Helium II Film Transfer

A THESIS

presented by

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I hereby declare that this thesis has been composed by me, that the work of which it is a record has been carried out by me, and that it has not been accepted in any previous application for a Higher Degree. The work was conducted in the Physical Laboratories of the University of St. Andrews under the supervision of Professor J.F.Allen, F.R.S.

I certify that Christopher Cameron Matheson, B.Sc. has spent 9 terms as Research Student in the United College of St. Salvator and St. Leonard, that he has fulfilled the conditions of Ordinance Number 16 of the University Court of St. Andrews and that he is qualified to submit the accompanying thesis in application for the Degree of Doctor of Philosophy.

(Supervisor)

CAREER.

I matriculated in October 1958 in the United College of St. Salvator and St. Leonard in the University of St. Andrews and graduated as Bachelor of Science with First Class Honours in Natural Philosophy in June 1962. In October 1962 I was admitted by the Faculty of Science of St. Andrews University as a Research Student in the Department of Natural Philosophy in the same college under the supervision of Professor J.F. Allen. In June 1962 the Executive Committee of the Carnegie Trust for the Universities of Scotland awarded me a Carnegie Scholarship for the following three years.

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"... an unstable Victorian semi-detached house."

(Chapter 6)

PREFACE

Helium, the lightest of the noble gases, is the most difficult of all to liquefy. Despite this, since the first liquefaction of helium by Kamerlingh Onnes (1908) the properties of the liquid have been extensively investigated. The continued interest in the liquid after almost ~~three~~ six decades is due to its unique properties, the most striking of which is the superfluidity exhibited below 2.19°K. One of the most clear-cut examples of superfluid flow is that of the saturated helium film which is formed on all solid surfaces in contact with the liquid. Although fluid transfer by the film has been the subject of a great many experimental and theoretical scrutinies, many aspects of the phenomenon have remained enigmatic. In particular, the variability of the rate of superfluid transfer through the film under various experimental conditions has never been satisfactorily explained. The present study was undertaken to clarify this situation, and to provide information on which to base a hydrodynamic theory of the film. Part of the research has already been reported (Matheson and Tilley, 1965; Allen and Matheson, 1965).

Both a knowledge of the salient properties of liquid helium and an understanding of the various theories of superfluidity are necessary if the idiosyncracies of the helium film are to be resolved. It would seem appropriate, therefore, to review these particular facets of the helium problem so that the ensuing investigation of the film can be seen in its proper perspective.

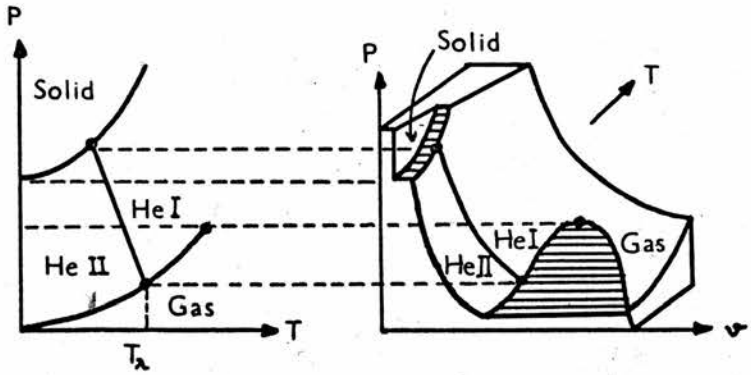

Figure 1.1.1 Schematic Diagram of the Phases of He^4.

CHAPTER 1.

Introduction.

1.1 Equilibrium Properties of Liquid Helium.

The first indication that the behaviour of liquid helium is unusual stemmed from early measurements of the density of the liquid. Kamerlingh Onnes (1911) reported that there was a maximum in the density function at a temperature of 2.19°K. Also at this temperature a discontinuity in the dielectric constant was found (Wolfke and Keesom, 1928). These results, taken together with specific heat measurements in the same temperature region, led Keesom and Wolfke (1928) to suggest that liquid helium undergoes some sort of modification at a temperature of 2.19°K. They put forward the idea of a phase transition which divides liquid helium into two separate entities. Although these authors were mistaken in classing the transformation of phase as first order, their nomenclature for the liquid phases of helium has remained; at temperatures above the phase transition the liquid is referred to as He I, while that below the transition is He II.

The technique of obtaining very low temperatures by adiabatic demagnetization has enabled the relation between the various phases of helium to be fully explored. The P-T diagram and the equation of state surface are shown in Figure 1.1.1. In addition to there being no triple point, the liquid has been found to exist even at temperatures below 0.1°K. Below about 1°K the slope of the melting-pressure curve tends to zero approximately as the eighth

power of the temperature, and extrapolation of the curve to the absolute zero indicates that the solid state can only be realised near the absolute zero by the application of pressures in excess of 25 atmospheres. Thus, by the Clausius-Clapeyron equation,

$$\left(\frac{dP}{dT}\right)_{m.p.c} = \frac{S_{liquid} - S_{solid}}{V_{liquid} - V_{solid}}, \qquad (1.1.1)$$

the entropy difference between solid and liquid vanishes at the absolute zero, as indeed it must by the Third Law of Thermodynamics. At the absolute zero the state of liquid helium is one of some kind of perfect order. Early theories of liquid helium relied broadly on one of two principles, either ordering of helium atoms in geometrical space, or ordering in momentum space. It should be noted, however, that theories involving static spatial structures (e.g. Frohlich, 1937) are now disregarded in favour of the condensation phenomenon of a fluid obeying Bose-Einstein statistics. The weakness of theories involving spatial order follows from the small mass of helium and the weakness of the helium-helium interaction. The resulting high zero-point motion of helium atoms (Simon, 1934) means that it is impossible to consider the atoms as localized at particular lattice sites.

In view of the behaviour of the phases of helium, it is not surprising that many other properties of the liquid are unusual. Along the saturated vapour pressure curve the specific heat becomes logarithmically infinite as the temperature of the liquid phase change is approached from either side (e.g. Fairbank, Buckingham and Kellers, 1958). From the shape of this curve the temperature

at which the specific heat anomaly occurs has been termed the lambda point. Since there is no latent heat associated with the lambda transition, both the entropy and density of the liquid are continuous over this temperature range. In addition, the variation of the density of liquid helium in the neighbourhood of the lambda point indicates that there is a discontinuity in the expansion coefficient. The phase transition He I -> He II is thus of the second order: first order derivatives of the Gibbs function, volume and entropy, suffer no discontinuity at the lambda point, while second order derivatives, specific heat at constant pressure and coefficient of expansion, are discontinuous.

1.2 Transport Properties of Liquid Helium.

The years 1936 - 1939 were probably the most rewarding period as far as clarification of the basic properties of liquid helium was concerned. Keesom and Miss Keesom (1936) and independently Rollin (1936) found a dramatic change in the heat conductivity of the liquid at the lambda point. Measurements by Allen, Peierls and Uddin (1937) revealed that the heat current was not proportional to the temperature gradient, but was rather a function of grad T (Keesom and Saris, 1940). It was also shown (Allen and Jones, 1938) that the heat conductivity increased as the characteristic size of the flow channel decreased. It was apparent from these experiments that the usual definition of a coefficient of thermal conductivity was meaningless when applied to He II. This was the first hint that the normally accepted differential equations describing the transport

of matter would not suffice for an understanding of the helium problem.

Early measurements of the thermal properties of the bulk liquid were complicated initially by the discovery by Allen and Jones (1938) that a temperature difference between two vessels connected by a thin capillary containing He II always resulted in a level difference between the vessels, the liquid in the hotter vessel standing at the greater height. The gravitational potential developed (thermomechanical or fountain effect) was dependent on both the heat input and the ambient temperature. The reverse effect, the mechanocaloric effect, has been observed by Daunt and Mendelssohn (1939c).

A further departure from the normal behaviour of liquids was demonstrated by the viscosity measurements carried out on liquid helium. While the viscosity of He I behaved in a classical manner, that of He II defied understanding, at least from the view point of classical hydrodynamics. Allen and Misener (1939), studying the isothermal flow of He II through very narrow channels, observed that He II could flow with great ease through the narrowest of channels which were effectively blocked to the liquid at temperatures above the lambda point. The nature of the flow of He II was neither laminar nor turbulent in the accepted sense; indeed if the channel were made small enough (say 10^{-5}cm in width) the flow was independent of the driving pressure. Clearly this type of flow - superfluidity - does not fit within the framework of the usual equations of macroscopic phyaics. It only proved possible from

the experiments of Allen and Misener to set an upper limit on the viscosity of He II of the order of 10^{-5} μ poise. On the other hand, a disc suspended in He II and executing an oscillatory motion suffered severe damping (Keesom and MacWood, 1938), indicative of a viscosity at least a million times greater than that observed by Allen and Misener.

1.3 The Surface Film; Early Observations

The first indication of surface film formation in liquid helium below the lambda point arose during the experiments of Kamerlingh Onnes (1922). Working with two concentric Dewar vessels, he commented that any difference of liquid levels always disappeared until the levels lay in the same horizontal plane. Although he proposed that this adjustment of levels could be explained by a distillation process, the rate of decay of the level difference was too great for it to be a normal distillation process.

Kürti, Rollin and Simon (1936) suggested that their observations of an anomalously high heat flow into a vessel containing He II could be accounted for by the existence of a liquid layer on the walls of the tube connecting the experimental chamber to warmer regions in the cryostat. This idea was confirmed by Rollin and Simon (1939) in further experiments on the anomalous evaporation from He II chambers. They deduced that a liquid film is formed on all surfaces in contact with bulk He II, and that the nature of the surface is unimportant. Since neither the mechanism of formation of the film, nor indeed the reason for its existence was

clear, they took the film as a given phenomenon and designed further experiments to discover its properties. They put forward two possible mechanisms whereby heat could be transported by such a film. Either the film was stationary and possessed a high heat conductivity, of the same order as that in the bulk liquid, or the film moved steadily from regions of low to high temperature, evaporating in the latter region. It then followed from the invariance of the evaporation rate when the efflux of gas from the He II chamber was prevented, that the only tenable mechanism was that of a 'creeping' film of helium. Rollin and Simon also noted that the rate of transfer by the film was limited by the narrowest constriction in its flow path.

Although transfer rates of liquid by the film were not explicitely stated by these authors, it has been pointed out (Fairbank and Lane, 1949) that an estimate can be made from their published data. At the temperature concerned (1.6°K) the flow rate (8×10^{-5} cm^2 sec^{-1}) agrees with that of later work.

Kikoin and Laserev (1938, 1939) confirmed the presence of the helium film on the walls of a tube in contact with the bulk liquid. They also noted the ease with which the film can be destroyed by incident radiation which probably explains why the film remained unobserved for so long.

The results of the first systematic attempt to study the helium film (Daunt and Mendelssohn, 1939a, b) also supported Rollin and Simon's conclusion that liquid is transferred by the film in a thick adsorbed layer which moves towards regions of higher tempera-

ture. Daunt and Mendelssohn were also able to show that the film did not have an anomalously high heat conductivity. Moreover, they observed that a thermomechanical pressure head could be developed between two vessels connected only by the helium film. This argues in favour of the liquid in the film being of the same composition, on the average, as that in narrow channels.

The occurrence of the helium film, perhaps more than any other single property of the liquid, serves to emphasize the non-classical nature of transport phenomena below the lambda point.

