Compensatory and Noncompensatory Consumer Strategies in a Monopolistic Screening Model

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Abstract

A monopolist supplies a multi-attribute good and does not know whether the consumer makes or avoids tradeoffs between attributes. We illustrate a form of exploitation to which the tradeoff-avoiding consumer is vulnerable. (JEL: D6, D8, L1, L2)

Keywords: Bounded Rationality, Compensatory Strategy, Noncompensatory Strategy, Monopolistic Screening, Satisficing.

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1 Introduction

Ellison (2006) identifies three distinct traditions in the literature on bounded rationality and industrial organization. The first one is called *rule-of-thumb* approach. This tradition, rather than characterizing equilibrium behavior, assumes that economic agents behave in some simple way. The second one is called *explicit bounded rationality* approach, assumes that cognition is costly, and derives second-best behaviors, given the costs. The third one models economic agents as individuals subject to biases typically detected in experimental economics and psychology and examines what happens in a market in which consumers or firms exhibit such a bias. Our paper belongs to the third category.

A decision strategy is called *compensatory* whenever the decision-maker makes tradeoffs between attributes and *noncompensatory* whenever she does not (Payne, Bettman and Johnson, 1993). Imagine that a consumer, who wants to purchase a new car, is interested in only two attributes: price and level of emissions. Assume that she prefers to spend as little as possible and to own a car that does not pollute that much. Whether she follows either a compensatory or a noncompensatory strategy has an effect on her choice behavior. For instance, if she follows a compensatory decision strategy, then she may be indifferent between $\alpha = (p_\alpha, q_\alpha)$ and $\beta = (p_\beta, q_\beta)$, where p stands for price and q for quality in terms of level of emissions with $p_\alpha < p_\beta$ and $q_\alpha < q_\beta$.[1] On the contrary, if she follows a noncompensatory strategy, then she might judge one attribute more important than the other. For example, if price is more important than level of emissions, then she prefers α over β for any α, β such that $p_\alpha < p_\beta$. Standard utility maximization is an example of the former and the *lexicographic heuristic* of the latter.

Experimental evidence is consistent with both kinds of strategy. More precisely, when time pressure is relatively low (resp., high), subjects tend to rely on a compensatory (resp., noncompensatory) strategy (Payne, Bettman

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[1]We interpret $q_\alpha < q_\beta$ if and only if α's level of emissions is greater than β's.

and Johnson, 1993; Rieskamp and Hoffrage, 1999; 2008). The intuition for these findings is that compensatory strategies require more cognitive effort than noncompensatory ones, whereas compensatory strategies are more accurate.[2]

We propose a monopolistic screening model in which a monopolist supplies a two-attribute good and does not know whether the consumer follows either a compensatory or a noncompensatory strategy. We assume that the consumer that makes tradeoffs is an expected-utility maximizer. On the other hand, as noncompensatory strategy we employ the *satisficing heuristic* (Simon, 1955), according to which the decision-maker has an aspiration level for each attribute and judges satisfactory all those alternatives that meet each threshold. This choice is motivated by the fact that there is experimental evidence supporting Simon's idea of bounded rationality and satisficing (Caplin, Dean and Martin, 2009; Reutskaja et al., 2011). Our goal is to investigate the effects of monopolist's behavior on consumer's welfare.

Our main finding is that the boundedly rational consumer is subject to exploitation. That is, the monopolist exploits the fact that the boundedly rational consumer is unwilling to make tradeoffs between attributes to always supply to her an alternative whose attributes are never better than her aspiration levels. This holds independently of the probability that the consumer is an expected-utility maximizer. The intuition is that the fact that the boundedly rational consumer does not make tradeoffs between attributes (and the expected utility-maximizer does) always allows the monopolist to supply the 'minimum' to the boundedly rational consumer and, by properly combining the attributes, an unsatisfactory alternative that yields the reservation utility to the other type. We argue that one way to reduce this form of exploitation is to promote policies aimed at discouraging the fully rational consumer from considering 'extreme' alternatives.

