Practical Techniques for the Determination of Minor Planet Orbits

ABSTRACT

This thesis is concerned with the practice of orbit determination for minor planets. All aspects of the work are dealt with, but particular emphasis is placed on computational procedures and a new library of FORTRAN IV computer programs designed specifically for minor planet work is described.

The library, which is denoted by the generic name ORBIT, was developed primarily to meet the requirements of investigators working in the field of minor planet orbits at St Andrews. It is restricted to the solution of the two-body problem, no account having been taken of perturbations, and it would be particularly suitable for use in the initial determination of the orbit of a newly-discovered planet.

The thesis falls naturally into two parts, the first being primarily concerned with the ORBIT programs themselves and the second describing the non-computational procedures involved. These latter procedures are presented by means of an example, a detailed account being given of a determination of the orbit of the minor planet 16 Psyche from original observations.

After the Introduction, which gives a brief account of the work in its historical context, Part 1 of the thesis begins by outlining the general programming considerations observed in the development of the ORBIT library. Some basic subprograms are also described here. The subsequent chapters of Part 1 then describe in turn each of the computer solutions in the library, namely (i) the reduction of plate measurements; (ii) the determination of a preliminary orbit; (iii) the computation of an ephemeris and (iv) the improvement of the orbit.

The second part of the thesis begins by describing the series of 54 photographic observations of 16 Psyche made during its apparition of 1970–71, and this is followed by an account of the plate measurement procedures used. Part 2 ends with a discussion of the computer reduction

of the observations which not only attempts to evaluate the overall effectiveness of the various techniques in practice, but also to examine the effects of planetary perturbations on the orbit of 16 Psyche. Finally, the Conclusion suggests various improvements which could be made to the ORBIT programs and some possible future developments.

Practical Techniques for the Determination of Minor Planet Orbits

by

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A Thesis presented for the Degree of Master of Science

in the University of St. Andrews

June 1975

DECLARATION

I hereby declare that the following Thesis is the result of work carried out by me, that the Thesis is my own composition, and that it has not previously been presented for a Higher Degree. The research was carried out at the University Observatory, St. Andrews.

F. G. Watson.

F G Watson

CERTIFICATE

I hereby certify that Frederick G Watson has spent seven terms at research work in the University Observatory, St. Andrews, that he has fulfilled the conditions of Ordinance No. 51 (St. Andrews) and that he is qualified to submit the accompanying Thesis in application for the Degree of Master of Science.

T. Slebarski

T B Slebarski

PREFACE

This thesis is concerned with practical techniques developed for the determination of the orbits of minor planets. Particular emphasis is placed on computational procedures and a new library of FORTRAN IV computer programs designed specifically for the reduction of minor planet observations is described.

The library, which is denoted by the generic name ORBIT, was developed primarily to meet the requirements of investigators working in the field of minor planet orbits at St. Andrews. It is restricted to the solution of the two-body problem , no account having been taken of perturbations, and it is particularly suitable for use in the initial determination of the orbit of a newly-discovered planet.

The thesis falls naturally into two parts, the first being primarily concerned with the ORBIT programs themselves and the second describing the non-computational procedures involved in the observation of minor planets, measurement of photographic plates and reduction of data. These latter techniques are presented by means of an example, a detailed account being given of a determination of the orbit of the minor planet 16 Psyche from original observations. This section also serves to illustrate the use of the ORBIT library in practice.

In writing this thesis I should like to record my indebtedness to my supervisor, Mr T B Slebarski, whose knowledge and experience in minor planet work was a constant source of inspiration and whose helpful advice and guidance were always freely given, and to Professor D W N Stibbs, Director of the University Observatory, without whose help the work would have been impossible. Thanks are also due to the Teaching Staff and Research Students of the University Observatory for helpful discussions (particularly to Dr D Kilkenny for the use of his manual computation of the orbit of 1361 Leuschneria), to the Technical Staff for their ready assistance and to the Staff of the University Computing Laboratory for their cooperation throughout.

It is also a pleasure to thank Dr A T Sinclair of the Royal Greenwich Observatory (who provided the values of theoretical perturbations on the orbit of 16 Psyche) for many constructive suggestions. I am grateful to Miss S Bowen, Mrs G Chubb, Miss J Gaydon, Miss L Harris and Mrs V Randall who typed the thesis.

Finally, I should like to thank my wife Carol for her continual and unselfish support throughout the period of the work, especially as no external financial aid was available.

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INTRODUCTION

The discovery of the first minor planet on 1801 January 1 presented the mathematicians of the time with a problem for which there had hitherto been no need for a solution; that is, the determination of the orbit of a body moving in an ellipse about the sun from a limited number of observations over a short arc, and with sufficient precision for the body to be reacquired at its next apparition. Previous orbit determination methods had been concerned with comet orbits (for which the eccentricity is often essentially unity thus simplifying the problem) and the general case where no assumption is made with regard to the eccentricity had not been attempted with anything other than very low precision. The solution of the immediate problem, the reacquisition of 1 Ceres after its "first" conjunction with the sun, was dramatically provided by Gauss and it is remarkable that his work carried out at the very dawn of minor planet orbit studies should continue to dominate the field even until recent times.

Gauss's techniques for minor planet orbit determination were improved upon by other astronomers throughout the last century, and the introduction of mechanical calculators in the early part of this century led to further refinements. It is on these foundations that the present work is based, for we have now carried the development a step further by the application to the problem of the high-speed electronic digital computer, whose widespread introduction in recent years has revolutionized virtually all branches of science. Unlike some modern techniques in the field of orbit determination, the work to be described here is based firmly on traditional methods and only minor improvements have been made in adapting them for computer reduction. These have been mainly concerned with ensuring that the computations proceed correctly under automatic control.

The primary aim behind the work has been to provide a convenient package of computer programs (the ORBIT library) with some associated practical techniques which together may be used to provide rapid solutions in the orbit determination process for minor planets. Of course, there are many different reduction methods available, each one suited to particular conditions, but for the problems dealt with by the ORBIT library we have endeavoured to use the most widely applicable methods.

The development of a suite of programs with these aims in mind is, of course, by no means unique, the tendency being for individual establishments to develop their own. This is particularly true of institutions which specialize in minor planet studies such as the computing centres at Cincinnati and Leningrad. We would also mention the work of Sitarski and Ziołkowski (1969) at the Polish Academy of Sciences in the development of a library of ALGOL programs intended primarily for the investigation of comet orbits. These programs differ from the ORBIT programs in that planetary perturbations are taken into account; nevertheless, there are certain areas of overlap where interesting comparisons are possible. For example, we remark on the contrast in approach between the ALGOL procedure KEPLER (Sitarski (1968)) and the subroutine KEPLER described in Chapter 1.1 of the present work, both of which have the same function.

Since there are varying degrees of determinateness of an orbit it is necessary to define at this point what we mean by each of the stages in the solution of the orbit determination problem (excluding for the moment the fundamentally separate process of the reduction of plate measurements). The definition of these stages which we adopt is that generally used, the stages being summarised by Dubyago (1961, p.3) as

"1. The determination of the elements of a preliminary orbit, without taking into consideration the perturbations; in other words, the solution of the problem of two bodies. For this purpose, three and occasionally four observations of the body under investigation are taken, covering an interval of several days or weeks or, under special circumstances only, of several months. It is desirable to obtain the preliminary orbit of a planet or a comet as soon as possible after discovery to permit a quicker preduction of its future positions for further observations.

"2. The improvement of a preliminary orbit, which is done after the accumulation of a longer series of observations. It is also possible in the majority of calculations of improved orbits to neglect perturbations.

"3. Computation of the definitive orbit. The definitive orbit is the one which agrees, in the most probable manner, with all the obtained observations .... In the determination of a definitive orbit, it is almost always necessary to take into account the perturbations, at least those that are due to the principal planets."

As we have already implied, no attempt is made in the present work to undertake stage (3) above, the solutions given being restricted solely to the problem of two bodies. We would remark that stages (1) and (2) are retained as separate entities in the work as they have traditionally been in the past; the modern trend seems to be towards the use of a large number of observations in the determination of the preliminary orbit, thus effectively combining (1) and (2) (see, for example, Herget (1965)). This trend is clearly due to the increased volume of calculation which can now be undertaken with computers.

In the present work, computer solutions are given for the following problems: (i) the reduction of plate measurements; (ii) the determination of a preliminary orbit; (iii) the computation of an ephemeris and (iv) the improvement of the orbit. Details of these are given in Part 1 of the Thesis. In order to avoid the need for detailed descriptions of the purely mechanical aspects of each of these programs, the first chapter gives a brief account of the general programming considerations observed in their development. Some basic subprograms are also described here. The subsequent chapters then describe in turn each of the computer solutions in the ORBIT library, these chapters being to some extent self-contained so as to form working guides to the programs. Listings of the main programs are given at the end of their respective chapters, those of the subprograms being inserted at the appropriate points in the text.

The second part of the Thesis gives an account of a determination of the orbit of the minor planet 16 Psyche in order to demonstrate the use of the programs as a unified computational package. Descriptions of the observation of the planet and plate measurement procedures are included and considerable detail is given since the practical and computational procedures are very closely bound together, being merely different aspects of the same process.

The nomenclature and notation employed throughout the work is, in general, the same as that to be found in the literature and considerable use is made of vector notation. Finally, it should be noted that the convention is followed of true, eccentric and mean anomalies being measured from 0 to 2π radians rather than the common alternative of $-\pi$ to $+\pi$.

Part 1

REDUCTION METHODS AND COMPUTER PROGRAMS

Chapter 1.1 General Considerations and Basic Subprograms

We begin our account of the computer programs which constitute the ORBIT library with an outline of some of the programming considerations underlying the work together with some of the features which are common throughout. The programs are all coded in the FORTRAN IV language as defined and described in IBM (1968c) for execution under the IBM System/360 Model 44 Programming System (see Chapter 2.3 of this Thesis). Input of data is by card reader (data set reference number 5) and output is by line-printer (data set reference number 6). The output formats are designed for use with a line-width of 130 characters and a page-length of 44 lines.

One of the primary aims in the design of the programs has been to minimise execution times as far as possible by avoiding unnecessary computation. For example, a calculation is generally performed with a specific data value no more than once even if this has necessitated the introduction of a new symbolic name as an intermediate variable. Thus economy of execution time has taken some precedence over economy of core storage, although due regard to the latter has, of course, been paid. In general, the programming considerations outlined by IBM (1968d, pp68 - 71) have been borne in mind.

In view of the nature of the problems dealt with by the programs every effort has been made to preserve a high degree of precision throughout and because of the large amount of core storage available it has been possible to use double-length for all real variables. This has kept to a minimum the cumulative errors to which long programs are especially susceptible, and it has meant that the accuracy of the results is limited not by the computer so much as the reduction methods used to obtain them. In some cases, such as in iterative procedures, it is clearly possible to exercise some control over the precision of the results, but in general the accuracy of the reduction methods tends to be dependent on the validity of approximations made within them (particularly in relation to the data being used). It is thus very difficult to place theoretical limits on the probable overall accuracy of the programs and no attempt has been made to do this. Assessments of the performance of the programs in practice can be obtained, however, and this is discussed both in the descriptions of the individual reduction methods and in the analysis in Part 2 of this Thesis. We may remark at this point that in the interests of straightforward programming the output data formats are generally standardised as regards precision throughout the library, and so no conclusion as to the accuracy of a given set of results should be drawn from the number of significant figures to which the results are printed out.

With regard to the actual coding of the programs, normal FORTRAN procedures have been adopted for the most part. One minor exception to this has been in the naming of variables within the programs, no initial letter type-convention having been followed for real variables (the variable types being explicitly declared) although the normal convention has been followed for integer variables. Concerning symbolic names in general, we remark that most have an obvious interpretation and we do not list the meaning of all names used in the programs. Many variable names refer to the same quantity throughout the library, for example:

CONV	=	number of seconds of arc in one radian;
TCONV	=	number of seconds of time in one radian;
DCONV	=	number of seconds of time in one day;
IDENTN(6)	=	identification or name of an object as 24 alphameric characters;
IRDATE	=	date of reduction in integer form;
IDEHO	=	integral degrees or hours;
IMIN	=	integral minutes;
SEC	=	remaining seconds,

the latter three variables being used whenever an angular quantity or time is being read into or written out of a program. The naming of the ORBIT programs themselves also follows fairly obvious lines except that the four main programs to be described are named ORBIT 1 (preliminary orbit determination), ORBIT 2 (reduction of plate measurements), ORBIT 3 (ephemeris generation) and ORBIT 4 (orbit improvement).

The layout of the programs again follows normal conventions for the most part, but we may make one or two remarks concerning the arrangement of branching. The nature of the problems dealt with by the ORBIT library is such that the testing of values is continually taking place to ensure that no anomalous condition has arisen, and because of the many places in each of the main programs where such a condition could cause termination of execution it is desirable to have error messages to distinguish one possible termination point from another. These are introduced by reversing the normal logic of the test situation as shown in Fig. 1(a) where, instead of testing for the presence of an error condition, we test for its absence so that an error message can be easily introduced. (The statement GO TO 1000 in all the main programs is synonymous with STOP, statement 1000 being merely a write instruction causing a page-throw at the end of the output.) This technique is carried over into the situation where such a branch occurs in a subroutine, necessitating the use of multiple returns. The ordinary RETURN statement is used when an error condition has arisen, whilst RETURN1 is used for normal exit from the subroutine. Iterative procedures are used frequently in the ORBIT library and their layout is invariably as shown in Fig. 1(b), where again the arrangement of the logic allows for the inclusion of an error message in the appropriate branch. These procedures are often written as subroutines, multiple returns being used as above. The error messages themselves are then written from the main program since all ORBIT subprograms have been kept free of read/write statements.

In writing the ORBIT library it was found convenient to deal with certain routine computations by packaging them as subprograms in such a way as to enable them to be readily inserted whenever they were required. These

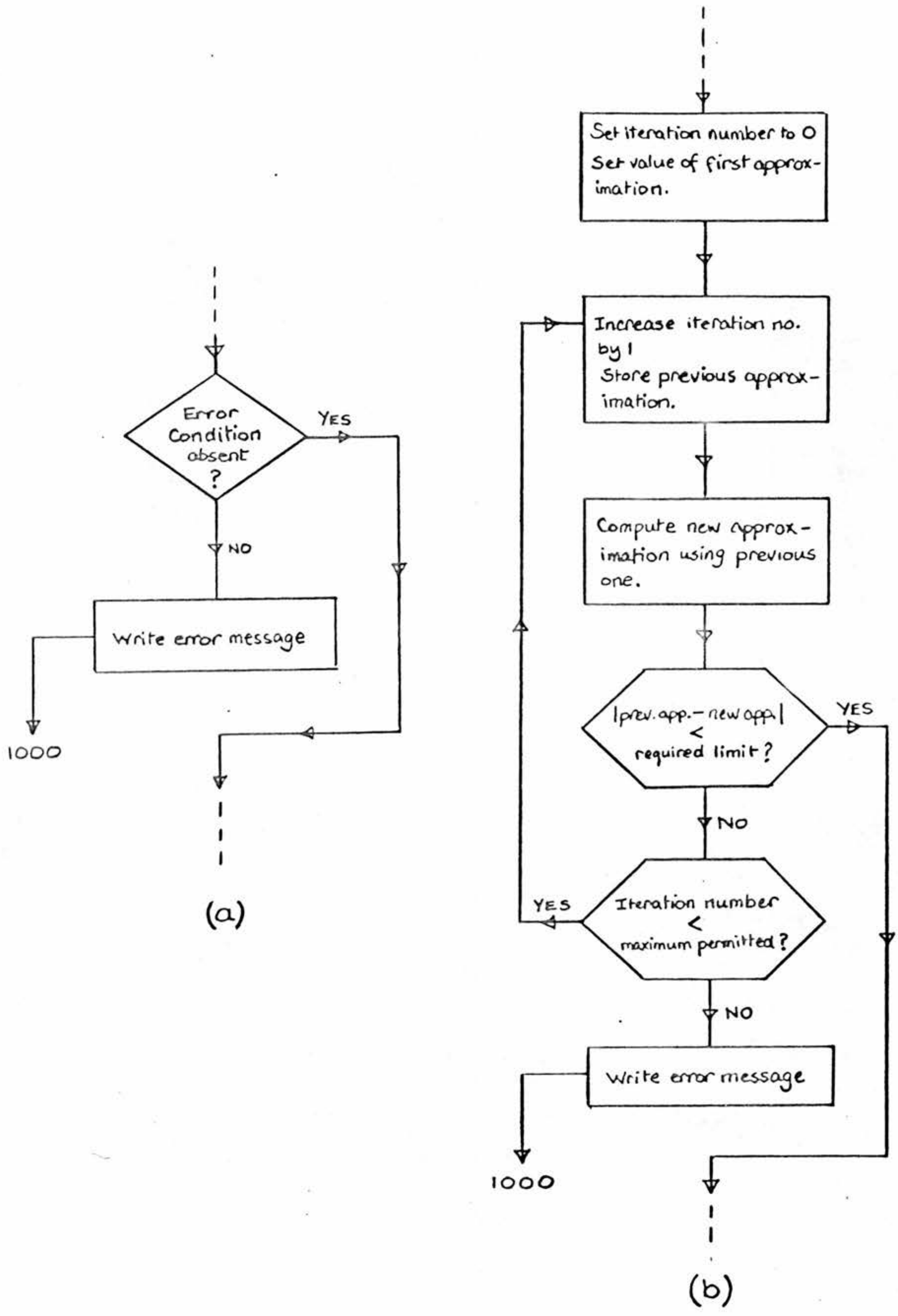

Error Condition absent ?
YES
NO
Write error message
1000
(a)
Set iteration number to 0
Set value of first approx-imation.
Increase iteration no. by 1
Store previous approx-imation.
Compute new approx-imation using previous one.
|prev. app. − new app.| < required limit ?
YES
NO
Iteration number < maximum permitted ?
YES
NO
Write error message
1000
(b)

Fig. 1

basic subprograms are thus of a fairly general nature in their application (so far as the present work is concerned) and the remainder of this chapter is given over to a brief description of each of them. Subprograms which are not basic in the sense that they are applicable in only one particular situation are described within the context for which they were written.

DICOS

From the R.A. and Dec. (α, δ in radians) of an object, DICOS computes the direction cosines λ, μ, ν of the object using the geometrical relationships

$$\lambda = \cos\delta \cos\alpha$$
$$\mu = \cos\delta \sin\alpha$$
$$\nu = \sin\delta$$

DICOS also returns the sum of the squares of the direction cosines (which should equal exactly unity) for checking purposes.

COVECT, SCALAR

These two FUNCTION subprograms provide respectively the vector and scalar products of two vectors. If the two vectors are denoted by $\underline{a} \equiv (a_x, a_y, a_z)$ and $\underline{b} \equiv (b_x, b_y, b_z)$ then

$$\underline{a} \times \underline{b} \equiv (a_y b_z - a_z b_y,\ a_z b_x - a_x b_z,\ a_x b_y - a_y b_x)$$

and $$\underline{a}.\underline{b} = a_x b_x + a_y b_y + a_z b_z.$$

COVECT gives one of the three components of $\underline{a} \times \underline{b}$; thus by using COVECT three times with the appropriate components of $\underline{a}$ and $\underline{b}$ as arguments, all the components of $\underline{a} \times \underline{b}$ can be obtained. SCALAR gives $\underline{a}.\underline{b}$ directly.

We remark that COVECT and SCALAR are often used for evaluating expressions

```
      SUBROUTINE DICOS(RA,DE,LAMBDA,MU,NU,SUMSQ)
C
C     COMPUTATION OF THE DIRECTION COSINES OF A CELESTIAL OBJECT
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, MARCH 1971
C
      REAL*8 RA,DE,LAMBDA,MU,NU,SUMSQ,DCOS,DSIN
C
      LAMBDA=DCOS(DE)*DCOS(RA)
      MU=DCOS(DE)*DSIN(RA)
      NU=DSIN(DE)
C
      SUMSQ=LAMBDA*LAMBDA+MU*MU+NU*NU
C
      RETURN
      END
```

```
      REAL FUNCTION COVECT*8(AY,AZ,BY,BZ)
C
C     X (OR Y OR Z) COMPONENT OF THE VECTOR PRODUCT OF TWO VECTORS
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, MARCH 1971
C
      REAL*8 AY,AZ,BY,BZ
      COVECT=AY*BZ-AZ*BY
C
      RETURN
      END
```

```
      REAL FUNCTION SCALAR*8(AX,AY,AZ,BX,BY,BZ)
C
C     SCALAR PRODUCT OF TWO VECTORS
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, MARCH 1971
C
      REAL*8 AX,AY,AZ,BX,BY,BZ
      SCALAR=AX*BX+AY*BY+AZ*BZ
C
      RETURN
      END
```

of the above form outside the vector context (i.e. when the arguments are not vector components).

ANGLE

It is convenient to describe the ANGLE group of subprograms as a whole since their functions are generally related. As their name implies, they are designed to deal as conveniently as possible with the handling of angular data, particularly where this is being input to or output from a main program.

ANGLE1, ANGLE2, ANGLE4 and ANGLE6 deal solely with this input/output aspect. ANGLE1 is a conversion program to change angles (or times) expressed in degrees (or hours), minutes and seconds into the "internal units" used in the programs, usually radians for angles, and radians or days for times. The subprogram first expresses the angle in seconds and then converts this to the units required by dividing it by FACTOR, the number of seconds in each required unit (eg the number of arc seconds in one radian). ANGLE2 performs the inverse operation of ANGLE1. When angles which may take negative values are involved (such as the declination of an object) further steps have to be taken since ANGLE1 and ANGLE2 are invalid for negative values. Furthermore, since positive numbers have their signs suppressed on print-out, it is desirable to hold the sign as a separate alphameric quantity when printing out signed angles. ANGLE4 and ANGLE6 deal with the signs, ANGLE4 combining a positive quantity and an alphameric sign to make a positive or negative quantity, and ANGLE6 performing the inverse operation of separating a quantity into an alphameric sign and a positive number. (ANGLE4 is a multiple return subprogram arranged so that an error message can be given if the alphameric sign read in is anything other than plus, blank or minus.)

When angles in the range 0 to 2π radians (such as the right ascension of an object) are being dealt with in a program (and in particular when they

```
      REAL FUNCTION ANGLE1*8(FACTOR,IDEHO,IMIN,SEC)
C
C     POSITIVE ANGLE OR TIME IN DEGREES, MINS, SECS OR HOURS, MINS, SECS
C     EXPRESSED IN INTERNAL UNITS
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, JULY 1971
C
      REAL*8 FACTOR,SEC,SECOND,DFLOAT
      SECOND=SEC+DFLOAT(60*(IMIN+60*IDEHO))
      ANGLE1=SECOND/FACTOR
C
      RETURN
      END



      SUBROUTINE ANGLE2(FACTOR,ANGLE,IDEHO,IMIN,SEC)
C
C     CONVERSION OF POSITIVE ANGLE OR TIME FROM INTERNAL UNITS
C     INTO DEGREES, MINS, SECS OR HOURS, MINS, SECS
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, JULY 1971
C
      REAL*8 FACTOR,ANGLE,SECOND,SEC,DFLOAT
      SECOND=ANGLE*FACTOR
C
      INTMIN=IDINT(SECOND/60.0D0)
      IDEHO=INTMIN/60
      IMIN=INTMIN-60*IDEHO
      SEC=SECOND-DFLOAT(60*INTMIN)
C
      RETURN
      END



      REAL FUNCTION ANGLE3*8(ANGLE)
C
C     ANGLE IN RADIANS EXPRESSED AS ANGLE MODULO +2PI
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, JULY 1971
C
      REAL*8 ANGLE,TWOPI,DMOD
      TWOPI=6.283185307179586476 9
C
      ANGLE=DMOD(ANGLE,TWOPI)
C
      IF(ANGLE.GE.0.0D0)GO TO 10
      ANGLE=ANGLE+TWOPI
C
   10 ANGLE3=ANGLE
C
      RETURN
      END
```

```
      SUBROUTINE ANGLE4(ISIGN,ANGLE,*)
C
C     ALLOCATION OF SIGN TO POSITIVE ANGLE IN INTERNAL UNITS
C     ACCORDING TO VALUE OF ALPHAMERIC SIGN SUPPLIED
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, JULY 1971
C
      INTEGER ASIGN(3)/'+',' ','-'/
      REAL*8 ANGLE,NSIGN(3)/2*1.0D0,-1.0D0/
C
      DO 10 I=1,3
      IF(ISIGN.NE.ASIGN(I))GO TO 10
      ANGLE=ANGLE*NSIGN(I)
      RETURN1
   10 CONTINUE
C
      RETURN
      END

      REAL FUNCTION ANGLE5*8(SINE,COSINE)
C
C     TRUE VALUE OF AN ANGLE IN THE RANGE 0 TO 2PI RADIANS
C     COMPUTED FROM ITS SINE AND COSINE
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, JULY 1971
C
      REAL*8 SINE,COSINE,TWOPI,ANGLE,DATAN2
      TWOPI=6.283185307179586769
C
      ANGLE=DATAN2(SINE,COSINE)
C
      IF(ANGLE.GE.0.0D0)GO TO 10
      ANGLE=ANGLE+TWOPI
C
   10 ANGLE5=ANGLE
C
      RETURN
      END

      SUBROUTINE ANGLE6(ISIGN,ANGLE)
C
C     TRANSFORMATION OF POSITIVE OR NEGATIVE ANGLE IN INTERNAL UNITS
C     INTO POSITIVE ANGLE IN INTERNAL UNITS WITH ALPHAMERIC SIGN
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, AUGUST 1971
C
      INTEGER ASIGN(3)/'+',' ','-'/
      REAL*8 ANGLE,DABS
C
      IF(ANGLE)10,20,30
   10 I=3
      GO TO 40
   20 I=2
      GO TO 40
   30 I=1
C
   40 ISIGN=ASIGN(I)
      ANGLE=DABS(ANGLE)
C
      RETURN
      END
```

are being printed out) care must be taken to ensure that the angle does in fact lie within this range. For example, a computation may produce a value of an R.A. equal to $-\pi$ radians (-12^h) which must be rectified to lie between the above limits. This is accomplished by means of ANGLE3 which expresses the angle modulo 2π and then ensures that it is greater than zero, the latter being necessary because the FORTRAN function DMOD alone would give values in the range -2π to $+2\pi$.

Certain difficulties can arise when the difference between two similar angles of this type is obtained, even if both have been previously rectified to lie within the correct range. For example, subtraction of an R.A. of 23^h from one of 2^h should give 3^h but the computer would obtain -21^h; similarly, subtraction of an R.A. of 2^h from one of 23^h should give -3^h but the computer would obtain 21^h. This difficulty is overcome by means of ANGLE8 which ensures that the smallest numerical difference between the two angles is obtained and that the difference has the correct sign. It is always used when angles of this type are subtracted.

It is often necessary to determine the value of an angle in the range 0 to 2π radians from its sine and cosine (both being required to determine the correct quadrant) and this is accomplished by means of ANGLE5. The program obtains the value of the angle using the FORTRAN function DATAN2 (which evaluates $\tan^{-1}(^{\sin}/\cos)$) and then, since DATAN2 gives values in the range $-\pi$ to $+\pi$, modifies the result to lie between 0 and 2π.

The final ANGLE subprogram, ANGLE7, is of much more limited application than the others. It was written to solve the geometrical relationship

$$\tan \frac{E}{2} = \left(\frac{1 - e}{1 + e}\right)^{\frac{1}{2}} \tan \frac{v}{2}$$

between the eccentric anomaly E and the true anomaly v of a planet in its orbit (e being the eccentricity) and as such it will evaluate any expression

```
      REAL FUNCTION ANGLE7*8(A,ANGLE)
C
C     TRUE VALUE OF AN ANGLE IN THE RANGE 0 TO 2PI RADIANS
C     COMPUTED FROM AN EQUATION OF THE FORM
C     TAN(ANGLE7/2) = A*TAN(ANGLE/2)
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, JULY 1971
C
      REAL*8 A,ANGLE,PI,HALF7,DTAN,DATAN
      PI=3.14159265358979323895
C
      HALF7=DATAN(A*DTAN(ANGLE/2.0D0))
C
      IF(HALF7.GE.0.0D0)GO TO 10
      HALF7=HALF7+PI
C
   10 ANGLE7=2.0D0*HALF7
C
      RETURN
      END
```

```
      REAL FUNCTION ANGLE8*8(ANG2,ANG1)
C
C     SMALLEST DIFFERENCE OF TWO ANGLES EACH IN THE RANGE
C     0 TO 2PI RADIANS
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, AUGUST 1971
C
      REAL*8 ANG2,ANG1,PI,TWOPI,DIFF
      PI=3.1415 92653 58979 32384 62643
      TWOPI=6.2831 85307 17958 64769 25286
C
      DIFF=ANG2-ANG1
C
      IF(DIFF.GT.PI)GO TO 10
      IF(DIFF.LT.-PI)GO TO 20
      GO TO 30
C
   10 DIFF=DIFF-TWOPI
      GO TO 30
   20 DIFF=DIFF+TWOPI
C
   30 ANGLE8=DIFF
C
      RETURN
      END
```

of the form

$$2 \tan^{-1} (A \tan \frac{\Theta}{2})$$

where both Θ and the result lie in the range 0 to 2π radians. It is invalid in the case where $\Theta = \pi$.

VECTOR, ORBEL

These two subroutines are concerned with the transformation of the orbital elements

ω = orientation of major axis (argument of perihelion),

i = inclination of orbit,

Ω = longitude of ascending node

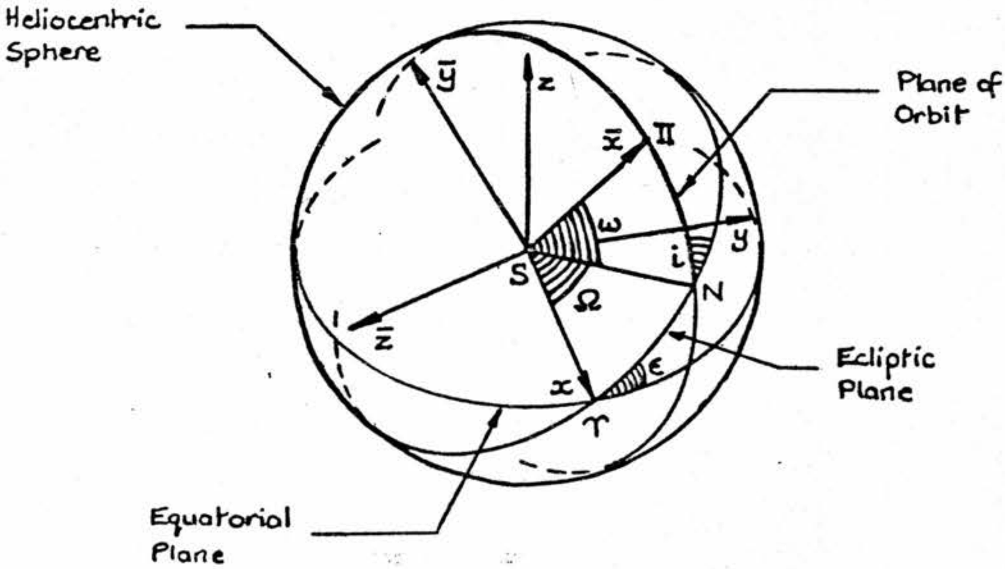
into the vectorial equatorial constants P_x, P_y, P_z, Q_x, Q_y, Q_z, and vice versa.

To understand the mechanics of the transformation we must consider first the heliocentric rectangular coordinate systems shown in Fig. 2; viz. the equatorial system S x y z and the orbital system S $\bar{x}\ \bar{y}\ \bar{z}$. We can write the transformation from one of these to the other as

$$[x\ y\ z] = [\bar{x}\ \bar{y}\ \bar{z}] \quad r(-\omega)\ p(-i)\ r(-\Omega)\ p(-\varepsilon)$$

where

$$r(-\Phi) = \begin{bmatrix} \cos\Phi & \sin\Phi & 0 \\ -\sin\Phi & \cos\Phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and } p(-\Phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Phi & \sin\Phi \\ 0 & -\sin\Phi & \cos\Phi \end{bmatrix}$$



♈ - Vernal Equinox

N - Ascending Node of Orbit

Π - Perihelion

Fig. 2

are clockwise rotations about the $\bar{z}$ - axis and SN respectively.

Now the unit vectors $\hat{\underline{P}}$, $\hat{\underline{Q}}$ and $\hat{\underline{R}}$ along the $\bar{x}$, $\bar{y}$ and $\bar{z}$ axes have components in the equatorial system of (P_x, P_y, P_z), (Q_x, Q_y, Q_z) and (R_x, R_y, R_z) respectively, so we can write

$$[x\ y\ z] = [\bar{x}\ \bar{y}\ \bar{z}] \begin{bmatrix} P_x & P_y & P_z \\ Q_x & Q_y & Q_z \\ R_x & R_y & R_z \end{bmatrix}$$

Combining this and the above matrix equations gives us

$$\begin{bmatrix} P_x & P_y & P_z \\ Q_x & Q_y & Q_z \\ R_x & R_y & R_z \end{bmatrix} = r(-\omega)\ p(-i)\ r(-\Omega)\ p(-\varepsilon)$$

which is essentially the transformation we seek. Upon expansion this gives P_x, P_y, P_z, Q_x, Q_y, Q_z directly in terms of ω, i, Ω and ε (the obliquity of the ecliptic) and these are the expressions used in subroutine VECTOR. The arrangement of these in the subroutine is as given by Herget (1948 , p. 50) using the intermediate variables p and q. We note that ω, i, Ω and ε are represented in the subroutine by OMEGA, SI, OMEGA1 and OBL respectively, all in units of radians.

Rearrangement of the expressions gives

$$\sin\omega\ \sin i = P_z \cos\varepsilon - P_y \sin\varepsilon$$
$$\cos\omega\ \sin i = Q_z \cos\varepsilon - Q_y \sin\varepsilon$$
$$\sin\Omega = (P_y \cos\omega - Q_y \sin\omega)/\cos\varepsilon$$
$$\cos\Omega = P_x \cos\omega - Q_x \sin\omega$$

Division of the first two equations gives tan ω, squaring them and adding

```
      SUBROUTINE VECTOR(OBL,OMEGA,SI,OMEGA1,UPX,UPY,UPZ,UQX,UQY,UQZ)
C
C     COMPUTATION OF VECTORIAL EQUATORIAL CONSTANTS USING
C     CLASSICAL ORBITAL ELEMENTS
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, MAY 1972
C
      REAL*8 OBL,OMEGA,SI,OMEGA1,A,B,C,D,E,F,G,H,EG,EH,FD,FC,P,Q,
     *UPX,UPY,UPZ,UQX,UQY,UQZ,
     *DCOS,DSIN
C
      A=DCOS(OBL)
      B=DSIN(OBL)
      C=DCOS(OMEGA)
      D=DSIN(OMEGA)
      E=DCOS(SI)
      F=DSIN(SI)
      G=DCOS(OMEGA1)
      H=DSIN(OMEGA1)
C
      EG=E*G
      EH=E*H
      FD=F*D
      FC=F*C
C
      P=D*EG+C*H
      Q=C*EG-D*H
C
      UPX=-(D*EH-C*G)
      UPY=A*P-B*FC
      UPZ=A*FD+B*P
C
      UQX=-(C*EH+D*G)
      UQY=-(B*FC-A*Q)
      UQZ=B*Q+A*FC
C
      RETURN
      END
```

```
      SUBROUTINE ORBEL(OBL,UPX,UPY,UPZ,UQX,UQY,UQZ,OMEGA,SI,OMEGA1)
C
C     COMPUTATION OF THE CLASSICAL ORBITAL ELEMENTS OMEGA, I AND OMEGA1
C     USING VECTORIAL EQUATORIAL CONSTANTS
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, JUNE 1973
C
C
C     ORBIT LIBRARY SUBPROGRAMS CALLED - COVECT,ANGLE5
C
C
      REAL*8 OBL,UPX,UPY,UPZ,UQX,UQY,UQZ,COSOBL,SINOBL,SOMSIP,COMSIP,
     *OMEGA,SI,COSOM,SINOM,SINOM1,COSOM1,OMEGA1,
     *DCOS,DSIN,DARSIN,DSQRT,COVECT,ANGLE5
C
      COSOBL=DCOS(OBL)
      SINOBL=DSIN(OBL)
      SOMSIP=COVECT(UPZ,UPY,SINOBL,COSOBL)
      COMSIP=COVECT(UQZ,UQY,SINOBL,COSOBL)
C
      OMEGA=ANGLE5(SOMSIP,COMSIP)
C
      SI=DARSIN(DSQRT(SOMSIP*SOMSIP+COMSIP*COMSIP))
C
      COSOM=DCOS(OMEGA)
      SINOM=DSIN(OMEGA)
      SINOM1=COVECT(UPY,UQY,SINOM,COSOM)/COSOBL
      COSOM1=COVECT(UPX,UQX,SINOM,COSOM)
C
      OMEGA1=ANGLE5(SINOM1,COSOM1)
C
      RETURN
      END
```

gives $\sin^2 i$, and division of the last two gives $\tan \Omega$; thus we obtain ω, i and Ω in terms of P_x, P_y, P_z, Q_x, Q_y, Q_z and ε. This is the method used in subroutine ORBEL to obtain the inverse transformation of VECTOR.

It should be noted that the three vectorial constants R_x, R_y, R_z are not dealt with by these subroutines as they are often not required. They can in any case be obtained from the other six constants.

GEOS

In orbit computations it is frequently necessary to know the geocentric equatorial rectangular coordinates of the sun at a particular instant. These coordinates (X, Y, Z) in astronomical units, referred to either the mean equinox and equator of 1950.0 or that of the year concerned, are tabulated with first and second differences for 0^h E.T. each day in the "Astronomical Ephemeris" and it is the function of subroutine GEOS to interpolate the coordinates to the required instant. The program uses Everett's interpolation formula

$$f_p = (1 - p)\, f_0 + pf_1 + E_2 \delta_0^{\,2} + F_2 \delta_1^{\,2}$$

where p is the required fraction of the interval between the given function values f_0 and f_1, $\delta_0^{\,2}$ and $\delta_1^{\,2}$ are second differences, and

$$E_2 = -\tfrac{1}{6}\, p(p - 1)\,(p - 2)$$
$$F_2 = \tfrac{1}{6}\,(p + 1)\, p\, (p - 1)$$

In this case, p is the required E.T. in days.

Although this formula uses only second differences, it is essentially a third-degree formula and can therefore be used if fourth differences are negligible, as they are in this case (see "Interpolation and Allied Tables", p. 19). It is derived in Baker and Makemson(1967, p. 355).

```
      SUBROUTINE GEOS(P,X0,D2X0,Y0,D2Y0,Z0,D2Z0,X1,D2X1,Y1,D2Y1,Z1,D2Z1,
     *X,Y,Z)
C
C     COMPUTATION OF THE GEOCENTRIC EQUATORIAL RECTANGULAR COORDINATES
C     OF THE SUN AT A GIVEN INSTANT BY INTERPOLATION
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, MARCH 1971
C
      REAL*8 P,X0,D2X0,Y0,D2Y0,Z0,D2Z0,X1,D2X1,Y1,D2Y1,Z1,D2Z1,E2,F2,A,
     *X,Y,Z,EVERET,F0,F1,D2F0,D2F1
C
      EVERET(F0,F1,D2F0,D2F1)=A*F0+P*F1+E2*D2F0+F2*D2F1
C
      E2=-P*(P-1.0D0)*(P-2.0D0)/6.0D0
      F2=(P+1.0D0)*P*(P-1.0D0)/6.0D0
      A=(1.0D0-P)
C
      X=EVERET(X0,X1,D2X0,D2X1)
      Y=EVERET(Y0,Y1,D2Y0,D2Y1)
      Z=EVERET(Z0,Z1,D2Z0,D2Z1)
C
      RETURN
      END
```

```
      SUBROUTINE TOPOS(LONG,LAT,EPOCH,YDATE,GSTOD,UTD,X,Y,Z,RX,RY,RZ)
C
C     REDUCTION OF THE GEOCENTRIC EQUATORIAL RECTANGULAR COORDINATES
C     OF THE SUN TO TOPOCENTRIC COORDINATES
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, MARCH 1971
C
      REAL*8 LONG,LAT,EPOCH,YDATE,GSTOD,UTD,X,Y,Z,RHO,DVSFMD,JCONV,
     *TCONV,LST,T,MBAR,THETA,DXY,RX,RY,RZ,DCOS,DSIN
C
      RHO=0.00004263
      DVSFMD=1.0027379166666666
      JCONV=0.1591549430
      TCONV=13750.9870830951
C
      LST=(GSTOD+UTD*DVSFMD-LONG)/JCONV
      T=(YDATE+EPOCH-3800.0D0)/200.0D0
      MBAR=(3.07234D0+0.00186D0*T)/TCONV
      THETA=LST+MBAR*(EPOCH-YDATE)
C
      DXY=-RHO*DCOS(LAT)
C
      RX=X+DXY*DCOS(THETA)
      RY=Y+DXY*DSIN(THETA)
      RZ=Z-RHO*DSIN(LAT)
C
      RETURN
      END
```

TOPOS

Observations of minor planets give R.A. and Dec. as topocentric coordinates. In orbit computations, rather than reduce the R.A. and Dec. to geocentric coordinates (which requires a knowledge of the earth-planet separation) we reduce the sun's geocentric equatorial rectangular coordinates (X, Y, Z) to topocentric coordinates (R_x, R_y, R_z) (not to be confused with the equatorial components of $\underline{\hat{R}}$). This is accomplished in ORBIT by means of subroutine TOPOS which computes and applies the corrections to (X, Y, Z) to obtain (R_x, R_y, R_z).

In Fig. 3 the geocentric equatorial coordinates (ξ, η, ζ) of the observer O on the surface of the earth are given by

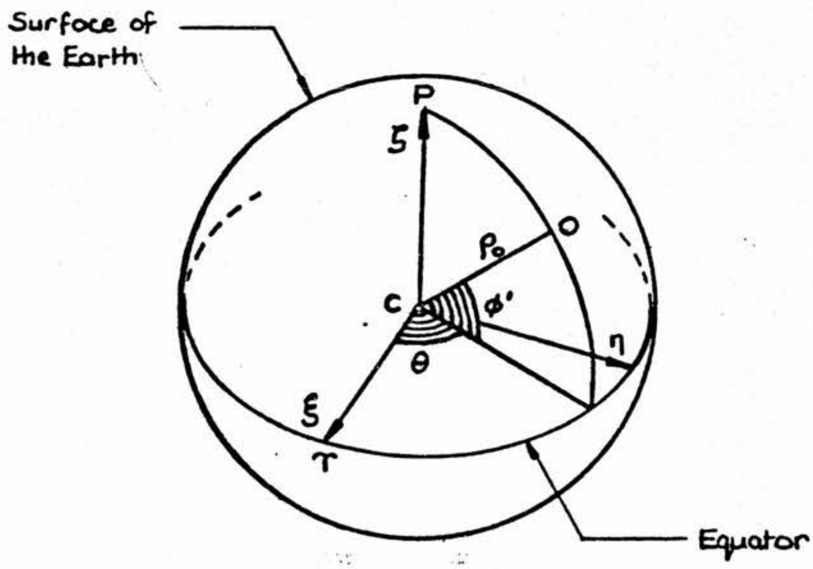
$$\xi = \rho_0 \cos\phi' \cos\theta$$
$$\eta = \rho_0 \cos\phi' \sin\theta$$
$$\zeta = \rho_0 \sin\phi'$$

where ρ_0 is the earth's radius in astronomical units (regarded as constant without loss in accuracy), ϕ' is the observer's geocentric latitude and θ is the local sidereal time at the instant of the observation. These coordinates are subtracted from the solar coordinates (X, Y, Z) to give (R_x, R_y, R_z).

The local (mean) sidereal time of the observation is obtained from

LMST = GMST at 0^hU.T. + elapsed sidereal time since 0^hU.T. - observer's longitude

The GMST at 0^hU.T. on the day of the observation is obtained from the "Astronomical Ephemeris" and the elapsed sidereal time since 0^hU.T. is obtained by expressing the U.T. of the observation in sidereal time units. In subroutine TOPOS, the GMST at 0^hU.T. (GSTOD), the U.T. of the observation (UTD) and the observer's longitude (LONG) are all in units of days. Since the elapsed sidereal time is obtained by



♈ - Vernal Equinox

P - North Pole

C - Centre of Earth

O - Position of observer

Fig. 3

multiplication of the U.T. by 1+ the daily variation of sidereal time from mean time in days (= DVSFMD), this too is in days, and thus so is the resulting LST. The LST is converted to radians using the conversion factor JCONV since it is to be used as the argument of trigonometrical functions.

First, however, a correction for precession has to be applied since the LST we have obtained is with respect to the mean equinox of date and we require it referred to the mean equinox of the rest of the data (usually that of 1950.0). The annual precession in R.A. is given by

$$\overline{m} = 3\overset{s}{.}07234 + 0\overset{s}{.}00186T$$

where T is the median time in centuries between the time of observation and the epoch of the required mean equinox;

ie. if (as in TOPOS) YDATE = time of observation in years

EPOCH = epoch of required mean equinox in years

then $$T = \frac{(\text{YDATE} - 1900.0) + \frac{1}{2}(\text{EPOCH} - \text{YDATE})}{100}$$

or $$T = \frac{\text{YDATE} + \text{EPOCH} - 3800}{200}$$

The use of the median time simplifies the expansion for $\overline{m}$ and it is sufficiently accurate for values of T < 0.5 centuries.

Once $\overline{m}$ has been obtained (in the same units as LST, viz. radians) we can write

$$\theta = \text{LST} + \overline{m}\,(\text{EPOCH} - \text{YDATE})$$

since $\overline{m}$ is annual precession.

In TOPOS, the observer's latitude (LAT) is entered into the subroutine in units of radians and so, having computed θ in radians, the expressions for (ξ, η, ζ) can be evaluated immediately and the topocentric solar coordinates (R_x, R_y, R_z) obtained.

KEPLER

According to Herget (1948, p. 33), literally hundreds of methods have been given for the solution of Kepler's equation

$$M = E - e \sin E$$

where M and E are the mean and eccentric anomalies of a planet in its orbit and e is the eccentricity of the orbit. An iterative procedure is generally used and the determination of a satisfactory first approximation to E has often been the main point of new methods.

The method used in subroutine KEPLER is due to Encke and the first approximation E_1 is obtained as follows:

define y so that $\tan y = \dfrac{e \sin M}{1 - e \cos M}$,

then if $\eta = \sin y$

we have $E_1 = M + \eta - \tfrac{1}{6}\eta^4 \cot M.$

The second and subsequent approximations are obtained as described by Danby (1962, p. 149), viz., for the jth approximation:

let $M_{j-1} = E_{j-1} - e \sin E_{j-1}$

then $\Delta E_{j-1} = \dfrac{M - M_{j-1}}{1 - e \cos E_{j-1}}$

and $E_j = E_{j-1} + \Delta E_{j-1}$

It should be noted that in this subroutine e, M and E are represented by E, M and CE respectively, and that M and CE are in radians. The

```
      SUBROUTINE KEPLER(E,M,CE,*)
C
C     SOLUTION OF KEPLER'S EQUATION BY ENCKE'S METHOD
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, MAY 1972
C
C     UPDATED JUNE 1972
C
C
C     ORBIT LIBRARY SUBPROGRAMS CALLED - ANGLE3,ANGLE8
C
C
      REAL*8 E,M,ETA,CE,TESTM,DELTAE,
     *DSIN,DATAN2,DCOS,DTAN,DABS,ANGLE3,ANGLE8
C
      ITNUM=0
      ETA=DSIN(DATAN2(E*DSIN(M),1.0D0-E*DCOS(M)))
      CE=ANGLE3(M+ETA-(ETA**4)/(6.0D0*DTAN(M)))
C
   10 ITNUM=ITNUM+1
      TESTM=ANGLE3(CE-E*DSIN(CE))
      DELTAE=ANGLE8(M,TESTM)/(1.0D0-E*DCOS(CE))
      CE=ANGLE3(CE+DELTAE)
C
      IF(DABS(DELTAE).LT.1.0D-8)RETURN1
      IF(ITNUM.GT.50)RETURN
      GO TO 10
C
      END
```

subroutine follows the general layout of iterative procedures discussed earlier in this chapter, and it is arranged with multiple returns so that an error message can be given in the event of the number of iterations exceeding 50. The subroutine terminates iteration normally when successive approximations to E agree to within 10^{-8} radians $= 0\overset{''}{.}002$.

Chapter 1.2 The Reduction of Plate Measurements

The reduction of plate measurements to obtain the celestial coordinates of objects is the fundamental problem of photographic astrometry. Basically the problem consists of establishing a transformation between the measured coordinates of images on a photographic plate and the positions in the sky of the objects which produced the images. This is normally done by taking into account measurements of reference stars whose celestial coordinates are accurately known; once the transformation has been established, the unknown celestial coordinates of an object such as a minor planet can be deduced from its coordinates on the plate.

In practice the transformation between the measured and celestial coordinates is not a straightforward geometrical one because of such factors as atmospheric refraction, aberration and effects (such as plate tilt) due to errors in the instrument used to obtain the plate. The way in which the transformation is normally established is as follows: we define the standard coordinates (ξ,η) of a celestial object to be the coordinates on a plane tangent to the celestial sphere relative to the tangent point as origin of that point on the plane which is a central (or gnomonic) projection of the object. The ξ and η axes are arranged to correspond in orientation with RA and Dec; thus the relationship between the standard coordinates and the RA and Dec (relative to the RA and Dec of the tangent point) is the purely geometrical one mentioned above. Details of this relationship are given, for example, by König (1962).

