Combining Cartel Penalties and Private Damage Actions: The Impact on Cartel Prices*

Yannis Katsoulacos‡
Evgenia Motchenkova†
David Ulph†

Abstract

In many countries antitrust enforcement by Competition Authorities through prosecution and the imposition of penalties is complemented in price-fixing cases by private damage actions, which should affect both cartel deterrence and the prices set by those cartels that do form. We show that the impact of combining penalties and damages on cartel prices is not clearcut, and depends on both the nature of the penalty regime and the way that damages are calculated. We demonstrate this by focusing on two ways of calculating damages that have been advocated in practice and two different forms of the widely used revenue-based penalty regime. When the simple form of revenue-based penalties is in force, the standard method of calculating damages worsens its harmful pricing effects, whereas the proposed alternative method of calculating damages can overturn them. When a more sophisticated form of revenue-based penalties is in operation, imposing damages will improve its beneficial pricing effects under both methods of damage calculation, but the alternative method is more effective. In all cases combining penalties and damages improves deterrence.

JEL Classification: L4 Antitrust Policy, K21 Antitrust Law, D43 Oligopoly and Other Forms of Market Imperfection.

Keywords: Cartel Damages, Antitrust Penalties, Antitrust Enforcement, Antitrust Law, Cartels.

* For helpful comments on this version and previous drafts we thank Yossi Spiegel, Eric van Damme, Maarten Pieter Schinkel, Thomas Ross, Joe Harrington, Peter Dijkstra and participants at the ACM workshop (2018) and CRESSE (2018) conference. We also thank two anonymous referees for their very useful comments and suggestions, and for their prompt responses given the tight deadlines. We are grateful to the Tinbergen Institute, Vrije Universiteit Amsterdam, Short-term Visitor Program for financial support.
‡ Department of Economic Science, Athens University of Economics and Business, Patission 76, Athens 104 34, Greece. Email: yanniskatsoulacos@gmail.com
† Department of Economics, Vrije Universiteit Amsterdam, TILEC and Tinbergen Institute, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands. Email: emotchenkova@feweb.vu.nl
‡ School of Economics and Finance, University of St Andrews, St Andrews, KY16 9AR, Scotland. Email: du1@st-andrews.ac.uk
1. Introduction

Following the US and Canada, in many countries private damage actions against companies violating competition law, particularly by price fixing, are now common practice. This is increasingly so in Europe following the publication of the European Directive on Antitrust Damages Actions in 2014.¹ Public enforcement sanctioning and private damage actions serve primarily different purposes. Public enforcement’s main objective is to bring existing cartel activity to an end and to deter future violations by imposing sanctions on companies violating competition law. On the other hand, private damages focus on compensating those who have suffered from price-fixing. While the two policies are complementary, each can contribute to the objectives of the other.² Public enforcement can increase the chances of successful private damage actions and the possibility of obtaining private damage actions could enhance detection by giving incentives to customers to discover and report price-fixing.

There is an extensive literature analysing the effects of public cartel enforcement in isolation from private damages, see e.g. Rey (2003), Buccirossi and Spagnolo (2007), Schinkel (2007), Bageri et al. (2013), Katsoulacos and Ulph (2013), Bos et al. (2018), Katsoulacos et al. (2015, 2019). Similarly, the isolated effects of treble damage awards in private antitrust law suits have been analysed in Salant (1987), Baker (1988), Block et al. (1981) and Besanko and Spulber (1990). The interplay of public and private antitrust enforcement has been discussed in e.g. Landes (1983), Segal and Whinston (2006), McAfee et al. (2008) and Wils (2017), but the focus has been mainly on deterrence effects.

The literature on the cartel pricing effects of combining public penalties and private damages is more sparse. Hunold (2013) analyses the effects of private damage actions on cartel pricing incentives in the setting where downstream firms, buying at excessive prices, are compensated for the lost profits or overcharges. In the context of industries selling to final consumers, Harrington (2004a, 2005) analysed the cartel pricing effects of combining private damages with a fixed public penalty.³ However, currently most jurisdictions, including the US

²Furthermore, each method is costly for firms: not only do lawyers and managerial attention create additional costs, but also the bad publicity the firm gets.
³There are two important differences between our paper and the work of Harrington (2004a, 2005). The first is that he considers the case where the probability of detection depends on the cartel price, whereas we assume a constant detection probability. The second is that Harrington considers a combination of a fixed public penalty with what we call the traditional method of damage calculation based on the absolute overcharge, while we consider two forms of public penalty structures and an additional method of damage calculation based on the proportional overcharge. Taken together these differences allow us to isolate the price effects that arise solely due to the design of the sanctioning system, whilst in Harrington’s work price effects are also driven by the assumed relationship between cartel price and the probability of detection.
and the EU, employ penalties based on cartel revenue.\textsuperscript{4} In this paper we focus on the implications for cartel pricing of combining private damages with these revenue-based penalties, under which the financial penalty to be imposed on a cartel is determined by applying a penalty rate to the cartel’s revenue. We show that the implications depend crucially on the details of how the penalty rate is determined and on the method used for calculating damages. In some cases combining private damages with public penalties exacerbates the adverse pricing effects of penalties in isolation. In other cases combining the two can improve the already good pricing effects of the penalty regime in isolation.

Rather than aiming at very general results we establish these conclusions by focusing on just two forms of revenue-based penalties and two methods of calculating damages.\textsuperscript{5} These are theoretical characterisations of penalty regimes and penalty calculation methods that are used or have been advocated for use in practice, and allow us to demonstrate the range of conflicting conclusions mentioned above. Although our focus is on cartel pricing behaviour, we also examine the combined impact of penalties and damages on deterrence, and confirm the unsurprising conclusion that, in all cases, deterrence is higher when public penalties are combined with private damages, than under public penalties alone.

Before setting out the two forms of revenue-based penalties that we consider, we note that these penalty regimes typically take the form of a baseline penalty rate that is imposed for the anti-competitive offense of forming a cartel and raising price above the but-for price. As discussed below this penalty rate may or may not be set in a way that takes account of the severity of the offence – essentially the size of the overcharge. This rate is subsequently adjusted to reflect a variety of aggravating and mitigating factors such as how co-operative the parties were in the investigations, recidivism, proportionality, bankruptcy concerns etc., all of which are unrelated to the cartel price set in the particular offense that is being penalised.\textsuperscript{6} Moreover these behaviours and considerations lie outside the scope of this paper, which focuses on how the penalty and damages system affects cartel pricing behaviour, so we follow

\textsuperscript{4}The EU, US and the majority of OECD countries employ revenue-based penalties (or penalties formed as a fraction of cartel turnover or sales).

\textsuperscript{5}It should be noted that this paper does not intend to derive an optimal structure of the public and private enforcement instruments or to add to the already well developed literature on analysis of optimal penalty regimes in the tradition of Becker (1968). As such our paper is consistent with a wide range of recent literature that considers penalty setting as an inherently second-best exercise and recognises that the competition authorities in the real world are constrained by legal rules and restrictions and, therefore are bound to set second-best penalties. See for example: Bos and Schinkel (2006), Buccirossi and Spagnolo (2007), Schinkel (2007), Veljanovski (2007), Connor and Lande (2008), Harrington (2010), Allain et al. (2011), Bageri et al. (2013), Katsoulacos et al. (2015), Spagnolo and Marvão (2016), Dargaud et al. (2016), or Houba et al. (2018).

\textsuperscript{6}See see EC (2006), ICN Report (2017) or CMA Guidelines (2018) for an exposition of this penalty structure.
the extensive literature on how the penalty system affects cartel behaviour in essentially treating the penalty system as if it comprised just the baseline penalty rate\textsuperscript{7}.

The two forms of revenue-based penalty that we consider are (i) a simple revenue-based penalty under which the total size of the financial penalty is determined by multiplying cartel revenue by a fixed penalty rate that is the same across all cases and unrelated to the price actually set by the cartel; (ii) a linear sophisticated revenue-based penalty regime under which the penalty rate applied to cartel revenue varies directly in proportion to the proportional overcharge set by the cartel, where this factor of proportionality is the same across all cases and is also publicly known. Here the proportional overcharge is defined as the absolute overcharge (difference between the cartel price and the but-for price) divided by the but-for price. As we have argued elsewhere – Katsoulacos et al. (2015, 2019) – the first form of penalty regime is taken by many scholars who analyse revenue-based penalties to characterise the existing penalty regime in many countries.\textsuperscript{8} The second can be thought of as characterising, albeit in a very stylised way, the more recent guidance by the UK Competition and Market Authority (CMA 73, 2018) under which the baseline penalty rate can vary between 0 % and 30% depending on the seriousness of the infringement. From the existing literature we know that, in the absence of private damages, under the first form of revenue-based penalty the cartel price will be above the monopoly price - see for example Bageri et al. (2013), Katsoulacos and Ulph (2013), and Katsoulacos et al. (2015) – while under the second it will be below the monopoly price – Katsoulacos et. al. (2019).

Turning to the two methods for calculating damages, the first is what we call the traditional measure of damages applied by courts. Here the actual quantity purchased is multiplied by the absolute overcharge - the difference between the actual price and the but-for price (see e.g. Harrington (2004b) or Brander and Ross (2017)). This method is widely employed by the Canadian and US Courts, for example, \textit{Sun-Rype Products Ltd. v. Archer Daniels Midland Company}, 2013 SCC 58 [\textit{Sun-Rype}] in Canada.\textsuperscript{9} However, as noted by many authors - Fisher (2006), Han et al. (2008), Basso and Ross (2010), Paha (2012), Rosenboom et al. (2017), and Franck and Peitz (2017) – this traditional measure of damages underestimates

\textsuperscript{7} This literature is referred to in footnote 5.

