- If $t_1, ..., t_n$ are terms, i.e., variables or constants, and P is any predicate, then $Pt_1...t_n$ is an atomic formula (provided that the arity of P is respected).
- If φ and ψ are wffs, so are: $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, $\varphi \rightarrow \psi$, $\varphi \leftrightarrow \psi$, $\Box\varphi$, $\Diamond\varphi$.
- If φ is a wff and x is any variable, so are: $\forall x\varphi$, $\exists x\varphi$.

Further syntactic characteristics of the language of QML that is worth pointing out are the following:

- If φ is a wff containing any variable x, x is said to be *bound* in φ if it is next to a quantifier or within the scope of one associated with x. Otherwise, x is said to be *free* in φ. A wff is said to be *open* if it contains at least one free variable, and *closed* if it contains none.
- For any formula φ, given a variable x, φ^x_y is the result of substituting *all* free occurrences of x in φ with y. If it is not required for every occurrence of x to be substituted with y, we write $\varphi\binom{x}{y}$.
- Although the lambda operator, λ, is not included in the formal language, I will sometimes make use of it for ease of explanation. The lambda operator is a variable-binding operator borrowed from the lambda calculus allowing us to express functions from which predicates can be 'abstracted' – this is why λ is considered a 'predicate-forming' operator. In general terms, $\langle\lambda x.\varphi x\rangle$ is the predicate being 'abstracted' from a certain formula φx. If we have an argument to which it applies, we can write $\langle\lambda x.\varphi x\rangle(t)$, meaning that the relevant entity designated by t has $\langle\lambda x.\varphi x\rangle$ as a property. For instance, given that the monadic predicate M stands for 'is a musician', we can abstract the lambda term $\langle\lambda x.Mx\rangle$ and express 'Frank Zappa is a musician' as $\langle\lambda x.Mx\rangle(f)$, with f standing for 'Frank Zappa'. Intuitively, the lambda operator can be used to express more complex sentences, such as those involving predicates with a higher arity. For example, suppose that the predicate symbol L stands for the binary predicate 'loves'. THe lambda operator allows us to abstract the function $\langle\lambda x.\lambda y.Lxy\rangle$ and formalise the sentence 'Mary loves John' as $\langle\lambda x.\langle\lambda y.Lxy\rangle(m)\rangle(j)$, where m stands for 'Mary' and j for 'John'.[38]

2.5.2 Axiomatic system

There are several systems of modal logic one can work with. Depending on the axioms one decides to work with, the system might be weaker or stronger, but the basic system of QML – the one of which all the others are extensions – is known as '**System K**' (for Kripke). The following are the logical axioms characterising it:

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[38] I am aware that this discussion on the λ-operator is not exhaustive, but too much space and time would be needed to provide one. Unfortunately, there is no extensive literature in philosophy on the lambda calculus, but Fitting and Mendelsohn (1998, §9) and Gamut (1991, §4.4) provide a precise and complete overview of the λ-operator (and the λ-calculus more in general), so I refer back to them.

L1. $\varphi \to (\psi \to \varphi)$

L2. $(\varphi \to (\psi \to \chi)) \to ((\varphi \to \psi) \to (\varphi \to \chi))$

L3. $(\neg\psi \to \neg\varphi) \to ((\neg\psi \to \varphi) \to \psi)$

L4. $\forall x \varphi x \to \varphi^x_y$ (with y free for x in φ)

L5. $\forall x(\varphi \to \psi) \to (\varphi \to \forall x \psi)$ (with x bound in φ)

L6. $x = x$

L7. $x = y \to (\varphi \to \varphi\binom{x}{y})$ (with y free for x in φ)

L8. $\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$

L1-L5 represent one of the most common axiomatisations for first-order predicate calculus without equality (see, for instance, Mendelson (1964/2009, p.62)), to which two other clusters of axiom schemas are added. The first, which comprises L6 and 7, concerns axioms of identity. In this respect, it might be useful to highlight that L7 corresponds to the principle of the *Indiscernibility of Identicals*,[39] which states that identical objects are indiscernible, i.e., they share the same properties. L7 should not be confused with its converse principle, $(\varphi \to \varphi\binom{x}{y}) \to x = y$, known as *Identity of Indiscernibles*.[40] The joint work of these two principles is sometimes taken to define identity, but while the former is widely accepted as a logical truth, the latter is much more controversial. It is therefore important to keep them distinguished and remember that L7 corresponds to *Indiscernibility of Identicals* and not its converse. L1-L7 is a well-known axiomatisation for first-order predicate calculus with equality (see, again, Mendelson (1964/2009, p.88)). L8 is called '**K-schema**' and is the axiom securing the shift from FOL to QML. As we said above, system K is the weakest system of QML, but different axioms (and groups of axioms) can be added to it to obtain stronger systems. The most famous are (Sider, 2010, §6.3):

L9. $\Box\varphi \to \Diamond\varphi$

L10. $\Box\varphi \to \varphi$

L11. $\Diamond\Box\varphi \to \varphi$

L12. $\Box\varphi \to \Box\Box\varphi$

L13. $\Diamond\Box\varphi \to \Box\varphi$

L9 says that if something is necessary, then it is possible. The system resulting from the addition of L9 (called **D-schema)** to L8 is very weak. It is generally considered unsuited for representing metaphysical and logical necessity: if φ is a logical truth, i.e., it holds in all possible worlds, it should hold in the actual one too rather than only being possible. Nonetheless, system **KD** retains some philosophical interest as it seems it could represent *moral* necessity. Under this

[39] The *Indiscernibility of Identicals* is sometimes referred to as 'Leibniz Law', but here I shall avoid doing so given that, in the literature, the same label is also used to refer to its converse formula (the *Indentity of Indiscernibles*), or the compound of the two.

[40] Strictly speaking, a proper formulation of the Indiscernibility of Identicals and the Identity of Indiscernibles involves quantification over the φs and cannot, therefore, be formulated in first-order logic. With that proviso, I shall keep using these labels even though our logic setting is not second order.

suggestion, we might read L9 as 'if something is morally necessary, then it is morally permissible'. With L10, things start getting interesting. Often referred to as **T-schema**, L10 encapsulates the idea of alethic modality – if something is necessary, it is true. It is generally agreed that a system representing metaphysical necessity is at least as strong as **KT**. (It is well-known that L9 is provable in KT.) The next axiom schema (L11) is known as **B-schema** or 'Brouwerian axiom' (for the logician Brouwer). According to L11, if it is possible that φ is a necessary truth, then φ is a logical truth (in the actual world). System **KB** arises when L11 is added to L8. Adding both L10 and L11 to system K gives us system **KTB**. L12, or **4-schema**, says that if φ is necessary, then it is necessary that φ is necessary. When added to the basic axiom of modal logic, L8, we get the system **K4**. If L10 is also present in the axiomatisation, we get the famous **S4** system of modal logic. Finally, L13, or **5-schema**, states that if it possibly necessary that φ, then necessarily φ. Adding L13 to system only K gives us system **K5**. If we add L13 to system KT we obtain the strongest system of normal modal logics, system **S5**.[41]

It is worth highlighting that the list of systems I have presented above is not exhaustive. It does not include several other modal logics that can be derived by 'mixing and matching' the various axioms, such as *KDB* (L9+L11), *KD4* (L9+L12), and so on.

Turning to the rules of inference, they are:

R1. (Modus Ponens) $\dfrac{\phi \qquad \phi \to \psi}{\psi}$

R2. (Necessitation) $\dfrac{\phi}{\Box\phi}$

R3. (Universal Generalisation) $\dfrac{\varphi^x_y}{\forall x \varphi^x_y}$

Besides these fundamental inference rules, there is a fourth one that can be proved axiomatically thanks to joint work of R1, R2, and L8: [42]

R4. (Derived Rule of Regularity) $\dfrac{\phi \to \psi}{\Box\phi \to \Box\psi}$

2.5.3 Semantics

In QML, we must make at least a couple of crucial choices. The first one concerns the domain of discourse: should it remain constant across all possible worlds of the model, or should we allow it to change from world to world? The former option is known as constant domain semantics, while the latter as variable (or varying) domain semantics.

Constant domain semantics (henceforth, **CD**) represents the simplest and most straightforward option to make quantifiers and modal operators interact. It is associated with **possibilism**, a metaphysical modal thesis according to which *merely possible* entities also exist in the actual world, side by side with the chair I am sitting on and the cup of tea I have on the desk. Following

[41] Another famous formulation of *S5* is given by replacing L13 with two other schemas, namely L9 and L12.

[42] Following Fitting and Mendelsohn (1998, p.68):

(1)	$\varphi \to \psi$	any formula
(2)	$\Box(\varphi \to \psi)$	R2, 1
(3)	$\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$	L8, 2
(4)	$\Box\varphi \to \Box\psi$	R1, 3

classical logic, CD regards variables as ranging over existing objects rather than 'actually existing', i.e., existing in the actual world. 'To exist' here means, informally, something like 'to exist somehow' or 'to exist in some way'. This is why, in CD, if an object exists in some possible world, it exists in *all*. We do not need things to be actual for them to exist. The possibilist maintains that we need things to be actual in order to actually exist, but we do not need things to be actual to grant them existence. Otherwise put, being actual is a sufficient yet not necessary condition to exist – or, better, to be. *Possibilia* and mere *possibilia* are not actual entities, but still are.

On the other hand, **variable domain semantics** (henceforth, **VD**) allows us to 'free the logic' and thus discriminate between existing entities (i.e., à la Quine, those that are the values of a bound variable) and non-existing ones. It goes hand in hand with the modal doctrine of **actualism**, according to which only actually existing entities exist. In VD, different entities exist in different worlds because our ontological commitments are limited to *actually existing* entities. Pegasus does not exist here, in the actual world, so we cannot quantify over it, even though it might well exist in some other possible world. Evidently, this stands in stark opposition to a fundamental rule of inference of classical logic, Universal Instantiation, which grants that, from a universal proposition stating that all xs are φ, we can infer a particular instance of it (say, $\varphi(a)$). VD does not bite the bullet of classical logic for which *everything* that exists can be captured by the quantifiers – this is why Universal Instantiation fails in this framework. VD holds that the domain of quantification differs across possible worlds. Thus, if a is φ at some world, it might[43] fail to be so at another simply because a does not exist there. This is the fundamental reason why VD is not given in classical first-order logic but adopts a free logic instead.[44] All of this can be transposed in formal terms as follows. Starting with CD, an interpretation $\mathcal{I}$ for the language $\mathcal{L}$ is a structure $\langle D,\ W,\ R,\ v\rangle$. D is the non-empty constant domain of quantification, W is a non-empty set of possible worlds, and R is a binary accessibility relation between worlds. v is the evaluation function that assigns a member, $v(c)$, of D to each constant c of the language $\mathcal{L}$, and an extension, $v_w(P_n)$, i.e., an ordered set of n-tuples from D at each world, to each pair comprising a world w and a predicate P_n. Analogously to what happens in first-order predicate logic, we need to extend the language by adding a constant, k_d, for all $d \in D$, such that $v(k_d) = d$. What we obtain is the language of an interpretation, $\mathcal{L}(\mathcal{I})$, which we need to give all the truth conditions for QML. Now, v assigns a truth value, $v_w(\varphi)$, in each world w, to each (closed) wff of $\mathcal{L}$, and the truth conditions are as follows:

$$v_w(Pc_1...c_n) = \begin{cases} 1, \text{ if } \langle v(c_1), ..., (c_n)\rangle \in v_w(P); \\ 0, \text{ otherwise.} \end{cases}$$

$$v_w(\neg\varphi) = \begin{cases} 1, \text{ if } v_w(\varphi) = 0; \\ 0, \text{ otherwise.} \end{cases}$$

$$v_w(\varphi \wedge \psi) = \begin{cases} 1, \text{ if } v_w(\varphi) = v_w(\psi) = 1; \\ 0, \text{ otherwise.} \end{cases}$$

43 Some have argued that not only the domain of quantification in VD *might* differ, but that *must* differ, given that the same actual entity cannot exist at several possible worlds. I believe this is a strong critique, but I will not enter into this discussion.

44 See Priest (2008) for more details.

$$v_w(\varphi \vee \psi) = \begin{cases} 1, & \text{if } v_w(\varphi) = 1 \text{ or } v_w(\psi) = 0; \\ 0, & \text{otherwise.} \end{cases}$$

$$v_w(\varphi \to \psi) = \begin{cases} 1, & \text{if } v_w(\varphi) = 0 \text{ or } v_w(\psi) = 1; \\ 0, & \text{otherwise.} \end{cases}$$

$$v_w(\varphi \leftrightarrow \psi) = \begin{cases} 1, & \text{if } v_w(\varphi) = v_w(\psi); \\ 0, & \text{otherwise.} \end{cases}$$

$$v_w(\forall x \varphi) = \begin{cases} 1, & \text{if for } \textit{all } d \in D,\ v_w(\varphi_x(k_d)) = 1; \\ 0, & \text{otherwise.} \end{cases}$$

$$v_w(\exists x \varphi) = \begin{cases} 1, & \text{if for } \textit{some } d \in D,\ v_w(\varphi_x(k_d)) = 1; \\ 0, & \text{otherwise.} \end{cases}$$

$$v_w(\Box \varphi) = \begin{cases} 1, & \text{if for } \textit{all } w_x \text{ such that } wRw_x,\ v_{w_x}(\varphi) = 1; \\ 0, & \text{otherwise.} \end{cases}$$

$$v_w(\Diamond \varphi) = \begin{cases} 1, & \text{if for } \textit{some } w_x \text{ such that } wRw_x,\ v_{w_x}(\varphi) = 1; \\ 0, & \text{otherwise.} \end{cases}$$

Validity is defined as usual: $\Sigma \vDash \varphi$ iff for all worlds $w \in W$ of all interpretations $\langle D, W, R, v\rangle$, if $v_w(\psi) = 1$ for all the premises $\psi \in \Sigma$, then $v_w(\varphi) = 1$.

Moving to VD, an interpretation $\mathfrak{I}$ for the language $\mathcal{L}$ is a structure $\langle D, W, R, v\rangle$. Each component of the structure remains the same, but now, for every $w \in W$, we have that $v(w) \subseteq D$. We shall write the relevant subset of D at world w as D_w. Introducing a predicate, E, for *actual* existence,[45] and defining it as $Ex :\equiv \exists y\, x = y$,[46] we have that $v_w(E) = D_w$. However, for any other predicate, P_n, we have that $v_w(P_n) \subseteq D^n$ (and not D^n_w). The truth conditions remain the same as in the constant domain case, with the only exception of those for the quantifiers, which become:

$$v_w(\forall x \varphi) = \begin{cases} 1, & \text{if for } \textit{all } d \in D_w,\ v_w(\varphi_x(k_d)) = 1; \\ 0, & \text{otherwise.} \end{cases}$$

45 E is often referred to as 'existence predicate'. However, given the nature of the discussion that will follow, and the linguistic ambiguity of what is meant by 'existence', I prefer to call it '*actual* existence predicate'.

46 $\exists y\, x = y$ is a valid formula of FOL basically saying that, for any x, x is something (rather than nothing). It is trivially satisfied in CD because quantification is unrestricted there.

$$v_w(\exists x\varphi) = \begin{cases} 1, & \text{if for } \textit{some } d \in D_w,\ v_w(\varphi_x(k_d)) = 1; \\ 0, & \text{otherwise.} \end{cases}$$

Validity is defined as in the constant domain case.

The second issue on which we are required to take a stand concerns the relation governing accessibility across possible worlds. There are several constraints that it might or might not satisfy. A frame $\langle D, W, R\rangle$ is:[47]

1. **serial** if, for all $w_1 \in$ W, there is a w_2 such that w_1Rw_2.
2. **reflexive** if, for all $w \in$ W, wRw.
3. **symmetric** if, for all w_1, $w_2 \in$ W, if w_1Rw_2, then w_2Rw_1.
4. **transitive** if, for all w_1, w_2, $w_3 \in$ W, if w_1Rw_2 and w_2Rw_3, then w_1Rw_3.
5. **euclidean** if, for all w_1, w_2, $w_3 \in$ W, if w_1Rw_2 and w_1Rw_3, then w_2Rw_3.

Choosing constraints for the accessibility relation is the semantic equivalent of choosing axioms for the system. Thus, each of the above constraints corresponds to a particular axiom. A serial frame corresponds to L9; a reflexive one to L10; a symmetric one to L11; a transitive one to L12; and an euclidean one to L13. Accordingly to what happens with the axioms, different constraints might be satisfied in a certain frame. So, for instance, a given frame might both be reflexive and symmetric (thereby corresponding to *KTB*) or reflexive, symmetric, and transitive (thereby corresponding to *S5*).

2.5.4 Necessity *De Re* and Necessity *De Dicto*

As the literature on the *de re vs de dicto* modality has extensively stressed, modal sentences of natural language are seldom unequivocal. Coming back to our example concerning houseless people and what Orwell defined 'ordinary working' people, consider the following sentences:[48]

(1) Some houseless person might have been an ordinary working one.

(2) Some ordinary working person might have been a houseless one.

There are a couple of ways in which they can be formalised:

(1a) $\Diamond\exists x(Hx \wedge Ox)$

(1b) $\exists x(Hx \wedge \Diamond Ox)$.

(2a) $\Diamond\exists x(Ox \wedge Hx)$

(2b) $\exists x(Ox \wedge \Diamond Hx)$.

[47] See Fitting and Mendelsohn (1998, §4.6 and §4.7) for the specifics on constant and varible domain (augmented) frames.

[48] A famous example that is often used to show the relevance of the *de re vs de dicto* distinction concerns the number of planets in our solar system. See, among the others, Fitting and Mendelsohn (1998, p.87-88) and Quine (1953).

Evidently, (1a)-(2a) have a very different meaning from (1b)-(2b). Through QML, we can defuse the ambiguity of sentences like (1) and (2) by unveiling their possible interpretations. On a *de dicto* interpretation like (1a)-(2a), the scope of the modal operator is overarching: it is what is being said, the *dictum*, that is possible. Instead, on a *de re* reading like (1b)-(2b), the scope of the operator is restricted to the thing, the *res*, to which the property of possibly being such and such is predicated. It is, therefore, straightforward to see why (1b)-(2b) are the correct formalisations of (1) and (2). It is some *actual* houseless person who might have been an ordinary working person and, vice versa, it is some *actual* ordinary working person who might have been houseless. The ambiguity displayed by (1) and (2) is known as '*de re*/*de dicto*' ambiguity. The plot thickens, however, when we deal with sentences like the following:

(3) Every houseless person is necessarily experiencing a permanent lack of housing.

It displays a three-headed ambiguity:

(3a) $\Box\forall x(Hx \to Lx)$

(3b) $\forall x\Box(Hx \to Lx)$

(3c) $\forall x(Hx \to \Box Lx)$

(3a) is true because it is true, in any possible world, that if someone is houseless, she is experiencing a permanent lack of housing. The first of the two *de re* readings, (3b), also seems to be true: it is necessary for someone that if they are houseless then they are experiencing a lack of permanent housing. Contrarily, (3c) is false. If someone is houseless in the actual world, it does not follow that they experience a permanent lack of housing in any possible world. Alternatively put, being a houseless person in the actual world does not imply that one has the property of experiencing a permanent lack of housing essentially.

Finally, consider a singular term t like 'Muhammad Sarim Akhtar' (the guy in Figure 4):

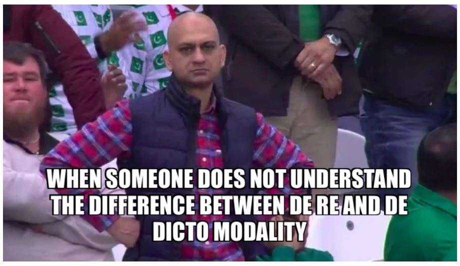

Figure 4: Muhammad Sarim Akhtar, whose expression of disappointment became a meme.

According to the information I found on his LinkedIn profile, he works as a manager for a multinational company. Thus, if we consider 'ordinary working people' as an umbrella term for anyone whose job would typically not be considered *sui generis*, i.e., following the usual capitalistic mechanisms for earning one's own living, he might be considered so. By means of the λ-calculus[49] we get:

(4a) $\Box\langle\lambda x.x \text{ is an ordinary working man }\rangle(t)$

[49] See §9 in Fitting and Mendelsohn (1998) for a discussion of how the lambda calculus allows us to treat singular terms in a modal context.

(4b) $\langle \lambda x.\Box(x \text{ is an ordinary working man})\rangle(t)$

(4a) displays a *de dicto* reading; it says that it is necessary that Muhammad Sarim Akhtar is an ordinary working man. And this is false since he might have been – pushing on the distinction made by Orwell – homeless. On the other hand, (4b) is *de re* and says that Muhammad Sarim Akhtar is necessarily a working man. This is true, as the reference for 'Muhammad Sarim Akhtar' has been fixed in the actual world, and, in the actual world, Muhammad Sarim Akhtar is an ordinary working man.

2.6 A (short) evaluation

Generally speaking, it is good practice to endorse the most neutral strategy. In our case, this translates into advocating for the strategy that better fits CEM's commitments. I take this strategy to be the modification of the underlying logical system. In light of what we have discussed, I think we can argue that it is the most neutral strategy for at least two main reasons. First, no revision of the basic mereological framework is required. What is valid in first-order CEM remains valid and equally powerful in modal CEM. Second, we can avoid quantifying over times and worlds in the object language because the meta-theory explains their import. The other two strategies require that CEM's axioms, definitions, and theorems are revised according to their respective frameworks. Besides being more neutral than the other two, the third strategy has two other advantages. The first one is that it is easier to distinguish between principles deriving from within CEM from the external ones. The second is that QML allows us to draw the distinction between *de re* and *de dicto* modality.

To conclude, I think the best way to deal with modality and time within mereology is to change its underlying logic. In saying so, I rely on the Wittgensteinian belief that logic (and mereology) takes care of itself.

Chapter 3

Modal CEM

Moving from FOL to QML brings about several novelties. Some are strictly mereological, others are more general and do not depend on the mereological theory we are endorsing. For clarity's sake, the discussion will be organised as follows. I will first present those implications that apply merely in virtue of the type of logic involved (let's call them logical implications) and I will then focus on the mereological ones, i.e., those that result from our endorsement of a specific mereological theory (CEM in our case).

3.1 System KTB and Constant Domain Modal Logic

In 2.5, we saw that there are two crucial choices one has to make when working with QML: (i) choosing whether to endorse a constant domain or a variable domain semantics and (ii) choosing which constraints to impose on R. The time has come for me to make a choice. Let's start with (i). As I have already anticipated, I endorse *CD*, but I still owe the reader an explanation for this. In what follows, I will provide two technical reasons that make it more convenient to work with *CD* rather than with *VD*.

First, working with *CD* is technically straightforward. No particular care is needed: all that one needs to do is to 'smash together' first-order predicate logic and normal propositional modal logic. As we have already seen, quantifiers rage over the same domain of quantification regardless of the possible world we are considering, so there is no need to depart from the usual rules of classical logic. Avoiding 'to free' the logic proves particularly useful for our purposes – investigating CEM in a modal scenario. Deciding to base CEM on a free logic could well be done; the problem is that 'what follows from the axioms, which is determined by the underlying logical theory, would need to be carefully assessed' (Cotnoir and Varzi, 2021, p. 263). Contrarily, endorsing *CD* allows us to maintain CEM in its original formulation. A second, even more crucial, reason to stick with *CD* can be found in a recently published article (Michalczenia, 2022). In Fitting and Mendelsohn (1998) it was argued that *CD* and *VD* can 'simulate' each other once some technical adjustments are made. Following the actualist taste, suppose that we have, in constant domain semantics, an existence predicate for *actually* existing entities. Let $\mathcal{E}$ be a unary predicate for actual existence. The following are the conditions given in under which a formula φ is existentially relativised:

- If $P_{c_1 \ldots c_n}$ is atomic, $P^{\mathcal{E}}_{c_1 \ldots c_n} = P_{c_1 \ldots c_n}$
- $(\neg\varphi)^{\mathcal{E}} = \neg(\varphi)^{\mathcal{E}}$

- Given any binary connective $*$, $(\varphi * \psi)^{\mathcal{E}} = (\varphi^{\mathcal{E}} * \psi^{\mathcal{E}})$
- $(\Box\varphi)^{\mathcal{E}} = \Box\varphi^{\mathcal{E}}$
- $(\Diamond\varphi)^{\mathcal{E}} = \Diamond\varphi^{\mathcal{E}}$
- $(\forall x\varphi)^{\mathcal{E}} = (\forall x(\mathcal{E}x \to \varphi^{\mathcal{E}}))$
- $(\exists x\varphi)^{\mathcal{E}} = (\exists x(\mathcal{E}x \wedge \varphi^{\mathcal{E}}))$

Given that, Fitting and Mendelsohn conclude (in Proposition 4.8.2):

> Let Φ be a sentence not containing the symbol $\mathcal{E}$. Then: Φ is valid in every varying domain domain model if and only if $\Phi^{\mathcal{E}}$ is valid in every constant domain model.

That would already be great news for possibilists, as it shows that adopting an actualist or a possibilist quantification is merely a choice of pre-theoretical taste, especially since they both seem to have the same explanatory power. Starting from this proposition, (Michalczenia, 2022) shows that it is not the case that the semantic machinery of *VD* is equivalent to that of *CD*. As it is, $VD \Vdash \varphi \Rightarrow CD \Vdash \varphi^{\mathcal{E}}$ does not hold. Only the other conditional, $CD \Vdash \varphi^{\mathcal{E}} \Rightarrow VD \Vdash \varphi$ is valid.[50] Michalczenia (2022) goes on by proving that we can, nonetheless, prove a weaker version of the theorem, but I refer back to the article for the technical details. Here, I want to highlight that this result is philosophically interesting as it shows that a possibilist quantification is richer than an actualist one. Therefore, adopting *CD* supplemented with $\mathcal{E}$ has the advantage of not only mimicking *VD* successfully but also of granting more expressive power to those who do not share an actualist standpoint.