Our main result is in line with the literature on bounded rationality and

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[2]We intend accuracy as it is defined by Payne, Bettman and Johnson (1993). That is, as how far is a given strategy from standard theory in prescribing a certain choice.

industrial organization. Rubinstein (1993), for instance, proposes a model in which a monopolist faces a population of heterogeneous consumers that differ in the ability to process information. Rubinstein (1993) shows that the monopolist can increase its profits by discriminating among consumers with different abilities. Spiegler (2006a), instead, defines boundedly rational consumers as individuals that follow an 'anedoctal' kind of reasoning. He shows that even market interventions aimed at pushing out low-quality firms do not improve welfare. Spiegler (2006b), on the other hand, proposes a market model in which profit-maximizing firms compete in a multidimensional pricing framework by defining boundedly rational consumers as individuals that have limited capacity of understanding complex objects. In that model each firm to an increase in competition responds by putting in practice a confounding strategy rather than a strategy of more competitive prices. Eliaz and Spiegler (2006) assume that individuals have dynamically inconsistent preferences and are heterogeneous in the sense that they have different abilities in forecasting potential changes in their future tastes. For instance, more sophisticated types are more capable of perceiving changes in their preferences. Eliaz and Spiegler (2006) find that more naive individuals are typically subject to an higher exploitation and are associated with higher profits for the principal. Finally, Rubinstein and Spiegler (2008) investigate vulnerability to exploitation of boundedly rational invididuals that have to decide whether or not to buy a lottery.[3]

The paper is organized as follows. Section 2 introduces the formal model; Section 3 contains the formal analysis; Section 4 discusses the results and concludes.

2　The Model

Let $X \subseteq \Re_+^0 \times \Re_+$. Each vector $x = (p_x, q_x) \in X$ represents a two-attribute alternative, where $p_x > 0$ is the price and $q_x \geq 0$ some attribute different

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[3]See Ellison (2006) and Spiegler (2011) for a survey of this literature.

from price.

We consider the simplest setting by assuming that there is a monopolist that produces good x whose profit function is $\pi(x) = p_x - \alpha q_x$, where $\alpha \in (0, 1)$. The goal of the monopolist is to maximize profits.

On the demand side there are two kinds of consumer: the fully and the boundedly rational consumer (FRC and BRC, respectively). The FRC is an expected utility maximizer whose Bernoulli utility function is defined as $u(x) = u(g(q_x) - p_x)$, where the function $g(q_x) = \ln(q_x + 1)$ measures how much the FRC values characteristic q_x in monetary units. The FRC is assumed to be risk averse and her reservation utility is $\bar{u} > u(0)$.

The BRC follows the satisficing heuristic: she judges a good $y \in X$ to be satisfactory whenever $p_y \leq \bar{p}$ and $q_y \geq \bar{q}$, where $\bar{p} > 0$ and $\bar{q} > 0$ are the aspiration levels for price and quality, respectively. Let $(\bar{p}, \bar{q}) \in X$ be the *minimal satisfactory alternative*. The BRC's goal is to get a satisfactory product.

From now on we denote by $x = (p_x, q_x)$ and $y = (p_y, q_y)$ the products that the monopolist supplies to the FRC and to the BRC, respectively.

3   Analysis

We first assume that the monopolist knows that the consumer is an FRC. In this case the monopolist maximizes its profits subject to the reservation-utility constraint.

PROBLEM 1
$\max_{p_x, q_x} p_x - \alpha q_x$
s.t. (i) $\ln(q_x + 1) - p_x \geq u^{-1}(\bar{u})$

Let the solution be denoted by $x^c = (p_x^c, q_x^c)$, where c stands for *certainty*. Proofs are given in the appendix.

Proposition 1. *The solution to problem 1 is characterized as follows:* $x^c = (ln(\frac{1}{\alpha}) - u^{-1}(\bar{u}), \frac{1}{\alpha} - 1)$.

Suppose now that the monopolist knows that consumers is a BRC. In this case the monopolist maximizes its profits subject to the constraint that the supplied product y has to be satisfactory, that is, $p_y \leq \bar{p}$ and $q_y \geq \bar{q}$.

PROBLEM 2

$$\max_{p_y, q_y} p_y - \alpha q_y$$
$$\text{s.t. (i) } p_y \leq \bar{p}$$
$$\text{(ii) } q_y \geq \bar{q}$$

Let the solution be denoted by $y^c = (p_y^c, q_y^c)$. We omit the proof as the result is immediate.

Proposition 2. *The solution to problem 2 is characterized as follows:* $y^c = (\bar{p}, \bar{q})$.