1.4 Theories of Liquid Helium.

(a) Bose-Einstein Condensation

Having shown from energetic considerations that a static spatial model of liquid helium was not possible, London (1938a, b) revived the idea of the Bose-Einstein condensation phenomenon for an ideal gas in the form first mentioned by Einstein (1924). London treated liquid helium as a gaslike assembly with a density appropriate to that of the liquid; moreover, since the total spin of the He^4 atom is integral, he regarded a non-interacting assembly of He^4 atoms as obeying Bose-Einstein statistics. London then suggested that the very close agreement between the experimental lambda-point of liquid helium and the condensation temperature of an ideal gas with the mass and molar volume of the helium atom was more than fortuitious. Although liquid helium is far from being an ideal gas, the large specific volume and small intermolecular forces led London to believe in at least a qualitative description

of the lambea phenomenon in terms of Bose-Einstein condensation. In this picture of liquiid helium as a degenerate Bose gas, an increasing fraction of atoms will condense into the state of smallest momentum as the temperature is reduced below the lambda point. Zero entropy (perfect order) is achieved at the absolute zero because all particles are in the ground state, In the uncondensed phase the remaining atoms are statistically distributed over the excited states and possess a normal momentum. Hence the viscosity of He II is due entirely to atoms in excited states.

As might be expected, there are difficulties in reconciling the ideal gas model with the observed characteristics of liquid helium. For instance, the phase transition of an ideal gas at the condensation temperature is third order, with no discontinuity of entropy or specific heat. Moreover, although the specific heat of He^4 between about 0.6°K and 1.4°K falls off as the sixth power of the temperature, the London model predicts a variation of C_v as $T^{3/2}$, Dingle (1952) has pointed out that to remove these discrepancies involves consideration of the interatomic forces which are disregarded in the ideal Bose-Einstein gas.

(b) The Two Fluid Model.

The phenomenological two-fluid model proposed by Tisza (1938) is based on the notion of a Bose-Einstein gas with an entropy-free condensate. It is useful to list the main assumptions on which the model is based:

(i) Liquid helium below the lambda point is a mixture of two coexisting fluids; the superfluid and the normal fluid. Two

separate velocity fields are necessary to describe the motion of the two interpenetrating fluids. The relative proportions of the two fluids depends only on the temperature; at the absolute zero all the liquid is superfluid, while at the lambda point the fluid is entirely normal.

(ii) From the equivalence of the superfluid with the condensed phase of a Bose-Einstein gas, it follows that the superfluid can carry no entropy. In addition, the superfluid has no viscosity and flows without friction, provided the velocity is less than a certain critical velocity.

(iii) The normal fluid alone is responsible for the viscosity of the liquid. The heat content of He II rests entirely in the normal fluid.

(c) Excitations in Liquid Helium.

Breaking away from the ideal gas approach, Landau (1941, 1944, 1947) put the two fluid model on an entirely different basis. Initially, without referring explicitely either to interatomic forces or the nature of the statistics obeyed by the He^4 atom, he endeavoured to lay a foundation on which a molecular theory of liquid helium could be built. Considering the liquid as a system of interacting particles he showed that direct quantization of the hydrodynamic variables led to stationary states with zero values of $\nabla \times \underline{v}$ (states of potential motion). In addition there are also states for which $\nabla \times \underline{v}$ attained non-zero values (vortex motions). The impossibility of continuous transitions between these two states followed by anaolgy with the commutation rules of

the quantum mechanical angular momentum. The excited state of liquid helium, therefore, consisted of two localized sets of excitations, separated by an energy gap.

On this picture of the liquid as a continuum, the quanta associated with the irrotational motion of the liquid, longitudinal sound waves, are accoustic phonons, while the elementary excitations of the rotational modes are referred to by Landau as rotons. The enerby spectrum for liquid helium can be described by the equation:

$$\varepsilon = cp, \quad p \ll p_0,$$
$$\varepsilon(\mathbf{p}) = \Delta + \frac{(p-p_0)^2}{2\mu}, \quad p \sim p_0, \tag{1.4.1}$$

where $\varepsilon(\mathbf{p})$ is the energy of an excitation of momentum $\mathbf{p}$, $p = |\mathbf{p}|$, c is the accoustic velocity in liquid helium, Δ is the minimum energy of a roton occurring at momentum p_0 and μ is its effective mass. The constants c, Δ, p_0 and μ are determined empirically to obtain agreement with the thermodynamic properties of liquid helium. Although the phonon and roton parts of the spectrum form a continuous curve, the final spectrum can be treated as two separate branches: the phonon spectrum starting from the ground state with slope c, and the roton branch situated in the region of the minimum of the curve around momentum p_0.

In the Landau theory the two fluids lose their identity as two different states in which the liquid can exist; to retain the two fluid concept it is necessary to identify the normal fluid with

the quasi-particles and the superfluid with the background medium through which the excitations move. The superfluid is thus an ideal fluid which can flow frictionlessly in a purely potential manner. The density of the normal fluid becomes the mass density of the excitations, and the superfluid density reduces to a mathematically convenient parameter. The Landau theory is essentially a low temperature theory which gives no satisfactory explanation of the lambda phenomenon. Nevertheless, many of the observed equilibrium and transport properties of liquid helium can be explained by it.

(d) Feynman's Theory.

There have been many attempts to derive the energy spectrum of liquid helium from first principles. A collective coordinate approach by Bogolyubov (1947) and a hard-sphere Bose gas model for the liquid (Brueckner and Sawada, 1957) have been among the most successful. But by far the most basic approach was that by Feynman (1955). Realising the impossibility of solving the Schrödinger equation for all the N atoms in the liquid, he made inspired guesses at what the solutions should be. The crux of his argument lies in the symmetry property of the wave functions of a system of Bose-Einstein particles undergoing various kinds of motion. For minimum energy the curvature of the wave function must be least. Hence the ground state configuration of the liquid is that of uniform density. Excited states result if this uniform distribution is disturbed. Such a perturbation is a compression wave or phonon with state energy cp which can be very small,

provided the wave length is long enough. By the symmetry of atoms obeying Bose-Einstein statistics, all large scale changes of configuration in the liquid can be synthesized by movements over distances of less than the interatomic spacing. Associated with large scale motion, therefore, is a high gradient of the wave function and hence a high energy. In this way Feynman argued that phonons are the lowest energy excitations and that all higher energy excitations (e.g. rotons) are the result of configurational changes unaccompanied by density variations.

On quantum-mechanical grounds there should be a minimum in the excitation curve at the order of one half the interatomic spacing, since the least possible curvature of the roton wave function occurs in this range. It follows also that the energy should be linear with p for small p , The Feynman energy spectrum qualitatively resembles that of Landau, although the energy gap is roughly twice that assumed by Landau from thermal data. This was remedied in later work (Feynman and Cohen, 1956) by a slight modification of the roton wave function to account for the return flow of atoms in the large scale stirring process.

The significance of the shape of the energy spectrum and the size of the energy gap become apparent when the flow of helium II is considered. It is because a tangent of zero gradient from the origin can never be drawn to the $\varepsilon = \varepsilon(p)$ curve that superfluidity exists. The implications of this will be discussed in the following chapter.

CHAPTER 2.

The Motion of Helium II.

2.1 Irrotationality of The Superfluid Velocity Field.

The movement of a classical liquid is completely described by two scalar fields and one vector field, evaluated at any point in the liquid. Knowledge of the state of He II, however, requires specification of a second vector field, usually taken, in accordance with the two fluid model, to be the superfluid velocity ($\underline{v}_s$). The properties of the superfluid velocity field will be briefly considered; discussion of the breakdown of superfluidity will be deferred until Section 2.4 of this chapter.

Landau's quantum hydrodynamic theory (1941) was the first attempt to characterize superfluid flow below the critical velocity by postulating that the motion of the superfluid was curl-free, i.e.

$$\nabla \times \underline{v}_s = 0. \qquad (2.1.1)$$

This restriction was demonstrated in another way by London (1954) who showed that absence of vortex motion in the superfluid follows if motion of the ground state is accompanied by adiabatic changes of the wave function.

Zilsel (1950) endeavoured to show that denial of vorticity to the superfluid is not necessarily an independent physical requirement of the motion, but merely a subsidiary condition on the velocity.

Experimentally, such potential motion would result in an apparent absence of viscosity in the superfluid; in the equation of motion of the superfluid the classically expected viscosity term $\eta \nabla \times \nabla \times \underline{v}_s$ would not appear. Thus irrotationality of the superfluid velocity field rather than absence of interactions was considered in interpreting early experiments. For example, the moment of inertia of a stack of discs oscillating in He II (Andronikashvili, 1946) would be unaffected by the presence of superfluid.

If the superfluid can undergo only pure potential motion, the shape of the free surface of rotating He II should be different from that of an ordinary viscous liquid. However, Osborne (1950) observed that the profile of liquid helium in a rotating vessel below the lambda point was indistinguishable from the classical paraboloid of revolution. It is thus inferred that the superfluid is entrained into rotation in some way. Meservey (1964) has extended Osborne's measurements down to rotational speeds of 0.29 rad/sec with identical results, thus refuting the idea that the observations are due to the critical velocity being exceeded. Moreover, Reppy and Lane (1961) found that the angular momentum collected when a rotating bucket of He II is suddenly brought to rest is that of the whole mass of fluid rotating classically.

2.2 The Quantization of Circulation.

A deeper insight into the flow properties of bulk superfluid stemmed from the work of Feynman (1955) who extended his method of discussing excited states in terms of suitable wave functions in

configuration space. He argued that the state of motion of the superfluid could be represented by wave functions of the type

$$\psi_{motion} = \Phi \exp\left[\frac{im}{\hbar} \underline{v} \cdot \sum_j \underline{r}_j\right], \qquad (2.2.1)$$

where Φ is the ground state wave function and $\underline{v}$ is the uniform velocity of flow. For slowly varying velocity fields the phase of the wave function can be written as $\sum_j S(\underline{r}_j)$, where $S(\underline{r})$ is a function that varies slowly with $\underline{r}$. The velocity of flow at any point is then

$$\underline{v} = \frac{\hbar}{m} \text{grad } S. \qquad (2.2.2)$$

Identifying this velocity field with that of the superfluid implies that in a simply connected region the superfluid circulation is zero.

It has been seen, however, that the rotational behaviour of helium requires a non-zero circulation. Feynman pointed out that this arises from Bose symmetry requirements on the wave function. In any multiply connected region the wave function must be single valued, and the phase S for each atom must change by an integral multiple of 2π around a closed path. Hence

$$\oint \text{grad } S \cdot d\underline{s} = 2\pi n, \qquad (2.2.3)$$

n being an integer, or

$$\chi = \oint \underline{v} \cdot d\underline{s} = 2\pi n\hbar/m, \qquad (2.2.4)$$

where χ is the circulation. Feynman therefore concluded that the circulation of the superfluid velocity field is quantized.

In the axially symmetric case the velocity is given by

$$v = n \frac{\hbar}{m} \cdot \frac{1}{r}, \qquad (2.2.5)$$

which is the velocity distribution of a vortex line in classical hydrodynamics. Thus, although the superfluid circulation in a multiply connected volume is quantized, zero curl of velocity is still preserved everywhere except on the line singularities of velocity. Such singularities, moving with the fluid velocity and having circulation not less than h/m, are quantized vortex lines.