Moving to (ii), **KTB** is the system I endorse. Importantly, several proofs will only require a weaker logic. To keep track of the system that needs to be in place for a given theorem to be derivable, I will specify which between *K*, *KT* and *KB* justifies the derivation. The reason for doing it is that L11 is considered a relatively controversial axiom. However, there are different reasons why it might sound controversial. One is only misguided, but the other is serious. As for the former, think about someone in your life who is very important to you. Now, L11 says that if it is possible that, necessarily, you never met that person, then you never met that person. That might sound a bit fishy. The $\Diamond$-version of the schema, $(\varphi \to \Box\Diamond\varphi)$, which is easily derivable in **KB**, seems acceptable though. Recalling famous examples like 'the snow is white' or 'the grass is green' or 'the cat is on the mat', the equivalent formulation of L11 states that if the snow is white, or the grass is green, or the cat is on the mat, is true, then it is necessarily possible that that is the case. And it seems obvious that that is the case. How could one version of the Brouwerian axiom be intuitive while the other is not? Garson (2021) explains the phenomenon in terms of natural language ambiguities, but I do not think that is a satisfactory explanation. Instead, I think that anticipating what we will see in the section on semantics might be of some help in explaining what is going on. L11 corresponds to the symmetry of accessibility. Roughly, this means that two possible worlds can see each other. Now, if the accessibility relation is symmetric, this means that, from $w_{@}$, we have access to another world, and vice versa. By L11, we have that what is possibly necessary is the case. And given the semantics, it is simply not possible that p (that you met that important person) and that $\neg p$ (that you never met that important person) coexist in $w_{@}$. Only

[50] For any φ not containing $\mathcal{E}$ in both cases.

one can remain. And which turns out to be the 'winner' crucially depends on which world we are in. If we are in $w_@$, L11 says that if it is possibly necessary that p, then p (if it is possibly necessary that you met that person, then you met that person). If we are in w_1, L11 says that if it is possibly necessary that $\neg p$, then $\neg p$ (if it is possibly necessary that you never met that person, then you never met that person). Here, notice, there is no 'cross-world' contamination. It is not the case that we are $w_@$, and L11 states that $\neg p$. But it is the case if we are in w_1. Coming to the serious worry, the problem at stake is the following. Under an epistemic interpretation to $\Box$ (generally rendered with 'K'), L11 says: $\neg K \neg K \varphi \rightarrow \varphi$.[51] That is: if it is not epistemically true that φ is not epistemically true (φ), then φ holds. But this sounds highly counter-intuitive. How could it be possible that if I am ignorant of my own ignorance about a certain fact, then that fact holds? For this reason, modal logics containing L11 as an axiom or theorem are generally taken to be unsuited as epistemic logics. However, under an alethic interpretation of $\Box$, L11 seems plausible.

Overall, there are two main reasons to endorse *KTB*. First, as we have already mentioned, it allows us to derive interesting theorems that a weaker logic like *KT* cannot prove without having to endorse a stronger logic like *S5*. Second, *KTB* is weaker than *S5*, but can be easily extended by only substituting L11 with L13. System *S5* is generally considered 'the appropriate system of modal logic for logical necessity' (Priest, 2008, p.46), so one might well prefer endorsing *S5* rather than *KTB*. However, by being an extension of *KTB*, *S5* will still inherit all the valid theorems that are provable in *KTB*. My aim is to keep the commitments as low as possible and show that we can prove a bunch of interesting logical and mereological theorems without necessarily endorsing *S5*, which comes, after all, with a heavier burden than *KTB*.

3.2 Logical Implications

3.2.1 Barcan Formula and Converse Barcan Formula

In an article of 1946, Ruth Barcan Marcus added the axiom schema $\Diamond(\exists\alpha)A \mathrel{-\!\!3} (\exists\alpha)\Diamond A$ (labelled axiom schema 11)[52] to her axiomatisation for a possible quantified extension of Lewis Calculus S2. Nowadays, the Barcan Formula (BF), as it has come to be called, is simplified as $\Diamond\exists x\varphi x \rightarrow \exists x\Diamond\varphi x$ and, together with its Converse (CBF), $\exists x\Diamond\varphi x \rightarrow \Diamond\exists x\varphi x$, represent the most straightforward way to make modal operators interact with quantifiers. At the time of Marcus, it was still unclear how (and whether) it was possible to combine the two: her work is one of the first ones dealing with QML. Given her strictly syntactic approach, BF and CBF's logical and metaphysical import remained obscure until Kripke's pivotal article of 1963, which gave semantics for QML by adopting a model-theoretic framework. Kripke's work revealed that, implicit to Marcus's work, (i) there was the assumption of a constant domain of quantification and that (ii) this was why both BF and CBF turned out valid in QML. Let's have a look at the axiomatic proof of both BF and CBF by making use of their universally quantified equivalent formulations:

T8. Converse Barcan Formula: $\Box\forall x\varphi x \rightarrow \forall x\Box\varphi x$.

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[51] Given the definition of $\Diamond$.

[52] The 'fish hook' symbol $\mathrel{-\!\!3}$ stands for 'strict implication', defined as $\Box(\varphi \rightarrow \psi)$.

(1)	$\forall x\varphi x \to \varphi x$	L4
(2)	$\Box(\forall x\varphi x \to \varphi x)$	R2, 1
(3)	$\Box\forall x\varphi x \to \Box\varphi x$	L8, 2
(4)	$\forall x(\Box\forall x\varphi x \to \Box\varphi x)$	R3, 3
(5)	$\Box\forall x\varphi x \to \forall x\Box\varphi x$	L5, 4

T9. Barcan Formula: $\forall x\Box\varphi x \to \Box\forall x\varphi x$.

(1)	$\forall x\Box\varphi x \to \Box\varphi x$	L4
(2)	$\forall x(\forall x\Box\varphi x \to \Box\varphi x)$	R3, 1
(3)	$\Box\forall x(\forall x\Box\varphi x \to \Box\varphi x)$	R2, 2
(4)	$\forall x\Box(\forall x\Box\varphi x \to \Box\varphi x)$	T8, 3
(5)	$\forall x(\Diamond\forall x\Box\varphi x \to \Diamond\Box\varphi x)$	K $(\Box(\varphi \to \psi) \to (\Diamond\varphi \to \Diamond\psi))$, 4
(6)	$\forall x(\Diamond\forall x\Box\varphi x \to \varphi x)$	KB $(\Diamond\varphi \to \Diamond\Box\psi) \to (\Diamond\varphi \to \psi))$, 5
(7)	$\Diamond\forall x\Box\varphi x \to \forall x\varphi x$	L5, 6
(8)	$\Box\Diamond\forall x\Box\varphi x \to \Box\forall x\varphi x$	R4, 7
(9)	$\forall x\Box\varphi x \to \Box\forall x\varphi x$	KB $(\Box\Diamond\varphi \to \Box\psi) \to (\varphi \to \Box\psi))$, 8

T9 says that if everything necessarily satisfies a certain condition, then it is necessary that everything satisfies that condition; T8 says that if it is necessary that everything satisfies a certain condition, then everything necessarily satisfies that condition. This might all sound a bit obscure, so let's consider the equivalent formulations we stated at the beginning of this section. $\Diamond\exists x\varphi x \to \exists x\Diamond\varphi x$ (corresponding to T9) states that if it is possible that there is something satisfying a certain condition, then there is something possibly satisfying it. In other words, *ceteris paribus*, nothing comes into existence. $\exists x\Diamond\varphi x \to \Diamond\exists x\varphi x$ (corresponding to T8) states that if there is something that possibly satisfies a certain condition, then it is possible that there is something satisfying it. Alternatively, *ceteris paribus*, nothing comes out of existence. It is worth highlighting that, contrarily to T8, T9 could only be axiomatically derived in a logic that is at least as strong as *KTB*, but it is valid even in the simplest system *K*.

Given T8 and T9, the following theorem – encoding the disappearance of the *de re* - *de dicto* distinction – immediately follows:

T10. Ibn-Sina's Principle: $\forall x\Box\varphi x \leftrightarrow \Box\forall x\varphi x$.[53]

Now, T8 and T9 (and, therefore, T10) were considered inescapable consequences of QML until, as we have already anticipated, Kripke (1963, p. 88-89) argued that Step (2) is flawed (in both proofs). R2 is not only applied to bound variables but to free variables, too, thereby causing a loss of generality. To avoid so, he suggests (*à la* Quine) that no formula containing free variables should be involved. Alternatively, we have to adopt a 'generality interpretation' (p.89) of the free variables. The crucial assumption here is, crudely, that free variables are just implicitly quantified variables, as 'assertion of formulae containing free variables is at best a convenience; assertion of $A(x)$ with free x can always be replaced by assertion of $(x)A(x)$.' (p.89). Thus, x is required to

53 The label comes from Williamson (2013). The Persian philosopher Ibn-Sina, Williamson writes, argued for the equivalence between the *de re* and *de dicto* reading of possibility 'more than nine centuries before Barcan Marcus's 1946 paper' and, although 'Barcan Marcus came on the principle independently of Ibn-Sina, [...] we should give credit where it is due.' (2013, p.45).

pick out something from the domain of quantification that is in place in the possible world we are considering.

To see the point more clearly, consider that CBF corresponds to monotonicity and BF to anti-monotonicity.[54]. A model is *monotonic* if, given that $w_0, w_1 \in W$, $w_0 R w_1$ implies $D(w_0) \subseteq D(w_1)$. This means that we have a counterexample to CBF as soon as $D(w_0) \nsubseteq D(w_1)$, and this is precisely the line Kripke's (1963) adopts for proving that CBF turns out to be invalid if we allow the domain D in the frame to change across possible worlds. Were we unhappy with the result and want monotonicity back, we have to require that $w_0 R w_1$ so that $D(w_0) \nsubseteq D(w_1)$ does not hold anymore. On the other hand, a model is *anti-monotonic* if, provided that $w_0, w_1 \in W$, $w_0 R w_1$ implies $D(w_1) \subseteq D(w_0)$. Similarly to what happens for a monotonic model, an anti-monotonic one ceases to be so as soon as $D(w_1) \nsubseteq D(w_0)$, and we can reinstate it only by imposing that $w_1 R w_0$. Thus, while a constant domain augmented frame $\langle D,\ W,\ R\rangle$ proves to be both monotonic and anti-monotonic because D remains the same when moving from world to world,[55] a variable domain augmented frame fails to satisfy both (unless we have a *locally* constant domain frame).[56] It is possible for a variable domain augmented frame to satisfy monotonicity or anti-monotonicity – in the former case, we talk about *nested* domain models, whereas, in the latter, about *shrinking* domain models – yet not both together. In other words, while BF and CBF are always valid in a constant domain augmented frame, things are more complex in the case of a variable domain augmented frame. In the latter, D does not have to satisfy any particular condition. This means that it *might* satisfy some, but it is not mandatory. Monotonicity and antimonotonicity represent possible conditions that a variable domain augmented frame might happen to satisfy: if monotonicity is satisfied, then CBF is valid in every model based on that frame; if antimonotonicity is satisfied, then BF is valid in every model based on that frame. And since Kripke's semantics invalidates both BF and CBF, it is neither monotonic nor anti-monotonic.

3.2.2 Barcan Formula, *Possibilia*, and Mere *Possibilia*

When discussing T8 and T9 – either with other philosophers or with that friend of yours you generally submit your philosophical ideas to – likely, you will not get a cheering reaction. Things might be okay with T8 as it is normally accepted that if there is something that is possibly φ, then it is possible that there is something that is φ, yet not with T9. We will see in the next section that T8 is not entirely free from reproach too, but let's now focus on T9.

Semantically, T9 states that if it is possible that there is something satisfying a certain φ condition, then there is something that potentially satisfies φ. Following Williamson (2000, 2013), we can identify two possible readings of a *de re* sentence like 'x is a possible thief' ($\exists x \Diamond Tx$, where T stands for 'is a thief'): a predicative and an attributive one. The *predicative* reading interprets 'x is a possible thief' as 'x is an thief and x could have existed' ($\exists x(Tx \wedge \Diamond Ex)$, where E is the

[54] The credits for showing the correspondence of CBF and BF to monotonicity and anti-monotonicity respectively belong (again) to Kripke. Fitting and Mendelsohn (1998) and Fitting (1999) provide a good insight on this, so I refer back to them for a more thorough discussion of the issue.

[55] Importantly, as Fitting (1999, p.4) underlines, the converse is not true. Having both monotonicity and anti-monotonicity in place merely assures us that if the accessibility relation between two possible worlds is symmetric, then they have the same domains. In order to have constant domains, we need to meet another condition, namely that $D(w_0) = D(w_1)$ for all $w_0, w_1 \in D$.

[56] See Fitting and Mendelsohn (1998, p. 112-113) for more details on locally constant domain frames.

actual existence predicate). The underlying assumptions here are that only what exists in the actual world can be quantified over – an assumption which echoes the actualist taste – and that whatever is could have been. As Williamson emphasises, in this reading, it is trivially true that all possible φxs are φxs. All possible thieves are already actual thieves! On the other hand, the *attributive reading* reads 'x is a possible thief' just as 'x could be a thief' ($\exists x \Diamond Tx$). This reading allows possible φxs not to be φxs, so it does not trivialise our ontology. Everyone is potentially a thief, but not all possible thieves are thieves! The point here is the following. On both readings of 'x is a possible φ', all φxs are possible φxs – all thieves are possible thieves. Thus, T9 and the *possibilia* it generates does not seem to create too much trouble when *possibilia* are involved. The relevant instance of T9, $\Diamond \exists x Tx \to \exists x \Diamond Tx$, would be interpreted differently by the actualist and the possibilist – the former would adopt a predicative reading, while the latter an attributive one – but both would recognise that there are possible thieves and *possibilia* more in general.

Things drastically change when *merely* possible entities are involved. Something is a merely possible entity iff it may hold, but it does not. Given what we have said above, it is predictable that under a predicative reading, there are none: if x does not exist, how can is it possible to predicate something to it? Consider the famous example of Wittgenstein's possible son.[57] Under a predicative reading, we would have that 'x is Wittgenstein's possible son, and x could have existed' ($\exists x(S_W x \land \neg Ex \land \Diamond Ex)$, where S_W stands for 'is Wittgenstein's son'). Evidently, there is no Wittgenstein's son in the actual world (biological, at least), so the formula cannot be satisfied. The mere non-existence of Wittgenstein's possible son in the actual world suffices to invalidate T9. On the contrary, under an attributive reading, there can be an x that could have been but is not. There is an x that could have been Wittgenstein's son but is not ($\exists x(\neg S_W x \land \Diamond S_W x)$). That Wittgenstein's possible son does not exist in the actual world does not threaten T9 in this case. There is still something out there that could have been his son. Let me stop here for a quick remark. In implying a *de re* possibility from a *de dicto* one, T9 is not talking about a specific individual entity, viz., it does not say that there is one particular person who is potentially Wittgenstein's son. What it states, strictly speaking, is that there is *something* which is potentially Wittgenstein's son. Failures to understand this crucial point might lead to fatal misunderstandings. Nonetheless, this is not the core of the matter. What is protested against those endorsing constant domain semantics is precisely that T9 grants some sort of existence to merely possible entities like Wittgenstein's son and Pegasus (but not, remember, that there are possible entities). Replies against these allegations are well-known, so I will not indulge too much on this issue. Moreover, although mere possibilia are an ontological by-product of endorsing *CD*, were we to desire an actualist grasp on this issue, we have seen that we can easily replicate *VD* and thereby exclude them from our ontological catalogue of commitments. All that it would take is to relativise T9 accordingly.

Williamson has famously argued that actualists are committed to mere *possibilia* despite the fact that T8 and T9 are not provable in *VD*. Menzel (2022) provides a handy version of it. It goes as follows. Suppose, accordingly to the actualist taste, that (i) modal operators quantify over possible worlds, and (ii) mere *possibilia* are kept separated from actually existing entities. We can formalise the thought that, although it is possible that Wittgenstein could have had a child, it is not possible that there is such a possible entity in the actual world as $\Diamond \exists x S_W x \land \neg \exists x \Diamond S_W x$. What happens is that to state the latter, the actualist must quantify over non-actual entities. In fact, for the first condition to be true, there must be an accessible possible world with an entity that

[57] This example was first introduced in the literature by Williamson (1998).

satisfies $S_W x$, but the second conjunct prevents an actual entity from doing so. Therefore, it seems that the actualist is committed to merely possible (and possible) entities at the exact same time that she denies their existence. At this point, someone might wonder whether stating the thought behind $\Diamond\exists x S_W x \wedge \neg\exists x \Diamond S_W x$ in terms of singular predication might avoid the commitment. By means of the λ-operator we would get: $\Diamond\langle\lambda x.(\exists y)(y = x)\rangle(s) \wedge \neg\langle\lambda x.\Diamond(\exists y)(y = x)\rangle(s)$, where s is a singular term for 'Wittgenstein's son'. Now, it is a generally accepted norm that if a term fails to designate, then all the predicate abstractions in which it appears are false,[58] so, given the previous argument, the actualist can neither state the λ-version of the intuitive idea that it is possible that Wittgenstein could have had a child, but it is not possible that there is an entity being his possible child.

3.2.3 Necessity of Existence

Another controversial theorem of QML is the following:

T11. Necessity of Existence: $\Box\forall x\exists y\, x = y \rightarrow \forall x\Box\exists y\, x = y$.[59]

T11 says that if it is necessary that everything exists, then everything exists necessarily. T11 is just a substitution instance of T8 for $\exists y\, x = y$. As we have already seen, $\exists y\, x = y$ is a valid formula of FOL stating that, for any x, x is something rather than nothing. Because of the unrestricted quantification in place in *CD*, the formula is trivially satisfied there, but what it is interesting to see is that T11 is valid also within an actualist framework – the difference lying on the restricted quantification in place in this case. The same approach can be virtually applied to T9, from which we obtain $\forall x\Box\exists y\, x = y \rightarrow \Box\forall x\exists y\, x = y$, but this result is less philosophically interesting. It is, in general, widely accepted that a *de dicto* necessity follows from a *de re* one; so it is relatively uncontroversial that if something exists necessarily, then it is necessary that it exists.

Given that the antecedent of T11 is a valid formula in the system, by R1, we get:

T12. Necessary Existence: $\forall x\Box\exists y\, x = y$.

Hence, by R2 we have

T13. Necessary Necessity of Existence: $\Box(\forall x\Box\exists y\, x = y)$.

T12 states that everything is necessarily something or, alternatively, that there cannot be anything which is nothing. T13 makes necessary existence necessary. (Parmenides cheers). Intuitively, possibilists and actualists interpret T12 and T13 differently. We have discussed at length the different commitments characterising the two positions, so it should not come as a surprise that while the possibilists permit T13 and T13 to apply also to merely possible entities, actualists abhor that. Thus, the former recognises the necessary existence of Wittgenstein's possible son ($\forall x\langle\lambda x.\Box\exists y\, x = y\rangle(t)$, where t denotes Wittgenstein's possible son) and its necessary nature ($\Box\forall x\langle\lambda x.\Box\exists y\, x = y\rangle(t)$), while the latter rejects all of that. Nothing new so far, then. To reach more interesting shores, we must consider the actualist perspective. Under restricted quantification, T12 and T13 will apply to actually existing entities only. Let t denote me, and consider what T12 and T13 gives us: $\Box\exists y\, t = y$ (or, equivalently, $\Box Et$) and $\Box(\Box\exists y\, t = y)$ (or, equivalently, $\Box\Box Et$) respectively. $\Box Et$ says that I necessarily exist, and $\Box(\Box Et)$ reinforces the ineluctability

58 See Fitting (1991) for a useful discussion on predicate abstraction and non-denoting terms.

59 Equivalently: $\exists x\Diamond\exists y\, x = y \rightarrow \Diamond\exists x\exists y\, x = y$.

of my existence by affirming that it is necessary that I necessarily exist. Here is where a new philosophical figure, the contingentist, enters the scene. Necessitism stands in stark opposition to contingentism, which holds that existence is a matter of contingency – that it is contingent that there is what there is. Quoting from Williamson's *Modal Logic as Metaphysics*:

> Call the proposition that it is necessary what there is *necessitism*, and its negation *contingentism*. In slightly less compressed form, necessitism says that necessarily everything is necessarily something; still more long-windedly: it is necessary that everything is such that it is necessary that something is identical with it. In a slogan: ontology is necessary. Contingentism denies that necessarily everything is necessarily something. In a slogan: ontology is contingent. Necessitists need not deny that it is contingent which *kinds* of thing are instantiated, at least for some kinds. Thus they can happily accept that it is contingent that there are animals. Since there could have been no animals, by their lights an animal is only contingently an animal. But it does not follow that an animal is only contingently something. For the necessitist, it could have been something without being an animal. (Williamson, 2013, p.2, author's emphasis)

Importantly, it should not be inferred from all of the above that necessitists – possibilists and actualists alike – reject the notion of a contingent entity. As a matter of fact, they both agree that there are contingent entities. There is no incompatibility between this claim and theorems like T12 and T13. As Williamson repeatedly points out in the book, necessitism holds that while it is not contingent *what* there is, it is contingent *how* it is what there is. Thus, possibilists and actualists can agree that there are contingent beings, i.e., entities for which existing and non-existing is equally possible. The formal definition of contingent entity might be given as

D11. Contingent Actual Entity $C_E y :\equiv \Diamond Ex \wedge \Diamond \neg Ex$.[60]

P1. Existence of Contingent Actual Entities: $\exists y\, C_E y$.

Importantly, P1 is not a logically derivable theorem of the system. It is a theorem that the system *might* accommodate given our metaphysical preferences. Still, if we want to derive them as theorems, we need to make the relevant changes in the system. QML does not directly address the issue of contingentism, but since contingency is necessity's other side of the coin, our system has something to say about contingent entities. Many philosophers disagree with this statement, however. Arthur Prior is arguably the foremost representative of such a skeptical attitude. His quest for a satisfactory treatment of contingent beings arose from the belief that 'ordinary modal logic is haunted by the myth that whatever exists exists necessarily' (1957, p. 48). That the system proves T12 and T13 seems incompatible with the commonsensical intuition that we are, as human beings, contingent entities. I exist, but I might not have if my parents never met or, somewhat more crudely, if a different sperm reached the egg. A long time ago we became aware of our finitude as beings and embraced the contingency of our own existence ever since. T12 and T13 seem to tell us the opposite. To the contingentist, they look like the Sirens Odysseus encounters on his way back to Ithaca: by digging deeper into our existential desires, they promise us what lies in our deepest recesses. Or so tells us the famous story of Qin Shi Huang, the first Emperor

60 One might wonder why the definition of a contingent entity is given in actualist terms. The reason is straightforward. Defining a contingent entity as $Cy :\equiv \Diamond \exists x\, x = y \wedge \Diamond \neg \exists x\, x = y$ clashes with the antecedent of T11, which is a valid formula. Therefore, the second conjunct of such a definition could never be true.

of China, who died poisoned by that same mercury he thought would give him immortality. It is only by knowing ourselves, i.e., by recognising our contingent nature, that we can avoid fooling ourselves. Thereby the rejection of T12 and T13 as a mere myth feeding our egoistic desires.

Things are, however, more complex than what seems *prima facie*. How QML and contingency interact is as philosophically interesting as intricate, and it requires careful analysis and investigation to be untangled (Deutsch, 1990 offers an insightful overview on this). In the final part of this section, I hope to clarify how contingentism interacts with actualism and possibilism and why there is room for contingent entities within a possibilist framework. Let's deal with the former issue first. Consider the actualist version of T12 and T13:

T14. Necessary Actual Existence: $\forall y \Box Ey$.

T15. Necessary Necessity of Actual Existence: $\Box \forall y \Box Ey$.

T14 says that every actual entity necessarily exists, and T15 makes that necessity necessary. If something actually exists, its existence is necessary, and this necessity is necessary. Now, how can the actualist make sense of contingent entities given that everything that could exist already exists necessarily? In fact, it cannot. There is no space for contingent entities within an actualist framework (see Menzel, 2022). Here is another occasion in which possibilism proves better off. Under a possibilist interpretation, T14 and T15 are perfectly compatible with P11! The possibilist allows for there to be things which are not actual, thereby recognising that there are things which could have existed but do not. Moreover, regardless of whether an entity is actual, possible, or merely possible, possibilism recognises that it is necessary. Thus, it recognises that contingent entities exist and that their being something (rather than nothing) is necessary. The underlying thought is this: it does not matter whether x is actual or not; there could still have been an x.

To conclude, the following is another valid theorem of QML:

T16. Ibn-Sina's Principle on Existence: $\forall x \Box \exists y\, x = y \leftrightarrow \Box \forall x \exists y\, x = y$.[61]

T16 tells us that a *de dicto* necessity of existence does not float free from the necessary existence of particular entities. However, retaining so does not mean that I am a necessary entity *tout court*. It says that my being something rather than nothing is necessary.

3.2.4 Necessity of Identity

The final logical consequence of QML we will discuss involves identity. Given what we have sees in the previous section, one might wonder whether every entity in the domain is necessarily identical to itself, besides being necessarily existing. The answer is affirmative. The system proves the following theorem:

T17. Necessity of Identity: $\forall x \forall y\, (x = y \to \Box x = y)$.