If we now consider the measured coordinates (x, y) of the object on the plate, we find we can relate them to the standard coordinates by the linear formulae

$$\xi - x = ax + by + c$$
$$\eta - y = dx + ey + f$$

where a,b, ..., f are constants dependent in a composite way on the first order effects of refraction, aberration, misorientation between the (x,y) and (ξ,η) axes, non-perpendicularity of the x and y axes and non-coincidence of the origins of the (x,y) and (ξ,η) axes (see Smart (1965, pp 287-300)). The small effect of tilt between the (x,y) and (ξ,η) axes is neglected. The quantities a,b,...,f are called plate constants, being constant for each set of measurements of a particular plate, and they can be determined by the substitution in the formulae of the measured coordinates (x,y) and standard coordinates (ξ,η) for three reference stars of known RA and Dec (α,δ) (from which (ξ,η) are derived) resulting in three sets of equations which may be solved simultaneously for a,b,...,f. In practice, a large number N of known stars is measured, yielding N sets of equations of the above form from which a,b,...,f are obtained by a least-squares solution. Knowing the plate constants we can then use the measured coordinates (x_o,y_o) of an unknown object to obtain its standard coordinates (ξ_o,η_o) and thus its RA and Dec (α_o,δ_o).

Several modifications of this basic method have been introduced, of which the most important to us is the method of dependences developed by F. Schlesinger, originally for stellar parallax determinations (Schlesinger (1911)) but later in modified form for the determination of asteroid and comet positions (Schlesinger (1926)). A lucid exposition of this latter method is given by Wood (1929); alternative treatments are given by Smart (1965, pp 404-413) and van de Kamp (1967, pp 102-113).

In its simplest and most widely used form, the method employs measurements of three known reference stars and consists in utilising an imaginary point known as the dependence centre whose exact coordinates can be computed, and which coincides very nearly with the image of the minor planet or comet. The small discrepancy between the position of the dependence centre and that of the object is taken into account in the computation by a linear transformation requiring both a knowledge of the plate scale and a close coincidence in the orientations of the measured and standard coordinate systems. The dependences D_i ($i = 1,2,3$) are the barycentric coordinates of the dependence centre with respect to the three reference stars, ie if the three stars have measured coordinates (x_i,y_i) ($i = 1,2,3$) and the dependence centre has coordinates (x_A,y_A) then

$$x_A = \sum_{i=1}^{3} D_i x_i; \qquad y_A = \sum_{i=1}^{3} D_i y_i \qquad \text{and} \qquad \sum_{i=1}^{3} D_i = 1$$

More than one method is available for the determination of the dependences and once they have been obtained, spherical transformations are used to determine the RA and Dec of the object in terms of the dependences, the RA's and Dec's of the reference stars and the plate centre, and the small discrepancy mentioned above (known as the plate solution or offset). In practice, one of the three reference stars is adopted as the origin (or base) for these transformations instead of the plate centre as this results in considerable simplification and there is no serious loss in accuracy. The evaluation of the plate constants is entirely eliminated.

The method chosen for the reduction of plate measurements in the present work is a modification of the above due to Comrie (1929). In this method the dependences are computed more precisely than in others, and this results in the exact coincidence of the dependence centre with the image of the asteroid (ie plate solution = 0). This in turn leads to the principal advantages of the method, viz that no knowledge of the plate scale is required and that the orientation in which the plate is measured is immaterial.

The method is used as follows: three reference stars having known RA and Dec (α_i,δ_i) ($i = 1,2,3$) are selected on the plate, preferably in such a way as to enclose the asteroid within the triangle that they form (which should be as small as possible) but in any case so that the asteroid is away from the sides of the triangle. (The asteroid being outside the triangle merely results in one or two of the D_i being negative; however, the configuration should still be as compact as possible.) Let the measured coordinates of the reference stars be (x_i,y_i) $(i = 1,2,3)$ and those of the asteroid or other object be (x_o,y_o). We then compute

$$N_1 = (x_2 - x_o)(y_3 - y_o) - (y_2 - y_o)(x_3 - x_o)$$
$$N_2 = (x_3 - x_o)(y_1 - y_o) - (y_3 - y_o)(x_1 - x_o)$$
$$N_3 = (x_1 - x_o)(y_2 - y_o) - (y_1 - y_o)(x_2 - x_o)$$

$$N = \sum_{i=1}^{3} N_i$$

and obtain the dependences as

$$(i = 1,2,3) \quad D_i = \frac{N_i}{N}$$

The values obtained for the dependences can be checked using the relationships

$$\sum_{i=1}^{3} D_i = 1; \qquad \sum_{i=1}^{3} D_i x_i = x_o; \qquad \sum_{i=1}^{3} D_i y_i = y_o$$

given earlier.

We next compute

$$(i = 1,2,3) \quad \Delta\alpha_i = \alpha_i - \alpha; \; \Delta\delta_i = \delta_i - \delta$$

where (α,δ) are the RA and Dec of the base of the plate which, as we have already seen, may be taken as one of the reference stars, this resulting in one $\Delta\alpha_i$ and one $\Delta\delta_i$ becoming zero. In practice we take the first reference star $(i = 1)$ as the base of the plate.

The RA and Dec of the object (α_o, δ_o) are then obtained directly from

$$\alpha_o = \alpha + \sum_{i=1}^{3} D_i\Delta\alpha_i + \left[\left(\sum_{i=1}^{3} D_i\Delta\alpha_i\right)\left(\sum_{i=1}^{3} D_i\Delta\delta_i\right) - \sum_{i=1}^{3} D_i\Delta\alpha_i\Delta\delta_i\right] \tan\delta;$$

$$\delta_o = \delta + \sum_{i=1}^{3} D_i\Delta\delta_i + \tfrac{1}{4}\left[\sum_{i=1}^{3} D_i(\Delta\alpha_i)^2 - \left(\sum_{i=1}^{3} D_i\Delta\alpha_i\right)^2\right] \sin 2\delta$$

where the second order terms in square brackets are small quantities representing the corrections involved in the conversion into spherical coordinates. In these expressions all the angular quantities are in radians. With the exception of the inclusion of the computational checks, the method is used exactly as given here in the ORBIT subroutine COMRIE which forms the basis of the plate measurement reduction program ORBIT 2.

In his account of this method, Comrie discusses the necessary safeguards required to ensure that the (α_o, δ_o) obtained are not in error due to incorrect plate measurements or faulty reference star positions. The conclusion he reaches is that the best protection against such a possibility is to repeat the entire procedure, measuring three different reference stars and carrying out an independent reduction to obtain a second value of (α_o, δ_o) which may then be compared with the first. The programming of this method as a subroutine means that it can be easily incorporated into a main program in such a way as to permit this repetition of the process and, in fact, ORBIT 2 is arranged so as to be able to execute up to nine such independent reductions, a mean value of (α_o, δ_o) being obtained from the results.

Having deduced the equatorial coordinates of the object, it is often necessary to correct them for precession. Obviously, the RA and Dec obtained by the preceding method will be referred to the same equator and equinox as the equatorial coordinates of the reference stars (which must, of course, be referred to a common equator and equinox) and to reduce these to a different equator and equinox we use Newcomb's method.

A full account of this is given in the "Explanatory Supplement to the

```
      SUBROUTINE COMRIE(X,Y,RA,DE,XO,YO,RAO,DEO)
C
C     REDUCTION OF THE POSITION OF AN OBJECT FROM PLATE
C     MEASUREMENTS BY COMRIE'S THREE-STAR METHOD
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, SEPTEMBER 1971
C
C
C     ORBIT LIBRARY SUBPROGRAMS CALLED - COVECT,ANGLE8
C
C
      REAL*8 X(3),Y(3),RA(3),DE(3),XO,YO,XDIFF(3),YDIFF(3),N(3),NSUM,
     *D(3),RADIFF,DEDIFF,DRA(3),DDE(3),DRARA(3),DRADE(3),SDRA,SDDE,
     *SDRARA,SDRADE,RAO,DEO,
     *DTAN,DSIN,COVECT,ANGLE8
C
C     COMPUTATION OF THE DEPENDENCES D(I)
C
      DO 10 I=1,3
      XDIFF(I)=X(I)-XO
   10 YDIFF(I)=Y(I)-YO
C
      N(1)=COVECT(XDIFF(2),YDIFF(2),XDIFF(3),YDIFF(3))
      N(2)=COVECT(XDIFF(3),YDIFF(3),XDIFF(1),YDIFF(1))
      N(3)=COVECT(XDIFF(1),YDIFF(1),XDIFF(2),YDIFF(2))
      NSUM=N(1)+N(2)+N(3)
C
      DO 20 I=1,3
   20 D(I)=N(I)/NSUM
C
C     COMPUTATION OF THE OBJECT'S EQUATORIAL COORDINATES (RAO,DEO)
C
      DO 30 I=1,3
      RADIFF=ANGLE8(RA(I),RA(1))
      DEDIFF=DE(I)-DE(1)
C
      DRA(I)=D(I)*RADIFF
      DDE(I)=D(I)*DEDIFF
      DRARA(I)=DRA(I)*RADIFF
   30 DRADE(I)=DRA(I)*DEDIFF
C
      SDRA=DRA(2)+DRA(3)
      SDDE=DDE(2)+DDE(3)
      SDRARA=DRARA(2)+DRARA(3)
      SDRADE=DRADE(2)+DRADE(3)
C
      RAO=RA(1)+SDRA+(SDRA*SDDE-SDRADE)*DTAN(DE(1))
      DEO=DE(1)+SDDE+0.25D0*(SDRARA-SDRA*SDRA)*DSIN(2.0D0*DE(1))
C
      RETURN
      END
```

Astronomical Ephemeris", pp. 29-31; briefly the method is as follows: it is required to transform (α_o, δ_o) at epoch T_o to (α, δ) at epoch T_1 where T_o, T_1 are in tropical centuries since 1900.0. Let $T = T_1 - T_o$, then

$$J = (2004''.682 - 0''.853T_o)\ T - 0''.426T^2 - 0''.042T^3$$

$$\zeta_o = (2304''.250 + 1''.396T_o)\ T + 0''.302T^2 + 0''.018T^3$$

$$z = \zeta_o + 0''.791\ T^2$$

where J (denoted by θ in the "Explanatory Supplement") is the inclination of the equator of T_1 to the equator of T_o, $90^\circ - \zeta_o$ is the right ascension of the ascending node of the equator of T_1 on the equator of T_o reckoned from the equinox of T_o, and $90^\circ + z$ is the right ascension of the node reckoned from the equinox of T_1.

By spherical trigonometry we can show that these quantities and (α, δ), (α_o, δ_o) are related by

$$\tan\ (\alpha - \alpha_o - \zeta_o - z) = \frac{q \sin\ (\alpha_o + \zeta_o)}{1 - q \cos\ (\alpha_o + \zeta_o)}$$

where $q = \sin J\ [\tan \delta_o + \cos\ (\alpha_o + \zeta_o) \tan \tfrac{1}{2} J]$;

and $\tan \tfrac{1}{2}\ (\delta - \delta_o) = \dfrac{\tan \frac{1}{2} J \cos \{ (\alpha_o + \zeta_o) + \frac{1}{2} (\alpha - \alpha_o - \zeta_o - z)\}}{\cos \frac{1}{2} (\alpha - \alpha_o - \zeta_o - z)}$

Thus, knowing J, ζ_o, z and (α_o, δ_o) we can obtain (α, δ). These formulae are rigorous and can thus be used for any interval of time.

Newcomb's method is used in exactly this form in the ORBIT subroutine NEWCMB, in which it should be noted that EPOCHO and EPOCH are the initial

```
      SUBROUTINE NEWCMB(EPOCHC,EPOCH,RA,DE)
C
C     REDUCTICN CF THE EQUATORIAL COCRCINATES (RA,DE) FROM THE
C     EQUATOR AND EQUINOX OF EPOCHO TO THE EQUATCR AND EQUINCX
C     CF EPCCH BY NEWCOMB'S METHOD
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY CBSERVATORY, SEPTEMBER 1971
C
      REAL*8 EPOCHO,EPOCH,RA,DE,CONV,TO,T,J,ZETAC,Z,A,Q,B,C,
     *CNTURY,DATE,DSIN,DCOS,CTAN,CATAN
C
      CNTURY(DATE)=(DATE-1900.CDO)/10C.0DO
      CONV=206264.80624 64262
C
      TO=CNTURY(EPOCHO)
      T=CNTURY(EPOCH)-TO
C
      J=(2C04.682DO-0.853DO*TO)*T-0.426DO*T**2.0DO-0.042DO*T**3.0DO
      ZCTAO=(2304.250DO+1.396DC*TO)*T+C.3C2DC*T**2.0DO+0.018DO*T**3.0CO
      Z=ZETAO+0.791CO*T**2.0CO
C
      J=J/CONV
      ZETAO=ZETAO/CONV
      Z=Z/CCNV
C
      A=RA+ZETAO
      Q=CSIN(J)*(CTAN(DE)+DCOS(A)*DTAN(0.5DO*J))
C
      B=DATAN((Q*DSIN(A))/(1.0DO-Q*DCCS(A)))
      C=CATAN((DTAN(0.5DO*J)*DCOS(A+0.5DC*B))/DCOS(0.5DO*B))
C
      RA=RA+ZETAC+Z+B
      DE=DE+2.0DC*C
C
      RETURN
      END
```

and final epochs expressed in years (eg 1950.0) and that RA, DE (in radians) enter the subroutine as (α_o, δ_o) and are returned as (α, δ).

Having described the basic reduction methods used in ORBIT 2 we give now a brief account of the working of the program itself. Although intended primarily as a plate reduction program, ORBIT 2 is arranged so that it can if necessary be used merely to correct known equatorial coordinates for precession; ie direct access to subroutine NEWCMB can be obtained without subroutine COMRIE being first executed. This facility is occasionally useful, though it is somewhat wasteful of core storage space as less than half the program is being used.

The program first reads in preliminary identification data and an alphameric access code IACODE. If this code is NCM (for NEWCMB) then the program skips to statement 260, reads in the equatorial coordinates to be corrected for precession in the format given on page 46, corrects them by calling subroutine NEWCMB, prints out the results as shown in the sample output on page 47 and stops.

If the access code is ALL (for the reduction of plate measurements and, if necessary, the correction for precession) the program reads in the required data in the format given on page 48. This includes the number of independent reductions to be undertaken (N), the maximum permissible discrepancy in RA and Dec between the results of the reductions (ERROR), the measured coordinates of the object and, for each reduction, the measured coordinates and catalogue positions of the reference stars. Since these positions have to be corrected for proper motion, the annual proper motions in RA and Dec (MU(1), MU(2)) are included in the data. Although the program was designed for use with the Smithsonian Astrophysical Observatory Star Catalogue this does not preclude its use with other catalogues, but care must be taken as some catalogues do not include the effect of proper motion in the RA and Dec. In these cases an original epoch is given with each annual proper motion and the correction must be made from the original epoch ODATE (rather than the precessional epoch of the catalogue, EPOCHO) to the epoch of the plate YDATE, the notation being that of ORBIT 2. For this reason original epoch values for both RA and Dec are included in the data. When the SAO Star Catalogue is used, a choice of original epoch or epoch 1950.0 is available; it is preferable to use the latter data, the ODATE values being entered into the program as 1950.0.

Having read in the data, the program then executes the N reductions to obtain K independent values of the RA and Dec of the object. (Normally, unless some error occurs, K = N.) In each reduction the proper motion corrections are applied to the star positions by means of the statement function PMCORR before these positions and the plate measurements are used by subroutine COMRIE to produce the Jth set of values of the asteroid's coordinates, this being then printed out (see the sample output on page 49).

When all N reductions have been completed, the K sets of results obtained from them are checked to ensure that they agree to within the value of ERROR and mean values of the asteroid's coordinates are taken. This final result is then corrected for precession (unless EPOCH = EPOCHO when the correction is omitted) and printed out, thus completing the execution of the program.

A typical execution time for the program under the Model 44 Programming System is 1.4 seconds, two independent reductions of plate measurements being carried out in this time.

When the program first became operational numerous checks were carried out by manual computation to ensure that no errors were present and to attempt to ascertain the accuracy of the program. Manual computations using the same method and data invariably produced results which agreed with the program to better than 0".005 indicating that the computation was sufficiently precise. Checks made on the accuracy of the method by comparing the results of independent reductions on the same plate gave agreement well within 0".5 when the geometrical requirements of the method with respect to the images on the plates were adequately satisfied. Comparison of the results obtained by interchanging the three reference stars in a reduction in order to use each in turn as the base of the plate surprisingly produced variations of up to $0\overset{''}{.}4$ due to the varying distance of the base star from the plate centre. This large value can almost certainly be explained in terms of the wide field of the plates used in this work; any inaccuracy thus produced can be minimised by ensuring that the images of the asteroid and reference stars (particularly that of the base star) are as near the plate centre as possible.

An improvement to this program not unconnected with the above remarks would be the modification of subroutine COMRIE so that in addition to the equatorial coordinates of the object it would return to the main program a weighting factor related to the probable accuracy of the coordinates. This would be possible because of the evident dependence of the probable accuracy on the size and geometry of the configuration of images on the plate together with the distances of the images from the plate centre, other things such as measurement accuracy being equal. An explicit statement of such a relationship would need to be developed and incorporated in COMRIE; the resulting weighting factor could then be applied in the main program so that reductions obtained using the most ideal configuration of images would receive the greatest weight. The resulting mean values of the coordinates of the object would then be considerably more realistic than the present unweighted means, particularly when small numbers of reductions are involved.

```
C
C     ORBIT 2 VERSION 1
C
C
C     REDUCTION OF THE POSITION OF AN OBJECT FROM PLATE
C     MEASUREMENTS BY COMRIE'S METHOD AND/OR REDUCTION
C     TO A GIVEN EQUATOR AND EQUINOX BY NEWCOMB'S METHOD
C
C
C     CODED IN FORTRAN IV FOR THE IBM SYSTEM/360 MODEL 44
C     PROGRAMMING SYSTEM
C
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, SEPTEMBER 1971
C
C
C     UPDATED JANUARY 1972
C
C
C     ORBIT LIBRARY SUBPROGRAMS CALLED - COMRIE,NEWCMB,(COVECT),ANGLE1,
C     ANGLE2,ANGLE3,ANGLE4,ANGLE6,ANGLE8
C
C
      INTEGER*4 IRDATE,IDEHC(2),IMIN(2),N,I,J,K,ICATNO(3),JCATNO(3),
     *ISIGN(2),IDENTN(6),IDENTD(14),IDENTO(20),IACODE,IDENTP(3),
     *IALL/' ALL'/,INCM/' NCM'/,IORD(9)/'ST','ND','RD',6*'TH'/
C
      REAL*8 CONV,TCONV,SEC(2),EPOCH,ERROR,YDATE,XO,YO,EPOCHO,X(3),Y(3),
     *MU(2),DCATE(2),RA(3),DE(3),RAO(9),DEO(9),RADIFF,RASUM,DESUM,RK,
     *RAOBJ,DEOBJ
C
      REAL*8 PMCORR,COORD,APM,DATE2,DATE1,
     *DABS,DCOS,DFLOAT,
     *ANGLE1,ANGLE3,ANGLE8
C
      PMCORR(COORD,APM,DATE2,DATE1)=COORD+APM*(DATE2-DATE1)
C
    1 FORMAT('1',4X,23HFGW / ORBIT 2 VERSION 1,83X,4HDATE,I7)
    2 FORMAT(////25X,81HREDUCTION OF THE POSITION OF AN OBJECT FROM PLAT
     *E MEASUREMENTS BY COMRIE'S METHOD,///25X,64HAND REDUCTION TO A GIV
     *EN EQUATOR AND EQUINOX BY NEWCOMB'S METHOD)
    3 FORMAT(////25X,89HREDUCTION OF THE POSITION OF AN OBJECT TO A GIVE
     *N EQUATOR AND EQUINOX BY NEWCOMB'S METHOD)
```

```
C
C     INPUT OF PRELIMINARY DATA
C
C
      CONV=206264.80624 64262
      TCONV=13750.98708 30951
C
      READ(5,10)(IDENTN(I),I=1,6),(IDENTD(I),I=1,14),(IDENTO(I),I=1,2C),
     *IRDATE,IACCDE,EPOCH
   10 FORMAT(6A4,14A4,/,20A4,/,I6,/,6X,A4,F7.1)
C
      WRITE(6,1)IRDATE
C
      IF(IACODE.EQ.IALL)GO TO 30
      IF(IACCDE.EQ.INCM)GO TO 260
      WRITE(6,20)
   20 FORMAT(////,5X,'INVALID ACCESS CODE - EXECUTION HAS BEEN TERMINATE
     *D')
      GO TC 1000
C
C
C     EXECUTION OF N INDEPENDENT REDUCTIONS OF POSITION
C     BY COMRIE'S METHOD
C
C
   30 READ(5,40)(IDENTP(I),I=1,3),N,ERROR,YDATE,X0,Y0,EPOCHC
   40 FORMAT(6X,3A4,12X,I1,6X,F8.2,/,F6.1,2F10.4,/,F6.1)
C
      WRITE(6,2)
      WRITE(6,50)(IDENTN(I),I=1,6)
   50 FORMAT(/////,5X,'NAME CF OBJECT - ',6A4)
      WRITE(6,60)(IDENTD(I),I=1,14)
   60 FORMAT(///,5X,'TIME OF OBSERVATION - ',14A4)
      WRITE(6,70)(IDENTO(I),I=1,20)
   70 FORMAT(///,5X,'PLACE OF OBSERVATION - ',20A4)
      WRITE(6,80)(IDENTP(I),I=1,3)
   80 FORMAT(///,5X,'PLATE NUMBER - ',3A4)
      WRITE(6,90)EPOCHO
   90 FORMAT(///,5X,'TOPOCENTRIC EQUATORIAL COORDINATES OF OBJECT (EPOCH
     *',F7.1,') -')
C
      K=0
      DO 170 J=1,N
C
      DO 100 I=1,3
  100 READ(5,110)ICATNO(I),X(I),Y(I)
  110 FORMAT(I6,2F10.4)
C
```

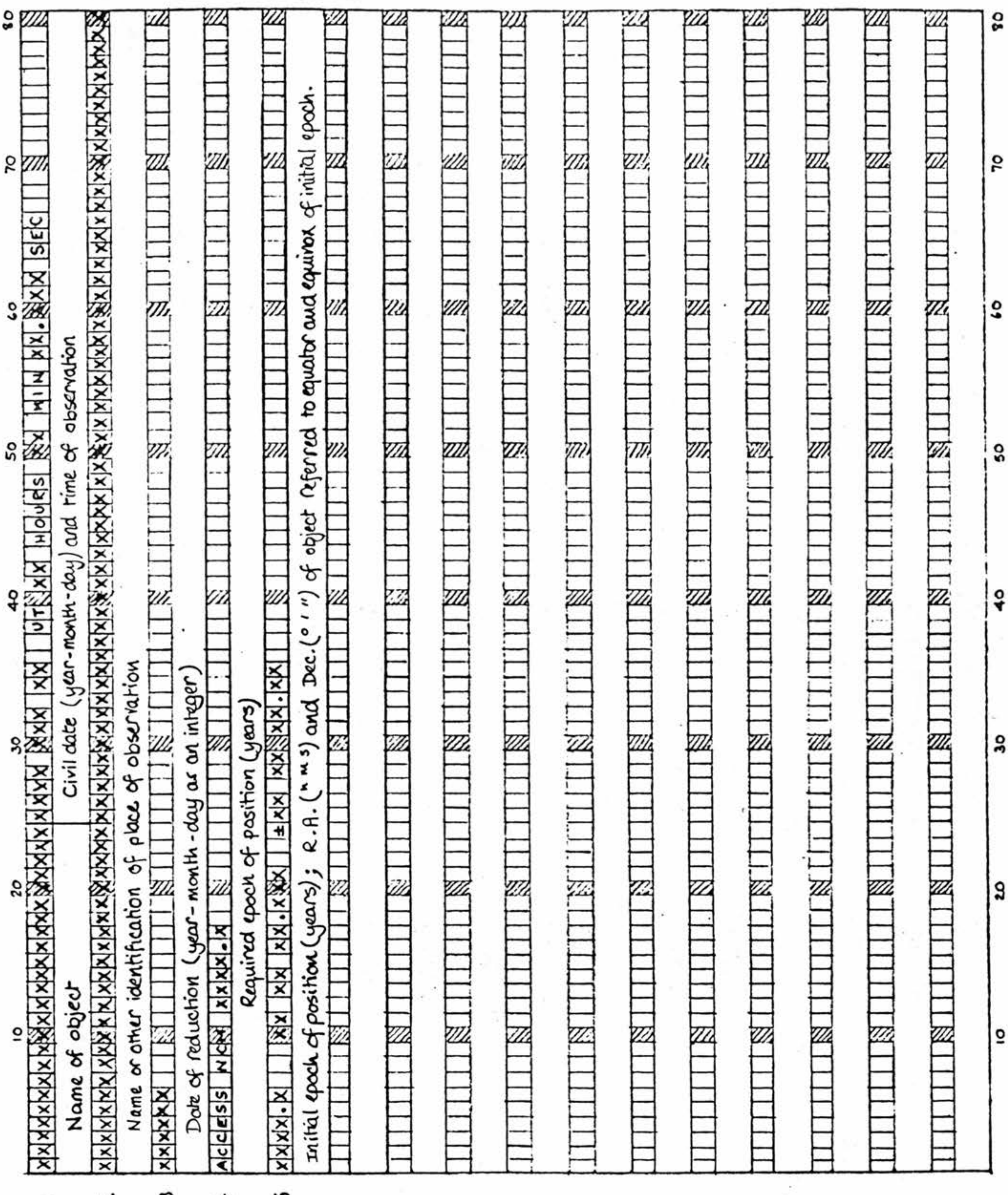
```
      DO 130 I=1,3
      READ(5,120)JCATNO(I),IDEHO(1),IMIN(1),SEC(1),MU(1),ODATE(1),
     *ISIGN(2),IDEHO(2),IMIN(2),SEC(2),MU(2),ODATE(2)
  120 FORMAT(I6,2X,2I3,F7.3,2X,F7.4,2X,F6.1,2X,A1,I2,I3,F6.2,2X,F6.3,2X,
     *F6.1)
C
      RA(I)=ANGLE1(TCONV,IDEHO(1),IMIN(1),SEC(1))
C
      DE(I)=ANGLE1(CONV,IDEHO(2),IMIN(2),SEC(2))
      CALL ANGLE4(ISIGN(2),DE(I),£125)
      WRITE(6,123)J,IORD(J)
  123 FORMAT(//,10X,I1,A2,' REDUCTION',10X,'INVALID SIGN GIVEN WITH CATA
     *LOGUE DATA - REDUCTION OMITTED')
      GO TO 170
C
  125 MU(1)=MU(1)/TCONV
      MU(2)=MU(2)/CONV
C
      RA(I)=PMCORR(RA(I),MU(1),YDATE,ODATE(1))
  130 DE(I)=PMCORR(DE(I),MU(2),YDATE,ODATE(2))
C
      DO 150 I=1,3
      IF(JCATNO(I).EQ.ICATNO(I))GO TO 150
      WRITE(6,140)J,IORD(J)
  140 FORMAT(//,10X,I1,A2,' REDUCTION',10X,'CATALOGUE DATA AND PLATE DAT
     *A DO NOT CORRESPOND - REDUCTION OMITTED')
      GO TO 170
  150 CONTINUE
C
      K=K+1
      CALL COMRIE(X,Y,RA,DE,XO,YO,RAO(K),DEO(K))
C
      RAO(K)=ANGLE3(RAO(K))
      CALL ANGLE6(ISIGN(2),DEO(K))
      CALL ANGLE2(TCONV,RAO(K),IDEHO(1),IMIN(1),SEC(1))
      CALL ANGLE2(CONV,DEO(K),IDEHO(2),IMIN(2),SEC(2))
C
      WRITE(6,160)J,IORD(J),IDEHO(1),IMIN(1),SEC(1),ISIGN(2),IDEHO(2),
     *IMIN(2),SEC(2)
  160 FORMAT(//,10X,I1,A2,' REDUCTION',10X,'ALPHA =',I3,' HOURS',I3,' MI
     *N',F7.3,' SEC',5X,'DELTA = ',A1,I2,' DEG',I3,' MIN',F6.2,' SEC')
C
      CALL ANGLE4(ISIGN(2),DEO(K),£170)
  170 CONTINUE
```

```
C
C
C     COMPUTATION OF THE UNWEIGHTED MEAN OF THE K POSITIONS OBTAINED
C     FROM THE N ATTEMPTED REDUCTIONS (K LESS THAN OR EQUAL TO N)
C
C
      IF(K.GT.0)GO TO 190
      WRITE(6,180)
  180 FORMAT(//,10X,'NO REDUCTIONS HAVE BEEN COMPLETED - EXECUTION HAS B
     *EEN TERMINATED')
      GO TO 1000
C
  190 IF(K.GT.1)GO TO 200
      IF(EPOCH.EQ.EPOCHO)GO TO 1000
      WRITE(6,90)EPOCH
      RAOBJ=RAO(1)
      DEOBJ=DEO(1)
      GO TO 290
C
  200 ERROR=ERROR/CONV
      RASUM=0.0D0
      DESUM=0.0D0
C
      DO 230 I=1,K
      RADIFF=ANGLE8(RAO(I),RAO(1))
      IF((DABS(RADIFF*DCOS(DEO(I))).LT.ERROR).AND.(DABS(DEO(I)-DEO(1))
     *.LT.ERROR))GO TO 220
      WRITE(6,210)
  210 FORMAT(//,10X,'AGREEMENT IS UNACCEPTABLE - EXECUTION HAS BEEN TERM
     *INATED')
      GO TO 1000
C
  220 RAO(I)=RAO(1)+RADIFF
      RASUM=RASUM+RAO(I)
  230 DESUM=DESUM+DEO(I)
C
      WRITE(6,240)
  240 FORMAT(//,10X,'AGREEMENT IS ACCEPTABLE')
      WRITE(6,250)EPOCH
  250 FORMAT(///,5X,'UNWEIGHTED MEAN OF THE ABOVE COORDINATES     (EPOCH
     *',F7.1,') -')
C
      RK=DFLOAT(K)
      RAOBJ=RASUM/RK
      DEOBJ=DESUM/RK
C
      IF(EPOCH.EQ.EPOCHO)GO TO 300
      GO TO 290
```

```
C
C
C      INPUT OF POSITION OF OBJECT FOR REDUCTION BY NEWCOMB'S METHOD
C      WHEN REDUCTION FROM PLATE MEASUREMENTS BY COMRIE'S METHOD
C      IS NOT REQUIRED
C
C
  260 READ(5,270)EPOCHO,IDEHO(1),IMIN(1),SEC(1),ISIGN(2),IDEHO(2),
     *IMIN(2),SEC(2)
  270 FORMAT(F6.1,2X,2I3,F7.3,2X,A1,I2,I3,F6.2)
C
      WRITE(6,3)
      WRITE(6,50)(IDENTN(I),I=1,6)
      WRITE(6,60)(IDENTD(I),I=1,14)
      WRITE(6,70)(IDENTO(I),I=1,20)
      WRITE(6,90)EPOCH
C
      RAOBJ=ANGLE1(TCONV,IDEHO(1),IMIN(1),SEC(1))
C
      DEOBJ=ANGLE1(CONV,IDEHO(2),IMIN(2),SEC(2))
      CALL ANGLE4(ISIGN(2),DEOBJ,£290)
      WRITE(6,280)
  280 FORMAT(//,10X,'INVALID SIGN GIVEN WITH ANGULAR DATA - EXECUTION HA
     *S BEEN TERMINATED')
      GO TO 1000
C
C
C      REDUCTION OF POSITION OF OBJECT TO EQUATOR AND EQUINOX OF EPOCH
C      BY NEWCOMB'S METHOD
C
C
  290 CALL NEWCMB(EPOCHO,EPOCH,RAOBJ,DEOBJ)
C
  300 RAOBJ=ANGLE3(RAOBJ)
      CALL ANGLE6(ISIGN(2),DEOBJ)
      CALL ANGLE2(TCONV,RAOBJ,IDEHO(1),IMIN(1),SEC(1))
      CALL ANGLE2(CONV,DEOBJ,IDEHO(2),IMIN(2),SEC(2))
C
      WRITE(6,310)IDEHO(1),IMIN(1),SEC(1),ISIGN(2),IDEHO(2),IMIN(2),
     *SEC(2)
  310 FORMAT(//,33X,'ALPHA =',I3,' HOURS',I3,' MIN',F7.3,' SEC',5X,'DELT
     *A = ',A1,I2,' DEG',I3,' MIN',F6.2,' SEC')
C
C
C      UNIQUE TERMINATION POINT
C
C
 1000 WRITE(6,1010)
 1010 FORMAT('1')
      WRITE(6,1010)
C
      STOP
      END
```

DATA FORMAT FOR ORBIT 2

(Precession correction only)



GW / ORBIT 2 VERSION 1 DATE 710916

REDUCTION OF THE POSITION OF AN OBJECT TO A GIVEN EQUATOR AND EQUINOX BY NEWCOMB'S METHOD

NAME OF OBJECT - 1361 LEUSCHNERIA

TIME OF OBSERVATION - 1935 AUGUST 30 UT 00 HOURS 00 MIN 51.84 SEC

PLACE OF OBSERVATION - ROYAL OBSERVATORY, UCCLE, BELGIUM

TOPOCENTRIC EQUATORIAL COORDINATES OF OBJECT (EPOCH 1950.0) -

ALPHA = 23 HOURS 6 MIN 6.364 SEC DELTA = - 3 DEG 41 MIN 27.45 SEC

DATA FORMAT FOR ORBIT 2

(Reduction of Plate Measurements)

Line	Columns 1–80	Description
1	XXXXXXXXXXXXXXXXXXXXXXXXX XXXX XX XX UT XX HOURS XX MIN XX.XXX SEC	Name of object; Civil date (year-month-day) and time of observation
2	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX	Name or other identification of place of observation
3	XXXXXX	Date of reduction (year-month-day as an integer)
4	ACCESS ALL XXXX.X	Required epoch of final position (years)
5	PLATE XXXXXXXXXXXX, REDUCTIONS X, ERROR XXXX.XX	Plate identification; no. of reductions N to be carried out; max. permissible discrepancy between reductions (arc seconds)
6	XXXX.X ±XXX.XXXX ±XXX.XXXX	Date of observation (years); measured coords. X_0, Y_0 of object
7	XXXX.X	Epoch of equator and equinox to which ref. star catalogue is referred
8	XXXXXX ±XXX.XXXX ±XXX.XXXX	Cat. no. (integer); measured coords. X, Y of 1st reference star
9	XXXXXX ±XXX.XXXX ±XXX.XXXX	— " — ; — " — of 2nd reference star
10	XXXXXX ±XXX.XXXX ±XXX.XXXX	— " — ; — " — of 3rd reference star
11	XXXXXX XX XX XX.XXX ±X.XXXX XXXX.X ±XX XX XX.XX ±X.XXX XXXX.X	Cat. no. (integer); R.A. ($^h\ ^m\ ^s$); μ_α (^{s}); Original epoch of R.A. (years); Dec. (° ′ ″); μ_δ (″); Orig. epoch of Dec. for 1st ref. star
12	XXXXXX XX XX XX.XXX ±X.XXXX XXXX.X ±XX XX XX.XX ±X.XXX XXXX.X	— " — ; — " — ; — " — ; — " — ; — " — ; — " — ; — " — for 2nd ref. star
13	XXXXXX XX XX XX.XXX ±X.XXXX XXXX.X ±XX XX XX.XX ±X.XXX XXXX.X	— " — ; — " — ; — " — ; — " — ; — " — ; — " — ; — " — for 3rd ref. star

Lines 8-13 are repeated for each additional reduction, being 14-19 for 2nd red'n, 20-25 for 3rd etc. Note that $1 \leqslant N \leqslant 9$.

FGW / ORBIT 2 VERSION 1 DATE 720207

REDUCTION OF THE POSITION OF AN OBJECT FROM PLATE MEASUREMENTS BY COMRIE'S METHOD

AND REDUCTION TO A GIVEN EQUATOR AND EQUINOX BY NEWCOMB'S METHOD

NAME OF OBJECT - 22 KALLIOPE

TIME OF OBSERVATION - 1966 NOVEMBER 9 UT 00 HOURS 19 MIN 00 SEC

PLACE OF OBSERVATION - ST.ANDREWS UNIVERSITY OBSERVATORY

PLATE NUMBER - TBS/171

TOPOCENTRIC EQUATORIAL COORDINATES OF OBJECT (EPOCH 1950.0) -

1ST REDUCTION	ALPHA = 3 HOURS 38 MIN 22.984 SEC	DELTA = +13 DEG 48 MIN 52.09 SEC
2ND REDUCTION	ALPHA = 3 HOURS 38 MIN 22.995 SEC	DELTA = +13 DEG 48 MIN 52.22 SEC
3RD REDUCTION	ALPHA = 3 HOURS 38 MIN 22.963 SEC	DELTA = +13 DEG 48 MIN 51.81 SEC
4TH REDUCTION	ALPHA = 3 HOURS 38 MIN 23.040 SEC	DELTA = +13 DEG 48 MIN 51.71 SEC
5TH REDUCTION	ALPHA = 3 HOURS 38 MIN 23.008 SEC	DELTA = +13 DEG 48 MIN 52.06 SEC
6TH REDUCTION	ALPHA = 3 HOURS 38 MIN 23.021 SEC	DELTA = +13 DEG 48 MIN 51.92 SEC

AGREEMENT IS ACCEPTABLE

UNWEIGHTED MEAN OF THE ABOVE COORDINATES (EPOCH 1975.0) -

ALPHA = 3 HOURS 39 MIN 46.569 SEC DELTA = +13 DEG 53 MIN 41.00 SEC

Chapter 1.3 The Determination Of The Preliminary Orbit

The initial determination of the orbit of a minor planet to obtain the preliminary orbit is the fundamental problem of the present work. It consists in the reduction of a set of observed topocentric coordinates (i = 1, 2, 3) (α_i, δ_i) of the minor planet to obtain a complete knowledge of the planet's orbit in the form of the orbital elements e, a, M_o, ω, i, Ω where

- e = eccentricity of the orbit;
- a = semi-major axis of the orbit;
- M_o = mean anomaly of the planet at some epoch t_o;
- ω = orientation of the major axis (argument of perihelion);
- i = inclination of the orbit;
- Ω = longitude of the ascending node.

Essentially, the problem of heliocentric orbit determination reduces to the following: let (i = 1, 2, 3) $\underline{r}_i \equiv (r_{xi}, r_{yi}, r_{zi})$ be the heliocentric radius vectors of the planet at three instants t_1, t_2, t_3; similarly, let (i = 1, 2, 3) $\underline{R}_i \equiv (R_{xi}, R_{yi}, R_{zi})$ and $\underline{\rho}_i \equiv (\rho_{xi}, \rho_{yi}, \rho_{zi})$ be the topocentric radius vectors of the sun and the planet respectively at the same instants. From a knowledge of the solar radius vectors $\underline{R}_i$ and the planet's direction cosines $\underline{\hat{\rho}}_i$ (where $\underline{\rho}_i = \rho_i\underline{\hat{\rho}}_i$) we wish to determine (i = 1, 2, 3) $\underline{r}_i$ and hence the orbital elements. This can only be accomplished by the introduction into the computation of certain geometrical and dynamical constraints and, in general, it is the manner in which these are introduced that distinguishes one method of reduction from another.

The basic method used in the present work is that due to Gauss (1857) which was developed by him shortly after the discovery of the first minor planet and which is eminently suitable for the determination of their elliptical orbits. The method is used in the modified form given by Merton (1925 and 1929) for machine computation; as might be expected, it was found to be readily adaptable to computer reduction. Numerous accounts giving a full development of the method exist in the literature (for example in Williams (1934, Chapter 8), Herget (1948, Chapter 5), Escobal (1965, Section 7.3) and Baker (1967, Sections 1.3.1 and 1.3.7)) and we here confine ourselves to giving a brief outline of the method as it is used in the ORBIT 1 computer program.

We begin by considering the vectors $\underline{r}_i$, $\underline{R}_i$ and $\underline{\rho}_i$ defined above; clearly these are related by

$$(i = 1, 2, 3) \quad \underline{r}_i = \underline{\rho}_i - \underline{R}_i$$

Furthermore, since the planet moves in a plane passing through the sun (the above-mentioned geometrical condition) we can write

$$\underline{r}_2 = c_1\underline{r}_1 + c_3\underline{r}_3$$

where $\quad c_1 = \dfrac{[r_2, r_3]}{[r_1, r_3]}, \qquad c_3 = \dfrac{[r_1, r_2]}{[r_1, r_3]}$

and, in general, $[r_i, r_j]$ is the area of the triangle formed by the sun and the positions of the planet at times t_i and t_j.

Combining these two expressions to eliminate the $\underline{r}_i$ we obtain Gauss's fundamental equation

$$c_1\underline{\rho}_1 - \underline{\rho}_2 + c_3\underline{\rho}_3 = c_1\underline{R}_1 - \underline{R}_2 + c_3\underline{R}_3$$

and if we now scalar multiply both sides of this by $(\hat{\underline{\rho}}_1 \times \hat{\underline{\rho}}_3)$ we obtain

$$-\rho_2\hat{\underline{\rho}}_2 . (\hat{\underline{\rho}}_1 \times \hat{\underline{\rho}}_3) = c_1\underline{R}_1 . (\hat{\underline{\rho}}_1 \times \hat{\underline{\rho}}_3) - \underline{R}_2 . (\hat{\underline{\rho}}_1 \times \hat{\underline{\rho}}_3) + c_3\underline{R}_3 . (\hat{\underline{\rho}}_1 \times \hat{\underline{\rho}}_3)$$

which, since the $\hat{\underline{\rho}}_i$ and $\underline{R}_i$ are known, reduces to an equation of the form

$$E\rho_2 = F_1 c_1 - F_2 + F_3 c_3 \qquad \text{(A)}$$

Consider now the triangle formed by the sun, the observer and the planet at t_2; this has sides of length r_2, R_2 and ρ_2 and thus we can write

$$r_2^{\ 2} = \rho_2^{\ 2} - 2\rho_2 R_2 \cos\psi_2 + R_2^{\ 2}$$

where ψ_2 is the angle subtended at the observer by the sun and the planet at instant t_2; ie the angle between $\underline{\rho}_2$ and $\underline{R}_2$. Since

$$\cos\psi_2 = \frac{\underline{\rho}_2 \cdot \underline{R}_2}{\rho_2\ R_2}$$

we can rewrite the equation as

$$r_2^{\ 2} = \rho_2^{\ 2} - 2\rho_2 \hat{\underline{\rho}}_2 \cdot \underline{R}_2 + R_2^{\ 2}$$

which, since $\hat{\underline{\rho}}_2$ and $\underline{R}_2$ are known, is an equation of the form

$$r_2^{\ 2} = \rho_2^{\ 2} - G\rho_2 + H \qquad \text{(B)}$$

Thus the first step in the computation of the orbit is to use the topocentric direction cosines of the planet $(\hat{\rho}_{xi}, \hat{\rho}_{yi}, \hat{\rho}_{zi}) \equiv \hat{\underline{\rho}}_i$ obtained from its observed coordinates (α_i, δ_i) and the topocentric rectangular coordinates of the sun $(R_{xi}, R_{yi}, R_{zi}) \equiv \underline{R}_i$ obtained from geocentric tables and corrected for parallax to evaluate the coefficients E, F_1, F_2, F_3, G and H for the equations (A) and (B). These equations having been thus set up we can then proceed to obtain first approximations to the $\underline{r}_i$.

In order to do this we require a knowledge of c_1 and c_3 and in the first instance we obtain approximate values for these from the equations

$$c_1 = \frac{\tau_1}{\tau_2}\left\{1 + \frac{\tau_2^2 - \tau_1^2}{6r_2^3}\right\}; \quad c_3 = \frac{\tau_3}{\tau_2}\left\{1 + \frac{\tau_2^2 - \tau_3^2}{6r_2^3}\right\} \qquad (*)$$

where $\tau_1 = k(t_3 - t_2)$, $\tau_2 = k(t_3 - t_1)$, $\tau_3 = k(t_2 - t_1)$ are normalised times, k being the Gaussian constant = 0.01720209895 day^{-1} and t_i the instants of the observations in days. (These expressions represent a first approximation to the dynamical constraint mentioned earlier.)

Clearly, since τ_1, τ_2 and τ_3 are known, the elimination of c_1 and c_3 in equation (A) by substitution of the above expressions (*) will result in an equation of the form

$$\rho_2 = A - \frac{B}{r_2^3} \qquad (A_1)$$

Reintroducing equation (B) as

$$r_2^2 = \rho_2^2 - G\rho_2 + H \qquad (B_1)$$

we see that we can obtain values for ρ_2 and r_2 by simultaneous solution of the two equations. This is accomplished by means of successive approximations, r_2 = 2.8 a.u. being taken as the first approximation since this is the mean heliocentric distance of the minor planets. This value is used in equation (A_1) to obtain a value of ρ_2 which is then inserted into (B_1) to obtain a second approximation of r_2. This is resubstituted into (A_1) and the process is continued until successive approximations of ρ_2 and r_2 are identical to the required accuracy. In the ORBIT library, the solution of these equations is accomplished in exactly this way by the subroutine SOLVAB. The subroutine is arranged to terminate when successive approximations agree to within 10^{-8} a.u. and it uses multiple returns arranged in the way already described for iterative subroutines in Chapter 1.1. It should be noted that the value of the first approximation to r_2 is entered into the subroutine from the main program.

The r_2 and ρ_2 obtained by the solution of these equations are, of course, only approximate since the coefficients A and B in equation (A_1) have been obtained using approximate expressions for c_1 and c_3. It should be noted, however, that the **coefficients** G and H in equation (B_1) are exact and thus do not change with successive approximations to the $\underline{r}_i$ in the subsequent computation.

The next step in the reduction is to use the value of r_2 to obtain first approximations for c_1 and c_3 by means of the expressions (*) above and, this having been done, we use Gauss's fundamental equation

$$c_1\underline{\rho}_1 - \underline{\rho}_2 + c_3\underline{\rho}_3 = c_1\underline{R}_1 - \underline{R}_2 + c_3\underline{R}_3$$

to obtain ρ_1 and ρ_3. The equation is used in the form

$$c_1\,\hat{\rho}_{x1}\,\rho_1 + c_3\,\hat{\rho}_{x3}\,\rho_3 = c_1\,R_{x1} - R_{x2} + c_3\,R_{x3} + \hat{\rho}_{x2}\,\rho_2$$

$$c_1\,\hat{\rho}_{y1}\,\rho_1 + c_3\,\hat{\rho}_{y3}\,\rho_3 = c_1\,R_{y1} - R_{y2} + c_3\,R_{y3} + \hat{\rho}_{y2}\,\rho_2$$

$$c_1\,\hat{\rho}_{z1}\,\rho_1 + c_3\,\hat{\rho}_{z3}\,\rho_3 = c_1\,R_{z1} - R_{z2} + c_3\,R_{z3} + \hat{\rho}_{z2}\,\rho_2$$

and since ρ_1 and ρ_3 are the only unknowns we require only two of these component equations to obtain a solution. Traditionally, the equation in z has therefore been used to provide a numerical check on the values of ρ_1 and ρ_3 obtained and, although this is generally unnecessary in computer reduction, it has been retained in the present work.

```
      SUBROUTINE SOLVAB(CONSAN,CSR2M3,CRHO2B,CONSBN,SR2,RHO2,ITNUM,*)
C
C     SOLUTION OF EQUATIONS AN AND BN BY SUCCESSIVE APPROXIMATIONS
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, APRIL 1971
C
      REAL*8 CONSAN,CSR2M3,CRHO2B,CONSBN,SR2,PREVSR,PREVRO,RHO2,DSQRT,
     *DABS
C
      PREVSR=0.0D0
      PREVRO=0.0D0
      ITNUM=0
C
   10 ITNUM=ITNUM+1
      RHO2=CONSAN+CSR2M3/SR2**3.0D0
      SR2=DSQRT(RHO2**2.0D0+CRHO2B*RHO2+CONSBN)
C
      IF((DABS(PREVRO-RHO2).LT.1.0D-8).AND.(DABS(PREVSR-SR2).LT.1.0D-8))
     *RETURN1
      IF(ITNUM.GT.20)RETURN
C
      PREVRO=RHO2
      PREVSR=SR2
      GO TO 10
C
      END
```

The equations are solved in the normal way in the ORBIT library by means of subroutine GAUSS. The right hand side of each equation is first evaluated using the statement function RHS and the first two equations are then solved simultaneously to obtain ρ_1 and ρ_3. These values are then used to evaluate the left hand side of the equation in z and the agreement between this and the right hand side is checked, a multiple return arrangement again being used to allow for the (remote) possibility of disagreement.