The actual conditions in rotating helium are likely to be represented by the model of lowest energy. For an ordinary liquid, rotating at angular velocity Ω, minimization of the free energy shows that the state of lowest energy is that of solid body rotation with a velocity at every point of $\underline{v} = \Omega \underline{r}$ and $\text{curl}\,\underline{v} = 2\Omega$ everywhere. A similar calculation for liquid helium shows that the minimum energy state, compatible with zero curl of velocity, corresponds to a uniform array of minimum strength vortex lines all parallel to the axis of rotation. The lines are distributed such that the average circulation per unit area is equal to the average curl of the velocity in true solid body rotation.

If the region under consideration encloses N unit vortices,

$$\chi = \oint \underline{v} \cdot d\underline{s} = N \cdot \frac{2\pi\hbar}{m} = [\text{curl}\,\underline{v}]_{\text{average}} = 2\Omega. \qquad (2.2.6)$$

Hence, in a rotating vessel of He II, there is a spatially uniform line density of

$$A = \frac{2m\Omega}{h} = 2{\cdot}1 \times 10^3 \Omega \text{ Lines/cm}^2. \qquad (2.2.7)$$

Macroscopically, the angular velocity distribution can be considered little different from uniform, and the free surface should appear to be parabolic as was observed by Osborne.

It must be pointed out that the quantization condition (2.2.4) is a means whereby the superfluid can rotate without dissipation of energy. Feynman realized that quantum vortices played a different and completely separate role in the breakdown of superfluidity.

2.3 Experimental evidence for quantized vortices.

Hall and Vinen (1956a, b) observed that the collective effect of a large number of quanta of circulation is such as to introduce an anisotropy into the excess attenuation of second sound depending on the relative orientations of the propagation of second sound and the axis of rotation. Assuming that the strong velocity fields surrounding the vortex lines act as scattering centres for thermal excitations, they interpreted their results in terms of a frictional force between the normal and superfluid components of He II.

Additional evidence for an array of vortices in the liquid was provided by Hall (1958) who performed an Andronikashvili pile of discs experiment in a rotating vessel. The variation of the period of oscillation of the disc system could be explained by the

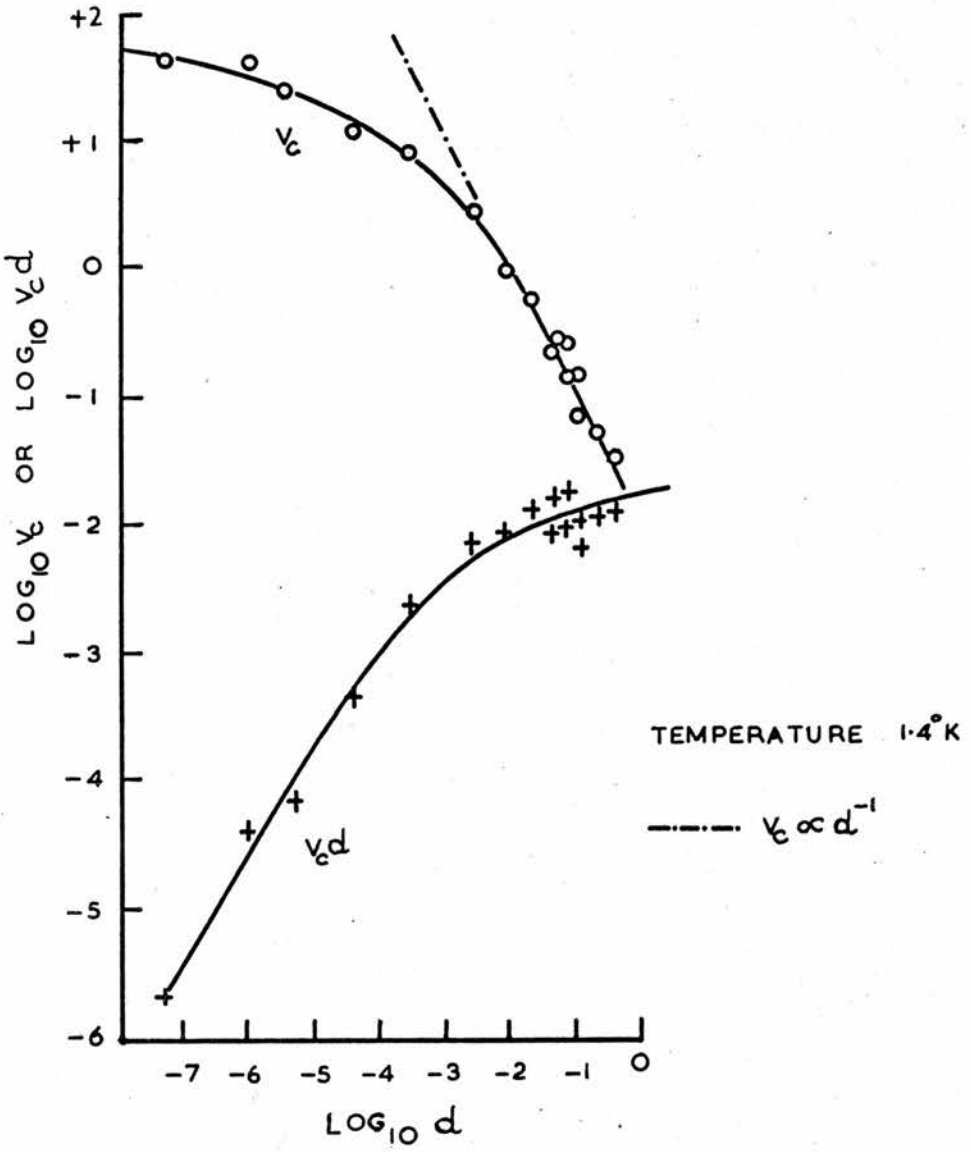

Figure 2.4.1 Observed Critical Velocity and Critical Transfer Rate as a Function of d.

presence of circularly polarized transverse waves on the vortex lines attached to the lower disc.

Isolation of discrete vortex lines was first accomplished by Vinen (1961). The superfluid circulation round a wire situated on the axis of rotation of a volume of helium was obtained by measuring the rate of precession of the plane of transverse vibration of the wire. It appears from Vinen's experiment that the most preferred value for the circulation is h/m and that this is never exceeded. In addition he showed that such circulation should persist indefinitely following cessation of rotation.

2.4 Hydrodynamic Stability of Superfluid Flow.

At velocities greater than a certain critical velocity (v_c) the flow of the superfluid component of liquid helium loses its frictionless character and becomes irreversible. In the helium film the transfer is thought to occur at the critical velocity. In view of this, it is of interest to consider the origin and behaviour of the instability of superflow.

The concept of energy dissipation in the superfluid has frequently been invoked to explain diversified flow phenomena. Atkins (1959, p200) has collated measurements of critical velocity from a wide range of experiments. Figure 2.4.1 shows the observed behaviour of the critical velocity at 1.4°K in a logarithmic plot of v_c against the characteristic length d involved in a particular experiment. Increasing from less than 3×10^{-2} cm sec^{-1}, v_c ranges over three orders of magnitude as the characteristic dimension of the flow channel decreases from several mm to tens of

Angstrom units. The rate of increase of v_c becomes less pronounced for $d < 10^{-3}$cm. It has been suggested by Atkins (1959, p201) that the critical velocity may assume a constant value for $d \leq 10^{-6}$cm. This view is supported by recent work* on superfluid flow in narrow wire-filled channels which indicates the extreme insensitivity of the critical velocity to the characteristic dimensions when the latter is of the order of 300 Å.

It remains to establish the reason for the breakdown of superfluidity at the critical velocity. Landau (1941) argued that the only way in which energy could be lost by the superfluid is by production of thermal excitations. Loss of momentum in any other manner would violate the restriction $\text{curl}\, v_s = 0$. It follows from conservation of energy and momentum that creation of an excitation in the superfluid is only possible if

$$\left|\frac{\varepsilon(\mathbf{p})}{\mathbf{p}}\right| \geq v_s . \qquad (2.4.1)$$

Thus superfluid flow is frictionless only below a velocity given by

$$v_c = \left|\frac{\varepsilon(\mathbf{p})}{\mathbf{p}}\right|_{min} , \qquad (2.4.2)$$

where $\left|\frac{\varepsilon(\mathbf{p})}{\mathbf{p}}\right|_{min}$ is the minimum value of the ratio of energy to momentum (or impulse) for any value of $\mathbf{p}$.

Using the criterion, the velocity required to create a phonon or a roton is 2.39×10^4cm sec^{-1} or 6.0×10^3cm sec^{-1} respectively. Since experimental critical velocities are never greater than 50 cm sec^{-1} (and usually much less) it is clear that neither phonons nor

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Footnote*. D. J. Watmough, to be published.

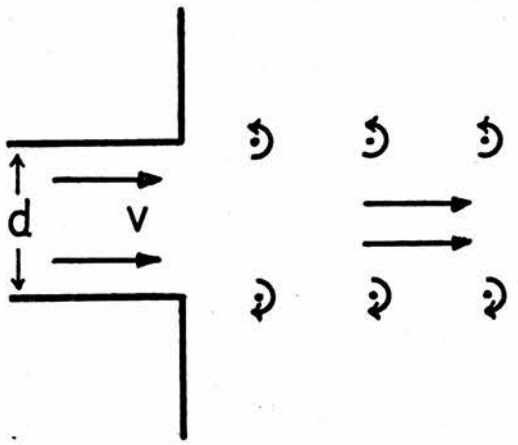


Figure 2.4.2 Feynman's Visualization of the Creation of Vorticity at an Orifice.

rotons play a role in the onset of resistance to superfluid flow.

Believing Landau's explanation was essentially correct, Ginsburg (1949) commented that low critical velocities would result if other excitations with smaller values of $\left|\frac{\varepsilon(p)}{p}\right|_{min}$ could be found.

Accordingly, Feynman pictured quantized vortex lines as a class of excitation to which the Landau criterion (2.4.2) could be applied. He put forward a theory of critical velocity in narrow channels. Initially he did not use the Landau criterion, but considered the continuous production of a Karman street of ring vortices at the exit of the superfluid flow channel to be due to the fierce acceleration of the irrotational flow at the corner of the orifice. The flow pattern of the emerging superfluid is shown in Figure 2.4.2. Equating the kinetic energy available in the fluid stream to the energy required for the creation of vortices leads to a minimum velocity for this type of excitation of

$$v_c = \frac{\hbar}{md} \ln\left(\frac{d}{a}\right), \qquad (2.4.3)$$

where a is the vortex core size.

A more general argument, based on the Landau criterion, is given by Atkins (1959, p116). The energy of a vortex ring of radius R in bulk He II is

$$\varepsilon = 2\pi^2 R \rho_s \left(\frac{\hbar}{m}\right)^2 \ln\left(\frac{R}{a}\right). \qquad (2.4.4)$$

The impulse perpendicular to the plane of the ring is

$$p = 2\pi^2 R^2 \rho_s \frac{\hbar}{m}. \qquad (2.4.5)$$

Thus the critical velocity is

$$v_c = \left|\frac{\varepsilon}{p}\right| = \frac{\hbar}{m}\cdot\frac{2}{d}\cdot\ln\left(\frac{d}{2a}\right), \qquad (2.4.6)$$

since the maximum attainable value of R is $d/2$.

The velocity of the onset of dissipation predicted by the vorticity model agrees well with experiment for $d > 10^{-3}$cm. However, the magnitude of the critical velocity is an order of magnitude too high when calculated for characteristic dimensions of the order of film thickness. More sophisticated analyses based on vortex creation (Peshkov, 1961 : Fineman and Chase, 1963) have failed to resolve this discrepancy. It has been suggested (Vinen, 1957: Townsend, 1963) that the concept of vorticity may be irrelevant to the problem of critical velocities, at least in very narrow channels. The reasons for the breakdown of the superfluid regime in the film are discussed in Chapter 3.

