



# Hybrid Modal Realism Debugged

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## Abstract

In this paper, I support a hybrid view regarding the metaphysics of worlds. I endorse Lewisian Modal Realism for possible worlds (LMR). My aim is to come up with a hybrid account of impossible worlds that provides all the plenitude of impossibilities for all fine-grained intentional contents. I raise several challenges for such a plenitudinous hybrid theory. My version of hybrid modal realism builds impossible worlds as set-theoretic constructions out of genuine individuals and sets of them, that is, as set-theoretic constructions from parts and sets of parts of genuine Lewisian worlds. *Structured worlds* are defined as sets of tuples: structured entities built out of Lewisian ‘raw material’. These structured worlds are ersatz worlds, some of which are impossible. I claim that propositions must be sets of worlds rather than members of sets. Once the construction is in place, I evaluate the proposal and show that my hybrid account is able to supply a plenitude of impossibilities and thus giving the resources to make all the hyperintensional distinctions we are looking for, whilst remaining Lewisian-conservative.

## 1 The Search for a Plenitude of Impossibilities

From non-normal modal logics to inconsistent or impossible beliefs, non-trivial counterpossibles and intuitively distinct necessarily or impossible propositions, the motivations for amending the possible worlds account of the corresponding notions (non-normal modalities, doxastic contents, counterfactuals, propositions) with the introduction of impossible worlds are numerous.<sup>1</sup> The possible worlds framework has the inability to make *hyperintensional* distinctions: to discriminate between

<sup>1</sup> See e.g. (Nolan, 1997, 2013, 2021; Berto and Jago, 2019, 2018).

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necessary equivalents. It provides us with analyses which are not fine-grained enough. This is the now familiar ‘Granularity Problem’.<sup>2</sup>

The main shortcomings stem from the identification of propositions with sets of possible worlds. Take the impossible propositions *that it is raining and it is not raining*, *that  $2+2=5$*  and *that Fermat’s Last theorem is false*. They are all true at no possible world. Therefore are all identical to the empty set. Yet, it seems that they are about different things, and that we can believe some but not all. The possible worlds framework is thus unable to account for the difference in aboutness, or to make sense of agents with impossible or inconsistent but non-trivial beliefs.

The motivations for introducing impossible worlds in metaphysics and semantics largely come from these conflicts between the intuitions on the one hand, and the need for adequate semantics for intentional contents on the other hand. This paper aligns itself with the ambition of reaching more fine-grained analyses of contents by enriching the possible worlds framework with impossible worlds. Along these lines, I aim here to offer a *hybrid* account that keeps the benefits of a specific theory of modality, namely Lewisian Modal Realism (LMR), and that encompasses impossible worlds in order to supply the desired plenitude of impossibilities.

What are we looking for? A first requirement for an account to be ‘fine-grainy adequate’ is that it must be able to fulfil with a suitable content all intentional attitudes:

‘plenitude’ means the provision of enough non-trivial content for any possibly held intention. Only a semantics that is plenitudinous in this sense offers a solution to the Granularity Problem, viz. the failure to semantically distinguish intuitively distinct content. (Reinert, 2017, p.135)

In evaluating whether an account of impossible world does its job, we must examine to what extent it affords the plenitudinous representations of impossibilities. This give us a first guideline: investigations into the nature of impossible worlds must be led by the questions surrounding the granularity problem. With this in mind, it is advisable to adopt the following principle set out by Jago:

The answer to the granularity question depends on what we want worlds (and sets of worlds) to do in our theory. It might be that different kinds of worlds, with differing granularities, are required... for these applications. A full semantic theory will need to analyse all of these notions and more. *So we should adopt the principle that the application demanding the finest grain should determinate the granularity of worlds in general.* (Jago, 2015, pp.588-89. My italics)

In what follows I introduce the hybrid theories of impossible worlds and raise challenges for a fine-grained hybrid account (Sect. 2); I then set out my hybrid proposal, ‘Structural Hybrid Modal Realism’ (SHMR) (Sect. 3); I evaluate SHMR with respect to the challenges raised previously (Sect. 4), and conclude (Sect. 5).

<sup>2</sup> As named by Barwise (1997).

## 2 Being Hybrid About Impossible Worlds

I have two main ambitions: (i) supplying the resources to reach the plenitude of impossibilities in order to provide very fine-grained accounts of hyperintensional notions and (ii) retaining the benefits of Lewisian Modal Realism (LMR), above all its reductive account of modality. In order to satisfy (i) and (ii), I will support a hybrid account of worlds in the line of Berto's Hybrid Modal Realism (HMR). Standing for a hybrid theory is standing 'against the parity thesis' according to which possible worlds and impossible worlds are on a par regarding their metaphysical status. Berto defines what 'being hybrid' means:

(1) go realist when it is about possible worlds, and (2) exploit the set-theoretic machinery of modal realism to represent different impossible worlds and impossible propositions as distinct ersatz abstract constructions. (Berto, 2010, p.481)

Here is an important constraint on a hybrid construction of worlds: extra ontological costs or intensional entities should not appear at any stage of the proposal. Impossible worlds, and more generally ersatz worlds, will have to be set-theoretic constructions out of the LMR 'raw material'<sup>3</sup> and call upon set-theoretic constructions and the LMR ontology only.

Ersatzists are quick to object against the LMR background. I will not address those objections here. If the reader is too averse to LMR, she will hardly be happy with my proposal. But if the reader thinks LMR is a worthy view (although she has a hard time believing in it) as nearly everyone does, then she might be interested in what I offer.

### 2.1 Hybrid Views

A range of hybrid views have been offered, from 'overtly' hybrid proposals (Berto, 2010; Reinert, 2017, 2018), to 'hybrid-compatible' ones (Mares, 1997; Restall, 1997; Krakauer, 2012, 2013). I call *hybrid-compatible* a view that builds impossible worlds as set-theoretic constructions out of possible worlds. On the other hand, I call *overtly hybrid* an approach that additionally endorses LMR about possible worlds. A hybrid-compatible account can be turned into an overtly hybrid in choosing the Lewis genuine worlds as the machinery from which to build impossible worlds.

Different classifications would be possible. One could categorize the views regardless of the underlying nature of the possible worlds they accept, but instead focus on how the ersatz worlds are constructed: Mares's and Reinert's proposals meet in making use in their constructions of notions that come from situation

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<sup>3</sup> Lewis (1986, p. 186): 'I could construct excellent ersatz worlds in ever so many ways, drawing on the genuine worlds for raw material.'

semantics. In contrast, Restall's and Berto's constructions are limited to sets of possible worlds, and sets of sets.<sup>4</sup>

I start by briefly reviewing Berto's theory. The limitations of the approach and the challenges that hybrid theorists face will be discussed in the next section.

Berto (2010) gives the Granularity Problem as a motive for the introduction of impossible worlds. He is concerned with the metaphysical status of worlds and the underlying theory of the possible worlds. Berto's 'Hybrid Modal Realism' (HMR) aims to

- (a) refining the possible-worlds apparatus in order to deal with impossibilities more satisfactorily, while at the same time (b) retaining the alleged capacity of modal realism to provide a fully reductionist account of modalities. (Berto, 2010, p. 475)

Berto constructs ersatz 'world-books' out of genuine Lewisian possible worlds. This idea of impossible world-books originates from Divers (2002). World-books are sets of atomic propositions, that is sets of sets of genuine worlds. A world-book is inconsistent - and so represents an impossible situation - when no genuine world is a member of each of its subsets, that is, when the conjunction of its members cannot be true at any possible world. A world-book of which every member contains several possible worlds represents an incomplete situation. A world-book that contains all and only the atomic propositions true at one world is a representation of this possible world. A world-book is a representation of our world if it contains all and only the atomic propositions true at the actual world, that is if the actual world is the only world that appears in every member set (Berto, 2010, p. 483).

This allows to differentiate outright and logically complex contradictions. If  $[A]$  and  $[B]$  are two distinct possible *contingent* propositions, then they are two distinct non-empty sets of possible worlds. That is,  $[A] = X$  and  $[B] = Y$  where  $X, Y \subset W$ ,  $W$  the set of all the possible worlds, and  $X \neq Y$ . Berto takes the impossible proposition  $[A \wedge \neg A]$  to be the set  $\{[A], [\neg A]\} = \{X, W \setminus X\}$ . Likewise,  $[B \wedge \neg B] = \{[B], [\neg B]\} = \{Y, W \setminus Y\}$ . For the sake of illustration, Berto invites us to consider a simplified model (I change the notation):  $W = \{w_1, w_2, w_3, w_4, w_5\}$ ,  $[A] = X = \{w_1, w_2\}$ ,  $[B] = Y = \{w_2, w_3, w_4\}$ . The impossible proposition  $[A \wedge \neg A]$  is then the set of sets of genuine worlds  $\{\{w_1, w_2\}, \{w_3, w_4, w_5\}\}$ , whereas  $[B \wedge \neg B]$  is the set of sets  $\{\{w_2, w_3, w_4\}, \{w_1, w_5\}\}$ . Given that  $[A \wedge \neg A] \neq [B \wedge \neg B]$ , the two impossible (no genuine world belongs to both member-sets) propositions are different and differentiated.