Finally assume that the monopolist does not know with certainty whether the consumer is an FRC or a BRC. Let $\rho \in (0, 1)$ be the probability that the consumer is an FRC. The monopolist has to find two goods $x, y \in X$ such that expected profits are maximized, where x yields the reservation utility and y is satisfactory. However, the optimal solution must satisfy also the incentive-compatible constraints. That is, first, alternative x must yield at least the same level of utility of good y. Second, good x does not have to be more than satisfactory. In order to ensure the second incentive-compatible constraint to hold, we impose $p_x \geq \bar{p}$.[4]

PROBLEM 3

$$\max_{p_x, p_y, q_x, q_y} \rho (p_x - \alpha q_x) + (1 - \rho) (p_y - \alpha q_y)$$

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[4]Alternatively, we could have assumed $q_x \leq \bar{q}$ and the results would have been qualitatively the same.

s.t. (i) $\ln(q_x + 1) - p_x \geq u^{-1}(\bar{u})$

(ii) $p_y \leq \bar{p}$

(iii) $q_y \geq \bar{q}$

(iv) $\ln(q_x + 1) - p_x \geq \ln(q_y + 1) - p_y$

(v) $p_x \geq \bar{p}$

Let the solution be denoted by $x^u = (p_x^u, q_x^u)$ and $y^u = (p_y^u, q_y^u)$, where 'u' stands for *uncertainty*.

Proposition 3. *The solution to problem 3 is characterized as follows.*

1. *Assume $\bar{u} \geq u(\bar{p}, \bar{q})$.*

   (a) *if $\bar{p} < p_x^c$, then $x^u = \left(\ln\left(\frac{1}{\alpha}\right) - u^{-1}(\bar{u}), \frac{1}{\alpha} - 1\right)$ and $y^u = (\bar{p}, \bar{q})$;*

   (b) *if $\bar{p} \geq p_x^c$, then $x^u = \left(\bar{p}, e^{\bar{p} + u^{-1}(\bar{u})} - 1\right)$ and $y^u = (\bar{p}, \bar{q})$.*

2. *Assume $\bar{u} < u(\bar{p}, \bar{q})$.*

   (a) *if $\bar{p} < p_x^c$, then $x^u = \left(\ln\left(\frac{1}{\alpha}\right) - \ln(\bar{q} + 1) + \bar{p}, \frac{1}{\alpha} - 1\right)$ and $y^u = (\bar{p}, \bar{q})$.*

   (b) *if $\bar{p} \geq p_x^c$, then $x^u = (\bar{p}, \bar{q})$ and $y^u = (\bar{p}, \bar{q})$.*

4   Discussion and Conclusion

We draw several conclusions from the previous section. First, the FRC always gets at least the reservation utility. However, she is better off under uncertainty, because if the thresholds $\bar{p}$ and $\bar{q}$ are particularly high, then she gets more than $\bar{u}$.

Second, the BRC gets nothing more than the minimal satisfactory alternative independently of whether the monopolist is certain or uncertain about the consumer's type.

Third, the fact that the FRC makes trade-offs between attributes always allows the monopolist to give the minimum to the BRC and by properly

combining the attributes an unsatisfactory good that yields the reservation utility to the FRC. Figure 1 graphically illustrates this intuition.
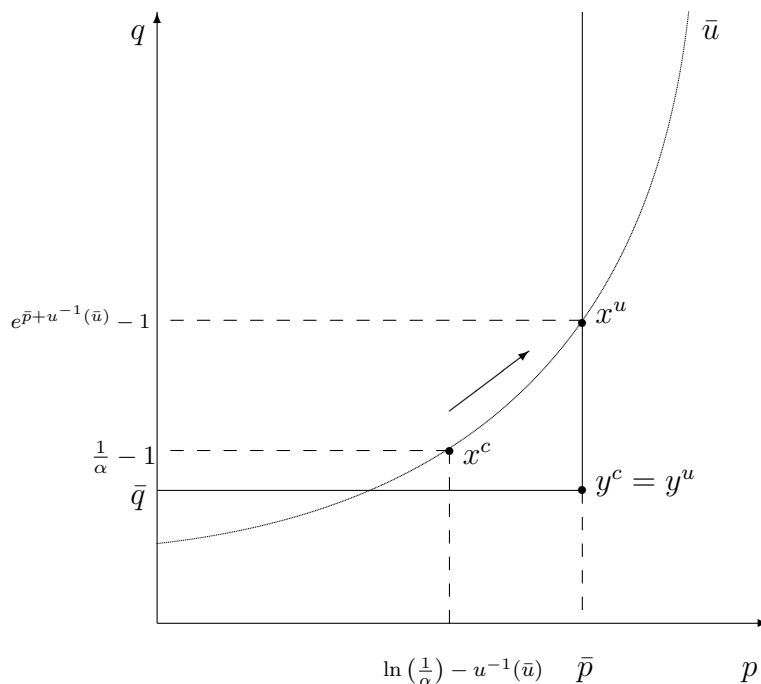