(1)	$x = x$	L6
(2)	$\Box x = x$	R2, 1
(3)	$x = y \to (\Box x = x \to \Box x = y)$	L7, 2
(4)	$x = y \to \Box x = y$	PL $(((\varphi \to (\psi \to \chi)) \wedge \psi) \to (\varphi \to \chi))$, 2, 3
(5)	$\forall x \forall y (x = y \to \Box x = y)$	R3, 4

[61] Equivalently: $\exists x \Diamond \exists y\, x = y \leftrightarrow \Diamond \exists x \exists y\, x = y$.

T17 says that if two entities are identical, then they are necessarily identical. The proof I gave here is similar to Kripke's (1971, p.162-163), although the formula was first discovered by Barcan Marcus (1947). By R2, we then obtain a stronger theorem codifying that it is necessary for two things to be necessarily identical, if they already are:

T18. Necessary Necessity of Identity: $\Box\forall x\forall y(x = y \rightarrow \Box x = y)$.

At this point, one might wonder whether the necessity of distinctness is also valid in the system. After all, if two things that are identical are necessarily identical, it seems to make sense that the opposite should hold too, namely that if two things are distinct, then they are necessarily distinct. And, in fact, this proves to be the case:

T19. Necessity of Distinctness: $\forall x\forall y\,(x \neq y \rightarrow \Box x \neq y)$.

(1)	$\forall x\forall y(\neg\Box x = y \rightarrow \neg\, x = y)$	contrapositive of T17
(2)	$\forall x\forall y(\Diamond x \neq y \rightarrow x \neq y)$	equivalent formulation of 1
(3)	$\forall x\forall y(\Box\Diamond x \neq y \rightarrow \Box x \neq y)$	R4, 2
(4)	$\forall x\forall y(x \neq y \rightarrow \Box x \neq y)$	KB $(\Box\Diamond\varphi \rightarrow \Box\psi) \rightarrow (\varphi \rightarrow \Box\psi))$, 3

And, unsurprisingly:

T20. Necessary Necessity of Distinctness: $\Box\forall x\forall y(x \neq y \rightarrow \Box x \neq y)$.

Although T19 (and T20) are derivable in the system we are endorsing, system KTB, they are not provable in weaker systems that do not have L11 among their axioms, like KT. That is the crucial difference between T17 and T19: while T17 is easily derivable in any K system, T19 turns out to be axiomatically provable only in systems that are at least as strong as KB. This is a very interesting result. Identity and distinctness seem so closely related to each other that you would expect them to display analogous, albeit opposite behaviours. You would expect them to be liable to receive the same treatment, but logic proves you wrong. Model-theoretically, T19 is valid in any K system – that is, the associated tableaux is closed – but becomes axiomatically provable only in systems committed to L11. The question naturally arises: why do $x = y$ and $x \neq y$ display such differences?

The role identity plays in modal contexts is highly controversial and has generated an intense debate since the dawn of QML.[62] Quine has notoriously been one of the earliest and fiercest opponents of QML and his criticism towards T17 is paradigmatic of the dissatisfaction about the way equality works in QML, regardless of the semantics adopted (constant or variable domains).[63] Quine's argument has become known as an argument against essentialism and focuses on the substitution of equals for equals in a modal context, which he aims to show it relentlessly fails. Given the complexity of the issue at stake, I will not attempt to provide an exhaustive analysis, but rather provide a taste of it. Consider the following modal sentence:

(5) If Hesperus is identical to Phosphorus, then necessarily Hesperus is identical to Phosphorus.[64]

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62 As a side note, it might be interesting to point out that the famous Quine-Barcan Marcus dispute seems to have involved more than just logical and metaphysical disagreement. There are several resources on this respect, but a particularly interesting one is Max Cresswell's talk Why did W.V.O. Quine hate Ruth Barcan Marcus?.

63 A good reference is perhaps Quine (1953).

64 This modal version of Frege's puzzle is often found in manuals of modal logic. See, for instance, Braüner and Ghilardi (2007) and Fitting and Mendelsohn (1998).

(5) should be valid instance of T17, but it is not. Hesperus is identical to Phosphorus in the actual world, but nothing prevents them to be distinct in another possible world. That this is a 'referentially opaque context' (Quine, 1953, p. 162) can be easily seen if, letting a and b to stand for 'Hesperus' and 'Phosphorus' respectively, we distinguish between a *de re* and *de dicto* reading of (5):

(5a) $\langle \lambda x, y.(x = y)\rangle(a, b) \to \langle \lambda x, y.\Box(x = y)\rangle(a, b)$

(5b) $\langle \lambda x, y.(x = y)\rangle(a, b) \to \Box\langle \lambda x, y.(x = y)\rangle(a, b)$

(5a) is true because, in the actual world, the reference of Hesperus and Phosphorus is fixed to be the same for both, i.e., the planet Venus. On the contrary, (5b) is false because 'Hesperus' and 'Phosphorus' do not designate the same object in all possible worlds. It is possible, in fact, that in another world Venus is not Hesperus (or Phosphorus). A version of Quine's argument against essentialism roughly runs as follows:[65]

1. 'Hesperus' and 'Phosphorus' both refer to Venus.
2. It is necessary that Hesperus is the evening star.
3. It is not necessary that Phosphorus is the evening star.
4. Therefore, it is pointless to ask of Venus whether it necessarily is the evening star.

Kripke's (1980) reply is ingenious. He argues that there is a crucial difference between 'Venus' and 'Hesperus' and 'Phosphorus': while 'Venus' rigidly designates, the other two do not. Following Kripke, something is ' a *rigid designator* if in every possible world it designates the same object, a *nonrigid* or *accidental designator* if that is not the case.' (1980, p. 48, author's emphasis) At this point, one might wonder what counts as a rigid designator and what does not. Historically, this issue has caused a lot of controversies. Kripke (and many others) treat proper names as rigid designators, but there are alternatives. The Russellian analysis of proper names as definite descriptions prevents us from distinguishing 'Venus' and 'Hesperus' and 'Phosphorus' because they are all regarded as disguised definite descriptions. The descriptivist theory of names defended by Russell takes indexicals as the only *logical* proper names and regards *ordinary* proper names like Venus as abbreviated definite descriptions. Kripke has notoriously argued against descriptivism, instead defending a causal theory of reference. It is important to note that Kripke's appeal to the notion of rigid designation is crucially linked to his preferred theory of reference. His solution might not work if his causal theory of reference is not endorsed.[66]

Coming back to our discussion, Kripke's thesis that proper names are rigid designators allows us to make sense of the failure of T17 in (5). T17 fails in that case because the terms involved are intensional, nonrigid designators. Alternatively put, T17 can only make sense of *extensionally* equivalent terms. To ask for *intensional* equivalence is to raise the stakes to the level of synonymy, and that is not what T17 is meant to do. All it says is that if two entities have the same extension, they necessarily have the same extension.

The following are other theorems concerning equality that the system validates:

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65 In his paper, the example used concerns the number of planets.

66 See Stanley (2017) for a crisp overview on the issue.

T21. De Re Necessity of Identity: $\Box\forall x\forall y\, x = y \rightarrow \forall x\forall y\, \Box x = y$.[67]

T22. De Dicto Necessity of Identity: $\forall x\forall y\, \Box x = y \rightarrow \Box\forall x\forall y\, x = y$. [68]

T23. Ibn-Sina's Principles on Identity: $\forall x\forall y\, \Box x = y \leftrightarrow \Box\forall x\forall y\, x = y$.

T21 and T22 prove, respectively, that if it is necessary that two entities are identical, then they are necessarily identical, and that if two entities are necessarily identical, then it is necessary that they are identical. From the combination of these theorems, the equivalence between the *de re* and *de dicto* necessity of identity is proved (T23).

3.3 Mereological implications

Once the logical implications are evaluated, and we need to enter the domain of mereology, we face a fork in the road. One possibility is to necessitate CEM, i.e., apply R2 to its axiom system and see which theorems become derivable. The other is not to necessitate any axiom of CEM and simply allow the theorems of QML to interact with the standard non-modal version of CEM.[69] One should think carefully about which commitments she is willing to take. Although we set up the axiomatic system (2.5) in such a way that the φs need not only be valid formulas of the logic – they can also be mereological – deciding to necessitate an axiom means committing oneself to the *metaphysical impossiblity* that such a principle could fail in any possible world. The semantics tells us that if a certain φ is necessarily true, it is true in all (accessible) possible worlds. Moreover, as Simons (1987) writes, 'rather few axioms of non-modal mereology are secure from all reproach' (p.263), and the worries one might have with the non-modal version of A1-A5 could only be exacerbated once they are necessitated. In this respect, A2 and A5 are the most problematic axioms to be necessitated, but the latter causes the most philosophical discomfort. We will see there are several ways to deal with it in a modal scenario. In what follows, I shall start by presenting the kind of theorems we can derive if no axiom of CEM is necessitated. After evaluating their philosophical import, I shall discuss the most relevant alternatives we have once we decide to necessitate CEM.

For the consequences of combining QML with a non-necessitated version of CEM (QML**+**A1-A5) we can be brief. Given T17 and the extensionality principles (T4-T7), by PL we easily get:

T24. NI+PPE: $\forall x(\exists w PPwx \rightarrow \forall y(\forall z(PPzx \leftrightarrow PPzy) \rightarrow \Box x = y))$

T25. NI+PE: $\forall x\forall y(\forall z(Pzx \leftrightarrow Pzy) \rightarrow \Box x = y)$

T26. NI+OE: $\forall x\forall y(\forall z(Ozx \leftrightarrow Ozy) \rightarrow \Box x = y)$

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67 Proof:

(1)	$\forall x\forall y\,(x = y \rightarrow \Box x = y)$	T17
(2)	$\forall x\forall y\, x = y \rightarrow \forall x\forall y\, \Box x = y$	equivalent formulation of 1
(3)	$\Box\forall x\forall y\, x = y \rightarrow \Box\forall x\forall y\, \Box x = y$	R4, 2
(4)	$\Box\forall x\forall y\, x = y \rightarrow \forall x\forall y\, \Box x = y$	K $((\Box\forall x\varphi \rightarrow \Box\forall x\Box\varphi) \rightarrow (\Box\forall x\varphi \rightarrow \forall x\Box\varphi))$, 3

68 Proof:

(1)	$\forall x\forall y\,(x = y \rightarrow \Box x = y)$	T17
(2)	$\forall x\forall y\, x = y \rightarrow \forall x\forall y\, \Box x = y$	equivalent formulation of 1
(3)	$\Box\forall x\forall y\, x = y \rightarrow \Box\forall x\forall y\, \Box x = y$	R4, 2
(4)	$\forall x\forall y\, \Box x = y \rightarrow \Box\forall x\forall y\, x = y$	K $((\Box\forall x\varphi \rightarrow \Box\forall x\Box\varphi) \rightarrow (\forall x\Box\varphi \rightarrow \Box\forall x\varphi))$, 3

69 I am grateful to my primary supervisor, Dr Aaron J. Cotnoir, for stressing this point.

T27. NI+FU: $\forall z \forall w((F\varphi z \wedge F\varphi w) \rightarrow \Box z = w)$

Any of the principles above grant that if two things are extensionally equivalent (whether in terms of proper parts, or overlap, etc.), they are necessarily identical. As we know, T4-T7 bear a special relation with A2 – with T5 even being equivalent to it in the presence of reflexivity (A1) and transitivity (A3) – so it does not come with surprise that we also get:

T28. NI+A2: $\forall x \forall y((Pxy \wedge Pyx) \rightarrow \Box x = y)$

These principles are not of much philosophical interest. Let's take T26 as the representative of the extensionality principles and consider the puzzle of Tib and Tibbles again. T26 says that if Tib and Tibbles overlap all the same things, they are identical. Thus, the interaction of QML with non-modalised CEM tells us that Tib cannot be identical to Tibbles in all possible worlds if they do not overlap all the same objects in the actual world. In other words, if there is a possible world in which Tib and Tibbles are not identical, they do not overlap all the same things in the actual world. In this scenario, however, we have no idea whether Tib and Tibbles' mereological composition might change across possible worlds. T26 and analogous principles merely say that if Tib and Tibbles are coextensional, then they are necessarily identical, thereby granting their necessary identity regardless of the mereological composition they might have in different worlds. That is, there might exist a possible world in which, rather than losing his tail, Tibbles loses one of his legs, thereby becoming identical with another proper part of him, which we shall call Ti. Now, since in the actual world Tibbles and Tib are coextensional, T26 tells us that they are identical in all possible worlds – completely disregarding the fact that Tibbles is also identical to Ti in another possible world. What emerges is that Tibbles might change his mereological composition across possible worlds and remain identical to Tib as long as they are coextensional in the actual world. But there is no guarantee that they remain coextensional in all possible worlds. All we know is that if they are coextensional in the actual world, then they are necessarily identical.

While not necessitating CEM allows us to enter the modal (and possibly temporal) with a 'hands-off' attitude, the onto-mereological lunch is rather poor. It is by no surprise that Hovda (2013) and Uzquiano (2014) – the only two articles that, to my knowledge, deal with the relationship between CEM, time, and modality – decide to necessitate CEM. Uzquiano (2014) focuses on the modal dimension, and Hovda (2013) on the temporal one, but their considerations apply to both dimensions nonetheless. I shall refer back to both of them, and especially Uzquiano (2014), in what follows. The issue I want to address now is a preliminary one: which axioms should we necessitate?

3.3.1 Necessitating Mereological Axioms

It is widely recognised that CEM's axioms are not intuitively on par. The ordering axioms, or at least some of them, are often argued to be analytically true, whereas composition and decomposition axioms are much more controversial. That A1-A5 are not equally loaded even in the standard non-modal scenario gets naturally amplified in a modal one. As we have anticipated at the beginning of this section, A2 and A5 raise the most concerns. The literature on non-extensional mereologies has clearly shown that anti-symmetry (A2) is undeniably a controversial axiom. In the anti-extensionalist realm, anti-symmetry is often rejected (and for good reasons indeed).[70] Cotnoir

70 See, for instance, Cotnoir (2010), Cotnoir and Baxter (2014a), Thomson (1983). For an useful overview of the reasons to reject A2 see Cotnoir (2013b).

and Varzi (2021, §3.2) offer an in-depth analysis of the anti-symmetry axiom within CEM and discuss its relation with non-classical mereologies, so I refer back to them for discussion about these issues. The minimal point I want to highlight here – one that emerges from their analysis – is that what seems really concerning about A2 is that it is closely entwined with extensionality, which is one the most controversial theses of CEM alongside universalism. There are direct counterexamples to A2 mainly in the form of *exotica* (like Borges's Aleph),[71] but it seems that A2 mainly faces *indirect* critiques stemming from the dissatisfaction with some of the consequences it brings along, namely the extensionality principles. Thus, A2 does not seem particularly controversial insofar as we are happy for the mereology to be extensional – as in the case of CEM. Moreover, a necessitated version of A2 is crucial in most of the proofs we will soon see. All the extensionality principles intrinsically depend on A2. By deciding not to necessitate A2, it turns out that the only theorems we can derive are the ones we have just encountered in the section above (with a few irrelevant additions that I will highlight as we proceed). Given all of the above, it seems that we can reasonably apply R2 to A1-A4:

T29. $\Box$**A1**: $\Box \forall x Pxx$

T30. $\Box$**A2**: $\Box \forall x \forall y ((Pxy \wedge Pyx) \rightarrow x = y)$

T31. $\Box$**A3**: $\Box \forall x \forall y \forall z ((Pxy \wedge Pyz) \rightarrow Pxz)$

T32. $\Box$**A4**: $\Box \forall x \forall y (\neg Pxy \rightarrow \exists z (Pzx \wedge Dzy))$

Now, what about A5? Contrarily to A2, A5 is more problematic *per se*. Commonly referred to as Unrestricted Composition or Universalism, its metaphysical necessity has been extensively defended in the past under the belief that if it is metaphysically true, then it is necessarily so.[72] Nonetheless, that A5 is necessary always had its dissenters. The standard argument against it revolves around the idea that A5 is too permissive, i.e., that it allows for way too many fusions. It guarantees that there are the most scattered and gerrymandered fusions one can think of. Examples abound in the literature: from the famous aforesaid trout-turkey, to the 'the right half of my left shoe plus the Moon plus the sum of all Her Majesty's ear-rings' (Lewis, 1986, p. 213), to 'the color blue and me' (Van Inwagen, 1987, p. 35), to 'my left tennis shoe, W. V. Quine, and the Taj Mahal' (Rea, 1998, p.348), to 'the moon and the six pennies scattered across my desktop' (Van Cleve, 2008, p. 321). Supporters of this permissivist view of composition are generally not moved by this sort of objection.[73] Consider, one for all, Lewis's view on the matter:

> I never said, of course, that a trout-turkey is no different from an ordinary, much-heard-of thing. It is inhomogeneous, disconnected, and not in contrast with its surroundings. (Not along some of its borders.) It is not cohesive, not causally integrated, not a causal unit in its impact on the rest of the world. It is not carved at the joints. But none of that has any bearing on whether it exists. If you wish to ignore it, of course you may. Only if you speak with your quantifiers wide open must you affirm the trout-turkey's existence. (1991, p.80)

[71] This counterexample is originally due to Sanford (1993), but see again Cotnoir and Varzi (2021) for a concise discussion of the example and the related replies.

[72] See, among the others, Lewis (1986, 1991), Rea (1998), Sider (2007), and Van Cleve (2008).

[73] See Korman (2015).

More recently, however, a much more serious objection, known as 'the junk objection', has been put forward by Bohn (2009). It stems from the metaphysical possibility of junky words, namely words in which everything is a proper part of something else ($\forall x \exists y(PPxy)$). They contrast with so-called 'gunky worlds', worlds in which everything has something as a proper part ($\forall x \exists y(PPyx)$). The possibility of both junky and gunky worlds threatens A5, albeit for different reasons, but the possibility of a junky world is the most endangering of the two. Bohn's argument runs as follows:

1. If A5 is necessarily true, then the universal fusion, u (D9), must exist in all worlds.

2. Junky worlds are metaphysically possible.

3. If a world is junky, then u does not exist at that world.

4. Therefore, A5 is either metaphysically contingent or necessarily false.

Interestingly, this argument might be seen as a variant of Russell's Paradox. Given $\Box A5$, u is necessarily the universal fusion, i.e., the fusion of absolutely everything that there is. If there are junky worlds, however, u must be the proper part of something there. But how can u be and not be a proper part of something else?

Most of the objections to Bohn's argument focus on the second premise, and rightly so. Cotnoir (2014) showed that attacking the second premise is the only viable option to reject the argument. Without entering the debate, the take-home message is that it does not seem wise to necessitate A5. However, given the philosophical endorsement A5 and $\Box$A5 historically received, I will present the consequences of having $\Box$A5 in a system. What I want to highlight now is that adopting or rejecting $\Box$A5 are not the only two options we have because, in fact, we can also defend a weaker version of A5 and $\Box$A5. We will discuss them alongside A5 and $\Box$A5 in 3.3.5. Before getting there though, we will discuss other consequences of modal CEM, namely the necessity of extensionality (3.3.2), the necessity of non-extensionality (3.3.3), and how necessary extensionality connects to the issue of mereological essentialism (3.3.4).

3.3.2 Necessity of Extensionality

In Uzquiano (2014), the following theorem is proven:

T33. Necessity of O-Extensionality: $\forall x \forall y(\forall z(Ozx \leftrightarrow Ozy) \rightarrow \Box \forall z(Ozx \leftrightarrow Ozy))$.

T33 states that if two things overlap all the same things, then they do so necessarily. In contrapositive terms: it is possible that two things do not overlap all the same things only if they do not *actually* overlap them. In other words, T33 allows for something to overlap differently only if it already does. Evidently, this is an interesting modal theorem of CEM. Consider the following axiomatic proof, which is a formalised version of the proof Uzquiano gives in his paper (p.39-40):[74]

[74] I encourage the reader to have a look at the original formulation of this proof as mine is an axiomatic transposition of Uzquiano's. I should also point out that Uzquiano adopts system KT in his paper.

(1)	$\forall x \forall y(\forall z(Ozx \leftrightarrow Ozy) \rightarrow x = y)$	T6
(2)	$\forall x \forall y\,(x = y \rightarrow \forall z(Ozx \leftrightarrow Ozy))$	L7
(3)	$\forall x \forall y(\forall z(Ozx \leftrightarrow Ozy) \leftrightarrow x = y)$	PL $((\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)) \rightarrow (\varphi \leftrightarrow \psi)$ 1, 2
(4)	$\Box \forall x \forall y(\forall z(Ozx \leftrightarrow Ozy) \leftrightarrow x = y)$	R2, 3
(5)	$\forall x \forall y\,(x = y \rightarrow \Box x = y)$	T17
(6)	$\forall x \forall y(\forall z(Ozx \leftrightarrow Ozy) \rightarrow \Box x = y)$	PL $(((\varphi \leftrightarrow \psi) \wedge (\psi \rightarrow \chi)) \rightarrow$ $\rightarrow (\varphi \rightarrow \chi))$, 3, 5
(7)	$\forall x \forall y(\forall z(Ozx \leftrightarrow Ozy) \rightarrow \Box \forall z(Ozx \leftrightarrow Ozy))$	K $((\Box(\varphi \leftrightarrow \psi) \wedge (\varphi \rightarrow \Box \psi)) \rightarrow$ $\rightarrow (\varphi \rightarrow \Box \varphi))$, 4, 6

As the reader can easily tell from the proof above, we do not even need to endorse *KT* to prove T33, as no axiom or theorem of *KT* is used. All we need is system *K*. This is relevant. It means that even while keeping our commitments at the lowest, we are still above to prove this theorem. All that it takes is to endorse QML and have a necessitated version of A2 and A4, which together prove the necessitated version of the extensionality of overlap. The validity T17 plays a profound role in this proof. Parthood and identity are tied by bonds that are as strong as blurred. While, as we will see, the strongest version of the necessity of parthood, a.k.a. mereological essentialism, is not provable unless a further axiom is added, the interplay between T17 and the extensionality principles of CEM delivers several interesting modal theorems. T33 is one of them, and the procedure I used to prove it can be adapted to the other extensionality principles we encountered in 1.2:

T34. Necessity of P-Extensionality: $\forall x \forall y(\forall z(Pzx \leftrightarrow Pzy) \rightarrow \Box \forall z(Pzx \leftrightarrow Pzy))$.[75]

T35. Necessity of PP-Extensionality: $\forall x(\exists w PPwx \rightarrow \forall y(\forall z(PPzx \leftrightarrow PPzy) \rightarrow$ $\rightarrow \Box \forall z(PPzx \leftrightarrow PPzy)))$.[76]

T36. Necessity of Fusion Uniqueness: $\forall z \forall w((F\varphi z \wedge F\varphi w) \rightarrow \Box(F\varphi z \wedge F\varphi w))$.[77]

T34 says that if two entities are part of each other, they necessarily are. T35 rules out the possibility of two distinct things not sharing the same proper parts they share in the actual world

[75] Proof:

(1)	$\forall x \forall y(\forall z(Pzx \leftrightarrow Pzy) \rightarrow x = y)$	T5
(2)	$\forall x \forall y(\forall z(Pzx \leftrightarrow Pzy) \leftrightarrow x = y)$	L7, 1
(3)	$\Box \forall x \forall y(\forall z(Pzx \leftrightarrow Pzy) \leftrightarrow x = y)$	R2, 2
(4)	$\forall x \forall y\,(x = y \rightarrow \Box x = y)$	T17
(5)	$\forall x \forall y(\forall z(Pzx \leftrightarrow Pzy) \rightarrow \Box x = y)$	PL $(((\varphi \leftrightarrow \psi) \wedge (\psi \rightarrow \chi)) \rightarrow$ $\rightarrow (\varphi \rightarrow \chi))$, 2, 4
(6)	$\forall x \forall y(\forall z(Pzx \leftrightarrow Pzy) \rightarrow \Box \forall z(Pzx \leftrightarrow Pzy))$	K $((\Box(\varphi \leftrightarrow \psi) \wedge (\varphi \rightarrow \Box \psi)) \rightarrow$ $\rightarrow (\varphi \rightarrow \Box \varphi))$, 3, 5

[76] Proof:

(1)	$\forall x(\exists w PPwx \rightarrow \forall y(\forall z(PPzx \leftrightarrow PPzy) \rightarrow x = y))$	T4
(2)	$\forall x(\exists w PPwx \rightarrow \forall y(\forall z(PPzx \leftrightarrow PPzy) \leftrightarrow x = y))$	L7, 1
(3)	$\Box \forall x(\exists w PPwx \rightarrow \forall y(\forall z(PPzx \leftrightarrow PPzy) \leftrightarrow x = y))$	R2, 2
(4)	$\forall x \forall y\,(x = y \rightarrow \Box x = y)$	T17
(5)	$\forall x(\exists w PPwx \rightarrow \forall y(\forall z(PPzx \leftrightarrow PPzy) \rightarrow \Box x = y))$	PL $(((\varphi \leftrightarrow \psi) \wedge (\psi \rightarrow \chi)) \rightarrow$ $\rightarrow (\varphi \rightarrow \chi))$, 2, 4
(6)	$\forall x(\exists w PPwx \rightarrow \forall y(\forall z(PPzx \leftrightarrow PPzy) \rightarrow$ $\rightarrow \Box \forall z(PPzx \leftrightarrow PPzy)))$	K $((\Box(\varphi \leftrightarrow \psi) \wedge (\varphi \rightarrow \Box \psi)) \rightarrow$ $\rightarrow (\varphi \rightarrow \Box \varphi))$, 3, 5

[77] Proof:

in all possible worlds. T36 guarantees that if something is a fusion of some other things in the actual world, then it will be so in all possible worlds. As it is evident from the chain of dependence in the relative proofs, T33-T36 rely on □A2. □A2, alongside □A3 (for T34) and □A4 (for T33, T35, and T36), guarantees their provability.

