Force-free Collisionless Current Sheets: A Systematic Method for Adding Asymmetries

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Current sheets are regions of space in which the current density is very localised and varies strongly in one spatial direction. Equilibrium particle distribution functions are known for force-free current sheets and lead to spatial density and temperature structures which are either constant in space or vary symmetrically. Recent observations of current sheets in the solar wind have shown systematic asymmetries in particle density and temperature while the pressure remains constant (further references in [1]). In this contribution we describe a systematic approach to finding distribution functions for this specific case. This mathematical foundation has been used to show why examples mentioned in Neukirch et al. (2020) [1] succeed and how it can be used to find new ones. The latter is not a straightforward process: even if a function satisfies all mathematical requirements, it can still be unreasonable in a physical sense.

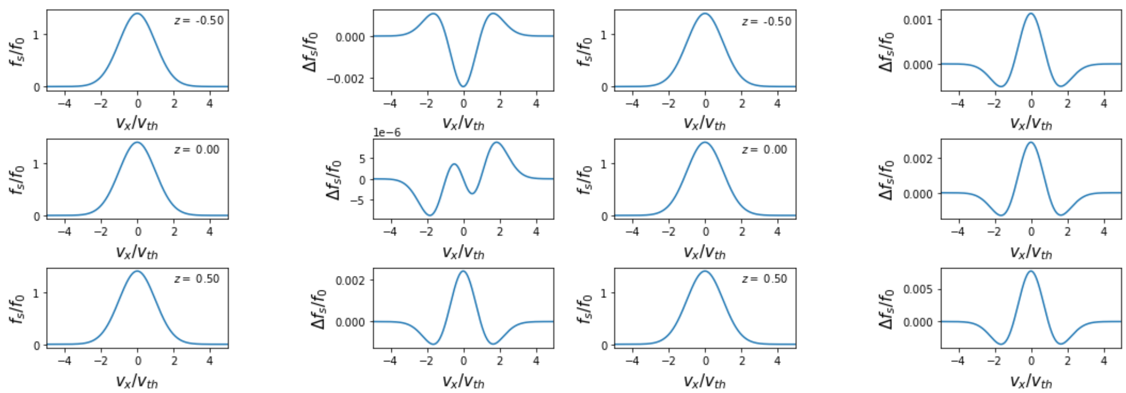

Figure 1: *Dependence of the full particle distribution function $f_s = f_{ff,s} + \Delta f_s$ on v_x (for $v_y = v_z = 0$) at three different positions $z/L = -0.5$ (top row), $z/L = 0.0$ (middle row) and $z/L = 0.5$ (bottom row) for g_2 with k_2 (far left) and k_3 (second from right). The same for Δf_s alone (k_2 second from left, k_3 far right). Here $\varepsilon = 0.01$ and $u_0/v_{th} = -3.9 \cdot 10^{-3}$.*

Due to the one-dimensional nature of the current sheet models one can find equilibrium distribution functions of the form

$$f \equiv f(H, p_x, p_y)$$

where $H = \frac{1}{2}mv^2 + q\phi$ is the Hamiltonian with $v^2 = v_x^2 + v_y^2 + v_z^2$ and $p_x = mv_x + qA_x$ and $p_y = mv_y + qA_y$ are the canonical momenta in x and y direction. We start from a known distribution function for the force-free Harris sheet [2]. In order to model the observed asymmetric variations in density and temperature Neukirch et al. (2020) [1] have added a term Δf to the force-free Harris sheet distribution function, leading to

$$f = f_{ff} + \Delta f$$

where Δf adds a spatial asymmetry to the number density, but does not contribute to the current density, i.e. satisfies

$$\int \Delta f d^3v \neq 0,$$

and

$$\int \mathbf{v}\Delta f d^3v = 0. \tag{1}$$

As seen in Neukirch et. al (2020) [1] one possible class of functions for Δf is of separable form, such that $\Delta f \equiv \Delta f(H, p_x) = g(H)k(p_x)$ with $\partial \Delta f/\partial p_x = g(H)k'(p_x)$. Different approaches to find suitable pairs of $g(H)$ and $k(p_x)$ exist of which we have focussed on an ansatz using Fourier transformation. Let $\phi = 0$ and $G \in C^2(\mathbb{R})$ with $g(H) = G''(H)$. Let F be an appropriate function with $F(qA_x - p_x) = G\left((p_x - qA_x)^2/(2m)\right)$, then condition (1) can be written as

$$0 = \int_{-\infty}^{\infty} k'(p_x)F(qA_x - p_x)dp_x$$

which is an integral of convolution type. Therefore, it's Fourier transform is given by the product of the Fourier transforms of k' and F individually:

$$\hat{k}'(u) \cdot \hat{F}(u) = 0 \tag{2}$$

where the Fourier transform is indicated by ˆ. Neukirch et al. (2020) [1] have proposed combinations of $k_1(p_x) = C_1 p_x$, $k_2(p_x) = \frac{1}{\omega}\sin(\omega p_x)$ and $k_3(p_x) = \frac{1}{\omega}\exp(\omega p_x)$ with $g_1(H) = K(e^{-aH} - ce^{-bH})$ and $g_2(H) = K(a - bH)e^{-bH}$. The Fourier transform approach can be used to derive these functions but we have shown that trigonometric and exponential forms of k imply asymmetries that are not consistent with observations (see [1]). Figure 1 shows the dependence of the full particle distribution functions and its asymmetric contribution on v_x for specific values of z and Figure 2 shows the related asymmetric density and temperature profiles.