\textsuperscript{8} See for example Bos and Schinkel (2006), Schinkel (2007), Buccirossi and Spagnolo (2007), Bageri et al. (2013), Katsoulacos and Ulph (2013), or Smuda (2013). These authors sometimes use an illustrative value of 10% for this fixed penalty rate, reflecting \textit{EU Fining Guidelines} that fines cannot exceed 10% of the total annual turnover of the company - see EC (2006) or ICN Report (2017). However this limit relates to the final penalty rate rather than the baseline penalty rate.

\textsuperscript{9} In addition, Harrington (2004b), Paha (2012) and Brander and Ross (2017), among others, make multiple references to the cases where damages were calculated as the product of overcharge per unit and the number of units purchased.
the true total harm, since it does not account for the dead-weight loss arising because some purchases, which would have taken place at the but-for price, do not occur at the collusive price.

A variant of this traditional method is that proposed by Brander and Ross (2017) in which the cartel revenue is multiplied by the proportional overcharge defined as the absolute overcharge divided by the cartel price. Although formally equivalent to the traditional measure, Brander and Ross (2017) argue that it can be easier to calculate in situations when the same product is sold for different prices to different consumers; or when there are multiple variants of at-issue products commanding different prices; or where products are bundled and it is hard to differentiate between the prices and the quantities of each particular good in the bundle.10

Building on this approach we define an alternative method of calculating damages that we call the proportional overcharge method in which cartel revenue is multiplied by the proportional overcharge defined, as in the case of the linear sophisticated revenue-based penalty, as the absolute overcharge divided by the but-for price. This requires exactly the same information as the Brander and Ross measure and so has the same computational advantages as the measure they propose. Moreover, as explained below, it has two other advantages.

Our major conclusions are as follows. When private damages are combined with the simple form of revenue-based public penalties, then, if damages are calculated in the traditional way, the cartel price will be even further above the monopoly price than would be the case under the penalty regime alone. However, if, under simple revenue-based penalties, damages are calculated by the alternative method, then there is a range of cases where the cartel price will not only be lower than that under the penalty regime alone but will lie below the monopoly price and so totally eliminate the distortion caused by the penalty regime. When private damages are combined with a linear sophisticated form of revenue-based public penalties then, under both methods of calculating damages, the cartel price will be further below the monopoly price than would be the case using penalties alone. However, the lowest cartel price is achieved when the alternative method of calculating damages is used.

This suggests that there is a strong case for using the alternative method of calculating damages more widely, on the grounds that, in addition to mitigating the under-compensation property of the traditional method, it also has better pricing properties. A more general conclusion is that it can be misleading to examine the effects of damage claims and public

---

10This type of situation arose in the Microsoft Case where there were multiple products at issue including Word, Excel, Office, and Windows, and there are many variants of each of these products. Also different customers may have paid different prices for the same product based on volume discounts, student status, etc.
enforcement in isolation from one another: their interaction is important but depends on the precise details of each of the two policies.

The structure of the paper is as follows. Section 2 outlines the model set-up. Section 3 provides a detailed analysis of the price effects and the deterrence properties of the two types of revenue-based penalty regimes combined with the two main methods used for calculating recoverable damages. Section 4 examines the robustness of our conclusions to several extensions of the workhorse model. Section 5 concludes.

2. The Model

We analyse an infinitely repeated model of cartel formation and pricing behaviour under complete information as developed in Katsoulacos et al. (2015, 2019). Specifically, we employ a homogeneous products constant marginal costs symmetric Bertrand framework. Though this setting is somewhat specific, it captures the key competitive and collusive forces and, in particular, allows for a clean comparison of different enforcement regimes and their impact on oligopoly pricing.

Consider an economy comprising a range of types of industry, in each of which there is a homogeneous product with a decreasing and weakly concave demand function \( Q(p) \), while production by any firm in that industry is carried out under constant unit costs \( c > 0 \). The associated elasticity of demand is given by \( \eta(p) = -\frac{pQ'(p)}{Q(p)} > 0 \). We assume that for all types of industry:

\[
\eta(p) \text{ is non-decreasing in } p \text{ and } \exists \bar{p} \geq 0 \text{ s.t. } \eta(p) > 1 \forall p > \bar{p}. \tag{1}
\]

So an industry type is characterised by \( c, Q(\cdot) \). Because the number of firms that will be able to form a stable cartel in any given type of industry is endogenous, and, in particular, will depend on the system of penalties and fines in operation, we assume that for each industry type there is a range of industries that differ in the number of firms, \( n \geq 2 \), operating in that industry.

The “but-for” price, denoted by \( p^b \), is the price set in a competitive equilibrium. In the main part of the paper we take this to be the symmetric Bertrand equilibrium, and so, given our other assumptions: (i) \( p^b = c \); (ii) for a cartel to be able to effectively raise price above the “but-for” level all firms in an industry have to join the cartel.\(^{11}\) If a cartel forms and sets a

---

\(^{11}\) In Section 4 we check the robustness of our conclusions by considering the setting where the symmetric but-for prices are higher than marginal costs, i.e. \( p^b > c \).
common price \( p > p^B \), the associated industry operating profits and revenue will be 
\[ \pi(p) = (p - c)Q(p) \quad \text{and} \quad R(p) = pQ(p). \]

The price that maximises joint profits is the monopoly price 
\[ p^M = \arg \max \pi(p). \]

There is a Competition Authority (CA) that investigates, discovers, prosecutes and penalises cartels. Following the current EU sentencing guidelines for fine imposition (see EC (2006)) the penalty imposed by the CA is on cartel revenue \( R(p) \). We examine two forms that such a revenue-based penalty might take. The first is the standard simple revenue-based penalty regime with constant penalty rate \( \rho, 0 \leq \rho \leq 1 \), so the fine paid is given by 
\[ F_\rho(p) = \rho R(p). \]

The value of \( \rho \) is public knowledge. The second is the linear sophisticated revenue-based regime introduced in Katsoulacos et al. (2019). Here the penalty rate applied to a cartel’s revenue varies in direct proportion to the cartel’s proportional overcharge. So the fine paid is given by 
\[ F_{SS}(p) = \sigma \left( \frac{p - p^B}{p^B} \right) R(p), \]

where \( (p - p^B)/p^B \) is the proportional overcharge, and \( \sigma, 0 \leq \sigma \leq 1 \) is the rate at which the penalty rate changes with the overcharge. The proportionate structure of this regime as well as the value of \( \sigma \) are public knowledge. Thus the penalty rate increases in a very simple and transparent way with the severity of the offence, as measured by the proportional overcharge.\(^{12}\)

Let \( \beta, \ 0 \leq \beta < 1 \), denote the probability that in each period a cartel is detected, successfully prosecuted and penalised according to one of the penalty schedules specified above. To keep the analysis tractable we assume that \( \beta \) is independent of \( p \), and, moreover, its value is common knowledge.\(^{13}\) Further we assume that if a cartel is successfully prosecuted by a CA then (i) any private damages claim will succeed with probability 1; (ii) the fraction of damages that are actually recovered through law suits is given by \( \gamma \), \( 0 \leq \gamma \leq 1 \). This reflects the fact that only a fraction of total damage is actually recoverable because not all consumers are represented in a class action. This factor could vary across industries.

\(^{12}\)According to the ICN report (2017), 26 responding agencies indicated that, while determining the size of the penalties, they also consider various factors including the gravity/seriousness of the infringement and that the penalty rates are adjusted in the light of those factors, including severity of the offence.

\(^{13}\)To capture the fact that higher overcharges can increase suspicions by CA and, hence, also increase the probability of detection and prosecution, one can assume that \( \beta \) is increasing in \( p \). However, incorporating this feature without specifying the exact functional form of the \( \beta(p) \) function makes the model and its solution in the current paper intractable. However, it has been shown before (e.g. in Katsoulacos and Ulph (2013) and in Houba et al. (2010)) that in a setting without damages, endogenizing the \( \beta(p) \) function would imply lower unconstrained cartel overcharges for both revenue-based and illegal gains-based structures of public penalties.
The traditional method of calculating damages, as discussed in e.g. Brander and Ross (2006, 2017), Paha (2012) or Harrington (2014), is the product of the absolute overcharge and actual cartel output, and given by the formula \( D_A(p) = (p - p^B)Q(p) \). The alternative method of damage calculation is the product of the proportional overcharge and cartel revenue and so is given by the formula \( D_P(p) = \frac{p - p^B}{p^B}R(p) \). Notice that this method of calculation is equivalent to a linear sophisticated penalty regime with \( \sigma = 1 \). From the formulae we have

\[
D_A(p) = \frac{p - p^B}{p^B}R(p) = \frac{p - p^B}{p^B}pQ(p) = \frac{p}{p^B}\left[(p - p^B)Q(p)\right] > D_A(p)
\]

and so, as noted in the introduction, this method of calculating damages produces a higher level of compensation than the traditional method.

We assume that, following a successful prosecution, the cartel immediately re-establishes. In addition, as is natural in our modelling framework, we assume that there is equal profit-sharing at equal prices. Given all these assumptions, then, in an industry characterised by \((n,c,Q(p))\), the expected present value of profits for a single firm that is a member of a cartel that has set a price \(p\), and faces the penalty regime \(r \in \{R,SR\}\) and damage calculation method \(d \in \{A,P\}\) is given by

\[
V_{r,d}(p) = \frac{\pi(p) - \beta(F_r(p) + \gamma D_d(p))}{n(1-\delta)}
\]

where \(\delta, 0 < \delta < 1\) is the discount factor.