As for T33, the underlying intuition of all these modal extensional theorems of CEM is that mereological modifications across possible worlds 'are subject to stringent coordination constraints' (Uzquiano, 2014, p.40). Take the Mona Lisa. Given T33-T36, modal CEM rejects the possibility that the Mona Lisa *could be* composed differently than how it is: take one single stroke away, and the painting is not the same anymore. Importantly, this should not be taken as a denial of the possibility that the Mona Lisa *could have been* composed differently. Saying that something could have been different is crucially different from saying that something could be different. Drawing a parallel with ordinary language, the former corresponds to a third conditional, whereas the latter to a second conditional. The difference between the two rests on the grade of possibility involved. A second conditional describes a situation that could happen in the future, whereas a third conditional depicts a circumstance that did not occur in the past but could have. And since past events are ordinarily regarded as unchangeable, there is no possibility that that past circumstance could be altered (although the possibility of time travel is something we wonder about). Translating conditional talks of ordinary language into metaphysical terms, a sentence like 'I could have had a brother' implies that there is a possible world in which I exist and I have a brother. That seems metaphysically plausible. On the other hand, a sentence like 'I could have a brother' implies that *in the actual world* there is still the possibility that my parents have a son, but that is biologically and metaphysically implausible. If I were to utter that sentence when I was, say, 5, it would have been possible that I would have had a brother, but not now. Coming back to our example of the Mona Lisa, consider T36. T36 says that the Mona Lisa is, in all possible worlds, the fusion of the xs it fuses in the actual world, i.e., there is no other fusion in any possible world which fuses exactly the same xs. Alternatively, consider T35. T35 affirms that the Mona Lisa has exactly the same proper parts in all possible world, or, looking at the contrapositive, that the Mona Lisa could have different proper parts – one more stroke, a fingerprint of Leonardo da Vinci and so on – only if it already does.

3.3.3 Necessity of non-Extensionality

Now that we have discussed the theorems of modal CEM deriving from T17, it is time to have a look at the other side of the coin: theorems of modal CEM stemming from the interplay between

(1)	$\forall y(Oyz \leftrightarrow \exists x(\varphi x \land Oyx))$	D7
(2)	$\forall y(Oyw \leftrightarrow \exists x(\varphi x \land Oyx))$	D7
(3)	$\forall z \forall w(z = w \to \forall y((Oyz \leftrightarrow \exists x(\varphi x \land Oyx) \land Oyw \leftrightarrow \exists x(\varphi x \land Oyx))))$	L7, 1, 2
(4)	$\forall z \forall w\, (z = w \to (F\varphi z \land F\varphi w))$	D7
(5)	$\forall z \forall w\, ((F\varphi z \land F\varphi w) \to z = w)$	T7
(6)	$\forall z \forall w\, ((F\varphi z \land F\varphi w) \leftrightarrow z = w)$	PL $(((\varphi \to \psi) \land (\psi \to \varphi)) \to$ $\to (\varphi \leftrightarrow \psi))$, 4, 5
(7)	$\Box \forall z \forall w\, ((F\varphi z \land F\varphi w) \leftrightarrow z = w)$	R2, 6
(8)	$\forall x \forall y\, (x = y \to \Box x = y)$	T17
(9)	$\forall z \forall w((F\varphi z \land F\varphi w) \to \Box z = w)$	PL $(((\varphi \leftrightarrow \psi) \land (\psi \to \chi)) \to$ $\to (\varphi \to \chi))$, 6, 8
(10)	$\forall z \forall w((F\varphi z \land F\varphi w) \to \Box(F\varphi z \land F\varphi w))$	K $((\Box(\varphi \leftrightarrow \psi) \land (\varphi \to \Box\psi)) \to$ $\to (\varphi \to \Box\varphi))$, 7, 9

distinctness – as codified in T19 – and extensionality. Let's follow again Uzquiano (2004). After proving T33, he writes that the necessity of non-overlap is a theorem of modal CEM too:

T37. Necessity of non-O-Extensionality: $\forall x \forall y(\neg\forall z(Ozx \leftrightarrow Ozy) \rightarrow \Box\neg\forall z(Ozx \leftrightarrow Ozy))$.

The fundamental difference between T37 and T33 lies in the logic that must be endorsed to prove each of them. While T33, as we have already said, is derivable in any system of logic K, T37 needs a logic that is at least as strong as KB to become provable. Besides that, the necessitated axioms that need to be in place are the same, viz., $\Box$ A2 and $\Box$ A4.

We can prove this theorem by means of the strategy we have been using so far:

(1)	$\forall x \forall y(x \neq y \rightarrow (\neg\forall z\, Ozx \leftrightarrow Ozy))$	contrapositive of T6
(2)	$\forall x \forall y(x \neq y \leftrightarrow (\neg\forall z\, Ozx \leftrightarrow Ozy))$	contrapositive of L7, 1
(3)	$\Box\forall x \forall y(x \neq y \leftrightarrow (\neg\forall z\, Ozx \leftrightarrow Ozy))$	R2, 2
(4)	$\forall x \forall y\,(x \neq y \rightarrow \Box x \neq y)$	T19
(5)	$\forall x \forall y((\neg\forall z\, Ozx \leftrightarrow Ozy) \rightarrow \Box x \neq y)$	PL $(((\varphi \leftrightarrow \psi) \wedge (\varphi \rightarrow \chi)) \rightarrow$ $\rightarrow (\psi \rightarrow \chi))$, 2, 4
(6)	$\forall x \forall y(\neg\forall z(Ozx \leftrightarrow Ozy) \rightarrow \Box\neg\forall z(Ozx \leftrightarrow Ozy))$	K $((\Box(\varphi \leftrightarrow \psi) \wedge (\psi \rightarrow \Box\varphi)) \rightarrow$ $\rightarrow (\psi \rightarrow \Box\psi))$, 3, 5

T37 says that if two things do not overlap all the same things, then they never do so. Recalling the example of Tibbles, the Cat, T37 states that Tibbles and Tib can never overlap all the same things. This might sound confusing if not contradictory, so I would like to stop here to (hopefully) clarify what is going on. In the eyes of CEM, Tib is a proper part of Tibbles and, by A4, we know that a proper part cannot be identical to the whole, no matter how big that proper part is. The reason why the implications of T37 leave us perplexed is, I think, our inner tendency to see diachronic (or, in modal terms, trans-world) identity where, strictly speaking, there is none. This thought has a strong humean aftertaste:

> We have a distinct idea of an object, that remains invariable and uninterrupted thro' a suppos'd variation of time; and this idea we call that of *identity* or *sameness*. We have also a distinct idea of several different objects existing in succession, and connected together by a close relation; and this to an accurate view affords as perfect a notion of *diversity*, as if there was no manner of relation among objects. But tho' these two ideas of identity, and a succession of related objects be in themselves perfectly distinct, and even contrary, yet 'tis certain, that in our common way of thinking they are generally confounded with each other. That action of the imagination, by which we consider the uninterrupted and invariable object, and that by which we reflect on the succession of related objects, [...] is the cause of the confusion and mistake, and makes us substitute the notion of identity, instead of that of related objects. (Hume, 1739/1960, book I, part IV, section VI, pp. 253-254, author's emphasis)

Although we might be tempted to think that, after losing his tail, Tibbles becomes identical to Tib, we must bear in mind that they are so only in a loose, commonsensical way. When we do so, we are misled by two different conceptions of Tibbles we have: the cat who, albeit losing his tail, is still the same, and the cat Tibbles was before losing his tail. Our desire to keep these two thoughts together arguably leads us to say 'Tibbles has become identical to Tib', which, unpacked, means something like 'Tibbles lost his tail, and since Tib is Tibbles minus his tail, he is now identical to

Tib, but Tib is still Tibbles'. It is no mystery that personal identity is a tricky issue. And that is all I will say about it. The crucial point here is that T37 draws a sharp mereological distinction between Tibbles and Tib regardless of our concerns about personal identity. The crucial point here is that T37 draws a sharp mereological distinction between Tibbles and Tib regardless of our concerns about personal identity. Tibbles overlaps different xs from Tib – the simple fact of having Tib as a proper part, while Tib cannot, is exemplary – and this is a sufficient condition for them not to be identical in any possible world. Full stop.

Following the same tactic, the necessity of non-extensionality can be derived for all the other extensionality principles:

T38. Necessity of non-P-Extensionality: $\forall x \forall y(\neg\forall z(Pzx \leftrightarrow Pzy) \rightarrow \Box\neg\forall z(Pzx \leftrightarrow Pzy))$[78]

T39. Necessity of non-PP-Extensionality: $\forall x \forall y(\neg(\exists w PPwx \rightarrow \forall z(PPzx \leftrightarrow PPzy)) \rightarrow$
$\rightarrow \Box\neg(\exists w PPwx \rightarrow \forall z(PPzx \leftrightarrow PPzy)))$[79]

T40. Necessity of non-Fusion-Uniqueness: $\forall z \forall w(\neg(F\varphi z \wedge F\varphi w) \rightarrow \Box\neg(F\varphi z \wedge F\varphi w))$[80]

T38 says that if two things are not part of each other, then, necessarily, they are not. T39 prohibits two different things not sharing the same proper parts in the actual world from doing so in some other possible world: it is not possible that the same proper parts compose different things in different possible worlds unless they already do. T40 guarantees that if two objects are not the fusions of the same φs, they are necessarily not so. In other words, if something is not a fusion of the φs, then it is not their fusion in any possible world.

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78 Proof:

(1)	$\forall x \forall y(x \neq y \rightarrow (\neg\forall z Pzx \leftrightarrow Pzy))$	contrapositive of T5
(2)	$\forall x \forall y(x \neq y \leftrightarrow (\neg\forall z Pzx \leftrightarrow Pzy))$	contrapositive of L7, 1
(3)	$\Box\forall x \forall y(x \neq y \leftrightarrow (\neg\forall z Pzx \leftrightarrow Pzy))$	R2, 2
(4)	$\forall x \forall y\ (x \neq y \rightarrow \Box x \neq y)$	T19
(5)	$\forall x \forall y((\neg\forall z\ Pzx \leftrightarrow Pzy) \rightarrow \Box x \neq y)$	PL $(((\varphi \leftrightarrow \psi) \wedge (\varphi \rightarrow \chi)) \rightarrow$ $\rightarrow (\psi \rightarrow \chi))$, 2, 4
(6)	$\forall x \forall y(\neg\forall z(Pzx \leftrightarrow Pzy) \rightarrow \Box\neg\forall z(Pzx \leftrightarrow Pzy))$	K $((\Box(\varphi \leftrightarrow \psi) \wedge (\psi \rightarrow \Box\varphi)) \rightarrow$ $\rightarrow (\psi \rightarrow \Box\psi))$, 3, 5

79 Proof:

(1)	$\forall x \forall y(x \neq y \rightarrow \neg(\exists w PPwx \rightarrow \forall z(PPzx \leftrightarrow PPzy)))$	contrapositive of T4
(2)	$\forall x \forall y(x \neq y \leftrightarrow \neg(\exists w PPwx \rightarrow \forall z(PPzx \leftrightarrow PPzy)))$	contrapositive of L7, 1
(3)	$\Box\forall x \forall y(x \neq y \leftrightarrow \neg(\exists w PPwx \rightarrow \forall z(PPzx \leftrightarrow PPzy)))$	R2, 2
(4)	$\forall x \forall y\ (x \neq y \rightarrow \Box x \neq y)$	T19
(5)	$\forall x \forall y(\neg(\exists w PPwx \rightarrow \forall z(PPzx \leftrightarrow PPzy)) \rightarrow \Box x \neq y)$	PL $(((\varphi \leftrightarrow \psi) \wedge (\varphi \rightarrow \chi)) \rightarrow$ $\rightarrow (\psi \rightarrow \chi))$, 2, 4
(6)	$\forall x \forall y(\neg(\exists w PPwx \rightarrow \forall z(PPzx \leftrightarrow PPzy)) \rightarrow$ $\rightarrow \Box\neg(\exists w PPwx \rightarrow \forall z(PPzx \leftrightarrow PPzy)))$	K $((\Box(\varphi \leftrightarrow \psi) \wedge (\psi \rightarrow \Box\varphi)) \rightarrow$ $\rightarrow (\psi \rightarrow \Box\psi))$, 3, 5

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(1)	$\forall z \forall w(z \neq w \rightarrow \neg\,(F\varphi z \wedge F\varphi w))$	contrapositive of T7
(2)	$\forall z \forall w(z \neq w \leftrightarrow \neg\,(F\varphi z \wedge F\varphi w))$	contrapositive of L7, 1
(3)	$\Box\forall z \forall w(z \neq w \leftrightarrow \neg\,(F\varphi z \wedge F\varphi w))$	R2, 2
(4)	$\forall x \forall y\ (x \neq y \rightarrow \Box x \neq y)$	T19
(5)	$\forall z \forall w(\neg(F\varphi z \wedge F\varphi w) \rightarrow \Box z \neq w)$	PL $(((\varphi \leftrightarrow \psi) \wedge (\varphi \rightarrow \chi)) \rightarrow$ $\rightarrow (\psi \rightarrow \chi))$, 2, 4
(6)	$\forall z \forall w(\neg(F\varphi z \wedge F\varphi w) \rightarrow \Box\neg(F\varphi z \wedge F\varphi w))$	K $((\Box(\varphi \leftrightarrow \psi) \wedge (\psi \rightarrow \Box\varphi)) \rightarrow$ $\rightarrow (\psi \rightarrow \Box\psi))$, 3, 5

3.3.4 Necessary extensionality and mereological essentialism

In system *KTB*, the following is a mereological instance of T10:

T41. De Re/De Dicto O-Extensionality: $\forall x \forall y \forall z \Box(Ozx \leftrightarrow Ozy) \leftrightarrow \Box \forall x \forall y \forall z(Ozx \leftrightarrow Ozy)$

T41 draws an equivalence between *de re* and a *de dicto* overlap by stating that it is necessary that something overlaps some other things *de re* if, and only if, it necessarily overlaps them. By means of T33 and T41, we can then prove the following theorem:

T42. Necessary O-Extensionality: $\forall x \forall y(\forall z(Ozx \leftrightarrow Ozy) \rightarrow \forall z \Box(Ozx \leftrightarrow Ozy))$[81]

T42 is the *de re* version of T33. It says that if two things overlap all the same things, then they necessarily do so. Importantly, both T41 and T42 are valid in mereological systems endorsing at least *KB*. This is due to the fact that they rely on T10, which we have seen follows from the interaction of BF (T9) and CBF (T8) – to recall, BF (T9) is provable in systems endorsing the Brouwerian axiom (L11). Now, since both T41 and T42 rely on an extensionality theorem, T6, the same strategy could be used to derive similar modal extensionality theorems:

T43. De Re/De Dicto P-Extensionality: $\forall x \forall y \forall z \Box(Pzx \leftrightarrow Pzy) \leftrightarrow \Box \forall x \forall y \forall z(Pzx \leftrightarrow Pzy)$

T44. Necessary P-Extensionality: $\forall x \forall y(\forall z(Pzx \leftrightarrow Pzy) \rightarrow \forall z \Box(Pzx \leftrightarrow Pzy))$[82]

T45. De Re/De Dicto PP-Extensionality: $\forall x \Box(\exists w PPwx \rightarrow \forall y \forall z(PPzx \leftrightarrow PPzy)) \leftrightarrow$
$\leftrightarrow \Box \forall x(\exists w PPwx \rightarrow \forall y \forall z(PPzx \leftrightarrow PPzy))$

T46. Necessary PP-Extensionality: $\forall x(\exists w PPwx \rightarrow \forall y(\forall z(PPzx \leftrightarrow PPzy) \rightarrow$
$\rightarrow \forall z \Box(PPzx \leftrightarrow PPzy)))$[83]

T44 says that if two distinct objects are part of each other, then they necessarily are. T46 says that if two distinct objects have the same proper parts, they necessarily do. Now, given the equivalence between the *de dicto* and *de re* interpretation in *KB*, T36 can also be given a *de re* reading: if two distinct objects are coextensional fusions, then they necessarily are. The underlying thought of all these theorems is that two actual coextensional entities can never fail to be so. Coextensional entities remain coextensional in every possible world. Contrarily to the extensionality of overlap, parthood, and proper parthood, fusion uniqueness does not present two distinct formulations for the *de dicto* and *de re* interpretation. The same theorem, $\forall z \forall w((F\varphi z \wedge F\varphi w) \rightarrow \Box(F\varphi z \wedge F\varphi w))$, can be read in both ways, then.

[81] Proof:

(1)	$\forall x \forall y(\forall z(Ozx \leftrightarrow Ozy) \rightarrow \Box \forall z(Ozx \leftrightarrow Ozy))$	T33
(2)	$\forall x \forall y \forall z \Box(Ozx \leftrightarrow Ozy) \leftrightarrow \Box \forall x \forall y \forall z(Ozx \leftrightarrow Ozy)$	T41
(3)	$\forall x \forall y(\forall z(Ozx \leftrightarrow Ozy) \rightarrow \forall z \Box(Ozx \leftrightarrow Ozy))$	PL $(((\varphi \rightarrow \psi) \wedge (\psi \leftrightarrow \chi)) \rightarrow (\varphi \rightarrow \chi))$, 1, 2

[82] Proof:

(1)	$\forall x \forall y(\forall z(Pzx \leftrightarrow Pzy) \rightarrow \Box \forall z(Pzx \leftrightarrow Pzy))$	T34
(2)	$\forall x \forall y \forall z \Box(Pzx \leftrightarrow Pzy) \leftrightarrow \Box \forall x \forall y \forall z(Pzx \leftrightarrow P)$	T43
(3)	$\forall x \forall y(\forall z(Pzx \leftrightarrow Pzy) \rightarrow \forall z \Box(Pzx \leftrightarrow Pzy))$	PL $(((\varphi \rightarrow \psi) \wedge (\psi \leftrightarrow \chi)) \rightarrow (\varphi \rightarrow \chi))$, 1, 2

[83] Proof:

(1)	$\forall x(\exists w PPwx \rightarrow \forall y(\forall z(PPzx \leftrightarrow PPzy) \rightarrow \Box \forall z(PPzx \leftrightarrow PPzy)))$	T35
(2)	$\forall x \Box(\exists w PPwx \rightarrow \forall y \forall z(PPzx \leftrightarrow PPzy)) \leftrightarrow$ $\leftrightarrow \Box \forall x(\exists w PPwx \rightarrow \forall y \forall z(PPzx \leftrightarrow PPzy))$	T45
(3)	$\forall x(\exists w PPwx \rightarrow \forall y(\forall z(PPzx \leftrightarrow PPzy) \rightarrow \forall z \Box(PPzx \leftrightarrow PPzy)))$	PL $(((\varphi \rightarrow \psi) \wedge (\psi \leftrightarrow \chi)) \rightarrow (\varphi \rightarrow \chi))$, 1, 2

Given the crucial role A2 plays in the theorems we proved so far, it does not come as a surprise that the necessity of mutual parthood is another theorem of the system:

T47. Necessary Mutual Parthood: $\forall x \forall y((Pxy \wedge Pyx) \rightarrow \Box(Pxy \wedge Pyx))$.[84]

With the *de re* version of the extensionality theorems, we reach the bottom line of what modal CEM proves under the conditions we stated at the beginning. It is now time to formally prove why modal CEM does not imply mereological essentialism (ME) despite coming close. Consider the the following passage from Chisholm (1973) encoding ME:

> The puzzle pertains to what I shall call the principle of mereological essentialism. The principle may be formulated by saying that, for any whole x, if x has y as one of its parts, then y is part of x in every possible world in which x exists. The principle may also be put by saying that every whole has the parts that it has necessarily, or by saying that if y is part of x then the property of having y as one of its parts is essential to x. If the principle is true, then if y is ever part of x, y will be part of x as long as x exists. (p.582)

This quote states that if x is part of y at the actual world, then it is so in every possible world. We can formalise this thought as follows:

P2. ME: $\forall x \forall y(Pxy \rightarrow \Box Pxy)$

If we wish to take actuality into consideration, we can also formalise ME as follows:[85]

P3. *E*ME: $\forall x \forall y(Pxy \rightarrow \Box(Ey \rightarrow Pxy))$[86]

P3 specifies that if x is part of y, then it is necessary that if y *actually* exists, x is part of it. Otherwise put, x is necessarily part of y only insofar as y is an actual entity. However, given our present circumstances, we need not discuss this more common actualist formulation of ME. If we can prove that T47 is distinct from P2, its actualist relativisation will be as well.

Prima facie, T47 looks dangerously similar to P2. At a closer look, however, the fundamental difference between the two easily comes to light. While $\forall x(\varphi x \rightarrow \psi x)$ entails $\forall x \varphi x \rightarrow \forall x \psi x$, the converse is generally not valid. In the case at stake, for instance, it is not. But consider, for

[84] Proof:

(1)	$\forall x \forall y(Pxy \wedge Pyx) \rightarrow x = y$	A2
(2)	$\forall x Pxx$	A1
(3)	$\forall x \forall y(x = y \rightarrow (Pxy \wedge Pyx))$	FOL $((\forall x \forall y((\varphi xy \wedge \varphi yx) \rightarrow x = y) \wedge \forall x \varphi xx) \rightarrow$ $\rightarrow \forall x \forall y(x = y \rightarrow (\varphi xy \wedge \varphi yx)))$, 1, 2
(4)	$\forall x \forall y(Pxy \wedge Pyx) \leftrightarrow x = y$	PL $(((\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)) \rightarrow \varphi \leftrightarrow \psi)$, 1, 3
(5)	$\Box(\forall x \forall y(Pxy \wedge Pyx) \leftrightarrow x = y)$	R2, 4
(6)	$\forall x \forall y\ (x = y \rightarrow \Box x = y)$	T17
(7)	$\forall x \forall y((Pxy \wedge Pyx) \rightarrow \Box x = y)$	PL $(((\varphi \leftrightarrow \psi) \wedge (\psi \rightarrow \chi)) \rightarrow$ $\rightarrow (\varphi \rightarrow \chi))$, 4, 6
(8)	$\forall x \forall y((Pxy \wedge Pyx) \rightarrow \Box(Pxy \wedge Pyx))$	K $((\Box(\varphi \leftrightarrow \psi) \wedge (\varphi \rightarrow \Box\psi)) \rightarrow$ $\rightarrow (\varphi \rightarrow \Box\varphi))$, 5, 7

[85] Alternative formulations include, for instance, $\exists x \bigvee_{1 \leq k \leq n} x = y_k \rightarrow \exists y \Box Fu(y, [\bigvee_{1 \leq k \leq n} x = y_k])$ (Uzquiano, 2014).

[86] Adapted from the the WME principle in Simons (1987, p.272). The box in front of the formula has been removed, quantifiers have been added, and the tensed vocabulary has been omitted. An analogous formulation can also be found in Cotnoir and Varzi (2021, p.260).

instance, the following valid formula: $\forall x(Px \wedge Qx) \rightarrow \forall xPx$. The following is a valid entailment: $\forall x(Px \wedge Qx) \rightarrow \forall xPx \vDash \forall x((Px \wedge Qx) \rightarrow Px)$. The entailment holds the other way around too, so we have a bidirectional distribution of the universal quantifier. Alternatively, we can consider a modal formula involving the identity predicate. The universal quantifier distributes in both directions again: $\forall x \forall y\, x = y \rightarrow \forall x \forall y\, \Box x = y \vDash \forall x \forall y(x = y \rightarrow \Box x = y)$, and *vice versa*. Why does the universal quantifier behave so differently in these two examples? A partial explanation comes from the different nature of the formulas involved. While $\forall x(Px \wedge Qx) \rightarrow \forall xPx$ and $\forall x \forall y\, x = y \rightarrow \forall x \forall y\, \Box x = y$ are tautologically valid, T47 is not. A second crucial element emerges after remembering that it is a modal scenario we are considering. Predicates change their extensions across possible worlds, the only exception (in our system) being precisely the identity predicate. Thus, while the extension of the identity predicate stays fixed in $\forall x \forall y\, x = y \rightarrow \forall x \forall y \Box x = y$, there is no alike guarantee for the parthood predicate in T47. We would get that guarantee if we were to grant the parthood relation the same status as the identity relation, but that is not a lightweight decision to make. Regardless of the arguments raised in favour of the Composition As Identity thesis (CAI), the philosophical fog is still too thick to cut with a knife and, therefore, it would be unwise to treat parthood as identity.

The universal quantifier distributes only in one direction in our case because T47 is neither a tautology nor the predicate it contains stays fixed across possible worlds. Thus, P2 entails T47, but T47 does not entail P2. That is why P2 is not derivable in the system unless we add some further axiom or modify one of the existing ones (A5 most likely). Many philosophers recognise this fact in the literature. Uzquiano (2014), for instance, adopts the latter strategy and puts forward a rigidified version of A5 that makes P2 provable (see p.44-46). Van Cleve (1985), on the other hand, shows that we can prove P2 if, alongside $\Box$A5 and a *de re* version of uniqueness of composition (T36), we also add a separation principle (see p.596-599). Both these approaches allow us to derive P2 provided that the relevant modifications are made. However, my current interest is investigating what happens to CEM *as it is* once QML becomes its underlying logical system. As if we were on a beach, looking at what the sea has left on the shore, I want to see what CEM gives us back when interacting with QML. We have pointed out that CEM is not committed to it already in 2.2, and we have just given a formal proof of why our intuitions were correct. Let's repeat it one more time, then: CEM does not lead to mereological essentialism. Proving P2 within CEM is possible, but it is *not* a thesis that CEM *logically* implies and that you can therefore 'get for free'. What we get for free, though, are theorems involving the necessity of extensionality.

