

A Note on Equiprobability Prior to 1500

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Abstract

Rudolf Schuessler has argued that sixteenth-century thinkers developed a concept of equal probability that was virtually absent before 1500 and that may have contributed to the birth of mathematical probability shortly after 1650. This note uses additional textual evidence to argue that the concept of equal probability was in fact generally available to medieval thinkers. It is true that ascriptions of equal probability are comparatively rare in medieval texts, but this can be explained without positing a conceptual blind spot.

Keywords

history of probability – probable opinion – scholasticism – medieval – Latin

The history of probability, broadly conceived, stands to benefit greatly from Rudolf Schuessler's detailed engagement with early modern scholastic sources in a project that has now culminated in a rich and valuable monograph.¹ Schuessler's new body of work comes with a number of innovative contentions, one of them being that mathematical probability had a longer and more complex gestation before its birth in the mid-seventeenth century than historians have previously recognized. This note provides a corrective to one aspect of this contention: the claim, originally aired in the pages of this journal, that

1 Rudolf Schuessler, *The Debate on Probable Opinions in the Scholastic Tradition* (Leiden, 2019). To avoid needless complication, I follow Schuessler's practice of translating '*probabilis*' as 'probable' "with the caveat that it should not be confused with modern numerical probability" (2; cf. 37–38). I also follow his practice of writing his surname, originally 'Schüßler,' in "the international, machine legible spelling" throughout (9 n. 12).

sixteenth-century thinkers developed a concept of *equal* probability that was “virtually absent from medieval thought” and that may have “facilitated the emergence of numerical representations of probability.” Schuessler himself had only found three explicit uses of this concept “in a sizeable number of medieval sources,” and although he acknowledged that others might be unearthed, he thought it “unlikely that the picture will fundamentally change following further reviews of the texts.”² I first present some additional evidence suggesting that the concept of equal probability was in fact generally available to medieval thinkers; I then propose an alternative explanation for the rarity of its deployment in medieval texts; and I conclude with an appraisal of the significance of my findings for Schuessler’s overall account.

1 Some Additional Evidence

Schuessler highlighted the absence of equiprobability in medieval translations of Aristotle, which he contrasted with its presence in sixteenth-century commentaries on (and eventually translations of) the *Topics* and *Sophistical Refutations*. In particular, he cited four occurrences of ‘ὀμοίως ἔνδοξον’ (equally reputable/acceptable) that Boethius had translated as ‘*similiter probabile/opinabile*’ and that were not retranslated as ‘*aeque probabile*’ before 1550, so that medieval scholastics “apparently did not derive the idea of equal probability from the Aristotelian corpus.”³ I should therefore begin by noting that equiprobability can be found eleven times in the same locations in the expository paraphrases by Albert the Great (1260s), who at one point even glossed ‘*similiter*’ as ‘*equaliter*.’⁴ Likewise, William of Ockham (early 1320s) mentioned equally probable premisses twice in his commentary on the same passage of the *Sophistical Refutations*.⁵ For that matter, both authors interpreted this

2 Rudolf Schuessler, “Equi-Probability Prior to 1650,” *Early Science and Medicine*, 21 (2016), 54–74. The argument is concisely reprised in Schuessler, *The Debate*, 64–66 and 480–483.

3 Schuessler, “Equi-Probability,” 61, 70; idem, *The Debate*, 65.

4 Albertus Magnus, *Topicorum* III. 3.1 (ad 119a36–119b16), in *Logica* (Venice, 1494), 169vb–170ra: “eque probabile illi erit quod aliqua insensibilitas non est in ipso naturalis,” plus seven occurrences of ‘(sequitur) eque probabiliter’; ibid. VIII. 2.7 (ad 161b34–37), 200va: “non oportet dicere syllogismos similiter, hoc est equaliter, esse probabiles et verisimiles.” *Elenchorum* II. 5.1 (ad 182b37–183a4), in *Logica*, 242rb: “affirmativa et negativa in talibus equaliter sunt probabiles, propter quod necessarium est dubitare respondentem quid eligat ad respondendum; maxime ergo [...] est huiusmodi oratio acris [...] que facit conclusionem ex [...] premissis que sunt ex equo probabiles.”