We now have approximate values for ρ_1, ρ_2 and ρ_3 and we use these together with the known values of $\underline{R}_i$ and $\underline{\hat{\rho}}_i$ to obtain the first approximations to the $\underline{r}_i$ thus:

$$(i = 1,2,3) \quad \underline{r}_i = \rho_i \, \underline{\hat{\rho}}_i - \underline{R}_i$$

The second approximations to the $\underline{r}_i$ are obtained in a similar manner to the first with the exception of the computation of c_1 and c_3, for which more sophisticated expressions can now be used. These are the formulae of Gibbs (1888):

$$c_1 = \frac{\tau_1}{\tau_2} \left\{ \frac{1 + B_1 r_1^{-3}}{1 - B_2 r_2^{-3}} \right\} \; ; \; c_3 = \frac{\tau_3}{\tau_2} \left\{ \frac{1 + B_3 r_3^{-3}}{1 - B_2 r_2^{-3}} \right\}$$

where

$$B_1 = 1/12 \, (\tau_3^2 + \tau_1\tau_3 - \tau_1^2)$$

$$B_2 = 1/12 \, (\tau_1^2 + 3\tau_1\tau_2 + \tau_3^2)$$

$$B_3 = 1/12 \, (\tau_1^2 + \tau_1\tau_3 - \tau_3^2)$$

and again $\tau_1 = k(t_3 - t_2)$, $\tau_2 = k(t_3 - t_1)$, $\tau_3 = k(t_2 - t_1)$. The t_i should now be corrected to Ephemeris Time since we are aiming for higher precision than in the first approximation and, furthermore, planetary aberration should now be taken into account. This is done by subtracting

```
      SUBROUTINE GAUSS(RX1,RX2,RX3,RY1,RY2,RY3,RZ1,RZ2,RZ3,URHOX1,
     *URHOX2,URHOX3,URHOY1,URHOY2,URHOY3,URHOZ1,URHOZ2,URHOZ3,RHO1,RHO2,
     *RHO3,C1,C3,RHSZ,LHSZ,*)
C
C     SOLUTION OF GAUSS'S FUNDAMENTAL EQUATION FOR RHO1 AND RHO3
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, APRIL 1971
C
      REAL*8 RX1,RX2,RX3,RY1,RY2,RY3,RZ1,RZ2,RZ3,URHOX1,URHOX2,URHOX3,
     *URHOY1,URHOY2,URHOY3,URHOZ1,URHOZ2,URHOZ3,RHO1,RHO2,RHO3,C1,C3,
     *RHSZ,LHSZ,RHSX,RHSY,CRHO1X,CRHO3X,CRHO1Y,CRHO3Y,CRHO1Z,CRHO3Z,A,B,
     *C,DABS,RHS,R1,R2,R3,URHO2
C
      RHS(R1,R2,R3,URHO2)=C1*R1-R2+C3*R3+URHO2*RHO2
C
      RHSX=RHS(RX1,RX2,RX3,URHOX2)
      RHSY=RHS(RY1,RY2,RY3,URHOY2)
      RHSZ=RHS(RZ1,RZ2,RZ3,URHOZ2)
C
      CRHO1X=C1*URHOX1
      CRHO3X=C3*URHOX3
      CRHO1Y=C1*URHOY1
      CRHO3Y=C3*URHOY3
      CRHO1Z=C1*URHOZ1
      CRHO3Z=C3*URHOZ3
C
      A=CRHO1X*CRHO3Y
      B=RHSX/CRHO1X-CRHO3X*RHSY/A
      C=1.0D0-CRHO3X*CRHO1Y/A
C
      RHO1=B/C
      RHO3=RHSY/CRHO3Y-CRHO1Y*RHO1/CRHO3Y
C
      LHSZ=CRHO1Z*RHO1+CRHO3Z*RHO3
C
      IF(DABS(LHSZ-RHSZ).LT.1.0D-6)RETURN1
C
      RETURN
      END
```

from the E.T. the light time for the distance between the planet and the observer, ie.

$$(i = 1,2,3) \text{ corrected } t_i = t_i - 0.00577560\ \rho_i$$

where the numerical coefficient of the ρ_i is called the light time for unit distance, being the time in days during which light travels one a.u. Thus the corrected t_i represent the E.T.'s at the instants at which the observed rays of light left the minor planet.

Gibbs' formulae are used in such a way as to allow improved values of r_2 and ρ_2 to be found in much the same way as before. Having used the formulae to obtain values of c_1 and c_3 from the r_i obtained in the previous approximation, we then compute

$$\bar{\nu}_1 = \left(c_1 - \frac{\tau_1}{\tau_2}\right) r_2^{\,3} \ ; \ \bar{\nu}_3 = \left(c_3 - \frac{\tau_3}{\tau_2}\right) r_2^{\,3}$$

again using the r_2 from the previous approximation. We can now rearrange these equations as

$$c_1 = \frac{\tau_1}{\tau_2} + \frac{\bar{\nu}_1}{r_2^{\,3}} \ ; \ c_3 = \frac{\tau_3}{\tau_2} + \frac{\bar{\nu}_3}{r_2^{\,3}} \qquad (**)$$

in which the τ_i, $\bar{\nu}_1$ and $\bar{\nu}_3$ are known and r_2, c_1 and c_3 are unknowns of the next approximation.

Substituting these expressions in equation (A) to eliminate c_1 and c_3 we again obtain an equation of the form

$$\rho_2 = A - \frac{B}{r_2^{\,3}} \qquad (A_2)$$

which may again be solved with equation (B)

$$r_2^{\,2} = \rho_2^{\,2} - G\rho_2 + H \qquad (B_2)$$

to obtain new values of r_2 and ρ_2. As before, the solution is obtained by means of subroutine SOLVAB, the value of r_2 obtained in the previous approximation being taken as the initial value for the iteration.

The remainder of this second approximation to the $\underline{r}_i$ now proceeds in exactly the same way as the first, c_1 and c_3 being determined from the expressions (**) above and Gauss's equation being solved (using subroutine GAUSS) to obtain new values of ρ_1 and ρ_3 which are then used with ρ_2 to obtain the $\underline{r}_i$ by means of

$$(i = 1,2,3) \quad \underline{r}_i = \rho_i\, \underline{\hat{\rho}}_i - \underline{R}_i$$

Having completed the second approximation we must now examine the convergence of the values being obtained since, although it is unlikely that the second approximation will be the final one, all subsequent approximations will be obtained by exactly the same method and can therefore be computed by the same section of program. Thus the agreement between successive approximations must be checked at this point; it is checked firstly between successive values of c_1 and c_3, and if these agree to better than 10^{-6}, the agreement between the successive approximations to the $\underline{r}_i$ are checked. If these agree sufficiently well, the iteration is terminated and the latest approximations to the $\underline{r}_i$ are adopted as the final values. If the agreement between successive values is not good enough, and if less than nine approximations have been obtained, another iteration is executed.

As we have already stated, the third and subsequent approximations are obtained in exactly the same way as the second. Thus, in the nth approximation, Gibbs' formulae are used with the latest light time correction applied to the t_i to obtain the c_1 and c_3 which are then used to determine $\overline{\nu}_1$ and $\overline{\nu}_3$. The equations

$$c_1 = \frac{\tau_1}{\tau_2} + \frac{\overline{\nu}_1}{r_2^{\,3}} \;;\; c_3 = \frac{\tau_3}{\tau_2} + \frac{\overline{\nu}_3}{r_2^{\,3}} \qquad (*\ldots*)$$

are then formed and substituted in equation (A) to obtain

$$\rho_2 = A - \frac{B}{r_2^{\,3}} \qquad (\mathbf{A_n})$$

which is solved with

$$r_2^{\,2} = \rho_2^{\,2} - G\rho_2 + H \qquad (\mathbf{B_n})$$

to give r_2 and ρ_2. Equations $(*\ldots.*)$ are then evaluated giving c_1 and c_3, and Gauss's equation is solved for ρ_1 and ρ_3. Finally, the $\underline{r}_i$ are obtained from

$$(i = 1,2,3)\ \underline{r}_i = \rho_i\,\hat{\underline{\rho}}_i - \underline{R}_i.$$

It will be noted that the whole process of successive approximations for obtaining the final values of the $\underline{r}_i$ is merely a large-scale example of Fig. 1(b). The convergence is normally very rapid due to the high degree of refinement in the method, and as a rule, no more than three or four approximations to the $\underline{r}_i$ are required. A numerical check on the final $\underline{r}_i$ can be carried out using the geometrical relation

$$c_1\underline{r}_1 + c_3\underline{r}_3 = \underline{r}_2$$

and this check is included in the present work although again it should not be necessary in computer reduction.

The next stage in the computation is the determination of the ratio of the area of the sector to that of the triangle in the orbit. This important concept is fundamental to the Gaussian method and its determination forms an intermediate stage between the calculation of the $\underline{r}_i$ and the evaluation of the orbital elements.

Consider the planet at two instants $t_{(1)}$ and $t_{(2)}$ when its heliocentric distances, eccentric anomalies and true anomalies are $r_{(1)}$, $r_{(2)}$, $E_{(1)}$, $E_{(2)}$ and $v_{(1)}$, $v_{(2)}$ respectively. We define

$$2f = v_{(2)} - v_{(1)};$$

$$2g = E_{(2)} - E_{(1)}.$$

Now the area of the sector bounded by the orbit and the radius vectors of the planet at $t_{(1)}$ and $t_{(2)}$ is $1/2\ (\mu p)^{\frac{1}{2}}\ (t_{(2)} - t_{(1)})$ where $p = a\ (1 - e^2)$ and $\mu = k^2$ (for a minor planet). Since $k\ (t_{(2)} - t_{(1)}) = \tau$ (normalised time) we have

$$\text{area of sector} = 1/2\ \tau\ \sqrt{p}.$$

The area of the triangle formed by the sun and the positions of the planet at the two instants is $1/2\ r_{(1)}\ r_{(2)}\ \sin\ (v_{(2)} - v_{(1)})$, ie

$$\text{area of triangle} = 1/2\ r_{(1)}\ r_{(2)}\ \sin 2f.$$

Thus, if $\overline{y}$ is the ratio of the area of the sector to the area of the triangle, then

$$\bar{y} = \frac{\tau\sqrt{p}}{r_{(1)}\, r_{(2)} \sin 2f} \quad :$$

The elimination of p from this expression leads eventually to Gauss's equations for the determination of $\bar{y}$:

$$\left.\begin{aligned} &\bar{y}^2 = \frac{m^2}{l + \sin^2\tfrac{1}{2}g} \quad ; \quad \bar{y}^3 - \bar{y}^2 = m^2 \left\{\frac{2g - \sin 2g}{\sin^3 g}\right\} \\ &\text{where} \quad m^2 = \frac{\tau^2}{(2\sqrt{r_{(1)}\, r_{(2)}} \cos f)^3} \; ; \quad l = \frac{r_{(1)} + r_{(2)}}{4\sqrt{r_{(1)}\, r_{(2)}} \cos f} - \frac{1}{2}. \end{aligned}\right\} \quad \text{(C)}$$

(An extensive account of the development of these and the following equations is given by Dubyago (1961, Section 41).)

If g is small, as is the case in preliminary orbit determination, we can use the cubic equation in $\bar{y}$ derived by Gauss using series expansions:

$$\left.\begin{aligned} &\bar{y}^3 - \bar{y}^2 - h\bar{y} - \tfrac{1}{9}h \doteq 0 \\ &\text{where} \quad h = \frac{m^2}{\frac{5}{6} + l + \xi}, \quad \xi = \frac{2}{35}x^2 + \frac{52}{1575}x^3 + \ldots. \\ &\text{and} \quad x = \frac{m^2}{\bar{y}^2} - l \quad (= \sin^2\tfrac{1}{2}g). \end{aligned}\right\} \quad \text{(D)}$$

Furthermore, if we define K by $K^2 = r_{(1)}\, r_{(2)} + \underline{r}_{(1)}\cdot\underline{r}_{(2)}$ $(= 2r_{(1)}\, r_{(2)} \cos^2 f)$ and substitute this in equations (C) we obtain

$$m^2 = \frac{\tau^2}{2\sqrt{2}\,K^3} \quad ; \quad l = \frac{r_{(1)} + r_{(2)}}{2\sqrt{2}\,K} - \tfrac{1}{2} \qquad \text{(E)}$$

Thus the procedure for determining $\overline{y}_i$ (i = 1,2,3) for the three observations is as follows: firstly we evaluate the K_i from

$$K_1^2 = r_2 r_3 + \underline{r}_2 \cdot \underline{r}_3$$
$$K_2^2 = r_1 r_3 + \underline{r}_1 \cdot \underline{r}_3$$
$$K_3^2 = r_1 r_2 + \underline{r}_1 \cdot \underline{r}_2$$

using the final values of the $\underline{r}_i$ obtained in the previous section of the work. We next compute the m_i^2 and l_i using equations (E):

$$(i = 1,2,3) \quad m_i^2 = \frac{\tau_i^2}{2\sqrt{2}\, K_i^3} \quad ; \quad l_i = \frac{(r_{sum})_i}{2\sqrt{2}\, K_i} - \frac{1}{2}$$

where

$$(r_{sum})_1 = r_2 + r_3$$
$$(r_{sum})_2 = r_1 + r_3$$
$$(r_{sum})_3 = r_1 + r_2$$

and the τ_i are the final values (corrected for light time) obtained in the previous section.

The equations (D) are then solved by successive approximations, the actual cubic in $\overline{y}$ (which has only one positive root) being solved by Hansen's continued fraction

$$\overline{y} = 1 + \frac{10}{11} \cfrac{\frac{11h}{9}}{1 + \cfrac{\frac{11h}{9}}{1 + \frac{11h}{9} \cdots}}$$

which is valid for small h. The first approximations for the h_i are obtained by neglecting the ξ_i:

$$(i = 1,2,3)\quad h_i = \frac{m_i^2}{\frac{5}{6} + l_i}$$

and these values are substituted into the continued fraction to obtain the first approximations to the $\bar{y}_i$. These are then used to compute

$$(i = 1,2,3)\quad x_i = \frac{m_i^2}{\bar{y}_i^2} - l_i$$

and $$(i = 1,2,3)\quad \xi_i = \frac{2}{35}\, x_i^2 + \frac{52}{1575}\, x_i^3$$

which are used to obtain second approximations to the h_i:

$$(i = 1,2,3)\quad h_i = \frac{m_i^2}{\frac{5}{6} + l_i + \xi_i}$$

Second approximations to the $\bar{y}_i$ are then obtained, and the process is repeated until the successive values of the h_i and $\bar{y}_i$ agree sufficiently well. This iterative procedure for solving equations (D) is incorporated into the computer reduction as subroutine SOLVD which follows exactly the above method, terminating when successive values of the h_i and $\bar{y}_i$ agree to within 10^{-8}. As a rule, this is after the second approximation, the effect of the ξ_i being very small. SOLVD is arranged with multiple returns in the same way as the other iterative subroutines in ORBIT.

A check on the values obtained for $\bar{y}_i$ is afforded by Kepler's second law, which can be written in the form

$$c_1 = \frac{\tau_1}{\tau_2}\frac{\bar{y}_2}{\bar{y}_1} \quad ; \quad c_3 = \frac{\tau_3}{\tau_2}\frac{\bar{y}_2}{\bar{y}_3}$$

```
      SUBROUTINE SOLVD(MSQ,L,H,YBAR,JAPPRX,*)
C
C     SOLUTION OF EQUATIONS D BY SUCCESSIVE APPROXIMATIONS
C
C     F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, MAY 1971
C
      REAL*8 MSQ,L,XI,PREVH,PREVY,H,HTERM,YBAR,X,DABS
C
      XI=0.0D0
      PREVH=0.0D0
      PREVY=0.0D0
      JAPPRX=0
C
   10 JAPPRX=JAPPRX+1
      H=MSQ/(0.8333333333333333D0+L+XI)
      HTERM=1.2222222222222222D0*H
      YBAR=1.0D0+1.1111111111111111D0*H/(1.0D0+HTERM/(1.0D0+HTERM))
C
      IF((DABS(PREVH-H).LT.1.0D-8).AND.(DABS(PREVY-YBAR).LT.1.0D-8))
     *RETURN1
      IF(JAPPRX.GT.5)RETURN
C
      X=MSQ/YBAR**2.0D0-L
      XI=0.05714285714285700*X**2.0D0+0.03301587301587301D0*X**3.0D0
C
      PREVH=H
      PREVY=YBAR
      GO TO 10
C
      END
```

The values of c_1 and c_3 obtained from these expressions are compared with the values obtained in the final approximation to the $\underline{r}_i$; there should be good agreement between the two sets of values if the accuracy has been maintained.

We now have values of $\underline{r}_i$, τ_i and $\overline{y}_i$ ($i = 1,2,3$) and can thus proceed to the computation of the orbital elements. We begin with the evaluation of the parameter of the orbit p ($= a(1-e^2)$). We could obtain this directly from

$$\overline{y} = \frac{\tau\sqrt{p}}{r_{(1)}\, r_{(2)} \sin 2f}$$

using any one of the $\overline{y}_i$ and the corresponding τ_i, r_i and f_i (preferably $\overline{y}_2$, τ_2, etc. since this would give the greatest accuracy due to the larger values of τ_2 and $2f_2 = v_3 - v_1$). In practice, however, it is normal to proceed by defining

$$\sigma = \frac{\underline{r}_1 . \underline{r}_3}{r_1^2} \quad \text{and } \underline{r}_o = \underline{r}_3 - \sigma\underline{r}_1$$

whence
$$p = \frac{(r_1 r_o)^2}{\tau_2^2}\, \overline{y}_2^{\,2}$$

since
$$r_o = r_3 \sin (v_3 - v_1)$$

(see Dubyago (1961, p. 148)).

We next obtain the eccentricity e and true anomalies v_1 and v_3 by means of the polar equations

$$r_1 = \frac{p}{1 + e \cos v_1} \quad ; \quad r_3 = \frac{p}{1 + e \cos v_3}$$

From these equations we obtain

$$e \cos v_1 = \frac{p}{r_1} - 1; \; e \cos v_3 = \frac{p}{r_3} - 1$$

But $$e \cos v_3 = e \cos (v_1 + (v_3 - v_1))$$

$$= e \cos v_1 \cos (v_3 - v_1) - e \sin v_1 \sin (v_3 - v_1)$$

$$= \frac{p}{r_3} - 1$$

whence $$\left.\begin{aligned} e \sin v_1 &= \frac{\left(\frac{p}{r_1} - 1\right) \cos (v_3 - v_1) - \left(\frac{p}{r_3} - 1\right)}{\sin (v_3 - v_1)} \\ e \cos v_1 &= \frac{p}{r_1} - 1 \end{aligned}\right\}$$

We thus proceed in the computation by evaluating

$$\cos (v_3 - v_1) = \frac{\underline{r}_1 . \underline{r}_3}{r_1 \, r_3}$$

and $$\sin (v_3 - v_1) = \sqrt{(1 - \cos^2 (v_3 - v_1))}$$

and thence the right hand sides of the above bracketed expressions. Squaring and adding these latter gives e^2 and dividing them gives $\tan v_1$. The value of v_3 is obtained using $\cos (v_3 - v_1)$ evaluated above.

The semi-major axis a and mean daily motion n are obtained directly from

$$a = \frac{p}{(1 - e^2)}$$

and

$$n = \frac{k}{a^{3/2}}$$

where k is the Gaussian constant. The eccentric anomalies E_1 and E_3 are obtained from v_1 and v_3 by the equations

$$\tan \frac{E_1}{2} = \sqrt{\frac{1 - e}{1 + e}} \tan \frac{v_1}{2} \quad ; \quad \tan \frac{E_3}{2} = \sqrt{\frac{1 - e}{1 + e}} \tan \frac{v_3}{2}$$

using the function ANGLE7 described in Chapter 1.1, a numerical check on the results being provided by the evaluation of each side of

$$a(1 - e^2)^{\frac{1}{2}} \sin \tfrac{1}{2} (E_3 - E_1) = (r_1 r_3)^{\frac{1}{2}} \sin \tfrac{1}{2} (v_3 - v_1)$$

Kepler's equation

$$M = E - e \sin E$$

is then used to obtain the mean anomalies M_1 and M_3, and these are checked by recomputing the mean daily motion n from

$$n = \frac{M_3 - M_1}{t_3 - t_1}$$

where the t_i are in E.T. corrected for planetary aberration.

Although it is not normally quoted as an element for elliptical orbits, the time of perihelion passage T (in E.T.) is computed using

$$T = t_1 - \frac{M_1}{n}$$

This gives a value of T for the last perihelion passage to occur before the first observation. The value is checked by recomputing M_3 using

$$M_3 = n(t_3 - T)$$

The element which normally replaces T in elliptical orbits is the mean anomaly M_o at some specific epoch t_o (E.T.) usually chosen to be near the times of the observations since it is generally taken as the epoch to which all the elements refer. Three alternative ways of computing M_o are readily available and, although it is unnecessary in computer reduction, all are included in the present work as they would be in a manual computation. The three alternative expressions for M_o are

$$M_o = n\,(t_o - T)$$
$$M_o = M_1 + n\,(t_o - t_1)$$
$$M_o = M_3 + n\,(t_o - t_3)$$

where, again, the t_i are in E.T. corrected for planetary aberration.

To find the remaining three elements ω, i and Ω, we require a knowledge of the vectorial equatorial constants P_x, P_y, P_z, Q_x, Q_y, Q_z which are the components in the equatorial system of the unit vectors $\underline{\hat{P}}$ and $\underline{\hat{Q}}$ directed along the major axis and (passing through the sun) parallel to the minor axis of the orbit respectively. The equations

$$\underline{r}_1 = \underline{\hat{P}}\, r_1 \cos v_1 + \underline{\hat{Q}}\, r_1 \sin v_1$$

and

$$\underline{r}_3 = \underline{\hat{P}}\, r_3 \cos v_3 + \underline{\hat{Q}}\, r_3 \sin v_3$$

give us

$$\underline{\hat{P}} = \frac{\underline{r}_1\, r_3 \sin v_3 - \underline{r}_3\, r_1 \sin v_1}{r_1\, r_3 \sin (v_3 - v_1)}$$

and

$$\underline{\hat{Q}} = \frac{\underline{r}_3\, r_1 \cos v_1 - \underline{r}_1\, r_3 \cos v_3}{r_1\, r_3 \sin (v_3 - v_1)}$$

from which $\underline{\hat{P}}$ and $\underline{\hat{Q}}$ could be evaluated directly, but we again find it more convenient to proceed using

$$\sigma = \frac{\underline{r}_1 \cdot \underline{r}_2}{r_1^2} \quad \text{and} \quad \underline{r}_o = \underline{r}_3 - \sigma\, \underline{r}_1$$

whence
$$\hat{\underline{P}} = \frac{\cos v_1}{r_1}\, \underline{r}_1 - \frac{\sin v_1}{r_o}\, \underline{r}_o$$

and
$$\hat{\underline{Q}} = \frac{\sin v_1}{r_1}\, \underline{r}_1 + \frac{\cos v_1}{r_o}\, \underline{r}_o$$

since
$$r_o = r_3 \sin (v_3 - v_1)$$

(see Dubyago (1961, p. 151)). Having obtained $\hat{\underline{P}}$ and $\hat{\underline{Q}}$ it is usual to evaluate the sums of the squares of their components (which we expect to be unity) and their scalar product $\hat{\underline{P}}.\hat{\underline{Q}}$ (which should, of course, be zero since the vectors form an orthogonal system).

It now only remains to compute the values of ω, i and Ω from P_x, P_y, P_z, Q_x, Q_y, Q_z and the transformations for this have already been given in Chapter 1.1. The subroutine ORBEL is not used in the ORBIT 1 program, although the identical method is used with the addition of a numerical check on the value of i using

$$\cos i = -(P_x \sin \omega + Q_x \cos \omega) \operatorname{cosec} \Omega$$

being included. It should be noted that no provision has been made for the inclination i exceeding 90° since there are no known minor planets with retrograde orbits.

As we stated at the outset, the ORBIT 1 computer program follows exactly the method we have given here to obtain the preliminary orbital elements. In addition to the computation we have described, there is a short introductory section at the beginning of the program concerned with the input of data, its transformation into the required form, and elementary checks on the data to ensure that no obvious errors are present. The input data format is given on page 90, and it will be seen that the data can be broadly divided into preliminary data (lines 1-7), observed data (lines 8-10) and data extracted from the "Astronomical Ephemeris" (lines 11-19). The value taken for ΔT (the difference between Ephemeris Time and Universal Time) should be a mean value for the period covered by the observations; since ΔT is an observed quantity it may not be known for the time of the observations and in this case it must be extrapolated from known values.

Having read in and checked the data, the program then executes preliminary computations to obtain the topocentric coordinates of the sun and the direction cosines of the minor planet at the instant of each observation before proceeding to the computation proper. The subroutines used in these preliminary computations have already been described in Chapter 1.1.

The output of the program is as given in the sample on pages 92 to 104. It will be seen that the program differs from normal practice in that virtually all the intermediate results are presented, in much the same way as in a manual computation. Also, an attempt has been made to give some explanation in the output of the main computational processes involved in the reduction, the "Notes on the Equations Quoted" at the beginning of the output with the references to Baker (1967) and Dubyago (1961) being the primary example of this. This style of computing was adopted so that a user would be able to follow the progress of the computation without having to consult the program itself and thus without needing to be familiar with the FORTRAN language. A particular example of this would be in the use of the program to check manual computations (by students, for example), this being the reason why most of the numerical checks on the work have been retained.

In contrast to the week or so required by an experienced computer for the manual computation of an orbit by Gauss's method, the execution time of ORBIT 1 under the Model 44 Programming System is 8.0 seconds.

The accuracy of the program in terms of the precision with which it will compute orbital elements from a given set of observations depends partly on the limits set on the agreement of successive values in the iterative procedures, and partly on the validity of the approximations made in the method. Preliminary orbit methods in general are particularly susceptible to loss in accuracy due to the approximations made within them (but this does not necessarily detract from their value since the orbits obtained usually undergo subsequent correction anyway). However, the approximations made within the Gaussian method as described here are such that a high degree of precision is maintained throughout unless critical conditions such as near-coincidence of the orbit plane with the ecliptic are encountered.

The program was checked for errors and general accuracy on its completion by comparing the results obtained from it with those obtained from a manual computation by the same method, the same initial data being used in both cases. The example chosen was a determination of the orbit of 1361 Leuschneria from observations made by Herget and Arend (1936), the manual computation having been undertaken by D Kilkenny (1968). The elements obtained by Kilkenny were

Epoch 1935 July $17^{d}.0$ E.T.	Equator and equinox 1950.0
$e = 0.1215504$	$i = 21^{\circ}30'\ 23''.72$
$a = 3.087924$ a.u.	$\omega = 170^{\circ}00'\ 18''.71$
$M_{o} = 357^{\circ}12'\ 41''.47$	$\Omega = 165^{\circ}26'\ 35''.19$

The ORBIT 1 output for this example is given as the sample output for the program on pages 92 to 104, the elements obtained being

Epoch 1935 July $17^{d}.0$ E.T. Equator and equinox 1950.0

$e = 0.12155311$ $i = 21^{\circ}30'\ 30''.69$

$a = 3.08798396$ a.u. $\omega = 169^{\circ}56'\ 53''.41$

$M_{o} = 357^{\circ}15'\ 22''.69$ $\Omega = 165^{\circ}26'\ 34''.68$

Clearly, the agreement is very good, particularly in view of the fact that Kilkenny was working to an accuracy several orders of magnitude less than the computer program. A similar check was later carried out using a manual computation of the orbit of 173 Ino made by Dr I G van Breda, the results again showing very good agreement. We thus conclude that the accuracy of a reduction by ORBIT 1 is at least as good as, and probably very much better than the equivalent manual reduction.

```
C        ORBIT 1 VERSION 1
C
C
C        DETERMINATION OF PRELIMINARY ORBITAL ELEMENTS FOR A
C        MINOR PLANET FROM THREE OBSERVATIONS USING GAUSS'S METHOD
C
C
C        CODED IN FORTRAN IV FOR THE IBM SYSTEM/360 MODEL 44
C        PROGRAMMING SYSTEM
C
C
C        F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, JULY 1971
C
C
C        UPDATED JANUARY 1972
C
C
C
C        ORBIT LIBRARY SUBPROGRAMS CALLED - GEOS,TOPOS,DICOS,SOLVAB,GAUSS,
C        SOLVC,COVECT,SCALAR,ANGLE1,ANGLE2,ANGLE3,ANGLE4,ANGLE5,ANGLE7
C
C
C
      INTEGER*4 IRDATE,IDEHO(5),IMIN(5),IT,I,J,IPAGE,IAPPRX,ITNUM,
     *JAPPRX,JPAGE,ISIGN(3),IDENTN(6),IDENTD(20),IDCE(13),LTRX/'X'/,
     *LTRY/'Y'/,LTRZ/'Z'/,LTRP/'P'/,LTRE/'E'/,LTRV/'V'/,LTRA/'A'/,
     *LTRM/'M'/,LTRQ/'Q'/
C
      REAL*8 DCONV,CONV,TCONV,JDZERO,K,LITIME,SEC(5),JDCE,DTSEC,EPOCH,
     *LONG,LAT,OBL,DT,CEDATE,YDATE(3),JD(3),UTD(3),RA(3),DE(3),ETD(3),
     *DATE,UTDATE(3),ETDATE(3),JD0(3),X0(3),D2X0(3),Y0(3),D2Y0(3),Z0(3),
     *D2Z0(3),JD1(3),X1(3),D2X1(3),Y1(3),D2Y1(3),Z1(3),D2Z1(3),JDGST(3),
     *GSTOD(3),DIFF1,DIFF2,X(3),Y(3),Z(3),RX(3),RY(3),RZ(3),URHOX(3),
     *URHOY(3),URHOZ(3),SUMSQ,U1C3X,U1C3Y,U1C3Z,CRHO2A,CRHO2B,CC1,
     *CONSTA,CC3,CONSTB,TAU(3),AC1,AC3,TAU2SQ,BC1,BC3,CONSA1,CSR2M3,
     *SR(3),RHO(3),SR2M3,C(3),RHSZ,LHSZ,SRX(3),SRY(3),SRZ(3),PREVC1,
     *PREVC3,PRVSRX(3),PRVSRY(3),PRVSRZ(3),CTDATE(3),TAU1SQ,TAU3SQ,
     *TAU13P,B(3),SR2CU,DENOM,CONSAN,LHSX,LHSY,SRSP(3),SRSUM(3),CK(3),
     *DENOM2(3),MSQ(3),L(3),H(3),YBAR(3),CHECKC(3),SR1SC,SIGMA,SRX0,
     *SRY0,SRZ0,SR0SC,P,SR13P,CV3MV1,SV3MV1,ECOSV1,ESINV1,E,V(3),AE,A,N,
     *BE,CE(3),LHS,RHS,M(3),CHECKN,NRAD,T,RT,ETTD,JDT,CHCKM3,M0(3),SR0,
     *AP,BP,AQ,BQ,UPX,UPY,UPZ,UQX,UQY,UQZ,COSOBL,SINOBL,SOMSIP,COMSIP,
     *OMEGA,SI,COSOM,SINOM,SINOM1,COSOM1,OMEGA1,CHECKI
C
      REAL*8 NTIME,TA,TB,SRFUNC,RO,URO,R,BFUNC,TASQ,TATB,TBSQ,KFUNC,SRA,
     *SRB,SRABSP,
     *DSQRT,DABS,DARCOS,DTAN,DSIN,DFLOAT,DCOS,DARSIN,
     *COVECT,SCALAR,ANGLE1,ANGLE3,ANGLE5,ANGLE7
C
      NTIME(TA,TB)=K*(TA-TB)
      SRFUNC(RO,URO,R)=RO*URO-R
      BFUNC(TASQ,TATB,TBSQ)=(TASQ+TATB+TBSQ)/12.0D0
      KFUNC(SRA,SRB,SRABSP)=DSQRT(SRA*SRB+SRABSP)
C
    1 FORMAT('1',4X,23HFGW / ORBIT 1 VERSION 1,83X,4HDATE,I7,//////////,
     *44X,45HDETERMINATION OF PRELIMINARY ORBITAL ELEMENTS,///,50X,21HFO
     *R THE MINOR PLANET ,6A4,///,44X,44HFROM THREE OBSERVATIONS USING G
     *AUSS'S METHOD,////////////////////////,50X,33HST.ANDREWS UNIVERSITY
     * OBSERVATORY)
    2 FORMAT('1',4X,50HGAUSSIAN DETERMINATION OF THE ORBITAL ELEMENTS OF
     * ,6A4,8X,4HDATE,I7,19X,4HPAGE,I3)
C
```

```
  3 FORMAT(/////,5X,29HNOTES ON THE EQUATIONS QUOTED,///,10X,39H(1)
   *EQUATIONS A AND B ARE OF THE FORM,//,38X,49HE*RHO2 = F1*C1 - F2 +
   *F3*C3 . . . . . . . . . . (A),//,33X,54HSR2 SQUARED = RHO2 SQUARED -
   * G*RHO2 + H  . . . . . . (B),//,16X,49H(BAKER (1967), EQUATIONS (1.3
   *2), (1.30), PAGE 25))
  4 FORMAT(//,10X,66H(2)   EQUATIONS AN ARE DERIVED FROM EQUATION A AN
   *D ARE OF THE FORM,//,40X,22HRHO2 = A - B/SR2 CUBED,//,16X,45HEQUAT
   *IONS BN ARE EACH IDENTICAL TO EQUATION B,//,16X,49H(BAKER (1967),
   *EQUATIONS (1.33), (1.30), PAGE 25))
  5 FORMAT(//,10X,54H(3)   GAUSS'S FUNDAMENTAL EQUATION IS USED IN THE
   * FORM)
  6 FORMAT(/,27X,'C1*URHO',A1,'1*RHO1 + C3*URHO',A1,'3*RHO3 = C1*R',A1
   *,'1 - R',A1,'2 + C3*R',A1,'3 + URHO',A1,'2*RHO2')
  7 FORMAT(///
   *        //,10X,109H(4)   EQUATIONS E FOR M SQUARED AND L FOLLOW IMM
   *EDIATELY FROM GAUSS'S EQUATIONS FOR THE DETERMINATION OF YBAR,//,1
   *6X, 62H(DESIGNATED C BUT NOT USED AS SUCH) BY THE CYCLIC SUBSTITUT
   *ION,//,27X,70HRCOT(2*SR2*SR3)*COS(SF1) = CK1 = RCOT(SR2*SR3 + VECT
   *OR SR2.VECTOR SR3),//,16X, 99H(DUBYAGO (1961), EQUATIONS (5-68), (
   *5-69), PAGE 142 ARE EQUATIONS C. NOTE THAT THIS AUTHOR WRITES M,//
   *,16X,14HFOR M SQUARED))
  8 FORMAT(//,10X,105H(5)   EQUATIONS D ARE GAUSS'S CUBIC EQUATION IN
   *YBAR AND THE EXPRESSIONS GIVING THE VALUE OF H. THE CUBIC,//,16X,1
   *07HIS SOLVED BY HANSEN'S CONTINUED FRACTION, AND THE QUANTITY XI I
   *S NEGLECTED IN THE FIRST APPROXIMATION FOR H,//,16X, 98H(DUBYAGO (
   *1961), EQUATIONS (5-78), (5-77), (5-74), (5-75), PAGE 144 AND EQUA
   *TION (5-80), PAGE 146))
  9 FORMAT(/////,5X,10HREFERENCES,///,10X,101HBAKER, R.M.L.,JR. (1967)
   *,'ASTRODYNAMICS - APPLICATIONS AND ADVANCED TOPICS.' ACADEMIC PRES
   *S, NEW YORK,//,10X,71HDUBYAGO, A.D. (1961),'THE DETERMINATION OF O
   *RBITS.' MACMILLAN, NEW YORK)
C
C
C     1. INPUT OF DATA
C
C
C     INPUT OF PRELIMINARY DATA
C
      DCONV=86400.0
      CONV=206264.8062464262
      TCONV=13750.9870830951
      JDZERO=2428000.5
      K=0.01720209895
      LITIME=0.00577560
C
      READ(5,10)(IDENTN(I),I=1,6),(IDENTD(I),I=1,20),IRDATE,JDCE,
     *(IDCE(I),I=1,13)
   10 FORMAT(6A4,/,20A4,/,I6,/,F9.1,2X,13A1)
C
      WRITE(6,1)IRDATE,(IDENTN(I),I=1,6)
C
      IPAGE=1
      WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
      WRITE(6,3)
      WRITE(6,4)
      WRITE(6,5)
      WRITE(6,6)LTRX,LTRX,LTRX,LTRX,LTRX,LTRX
      WRITE(6,6)LTRY,LTRY,LTRY,LTRY,LTRY,LTRY
      WRITE(6,6)LTRZ,LTRZ,LTRZ,LTRZ,LTRZ,LTRZ
C
```

```
      IPAGE=2
      WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
      WRITE(6,7)
      WRITE(6,8)
C
      IPAGE=3
      WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
      WRITE(6,9)
C
      IPAGE=4
      WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
      WRITE(6,15)(IDENTD(I),I=1,20)
   15 FORMAT(/////,5X,'DATES OF OBSERVATION -',///,10X,20A4)
C
      READ(5,20)ISIGN(1),IDEHO(1),IMIN(1),SEC(1),ISIGN(2),IDEHO(2),
     *IMIN(2),SEC(2),DTSEC,EPOCH,IDEHO(3),IMIN(3),SEC(3)
   20 FORMAT(A1,I2,I3,F7.3,2X,A1,I2,I3,F6.2,/,F8.3,/,F6.1,2X,2I3,F6.2)
C
      LONG=ANGLE1(DCONV,IDEHO(1),IMIN(1),SEC(1))
      CALL ANGLE4(ISIGN(1),LONG,£25)
      WRITE(6,23)
   23 FORMAT(///,10X,'INVALID SIGN GIVEN WITH ANGULAR DATA - EXECUTION H
     *AS BEEN TERMINATED')
      GO TO 1000
C
   25 LAT=ANGLE1(CONV,IDEHO(2),IMIN(2),SEC(2))
      CALL ANGLE4(ISIGN(2),LAT,£28)
      WRITE(6,23)
      GO TO 1000
C
   28 OBL=ANGLE1(CONV,IDEHO(3),IMIN(3),SEC(3))
C
      DT=DTSEC/DCONV
      CEDATE=JDCE-JDZERO
C
C     INPUT OF OBSERVED DATA
C
      DO 40 I=1,3
      READ(5,30)YDATE(I),JD(I),IDEHO(1),IMIN(1),SEC(1),IDEHO(2),IMIN(2),
     *SEC(2),ISIGN(3),IDEHO(3),IMIN(3),SEC(3)
   30 FORMAT(F8.3,2X,F9.1,2(2X,2I3,F7.3),2X,A1,I2,I3,F6.2)
C
      UTD(I)=ANGLE1(DCONV,IDEHO(1),IMIN(1),SEC(1))
      RA(I)=ANGLE1(TCONV,IDEHO(2),IMIN(2),SEC(2))
C
      DE(I)=ANGLE1(CONV,IDEHO(3),IMIN(3),SEC(3))
      CALL ANGLE4(ISIGN(3),DE(I),£35)
      WRITE(6,23)
      GO TO 1000
C
   35 ETD(I)=UTD(I)+DT
      DATE=JD(I)-JDZERO
      UTDATE(I)=DATE+UTD(I)
   40 ETDATE(I)=DATE+ETD(I)
```

```
C
C      INPUT OF EPHEMERIS DATA
C
       DC 45 I=1,3
    45 READ(5,50)JDO(I),XO(I),D2XO(I),YO(I),D2YO(I),ZO(I),D2ZO(I),JD1(I),
      *X1(I),D2X1(I),Y1(I),D2Y1(I),Z1(I),D2Z1(I)
    50 FORMAT(F9.1,6(1X,F10.7),/,F9.1,6(1X,F1C.7))
C
       DO 7C I=1,3
       READ(5,60)JDGST(I),IDEHO(1),IMIN(1),SEC(1)
    60 FORMAT(F9.1,1X,2I3,F7.3)
C
    70 GSTOD(I)=ANGLE1(DCONV,IDEHO(1),IMIN(1),SEC(1))
C
C      DATA ERRCR FILTER
C
       DO 160 I=1,3
       IF(JCGST(I).EQ.JD(I))GO TO 90
       WRITE(6,80)I
    80 FORMAT(///,10X,'ERROR IN DATA FOR OBSERVATION NO.',I2,//,11X,'- GI
      *VEN VALUE CF GST AT 00 HOURS UT HAS DATE DIFFERENT FROM THAT OF OB
      *SERVATION',//,11X,'- EXECUTION HAS BEEN TERMINATED')
       GO TO 1000
C
    9C DIFF1=JD1(I)-JDO(I)
       IF(DIFF1.EQ.1.ODO)GO TO 110
       WRITE(6,100)I
   100 FORMAT(///,10X,'ERRCR IN DATA FOR OBSERVATION NO.',I2,//,11X,'- IN
      *TERVAL BETWEEN GIVEN TWC SETS OF SOLAR CCORDINATES IS DIFFERENT FR
      *OM ONE DAY',//,13X,'RENDERING INTERPOLATION INVALID',//,11X,'- EXE
      *CUTICN HAS BEEN TERMINATED')
       GO TO 1CCO
C
   110 DIFF2=JDO(I)-JD(I)
       IF(DIFF2.EQ.0.ODO)GO TO 140
       IF(DIFF2.EQ.1.ODO)GO TO 150
   120 WRITE(6,130)I
   130 FORMAT(///,10X,'ERRCR IN DATA FOR OBSERVATION NO.',I2,//,11X,'- EP
      *HEMERIS TIME OF OBSERVATICN IS OUTSIDE RANGE OF GIVEN TWO SETS OF
      *SOLAR COORDINATES',//,13X,'RENDERING INTERPOLATION INVALID',//,11X
      *,'- EXECUTICN HAS BEEN TERMINATED')
       GO TO 1CCO
C
   140 IF(ETD(I).GT.1.ODO)GO TO 120
       GO TO 16C
   150 IF(ETD(I).LT.1.ODO)GO TO 120
   160 CCNTINUE
C
C
C      2. PRELIMINARY COMPUTATIONS
C
C
C      INTERPOLATION OF GEOCENTRIC SOLAR CCORDINATES BY SUBRCUTINE GEOS
C
       WRITE(6,170)
   170 FORMAT(////,5X,'GEOCENTRIC EQUATCRIAL RECTANGULAR COORDINATES OF T
      *HE SUN -')
C
       DO 175 I=1,3
       CALL GEOS(ETD(I),XO(I),D2XO(I),YC(I),D2YO(I),ZO(I),D2ZO(I),X1(I),
      *D2X1(I),Y1(I),D2Y1(I),Z1(I),D2Z1(I),X(I),Y(I),Z(I))
   175 WRITE(6,180)I,X(I),Y(I),Z(I)
   180 FORMAT(//,10X,'AT OBSERVATION NO.',I2,' , X =',F11.8,' , Y =',F11.
      *8,' , Z =',F11.8)
```

```
C
C     REDUCTION TO TOPOCENTRIC SCLAR CCORDINATES BY SUBRCUTINE TOPOS
C
      WRITE(6,190)
  190 FORMAT(////,5X,'TOPOCENTRIC EQUATORIAL RECTANGULAR COORDINATES OF
     *THE SUN -')
C
      DO 195 I=1,3
      CALL TOPOS(LONG,LAT,EPOCH,YDATE(I),GSTOD(I),UTD(I),X(I),Y(I),Z(I),
     *RX(I),RY(I),RZ(I))
  195 WRITE(6,200)I,RX(I),RY(I),RZ(I)
  200 FORMAT(//,10X,'AT OBSERVATION NC.',I2,' , RX =',F11.8,' , RY =',F1
     *1.8,' , RZ =',F11.8)
C
C     COMPUTATION OF DIRECTION COSINES OF ASTEROID BY SUBROUTINE DICOS
C
      IPAGE=5
      WRITE(6,2)(IDENTN(I), I=1,6),IRDATE,IPAGE
      WRITE(6,210)
  210 FORMAT(/////5X,'TOPOCENTRIC DIRECTION CCSINES OF THE MINOR PLANET
     *-')
C
      DO 215 I=1,3
      CALL DICOS(RA(I),DE(I),URHOX(I),URHOY(I),URHOZ(I),SUMSQ)
  215 WRITE(6,220)I,URHOX(I),URHOY(I),URHOZ(I),SUMSQ
  220 FORMAT(//,10X,'AT OBSERVATION NC.',I2,' , URHOX =',F11.8,' , URHOY
     * =',F11.8,' , URHOZ =',F11.8,' , SUM CF SQUARES =',F11.8)
C
C     COMPUTATICN OF THE COEFFICIENTS FOR THE EQUATIONS A AND B
C
      IPAGE=6
      WRITE(6,2)(IDENTN(I), I=1,6),IRDATE,IPAGE
      WRITE(6,230)
  230 FORMAT(/////,5X,'COEFFICIENTS FOR THE EQUATIONS A AND B -')
C
      U1C3X=COVECT(URHOY(1),URHOZ(1),URHOY(3),URHOZ(3))
      U1C3Y=COVECT(URHOZ(1),URHOX(1),URHOZ(3),URHOX(3))
      U1C3Z=COVECT(URHOX(1),URHOY(1),URHOX(3),URHOY(3))
C
      CRHO2A=-SCALAR(URHOX(2),URHOY(2),URHOZ(2),U1C3X,U1C3Y,U1C3Z)
C
      CRHO2B=-2.0D0*SCALAR(URHOX(2),URHOY(2),URHOZ(2),RX(2),RY(2),RZ(2))
C
      CC1=SCALAR(RX(1),RY(1),RZ(1),U1C3X,U1C3Y,U1C3Z)
C
      CONSTA=-SCALAR(RX(2),RY(2),RZ(2),U1C3X,U1C3Y,U1C3Z)
C
      CC3=SCALAR(RX(3),RY(3),RZ(3),U1C3X,U1C3Y,U1C3Z)
C
      CONSTB=SCALAR(RX(2),RY(2),RZ(2),RX(2),RY(2),RZ(2))
C
      WRITE(6,240)CRHO2A,CC1,CC3,CONSTA
  240 FORMAT(///,10X,'EQUATION A',///,10X,'COEFFICIENT OF RHO2 =',F13.10
     *,///,10X,'COEFFICIENT CF C1 =',F13.10,///,10X,'COEFFICIENT OF C3 =
     *',F13.10,///,10X,'CONSTANT TERM =',F13.10)
C
      WRITE(6,250)CRHO2B,CONSTB
  250 FORMAT(///,10X,'EQUATION B',///,10X,'COEFFICIENT CF RHO2 =',F13.10
     *,///,10X,'CONSTANT TERM =',F13.10)
```

```
C
C
C     3. FIRST APPROXIMATION OF THE SUN - PLANET VECTORS
C
C
      IPAGE=7
      WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
      IAPPRX=1
      WRITE(6,260)IAPPRX
  260 FORMAT(/////,5X,'COMPUTATION OF THE SUN - PLANET VECTORS (APPROXIM
     *ATION NO.',I2,')')
C
C     COMPUTATION OF THE COEFFICIENTS FOR THE EQUATIONS A1 AND B1
C
      WRITE(6,270)IAPPRX,IAPPRX
  270 FORMAT(////,5X,'COEFFICIENTS FOR THE EQUATIONS A',I1,' AND B',I1,'
     * -')
C
      TAU(1)=NTIME(UTDATE(3),UTDATE(2))
      TAU(2)=NTIME(UTDATE(3),UTDATE(1))
      TAU(3)=NTIME(UTDATE(2),UTDATE(1))
C
      AC1=TAU(1)/TAU(2)
      AC3=TAU(3)/TAU(2)
      TAU2SQ=TAU(2)**2.0D0
      BC1=AC1*(TAU2SQ-TAU(1)**2.0D0)/6.0D0
      BC3=AC3*(TAU2SQ-TAU(1)**2.0D0)/6.0D0
C
      CONSA1=(CC1*AC1+CONSTA+CC3*AC3)/CRHO2A
C
      CSR2M3=(CC1*BC1+CC3*BC3)/CRHO2A
C
      WRITE(6,280)IAPPRX,CSR2M3,CONSA1
  280 FORMAT(///,10X,'EQUATION A',I1,///,10X,'COEFFICIENT OF (1/SR2) CUB
     *ED =',F13.10,///,10X,'CONSTANT TERM =',F13.10)
C
      WRITE(6,290)IAPPRX,CRHO2B,CONSTB
  290 FORMAT(///,10X,'EQUATION B',I1,///,10X,'COEFFICIENT OF RHO2 =',F13
     *.10,///,10X,'CONSTANT TERM =',F13.10)
C
C     SOLUTION OF EQUATIONS A1 AND B1 BY SUBROUTINE SOLVAB
C
      WRITE(6,300)IAPPRX,IAPPRX
  300 FORMAT(////,5X,'SOLUTION OF THE EQUATIONS A',I1,' AND B',I1,' BY S
     *UCCESSIVE APPROXIMATIONS -')
C
      SR(2)=2.8D0
      CALL SOLVAB(CONSA1,CSR2M3,CRHO2B,CONSTB,SR(2),RHO(2),ITNUM,£320)
C
      WRITE(6,310)ITNUM,RHO(2),SR(2)
  310 FORMAT( //,10X,'SUCCESSIVE APPROXIMATIONS FAILED TO CONVERGE SUFFI
     *CIENTLY AFTER',I3,' ITERATIONS',//,11X,'- EXECUTION HAS BEEN TERMI
     *NATED', //,10X,'VALUES AT TERMINATION WERE  RHO2 =',F13.10,' , SR2
     * =',F13.10)
      GO TO 1000
C
  320 WRITE(6,330)ITNUM,RHO(2),SR(2)
  330 FORMAT( //,10X,'SUCCESSIVE APPROXIMATIONS CONVERGED SUFFICIENTLY A
     *FTER',I3,' ITERATIONS - PROCESS TERMINATED',///,10X,'VALUES AT TER
     *MINATION WERE  RHO2 =',F13.10,' , SR2 =',F13.10)
```

```
C
C     COMPUTATION OF RHO(1) AND RHO(3) BY SUBROUTINE GAUSS
C
      IPAGE=8
      WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
      WRITE(6,340)
  340 FORMAT(/////,5X,81HVALUES OF C1, C3 AND SOLUTION OF GAUSS'S FUNDAM
     *ENTAL EQUATION FOR RHO1 AND RHO3 -)
C
      SR2M3=1.0D0/SR(2)**3.0D0
      C(1)=AC1+BC1*SR2M3
      C(3)=AC3+BC3*SR2M3
C
      WRITE(6,350)C(1),C(3)
  350 FORMAT(//,10X,'C1 =',F13.10,' , C3 =',F13.10)
C
      CALL GAUSS(RX(1),RX(2),RX(3),RY(1),RY(2),RY(3),RZ(1),RZ(2),RZ(3),
     *URHOX(1),URHOX(2),URHOX(3),URHOY(1),URHOY(2),URHOY(3),URHOZ(1),
     *URHOZ(2),URHOZ(3),RHO(1),RHO(2),RHO(3),C(1),C(3),RHSZ,LHSZ,£370)
C
      WRITE(6,380)RHO(1),RHO(3),LHSZ,RHSZ
      WRITE(6,360)
  360 FORMAT(//,10X,'AGREEMENT IS UNACCEPTABLE - EXECUTION HAS BEEN TERM
     *INATED')
      GO TO 1000
C
  370 WRITE(6,380)RHO(1),RHO(3),LHSZ,RHSZ
  380 FORMAT(//,10X,'RHO1 =',F13.10,' , RHO3 =',F13.10,///,10X,53HUSING
     *GAUSS'S EQUATION IN Z AS A CHECK WE HAVE  LHS =,F13.10,' , RHS =',
     *F13.10)
      WRITE(6,390)
  390 FORMAT(//,10X,'AGREEMENT IS ACCEPTABLE')
C
C     COMPUTATION OF THE SUN - PLANET VECTORS (SRX(I),SRY(I),SRZ(I))
C
      WRITE(6,400)IAPPRX
  400 FORMAT(////,5X,'SUN - PLANET VECTORS (APPROXIMATION NO.',I2,') -')
C
      DO 410 I=1,3
      SRX(I)=SRFUNC(RHO(I),URHOX(I),RX(I))
      SRY(I)=SRFUNC(RHO(I),URHOY(I),RY(I))
      SRZ(I)=SRFUNC(RHO(I),URHOZ(I),RZ(I))
C
  410 WRITE(6,420)I,SRX(I),SRY(I),SRZ(I)
  420 FORMAT(//,10X,'AT OBSERVATION NO.',I2,' , SRX =',F11.8,' , SRY =',
     *F11.8,' , SRZ =',F11.8)
C
C
C     4. SECOND AND SUBSEQUENT APPROXIMATIONS OF SUN - PLANET VECTORS
C
C
  430 IPAGE=IPAGE+1
      WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
      IAPPRX=IAPPRX+1
      WRITE(6,260)IAPPRX
C
      PREVC1=C(1)
      PREVC3=C(3)
      DO 440 I=1,3
      PRVSRX(I)=SRX(I)
      PRVSRY(I)=SRY(I)
  440 PRVSRZ(I)=SRZ(I)
```

```
C
C     COMPUTATION OF THE COEFFICIENTS FOR THE EQUATIONS AN AND BN
C
      WRITE(6,270)IAPPRX,IAPPRX
C
      DO 450 I=1,3
  450 CTDATE(I)=ETDATE(I)-LITIME*RHO(I)
C
      TAU(1)=NTIME(CTDATE(3),CTDATE(2))
      TAU(2)=NTIME(CTDATE(3),CTDATE(1))
      TAU(3)=NTIME(CTDATE(2),CTDATE(1))
C
      TAU1SQ=TAU(1)**2.0D0
      TAU3SQ=TAU(3)**2.0D0
      TAU13P=TAU(1)*TAU(3)
C
      B(1)=BFUNC(TAU3SQ,TAU13P,-TAU1SQ)
      B(2)=BFUNC(TAU1SQ,3.0D0*TAU13P,TAU3SQ)
      B(3)=BFUNC(TAU1SQ,TAU13P,-TAU3SQ)
C
      DO 460 I=1,3
  460 SR(I)=DSQRT(SRX(I)*SRX(I)+SRY(I)*SRY(I)+SRZ(I)*SRZ(I))
C
      AC1=TAU(1)/TAU(2)
      AC3=TAU(3)/TAU(2)
      SR2CU=SR(2)**3.0D0
C
      DENOM=1.0D0-B(2)/SR2CU
      C(1)=AC1*(1.0D0+B(1)/SR(1)**3.0D0)/DENOM
      C(3)=AC3*(1.0D0+B(3)/SR(3)**3.0D0)/DENOM
C
      BC1=(C(1)-AC1)*SR2CU
      BC3=(C(3)-AC3)*SR2CU
C
      CONSAN=(CC1*AC1+CONSTA+CC3*AC3)/CRHO2A
C
      CSR2M3=(CC1*BC1+CC3*BC3)/CRHO2A
C
      WRITE(6,280)IAPPRX,CSR2M3,CONSAN
      WRITE(6,290)IAPPRX,CRHO2B,CONSTB
C
C     SOLUTION OF EQUATIONS AN AND BN BY SUBROUTINE SOLVAB
C
      WRITE(6,300)IAPPRX,IAPPRX
C
      CALL SOLVAB(CONSAN,CSR2M3,CRHO2B,CONSTB,SR(2),RHO(2),ITNUM,£470)
C
      WRITE(6,310)ITNUM,RHO(2),SR(2)
      GO TO 1000
C
  470 WRITE(6,330)ITNUM,RHO(2),SR(2)
C
C     COMPUTATION OF RHO(1) AND RHO(3) BY SUBROUTINE GAUSS
C
      IPAGE=IPAGE+1
      WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
      WRITE(6,340)
C
      SR2M3=1.0D0/SR(2)**3.0D0
      C(1)=AC1+BC1*SR2M3
      C(3)=AC3+BC3*SR2M3
C
      WRITE(6,350)C(1),C(3)
C
```

```
      CALL GAUSS(RX(1),RX(2),RX(3),RY(1),RY(2),RY(3),RZ(1),RZ(2),RZ(3),
     *URHOX(1),URHOX(2),URHOX(3),URHOY(1),URHOY(2),URHOY(3),URHOZ(1),
     *URHOZ(2),URHOZ(3),RHO(1),RHO(2),RHO(3),C(1),C(3),RHSZ,LHSZ,£480)
C
      WRITE(6,380)RHO(1),RHO(3),LHSZ,RHSZ
      WRITE(6,360)
      GO TO 1000
C
  480 WRITE(6,380)RHO(1),RHO(3),LHSZ,RHSZ
      WRITE(6,390)
C
C     COMPUTATION OF THE SUN - PLANET VECTORS (SRX(I),SRY(I),SRZ(I))
C
      WRITE(6,400)IAPPRX
C
      DO 490 I=1,3
      SRX(I)=SRFUNC(RHO(I),URHOX(I),RX(I))
      SRY(I)=SRFUNC(RHO(I),URHOY(I),RY(I))
      SRZ(I)=SRFUNC(RHO(I),URHOZ(I),RZ(I))
  490 WRITE(6,420)I,SRX(I),SRY(I),SRZ(I)
C
C     APPROXIMATION LOOP TERMINATION GATE
C
      IF((DABS(PREVC1-C(1)).LT.1.0D-6).AND.(DABS(PREVC3-C(3)).LT.1.0D-6)
     *)GO TO 520
C
  500 IF(IAPPRX.LT.9)GO TO 430
      WRITE(6,510)IAPPRX
  510 FORMAT(////,5X,'SUCCESSIVE APPROXIMATIONS OF THE SUN - PLANET VECT
     *ORS HAVE FAILED TO CONVERGE SUFFICIENTLY AFTER',I2,' ITERATIONS',/
     *//,10X,'- EXECUTION HAS BEEN TERMINATED')
      GO TO 1000
C
  520 DO 530 I=1,3
      IF((DABS(PRVSRX(I)-SRX(I)).GT.1.0D-5).OR.(DABS(PRVSRY(I)-SRY(I)).G
     *T.1.0D-5).OR.(DABS(PRVSRZ(I)-SRZ(I)).GT.1.0D-5))GO TO 500
  530 CONTINUE
C
      WRITE(6,540)IAPPRX
  540 FORMAT(////,5X,'SUCCESSIVE APPROXIMATIONS OF THE SUN - PLANET VECT
     *ORS HAVE NOW CONVERGED SUFFICIENTLY - NO.',I2,' IS FINAL APPROXIMA
     *TION')
C
C     CHECK ON FINAL VALUES OF SUN - PLANET VECTORS
C
      IPAGE=IPAGE+1
      WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
      WRITE(6,550)
  550 FORMAT(/////5X,'CHECK ON FINAL VALUES OF SUN - PLANET VECTORS USIN
     *G THE EQUATION  C1*VECTOR SR1 + C3*VECTOR SR3 = VECTOR SR2 -')
C
      LHSX=COVECT(C(1),-C(3),SRX(3),SRX(1))
      LHSY=COVECT(C(1),-C(3),SRY(3),SRY(1))
      LHSZ=COVECT(C(1),-C(3),SRZ(3),SRZ(1))
C
      IF((DABS(LHSX-SRX(2)).LT.1.0D-6).AND.(DABS(LHSY-SRY(2)).LT.1.0D-6)
     *.AND.(DABS(LHSZ-SRZ(2)).LT.1.0D-6))GO TO 560
      WRITE(6,570)LHSX,SRX(2),LHSY,SRY(2),LHSZ,SRZ(2)
      WRITE(6,360)
      GO TO 1000
C
```

```
  560 WRITE(6,570)LHSX,SRX(2),LHSY,SRY(2),LHSZ,SRZ(2)
  570  FORMAT(//,10X,'EQUATION IN X GIVES  LHS =',F13.10,' , RHS =',F13.
     *10,///,10X,'EQUATION IN Y GIVES  LHS =',F13.10,' , RHS =',F13.10,/
     *//,10X,'EQUATION IN Z GIVES  LHS =',F13.10,' , RHS =',F13.10)
      WRITE(6,390)
C
C
C     5. RATIO OF THE SECTOR TO THE TRIANGLE IN THE ORBIT
C
C
      IPAGE=IPAGE+1
      WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
      WRITE(6,580)
  580 FORMAT(/////,5X,'COMPUTATION OF THE RATIO YBAR OF THE AREA OF THE
     *SECTOR TO THE AREA OF THE TRIANGLE IN THE ORBIT')
C
C     COMPUTATION OF M SQUARED AND L BY EQUATIONS E
C
      WRITE(6,590)
  590 FORMAT(////,5X,'VALUES OF M SQUARED AND L FROM EQUATIONS E -')
C
      DO 600 I=1,3
  600 SR(I)=DSQRT(SRX(I)*SRX(I)+SRY(I)*SRY(I)+SRZ(I)*SRZ(I))
C
      SRSP(1)=SCALAR(SRX(2),SRY(2),SRZ(2),SRX(3),SRY(3),SRZ(3))
      SRSP(2)=SCALAR(SRX(1),SRY(1),SRZ(1),SRX(3),SRY(3),SRZ(3))
      SRSP(3)=SCALAR(SRX(1),SRY(1),SRZ(1),SRX(2),SRY(2),SRZ(2))
C
      SRSUM(1)=SR(2)+SR(3)
      SRSUM(2)=SR(1)+SR(3)
      SRSUM(3)=SR(1)+SR(2)
C
      CK(1)=KFUNC(SR(2),SR(3),SRSP(1))
      CK(2)=KFUNC(SR(1),SR(3),SRSP(2))
      CK(3)=KFUNC(SR(1),SR(2),SRSP(3))
C
      DO 610 I=1,3
      DENOM2(I)=2.8284271247461 82D0*CK(I)
      MSQ(I)=TAU(I)**2.0D0/(DENOM2(I)*CK(I)**2.0D0)
      L(I)=SRSUM(I)/DENOM2(I)-0.5D0
C
  610 WRITE(6,620)I,MSQ(I),L(I)
  620 FORMAT(//,10X,'FOR OBSERVATION NO.',I2,' , M SQUARED =',F14.11,' ,
     * L =',F14.11)
C
C     SOLUTION OF EQUATIONS D BY SUBROUTINE SOLVD
C
      IPAGE=IPAGE+1
      WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
      WRITE(6,630)
  630 FORMAT(/////5X,'SOLUTION OF EQUATIONS D BY SUCCESSIVE APPROXIMATIO
     *NS -')
C
      DO 650 I=1,3
      CALL SOLVD(MSQ(I),L(I),H(I),YBAR(I),JAPPRX,£650)
C
      WRITE(6,640)I,JAPPRX,H(I),YBAR(I)
  640 FORMAT(///,10X,'FOR OBSERVATION NO.',I2,', SUCCESSIVE APPROXIMATIO
     *NS FAILED TO CONVERGE SUFFICIENTLY AFTER',I2,' ITERATIONS',//,11X,
     *'- EXECUTION HAS BEEN TERMINATED',///,10X,'VALUES AT TERMINATION W
     *ERE  H =',F14.11,' , YBAR =',F14.11)
      GO TO 1000
C
```

```
  650 WRITE(6,660)I,JAPPRX,H(I),YBAR(I)
  660 FORMAT(///,10X,'FOR OBSERVATION NO.',I2,', SUCCESSIVE APPROXIMATIO
     *NS CONVERGED SUFFICIENTLY AFTER',I2,' ITERATIONS',///,10X,'VALUES
     *AT TERMINATION WERE  H =',F14.11,' , YBAR =',F14.11)
C
C     CHECK ON VALUES OF YBAR BY RECOMPUTATION OF C1 AND C3
C
      JPAGE=IPAGE-3
      WRITE(6,670)JPAGE
  670 FORMAT(////,5X,'CHECK ON VALUES OF YBAR BY RECOMPUTATION OF C1, C3
     * AND COMPARISON WITH C1, C3 OBTAINED ON PAGE',I3,' -')
C
      CHECKC(1)=AC1*YBAR(2)/YBAR(1)
      CHECKC(3)=AC3*YBAR(2)/YBAR(3)
C
      IF((DABS(CHECKC(1)-C(1)).LT.1.0D-6).AND.(DABS(CHECKC(3)-C(3)).LT.1
     *.0D-6))GO TO 680
      WRITE(6,350)CHECKC(1),CHECKC(3)
      WRITE(6,360)
      GO TO 1000
C
  680 WRITE(6,350)CHECKC(1),CHECKC(3)
      WRITE(6,390)
C
C
C     6. COMPUTATION OF THE ORBITAL ELEMENTS
C
C
      IPAGE=IPAGE+1
      WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
      WRITE(6,690)
  690 FORMAT(/////,5X,'COMPUTATION OF THE ORBITAL ELEMENTS')
C
C     COMPUTATION OF THE PARAMETER OF THE ORBIT P
C
      WRITE(6,700)
  700 FORMAT(////,5X,'PARAMETER OF THE ORBIT P -')
C
      SR1SQ=SR(1)**2.0D0
      SIGMA=SRSP(2)/SR1SQ
C
      SRX0=-SRFUNC(SIGMA,SRX(1),SRX(3))
      SRY0=-SRFUNC(SIGMA,SRY(1),SRY(3))
      SRZ0=-SRFUNC(SIGMA,SRZ(1),SRZ(3))
      SR0SQ=SRX0*SRX0+SRY0*SRY0+SRZ0*SRZ0
C
      P=SR0SQ*SR1SQ*(YBAR(2)/TAU(2))**2.0D0
      WRITE(6,710)LTRP,P
  710 FORMAT(//,10X,A1,' =',F11.8)
C
C     COMPUTATION OF ECCENTRICITY E AND TRUE ANOMALIES V1, V3
C
      WRITE(6,720)
  720 FORMAT(////,5X,'ECCENTRICITY E AND TRUE ANOMALIES V1, V3 -')
C
      SR13P=SR(1)*SR(3)
C
      CV3MV1=SRSP(2)/SR13P
      SV3MV1=DSQRT(1.0D0-CV3MV1**2.0D0)
      ECOSV1=P/SR(1)-1.0D0
      ESINV1=(ECOSV1*CV3MV1-P/SR(3)+1.0D0)/SV3MV1
C
```

```
      E=DSQRT(ECOSV1*ECOSV1+ESINV1*ESINV1)
      WRITE(6,710)LTRE,E
C
      V(1)=ANGLE5(ESINV1,ECOSV1)
      V(3)=ANGLE3(DARCOS(CV3MV1)+V(1))
C
      DO 730 I=1,3,2
      CALL ANGLE2(CONV,V(I),IDEHO(1),IMIN(1),SEC(1))
  730 WRITE(6,740)LTRV,I,IDEHO(1),IMIN(1),SEC(1)
  740 FORMAT(//,10X,A1,I1,' =',I4,' DEG',I3,' MIN',F6.2,' SEC')
C
C     COMPUTATION OF SEMI-MAJOR AXIS A AND MEAN DAILY MOTION N
C
      WRITE(6,750)
  750 FORMAT(////,5X,'SEMI-MAJOR AXIS A AND MEAN DAILY MOTION N -')
C
      AE=1.0D0-E**2.0D0
      A=P/AE
      N=K*CONV/DSQRT(A**3.0D0)
C
      WRITE(6,710)LTRA,A
      WRITE(6,760)N
  760 FORMAT(//,10X,'N =',F11.5,' SECONDS OF ARC PER DAY')
C
C     COMPUTATION AND CHECK OF ECCENTRIC ANOMALIES CE1, CE3
C
      IPAGE=IPAGE+1
      WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
      WRITE(6,770)
  770 FORMAT(/////,5X,'ECCENTRIC ANOMALIES E1, E3 -')
C
      BE=DSQRT((1.0D0-E)/(1.0D0+E))
C
      DO 780 I=1,3,2
      CE(I)=ANGLE7(BE,V(I))
      CALL ANGLE2(CONV,CE(I),IDEHO(1),IMIN(1),SEC(1))
  780 WRITE(6,740)LTRE,I,IDEHO(1),IMIN(1),SEC(1)
C
      LHS=A*DSQRT(AE)*DSIN(ANGLE3(CE(3)-CE(1))/2.0D0)
      RHS=DSQRT(SR13P)*DSIN(ANGLE3(V(3)-V(1))/2.0D0)
      WRITE(6,790)LHS,RHS
  790 FORMAT(//,10X,'USING  A*ROOT(1-E SQUARED)*SIN((E3-E1)/2) = ROOT(SR
     *1*SR3)*SIN((V3-V1)/2)  AS A CHECK WE HAVE',///,10X,'LHS =',F11.8,'
     * , RHS =',F11.8)
C
      IF (DABS(LHS-RHS).LT.1.0D-6)GO TO 800
      WRITE(6,360)
      GO TO 1000
C
  800 WRITE(6,390)
```