Now, what is the difference between them and P2? P2 says that once the mereological arrangement is given, it cannot be altered: everything actually existing is mereologically rigidly designated. Here is, again, Chisholm:

> [...] that particular table is *necessarily* made up of that particular stump and that particular board. [...]. We are saying, in application to our example of the table, that there exists an x, a y, and a z such that: x is identical with this table, y is identical with this stump, z is identical with this board, and x is such that, in every possible world in which x exists, it is made up of y and z. Our statement says nothing whatever about the way in which human beings may happen to conceive or to look upon such things as this table. (1973, p.583)

As we have already said, the extensional theorem corresponding to P2 is T47. T47 says that, for any x and y, if x is part of y and y is part of x, then they are part of each other at every possible world. The philosophical weight carried by T47 and alike theorems is much weaker than that of P2. While the former guarantees that coextensional entities remain so across possible worlds, the latter warrants the stronger claim that if something is part of something else, it remains so across all possible worlds. In other words, it takes less to satisfy T47 because all it is required is that x and y remain coextensional, regardless of whether they stay the same. No constraint is imposed on them being the exact *same* coextensional entities they are in the actual world. Contrarily, P2 rules out the possibility of mereological modal change. Whatever the actual mereological arrangement, it holds necessarily. The following model, $\langle W, R, D, I \rangle$, inspired by the passage above, makes a compelling case:

W={0,1}

R=$\{\langle 0,0\rangle, \langle 0,1\rangle, \langle 1,0\rangle, \langle 1,1\rangle\}$

D=$\{x, y, z, w\}$

I(P)$_{w_0}$=$\{< x, x >, < y, y >, < y, x >, < z, z >, < z, x >\}$

I(P)$_{w_1}$=$\{< w, w >, < y, y >, < y, w >, < z, z >, < z, w >\}$

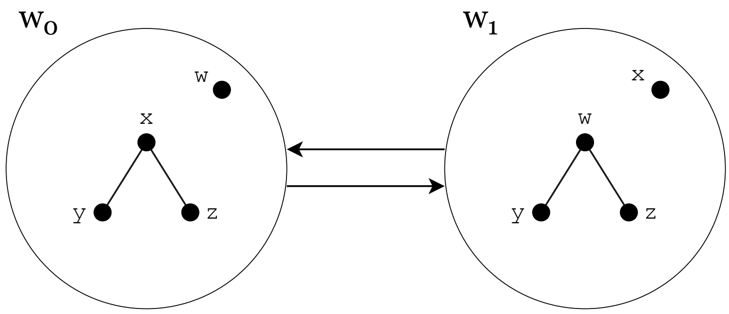

Figure 5: A model invalidating P2 that yet validates T47 and alike theorems.

Figure 5 graphically shows[87] that while T47 admits that the same predicate might be satisfied by different objects at different possible worlds, P2 abhors that. The former allows 'that particular stump' (y) and 'that particular board' (z) to fail to compose 'that particular table' (x) in some possible worlds and instead compose another one, w. In other words, our rudimentary table x might not exist in a different possible world notwithstanding the existence of y and z. The latter rules all this out. It states that if our table is composed of that particular stump and board, they compose it *necessarily*, i.e., there cannot exist a world in which x exists and y and z are not its parts. There is a core difference between T47 and P2 that makes them have different implications: while T47 codifies the *necessity of the extensionality of parthood*, P2 codifies the *necessity of parthood*. Talking about the 'necessity of parthood' is indeed ambiguous. It can refer to the extensionality of parthood (as in T47) but also to parthood itself (as in P2), and this is, in my opinion, where waters get muddy. But waters are shallow here. We only need to step back and recollect how T47 and alike theorems are proven. By doing so, we easily see that mereology-wise all those proofs get extensionality theorems as inputs and give back a corresponding bundle of modal extensionality

[87] The arrows for reflexivity have been omitted.

theorems. That we proved modal theorems through the extensionality principles of CEM is crucial not only to understand why the model in Figure 5 validates modal extensionality theorems like T47 yet invalidates other modal mereological theorems like P2 and P3, but also to make sense of the 'two seemingly contrasting thoughts' (Cotnoir and Varzi, 2021, p. 261) that our system proves, namely the necessity of identity and the non-necessity of parthood. Take a statue and the lump of clay it is made of. They are part of each other in the actual world, but while P2 rules that they remain so across all possible worlds, T47 allows the lump of clay to be part of, say, a vase or three distinct pottery mugs. As long as coextensionality is respected, there is no restriction on what is part of what across possible worlds. As long as the variables are not rigidly designated, everything can happen when moving from one world to another. Recall the model above. a can be the fusion of b and c at w_0 (under $v_{w_0}(x) = a$, $v_{w_0}(y) = b$, $v_{w_0}(z) = c$), and yet fail to be so at w_1 because d is the fusion of b and c instead (under $v_{w_0}(w) = d$, $v_{w_0}(y) = b$, $v_{w_0}(z) = c$). As Uzquiano points out, 'even if we assume variables such as x and y are rigid designators, what reason is there to assume the complex term $x+y$ to be rigid as well?' (2014, p.42). Summing up, T47 proves that if x and w are coextensional entities at w_0, then they are so in every possible world. Given that they are coextensional, a statue and its lump of clay can never change their parts (unless they already do). But, in other possible worlds, the lump of clay complies with the necessity of extensionality even if it is part of a vase rather than a statue. P2, on the other hand, imposes that the clay is only part of the statue across possible worlds.

3.3.5 Unrestricted Composition in a Modal Scenario

The time has come to discuss the issue of unrestricted composition in a modal scenario. As I have already anticipated, endorsing either A5 or $\Box$A5 are not the only options we have. By making use of the existential relativisation procedure we encountered in 3.1, A5 can be suitably relativised to actual existents as follows:

$$(\exists x\varphi x \to \exists z F\varphi z)^{\mathcal{E}} \equiv (\exists x\varphi x)^{\mathcal{E}} \to (\exists z F\varphi z)^{\mathcal{E}} \equiv (\exists x(\mathcal{E}x \land \varphi^{\mathcal{E}}x)) \to (\exists z(\mathcal{E}z \land F\varphi^{\mathcal{E}}z))$$

The following is the relativised version of A5 we thereby obtain:

A6. A5$^{\mathcal{E}}$: $\exists x(\mathcal{E}x \land \varphi^{\mathcal{E}}x) \to \exists z(\mathcal{E}z \land F\varphi^{\mathcal{E}}z)$.

A6 states that if there is at least one *actually* existing entity satisfying a certain *actual* condition φ, then there is an actual entity which is the fusion of all those entities *actually* satisfying that condition. The different metaphysical import A6 bears compared to A5 is evident. While A5 grants the existence of fusions regardless of their ontological status – actual, possible, merely possible – A6 restricts unrestricted composition to range over actual entities satisfying a certain actualist relativisation of φ. In *CD*, A5 inevitably becomes stronger than in the usual non-modal scenario. In the hands of a possibilist, A5 makes our ontology expand exponentially by warranting the existence of fusions that standard CEM itself would not recognise. CEM is *extensional*, meaning that it deals with the set of all *actual* things satisfying that particular predicate which is parthood. Mereology is not, strictly speaking, involved in the ontological inquiry of drawing up a catalogue of what there is:

> [...] mereology is meant to be a general theory of parthood relations. And as a general theory, its commitment to extensionality pertains to entities of *any* kind. Do x and y have the same parts? Identical. Do they overlap the same things? Identical. Are they disjoint from the same things? Identical. (Cotnoir and Varzi, 2021, p. 75)

Thus, mereology ranges over all the entities ontology recognises. In *CD*, where the domain remains the same across possible worlds, A5 let the possibilist play freely: there is a fusion of Wittgenstein's possible son, the moon, trout-turkeys, W. V. Quine, all the pizzas I had in my life, the Great Sphinx of Giza, and Pegasus, for example. When FOL is the underlying logical setting of CEM, the extension of the parthood predicate is restricted to the set of all actual entities. But when QML becomes the logical framework of CEM, which entities fall under the parthood predicate crucially depends on whether *CD* or *VD* is adopted. If we allow for unrestricted quantifiers, the existence of weird fusions is guaranteed by A5. A6 seems to bring some order to this scenario by bringing the commitments down to CEM's standard level: only things that exist in the actual world can be fused together. That seems a plausible and desirable principle to have. But this is not, strictly speaking, what A6 says. By imposing an existential relativisation to φ, A6 is explicitly requiring that all the quantifiers involved in φ are restricted to actual entities.[88] A6 only grants the existence of fusions fusing together actual entities which satisfy an $\mathcal{E}$-restricted version of φ. Take the predicate 'being married', M, for example, and let φ be $\exists x \exists y\, Mxy$. We already know that if A5 is endorsed, the fusion of the xs satisfying φ includes, say, Ceasar and Cleopatra, and Anna Karenina and Alexei Alexandrovich Karenin. We already know that they are automatically excluded from that fusion if A6 is adopted because they are not actual entities and thus the first conjunct of the antecedent cannot be satisfied. Nothing new so far, then. But consider the existential relativisation of φ, $\varphi^{\mathcal{E}}$, that A6 requires: $\exists x \exists y(\mathcal{E}x \wedge \mathcal{E}y \wedge M^{\mathcal{E}}xy)$. 'To be married to someone', M, becomes 'to be married to someone actually existing', $M^{\mathcal{E}}$, which we might also render as 'to be actually married'. This means that no actual individual who is not married to another actual individual can ever be part of the fusion A6 recognises. This, too, seems a desirable result A6 brings about. There is a good reason if it is legally recognised that, when a partner dies, the other spouse changes their status from 'married' to 'widowed' and can, ultimately, remarry. Some people, however, might still want to consider themselves married even after losing their partner. Evidently, A6 does not take such individual feelings into consideration. You are either married to an actually existing entity, and, therefore, part of the relevant fusion, or you are not and therefore excluded from it. A5, on the other hand, will grant you a spot among the xs because the requirement of an *actual* spouse is rescinded there.

Thinking more carefully about $\exists x \exists y(\mathcal{E}x \wedge \mathcal{E}y \wedge M^{\mathcal{E}}xy)$, however, it becomes clear that $M^{\mathcal{E}}xy$ might be redundant. If x and y are required to be *actual* entities, it automatically follows that if x is married at all, then it is married to an actual entity. In other words, that an individual is married to another *by default* implies that it is married to an actual individual. If the only entities admitted in the domain are the actually existing ones, then any relation whatsoever involves actually existing individuals. Given the ontological commitments already in place, it is not possible that an individual could be married to a non-actual one. To see the point more clearly, we might want to consider a limit-case scenario of A6 in which $\varphi^{\mathcal{E}}$ stands for 'not to be an actual entity' $(\neg\mathcal{E}^{\mathcal{E}}x)$. We get an empty set: no entity in the domain satisfies that condition precisely because every entity is actual. Now, what happens if we 'unload' φ from the actual existence commitment $(\neg\mathcal{E}x)$? Again, no individual in the domain satisfies it: the relevant set is empty. Short explanation: the actual subset of actually existing entities just is the subset of actually existing entities. Long explanation: what is not an actual entity is not an entity at all from an actualist perspective, so shifting from 'not to be an actual entity' to 'not to be an entity' is moving in shallow waters.

[88] I am grateful to my primary supervisor, Dr Aaron J. Cotnoir, for emphasising this crucial element of A6.

If we postulate that individuals must actually exist – as A6 does – then there is no substantial difference between saying that those individuals satisfy a certain actual condition or that they satisfy a certain condition *tout-court*. This is why specifying that x and y satisfy $M^{\mathcal{E}}$ appears to be redundant. They satisfy $M^{\mathcal{E}}$ in the same way as they satisfy M. Satisfying a certain φ instead of its existential relativisation does not make any difference to actual entities because $\varphi^{\mathcal{E}}$ *just is* φ, and *vice versa*. Therefore, my suggestion is to drop the existential relativisation on the φ and consider the following as the relevant alternative to A5:

A7. $\mathcal{E}$A5: $\exists x(\mathcal{E}x \wedge \varphi x) \to \exists z(\mathcal{E}z \wedge F\varphi z)$.

Resembling A6, A7 says that if there are some actual entities satisfying a certain condition φ, then there is some actual entity fusing them together. Besides its intuitiveness, A7 is an appealing principle to have also because its necessitated version does not look as scary as $\Box$A5:

T48. $\Box\mathcal{E}$A5: $\Box(\exists x(\mathcal{E}x \wedge \varphi x) \to \exists z(\mathcal{E}z \wedge F\varphi z))$.

T48 says that, necessarily, if some actual xs satisfy a given condition, then there is an actual fusion of them. Surely, in the eyes of CEM, it is necessary that for any actual entity satisfying a certain φ, there is an actually existing entity fusing it. T48 does not admit more fusions than those recognised by CEM in a non-modal framework. That is why we might want to resort to A7: it is a better fit with our *desiderata* than A5. $\Box$A5 grants the existence of fusions that we might not want to have around, but $\Box$A7 escapes such ontological escalation, bringing the commitments down to the ones CEM already has in a non-modal scenario. But let's now discuss the most significant results that $\Box$A7 produces. From T48, two interesting theorems of modal CEM emerge:

T49. $\mathcal{E}$A5-Possibility of Fusion: $\Diamond\exists x(\mathcal{E}x \wedge \varphi x) \to \Diamond\exists z F\varphi z$.

(1)	$\Box(\exists x(\mathcal{E}x \wedge \varphi x) \to \exists z(\mathcal{E}z \wedge F\varphi z))$	T48
(2)	$\Diamond\exists x(\mathcal{E}x \wedge \varphi x) \to \Diamond\exists z(\mathcal{E}z \wedge F\varphi z)$	K $(\Box(\varphi \to \psi) \to (\Diamond\varphi \to \Diamond\psi))$, 1
(3)	$\Diamond\exists x(\mathcal{E}x \wedge \varphi x) \to \Diamond\exists z F\varphi z$	K $((\Diamond(\varphi \wedge \psi) \to \Diamond(\chi \wedge \xi)) \to (\Diamond(\varphi \wedge \psi) \to \Diamond\xi))$, 2

T50. $\mathcal{E}$A5-Possible Fusion: $\Diamond\exists x(\mathcal{E}x \wedge \varphi x) \to \exists z\Diamond F\varphi z$.

(1)	$\Diamond\exists x(\mathcal{E}x \wedge \varphi x) \to \Diamond\exists z F\varphi z$	T49
(2)	$\Diamond\exists x(\mathcal{E}x \wedge \varphi x) \to \exists z\Diamond F\varphi z$	K $((\psi \to \Diamond\exists x\varphi x) \to (\psi \to \exists x\Diamond\varphi)x)$, 1

T49 says that if it is possible that there are some actual entities satisfying a certain φ, then there is a *de dicto* possibility that there is a fusion of them. T50, on the other hand, grants that that possibility is *de re*: there is, in the actual world, a possible fusion of the actual xs. Note that the actuality constraint on the fusion has been dropped in both theorems. One might well wish to keep the existential relativisation on the consequent, but undeniably, the theorems would be less remarkable. Moreover, the system would still prove T49 and T50. It comes without surprise that the fusion of some actual entities is itself an actual entity. What does come with surprise is instead that that fusion might not be an actual entity despite involving only actually existing entities. Moreover, if the logic adopted is at least reflexive – i.e., has L10 among its axioms – the following stronger theorems are easily derivable:

T51. $\mathcal{E}$A5-Possibility of Fusion*: $\exists x(\mathcal{E}x \wedge \varphi x) \to \Diamond\exists z F\varphi z$.

(1)	$\Diamond\exists x(\mathcal{E}x \wedge \varphi x) \to \Diamond\exists z F\varphi z$	T49
(2)	$\exists x(\mathcal{E}x \wedge \varphi x) \to \Diamond\exists z F\varphi z$	KT $((\Diamond\varphi \to \Diamond\psi) \to (\varphi \to \Diamond\psi))$, 1

T52. $\mathcal{E}$A5-Possible Fusion*: $\exists x(\mathcal{E}x \land \varphi x) \to \exists z \Diamond F\varphi z$.

(1) $\Diamond\exists x(\mathcal{E}x \land \varphi x) \to \exists z \Diamond F\varphi z$ T50
(2) $\exists x(\mathcal{E}x \land \varphi x) \to \exists z \Diamond F\varphi z$ KT $((\Diamond\varphi \to \psi) \to (\varphi \to \psi))$, 1

Although T51 is already significant, T52 is the real star here. T51 proves that if some actual entities satisfy a certain φ, then there may be a fusion (even merely possible) of those entities. T52 goes even further. It says that for those actual entities, there is something – not necessarily actual, but somewhere in the actual world – which is a possible fusion of them. As for T49 and T50, it is worth highlighting that, on the one hand, T51 and T52 require the xs to be actual entities but, on the other, they do not have the same constraint on the fusion composed by those actual entities. All these theorems seem to be suggesting is that, if we have a fusion of all the actual xs satisfying M, that fusion need not be actual to exist. Was it not to be actual, it will still exist somehow (à la possibilist). That would not be an exciting result under $\Box$A5 because it would be already among its underlying commitments that there are merely possible fusions, but it becomes so when endorsing $\Box$A7 (T48). $\Box$A7 starts with the opposite commitment, but it ends up recognising the existence of non-actual fusions. In a nutshell, the core reason why all of this matters is that starting with an actualist version of unrestricted composition does not undermine the fact that fusions might not be actual. And that result is noteworthy. Consider the modal condition 'being a possible Supreme Court judge' ($\Diamond J$). T52 affirms that for all those actual entities satisfying $\Diamond J$, i.e., all those entities which might be Supreme Court judges, there is a fusion – actual or not – of them. That T52 does not impose that the fusion must be actual (like the entities it fuses together) is relevant because we might not want to say that there is an *actual* fusion of possible Supreme Court judges. There might well be actual individuals who are possible Supreme Court judges, but why should we want to have that there is a fusion of them rambling around in the actual world too? There is a fusion of those entities, but it does not retain the same ontological weight as the entities composing it.

To recap, there are four main alternative systems we may want to consider in a modal CEM scenario:

(i) QML**+**$\Box$A1-$\Box$A4**+**A5

(ii) QML**+**$\Box$A1-$\Box$A4**+**$\Box$A5

(iii) QML**+**$\Box$A1-$\Box$A4**+**$\mathcal{E}$A5

(iv) QML**+**$\Box$A1-$\Box$A4**+**$\Box\mathcal{E}$A5

The theorems that we discussed in 3.3.2, 3.3.3, and 3.3.4 are valid in all of them. What tears (i)-(iv) apart is their respective approach toward unrestricted composition. (i) and (ii) endorse the canonical version of that thesis, with the latter necessitating it. (iii) and (iv), on the other hand, resort to an existentially relativised version of the standard axiom, with (iv) necessitating it.

Chapter 4

Formal Mereology and Applied Mereology

The investigation we carried out in the previous chapters deals with the formal notion of parthood. From a Husserlian perspective, this means that we are laying down the fundamental features that *every* mereological relation must have in order to qualify as such. However, this is not the only way to work with mereology. A fascinating and still under-developed area is what Johansson (2015) defined as 'applied mereology', the study of domain-specific parthood relations. Arguably, there are as many applied mereologies as material ontologies. Applied mereologies (or 'material mereologies') study parthood relations in particular portions of being rather than in the Being *qua* Being. Examples of this abound: atoms in a molecule (chemistry), cells in an organism (biology), thermodynamic equilibrium in a system (physics), the Battle of Waterloo in the Napoleonic Wars (history), mass nouns in a sentence (linguistics), emotions in an individual (psychology), a minority group in society (sociology), and so on.[89] Intuitively, each of them displays a different declination of the notion of parthood, such that they do not seem to share all the same features despite using the same lexical item. Even more importantly, they do not seem to conform to the features established by CEM. For instance, CEM requires the parthood relation to be transitive, and

> [...] this seems fairly obvious when we deal with the parts of an organism's body. The Pope's left ventricle is a part of his heart, and any cell in that ventricle is a part of that heart and part of the Pope. [...]. However, when we shift contexts, and ask similar questions about the relationships within such a composite whole as the Roman Catholic Church and such organisms as the Pope of which it is composed, we get an apparent paradox. It seems counter-intuitive to say that there is a whole-part relationship below the organismal level, for ordinarily we do not consider the Pope's heart or any other organ, cell or atom in his body to be a part of the Roman Catholic Church. (Ghiselin, 2007, p. 285)

We will dig deeper into this issue in a few pages. What is worth highlighting for now is the evident tension between ordinary uses of 'part' and the characterisation CEM gives. At this point, three questions naturally arise: How can we construct domain-specific mereologies? What use can

89 Calosi and Graziani (2014) provide a good overview of the different mereological relations in the sciences. Unfortunately, there is no analogous collection for the social sciences and humanities.

formal mereology be if it does not have any 'practical' application? What is, if any, the relationship between formal and material mereology?

These are all tricky questions that have not yet received the philosophical attention they deserve. In what follows, I will actively engage with them after setting the stage for their investigation.

4.1 Parthood and φ-parthood

In the literature, it has become common practice to distinguish between the notion of parthood, which refers to the basic, rawest sense of parthood CEM aims at grasping, and 'φ-parthood, which through a predicate modifier, φ, narrows down the interpretation of the relation by adding further conditions on it.[90] Following Cotnoir and Varzi (2021), we can provide two distinct definitions of φ-parthood:

D12. φ-Parthood 1: $P_\varphi xy :\equiv Pxy \wedge \varphi x$

D13. φ-Parthood 2: $P_\varphi xy :\equiv Pxy \wedge \varphi xy$

The core difference between D12 and D13 lies in the nature of the conditions involved.[91] While the former ascribes a *monadic* property to x, the latter grasps a *relational* qualification. To say that the handle of the knife is red – which means, unpacked, that the handle is part of the knife and that the handle is red – is crucially different from saying that the handle is a functional part of the knife. In this latter case, the handle serves a specific purpose within a bigger whole. Its relevant feature is relational because it only exists in light of its relationship with the whole: relational properties only exist if their relata exist. In the knife case, the handle can be a functinal part of the knife only insofar as the knife exists. The knife might not exist, but the handle could still exist and be red but fail to be a functional part (provided that it is not the functional part of something else, like another knife). The issue at stake here might seem, *prima facie*, that the handle as a functional part *ontologically depends* on the knife to be so. However, this is incorrect. The elephant in the room is the relation of ontological dependence between parthood and φ-parthood, as we will soon see.

The issue of ontological dependence is as intricate as important. There are several ways in which some x might depend on some y, but here we are interested in ontological dependence only. Ontological dependence is often regarded as an umbrella term to indicate a category of asymmetric relations in which one entity existentially depends on another.[92] It is widely recognised among contemporary metaphysicians that there are, fundamentally, two types of analysis of ontological dependence: the so-called 'modal-existential analysis' and the 'essentialist analysis' (see, for instance, Tahko and Lowe (2020)). The modal-existential analysis interprets ontological dependence in terms of possible worlds. It says that x is ontologically dependent on y if there is no possible world in which x exists, but y does not, i.e., if it is a *de re* necessity that x exists only if y exists.

90 The notion of φ-parthood has been most famously discussed by Simons (1987), A. C. Varzi (2006), Casati and Varzi (1999), and Cotnoir and Varzi (2021).

91 The debate between realism and nominalism about properties and relations is arguably one of the most trodden moot points of metaphysics; it would be dumb of me to attempt to defend one specific position over many others. Thus, I shall not take a stance on this issue.

92 The vastity and complexity of the issue at hand do not allow me to discuss it with the depth it deserves. Thus, I refer back to Correia (2008), Koslicki (2013), and Tahko and Lowe (2020) for useful surveys on the matter.

This conception of ontological dependence used to be very popular in the past,[93] but has recently come under attack, most famously by Kit Fine and other neo-Aristotelians. In turn, they defend a non-modal conception of ontological dependence, which has the notion of essence, rather than existence, at its core.[94] The essentialist analysis traces ontological dependence back to the essence of entities, namely, to the conditions that a certain thing must satisfy to be what it is. Besides the evident differences, however, both analyses classify ontological dependence among the most fundamental metaphysical relations. An interesting feature of fundamental metaphysical relations is that they seem interconnected. Consider the interaction between ontological dependence and parthood, itself considered a fundamental relation. When parthood occurs, ontological dependence is also there, and *vice versa*. One particular instance of this interplay between parthood and ontological dependence involves φ-parthood. We ask: is φ-parthood ontologically dependent on parthood? Arguably, yes. Most, if not all, of those who wrote on φ-parthood clarify that it arises once extra-mereological conditions get imposed on the parthood relation, namely, once a narrower sense of 'part' is required. While those other conditions narrow down the original interpretation of parthood, the φ-parthood relation that emerges is still intrinsically dependent on its parent relation. For this reason, many think that what is true of parthood must also be true of φ-parthood, otherwise the emergence of an 'odd subsumption relation' (see below):

> All the φs in question are said to specify (Simons) or modify (Casati and Varzi) a "broader notion of parthood." Therefore, the relational predicate '$<^{\varphi}$' ought to be to the relational predicate '$<$' what property predicates such as 'light red' and 'quickly running' are to the more general property predicates 'red' and 'running', respectively.7 What is true of 'red' is necessarily also true of the 'light red' which it subsumes, what is true of 'running' is necessarily also true of 'running quickly', and what is true of 'x $<$ y' ought necessarily be true of '$<^{\varphi}$'. Since 'x $<$ y' is transitive, '$<^{\varphi}$' ought to be so as well. But according to the Simons-Casati-Varzi analysis, the latter predicate is non-transitive. I do not think one can make sense of such an *odd subsumption relation*, and nor have the philosophers mentioned tried to. They seem simply not to have noted the issue that I have raised. (Johansson, 2004, p.165, my emphasis)

Leaving the transitivity issue aside (for now), the point Johansson is raising here is that the relation between parthood and φ-parthood turns out problematic because it fails to respect the subsumption principle. The subsumption principle states that 'when a concept is more specific than some other concept, it inherits the properties of the more general one.' (Nardi and Brachman, 2003, p.9). Putting things a bit differently, we could say that Johansson is characterising the ontological dependence between parthood and φ-parthood in terms of subsumption. In other words, he is treating the subsumption relation between φ-parthood and parthood as the one characterising the dependence of φ-parthood to parthood.