5 Guillelmus de Ockham, *Expositio super libros Elenchorum* II. 18.6 (ad 182b37–183a4), ed. Francesco del Punta (St. Bonaventure, NY, 2001), 314–315: “si etiam praemissae essent aequae

passage as saying that equiprobability leads to doubt (*ἀπορεῖν*, *dubitare*), contrary to Schuessler's remark that "a verbatim ascription of equal probability was apparently not connected to doubt before 1500."⁶

The implications of equiprobability for doxastic psychology were also raised in a different context by Siger of Brabant (ca. 1275). Commenting on *Metaphysics* IV. 5, where Aristotle had dealt with denials of the principle of non-contradiction based on honestly mistaken reasoning, Siger asked whether someone with "equally probable" arguments for two contradictory propositions had to believe both of them. His discussion survives in two full versions and a summary; equal probability appears three times in each of the full versions and once in the summary.⁷

Nor are these the earliest mentions of equiprobability in Latin. Contrary to Schuessler's suggestion that the notion "was not used in antiquity," Augustine himself had conceded that his numerological speculations might have "equally probable" rivals (*aeque probabiles*).⁸ The notion can also be found in two popular texts from the first half-century or so of the scholastic tradition. John of Salisbury in the *Policraticus* (1159) denounced scepticism as obstructive,

probabiles, difficile esset solvere eam, et contingeret facere duos syllogismos ex opposito aequae acris"; *ibid.* II. 18.7 (ad 183a4–7), 315: "oratio secunda acris est illa [...] quae est ex praemissis aequae probabilibus, et scitur quod aliqua illarum praemissarum est interimenda, sed nescitur quae. Talis enim oratio facit dubitare."

6 Schuessler, *The Debate*, 41 n. 42.

7 Siger de Brabant, *Quaestiones in Metaphysicam* (Munich) q. 30, ed. William Dunphy (Louvain-la-Neuve, 1981), 223–224: "habens rationes aequae probabiles ad utramque partem contradictionis <non> habet opinari utramque [...] nec habet opinari alterum tantum, quia rationes quae facerent opinari alteram sunt aequae probabiles; qua igitur ratione magis opinaretur unam partem quam aliam? [...] Potest enim ratio probabilis considerari dupliciter: absolute et in se, et sic inducit effectum suum; vel ut refertur ad rationem aequae probabilem ad oppositum, et sic impeditur ab effectu per istam." *Quaestiones in Metaphysicam* (Cambridge) q. 31, ed. Armand Maurer (Louvain-la-Neuve, 1983), 175–176: "quaeritur utrum, cum aliquis habeat rationes aequae probabiles ad utramque partem contradictionis, de necessitate utramque partem opinetur. [...] dico primo quod cum aliquis habeat rationes aequae probabiles ad utramque partem contradictionis, utramque partem credere non potest [...]. Nec credit alteram determinate; cum enim rationes ad utramque partem aequae probabiles habeat, non est maior ratio quare plus determinetur ad unum quam ad alterum." *Quaestiones in Metaphysicam* (Paris) q. 19, ed. Armand Maurer (Louvain-la-Neuve, 1983), 428: "impedire potest unum syllogismum probabilem ratio aequae probabilis, ad quam cum refertur non generabit opinionem." On the different versions, see the joint review by Steven P. Marrone, *Speculum*, 61 (1986), 1005–1007.