Following the standard grim-trigger strategy profile firms collude on cartel price, \(p\), in the first period and continue setting \(p\) as long as no firm defects. If a firm defects from the cartel it sets a price \(p^D\), below the cartel price, and, for a single period obtains deviation profit \(\pi^D(p)\). Any deviation by any firm leads to breakdown of trust and competition at price \(p^B\) for ever more. To keep the analysis tractable we take defecting firms to be immune from any future prosecution by the CA. Then for collusion to be stable the following cartel stability condition has to be satisfied:

---

\(^{14}\)In the US, where treble damages can be claimed, recoverable damages will be \(D_A(p) = 3(p - p^B)Q(p)\).

\(^{15}\)This assumption is adopted in Katsoulacos et al. (2015) - and also, for example, Motta and Polo (2003) and Chen and Rey (2013). In Section 4 we provide an extension showing that relaxing this assumption does not affect the qualitative nature of our results.

\(^{16}\)While we recognise that in practice deviating firms can be subject to both penalties and damages, here we are adopting the assumption of our earlier paper that we are now extending to include damages. A similar assumption is also made by Motta and Polo (2003) and Houba et al. (2018). Further support for the assumption comes from
\[ V_{r,d}(p) \geq \pi^D(p). \]  

(3)

Hence, the price set by a cartel facing penalty regime, \( r \), and method of damage calculation, \( d \), is that which maximises \( V_{r,d}(p) \) subject to \( p \geq p^B \) and the stability condition (3). We denote this by \( p^C_{r,d} \). Note that if the stability condition does not bind then this price is equal to unconstrained cartel price, \( \tilde{p}^C_{r,d} \):

\[ p^C_{r,d} = \tilde{p}^C_{r,d} = \arg \max \left[ \pi(p) - \beta(F_r(p) + \gamma D_d(p)) \right]. \]  

(4)

On the other hand if the stability condition binds then \( p^C_{r,d} \) is the solution to

\[ \pi(p) = \beta(F_r(p) + \gamma D_d(p)) + n(1-\delta)\pi^D(p). \]  

(5)

Having set out the framework, we can now investigate how both the cartel price and the degree of deterrence vary depending on which of the combinations of public revenue-based penalties and methods for calculating private damages are employed.

### 3. Analysis

In this section we formally analyse the price and deterrence effects when public enforcement is complemented by private damage actions. In section 3.1 we focus on the pricing behaviour of those cartels that do form, i.e. for which the cartel stability condition is satisfied. So we focus on the unconstrained collusive prices and examine the effects of both private damages and public penalties when used in isolation, before going on to consider the implications of combining them. Section 3.2 analyses the deterrence properties of various penalty and damage regimes both in isolation and in combination, and so we need to consider when the cartel stability condition binds, and what happens when it does.

#### 3.1. Analysis of Unconstrained Pricing

To establish a baseline framework, we start by looking at the effects of penalties in isolation and the effects of private damages in isolation.

##### 3.1.1. Effects of Public Revenue-Based Penalties in Isolation

the work of Spagnolo (2004) and Buccirossi et al. (2018) that suggests that the best policy is to avoid penalising price-deviating firms, including exemption from follow-on damages. This would increase the incentives to deviate to the maximum and destabilize cartels.
In this sub-section we set out the effect on cartel prices of using our two variants of a revenue-based penalty regime in the absence of private damages. The first variant is the simple revenue-based penalty regime as described above. Result 1 is taken from Katsoulacos et al. (2015) and re-establishes the distortive price effects of this penalty regime first analysed in Bageri et al. (2013) and Katsoulacos and Ulph (2013). The proof of this result and all other proofs can be found in the Appendix.

**Result 1:**

(i) Under a simple revenue-based penalty regime the cartel price is set above the monopoly price, i.e. $\hat{p}_R^C > p^M$.

(ii) Moreover, the higher the penalty rate the higher the cartel price.

The intuition is that this penalty structure acts like a distortive revenue tax and so induces firms to act as if costs are higher than they really are. So the price set by the cartel is above the simple monopoly level. Moreover, the tougher the tax the bigger the distortion.

The second variant of the revenue-based penalty regime is the linear sophisticated penalty regime, first introduced in Katsoulacos et al. (2019), from which the next result is drawn. Here the penalty to be imposed on a cartel is calculated by multiplying its revenue by a penalty rate that varies directly in proportion to the proportional cartel overcharge, with factor of proportionality $\sigma, 0 < \sigma < 1$. The structure of this penalty regime as well as the value of $\sigma$ are public knowledge.

**Result 2:**

(i) Under a linear sophisticated revenue-based penalty regime the cartel price is set below the monopoly price, i.e. $\hat{p}_{Sr}^C < p^M$ for all $p^B \geq c$.

(ii) Moreover, the higher the factor of proportionality, $\sigma$, the lower is the cartel price.

The intuition behind this result is that raising the penalty rate in proportion to the cartel overcharge is sufficient to more than offset the incentives to set prices above the monopoly level that arise from using cartel revenue as the penalty base – as shown in Result 1.

### 3.1.2. Effects of Private Damage Actions in Isolation

---

17The original result in Katsoulacos et al. (2015) was discussed in the homogeneous products setting with $p^B = c$. In the Appendix we show that this result extends to a more general setting with $p^B > c$.

18In this sub-section we are just reporting conclusions that we, and others, have established elsewhere, so we refer to the conclusions as Results. In the rest of the paper our conclusions are stated as Propositions.
We start with the price effects of the traditional method for calculating damages, which is based on the absolute overcharge and is given by the formula \( D_A(p) = (p - p^B)Q(p) \). In the absence of public penalties \( F_r(p) = 0 \), so the value function in (2) will have the form:

\[
V_A(p) = \frac{\pi(p) - \beta \gamma (p - p^B)Q(p)}{n(1 - \delta)}.
\]

Maximising the expression in (6) with respect to \( p \) gives the following result.

**Proposition 1:** In the absence of public penalties, the traditional method for calculating damages induces the cartel to set a price \( \hat{p}_A^C = p^M \).\(^{19}\)

So the standard method of calculating recoverable damages is not distortive, i.e. does not induce the cartel to set a price above the monopoly price which it would set in the absence of any intervention. The result is intuitively obvious, since, given our assumption that \( p^B = c \), the standard method for calculating recoverable damages is essentially acting like a probabilistic profit tax. This price neutrality of the standard method for calculating recoverable damages can be related to the work of Salant (1987), Baker (1988) and Hunold (2013). Salant (1987) shows that (under some plausible conditions) imposition of a treble-damage regime has no effect on output or aggregate surplus and raises the market price. Furthermore, Hunold (2013) shows that, on the same assumption that \( p^B = c \), the standard overcharge compensation is price neutral both in the presence and in the absence of a competitive fringe, in the setting where a downstream firm buys from the infringer and then sells the product to the final consumers.

The alternative method for calculating damages is based on the proportional overcharge and is given by the formula \( D_p(p) = \left( \frac{p - p^B}{p^B} \right)pQ(p) \). This method has been mentioned in e.g. Brander and Ross (2017) and Smuda (2013), though, to the best of our knowledge, its price effects have not been formally analysed before.

In the absence of public penalties \( F_r(p) = 0 \), the value function in (2) will have the form:

\[
V_p(p) = \frac{\pi(p) - \beta \gamma \left( \frac{p - p^B}{p^B} \right)pQ(p)}{n(1 - \delta)}.
\]

\(^{19}\) We show in the Appendix that in the more general model with \( p^B > c \) we get \( \hat{p}_A^C < p^M \). However this does not affect our conclusion that the standard method of calculating recoverable damages in isolation is not distortive, i.e. it does not induce prices above the simple monopoly level.
This leads to:

**Proposition 2:** In the absence of public penalties, the alternative method for calculating damages induces the cartel to set a price $\hat{p}_C^p < p^M$.  

This result is intuitive and follows immediately from Result 2, since, as noted in Section 2, this method of calculating damages is equivalent to the linear sophisticated revenue-based penalty.

Thus, in comparison with the traditional method of calculating damages, the method based on the proportional overcharge benefits consumers in two ways: (i) it induces lower cartel prices and so reduces the harm suffered by consumers; (ii) it provides a higher level of compensation for this harm. Moreover, there are no potential obstacles to implementing this damage estimation method in practice, since it requires exactly the same information as the more widely employed method based on the absolute overcharge.

### 3.1.3. Combined Effects of Private Damage Actions and Public Penalties

Our next result shows that the interaction of private damages calculated in the traditional way with a simple revenue-based public penalty structure amplifies the adverse price effects of the latter, by pushing the cartel price even further above the monopoly price. This is despite the fact that, as Proposition 1 shows, when damages, calculated this way, operate in isolation, they are price neutral.

In this case the value function in (2) can be written as

$$V_{R,A}(p) = \pi(p) - \beta(\rho R(p) + \gamma(p - p^g)Q(p)) \frac{n(1-\delta)}{\delta}.$$  \hspace{1cm} (8)

Maximization of the expression in (8) with respect to $p$ gives the following result.

**Proposition 3:**

(i) Combining public simple revenue-based penalties and the standard method for calculating recoverable damages based on absolute overcharge implies an increase in the cartel price: $\hat{p}_A^C = p^M < \hat{p}_R^C < \hat{p}_{R,A}^C$.

(ii) Moreover, the cartel price gets larger the greater the extent to which private damages can be recovered.