Berto's world-books can also represent situations impossible due to the joint incompatibility of three or more propositions, otherwise possibly two-by-two compatible. He makes use of Lewis's example of incompatible beliefs or 'inconsistent corpus'.<sup>5</sup> Take (I rename)  $[N] = \text{that Nassau-street runs East-West}$ ,  $[R] = \text{that the Railroad nearby runs North-South}$  and  $[P] = \text{that Nassau Street is parallel to the Railroad}$

<sup>4</sup> One of my motivations for taking over Berto's HMR, and meeting the challenges it faces, is that I would like to avoid resorting to the situation semantics devices that Mares and Reinert are led to use. I detail Mares's and Reinert's proposals in comparison to my own account in (2022c).

<sup>5</sup> (Lewis, 1982, p. 436; Berto, 2010, p. 482).

*nearby*. These three propositions constitute an inconsistent triple which, as in Lewis's example, can form an inconsistent content of belief. HMR makes room for such content: In the above simplified models, take  $[N] = \{w_1, w_5\}$ ,  $[R] = \{w_2, w_3, w_5\}$  and  $[P] = \{w_1, w_2, w_3, w_4\}$ . The proposition *that N and R and P* can be represented by the set  $\{[N], [R], [P]\} = \{\{w_1, w_5\}, \{w_2, w_3, w_5\}, \{w_1, w_2, w_3, w_4\}\}$ . As wished -  $[N]$ ,  $[R]$  and  $[P]$  being jointly inconsistent - it is an impossible representation.

## 2.2 Challenges for a New Hybrid Theory

In the remaining of the paper, I use *blatant*<sup>6</sup> impossibility to designate any impossibility that is an *outright*, i.e. *explicit logically structured* contradiction in the form  $A \wedge \neg A$ . I call *subtle* impossibility any impossibility which is not blatant. Therefore, the following count as subtle impossibilities: - atomic impossibilities as '2 is odd'; - implicit contradictions or metaphysical complex impossibilities as 'this swan is both green and red all over', 'electron *e* is positively and negatively charged'; - conjunctions of three or more true atomic propositions jointly incompatible as the above example of 'Nassau Street runs East-West and the Railroad nearby runs North-South and Nassau Street is parallel to the Railroad nearby'.<sup>7</sup>

Hybrid accounts such as Berto's HMR world-books suffer with well-known limitations, pointed out by many impossible worlds theorists.<sup>8</sup> They fail to provide a plenitude of impossibilities.

In this paper, I focus on Berto's HMR deficiencies. The account works quite well when it comes to representing blatant and logically structured contradictions. Berto's construction seems unable to differentiate between intuitively distinct necessary propositions and impossible propositions that are non-conjunctive.

The HMR granularity deficiency gives rise to what I identify as seven challenges for any new hybrid proposal:

- (1) HMR is unable to differentiate between distinct *atomic impossibilities*. Any atomic impossible proposition is true at no genuine world. All atomic impossible propositions are therefore (in LMR) identified with the empty set and a world-book which represents one represents all as a set containing the empty set.

<sup>6</sup> David Lewis famously distinguished between *blatant* and *subtle* impossibilities in (Lewis, 2004).

<sup>7</sup> So I depart from Lewis's use of 'blatant' in the context of this example, which characterizes the conjunction of the three jointly incompatible propositions as 'blatantly inconsistent' (Lewis, 1982, p. 436.). Other accounts of the distinction between 'blatant' and 'subtle' impossibilities are offered in the literature. Jago suggests a more nuanced, degree-theoretic account of the distinction: '[S]ubtlety is a matter of degree (or at least, it is not always a determinate matter whether an inconsistent representation is subtly impossible)' (Jago, 2014, p. 195). According to Jago's proposal, a subtle impossibility is an impossibility which is not 'unwound' in a blatant contradiction after a small (but not determinate) number of steps. See (Jago, 2014, Chapter 7; Jago, 2013).

<sup>8</sup> For nice overviews of the benefits and limitations of each of these views, see (Kiourti, 2010, Chapter III; Reinert, 2018; Reinert 2017, Chapter 5).

- (2) HMR is unable to differentiate between *atomic necessities*. All atomic necessary propositions are true at all genuine worlds. They are (in LMR) identified with the set of all genuine worlds. Therefore, a world-book that represents one represents all as a set containing the set of all worlds.
- (3) HMR is unable to differentiate between *necessarily coextensive properties*. *Being triangular* and *being trilateral*, *being a bachelor* and *being an unmarried male*, or *being a vixen* and *being a female fox* are all pair of necessarily coextensive properties. The two properties of a pair have the same instances in all possible worlds. Therefore, the propositions *that t is triangular* and *that t is trilateral* are the same set of possible worlds and HMR world-books cannot differentiate between them, nor differentiate between the impossibilities *that that t is triangular and t is not triangular* and *that t is triangular and t is not trilateral*.
- (4) **Granularity issue** The issue relates to the logical closure of worlds. Under which principles or logical rules our impossible worlds have to be closed? Should the impossible worlds be closed under some rules?

If our impossible worlds are closed under a logical rule  $R$  and that  $C$  is  $R$ -entailed by  $A$ , then any impossible worlds that represents  $A$  will represent  $C$ . This seems an apparent shortcoming for the full fine-grainedness we are expecting of our impossible worlds. Our proposal should come up with impossible worlds that are not constrained by any closure principle other than identity. Berto's HMR faces such granularity restrictions: A HMR world-book represents a conjunction  $[A \wedge B]$  by the set containing the proposition  $[A]$  and the proposition  $[B]$  as its members. Therefore, a world-book represents *that A and B* if and only if it represents both *that A* and *that B*. So, HMR worlds are closed under the rule of adjunction.

- (5) **Coreferring names and necessary identities.** A potential challenge for hybrid accounts is raised by *coreferring proper names*. Berto and Jago (2019, p. 58) defend this idea. They hold that HMR conflates the contents of *that Hesperus is bright* and *that Phosphorus is bright*. Given the necessity of identity, necessarily, Hesperus is Phosphorus. So:

there is but one planet from which to construct ersatz impossible worlds.  
So no impossible world contains Hesperus but not Phosphorus, and no impossible world says that Hesperus is not Phosphorus.

This instance of the Frege's puzzle relies on theses about proper names and transworld identity. An adequate response to this fourth challenge depends on one's

particular underlying ontology, on one's way of dealing with names and with the modal status of identity statements. Anyway, the threat has to be addressed.

(6)&(7) Finally, Reinert (2017, 2018) raises two other challenges for any theory which aims to enhance LMR by adding impossible worlds while abiding by its reductive analysis of possibility. She calls these two challenges *extraordinary impossibilities* and *higher-order impossibilities* respectively.<sup>9</sup>

To keep this paper within decent word limits, I here focus on only the first three challenges. While applying the framework to these issues should give the reader some insight into how SHMR is able to address the remaining four challenges, I outline my proposed solutions in the final section. I do offer detailed answers to the Granularity Issue in (2022a), the challenge raised by Frege's puzzle cases in (2022b), and Reinert's challenges in (2022c).

### 3 Structural Hybrid Modal Realism (SHMR)

'Structural Hybrid Modal Realism' (SHMR) draws on ideas from Berto's (2010, pp. 485–486), Jago (2014, 2014, 2015) and Krakauer (2012, 2013) but combines them in a, hopefully, original manner.<sup>10 11</sup>

We start with a plurality of Lewisian worlds whose parts are concrete world-bound individuals.<sup>12</sup> My crucial insight: I hold that the limitations that HMR faces come from the building of its impossible worlds as set-theoretic constructions out of possible worlds themselves. To be able to make all the hyperintensional distinctions hybrid-built impossible worlds must be some set-theoretic constructions out of *parts* of genuine worlds. The 'building blocks' will be genuine (proper) *possibilia*: the individuals that inhabit genuine worlds and sets of those individuals. Our ersatz worlds will thereby be defined as set-theoretic constructions out of these individuals,

<sup>9</sup> 'Extraordinary and higher-order impossibilities challenge any theory that adds 'impossible worlds' to the Lewisian ontology.' (Reinert, 2018, p. 314) Reinert describes them as 'two kinds of syntactically inconspicuous impossibilities [which] arise in LMR, which are not even subtly contradictory.' (Reinert, 2018, p. 312). I detail these two challenges in (2022c).

<sup>10</sup> Jago supports a linguistic ersatzism for both possible and impossible worlds. I avail myself of his proposal and tools which I think can be accommodated by hybrid theorists, as Jago highlights himself on several occasions.

<sup>11</sup> Reinert's (2017, chapters 5 and 6; 2018) own hybrid theory aims to meet these challenges. She follows Mares's account in making use of idea taken from Barwise and Perry's situation semantics, and builds her worlds as sets of states of affairs conceived as information-states structures. I will not argue here against such approach. I think that Reinert's improvements of Mares's construction succeed in overtaking many of the limitations of previous hybrid theories.