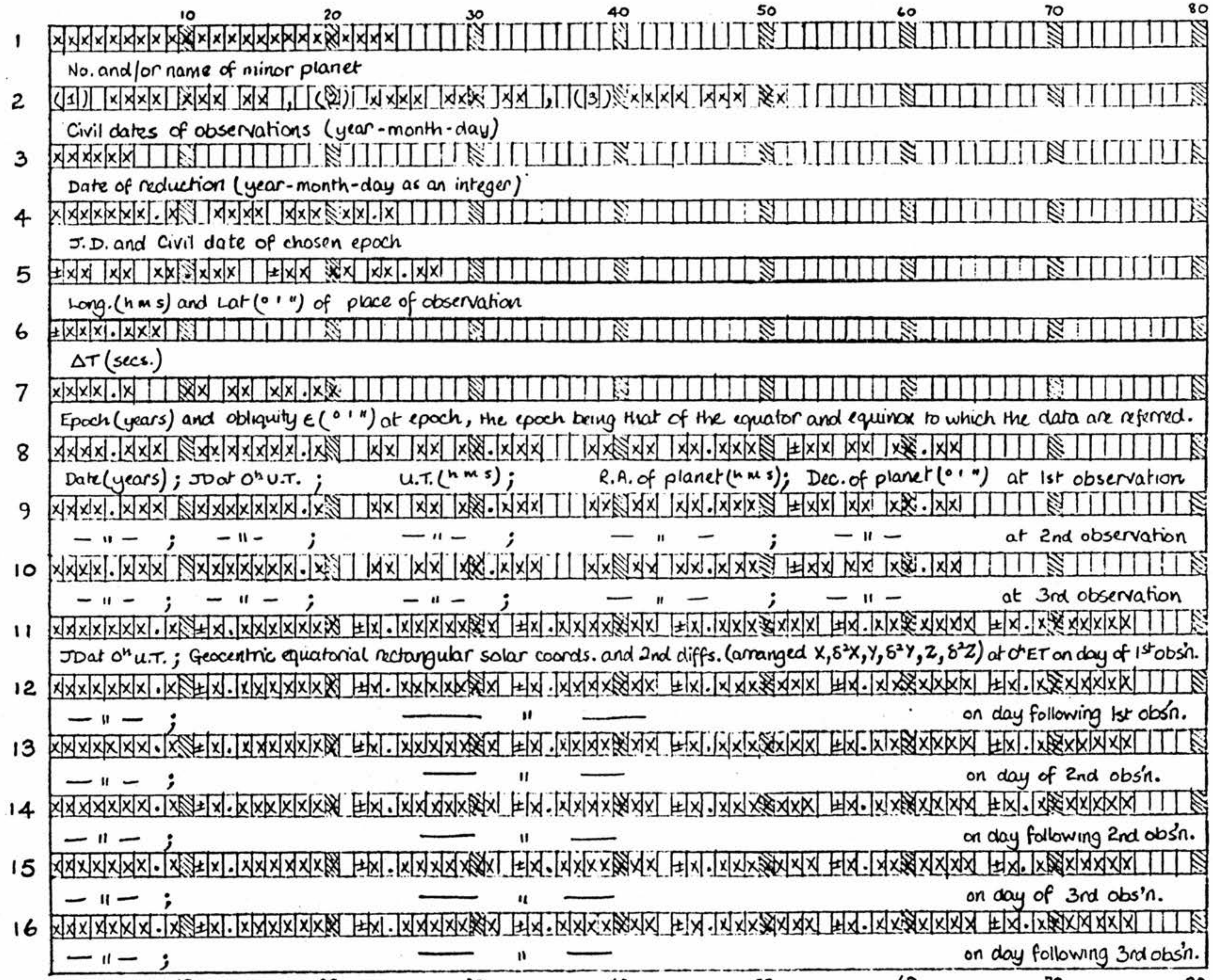
```
C
C     CCMPUTATICN ANC CHECK OF MEAN ANOMALIES M1, M3
C
      WRITE(6,81C)
  810 FORMAT(////,5X,'MEAN ANOMALIES M1, M3 -')
C
      DO 820 I=1,3,2
      M(I)=ANGLE3(CE(I)-E*DSIN(CE(I)))
      CALL ANGLE2(CONV,M(I),ICEHO(1),IMIN(1),SEC(1))
  82C WRITE(6,74C)LTRM,I,IDEHO(1),IMIN(1),SEC(1)
C
      CHECKN=(ANGLE3(M(3)-M(1))/(CTDATE(3)-CTDATE(1)))*CONV
      WRITE(6,83C)
  830 FORMAT(//,10X,'RECOMPUTATION OF N AND CCMPARISON WITH N CBTAINEC P
     *REVICUSLY AS A CHECK ON VALUES OF M1, M3 GIVES')
      WRITE(6,760)CHECKN
C
      IF(DABS(CHECKN-N).LT.1.CD-1)GO TO 84C
      WRITE(6,360)
      GO TC 1CCO
C
  840 WRITE(6,390)
C
C     DETERMINATION AND CHECK OF TIME OF PERIHELICN PASSAGE T
C
      IPAGE=IPAGE+1
      WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
      WRITE(6,850)
  850 FORMAT(/////,5X,'TIME CF PERIHELION PASSAGE T -')
C
      NRAD=N/CONV
      T=CTCATE(1)-M(1)/NRAC
      IF(T.GE.0.0D0)GO TO 870
      WRITE(6,860)JDZERO
  860 FORMAT(//,10X,'LAST PERIHELION PASSAGE OCCURRED BEFORE JD',F10.1,'
     * - EXECUTICN HAS BEEN TERMINATEC')
      GO TO 1C00
C
  870 IT=ICINT(T)
      RT=DFLCAT(IT)
      ETTD=T-RT
      JDT=JDZERO+RT
C
      CALL ANGLE2(DCCNV,ETTC,ICEHO(1),IMIN(1),SEC(1))
      WRITE(6,880)JDT,IDEHC(1),IMIN(1),SEC(1)
  880 FORMAT(//,10X,'T = JD',F10.1,' ,',I3,' HOURS',I3,' MIN',F7.3,' SEC
     * ET')
      WRITE(6,885)
  885 FORMAT(//,10X,'(T IS THE TIME OF THE LAST PERIHELICN PASSAGE TC CC
     *CUR BEFCRE THE CATE OF THE FIRST OBSERVATION)')
C
      CHCKM3=ANGLE3(NRAD*(CTDATE(3)-T))
      CALL ANGLE2(CONV,CHCKM3,IDEHO(1),IMIN(1),SEC(1))
      WRITE(6,890)
  89C FORMAT(//,10X,'RECOMPUTATICN CF M3 AND COMPARISON WITH M3 OBTAINED
     * PREVIOUSLY AS A CHECK ON VALUE OF T GIVES')
      I=3
      WRITE(6,74C)LTRM,I,IDEHC(1),IMIN(1),SEC(1)
C
```

```
      IF(DABS(CHCKM3-M(3)).LT.1.0D-5)GC TC 900
      WRITE(6,360)
      GC TC 1000
C
  9CC WRITE(6,39C)
C
C     TRIPLICATE COMPUTATION OF MEAN ANOMALY MO AT CHOSEN EPOCH
C
      WRITE(6,910)
  910 FORMAT(////,5X,'MEAN ANOMALY MO AT A CHOSEN EPOCH COMPUTED IN THRE
     *E CIFFERENT WAYS AND COMPARED -')
      WRITE(6,92C)JDCE,(IDCE(I),I=1,13)
  920 FORMAT(//,10X,'CHOSEN EPOCH = JD',F1C.1,' ET = ',13A1,' ET')
C
      MO(2)=ANGLE3(NRAD*(CECATE-T))
      DO 930 J=1,3,2
  930 MO(J)=ANGLE3(M(J)+NRAD*(CEDATE-CTDATE(J)))
C
      I=C
      DO 940 J=1,3
      CALL ANGLE2(CONV,MO(J),IDEHO(4),IMIN(4),SEC(4))
  940 WRITE(6,740)LTRM,I,IDEHC(4),IMIN(4),SEC(4)
C
      IF((CABS(MO(1)-MO(2)).LT.1.0D-5).AND.(DABS(MO(2)-MO(3)).LT.1.0D-5)
     *)GC TC 950
      WRITE(6,360)
      GC TC 1000
C
  95C WRITE(6,39C)
C
C     CCMPUTATICN OF VECTORIAL EQUATORIAL CONSTANTS
C
      IPAGE=IPAGE+1
      WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
      WRITE(6,960)
  960 FORMAT(/////5X,'VECTORIAL EQUATORIAL CCNSTANTS -')
C
      SRO=DSQRT(SROSQ)
C
      AP=DCOS(V(1))/SR(1)
      BP=DSIN(V(1))/SRO
      AQ=SRO*BP/SR(1)
      BQ=-SR(1)*AP/SRO
C
      UPX=COVECT(AP,BP,SRXO,SRX(1))
      UPY=COVECT(AP,BP,SRYO,SRY(1))
      UPZ=CCVECT(AP,BP,SRZO,SRZ(1))
C
      SUMSQ=UPX*UPX+UPY*UPY+UPZ*UPZ
      WRITE(6,970)LTRP,UPX,LTRP,UPY,LTRP,UPZ,SUMSQ
  970 FORMAT(//,10X,'U',A1,'X =',F11.8,' , U',A1,'Y =',F11.8,' , U',A1,'
     *Z =',F11.8,' , SUM CF SQUARES =',F11.8)
C
      UQX=COVECT(AQ,BQ,SRXO,SRX(1))
      UQY=CCVECT(AQ,BC,SRYO,SRY(1))
      UQZ=COVECT(AQ,BQ,SRZO,SRZ(1))
C
      SUMSC=UQX*UQX+UQY*UQY+UQZ*UQZ
      WRITE(6,970)LTRQ,UQX,LTRQ,UQY,LTRQ,UQZ,SUMSQ
C
      SUMSC=SCALAR(UPX,UPY,UPZ,UQX,UQY,UQZ)
      WRITE(6,98C)SUMSQ
  980 FORMAT(//,10X,'SCALAR PRODUCT OF (ORTHOGONAL) UNIT VECTORS P ANC Q
     * =',F11.8)
```

```
C
C     DETERMINATION OF ORIENTATION OF MAJOR AXIS OMEGA
C
      WRITE(6,990)
  990 FORMAT(//// ,5X,'ORIENTATION OF MAJOR AXIS OMEGA -')
C
      COSOBL=DCOS(OBL)
      SINOBL=DSIN(OBL)
      SOMSIP=COVECT(UPZ,UPY,SINOBL,COSOBL)
      COMSIP=COVECT(UQZ,UQY,SINOBL,COSOBL)
C
      OMEGA=ANGLE5(SOMSIP,COMSIP)
      CALL ANGLE2(CONV,OMEGA,IDEHO(1),IMIN(1),SEC(1))
      WRITE(6,1020)IDEHO(1),IMIN(1),SEC(1)
 1020 FORMAT(//,10X,'OMEGA =',I4,' DEG',I3,' MIN',F6.2,' SEC')
C
C     DETERMINATION OF INCLINATION OF ORBIT SI
C
      WRITE(6,1030)
 1030 FORMAT(////,5X,'INCLINATION OF ORBIT I -')
C
      SI=DARSIN(DSQRT(SOMSIP*SOMSIP+COMSIP*COMSIP))
      CALL ANGLE2(CONV,SI,IDEHO(2),IMIN(2),SEC(2))
      WRITE(6,1040)IDEHO(2),IMIN(2),SEC(2)
 1040 FORMAT(//,10X,'I =',I4,' DEG',I3,' MIN',F6.2,' SEC')
C
C     DETERMINATION AND CHECK OF LONGITUDE OF ASCENDING NODE OMEGA1
C
      IPAGE=IPAGE+1
      WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
      WRITE(6,1050)
 1050 FORMAT(/////5X,'LONGITUDE OF ASCENDING NODE OMEGA1 -')
C
      COSOM=DCOS(OMEGA)
      SINOM=DSIN(OMEGA)
      SINOM1=COVECT(UPY,UQY,SINOM,COSOM)/COSOBL
      COSOM1=COVECT(UPX,UQX,SINOM,COSOM)
C
      OMEGA1=ANGLE5(SINOM1,COSOM1)
      CALL ANGLE2(CONV,OMEGA1,IDEHO(3),IMIN(3),SEC(3))
      WRITE(6,1060)IDEHO(3),IMIN(3),SEC(3)
 1060 FORMAT(//,10X,'OMEGA1 =',I4,' DEG',I3,' MIN',F6.2,' SEC')
C
      CHECKI=DARCOS(-(UPX*SINOM+UQX*COSOM)/SINOM1)
      CALL ANGLE2(CONV,CHECKI,IDEHO(5),IMIN(5),SEC(5))
      WRITE(6,1070)
 1070 FORMAT(//,10X,'RECOMPUTATION OF I AND COMPARISON WITH I OBTAINED P
     *REVIOUSLY AS A CHECK ON VALUE OF OMEGA1 GIVES')
      WRITE(6,1040)IDEHO(5),IMIN(5),SEC(5)
C
      IF(DABS(CHECKI-SI).LT.1.0D-5)GO TO 1080
      WRITE(6,360)
      GO TO 1000
C
 1080 WRITE(6,390)
C
      WRITE(6,1085)
 1085 FORMAT(////,5X,'THE DETERMINATION OF THE ORBIT HAS NOW BEEN COMPLE
     *TED. A SUMMARY OF THE CLASSICAL ELEMENTS APPEARS ON THE FOLLOWING
     *PAGE')
```

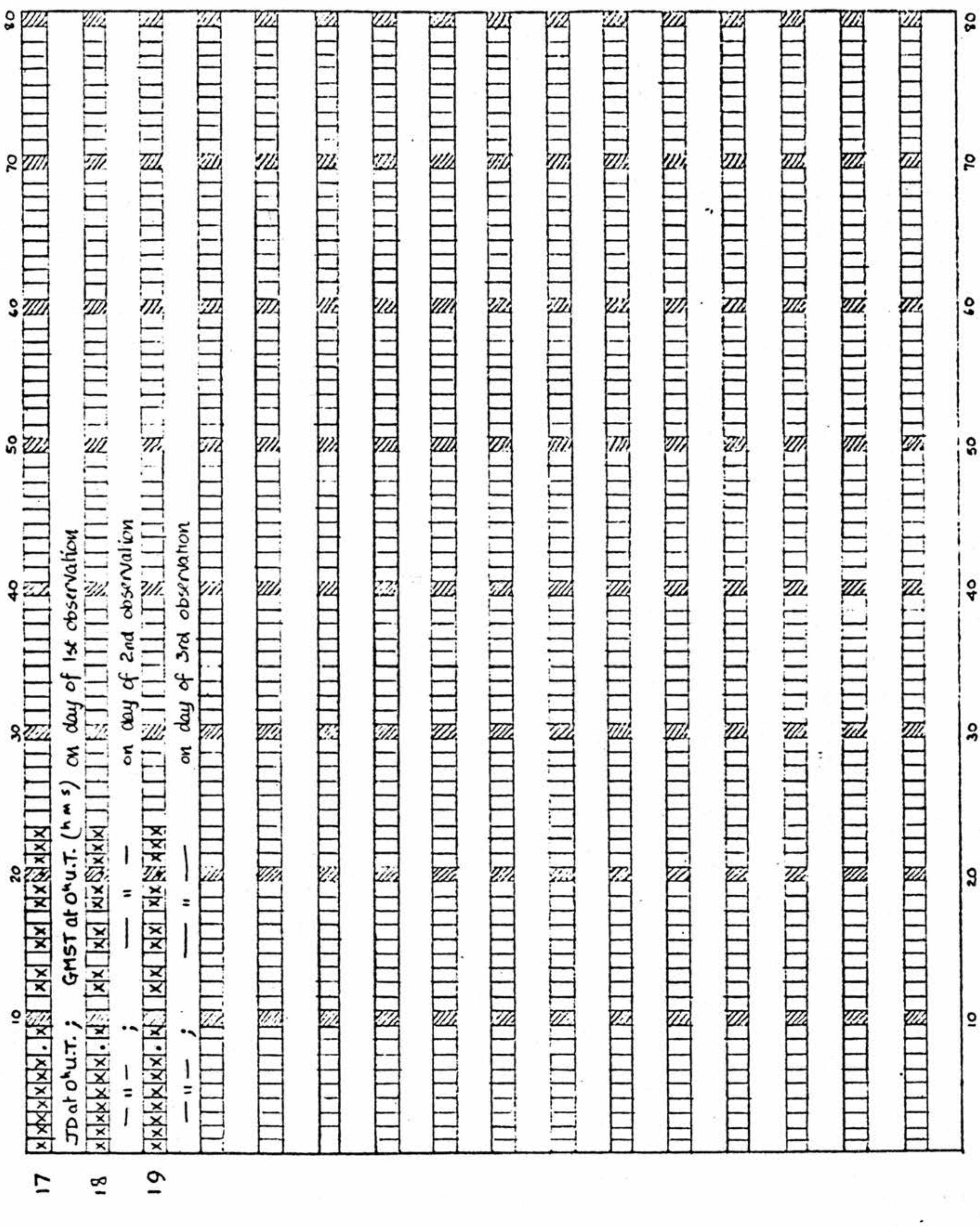
```
C
C      SUMMARY OF ELEMENTS OBTAINED
C
       IPAGE=IPAGE+1
       WRITE(6,2)(IDENTN(I),I=1,6),IRDATE,IPAGE
       WRITE(6,1090)EPCCH
 1090 FORMAT(/////,5X,'SUMMARY CF THE CRBITAL ELEMENTS (REFERRED TO THE
     *EQUATOR AND EQUINOX OF',F7.1,') -')
C
       WRITE(6,1100)E
 11CC FORMAT(////,10X,'ECCENTRICITY E =',F11.8)
       WRITE(6,1110)A
 1110 FORMAT(///,10X,'SEMI-MAJOR AXIS A =',F11.8,' ASTRONOMICAL UNITS')
       WRITE(6,1120)(IDCE(I),I=1,13),ICEHC(4),IMIN(4),SEC(4)
 1120 FORMAT(///,10X,'MEAN ANOMALY MC AT EPOCH ',13A1,' ET =',I4,' DEG',
     *I3,' MIN',F6.2,' SEC')
       WRITE(6,1130)IDEHO(1),IMIN(1),SEC(1)
 1130 FORMAT(///,10X,'ORIENTATION OF MAJOR AXIS CMEGA =',I4,' DEG',I3,'
     *MIN',F6.2,' SEC')
       WRITE(6,1140)ICEHC(2),IMIN(2),SEC(2)
 1140 FORMAT(///,10X,'INCLINATION OF CRBIT I =',I4,' DEG',I3,' MIN',F6.2
     *,' SEC')
       WRITE(6,1150)ICEHC(3),IMIN(3),SEC(3)
 1150 FORMAT(///,10X,'LONGITUDE OF ASCENDING NODE OMEGA1 =',I4,' DEG',I3
     *,' MIN',F6.2,' SEC')
C
C      TERMINATION POINT FOR COMPLETE AND INCOMPLETE EXECUTION
C
 1000 IPAGE=IPAGE+1
       WRITE(6,2)(IDENTN(I),I=1,6),IRCATE,IPAGE
       WRITE(6,10C5)
 1005 FORMAT(/////,5X,'REMARKS')
C
       WRITE(6,1010)
 1010 FCRMAT('1')
       WRITE(6,1010)
C
       STOP
       END
```

DATA FORMAT FOR ORBIT 1



(contd. overleaf)

DATA FORMAT FOR ORBIT 1 (contd.)



FGW / ORBIT 1 VERSION 1 DATE 720221

DETERMINATION OF PRELIMINARY ORBITAL ELEMENTS

FOR THE MINOR PLANET 1361 LEUSCHNERIA

FROM THREE OBSERVATIONS USING GAUSS'S METHOD

ST.ANDREWS UNIVERSITY OBSERVATORY

NOTES ON THE EQUATIONS QUOTED

(1) EQUATIONS A AND B ARE OF THE FORM

F*RHO2 = F1*C1 - F2 + F3*C3 . . . . (A)

SR2 SQUARED = RHO2 SQUARED - G*RHO2 + H . . . . . (B)

(BAKER (1967), EQUATIONS (1.32), (1.30), PAGE 25)

(2) EQUATIONS AN ARE DERIVED FROM EQUATION A AND ARE OF THE FORM

RHO2 = A - B/SR2 CUBED

EQUATIONS BN ARE EACH IDENTICAL TO EQUATION B

(BAKER (1967), EQUATIONS (1.33), (1.30), PAGE 25)

(3) GAUSS'S FUNDAMENTAL EQUATION IS USED IN THE FORM

C1*URHOX1*RHO1 + C3*URHOX3*RHO3 = C1*RX1 - RX2 + C3*RX3 + URHOX2*RHO2

C1*URHOY1*RHO1 + C3*URHOY3*RHO3 = C1*RY1 - RY2 + C3*RY3 + URHOY2*RHO2

C1*URHOZ1*RHO1 + C3*URHOZ3*RHO3 = C1*RZ1 - RZ2 + C3*RZ3 + URHOZ2*RHO2

(4) EQUATIONS E FOR M SQUARED AND L FOLLOW IMMEDIATELY FROM GAUSS'S EQUATIONS FOR THE DETERMINATION OF YBAR (DESIGNATED C BUT NOT USED AS SUCH) BY THE CYCLIC SUBSTITUTION

ROOT(2*SR2*SR3)*COS(SF1) = CK1 = ROOT(SR2*SR3 + VECTOR SR2.VECTOR SR3)

(DUBYAGO (1961), EQUATIONS (5-68), (5-69), PAGE 142 ARE EQUATIONS C. NOTE THAT THIS AUTHOR WRITES M FOR M SQUARED)

(5) EQUATIONS D ARE GAUSS'S CUBIC EQUATION IN YBAR AND THE EXPRESSIONS GIVING THE VALUE OF H. THE CUBIC IS SOLVED BY HANSEN'S CONTINUED FRACTION, AND THE QUANTITY XI IS NEGLECTED IN THE FIRST APPROXIMATION FOR H

(DUBYAGO (1961), EQUATIONS (5-78), (5-77), (5-74), (5-75), PAGE 144 AND EQUATION (5-80), PAGE 146)

REFERENCES

BAKER, R.M.L.,JR. (1967),'ASTRODYNAMICS - APPLICATIONS AND ADVANCED TOPICS.' ACADEMIC PRESS, NEW YORK

DUBYAGO, A.D. (1961),'THE DETERMINATION OF ORBITS.' MACMILLAN, NEW YORK

DATES OF OBSERVATION -

(1) 1935 AUG 30 , (2) 1935 SEP 23 , (3) 1935 OCT 21

GEOCENTRIC EQUATORIAL RECTANGULAR COORDINATES OF THE SUN -

AT OBSERVATION NO. 1 , X =-0.92171169 , Y = 0.37827056 , Z = 0.16406096

AT OBSERVATION NO. 2 , X =-1.00322055 , Y =-0.00143706 , Z =-0.00062710

AT OBSERVATION NO. 3 , X =-0.88110324 , Y =-0.42451768 , Z =-0.18412758

TOPOCENTRIC EQUATORIAL RECTANGULAR COORDINATES OF THE SUN -

AT OBSERVATION NO. 1 , RX =-0.92173732 , RY = 0.37827888 , RZ = 0.16402792

AT OBSERVATION NO. 2 , RX =-1.00324123 , RY =-0.00141979 , RZ =-0.00066013

AT OBSERVATION NO. 3 , RX =-0.88112860 , RY =-0.42450858 , RZ =-0.18416062

TOPOCENTRIC DIRECTION COSINES OF THE MINOR PLANET -

AT OBSERVATION NO. 1 , URHOX = 0.97046067 , URHOY =-0.23251232 , URHOZ =-0.06437470 , SUM OF SQUARES = 1.00000000

AT OBSERVATION NO. 2 , URHOX = 0.94369322 , URHOY =-0.29283719 , URHOZ =-0.15391389 , SUM OF SQUARES = 1.00000000

AT OBSERVATION NO. 3 , URHOX = 0.92096454 , URHOY =-0.31883531 , URHOZ =-0.22398293 , SUM OF SQUARES = 1.00000000

COEFFICIENTS FOR THE EQUATIONS A AND B -

EQUATION A

COEFFICIENT OF RHO2 = 0.0018493304

COEFFICIENT OF C1 = 0.0150850422

COEFFICIENT OF C3 =-0.0773620853

CONSTANT TERM = 0.0318176514

EQUATION B

COEFFICIENT OF RHO2 = 1.8924691488

CONSTANT TERM = 1.0064954085

COMPUTATION OF THE SUN - PLANET VECTORS (APPROXIMATION NO. 1)

COEFFICIENTS FOR THE EQUATIONS A1 AND B1 -

EQUATION A1

COEFFICIENT OF (1/SR2) CUBED =-1.5236597559

CONSTANT TERM = 1.8372039837

EQUATION B1

COEFFICIENT OF RHO2 = 1.8924691488

CONSTANT TERM = 1.0064954085

SOLUTION OF THE EQUATIONS A1 AND B1 BY SUCCESSIVE APPROXIMATIONS -

SUCCESSIVE APPROXIMATIONS CONVERGED SUFFICIENTLY AFTER 8 ITERATIONS - PROCESS TERMINATED

VALUES AT TERMINATION WERE RHO2 = 1.7622276055 , SR2 = 2.7289014124

VALUES OF C1, C3 AND SOLUTION OF GAUSS'S FUNDAMENTAL EQUATION FOR RHO1 AND RHO3 -

C1 = 0.5319855135 , C3 = 0.4728902981

RHO1 = 1.7242966367 , RHO3 = 1.9953572434

USING GAUSS'S EQUATION IN Z AS A CHECK WE HAVE LHS =-0.2703979785 , RHS =-0.2703979785

AGREEMENT IS ACCEPTABLE

SUN - PLANET VECTORS (APPROXIMATION NO. 1) -

AT OBSERVATION NO. 1 , SRX = 2.59508968 , SRY =-0.77919677 , SRZ =-0.27502850

AT OBSERVATION NO. 2 , SRX = 2.66624347 , SRY =-0.51462599 , SRZ =-0.27057118

AT OBSERVATION NO. 3 , SRX = 2.71878187 , SRY =-0.21168177 , SRZ =-0.26276534

COMPUTATION OF THE SUN - PLANET VECTORS (APPROXIMATION NO. 2)

COEFFICIENTS FOR THE EQUATIONS A2 AND B2 -

EQUATION A2

COEFFICIENT OF (1/SR2) CUBED =-1.6849724423

CONSTANT TERM = 1.8367143608

EQUATION B2

COEFFICIENT OF RHO2 = 1.8924691488

CONSTANT TERM = 1.0064954085

SOLUTION OF THE EQUATIONS A2 AND B2 BY SUCCESSIVE APPROXIMATIONS -

SUCCESSIVE APPROXIMATIONS CONVERGED SUFFICIENTLY AFTER 7 ITERATIONS - PROCESS TERMINATED

VALUES AT TERMINATION WERE RHO2 = 1.7529556024 , SR2 = 2.7196990916

VALUES OF C1, C3 AND SOLUTION OF GAUSS'S FUNDAMENTAL EQUATION FOR RHO1 AND RHO3 -

C1 = 0.5320180305 , C3 = 0.4731176997

RHO1 = 1.7155036147 , RHO3 = 1.9840884654

USING GAUSS'S EQUATION IN Z AS A CHECK WE HAVE LHS =-0.2690079252 , RHS =-0.2690079252

AGREEMENT IS ACCEPTABLE

SUN - PLANET VECTORS (APPROXIMATION NO. 2) -

AT OBSERVATION NO. 1 , SRX = 2.58656610 , SRY =-0.77715461 , SRZ =-0.27446309

AT OBSERVATION NO. 2 , SRX = 2.65749354 , SRY =-0.51191080 , SRZ =-0.26914409

AT OBSERVATION NO. 3 , SRX = 2.70840373 , SRY =-0.20808889 , SRZ =-0.26024132

COMPUTATION OF THE SUN - PLANET VECTORS (APPROXIMATION NO. 3)

COEFFICIENTS FOR THE EQUATIONS A3 AND B3 -

EQUATION A3

COEFFICIENT OF (1/SR2) CUBED =-1.6857922955

CONSTANT TERM = 1.8367180803

EQUATION B3

COEFFICIENT OF RHO2 = 1.8924691488

CONSTANT TERM = 1.0064954085

SOLUTION OF THE EQUATIONS A3 AND B3 BY SUCCESSIVE APPROXIMATIONS -

SUCCESSIVE APPROXIMATIONS CONVERGED SUFFICIENTLY AFTER 5 ITERATIONS - PROCESS TERMINATED

VALUES AT TERMINATION WERE RHO2 = 1.7529148264 , SR2 = 2.7196586232

VALUES OF C1, C3 AND SOLUTION OF GAUSS'S FUNDAMENTAL EQUATION FOR RHO1 AND RHO3 -

C1 = 0.5320181749 , C3 = 0.4731187026

RHO1 = 1.7154649222 , RHO3 = 1.9840389095

USING GAUSS'S EQUATION IN Z AS A CHECK WE HAVE LHS =-0.2690018102 , RHS =-0.2690018102

AGREEMENT IS ACCEPTABLE

SUN - PLANET VECTORS (APPROXIMATION NO. 3) -

AT OBSERVATION NO. 1 , SRX = 2.58652855 , SRY =-0.77714561 , SRZ =-0.27446060

AT OBSERVATION NO. 2 , SRX = 2.65745506 , SRY =-0.51189886 , SRZ =-0.26913782

AT OBSERVATION NO. 3 , SRX = 2.70835809 , SRY =-0.20807309 , SRZ =-0.26023023

COMPUTATION OF THE SUN - PLANET VECTORS (APPROXIMATION NO. 4)