2.5 The Hydrodynamic Equations for Helium II.

Starting from the two fluid concept developed by London, Tisza and Landau, current hydrodynamic theories of liquid helium are based on a continuum approach to the behaviour of an assembly of atoms. The hydrodynamic equations will be given in a form consistent with later discussion on the helium film. The only frictional term introduced will be a viscosity term appearing in the equation of motion for the normal fluid. Mean free path effects are ignored. The equations, therefore, do not apply to velocities greater than v_c, when additional dissipative terms,

such as Gorter-Mellink mutual friction (1949), must be taken into account.

In the limit of small velocities (less than the velocity of sound) and small temperature differences (less than the absolute temperature), compressibility and thermal expansion of the liquid can be neglected. The macroscopic mass flow of the liquid is given by the hydrodynamic current density

$$\underline{j} = \rho_s \underline{v}_s + \rho_n \underline{v}_n , \tag{2.5.1}$$

where $\underline{v}_n$ is the normal fluid velocity.

The total density of He II is the sum of the densities of the separate components,

$$\rho = \rho_n + \rho_s . \tag{2.5.2}$$

Conservation of mass is satisfied if

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \underline{j} = 0 . \tag{2.5.3}$$

In addition, entropy must be conserved. Hence

$$\frac{\partial (\rho S)}{\partial t} + \operatorname{div} (\rho S \underline{v}_n) = 0 , \tag{2.5.4}$$

since no entropy is carried by the superfluid.

Conservation of momentum for each fluid is expressed by the equations

$$\rho_s \frac{\partial \underline{v}_s}{\partial t} + \rho_s \underline{v}_s \cdot \nabla \underline{v}_s = - \frac{\rho_s}{\rho} \nabla P + \rho_s S \nabla T \tag{2.5.5}$$

and

$$\rho_n \frac{\partial \underline{v}_n}{\partial t} + \rho_n \underline{v}_n \cdot \nabla \underline{v}_n = -\frac{\rho_n}{\rho} \nabla P - \rho_n S \nabla T + \eta_n \left[\nabla^2 \underline{v}_n + \tfrac{1}{3} \nabla . \nabla \underline{v}_n \right], \quad (2.5.6)$$

where P is the pressure, and η_n is the viscosity associated with the normal component. For steady flow of the superfluid, the equation (2.5.6) reduces to

$$\nabla P = \rho S \nabla T, \quad (2.5.7)$$

which is the equation first derived by London (1938c; 1954, p72) for the fountain pressure in very narrow capillaries. Thus the terms in grad T in the momentum equations express the ability of the superfluid and the normal fluid to move under a temperature gradient. Apart from these terms accounting for thermal effects, the momentum equations are similar to the equations of classical hydrodynamics; in particular, the equation for normal fluid motion can be formally identified with the Navier-Stokes equation for a liquid of density ρ_n and viscosity η_n.

It is seen, therefore, that the motion of He II, at least when $\underline{v}_s$ and $\underline{v}_n$ are not too large, can be resolved into two separate motions each describable by classical hydrodynamics. In addition to the hydrodynamic equations, boundary conditions must be impressed on $\underline{v}_s$ and $\underline{v}_n$ for a complete solution of any problem. In the analysis of normal fluid motion in thick helium films in Appendix A it will be assumed that the behaviour of the normal fluid is governed by the solution of equation (2.5.6) with the usual classical boundary conditions on $\underline{v}_n$.

CHAPTER 3.

The Properties of the Saturated Helium Film.

3.1 Definition of a 'Normal' Helium Film.

The work to be described in this thesis shows that the properties of a helium film can depend on the way in which the film has been formed. It is desirable, therefore, in any critical review of the properties of the film to state explicitely the mode of formation of the film in the case under consideration. In many cases this is impossible, since little importance has hitherto been attached to the pre-history of the film. However, unless otherwise stated, this chapter will be concerned with the film formed by superfluid creep over a previously unwetted substrate. This film we define as the normal film.

3.2 Measurements of Film Thickness.

Kikoin and Lasarev (1939) and, independently, Daunt and Mendelssohn (1939b) made the first assessment of film thickness. Both experiments consisted of evaporating the film from a metal surface of known area and measuring the volume evaporated. In this way an average film thickness was obtained, assuming that the density in the film was identical with that of the bulk liquid. Daunt and Mendelssohn found little variation in film thickness between 2.1°K and 1.5°K, the mean thickness being 3.5×10^{-6}cm. Above the lambda point the thickness was less than 10^{-7}cm. At an unstated temperature, Kikoin and Lasarev estimated the thickness to be between 2 and 3×10^{-6}cm.

Table 3.2.1

Author	$k \times 10^{-6}$	z	Temperature (°K)
Bowers	11.8	2	Independent
Ham and Jackson	3.15	2.3	1.32
	2.96	2.6	2.05
Atkins	1.5	7.1	1.47
	2.2	7.1	2.0
Picus	2.5	1 ; 2	Approx. independent

Bowers (1953) also measured the average thickness of the film. A microbalance was used to determine the effective increase in weight of an aluminium foil as it was covered by the film. Most theories of the film show that the thickness (d) of the film at height H above the bulk liquid level can be expressed as

$$d = K/H^{1/n}, \qquad (3.2.1)$$

where K and n are constants. Assuming a profile of this general form, Bowers obtained values of K and n by expressing the weight of the film as an integral of film thickness over the height of the specimen. His values are shown in Table 3.2.1. Below the lambda point the thickness was independent of temperature.

The most detailed investigation of film thickness has been made by Jackson and his students using an optical method (Burge and Jackson, 1951: Jackson and Henshaw, 1953: Ham and Jackson, 1954, 1957). When plane polarized light is reflected from a bare polished metal surface the reflected light is elliptically polarized. If the substrate is then covered by a helium film the resulting change in ellipticity and orientation of the ellipse can be related to film thickness. The method relies on the assumption that the refractive index of the film is identical with that of the bulk liquid. The technique can also be used to measure the rate of extension of the film over a dry substrate (Jackson and Henshaw, 1950). Only the later measurements of Ham and Jackson will be reported here, as earlier results were found to be incorrect owing to birefringent effects in the glass Dewars. Typical values of K and n are shown in Table 3.2.1. The thickness decreased only slightly with falling

temperature. The apparent absence of the film above the lambda point which had hitherto been noted (e.g. Burge and Jackson, 1951) was shown to be consequent upon the difficulty of replenishing a Helium I film from the bulk liquid. Apart from this, Ham and Jackson showed that the film thickness does not alter radically on passing through the lambda point. Since theories of the film based on van der Waals attractive forces do not predict a discontinuity of thickness at the lambda point, this observation removes one of the main obstacles to such theories.

The helium film in motion has been studied by Atkins (1950a). When the liquid level within a capillary approaches the level of the outer bath as a result of film flow, the inner meniscus oscillates about the equilibrium position. If simplifying assumptions regarding the hydrodynamics of the flow are made, the period of oscillation can be directly related to film thickness. The variation of the period with film height enables the constants in the general profile equation to be estimated as shown in Table 3.2.1. The value of z is much greater than that of other methods, but appears to diminish at film heights below 1 cm. The validity of the assumptions used in deriving the period of oscillation of the film must be closely examined when comparing Atkins' results with other thickness measurements. A critical appraisal, however, is difficult in view of the scant information which exists on the nature of the flowing film.

A further study of film oscillations at low level differences by Picus (1954) supported Atkins' estimates of k, but showed further

that k increases when the film is in motion. Picus also found that z was dependent on the state of motion of the film, the static film having a greater value of z than the moving film. Picus concluded that the profile of the moving film might be different from that of the static film.

Gribbon and Jackson (1963) have also investigated the effect of motion on the thickness of the film. Using an optical method based on that of Ham and Jackson (1957), they found that the thickness of the moving film was of the order of 5% less than that of the stationary film at a temperature of 1.68°K and a height of 1 cm.

On the other hand, a 20% increase in thickness over the static value was observed by Burge and Jackson (1951). In their experiment, however, the flow was initiated by a thermal gradient generated by a small heater at one end of the film. It is probable that the observed change of profile with motion is due to a fountain effect.

3.3 Theories of Formation of the Film.

Frenkel (1940) and Schiff (1941) showed that the existence of the helium film follows simply from the theory of interatomic forces. They considered the van der Waals attraction between helium atoms and the wall, and between the helium atoms themselves. The potential energy of attraction between any two atoms varies as the inverse sixth power of their separation. Integrating over the semi-infinite substrate involved, leads to the van der Waals contribution to the potential energy varying as the inverse third power of the distance (x) from the wall. The total potential

energy of an atom in the film at a height y above the bulk liquid level is therefore

$$\phi = mgy - \frac{m\alpha}{x^3}, \qquad (3.3.1)$$

where m is the mass of the helium atom and α is a positive constant specifying the strength of the adhesive force. Minimization of ϕ leads to the equation of the free surface of the film

$$d = \kappa/H^{1/3}, \qquad (3.3.2)$$

where d is the film thickness at height H and $\kappa = (\alpha/g)^{1/3}$. Thus, a theory based solely on van der Waals attractive forces between atoms leads to a value of 3 for z in the general profile equation (3.2.1). κ-values for various substrates have been estimated by Schiff. For non-metals, κ increases from 2.2 (rock salt) to 4 (glass), while for metals κ varies between 4.3 (copper) and 4.7 (silver).

Considering the helium film as an adsorbed layer close to saturation, Rice and Widom (1953), Atkins (1954) and Meyer (1955a) have used adsorption thermodynamics to calculate the film profile. The Schiff-Frenkel result obtains and, in addition, it is shown that κ is independent of temperature.

McCrum and Eisenstein (1955) suggested that a van der Waals type of film theory can usefully be extended if assumptions concerning the nature of the substrate are made. Contamination of the substrate by a polar impurity will affect the van der Waals

interaction. The potential energy (3.3.1) from which the film profile is derived is modified by including an electrostatic energy term proportional to the product of the local electric field and the polarisability of helium.

Adopting a continuum approach to the liquid led Atkins (1954) to consider the implications of the variation of hydrostatic pressure within the film. The pressure (P) within the potential field (3.3.1) is

$$P = P_0 + \rho\left(\frac{\alpha}{x^3} - gy\right), \qquad (3.3.3)$$

where P_0 is the saturated vapour pressure. Since the pressure at the free surface of the film is P_0, equation (3.3.2) obtains for the film profile. The pressure within the film can therefore be rewritten as

$$P = P_0 + \rho\alpha\left(\frac{1}{x^3} - \frac{1}{d^3}\right). \qquad (3.3.4)$$

If P is equated to the solidification pressure for liquid helium it follows that for $x \leq 7 \times 10^{-8}$ cm the film is in the solid state. In addition, at temperatures above 1.75°K the existence of a layer of helium I within the film can be inferred from the pressure distribution (3.3.4) together with the λ-curve shown in figure 1.1.1. However, the decrease in channel available for superflow due to these modifications need only be considered for temperatures within a few millidegrees of the lambda point.