<sup>12</sup> In what follows, I will use the qualifying terms 'genuine' and 'Lewisian' to describe possible worlds as spatio-temporally disconnected mereological sums as defined and endorsed by Lewis. Besides, and in line with my commitment to Lewisian Modal Realism, 'possible worlds' will always refer (except if indicated otherwise) to Lewisian worlds.

properties and relations between individuals. Some of those ersatz worlds will be possible, and will represent Lewisian worlds (in a way to be specified) and genuine possibilities, and some will be impossible (in a way to be specified as well), and will give us the resources to represent impossibilities. Those ersatz worlds, which I will call ‘structured worlds’<sup>13</sup> will be defined as sets of *tuples*.

### 3.1 Tuples

I call ‘unstructured proposition’ a proposition as set-of-possible-worlds. With each sentence ‘A’ that expresses a certain proposition *that A*, we associate the unstructured proposition-*qua*-set of genuine worlds [A] at which *that A* is true.

We also associate ‘A’ with a structured entity: a set-theoretic construction that reflects the syntactic structure of ‘A’. I call this structured entity a *tuple*. This likely reminds the reader of well-known structural approaches to propositions which share the common idea that propositions *are (identified with)* complex structured entities whose structure mirrors the structure of the sentences expressing them and in which lexical items are replaced by semantic values.<sup>14</sup> There are several ways to construct tuples and the different existing theories of structured propositions may provide or indicate equally satisfactory alternative ways to proceed. I will rely on the structuralist strategy outlined by Lewis himself (1986, pp. 55-59). Thus, a tuple is an ordered set whose constituents are the semantic values assigned to the lexical items of the sentence. What is a semantic value?:

Semantic values may be anything, so long as their jobs get done. (Lewis, 1980, p. 83)

The present account constructs tuples out of Lewisian ‘raw material’ provided by the plurality of concrete worlds. Semantic values in tuples will be made up by genuine individuals, including merely possible ones. In other words, tuples will be adequate sets (or classes) of *possibilia*.

I do not take tuples to be *propositions*. A tuple is associated with an unstructured proposition, in the sense that a tuple mimics the syntactic structure of sentence that expresses a proposition, proposition which can be identified with a set of possible worlds. But a tuple is not itself the proposition expressed by the sentence of which it echoes the structure (I go back to my distinction between tuples and propositions below).

There are well-known sets of questions raised by structural set-theoretic constructions of this kind. Some are linked to their identification with propositions and, for that reason, do not apply to the present account.<sup>15</sup> Others more specifically relate

<sup>13</sup> I borrow this denomination from Krakauer (2012, 2013)

<sup>14</sup> There are distinct structural accounts of propositions in the philosophical literature. See e.g. King (2008, 1995, 1996) and Soames (1987).

<sup>15</sup> Examples of issues of this first type are, e.g.: If propositions are some ordered set, in what order should the constituents be and why? How are the constituents bound together (what is the ‘metaphysical glue’) to make up the structured entity which is a proposition? ‘What determines the structure in which [the constituents] figure’? I will therefore not consider further the questions relating to the ‘making up’ of the structured entities, nor will I discuss the nature of the relation which binds together the constituents, nor the unity of the relevant structure.

to the construction of an ordered set from a sentence, and can arguably be seen as relevant problems. I will not delve into this: any response that structural theories of propositions can formulate should do the trick. Anyway, the account I am offering cannot be undermined by these sets of issues, since tuples do not have the responsibilities that propositions can have.

What matters is: (1) The existence and feasibility of ‘matching’ an unstructured proposition expressed by a sentence with a structured entity as a tuple; (2) The nature of constituents of the structured entities. Some clarification of (1). I assume that there is an adequate way to build structured entities whose structure and constituents mirror sentences and ‘correspond to’ or ‘match’ the propositions expressed by the sentences mirrored. This matching relation is just a function. I do not assume anything representational between an unstructured proposition-*qua* set of possible worlds and a tuple. A tuple can be extracted from a sentence which expresses a proposition. This matching is an association between a tuple and a proposition -*qua* set of worlds that can be linked to a sentence. Take a proposition  $[A]$ , it is a certain set of Lewisian worlds. This proposition can be expressed by at least one sentence ‘ $A$ ’. From ‘ $A$ ’, we can build the tuple  $\langle A \rangle$ . Conversely, take the tuple  $\langle A \rangle$ . It mirrors the structure of a sentence ‘ $A$ ’, which expresses a proposition *that*  $A$ , identified with the set of worlds  $[A]$ . Regarding (2), I have already mentioned that the constituents of tuples are semantic values and that semantic values must be, in a way or another, made of Lewisian raw material: genuine individuals (including *possibilia*) and sets thereof. I will come back to that point later. The specific semantic values we assign to names<sup>16</sup> and predicates are decisive for the granularity questions.

Thereby, I hold that the tuple corresponding to a sentence can be obtained by substituting the lexical items which appear in the syntactic structure with their semantic values, and by representing that structure by a nested ordered set. For the sake of illustration, let us assume temporarily a Russellian stance: the semantic values of predicates are the properties and the relations expressed by the predicates, and the semantic value of a name is the individual to which the name refers.<sup>17</sup> In all the following, for any lexical item  $l$ ,  $\ulcorner l \urcorner$  will represent the semantic value of  $l$ . If  $\ulcorner$  ‘Robin’ $\urcorner$  stands for the semantic value of the name ‘Robin’,  $\ulcorner$  ‘Robin’ $\urcorner$  is the individual, Robin, and  $\ulcorner$  ‘reading’ $\urcorner$  is the property of *reading*.

Each atomic proposition,  $[Ra_1\dots a_n]$ , with  $n$  the arity of  $R$ , will be represented by an ordered set built out of individuals, properties and relations. The proposition  $[Ra_1\dots a_n]$  is therefore represented by the tuple  $\langle a_1, \dots, a_n, \text{being } R\text{-related} \rangle$  where  $a_1, \dots, a_n$  are the individuals named by the name ‘ $a_1$ ’, ‘ $a_2$ ’, ‘ $a_n$ ’ and *being R-related* is the relation expressed by the predicate  $R$ . Nothing hangs on the choice of the order of the semantic values for individuals and the semantic values for predicates in

<sup>16</sup> In what follows, I use ‘name’ for ‘proper name’.

<sup>17</sup> For an example of this Russellian approach for semantic values of lexical items and sentences, see for example Soames (1987). For the sake of the presentation, I temporarily assume that the semantic value of a name is the concrete individual, actual or merely possible, referred by the name. Further specifications are obviously necessary to get an account of tuples able to make all the expected hyperintensional distinctions. It is worth stressing again, though, in the construction I offer, tuples are only made of individual (including *possibilia*) and properties and relations (as sets of *possibilia*).

making up the *atomic tuples*, and by stipulation I opt for writing the semantic values of names before the semantic values of predicates.<sup>18</sup>

Compound propositions can be expressed by tuples in the same way by means of relations, between unstructured propositions (sets of genuine worlds) or between properties and worlds, which are the semantic values for connectives and quantifiers. For instance, we may define *NEG* as a relation which holds between a set of worlds and its complement, *CONJ* as a relation which holds between two sets of worlds and their intersection, and *SOME* as a relation that holds between some property and the worlds where something instantiates that property (instead of taking the semantic values of connectives and quantifiers to be relations we may equally defined them as appropriate functions).

I write the semantic values of the connectives or quantifiers first in tuples. For example, from the atomic impossibility expressed by the proposition [some bachelor is married],<sup>19</sup> we can build the tuple  $\langle \text{SOME}, \langle B, M \rangle \rangle$ , where *SOME* is the relation just defined, *B* is the unstructured property (set of individuals) of being a bachelor, and *M* is the unstructured property of being married. Similarly, the subtle metaphysical impossibility *that swan a is both red and green all over* can be represented by the tuple  $\langle \text{CONJ}, \langle a, S \rangle, \langle \langle a, R \rangle, \langle a, G \rangle \rangle \rangle$ , with *a* the individual denoted by ‘a’ and *S*, *R*, and *G* the suitable properties *being a swan*, *being red* and *being green* respectively.

The order in complex tuples matters: ‘swan *a* is red and green’ and ‘swan *a* is green and red’ are mirrored by two distinct tuples:  $\langle \text{CONJ}, \langle a, S \rangle, \langle \langle a, R \rangle, \langle a, G \rangle \rangle \rangle$  and  $\langle \text{CONJ}, \langle a, S \rangle, \langle \langle a, G \rangle, \langle a, R \rangle \rangle \rangle$  respectively. More generally, the tuples  $\langle A \wedge B \rangle$  and  $\langle B \wedge A \rangle$  will be distinct, as soon as *A* and *B* are, and likewise for  $\langle A \vee B \rangle$  and  $\langle B \vee A \rangle$ .