Figure 1: Noncompensatory preferences prevent the BRC from getting something more than $(\bar{p}, \bar{q})$

Assume that $\bar{p} > p_x^c$ and that $\bar{q} < q_x^c$, so that both x^c and y^c are satisfactory and x^c is Pareto-superior to y^c. That is, x^c is a satisfactory alternative and unsatisfactory alternatives are relatively 'extreme' from the FRC's point of view. In this case, one expects that under uncertainty the monopolist makes the BRC better off by supplying to both the FRC and the BRC a good that yields $\bar{u}$. On the contrary, the optimal solution is $x^u = \left(\bar{p}, e^{\bar{p}+u^{-1}(\bar{u})} - 1\right)$ and $y^u = (\bar{p}, \bar{q})$. That is, the monopolist *pushes* x towards north-east along the indifference curve that yields $\bar{u}$ until it reaches the border of the satisficing area. In this way x^u is unsatisfactory and yields the reservation utility and the monopolist is free to supply the minimal satisfactory alternative to

the BRC.[5] Notice that the monopolist can adopt this strategy precisely because, on the one hand, the FRC is perfectly able to make tradeoffs between attributes, and, on the other hand, the BRC never exchanges satisfactory with unsatisfactory alternatives. We believe that this is an interesting result, because it provides a clear intuition of why independently of the probability ρ the fact that there is uncertainty about the consumer's type does not increase BRC's welfare. This result is in line with the literature on bounded rationality and industrial organization discussed in the introduction that suggests that the BRC is subject to exploitation.

Fourth, one way of improving BRC's welfare in the case depicted in figure 1 is to implement policies aimed at making unsatisfactory alternatives unattractive to the FRC. This would diminish the probability that the FRC considers 'extreme' alternatives and, in turn, reduce the monopolist's production space to satisfactory alternatives only. In this case the monopolist would be forced to supply to both types an alternative that yields the reservation utility, making the BRC better off.

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[5]Technically, x^u is satisfactory. However, since we are on the continuum, we interpret x^u as unsatisfactory, where $p_x^u = \bar{p} + \epsilon$.

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A Proofs

Proof of Proposition 1. Problem 1 can be rewritten as follows.

$\max_{p_x,q_x} p_x - \alpha q_x$

s.t. (i) $p_x - \ln(q_x + 1) \leq -u^{-1}(\bar{u})$

Here are the Lagrangian and the Kuhn-Tucker conditions.

$$\mathcal{L}(p_x, q_x, \lambda) = p_x - \alpha q_x - \lambda(p_x - \ln(q_x + 1) + u^{-1}(\bar{u}))$$

$$\frac{\partial \mathcal{L}}{\partial p_x} = 1 - \lambda = 0 \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial q_x} = -\alpha + \frac{\lambda}{q_x + 1} \leq 0 \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \ln(q_x + 1) - p_x - u^{-1}(\bar{u}) \geq 0 \tag{3}$$

together with $p_x > 0$, $q_x, \lambda \geq 0$, $q_x \left(\frac{\partial \mathcal{L}}{\partial q_x} \right) = 0$, and $\lambda \left(\frac{\partial \mathcal{L}}{\partial \lambda} \right) = 0$.

By condition A.1, $\lambda^* = 1$. Therefore, constraint (i) must be binding. Next, suppose, by contradiction, that $q_x = 0$. This implies that $p_x = -u^{-1}(\bar{u}) < 0$, which leads to a contradiction. Therefore, $q_x > 0$ and condition A.2 must be binding. Plugging $\lambda^* = 1$ into condition A.2 and solving for q_x, we get $q_x^* = \frac{1}{\alpha} - 1$. Next, plugging q_x^* into condition A.3 and solving for p_x, we get $p_x^* = \ln(\frac{1}{\alpha}) - u^{-1}(\bar{u})$. Let this solution be denoted by x^c.∎

Proof of Proposition 3. Problem 3 can be rewritten as follows.