---

20Contrary to Proposition 1, this result holds for all possible levels of the but-for price, $p^B \geq c$. A formal proof can be found in the Appendix.
To understand the intuition behind this result, notice that we can re-write (8) as:

\[ V_{R,A}(p) = \frac{\pi(p) - \beta(p + \gamma)R(p) + \beta\gamma p^\beta Q(p)}{n(1-\delta)}. \]

So introducing private damages, as calculated in the standard way, has two effects: (i) it acts like an increase in the penalty rate being applied to revenue; (ii) it acts like an output subsidy. From Result 1(ii) the first effect unambiguously increases the cartel price. While the second effect provides an incentive to cartels to expand output and so moderate the extent to which they push up price, this is insufficient to overcome the increased distortion created by the first effect.

We now turn to the effects of combining a simple revenue-based penalty regime with private damages calculated in the alternative way based on the proportional overcharge. Despite the fact that, when operated in isolation, private damages, calculated in this way, reduce cartel prices, we have been unable to show that, when operated in combination with a simple revenue-based penalty regime, these always result in a price below that which would have prevailed under the penalty alone. This is because when damages are calculated this way they act like a tougher revenue-based penalty regime and so, as in Result 1(ii), there will be a tendency to increase price. Nevertheless we can show that there is a class of cases in which combining the simple revenue-based penalty regime with private damages, calculated by the alternative method, will induce the cartel to set a price that is not only lower than that under the penalty alone, but below the monopoly price. So introducing private damages, calculated this way, can eliminate the distortionary nature of the simple revenue-based penalty regime in isolation.

When the simple revenue-based penalty regime is combined with private damages calculated in the alternative way the value function in (2) becomes:

\[ V_{R,R}(p) = \frac{\pi(p) - \beta(p + \gamma)R(p) + \beta\gamma p^\beta Q(p)}{n(1-\delta)}. \]

Choosing \( p \) to maximise the above expression leads to:

**Proposition 4:** Combining public simple revenue-based penalties with private damages calculated using the alternative method will induce the cartel to set a price \( \hat{p}_{R,R} < p^M < \hat{p}_R^C < \hat{p}_{R,A}^C \) whenever \( (p^M-c)/\gamma > \sqrt{\beta / \gamma} \).
Notice that the left hand side of (9), the monopoly overcharge \( (p^M - c) / c \), is independent of any features relating to either penalties or damages and will vary across industries depending on their fundamental demand and cost characteristics. On the other hand, the right hand side of (9) depends solely on the penalty rate and the fraction of damages that are recoverable. So this result shows that, when damage compensation is calculated using the alternative method, then, for industries where parameters of demand and cost are such that \( (p^M - c) / c > \sqrt{\rho / \gamma} \), the combination of simple revenue-based penalties with private damage actions will lead to cartels setting price below that which would have arisen in the absence of any interventions. \textit{A fortiori} the cartel price will be below those that would have arisen under the simple revenue-based penalty alone and under the penalty when damages are calculated using the absolute overcharge. The range of industries for which this is true is greater the larger is the fraction of damages that are likely to be recovered, \( \gamma \), and the smaller is the penalty rate, \( \rho \).

The intuition for this result is that given the equivalence between damages calculated on the basis of the proportional overcharge and the linear sophisticated revenue-based penalty regime, cartels effectively face a combination of two revenue-based penalty regimes – the simple revenue-based one and the linear sophisticated one. From Results 1 and 2 we know that the former induces cartel prices above the monopoly price and the latter induces cartel prices below the monopoly price. The second effect dominates when the relative weight given to this regime is sufficiently large in the sense specified in the Proposition.

In the next Proposition we show that with a linear sophisticated revenue-based penalty structure both methods of damage calculation lead to cartel prices below those that would have arisen under the penalty alone and so, \textit{a fortiori}, below the monopoly price. Furthermore, prices are lower when damages are calculated using the alternative method rather than the traditional method.

\textbf{Proposition 5:}

(i) \textit{In the presence of public enforcement with linear sophisticated revenue-based penalties, both methods for calculating recoverable damages enhance the effectiveness of linear sophisticated revenue-based penalties in reducing cartel price-overcharge below the simple monopoly level. More specifically we have} \( \hat{p}_{SR,P}^c < \hat{p}_{SR,A}^c < \hat{p}_{SR}^c < p^M \).

(ii) \textit{Moreover, under both methods the cartel price is smaller the greater the extent to which private damages can be recovered.}
The intuition is straightforward. Introducing damages calculated on the basis of the absolute overcharge amplifies whatever distortive/non-distortive properties the penalty regime alone has – cf. Proposition 3. Introducing damages calculated on the basis of the proportional overcharge is effectively just like having a linear sophisticated penalty regime in which the penalty rate rises more steeply in relation to the cartel overcharge and so unambiguously lowers the cartel price – cf. Result 2.

Taking all these results together, two important policy conclusions emerge. The first is that the pricing effects of private damage actions cannot be fully assessed in isolation - one needs to assess them in conjunction with the underlying penalty regime. Moreover the conclusions depend sensitively on the nature of both the penalty regime and the method for calculating damages. The second is that there is a significant double benefit to consumers from switching the way in which private damages are calculated from the traditional method based on the absolute overcharge to the less widely used alternative method. For not only will it result in greater compensation for damages which better approximates true harm, but it can also lower collusive prices.

3.2. Analysis of Deterrence Properties

In this section we focus on the deterrence properties, which requires analysing the constrained maximization problem, where the cartel stability condition binds. As in Katsoulacos et al. (2019), $\Delta = n(1 - \delta)$ denotes the intrinsic difficulty of holding the cartel together and for any given industry type $\{c, Q(.)\}$ there is continuum of possible industries $\{c, Q(.), \Delta\}$, where $\Delta$ is distributed on $[0, \Delta_{\text{max}}]$.

The cartel is stable when (3) is satisfied. Then in the absence of a competition authority (3) can be rewritten as $\frac{\pi(p)}{\Delta} \geq \pi^D(p)$ and the maximum critical value of $\Delta$ would be $\Delta_{\text{max}} = \frac{\pi(p)}{\pi^D(p)}$. Whereas, once there is an active CA enforcing penalties on non-defecting cartel members, for cartel stability we must have

$$\frac{\pi(p) - \beta(F_r(p) + \gamma D_d(p))}{\Delta} \geq \pi^D(p).$$

Expression (11) below shows that normalizing $\Delta_{\text{max}}$ to 1 and assuming a uniform distribution makes it easier to translate statements about $\Delta_{\text{max}}$ into statements about the proportion of industries where cartels are deterred due to CA interventions.
Using (10) we can define
\[
\Lambda_{r,d} = \frac{\pi(p) - \beta(F_r(p) + \gamma D_r(p))}{\pi^D(p)} < \Delta_{\text{max}}. \tag{11}
\]
\(\Lambda_{r,d}\) is the maximum critical value of \(\Delta\) such that, under penalty regime \(r\) and method of damage calculation \(d\), either the stability condition binds or the non-negative overcharge constraint binds.\(^{22}\)

Next, we define the degree of deterrence achieved by the penalty regime \(r\) accompanied by the method of damage calculation \(d\), as \(D_{r,d}\). It represents the fraction of industries in which cartels would have formed in the absence of a CA in which they do not form given the presence of a CA operating penalty regime \(r\) and the method of damage calculation \(d\). Formally:
\[
D_{r,d} = 1 - \Lambda_{r,d}. \tag{12}
\]
Expressions (13) and (14) below give the maximum critical values of difficulty of holding the cartel together in the absence of private damage actions under simple revenue-based and under linear sophisticated revenue-based regimes, \(\Lambda_{R}\) and \(\Lambda_{SR}\) respectively. Formally, they are
\[
\Lambda_{R} = \frac{Y(\beta \rho)}{Y(0)} < 1, \tag{13}
\]
\[
\Lambda_{SR} = 1 - \beta \sigma < 1. \tag{14}
\]
To see this, observe that the expression in (13) is obtained by introducing \(Y(z) = MAX \frac{\pi(p) - z R(p)}{p}\), evaluating it and substituting into (11). Note that by the Envelope Theorem, \(Y(z)\) is a strictly decreasing function of \(z\), so the ratio in (13) is strictly less than 1 and a strictly decreasing function of \(\beta \rho\). Note also that under the simple revenue based system the unconstrained cartel price is \(\hat{p}_C^C > p^M\), so the optimal deviation is to set the simple monopoly price \(p^M\) and \(\pi^D(p) = \pi(p^M)\), which is equal to \(Y(0)\). The expression in (14) is obtained by substituting the relevant functions into expression (11) and noting that since under sophisticated revenue-based system the unconstrained cartel price is \(\hat{p}_C^S < p^M\), the optimal deviation is to set the price just below the collusive price and, hence, \(\pi^D(p) = \pi(p)\). This gives \(\Lambda_{SR} = 1 - \beta \sigma \frac{p}{c}\), which takes the value \(\Lambda_{SR} = 1 - \beta \sigma\), when \(p = c\). So \(\Lambda_{SR}\) is the value at which constrained overcharge is driven to zero.

\(^{22}\)The maximum critical level of difficulty, \(\Delta\), is the direct analogue of the minimum critical discount rate used in much of the literature.
Finally, we derive the degree of deterrence $D_{r,d}$ for each combination of regimes. Using the stability condition in (11), expressions (12), (13) and (14) and taking a first-order Taylor approximation to $Y(\beta p)$ around zero we obtain

$$D_r = \beta p \frac{P^M}{(P^M - c)} \quad \text{and} \quad D_{sr} = \beta \sigma.$$  \hspace{1cm} (15)

Similarly, one can show that the levels of degree of deterrence achieved by each combination of the public penalty regimes and private damage actions are given by

$$D_{r,a} = \beta p \frac{P^M}{(P^M - c)} + \beta \gamma, \quad D_{sr,a} = \beta \sigma + \beta \gamma,$$

$$D_{r,p} = \beta p \frac{P^M}{(P^M - c)} + \beta \gamma \frac{P^M}{c}, \quad D_{sr,p} = \beta \sigma + \beta \gamma.$$  \hspace{1cm} (16)

Analysis of the levels of the degree of deterrence in (16) and comparison to the benchmark levels in (15) give rise to the following proposition:

**Proposition 6:** Under both simple revenue-based penalties and linear sophisticated revenue-based penalties, both methods for calculating recoverable damages enhance the deterrence power of the corresponding system of public enforcement. More specifically we have

$$D_{r,a} > D_r \quad \text{and} \quad D_{r,p} > D_r,$$

$$D_{sr,a} > D_{sr} \quad \text{and} \quad D_{sr,p} > D_{sr}.$$  

Also for all combinations, the degree of deterrence is higher the larger the fraction of damages recovered, $\gamma$.