However, different sentences can be represented by a same tuple, so that a given tuple corresponds to a class of sentences. If two sentences differ by two different lexical items that have the same semantic values, the sentences are represented by the same tuple. This is an important feature of the account I offer: the construction is not just copying the syntax of the language. Three separate points I want to make: (1) the account is properly semantic, not syntactic, in that distinct sentences can be assigned the same content; (2) the account cuts at *most as* fine grainedly as equivalence classes of synonyms. However, a semantics that starts from the syntax and builds equivalence classes can be seen as explanatorily deficient. Moreover, when we build equivalence classes of synonyms, we are already presupposing the semantic notion of synonymy. It is better to have a characterization of synonymy that flows out of a constructions that starts with non-linguistic entities. Finally, (3) even if the result turns out to be *just as* fine-grained as equivalent classes of synonyms, (i.e. if it makes exactly the same distinctions as the equivalence classes approach), the account I offer gains in explanatory value insofar as contents are built out of non-linguistic entities only; namely, out of worldly ‘raw material’. This is surely more

<sup>18</sup> I.e.  $\langle a_1, \dots, a_n, \text{being } R\text{-related} \rangle$  instead of  $\langle \text{being } R\text{-related}, a_1, \dots, a_n \rangle$ .

<sup>19</sup> Following Krakauer (2012, p. 15)

properly semantic than building contents out of equivalence classes of pieces of language.<sup>20</sup>

We have appropriate ways to construct tuples, so that to each unstructured proposition  $[A]$ -*qua*-set of possible worlds expressed by a sentence ‘ $A$ ’ it corresponds a *tuple*  $\langle A \rangle$  which reflects the structure of the sentence in an adequate way. I say that the tuple  $\langle A \rangle$  *picks out* or *matches with* (or in an even looser way, as just written, *corresponds to*) the proposition  $[A]$ ; or, alternatively that every unstructured proposition  $[A]$  expressed by a sentence ‘ $A$ ’ can be matched with or picked out by a *tuple*  $\langle A \rangle$  which mirrors the structure of the sentence in an adequate way.<sup>21</sup>

### 3.2 Structured Worlds

Any arbitrary collection of tuples makes up a structured world. Structured worlds are ersatz worlds: they are ‘mere representations that such-and-such is the case’ (Jago, 2015, p. 6). Unlike Lewisian worlds, they do not represent *genuinely*, by having as parts what they represent. Representation by structured worlds is defined by tuple-inclusion. In particular, the representational capabilities of a structured world  $w_S$  depends on the tuples it contains:  $w_S$  represents *that*  $A$  if and only if  $\langle A \rangle \in w_S$ . Structured world-representation is as fine-grained as the tuples are.<sup>22</sup>

Some structured worlds are possible, others are impossible. A possible structured world represents a genuine world. The correspondence between structured worlds and genuine worlds works thus: Consider  $w_S$  a structured world (possible or impossible),  $W$  the space of all genuine possible worlds,  $w \in W$ , *that*  $A$  any proposition picked out by the tuple  $\langle A \rangle$ ,  $[A]$  the corresponding set of worlds in  $W$  (possibly empty). Now, a Lewisian world  $w$  is represented by a structured world  $w_S$  if:

- (1) For all tuples  $\langle A \rangle$ ,  $\langle A \rangle \in w_S$  if and only if  $w \in [A]$ .

In other words, a possible world  $w$  is represented by a structured world  $w_S$  if for all  $\langle A \rangle$ ,  $\langle A \rangle \in w_S$  iff the proposition *that*  $A$  is true at  $w$ .

Equivalently, we know that each structured world is a set of tuples, and each tuple picks out an unstructured proposition, i.e. a set of genuine worlds. Therefore, each structured world defines a certain set of sets of genuine worlds. Let  $S$  be the set of sets of genuine worlds corresponding to  $w_S$ . Then,  $w_S$  represents the possible world  $w$  if and only if  $\bigcap S = \{w\}$ . If we unfold a little more we have:

<sup>20</sup> For more, see e.g. my (2022b) on the semantics value for proper names.

<sup>21</sup> Note that to the same set of possible worlds correspond many different tuples, so it is in no way a one-to-one correspondence.

<sup>22</sup> In many ways this account is close to Jago (2015) proposal. However, Jago, unwilling to accept Lewisian modal realism, follows a *linguistic ersatzism* path. To form worlds as arbitrary sets of certain sentences, linguistic ersatzers have to specify a language for these sentences. This world-building or world-making language is generally taken to be similar to a *Lagadonian language*. Such linguistic ersatzism faces the well-known problems displayed by Lewis. LMR eliminates the need for such a language. Tuples are built out of (sets of) individuals, properties and relations provided by the Lewisian ontology which is at our disposal. See Lewis (1986, p. 142sq) on linguistic ersatzism.

- (2) For all  $\langle A \rangle \in w_S$ ,  $[A]$  the unstructured proposition-*qua* -set of possible worlds picked out by  $\langle A \rangle$ ,  $w_S$  represents a genuine possible world  $w$  iff  $\bigcap_{\langle A \rangle \in w_S} [A] = \{w\}$ .

A structured world  $w_S$  is impossible when no one genuine world belongs to all the unstructured propositions represented by the tuples which are the members of  $w_S$ . (1) and (2) guarantee that a structured world that contains a tuple representing an atomic impossibility or an outright contradiction (a contradictory proposition of the form  $[B \wedge \neg B]$ ) is an impossible world, as well as any structured world that contains tuples which represent mutually inconsistent propositions as their intersection is the empty set.

If the proposition *that A* is an atomic impossibility or a proposition of the form  $[B \wedge \neg B]$  such as  $\langle A \rangle \in w_S$ , then  $[A]$  is the empty set. So  $\bigcap_{\langle A \rangle \in w_S} [A] = \emptyset$  and  $w_S$  does not represent a genuine world. Now, take  $w_S$  a structured world such that for some proposition *that A*, both the tuples  $\langle A \rangle$  and  $\langle \neg A \rangle$  are in  $w_S$  and suppose  $w_S$  represents a genuine world. Then, by (1),  $w$  is a member of both  $[A]$  and  $[\neg A]$ , i.e.  $[A]$  is true at  $w$  and  $[\neg A]$  is true at  $w$ , i.e. the proposition  $[A]$  is both true and false at  $w$ . As by hypothesis  $w$  is a genuine world, this is impossible. Alternatively, if  $[A]$  is the set of genuine worlds at which *that A* is true,  $[\neg A] = W \setminus [A]$ . So if both  $\langle A \rangle$  and  $\langle \neg A \rangle$  are in  $w_S$ ,  $\bigcap_{\langle A \rangle \in w_S} [A] = \emptyset$ . Therefore,  $w_S$  cannot represent a genuine world. Recall Nassau

Street and the Railroad nearby: we have three propositions all possible, in pairs compatible, but jointly incompatible. The intersection of the three sets of worlds at which each is respectively true is the empty set. So, a structured world that contains the three corresponding tuples does not represent a genuine world.

One way for a structured world  $w_S$  to be impossible is therefore to be *inconsistent*: there are two of its members that pick out mutually disjoint propositions-*qua* -sets of genuine worlds.<sup>23</sup> This latter way of setting inconsistency covers the case of a structured world representing an atomic impossibility or an outright contradiction as both correspond to the empty set of possible worlds. For every pair of tuples in  $w_S$  of which one of them represents an atomic impossibility or an outright contradiction, the corresponding unstructured propositions are mutually disjoint (their intersection is empty).

A structured world  $w_S$  is also impossible when it is *incomplete*: there is a proposition  $[A]$  such that no tuple in  $w_S$  represents either  $[A]$  or its complement  $[\neg A] = W \setminus [A]$ . That is, neither  $\langle A \rangle$  nor  $\langle \neg A \rangle$  belong to  $w_S$ . In (2) the incompleteness is given by the fact that the intersection is more than one genuine world.

In sum, structured worlds can be *glutty* or *gappy*: A world  $w_S$  is *glutty* when a proposition and its negation are represented by the world. It contains a glut: for some proposition  $[A]$ , both  $\langle A \rangle$  and  $\langle \neg A \rangle$  are members of  $w_S$ . A world  $w_S$  is *gappy* when neither a proposition nor its negation is represented by the world. It has a gap: neither  $\langle A \rangle$  nor  $\langle \neg A \rangle$  is a member of  $w_S$ . Conversely, a structured world which is neither

<sup>23</sup> See Kiourti (2010, p. 81) who cites Divers (2002, pp. 313-314, fn. 19), and Berto (2010, p. 482).

incomplete nor inconsistent is a possible structured world and, as we have seen, its ‘members together truly describe a genuine Lewis-world’ (Kiourti, 2010, p. 84).

Two virtues of hybrid theories that this account preserves: first, the Lewisian definition of possibility still holds with no primitive modality. That is, we still have that:

(P) It is possible *that*  $A$  iff there is a world  $w$  such as, at  $w$ ,  $A$

with no undue modality in the definition as soon as the quantification in the right-hand side only ranges over Lewisian genuine worlds.