$\max_{p_x,p_y,q_x,q_y} \rho \left(p_x - \alpha q_x \right) + (1 - \rho) \left(p_y - \alpha q_y \right)$

s.t. (i) $p_x - \ln(q_1 + 1) \leq -u^{-1}(\bar{u})$

  (ii) $p_y - \bar{p} \leq 0$

  (iii) $\bar{q} - q_y \leq 0$

  (iv) $p_x - \ln(q_x + 1) - p_y + \ln(q_y + 1) \leq 0$

(v) $\bar{p} - p_x \leq 0$

Here are the Lagrangian and the Kuhn-Tucker conditions.

$\mathcal{L}(p_x, p_y, q_x, q_y, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6) = \rho(p_x - \alpha q_x) + (1 - \rho)(p_y - \alpha q_y) - \lambda_1(p_x - \ln(q_x + 1) + u^{-1}(\bar{u})) - \lambda_2(p_y - \bar{p}) - \lambda_3(\bar{q} - q_y) - \lambda_4(p_x - \ln(q_x + 1) - p_y + \ln(q_y + 1)) - \lambda_5(\bar{q} - p_x)$

$$\frac{\partial \mathcal{L}}{\partial p_x} = \rho - \lambda_1 - \lambda_4 + \lambda_5 = 0 \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial p_y} = (1 - \rho) - \lambda_2 + \lambda_4 = 0 \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial q_x} = -\rho\alpha + \frac{\lambda_1}{q_x + 1} + \frac{\lambda_4}{q_x + 1} \leq 0 \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial q_y} = -(1 - \rho)\alpha + \lambda_3 - \frac{\lambda_4}{q_y + 1} \leq 0 \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = \ln(q_x + 1) - p_x - u^{-1}(\bar{u}) \geq 0 \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = \bar{p} - p_y \geq 0 \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_3} = q_y - \bar{q} \geq 0 \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_4} = \ln(q_x + 1) - p_x - \ln(q_y + 1) + p_y \geq 0 \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_5} = p_x - \bar{p} \geq 0 \tag{12}$$

together with $p_x, p_y > 0$, $q_x, q_y, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \geq 0$, $q_x \left(\frac{\partial \mathcal{L}}{\partial q_x} \right) = 0$, $q_y \left(\frac{\partial \mathcal{L}}{\partial q_y} \right) = 0$, $\lambda_1 \left(\frac{\partial \mathcal{L}}{\partial \lambda_1} \right) = 0$, $\lambda_2 \left(\frac{\partial \mathcal{L}}{\partial \lambda_2} \right) = 0$, $\lambda_3 \left(\frac{\partial \mathcal{L}}{\partial \lambda_3} \right) = 0$, $\lambda_4 \left(\frac{\partial \mathcal{L}}{\partial \lambda_4} \right) = 0$, and $\lambda_5 \left(\frac{\partial \mathcal{L}}{\partial \lambda_5} \right) = 0$.

Notice that since $\bar{q} > 0$, then, by condition A.10, $q_y > 0$, and, therefore, condition A.7 must be binding. Therefore, it must be that $\lambda_3 > 0$. This implies that $q_y^* = \bar{q}$. Next, condition A.5 implies that $\lambda_2 > 0$. Hence, $p_y = \bar{p}$. Further, assume by contradiction that $q_x = 0$. Then, by condition A.8,

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$p_x \leq -u^{-1}(\bar{u})$. Since $u^{-1}(\bar{u}) > 0$, then $p_x < 0$, which leads to a contradiction. Therefore, $q_x > 0$ and condition A.6 must be binding. Finally, notice that it cannot be that $\lambda_1 = \lambda_4 = 0$, because otherwise, by condition A.4, $\lambda_5 = -\rho$, which leads to a contradiction.

- Assume first that $\lambda_5 = 0$. This implies that $p_x > \bar{p}$.