8 Schuessler, "Equi-Probability," 61 n. 21. Augustinus, *De trinitate* IV. 6, ed. William J. Mountain and François Glorie (Turnhout, 1968): "horum [...] numerorum causas cur in scripturis sanctis positi sint potest alius alias indagare, uel quibus istae quas ego reddidi praeponendae sint, uel aequae probabiles, uel istis etiam probabiliores."

complaining that nothing could be proved “for someone to whom all things are equally probable” (*aeque probabilia*).⁹ And Peter the Chanter in the *Verbum abbreviatum* (1180s) denounced human justice as inconstant, citing the alleged remark by Pope Alexander III that whenever he had given a positive verdict, he could have given a negative one “if prompted by equivalent arguments and equal probabilities” (*equis probabilitatibus*).¹⁰

I must confess, belatedly, that my examples so far have all been taken from the Library of Latin Texts.¹¹ As of the latest update (28 December 2020), this database yields nine further occurrences in medieval scholastic texts, and readers may perhaps be relieved if I list the four authors without going into details: Peter John Olivi (ca. 1280s),¹² Hervaeus Natalis (ca. 1300s),¹³ Peter of Ailly (1377–1378),¹⁴ and John Wyclif (late 1370s–early 1380s).¹⁵ Of course, although

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- 9 Ioannes Saresberiensis, *Policraticus* VII. 7, ed. Clement C.J. Webb, 2 vols. (Oxford, 1909), 2: 116: “Ei namque cui omnia aeque probabilia sunt nichil probari potest.”
- 10 Petrus Cantor, *Verbum abbreviatum. Textus prior* 46, ed. Monique Boutry (Turnhout, 2012), 304: “asseritur Alexandrum papam dixisse se, si paribus rationibus et equis probabilitatibus moueretur, paratum esse iudicare pro negatiua quotiens iudicatum est ab eo pro affirmatiua”; the phrase is absent from the *Textus alter* but retained in the *Textus conflatus* I. 51, ed. Monique Boutry (Turnhout, 2004): “asseritur sepius dixisse quod numquam se astrinxit humanarum legum regulis uel decretis, sed secundum proprium motum mentis sic semper iudicauit in affirmatiua ut idem iudicare posset in negatiua si paribus rationibus et equis probabilitatibus moueretur.”
- 11 The LLT is a subscription-only database; see <www.brepolis.net>.
- 12 Petrus Johannes Olivi, *Lectura super Lucam* ad 24:12, ed. Fortunato Iozzelli (Grottaferrata, 2010), 648: “Nec mireris si uariis modis hoc potuisse fieri et concordari dicimus, licet aliquando plus alterum aprobemus, aliquando uero utrumque eque probabilem iudicemus, quia narrationes euangelistarum aliquando sic se habent ut eque possibiliter et probabiliter modis pluribus concordentur, et aliquando sic quod nequeunt nisi unico modo.”
- 13 Hervaeus Natalis, *De unitate formae substantialis in eodem supposito* 2.2, ed. Lambertus M. de Rijk (Turnhout, 2011), 210: “Illa evasio nulla est [...] secundum quam etiam forma accidentalis potest sustineri esse substantialis eque probabiliter sicut potest sustineri illa secunda forma quam ipsi ponunt esse substantialem. Sed secundum istam evasionem posset eque probabiliter sustineri formam accidentalem, saltem quantum ad accidentia que necessario consequuntur rem, esse formam substantialem sicut formam secundam quam ipsi ponunt.”
- 14 Petrus de Alliaco, *Questiones super primum, tertium et quartum librum Sententiarum* prolog., ed. Monica Brinzei (Turnhout, 2013), 194: “Ista autem propositio est probabilis, sed videtur michi quod eius oppositum eque probabiliter posset poni.”
- 15 Johannes Wyclif, *Tractatus de veritate sacrae scripturae* 17, ed. Rudolf Buddensieg, vol. 2 (London, 1906), 55: “quilibet enim sensus scripture plus pius et eque probabilis est sibi adhibendus.” *Tractatus de ecclesia* 23, ed. Johann Loserth (London, 1886), 568: “eque probabiliter contingit sompnare quod <sancti> [...] fecerunt ista [...] pro illo qui eis participet, sicut quod <deus> reliquit †papam [recte pape] quod ipsa distribuat; ideo videtur peti ignotum per ignocius”; *ibid.* 575: “eque probabiliter posset unum fingi ut reliquum, cum toti isti materie deficit fundamentum.” *Tractatus de blasphemia* 11, ed. Michael Henry

the Library of Latin Texts is a wonderful resource, now 40 per cent larger than it was when Schuessler conducted his research, it still contains only a tiny fraction of the medieval texts that have been printed, let alone the ones that have not. No surprise, then, to find another example in my own much smaller digital collection: William Heytesbury (1330s).¹⁶