Then, Lewis’s objection against (genuine) impossible worlds does not apply.<sup>24</sup> There is no risk for a contradiction at a structured world, being an ersatz construction, to spread into a contradiction at the actual world. ‘At  $w_s$ ,  $A \wedge \neg A$ ’ amounts to  $\langle \text{CONJ}, \langle A, \langle \text{NEG}, A \rangle \rangle \rangle \in w_s$ . This does not involve that  $\langle A \rangle$  and  $\langle \text{NEG}, \langle A \rangle \rangle$  are in  $w_s$ . But, even if we had ‘at  $w_s$ ,  $A$  and at  $w_s$ ,  $\neg A$ ’, it would not follow that ‘at  $w_s$ ,  $A$  and it is not the case that at  $w_s$ ,  $A$ ’. This latter claim amounts to say that both  $\langle A \rangle$  is in  $w_s$  and  $\langle A \rangle$  is not in  $w_s$ . ‘At a structured world’ must be understood as ‘according to a structured world’, which is analysed as inclusion of tuples. Therefore, ‘at structured world  $w_s$ ’ does not work as a restricting modifier, unlike ‘at genuine world  $w$ ’ where ‘at  $w$ ’ restricts the quantification to parts of  $w$ .

### 3.2.1 Propositions and truth-at-a structured world

As already stressed, tuples are not propositions. Tuples are merely lists. For the list  $\langle \ulcorner \text{Macron} \urcorner, \text{being German} \rangle$  to be the proposition *that Macron is German* we would need another element, something that unifies or binds together the elements of the list to ‘make a proposition emerge’.<sup>25</sup>

Thus, the truth-at-a structured world *of tuples* is not defined: tuples are not truth (or falsity) bearers.<sup>26</sup> A list is not true or false.

I take propositions to be the content of meaningful sentences. I also take propositions to be objects of attitudes (contents of intentional states) and primary truth and falsity bearers. Propositions are suitable to have modal properties: they can be possible, impossible, contingent, necessary and they can entail one another. My stance

<sup>24</sup> Lewis has notoriously argued against the idea of an extension to genuine impossible worlds. The idea of Lewis’s unwillingness to admit impossible worlds is grounded in the behaviour of the phrase ‘at world  $w$ ’ in his analysis of modality. The phrase behaves as a restrict modifier, as function our uses of expressions like ‘in the fridge’ or ‘on the mountain’. Such restrict modifier commutes with connectives so that if we have an impossible world  $w$  at which a contradiction holds then this contradiction spreads out into a contradiction at the actual world. More precisely, ‘at  $w$ :  $(A \wedge \neg A)$ ’ is equivalent to the overt contradiction ‘at  $w$ :  $A$  and  $\neg$  (at  $w$ :  $A$ )’, as are equivalent ‘on the mountain both  $A$  and not  $A$ ’ and ‘on the mountain  $A$  and not: on the mountain  $A$ ’. See Lewis (1986, fn.3, p. 7).

<sup>25</sup> Not only the tuples do not comprise such unifying element, but this element, that metaphysical glue, is arguably very mysterious. It is the usual debate on the unity of proposition and a well-known metaphysical conundrum I do not want to get into. Gaskin’s (2008) book is devoted to the issue. See also footnote 15.

<sup>26</sup> Pace Krakauer, for whom ‘a structured proposition is true at a structured world iff that proposition is a member of the set that composes the world.’ (Krakauer, 2012). Krakauer calls ‘structured propositions’ what I call ‘tuples’.

is that ‘proposition’<sup>27</sup> is a label which should designate entities that *by their nature* play certain theoretical roles, to wit, being objects of attitudes and primary truth-bearers. That the entities that fulfil these theoretical roles are named ‘propositions’ is the object of a stipulation.<sup>28</sup> If one does not think that a single kind of entity is able to play all the roles I have mentioned at once, they are free to use ‘proposition’ to call only, say, the entities which are truth and falsity bearers.<sup>29</sup>

I claim that tuples are unable to play the theoretical roles I attribute to entities I call ‘propositions’. By their nature, tuples are not the kind of entities that can be objects of attitude or truth-bearers, nor are suitable to have modal properties. An adequate fine-grained, hyperintensional notion of proposition is rather achieved by still identifying propositions with sets of worlds. The reason rests on the view, which I endorse, that propositions are better seen as sets of worlds than as members of worlds. Why cannot propositions be members of worlds? What is at stake is the nature of propositions as well as the nature of worlds. In ‘Hyperintensional propositions’, Jago (2015) sets out metaphysical reasons for preferring the set-of-worlds approach over the set-membership view for propositions, appealing to what he calls the *nature-of-sets thesis*.<sup>30</sup>

It has been well known since Fine’s ‘Essence and modality’ (1994) that, although it is part of the essence of the singleton {Socrate} that Socrate belongs to it, the converse does not hold.<sup>31</sup> The *nature-of-sets thesis* rests on this asymmetry between the nature of a set and the nature of its members or proper subsets. The idea is that a member or a proper subset  $X$  of a set  $Y$  ‘fixes the identity’, determines the nature of  $Y$  by being its member or subset while the nature of  $X$  is not determined by this relation of membership or inclusion (Jago, 2015, p. 598).

Now, suppose that tuples are propositions. Since structured worlds are sets of tuples, then structured worlds would be sets of propositions. We would have to define ‘truth-at-a world’ as set-membership, in the same way we define representation by a world, i.e. a proposition *that*  $A$  is true-at-world- $w_s$  iff  $\langle A \rangle^{32} \in w_s$ . By the *nature-of-sets thesis*, it would follow that it is not in the nature of a proposition to be a member of a world  $w_s$ , and so it is not in the nature of a proposition to be true-at-a-world- $w_s$ .

Similar arguments show that the identification of tuples-*qua*-members of structured worlds with propositions leads to the upshot that propositions are not the

<sup>27</sup> (Jago, 2015, 2018).

<sup>28</sup> See e.g. (McGrath and Devin, 2018; Jago, 2018, chapter 8).

<sup>29</sup> It is Jago’s path, see Jago (2018, p. 238):

‘I’ve already reserved the term ‘proposition’ for the entities that occupy the first of these theoretical roles: propositions are the bearers of truth and falsity. So, on this usage, attitudes such as belief are not relations between agents and propositions’.

<sup>30</sup> I will also rely on Jago’s book *What truth is* (2018, Chapter 8), although its author offers there quite a different account of the nature of propositions. See (Jago, 2017, 2018), where propositions are analysed in terms of their possible truthmakers, and identified with truthmarker conditions. For more on the nature and theoretical roles of propositions, see my (2022a).

<sup>31</sup> (Fine, 1994, pp. 4-5).

<sup>32</sup> *qua*-proposition, by hypothesis.

kind of entity which by nature is able to be possible or necessary, nor to entail one another. If we grant that we expect propositions to play these theoretical roles, we must prefer a concept of proposition as sets of worlds rather than tuples.<sup>33</sup>

We still need to provide an account of propositions as more fine-grained entities than are propositions-*qua*-sets of genuine worlds. The idea is to define an enriched notion of propositions as sets of *structured* worlds, that is sets of sets of tuples. Call these *extended propositions*.<sup>34</sup>

They are defined as follows. The extended proposition  $[A]_E$ -*qua*-set of structured worlds is the maximal set of structured worlds  $X$  for which  $\bigcap X = \langle A \rangle$ . It is the set of all the structured worlds to which  $\langle A \rangle$  belongs:  $\{w_S | \langle A \rangle \in w_S\}$ .<sup>35</sup>

There is a direct correspondence between tuples and extended propositions. Consider two tuples  $\langle A \rangle$  and  $\langle B \rangle$ . The extended propositions  $[A]_E$  and  $[B]_E$  are the sets of all the structured worlds that contain the relevant tuples,  $\{w | \langle A \rangle \in w\}$  and  $\{w | \langle B \rangle \in w\}$  respectively, so that  $\bigcap \{w | \langle A \rangle \in w\} = \{\langle A \rangle\}$  and  $\bigcap \{w | \langle B \rangle \in w\} = \{\langle B \rangle\}$ . Therefore, as soon as  $\langle A \rangle$  and  $\langle B \rangle$  can be discriminated,  $[A]_E$  and  $[B]_E$  are distinct. Extended propositions inherit the granularity of tuples, and provide us with the resources to make the hyperintensional distinctions unstructured propositions fail to deliver.

How do extended propositions behave *vis-à-vis* truth? With  $W_S$  the set of all the structured worlds,

$$[A]_E \text{ is true-at-} w_S \text{ iff } w_S \in [A]_E = \{w | \langle A \rangle \in w, w \in W_S\}$$

Truth-at-a-structured world is therefore defined in terms of world-membership of tuples: the proposition *that A* is *true-at- $w_S$*  iff  $w_S \in [A]_E$  iff  $\langle A \rangle \in w_S$ . This is in accordance with the nature-of-sets thesis: it is in the nature of an extended proposition  $[A]_E$  to be true-at- $w_S$  when  $w_S$  is a member of  $[A]_E$ , when  $\langle A \rangle$  is a member of  $w_S$ .<sup>36</sup>

We end up with two notions or levels of propositions-*qua*-sets of worlds, as unstructured and as extended. This raises questions. Do I want to analyse all contents as extended propositions or only, say, the impossible contents? I hold that the notion of propositions used in an analysis of contents should not depend on the modal status of the content, but rather on the notion of content under analysis. Thus, I maintain that all intentional contents, which require a very fine-grained account, must be analysed uniformly using extended propositions. Do we then still need a notion of proposition-*qua*-set of genuine worlds? I tend to think that for some specific purposes the coarse-grained notion of propositions-*qua*-genuine worlds will be sufficient. Although further

<sup>33</sup> See Jago (2015, pp. 598-600); in particular, for detailed definitions of what it is for propositions to be possible and necessary, or to entail one another, in the sets-of-worlds approach, see e.g., Jago (2015, pp. 598sq).