COEFFICIENTS FOR THE EQUATIONS A4 AND B4 -

EQUATION A4

COEFFICIENT OF (1/SR2) CUBED =-1.6857959100

CONSTANT TERM = 1.8367180968

EQUATION B4

COEFFICIENT OF RHO2 = 1.8924691488

CONSTANT TERM = 1.0064954085

SOLUTION OF THE EQUATIONS A4 AND B4 BY SUCCESSIVE APPROXIMATIONS -

SUCCESSIVE APPROXIMATIONS CONVERGED SUFFICIENTLY AFTER 3 ITERATIONS - PROCESS TERMINATED

VALUES AT TERMINATION WERE RHO2 = 1.7529146467 , SR2 = 2.7196584448

VALUES OF C1, C3 AND SOLUTION OF GAUSS'S FUNDAMENTAL EQUATION FOR RHO1 AND RHO3 -

C1 = 0.5320181756 , C3 = 0.4731187071

RHO1 = 1.7154647515 , RHO3 = 1.9840386910

USING GAUSS'S EQUATION IN Z AS A CHECK WE HAVE LHS =-0.2690017833 , RHS =-0.2690017833

AGREEMENT IS ACCEPTABLE

SUN - PLANET VECTORS (APPROXIMATION NO. 4) -

AT OBSERVATION NO. 1 , SRX = 2.58652839 , SRY =-0.77714557 , SRZ =-0.27446059

AT OBSERVATION NO. 2 , SRX = 2.65745489 , SRY =-0.51189881 , SRZ =-0.26913779

AT OBSERVATION NO. 3 , SRX = 2.70835789 , SRY =-0.20807302 , SRZ =-0.26023018

SUCCESSIVE APPROXIMATIONS OF THE SUN - PLANET VECTORS HAVE NOW CONVERGED SUFFICIENTLY - NO. 4 IS FINAL APPROXIMATION

CHECK ON FINAL VALUES OF SUN - PLANET VECTORS USING THE EQUATION C1*VECTOR SR1 + C3*VECTOR SR3 = VECTOR SR2 -

EQUATION IN X GIVES LHS = 2.6574548947 , RHS = 2.6574549947

EQUATION IN Y GIVES LHS =-0.5118988084 , RHS =-0.5118988084

EQUATION IN Z GIVES LHS =-0.2691377882 , RHS =-0.2691377882

AGREEMENT IS ACCEPTABLE

COMPUTATION OF THE RATIO YBAR OF THE AREA OF THE SECTOR TO THE AREA OF THE TRIANGLE IN THE ORBIT

VALUES OF M SQUARED AND L FROM EQUATIONS E -

FOR OBSERVATION NO. 1 , M SQUARED = 0.00143902288 , L = 0.00080181665

FOR OBSERVATION NO. 2 , M SQUARED = 0.00521322651 , L = 0.00288399548

FOR OBSERVATION NO. 3 , M SQUARED = 0.00114491767 , L = 0.00063964334

SOLUTION OF EQUATIONS D BY SUCCESSIVE APPROXIMATIONS -

FOR OBSERVATION NO. 1, SUCCESSIVE APPROXIMATIONS CONVERGED SUFFICIENTLY AFTER 2 ITERATIONS

VALUES AT TERMINATION WERE H = 0.00172516749 , YBAR = 1.00191282798

FOR OBSERVATION NO. 2, SUCCESSIVE APPROXIMATIONS CONVERGED SUFFICIENTLY AFTER 2 ITERATIONS

VALUES AT TERMINATION WERE H = 0.00623429402 , YBAR = 1.00687500408

FOR OBSERVATION NO. 3, SUCCESSIVE APPROXIMATIONS CONVERGED SUFFICIENTLY AFTER 2 ITERATIONS

VALUES AT TERMINATION WERE H = 0.00137284742 , YBAR = 1.00152283510

CHECK ON VALUES OF YBAR BY RECOMPUTATION OF C1, C3 AND COMPARISON WITH C1, C3 OBTAINED ON PAGE 14 -

C1 = 0.5320178638 , C3 = 0.4731189994

AGREEMENT IS ACCEPTABLE

COMPUTATION OF THE ORBITAL ELEMENTS

PARAMETER OF THE ORBIT P -

P = 3.04235851

ECCENTRICITY E AND TRUE ANOMALIES V1, V3 -

E = 0.12155311

V1 = 6 DEG 44 MIN 42.18 SEC

V3 = 19 DEG 1 MIN 11.23 SEC

SEMI-MAJOR AXIS A AND MEAN DAILY MOTION N -

A = 3.08798396

N = 653.87393 SECONDS OF ARC PER DAY

ECCENTRIC ANOMALIES E1, E3 -

E1 = 5 DEG 58 MIN 15.33 SEC

E3 = 16 DEG 51 MIN 58.06 SEC

USING A*ROOT(1-E SQUARED)*SIN((E3-E1)/2) = ROOT(SR1*SR3)*SIN((V3-V1)/2) AS A CHECK WE HAVE

LHS = 0.29098515 , RHS = 0.29098515

AGREEMENT IS ACCEPTABLE

MEAN ANOMALIES M1, M3 -

M1 = 5 DEG 14 MIN 47.23 SEC

M3 = 14 DEG 50 MIN 43.72 SEC

RECOMPUTATION OF N AND COMPARISON WITH N OBTAINED PREVIOUSLY AS A CHECK ON VALUES OF M1, M3 GIVES

N = 653.87392 SECONDS OF ARC PER DAY

AGREEMENT IS ACCEPTABLE

TIME OF PERIHELION PASSAGE T -

T = JD 2426015.5 , 2 HOURS 32 MIN 24.260 SEC ET

(T IS THE TIME OF THE LAST PERIHELION PASSAGE TO OCCUR BEFORE THE DATE OF THE FIRST OBSERVATION)

RECOMPUTATION OF M3 AND COMPARISON WITH M3 OBTAINED PREVIOUSLY AS A CHECK ON VALUE OF T GIVES

M3 = 14 DEG 50 MIN 43.72 SEC

AGREEMENT IS ACCEPTABLE

MEAN ANOMALY MO AT A CHOSEN EPOCH COMPUTED IN THREE DIFFERENT WAYS AND COMPARED -

CHOSEN EPOCH = JD 2428000.5 ET = 1935 JUL 17.0 ET

MO = 357 DEG 15 MIN 22.69 SEC

MO = 357 DEG 15 MIN 22.69 SEC

MO = 357 DEG 15 MIN 22.69 SEC

AGREEMENT IS ACCEPTABLE

VECTORIAL EQUATORIAL CONSTANTS -

UPX = 0.91222662 , UPY =-0.39670910 , UPZ =-0.10229606 , SUM OF SQUARES = 1.00000000

UQX = 0.39918731 , UQY = 0.91686009 , UQZ = 0.00413061 , SUM OF SQUARES = 1.00000000

SCALAR PRODUCT OF (ORTHOGONAL) UNIT VECTORS P AND Q =-0.00000000

ORIENTATION OF MAJOR AXIS OMEGA -

OMEGA = 169 DEG 56 MIN 53.41 SEC

INCLINATION OF ORBIT I -

I = 21 DEG 30 MIN 30.69 SEC

LONGITUDE OF ASCENDING NODE OMEGA1 -

OMEGA1 = 165 DEG 26 MIN 34.68 SEC

RECOMPUTATION OF I AND COMPARISON WITH I OBTAINED PREVIOUSLY AS A CHECK ON VALUE OF OMEGA1 GIVES

I = 21 DEG 30 MIN 30.69 SEC

AGREEMENT IS ACCEPTABLE

THE DETERMINATION OF THE ORBIT HAS NOW BEEN COMPLETED. A SUMMARY OF THE CLASSICAL ELEMENTS APPEARS ON THE FOLLOWING PAGE

GAUSSIAN DETERMINATION OF THE ORBITAL ELEMENTS OF 1361 LEUSCHNERIA DATE 720221 PAGE 23

SUMMARY OF THE ORBITAL ELEMENTS (REFERRED TO THE EQUATOR AND EQUINOX OF 1950.0) -

ECCENTRICITY E = 0.12155311

SEMI-MAJOR AXIS A = 3.08798396 ASTRONOMICAL UNITS

MEAN ANOMALY MO AT EPOCH 1935 JUL 17.0 ET = 357 DEG 15 MIN 22.69 SEC

ORIENTATION OF MAJOR AXIS OMEGA = 169 DEG 56 MIN 53.41 SEC

INCLINATION OF ORBIT I = 21 DEG 30 MIN 30.69 SEC

LONGITUDE OF ASCENDING NODE OMEGA1 = 165 DEG 26 MIN 34.68 SEC

GAUSSIAN DETERMINATION OF THE ORBITAL ELEMENTS OF 1361 LEUSCHNERIA DATE 720221 PAGE 24

REMARKS

Chapter 1.4 The Generation of an Ephemeris

In the computation of an ephemeris for a minor planet we are essentially solving the inverse problem to that dealt with in the previous chapter. That is, given the elements of the planet's orbit, we wish to determine its apparent position in the sky at a series of specific instants. The solution of this problem is, however, very much more straightforward than the previous one since the orbit is now completely defined in time and space and the computation of the object's RA, Dec. and distance is a transformation from one three-dimensional coordinate system to another rather than from a two-dimensional system to a three-dimensional one as in the orbit determination process.

The purpose of generating an ephemeris for a minor planet is normally to aid observers in finding the object rather than to give computed positions to be compared with observations for orbit improvement, a technique which will be dealt with in the next chapter. In view of this we can simplify the procedure in two ways: firstly by computing the coordinates of the planet at only that time of day for which the geocentric rectangular coordinates of the sun are tabulated in the "Astronomical Ephemeris" (viz. 0^h E.T.) thus avoiding interpolation of the solar coordinates, and secondly by giving the planet's geocentric coordinates to avoid correcting for parallax. Should more exact coordinates of the planet be required for an instant between the tabulated ones, the geocentric coordinates of the planet at that instant can be obtained by interpolation and these can then be reduced to topocentric coordinates by the application of a small parallax correction. Furthermore, since the ephemeris gives the planet's coordinates referred to the same equator and equinox as the orbital elements, a precession correction may be necessary to reduce the coordinates either to the standard equinox of a star chart or catalogue or to the mean equinox of date depending on whether a finding chart or setting circles are to be used to locate the object. In practice, however, refinements such as these are rarely necessary, the geocentric positions at 0^h E.T. normally being perfectly adequate.

The method used to obtain the planet's coordinates can be developed very simply. We begin by considering the transformation between the orbital and equatorial heliocentric rectangular coordinate systems S $\bar{x}\ \bar{y}\ \bar{z}$ and S x y z given in Chapter 1.1:

$$[x\ y\ z] = [\bar{x}\ \bar{y}\ \bar{z}] \begin{bmatrix} P_x & P_y & P_z \\ Q_x & Q_y & Q_z \\ R_x & R_y & R_z \end{bmatrix}$$

where P_x, P_y,..., R_z are the vectorial equatorial constants. Writing this equation in the form

$$x = P_x\bar{x} + Q_x\bar{y} + R_x\bar{z}$$

$$y = P_y\bar{x} + Q_y\bar{y} + R_y\bar{z}$$

$$z = P_z\bar{x} + Q_z\bar{y} + R_z\bar{z}$$

and noting that the position of a body in the (elliptical) orbit is given by

$$\bar{x} = a(\cos E - e)$$

$$\bar{y} = b \sin E$$

$$\bar{z} = 0$$

(where e, a, b and E have their usual meanings) we obtain

$$x = aP_x (\cos E - e) + bQ_x \sin E$$

$$y = aP_y (\cos E - e) + bQ_y \sin E$$

$$z = aP_z (\cos E - e) + bQ_z \sin E$$

as the heliocentric equatorial coordinates of the planet in its orbit. Clearly, these coordinates are none other than the equatorial components of the planet's heliocentric radius vector $\underline{r}$; ie, in the notation of the previous chapter,

$$[x\ y\ z] = [r_x\ r_y\ r_z]$$

If we now introduce the geocentric radius vectors $\underline{R} \equiv (X, Y, Z)$ and $\underline{\rho} \equiv (\xi, \eta, \zeta)$ of the sun and planet respectively (which should not be confused with the equivalent topocentric vectors $\underline{R} \equiv (R_x, R_y, R_z)$ and $\underline{\rho} \equiv (\rho_x, \rho_y, \rho_z)$ used in the previous chapter) we have

$$\underline{\rho} = \underline{R} + \underline{r};$$

ie

$$\xi = X + x$$

$$\eta = Y + y$$

$$\zeta = Z + z$$

In terms of the equatorial coordinates (α, δ) and distance ρ of the planet, its geocentric equatorial rectangular coordinates are

$$\xi = \rho \cos \alpha \cos \delta$$

$$\eta = \rho \sin \alpha \cos \delta$$

$$\zeta = \rho \sin \delta$$

Rearrangement of these expressions then gives us

$$\tan \alpha = \frac{\eta}{\xi}$$

$$\sin \delta = \frac{\zeta}{\rho}$$

where

$$\rho^2 = \xi^2 + \eta^2 + \zeta^2$$

Thus the method used to obtain the planet's coordinates (α, δ, ρ) in the ORBIT 3 computer program is as follows: from the classical orbital elements e, a, ω, i, Ω we obtain values of the mean daily motion n $(= k/a^{3/2})$, the semiminor axis b $(= a\sqrt{1 - e^2})$ and the vectorial equatorial constants P_x, P_y, P_z, Q_x, Q_y, Q_z (using subroutine VECTOR). The products aP_x, aP_y, aP_z, bQ_x, bQ_y, bQ_z are formed. The geocentric equatorial rectangular coordinates of the sun X, Y, Z at the instant t for which the planet's position is required (t being at 0^h E.T.) are then read in, and the planet's mean anomaly M at t is computed from

$$M = M_o + n\ (t - t_o)$$

where M_o is the mean anomaly at t_o. Kepler's equation

$$M = E - e \sin E$$

is then solved using subroutine KEPLER to obtain the planet's eccentric anomaly E at t, and this is used as the argument in the equations

$$\xi = X + aP_x\ (\cos E - e) + bQ_x \sin E$$

$$\eta = Y + aP_y\ (\cos E - e) + bQ_y \sin E$$

$$\zeta = Z + aP_z\ (\cos E - e) + bQ_z \sin E$$

to obtain the planet's geocentric rectangular coordinates at t directly. We then compute

$$\rho = \sqrt{(\xi^2 + \eta^2 + \zeta^2)}$$

and finally

$$\alpha = \tan^{-1} \frac{\eta}{\xi}$$

$$\delta = \sin^{-1} \frac{\zeta}{\rho}$$

The values of α and δ thus obtained are the true geocentric coordinates of the planet in the sky at the instant t. However, the light reaching the observer at t left the planet appreciably earlier when its coordinates were some other values of α and δ, and it is these values, viz. the planet's apparent geocentric coordinates at t, that we require. In order to obtain them we apply a correction for the light time to the true coordinates. If (α_A, δ_A) are the required apparent coordinates and (α, δ) are the true (geometrical) coordinates, then we have

$$\alpha_A = \alpha - \text{light time in days x instantaneous daily motion in } \alpha$$

$$\delta_A = \delta - \text{light time in days x instantaneous daily motion in } \delta$$

assuming the object's apparent motion is constant and linear over the period of the light time ("Explanatory Supplement to the Astronomical Ephemeris", pp. 49-51 and 125). The value of the light time is 0.0057683ρ where ρ is the true geocentric distance of the planet and the numerical coefficient is the light time in days for unit distance (a less recent value than that used in the previous chapter: see "Supplement to the A.E. 1968", p. 20s). The instantaneous daily motion in each coordinate is obtained to a first approximation by taking the mean of the daily motions for the day before and the day after the instant t for which the coordinates are required; ie.

$$\text{instantaneous daily motion in } \alpha = \frac{(\alpha_{t+1} - \alpha_t) + (\alpha_t - \alpha_{t-1})}{2}$$

$$\text{instantaneous daily motion in } \delta = \frac{(\delta_{t+1} - \delta_t) + (\delta_t - \delta_{t-1})}{2}$$

The corrections to the true coordinates then become

$$-0.00288415\rho\ (\alpha_{t+1} - \alpha_{t-1})$$

$$-0.00288415\rho\ (\delta_{t+1} - \delta_{t-1})$$

Clearly, a consequence of using this method to obtain the light time corrections is that for each (α, δ, ρ) obtained (for 0^h E.T.) we must compute (α, δ, ρ) for 0^h E.T. on the previous and following days. If the ephemeris has an interval of one day then these values will be required anyway, but when the interval is more than one day the values must be specially computed. The ORBIT 3 program deals with this by ensuring that (α, δ, ρ) have been computed for three consecutive days and stored in arrays as (i = 1,2,3) $(\alpha_i,\ \delta_i,\ \rho_i)$ before computing the apparent $(\alpha_A,\ \delta_A)$ for the middle date by applying the light time correction to $(\alpha_2,\ \delta_2)$, this result being then printed out. Thus in the input format for the program given on page 116, line 11 (which gives the Julian Date for which the planet's (α, δ, ρ) at 0^h E.T. are required together with the rectangular solar coordinates for the same instant) must appear at least three times giving values for three successive days.

Having computed and printed $(\alpha_A,\ \delta_A)$ for the middle date, ORBIT 3 then reads the next card in the format of line 11, computes the (α,δ,ρ) for the date given and enters these into the above-mentioned arrays as $(\alpha_3,\ \delta_3,\ \rho_3)$, the previous $(\alpha_3,\ \delta_3,\ \rho_3)$ becoming $(\alpha_2,\ \delta_2,\ \rho_2)$ and so on. The values of the corresponding times t_i (i = 1,2,3) are then checked to determine whether the interval between each is one day; if it is then the apparent coordinates for the new middle date are computed and printed out. If it is not, the program reads in the next card in the format of line 11 and computes (α,δ,ρ), repeating the procedure until the arrays of (i = 1,2,3) $(\alpha_i,\ \delta_i,\ \rho_i)$ again contain data for three successive days whereupon the $(\alpha_A,\ \delta_A)$ for the middle date are computed and printed out. (The output from the program is as given in the sample on pages 117 and 118.)

Thus in order to compute an ephemeris with an interval of one day covering a specific period, a solar data card in the format of line 11 must be given (i) for the day before the first date in the period; (ii) for each day in the period covered by the ephemeris, and (iii) for the day after the last date in the period. The positions of the asteroid on dates (i) and (iii) are not given in the output; they are required merely to compute the light time corrections for the first and last dates in the period of the ephemeris. There is no limit to the number of positions which may be computed by repetition of line 11 for successive days; discontinuities in the time interval are acceptable but no positions will be given for the last date before the discontinuity and the first date after it. This means that an ephemeris with a time interval of, say, ten days may be generated by the input of solar data for $t-1^d$, t, $t+1^d$, $t+9^d$, $t+10^d$, $t+11^d$, etc., the asteroid's position being printed out for t, $t+10^d$, etc.

The computation of the ephemeris is terminated when the end card in the format of line 12 (page 116) is read in. It may be noted that if this appears immediately after line 10, ORBIT 3 will merely compute and print the vectorial equatorial constants of the orbit and then stop; thus the program has a (somewhat trivial) secondary function. It is important to note that in the output of the asteroid's coordinates (α, δ, ρ), only α and δ are corrected for light time, the geocentric distance given (ρ) being the true distance of the minor planet at the tabulated instant.

Under the Model 44 Programming System, a typical execution time for ORBIT 3 is 14.5 seconds for ten pages of daily ephemeris (180 asteroid positions) together with the preliminary computations.

The program was tested on its completion by the generation of an ephemeris for the minor planet 16 Psyche based on orbital elements obtained from ORBIT 1 using observations made by the writer (described fully in Part 2 of this work). The sample output for the program given on page 118 is part of this ephemeris. The geocentric coordinates of the minor planet for the three instants at which the original observations were made were obtained from the ephemeris by linear interpolation. These were then reduced to topocentric coordinates by the application of a parallax correction obtained as described in the "Explanatory Supplement to the Astronomical Ephemeris", p. 63. The results were then compared with the original observations, as follows:

U.T of observation	Observed RA and Dec. (1950.0)	Computed RA and Dec. (1950.0)
1970 Sep. $01^d\ 03^h\ 28^m\ 30^s$	$4^h\ 42^m\ 34^s.96$	$4^h\ 42^m\ 34^s.76$
(JD 2440830.6448)	$+19^\circ\ 01'\ 07''.8$	$+19^\circ\ 01'\ 08''.5$
1970 Sep. $16^d\ 03^h\ 27^m\ 30^s$	$4^h\ 57^m\ 14^s.21$	$4^h\ 57^m\ 14^s.29$
(JD 2440845.6441)	$+19^\circ\ 07'\ 30''.7$	$+19^\circ\ 07'\ 32''.1$
1970 Oct. $01^d\ 02^h\ 16^m\ 15^s$	$5^h\ 07^m\ 13^s.62$	$5^h\ 07^m\ 14^s.03$
(JD 2440860.5946)	$+19^\circ\ 01'\ 19''.6$	$+19^\circ\ 01'\ 21''.5$

Clearly the agreement is reasonably good, particularly in view of the rough linear interpolation used to obtain the computed coordinates from the ephemeris positions, ΔT (= $+41^s$ in 1970) having been neglected.

In fact, due to the linear interpolation method used to obtain the light time corrections within ORBIT 3, we would not expect the accuracy of an ephemeris produced by the program to be much better than 0'.1 (depending on the shape of the apparent orbit). We would therefore reiterate at this point the remark made in Chapter 1.1 that the precision given in the output of the ORBIT programs is standardised and thus does not reflect in any way the accuracy of the results, as is obviously the case here.

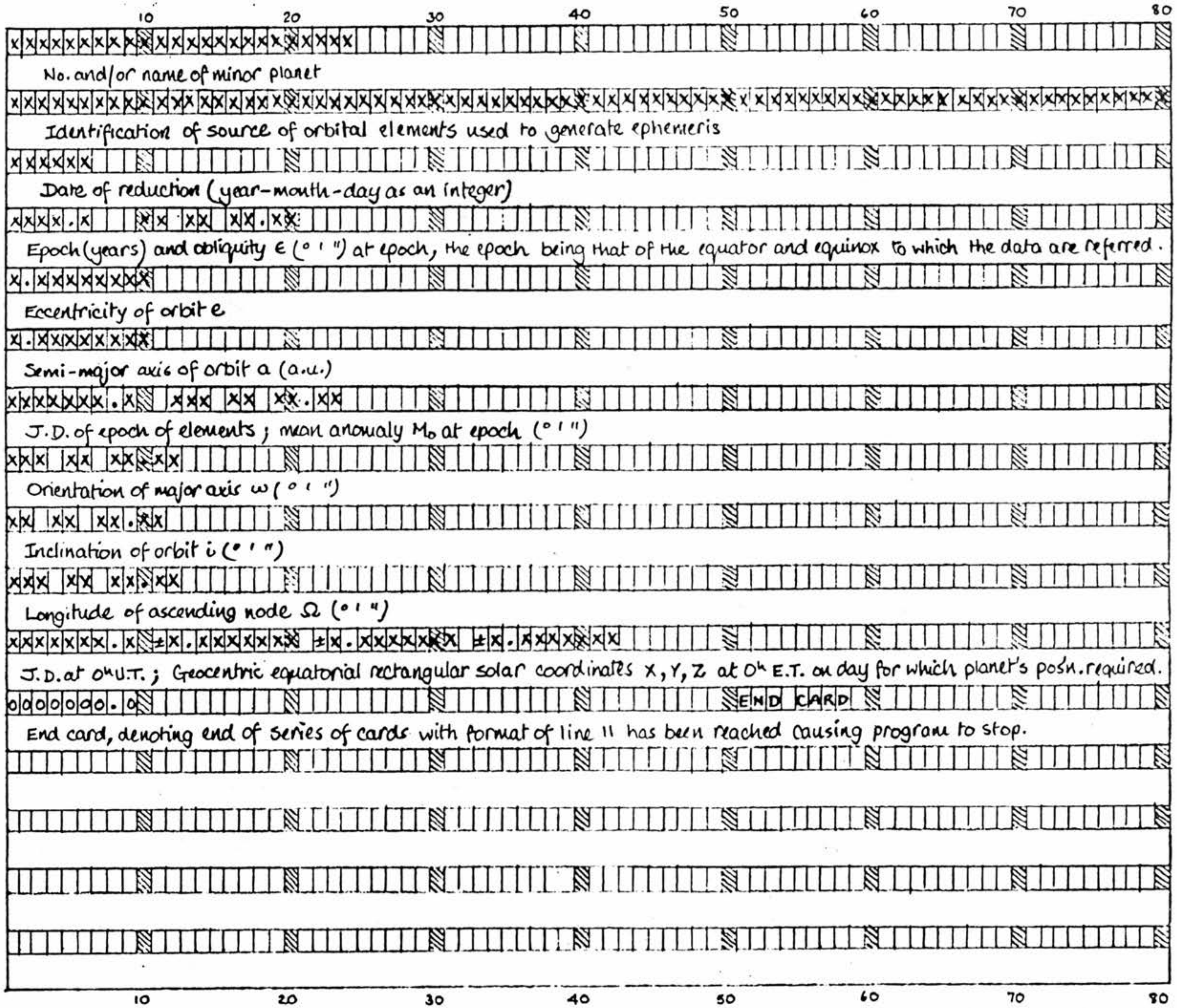
Finally, we remark that further insight into the accuracy given by ORBIT 3 and ORBIT 1 under varying conditions could be gained by supplying ORBIT 3 with hypothetical elements and using values from the resulting ephemeris to regain the elements by means of ORBIT 1. This was not in fact carried out in detail, although some ORBIT 3 runs with hypothetical orbital elements were made in order to obtain a preliminary assessment of the program's performance with high eccentricity orbits.

```
C       ORBIT 3 VERSION 1
C
C
C       GENERATION OF AN EPHEMERIS FOR A MINOR PLANET
C       USING THE CLASSICAL ORBITAL ELEMENTS
C
C
C       CODED IN FORTRAN IV FOR THE IBM SYSTEM/360 MODEL 44
C       PROGRAMMING SYSTEM
C
C
C       F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, APRIL 1972
C
C
C
C       ORBIT LIBRARY SUBPROGRAMS CALLED - VECTOR,KEPLER,ANGLE1,ANGLE2,
C       ANGLE3,ANGLE5,ANGLE6,ANGLE8
C
C
C
        INTEGER*4 I,IRDATE,IDEHO(3),IMIN(3),LCOUNT,IPAGE,
       *IDENTN(6),IDENTS(20),ISIGN2
C
        REAL*8 CONV,TCONV,K,LTIME2,EPOCH,SEC(3),OBL,E,A,JDCE,MO,NRAD,B,
       *OMEGA,SI,OMEGA1,UPX,UPY,UPZ,UQX,UQY,UQZ,SUMSQ(2),JD(4),X,Y,Z,M,
       *CE,CTERM,STERM,XI,ETA,ZETA,RHO(4),ALPHA(4),DELTA(4),FACTOR,
       *AALPHA,ADELTA
C
        REAL*8 DSQRT,DCOS,DSIN,DARSIN,
       *ANGLE1,ANGLE3,ANGLE5,ANGLE8
C
      1 FORMAT('1',4X,23HFGW / ORBIT 3 VERSION 1,83X,4HDATE,I7)
      2 FORMAT(////,35X,'GEOCENTRIC EPHEMERIS FOR THE MINOR PLANET ',6A4)
      3 FORMAT('1',4X,'APPARENT GEOCENTRIC COORDINATES OF ',6A4,50X,'PAGE'
       *,I3)
      4 FORMAT(///,8X,'DATE (0 HOURS ET)',14X,'ALPHA (',F6.1,')',21X,
       *'DELTA (',F6.1,')',17X,'TRUE DISTANCE',/)
C
C
C       PRELIMINARY COMPUTATIONS
C
C
        CONV=206264.80624 64262
        TCONV=13750.98708 30951
        K=0.01720 20989 5
        LTIME2=0.00288 415
C
        READ(5,10)(IDENTN(I),I=1,6),(IDENTS(I),I=1,20),IRDATE,
       *EPOCH,IDEHO(1),IMIN(1),SEC(1)
     10 FORMAT(6A4,/,20A4,/,I6,/,F6.1,2X,2I3,F6.2)
C
        WRITE(6,1)IRDATE
        WRITE(6,2)(IDENTN(I),I=1,6)
C
        WRITE(6,20)(IDENTS(I),I=1,20)
     20 FORMAT(/////,5X,'SOURCE OF ORBITAL ELEMENTS -',///,10X,20A4)
        WRITE(6,30)EPOCH
     30 FORMAT(///,5X,'ELEMENTS USED (EPOCH',F7.1,') -')
C
        OBL=ANGLE1(CONV,IDEHO(1),IMIN(1),SEC(1))
C
        READ(5,40)E,A,JDCE,IDEHO(1),IMIN(1),SEC(1)
     40 FORMAT(F10.8,/,F10.8,/,F9.1,2X,2I3,F6.2)
C
```

```
      WRITE(6,50)E
   50 FORMAT(// ,10X,'ECCENTRICITY E =',F11.8)
      WRITE(6,60)A
   60 FORMAT(/,10X,'SEMI-MAJOR AXIS A =',F11.8,' ASTRONOMICAL UNITS')
      WRITE(6,70)JDCE,IDEHO(1),IMIN(1),SEC(1)
   70 FORMAT(/,10X,'MEAN ANOMALY MO AT EPOCH JD',F10.1,' ET =',I4,' DEG'
     *,I3,' MIN',F6.2,' SEC')
C
      MO=ANGLE1(CONV,IDEHO(1),IMIN(1),SEC(1))
C
      NRAD=K/DSQRT(A**3.0D0)
      B=A*DSQRT(1.0D0-E**2.0D0)
C
      READ(5,80)IDEHO(1),IMIN(1),SEC(1),IDEHO(2),IMIN(2),SEC(2),
     *IDEHO(3),IMIN(3),SEC(3)
   80 FORMAT(2I3,F6.2,/,I2,I3,F6.2,/,2I3,F6.2)
C
      WRITE(6,90)IDEHO(1),IMIN(1),SEC(1)
   90 FORMAT(/,10X,'ORIENTATION OF MAJOR AXIS OMEGA =',I4,' DEG',I3,
     *' MIN',F6.2,' SEC')
      WRITE(6,100)IDEHO(2),IMIN(2),SEC(2)
  100 FORMAT(/,10X,'INCLINATION OF ORBIT I =',I4,' DEG',I3,' MIN',F6.2,
     *' SEC')
      WRITE(6,110)IDEHO(3),IMIN(3),SEC(3)
  110 FORMAT(/,10X,'LONGITUDE OF ASCENDING NODE OMEGA1 =',I4,' DEG',I3,
     *' MIN',F6.2,' SEC')
C
      OMEGA=ANGLE1(CONV,IDEHO(1),IMIN(1),SEC(1))
      SI=ANGLE1(CONV,IDEHO(2),IMIN(2),SEC(2))
      OMEGA1=ANGLE1(CONV,IDEHO(3),IMIN(3),SEC(3))
C
      CALL VECTOR(OBL,OMEGA,SI,OMEGA1,UPX,UPY,UPZ,UQX,UQY,UQZ)
C
      SUMSQ(1)=UPX*UPX+UPY*UPY+UPZ*UPZ
      SUMSQ(2)=UQX*UQX+UQY*UQY+UQZ*UQZ
C
      WRITE(6,120)
  120 FORMAT(///,5X,'DERIVED VECTORIAL EQUATORIAL CONSTANTS -')
      WRITE(6,130)UPX,UPY,UPZ,SUMSQ(1)
  130 FORMAT(// ,10X,'UPX =',F11.8,' , UPY =',F11.8,' , UPZ =',F11.8,
     *' , SUM OF SQUARES =',F11.8)
      WRITE(6,140)UQX,UQY,UQZ,SUMSQ(2)
  140 FORMAT(/,10X,'UQX =',F11.8,' , UQY =',F11.8,' , UQZ =',F11.8,
     *' , SUM OF SQUARES =',F11.8)
C
      UPX=A*UPX
      UPY=A*UPY
      UPZ=A*UPZ
C
      UQX=B*UQX
      UQY=B*UQY
      UQZ=B*UQZ
C
C
C      GENERATION OF THE EPHEMERIS
C
C
      LCOUNT=17
      IPAGE=0
C
      DO 150 I=1,3
  150 JD(I)=0.0D0
C
```

```
  160 READ(5,170)JD(4),X,Y,Z
  170 FORMAT(F9.1,3(1X,F10.7))
C
      IF(JD(4).EQ.0.0D0)GO TO 1000
      M=ANGLE3(M0+NRAD*(JD(4)-JDCE))
C
      CALL KEPLER(E,M,CE,£190)
      WRITE(6,180)
  180 FORMAT(/,10X,83HFAILURE HAS OCCURRED IN THE SOLUTION OF KEPLER'S E
     *QUATION - ORBIT 3 CANNOT CONTINUE)
      GO TO 1000
C
  190 CTERM=DCOS(CE)-E
      STERM=DSIN(CE)
C
      XI=X+UPX*CTERM+UQX*STERM
      ETA=Y+UPY*CTERM+UQY*STERM
      ZETA=Z+UPZ*CTERM+UQZ*STERM
C
      RHO(4)=DSQRT(XI*XI+ETA*ETA+ZETA*ZETA)
      ALPHA(4)=ANGLE5(ETA,XI)
      DELTA(4)=DARSIN(ZETA/RHO(4))
C
      DO 200 I=1,3
      JD(I)=JD(I+1)
      RHO(I)=RHO(I+1)
      ALPHA(I)=ALPHA(I+1)
  200 DELTA(I)=DELTA(I+1)
C
      IF(((JD(3)-JD(2)).NE.1.0D0).OR.((JD(2)-JD(1)).NE.1.0D0))GO TO 160
      LCOUNT=LCOUNT+1
C
      IF(LCOUNT.LE.17)GO TO 210
      LCOUNT=0
      IPAGE=IPAGE+1
      WRITE(6,3)(IDENTN(I),I=1,6),IPAGE
      WRITE(6,4)EPOCH,EPOCH
C
  210 FACTOR=LTIME2*RHO(2)
      AALPHA=ALPHA(2)-FACTOR*ANGLE8(ALPHA(3),ALPHA(1))
      ADELTA=DELTA(2)-FACTOR*(DELTA(3)-DELTA(1))
C
      AALPHA=ANGLE3(AALPHA)
      CALL ANGLE6(ISIGN2,ADELTA)
      CALL ANGLE2(TCONV,AALPHA,IDEHO(1),IMIN(1),SEC(1))
      CALL ANGLE2(CONV,ADELTA,IDEHO(2),IMIN(2),SEC(2))
C
      WRITE(6,220)JD(2),IDEHO(1),IMIN(1),SEC(1),ISIGN2,IDEHO(2),IMIN(2),
     *SEC(2),RHO(2)
  220 FORMAT(/,10X,'JD',F10.1,10X,I3,' HOURS',I3,' MIN',F7.3,' SEC',10X,
     *A1,I2,' DEG',I3,' MIN',F6.2,' SEC',10X,F12.8,' AU')
      GO TO 160
C
C
C     TERMINATION POINT
C
C
 1000 WRITE(6,1010)
 1010 FORMAT('1')
C
      STOP
      END
```

DATA FORMAT FOR ORBIT 3



Lines 5-10 are the orbital elements used. Line 11 is repeated (at least 3 times) to generate the ephemeris — see text.

FGW / ORBIT 3 VERSION 1 DATE 720608

GECCENTRIC EPHEMERIS FOR THE MINCR PLANET 16 PSYCHE

SOURCE OF ORBITAL ELEMENTS -.

ORBIT 1 RESULTS 720329/1

ELEMENTS USEC (EPOCH 195C.0) -

ECCENTRICITY E = 0.14501944

SEMI-MAJCR AXIS A = 2.93994782 ASTRCNCMICAL UNITS

MEAN ANCMALY MO AT EPOCH JD 2440RCC.5 ET = 17 DEG 13 MIN 55.77 SEC

ORIENTATION OF MAJCR AXIS OMEGA = 227 DEG 21 MIN 24.55 SEC

INCLINATION OF ORBIT I = 3 DEG 5 MIN 33.77 SEC

LONGITUCE OF ASCENDING NODE OMEGA1 = 150 CEG 14 MIN 56.91 SEC

DERIVED VECTORIAL EQUATURIAL CONSTANTS -

UPX = 0.95262757 , UPY = 0.29243336 , UPZ = 0.08356703 , SUM OF SQUARES = 1.0CCCCCOO

UQX =-0.3C295R67 , UQY = 0.88821960 , UQZ = 0.34537225 , SUM OF SQUARES = 1.0CCCCCCO

APPARENT GEOCENTRIC COORDINATES OF 16 PSYCHE — PAGE 1

DATE (0 HOURS ET)	ALPHA (1950.0)	DELTA (1950.0)	TRUE DISTANCE
JD 2440829.5	4 HOURS 41 MIN 17.937 SEC	+19 DEG 0 MIN 5.96 SEC	2.43199244 AU
JD 2440830.5	4 HOURS 42 MIN 25.103 SEC	+19 DEG 1 MIN 3.21 SEC	2.42017638 AU
JD 2440831.5	4 HOURS 43 MIN 31.303 SEC	+19 DEG 1 MIN 56.33 SEC	2.40835030 AU
JD 2440832.5	4 HOURS 44 MIN 36.521 SEC	+19 DEG 2 MIN 45.37 SEC	2.39651664 AU
JD 2440833.5	4 HOURS 45 MIN 40.738 SEC	+19 DEG 3 MIN 30.38 SEC	2.38467738 AU
JD 2440834.5	4 HOURS 46 MIN 43.936 SEC	+19 DEG 4 MIN 11.40 SEC	2.37283470 AU
JD 2440835.5	4 HOURS 47 MIN 46.100 SEC	+19 DEG 4 MIN 48.51 SEC	2.36099096 AU
JD 2440836.5	4 HOURS 48 MIN 47.210 SEC	+19 DEG 5 MIN 21.73 SEC	2.34914834 AU
JD 2440837.5	4 HOURS 49 MIN 47.251 SEC	+19 DEG 5 MIN 51.13 SEC	2.33730903 AU
JD 2440838.5	4 HOURS 50 MIN 46.207 SEC	+19 DEG 6 MIN 16.76 SEC	2.32547529 AU
JD 2440839.5	4 HOURS 51 MIN 44.059 SEC	+19 DEG 6 MIN 38.68 SEC	2.31364550 AU
JD 2440840.5	4 HOURS 52 MIN 40.792 SEC	+19 DEG 6 MIN 56.93 SEC	2.30183356 AU
JD 2440841.5	4 HOURS 53 MIN 36.388 SEC	+19 DEG 7 MIN 11.56 SEC	2.29002976 AU
JD 2440842.5	4 HOURS 54 MIN 30.833 SEC	+19 DEG 7 MIN 22.63 SEC	2.27824005 AU
JD 2440843.5	4 HOURS 55 MIN 24.108 SEC	+19 DEG 7 MIN 30.19 SEC	2.26646646 AU
JD 2440844.5	4 HOURS 56 MIN 16.195 SEC	+19 DEG 7 MIN 34.28 SEC	2.25471109 AU
JD 2440845.5	4 HOURS 57 MIN 7.075 SEC	+19 DEG 7 MIN 34.95 SEC	2.24297581 AU
JD 2440846.5	4 HOURS 57 MIN 56.730 SEC	+19 DEG 7 MIN 32.25 SEC	2.23126248 AU

APPARENT GEOCENTRIC COORDINATES OF 16 PSYCHE — PAGE 2

DATE (0 HOURS ET)	ALPHA (1950.0)	DELTA (1950.0)	TRUE DISTANCE
JD 2440847.5	4 HOURS 58 MIN 45.138 SEC	+19 DEG 7 MIN 26.23 SEC	2.21957353 AU
JD 2440848.5	4 HOURS 59 MIN 32.278 SEC	+19 DEG 7 MIN 16.93 SEC	2.20791090 AU
JD 2440849.5	5 HOURS 0 MIN 18.129 SEC	+19 DEG 7 MIN 4.39 SEC	2.19627729 AU
JD 2440850.5	5 HOURS 1 MIN 2.669 SEC	+19 DEG 6 MIN 48.67 SEC	2.18467508 AU
JD 2440851.5	5 HOURS 1 MIN 45.875 SEC	+19 DEG 6 MIN 29.82 SEC	2.17310693 AU
JD 2440852.5	5 HOURS 2 MIN 27.726 SEC	+19 DEG 6 MIN 7.90 SEC	2.16157581 AU
JD 2440853.5	5 HOURS 3 MIN 8.199 SEC	+19 DEG 5 MIN 42.94 SEC	2.15008469 AU
JD 2440854.5	5 HOURS 3 MIN 47.274 SEC	+19 DEG 5 MIN 15.04 SEC	2.13863650 AU
JD 2440855.5	5 HOURS 4 MIN 24.930 SEC	+19 DEG 4 MIN 44.22 SEC	2.12723466 AU
JD 2440856.5	5 HOURS 5 MIN 1.146 SEC	+19 DEG 4 MIN 10.56 SEC	2.11588223 AU
JD 2440857.5	5 HOURS 5 MIN 35.902 SEC	+19 DEG 3 MIN 34.11 SEC	2.10458263 AU
JD 2440858.5	5 HOURS 6 MIN 9.179 SEC	+19 DEG 2 MIN 54.93 SEC	2.09333927 AU
JD 2440859.5	5 HOURS 6 MIN 40.958 SEC	+19 DEG 2 MIN 13.09 SEC	2.08215552 AU
JD 2440860.5	5 HOURS 7 MIN 11.220 SEC	+19 DEG 1 MIN 28.65 SEC	2.07103503 AU
JD 2440861.5	5 HOURS 7 MIN 39.947 SEC	+19 DEG 0 MIN 41.68 SEC	2.05998150 AU
JD 2440862.5	5 HOURS 8 MIN 7.124 SEC	+18 DEG 59 MIN 52.22 SEC	2.04899834 AU
JD 2440863.5	5 HOURS 8 MIN 32.732 SEC	+18 DEG 59 MIN 0.37 SEC	2.03808951 AU

Chapter 1.5 The Improvement of the Orbit

The final problem dealt with by the ORBIT library is the improvement of an orbit by the differential correction of the elements. Essentially, the technique consists in using the preliminary orbital elements to compute the object's equatorial coordinates (α_c, δ_c) for a series of instants at which its observed coordinates (α_o, δ_o) have been obtained; the residuals $d\alpha = \cos\delta_c (\alpha_o - \alpha_c)$ and $d\delta = (\delta_o - \delta_c)$ are then used to determine corrections to the orbital elements which will result in the corrected orbit being the one which best fits the observations, ie a second set of residuals obtained using the corrected orbit will contain randomly distributed minimum values. Thus, whilst the corrected orbit is still Keplerian (ie a two-body orbit, perturbations having been neglected) it is nevertheless the best-fitting mean orbit for the period of the observations.

The method used for the differential correction in the present work is that given by Porter (1949) and the ORBIT 4 computer program follows Porter's method exactly with the addition of two small improvements later suggested by him (see Dinwoodie (1972, p. 11)). Porter's paper gives in full the derivation of the method from first principles and, rather than repeat this development here, we confine ourselves to giving an outline of the salient points together with an account of the method as it is used in ORBIT 4.

Porter begins by selecting e, n (mean daily motion) and M_o (at t_o) together with the equatorial components of $\hat{\underline{P}}$, $\hat{\underline{Q}}$ and $\hat{\underline{R}}$ as the form of the orbital elements most suitable for the differential correction; thus the corrections required will be de, dn, dM_o and the quantities dp, dq, ds which are rotations about the vectors $\hat{\underline{P}}$, $\hat{\underline{Q}}$ and $\hat{\underline{R}}$ respectively. He then goes on to develop relationships between these quantities and the residuals $d\alpha$, $d\delta$, obtaining the so-called "equations of condition" in the form

$$A = B_1 dM_o + B_2 dn + B_3 de + B_4 ds$$

$$D = C_1 dM_o + C_2 dn + C_3 de + C_4 ds + C_5 dp + C_6 dq$$

where A and D are functions of both $d\alpha$ and $d\delta$. Clearly, for each observation of the object, we can obtain one such pair of equations in which A, D, the B_i and the C_i are known. Thus, to obtain a solution for the differential corrections, we require a minimum of four observations yielding four pairs of equations, the values of dM_o, dn, de, ds being obtained from the equations in A and these being then substituted into the equations in D to obtain dp and dq. In practice it is normal to obtain more than the minimum number of equations so that a least-squares or some alternative method (such as the one given by Porter in the present case) may be used for their solution.

Once the differential corrections have been obtained they can then be applied to the orbital elements; de, dn and dM_o are applied directly but dp, dq and ds must first be transformed into corrections dP_x, dP_y, dP_z, dQ_x, dQ_y, dQ_z, dR_x, dR_y, dR_z to the vectorial equatorial constants.

Although we do not make use of it here, it is worthwhile remarking that Porter gives an alternative method for use when orbital data from a previous apparition of the object is available. This involves the elimination of dM_o from the equations of condition by means of the substitution

$$dM_o = (t_o - T)\, dn$$

where T is the time of the previous perihelion passage, the equations of condition then becoming

$$A = B'_2 dn + B_3 de + B_4 ds$$

$$D = C'_2 dn + C_3 de + C_4 ds + C_5 dp + C_6 dq$$

which may be solved when a minimum of three observations are available.

Since in the present work we assume no previous knowledge of the orbit other than the preliminary orbital elements required to be corrected, we retain dM_o as an unknown in the equations of condition.

The principal steps of the reduction as it is used in the ORBIT 4 computer program are as follows: in the same way as in ORBIT 3 the (uncorrected) classical orbital elements e, a, M_o (at t_o), ω, i, Ω are read (in the format given on page 147) and ω, i, Ω are used to obtain values for the vectorial equatorial constants P_x, P_y, P_z, Q_x, Q_y, Q_z (by subroutine VECTOR). The equatorial components of $\hat{\underline{R}}$ are then obtained from

$$R_x = P_yQ_z - P_zQ_y$$

$$R_y = P_zQ_x - P_xQ_z$$

$$R_z = P_xQ_y - P_yQ_x$$

(As we have remarked before, these should not be confused with the equatorial components R_x, R_y, R_z of the topocentric solar radius vector $\underline{R}$ which will also be required in this chapter.) The values of the angle of eccentricity ϕ (= $\sin^{-1}e$), semi-minor axis b (= a cos ϕ) and mean daily motion n (= $k/a^{3/2}$) are then computed, together with K(= $2/_{3n}$) defined by the differential relation da = -Kadn. The products aP_x, aP_y, aP_z, bQ_x, bQ_y, bQ_z are formed.

The computation of the residuals dα, dδ and the coefficients for the equations of condition is then carried out for each observation in turn. A check is included at this point to ensure that the number of observations to be taken into account (N) is not less than four and (rather arbitrarily) not greater than twenty. For each observation the program first reads in the data given in lines 12-17 of the input data format, this being required to compute precisely the topocentric RA and Dec of the object (α_c, δ_c) at the instant of the observation, and thence the residuals dα, dδ. The longitude and latitude of the observing station (required for the correction of the rectangular solar coordinates to topocentric values) are read in with the data for each observation so that observations from more than one station may be taken into account. Similarly, a new value of ΔT (= E.T. - U.T.) is given with each observation to obtain maximum precision. It may be remarked at this point that the format of lines 12-17 is the same as for the corresponding data in ORBIT 1 to simplify any interchange between the two programs and, in fact, the computation now proceeds in much the same way as in ORBIT 1, the data being checked before subroutines GEOS and TOPOS are used for the interpolation of the geocentric solar coordinates to the instant of the observation, and their subsequent correction to topocentric coordinates R_x, R_y, R_z.

We now compute the object's R.A. and Dec. (α_c, δ_c) in much the same way as in ORBIT 3, viz. the mean anomaly M at the E.T. of the observation t is obtained from

$$M = M_o + n(t - t_o)$$

and this is used in Kepler's equation

$$M = E - e \sin E$$

to obtain E at t by means of subroutine KEPLER. The object's topocentric rectangular coordinates ξ, η, ζ (denoted in previous chapters by ρ_x, ρ_y, ρ_z) are then obtained from

$$\xi = R_x + aP_x (\cos E - e) + bQ_x \sin E$$

$$\eta = R_y + aP_y (\cos E - e) + bQ_y \sin E$$

$$\zeta = R_z + aP_z (\cos E - e) + bQ_z \sin E$$

(where R_x, R_y, R_z are the topocentric coordinates of the sun) and its topocentric distance ρ is obtained from

$$\rho = \sqrt{(\xi^2 + \eta^2 + \zeta^2)}$$

We now depart from the procedure of ORBIT 3, however, by the introduction of the correction for light time. This is because the manner in which the correction was obtained in ORBIT 3 is neither suitable nor sufficiently precise for use here due to its dependence on the linear interpolation of successive (α_c, δ_c) values, and so we employ a different method as follows. Knowing as we do at this point the topocentric distance ρ of the object at the actual time of the observation t, we can determine the light time $0^d.00577560\rho$ and subtract this from t to obtain a first approximation of the time at which the light left the planet (the true time of the observation). If this value is now used as t in the above computation we can obtain a new value of ρ and subsequently iterate until successive values of ρ converge, the final (ξ, η, ζ) representing the object's coordinates at exactly the instant at which the observed light left it. (No successive corrections to the sun's coordinates are required, of course, since it is the true coordinates that are tabulated and are thus obtained when the solar ephemeris is entered with the actual time of the observation.)