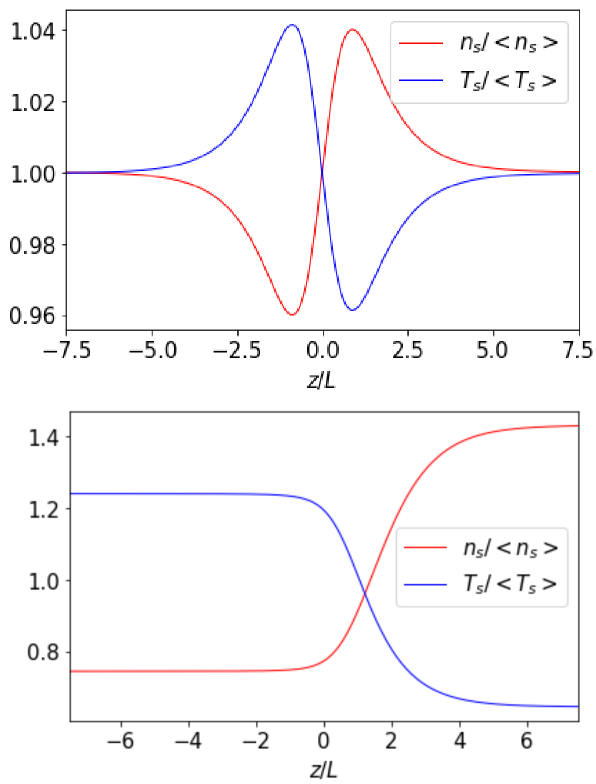

Figure 2: *Asymmetric density and temperature profiles resulting from the theoretical model for k_2 on the top and k_3 on the bottom (identical for with g_1 and g_2 in both cases). ($\varepsilon = 0.01$)*

We can use condition (2) to find new pairs of functions. The function $\delta(H_0 - H)$ has been studied as the H-dependent part of the distribution function in Wilson and Neukirch (2011) [3]. Inspired by this we set $g(H) = \delta(H_0 - H)$ in our separable approach and find $\hat{F} = ((4m\pi^3)^{-1}\sin(2\pi\gamma u) - \gamma(2m\pi^2)^{-1}u\cos(2\pi\gamma u))/u^3$. Then we choose $\hat{k}'$ such that equation (2) vanishes. Avoiding the singularity of $\hat{F}$ at $u = 0$ we focus on the case $u \neq 0$. In this case the roots of $\hat{F}$ are defined by $(2\pi)^{-1}\tan(2\pi\beta u) - \beta u = 0$. Solutions for this equation exists but cannot be determined analytically, so we remain by saying that if u^* is one of these roots we can choose $\hat{k}'(u) = \delta(u - u^*)$ and hence we obtain $k(p_x) = \frac{1}{2\pi i u^*}\exp(2\pi i u^* p_x)$. We note that major problems of this example are caused by the derivatives of the delta dirac function that can lead to the distribution function not being zero and that prevents the boundary conditions from being satisfied. We clearly have Δf larger than zero at all times, which using an alternative k of trigonometric origin would not be given. Substituting the dirac delta function and the exponential directly into the condition on the current density we notice that this condition can never be satisfied. If g is a dirac delta function our condition can be written as

$$\int_{-\infty}^{\infty} k'(p_x)(H_0 - \overline{H}_{min})\theta(H_0 - \overline{H}_{min})dp_x = 0$$

which can, in the case $k' \equiv \exp$, never vanish because $\theta(x) > 0$ for all x and $(H_0 - \overline{H}_{min})$ has the same sign for all p_x. So possibilities for k' only include functions that are odd with respect to v_x, but if k attains negative values the full particle distribution function might not remain positive everywhere. In conclusion this example is not suitable as Δf in a physical sense, even though the mathematical side can be worked out fully by numerical methods.

Alternatively, we can choose both Fourier transforms in condition (2) directly and then determine g and k. This "inverse" approach is more difficult to apply compared to the one described above and it leaves it more unpredictable if the resulting pair of functions will make sense physically. As an example we combined Heaviside functions with an exponential function and an arbitrary function h given by $\hat{F}(u) = \theta(u - u_0)e^{-au^2}$ and $\hat{k}'(u) = \theta(u_1 - u)h(u)$ with u_0 and u_1 such that equation 2 is satisfied. This choice leads to k and g being defined through $\exp(x)$, erf(x) and Ei(x). In this case particle density and pressure tensor contribution of Δf include integrating products of these special functions, which either do not exist or in the best case need to be determined numerically.

We have introduced a mathematically systematic approach to finding distribution functions that lead to asymmetries in density and temperature. Using a separable representation and Fourier transformation we have found examples of distribution functions that are mathematically correct but do not fulfil all physical requirements, e.g. boundary conditions. In this context numerical methods to solve special function integrals and approximations of functions such as the delta function could be tried.

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References

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