A. C. Varzi (2006) offers a prompt reply. To prove that Johansson's objection 'misfires', Varzi articulates his short yet compelling counter-argument into two main points, one concerning the validity, the other the scope of the subsumption principle. Regarding the former, Varzi argues that,

93 Among many others, Peter Simons and Ruth Barcan Markus have famously defended the modal-existential analysis of dependence. See, for instance, Simons (1987, §8, p. 294-295 in particular), and Marcus (1967).

94 As Correia (2008) points out, the distinction between analyses based on existence and analyses based on essence might be blurred. It is not unusual that ontological dependence falls into essential dependence and *vice versa*.

however intuitively appealing, the subsumption principle should not be considered unconditionally valid. Consider, Varzi continues, two concepts of which one is subsumed by the other, like 'red' and 'light red'. Now, 'light red' undeniably inherits from 'red' the property of being a colour, yet while the latter can be legitimately predicated on some wine, the former cannot. Thus, that φ-parthood fails to inherit all the features characterising parthood is not a good enough reason to talk of 'odd subsumption relation' between them. This phenomenon occurs between properties standing in a parenthood relation as much as between relations of a similar kind. The second, and even more crucial, point Varzi makes is the following. Properties of concepts should be kept distinct from the properties of those entities falling under them. This second point has a devastating impact on Johansson's objection. Through this distinction, we can see that while the subsumption principle applies to properties held by entities, it might fail to hold in the case of properties of concepts. Recalling the example given by Varzi, suppose that every red object is round. The particular shade of red an object might have (lighter or darker) is irrelevant: it will be round. That happens because the object characterised by the more specific concept inherits the properties of the more general one. However, when we deal with concepts, such inheritance is less straightforward. While 'light red' is subsumed by 'red', it does not inherit the property of subsuming 'dark red', for instance.

What does this mean for the relation between parthood and φ-parthood? At least two noteworthy results emerge from Varzi's analysis. First, by being an extensional restriction of parthood, φ-parthood adds further constraints on mereological entities. However, those entities that satisfy the φ-parthood requirements inherit all parthood's features (namely, those codified by CEM, given our current commitments). Second, precisely because parthood is a concept, the properties characterising it need not be inherited by φ-parthood. Both these points are reinforced by Varzi's distinction between monadic and relational properties, which is, in my own opinion, the most significant contribution of the article. Coming back to the difference between D12 and D13, what we have said so far would not work if φ-parthood is codified as D12. In that case, the argument proposed by Varzi collapses because every property associated with parthood must hold for φ-parthood as well. If the property is monadic, what is modified by the φ condition is one of the two relata of the parthood relation, not the relation itself. Consequently, the resulting parthood relation does not shed its skin but rather, so to say, paints on its old one. To say that the handle is part of the knife and that the handle is red adds the property of redness to the handle, leaving the parthood relation untouched. To say that the handle is part of the knife and that the handle is red, or that the handle is part of the knife (without any further monadic specification) has no implications on parthood. Contrarily, to say that the handle is a functional part of the knife predicates a certain condition to the parthood relation itself so that the relationship involved is not merely parthood anymore but φ-parthood instead. In a nutshell, only when the parthood relation itself is modified – to repeat once more, thanks to φ conditions that narrow its extension down – we can legitimately talk of φ-parthood.

4.2 Which φ-conditions?

At this point, a question about the nature of these φ-conditions naturally arises: are they chosen within or outside mereology? The answer we get from the literature is that they are chosen outside mereology.[95] This is intuitive. Mereology aims at uncovering the fundamental features of the parthood relation regardless of the specific domains in which we commonly divide reality. Thus, it automatically follows that it cannot provide a set of necessary and sufficient conditions for characterising a certain φ-parthood like social group membership. That formal mereology cannot provide an explanatory theory of φ-parthood relations should not come as a surprise: they involve notions external to formal mereology. It is rather applied mereology that is supposed to do the job. Applied mereology includes in its theorising extra-mereological concepts that formal mereology necessarily leaves aside. One exception is represented by the relation of immediate parthood, which Simons (1987, p.107-108) has famously shown to be directly derivable from CEM. The following is the definition of 'immediate part' he presents (notation adapted)[96]:

D14. Immediate Parthood: $IPxy :\equiv PPxy \wedge \neg\exists z(PPzx \wedge PPzy)$.

Evidently, immediate parthood is not transitive. If a is an immediate part of b, and b is an immediate part of c, it does not follow that a is also an immediate part of c. Is this problematic? Since immediate parthood has been defined within CEM without needing any non-mereological concept, shouldn't it inherit all the features CEM ascribes to parthood? Cotnoir and Varzi (2021) promptly clear the air. In characterising immediate parthood this way, '[...] we are simply narrowing the relation of proper part by a condition φ – expressed by the negative existential $\neg\exists z(PPzx \wedge PPzy)$ – that does not distribute over the broader notion.' (p.82).

Besides immediate parthood, however, any other φ-parthood relation employs conditions based on notions that are extrinsic to mereology and on which, consequently, formal mereology remains silent. Failing to see this has led many to think that philosophical accounts based on mereological theories (especially those based on CEM) are hopelessly doomed. The general pattern of most such critiques is the following: take a particular mereological theory (CEM is the preferred target of recent attacks), consider a specific domain of reality, apply the chosen theory to that domain, and show how poorly it performs. However, this way of proceeding is not only deceitful (despite the popularity it gained in the literature) but also shows a lack of philosophical discernment and critical thinking. I hope to have given sufficient evidence that this approach rests on the (mistaken) assumption that φ-parthood relations can be treated as general parthood. Far from being equated to the latter, the former presents a more specific and mereologically pre-determinate structure that needs extra-mereological investigation to be comprehended.

This should not be taken to mean that formal mereology has nothing to say once an applied theory of mereology is in place. The connection between formal and applied mereology is likely to be as tricky as that between pure and applied mathematics.[97] Nonetheless, the dichotomy between formal and applied mereology should not lead to a ghettoisation of the former in the name of its apparent disengagement with 'worldly things'. From a philosophical perspective, I think formal mereology can illuminate the fundamental structure of things on which superstructures have been imposed. Countless other relations investigate the interplay between a whole and its parts, yet they all are φ-parthood relations originating from a specific, presupposed understanding of the

95 To the best of my knowledge, no one has yet defended the opposite view, namely that mereology alone can specify the further conditions characterising φ-parthood. Those who have worked on φ-parthood so far (see footnote 90) have argued that φ-conditions are extrinsic to mereology as a formal theory.

96 An analogous formulation can be found in Cotnoir and Varzi (2021, p.81).

97 A very interesting article concerning the dichotomy between pure and applied mathematics is Pérez-Escobar and Sarikaya (2022). The thesis they defend is, in spirit, very similar to the one I defend here.

parthood relation. Although this is particularly evident when social sciences are involved, I think the same applies to natural sciences too.[98] Let us give an example. Suppose I want to construct a mereological theory suitable for bio-medical practices. In that case, I will take the *already* existing empirical evidence and develop a system that best accounts for it. The mereological relation between different bio-medical structures crucially relies on scientific discoveries, such that the system I propose must be in line with the pre-conditions imposed by the discipline. That the facial bones of the skull are not part of the cranial cavity is something that only the discipline of anatomy can decide.[99]

4.3 Meta-theoretical considerations

Setting formal and applied mereology apart might seem a simple and relatively innocuous solution, but it does not come immaculate. Two issues are, in my opinion, particularly pressing. The first, which is also the less problematic one, might be summed up as follows: what can formal mereology say about those things on which an applied ontology seems to be more helpful? Since formal mereology does not equip us with the kind of analysis we are used to considering valid and, moreover, its outputs contrast with our desiderata, formal mereology appears useless, if not harmful. The second issue is even trickier. We have said earlier that φ-parthood is a specification of the general parthood-relation. But how do we even know that the characterisation given by CEM really grasps parthood in its most general form, rather than another φ-parthood? In what follows, I shall deal with these questions by considering a particular φ-parthood, namely the membership relation in social groups, which will also be my case study in the next chapter. I hope this will clarify my points and provide a meta-theoretical foundation for the upcoming issues concerning social groups. Let us start by considering the former question first.

How would you react if you were to hear someone comparing social groups to mere aggregates of molecules? A similar statement, I believe, would strike many as wrong. However, there is a deep sense in which a chemist might be justified in talking of social groups in those terms. Every material object is, after all, ultimately composed of molecules. Yet, when we think about the social world, this way of looking at things does not seem to do justice to the reality we experience daily. Go tell someone who just got their house seized that the bank enforcing the seizure is an aggregate of molecules. As far as the social world is concerned, there seems to be a further level that cannot be grasped by analyses disclosing the 'true' structure beyond the simulacra of our everyday experiences. We might call this further layer 'the social dimension' or 'the social reality'. In his landmark book *The Construction of Social Reality* (1995), John R. Searle writes:

> [...] the complex structure of social reality is, so to speak, weightless and invisible. The child is brought up in a culture where he or she simply takes social reality for granted. We learn to perceive and use cars, bathtubs, houses, money, restaurants, and schools without reflecting on the special features of their ontology and without being aware that they have *a special ontology.* They seem natural to us as stones and water and trees. Indeed, if anything, in most cases it is harder to see objects as just natural

98 The views of the latter tend to be similar and therefore more compatible with those of CEM. Thus, in some cases, the scientific explanation runs along the same lines as the mereological one (think about the Pauli exclusion principle, for example).

99 For a practical example of this strategy see, for instance, Schulz et al. (2006).

> phenomena, stripped of their functional roles, than it is to to see our surroundings in terms of their *socially defined functions.* [...]. Cars are for driving; dollars for earning, spending, and saving; bathtubs for taking a bath. But once there is no function, no answer to the question, What's it for? we are left with a harder intellectual task of identifying things in terms of their intrinsic features without reference to our interests, purposes, and goals. (p.4-5, my emphasis)

Social phenomena are intrinsically characterized by their function, which we, as a society, choose for them. We might thus perceive social facts as if they were 'natural', i.e., mind-independent and, therefore, already given. However, they are not.[100] This is why they have a 'special ontology', i.e., a *sui generis* ontology that cannot be brought back to what is sometimes called 'brute facts'. It is a widespread mistake to attempt to explain a social fact without remembering that it is social, so humanly constructed despite objective. That the money market functions how it functions, that the dollar is a valid currency and that if you don't repay your debt, the bank will take your house are all social facts displaying a common feature: they are this way because we decided so. If I am stabbed through the heart, I die within a few minutes, if not seconds. Contrarily to the unavoidability of me dying if I am stabbed through the heart, the money market might function differently from how it does, the dollar might not be a currency at all, and your house might not be taken despite your failure to repay the debt. This is, I think, the fundamental underlying intuition beyond Searle's thought. But how does all of this connect to the question we are considering?

The φ-conditions of φ-parthood seem to express the unique features of mereological relations as they appear to be in specific domains of reality. In the present case, they express socially constructed relations characterising social reality. Relations like 'being a member of', 'being the grandfather of', 'being an investor of', 'being a customer of', etc., are socially constructed because society made them what they are. An interesting case comes from the different ways in which different cultures look at kinship relations. In Hindi, 'being the mother of the mother' and 'being the mother of the father' are different relations for which different terms are used. English, instead, does not linguistically draw the distinction. There are states across the world, like Afghanistan, Pakistan, Bangladesh, Iran, and Egypt, in which polygamy is legally recognised so that the features of 'being the wife of' and 'being the husband of' radically change from the ones we are used to in the Western world. Child fostering is a well-spread practice in West Africa involving a conception of parenthood that significantly contrasts with the biological/adoptive one. Even more interestingly, communities like that of the Baatombu do not have a term for distinguishing between biological and foster parents and, therefore, between 'being the biological mother of' and 'being the foster mother of'.[101] The point is that the φ-conditions we might build into a specific mereological relation are arbitrarily chosen and vary across cultures and groups of people. I take this as the point where the parthood relation becomes philosophically crucial. On the one hand, realising that the specific φ-parthood we are considering is a specification of the parthood relation can allow us to see more clearly that it is socially constructed and, therefore, intrinsically dependent on our

[100] It is contentious whether there is a distinction between what is socially constructed and what is not. That is a fascinating yet thorny issue I cannot satisfactorily confront here. For the present purposes, no commitment to such a dichotomy between 'human' and 'natural' is needed because the question I am attempting to answer does not require any preexisting foundational commitment. Moreover, my view on the matter is compatible with such dualism as well as the opposite view.

[101] This ethnographic case study can be found, for instance, in Alber (2003).

conceptual and representational schemes. We often take the φ-parthood relations of the *status quo* as the only possible alternatives. However, the fact that we, as a society, decided how to fill those φ-conditions should lead us to think that nothing is written in stone – especially given that language is another human product – and that we could, therefore, virtually change many things belonging to the social sphere. In short, we take x to be x, with all its features and meanings, because we *decided* so, and because society *recognises* x as such.[102]

On the other hand, parthood represents the fundamental mereological relation of the being *qua* being, and the features derived through CEM hold at the heart of every part and whole. By remembering that the most general and universal features of parthood are such and such, it seems that we can see what surrounds us with different eyes as if we could grasp an alternative structure alongside the one we used to see. Each of us individually confronts the fact that, far from being a monadic, independent entity, we are ultimately part of bigger wholes (because of A5), for example. Or that I must be composed of the exact parts I am composed of (given T35), regardless of how myself or others might judge them – think about past experiences, sexual orientations, physical features, etc. Or that many off-shore companies are mere 'ghost entities' whose economic transactions are, in fact, part of the economic transactions of one particular individual (who should, therefore, be accountable for them).[103] Thus, parthood is, as it were, a memorandum of (i) how things are before the superimposition of any φ-condition, and (ii) the fact that we have to look *outside* mereology in order to know what conditions need to be filled in.

Let's now move to the second issue we raised at the beginning of this section: the social construction of parthood itself. The *locus classicus* of this sort of issue is the question 'Is math invented or discovered?'. That is a pressing question indeed. It seems to arise from the contrasting intuitions we face when doing math. As a form of human knowledge, it makes sense to say that it is invented, i.e., constructed. Yet, we get a sense of perfection out of it – no wonder God and mathematics have been associated throughout history – making us think it is independent of any human construction. With a great deal of simplification, the former intuition prevails in those endorsing an 'anti-realist' position, the latter in those with a 'realist' standpoint. These alternative views characterise many other disciplines, mereology arguably being one of them. Furthermore, as no definitive argument has been put forward for either realism or anti-realism in mathematics, the same holds for mereology. In other words, we can neither affirm nor deny that formal mereology is humanly constructed. As it stands, it all depends on individual pre-theoretical assumptions. Given that, it seems natural to ask how to draw the line between parthood and φ-parthood given that they might both be humanly constructed and, even more fundamentally, whether we should be drawing that line at all. As I have already anticipated in footnote 100, I shall not take a stand on the issue. Instead, I want to approach the issue by looking again at the different goals and characters of parthood and φ-parthood. While parthood aims at grasping the fundamental, overarching features of any part-whole relation, φ-parthood is interested in looking at the features of mereological relations in restricted domains of reality. In the words of Husserl, whose philosophical conceptions have been silently guiding us throughout this dissertation:

> A part *as such* cannot exist at all without a whole whose part it is. On the other hand

[102] The phenomenon of social recognition is fascinating and intriguing, but I will not discuss it here.

[103] Thanks to websites like the International Consortium of Investigative Journalists, it is possible to 'find out who is behind more than 810,000 off-shore companies, foundations and trusts from the Pandora Papers, Paradise Papers, Bahamas Leaks, Panama Papers and Offshore Leaks investigations'.

> we say, with an eye to *independent* parts: A part often *can* exist without a whole whose part it is. Obviously this involves no contradiction. What we mean is that, if the part is treated in respect of its *internal content*, its own essence, then a thing having this same content can exist without a whole *in* which it exists; it can exist by itself, not associated with anything else, and will not then be a part. Change in, or complete elimination of associations, does not here affect the part's own, peculiarly qualified content, and does not eliminate its existence: only its relations fall away, the fact that it *is* a part. The contrary holds of other sorts of parts: without any association, as non-parts, they are unthinkable, *in virtue of their very content.* (Husserl, 1900-1901/2001, Investigation III, §11, p.20, author's emphasis)

That formal mereology studies independent parts, while applied mereology dependent ones, is the only explanatory claim we need to justify their difference. (Moreover, that claim holds regardless of whether we endorse a realist or anti-realist perspective). Thus, whether parthood, as codified by CEM, is humanly constructed is an interesting yet futile question in our situation. We are not trying to argue that while φ-parthood is socially constructed, parthood is not. All we have tried to defend is that they should be kept distinct, regardless of their origin, because they are intrinsically directed towards different scopes and domains.

Chapter 5

A Case Study: The Ontology of Social Groups

The investigation of the social world has often proven to be tricky. We live surrounded by social facts (like our native language), events (like the Olympic games), objects (like Tesla), etc., and yet, when questioned about their nature, properties and even existence, we struggle with the answer. What is particularly intriguing about the social world is that – as we have already seen in the previous chapter – it seems to contrast with the so-called 'natural world'.[104] The latter, contrarily to the former, is what we think would exist even if the entire humanity was to disappear. This is the fundamental intuition guiding our attribution of mind-independence to it. While Mount Everest would have existed regardless of any mind thinking of it, what is it left of marriages, pounds, and the Colosseum, if the *Homo sapiens sapiens* would not exist?[105] In this sense, social entities are mind-dependent. But we should be careful in how to interpret this statement. While social entities do not depend on a single human mind, they still rely on what we might call 'the *collective* mind'. The collective mind (also called 'collective consciousness') is a sociological concept that refers to the set of beliefs, values, norms, and attitudes that most individuals share in a society or social group. Emile Durkheim, one of the founding fathers of sociology, says the collective mind is 'the totality of beliefs and sentiments common to the average members of a society forms a determinate system with a life of its own.' (Durkheim, 1893/1984, p.38-39) If you go to the bank to withdraw the amount of money your new landlord requires as a cash deposit, you expect to be given notes (which, moreover, have to amount to a specific sum) and not, say, pebbles. The reason why a situation like this does not happen is due to the fact that everyone involved in the transaction knows what 'money' stands for. No social interaction would be possible if the referents of our linguistic expressions did not exist. The collective mind grants social entities an ontological status, such that the semantic value of any proposition concerning the social world depends on what is

[104] On the problem of how to demarcate social ontology see, for example, Epstein (2018).

[105] This is a contentious claim. Philosophers belonging to the idealist tradition – most prominently Hegel and Gentile – have powerfully argued that not even Mount Everest could have existed without an 'Absolute Spirit' or 'Absolute Consciousness' thinking of it. Starting from Descartes, the onto-epistemic problem concerning the relationship between reality and human knowledge has been the focal point of most modern philosophy and beyond. Nevertheless, most contemporary analytic philosophers frame the debates on the assumption that there is a mind-independent reality and that we can access it. Although I have my own preferred view on the issue, I shall remain silent as taking such a commitment would be superfluous in the present circumstances.

recognized as existent.

In this framework, social groups are particularly important. They permeate many, if not most, aspects of society and individual lives: we talk about political parties and governments, we blame a certain company for polluting a river, we feel we belong to a social group rather than another, and so on. The specific account I will defend is known in the literature as fusionism, but I will call it 'Classical Extensional Mereological account of social groups' (henceforth, $\mathrm{CEM_{SG}}$). Looking at the literature on social ontology, we can easily find out that $\mathrm{CEM_{SG}}$ is highly unpopular among social ontologists, who generally discuss it just to dismiss it. There are clear reasons for that, which we will soon see. For now, suffices to say that, in its canonical version, $\mathrm{CEM_{SG}}$ holds that the relation bearing between a social group and its members is the parthood relation as CEM outlines it. But, I shall argue, this is incorrect. The aim of this final chapter is to defend what I take to be the correct version of $\mathrm{CEM_{SG}}$ by means of what I have been arguing for in the previous chapters and, ultimately, show that many philosophers got $\mathrm{CEM_{SG}}$ (and CEM more generally) wrong.

5.1 Framing the debate

5.1.1 Ontological realism

Generally speaking, the attitude to which we engage with social groups is realist: we think that social groups are existing entities with which we can interact. This commonsensical perspective concerning the ontological status of social groups is known in the literature as 'group realism'. It contrasts with 'group eliminativism', which affirms that social groups are nothing more than *façon de parler*, mere figurative expressions lacking any ontological overlay.[106] Some people occupy a middle ground position arguing that while 'social group' is a term of art, that social groups like political parties, sport teams, etc. exist, should not even be a question.[107] It is a debated issue whether social groups exist or not, but group realism is (predictably) the most popular position among social ontologists. This position offers at least three advantages. First, it goes hand in hand with common sense and ordinary linguistic discourses; second, it simplifies the picture of the social world and allows social groups to fit more consistently in the general framework; third, it allows us to make sense of our widespread practice of belief attribution – crucial in attributing epistemic or moral responsibility to governments, companies, communities, etc. In what follows, I shall follow suit and presuppose that group realism is onto something here.

As highlighted by, among others, Epstein (2018), there are two main kinds of enquiries to engage with within group realism. The first one is the ontological question 'What are social groups?'; the second, the organisational 'How can social groups be categorised?'. While the latter is an interesting one – having a taxonomy of social groups would be useful indeed – I shall not deal with with it. My interest here is to dig deeper into the ontological status of social groups, rather than attempting to classify them, whatever they are. Having that said, let's briefly present the most relevant realist accounts of social groups one can find in the literature:

1. Groups as non-singular pluralities: groups are their members taken together yet not joined together. Given that groups are nothing over and above their members, they are not further entities.

[106] See Chisholm (1973), Van Inwagen (1990), and Unger (1979).

[107] See, for instance, Thomasson (2019).

2. Groups as aggregates: groups are aggregates characterized by an extensional yet intransitive 'member-componenthood' relation that is restricted to individuals.

3. Groups as sets: groups are sets of individuals.

4. Groups as fusions: groups are mereological sums characterized by the parthood relation.

5. Groups as hylomorphic compounds: groups are compounds of matter and form (or matter and structure, as it might be more convenient to rephrase it for the present discussion).

6. Groups as sui generis entities: groups are entities that cannot be identified with entities of any other category (sets, properties, etc.).

This subdivision is very rough and does not do justice to the complexity of each position. See Ritchie (2015) for a more detailed overview of accounts (1)-(5). For a recent defence of (1), see Horden and López de Sa (2021). (2) has been prominently endorsed by Burge (1977) on set-theoretic grounds. Effingham (2010) presents the standard conception of (3) and develops an interesting new account of setism that can solve the problematic features of the standard one. Standard versions of (4) take atemporal notions like parthood or proper parthood as primitive, but Sider (2001) offers a variation of fusionism that takes parthood-at-t as primitive and compounds it with four-dimensionalism. See Ritchie (2015) and Sheehy (2006b) as prominent examples of (5), and Uzquiano (2004) for a defence of (6). As I have already anticipated, the specific position I will defend is an alternative account of fusionism. While the canonical account regards social groups as mereological sums characterised by the parthood relation, I defend the idea that social groups are mereological sums characterised by a φ-parthood relation.

5.1.2 Short history of $\mathbf{M_{SG}}$

Hawley (2017) crisply described the fortune CEM_{SG} experienced throughout the years and the objections it has raised. In what follows, I will take her narrative as the starting point of our discussion, but my narrative will be different in terms of scope as well as goals. Before presenting those differences, a fundamental caveat is in place. Mereology is the philosophocal discipline investigating the relationship between (i) a whole and its parts, and (ii) its parts themselves. In some recent literature, however, 'mereology' (M) has also been used to refer to the particular mereological thesis of CEM. But this is a conceptual mistake. M is not intrinsically committed to any specific theory but allows for different theorizations instead. CEM is the most famous mereological account on the market, but it is by no means the only possible one. While CEM is committed to, say, universality of composition, M remains silent on whether that is the case or not. It follows that CEM_{SG} is a particular mereological account of M_{SG}. M_{SG} affirms, minimally, that the relation between a social group and its members is the parthood relation, so that a social group is a whole of which its members are parts. The features of the parthood relation depend on the eventual characterisation that is given to it through the axiomatisation, but that might well be different (even very different) from that of CEM. In short, CEM implies M, but not viceversa. This is why it is crucial to keep them distinct. This distinction plays a crucial role in Hawley's analysis of the mereological account of social groups. She aims at defending M_{SG} by arguing that the mereological theory one decides to adopt – i.e., in whatever way one characterises M – is not relevant as long as it can address all the objections she presents. In other words, what she argues for encompasses any personal taste for a particular mereological theory. Since I defend a mereological account based on CEM instead, my approach is more committed than hers. Yet, our approaches remain compatible. My aim is to defend a particular version of $\mathrm{CEM_{SG}}$ by showing that it can reject the long-lasting objections that have been raised against it and that led to its marginalisation. The following is the story of $\mathrm{M_{SG}}$ that Hawley (2017) tells us.