To sum up, whereas Schuessler had only found three occurrences of equal probability in two minor authors and one positively obscure author – Simon of Faversham (ca. 1280), Hugh of Newcastle (1310s) and Stephen Patrington (1380) respectively¹⁷ – the ten authors responsible for my 28 additional medieval occurrences include some of the most famous and influential figures from the twelfth to the fourteenth centuries: John of Salisbury, Peter the Chanter, Albert the Great, Siger of Brabant, Peter John Olivi, Hervaeus Natalis, William of Ockham, William Heytesbury, Peter of Ailly, and John Wyclif. What's more, the conspicuous absence of fifteenth-century authors from this electronic harvest has a simple explanation: the neglect of the period by modern scholars has resulted in a lack of searchable texts.¹⁸ Accordingly, if we dust off an incunabular edition of the *Sophistical Refutations* commentary by Johannes Versoris (ca. 1440) and turn to the usual passage, we will find two more occurrences.¹⁹

Dziewicki (London, 1893), 159: "Habet [...] ecclesia romana usum suum, et ecclesia anglicana usum disparem plus vel eque probabilem."

- 16 Guilelmus Hentisberus, *Sophismata* 9, ed. Ludovicus de Carera and Pasius de Bisiolis (Venice, 1491), 56ra: "eque probabile apparet ponere aliquid esse ultimum instans in quo a et d apparebunt equalia [...] sicut ponere primum instans in quo apparebunt inequalia [...]" (discovered via Fabienne Pironet's transcription of the 1494 edition).
- 17 Schuessler, "Equi-Probability," 61–62. As he notes, only the occurrence in Simon of Faversham's *Sophistical Refutations* commentary was his own discovery; the others had been cited in James Franklin, *The Science of Conjecture: Evidence and Probability before Pascal* (Baltimore, MD, 2001), 209, 440 n. 59. The quotation from Hugh of Newcastle's *Sentences* commentary inherits Michalski's mistranscription 'circa' for 'contra' (against this way of arguing). The quotation from Patrington's notebook probably inherits the scribal error 'eo' for 'alio' (any other absolute); it is certainly distorted by mistranslating 'capio istam propositionem eque probabilem' (I take this proposition, equally probable) as 'I take this proposition as equally probable.' Patrington took the passage from a *Sentences* commentary from the early 1320s: *Robert Greystones on Certainty and Skepticism: Selections from His Works*, ed. Robert Andrews, Jennifer Ottman and Mark Henninger (Oxford, 2020), liv–lvi and 130–131.
- 18 For some relevant discussion, see Rudolf Schuessler, "Was There a Downturn in Fifteenth-Century Scholastic Philosophy?," *Studia Neoaristotelica*, 15 (2018), 5–38.
- 19 Johannes Versoris, *Super libros Elenchorum* 11. 4 (ad 183a4–7), in *Super omnes libros nove logice* (Cologne, ca. 1487), sig. C vii va: "oratio dicitur minus acris quando infert improbabile ex premissis eque probabilibus ita quod non sunt multum probabiles, et tunc respondens percipit quod talis oratio debet solvi per interempcionem, sed non bene scit per cuius interempcionem solvatur propter equalitatem premissarum in probabilitate."