This proposition confirms the standard intuition in the literature – Whinston (2006), McAfee et al. (2008). The result is driven by the fact that in our model any expected threat of paying private damages reduces the left hand side of the stability condition in (10), while it does not affect the right hand side of this condition. Thus the constraint becomes tighter and cartel stability is reduced. The intuition is pretty obvious: making firms face a greater financial outlay

---

23Detailed derivations of the levels of the degree of deterrence in (15) and (16) can be found in the Appendix. Note that the result for $D_{r,p}$ in (16) holds when $\hat{p}_{r,p} > P^M$. In the Appendix we show that when $\hat{p}_{r,p} < P^M$, $D_{r,p}$ has slightly different structure. However, the results of comparison are similar and $D_{r,p} > D_r$ under the condition of Proposition 4.

24Formal analysis of the model, where price-deviating firms can be penalized and where partial leniency is available, is outside of the scope of this paper. Although it may affect deterrence results depending on how well the leniency applicants are protected from the follow up damages claims. We leave this extension to future research.
should they be prosecuted for forming a cartel makes them less likely to want to form a cartel in the first place.

A similar effect has been discussed in Whinston (2006) and Segal and Whinston (2006), who point out that consumers can benefit from the additional increase in deterrence as a result of treble damages. This discussion relates to the large literature on the effects of treble damages awards, which starts with the work of Breit and Elzinga (1974), who emphasised that the prospect of treble damage awards can create a “perverse incentive” for buyers to purchase more and encourages victims to “get damaged”. This idea was formalized later in Salant (1987) and Baker (1988), who assume that the customers anticipate with some probability that they will receive damage compensation and adjust their demand upwards. They also observe that the size of the damage multiple in treble damages is “neutral” and does not affect consumer welfare. Block et al. (1981) emphasized another important effect of treble damage awards: namely that the prospect of such damages awards can also affect strategic pricing on the seller’s side. Our model does not capture customers’ incentives to “get damaged”, however it incorporates the strategic pricing effect identified in Block et al. (1981) as well as the positive deterrence effect mentioned in Segal and Whinston (2006).

This last effect is based on the property that in our setting the profitability of collusion is always lower under both methods of damages calculation compared to the no compensation situation. This result is different from Hunold (2013), where in a model with downstream intermediaries it is shown that private damages actions can increase the cartel profits, indicating that deterrence may decrease. In our framework the mechanism is different. Since there are no downstream intermediaries and the final consumers are not active, the increase in price is purely due to the change in the optimal response of cartel members induced by the change in the combined penalty structure due to the possibility of private damages.

4. Extensions and Discussion

In this section we discuss the robustness of our main results to the modelling assumptions made in Section 2 that underpin the analysis and conclusions of Section 3. We

25Note that under the EU system, where only single damages can be claimed, this effect will be less prominent.

26In a vertical setting with downstream intermediaries Hunold (2013) shows that the profits of a cartel upstream can actually increase when downstream firms are entitled to Lost Profit Compensation (LPC) claims against upstream cartelists. The reason for this is that if a downstream firm expects a positive LPC, it is willing to purchase from the cartelists at input prices above the competitive level. This translates in an increase in consumer prices as well. The reason the expected cartel profits increase in Hunold (2013) is that the LPC claims relax the constraints of the contracting problem between the upstream cartelists and each downstream firm. This increases industry profits, so that cartelists can appropriate part of it.
consider four sets of issues: alternative market conditions, asymmetries, probabilistically re-establishing cartels and enforcement in the presence of leniency programs.

4.1. Alternative Market Conditions

The analysis conducted above has been based on the assumptions that industries are characterised by homogeneous products, firms have constant and identical marginal costs and compete in prices - though we have assumed very general demand functions. When these assumptions fail to hold – for example when there is quantity competition, or differentiated products then both the underlying but-for equilibrium and the analysis of cartel formation will differ (see e.g. Harrington (1991), Ross (1992) or Häckner (1994)). In particular, it is also possible that not all firms will join the cartel, so that cartel stability conditions will take a different form from those considered here (see e.g. Bos and Harrington (2010, 2015), Kalb (2018)). Such settings may also have serious implications for damage computations, particularly if but-for prices can differ for differentiated products, though, as noted in the introduction, Brander and Ross (2017) have pointed out that one of the attractions of the proportional overcharge method of calculating damages is that it is often easier to undertake in some of these richer and potentially asymmetric environments.

Obtaining tractable analytical solutions in such a framework with general demand functions is extremely difficult and one may need to resort to more specific functional forms and/or resort to numerical solutions. Nevertheless we can obtain some conclusions. One implication of these richer market structures is that typically the but-for price will be above marginal cost. By way of a partial attempt to recognise this richer set of market conditions we have explored how far our results generalise to the case where the symmetric but-for price is above marginal costs, and can show that our key pricing results will largely go through. The results in propositions 1, 2, 4 and 5 extend straightforwardly to the case with \( p^B > c \). Detailed derivations are provided in the Appendix. The result in Proposition 3 requires further elaborations, and Proposition 7 below sets out how Proposition 3 extends to this more general setting.

Proposition 7:

27Examples of the contributions taking this approach are e.g. Rothschild (1999) who in a linear demand setting points out how analysis of cartel stability alters when costs are heterogeneous, Brandão et al. (2014) who in a linear demand setting analyse collusion in markets with entry, increasing production costs and cost asymmetries or Kalb (2018) who showed that in a setting with differentiated goods the pricing effects of simple revenue-based penalties analysed in this paper will largely generalise.
Combining public enforcement with simple revenue-based penalties and the standard method for calculating recoverable damages based on the absolute overcharge implies an increase in the cartel price above that under the simple revenue-based penalties for \( c \leq p^b < \frac{c}{1 - \beta \rho} \).

More specifically we have:

\[
\hat{p}_A^C = p^M < \hat{p}_R^C < \hat{p}_{R,A}^C \quad \text{for} \quad p^b = c,
\]

\[
\hat{p}_A^C < p^M < \hat{p}_R^C < \hat{p}_{R,A}^C \quad \text{for} \quad c < p^b < \frac{c}{1 - \beta \rho},
\]

\[
\hat{p}_A^C < p^M < \hat{p}_{R,A}^C < \hat{p}_R^C \quad \text{for} \quad \frac{c}{1 - \beta \rho} < p^b < \frac{c(\gamma + \rho)}{\gamma},
\]

\[
\hat{p}_A^C < \hat{p}_{R,A}^C < p^M < \hat{p}_R^C \quad \text{for} \quad \frac{c(\gamma + \rho)}{\gamma} < p^b < \frac{c}{\gamma \beta}.
\]

So, for a wide range of industries, where the but-for price is not too far above marginal cost, i.e. \( c \leq p^b < c/(1 - \beta \rho) \), the results of Proposition 3 generalise and when private damages, as calculated by the standard method, are combined with the simple revenue-based penalty system the resulting cartel price will be above that arising under the penalty alone. However, when the but-for price is further above marginal cost then combining penalties and damages can result in cartel prices below that which emerges solely under the penalty. Indeed, if the but-for price is very much higher than marginal costs the combined system of damages and penalties can produce a cartel price below the monopoly price.

Although we have been able to get analytical results that largely generalise our conclusions regarding cartel pricing, it has not been possible to get analogous generalisations of our results on deterrence. This is in part because the but-for price enters into the expressions for profits on both sides of the cartel stability condition and in part because of the complexity of the results on pricing as shown in Proposition 7. However, it is hard to believe that any results we could establish would overturn our rather obvious conclusion in Section 3.2 that there will be greater deterrence when potential cartels face the threat of both public penalties and private damages than when they face the threat of either in isolation.

4.2. Cost Asymmetries

In this sub-section we offer a brief reflection on how our conclusions might be affected if firms had constant marginal costs but now these differ across firms.

We start with the case where we retain the assumptions of a homogeneous product, Bertrand competition, and, for simplicity consider the case of a duopoly. Provided the cost of the high cost firm is below the monopoly price of the low cost firm the Betrand equilibrium
will be such that the low-cost firm sets its price just below the cost of the high cost firm. The high-cost firm would be inactive and the but-for price would effectively be the marginal cost of the high cost firm. By forming a cartel the low-cost firm can sell its product at a higher price but the high cost firm can now get some market share.

The cartel price will now emerge as a result of negotiations over the level of price and market shares that generate an efficient sharing of profits recognising the outside options that firms have in the but-for situation. Having a simple revenue-based penalty regime does not alter the analysis except to the extent that it is profits net of the penalty that are being bargained over.

However introducing damages and/or sophisticated revenue-based penalties alters the bargaining strength of the parties as there is a definite advantage of having the high-cost firm in the cartel since that raises the but-for price and so lowers the penalty that both firms face. So introducing asymmetries means that it is not just price but market shares that could change under different forms of penalty and damage regimes, and the effects on market shares could potentially be quite significant.