<sup>34</sup> For lack of a better designation.

<sup>35</sup> As usual,  $\langle A \rangle$  is the tuple formed from the sentence 'A' which expresses *that A*. Here again, see Jago (2015):

'[...] the sets-of-worlds proposition that A is to be defined as the maximal set of worlds for which  $\bigcap X = T_A$ , i.e.,  $\{w | T_A \in w\}$ . (Jago, 2015, p. 594. Jago writes  $T_A$  what I write  $\langle A \rangle$ )'

<sup>36</sup> (Jago, 2015, p. 598).

investigation through the roles that propositions must play would be required, I think that the twofold notion of proposition is by no means problematic.<sup>37</sup>

## 4 Evaluation of SHMR

I now set out how the SHMR construction deals with challenges 1–3 from 2.2. I only outline solutions for challenges 4–7 below: a reader interested in detailed answers can consult my (2022a, b, c).

**SHMR scores at least as well as Berto’s HMR** Structured worlds have the capacities to represent (0.i) outright contradictions; (0.ii) multiple incompatible propositions; (0.iii.) subtle ‘broadly logical’ complex impossibilities:

- (0.i.) Take the contradiction of the form  $[A \wedge \neg A]$ . The corresponding tuple is  $\langle \text{CONJ} \langle A, \langle \text{NEG} A \rangle \rangle \rangle$ . Let  $w_S$  a world such that  $\langle \text{CONJ} \langle A, \langle \text{NEG} A \rangle \rangle \rangle \in w_S$ .  $w_S$  is an impossible structured world that represents *that A and not A*. At  $w_S$ ,  $[A \wedge \neg A]_E$  is true. Now take  $[B]$  such as  $\langle B \rangle \neq \langle A \rangle$ .  $\langle B \wedge \neg B \rangle = \langle \text{CONJ} \langle B, \langle \text{NEG} B \rangle \rangle \rangle \neq \langle \text{CONJ} \langle A, \langle \text{NEG} A \rangle \rangle \rangle = \langle A \wedge \neg A \rangle$ . The structured world  $w_{S^*}$  such that  $\langle B \wedge \neg B \rangle \in w_{S^*}$  and  $\langle A \wedge \neg A \rangle \notin w_{S^*}$  represents *that B and not B* but does not represent *that A and not A*. Since  $w_{S^*} \in [B \wedge \neg B]_E$ , *that B and not B* is true at  $w_{S^*}$ . Finally  $w_{S^*} \notin [A \wedge \neg A]_E = \{w \mid \langle A \wedge \neg A \rangle \in w\}$ . SHMR provides worlds which represent and distinguish between outright contradictions.
- (0.ii.) Take the incompatible propositions  $[N]$ ,  $[R]$  and  $[P]$  introduced above. A structured world  $w_S$  that contains  $\langle N \rangle$ ,  $\langle R \rangle$  and  $\langle P \rangle$  represents the situation made impossible by the jointly incompatible propositions. Now, take the intersection  $\bigcap_{\langle A \rangle \in w_S} [A]$  and let  $[N]$  being the set of the genuine worlds at which it is true *that N*, and likewise for  $R$  and  $P$ .  $[N] \cap [R] \cap [P] = \emptyset$ , so  $\bigcap_{\langle A \rangle \in w_S} [A] = \emptyset$ . Therefore  $w_S$  is an impossible world.
- (0.iii.) A structured world  $w_S$  such as  $\langle \text{CONJ}, \langle \langle a, S \rangle, \langle \langle a, R \rangle, \langle a, G \rangle \rangle \rangle \rangle \in w_S$  represents *that swan a is both red and green all over*. Therefore, SHMR can

<sup>37</sup> Let me quote Lewis (1970, p.32) about intensions and meaning:

‘But this difficulty does not worry me : we will have both intensions and what I call meanings, and sometimes one and sometimes the other will be preferable as explication of our ordinary discourse about meanings. Perhaps some entities of intermediate fineness can also be found, but I doubt that there s any uniquely natural way to do so’.

represent the ‘broadly logical’ impossibilities of the form  $Pa \wedge Qa$  where  $P$  and  $Q$ ,  $Q \neq \neg P$ ,<sup>38</sup> are predicates for *incompatible properties*.<sup>39</sup>

**Atomic impossibilities and necessities** Challenges (1) and (2) are similarly easily met:

- (1) The impossible atomic proposition expressed by ‘2 is odd’ is represented by the ordered set  $\langle 2, \textit{being odd} \rangle$ . The structured world  $w_s, \langle 2, \textit{being odd} \rangle \in w_s$ , is an impossible world which represents that 2 is odd. Now consider the propositions *that four is odd* and *that four is prime*. Both propositions are mathematical falsehoods. The two relevant tuples are  $\langle 4, \textit{being odd} \rangle$  and  $\langle 4, \textit{being prime} \rangle$ . If we identify properties with the sets of their actual and possible instances. Since the set of individuals that instantiate *being odd* and the set of individuals that instantiate *being prime* are distinct, the tuples  $\langle 4, \textit{being odd} \rangle$  and  $\langle 4, \textit{being prime} \rangle$  are too. Hence, there are *different* structured worlds that represent *that four is odd* or *that four is prime*, and still other structured worlds that represent both.<sup>40</sup> Therefore, the account discriminates between atomic impossibilities.
- (2) The symmetrical case of the necessary propositions is dealt similarly and is easy to check. The tuples built out of sentences expressing necessary propositions make up structured worlds which enable to differentiate between these propositions true at all the genuine worlds.

**Properties** I have assumed that the semantic values of predicates which constitute tuples are the properties expressed by the predicates. What are these properties? To remain ontological conservative with the Lewisian backdrops, I endorse an abundant conception identifying a property expressed by a predicate with the set of the individuals, actual and possible, instantiating the property. As noticed above, such a conception might seem to prevent from making certain hyperintensional distinctions. Since properties are individuated by their extensions, all necessarily coextensive properties are conflated, as well as all potential impossible properties.

Against defenders of concrete impossible worlds like Kiourti (2010, p. 89) and Vacek (2013, p. 298), I argue that the LMR ontological base *does* provide enough material to make the distinctions we want. The limitations of previous hybrid proposals stem from their ways of building impossible worlds and *not* from the refusal to admit genuine *impossibilia*.

<sup>38</sup> The case  $Q = \neg P$  is a case of an outright contradiction.

<sup>39</sup> Likewise for relation e.g. impossibilities of the form  $Rab \wedge Rba$ , where  $R$  is the predicate for a binary asymmetric relation. See Kiourti (2010, p. 82).

<sup>40</sup> See Kiourti (2010, p. 86).

The problematic cases are actually few, and only related to *atomic* necessarily coextensive properties and *atomic* impossible properties. I consider each of these case in turn.

- (3) **Necessarily coextensive properties** I have introduced the challenge with the paradigmatic cases of necessarily coextensive properties *being triangular* and *being trilateral*, *being a vixen* and *being a female fox* and *being a bachelor* and *being an unmarried man*.

Not all cases are equally problematic or at least do not raise the same kind of puzzle. First, as suggested by Kiourti (2010, p. 88), I am willing to concede that *even though* our account would still conflate these kinds of pairs of properties, it would not be a shortcoming. That is:

*being a vixen* simply is the property of *being a female fox*, for these are simply different terms for what is intuitively the very same property. Then it only stands to reason that any world where a vixen is not a female fox is a world where a vixen is not a vixen; and similarly with cases involving the property of *being a bachelor* and thereby arguably the property of being an *unmarried male*. (Kiourti, 2010, p. 88)

‘Unmarried male’ is only the meaning of ‘bachelor’, ‘female fox’ is only the meaning of ‘vixen’. Could someone really believe (or be in whatever representational state such) *that v is a vixen but not a female fox* while not believing *that v is a vixen but not vixen*, unless they do not grasp the meaning of the terms? I am far from convinced that there are such doxastic possibilities.

I rather think that if one does not know that these phrases mean the same, the issue is of linguistic competence, not of mental content: they do not know what some expression of English means. Here, I am interested in modeling thinkers who think impossibilities but are otherwise competent meaning-wise. My structured constructions are less fine-grained than the syntax of language: even fine-grained contents of attitude ascriptions and intentional states should not take into account the redundancies of language.

