Coronal Magnetic Field Extrapolation Using a Specific Family of Analytical 3D MHS Equilibria

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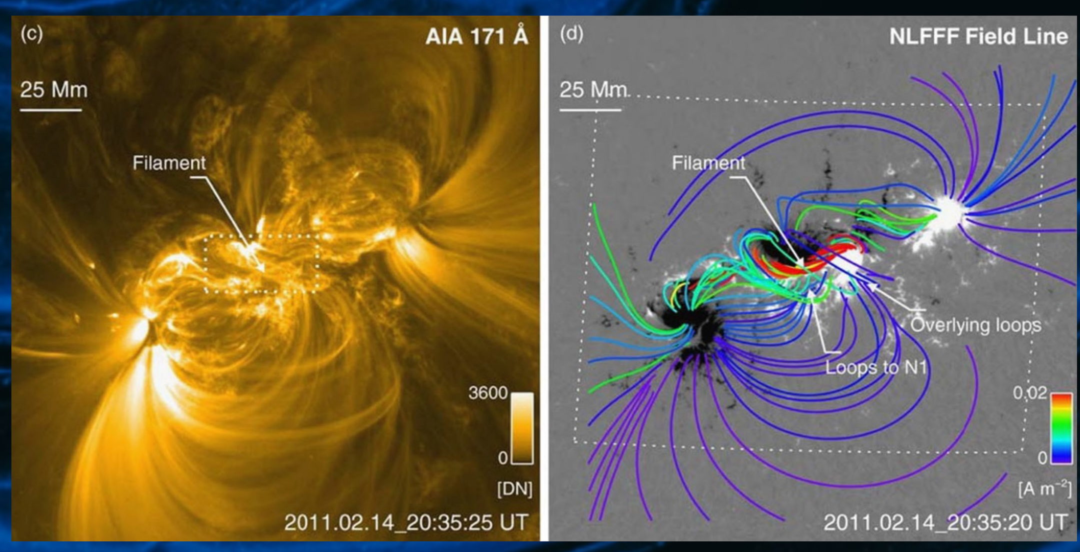
I. INTRODUCTION

- Coronal magnetic field models have to rely on extrapolation methods using photospheric magnetograms as boundary conditions
- The non-force-free lower regions of the solar atmosphere require magnetohydrostatic (MHS) field models instead of force-free extrapolation methods



[Wiegelmann et al. (2017)]

II. THEORY

- Numerical methods to calculate MHS solutions can deal with non-linear problems and provide accurate models
- Analytical three-dimensional MHS equilibria can be used as a numerically "cheaper" complementary method
- We discuss a family of analytical MHS equilibria that allows for a transition from a non-force-free region to a force-free region

III. MAGNETOHYDROSTATIC EQUATIONS

Magnetohydrostatic equation:	$j \times B - \nabla p - \rho \nabla \Psi = 0$
Ampere's Law:	$\nabla \times B = \mu_0 j$
Solenoidal constraint:	$\nabla \cdot B = 0$
Force-free fields:	$j \times B = 0 \Rightarrow \mu_0 = \alpha(r) B$
Non-force free fields:	Current density has a component perpendicular to the magnetic field vector!

A transition from non-force-free to force-free (photosphere to corona) with increasing height z can be modelled by incorporating a function $F = f(z) B_z$ into the current density:

$$\mu_0 j = \alpha B + \nabla \times (F \hat{z})$$

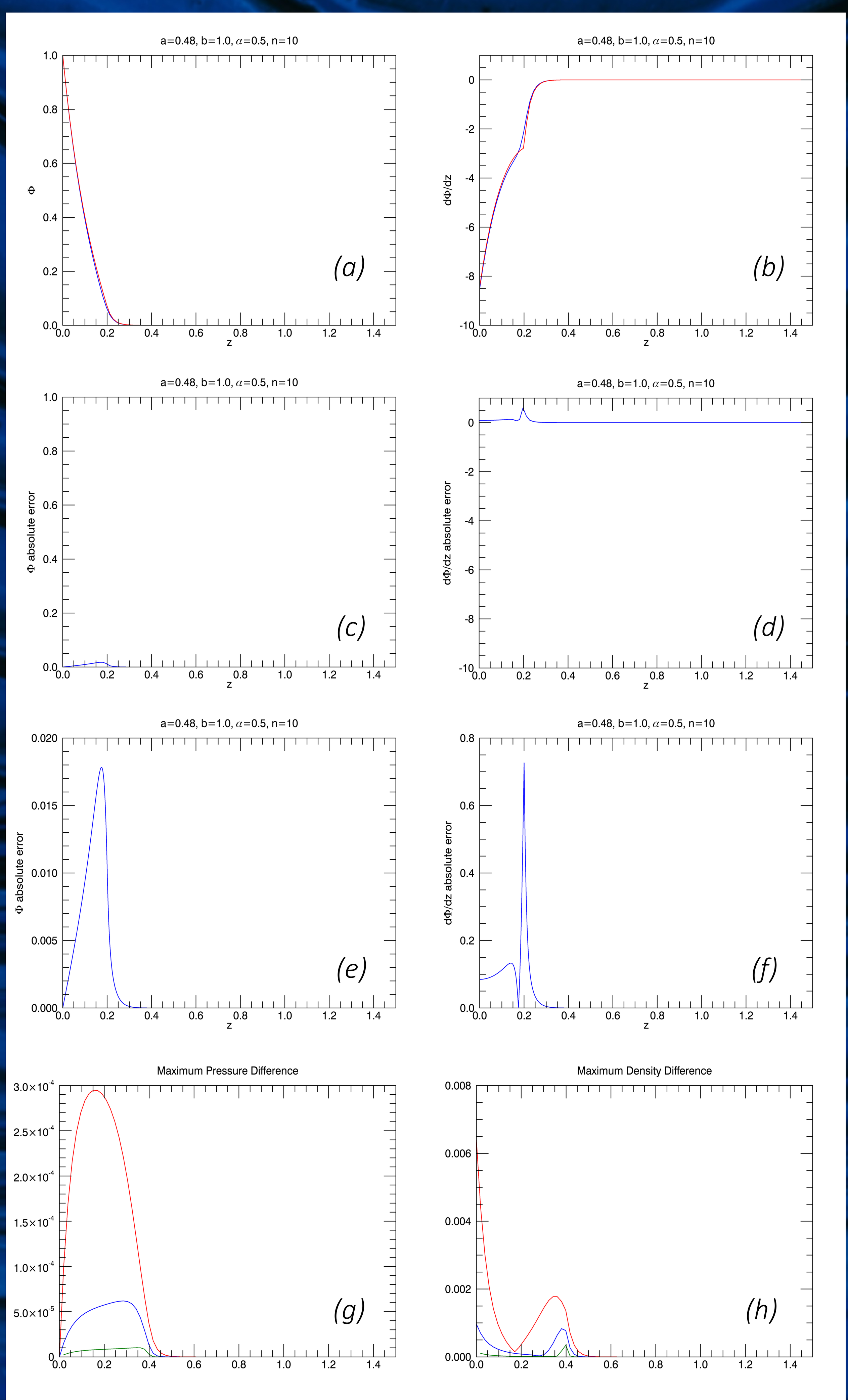
In our case we use a hyperbolic tangent height profile as "switch-off"-function:

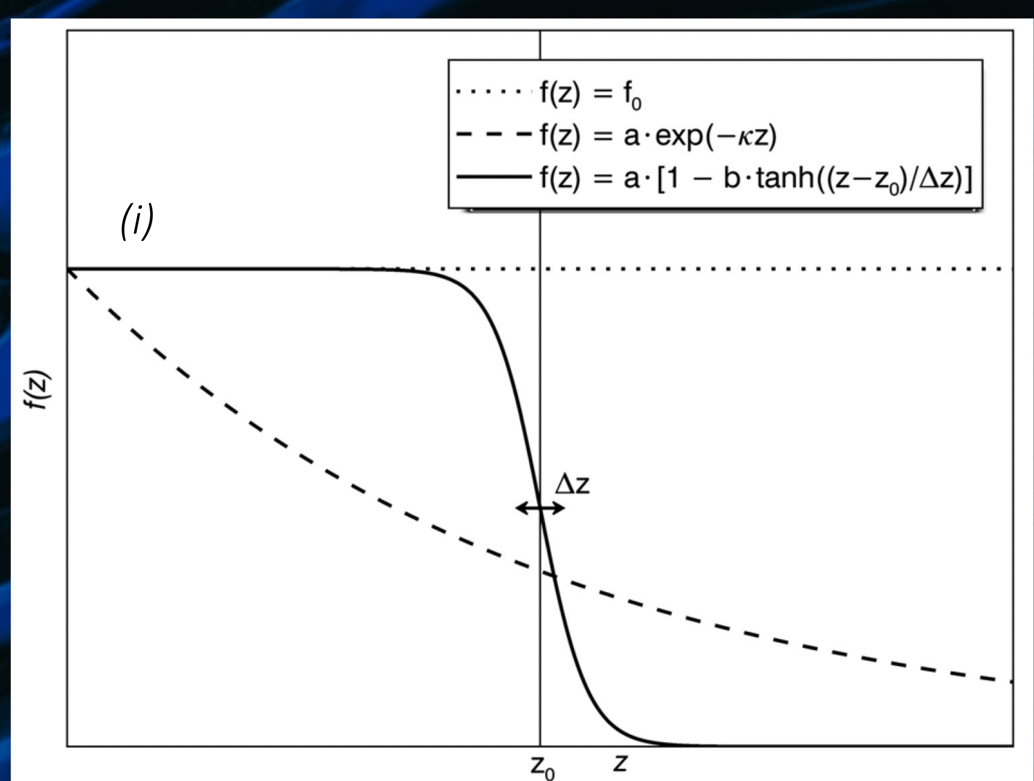
$$f(z) = a\left[1 - b \tanh\left(\frac{z - z_0}{\Delta z}\right)\right]$$

Plot (i) below [Neukirch and Wiegelmann (2019)] shows this height profile and also alternative (linear and exponential) versions of f as used by e.g. Low (1991).

IV. METHOD

- The analytical solution of the MHS model using a current density as defined on the left involves special functions (hypergeometric) [see Neukirch and Wiegelmann (2019)]
- Routines for the calculation of these are available, but can affect both the speed and the numerical accuracy of the calculations
- The asymptotic behaviour of this solution can be used to numerically approximate it through exponential functions aiming to improve the numerical efficiency





(a), (b): Analytical (red) and asymptotic (blue) version of function $\bar{\Phi}$ and its first derivative w.r.t. to z plotted for a single Fourier mode. The x-axis displays height z from photosphere into the corona.

(c), (d): Aboslute error between the red and the blue function from above. We see the greatest error occuring around z_0.

(e), (f): Error plots from above zoomed .

(g), (h): Maximum difference in plasma pressure and density for different choices of Δz. In red $\Delta z = 0.1$, in blue $\Delta z = 0.05$ and in green $\Delta z = 0.02$

V. RESULTS

- Model includes transition from non-force-free to force-free using a special function that allows for more flexibility
- Asymptotic approximation of hypergeometric function performs well
- Error in ρ and p small in relevant parameter regimes [see (g), (h)], in B of the order of 10^{-6}
- Asymptotic calculation of the magnetic field improves running time of code

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