Bijl, de Boer and Michels (1941) developed a quantum mechanical theory of the film phenomenon. Due to the small thickness of the

film, zero point energy might be expected to dominate interatomic forces. For an ideal Bose-Einstein gas below its condensation temperature the zero point energy contribution to the total energy of the film is $h^2/8md^2$ per atom. If the sum of this energy and the gravitational energy is minimized with respect to thickness, the equilibrium thickness of the film is given by

$$d = \kappa/H^{1/2}, \tag{3.3.5}$$

where $\kappa = (h^2/8m^2g)^{1/2}$.

On this model, film thickness would decrease rapidly at temperatures above 1.5°K as the number of atoms in the ground state diminishes. Since atomic interactions are ignored in this derivation of film profile, the manner of film formation in the first place is somewhat obscure. Moreover, Mott (1949) has criticized the theory on the grounds that the choice of wave function (implicit upon the form of the zero point energy) gives rise to an alarming variation of density across the film.

Atkins (1954) derived an equation for the free surface of the film on the assumption that the zero point energy of atoms in the film is thickness dependent. Only the contribution to zero point energy arising from longitudinal Debye modes is amenable to calculation. Unlike Bijl et al, Atkins takes account of the van der Waals energy. The total energy is minimized with respect to both thickness and density and the profile is given by

$$H = \left(\frac{4\times10^{-6}}{d}\right)^3 + \left(\frac{4\times10^{-6}}{d}\right)^2. \tag{3.3.6}$$

Depending on the film thickness, values of z (equation 3.2.1) are expected to lie between 2 and 3. Dzyaloshinskii, Lifshitz and Pitaevskii (1960) criticise the term in d^{-2} in Atkins' expression as resulting from an incorrect choice of cut-off frequency in the spectrum of normal Debye modes. From a theory in which the van der Waals energy of the film is related to the dielectric properties of the substrate, the film and the vapour, they propose a term, additional to the van der Waals term, in d^{-4}. This, it is said, is due to acoustic fluctuations. Be this as it may, it should be noted (Franchetti, 1960) that terms in d^{-2} can never arise from the theory of Dzyaloshinskii et al since they treat the liquid as a continuum and thereby dispense with the need for a cut-off frequency. Another objection to expression (3.3.6) has been put forward by Matsuda and van den Meijdenberg (1960) who state that the coefficient of the term in d^{-2} as derived by Atkins is in fact negative. A positive coefficient can only arise if a non-uniform density distribution within the film is considered.

Starting from a one-dimensional Schrödinger equation and using the variational principle, Temperley (1949) looked at the energy levels of a helium atom moving in the combined inverse cube potential of the wall and the layer of atoms covering it. The change in gravitational energy with height is balanced against the change of binding energy of atoms in the film as the thickness diminishes with height. The equation of the film surface is found to be

$$mgH - \frac{h^2}{32\pi^2 m}\left(a + b/d^2\right)^2 = \text{constant}, \qquad (3.3.7)$$

where a and b are constants. No more than rough comparison of this profile with experimental data can be made because of the presence of the adjustable parameters a and b. Temperley also predicted a variation of film profile with temperature. Depending on the assumptions made with respect to the energy difference between the film and the bulk liquid, there are three possibilities: film thickness can remain constant below the lambda point, it can decline to zero at the absolute zero or it can pass through a maximum at 1.8°K and vanish at the lambda point. Temperley feels that there is slight evidence for the third possibility.

Franchetti (1956, 1957a,b) investigated the difference in free energy (ΔF) between the liquid in the film and in the bulk. As in the Frenkel-Schiff approach, there are terms in ΔF arising from van der Waals forces and the gravitational energy change. Franchetti also considers contributions stemming from the extreme thinness of the film. A zero point energy thus arises in much the same way as that considered by Atkins (1954). In addition, the free energy changes involved when phonons and rotons are confined within a surface film are analysed. The variation of thickness with height in the film is finally found to be

$$H = \frac{A}{d^3} + \frac{B + C(T)}{d^2}, \qquad (3.3.8)$$

where A and B are constants and $C(T)$ is a temperature dependent function, the coefficient of which must be obtained from experimental data on film thickness. In general, an increase of film thickness with increasing temperature is expected. If

Franchetti's result is expressed in terms of the general profile equation (3.2.1) then $2 < \tau < 3$, depending on the nature of the substrate.

As has been seen, a certain amount of controversy exists concerning the thickness of the moving film as compared with that of the static film. The problem to be resolved theoretically is whether the film shall thin or thicken as a result of its motion. Although Picus (1954) suggested that the addition of the kinetic energy of the moving film to the free energy might affect the static profile, he makes no statement regarding the sign of the thickness change. Adopting the suggestion made by Picus, Meyer (1955b) calculated the differential free energy change resulting from motion of the film and found that the static film should be thinner than the moving film. Franchetti (1958) considered the variation of chemical potential when the film is set into motion, assuming the flow process to be reversible. He predicts an increase of thickness for the flowing film. On the other hand, the situation has been analysed by Kontorovitch (1956) and Arkihipov (1958) who find that thickness decreases with motion.

These conflicting theories have been examined by Tilley (1964). The Meyer-Franchetti result is shown to be inconsistent with the assumptions of the two fluid model; in particular, motion of the superfluid ought not to influence such thermodynamic functions as the free energy. Moreover, it was pointed out that Franchetti derives the kinetic energy of the film in a fixed frame of reference and proceeds to treat it as if it had been derived substantially.

If a local Galilean transformation of the Gibbs free energy in Franchetti's calculation is made then his results confirm those of Kontorovitch and Arkhipov who consistently use a Eulerian description of the motion.

The theoretical decrease in thickness subsequent upon motion of the helium film amounts to but a few per cent of the total thickness. For the sake of simplicity, therefore, this change can be neglected. We shall assume that the free surface of a normal helium film, whether static or in motion, is given in general by the profile equation 3.2.1.

3.4 Normal Film Transfer.

3.4.1. Basic Properties.

Daunt and Mendelssohn (1939a,b) established the main properties of helium film transfer by observing the rate of decay of the gravitational potential difference between two quantities of bulk helium connected by the film. The rate at which a cylindrical glass beaker, partially immersed in a helium bath, filled by film transfer was compared with its rate of emptying. Volume efflux rates were also obtained for the situation in which a partially filled beaker was raised completely above the bath. In this case, the formation of a steady stream of drops at the base of the beaker as it emptied was noted. Daunt and Mendelssohn concluded that the rate of transfer by the film is almost independent of both the driving pressure head and the film path length. A slight variation of flow rate associated with low pressure heads and proximity of the inner meniscus to the rim was commented on.

Discussion of the latter effect is postponed until Chapter 4.

Pressure head effects have been investigated by Atkins (1950b). For level differences less than 0.5 cm the rate of transfer decreases slightly. Eselson and Lasarev (1952) suggested that this decrease results from the temperature difference developed between the beaker and the bath due to the tardiness of heat exchange. Atkins, Rosenbaum and Seki (1959) dispute this, however, on the grounds that rapid distillation must limit such temperature differences to less than 10^{-6}°K. The associated thermo-mechanical pressure head is then negligable compared with the pressure head at which the decrease in the flow occurs.

As will be discussed in section 5 of this chapter, the pressure head necessary to maintain a state of motion in the film is a direct measure of the frictional forces opposing the flow. Of fundamental importance, therefore, are experiments relating to the behaviour of film transfer in the limit of zero pressure head. Such sub-critical flow was first observed by Daunt and Mendelssohn (1946). If two coaxial beakers are filled or emptied by the film, the inner meniscus and the surrounding annular meniscus always remain, on average, in the same horizontal plane. ~~This is clear evidence that film transfer, at least over the inner connecting surface, is not associated with a gravitational potential.~~ Chandrasekhar and Mendelsshon (1955) showed, moreover, that sub-critical transfer initiated by a thermal potential is also a frictionless flow.

Film flow at low pressure heads can also be studied by displacing the inner level of a beaker of helium with a slowly moving plunger. Using this technique Picus (1954) reported that steady film flow was maintained for pressure heads as low as tens of microns of helium. Seki (1962) pointed out that the significance of this small residual pressure head must be weighed against the possibility of thermo-mechanical effects.

From the small damping of film oscillations, Atkins (1950b) concluded that the resistance to flow through the film is negligible, if not zero. Osborne (1962) suggested that momentum exchange between atoms in the liquid and the vapour might account for the slight damping observed by Atkins.

For pressure heads between 0.5 cm and 4 cm, the observed difference between the film-filling rate and the film-emptying rate was discussed by Atkins (1950b). Whereas the emptying rate decreases continuously as the level difference diminishes, the filling rate remains practically constant, its magnitude depending only on the distance between the bath level and the rim of the beaker. This behaviour can be understood if the section of the film which controls the flow is that at the rim of the beaker, above the upper liquid level. While a beaker is emptying by film flow, the thickness at the governing section diminishes as the inner level falls. The apparent dependence of emptying rate on level difference is, therefore, reducible to a dependence on the depth of the falling meniscus below the beaker rim. On the other hand, the film thickness at the beaker rim during filling is

essentially constant, but can vary from run-to-run if the depth of the bath level below the rim is altered.

For film flow in a vessel of arbitrary shape, Daunt and Mendelssohn (1939a) found the governing section for flow to occur at the narrowest perimeter in the flow path above the upper of two liquid levels connected by the film. Thus bulk liquid can be formed by the film if there is a constriction below the upper level (Chandrasekhar and Mendelssohn, 1955). More generally, Miss Walker (1962) has shown that the governing section occurs at a place above the upper liquid level where the product of geometrical perimeter and film thickness is minimum.

Following their observations of a thermo-mechanical effect in the film, Daunt and Mendelssohn (1950) investigated the flow of the film under a thermal gradient. If heat is supplied to the bulk liquid within a small Dewar vessel a steady level difference develops between the inside and outside of the vessel. In this equilibrium state the volume of helium evaporating is continuously replenished by film flow. The total heat supplied per unit time is

$$\dot{Q} = n\sigma\rho(L+TS), \qquad (3.4.1)$$

where n is the total perimeter available for film flow, σ is the volume rate of transfer per cm of perimeter, L is the latent heat of evaporation of the bulk liquid and TS is the energy of conversion of superfluid in the film to normal fluid in the bulk. Since $TS \ll L$, film transfer is proportional to heat input. However, above a certain critical heat current $\dot{Q}_c$, the film

cannot compensate for loss of fluid by evaporation. Thus, the level difference continuously decays as long as the super-critical heat current is maintained. Assuming the entire resistance to motion of the film and vapour to rest entirely in the viscous flow of the vapour, the calculated value of $\dot{Q}_c$ agrees well with the observed value. From this it is inferred that film flow under a thermal potential is limited by a critical flow rate identical with that observed under a gravitational potential. Brown and Mendelssohn (1950) found no variation of transfer rate with temperature gradient.

3.4.2 Variation of Transfer Rate with Temperature above 1.2°K.

Under isothermal conditions the rate at which liquid helium is transported by the film is a function of the temperature. In their investigation, Daunt and Mendelssohn (1939b) found that the rate increased from zero at the lambda point to a maximum value of about 7.5×10^{-5} cm^2 sec^{-1} at 1.5°K. For $1.1^\circ < T < 1.5$°K, there was no appreciable change in rate, although it appeared to decline very slightly as the temperature fell. Most reports on the helium film contain data revealing a similar trend for the temperature dependence, although the decline in rate below 1.5°K has seldom been reproduced. Of interest is the work of Mendelssohn and White (1950) who find that the variation of rate with temperature can be described by a function proportional to

$$\left[1 - \left(\frac{T}{T_\lambda}\right)^\alpha\right], \qquad (3.4.1)$$

where T_λ is the lambda temperature and α is a constant.