The final values of (ξ, η, ζ) and ρ having been obtained, we compute

$$\alpha_c = \tan^{-1} \frac{\eta}{\xi}$$

$$\delta_c = \sin^{-1} \frac{\zeta}{\rho}$$

which represent the apparent topocentric coordinates of the object at the (actual) instant of the observation and can thus be legitimately compared with the observed coordinates (α_o, δ_o) to obtain the residuals

$$d\alpha = \cos\delta_c\ (\alpha_o - \alpha_c)$$

$$d\delta = (\delta_o - \delta_c)$$

(For the purpose of this argument we assume that the reduction of the observed coordinates (α_o, δ_o) has already taken account of stellar aberration, atmospheric refraction, precession and nutation, as was implicitly assumed in the case of the observed coordinates for the determination of the preliminary orbit. In both cases the effect of diurnal aberration can be considered small enough to be neglected.) The residuals for the observation are printed out (see the sample output on pages 149 to 151) and we then go on to compute the coefficients in the equations of condition for the same observation.

We begin by computing the object's direction cosines

$$\lambda = \cos\delta_o \cos\alpha_o$$

$$\mu = \cos\delta_o \sin\alpha_o$$

$$\nu = \sin\delta_o$$

(without the use of subroutine DICOS, which would be inefficient in this context) and the auxiliary quantities

$$R_1 = R_y \cos\alpha_o - R_x \sin\alpha_o$$

$$R_2 = R_z \cos\delta_o - \sin\delta_o\ (R_y \sin\alpha_o + R_x \cos\alpha_o)$$

where R_x, R_y, R_z represent the equatorial components of $\underline{\hat{R}}$ (as they do throughout the remainder of this section). We then compute

$$P = \lambda P_x + \mu P_y + \nu P_z$$

$$Q = \lambda Q_x + \mu Q_y + \nu Q_z$$

$$R = \lambda R_x + \mu R_y + \nu R_z$$

and, using the final values of ρ and E obtained in the computation of the residuals,

$$(r/a) = 1 - e \cos E$$

$$\sin v = \frac{\cos \phi \sin E}{(r/a)}$$

$$\cos v = \frac{\cos E - e}{(r/a)}$$

$$G_C = \frac{a}{\rho} P \; ; \quad G_S = \frac{a}{\rho} Q$$

and finally

$$F_C = G_S \sin v + G_C \cos v ; \; F_S = G_S \cos v - G_C \sin v$$

where v is the true anomaly of the object, r being its heliocentric distance.

The coefficients for the equations of condition are then obtained from

$$A = R_2 d\alpha - R_1 d\delta$$

$$D = R_1 d\alpha + R_2 d\delta$$

$$B_1 = F_C \sec \phi + G_C \tan \phi$$

$$B_2 = B_1 (t - t_o) + K (r/a) F_S$$

$$B_3 = F_C \sec \phi \sin E + G_S$$

$$B_4 = F_C (r/a)$$

$$C_1 = -(F_S \sec\phi + G_S \tan\phi)\,R$$

$$C_2 = C_1\,(t - t_o) + K\,(r/a)\,F_C\,R$$

$$C_3 = -(F_S \sec\phi \sin E - G_C)\,R$$

$$C_4 = -F_S\,(r/a)\,R$$

$$C_5 = (a/\rho)\cos\phi \sin E\,(1 - R^2)$$

$$C_6 = -(a/\rho)\,(\cos E - e)\,(1 - R^2)$$

where t is the true time of the observation.

These coefficients having been evaluated and stored in arrays, the whole process of computing the residuals and the coefficients is repeated for the next observation. When this has been carried out for all N observations we then have coefficients for N sets of equations of condition in the form

$$A = B_1 dM_o + B_2 dn + B_3 de + B_4 ds$$

$$D = C_1 dM_o + C_2 dn + C_3 de + C_4 ds + C_5 dp + C_6 dq$$

and the next step in the reduction is the solution of these to obtain the differential corrections de, dn, dM_o, dp, dq, ds. Concerning this, Porter argues that a least-squares method of solution is to some extent unjustifiable since the residuals will not be normally distributed and he suggests an alternative method which we follow here.

Disregarding Porter's changes of scale (which are merely for convenience in manual computation) and implementing his later suggestion of eliminating the variables from the equations in A in the order ds, de, dn, dM_o rather than the reverse, the method of solution is basically as follows. The equations in A are solved first, and the signs of the coefficients A, B_1, B_2, B_3, B_4 in each equation in the set are (if necessary) changed so as to make all the coefficients B_4 of ds positive. The equations are then summed to give a new equation

$$\Sigma A = \Sigma B_1 \, dM_o + \Sigma B_2 dn + \Sigma B_3 de + \Sigma B_4 ds$$

(where the Σ represents a summation over N) and this is then used to eliminate ds from the original equations, giving N equations of the form

$$A' = B_1' dM_o + B_2' dn + B_3' de$$

We now change the signs so that all the coefficients B_3' of de are positive, sum the equations to obtain

$$\Sigma A' = \Sigma B_1' dM_o + \Sigma B_2' dn + \Sigma B_3' de$$

and use this to eliminate de from the equations in A', giving them the form

$$A'' = B_1'' dM_o + B_2'' dn$$

Repeating the process again by changing the signs so that all the B_2'' are positive and summing, we obtain

$$\Sigma A'' = \Sigma B_1'' dM_o + \Sigma B_2'' dn$$

which is used to eliminate dn from the equations in A'' yielding

$$A''' = B_1''' dM_o$$

Finally, the signs are changed so as to render the coefficients B_1''' positive and the equations are summed to give

$$\Sigma A''' = \Sigma B_1''' dM_o$$

from which the value of dM_o is obtained, this being then substituted into

$$\Sigma A'' = \Sigma B_1'' dM_o + \Sigma B_2'' dn$$

$$\Sigma A' = \Sigma B_1' dM_o + \Sigma B_2' dn + \Sigma B_3' de$$

$$\Sigma A = \Sigma B_1 dM_o + \Sigma B_2 dn + \Sigma B_3 de + \Sigma B_4 ds$$

to obtain, successively, dn, de and ds.

An identical method is followed for the solution of the equations in D, having first substituted into them the values of dM_o, dn, de, ds obtained above to give N equations of the form

$$D_1 = C_5 dp + C_6 dq$$

The equations

$$\Sigma D_1 = \Sigma C_5 dp + \Sigma C_6 dq$$

and (on the elimination of dq)

$$\Sigma D_1' = \Sigma C_5' dp$$

are obtained in the same way as before, these yielding the values of dp and dq. The solution of the equations of condition is thus completed and we can now apply the corrections obtained to the original orbital elements.

In applying the corrections dp, dq, ds it is first necessary to transform them into corrections dP_x, dP_y, dP_z, dQ_x, dQ_y, dQ_z, dR_x, dR_y, dR_z to the vectorial equatorial constants and it is here that we employ the second of Porter's suggested improvements. The actual relationships between the corrections to the vectorial constants and the rotations dp, dq, ds are

$$
\begin{aligned}
dP_x &= Q_x ds - R_x dq \\
dP_y &= Q_y ds - R_y dq \\
&\vdots \\
dQ_x &= R_x dp - P_x ds \\
&\vdots \\
dR_z &= P_z dq - Q_z dp
\end{aligned}
$$

and these are used with the original values of P_x, P_y, .., Q_x, .., R_z to obtain first approximations to dP_x, dP_y, .., dQ_x, .., dR_z. The values thus obtained are then used to form $P_x + \frac{1}{2}dP_x$, $P_y + \frac{1}{2}dP_y$, .., $Q_x + \frac{1}{2}dQ_x$, ..., $R_z + \frac{1}{2}dR_z$ which are substituted for P_x, P_y, ..., Q_x, ..., R_z in the above expressions. Thus an iterative procedure is set up to obtain refined values of dP_x, dP_y, .., dQ_x, .., dR_z and when successive approximations to these have converged satisfactorily they are applied directly to the original vectorial constants to obtain the corrected values $P_x + dP_x$, $P_y + dP_y$, ..., $Q_x + dQ_x$, ..., $R_z + dR_z$.

The corrections dM_o, dn, de are applied directly to the original elements to obtain the corrected values $M_o + dM_o$, n + dn, e + de and the corrected values of a $(= (k/n)^{2/3})$ and b (= a cos ϕ where $\phi = \sin^{-1} e$) are then computed. Finally, subroutine ORBEL is used to transform P_x, P_y, ..., Q_z into new values of ω, i, Ω; thus we now have corrected values for the elements in the same form as at the beginning (viz. e, a, M_o (at t_o), ω, i, Ω) and the improvement of the orbit is completed.

It now only remains to ascertain the extent to which the new elements fit the observations by means of the computation of a second set of residuals and this is carried out in exactly the same way as for the first set, thus ending the reduction.

Under the Model 44 Programming System the ORBIT 4 program will execute the whole of this computation in (typically) 6 seconds, this particular time being for an orbit improvement in which twelve observations were taken into account.

The program was tested by using a selection of twelve observations of 16 Psyche taken from the series described in Part 2 of this thesis, the original orbital elements for the improvement having been computed by the ORBIT 1 program. The results of the first test run are given as the sample output for the program on pages 149 to 151 . It will be seen that a large reduction in the magnitude of the residuals has been accomplished, the maximum residuals before correction being $-175''.69$ in α, $-30''.91$ in δ, and after correction $+3''.18$ in α, $+1''.22$ in δ. However, it is also apparent from the output that the second set of residuals does not display the random distribution that we should expect if the program is to be regarded as fully effective. This is clearly due to the extreme conditions involved in this case, viz. a relatively large number of observations combined with some very large initial residuals, and in order to overcome this a second run of the program beginning with the corrected elements was carried out. This run produced the following values for the orbital elements:

Epoch 1970 Aug. $02^{d}.0$ E.T.	Equator and equinox 1950.0
$e = 0.13914292$	$i = 3^{\circ}\ 05'\ 29''.99$
$a = 2.92094523$ a.u.	$\omega = 227^{\circ}\ 33'\ 07''.00$
$M_{o} = 17^{\circ}\ 21'\ 54''.64$	$\Omega = 150^{\circ}\ 10'\ 14''.86$

which yielded the following set of residuals:

$d\alpha$	$d\delta$
-0".53	+0".94
-0.12	+1.04
+0.36	+0.08
-0.14	+0.25
+0.71	-0.46
-0.07	-0.61
+0.08	-1.22
-0.17	-0.29
+0.11	+0.02
+0.22	-0.15
+0.19	+0.16
-0.56	+0.22

Clearly, these show a marked improvement over the previous set.

In order to determine whether the orbit could be improved still further, a third run of the program was carried out beginning with the above elements but this produced values of the corrections de, dn, dM_o, dp, dq, ds which were essentially zero and so the elements remained unchanged. We thus conclude that the above orbit is the one which best fits the twelve observations used.

It will be noted that in carrying out these repeated runs of ORBIT 4 we have essentially set up an iterative procedure and an obvious improvement would be to make provision for this within the program itself, the iterative loop being over virtually the entire computation. A technique such as this would have presented a formidable task using manual calculation, but it becomes a realistic proposal when the computer is used.

The ORBIT 4 results presented above lead us to the conclusion that the performance of the program is satisfactory and that the method is effective in fitting an orbit to a given series of observations. An attempt to evaluate the overall accuracy of the program using further observations of 16 Psyche was subsequently made, and this is described in Part 2 of this thesis.

```
C       ORBIT 4 VERSION 1
C
C
C       IMPROVEMENT OF ORBITAL ELEMENTS BY DIFFERENTIAL CORRECTION
C       USING PORTER'S METHOD
C
C
C       CODED IN FORTRAN IV FOR THE IBM SYSTEM/360 MODEL 44
C       PROGRAMMING SYSTEM
C
C
C       F.G.WATSON, ST.ANDREWS UNIVERSITY OBSERVATORY, JUNE 1973
C
C
C
C       ORBIT LIBRARY SUBPROGRAMS CALLED - GEOS,TOPOS,VECTOR,KEPLER,
C       ORBEL,COVECT,SCALAR,ANGLE1,ANGLE2,ANGLE3,ANGLE4,ANGLE5,ANGLE6,
C       ANGLE8
C
C
C
        INTEGER*4 IRDATE,IDEHO(4),IMIN(4),IPAGE,N,I,J,IAPPRX,
       *IDENTN(6),IDENTS(20),ISIGN(4)
C
        REAL*8 CCONV,CONV,TCONV,K,LITIME,EPOCH,SEC(4),OBL,E,A,JDCE,MC,
       *OMEGA,SI,OMEGA1,UPX,UPY,UPZ,UQX,UQY,UQZ,URX,URY,URZ,SUMSQ(3),
       *PHI,COSPHI,SECPHI,TANPHI,B,NRAD,CK,AX,AY,AZ,BX,BY,BZ,CT,LONG,LAT,
       *YDATE,JD(20),UTD,OALPHA(20),ODELTA(20),ETD,JDC,XO,D2XO,YO,D2YO,ZO,
       *D2ZO,JD1,X1,D2X1,Y1,D2Y1,Z1,D2Z1,JDGST,GSTOD,X,Y,Z,RX(20),RY(20),
       *RZ(20),RHO,PREVRO,JDT,T,M,CE,CTERM,STERM,XI,ETA,ZETA,CALPHA,
       *CDELTA,DALPHA,DDELTA,DASEC,DDSEC,SINA,COSA,SIND,COSD,LAMBDA,ML,R1,
       *R2,BP,BC,BR,AUPRHO,CIFACT,RUPONA,CSPSNE,SINV,COSV,GC,GS,FC,FS,
       *FCSECP,FSSECP,CA(20),CD(20),B1(20),B2(20),B3(20),B4(20),C1(20),
       *C2(20),C3(20),C4(20),C5(20),C6(20),CASUM,B1SUM,B2SUM,B3SUM,B4SUM,
       *CA1ST,B11ST,B21ST,B31ST,CA2ND,B12ND,B22ND,CA3RD,B13RD,DMO,DN,DE,
       *DS,CDSUM,C5SUM,C6SUM,CD1ST,C51ST,DP,DQ,DMOSEC,DNSEC,DUPX,DUPY,
       *DUPZ,DUQX,DUQY,DUQZ,DURX,DURY,DURZ,PRDUPX,PRDUPY,PRDUPZ,PRDUQX,
       *PRDUQY,PRDUQZ,PRDURX,PRDURY,PRDURZ
C
        REAL*8 DVEC,VEC1,DVEC1,DROT2,VEC2,DVEC2,DROT1,
       *DARSIN,DCOS,DTAN,DSIN,DSQRT,DABS,
       *COVECT,SCALAR,ANGLE1,ANGLE3,ANGLE5,ANGLE8
C
        DVEC(VEC1,DVEC1,DROT2,VEC2,DVEC2,DROT1) =
       *    (VEC1+0.5D0*DVEC1)*DROT2-(VEC2+0.5D0*DVEC2)*DROT1
C
      1 FORMAT('1',4X,23HFGW / ORBIT 4 VERSION 1,83X,4HDATE,I7)
      2 FORMAT(///  ,40X, 28HIMPROVEMENT OF THE ORBIT OF ,6A4,
       *//  ,40X, 48HBY DIFFERENTIAL CORRECTION USING PORTER'S METHOD)
      3 FORMAT('1',4X, 40HDIFFERENTIAL CORRECTION OF THE ORBIT OF ,6A4,
       *45X,4HPAGE,I2)
```

```
C
C
C     PRELIMINARY COMPUTATIONS
C
C
      DCONV=86400.0
      CONV=206264.80624 64262
      TCONV=13750.98708 30951
      K=0.01720 20989 5
      LITIME=0.00577 560
C
      READ(5,10)(IDENTN(J),J=1,6),(IDENTS(J),J=1,20),IRDATE,
     *EPOCH,IDEHO(1),IMIN(1),SEC(1)
   10 FORMAT(6A4,/,20A4,/,I6,/,F6.1,2X,2I3,F6.2)
C
      WRITE(6,1)IRDATE
      WRITE(6,2)(IDENTN(J),J=1,6)
C
      WRITE(6,20)(IDENTS(J),J=1,20)
   20 FORMAT(////  ,5X,'SOURCE OF ORIGINAL ELEMENTS -',///,10X,20A4)
      WRITE(6,30)EPOCH
   30 FORMAT(///,5X,'ORIGINAL ELEMENTS (EPOCH',F7.1,') -')
C
      OBL=ANGLE1(CONV,IDEHO(1),IMIN(1),SEC(1))
C
      READ(5,40)E,A,JDCE,IDEHO(1),IMIN(1),SEC(1),IDEHO(2),IMIN(2),
     *SEC(2),IDEHO(3),IMIN(3),SEC(3),IDEHO(4),IMIN(4),SEC(4)
   40 FORMAT(F10.8,/,F10.8,/,F9.1,2X,2I3,F6.2,/,2I3,F6.2,/,I2,I3,F6.2,
     */,2I3,F6.2)
C
      WRITE(6,50)E
   50 FORMAT(//,10X,'ECCENTRICITY E =',F11.8)
      WRITE(6,60)A
   60 FORMAT(/,10X,'SEMI-MAJOR AXIS A =',F11.8,' ASTRONOMICAL UNITS')
      WRITE(6,70)JDCE,IDEHO(1),IMIN(1),SEC(1)
   70 FORMAT(/,10X,'MEAN ANOMALY MO AT EPOCH JD',F10.1,' ET =',I4,' DEG'
     *,I3,' MIN',F6.2,' SEC')
      WRITE(6,80)IDEHO(2),IMIN(2),SEC(2)
   80 FORMAT(/,10X,'ORIENTATION OF MAJOR AXIS OMEGA =',I4,' DEG',I3,
     *' MIN',F6.2,' SEC')
      WRITE(6,90)IDEHO(3),IMIN(3),SEC(3)
   90 FORMAT(/,10X,'INCLINATION OF ORBIT I =',I3,' DEG',I3,' MIN',F6.2,
     *' SEC')
      WRITE(6,100)IDEHO(4),IMIN(4),SEC(4)
  100 FORMAT(/,10X,'LONGITUDE OF ASCENDING NODE OMEGA1 =',I4,' DEG',I3,
     *' MIN',F6.2,' SEC')
C
```

```
      MO=ANGLE1(CONV,IDEHO(1),IMIN(1),SEC(1))
      OMEGA=ANGLE1(CONV,IDEHO(2),IMIN(2),SEC(2))
      SI=ANGLE1(CONV,IDEHO(3),IMIN(3),SEC(3))
      OMEGA1=ANGLE1(CONV,IDEHO(4),IMIN(4),SEC(4))
C
      CALL VECTOR(OBL,OMEGA,SI,OMEGA1,UPX,UPY,UPZ,UQX,UQY,UQZ)
C
      URX=COVECT(UPY,UPZ,UQY,UQZ)
      URY=COVECT(UPZ,UPX,UQZ,UQX)
      URZ=COVECT(UPX,UPY,UQX,UQY)
C
      SUMSQ(1)=SCALAR(UPX,UPY,UPZ,UPX,UPY,UPZ)
      SUMSQ(2)=SCALAR(UQX,UQY,UQZ,UQX,UQY,UQZ)
      SUMSQ(3)=SCALAR(URX,URY,URZ,URX,URY,URZ)
C
      WRITE(6,110)
  110 FORMAT(///,5X,'DERIVED VECTORIAL EQUATORIAL CONSTANTS -')
      WRITE(6,120)UPX,UPY,UPZ,SUMSQ(1)
  120 FORMAT(//,10X,'UPX =',F11.8,', UPY =',F11.8,', UPZ =',F11.8,
     *', SUM OF SQUARES =',F11.8)
      WRITE(6,130)UQX,UQY,UQZ,SUMSQ(2)
  130 FORMAT(/,10X,'UQX =',F11.8,', UQY =',F11.8,', UQZ =',F11.8,
     *', SUM OF SQUARES =',F11.8)
      WRITE(6,140)URX,URY,URZ,SUMSQ(3)
  140 FORMAT(/,10X,'URX =',F11.8,', URY =',F11.8,', URZ =',F11.8,
     *', SUM OF SQUARES =',F11.8)
C
      PHI=DARSIN(E)
      COSPHI=DCOS(PHI)
      SECPHI=1.0D0/COSPHI
      TANPHI=DTAN(PHI)
C
      B=A*COSPHI
      NRAD=K/DSQRT(A**3)
      CK=0.66666666666666666666666D0/NRAD
C
      AX=A*UPX
      AY=A*UPY
      AZ=A*UPZ
C
      BX=B*UQX
      BY=B*UQY
      BZ=B*UQZ
C
C
C     COMPUTATION OF THE FIRST SET OF RESIDUALS
C
C
      IPAGE=2
      WRITE(6,3)(IDENTN(J),J=1,6),IPAGE
C
      READ(5,150)N
  150 FORMAT(I2)
C
```

```
      IF((N.GE.4).AND.(N.LE.20))GO TO 170
      WRITE(6,160)
  160 FORMAT(/,10X,'NUMBER OF OBSERVATIONS IS OUTSIDE PERMITTED RANGE -
     *ORBIT 4 CANNOT CONTINUE')
      GO TO 1000
C
  170 WRITE(6,180)
  180 FORMAT(////   ,5X,'FIRST SET OF RESIDUALS (FROM UNCORRECTED ELEMENT
     *S) -')
      WRITE(6,190)EPOCH,EPOCH
  190 FORMAT(///   ,12X,'DATE OF',12X,'OBSERVED COORDINATES (',F6.1,')',
     *7X,'COMPUTED COORDINATES (',F6.1,')',10X,'RESIDUALS (O-C)',
     *//,10X,'OBSERVATION',13X,'ALPHA',12X,'DELTA',14X,'ALPHA',12X,
     *'DELTA',13X,'DALPHA',5X,'DDELTA',
     *//,14X,'JED',14X,'HR MIN  SEC',6X,'DEG MIN  SEC',7X,'HR MIN  SEC',
     *6X,'DEG MIN  SEC', 9X,'ARCSEC',5X,'ARCSEC',/)
C
      DO 400 I=1,N
C
      READ(5,200)ISIGN(1),IDEHO(1),IMIN(1),SEC(1),ISIGN(2),IDEHO(2),
     *IMIN(2),SEC(2),DT
  200 FORMAT(A1,I2,I3,F7.3,2X,A1,I2,I3,F6.2,/,F8.3)
C
      LONG=ANGLE1(DCONV,IDEHO(1),IMIN(1),SEC(1))
      CALL ANGLE4(ISIGN(1),LONG,£220)
      WRITE(6,210)
  210 FORMAT(/,10X,'INVALID SIGN GIVEN WITH ANGULAR DATA - ORBIT 4 CANNO
     *T CONTINUE')
      GO TO 1000
C
  220 LAT=ANGLE1(CONV,IDEHO(2),IMIN(2),SEC(2))
      CALL ANGLE4(ISIGN(2),LAT,£230)
      WRITE(6,210)
      GO TO 1000
C
  230 DT=DT/DCONV
C
      READ(5,240)YDATE,JD(I),IDEHO(1),IMIN(1),SEC(1),IDEHO(2),IMIN(2),
     *SEC(2),ISIGN(3),IDEHO(3),IMIN(3),SEC(3)
  240 FORMAT(F8.3,2X,F9.1,2(2X,2I3,F7.3),2X,A1,I2,I3,F6.2)
C
      UTD=ANGLE1(DCONV,IDEHO(1),IMIN(1),SEC(1))
      OALPHA(I)=ANGLE1(TCONV,IDEHO(2),IMIN(2),SEC(2))
      ODELTA(I)=ANGLE1(CONV,IDEHO(3),IMIN(3),SEC(3))
C
      CALL ANGLE4(ISIGN(3),ODELTA(I),£250)
      WRITE(6,210)
      GO TO 1000
C
  250 ETD=UTD+DT
C
```

```
      READ(5,260)JD0,X0,D2X0,YC,D2Y0,Z0,D2Z0,
     *              JD1,X1,D2X1,Y1,D2Y1,Z1,D2Z1
  260 FORMAT(F9.1,6(1X,F10.7),/,F9.1,6(1X,F10.7))
C
      READ(5,270)JDGST,IDEHO(1),IMIN(1),SEC(1)
  270 FORMAT(F9.1,1X,2I3,F7.3)
C
      GSTOD=ANGLE1(DCONV,IDEHC(1),IMIN(1),SEC(1))
C
      IF(JDGST.EQ.JD(I))GO TO 290
      WRITE(6,280)
  280 FORMAT(/,10X,'GIVEN VALUE OF GST AT 0 HOURS UT IS INVALID - ORBIT
     *4 CANNOT CONTINUE')
      GO TO 1000
C
  290 IF((JD1-JD0).EQ.1.0D0)GO TO 310
      WRITE(6,300)
  300 FORMAT(/,10X,'GIVEN SCLAR COORDINATES ARE INVALID - ORBIT 4 CANNOT
     * CONTINUE')
      GO TO 1000
C
  310 JD(I)=JD(I)+ETD
C
      IF((JD0.LE.JD(I)).AND.(JD(I).LE.JD1))GO TO 330
      WRITE(6,320)
  320 FORMAT(/,10X,'CORRECTION TO ET HAS INVALIDATED GIVEN SCLAR COORDIN
     *ATES - ORBIT 4 CANNOT CONTINUE')
      GO TO 1000
C
  330 CALL GEOS(ETD,X0,D2X0,YC,D2YC,ZC,D2ZC,
     *              X1,D2X1,Y1,D2Y1,Z1,D2Z1,
     *              X,Y,Z)
C
      CALL TOPOS(LONG,LAT,EPOCH,YDATE,GSTOD,UTD,X,Y,Z,RX(I),RY(I),RZ(I))
C
      IAPPRX=0
      RHO=0.0D0
C
  340 IAPPRX=IAPPRX+1
      PREVRO=RHO
C
      JDT=JD(I)-LITIME*PREVRO
      T=JDT-JDCE
      M=ANGLE3(M0+NRAD*T)
C
      CALL KEPLER(E,M,CE,£360)
      WRITE(6,350)
  350 FORMAT(/,10X, 55HKEPLER'S EQUATION IS UNSOLVED - ORBIT 4 CANNCT CC
     *NTINUE)
      GO TO 1000
C
  360 CTERM=DCOS(CE)-E
      STERM=DSIN(CE)
C
      XI=RX(I)+AX*CTERM+BX*STERM
      ETA=RY(I)+AY*CTERM+BY*STERM
      ZETA=RZ(I)+AZ*CTERM+BZ*STERM
C
      RHO=DSQRT(XI*XI+ETA*ETA+ZETA*ZETA)
C
```

```
      IF(DABS(PREVRO-RHO).LT.1.0D-6)GO TO 380
      IF(IAPPRX.LE.10)GO TO 340
      WRITE(6,370)
  370 FORMAT(/,10X,'SUCCESSIVE VALUES OF RHO DO NOT CONVERGE - ORBIT 4 C
     *ANNOT CONTINUE')
      GO TO 1000
C
  380 CALPHA=ANGLE5(ETA,XI)
      CDELTA=DARSIN(ZETA/RHO)
C
      DALPHA=DCOS(CDELTA)*ANGLE8(OALPHA(I),CALPHA)
      DDELTA=ODELTA(I)-CDELTA
C
      DASEC=DALPHA*CONV
      DDSEC=DDELTA*CONV
C
      CALL ANGLE6(ISIGN(4),CDELTA)
      CALL ANGLE6(ISIGN(1),DASEC)
      CALL ANGLE6(ISIGN(2),DDSEC)
C
      CALL ANGLE2(TCONV,CALPHA,IDEHO(1),IMIN(1),SEC(1))
      CALL ANGLE2(CONV,CDELTA,IDEHO(4),IMIN(4),SEC(4))
C
      WRITE(6,390)JD(I),
     *IDEHO(2),IMIN(2),SEC(2),ISIGN(3),IDEHO(3),IMIN(3),SEC(3),
     *IDEHO(1),IMIN(1),SEC(1),ISIGN(4),IDEHO(4),IMIN(4),SEC(4),
     *ISIGN(1),DASEC,ISIGN(2),DDSEC
  390 FORMAT( 9X,F13.5,8X,2(I2,2X),F6.3,3X,A1,2(I2,2X),F5.2,
     *                   5X,2(I2,2X),F6.3,3X,A1,2(I2,2X),F5.2,
     *                   7X,A1,F6.2,4X,A1,F6.2)
C
C
C     COMPUTATION OF THE COEFFICIENTS FOR THE EQUATIONS OF CONDITION
C
C
      SINA=DSIN(OALPHA(I))
      COSA=DCOS(OALPHA(I))
      SIND=DSIN(ODELTA(I))
      COSD=DCOS(ODELTA(I))
C
      LAMBDA=COSD*COSA
      MU=COSD*SINA
C
      R1=URY*COSA-URX*SINA
      R2=URZ*COSD-SIND*(URY*SINA+URX*COSA)
C
      BP=SCALAR(UPX,UPY,UPZ,LAMBDA,MU,SIND)
      BQ=SCALAR(UQX,UQY,UQZ,LAMBDA,MU,SIND)
      BR=SCALAR(URX,URY,URZ,LAMBDA,MU,SIND)
```

```
C
      AUPRHO=A/RHO
      CIFACT=AUPRHO*(1.0D0-BR**2)
C
      RUPONA=1.0D0-E*(CTERM+E)
      CSPSNE=COSPHI*STERM
C
      SINV=CSPSNE/RUPONA
      COSV= CTERM/RUPONA
C
      GC=AUPRHO*BP
      GS=AUPRHO*BQ
C
      FC=GS*SINV+GC*COSV
      FS=GS*COSV-GC*SINV
C
      FCSECP=FC*SECPHI
      FSSECP=FS*SECPHI
C
      CA(I)=R2*DALPHA-R1*DDELTA
      CD(I)=R1*DALPHA+R2*DDELTA
C
      B1(I)=FCSECP+GC*TANPHI
      B2(I)=B1(I)*T+CK*RUPONA*FS
      B3(I)=FCSECP*STERM+GS
      B4(I)=FC*RUPONA
C
      C1(I)=-(FSSECP+GS*TANPHI)*BR
      C2(I)=C1(I)*T+CK*B4(I)*BR
      C3(I)=-(FSSECP*STERM-GC)*BR
      C4(I)=-FS*RUPONA*BR
      C5(I)=CIFACT*CSPSNE
  400 C6(I)=-CIFACT*CTERM
C
      IPAGE=3
      WRITE(6,3)(IDENTN(J),J=1,6),IPAGE
C
      WRITE(6,410)
  410 FORMAT(////  ,5X,'COEFFICIENTS FOR THE EQUATIONS OF CONDITION -')
      WRITE(6,420)
  420 FORMAT(////  ,8X,'JED',13X,'A',8X,'B1',6X,'B2',6X,'B3',6X,'B4',
     *10X,'D',8X,'C1',6X,'C2',6X,'C3',6X,'C4',6X,'C5',6X,'C6',//)
C
      DO 430 I=1,N
  430 WRITE(6,440)JD(I),
     *CA(I),B1(I),B2(I),B3(I),B4(I),
     *CD(I),C1(I),C2(I),C3(I),C4(I),C5(I),C6(I)
  440 FORMAT(3X,F13.5,2X,F11.8,F8.4,F8.2,2F8.4,
     *                    2X,F11.8,6F8.4)
C
      IPAGE=4
      WRITE(6,3)(IDENTN(J),J=1,6),IPAGE
```

```
C
C
C     SOLUTION OF THE EQUATIONS OF CONDITION
C
C
      DO 450 I=1,N
      IF(B4(I).GE.0.0D0)GO TO 450
      CA(I)=-CA(I)
      B1(I)=-B1(I)
      B2(I)=-B2(I)
      B3(I)=-B3(I)
      B4(I)=-B4(I)
  450 CONTINUE
C
      CASUM=0.0D0
      B1SUM=0.0D0
      B2SUM=0.0D0
      B3SUM=0.0D0
      B4SUM=0.0D0
C
      DO 460 I=1,N
      CASUM=CASUM+CA(I)
      B1SUM=B1SUM+B1(I)
      B2SUM=B2SUM+B2(I)
      B3SUM=B3SUM+B3(I)
  460 B4SUM=B4SUM+B4(I)
C
      CA1ST=CASUM/B4SUM
      B11ST=B1SUM/B4SUM
      B21ST=B2SUM/B4SUM
      B31ST=B3SUM/B4SUM
C
      DO 470 I=1,N
      CA(I)=CA(I)-B4(I)*CA1ST
      B1(I)=B1(I)-B4(I)*B11ST
      B2(I)=B2(I)-B4(I)*B21ST
      B3(I)=B3(I)-B4(I)*B31ST
C
      IF(B3(I).GE.0.0D0)GO TO 470
      CA(I)=-CA(I)
      B1(I)=-B1(I)
      B2(I)=-B2(I)
      B3(I)=-B3(I)
  470 CONTINUE
C
      CASUM=0.0D0
      B1SUM=0.0D0
      B2SUM=0.0D0
      B3SUM=0.0D0
C
      DO 480 I=1,N
      CASUM=CASUM+CA(I)
      B1SUM=B1SUM+B1(I)
      B2SUM=B2SUM+B2(I)
  480 B3SUM=B3SUM+B3(I)
C
```

```
      CA2ND=CASUM/B3SUM
      B12ND=B1SUM/B3SUM
      B22ND=B2SUM/B3SUM
C
      DO 490 I=1,N
      CA(I)=CA(I)-B3(I)*CA2ND
      B1(I)=B1(I)-B3(I)*B12ND
      B2(I)=B2(I)-B3(I)*B22ND
C
      IF(B2(I).GE.0.0D0)GO TO 490
      CA(I)=-CA(I)
      B1(I)=-B1(I)
      B2(I)=-B2(I)
  490 CONTINUE
C
      CASUM=0.0D0
      B1SUM=0.0D0
      B2SUM=0.0D0
C
      DO 500 I=1,N
      CASUM=CASUM+CA(I)
      B1SUM=B1SUM+B1(I)
  500 B2SUM=B2SUM+B2(I)
C
      CA3RD=CASUM/B2SUM
      B13RD=B1SUM/B2SUM
C
      DO 510 I=1,N
      CA(I)=CA(I)-B2(I)*CA3RD
      B1(I)=B1(I)-B2(I)*B13RD
C
      IF(B1(I).GE.0.0D0)GO TO 510
      CA(I)=-CA(I)
      B1(I)=-B1(I)
  510 CONTINUE
C
      CASUM=0.0D0
      B1SUM=0.0D0
C
      DO 520 I=1,N
      CASUM=CASUM+CA(I)
  520 B1SUM=B1SUM+B1(I)
C
      DMO=CASUM/B1SUM
      DN=CA3RD-B13RD*DMO
      DE=CA2ND-B12ND*DMO-B22ND*DN
      DS=CA1ST-B11ST*DMO-B21ST*DN-B31ST*DE
C
```

```
      DO 530 I=1,N
      CD(I)=CD(I)-C1(I)*DMO-C2(I)*DN-C3(I)*DE-C4(I)*DS
C
      IF(C6(I).GE.0.0D0)GO TO 530
      CD(I)=-CD(I)
      C5(I)=-C5(I)
      C6(I)=-C6(I)
  530 CONTINUE
C
      CDSUM=0.0D0
      C5SUM=0.0D0
      C6SUM=0.0D0
C
      DO 540 I=1,N
      CDSUM=CDSUM+CD(I)
      C5SUM=C5SUM+C5(I)
  540 C6SUM=C6SUM+C6(I)
C
      CD1ST=CDSUM/C6SUM
      C51ST=C5SUM/C6SUM
C
      DO 550 I=1,N
      CD(I)=CD(I)-C6(I)*CD1ST
      C5(I)=C5(I)-C6(I)*C51ST
C
      IF(C5(I).GE.0.0D0)GO TO 550
      CD(I)=-CD(I)
      C5(I)=-C5(I)
  550 CONTINUE
C
      CDSUM=0.0D0
      C5SUM=0.0D0
C
      DO 560 I=1,N
      CDSUM=CDSUM+CD(I)
  560 C5SUM=C5SUM+C5(I)
C
      DP=CDSUM/C5SUM
      DQ=CD1ST-C51ST*DP
C
      DMOSEC=DMO*CONV
      DNSEC=DN*CONV
C
      WRITE(6,570)
  570 FORMAT(////   ,5X,'SOLUTION OF THE EQUATIONS OF CONDITION -')
      WRITE(6,580)DMOSEC,DNSEC,DE,DS,DP,DQ
  580 FORMAT(//,10X,'DMO =',F10.5,' ARC SECONDS, DN =',F10.5,' ARC SECCN
     *DS,',//,10X,'DE =',F11.8,', DS =',F11.8,', DP =',F11.8,', DQ =',
     *F11.8)
```

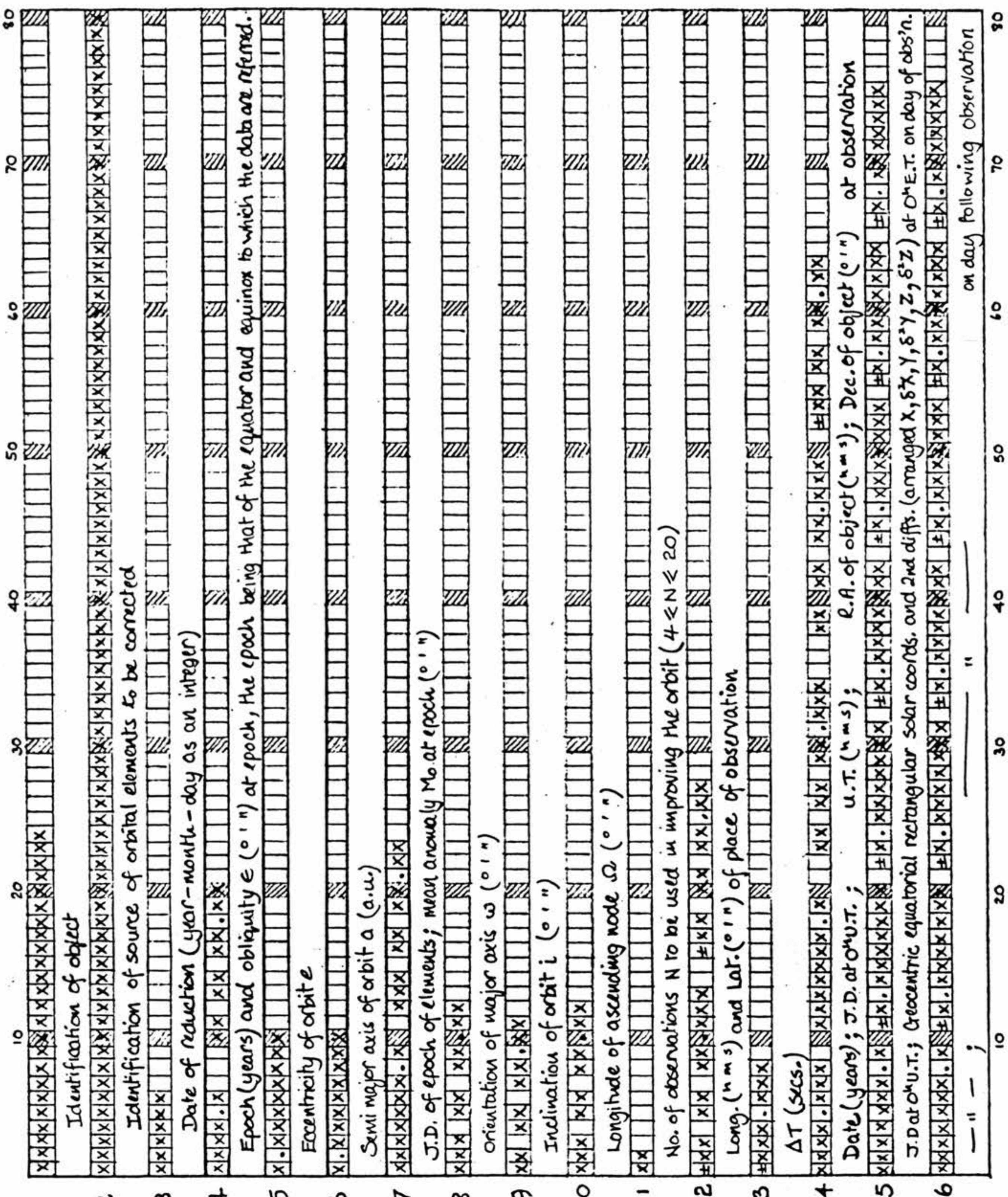
```
C
C
C      CORRECTICN OF THE CRBITAL ELEMENTS
C
C
       IAPPRX=0
C
       DUPX=0.CD0
       DUPY=0.0D0
       DUPZ=0.0D0
C
       DUQX=C.CD0
       DUQY=0.0D0
       DUQZ=0.0D0
C
       DURX=0.0D0
       DURY=0.0D0
       DURZ=0.0D0
C
  590  IAPPRX=IAPPRX+1
C
       PRDUPX=DUPX
       PRDUPY=DUPY
       PRDUPZ=DUPZ
C
       PRDUQX=DUQX
       PRDUQY=DUQY
       PRDUQZ=DUQZ
C
       PRDURX=DURX
       PRDURY=DURY
       PRDURZ=DURZ
C
       DUPX=DVEC(UQX,PRDUQX,DS,URX,PRDURX,DQ)
       DUPY=DVEC(UQY,PRDUQY,DS,URY,PRDURY,DQ)
       DUPZ=DVEC(UQZ,PRDUQZ,DS,URZ,PRDURZ,DQ)
C
       DUQX=DVEC(URX,PRDURX,DP,UPX,PRDUPX,DS)
       DUQY=DVEC(URY,PRDURY,DP,UPY,PRDUPY,DS)
       DUQZ=DVEC(URZ,PRDURZ,DP,UPZ,PRDUPZ,DS)
C
       DURX=DVEC(UPX,PRDUPX,DQ,UQX,PRDUQX,DP)
       DURY=DVEC(UPY,PRDUPY,DQ,UQY,PRDUQY,DP)
       DURZ=DVEC(UPZ,PRDUPZ,DQ,UQZ,PRDUQZ,DP)
C
       IF((DABS(PRDUPX-DUPX).LT.1.0D-6).AND.
     *    (DABS(PRDUPY-DUPY).LT.1.0D-6).AND.
     *    (DABS(PRDUPZ-DUPZ).LT.1.0D-6).AND.
     *    (DABS(PRDUQX-DUQX).LT.1.0D-6).AND.
     *    (DABS(PRDUQY-DUQY).LT.1.0D-6).AND.
     *    (DABS(PRDUQZ-DUQZ).LT.1.0D-6).AND.
     *    (DABS(PRDURX-DURX).LT.1.0D-6).AND.
     *    (DABS(PRDURY-DURY).LT.1.0D-6).AND.
     *    (DABS(PRDURZ-DURZ).LT.1.0D-6))GO TO 610
C
       IF(IAPPRX.LE.10)GO TO 590
       WRITE(6,600)
  600  FORMAT(/,10X,'SUCCESSIVE VALUES OF VECTOR CORRECTIONS DO NOT CONVE
     *RGE - ORBIT 4 CANNOT CONTINUE')
       GO TO 1000
```

```
C
  610 UPX=UPX+DUPX
      UPY=UPY+DUPY
      UPZ=UPZ+DUPZ
C
      UQX=UQX+DUQX
      UQY=UQY+DUQY
      UQZ=UQZ+DUQZ
C
      URX=URX+DURX
      URY=URY+DURY
      URZ=URZ+DURZ
C
      SUMSQ(1)=SCALAR(UPX,UPY,UPZ,UPX,UPY,UPZ)
      SUMSQ(2)=SCALAR(UQX,UQY,UQZ,UQX,UQY,UQZ)
      SUMSQ(3)=SCALAR(URX,URY,URZ,URX,URY,URZ)
C
      WRITE(6,620)
  620 FORMAT(///,5X,'CORRECTED VECTORIAL EQUATORIAL CONSTANTS -')
      WRITE(6,120)UPX,UPY,UPZ,SUMSQ(1)
      WRITE(6,130)UQX,UQY,UQZ,SUMSQ(2)
      WRITE(6,140)URX,URY,URZ,SUMSQ(3)
C
      E=E+DE
      NRAD=NRAD+DN
      MO=ANGLE3(MO+DMO)
C
      A=(K/NRAD)**0.66666666666666666666666D0
      B=A*DCOS(DARSIN(E))
C
      CALL ORBEL(OBL,UPX,UPY,UPZ,UQX,UQY,UQZ,OMEGA,SI,OMEGA1)
C
      CALL ANGLE2(CONV,MO,IDEHO(1),IMIN(1),SEC(1))
      CALL ANGLE2(CONV,OMEGA,IDEHO(2),IMIN(2),SEC(2))
      CALL ANGLE2(CONV,SI,IDEHO(3),IMIN(3),SEC(3))
      CALL ANGLE2(CONV,OMEGA1,IDEHO(4),IMIN(4),SEC(4))
C
      WRITE(6,630)EPOCH
  630 FORMAT(///,5X,'CORRECTED ELEMENTS (EPOCH',F7.1,') -')
      WRITE(6,50)E
      WRITE(6,60)A
      WRITE(6,70)JDCE,IDEHO(1),IMIN(1),SEC(1)
      WRITE(6,80)IDEHO(2),IMIN(2),SEC(2)
      WRITE(6,90)IDEHO(3),IMIN(3),SEC(3)
      WRITE(6,100)IDEHO(4),IMIN(4),SEC(4)
```

```
C
C
C     COMPUTATION OF THE SECOND SET OF RESIDUALS
C
C
      IPAGE=5
      WRITE(6,3)(IDENTN(J),J=1,6),IPAGE
C
      AX=A*UPX
      AY=A*UPY
      AZ=A*UPZ
C
      BX=B*UQX
      BY=B*UQY
      BZ=B*UQZ
C
      WRITE(6,640)
  640 FORMAT(////  ,5X,'SECOND SET OF RESIDUALS (FROM CORRECTED ELEMENTS
     *) -')
      WRITE(6,190)EPOCH,EPOCH
C
      DO 680 I=1,N
C
      IAPPRX=0
      RHO=0.0D0
C
  650 IAPPRX=IAPPRX+1
      PREVRO=RHO
C
      JDT=JD(I)-LITIME*PREVRO
      M=ANGLE3(MO+NRAD*(JDT-JDCE))
C
      CALL KEPLER(E,M,CE,&660)
      WRITE(6,350)
      GO TO 1000
C
  660 CTERM=DCOS(CE)-E
      STERM=DSIN(CE)
C
      XI=RX(I)+AX*CTERM+BX*STERM
      ETA=RY(I)+AY*CTERM+BY*STERM
      ZETA=RZ(I)+AZ*CTERM+BZ*STERM
C
      RHO=DSQRT(XI*XI+ETA*ETA+ZETA*ZETA)
C
      IF(DABS(PREVRO-RHO).LT.1.0D-6)GO TO 670
      IF(IAPPRX.LE.10)GO TO 650
      WRITE(6,370)
      GO TO 1000
C
```

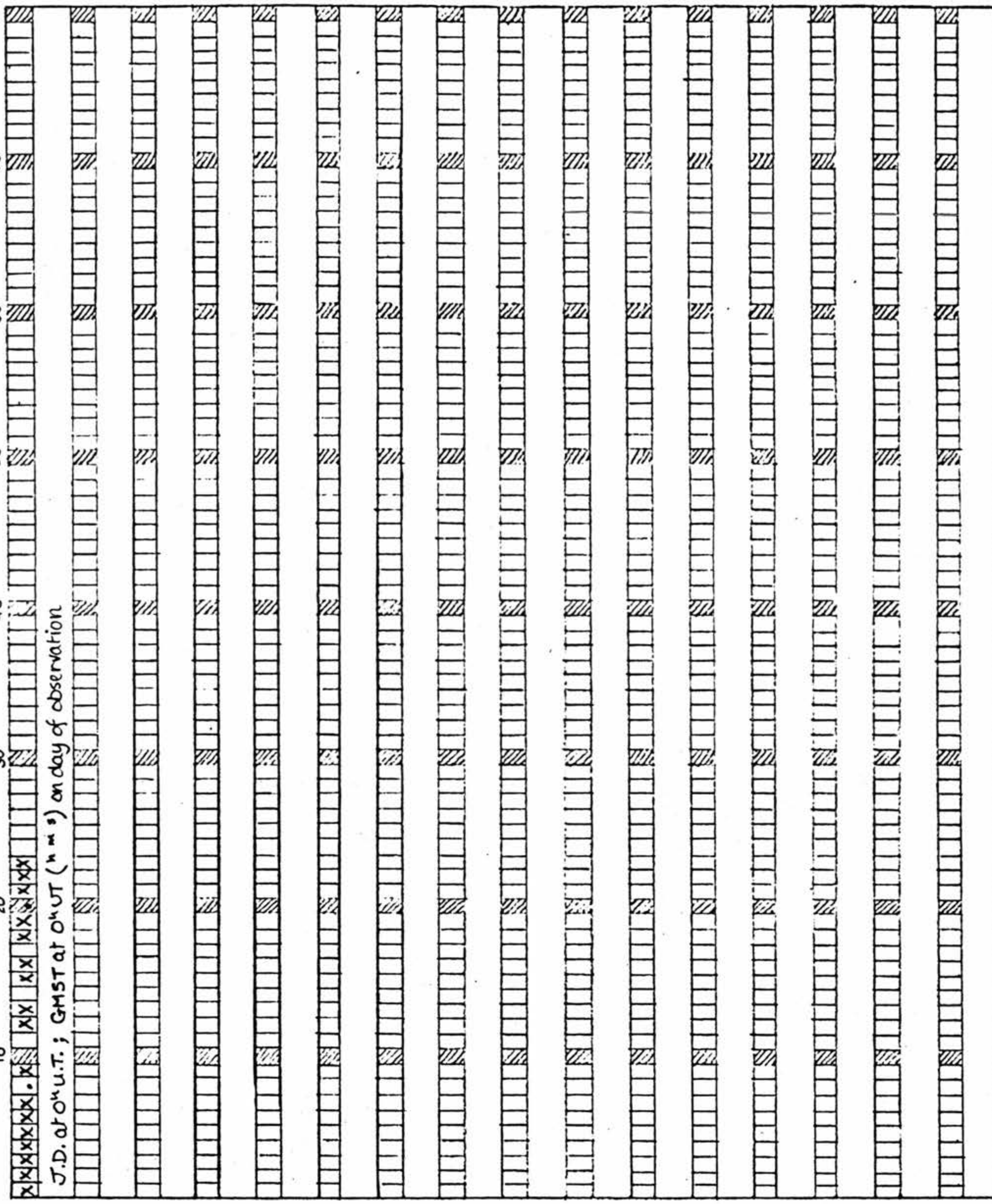
```
  670 CALPHA=ANGLE5(ETA,XI)
      CDELTA=DARSIN(ZETA/RHO)
C
      DASEC=DCOS(CDELTA)*ANGLE8(OALPHA(I),CALPHA)*CONV
      DDSEC=(ODELTA(I)-CDELTA)*CONV
C
      CALL ANGLE6(ISIGN(3),ODELTA(I))
      CALL ANGLE6(ISIGN(4),CDELTA)
      CALL ANGLE6(ISIGN(1),DASEC)
      CALL ANGLE6(ISIGN(2),DDSEC)
C
      CALL ANGLE2(TCONV,OALPHA(I),IDEHO(2),IMIN(2),SEC(2))
      CALL ANGLE2(CONV,ODELTA(I),IDEHO(3),IMIN(3),SEC(3))
      CALL ANGLE2(TCONV,CALPHA,IDEHO(1),IMIN(1),SEC(1))
      CALL ANGLE2(CONV,CDELTA,IDEHO(4),IMIN(4),SEC(4))
C
  680 WRITE(6,390)JD(I),
     *IDEHO(2),IMIN(2),SEC(2),ISIGN(3),IDEHO(3),IMIN(3),SEC(3),
     *IDEHO(1),IMIN(1),SEC(1),ISIGN(4),IDEHO(4),IMIN(4),SEC(4),
     *ISIGN(1),DASEC,ISIGN(2),DDSEC
C
C
C     TERMINATION POINT
C
C
 1000 WRITE(6,1010)
 1010 FORMAT('1')
C
      STOP
      END
```

DATA FORMAT FOR ORBIT 4



(contd. overleaf)   Lines 5-10 are the original orbital elements.