For more than a decade, $\mathrm{CEM_{SG}}$ had been appreciated, respected, and well-seen by many scholars (e.g., Mellor, (1982), Oppenheim and Putnam (1958), and Quinton (1975)). But David-Hillel Ruben was not among them. He thought that $\mathrm{CEM_{SG}}$ was nothing more than a siren song and thus decided to write an article in 1983, which was soon followed by an even more powerful book in 1985, where he dismantled $\mathrm{CEM_{SG}}$ piece after piece. He did not want to leave survivors, and that is what happened. That 'whatever relations human beings bear to social entities, the relation of being a part of it is not one of them' (Ruben, 1983, p.219) became a mantra repeated and perpetuated by almost everyone. The outcome was that $\mathrm{CEM_{SG}}$ fell out of philosophical favour for several decades. But here is the plot twist. During these years of oblivion, mereology – as a philosophical discipline – experienced a renewed interest among philosophers, especially since new, non-conventional approaches to mereology began to emerge. The development of non-classical mereologies that were not on the market at the time of Ruben made clear that Ruben (and everyone else who followed him) mistakenly identified M with CEM. By ruling out $\mathrm{CEM_{SG}}$, he erroneously thought to have ruled out $\mathrm{M_{SG}}$ as well. But that is not true. The development of non-classic mereologies clarified that CEM was not the only possible theory of mereology and, consequently, that $\mathrm{CEM_{SG}}$ was not the only viable mereological account of social groups. Many philosophers adopted a non-classical mereology and developed a non-classical account of social groups. An interesting outlier is Hawley (2017) who, as we have already said, argued for the tenability of $\mathrm{M_{SG}}$ on neutral grounds, viz., independently from the endorsement of a particular mereological theory. Approaches like those of Ritchie (2013, 2015) and Sheehy (2006a, 2006b) focus on the notion of structure and develop Neo-Aristotelian accounts describing social groups as compounds of matter and form (or better, structure) that are not individuated by their material parts, as CEM holds, but rather by their structure (or organisation).[108] The relations connecting the group members then constitute the functional structure of the group.[109] Fine (2020) has also recently entered the debate adopting the same methodological procedure but within the framework of his theory of embodiment. According to these authors, CEM (and, consequently, $\mathrm{CEM_{SG}}$) fails to appreciate the crucial role played by the notion of structure in a mereological theory. The mere existence of parts is insufficient in explaining the composition of a whole, and material objects are glaring examples of that. As Fine (1999) puts it, 'if the sandwich is to exist, it is not sufficient for the ingredients merely to be around. They must be appropriately assembled.' (p.63) I take this to be a very good point that I shall discuss later in this chapter. For now, I only want to present the framework in which my proposal inserts itself.

The substantive aim of this chapter is to investigate whether $\mathrm{CEM_{SG}}$ can be a viable account of social groups or deserves its bad reputation. My goal will thereby be to identify the main objects that have been raised against it and see whether they manage to hit the target. CEM used to be the dominant mereological theory in the past, but there are not many people defending it.[110] Thus, it

108 See Koslicki (2008).

109 For the close relation between structure and function, see again Koslicki (2008).

110 In the past decades, Lewis and Goodman have been prominent proponents of a full-blood CEM. More recently, Achille Varzi has greatly contributed to defending CEM and his works are thus often mentioned in this paper.

is unsurprising that supporters are nowhere to be found for CEM_{SG}. CEM_{SG} is a discredited view that no one seems willing to defend. With my proposal, I hope to show that an attractive CEM_{SG} is possible – one that is able to reject the traditional objections and satisfy all of the desiderata an account of social groups is generally asked to meet.

5.1.3 *Desiderata* in the literature

In the literature, philosophers recursively bring up at least nine *desiderata* that a satisfactory account of social groups should meet. They are:

(a) **Temporal Flexibility**: social groups display temporal flexibility, i.e., have different members at different times. A football team might be composed of some members at time t_1 but not at some further time t_2. The team can gain or lose members, for instance, by purchasing or selling a player in the transfer market, but it remains diachronically identical.

(b) **Modal Flexibility**: social groups display modal flexibility, i.e., have different members in different worlds. The Catholic Church would have had a different Pope today if (the actual) Pope Francis had not been elected by the College of Cardinals in 2013. *Pace* Divine Providence, there is a possible world in which (the human being that is referred to as) Pope Francis fails to be Pope because another Pope was elected.[111]

(c) **Temporariness**: social groups are temporary entities, i.e., exist at t_1 yet not at t_2. Alternatively put, they come in and out from existence rather than exist permanently. Pink Floyd did not exist before 1965 – when the band was formed – and does not exist now – although some fans might disagree – but existed for some time in between.

(d) **Contingency**: social groups are contingent entities, i.e., exist at one world and not at another. This means that they are non-Kripkean entities, namely entities that necessarily exist at every possible world. The Fed (very much) exists in the actual world, but there is a possible world in which it does not.

(e) **Reductionism**: social groups are not *sui generis* entities, i.e., they should not require any further ontological commitment exceeding those one already has. By Occam's Razor, we should not postulate the existence of entities whose existence could be explained otherwise. It follows that my family should be reducible to a type of entity to which we are already committed.

(f) **Non-Extensionality**: social groups can be extensionally coincident, i.e., can have the same members and yet be non-identical. Consider a family-run business. The family and the company share all their members, but remain different (in nature, purpose, etc.) and cannot be identified.

(g) **Selective Transitivity**: social groups might not be characterized by a transitive membership relation. Intuitively, while a nurse is a member of the hospital staff, her tongue is not. But there are also cases in which transitivity applies. For example, a member of the Royal Navy is also, by transitivity, a member of the British Army.

See also Lando (2017) for a recent defence of CEM.

[111] The labels 'temporal flexibility' and 'modal flexibility' are due to Hawley (2017).

(h) **Structure**: social groups are not random collections of people, but have some kind of structure instead. The UN, PETA or the group of LGBTQ+ people of colour are different entities from, say, a mere assemblage of holidaymakers on a camping site.

(i) **Selectivity**: Social groups cannot be composed of any entity whatsoever. A group of rocks, for instance, can never be a social group. Keeping our commitments at a minimum, we can say that whatever the entities that might potentially compose a social group, it is not possible to have a social group whose members are of any ontological kind. That is to say: it does not seem to make sense to define the group of red pens, or the group of fictional characters a social group.

5.2 CEM$_{SG}$: group membership as parthood

As already anticipated, the canonical version of CEM$_{SG}$ conceives the group membership relation as entirely reducible to the parthood relation. Consequently, the group membership relation inherits all the features that, under CEM, characterise parthood. Considering group membership as *just* parthood is very common in the literature on social ontology. By reducing members of a social group to its mereological parts, CEM$_{SG}$ easily satisfies (e). Given that social groups are mereological wholes having members as proper parts, there is no need to postulate new entities to account for them. Our ontological quest is already solved since they are ascribable to mereological wholes. Things are different for all the other *desiderata*, which it is generally agreed that CEM$_{SG}$ fails to satisfy. The following are the most relevant and, eventually, challenging features of social groups implied by CEM$_{SG}$:

Transitivity: By A3, social groups hold their members transitively. Given three different things, say a nose, a human being, and a particular social group, if the nose is part of the human being, and the human being is part of the social group, then the nose is part of that social group.

Extensionality: By the extensionality principles, if two social groups have all the same members, they are identical. No two different social can have the same members. If Alexis, Paul, and Millie are the only members of a reading group and a running group, then the reading group and the running group are identical. In other words, CEM$_{SG}$ imposes that social groups are unique: there is one and only one social group composed of Alexis, Paul, and Millie. The reading and running groups are identical or must differ in their mereological structure. A5, social groups are mereological wholes whose members satisfy a specific condition φ.

Unrestricted Composition: By A5, social groups are mereological wholes whose members satisfy a specific condition φ. But, given that φ can be any condition whatsoever, CEM$_{SG}$ grants that there is a social group composed of Jerome Powell, Ricky Gervais, and Slavoj Žižek. As long as a certain φ is satisfied by some individuals, there exists a social group composed by them.

As the reader might have already guessed, these features are problematic. They give rise to serious challenges to CEM$_{SG}$ that we will properly discuss in the next pages. Besides them, however, CEM$_{SG}$ is also haunted by a bunch of misguided objections arising from misconceptions of CEM's real implications. CEM_{SG} is indeed often presented in the literature as involving at least three other problematic features:

> **Essentialism**: Groups have their members essentially. Since no changes in membership are allowed in any world, the members of a social group are part of that social group in every possible world. There cannot be a possible world in which Jeff Bezos is not among the wealthiest people in the world – that is, he is not a member of the social group of the wealthiest people in the world. That social group counts him among his members in every possible world in which it exists.
>
> **Constancy**: Groups have their members permanently. A social group can persist only insofar as membership is not altered. Social groups can neither acquire nor lose members across time; otherwise, they cease to be those same groups. As long as the Laurel & Hardy comedy team exist, Stan Laurel and Oliver Hardy are part of it.
>
> **Composition as Identity**: Social groups are nothing over and above their parts. There is nothing special about social groups that makes them different from the sum of their members. The European Union is just the sum of all its members, and the sum of the European Union's members is the European Union.

However, these are illegitimate features of CEM_{SG}. In this chapter, my goal is twofold. First, bringing some clarity about which objections often raised against CEM_{SG} are well-founded and which are not. We will see in the next subsection (5.2.1) that CEM_{SG} is often assumed to imply theses it does not, in fact, endorse. There are, however, other objections that are genuine and represent a serious challenge to CEM_{SG}. We will present them afterwards.

5.2.1 Misguided objections

Adapting to CEM_{SG} what we have already said for CEM in 2.2, CEM_{SG} is not committed neither to essentialism, nor constancy, nor composition as identity. Given that we already labelled each of them as *ME*, *MC*, and *CAI* in 2.2, I shall follow suit. All we need to remember is that, in the present circumstances, they are to be interpreted within a social group scenario.

In 3.3.4, we formally proved that modal CEM does not entail *ME*, thus, modal CEM_{SG} does not entail it either. On the contrary, we showed that modal CEM allows for modal flexibility, albeit with very stringent conditions. Recalling the model in Figure 5, it is evident that a social group *can change* its members across different possible worlds. Suppose that g_1 and g_2 respectively correspond to the social group of the wealthiest people in the world and the social group of the poorest people in the world. To simplify, we assume that there are only four individuals in the domain, g_1 and g_2 and two human beings, a and b. At w_0, a and b are members of g_1 but, in w_1, they are members of g_2 instead. Importantly, g_1 and g_2 still exist in w_1 and w_0 respectively, regardless of the fact that they have no members there. In other words, the social group of the wealthiest people has no members in w_1 because a and b are members of the social group of the poorest people, and *vice versa* in w_0. Therefore, we can conclude that the same individuals can be part of different social groups across different possible worlds and, thus, that Jeff Bezos might well be among the poorest people in the world.

It is important to remember that all the necessity-claims CEM_{SG} makes are strictly extensional. If two social groups are coextensional, they remain so across possible worlds. That is, if the social group of the richest people in the world is coextensional with the social group of the richest people in the US, then they are identical. Evidently, this claim suffers of its own problems, but we will discuss them in the next subsection. For now, our focus is to point out that any objection attacking CEM$_{SG}$ under the assumption that it endorses ME is doomed to failure.

Now, although we have not explicitly discussed *MC*, there are no principled reasons for which what we have proved for *ME* should not hold for *MC* as well. To fully defend such a claim, however, one would need to endorse a temporal logic rather than a modal one. Keeping that in mind, we can still provide a good defense of the claim that CEM$_{SG}$ does not endorse *MC* either. We can do so by adopting a temporal interpretation of $\Box$: $\Box = A$ (always). Once this analogy is drawn, it is easy to see that all the modal theorems that we have proved in chapter 3 have an equivalent temporal interpretation. Thus, all that we have said above concerning social groups and *ME* can be transposed into a temporal setting. Thereby we get:[112]

P4. MC: $\forall x \forall y(Pxy \to A\, Pxy)$

T53. Eternal Mutual Parthood: $\forall x \forall y((Pxy \land Pyx) \to A(Pxy \land Pyx))$

P4 says that if x is part of y, then y *always* has x as a part throughout its existence. T53, on the other hand, says that if x and y are part of each other, then they will always be so. In other words, T53 proves that coextensional entities are always so: if x is coextensional with y then neither of them can ever lose or gain a part. This is why the model (resembling Figure 5) in Figure 6 below validates T53 but not P4: the system proves temporal *extensional* theorems of CEM.

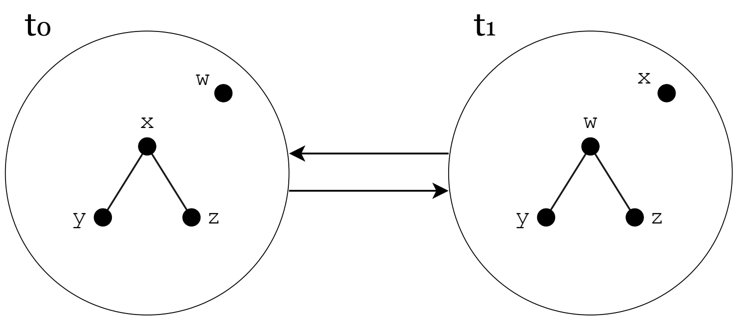

Figure 6: A model invalidating P4 that yet validates T53 and alike theorems.

T53 does not dare to prove that once two people are married they will always be, as P4 affirms. It (more humbly) says that if two people are married at t_0, there will always be something which is extensional to them (even if they get divorced at t_1). In a slogan: extensionality is forever. And this is a much less demanding principle than P4.

As regards *CAI*, we can be brief. On the one hand, endorsing or not endorsing it alone does not directly affect the fulfilment of any *desideratum*. On the other, because only the joint work of *CAI* and *ME* and *MC* respectively, would make CEM$_{SG}$ subject to objections from modal and temporal flexibility. If a football team has some members at t_1 and others at t_2, CEM$_{SG}$ proves that they are two different football teams if, and only if, we grant that members are constant to the group (*MC*) and that the group does not preserve identity over membership change because the sum of its members dictates its identity conditions (*CAI*). The same goes for modal flexibility.

112 I picked T47 for convenience, but any other extensionality theorem would do the same job.

CEM_{SG} proves that the Catholic Church is a different entity across possible worlds if, and only if, we endorse the thesis that Pope Francis is an essential member of the Catholic Church (*ME*) and that the Catholic Church does not retain its identity in a possible world in which someone else is Pope (*CAI*).

Therefore, given what we have said so far, CEM_{SG} can satisfy both (a) and (b). Besides them, however, it seems that CEM_{SG} can also account for (c) and (d). In 3.2.3, we have seen that *CD* is compatible with the existence of contingent entities. The Fed is an actual entity in the actual world, but there are other possible worlds in which it does not exist as an actual entity. Given the ontological commitments of *CD* it is still something – necessarily something – in those possible worlds in which it is not an actual entity, but its actual existence is contingent. According to *CD*, there are worlds in which the Fed is not an actual entity. And that the Fed is a contingently actual entity – that is, it does not actually exist in all worlds – is all we seem to care about. Again, the same goes for temporariness. Adapting all of the above to the temporal dimension, it seems that *CD* can account for the temporal nature of social groups. Pink Floyd were not an actual entity before 1965 and are not nowadays, but have been so sometime in between.

5.2.2 Objection from Transitivity

Consider the following inferences:

(1a) A biological subunit is part of a cell.

(1b) A cell is part of an organ.

(1c) A biological subunit is part of an organ.

(2a) The arm of Justice Breyer is part of Justice Breyer.

(2b) Justice Breyer is part of the Supreme Court.

(2c) The arm of Justice Breyer is part of the Supreme Court.[113]

Intuitions vary, but (1c) and (2c) are likely to strike many as wrong. While (1a), (1b), (2a), and (2b) do not sound odd – though, admittedly, (2a) would be hardly uttered – and can be easily assigned a semantic value (true), the same does not apply to (1c) and (2c). That transitivity does not hold in such cases is an evident problem for CEM. CEM takes the parthood relation to be a partial order and, thus, to be inherently transitive but, as (1) and (2) show, it fails to be so sometimes.[114] On the other hand, however, it seems wrong to say that (1c) and (2c) are semantically false. After all, there seems to be a mereological sense in which it is true that the biological subunit is part of an organ and that the arm of Justice Breyer is part of the Supreme Court. In other words, it seems that the parthood relation should be kept distinct from the membership relation. The tension originating from these two contrasting intuitions is what I shall call 'the transitivity dilemma'.

With transitivity, we are on the horns of a dilemma because, from one angle, we regard commonsense intuitions as trustworthy; consequently, we consider (1c) and (2c) false. From another

[113] The first example is taken from Rescher (1955), the second from Uzquiano (2004).

[114] There is an extensive literature concerning ordinary uses of parthood and transitivity lying at the intersection between ontology and linguistics. Among the others, see Rescher (1955), Cruse (1978), Lyons (1977).

angle, however, it seems that our *prima facie* intuitions are misleading and, therefore, that (1c) and (2c) are, in fact, true. In the former case, our job is to find the culprit of these fallacious inferences. We need to explain *what* causes inferences involving transitivity to produce undesirable outcomes: could it be transitivity itself not being constitutive of parthood? In the latter, we have to figure out *why* (1c) and (2c) strike us as odd, while (1a)-(1b) and (2a)-(2b) do not: are we interpreting 'parthood' in two different ways?

Objections to CEM$_{SG}$ targeting transitivity have followed either of these paths. Ruben (1983, 1985) and Uzquiano (2004), for instance, reject CEM$_{SG}$ on the basis of ordinary intuitions concerning parthood. They both argue that since there are clear cases in which the parthood relation is not transitive, transitivity cannot be analytic. The rejection of CEM (and CEM$_{SG}$), which has transitivity among its axioms, is inevitable. Other philosophers choose the second strategy. Epstein (2015), for instance, argues that there are cases (like in (1c) and (2c)) in which CEM$_{SG}$ picks out the wrong type of relationship. According to this line of thought, CEM$_{SG}$ is mistaken in considering the group membership relation as mere parthood. He writes:

> It does not seem right to say that Samuel Alito's right arm is one of the parts of the Supreme Court. (Or, at least, if Alito's right arm does count as a part of the Supreme Court, then 'parthood' is not the relation we are interested in. Rather, the relevant relation is that Alito is a member of the Supreme Court—and Alito's arm is certainly not that.)(p.144)

Considerations of these sorts are usually considered almost knock-down arguments against CEM$_{SG}$. Nevertheless, the latter is arguably the most challenging one. Following the suggestion, we might respectively label them 'Weak Transitivity Objection' (WTO) and 'Strong Transitivity Objection' (STO). We can summarise them as follows:

> **WTO**: There are instances of the parthood relation in which transitivity fails. If Justice Breyer is a member of the Supreme Court, it should not follow that his arm is a member of the Supreme Court too. Therefore, CEM$_{SG}$ cannot be the right account for group membership.

> **STO**: The parthood relation is inherently different from the membership relation. However, CEM$_{SG}$ reduces group membership to be just parthood. Therefore, CEM$_{SG}$ cannot be the right account for group membership.

5.2.3 Objection from Coextensionality

It is widely recognised that, along with the transitivity objection, the coextionality objection is one of the most demeaning for CEM$_{SG}$. Under CEM$_{SG}$'s assumption that membership just is parthood, we can define coextensional social groups as composite entities that are improperly part of each other (if we follow T5) or as composite objects having the same proper parts, i.e., the same members (if we follow T4).[115] Critiques have predominantly focused on the latter as there are clear cases in which different social groups happen to be composed of exactly the same members. For instance, the same members might compose a chess club and a nature club (Ritchie, 2013), a ruling class and a caste (Ruben, 1983), a theatrical troupe and a family (ibid.), the Supreme

[115] We could also define coextensional groups via T6 or T7, but using T5 and T4 might sound more intuitive given that we are discussing social groups.

Court and a hypothetical Special Committee on Judicial Ethics (Uzquiano, 2004). The problem for $\mathrm{CEM_{SG}}$ is that 4 rules this possibility out, as the following argument shows.

(1) Social groups have members as their proper parts.

(2) By T4, different social groups cannot have the same proper parts unless they are identical.

(3) If there exist non-identical social groups having the same members, then either T4 is false, or members are not proper parts of social groups.

(4) It is a matter of fact that non-identical social groups having the same members exist.

(5) Therefore, either T4 is false, or members are not proper parts of social groups.

Both disjuncts of the conclusion are detrimental for $\mathrm{CEM_{SG}}$. The first makes it collapse straight away because it undermines a crucial and undeniable theorem of CEM; the second forces $\mathrm{CEM_{SG}}$ to admit that members are not proper parts of social groups. The counter-intuitiveness of such a claim is evident. Besides that, however, $\mathrm{CEM_{SG}}$ cannot accommodate it in any case because it would clash with A5. It might be thought that a defendant of $\mathrm{CEM_{SG}}$ could avoid this objection by arguing that coextensional groups are, in fact, identical. But this is a non-starter. By the Indiscernibility of Identicals (L7), identical objects must share all their properties: this does not happen in the case of social groups. Intuitively, a family and a family-owned business are composed of the same members but remain distinct social groups since they differ in their properties. For example, while the family does not give receipts, the business does, and while affection-based relations do not govern the business, they do so in a family.

5.2.4 Objection from Structure

There is an intuitive ontological distinctiveness between organised and unorganised groups and mere assemblages of people. In the literature, groups like the UN and PETA are usually referred to as structured (or organised) groups, i.e., social groups having a specific structure in which the group and its members are organised. On the other hand, groups like LGBTQ+ people of colour are generally seen as unstructured (or unorganised) due to their lack of organisational structure. See Ritchie (2015) for a more detailed analysis of structured and unstructured groups, which she labels 'Type 1' and 'Type 2', respectively. This categorisation is intuitively appealing and, although it has been problematised by some authors,[116] it is helpful to see the crucial role that structure plays in the context of social groups. Even though, as Epstein (2018) underlines, the identity conditions seem to be relatively weak for some social groups and quite strong for others, what all social groups seem to have in common is *some sort* of structure. It might be more or less codified, but the structure is what distinguishes social groups from mere assemblages of people.

$\mathrm{CEM_{SG}}$ is unable to discriminate between them. $\mathrm{CEM_{SG}}$ only says, via A5, that as long as there are some xs satisfying a certain φ, there is a social group composed of them. This minimal claim guaranteeing the existence of a fusion of these xs does not provide us with any further information concerning the structure and organisational complexity that we take these different entities to

[116] Epstein (2019), for instance, affirms that social groups are too complex and diverse to be reduced to a dyadic classification. In place of it, he proposes to use four 'profiles' to discriminate between them. Thomasson (2019) takes a different attitude by not criticising Ritchie's distinction but calling for a less superficial analysis of both Type 1 and Type 2 groups.

display. In other words, CEM$_{\text{SG}}$ places organised and unorganised groups and mere assemblages of people on the same level, i.e., it can only recognise them as mereological wholes. The fact that CEM$_{\text{SG}}$ cannot account for the distinction we perceive between them leads to undesirable outcomes. For instance, it recognises that a whole composed of twelve people waiting at the bus station in Edinburgh and a whole like Facebook belong to the same ontological kind. And here is the clash between CEM$_{\text{SG}}$ and commonsense, which does not regard the two in the same way. The former would be easily recognised as a social group, yet not the latter. In the eyes of CEM$_{\text{SG}}$, however, all social groups are just mereological wholes and the structure they might have played no role in discriminating between them. Ritchie (2015) writes that CEM$_{\text{SG}}$

> [...] fails to capture the organisational component and required intentions for groups of Type 1. It also fails to capture that groups of Type 2 seem to involve a shared feature and are more difficult for members to join or leave. The view that groups are fusions fails for groups of Type 1 and Type 2. (p. 315)

5.2.5 Objection from Selectivity

The last objection we will consider concerns our last desideratum, (i). As for the previous objection, A5 is the target. Besides disregarding the structure that social groups might or might not have, A5 does not even consider the composition a social group might or might not have. A5 does not discriminate between entities beyond whether they satisfy a certain condition or not, and this has profound consequences on CEM$_{\text{SG}}$. It cannot make sense of the difference between, say, a pile of stones, a beehive, and a group of soldiers marching in a parade. In other words, CEM$_{\text{SG}}$ would grant the existence of too many social groups, leaving us without a reliable criterion to discriminate between which entities can be part of a social group and which cannot. That the group of soldiers is a social group sounds intuitive, but what about the pile of stones and the beehive? Under a broad conception of social groups, i.e., one permitting that non-human beings too can compose a social group, the beehive might be considered a social group, but definitely not a pile of stones.[117] Given CEM$_{\text{SG}}$, restricting the φ-conditions is not a viable solution. As an unavoidable axiom of CEM$_{\text{SG}}$, A5 grants the existence of the fusion of any entity whatsoever for any condition whatsoever. Thus, restricting the domain and the conditions to which A5 applies would be not only undeniably *ad hoc* but also unlawful. Attempting to limit the metaphysical import of A5 would simply dismantle CEM$_{\text{SG}}$ itself. (Notice that the same holds even under the existentially relativised versions of unrestricted composition, A6 and A7.) As Effingham (2010) put it:

> We might restrict the definition to certain sorts, e.g. x is a member$_G$ of group g at time $t =_{df} x$ is a person and x is a part of g at time t. But this illegitimately rules out non-persons from being member$_G$ of groups (for instance, team mascots which are animals). Trying to avoid this problem by broadening the sortal restriction to cover any living organism won't work as then the individual cells from each footballer will be member$_G$ (nor can it be avoided by allowing all living organisms except cells to be member$_G$, for not only is it ad hoc but some biologists may form a football team with an archaea as a mascot). (p. 255, author's emphasis)

[117] The view that we can talk of 'social groups' also in the case of non-human animals is not very popular among social ontologists. See, one for all, Gilbert (1992, §5.2).