The best representation of their results obtains if α is chosen to be six, although for an unbaked glass specimen a value of eight seemed preferable. Smith and Boorse (1955d), on the other hand, find a value of α equal to seven. The form of the function (3.4.1) allows comparison of the flow rate dependence on temperature with that of the superfluid fraction. From the experiments of Andronikashvili (1946) the concentration of superfluid component can be shown (London, 1954) to follow (3.4.1) with an α of 5.6. However, since the constant of proportionality in expression (3.4.1) is determined from a normalization procedure between theory and experiment, any similarity between the variation of flow rate and superfluid fraction with temperature may be fortuitious (Smith and Boorse, 1955d).

3.4.3. Effect of Contamination on Transfer.

It must be mentioned that the characteristic properties of film transfer are only reproducible when the film is formed over a 'clean' substrate. The behaviour of the flow rate, in particular, alters radically when the surface is contaminated by solid air or other impurity. For example, Atkins (1948) and de Haas and van den Berg (1949) obtained film transfer rates which, in addition to being many times greater than those observed by Daunt and Mendelssohn, were extremely dependent on the pressure head. This anomalous behaviour was shown by Bowers and Mendelssohn (1950) to be the result of condensed impurities on the substrate supporting the film. The extent of the departure of the rate from normal depends on the amount of impurity present;

there exists a saturation level of contamination above which the flow rate does not alter further. Even at this level, however, the degree of contamination is so slight as to be invisible. Bowers and Mendelssohn tentatively suggested that the geometrical perimeter controlling flow is increased by the granular nature of the solid deposit. It was later shown, however, (Ham and Jackson, 1954, 1957) that the Helium II film can attain thicknesses as great as 20×10^{-6} cm as a result of contamination of the substrate. In view of this, it seems probable that increased film thickness, rather than increased perimeter, is the dominant factor controlling flow over an 'impure' surface.

3.4.4 Relation Between Transfer and Substrate Material.

Despite the number of investigations of both film thickness and transfer rate over metal and dielectric surfaces as opposed to those of glass, theoretical predictions concerning a variation of film thickness with substrate material have not, on the whole, been borne out by experiment. Quite apart from the difficulty of achieving a contamination-free surface, there is the problem of separating any dependence on the chemical nature of the substrate from geometrical effects, such as surface roughness. Chandrasekhar and Mendelssohn (1952) found that film transfer on stainless steel of an exceptionally high surface finish is of the same order of that found by Mendelssohn and White (1950) on a baked-out glass surface. They support their conclusion that high transfer rates on metal surfaces (e.g. Mendelssohn and White, 1950) are due to micro-structure of the substrate by noting that the flow rate over

stainless steel increases three-fold if the smooth finish is destroyed by heating. In this connection, the statistical investigation of film transfer by Smith and Boorse (1955a,b,c,d) is probably the most definitive. They conclude that any systematic dependence on the composition of the substrate is invariably masked in practice by the background variation of transfer rate from run-to-run. Their estimate of the difference between the geometrical and microscopic controlling perimeters for flow suggests that the enormous variety of transfer rates associated with metal surfaces cannot be fully accounted for by an ~~uncontrollable~~ variation of surface roughness.

3.4.5 Experimental Difficulties Arising from Thermal Effects.

A particular danger with which any film experiment is fraught is that of thermal disturbance. In the first place, under isothermal conditions, the flowing superfluid by virtue of its heat defect creates a mechano-caloric effect. To prevent a gross build-up of temperature inhomogeneity between two thermally isolated reservoirs connected by the film, a distillation process must operate whereby the vapour gives up its heat upon condensation. Atkins (1950b) has shown that a temperature difference ΔT of 10^{-6}°K is sufficient for this process to occur. The thermal pressure head, $(S/g)\Delta T$, associated with a ΔT of this magnitude is of the order of 2×10^{-3}cm helium which is normally negligible in comparison with the gravitational pressure head.

Probably a more important thermal disturbance to the film is constituted by incident heat radiation. De Haas and van den Berg

(1949) suggested that the film transfer rate was increased in the absence of radiation. This, however, was refuted by Bowers and Mendelssohn (1950) who attributed high transfer rates to contamination. Bowers (1953) commented that the observed disparity in measured values of film thickness may be due to the sensitivity of the film to heat influx. Ham and Jackson (1954, 1957) showed that intense illumination completely removed the helium film, although re-formation to full thickness followed swiftly upon returning to normal conditions of illumination. If a temperature difference develops between the substrate containing the film and the bath, Atkins (1954) considers that the film will appear thinner than in isothermal conditions as a result of the variation of vapour pressure. Atkins (1950b) also attributed a slight difference of magnitude between film-filling and film-emptying rates to increased evaporation associated with excessive heat influx.

In view of the diversified results relating to film flow under heat influx, it is necessary to protect the film, as far as possible, from such disturbances. Filters of heat absorbing glass have proved successful for this purpose.

3.4.6. Film Transfer Below 1°K.

Since the ratio of ρ_s/ρ remains practically constant as the temperature falls below 1°K changes in flow rate in this temperature range cannot be attributed to variation of the superfluid fraction. Owing to the difficulty of obtaining and maintaining such temperatures, measurements are few in number. The results of

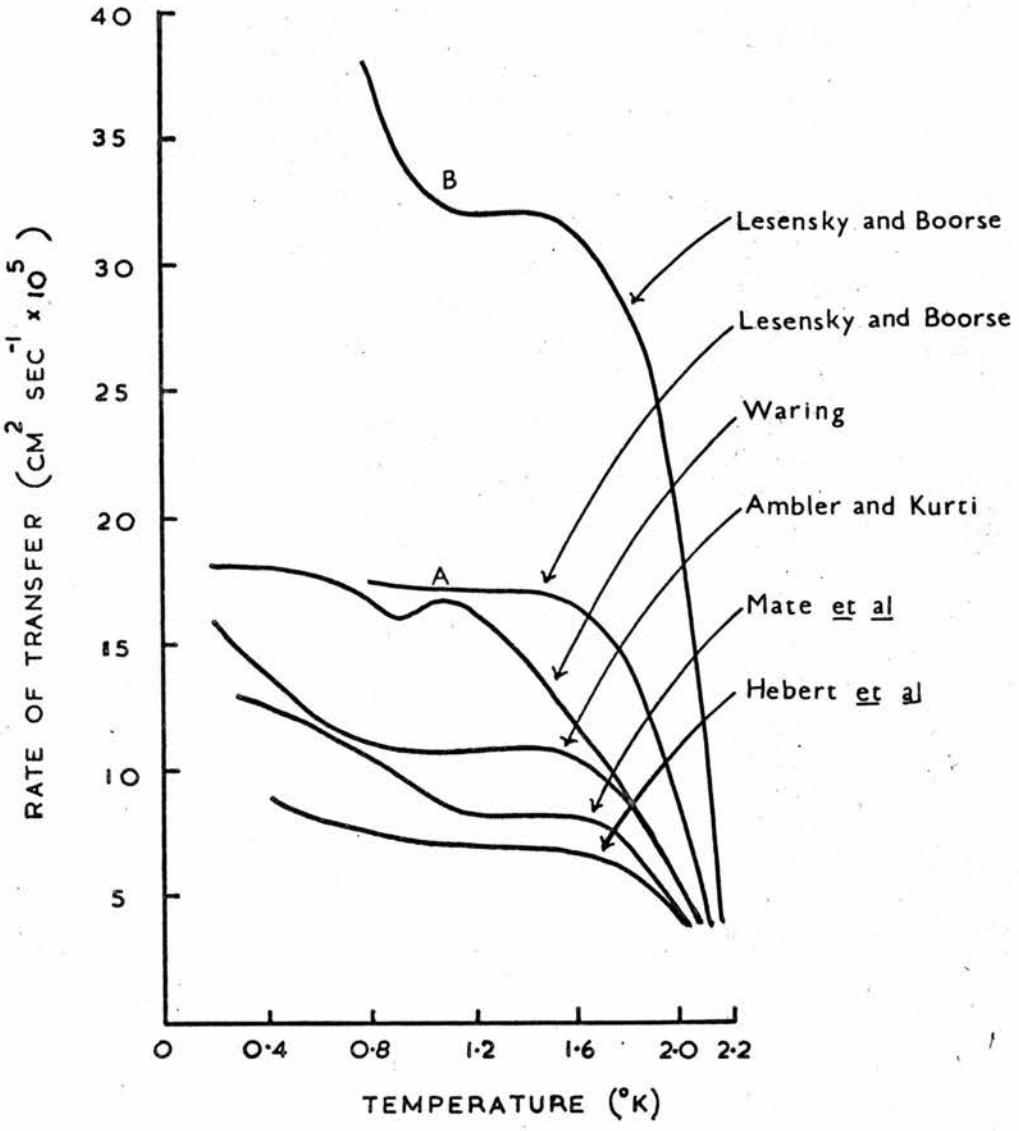


Figure 3.4.6.1 Helium Film Transfer Below 1°K.

various workers are shown in figure 3.4.6.1.

Temperatures down to 0.14°K were obtained by Ambler and Kurti (1952) by an adiabatic demagnetization technique. The rate of transfer of the film was determined from the rate of rise and fall of the helium meniscus within an open beaker. The rates above 1°K were higher than the accepted value for a glass substrate. From this it can be inferred that the substrate was contaminated. Since impurities can alter the temperature dependence of flow rate severe doubt must be cast on the validity of this experiment.

The effect of contamination on rates below 1°K was further shown by Lesensky and Boorse (1952) who compared the rate over a clean machined copper surface (curve A) with that over a contaminated surface (curve B). It appears that the presence of impurities steepens the change in rate as the temperature falls below 1.1°K. Unfortunately the heat input to the cryostat was too large to allow measurements to be extended below 0.7°K.

Film transfer under a thermal gradient was investigated by Waring (1955). Although 'extensive precautions' were taken to preclude contamination, flow rates were higher than the accepted value by a factor of two or more. In addition, transfer under a gravitational potential difference made with the same observation vessel exhibited a high rate. This experiment, therefore, must be viewed with caution.

If we adopt the criterion that contamination is present if the transfer rate over a glass substrate at 1.2°K is higher than, say, 8×10^{-5} cm^2 sec^{-1} then the results of Hebert, Chopra and

Brown (1957) are free from the objections raised against those of Ambler et al and Lesensky et al. Hebert et al find that the rate at 0.3°K has risen about 25% over its value at 0.9°K.

Mate, Harris-Lowe and Daunt (1965) also report flow rate measurements which are probably free from contamination. Their rate at 0.3°K is somewhat greater than that of Hebert et al. Moreover, the rise in rate is initiated at a slightly lower temperature ($\sim$1.15°K).

It appears, therefore, that although the flow rate below 1°K follows an upward trend, there is confusion as to the magnitude of the rise. Doubts as to the validity of certain of the experiments mitigates this confusion, but there is still a need for further investigation of this aspect of flow of the film.

3.5 Critical Velocity and the Mechanism of Film Transfer.

The normal fluid component within the flowing helium film is generally considered to remain locked to the substrate owing to its viscosity. Moreover, since the flow of the film between two liquid levels is essentially independent of both gravitational potential and thermal potential, motion of the superfluid must be frictionless. Thus, since the rate of flow of superfluid is both invariant and finite, it is implied that a limiting or critical velocity (v_c) is an essential property of the motion.