DATA FORMAT FOR ORBIT 4 (contd.)



Lines 12-17 are included N times, i.e. once for each observation used for the improvement of the orbit.

IMPROVEMENT OF THE ORBIT OF 16 PSYCHE

BY DIFFERENTIAL CORRECTION USING PORTER'S METHOD

SOURCE OF ORIGINAL ELEMENTS -

ORBIT 1 RESULTS 720329/1

ORIGINAL ELEMENTS (EPOCH 1950.0) -

ECCENTRICITY E = 0.14501944

SEMI-MAJOR AXIS A = 2.93994782 ASTRONOMICAL UNITS

MEAN ANOMALY MO AT EPOCH JD 2440800.5 ET = 17 DEG 13 MIN 55.77 SEC

ORIENTATION OF MAJOR AXIS OMEGA = 227 DEG 21 MIN 24.55 SEC

INCLINATION OF ORBIT I = 3 DEG 5 MIN 33.77 SEC

LONGITUDE OF ASCENDING NODE OMEGA1 = 150 DEG 14 MIN 56.91 SEC

DERIVED VECTORIAL EQUATORIAL CONSTANTS -

UPX = 0.95262757, UPY = 0.29743336, UPZ = 0.08356703, SUM OF SQUARES = 1.00000000

UQX =-0.30295867, UQY = 0.88821960, UQZ = 0.34537225, SUM OF SQUARES = 1.00000000

URX = 0.02677249, URY =-0.35432849, URZ = 0.93473769, SUM OF SQUARES = 1.00000000

FIRST SET OF RESIDUALS (FROM UNCORRECTED ELEMENTS) -

DATE OF OBSERVATION	OBSERVED COORDINATES (1950.0)						COMPUTED COORDINATES (1950.0)						RESIDUALS (C-O)	
	ALPHA			DELTA			ALPHA			DELTA			DALPHA	DDELTA
JED	HR	MIN	SEC	DEG	MIN	SEC	HR	MIN	SEC	DEG	MIN	SEC	ARCSEC	ARCSEC
2440868.59353	5	10	17.738	+18	53	56.23	5	10	17.738	+18	53	56.22	+ 0.00	+ 0.01
2440870.56182	5	10	47.176	+18	51	45.68	5	10	47.157	+18	51	45.81	+ 0.27	- 0.13
2440885.60256	5	10	52.405	+18	31	25.51	5	10	52.701	+18	31	29.31	- 4.21	- 3.80
2440896.45793	5	6	50.800	+18	13	47.25	5	6	51.876	+18	13	53.98	- 15.33	- 6.73
2440899.58664	5	5	7.346	+18	8	32.18	5	5	8.674	+18	8	40.69	- 18.93	- 8.51
2440910.58173	4	57	15.569	+17	49	39.28	4	57	18.452	+17	49	52.44	- 41.17	- 13.16
2440922.56830	4	46	37.725	+17	30	20.28	4	46	42.838	+17	30	39.91	- 73.14	- 19.63
2440956.49070	4	21	6.239	+17	10	8.73	4	21	17.553	+17	10	39.64	-162.14	- 30.91
2440972.36506	4	17	59.207	+17	28	43.45	4	18	11.487	+17	29	11.99	-175.69	- 28.54
2440984.37924	4	20	9.822	+17	54	9.57	4	20	21.815	+17	54	32.97	-171.18	- 23.40
2440999.295[illegible]4	4	27	47.808	+18	35	14.51	4	27	58.378	+18	35	27.75	-150.28	- 13.24
2441004.37334	4	31	28.985	+18	50	40.61	4	31	38.887	+18	50	49.91	-140.57	- 9.30

COEFFICIENTS FOR THE EQUATIONS OF CONDITION -

JED	A	B1	B2	B3	B4	C	C1	C2	C3	C4	C5	C6
2440868.59353	0.00000001	1.5132	190.65	2.1044	1.2270	0.00000006	0.0135	-3.6755	-0.0082	0.0086	0.8481	-0.9935
2440870.56182	0.00000123	1.5318	194.35	2.1384	1.2436	-0.00000076	0.0140	-3.8755	-0.0087	0.0089	0.8660	-0.9971
2440885.60256	-0.00002218	1.6818	220.05	2.3932	1.3752	-0.00001625	0.0170	-5.5196	-0.0131	0.0102	1.0057	-1.0166
2440896.49793	-0.00007735	1.7878	234.27	2.5564	1.4669	-0.00002442	0.0169	-6.7686	-0.0178	0.0094	1.1040	-1.0168
2440899.58664	-0.00009571	1.8150	237.30	2.5963	1.4903	-0.00003087	0.0164	-7.1107	-0.0194	0.0088	1.1303	-1.0139
2440910.58173	-0.00020577	1.8931	243.66	2.7043	1.5571	-0.00003902	0.0133	-8.1771	-0.0253	0.0057	1.2135	-0.9910
2440922.56830	-0.00036413	1.9294	241.96	2.7468	1.5882	-0.00004566	0.0078	-8.8318	-0.0312	0.0010	1.2785	-0.9414
2440956.49070	-0.00079996	1.7242	205.64	2.4763	1.4228	-0.00001159	-0.0038	-6.3564	-0.0280	-0.0068	1.2831	-0.6942
2440972.36506	-0.00086273	1.5499	188.52	2.2758	1.2849	0.00001434	-0.0037	-3.9651	-0.0178	-0.0053	1.2207	-0.5636
2440994.37924	-0.00083698	1.4217	180.42	2.1381	1.1856	0.00003251	-0.0023	-2.1811	-0.0098	-0.0031	1.1639	-0.4719
2440999.29504	-0.00072928	1.2807	176.97	1.9961	1.0791	0.00005572	-0.0003	-0.3106	-0.0014	-0.0005	1.0923	-0.3707
2441034.37334	-0.00068003	1.2381	174.23	1.9552	1.0475	0.00006335	0.0002	0.2206	0.0009	0.0003	1.0687	-0.3396

SOLUTION OF THE EQUATIONS OF CONDITION -

DMO = 489.12447 ARC SECONDS, DN = 6.88130 ARC SECONDS,

DE =-0.00588507, DS = 0.00198584, DP = 0.00006667, DQ = 0.00003652

CORRECTED VECTORIAL EQUATORIAL CONSTANTS -

UPX = 0.95202308, UPY = 0.29420956, UPZ = 0.08421865, SUM OF SQUARES = 1.00000000

UQX =-0.30484805, UQY = 0.88761348, UQZ = 0.34526798, SUM OF SQUARES = 1.00000000

URX = 0.02682753, URY =-0.35437697, URZ = 0.93471773, SUM OF SQUARES = 1.00000000

CORRECTED ELEMENTS (EPOCH 1950.0) -

ECCENTRICITY E = 0.13913437

SEMI-MAJOR AXIS A = 2.92094142 ASTRONOMICAL UNITS

MEAN ANOMALY MO AT EPOCH JD 2440800.5 ET = 17 DEG 22 MIN 4.89 SEC

ORIENTATION OF MAJOR AXIS OMEGA = 227 DEG 32 MIN 55.99 SEC

INCLINATION OF ORBIT I = 3 DEG 5 MIN 30.02 SEC

LONGITUDE OF ASCENDING NODE OMEGA1 = 150 DEG 10 MIN 14.67 SEC

SECOND SET OF RESIDUALS (FROM CORRECTED ELEMENTS) -

DATE OF OBSERVATION	OBSERVED COORDINATES (1950.0)						COMPUTED COORDINATES (1950.0)						RESIDUALS (C-O)	
	ALPHA			DELTA			ALPHA			DELTA			DALPHA	DDELTA
JED	HR	MIN	SEC	DEG	MIN	SEC	HR	MIN	SEC	DEG	MIN	SEC	ARCSEC	ARCSEC
2440868.59353	5	10	17.738	+18	53	56.23	5	10	17.649	+18	53	55.11	+ 1.26	+ 1.12
2440870.50182	5	10	47.176	+18	51	45.68	5	10	47.055	+18	51	44.46	+ 1.72	+ 1.22
2440885.60256	5	10	52.405	+18	31	25.51	5	10	52.225	+18	31	25.24	+ 2.56	+ 0.27
2440896.49793	5	6	50.800	+18	13	47.25	5	6	50.641	+18	13	46.78	+ 2.27	+ 0.47
2440899.59664	5	5	7.346	+18	8	32.18	5	5	7.123	+18	8	32.41	+ 3.18	- 0.23
2440910.56173	4	57	15.569	+17	49	39.28	4	57	15.392	+17	49	39.63	+ 2.53	- 0.35
2440922.50083	4	46	37.725	+17	30	20.28	4	46	37.534	+17	30	21.20	+ 2.73	- 0.92
2440956.49070	4	21	6.239	+17	10	8.73	4	21	6.080	+17	10	8.63	+ 2.28	+ 0.10
2440972.30506	4	17	59.207	+17	28	43.45	4	17	59.036	+17	28	43.02	+ 2.45	+ 0.43
2440984.37524	4	20	9.822	+17	54	9.57	4	20	9.645	+17	54	9.31	+ 2.52	+ 0.26
2440999.29504	4	27	47.808	+18	35	14.51	4	27	47.633	+18	35	13.95	+ 2.48	+ 0.56
2441004.37334	4	31	28.985	+18	50	40.61	4	31	28.863	+18	50	40.00	+ 1.74	+ 0.61

Part 2

A DETERMINATION OF THE ORBIT OF THE MINOR PLANET 16 PSYCHE

Chapter 2.1 Observation of the Planet

We now turn our attention to the practical use of the ORBIT library of programs in a determination of the orbit of the minor planet 16 Psyche. The choice of this planet as a subject for investigation was determined primarily by practical limitations rather than a particular interest in its orbit but this of course does not detract in any way from the intrinsic value of the observations obtained and their subsequent reduction.

The requirements to be fulfilled were as follows:

(1) since the observations were to be made during the observing season 1970 - 71 (approximately September to April at St Andrews) the object should be in opposition at the end of 1970;

(2) in order to keep exposure times short so as to minimise image trailing the object should be fairly bright (brighter than about 12^m for the particular telescope to be used);

(3) the orbit of the object should already be as well-determined as possible in order to provide a reliable check on the performance of the ORBIT programs;

(4) the orbit of the object should be generally representative of a "normal" minor planet orbit.

From the "Ephemerides of Minor Planets 1970" the object 16 Psyche was selected, having an opposition date of 1970 December 5 and an opposition magnitude of $10\overset{m}{.}0$. In addition, this planet has been observed since approximately 1850 and so, since its orbital elements display no abnormal peculiarities, the above conditions are all satisfied.

In order to gather as much data as possible, it was proposed that a long series of observations of the planet should be made extending throughout the whole observing season and, with this in mind, detailed preparations were made prior to the start of the season. These preparations (and, in fact, the whole observing procedure) were based upon experience gained during the previous winter when a short series of observations of 6 Hebe, 89 Julia and 129 Antigone was obtained. The preparations took the form of the production of a daily ephemeris, the selection of suitable guide stars and the preparation of a finding chart for the stars. The ephemeris and the finding chart are reproduced on the following pages (Figs 4 and 5).

The ephemeris was based on the positions given in the "Ephemerides of Minor Planets 1970". This annual publication gives the geocentric positions of minor planets in opposition during the year at 0^h E.T. on dates separated by an interval of ten days, these being the numbered dates in the daily ephemeris (see following page, column 5). The publication also gives the visual magnitude on the same dates for bright planets. The geocentric co-ordinates are referred to the mean equinox of 1950.0. To obtain the position (α,δ) of the minor planet on each day linear interpolation was used, this being also the case for its visual magnitude (m_v).

The guide stars were all selected from the Smithsonian Astrophysical Observatory Star Catalogue (epoch 1950.0) and were chosen so that the image of the minor planet would never be more than one degree from the centre of the plate, assuming that the image of the guide star coincided with the plate centre. Where a choice was possible, the brighter stars were selected for guiding and the duration of periods of guiding on faint stars was kept to a minimum.

Proper motion corrections for the period 1950.0 to 1971.0 were not applied to the positions of the guide stars since they would have been negligible in comparison with the setting accuracy of the telescope. The 1950.0 co-ordinates and visual magnitudes of the guide stars $(\alpha_*, \delta_*, m_{v*})$ are given in columns 9, 10 and 11 of the daily ephemeris and the number in column 8 is a reference number identifying the guide star on the finding chart.

The final column of the ephemeris was left for completion with plate numbers as plates were obtained. The symbol † appearing against a plate number indicated that for some reason (for example, bad visibility) the plate was obtained using a guide star other than the one given for that date. Thus the observer was provided not only with data for a particular

DAY	DATE (1970)	α h	α m	δ °	δ '	[illegible]	m_V m	RECOMMENDED GUIDE STAR SAO	GUIDE STAR CHART REF. NO.	α_* h	α_* m	δ_* °	δ_* '	m_{V*} m	PLATE NOS OBTAINED
TUE	SEPT. 1	4	42.2	+19	00	1	10.8	094112	1	4	43.3	+18	39	6.1	020
WED	2	4	43.2	+19	00		10.8	094112	1	4	43.3	+18	39	6.1	NONE
THUR	3	4	44.2	+19	01		10.8	094112	1	4	43.3	+18	39	6.1	021, 022
FRI	4	4	45.3	+19	01		10.8	094112	1	4	43.3	+18	39	6.1	NONE
SAT	5	4	46.3	+19	02		10.7	094164	2	4	48.4	+18	45	5.1	023, 024, 025
SUN	6	4	47.3	+19	03		10.7	094164	2	4	48.4	+18	45	5.1	NONE
MON	7	4	48.3	+19	03		10.7	094164	2	4	48.4	+18	45	5.1	NONE
TUE	8	4	49.3	+19	04		10.7	094164	2	4	48.4	+18	45	6.1	NONE
WED	9	4	50.4	+19	05		10.7	094199	3	4	52.0	+19	24	6.2	NONE
THUR	10	4	51.4	+19	05		10.7	094199	3	4	52.0	+19	24	6.2	NONE
FRI	11	4	52.4	+19	06	2	10.7	094199	3	4	52.0	+19	24	6.2	026
SAT	12	4	53.2	+19	06		10.7	094199	3	4	52.0	+19	24	6.2	NONE
SUN	13	4	54.1	+19	06		10.6	094199	3	4	52.0	+19	24	6.2	027, 028
MON	14	4	54.9	+19	06		10.6	094199	3	4	52.0	+19	24	6.2	029
TUE	15	4	55.8	+19	07		10.6	094199	3	4	52.0	+19	24	6.2	030, 031
WED	16	4	56.6	+19	07		10.6	094282	4	5	00.0	+19	27	8.9	032†, 033†
THUR	17	4	57.4	+19	07		10.6	094282	4	5	00.0	+19	27	8.9	NONE
FRI	18	4	58.3	+19	06		10.6	094282	4	5	00.0	+19	27	8.9	NONE
SAT	19	4	59.1	+19	06		10.6	094282	4	5	00.0	+19	27	8.9	NONE
SUN	20	5	00.0	+19	06		10.6	094306	5	5	02.6	+19	44	6.5	NONE
MON	21	5	00.8	+19	06	3	10.5	094306	5	5	02.6	+19	44	6.5	NONE
TUE	22	5	01.4	+19	05		10.5	094306	5	5	02.6	+19	44	6.5	NONE
WED	23	5	02.0	+19	05		10.5	094306	5	5	02.6	+19	44	6.5	NONE
THUR	24	5	02.6	+19	04		10.5	094306	5	5	02.6	+19	44	6.5	034
FRI	25	5	03.2	+19	04		10.5	094306	5	5	02.6	+19	44	6.5	035
SAT	26	5	03.8	+19	03		10.5	094306	5	5	02.6	+19	44	6.5	NONE
SUN	27	5	04.5	+19	03		10.5	094332	6	5	04.5	+18	35	5.0	NONE
MON	28	5	05.1	+19	02		10.5	094332	6	5	04.5	+18	35	5.0	036, 037, 038
TUE	29	5	05.7	+19	02		10.5	094332	6	5	04.5	+18	35	5.0	NONE
WED	30	5	06.3	+19	01		10.4	094332	6	5	04.5	+18	35	5.0	NONE
THUR	OCT. 1	5	06.9	+19	01	4	10.4	094332	6	5	04.5	+18	35	5.0	039
FRI	2	5	07.3	+19	00		10.4	094332	6	5	04.5	+18	35	5.0	040, 041
SAT	3	5	07.6	+18	59		10.4	094332	6	5	04.5	+18	35	5.0	042
SUN	4	5	08.0	+18	58		10.4	094332	6	5	04.5	+18	35	5.0	NONE
MON	5	5	08.3	+18	57		10.4	094422	7	5	12.9	+18	35	8.7	NONE
TUE	6	5	08.7	+18	56		10.4	094422	7	5	12.9	+18	35	8.7	NONE
WED	7	5	09.1	+18	55		10.4	094422	7	5	12.9	+18	35	8.7	NONE
THUR	8	5	09.4	+18	54		10.4	094422	7	5	12.9	+18	35	8.7	NONE
FRI	9	5	09.8	+18	53		10.3	094422	7	5	12.9	+18	35	8.7	043†
SAT	10	5	10.1	+18	52		10.3	094431	8	5	13.8	+18	23	7.5	NONE
SUN	11	5	10.5	+18	51	5	10.3	094431	8	5	13.8	+18	23	7.5	044
MON	12	5	10.6	+18	50		10.3	094431	8	5	13.8	+18	23	7.5	NONE
TUE	13	5	10.6	+18	48		10.3	094431	8	5	13.8	+18	23	7.5	NONE
WED	14	5	10.7	+18	47		10.3	094431	8	5	13.8	+18	23	7.5	NONE
THUR	15	5	10.8	+18	46		10.3	094431	8	5	13.8	+18	23	7.5	NONE
FRI	16	5	10.8	+18	44		10.3	094431	8	5	13.8	+18	23	7.5	NONE
SAT	17	5	10.9	+18	43		10.3	094431	8	5	13.8	+18	23	7.5	NONE
SUN	18	5	11.0	+18	42		10.3	094431	8	5	13.8	+18	23	7.5	NONE
MON	19	5	11.1	+18	41		10.2	094431	8	5	13.8	+18	23	7.5	NONE
TUE	20	5	11.1	+18	39		10.2	094431	8	5	13.8	+18	23	7.5	NONE
WED	21	5	11.2	+18	38	6	10.2	094431	8	5	13.8	+18	23	7.5	NONE
THUR	22	5	11.0	+18	36		10.2	094431	8	5	13.8	+18	23	7.5	NONE
FRI	23	5	10.8	+18	35		10.2	094431	8	5	13.8	+18	23	7.5	NONE
SAT	24	5	10.6	+18	34		10.2	094431	8	5	13.8	+18	23	7.5	NONE

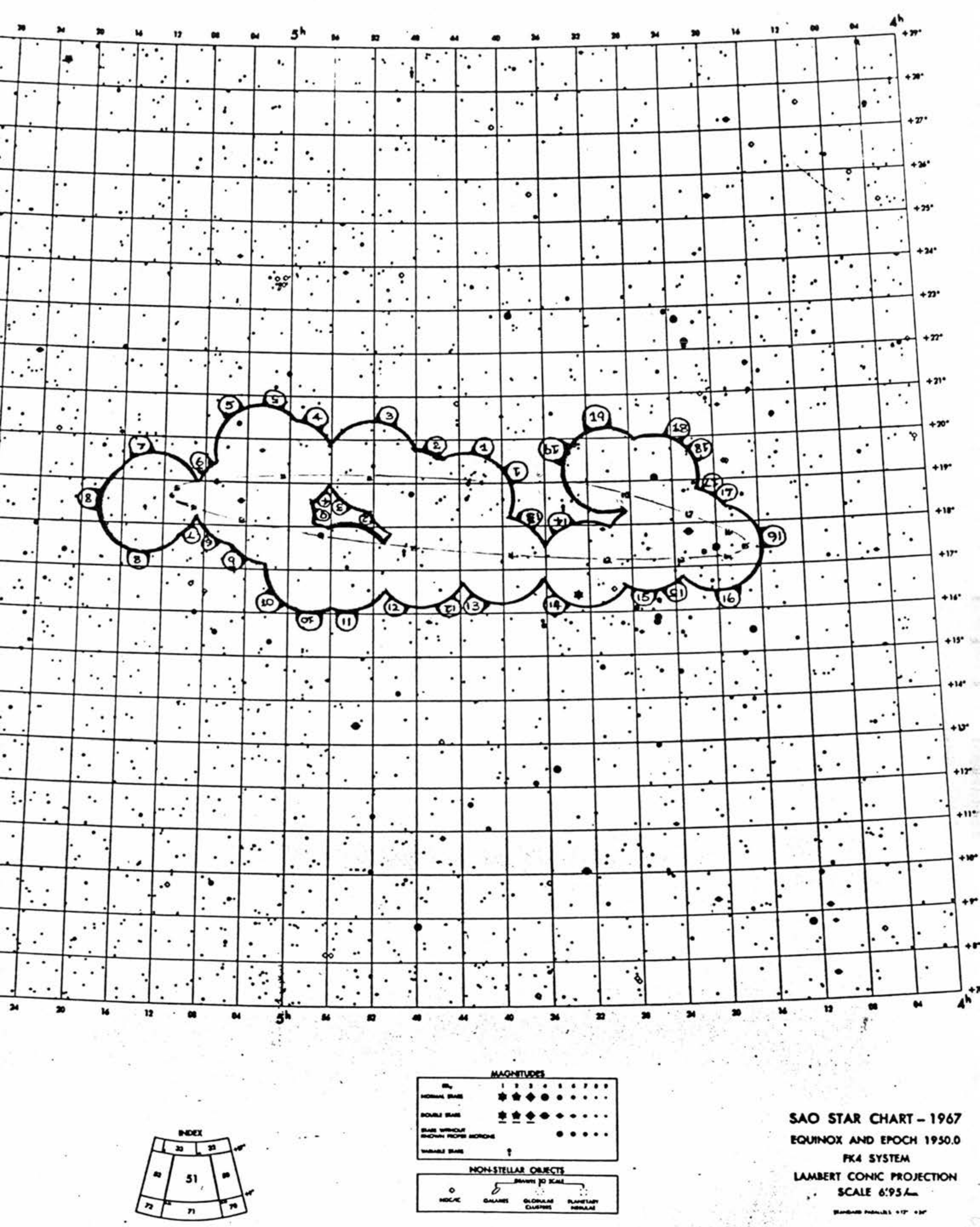
Fig. 4

DAY	DATE (1970)	α h	α m	δ °	δ '	[illegible]	M_V m	RECOMMENDED GUIDE STAR SAO	GUIDE STAR CHART REF. NO.	α* h	α* m	δ* °	δ* '	M_{V*} m	PLATE NOS OBTAINED
SUN	OCT. 25	5	10.4	+18	32		10.2	094431	8	5	13.8	+18	23	7.5	NONE
MON	26	5	10.2	+18	30		10.2	094431	8	5	13.8	+18	23	7.5	045
TUE	27	5	09.9	+18	29		10.2	094431	8	5	13.8	+18	23	7.5	046
WED	28	5	09.7	+18	28		10.2	094431	8	5	13.8	+18	23	7.5	NONE
THUR	29	5	09.5	+18	26		10.2	094431	8	5	13.8	+18	23	7.5	NONE
FRI	30	5	09.3	+18	24		10.1	094422	7	5	12.9	+18	35	8.7	NONE
SAT	31	5	09.1	+18	23	7	10.1	094422	7	5	12.9	+18	35	8.7	NONE
SUN	NOV. 1	5	08.6	+18	21		10.1	094422	7	5	12.9	+18	35	8.7	NONE
MON	2	5	08.1	+18	20		10.1	094332	6	5	04.5	+18	35	5.0	NONE
TUE	3	5	07.6	+18	18		10.1	094332	6	5	04.5	+18	35	5.0	NONE
WED	4	5	07.1	+18	16		10.1	094332	6	5	04.5	+18	35	5.0	047
THUR	5	5	06.6	+18	14		10.1	094332	6	5	04.5	+18	35	5.0	NONE
FRI	6	5	06.2	+18	13		10.1	094332	6	5	04.5	+18	35	5.0	048
SAT	7	5	05.7	+18	11		10.1	094332	6	5	04.5	+18	35	5.0	NONE
SUN	8	5	05.2	+18	09		10.1	094332	6	5	04.5	+18	35	5.0	NONE
MON	9	5	04.7	+18	08		10.1	094332	6	5	04.5	+18	35	5.0	049, 050
TUE	10	5	04.2	+18	06	8	10.1	094332	6	5	04.5	+18	35	5.0	051
WED	11	5	03.5	+18	04		10.1	094332	6	5	04.5	+18	35	5.0	NONE
THUR	12	5	02.8	+18	03		10.1	094332	6	5	04.5	+18	35	5.0	NONE
FRI	13	5	02.0	+18	01		10.1	094332	6	5	04.5	+18	35	5.0	NONE
SAT	14	5	01.3	+17	59		10.1	094294	9	5	01.9	+18	11	7.9	NONE
SUN	15	5	00.6	+17	58		10.1	094294	9	5	01.9	+18	11	7.9	NONE
MON	16	4	59.9	+17	56		10.0	094294	9	5	01.9	+18	11	7.9	NONE
TUE	17	4	59.2	+17	54		10.0	094294	9	5	01.9	+18	11	7.9	NONE
WED	18	4	58.4	+17	52		10.0	094294	9	5	01.9	+18	11	7.9	NONE
THUR	19	4	57.7	+17	51		10.0	094257	10	4	57.9	+17	03	9.0	NONE
FRI	20	4	57.0	+17	49	9	10.0	094227	11	4	54.5	+17	05	5.7	052, 053
SAT	21	4	56.1	+17	47		10.0	094227	11	4	54.5	+17	05	5.7	NONE
SUN	22	4	55.2	+17	46		10.0	094227	11	4	54.5	+17	05	5.7	NONE
MON	23	4	54.4	+17	44		10.0	094227	11	4	54.5	+17	05	5.7	NONE
TUE	24	4	53.5	+17	43		10.0	094227	11	4	54.5	+17	05	5.7	NONE
WED	25	4	52.6	+17	41		10.0	094227	11	4	54.5	+17	05	5.7	NONE
THUR	26	4	51.7	+17	39		10.0	094227	11	4	54.5	+17	05	5.7	NONE
FRI	27	4	50.8	+17	38		10.0	094227	11	4	54.5	+17	05	5.7	NONE
SAT	28	4	50.0	+17	36		10.0	094158	12	4	47.5	+17	07	7.2	NONE
SUN	29	4	49.1	+17	35		10.0	094158	12	4	47.5	+17	07	7.2	NONE
MON	30	4	48.2	+17	33	10	10.0	094158	12	4	47.5	+17	07	7.2	NONE
TUE	DEC. 1	4	47.3	+17	32		10.0	094158	12	4	47.5	+17	07	7.2	NONE
WED	2	4	46.4	+17	30		10.0	094158	12	4	47.5	+17	07	7.2	054
THUR	3	4	45.4	+17	29		10.0	094158	12	4	47.5	+17	07	7.2	NONE
FRI	4	4	44.5	+17	27		10.0	094158	12	4	47.5	+17	07	7.2	NONE
SAT	5	4	43.6	+17	26		9.9	094158	12	4	47.5	+17	07	7.2	NONE
SUN	6	4	42.7	+17	25		10.0	094080	13	4	40.1	+17	13	8.3	NONE
MON	7	4	41.8	+17	23		10.0	094080	13	4	40.1	+17	13	8.3	055†, 056†
TUE	8	4	40.8	+17	22		10.0	094080	13	4	40.1	+17	13	8.3	057†, 058†
WED	9	4	39.9	+17	20		10.0	094080	13	4	40.1	+17	13	8.3	NONE
THUR	10	4	39.0	+17	19	11	10.0	094080	13	4	40.1	+17	13	8.3	NONE
FRI	11	4	38.2	+17	18		10.0	094080	13	4	40.1	+17	13	8.3	NONE
SAT	12	4	37.3	+17	17		10.0	094080	13	4	40.1	+17	13	8.3	NONE
SUN	13	4	36.4	+17	16		10.0	094019	14	4	32.2	+17	06	7.1	NONE
MON	14	4	35.6	+17	15		10.1	094019	14	4	32.2	+17	06	7.1	NONE
TUE	15	4	34.8	+17	14		10.1	094019	14	4	32.2	+17	06	7.1	NONE
WED	16	4	33.9	+17	14		10.1	094019	14	4	32.2	+17	06	7.1	NONE
THUR	17	4	33.1	+17	13		10.1	094019	14	4	32.2	+17	06	7.1	NONE
FRI	18	4	32.2	+17	12		10.1	094019	14	4	32.2	+17	06	7.1	NONE
SAT	19	4	31.4	+17	11		10.1	094019	14	4	32.2	+17	06	7.1	NONE
SUN	20	4	30.5	+17	10	12	10.1	094019	14	4	32.2	+17	06	7.1	NONE
MON	21	4	29.8	+17	10		10.1	094019	14	4	32.2	+17	06	7.1	NONE
TUE	22	4	29.1	+17	09		10.1	094019	14	4	32.2	+17	06	7.1	NONE
WED	23	4	28.5	+17	09		10.2	094019	14	4	32.2	+17	06	7.1	NONE

Fig. 4 (contd.)

DAY	DATE (1970-71)	α h	α m	δ °	δ ′	[illegible]	MV m	RECOMMENDED GUIDE STAR SAO	GUIDE STAR CHART REF. NO.	α* h	α* m	δ* °	δ* ′	MV* m	PLATE NO. OBTAINED
THUR	DEC. 24	4	27.7	+17	09		10.2	093962	15	4	26.5	+17	26	7.3	NONE
FRI	25	4	27.1	+17	08		10.2	093962	15	4	26.5	+17	26	7.3	NONE
SAT	26	4	26.4	+17	08		10.2	093962	15	4	26.5	+17	26	7.3	NONE
SUN	27	4	25.7	+17	08		10.2	093962	15	4	26.5	+17	26	7.3	NONE
MON	28	4	25.1	+17	08		10.2	093962	15	4	26.5	+17	26	7.3	NONE
TUE	29	4	24.4	+17	07		10.2	093962	15	4	26.5	+17	26	7.3	NONE
WED	30	4	23.7	+17	07	13	10.2	093897	16	4	20.0	+17	26	3.9	NONE
THUR	31	4	23.3	+17	08		10.2	093897	16	4	20.0	+17	26	3.9	NONE
FRI	JAN. 1	4	22.8	+17	08		10.3	093897	16	4	20.0	+17	26	3.9	NONE
SAT	2	4	22.4	+17	08		10.3	093897	16	4	20.0	+17	26	3.9	NONE
SUN	3	4	21.9	+17	09		10.3	093897	16	4	20.0	+17	26	3.9	NONE
MON	4	4	21.5	+17	10		10.3	093897	16	4	20.0	+17	26	3.9	TBS/(i)†,(ii)†
TUE	5	4	21.1	+17	10		10.3	093897	16	4	20.0	+17	26	3.9	TBS/(iii)†
WED	6	4	20.6	+17	11		10.3	093897	16	4	20.0	+17	26	3.9	NONE
THUR	7	4	20.2	+17	11		10.3	093897	16	4	20.0	+17	26	3.9	NONE
FRI	8	4	19.7	+17	12		10.3	093897	16	4	20.0	+17	26	3.9	NONE
SAT	9	4	19.3	+17	12	14	10.3	093897	16	4	20.0	+17	26	3.9	NONE
SUN	10	4	19.1	+17	13		10.4	093897	16	4	20.0	+17	26	3.9	NONE
MON	11	4	19.0	+17	15		10.4	093897	16	4	20.0	+17	26	3.9	NONE
TUE	12	4	18.8	+17	16		10.4	093897	16	4	20.0	+17	26	3.9	NONE
WED	13	4	18.7	+17	17		10.4	093897	16	4	20.0	+17	26	3.9	NONE
THUR	14	4	18.5	+17	18		10.4	093897	16	4	20.0	+17	26	3.9	NONE
FRI	15	4	18.4	+17	20		10.4	093897	16	4	20.0	+17	26	3.9	NONE
SAT	16	4	18.2	+17	21		10.4	093897	16	4	20.0	+17	26	3.9	NONE
SUN	17	4	18.0	+17	22		10.4	093897	16	4	20.0	+17	26	3.9	NONE
MON	18	4	17.9	+17	24		10.5	093897	16	4	20.0	+17	26	3.9	NONE
TUE	19	4	17.7	+17	25	15	10.5	093897	16	4	20.0	+17	26	3.9	NONE
WED	20	4	17.8	+17	27		10.5	093897	16	4	20.0	+17	26	3.9	TBS/(iv)
THUR	21	4	17.9	+17	29		10.5	093897	16	4	20.0	+17	26	3.9	TBS/(v)
FRI	22	4	18.0	+17	31		10.5	093897	16	4	20.0	+17	26	3.9	NONE
SAT	23	4	18.1	+17	33		10.5	093897	16	4	20.0	+17	26	3.9	NONE
SUN	24	4	18.2	+17	34		10.5	093897	16	4	20.0	+17	26	3.9	NONE
MON	25	4	18.4	+17	36		10.6	093897	16	4	20.0	+17	26	3.9	TBS/(vi)
TUE	26	4	18.5	+17	38		10.6	093897	16	4	20.0	+17	26	3.9	NONE
WED	27	4	18.6	+17	40		10.6	093897	16	4	20.0	+17	26	3.9	NONE
THUR	28	4	18.7	+17	42		10.6	093897	16	4	20.0	+17	26	3.9	NONE
FRI	29	4	18.8	+17	44	16	10.6	093897	16	4	20.0	+17	26	3.9	NONE
SAT	30	4	19.2	+17	46		10.6	093897	16	4	20.0	+17	26	3.9	NONE
SUN	31	4	19.5	+17	49		10.6	093897	16	4	20.0	+17	26	3.9	TBS/(vii)
MON	FEB. 1	4	19.9	+17	52		10.6	093897	16	4	20.0	+17	26	3.9	NONE
TUE	2	4	20.2	+17	54		10.7	093897	16	4	20.0	+17	26	3.9	059,060
WED	3	4	20.6	+17	56		10.7	093897	16	4	20.0	+17	26	3.9	NONE
THUR	4	4	21.0	+17	59		10.7	093897	16	4	20.0	+17	26	3.9	NONE
FRI	5	4	21.3	+18	02		10.7	093897	16	4	20.0	+17	26	3.9	NONE
SAT	6	4	21.7	+18	04		10.7	093897	16	4	20.0	+17	26	3.9	NONE
SUN	7	4	22.0	+18	06		10.7	093897	16	4	20.0	+17	26	3.9	NONE
MON	8	4	22.4	+18	09	17	10.7	093923	17	4	22.6	+17	49	4.2	NONE
TUE	9	4	23.0	+18	12		10.8	093923	17	4	22.6	+17	49	4.2	NONE
WED	10	4	23.6	+18	15		10.8	093923	17	4	22.6	+17	49	4.2	NONE
THUR	11	4	24.2	+18	18		10.8	093954	18	4	25.7	+19	04	3.6	NONE
FRI	12	4	24.8	+18	21		10.8	093954	18	4	25.7	+19	04	3.6	NONE
SAT	13	4	25.4	+18	24		10.8	093954	18	4	25.7	+19	04	3.6	NONE
SUN	14	4	26.0	+18	26		10.8	093954	18	4	25.7	+19	04	3.6	NONE
MON	15	4	26.6	+18	29		10.8	093954	18	4	25.7	+19	04	3.6	NONE
TUE	16	4	27.2	+18	32		10.8	093954	18	4	25.7	+19	04	3.6	061,062
WED	17	4	27.8	+18	35		10.8	093954	18	4	25.7	+19	04	3.6	063
THUR	18	4	28.4	+18	38	18	10.9	093954	18	4	25.7	+19	04	3.6	NONE
FRI	19	4	29.2	+18	41		10.9	093954	18	4	25.7	+19	04	3.6	NONE
SAT	20	4	30.0	+18	44		10.9	093998	19	4	30.1	+19	15	7.8	NONE
SUN	21	4	30.8	+18	47		10.9	093998	19	4	30.1	+19	15	7.8	NONE

Fig.4 (contd.)



SAO STAR CHART – 1967
EQUINOX AND EPOCH 1950.0
FK4 SYSTEM
LAMBERT CONIC PROJECTION
SCALE 6.95
MAGNITUDES
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Fig. 5

evening's observation but also with a complete record of observations already obtained in the series.

Like the combined ephemeris and guide star list, the finding chart also represented something of a departure from normal practice. Instead of a sketch of the star field around each guide star being produced, the field of view of the finder telescope was represented by a circle of diameter two degrees centred on each guide star on a single low-scale chart of the region (duplicated from the Smithsonian Astrophysical Observatory Star Atlas). These circles were then marked with the reference number given in column 8 of the ephemeris (to avoid labelling each circle with the full SAO catalogue number of the guide star) and the overlapping portions of the circles were erased in order to prevent confusion. In addition, the apparent path of the asteroid was superimposed lightly on the chart, a number (corresponding to the numbered dates in the daily ephemeris) marking its position at ten-day intervals.

The ephemeris and finding chart in the form described were found to be extremely convenient during observation and, in general terms, the importance of careful preparation of this kind for minor planet observing cannot be overemphasized. It will be noted that, had it been operational at the time, ORBIT 3 could have been used to produce the daily ephemeris. The orbital elements required by the program would have been taken from the "Ephemerides of Minor Planets 1970" or, better, from a reduction of preliminary observations made immediately before the start of the main series. This would be preferable because, for this planet, the published elements refer to an epoch of osculation of 1951 December 20 and, since ORBIT 3 does not account for perturbations, large errors could be present if these were used.

The observations were all obtained using the Scott Lang Telescope of the University Observatory, St Andrews. This instrument is a 15-inch x 19-inch f/3 (effective f/3.9) Schmidt-Cassegrain telescope designed so as to give a flat distortion-free field approximately 5° in diameter. Detailed descriptions of the instrument are given by Finlay-Freundlich (1950, 1953) and a photograph appears in King (1955, p 376). The instrument has an equivalent focal length of 46.6 inches giving a plate scale of 170.54 arc seconds per mm; four-inch square plates are used. Despite the provision of a temperature compensated tube in the design of the telescope, secondary temperature variation effects are present and so the instrument is provided with thermometers and a graduated focussing ring, an empirical formula relating the focus setting to the tube temperature. The instrument is fitted with a two-inch finder telescope (field diameter approximately 2°) and a four-inch guide telescope with illuminated cross-wires (field diameter $\frac{1}{2}^{\circ}$).

Since the instrument had been in use for some twenty years when the present minor planet observations were carried out, some slight degree of maladjustment had arisen and so care was taken to use the central portion of the field as far as possible. The telescope is in fact now no longer operational, having been replaced in June 1973 by a 20-inch Ritchey-Chrétien reflector. It may be noted, however, that twenty-three years' service is remarkable for a telescope built purely as an experiment.

No rigorous criteria were applied to the choice of emulsion used since merely the positions of the images were required and almost any emulsion would provide this data. Also, the very fast telescope being used (f/3.9) rendered the speed of the emulsion immaterial to a large extent. In fact, a little over half the exposures were on Kodak OAO emulsion; the rest were on 103a-D. The exposures were all of the order of three minutes as this was found to produce the optimum size of the image of the minor planet for measuring. No change in exposure was found to be necessary to compensate for the planet's variation in magnitude (between $11^{m}.3$ and $10^{m}.0$) during the period of the observations but some changes in exposure were made under certain conditions of bright moonlight, twilight, etc.

Considerable care was taken in the determination of the instants of beginning and end of each exposure. The GPO "speaking clock" was used as the basis for the timing of the exposures, a good wristwatch running at the mean time rate being compared with the speaking clock before each observing session to determine the error of the watch. Since an accuracy of one second in the exposure times was aimed for and the dome was completely darkened during the exposures, a seconds timer in the dome (providing an audible beat at one second intervals) was used in conjunction with the wristwatch to time the observations. A wristwatch giving local sidereal time was also carried for convenience.

A standard procedure was followed during each observing session. The dome was opened and the telescope drive started approximately 30 minutes before the start of observing in order to allow the air in the dome to stabilise and the drive to warm up thoroughly. The telescope dust caps were removed at the same time. The first plate was then loaded into the telescope and, the hour circle having been set to the correct sidereal time, the instrument was set on the appropriate guide star using the ephemeris and finding chart described previously.

Immediately before the exposure was made, the thermometers were read and the focus set to the correct value. The dome lights were then extinguished and the dark slide and shutter were withdrawn to start the exposure. Guiding was carried on throughout in addition to counting off the elapsed seconds. At the end of the exposure, seconds were counted for a further minute while the lights were turned on and then the watch was read and the instants of beginning and end of the exposure deduced. The plate-holder was removed from the telescope and, in the darkroom, the plate was marked with a temporary identification number (using a diamond point) and placed in a cooled storage box. Unless observing was completed, the plate-holder was then reloaded.

An attempt was made to obtain at least two plates in an evening to facilitate location of the minor planet on the plate using the blink comparator. An exception to this was when the object was near its stationary points; at these times only one plate was necessary providing another one of the same field was obtained within a few days. A detailed record was kept of the observations obtained, all relevant data being noted.

The plates were normally processed the following day, being kept meanwhile in a cooled light-tight storage box. Normal processing procedures were used, development being by immersion in agitated Kodak D19b developer nominally at 68°F. The development time was five minutes in each case. No safelight was used.

After processing, the plates were marked with an adhesive tag giving the plate number and showing the correct orientation. The plates were stored in boxes having a capacity of 36 plates, each box having an index to its contents. Besides showing the number, date obtained and guide star for each plate, the index was arranged to indicate also the stage reached by the plate in its measurement and reduction.

By the end of the 1970-71 observing season, a total of 54 plates of 16 Psyche had been obtained on 37 nights. Of these plates, 45 were obtained by the writer; 7 were obtained by Mr. T.B. Slebarski and two by Mr. D. Kilkenny when the writer was unable to observe. Table I overleaf summarizes the data on the plates obtained.

TABLE I

Observations of 16 Psyche 1970-71

PLATE NO.	DATE	SHUTTER OPENED UT			SHUTTER CLOSED UT			GUIDE STAR SAO	EMULSION	REMARKS
		h	m	s	h	m	s			
FGW/020	1970 SEP 1	03	27	00	03	30	00	094112	OAO	Strong twilight
FGW/021	3	02	43	30	02	46	30	094112	OAO	Intermittent cloud
FGW/022	3	03	11	30	03	16	30	094112	OAO	
FGW/023	5	02	44	14	02	49	14	094164	OAO	Thin cloud
FGW/024	5	03	12	19	03	17	19	094164	OAO	Haze
FGW/025	5	03	41	00	03	45	00	094164	OAO	Twilight
FGW/026	11	02	39	05	02	42	05	094199	OAO	Intermittent cloud. High wind
FGW/027	13	02	46	30	02	50	00	094199	OAO	Thin cloud
FGW/028	13	03	17	05	03	20	05	094199	OAO	
FGW/029	14	03	30	30	03	33	30	094199	OAO	Twilight
FGW/030	15	02	49	30	02	52	30	094199	OAO	Moonlight
FGW/031	15	03	39	15	03	42	15	094199	OAO	
FGW/032	16	02	48	30	02	50	30	094199	OAO	Seeing very poor. Strong moonlight
FGW/033	16	03	26	30	03	28	30	094199	OAO	
FGW/034	24	03	45	35	03	47	35	094306	OAO	Thin cloud. Moonlight
FGW/035	25	04	42	40	04	44	40	094306	OAO	Some cloud obscuration. Twilight
FGW/036	28	01	50	34	01	53	34	094332	OAO	
FGW/037	28	02	37	55	02	40	55	094332	OAO	Sea mist obscuration

TABLE I (Continued)

PLATE NO.	DATE	SHUTTER OPENED UT			SHUTTER CLOSED UT			GUIDE STAR SAO	EMULSION	REMARKS
		h	m	s	h	m	s			
FGW/038	1970 SEP 28	03	22	38	03	25	38	094332	OAO	
FGW/039	OCT 1	02	14	45	02	17	45	094332	OAO	
FGW/040	2	02	01	40	02	04	40	094332	OAO	Very windy
FGW/041	2	02	57	20	03	00	20	094332	OAO	
FGW/042	3	02	12	45	02	15	45	094332	OAO	
FGW/043	9	02	12	30	02	15	30	094431	OAO	Rather hazy
FGW/044	11	01	26	50	01	29	50	094431	OAO	Sky excellent
FGW/045	26	02	25	30	02	28	30	094431	OAO	
FGW/046	27	00	11	15	00	14	15	094431	OAO	Thin cloud
FGW/047	NOV 4	01	36	20	01	39	20	094332	OAO	
FGW/048	5	23	54	50	23	57	50	094332	OAO	
FGW/049	9	02	02	35	02	05	35	094332	OAO	
FGW/050	9	03	10	37	03	13	37	094332	OAO	High wind making guiding difficult
FGW/051	10	03	16	25	03	19	25	094332	OAO	
FGW/052	20	01	01	45	01	04	45	094227	OAO	Moonlight
FGW/053	20	01	55	30	01	58	30	094227	103a-D	
FGW/054	DEC 2	01	36	10	01	39	10	094158	103a-D	Thin cloud
FGW/055	7	01	22	55	01	25	55	094158	103a-D	Seeing very poor
FGW/056	7	01	52	15	01	55	15	094158	103a-D	Seeing very poor
FGW/057	8	01	17	45	01	20	45	094158	103a-D	Moonlight

TABLE I (Continued)

PLATE NO	DATE	SHUTTER OPENED UT			SHUTTER CLOSED UT			GUIDE STAR SAO	EMULSON	REMARKS
		h	m	s	h	m	s			
FGW/058	1970 DEC 8	02	20	55	02	23	55	094158	103a-D	
TBS/(i)	1971 JAN 3	22	36	10	22	39	10	093907	103a-D	Moonlight
TBS/(ii)	4	00	24	55	00	27	55	093907	103a-D	Excellent sky. Guiding difficult
TBS/(iii)	4	23	44	25	23	47	25	093907	103a-D	Moonlight
TBS/(iv)	19	21	22	50	21	25	50	093897	103a-D	
TBS/(v)	20	20	43	30	20	46	30	093897	103a-D	
TBS/(vi)	24	22	12	15	22	15	15	093897	103a-D	Sky excellent
TBS/(vii)	30	23	39	14	23	42	14	093897	103a-D	
FGW/059	FEB 1	19	48	55	19	51	55	093897	103a-D	Moonlight. Guiding difficult
FGW/060	1	21	03	55	21	06	55	093897	103a-D	
FGW/061	15	20	33	03	20	36	03	093954	103a-D	
FGW/062	15	21	04	23	21	07	23	093954	103a-D	
FGW/063	16	19	02	40	19	05	40	093954	103a-D	
DK/(i)	21	20	23	35	20	26	35	093998	103a-D	Loaded incorrectly.Half plate is defocussed
DK/(ii)	21	20	55	25	20	58	25	093998	103a-D	
FGW/064	MAR 13	20	27	15	20	30	15	094164	103a-D	Cloud. Difficulty guiding.