The biggest complication in trying to understand how our pricing conclusions go through as a result of introducing these asymmetries arises from the fact that many of our conclusions relate to how the cartel price relates to the monopoly price, and, with asymmetric costs and a duopoly there are two monopoly prices. However we think that it is still likely that our conclusion that using the alternative way of measuring damages (based on the proportional overcharge) will result in a lower cartel price than under the traditional method of measuring damages (based on the absolute overcharge) will continue to hold.

If we drop the homogeneous product assumption and had, say, a geographically differentiated market, then there would be multiple but-for prices. However if the cartel just takes the form of an agreement to divide up the market leaving each firm with a localised monopoly, then, if the authorities can observe the different but-for prices and impose different penalties and damages on each member of the cartel, our results are likely to go through fairly intact since the cartel essentially comprises a set of monopolies – i.e. one firm cartels of the type analysed in the paper.

4.3. Probabilistically Re-establishing Cartels

Another assumption made in Section 2 is that after a cartel has been detected and penalised, it is re-established with probability 1. Alternative assumptions in the literature - e.g. Harrington (2005) or Houba et al. (2012) – are that such a cartel will dissolve either with
certainty or with some positive probability. In this section we generalize the model of Section 2 by assuming that following successful prosecution collusive activity will re-emerge with some probability lying between 0 and 1. More specifically, one can assume that, if the cartel is detected, then the firms will continue to collude at price $p$ with probability $\lambda, 0 \leq \lambda \leq 1$ and switch to competition at price $p_B$ with probability $1-\lambda$. Then the collusive value function in (2) can be re-written as $V_{c,d}(p) = \frac{\pi(p) - \beta(F_c(p) + \gamma D_c(p))}{n(1 - \delta + \beta \delta(1 - \lambda))}$. The value of deviation from cartel agreement is unaffected by this change.

This more general treatment produces more complex formulae for $V_{c,d}(p)$ but does not affect the main qualitative results of the paper. In particular, it is easy to see that one can replace the term $\Delta = n(1 - \delta)$ that appears in our analysis with the more general expression $\Delta_{\lambda} = n(1 - \delta + \beta \delta(1 - \lambda))$. Note that with this generalization we can perform similar analysis but with more general expression for maximum critical level of difficulty of holding the cartel together. Obviously, both the unconstrained cartel overcharges as well as the comparative analysis of deterrence properties of the relevant regimes and their combinations will not be affected by this change.

### 4.4. Enforcement in the Presence of Leniency Programs

Leniency programs grant total or partial immunity from fines to the firms that cooperate with CAs and reveal information about existing cartels, and are considered to be an effective instrument in shutting down cartels, deterring cartel formation, and reducing cartel prices. However, since filing a leniency application may still expose a firm to private law suits there are growing concerns in the law and economics literature that private actions may reduce the incentives to apply for leniency and, hence, diminish the positive deterrence effects of leniency programs. See for example, Cauffman (2011), Buccirossi et al. (2018) or Motchenkova and Spagnolo (2019) for an overview.

In our model we effectively assume that a deviating firm is immune from both penalties and private damages. This is in line with the recent work of Buccirossi et al. (2018). They point out that private damages claims will not reduce the effectiveness of leniency programs, if leniency applicants are optimally protected from follow-on damage claims and instead non-reporting cartel members are liable for damages caused by leniency applicants. Buccirossi et al. (2018) also point out that this more advanced structure has already been adopted in e.g. Hungary.
However, we recognise that this is an area that warrants further analysis. Such an analysis will require consideration of a range of possible collusive and non-collusive strategies involving deviating and/or reporting that will affect payoffs on both sides of the cartel stability condition – see Chen and Rey (2013) for a detailed overview of possible collusive strategies in the presence of leniency programs. This multiplicity of strategies and possible equilibrium solutions complicate the analysis and ideally should be a topic for a separate paper.

5. Concluding Remarks

This article provides a detailed analysis of the effects on both cartel prices and deterrence of combining private damages calculated by each of the two different methods used in practice with two variants of the widely used revenue-based penalty regime, these variants capturing important features of the systems in use. We show that conclusions relating to price are sensitive to both the details of how the penalty applied to cartel revenue is determined, and the method for calculating damages.

One important emerging conclusion is that there seems to be a strong case for using the alternative proportional overcharge method of calculating damages more widely, because it has the computational advantages of the variant of the traditional measure proposed by Brander and Ross (2017), while delivering two further benefits to consumers. First, in comparison with the traditional way of calculating damages it generates a higher level of compensation and so is less susceptible to the charge of under-compensating true harm. Second, it can induce cartels to set a lower price.

While we have carried out a number of robustness checks, extensions to settings of quantity competition, asymmetric firms, differentiated products, and to situations where leniency applications are possible all warrant further investigation.
Appendix

Proof of Result 1:

This result does not depend on the assumptions about the level of the but-for price as $p^b$ does not enter the cartel value expression.

If there were only public simple revenue-based penalties, then $p$ is chosen to maximise

$$(p-c)Q(p) - \beta p R(p) = (p-c)Q(p) - \beta \rho p Q(p).$$

Solution to the FOC, $\hat{p}_R^C$, is characterised by

$$\eta(p) = \frac{p(1-\beta\rho)}{p(1-\beta\rho)-c} = \frac{p}{p-\frac{c}{1-\beta\rho}} > \frac{p}{p-c} \text{ for all } p^b \geq c, \quad (A1)$$

where on the RHS we have the formula characterising the monopoly price-overcharge. Hence, we can conclude that $\hat{p}_R^C > p^M$. Note that for existence of unique interior solution we should have $\frac{p}{p-\frac{c}{1-\beta\rho}} > 0$. Moreover this ratio should be decreasing in $p$. These conditions are satisfied when $1-\beta\rho > 0$ and $p > \frac{c}{1-\beta\rho}$.

Moreover the whole functional relationship in (A1) shifts up and to the right as $\rho$ increases. So the tougher the penalty (the higher the penalty rate) the larger will be the cartel price-overcharge and, hence, also distortion. So we re-established here the results initially obtained in Bageri et al. (2013), Katsoulacos and Ulph (2013) or Katsoulacos et al. (2015).

Proof of Result 2:

The results of this proposition are identical for the setting where the but-for price is equal to the marginal cost and for the case, where $p^b > c$. So, we provide the general proof.

This proof re-establishes the result in Katsoulacos et al. (2019). If there were only public linear sophisticated revenue-based penalties, then $p$ is chosen to maximise

$$(p-c)Q(p) - \beta \sigma \left( \frac{p-p^b}{p^b} \right) p Q(p).$$

Assuming conditions for interior solution are satisfied the solution to the FOC, $\hat{p}_{sr}^C$, is characterised by
\[\eta(p) = \frac{p}{p-c} \left( \frac{p^b - \beta \sigma (2p - p^b)}{p - c} \right) < \frac{p}{p-c} \quad \text{for all } p^b \geq c. \quad (A2)\]

To see that the inequality above holds, note that
\[\frac{p^b - \beta \sigma (2p - p^b)}{p - c} < 1 \quad \text{for all } p^b \geq c \text{ and all} \quad p > p^b.\]

Hence, we can conclude that \( \hat{p}_C^* < p^M \).

Moreover the whole functional relationship in (A2) shifts down and to the left as \( \sigma \) increases. So the tougher the penalty the smaller will be the cartel price-overcharge.

**Proof of Proposition 1:**

We provide the general proof for the case \( p^b \geq c \). The proof for the setting where the but-for price is equal to marginal cost is a special case and is obtained by substituting \( p^b = c \) in the expressions below.

If there were only private damages determined according to the first method, then \( p \) is chosen to maximise
\[ (p-c)Q(p) - \beta \gamma (p - p^b)Q(p). \]

Solution to the FOC, \( \hat{p}_A^C \), is characterised by
\[ \eta(p) = -\frac{pQ'(p)}{Q(p)} = \frac{p(1 - \beta \gamma)}{p(1 - \beta \gamma) - c + \beta \gamma p^b} \leq \frac{p}{p-c} \quad \text{for all } p^b \geq c, \]

where on the right hand side (RHS) of the inequality we have the formula \( \frac{p}{p-c} \) characterising the monopoly price-overcharge in the absence of CA. Note that \( \eta(p) \) is increasing in \( p \) and expression \( \frac{p(1 - \beta \gamma)}{p(1 - \beta \gamma) - c + \beta \gamma p^b} \) is decreasing in \( p \) when \( c \leq p^b < \frac{c}{\beta \gamma} \) and \( 1 - \beta \gamma > 0 \). So there exists unique interior solution to the FOC given by the intersection of the two functions. Note that for existence of unique interior solution we should have \( \frac{p(1 - \beta \gamma)}{p(1 - \beta \gamma) - c + \beta \gamma p^b} > 0 \). Moreover this ratio should be decreasing in \( p \). These conditions are satisfied when \( 1 - \beta \gamma > 0 \) and \( c \leq p^b < \frac{c}{\beta \gamma} \).

Hence, we can conclude that \( \hat{p}_A^C < p^M \) for \( p^b > c \) and \( \hat{p}_A^C = p^M \) for \( p^b = c \), which establishes the result of Proposition 1.
Proof of Proposition 2:

The results of this proposition are identical for the setting where the but-for price is equal to marginal cost and for the case, where \( p^B > c \). So, we provide the general proof.