    - Suppose that $\lambda_1, \lambda_4 > 0$. Hence, constraints (i) and (iv) must be binding. Moreover, by condition A.4, $\lambda_1 + \lambda_4 = 1$. Using this result in condition A.6 and solving for q_x, we get $q_x^* = \frac{1}{\alpha} - 1$. Next, plugging q_x^* into condition A.8 and solving for p_x, we get $p_x^* = \ln\left(\frac{1}{\alpha}\right) - u^{-1}(\bar{u})$. By condition A.11, it must be that $\ln\left(\frac{1}{\alpha}\right) - \ln\left(\frac{1}{\alpha}\right) + u^{-1}(\bar{u}) = \ln(\bar{q}+1) - \bar{p}$, which implies that $\bar{u} = u(\bar{p}, \bar{q})$. Moreover, since $p_x > \bar{p}$ and $\bar{p} = \ln(\bar{q}+1) - u^{-1}(\bar{u})$, then $\frac{1}{\alpha} - 1 > \bar{q}$.

    - Assume $\lambda_1 > 0$ and $\lambda_4 = 0$. This implies that constraint (i) must be binding and constraint (iv) holds with strict inequality. Moreover, by condition A.4, $\lambda_1 = \rho$. Using this result in condition A.6 and solving for q_x, we get $q_x^* = \frac{1}{\alpha} - 1$. Next, plugging q_x^* into condition A.8 and solving for p_x, we get $p_x^* = \ln\left(\frac{1}{\alpha}\right) - u^{-1}(\bar{u})$. By condition A.11, it must be that $\ln\left(\frac{1}{\alpha}\right) - \ln\left(\frac{1}{\alpha}\right) + u^{-1}(\bar{u}) > \ln(\bar{q}+1) - \bar{p}$, which implies that $\bar{u} > u(\bar{p}, \bar{q})$. Moreover, since $p_x > \bar{p}$ and $\bar{p} > \ln(\bar{q}+1) - u^{-1}(\bar{u})$, then $\frac{1}{\alpha} - 1 > \bar{q}$.

    - Assume that $\lambda_1 = 0$ and $\lambda_4 > 0$. This implies that constraint (i) holds with strict inequality and constraint (iv) must be binding. Moreover, by condition A.4, $\lambda_4 = \rho$. Using this result in condition A.6 and solving for q_x, we get $q_x^* = \frac{1}{\alpha} - 1$. Next, plugging q_x^* into condition A.11 and solving for p_x, we get $p_x^* = \ln\left(\frac{1}{\alpha}\right) - \ln(\bar{q}+1) + \bar{p}$. By condition A.8, it must be that $\ln\left(\frac{1}{\alpha}\right) - \ln\left(\frac{1}{\alpha}\right) + \ln(\bar{q}+1) - \bar{p} - u^{-1}(\bar{u}) > 0$, which implies that $\bar{u} < u(\bar{p}, \bar{q})$. Moreover, since $p_x > \bar{p}$, then $\frac{1}{\alpha} - 1 > \bar{q}$.

- Suppose that $\lambda_5 > 0$. This implies that $p_x^* = \bar{p}$.

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- Suppose that $\lambda_1, \lambda_4 > 0$. This implies that constraints (i) and (iv) must be binding. Plugging p_x^* into condition A.8 and solving for q_x, we get $q_x^* = e^{\bar{p}+u^{-1}\bar{u}} - 1$. Next, by condition A.11, $\bar{p} + u^{-1}(\bar{u}) - \bar{p} = \ln(\bar{q}+1) - \bar{p}$, which implies that $\bar{u} = u(\bar{p}, \bar{q})$.

- Suppose that $\lambda_1 > 0$ and $\lambda_4 = 0$. This implies that constraint (i) must be binding and constraint (iv) holds with strict inequality. Plugging p_x^* into condition A.8 and solving for q_x, we get $q_x^* = e^{\bar{p}+u^{-1}\bar{u}} - 1$. Next, by condition A.11 $\bar{p} + u^{-1}(\bar{u}) - \bar{p} > \ln(\bar{q}+1) - \bar{p}$, which implies that $\bar{u} > u(\bar{p}, \bar{q})$.

- Suppose that $\lambda_1 = 0$ and $\lambda_4 > 0$. This implies that constraint (i) holds with strict inequality and constraint (iv) must be binding. Plugging p_x^* into condition A.11 and solving for q_x, we get $q_x^* = \bar{q}$. Next, by condition A.8 $\ln(\bar{q}+1) - \bar{p} - u^{-1}(\bar{u}) > 0$, which implies that $\bar{u} < u(\bar{p}, \bar{q})$

This concludes the proof.■