Whether such necessary coextensive properties correspond to genuine different doxastic possibilities or not, we may want impossible worlds that represent those properties as different to model potential non-doxastic contents. The present account *does* have the abilities to differentiate, say, a world where a vixen is not a female fox and a world where a vixen is not a vixen. The proposition *that a is a vixen but a is not a female fox* is distinguishable from the explicit contradiction *that a is a vixen but a is not a vixen*. Since tuples mimic the syntax of sentences expressing propositions, the tuple associated with the former proposition is undoubtedly distinct from the tuple associated with the latter. We can take these tuples to be  $\langle \text{CONJ}, \langle \Gamma 'v' \Gamma, \langle \text{being a vixen} \rangle \rangle \rangle$ ,  $\langle \text{NEG}, \langle \Gamma 'v' \Gamma, \langle \text{being a female, being a fox} \rangle \rangle \rangle$  and  $\langle \text{CONJ}, \langle \Gamma 'v' \Gamma,$

*being a vixen*),  $\langle \text{NEG}, \langle \ulcorner v \urcorner, \text{being vixen} \rangle \rangle$  respectively.<sup>41</sup> They are distinct tuples: the extension of *being a female*, *being a fox*, *being a vixen* are not identical sets, even if the intersection of the first two is equal to the third, neither is the structure of the two tuples. Likewise for the properties *being a bachelor* and *being an unmarried male*: we can represent that *b is an unmarried male* by the tuple  $\langle \ulcorner b \urcorner, \langle \text{being unmarried}, \text{being a male} \rangle \rangle$  and that *b is a bachelor* by  $\langle \ulcorner b \urcorner, \text{being a bachelor} \rangle$ . Providing that the two necessarily coextensive properties are not both atomic, we do not face conflations.

Things are less immediate with atomic necessarily coextensive properties like *being triangular* and *being trilateral*. The intuitive differentiation between these properties is stronger and conflating the propositions that *t is triangular* and that *t is trilateral* seems an objectionable limitation for a hyperintensional account of content. We would like some structured impossible worlds that make true the proposition that *t is triangular and t is not trilateral*. But hybrid theorists can use Lewis's own suggestion in *The Plurality of worlds*. As the previous analysis of *being a female fox* and *being an unmarried male* as structured properties built from unstructured ones, the idea is to construct atomic properties as structured properties in calling on 'higher-order unstructured relations that holds between properties and relations of individuals' (Lewis, 1986, p. 56). These higher-order properties and relations are 'constructed out of *possibilia* as much as first-order properties and relations of individuals are' (*ibid.*). As Lewis illustrates, such an approach enables to deal with the triangular/trilateral case: Take the relation of being an angle of, *A*, the relation of being a side of *S*, and a higher-order relation *T* for the higher-order (unstructured) relation between a (unstructured) property and a (unstructured) relation, both of individuals, which holds if and only if the former is the property of being something which exactly three things bear relation the latter to. The unique thing that bears *T* to *A* is the property of triangularity whilst the unique thing that bears *T* to *S* is the property of trilaterality:

Therefore let us take the structured property of triangularity as the pair  $\langle T, A \rangle$ , and the structured property of trilaterality as the pair  $\langle T, S \rangle$ . Since *S* and *A* differ, we have the desired difference between the two pairs that we took to be our two structured properties. (Lewis, 1986, p. 56)

This is easily extendable to other potential cases of atomic necessarily coextensive properties.

**Atomic impossible properties** What should be counted as impossible properties and which account should we provide for? We cannot identify impossible properties as sets of *possibilia*, for no possible individual can instantiate impossible properties. We also cannot appeal to *impossibilia*, as impossible individuals are not part of our LMR 'raw material'. We could try to construct *impossibilia* from *possibilia* and their instantiation of incompatible properties. For instance, as bundles of properties

<sup>41</sup> Where  $\langle \ulcorner v \urcorner, \langle \text{being a female}, \text{being a fox} \rangle \rangle$  shortens the co-bearing by *v* of the two properties in  $\langle \text{being a female}, \langle \text{being a fox} \rangle \rangle$ .

some of which are incompatible. Such constructions, though, not only are threatened by circularity as we then used the *impossibilia* so constructed for defining impossible properties, but would lack interest.<sup>42</sup> We can easily represent individuals instantiating metaphysically incompatible or contradictory properties, such as *that a is a round square* or *that a is round and not round*, and differentiate them by means of the structures of tuples.

I maintain that there is no such thing as an absolute atomic impossible property. A property is only impossible if it is of the form  $P \wedge \neg P$  or of the form  $P \wedge Q$ , where  $P$  and  $Q$  are incompatible. Accordingly, an individual will only be impossible by bearing properties that are impossible or incompatible. Either way, the present account cuts well and as finely as expected. The tuple that represents, say, *that a is P and is not P*,  $\langle Pa \wedge \neg Pa \rangle$ , is distinct from the tuple that represents *that a is Q and is not Q*,  $\langle Qa \wedge \neg Qa \rangle$ , as soon as  $P$  and  $Q$  stand for different properties. I am ready to challenge the reader: give me any putative atomic impossible property  $P$ , I will show you that it is of the kind ' $Q \wedge R$ ' or ' $Q \wedge \neg R$ '.

**Atomic impossibilities again** Are there any other atomic impossibilities that we cannot account for? Consider 'Robin squared the circle'. Is 'squaring the circle' an atomic impossibility? I do not think so. The impossibility of squaring the circle is the impossibility of carrying out a certain geometrical construction with the help of certain tools. More precisely 'squaring the circle' amounts to, for instance, 'constructing a square of the same area as a given circle using only a finite number of steps with compass and straightedge'. I hold that all such mathematical impossibilities are equivalent to impossible constructions or impossible calculations of some sort which are then structured. For example, that  $t$  is a Penrose triangle amounts for  $t$  to bearing properties that are incompatible in that their combination makes the existence of  $t$  impossible (at least in the three-dimensional Euclidean space). And my proposal is able to handle such impossibilities.<sup>43</sup>

**A sketch of the solutions to the remaining challenges** To stick within the world limits, the following briefly sketches a possible way to address each of the four remaining challenges. The full solutions are in my (2022a, b, c).

**(4) Granularity issue** Structured worlds defined in SHMR are not themselves structured. What is structured are tuples members of these worlds. A structured world does not need to obey any closure principle, nor to preserve any logical rule (except of  $A$  entailing  $A$ ). As an upshot, the highly fine-grained representation by structured worlds supplies all the resources necessary to address the Granularity Issue.

<sup>42</sup> See Kiourti (2010, p. 90).

<sup>43</sup> There is still one possible kind of atomic impossible properties that still need to be account for: the property of being on a certain fictional or mythical kind such as *being a unicorn*. I defend that and present in length how SHMR can deal with fictional and mythical kinds in (2022b).

The structured worlds account obeys Priest's two 'directives' on impossible worlds:<sup>44</sup>

- (1D) For any  $A$ , there is a world that represents *that*  $A$  and a world that does not represent *that*  $A$ .
- (2D) For any distinct  $A$  and  $B$ , there is a world that represents *that*  $A$  but does not represent *that*  $B$ .

Further, the space of structure worlds satisfies the "still stronger principle" offered by Berto and Jago (2019: p. 168):

- (NP+) If it is impossible that  $A_1, A_2, \dots$  but not  $C$ , then there is an impossible world that represents *that*  $A_1, A_2, \dots$  but not  $C$ .

It is straightforward to check that (1D) and (2D) hold in SHMR. For (NP+), just consider a set that contains the tuples  $\langle A_1 \rangle, \langle A_2 \rangle, \dots$  but which does not contain  $\langle C \rangle$ . Take for instance the incomplete world  $w_S := \{ \langle A_1 \rangle, \langle A_2 \rangle, \dots \}$  which has all and only the  $\langle A_i \rangle$  as members ( $i$  positive integer) and with  $A_i \neq C$  for all  $i$ . (NP+) tells us that:

[F]or any logical principle (other than  $A \models A$ ), there's a world which breaks that principle. (2019, p. 168)

The space of structured worlds includes *open worlds*: impossible worlds that are not closed under any consequence relation other than the identity relation.<sup>45</sup> Open worlds can supply all the plenitude and leeway needed to model intentional states and analyse counterpossibles.

**(5) Coreferring names and necessary identities** I have temporarily adopted a Russellian stance as to the semantic values of names whereby names have their referents as their semantic values. Such position comes with unwelcome confluences due to Frege puzzles: if it is possible to believe that Hesperus is Hesperus but not to believe that Hesperus is Phosphorus, SHMR must provide two distinct contents for the intentional (here doxastic) states and so differentiate between the propositions expressed by the two identity statements (H1) 'Hesperus is Hesperus' and (P1) 'Hesperus is Phosphorus'. However, if 'Hesperus' and 'Phosphorus' are rigid designators, they designate the same individual, namely Venus, in all possible worlds so that the true identity 'Hesperus=Phosphorus' is necessary. Berto and Jago raise this as a putative limitation of Berto's HMR: HMR seems unable to distinguish between propositions of the form *that Hesperus is F* and *that Phosphorus is F* owing to the

<sup>44</sup> See Priest (2005, pp. 187 and 190). I follow Berto and Jago's formulations of (but renumber) these two directives (2019, p. 167) who rephrase Priest's two directives as follows.