If there were only private damages determined according to the second method, then \( p \) is chosen to maximise

\[
(p-c)Q(p) - \beta \gamma \left( \frac{p-p^B}{p^B} \right) pQ(p).
\]

Solution to the FOC, \( \hat{p}_p^C \), is characterised by

\[
\eta(p) = \frac{p}{p-c} \left( p^B - \beta \gamma (2p-p^B) \right) < \frac{p}{p-c} \quad \text{for all } p^B \geq c. \tag{A3}
\]

In what follows, we assume that we are operating in the range of parameter values, where interior solutions exist for all combinations of instruments. Note that under the second method of damage calculation the expression \( \frac{p(p^B - \beta \gamma (2p-p^B))}{(p-c)(p^B - \beta \gamma p \frac{p-p^B}{p-c})} \) is greater than zero and is decreasing in \( p \). So there exists unique solution to the FOC given by the intersection of the two functions in (A3).

To see that the second inequality in (A3) holds, note that \( \frac{p^B - \beta \gamma (2p-p^B)}{p^B - \beta \gamma p \frac{p-p^B}{p-c}} < 1 \) for all \( p^B \geq c \), all \( c \) and all \( p \). Hence, we can conclude that \( \hat{p}_p^C < p^M \) for all \( p^B \geq c \).

Proof of Proposition 3:

Here we analyse the interaction of the simple revenue-based penalty and standard measure of recoverable damages in the setting where the but-for price is equal to marginal cost, i.e. \( p^B = c \).

The price is chosen to maximise

\[
(p-c)Q(p) - \beta \rho pQ(p) - \beta \gamma (p-c)Q(p).
\]

Solution, \( \hat{p}_{r,a}^C \), is characterised by

\[
\eta(p) = \frac{p(1 - \beta \rho - \beta \gamma)}{p(1 - \beta \rho - \beta \gamma) - c(1 - \beta \gamma)}. \tag{A4}
\]
Note that when \( 1 - \beta \rho - \beta \gamma > 0 \) the RHS is decreasing in \( p \), while \( \eta(p) \) is increasing in \( p \). So there exists unique solution to the FOC in (A4) given by the intersection of the two functions. If there are no private damages – so \( \gamma = 0 \) - then (A4) becomes:

\[
\eta(p) = \frac{p(1-\beta\rho)}{p(1-\beta\rho) - c},
\]

which is the formula determining cartel overcharge under simple revenue-based penalty regime and, hence, produces \( \hat{p}_R^C > p^M \).

Easy to see that for given parameters \( \beta, \rho, \gamma \) the expression in the RHS of (A4) is always greater than \( \frac{p(1-\beta\rho)}{p(1-\beta\rho) - c} \). Hence, we conclude that when \( p^B = c \), we have \( \hat{P}_{R,A}^C > \hat{p}_R^C > p^M \).

Moreover the whole functional relationship in (A4) shifts up and to the right as \( \gamma \) increases. So the cartel overcharge gets larger the greater the extent to which private damages can be recovered.

**Proof of Proposition 4:**

Here we compare the implications of the simple revenue-based penalty combined with the alternative measure of recoverable damages to the outcome where only a public revenue-based penalty is available and to the outcome in the absence of antitrust enforcement.

The results of this proposition are identical for the setting where the but-for price is equal to marginal cost and for the case, where \( p^B > c \). So, we provide the general proof.

Here \( p \) is chosen to maximise

\[
(p-c)Q(p) - \beta \rho p Q(p) - \beta \gamma \left( p - \frac{p^B}{p^B} \right) p Q(p).
\]

Assuming the conditions for interior solution are satisfied, solution, \( \hat{P}_{R,p}^C \), is characterised by

\[
\eta(p) = \frac{p \left( p^B (1-\beta \rho + \beta \gamma) - 2p \beta \gamma \right)}{(p-c) \beta \rho p^B \beta \rho p^B - \beta \gamma (p - p^B)p}. \tag{A5}
\]

If there are no private damages – so \( \gamma = 0 \) - then, once again, (A5) becomes:

\[
\eta(p) = \frac{p(1-\beta \rho)}{p(1-\beta \rho) - c},
\]

which is the expression (A1) determining cartel overcharge under the simple revenue-based penalty regime and, hence, produces \( \hat{p}_R^C > p^M \).
Note that the RHS of (A5) is smaller than \( \frac{p(1-\beta_p)}{p(1-\beta_p)-c} \) in (A1) when \( p > c / (1-\sqrt{\beta_p}) \). Also note that \( c / (1-\sqrt{\beta_p}) > c / (1-\beta_p) \) for all \( \beta_p < 1 \). So when condition for the existence of interior solution in (A1) is satisfied and \( p > c / (1-\sqrt{\beta_p}) \) the following rankings hold: \( \hat{p}_{R,p}^C < \hat{p}_R^C < \hat{p}_{R,a}^C \) and switching to the alternative structure with proportional overcharge can improve the outcome.

Next, note that the RHS of (A5) is smaller than \( \frac{p}{p-c} \) when \( p > c \left( \sqrt{\rho/\gamma} + 1 \right) \). Then combining this and the inequality derived above we can conclude that for \( p > c \left( \sqrt{\rho/\gamma} + 1 \right) > c / (1-\sqrt{\beta_p}) \) the following rankings hold: \( \hat{p}_{R,p}^C < \hat{p}_R^C < \hat{p}_{R,a}^C \) and switching to the alternative structure with proportional overcharge can reduce the prices below the simple monopoly level. Furthermore, for \( \hat{p}_{R,p}^C < p^M \) to hold we need the inequality above to hold at monopoly price, i.e. \( p^M > c \left( \sqrt{\rho/\gamma} + 1 \right) \).

Note that the above inequality can be rewritten in terms of the proportional overcharge as \( \left( p^M - c \right) / c > \sqrt{\rho/\gamma} \). This brings out more clearly the intuition behind Proposition 4. Since the left hand side is the endogenously determined cartel overcharge and can take any value, in principle. To make this inequality hold for a larger range of possible cartel overcharges we need to have sufficiently low right hand side. This can be achieved when \( \gamma \), the fraction of expected recoverable damages, is sufficiently large and \( \rho \) is on the contrary kept sufficiently low.

**Proof of Proposition 5:**

Again, we present the proof of this proposition for the general setting where \( p^B \geq c \). Conclusions for homogeneous products case are obtain by substituting \( c \) for \( p^B \) and will be identical to the conclusions obtained in more general setting.

In order to state the results of this proposition we have to compare two cases:

1. In the case of the linear sophisticated revenue-based penalty combined with the standard measure of recoverable damages, \( p \) is chosen to maximise

   \[
   (p-c)Q(p) - \beta\sigma \left( \frac{p-p^B}{p^B} \right) pQ(p) - \beta\gamma \left( p-p^B \right) Q(p).
   \]

   Solution, \( \hat{p}_{SR,a}^C \), is characterised by
\[ \eta(p) = \frac{p}{p-c} \left( \frac{p^g - \beta \gamma p^g - \beta \sigma (2p - p^u)}{p^g - \beta \sigma p \frac{p-p^g}{p-c} - \beta \gamma p^g \frac{p-p^g}{p-c}} \right). \]  

(A6)

If there are no private damages – so \( \gamma = 0 \) - then (A6) becomes:

\[ \eta(p) = \frac{p}{p-c} \left( \frac{p^g - \beta \sigma (2p - p^g)}{p^g - \beta \sigma p \frac{p-p^g}{p-c}} \right), \]

which is the formula determining cartel overcharge under linear sophisticated revenue-based penalty regime given in (A2) and produces \( \hat{p}_{SR}^C < p^M \). Also note that

\[ \frac{p^g - \beta \gamma p^g - \beta \sigma (2p - p^u)}{p^g - \beta \sigma p \frac{p-p^g}{p-c} - \beta \gamma p^g \frac{p-p^g}{p-c}} < \frac{p^g - \beta \sigma (2p - p^g)}{p^g - \beta \sigma p \frac{p-p^g}{p-c}} < 1 \]

for all \( p^g \geq c \) and all \( p > p^g \). Hence, we can conclude that \( \hat{p}_{SR,A}^C < \hat{p}_{SR}^C < p^M \).

2. In the case of the linear sophisticated revenue-based penalty combined with the alternative measure of recoverable damages, \( p \) is chosen to maximise

\[ (p-c)Q(p) - \beta \sigma \left( \frac{p-p^g}{p^g} \right) pQ(p) - \beta \gamma \left( \frac{p-p^g}{p^g} \right) pQ(p). \]

By analogy to Proposition 2, solution, \( \hat{p}_{SR,P}^C \), is characterised by

\[ \eta(p) = \frac{p}{p-c} \left( \frac{p^g + p^g \beta (\gamma + \sigma) - 2p \beta (\gamma + \sigma)}{p^g - \beta (\gamma + \sigma) p \frac{p-p^g}{p-c}} \right). \]  

(A7)

Also note that \( \frac{p^g + p^g \beta (\gamma + \sigma) - 2p \beta (\gamma + \sigma)}{p^g - \beta (\gamma + \sigma) p \frac{p-p^g}{p-c}} < 1 \) for all \( p^g, c, \) and \( p \). Hence, following the logic of Proposition 2 we can conclude that \( \hat{p}_{SR,P}^C < p^M \).

If there are no private damages – so \( \gamma = 0 \) - then once again (A7) becomes:

\[ \eta(p) = \frac{p}{p-c} \left( \frac{p^g - \beta \sigma (2p - p^g)}{p^g - \beta \sigma p \frac{p-p^g}{p-c}} \right), \]

which is the formula determining cartel overcharge under linear sophisticated revenue-based penalty regime and produces \( \hat{p}_{SR}^C < p^M \).

Notice also that the expression on the RHS of (A7) is strictly decreasing in \( \gamma \), which implies that \( \hat{p}_{SR,P}^C < \hat{p}_{SR}^C < p^M \).
Finally, comparison of expressions (A6) and (A7) implies that the locus on the RHS in (A6) will always be above the locus on the RHS in (A7), and, hence, we can conclude that $\hat{p}_{SR,P}^C < \hat{p}_{SR,A}^C < \hat{p}_{SR}^C < p^M$. Figure 1 illustrates the proofs.