Chapter 2.2 Measurement of the Plates

The observation of the minor planet having been completed, the next phase in the work was the measurement of the plates to obtain the position of the object at the instant of each observation. Before the actual measuring was reached, however, several intermediate stages had to be carried out, and it was found to be more convenient to execute each phase of the work on all the plates rather than complete the work from beginning to end on each plate individually.

The first task to be undertaken was the identification of the minor planet amongst the multitude of images on each plate and this was accomplished using a blink comparator. It was carried out on every plate in the series, no selection being applied at this stage.

The St Andrews blink comparator is a standard model produced by Messrs Grubb, Parsons of Newcastle on Tyne. The instrument consists of a heavy frame on which is supported a carriage movable in x and y and fitted with coarse scales and slow motions. The two plateholders rest on this carriage and are arranged so that one can be moved relative to the other in x – y, orientation and focus for the purposes of adjustment. The relative x – y motion is achieved via micrometer drums reading to 0.01mm (over a limited range) so that accurate comparative measures of plate co-ordinates are possible.

The plates are illuminated from above by two independent lamps and an optical system combines the two images and brings them to a single eye-piece at the front of the instrument. The "blinking" of the lamps is achieved electronically and the rate of blinking can be varied continuously from several blinks per second to one every few seconds by means of a knob on the control box. Switches are provided so that each lamp can be independently turned on or off, or blinked.

The procedure for using the comparator was very straightforward. The two plates to be compared were placed in the instrument and their relative position and orientation adjusted until the star fields appeared stationary when the lamps were blinked. The blink rate was then set to a comfortable level and the plates were inspected using the coarse x - y slow motions to move them. In practice it was necessary to scan only a limited area of the plate around the expected position of the asteroid rather than the whole plate, and the planet was usually very obvious by its motion when it was encountered. Once having discovered the asteroid, it was only necessary to mark its position on each plate by means of an ink circle drawn around it (on the glass side) and the operation was completed.

The blinking of the plates was carried out in the order in which they were obtained, the apparent direction of motion of the asteroid's image providing a check on the identification. When a number of consecutive plates had been obtained with the same guide star each plate was blinked with the one obtained before it, the asteroid's position having been already marked on the earlier plate.

The interval separating the pairs of plates which were compared varied from about half an hour to several days and consequently the amount of movement of the asteroid's image varied greatly. Such is the efficiency of the blinking method, however, that no difficulty was encountered in locating the object, even when one of the plates was badly defocussed (plate DK/(i) in comparison with DK/(ii)). Only one plate was obtained in isolation (viz. FGW/064) and this had to be blinked against another of the same field taken several months earlier.

It may be remarked here that the task of locating the asteroid on this series of plates without a blink comparator would have been tedious in the extreme and, moreover, no easy task due to the small size of the asteroid image and the very large number of star images on each of these plates. We also remark in passing that before undertaking the present work at St Andrews, the writer was involved with the manufacture of this blink comparator as an employee of Messrs Grubb, Parsons, being primarily concerned with the testing and alignment of the optical components.

The next stage in the work was the selection of the plates to be measured, it being neither practicable nor desirable to measure the whole series. This is primarily because many of the plates were obtained in pairs (or threes) separated by short intervals of time and there would have been little point in measuring both plates (or all three), bearing in mind the envisaged limits of the investigation.

Two criteria were applied in the selection of the plates. The first was that the plates chosen should form as nearly as possible an evenly spaced series throughout the observing season. Because of the unequal distribution of observations during the season caused by varying weather conditions this criterion could only be applied in very general terms but the aim was approached in some degree. The second criterion was that where a choice of plates was possible (each satisfying the first condition) those which would give the most accurate measurements were selected. Those with defects such as asymmetry of the images due to inaccurate guiding, large images caused by over-exposure, faint images caused by under-exposure and reduced contrast due to a bright sky background, all of which decrease the accuracy of the measurements, were rejected.

The result of this procedure was that 26 plates were selected for measurement, approximately half the total number obtained. At this stage in the work, the details of the 26 plates to be measured were entered on standard IBM 80-column data preparation forms (one for each plate) in the input format of the plate reduction program ORBIT 2 (see page 48). These forms, which we shall refer to as plate reduction data forms, were used to record all the data on each plate required for the computer reduction of the plate measurements. At this stage, the known data on each plate were:

name of object, date and time of observation (line 1);

place of observation (line 2);

ORBIT 2 access code ("ALL") and epoch required (line 4);

plate number, number of reductions and permissible error (line 5);

date of observation in years and decimals (line 6);

epoch of reference star co-ordinates (line 7)

and these were entered on each form.

It had been decided previously that two independent three-star reductions should be carried out on each plate. This would prevent the occurrence of any undetected gross errors in the results whilst the number of reference stars to be measured (six) would not be excessive. The tolerable discrepancy between the two reductions in RA and Dec ("permissible error") was set at 2.5 arc seconds, a fairly liberal value. The epoch of the reference star co-ordinates was entered as 1950.0 since the SAO Star Catalogue was to be used. The epoch required was also 1950.0.

The plates were then examined one by one and, by comparing them with the SAO Star Atlas, suitable reference stars were selected, bearing in mind the geometry of the images required by the method of dependences used in ORBIT 2. The six reference stars were marked on the plate in the same way as the asteroid (ink circles) and they were then labelled with identification letters A to F for use when measuring, the asteroid image being labelled X. The reference stars were then identified in the SAO Star Catalogue by means of the Star Atlas, and the catalogue numbers of the stars were entered on the plate reduction data form (lines 8, 9, 10 for first reduction; 14, 15, 16 for second) together with their identification letters in the margin. A note was also made on the form as to whether or not the image of the planet was within the triangle of reference stars for each of the two reductions, whether any of the stars were double and if so which component should be measured, and any other relevant data.

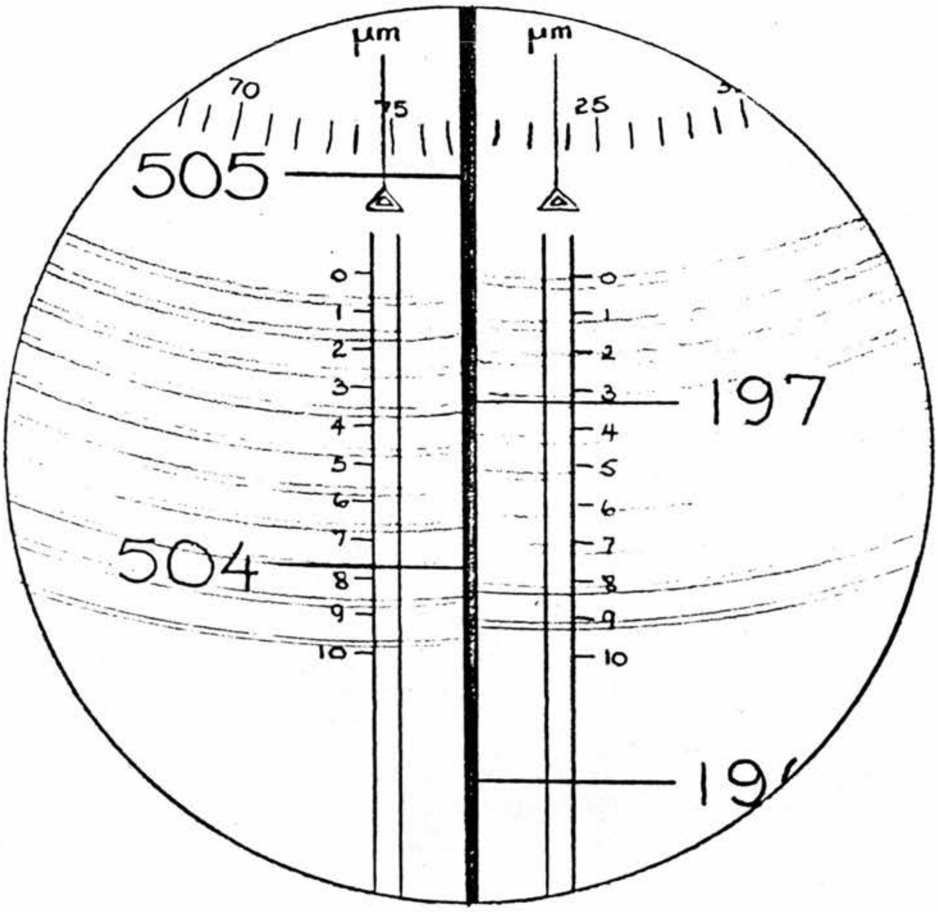
When reference stars had been selected for all the plates, the necessary data on them were extracted from the Star Catalogue and entered on the plate reduction data forms (lines 11, 12, 13 for first reductions; 17, 18, 19 for second) and this completed the preparations required for the measurement of the plates.

The plate measurement was carried out on the large Zeiss measuring machine at St Andrews. (Roth (1962, Plate XI) gives a photograph of an identical instrument.) This type of machine is capable of very high precision measurements and steps had been taken at St Andrews to ensure that this capability could be fully realized, the instrument being situated on a solid concrete floor and in a constant temperature environment.

The machine consists of a very heavy frame supporting an optically flat glass surface upon which runs the plate carriage. This can be moved directly or by means of slow motions for fine setting, and it is linked to two very accurately engraved glass scales which form the basis on which the measurements are made. These are engraved in millimetres and are arranged to be mutually perpendicular with a very high degree of precision, one providing the x-values and the other the y-values.

The plate is illuminated from above and an optical system brings the image to a binocular eyepiece located at the front of the instrument. Another optical system brings images of the x and y scales to the same eyepiece, selection of plate or scales being brought about by altering the illumination. The scales are read by means of double spirals engraved on two rotatable glass plates calibrated with circular scales which act in much the same way as a micrometer drum but with no mechanical connection to the main scales. When the scales are being viewed, the x-values are presented on the left of the field and the y-values on the right, their appearance being as shown in Figure 6. The large numerals are millimetres and these are engraved on the main scales. The two small vertical scales are tenths of a millimetre; they are fixed. The scales at the top of

ZEISS MEASURING MACHINE X - Y SCALES



$x = 504.7748$ mm $y = 197.3237$ mm

Fig. 6

the field show thousandths of a millimetre and they are on the spiral plates, segments of the spirals being visible in the field of view.

The controls of the machine include the plate/scale selector (which can be operated by means of a footswitch), the knobs for rotating the two spiral plates, star image brightness, magnification and focus controls, and an image rotator to turn the star field through 90 or 180 degrees. In use, the plate/scale selector is turned to "plate" and the star (or other object) to be measured is centred on an engraved graticule in the field of view by means of the setting slow motions. The selector is then turned to "scales" and the x - y co-ordinates of the star relative to an arbitrary but fixed zero can be read.

In reading the scales it is possible to estimate values to tenths of a micrometre but the overall accuracy of the machine is probably of the order of one or two micrometres. The main source of error is in the setting of the star image on the graticule, and the accuracy with which this can be done depends largely on the size and shape of the image. This is discussed in more detail at the end of the present chapter.

The procedure adopted for the measurement of the plates was developed as a result of trial measurements on earlier plates of other objects and, whilst it was fairly complex, it proved to be very efficient and extremely reliable. The trial measurements indicated that a certain amount of dark adaption was required for the comfortable use of the measuring machine over long periods, and the act of noting down a reading, apart from being time consuming, spoiled the dark adaption. A small portable tape recorder was therefore introduced as the means of noting the plate measurements, the microphone being placed so that the recording could be made without moving from the eyepiece.

The measuring procedure was as follows. First, the plate was loaded into the plateholder in the correct orientation and the retaining clips locked, care being taken to ensure that the plate was neither free to move in its holder nor under stress from the clips. No fine adjustment in the orientation of the plateholder was required since the reduction method to be used eliminated the need for this. The measuring sequence was then begun. For convenience, the images were always measured in the order A-B-C-D-E-F-X-A, the final A being a control measurement to indicate whether any movement of the plate in its holder had occurred during the measurements. Six (x, y) values were obtained for each image, the instrument being reset on the image for each (x, y) measurement and the star field being rotated through 180 degrees between each setting. This was to reduce the effect of any personal bias in setting when averaging over the six (x, y) measurements; the results nearly always indicated that such a bias was present in some degree. When the measurements for an image were complete the next image was begun immediately until the sequence of images was completed.

Leaving the plate undisturbed in the measuring machine in case any images required a repeat measurement, the tape recording was then played back and, for each image, the six x and y values were fed directly into two cumulative storage registers on an electronic desk calculator. The totals in these registers were then divided by six to obtain the mean x and y values for the image. This was carried out for all the images, the resulting mean (x, y) values being entered on the plate reduction data form (lines 6 (asteroid); 8, 9, 10 (stars for first reduction); 14, 15, 16 (stars for second reduction)). The control measurement of star A was averaged separately and noted on the form to facilitate comparison with the first star A measurement.

This process of entering the tape recorded data directly into the desk calculator was much faster than the alternative of transcribing the contents of the tape, but it had the disadvantage of the original data not being preserved since each recording was erased as soon as the next plate was measured. Therefore a check was required to ensure that the mean (x, y) values had been derived correctly from the data on the tape and that the original values themselves had all been correctly read from the measuring machine. This check was provided by the recording being played back a second time, the deviation of the original (x, y) values from the mean value for each image being mentally noted to lie within the expected limits. Any incorrectly read values or an incorrectly derived mean would be shown up by the large deviation of one or all the values from the mean (depending on the stage at which the error was made). Only errors in the fourth decimal place of millimetres would not be shown up, being of the same order as the scatter of the values used to compute the mean. When this check had been satisfactorily carried out, the plate was removed from the machine, the measuring process being completed.

This procedure was carried out for all the plates selected for measurement, the result being 26 completed plate reduction data forms in the correct format for punching onto cards for ORBIT 2 processing. Some details of the reference stars used in the plate measurement are presented in Table II.

Amongst these, in the penultimate column of the table, is given the discrepancy Δ between the two measurements of star A on each plate. This is deduced from $\Delta = \surd(\Delta x^2 + \Delta y^2)$ where Δx and Δy are the differences between the results of the main and control measurements of star A in x and y respectively. Assuming that there is no movement of the plate in its holder during the measurements we should expect Δ to vary randomly between zero and some small upper limit which would be indicative of the maximum difference which could occur between two separate measurements of the same object (each averaged over six values); that is, it would be indicative of the overall accuracy of the measurements. As will be seen from the table, this is in fact the case, very few of the values of Δ exceeding 25×10^{-4} mm. The notable exception is the value for plate FGW/024 which far exceeds the rest; we interpret this as representing an actual movement of the plate in its holder of $(45 \pm 25) \times 10^{-4}$mm whilst the measurements were being made.

TABLE II

MEASUREMENT OF PLATES OF 16 PSYCHE 1970-71

PLATE No	REFERENCE STARS (SAO Star Catalogue Nos)						STAR A	
	FIRST REDUCTION			SECOND REDUCTION			Δ (mm x 10^{-4})	m_v
FGW/020	094110	094122	094115	094103	094102	094092	31	9.2
FGW/022	094136	094112	094122	094110	094155	094115	17	6.8
FGW/024	094136	094164	094150	094122	094155	094143	45	6.8
FGW/026	094199	094164	094150	094243	094228	094210	12	6.2
FGW/028	094199	094164	094150	094243	094228	094210	21	6.2
FGW/033	094199	094164	094150	094243	094228	094210	1	6.2
FGW/034	094332	094370	094345	094306	076971	094369	3	5.0
FGW/038	094332	094370	094345	094306	076971	094369	3	5.0
FGW/039	094332	094422	094345	094306	076971	094369	15	5.0
FGW/042	094332	094422	094369	094306	094345	076971	21	5.0
FGW/043	094431	094422	094394	094408	094370	094369	17	7.5
FGW/044	094431	094422	094394	094408	094370	094369	14	7.5
FGW/045	094422	094394	094370	094431	094366	094369	9	7.5
FGW/047	094394	094370	094321	094332	094366	094422	5	5.0
FGW/048	094332	094366	094370	094351	094394	094321	8	5.0
FGW/049	094294	094366	094351	094313	094295	094321	25	7.9
FGW/053	094294	094257	094219	094229	094223	094227	22	7.9
FGW/054	094132	094120	094117	094138	094136	094157	18	8.1

TABLE II (Continued)

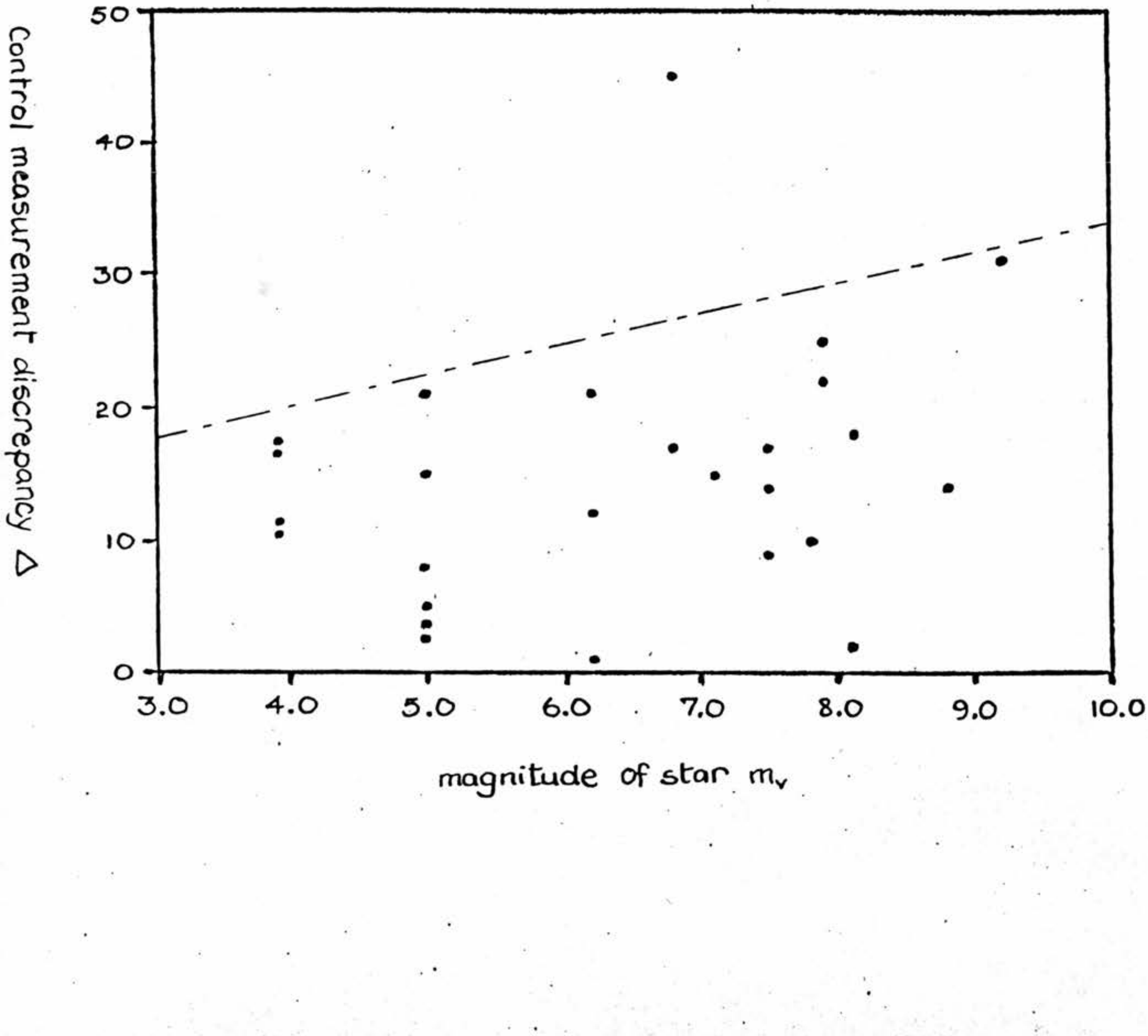
MEASUREMENT OF PLATES OF 16 PSYCHE 1970-71

PLATE No	REFERENCE STARS (SAO Star Catalogue Nos)						STAR A	
	FIRST REDUCTION			SECOND REDUCTION			Δ(mm x 10^{-4})	m_V
FGW/055	094132	094120	094117	094119	094136	094112	2	8.1
TBS/(iii)	093897	093927	093900	093886	093907	093913	11	3.9
TBS/(v)	093886	093883	093854	093882	093860	093880	14	8.8
TBS/(vi)	093897	093880	093860	093854	093882	093883	17	3.9
TBS/(vii)	093880	093882	093923	093883	093897	093907	11	3.9
FGW/060	093897	093923	093883	093907	093880	093915	17	3.9
FGW/063	093963	094002	093973	093954	093998	093953	15	7.1
DK/(ii)	093998	094017	094025	094002	094001	094032	10	7.8

(Star A on each plate is underlined)

An attempt was made to demonstrate the expected correlation between the accuracy of the measurements and the size of the image being measured by plotting Δ against the magnitude of the star used for the control measurement for all the plates. The magnitudes were extracted from the SAO Star Catalogue; m_v only was available (see Table II). The result of the plot is given in Figure 7. Whilst m_v from the catalogue is by no means a good measure of the image size on the plates and 26 points can hardly be regarded as sufficient, it does appear that there may be a slight upward trend in the upper limit of the scatter of the points as indicated by the broken line (ignoring the point for plate FGW/024). If this trend is real, it implies a decrease in setting accuracy with decreasing image size which is the converse of the correlation expected intuitively. A major factor in explaining this could well be the design of the setting graticule which, being essentially an open square design rather than a cross-wire, can probably be set most accurately on an image of the same order of size as the principal square. In the present case, this would correspond to the image size of a fairly bright star. Clearly these deductions are founded on the most flimsy evidence and cannot be regarded as conclusive. A full discussion of the relation between measurement accuracy and image size would have to take account of the quality and shape of the images as well as a much larger number of stars being used.

The procedures described in this chapter for the handling of plates were found to be extremely convenient in practice despite their apparent complexity, and the large amount of planned redundancy in the procedures meant that the measurements obtained were very reliable and had a high degree of precision. The development of these rigorous methods would almost certainly have been difficult to justify had only one or two plates been involved, but the relatively large number of plates being dealt with here ensured that the effort was worthwhile if only in terms of the amount of time saved on each plate measured.



DISCREPANCY/MAGNITUDE DIAGRAM FOR CONTROL MEASUREMENTS

Fig. 7

Chapter 2.3 Data Reduction

The reduction of the data was carried out using the IBM System/360 Model 44 computer installation at the University Computing Laboratory. Detailed descriptions of this type of machine are given by the IBM Corporation (1968 a, b). The St. Andrews installation had a core storage capacity of 128 kilobytes (upgraded to 256 k in 1972) and all the usual peripheral facilities were available. The machine was run in three distinct modes: (i) the Model 44 Programming System (44 PS); (ii) a remote-access timesharing system (RAX); (iii) the System/360 Operating System. Each system was available for a particular portion of each day according to an operating timetable.

The ORBIT programs were run under 44 PS and the very similar 44 MFT system (Multiprocessing with Fixed number of Tasks) which superseded 44 PS in 1972. Under these systems, a core storage capacity of 76k (112k from 1972) was available to the programmer but, in fact, none of the ORBIT programs used more than 37k.

The use of the Model 44 Programming System with FORTRAN programs is described in detail in IBM (1968 d). The FORTRAN program is normally translated into an object program (called a "module") by the FORTRAN compiler. This is then edited and linked with other modules required for the execution of the program (eg subroutines or FORTRAN-supplied subprograms) in order to produce an executable machine-language program known as a "phase". This is accomplished by means of the linkage editor LNKEDT. The phase can then be executed and, if the programmer so desires, he can arrange for the phase to be kept in a phase library so that it can be executed again at any time without prior compilation and linkage editing. In this case the phase must be given a name so that it can be referred to when execution is required. At St. Andrews, a public phase library was available and all the ORBIT programs were written into it as executable phases. This method of storing programs had the advantage of saving processing time and it also meant that the card decks containing the ORBIT source programs in FORTRAN were not damaged by being repeatedly passed through the card reader. Execution of a program was accomplished merely by the inclusion in a job of an "execute" statement specifying the phasename of the program required and following this with the input data to the program.

The first step in the ORBIT reduction was the transformation of the plate measurements into the topocentric RA and Dec of the minor planet for each of the 26 plates measured. As was described in the previous chapter, the measurements and other necessary data had been prepared in the format of the reduction program ORBIT 2 on the plate reduction data forms. Because of this, the process of transferring the data for each plate onto punched cards was accomplished with the minimum of delay and, since ORBIT 2 was already stored in the phase library, the results were obtained very quickly. These are given in Table III. For each plate, the results of both independent reductions are shown together with the resulting mean values of the coordinates. The time of observation is simply the mid-point of the exposure.

Since the SAO Star Catalogue was used to provide the equatorial coordinates of the reference stars, the topocentric coordinates of the minor planet are referred to the same frame of reference as the catalogue; that is, the equator and equinox of 1950.0 and in the FK4 system. This implies that first order corrections are automatically applied for atmospheric refraction and stellar aberration (although second order differential effects due to these causes will be present in the results) and that no correction will be required for nutation in the subsequent reduction of the data. The proper motions of the reference stars between 1950.0 (the epoch of the catalogue) and the instant of each observation are, of course, corrected for by ORBIT 2 and, had a change in the precessional epoch of the results been required, the program would have applied corrections accordingly. Thus,

TABLE III

REDUCED POSITIONS OF 16 PSYCHE 1970-71

PLATE NO	TIME OF OBSERVATION UT	TOPOCENTRIC EQUATORIAL COORDINATES (1950.0)							
		ALPHA				DELTA			
			First Reduction	Second Reduction	Unweighted Mean		First Reduction	Second Reduction	Unweighted Mean
FGW/020	1970 SEP 01^d 03^h 28^m 30^s	$04^h 42^m$	$34^s.938$	$34^s.974$	$34^s.956$	$+19^\circ$ 01'	07".12	08".52	07".82
FGW/022	03 03 14 00	04 44	45.383	45.372	45.377	+19 02	48.87	48.32	48.59
FGW/024	05 03 14 49	04 46	52.544	52.511	52.527	+19 04	13.29	13.83	13.56
FGW/026	11 02 40 35	04 52	46.971	47.051	47.011	+19 06	55.28	55.58	55.43
FGW/028	13 03 18 35	04 54	38.092	38.153	38.122	+19 07	20.44	20.90	20.67
FGW/033	16 03 27 30	04 57	14.228	14.194	14.211	+19 07	30.36	31.14	30.75
FGW/034	24 03 46 35	05 03	14.203	14.275	14.239	+19 05	35.63	33.59	34.61
FGW/038	28 03 24 08	05 05	40.351	40.421	40.386	+19 03	23.87	24.44	24.16
FGW/039	OCT 01 02 16 15	05 07	13.640	13.595	13.617	+19 01	19.39	19.88	19.63
FGW/042	03 02 14 15	05 08	09.124	09.188	09.156	+18 59	41.68	44.15	42.92
FGW/043	09 02 14 00	05 10	17.719	17.757	17.738	+18 53	56.66	55.79	56.23
FGW/044	11 01 28 20	05 10	47.128	47.224	47.176	+18 51	46.31	45.05	45.68
FGW/045	26 02 27 00	05 10	52.403	52.408	52.405	+18 31	25.77	25.24	25.51
FGW/047	NOV 04 01 37 50	05 07	47.861	47.835	47.848	+18 17	01.21	01.15	01.18
FGW/048	05 23 56 20	05 06	50.797	50.802	50.800	+18 13	47.03	47.47	47.25
FGW/049	09 02 04 05	05 05	07.295	07.396	07.346	+18 08	32.29	32.07	32.18
FGW/053	20 01 57 00	04 57	15.526	15.613	15.569	+17 49	38.56	40.00	39.28
FGW/054	DEC 02 01 37 40	04 46	37.795	37.654	37.725	+17 30	20.30	20.46	20.38
FGW/055	07 01 24 25	04 41	59.993	59.899	-	+17 23	25.77	20.34	-
TBS/(iii)	1971 JAN 04 23 45 55	04 21	06.242	06.236	06.239	+17 10	08.91	08.56	08.73
TBS/(v)	20 20 45 00	04 17	59.203	59.210	59.207	+17 28	43.46	43.45	43.45
TBS/(vi)	24 22 13 45	04 18	17.631	17.651	17.641	+17 36	20.54	20.27	20.41
TBS/(vii)	30 23 40 44	04 19	34.192	34.194	34.193	+17 49	36.91	36.89	36.90
FGW/060	FEB 01 21 05 25	04 20	09.805	09.840	09.822	+17 54	10.03	09.12	09.57
FGW/063	16 19 04 10	04 27	47.804	47.812	47.808	+18 35	14.57	14.46	14.51
DK/(ii)	21 20 56 55	04 31	28.990	28.980	28.985	+18 50	40.62	40.60	40.61

with the exception of diurnal aberration (which we regard as negligible), parallax and planetary aberration (both of which are corrected for in the orbit determination process), all the necessary corrections to the observed positions are accounted for and so the ORBIT 2 results may be used as input to the orbit determination programs.

Returning to Table III, it will be noted that the mean values for the equatorial coordinates are omitted for plate FGW/055. This is because the discrepancy in declination between the two reductions exceeded the maximum tolerable value of 2".5 and so ORBIT 2 terminated without computing the mean value. (In fact the program is arranged to terminate if either the discrepancy in declination $\Delta\delta$ or the discrepancy in right ascension $15'' \Delta\alpha \cos\delta$ exceed the maximum tolerable value. It would thus be possible for the overall angular discrepancy $\Delta' = \sqrt{\Delta\delta^2 + (15\,\Delta\alpha\cos\delta)^2}$ to have become as high as $\sqrt{(2''.5)^2 + (2''.5)^2} = 3''.54$ before stopping the program.) The value of $\Delta\delta$ for plate FGW/055 can be seen from Table III to be 5".43 and, since this is well in excess of both the permissible error and the values encountered for the other plates, an investigation was carried out to determine the cause of the failure.

The first possibility to be eliminated was that of an error being present in the plate measurements, although this was thought unlikely due to the checks built into the measuring procedure. The plate was therefore measured again but the results of the reduction showed similar discrepancies, thus confirming that the measurements were not in error. (In fact the value obtained for the overall discrepancy Δ' was exactly the same (5".59) in both cases.) The possibility of an error in the catalogue values for one of the reference stars then arose, although five out of the six stars were known to be correct as they had been used successfully on other plates. An ORBIT 2 reduction was carried out on the sixth star (SAO 094119) using its measured coordinates but its derived position was found to agree well with the catalogue values, thus eliminating this as the cause of error. Movement of the emulsion on the plate was next suspected, but investigation showed that this would be unlikely to be large enough to explain the observed error.

Finally, the actual configuration of the images on the plate was examined. It was found that the image of the asteroid was farther from the centre on this plate than on any other in the measured series, the actual distance being 28 mm corresponding to an angular distance of the asteroid from the plate centre of $1^{\circ}20'$. (This large value is due to the fact that the plate was obtained using a different (brighter) guide star from the one planned for that night because of bad conditions.) However, this in itself would not explain the discrepancy in δ because the effect of field distortion at a large distance from the plate centre would be to introduce a radial error and in this case the discrepancy was in a tangential direction. Furthermore, the theoretical distortion of the telescope at this distance off axis would be too small to account for the observed discrepancy and, in fact, it was demonstrated by running ORBIT 2 with the asteroid's plate measurements replaced by suitably chosen hypothetical measurements that the discrepancy in δ was largely independent of the position of the asteroid on the plate. Suspicion thus fell once more on the reference stars.

It was then noted that the triangle of reference stars used for the second reduction was itself relatively close to the edge of the plate. In particular, two of the stars (SAO 094136 and SAO 094112) were as much as 33mm and 38mm from the plate centre (corresponding to $1^{\circ}34'$ and $1^{\circ}48'$ off axis respectively) and furthermore their positions were such that a radial distortion on the plate would give rise to a declination error in each. An ORBIT 2 reduction was carried out on these stars using the other triangle as reference stars and the results obtained differed significantly from the catalogue values in declination. The computed values were respectively $1''.36$ and $1''.02$ less than the catalogue values and a subsequent calculation indicated that these differences would in fact account for the 5$''$.43 discrepancy in δ between the two reductions, primarily because of the relatively large distance between the image of the asteroid and the reference triangle used in the second reduction. It was thus concluded that distortion of these two star images near the edge of the plate was directly responsible for the large discrepancy, the planet's coordinates obtained from the second reduction being erroneous. The cause of the trouble having been established, the data on the plate was withdrawn from the subsequent reductions.

Returning again to Table III, we find that analysis of the results of the independent reductions sheds some light on the probable overall accuracy of the measurement-reduction procedure and hence on the likely accuracy of the computed equatorial coordinates of the planet. If we compute the overall angular discrepancy $\Delta' = \sqrt{\Delta\delta^2 + (15\ \Delta\alpha \cos\delta)^2}$ between the two independently reduced positions of the object for each plate we find that Δ' is less than 0$''$.5 in 32% of the cases, less than 1$''$.0 in 56% of the cases and less than 2$''$.0 in 88% of the cases. (This ignores the result from plate FGW/055 which we regard as anomalous.) Very roughly these values correspond to probabilities of 1/3, ½ and 9/10 that the results of two reductions on the same plate will agree to within 0$''$.5, 1$''$.0 and 2$''$.0 respectively. We may thus adopt a value of 1$''$.0 as the "probable error" (ie the discrepancy which is just as likely to be exceeded as not) and we remark that this is of the same order as the maximum errors which are thought to be present in the SAO Star Catalogue (see Eichhorn (1974, p. 209)). However, the source of the discrepancies above is more likely to be associated with differences in the geometrical configurations of the images on the plates.

The next stage in the reduction was the determination of a preliminary orbit for the minor planet using ORBIT 1. In fact five runs of the program were made in an attempt to obtain some evaluation of its performance with different sets of observations. The results of these runs (rounded to 1" and six places of decimals) are shown in Table IV. As is the case with all sets of elements derived from the present series of observations the mean anomalies are given for 1970 Aug. $02^{d}.0$ ET (= JD 2440800.5 ET) since this is the nearest date to the observations whose Julian Day Number less $0^{d}.5$ is exactly divisible by 400 (see the "Explanatory Supplement to the Astronomical Ephemeris", p. 97).

Without going into detail we note from Table IV that there is a fairly wide variation in the values of the elements obtained. While this may be due in part to the effect of differences in the spacing and symmetry of the observations on the idiosyncrasies of the Gaussian method, it is more likely to be caused by differences in the arcs of the orbit represented by each trio of observations. In this connection we remark that the central observations of the reductions are in order of date so that each reduction represents the orbit at a later stage than the previous one. It is thus of some interest to note the changes in successive values of the elements (in particular the mean longitude of the planet at epoch ($M_o + \omega + \Omega$) and eccentricity, both of which are monotonic) but since these results are of a preliminary nature we postpone detailed analysis.

The failure of ORBIT 1 to obtain a solution in reduction no. 4 was due to a marginal disagreement between the values of the triangle ratios c_1, c_3 obtained using the ratio $\overline{y}$ of the sector to the triangle, and the values obtained earlier in the program. This in turn was almost certainly due to the relatively wide spacing of the observations in this case and we may thus adopt 40 days as an approximate upper limit on this.

Although something of an academic exercise at this point, a daily ephemeris for the planet was then computed using ORBIT 3. The orbital elements from the first ORBIT 1 run (reduction no. 1 in Table IV) were used to generate the ephemeris which covered approximately the period of the observations (1970 Aug. 31 to 1971 Feb. 25). The first part is reproduced as the sample output of ORBIT 3 on pages 117 and 118.

TABLE IV

GAUSSIAN DETERMINATIONS OF THE ORBIT OF 16 PSYCHE

REDUCTION NO	OBSERVATIONS USED		ELEMENTS OBTAINED	
	Dates	Separation	Epoch 1970 Aug. $02^d.0$ ET	Equator and equinox 1950.0
1	1970 SEP 01 1970 SEP 16 1970 OCT 01	15^d 15^d	$e = 0.145019$ $a = 2.939948$ a.u. $M_o = 17^\circ\ 13'\ 56''$	$i = 3^\circ\ 05'\ 34''$ $\omega = 227^\circ\ 21'\ 25''$ $\Omega = 150^\circ\ 14'\ 57''$
2	1970 OCT 11 1970 OCT 26 1970 NOV 09	15^d 14^d	$e = 0.141740$ $a = 2.926010$ a.u. $M_o = 17^\circ\ 29'\ 47''$	$i = 3^\circ\ 05'\ 28''$ $\omega = 227^\circ\ 07'\ 22''$ $\Omega = 150^\circ\ 15'\ 50''$
3	1970 OCT 09 1970 NOV 05 1970 DEC 02	27^d 27^d	$e = 0.138287$ $a = 2.919775$ a.u. $M_o = 17^\circ\ 17'\ 42''$	$i = 3^\circ\ 05'\ 30''$ $\omega = 227^\circ\ 41'\ 01''$ $\Omega = 150^\circ\ 11'\ 25''$
4	1970 NOV 20 1971 JAN 04 1971 FEB 16	45^d 43^d	No result obtained	
5	1971 JAN 20 1971 FEB 01 1971 FEB 21	12^d 20^d	$e = 0.138000$ $a = 2.928492$ a.u. $M_o = 16^\circ\ 26'\ 48''$	$i = 3^\circ\ 05'\ 38''$ $\omega = 228^\circ\ 57'\ 14''$ $\Omega = 150^\circ\ 05'\ 43''$

The final stage in the reduction was the improvement of the orbit by means of ORBIT 4 using all the 25 observations in the series. Four sets of improved elements were obtained using different combinations of the observations and these are presented in Table V. Each set of elements was obtained using ORBIT 4 iteratively in the manner described at the end of Chapter 1.5 and so the elements represent the best-fitting orbit for the observations used to obtain them. The initial elements in all cases were those obtained from the first ORBIT 1 run and the residuals from the final elements were always of the same order and randomness as those quoted at the end of Chapter 1.5, rarely exceeding 1" in either coordinate.

Also included in Table V are two sets of published elements for comparison. These have been extracted from the "Ephemerides of Minor Planets", the elements designated PI first appearing in the edition for 1959 and those designated PII in the 1975 edition. Both these sets of elements were computed taking into account planetary perturbations and they refer to epochs of osculation of 1951 Dec. $20^{d}.0$ ET (set PI) and 1968 May $24^{d}.0$ ET (set PII) although in the table the mean anomalies are given for the epoch 1970 Aug. $02^{d}.0$ ET in order to facilitate comparison with the new elements.

The first two sets of new elements (I and II) were computed with the aim of assessing the accuracy of the ORBIT 4 program. They were arrived at by dividing the 25 observations into two groups, each group containing alternate observations in the series and thus representing essentially the same arc of the orbit, and obtaining improved orbital elements from each group. In order to equalize the numbers of observations in the groups one was omitted, that for 1971 Jan. 04 being selected as most suitable because its exclusion resulted in the two groups having a very similar distribution of observations over the arc. Clearly we should expect the elements obtained from these two groups to agree very closely and it will be seen

TABLE V

IMPROVED ORBITAL ELEMENTS OF 16 PSYCHE

SET NO	OBSERVATIONS USED	ELEMENTS Epoch 1970 Aug. $02^{d}.0$ ET	Equator and equinox 1950.0
I	Alternate observations 1970 Sep. 01 - 1971 Feb. 16 Total: 12 observations	$e = 0.139257$ $a = 2.920867$ a.u. $M_o = 17^\circ\ 23'\ 04''$	$i = 3^\circ\ 05'\ 30''$ $\omega = 227^\circ\ 30'\ 51''$ $\Omega = 150^\circ\ 10'\ 21''$
II	Alternate observations 1970 Sep. 03 - 1971 Feb. 21 Total: 12 observations	$e = 0.139223$ $a = 2.920810$ a.u. $M_o = 17^\circ\ 22'\ 49''$	$i = 3^\circ\ 05'\ 30''$ $\omega = 227^\circ\ 31'\ 00''$ $\Omega = 150^\circ\ 10'\ 39''$
III	All observations 1970 Sep. 01 - 1970 Oct. 26 Total: 13 observations	$e = 0.141061$ $a = 2.926488$ a.u. $M_o = 17^\circ\ 22'\ 01''$	$i = 3^\circ\ 05'\ 31''$ $\omega = 227^\circ\ 26'\ 07''$ $\Omega = 150^\circ\ 11'\ 02''$
IV	All observations 1970 Nov. 04 -1971 Feb. 21 Total: 12 observations	$e = 0.139221$ $a = 2.921026$ a.u. $M_o = 17^\circ\ 22'\ 52''$	$i = 3^\circ\ 05'\ 30''$ $\omega = 227^\circ\ 31'\ 47''$ $\Omega = 150^\circ\ 10'\ 01''$

PUBLISHED ELEMENTS

PI	Observations made at 5 oppositions 1950-1955	$e = 0.13530$ $a = 2.92282$ a.u. $M_o = 19^\circ\ 47'\ 06''$	$i = 3^\circ\ 05'\ 17''$ $\omega = 225^\circ\ 21'\ 50''$ $\Omega = 150^\circ\ 25'\ 26''$
PII	Observations made at 28 oppositions 1903-1970	$e = 0.139026$ $a = 2.919593$ a.u. $M_o = 17^\circ\ 12'\ 42''$	$i = 3^\circ\ 05'\ 31''$ $\omega = 227^\circ\ 44'\ 11''$ $\Omega = 150^\circ\ 10'\ 38''$

(In reckoning the alternate observations for sets I and II the observation of 1971 January 04 was omitted.)

from Table V that this is in fact the case, the differences between them being only in the fifth place of decimals and a few seconds of arc. We may thus assume that ORBIT 4 results in general have a reasonably high degree of reliability.

Elements III and IV were again obtained by splitting the observations into two groups, the division this time being into the first 13 and remaining 12 observations respectively. The elements thus obtained represent mean orbits for the periods 1970 September to October and 1970 November to 1971 February and may be compared in an attempt to examine changes in the orbit. In fact the differences between these two sets of elements are in general much larger than we should expect ($\Delta e = -0.0018$, $\Delta a = -0.0054$ a.u., $\Delta M_o = +51''$, $\Delta i = -1''$, $\Delta\omega = +5'\ 40''$, $\Delta\Omega = -1'\ 01''$). They are certainly too large to be accounted for by planetary perturbations and we speculate that the differences may be due in part to the unequal lengths of arc represented by the two sets of observations. In this connection we note that the elements for 1970 November to 1971 February (set IV) differ very little from those for the whole apparition (sets I and II). We also remark on the similarity of some of the values in set III with those of the earlier reductions in Table IV.

Whilst it is not within the scope of this work to draw any detailed conclusions about the long-term evolution of the orbit of 16 Psyche, it is nevertheless interesting to compare the mean elements obtained using the whole arc of the 1970-71 apparition (ie set I or set II) with the elements representing the orbit in 1951 (set PI). The differences between these elements are $\Delta e = +0.0040$, $\Delta a = -0.0020$ a.u., $\Delta M_o = -2^\circ\ 24'$, $\Delta i = +0'\ 13''$, $\Delta\omega = +2^\circ\ 09'$, $\Delta\Omega = -0^\circ\ 15'$. A calculation of the theoretical secular perturbations on the elements e, i, ω, Ω due to all the other planets (using Gauss's method for secular perturbations) gives values of $\Delta e = 0.0000$, $\Delta i = +0'\ 21''$, $\Delta\omega = +0^\circ\ 33'$, $\Delta\Omega = -0^\circ\ 16'$ as the changes in these elements over 19 years, from which we see that Δi and $\Delta\Omega$ agree fairly well with the observed values but Δe and $\Delta\omega$ do not. This disagreement could perhaps be due to the orbit of 16 Psyche being close to the 7:3 commensurability with Jupiter, which gives rise to effects not taken into account in the calculation of the perturbations. We may remark that some support is given to the validity of the large observed value of $\Delta\omega$ by the fact that the mean longitudes of the planet at epoch $(M_o + \omega + \Omega)$ obtained from the two sets of elements differ by only 30', of which 15' can be accounted for by the change in Ω.

Finally we consider the current published elements (PII in Table V). These represent the orbit at an epoch relatively close to that of the new elements and so we may compare sets I and II with PII. Once again we find that there is very satisfactory agreement and we thus conclude that the new elements constitute a fair representation of the orbit at the time of the observations, this emphasizing again the value of the two-body processes used to obtain them.

CONCLUSION

We conclude our account of the ORBIT library and its use by giving a brief outline of some possible future developments over and above those improvements already suggested in the text. As we have seen from Part 2 of the Thesis, the use of the ORBIT programs reduces the magnitude of the computational aspects of the orbit determination process to almost trivial proportions and we have thus accomplished in full the task upon which we originally set out. Nevertheless, there are areas where the convenience of the programs from the user's point of view could be increased still further.

These areas are mainly concerned with the form in which the data is required by the programs. For example, both ORBIT 1 and ORBIT 4 require the date of an observation to be provided as a Julian Date and also as a civil date expressed in years and decimals; a simple subprogram (or pair of subprograms) could be written to transform the conventional civil date in years, months and days to both these forms within the main program itself. Thus, only this conventional civil date would then need to be read in with each observation. Furthermore, if the inverse transformation of Julian Date to civil date were available in the form of a subroutine, values such as the time of perihelion passage could be output as a conventional civil date instead of (or as well as) the Julian Date as at present.

There are two sections of the ORBIT library where data is required to be punched onto cards which is already available in machine-readable form. The first of these is in the reduction of plate measurements which, as we have seen, has been exclusively carried out in this work with star positions from the SAO Star Catalogue. This catalogue is, in fact, available on magnetic tape and it would be a fairly straightforward matter to adapt ORBIT 2 so that the catalogue data could be read directly from the tape, only the measured data on the reference stars then needing to be punched. This would be particularly labour-saving if a large number of reductions was being carried out on each plate. Even further sophistication could perhaps be incorporated into this program by adapting it so that it used the values of the plate measurements themselves to recognise which reference stars had been chosen. This would require a knowledge of the SAO number of one or more of the stars and possibly of the plate scale and orientation, but it would avoid the very laborious task of identifying a large number of selected reference stars in the Star Catalogue. The development of this capability would, however, be far from straightforward.

The second set of data used in the ORBIT library which is available on magnetic tape is the geocentric data for the sun. The solar equatorial rectangular coordinates for each day together with first and second differences are available on tapes supplied by HM Nautical Almanac Office and these could, of course, be readily used by ORBIT 1, ORBIT 3 and ORBIT 4 with only minor modifications to the programs. They would be especially useful in the case of ORBIT 3 when lengthy ephemerides were being prepared.

An even more attractive idea for the acquisition of the solar coordinates (particularly in the case of ORBIT 1 and ORBIT 4 where they are required for non-tabular instants) is the actual computation of them within the program itself. HM Nautical Almanac Office has available a FORTRAN IV subroutine which produces the solar coordinates on the basis of Newcomb's theory for any specific instant, this having been extracted from the work of Mannino, Dall'Olio and D'Ascanio (1965). Other similar subroutines have more recently been developed elsewhere (for example at the US Naval Observatory) and, clearly, any one of these could be incorporated into ORBIT.

The results obtained at the end of Part 2 of this Thesis suggest that only comparatively low precision can be obtained in the determination of an orbit from three observations and that a larger number should be utilised wherever possible, ORBIT 4 being used to take into account the extra observations in the present case. This is very much in accordance with the current trend described at the outset. We have already mentioned the work of Herget (1965) in computing a preliminary orbit from more than the minimum number of observations and we would regard a process such as this (which does not differentiate between the "basic" observations and "extra" observations, but automatically takes all equally into account no matter how many are used) as the ideal preliminary orbit method.

It has been suggested by I G van Breda that a program such as ORBIT 4 could be modified to execute precisely this function by using the observations to correct initial elements which are merely guessed values and thence setting up an iterative process in much the same way as we described at the end of Chapter 1.5. In practice it is doubtful whether a sufficiently accurate guess could be made since anything other than a fairly close estimate of the elements would result in very large initial residuals with unpredictable consequences. A more realistic proposal might be to include in one and the same program a preliminary orbit method and an iterative improvement procedure, the initial orbit being deduced from three of the observations (say the first three) and the improvement taking into account all the observations. This proposal could very easily be applied to the ORBIT library and its development would form a worthwhile topic for further study. Such a program based on ORBIT 1 and ORBIT 4 would require a minimum of three observations to provide a solution and a minimum of four to provide an improved solution, but the maximum number which could be taken into account would be virtually unlimited.

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ANGLE2	Conversion of radians, etc. ($\geqslant 0$) to degrees/hours, mins., secs.	13
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ANGLE4	Alphameric sign applied to angle ($\geqslant 0$) in radians, etc.	14
ANGLE5	Angle in the range 0 to 2π radians from its sine and cosine.	14
ANGLE6	Angle ($\gtrless 0$) in radians, etc. separated into angle ($\geqslant 0$) and alphameric sign.	14
ANGLE7	Eccentric anomaly from true anomaly.	16
ANGLE8	Difference of two angles (in the range 0 to 2π radians) in the correct sense.	16
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