<sup>45</sup> See Priest (2005, pp. 21-22).

necessary identity of Hesperus and Phosphorus.<sup>46</sup> What about SHMR? Arguably, there must be structured worlds where Hesperus is not Phosphorus but where Hesperus is nevertheless still self-identical. Such structured worlds should contain the tuples  $\langle NEG, \langle =, \langle \ulcorner \text{'Hesperus'} \urcorner, \ulcorner \text{'Phosphorus'} \urcorner \rangle \rangle \rangle$  and  $\langle =, \langle \ulcorner \text{'Hesperus'} \urcorner, \ulcorner \text{'Hesperus'} \urcorner \rangle \rangle$ , with  $\ulcorner \text{'Hesperus'} \urcorner \neq \ulcorner \text{'Phosphorus'} \urcorner$ .<sup>47</sup>

There are two separate issues here. If we stick to a Russellian stance and assign names their (actual) referents as semantic values, if we in addition adopt the widespread view of names as rigid designators from which flows the necessary identity of Hesperus and Phosphorus,  $\langle NEG, \langle =, \langle \ulcorner \text{'Hesperus'} \urcorner, \ulcorner \text{'Phosphorus'} \urcorner \rangle \rangle \rangle$  is indeed just  $\langle NEG, \langle =, \langle \ulcorner \text{'Hesperus'} \urcorner, \ulcorner \text{'Hesperus'} \urcorner \rangle \rangle \rangle$ .

The challenge might then be to account for this impossibility: to provide an *impossible* structured world that represents, say, that Hesperus is not Phosphorus. Alternatively, we can question the necessary identity of Hesperus and Phosphorus. My LMR backdrop leads me to favour this Lewisian path. If it is not the case that Hesperus is Phosphorus in all possible worlds, we already have genuine worlds that represent Hesperus not being Phosphorus and so we also have *possible* structured worlds that represent Hesperus not being Phosphorus.

Both approaches require specifying what stands for proper names in tuples, since that could no longer be only the actual referents. Without further clarifications, the structured worlds construction, as is, still potentially conflates propositions that we want to be able to distinguish.

There are two main assumptions that I think need to be questioned: (1) That the semantic values of names are their (actual) referents; (2) That proper names are rigid designators, that is, they refer to the same individual in all possible worlds in a way that leads to the 'necessity of identity'. Hypothesis (2) falls promptly once the commitment to LMR plus counterpart theory (CT) is reiterated. Again, I present a full response to this challenge and show how SHMR can overcome it in (2022b).<sup>48, 49</sup>

**(6) Higher-order impossibilities** Satisfactory and rigorous solutions for the higher-order impossibilities and extraordinary impossibilities challenges call for a detailed semantics, which Reinert (2013, 2017, 2018) offers and which I will not even touch

<sup>46</sup> See the quotation above in, (5).

<sup>47</sup> I assume we have appropriately defined the semantic value for the relation of identity, and I write '=' for this value.

<sup>48</sup> Including the question of fictional entities and fictional names.

<sup>49</sup> Can we do better if we take semantic values to be possible worlds intensions as within the standard possible worlds semantics framework? Instead of following a Russellian path, the idea would be to consider the avenue of structured intensions or structured meanings (Lewis, 1970; Cresswell, 1985), and (King, 2008, §3.2.). The semantic value of a name would then be, for instance, a function from possible worlds to individuals, the function that maps a possible world  $w$  to the individual denoted by the name at  $w$ . If names are directly referential rigid designators, they designate the same individuals in every possible world in which the individuals exist. At first glance, we do not make much progress: the intension of 'Hesperus' and the intension of 'Phosphorus' do not differ more than their extensions do. If names are directly referential rigid designators, intension and extension coincide; the world of evaluation does not matter. The alternative is there again: either we figure out a better account of the semantics value of names, or we give up on the rigid designation of names.

upon here. The limited aim of this overview is to give an idea of how these impossibilities could be handled within the SHMR framework.

Higher-order impossibilities arise when we consider the ‘possibilization’ of impossible statements. Let  $A$  be an impossible proposition. Is there a world at which  $\Diamond A$  holds? There is no Lewisian world where  $A$  holds, as it is an impossible proposition. Therefore,  $\Diamond A$  is impossible too. Assuming  $\Diamond A$  possible, the LMR analyse of (*de dicto*) possibility statements (P) gives us a possible world  $w_1$  such as, at  $w_1$ ,  $\Diamond A$ . By (P) again, there is possible world  $w_2$  such as, at  $w_2$ ,  $A$ , which contradicts our assumption that  $A$  is impossible. Let then  $w_i$  be an impossible world at which  $\Diamond A$  is true. How to analyse  $\Diamond A$  at the impossible world  $w_i$ ?

The objection Reinert raises against Berto’s HMR is that, since there is no Lewisian world at which  $A$  holds, there is no world-book where  $\Diamond A$  holds either. One way to answer this issue would be to say: Well, a structured world  $w_S$  that contains the appropriate tuple  $\langle \Diamond A \rangle$  is a structured world at which  $\Diamond A$  is true, even though  $A$  is impossible. And if  $B$  is an impossibility distinct from  $A$ ,  $\langle \Diamond B \rangle$  is distinct from  $\langle \Diamond A \rangle$  and so our structured world is able to discriminate between the two. But it is likely not what Reinert means. The question is rather how to analyse the truth of  $\Diamond A$  at  $w_S$  when  $A$  is true at a structured world only. We can no longer use the clause for the analysis of possibilities: (P) still holds in SHMR, as we wish, but is restricted in the analysans to genuine worlds.

So far, I see no objection to adapting the semantics Reinert offers, building on Mares (1997) and Priest (2005), to the present account so that the alethic modalities can remain conservative at possible structured worlds and higher-order impossibilities can be validated at some (structured) impossible worlds. The feasibility of adapting Reinert’s semantics rests on the isomorphism between possible structured worlds and Lewisian worlds and the definition of structured worlds as *arbitrary* sets of tuples, which include open worlds as shown above.<sup>50</sup>

**(7) Extraordinary impossibilities** are impossibilities which ensue from extraordinary contents, that is, from modal claims that are false *at* any genuine world, albeit being true *from the standpoint of* all worlds.<sup>51</sup> Extraordinary impossibilities are negations of such extraordinary necessary truths. They include negations of LMR ontological necessities such as ‘there is a plurality of worlds’, and the negations of the necessary truths of the LMR theory.<sup>52</sup>

Reinert’s ‘extraordinary impossibilities’ challenge may be dealt with in SHMR as an instance of atomic impossibilities. LMR impossibilities like *that  $w_1$  is spatiotemporally related to  $w_2$  or that  $w$  is not concrete* (where ‘ $w$ ’, ‘ $w_1$ ’, ‘ $w_2$ ’ are names for genuine worlds) are easily represented by structured worlds that accommodate the suitable tuples, say  $\langle w_1, w_2, \text{being spatiotemporally related to} \rangle$  and  $\langle \text{NEG}, \langle w_1, \text{being concrete} \rangle \rangle$  respectively. A structured world  $w_S$  that contains one or more

<sup>50</sup> I sketch the idea of the semantics in the SHMR framework in (2022c).

<sup>51</sup> Extraordinary contents are modal claims which Divers (2002, p. 47 and around) calls ‘extraordinary cases’.

<sup>52</sup> (Reinert, 2017, pp. 43-44).

such of these tuples represents those impossibilities which are true at no genuine world nor true from the standpoint of any genuine world. The extended proposition *that  $w$  is not concrete* is true at the impossible structured world  $w_S$  if and only if  $w_S \in \{w \mid \langle \text{NEG}, \langle w, \text{being concrete} \rangle \rangle \in w\}$ . Similarly, SHMR provides structured impossible worlds that represent the negation of existential claims like *that there are no concrete worlds*.

Grant again that we can succeed in defining a suitable semantics. The semantics includes open worlds in its models. Truth and falsity are randomly assigned to all formulae, also complex, at these open worlds. There are therefore structured open worlds at which the existential claim of LMR theory are false and where the negation of the necessary truths of LMR are true.

## 5 Conclusion

Once the need for fine-grained accounts of content is acknowledged, bringing impossible worlds on board is an appealing move. But this is not so straightforward if one wants to (1) preserve the benefits of the possible world framework (2) provide enough fine-grainedness. Arguably, achieving a plenitude of impossibilities may be harder for a friend of Lewis's Modal Realism. But I have argued that following a hybrid path is both the most LMR conservative and the most promising when it comes to fine-grainedness. The deficiencies of previous hybrid theories like Berto's HMR stem from what LMR 'raw material' is chosen as the building blocks of the set-theoretic constructions which stand for impossible worlds. Those must be *parts* of genuine worlds rather than genuine worlds themselves. I have raised some challenges for a new hybrid view (Sect. 2), which my 'Structural Hybrid Modal Realism' (SHMR) proposal (Sect. 3) aims to meet. I have held that SHMR scores well with these challenges (Sect. 4). Several issues have been left for other occasions. But I am confident that a fully and satisfactory hybrid theory of impossible worlds is achievable.

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