![Figure 1: Unconstrained cartel overcharges for various combinations of revenue-based penalty regimes and methods for calculating recoverable damages](image)

**Detailed Derivations of Expressions in (16) and Proof of Proposition 6:**

In order to state the results of this proposition we have to compare six cases: $R; R,A; R,P; SR; SR,A; SR,P$.

1. **Case R**

To derive $\Delta R$ and $D_R$ we define $Y(z) = \max \pi(p) - zR(p)$ and take a first-order Taylor approximation to $Y(\beta \rho)$ around 0 and, bearing in mind that (i) by the Envelope Theorem $Y'(z) = -R(\hat{p}(z))$ where $\hat{p}(z)$ is the price that maximises $Y(z)$; (ii) when $z = 0$, $\hat{p}(0) = p^M$ we have $Y(\beta \rho) \approx Y(0) - \beta \rho R(p^M) = (p^M - c)Q(p^M) - \beta \rho p^M Q(p^M)$.

So $D_R = 1 - \frac{Y(\beta \rho)}{Y(0)} = 1 - \left\{ \frac{(p^M-c)Q(p^M) - \beta \rho p^M Q(p^M)}{(p^M-c)Q(p^M)} \right\} = \beta \rho \frac{p^M}{(p^M-c)}$.

2. **Case R,A**

To derive $\Delta_{R,A}$ and $D_{R,A}$ we take a first-order Taylor approximation to $Y\left(\beta \rho + \beta \gamma \frac{p-c}{p}\right)$ around 0. We get

$Y\left(\beta \rho + \beta \gamma \frac{p-c}{p}\right) \approx Y(0) - \left(\beta \rho + \beta \gamma \frac{p-c}{p}\right)R(p^M) = (p^M-c)Q(p^M) - \left(\beta \rho + \beta \gamma \frac{p-c}{p}\right)p^M Q(p^M)$.
3. Case R, P

When \( \hat{\beta}_c p_{R,P} > p^M \), in order to derive \( \bar{\Lambda}_{R,P} \) and \( D_{R,P} \) we take a first-order Taylor approximation to \( Y \left( \beta \rho + \beta \gamma \frac{p - c}{p} \right) \) around 0. We get

\[
Y \left( \beta \rho + \beta \gamma \frac{p - c}{c} \right) \approx Y(0) - \left( \beta \rho + \beta \gamma \frac{p - c}{c} \right) R(p^M) = (p^M - c) Q(p^M) - \left( \beta \rho + \beta \gamma \frac{p - c}{c} \right) p^M Q(p^M).
\]

So

\[
D_{R,P} = 1 - \frac{Y \left( \beta \rho + \beta \gamma \frac{p - c}{c} \right)}{Y(0)} = 1 - \left[ 1 - \left( \beta \rho + \beta \gamma \frac{p - c}{c} \right) \frac{p^M Q(p^M)}{(p^M - c) Q(p^M)} \right] = \beta \rho \frac{p^M}{(p^M - c)} + \beta \gamma.
\]

Note that this expression for \( D_{R,P} \) holds when \( \hat{\beta}_c p_{R,P} > p^M \). Next, we show that when \( \hat{\beta}_c p_{R,P} \leq p^M \) \( D_{R,P} \) will have slightly different structure. However, the results of comparison are similar and \( D_{R,P} > D_R \) under condition of Proposition 4.

When \( \hat{\beta}_c p_{R,P} \leq p^M \), in order to derive \( \bar{\Lambda}_{R,P} \) and \( D'_{R,P} \) we use stability condition in (10). This gives

\[
V(p) = \frac{(p - c) Q(p) - \beta \rho p Q(p) - \beta \gamma \frac{p - c}{c} p Q(p)}{\Delta} \geq (p - c) Q(p) = \pi^d(\theta).
\]

Simplifying this expression we get \( \bar{\Lambda}_{R,P} = 1 - \beta \rho \frac{p}{p - c} - \beta \gamma \frac{p}{c} \).

Hence, \( D'_{R,P}(p) = \beta \rho \frac{p}{p - c} + \beta \gamma \frac{p}{c} \). It is easy to see that for any \( c \leq p \leq p^M \) function \( D'_{R,P}(p) \) is concave, reaches its minimum at \( p = c \left( \sqrt{\rho / \gamma} + 1 \right) \) and is always above \( D_R(p) = \beta \rho \frac{p}{p - c} \) locus (including at \( p = p^M \)) for \( 0 < \gamma \leq 1 \). Hence, we can conclude that \( D'_{R,P} > D_R \).

4. Case SR

To derive \( \bar{\Lambda}_{SR} \) and \( D_{SR} \) we use stability condition in (10). This gives

\[
V(p) = \frac{(p - c) Q(p) - \beta \sigma \frac{p - c}{c} p Q(p)}{\Delta} \geq (p - c) Q(p) = \pi^d(\theta).
\]

Recall \( \bar{\Lambda}_{SR} \) is the value at which constrained overcharge is driven to 0. This implies that when \( p \to c \), we have \( \bar{\Lambda}_{SR} = 1 - \beta \sigma \) and \( D_{SR} = \beta \sigma \).
5. Case SR,A

Similarly, to derive $\overline{A}_{SR,A}$ and $D_{SR,A}$ we rewrite the stability condition in (10) as follows

$$V(p) = \frac{(p-c)Q(p) - \left(\beta\sigma\frac{p-c}{c} + \beta\gamma\frac{p-c}{p}\right)pQ(p)}{\Delta} \geq (p-c)Q(p).$$

This implies that when $p \to c$, we have $\overline{A}_{SR,A} = 1 - \beta\sigma - \beta\gamma$ and $D_{SR,A} = \beta\sigma + \beta\gamma$.

6. Case SR,P

Finally, to derive $\overline{A}_{SR,P}$ and $D_{SR,P}$ we rewrite the stability condition in (10) as follows

$$V(p) = \frac{(p-c)Q(p) - \left(\beta\sigma\frac{p-c}{c} + \beta\gamma\frac{p-c}{p}\right)pQ(p)}{\Delta} \geq (p-c)Q(p).$$

This implies that when $p \to c$, we have $\overline{A}_{SR,P} = 1 - \beta\sigma - \beta\gamma$ and $D_{SR,P} = \beta\sigma + \beta\gamma$.

**Proof of Proposition 7:**

Here we analyse the interaction of the simple revenue-based penalty and the standard measure of recoverable damages in the setting where the but-for price is greater than marginal cost, i.e. $p^B > c$.

The price is chosen to maximise

$$(p-c)Q(p) - \beta\rho pQ(p) - \beta\gamma(p-p^B)Q(p).$$

Solution, $\hat{p}_{R,A}$, is characterised by

$$\eta(p) = \frac{p(1-\beta\rho-\beta\gamma)}{p(1-\beta\rho-\beta\gamma)-c+\beta\gamma p^B}. \quad (A8)$$

Note that when $1 - \beta\rho - \beta\gamma > 0$ the RHS is positive and decreasing in $p$ for $c \leq p^B < \frac{c}{\beta\gamma}$, while $\eta(p)$ is increasing. So there exists unique solution to the FOC in (A8) given by the intersection of the two functions.

If there were no private damages – so $\gamma = 0$ - then (A8) becomes:

$$\eta(p) = \frac{p(1-\beta\rho)}{p(1-\beta\rho)-c},$$

which as before is the formula determining cartel overcharge under simple revenue-based penalty regime and, hence, produces $\hat{p}_{R} > p^{M}$. 

32
Easy to see that for given parameters $\beta, \rho, \gamma$ the expression in the RHS of (A8) is greater than $\frac{\rho(1 - \beta) - \rho}{\rho(1 - \beta) - \rho - c}$ when $c \leq p^b < \frac{c}{1 - \beta \rho}$. While it is smaller than $\frac{\rho(1 - \beta) - \rho}{\rho(1 - \beta) - \rho - c}$ for $\frac{c}{1 - \beta \rho} < p^b$.

Hence, we can conclude that for a wide range of parameter values we have $\hat{p}^C_{R,A} > \hat{p}^C_R > p^M$, which shows that conclusions of Proposition 3 can extend to the more general setting with but-for prices greater than marginal costs. To be more specific the complete analysis of all the possible cases implies the following four sets of inequalities:

1. $\hat{p}^C_A = p^M < \hat{p}^C_R < \hat{p}^C_{R,A}$ for $p^b = c$,
2. $\hat{p}^C_A < p^M < \hat{p}^C_R < \hat{p}^C_{R,A}$ for $c < p^b < \frac{c}{1 - \beta \rho}$,
3. $\hat{p}^C_A < p^M < \hat{p}^C_R < p^C$ for $\frac{c}{1 - \beta \rho} < p^b < \frac{c(\gamma + \rho)}{\gamma}$,
4. $\hat{p}^C_A < p^M < \hat{p}^C_{R,A} < \hat{p}^C_R$ for $\frac{c(\gamma + \rho)}{\gamma} < p^b < \frac{c}{\gamma \beta}$.

The first set of inequalities follows from comparison of (A4) and (A1) in Proposition 3, Result 1 and Proposition 1.

The next set of inequalities follows from comparison of (A8) and (A1), Result 1, and Proposition 1.

The last two sets of inequalities follow from comparison of (A8) to the formula characterising simple monopoly price-overcharge $\frac{p}{p - c}$ and Result 1.
References


CMA (73, 18 April 2018), *CMA’s guidance as to the appropriate amount of a penalty.*