Much attention has been directed to the mechanism whereby frictional forces in the film can vary with velocity (Atkins, 1950b, 1959, Ch.4). A classical fluid in laminar motion experiences a viscous force increasing linearly with the velocity. Gorter and

Mellink (1949) postulated a mutual friction force between normal fluid and superfluid proportional to the cube of the relative velocity. However, the preferred hypothesis concerning film flow is that the flow is completely frictionless below v_c, while above v_c the force of friction becomes immeasurably large. It is possible that this idealized mechanism is not realized in practice and that observed flow phenomena are due to a combination of frictionless flow together with flow exhibiting non-linear frictional forces.

If the velocity limited flow is identified with the critical transfer rate of the film then this rate can be written

$$\sigma = \frac{\rho_s}{\rho} v_s d_g, \qquad (3.5.1)$$

where d_g is the thickness of the film at the governing section. Although the gross variation of this rate with temperature above 1.2°K is roughly given by the factor ρ_s/ρ, possible variations of v_c and d_g with temperature must not be excluded. Thus, apart from theoretical estimates of the magnitude of v_c, it is vitally necessary to resolve the vexed question of whether the flow rate changes with film thickness.

Bijl, de Boer and Michels (1941) noted that the early experiments of Daunt and Mendelssohn on film transfer were well represented if the product of the average momentum of the superfluid moving at its critical velocity and the film thickness is of the order of $\hbar$. I.e.

$$(m v_c) d_g \sim \hbar . \qquad (3.5.2)$$

It is clear from this empirical relation that changes in thickness arising from the height dependence of film profile, nature of the substrate or indeed any other cause are compensated by a variation of v_c, with the result that the transfer rate maintains a unique value. Further justification of the constancy of the product $v_c d_g$ was given by Ginsburg (1949). In addition, Franchetti (1955) proposed that an expression of the form

$$(\overline{v^2})^{1/2} d_g \sim 3\times10^{-4} \text{ cm}^2 \text{ sec}^{-1} \qquad (3.5.3)$$

arises if energy dissipation sets in when the kinetic energy per atom of the collective adiabatic motion of the liquid reaches the value of the first standing wave having a $\underline{k}$-vector perpendicular to the wall. Since the velocity field (v) is varying, averages must be taken. This effectively reduces the product $(\overline{v^2})^{\frac{1}{2}} d_g$ to the order of magnitude of that given by Bijl et al.

The Landau criterion for superfluidity (1941) was discussed in Chapter 2. It was shown that neither the production of thermal excitations (phonons and rotons) nor the creation of vorticity can account for observed critical velocities in the helium film. However, Dash (1954a,b) has pointed out that the value of $\left|\frac{\varepsilon(\underline{p})}{\underline{p}}\right|_{min}$ can be reduced well below that appropriate to phonon creation provided a 'momentum cluster' of superfluid atoms is involved in the dissipation process. With suitable choice for the volume over which the cooperative phenomenon occurs, he finds

$$v_c d_g^{1/2} = \text{constant.} \qquad (3.5.4)$$

If the variation of film thickness with height is given by expression (3.2.1) it follows that

$$v_c d_g \propto H^{-1/2} . \qquad (3.5.5)$$

On these ground, therefore, the transfer rate σ is expected to vary as the height of the governing section above the upper liquid level. A similar proportionality between v_c and $d_g^{-1/2}$ was proposed by Mott (1949), although his model of a nucleus of stationary superfluid within a volume of moving superfluid is far removed from the mechanism envisaged by Landau and Dash.

Kuper (1956) considered quantized surface waves ("ripplons") as a class of elementary excitations to which the Landau criterion for critical velocity could be applied. $\left|\frac{\varepsilon(p)}{p}\right|_{min}$ is given, in this case, by the minimum phase velocity of combined gravity and capillary waves on a deep liquid with a density appropriate to that of helium II. The resulting critical velocity is $\sim$ 10 cm sec^{-1}, with a corresponding wavelength of 0.3 cm.

Atkins (1957) criticizes this theory on two grounds. Firstly, the body force per unit mass on the superfluid is not gravity but the van der Waals force. Thus the restoring force (f) in the film has magnitude $\sim 10^5$ times that assumed by Kuper. In the second place, Atkins realized the importance of modifying the phase velocity of the surface modes to take account of the finite depth of the film. These modifications lead to

$$v_c = (fd)^{1/2} . \qquad (3.5.6)$$

If the profile of the film is determined by a balance between van der Waals and gravitational potentials, the restoring force is

$$f = \frac{3\alpha}{d^4}, \tag{3.5.7}$$

where α is the van der Waals force constant. In addition, the variation of film thickness with height is given by (3.3.2). The critical velocity can therefore be rewritten as

$$v_c = \left(\frac{3\alpha}{d_g^3}\right)^{1/2} = (3gH)^{1/2}. \tag{3.5.8}$$

At a height of 1 cm the value of v_c is of the order of 50 cm sec^{-1}. Since experimental data suggests a slow increase of $(v_c d_g)$ with d_g, the monotonic decrease predicted by (3.5.8) is rather disturbing. The inclusion of terms in the profile equation arising from zero point energy merely raises v_c to about 70 cm sec^{-1} and, moreover, does not appreciably alter the dependence of the function $(v_c d_g)$ on d_g.

The instability of a superfluid film with respect to the building up of surface modes has also been considered by Arkhipov (1958) who obtained a critical flow rate an order of magnitude larger than observed values.

3.6 Wave Motion in the Normal Helium Film.

It is generally accepted that bulk helium II is capable of supporting two distinct types of wave motion. To the first order of approximation, in ordinary (first) sound the normal fluid and superfluid oscillate in unison, resulting in the propagation of

density waves at constant entropy. On the other hand, as a result of the adoption of two separate velocity fields to describe the hydrodynamic state of helium II, a temperature wave (second sound) is generated when the two fluids oscillate in anti-phase, their centre of gravity remaining at rest.

In stark contrast to the precise knowledge of the properties of first and second sound in the bulk liquid, wave motion in the film, until fairly recently, has been treated rather as a theoretical abstraction than a practical reality. For example, physical interpretation of the nature of the collective surface modes proposed by Kuper (1956) is a little difficult.

The role played by the normal fluid in wavy flow of the film has been examined by Tilley and Kuper (1965). Using the work of Kapitza (1948) on the inception of viscous waves on a classical liquid film they pointed out that conditions within the helium film are unfavourable as regards the appearance of a wave regime associated with the normal fluid. Such a regime is rather akin to wave motion in a vessel of syrup - the minimum wavelength in both cases is invariably larger than the dimensions of the apparatus. On these grounds, one must criticize the work of Hsieh (1964) on the stability of helium film flow at low Reynolds numbers.

Atkins (1959a) has greatly clarified the situation with his proposal that the vanishing viscosity of the superfluid component in the film allows it to execute an almost unattenuated oscillatory motion somewhat similar to that occurring in shallow water waves. The viscosity of the normal fluid precludes it from participating

in this type of motion which Atkins called third sound. The excess and defect of superfluid at the free surface of the film results in periodic temperature fluctuations along the surface. This, in turn, gives rise to a distillation process through the vapour and, moreover, to a restoring force of thermomechanical origin additional to that stemming from van der Waals attraction of the substrate. In the low frequency approximation (frequency < 10^3 sec $^{-1}$) the velocity of third sound is given by

$$u_3^2 \simeq \frac{\rho_s}{\rho} \alpha f \left[1 + \frac{TS}{L}\right]. \qquad (3.6.1)$$

Invariably TS << L and hence (3.6.1) differs from Atkins' expression for critical velocity (3.5.6) by the factor ρ_s/ρ. It is clear, therefore, that large fractional changes of thickness due to superfluid waves are possible without the superfluid velocity exceeding v_c. An important practical point is that in this frequency range the temperature oscillation associated with the motion is of the second order, and as such is immeasurably small.

So far, third sound has only been generated in a horizontal film by means of periodic heat pulses (Everitt, Atkins and Denenstein, 1962, 1964). Its existence has been demonstrated by using a polarized light technique to observe periodic variations in film thickness. It was found, for frequencies below 1300 sec^{-1}, that u_3 is independent of frequency. There was a slight difference between the observed dependence of u_3 on temperature and that predicted by (3.6.1). This discrepancy has yet to be explained. The attenuation of third sound appeared to depend on film thickness.

For $H > 10$ cm the attenuation was so slight that standing waves could be set up. Over a 1 cm height of film, however, the attenuation was about 0.5 cm^{-1} at a frequency of 100 cps.

Since the restoring force appearing in (3.6.1) is dependent on the value of z in expression (3.2.1), measurements of third sound can be used to give information regarding the nature of the forces on the fluid in the film. For $1 < H < 8$ cm, the observed variation of u_3 with H implies that f is given by equation (3.5.7). This supports theories of the profile equation arising from considerations of van der Waals forces.

CHAPTER 4.

Variable Film Transfer Rates.

4.1 Scatter of Normal Film Transfer.

The contention of the previous chapter was that under isothermal conditions and at a specified level of surface contamination the absolute magnitude of the film transfer rate is invariant at a particular temperature. There are, however, a limited number of observations which report gross variations in the transfer rate. Since these variations appear within a single experimental run, they cannot be related to changes of temperature, contamination or heat influx, all of which can reasonably be assumed to remain constant for the period of the run.

To establish the significance of this type of variation it is useful to consider the degree of reproducibility which has hitherto been achieved when observing the normal film transfer rate. It is generally found that errors of measurement cannot account for the scatter of a sequence of rates. For example, Mendelssohn and White (1950) found a maximum scatter of the order of 11% for the rate over baked-out glass. Mate, Harris-Lowe and Daunt (1964) noted that the scatter increases at lower temperatures, and that the scatter within a run can be as great as that from run-to-run. Their maximum scatter at 1.2°K is about 10%, while at 1.4°K it is 14%. Although the data of most experiments is too meagre to be of statistical significance, one is left with the strong impression that this scatter is unjustifiably high.