Some of these additional occurrences will no doubt be more significant than others for the history of probability. But a list based on such a limited selection of texts must be far from exhaustive, and of course all of these scholastics were writing in a tradition, not in isolation. On balance, then, I think we may safely reject Schuessler's claim that "non-occurrences" show that the concept of equal probability was virtually absent from medieval thought, and with it his suggestion that "an overwhelming majority of medieval scholars did not connect probability with a relation [viz. equality] that underlies its mathematization."²⁰

2 An Alternative Explanation

Schuessler is nonetheless right to say that "the order relations 'more' and 'less' [...] dominated comparisons of probability in the Middle Ages."²¹ By way of a quick illustration, a Library of Latin Texts search for '*magis probabil-*,' '*probabilior-*' and '*probabilius*' (more probable) gives 47 hits in Albert the Great, compared to the eleven for equal probability that we saw above; and the same search gives almost a hundred hits in Thomas Aquinas, compared to none at all for equal probability. The disparity is certainly striking, and if it is not the result of a conceptual blind spot, it needs to be accounted for in some other way.

My own explanation is simple: medieval authors wanted to say that X was more probable than Y more often than they wanted to say that X and Y were equally probable. This, in turn, needs explaining, but now the task is easier and readers may well be ahead of me. I suggest two factors, one psychological and the other dialectical. The first is that someone faced with conflicting opinions or arguments is comparatively unlikely to find them equally probable. The second is that someone presenting their own opinion or argument alongside an opposing opinion or argument is comparatively unlikely to want to present them as equally probable.

It will be noticed that neither part of my explanation mentions anything specific to *medieval* authors. I will now try to show that this is nothing to worry about. According to Google's Ngram Viewer – and with all the caveats that this entails – in books printed in the nineteenth and twentieth centuries 'more probable' outnumbered 'equally probable' in a given year by a ratio of around 15:1 on average. And if we exclude mathematical probability by looking at plausibility instead, we find that 'more plausible' outnumbered 'equally plausible'

²⁰ Schuessler, "Equi-Probability," 55.

²¹ Schuessler, *The Debate*, 41.

by a ratio of around 11:1 on average.²² This observation should reduce any initial surprise about the comparative rarity of equal probability in medieval texts. It also prompts another observation: it turns out that ‘less probable/plausible’ was only around 1.5 times as common as ‘equally probable/plausible.’ If we return to the Library of Latin Texts with this in mind, we find that ‘*minus probabil-*,’ ‘*improbabilior-*’ and ‘*improbabilius*’ were also quite rare, occurring only around twice as often as their equiprobable counterparts. This further disparity (unremarked by Schuessler) may also be explained in terms of the dialectical function of probability comparisons: they often serve to recommend the author’s own opinion as more probable.

3 Conclusion

Schuessler was surely wrong to claim that “[only] in the sixteenth century did probability explicitly become a concept for which three order relations, namely ‘greater than, smaller than, and equal’, were acknowledged as being adequate.” Even so, he may have been right to suggest that “the greater detail of early modern treatments of disagreement [...] increased the salience of the concept of equal probability and thus stimulated its use.”²³ Theoretically speaking, at least, there is also scope for context-specific variation in the second factor that I take to explain the rarity of its use in medieval texts, namely that authors are likely to want to present their own opinions as *more* probable than others. In any case, Schuessler explicitly did not claim that familiarity with the notion of equal probability was “the only or even the most important factor [leading] to the development of the probability calculus”;²⁴ I must therefore stress that this note only provides a minor corrective to his overall account.²⁵

22 These figures are derived from the Google Books Ngram Viewer using the “English 2019” corpus (February 2020). I have included ‘just as’ as a stylistic variant of ‘equally’ in an attempt to avoid overstating my case; without this tweak, the average yearly ratios are around 17:1 and 12:1 respectively.

23 Schuessler, *The Debate*, 65; idem, “Equi-Probability,” 73.

24 Schuessler, “Equi-Probability,” 55; see further idem, *The Debate*, 480–483.

25 I thank Rudolf Schuessler for his magnanimous comments on my original draft.