In conclusion, it seems that social groups are simply not composed of any entity whatsoever, contrary to what CEM_{SG} proves. A5 proves that there are all the mereological fusions – i.e., social groups, given that membership is just parthood – we are used to conceiving as such. However, it also suffers from an over-generation problem. CEM_{SG} recognises as social groups too many entities that we might want to keep distinct from actual social groups.

5.2.6 Score

The table below summarises our findings:

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
yes	yes	yes	yes	yes	no	no	no	no

The most relevant result is that, contrarily to a widespread belief among social ontologists, it is not true that a fusion has its parts essentially and permanently. On the contrary, CEM can account for modal and temporal flexibility, as well for contingent and temporary entities. Given that CEM_{SG} automatically satisfies (e), we can conclude that the first five *desiderata* are fulfilled. As regards the other four, however, CEM_{SG} is hopeless. Although the failure to meet these *desiderata* have generated serious objections, (g) and (f) are particularly problematic, and give strong reasons to reject CEM_{SG}.

5.3 φ-CEM_{SG}: group membership as φ-parthood

CEM_{SG} relies on the assumption that group membership is *just* parthood. However, this is a 'naïve mereological approach' (Strohmaier, 2018, p. 124) because it reduces being a member of a social group to being a part of it. In the article, Strohmaier follows many others in the literature who regard CEM_{SG} as the only account of social groups that can potentially arise from CEM and instead defend, following Hawley, 2017, non-CEM approaches. Although I disagree with that idea – I think that CEM_{SG} is not the only account of social groups that can arise from CEM – he is right in highlighting that CEM_{SG} is a naïve mereological approach. As a piece of formal ontology, mereology aims at investigating parthood in its deepest sense, not in its socially constructed one. I hold that CEM_{SG} is unable to account for group membership because it treats the *socially constructed* membership relation as if it was *just* parthood. But that is, I argue, a mistake. My proposal is to regard group membership as φ-parthood, namely, as parthood plus further conditions which narrow its interpretation down. According to this proposal, rather than being *just* mereological wholes, social groups are mereological wholes with a specified understanding of the parthood relation. Recalling what we have discussed in Chapter 4, there are two different ways to interpret φ-parthood:

D12: $P^1_\varphi xy :\equiv Pxy \wedge \varphi x$ (monadic φ-part)

D13: $P^2_\varphi xy :\equiv Pxy \wedge \varphi xy$ (relational φ-part)

The latter is the relevant interpretation of φ-parthood. While D12 attributes a monadic property to a given part, D13 modifies the parthood relation itself. That I am a member of my family is not the same as saying that I have dark hair, and, in the case of social groups, the membership relation is intrinsically relational. I cannot be a member of my family regardless of whether my family exists or not, but I can have dark hair regardless of any relation I might have with external entities. By narrowing down the parthood relation, membership emerges as an instance of φ-parthood. As such, membership stands in a dependence relation with parthood so that while every member of a social group must, in the first place, be part of it, not every part of a social group must be among its members. In a slogan: all members are parts, but not all parts are members. I shall label the proposed account 'φ-CEM$_{SG}$'. In what follows, I will often talk about 'φ-conditions' whenever φ-CEM$_{SG}$ is involved. By 'φ-conditions' I mean any possible condition that might characterize social groups but that is chosen from outside CEM. Those further conditions are crucial for my account, yet I will not attempt to provide a full characterization of them. I will rather keep my claims at a minimum and state that whatever these φ-conditions are, there must be some. There are two main reasons for which I think that this is a desirable approach. First, it allows for multiple implementations. The complexity and variety of social groups call for a flexible account that can make sense of the highest number of groups possible. Evidently, the more conditions are added, the more informative (and controversial) it becomes, yet for reasons independent of the account itself. Therefore, I prefer to let the reader plug in her own set of necessary and sufficient conditions – if she wants to – and judge φ-CEM$_{SG}$ on that basis. Second, my aim is to defend and redeem an account of social groups based on CEM. Finding what these φ-conditions are is a fascinating and crucial area of research in and outside philosophy, but engaging in such a project goes beyond the scope of this dissertation.

Now, φ-CEM$_{SG}$ retains the metaphysical firmness of CEM but allows for a more informative and socially interesting analysis of social groups than CEM$_{SG}$. Nothing prevents us from attributing values, functions, etc. to the worldly things around us, but we should always bear in mind that when trying to make sense of social groups that, although having the exact same members, are not identical, we are confusing parthood with membership. There is a deep, mereological sense in which they might be identical if they satisfy 4 (or any other coextensionality principle). Still, the same does not hold in the social realm. Hearing someone comparing social groups to aggregates of molecules is very likely to strike us as wrong, but there is a sense, a deep sense, in which a chemist might legitimately talk of social groups in those terms. As Fine (2020) significantly writes:

> The special character of social groups lies not in the fact of linkage but *in the nature of the links*. What normally accounts for the structural unity of an ordinary material object is some kind of physical cohesion. But what normally accounts for the unity of a social group is not physical cohesion but social cohesion [...]. (p. 89, my emphasis)

That social groups are mereological wholes characterised by special bonds between different members, and between each member and the group itself is why we have to reject CEM$_{SG}$, which does not take the specificity of that relation into consideration. φ-CEM$_{SG}$ allows us to account for that, despite retaining a solid link with parthood. Contrarily to what many think – think about Epstein's reasons to reject CEM$_{SG}$ that we discussed in 5.2.2 – membership is not entirely detached from parthood, far from it. By arising from further conditions narrowing the parthood relation down, group membership inherits all the features of parthood described by CEM while picking out a particular subset of parthood relations. It follows that what holds for CEM necessarily holds for φ-CEM$_{SG}$ too, but not *vice versa*. The advantages of φ-CEM$_{SG}$ in this respect will hopefully become even more apparent in the next subsection, where I discuss the replies that φ-CEM$_{SG}$ has to offer to the four objections to CEM$_{SG}$ we presented earlier.

5.3.1 Objection from Transitivity: A Reply

In 5.2.2, we saw that it is possible to distinguish between two different versions of the objection from transitivity: WTO and STO. In what follows, I will argue that we can offer a satisfactory reply to both if we endorse φ-CEM$_{\mathrm{SG}}$.

5.3.1.1 Rejecting WTO

This version of the transitivity objection stems from the realisation that there are legitimate instances of the parthood relation that yet fail to be transitive. CEM$_{\mathrm{SG}}$ proves that the parthood relation is always transitive, but this contradicts the evidence we have from ordinary language.

Simons (1987, p.108) effectively summarised the standard line of reply of CEM's friends against WTO. He argues that there might well be examples of parthood relations in which transitivity fails, but these are structurally more complex uses of 'part' that differ from CEM's. While CEM's interpretation of 'part' remains transitive, those 'structurally more complex' instances of the parthood relation are instances of what we have called 'φ-parthood', the parthood relation altered by the predicate-modifier. In short, Simons points out that 'part' is transitive, but 'φ-parthood' might not be so because of the further conditions imposed by the predicate modifier. When 'part' is employed in a modified, specific sense, it becomes, strictly speaking, 'φ-part'. Consequently, it might fail to be transitive because of the further conditions imposed by the predicate modifier. For instance, it strikes us as wrong that the arm of Justice Breyer is part of the Supreme Court because we are 'mixing up' parthood with a type of φ-parthood, namely group membership.

That is a compelling reply to WTO, yet not everyone is satisfied with it. According to some philosophers, there is a more serious threat not addressed by Simons concerning the relationship between parthood and φ-parthood, viz. STO.

5.3.1.2 Rejecting STO

In Chapter 4, we have seen that Johansson (2004) talks about the 'odd subsumption relation' that would emerge if φ-parthood fails to be transitive. The time has come to discuss transitivity, which we dismissed back then. The reply given by A. C. Varzi (2006) we discussed in Chapter 4 explicitly applies to transitivity: the subsumption principle does not force φ-parthood (the specific relation) to inherit the transitivity of parthood (the general relation). To see this more clearly, consider the following principle:

P5. Transitivity$_\varphi$: $\forall x \forall y \forall z((P_\varphi xy \wedge P_\varphi yz) \rightarrow (P_\varphi xz))$

Given D12 and D13, we can give two different versions of it.

P6. Monadic Transitivity$_\varphi$: $\forall x \forall y \forall z(((Pxy \wedge \varphi x) \wedge (Pyz \wedge \varphi y)) \rightarrow (Pxz \wedge \varphi x))$

P7. Relational Transitivity$_\varphi$: $\forall x \forall y \forall z(((Pxy \wedge \varphi xy) \wedge (Pyz \wedge \varphi yz)) \rightarrow (Pxz \wedge \varphi xz))$

Evidently, while P6 is a logical consequence of A3, P7 is not. As A. C. Varzi (2006) rightly points out, the conditional in P7 is independent of A3 and, therefore, P7 is not a theorem of CLM. From this, we can conclude that there might be cases in which φ-parthood predicates fail to be transitive – nothing forces the consequent of P7 to hold – and, therefore, not all φ-parthood predicates can be subsumed under parthood. φ-CEM$_{\mathrm{SG}}$ can account for the arm of Justice Breyer not being a member of the Supreme Court because although recognising that, in a strong mereological sense, *the arm is a part* of that mereological whole which we socially recognise as the Supreme Court, *that same arm is not a member* of the Supreme Court. The explanation is that while the arm satisfies the conditions for parthood, it does not satisfy the conditions for that φ-parthood which is group membership. Contrarily to his arm, Justice Breyer arguably satisfies both the conditions for parthood given by CEM and the conditions for membership outlined by φ-CEM$_{SG}$ and, thus, is a part as well as a member of the Supreme Court.

5.3.2 Objection from Coextensionality: A Reply

We have seen that the target of the coextensionality objection is CEM$_{SG}$'s incapability to account for coextensional yet qualitatively different social groups. T4 does not allow for two groups to be composed of the same proper parts unless they are identical. But are they really composed of the same parts? And which entities can count as parts of a social group? There are two main options available on the market. The first is that also non-human entities can be part of social groups; the second is that only human beings are parts of a social group. In what follows, I will argue in favour of the former, but I will show that my account can also accommodate the latter.

5.3.2.1 Non-human entities as parts too

Consider the nature and the chess club Ritchie (2013) talks about. Ted, Joe, Angelika and Terry are part of both social groups, but are they the only parts of the nature and chess club? We might be tempted to say 'yes' if we (i) interpret membership as parthood as CEM$_{SG}$ does, or (ii) think that only human beings can be parts. While we have already seen that (i) is a mistake, (ii) would force us to go against A5 and therefore reject CEM$_{SG}$.

Broadly speaking, what counts as part (or proper part) is determined by CEM$_{SG}$, whereas what counts as a member is determined by φCEM$_{SG}$ and, more specifically, by the further conditions supplementing it. Thus, we can affirm that Ted, Joe, Angelika, and Terry are not only proper parts of the chess and nature club but, intuitively,[118] also their members. What about, however, the chessboard (and its pieces) of the chess club and the plants of the nature group? It seems intuitive not to grant them the status of members, but should we dare to say that they are not even parts? I think we should not. They are (respectively) part of the chess and the nature club as legitimately as Ted, Joe, Angelika, and Terry. Non-human entities might not be members, but they can definitely be proper parts of social groups. This is particularly evident if we think about groups that heavily rely on technologies (like computers, virtual data sets, etc.). Companies like Microsoft, banks, universities, governments and many other social groups would exist differently if such non-human objects were not at least part of them. It is noteworthy that some of these technologies might even count as members in certain circumstances. For instance, a robot might be considered a work team member (see, e.g., Savela et al., 2021).[119]

Summing up, we can say that while it might be possible for two or more qualitatively different groups to have the same entities as members, it is not possible that they also have all the same proper parts.

118 Here, I rely on an intuitive understanding of the notion of membership. I hope the reader will be charitable with my intuitions.

119 In the scientific literature, there is an increasing interest in robots and their interaction with social groups. See Sebo et al. (2020) for an useful overview.

5.3.2.2 Human entities as parts only

Someone might object that there might well be cases in which the parts and members of a social group perfectly match. For instance, Ted, Joe, Angelika, and Terry could form two more groups, a political discussion group and a walking group, which seem to have only them as proper parts. Even though I think this could be challenged without too much trouble – it is always possible to find different proper parts between social groups – let us grant it to the opponent.

φCEM$_{\mathrm{SG}}$ can account for this view because it clearly distinguishes between parts and members. As we have already said, CEM rules on what counts as a part, whereas it is φ-parthood's business which parts count as members. Ted, Joe, Angelika, and Terry are parts of these social groups because they satisfy the parthood conditions imposed by CEM. However, they are also members of these social groups because they arguably satisfy the conditions imposed by φ-parthood. For instance, they might be members of a social group because they perform a specific function within the group or because they have been appointed in a relevant way. Social groups might be identical in respect to parthood (given the extensionality principles), but they are not in respect to membership. They are not identical, even though they share all their parts and members, because there is a subtle 'predicational shift' going on. The predicational shift allows different social groups to share the same proper parts and the same members and yet remain distinct. Building upon D13, we can give the following definition of φ-PP:

D15. Relational PP$_\varphi$: $PP^2_\varphi xy :\equiv Pxy \wedge \neg(x = y) \wedge \varphi xy$.

Consider now the following principle, resembling T4:

P8. PP$^2_\varphi$-Extensionality: $\forall x(\exists w PP^2_\varphi wx \to \forall y(\forall z(PP^2_\varphi zx \leftrightarrow PP^2_\varphi zy) \to x = y))$.

P8 highlights the different characterisations given to members of different social groups. The φ-condition characterising the membership relation of the political discussion group picks out different qualifications than the running group's. To be a member of the former is to satisfy a set of φ conditions different from that characterising the latter. Therefore, Ted, Joe, Angelika, and Terry are qualitatively different members, albeit being the same parts. That leads to the distinctiveness (or, more technically, the predicational shift) between 'being a member of the political discussion group' and 'being a member of the running group'. These two predicates express different properties that members of each group hold in virtue of being members of that particular social group. Evidently, this will fail to happen if we define φ-proper parthood following D12. Given the following (mistaken) definition

D16. Monadic PP$_\varphi$: $PP^1_\varphi xy :\equiv Pxy \wedge \neg(x = y) \wedge \varphi x$,

we get

P9. PP$^1_\varphi$-Extensionality: $\forall x(\exists w PP^1_\varphi wx \to \forall y(\forall z(PP^1_\varphi zx \leftrightarrow PP^1_\varphi zy) \to x = y))$.

While defining $\varphi-$PP *via* P16 forces us to admit that there might be counterexamples to T4 – that is, there might be cases of coextensionality between different social groups – D15 is independent of T4. Thus, in the latter case, we can legitimately affirm that two groups can have the same proper parts and the same members because the membership relation, thanks to φ, turns out to be different between groups. In other words, qualifying φ-proper parthood via D13 grants logical independence to P8, viz., P8 is not a theorem in CEM, such that instances of P8 are not counterexamples to T4. We might reinforce our point even more by considering (again) what L7 (the Indiscernibility of Identicals) says. It affirms that no two different things can have the same properties or, alternatively put, that identical things are indiscernible. Thus, according to L7, two social groups are identical if they have all the same properties in common. But this is not the case for our political discussion and running group as they might be indiscernible in respect to parthood yet not in respect to membership. They cannot be *socially* coextensional because each one picks out a different membership relation and, therefore, instantiates different properties.

One final remark before closing. Intuitively, the strategy I adopted can also be applied to other axioms and principles of CEM. For instance, the following φ-parthood principle resembling T5 might be interesting to look at:

P10. $\mathbf{P}^2_\varphi$-Extensionality: $\forall x \forall y(\forall z(P^2_\varphi zx \leftrightarrow P^2_\varphi zy) \rightarrow (x = y)$.

As with P8, if an instance of membership fails to satisfy P10 it does not represent a counterexample to T5. Thus, two distinct social groups might be identical in respect to parthood, but not in terms of membership - unless no predicational shift occurs, i.e., unless they satisfy L7.

5.3.3 Objection from Structure: A Reply

It is largely acknowledged in the literature that social groups have a structure – that is why (h) is among our *desiderata*. In this respect, Fine (2020) sheds light on a phenomenon that often goes unnoticed: structure can change. It is not just a matter of change in membership, as it seems that social groups can also persist through changes in structure.

A clear real-life case of change in the corporate structure is that of Alphabet, formerly called Google.[120] This feature of social groups is barely recognised in the literature. Ritchie (2013), for instance, thinks that group structures cannot be modified. This claim is problematic in at least two respects. Firstly, to say that a social group cannot change its structure implies that they are modally and temporally inflexible – thereby going against our *desiderata*. Secondly, group structures do not emerge out of a vacuum. They need some grounding before they can be realised. Although Ritchie (2020) has more recently admitted that social groups can resist changes in structure, the second issue still needs a satisfactory answer. Like Ritchie, many other social ontologists struggle to find a way out from the 'magic emergence' of social groups. It seems metaphysically fishy that a social group 'comes into existence' after a social ritual: 'Nothing up my sleeves, I nominate you, you, and you to be the new climate committee – do you accept? Good. Shazam! A new climate committee appears!' (Thomasson, 2019, p. 4831)

On the one hand, φ-CEM$_{SG}$ can account, by means of the φ-conditions, for all the different structures that different social groups might have – this replies to the objection from structure in 5.2.4. On the other, φ-CEM$_{SG}$ also provides an explanatory answer to the question 'How do social groups emerge?'.[121] Regarding the former, φ-CEM$_{SG}$ thinks of non-realised structures (to use Ritchie's terminology) as non-actual structures. To a question like 'What happens to the previous structure when a group structure changes?', φ-CEM$_{SG}$ replies that it does not go out of existence in a strong sense and, instead, becomes a non-actualised structure. Regarding the latter, φ-CEM$_{SG}$ argues that social groups are mereological fusions that are – or were, if we want

[120] See Hern (2015).

[121] See Epstein (2018) for an overview of the different positions on how social entities are created. See Ritchie (2018) for a discussion of social creationism and social groups.

to adopt a temporal perspective – possible social groups. Social groups emerge out of an entity that is already there somehow. Consider what Blakemore (2021) writes:

> In the 1990s, the longstanding bonds between lesbian, gay, and bisexual people in both daily life and liberation activism led to the widespread adoption of the LGB acronym (lesbian, gay and bisexual). But it took longer to gain acceptance for another term that is now part of the modern acronym: "transgender." *Though trans people have existed throughout history, the term only came into being in the 1960s.* (my emphasis)

When have transgender people been recognised as a social group? When we decided to impose some φ-conditions on that mereological fusion that was undeniably already there. The mereological fusion of transgender people *as* the social group of transgender people arguably came into existence once the society recognised their social identity, but, strictly speaking, the mereological fusion was there well before. The coming of social recognition did not attribute *ontological dignity* to it. Everything social recognition attributes is *social dignity*. Social dignity is undeniably vital, but we should keep it distinct from ontological dignity. Through its underlying mereological setting (CEM), φ-CEM$_{SG}$ grants ontological dignity to a specific whole – arguably, there have been people satisfying φ = 'is a transgender person' before the 1960s. Through the φ-conditions narrowing down the meaning of parthood to group membership, φ-CEM$_{SG}$ attributes social dignity to that whole. I consider this one of the proposed account's most interesting and powerful results. Recall the existentially relativised version of A5, A6. A6 says that if there are some actual entities actually satisfying a certain φ, there is something which is the actual fusion of them (therefore formally proving what I have informally defended above). However, we have more. T52, the theorem we proved at the end of 3.3.5, says that if there are some actual entities satisfying a certain φ, then there is something which is a possible fusion of them. This means that the fusion of the xs might be non-actual but still exists as a possible entity in the actual world. In the case of social groups, this means that even if the fusion of all actual xs satisfying φ = 'is a transgender person' might not have been an actual entity before the 1960s, there was still a possible fusion of them back then. The same holds for φ = 'is a possible member of the LGBTQ+ community'. There was no LGBTQ+ community in the Sixties, i.e., the LGBTQ+ community was not an actual fusion, but it was a possible fusion already back then because there were already actual entities satisfying the relevant φ. In other words, it does not really matter whether we consider a certain fusion to be actual or not. As long as there are actual entities satisfying the relevant φ, that possible fusion exists in the actual world.

5.3.4 Objection from Selectivity: A Reply

As regards φCEM$_{SG}$'s reply to the objection from selectivity, we can be brief. Which entities can be members of a social group and which conditions must be met for those entities to be members crucially relies on the φ- conditions we decide to implement in φCEM$_{SG}$. Several options available in the literature might be a good fit. For instance, φ-CEM$_{SG}$ could consider a mereological whole to be a social group if, and only if, some, or all, of its parts are agents and they are 'appropriately designed to contribute to the group's functioning' (Strohmaier, 2018, p.132). Somewhat closer to sociology, we might want to endorse Durkheim's inspired account Greenwood (2003) offers:

> Populations that constitute social groups are those populations whose members are bound by shared social forms of cognition, emotion, and behavior. Plausible examples

> of social groups constituted in this fashion would include the population of accountants, gays, historians, Gaelic speakers, the Azande, the Mafia, feminists, Protestants, Democrats, the cotizens of the city-state of Singapore (possibly) and of the United Kingdom (doubtfully). (p.102)

Another suggestion comes from Gilbert (1992), whose account is instead inspired by the work of the sociologist Georg Simmel. She writes:

> [...] human beings X, Y, Z, constitute a collectivity (social group) if and only if each appropriately thinks of himself and the others as 'us' or 'we'. 'We' is this sense refers to a set of people each of whom *shares with oneself* in some action, belief, attitude, or similar attribute. (p. 204)

The accounts above are by no means the only ones. There are many other relevant accounts in the literature one might wish to endorse and which φ-CEM$_{SG}$ might well accommodate. Given the present purposes, my goal is not to defend one particular set of φ-conditions for social groups. I have two other aims in mind instead. Firstly, to show that whatever our preferred set of necessary and sufficient conditions for social groups is, it can be implemented in φ-CEM$_{SG}$ as φ-conditions. Secondly, to highlight that these φ-conditions are, by their own nature, chosen from outside CEM. As discussed in the previous chapter, the special 'nature of the links' characterising social groups cannot be expected to arise within mereology. We need a cross-disciplinary approach to have a satisfactory account of social groups. In conclusion, we can say that φ-CEM$_{SG}$ makes some entities members of a social group but not others depending on which φ-conditions are put in place.

5.3.5 Score

The table below summarises our findings:

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
yes	yes	yes	yes	yes	yes	yes	yes	yes

Besides satisfying the same *desiderata* of CEM$_{SG}$, φ-CEM$_{SG}$ can also account for (f), (g), (h), and (i). Contrarily to what is commonly assumed, an account of social groups based on CEM is possible. The joint work of modal CEM and the notion of φ-parthood make that happen. Therefore, we can conclude that 'whatever relations human beings bear to social entities, the relation of being a part of it is not of them' (Ruben, 1983, p.219) should not be a mantra anymore – also in respect to CEM.

Conclusion

The spark for this thesis has been the recognition that

> Philosophers have sometimes taken "mereological fusions" to be things that *by definition* have certain properties that, in fact, bare theory CM [Classical Mereology] is neutral on. I believe that many have thought that, *by definition* a fusion has one or more of these three features: (1) it does not gain or lose parts; (2) it could not have had any parts other than those it in fact has; and even (3) it has its "identity conditions" determined by some ultra-thin kind or sortal like *mere aggregate of stuff/things.* (Hovda, 2013, p. 247, author's emphasis)

Paul Hovda, like Gabriel Uzquiano, is among the few whose work helped shed some light on the real implications of CEM in a modal and temporal scenario. With this thesis, I have aimed to contribute to that project by adopting the most neutral approach possible. To do that, I changed the underlying logical setting of CEM, switching from FOL to QML, without modifying the mereological theory itself. I was interested in seeing, given some necessary inputs, what the 'invisible hand' of logic would give us back when we allow QML to be the *substratum* of CEM. This project challenged conventional ideas about the implications of CEM by showing which theorems are actually provable and clarifying what distinguishes them from mereological essentialism, which is not provable. We have also seen that, in a modal scenario, the existentially restricted version of the axiom of unrestricted composition proves interesting results that call for further research.

Another way I tried to contribute with my work has been to apply the formal findings to social ontology by engaging with the issue of social groups. I defended the idea that the account of social groups based on CEM has been unfairly doomed to oblivion. On the one hand, I hope I have raised awareness of the real implications of such an account; on the other, I hope I have given good reasons why group membership should not be taken to be *just* parthood. After all, these final words are undeniably part of this thesis, but neither of us sees them as just 'parts'. They are (and we both know it) functional parts in a closing remark.

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