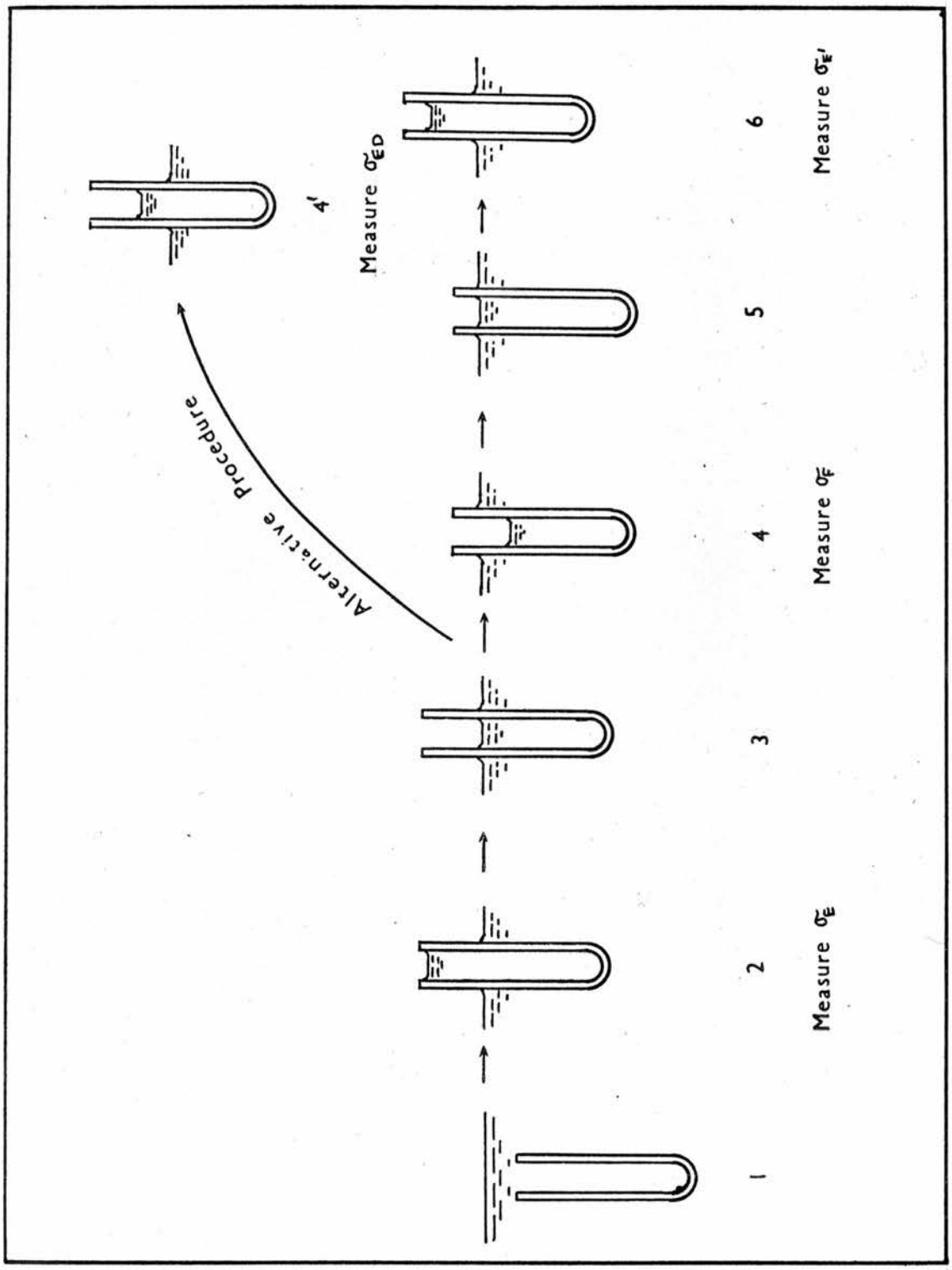


Figure 4.2.1 Filling by Complete Submersion and Subsequent Film Transfer Rates.

Possibly connected with this problem is the observation made by Gribbon and Jackson (1963) that measurements of moving film thickness are more uncertain than those of static thickness.

4.2 Enhanced Transfer Rates.

Apart from the 'unavoidable' scatter associated with the transfer process, the experiments of Eselson and Lasarev (1951, 1952) were the first indication that the notion of a characteristic rate of transfer might have to be reconsidered. The essence of their experiments was a comparison between film transfer out of a vessel after it had been filled by complete submersion in the liquid, and after liquid had been transferred into it by film flow alone. It was found that the film could transport liquid at two utterly different rates, depending on the pre-history of formation of the film.

The procedure whereby the different initial conditions for flow were achieved is shown in Figure 4.2.1. The vessel was a thick-walled glass beaker of internal diameter 2.54 mm. After the vessel had been completely immersed beneath the surface of the bath it was raised and the rate of emptying by the film (σ_E) determined. When the levels had equalized, as shown by stage 3, the beaker was depressed and the immediate filling rate (σ_F) observed. Further equalization of levels followed by lifting the beaker resulted in the final rate of efflux ($\sigma_{E'}$).

The main result of interest was that the rate of emptying after plunge-filling was invariably larger than the rate after filling by the film, i.e. $\sigma_E > \sigma_{E'}$. We have called the higher rate of

transfer an 'enhanced' rate. The degree of enhancement depends on the temperature. At 1.52°K the enhancement was of the order of 17%; at 1.69°K it was 10%; and at temperatures above 1.88°K it was impossible to distinguish two distinct rates. This behaviour has since been verified by Miss Walker (1962).

For the purpose of this dissertation, two other observations concerning the variation of emptying rate with procedure are vitally important. In the first place, the rate $\sigma_{E'}$ was independent of the time elapsed between stages 5 and 6 shown in the figure. In the second place, if some helium spilled out of the beaker while it was being raised from its submerged position, the subsequent rate was not enhanced.

The rate at which the beaker filled (stage 4) is also of interest. This rate (σ_F) was approximately equal to the subsequent rate of emptying $\sigma_{E'}$, but was in general somewhat less. However, as the temperature rose any difference between these rates decreased. In passing it should be noticed that both the rate of filling (σ_F) and of subsequent emptying ($\sigma_{E'}$) were slightly greater than we have accepted as normal in Chapter 3. Although it was inferred by Eselson and Lasarev that these rates are normal, it remains to establish whether this discrepancy is due to contamination or some more basic cause.

Eselson and Lasarev also pointed out that an alternative procedure can be adopted after the first equalization of levels (stage 3). This is shown by Stage 4' in figure 4.2.1. Instead of depressing the beaker, it can be raised and an emptying rate (σ_{E_D})

obtained. Since this is equivalent to breaking off measurements of σ_E and recommencing after a specified time τ, we shall refer to the rate σ_{ED} as a delay-time measurement of enhanced transfer. Frequently, although not invariably, for τ > 1 or 2 minutes it was found that σ_{ED} did not coincide with σ_E, but was rather of the order of $\sigma_{E'}$. For shorter delays, however, σ_{ED} was found to be identical with that following total immersion.

A further point which arises from this investigation is that the depth of the bath level below the beaker rim during filling can influence the subsequent rate of efflux. If the depth was small (few mm) the emptying rate lay between σ_E and $\sigma_{E'}$.

The physical conditions under which the observed phenomena occurred were explicitly commented on by Eselson and Lasarev. The maximum variation of transfer rate was only achieved when the beaker was placed within a constant level bath made up of a glass vessel surrounded by a copper screen with narrow viewing slits. No variation of transfer following the different methods of filling ensued if the flow took place in the extended bath of an ordinary helium Dewar.

Snyder and Donnelly (1959) reported that a beaker (i.d. 6.35mm) which had been filled to overflowing by the helium fountain exhibited variable flow rates.

Seki (1962), using a beaker of 1 cm i.d., reproduced the anomalous dependence of transfer rate on film history. In addition, he noted that occasionally the change from σ_E to $\sigma_{E'}$ did not occur. In such cases, if procedures 4 and 5 were repeated the

change immediately took place, σ_F being the first rate to alter. Seki also repeated the delay-time measurements and found, again in contrast to Eselson and Lasarev, that there was no significant change in rate for $\gamma \leq$ 40 minutes. Deliberate introduction of impurities into the cryostat had little effect on the rate of emptying after the beaker had been immersed and lifted, but tended to increase σ_F and $\sigma_{F'}$ slightly. Inferring from his measurements that there are possibly two states of the helium film, Seki concluded that the state produced immediately after raising the beaker from complete submergence is meta-stable since it frequently changes to another state which exhibits a lower transfer rate. The difference between the states was thought to be of internal origin rather than arising from changes of film thickness.

Allen (1960a,b) reported an enhancement of film transfer following plunge-filling of almost 70% at a temperature of 1.3°K. Moreover, he noted that rates of emptying after successive plunge-fills differed from each other, sometimes by as much as 15%. Allen (1963) also described how enhanced transfer could be achieved if the beaker had been filled with the bath level near the rim. During this type of filling process, the rate of influx is higher than normal and grows with decreasing depth of the bath level below the rim because of the variation of film thickness with height. This type of process will be referred to as "fast film-filling".

4.3 Vorticity and Film Transfer.

Seeking a reason for enhanced transfer, Allen (1960a,b) considered the role played by quantum vortex motion in the flow

properties of the film. He suggested that the superfluid was conveyed through the film on vortex "roller-bearing", the rate of motion being governed by the amount of vorticity present. It is probable that there is always sufficient vorticity available in the bulk liquid to allow film transfer to take place at least at the normal rate. Moreover, it can easily be imagined that enhanced transfer is obtained as a result of excessive amounts of vorticity produced, for example, by bulk liquid gushing into a totally immersed empty beaker.

As has been discussed in Chapter 3, the first few atomic layers in the film are firmly bound to the wall by van der Waals forces and do not take part in the motion of the film. If the remainder of the film is to flow in a purely potential manner, there must be a slip velocity for the first moving layer. Kuper (1960a,b) pointed out that the presence of an array of Feynman vortices in the film is consistent with such a velocity discontinuity. Developing this theme, he investigated the most stable configuration which will be achieved by a uniform array of vortices in the film. A single row of vortices all lying in the same plane with their axes perpendicular to the direction of motion is unstable. It is known, however, that a double row of vortex lines is stable, provided the rows are laterally displaced by an amount equal to one half the distance between successive vortex cores. Moreover, the vortices in the two rows must be of opposite sign. Such a Karman street of vortices classically occurs when a two dimensional flow is bounded on either side by parallel walls. If one wall together

with the adjacent row of vortices (up to the central corrugated stream-line of the pattern) is suppressed from this flow configuration the application to the helium film is obvious. Kuper's picture of the film, punctured by vortex lines running perpendicular to the direction of motion, and possessing a corrugated free surface, lends microscopic support to the rough model of film flow put forward by Allen.

For this mechanism to proceed satisfactorily, it is necessary that the vortices be small enough to be encompassed by the film. This restriction on what is effectively the spacing of the vortices leads to a minimum superfluid velocity of $\hbar/md$. This is of the same form as the critical velocity predicted, for example, by Bijl et al (1941). It must be emphasized, however, that the velocity derived from the vorticity model of film flow is a minimum velocity of flow below which there are not enough vortices to carry the film. Provided enough energy is available, higher velocities are possible, but the simple configuration of a single row of vortices is probably inadequate at velocities less than the minimum.

Vinen (1963) tentatively suggested that Kuper's reason for introducing ordered vorticity into the film, viz. a velocity discontinuity at the boundary of the liquid, is not necessarily valid.

Experimental support for the idea that film transfer is enhanced if an excess amount of quantum turbulence is present was cited by Allen (1960a,b; 1963). Turbulence was created within the

beaker by the up-and-down motion of a ferrite piston actuated by a solenoid situated outside the beaker. In addition to observing enhanced rates of efflux from a stirred beaker, Allen reported inhibited rates of filling. The absolute magnitude of the change in rate was dependent on the frequency of the mechanical agitation. An important observation was that stirring had no effect on the flow rate unless the meniscus within the beaker was visibly agitated. If stirring was unaccompanied by meniscus motion the transfer was found to be normal.

Since transfer could be both diminished and enhanced by stirring, Allen concluded that the rate of transfer depends on the specific amount of turbulence at both ends of the film. It was further inferred that there was a contribution to the chemical potential from turbulence. It is therefore expected that a static level difference should be maintained between two reservoirs connected by the film when there is a difference of specific turbulence between the reservoirs. An equilibrium level drop of 0.5 mm was observed by Allen.

Confirmation of Allen's results followed from the experiments of Selden and Dillinger (1964, 1965) who (by oscillating a tin sphere above a niobium coil) stirred the bulk liquid which supplied the film.

An intensive study of the variation of equilibrium level difference with the frequency of stirring has been carried out by Miss Walker (1962). In general there was a linear dependence of equilibrium level difference on frequency at constant stirring

amplitude. The irreproducible nature of the effect was attributed to the possibility that the stirrer could attain different modes of vibration.

4.4 The Rim Effect.

Another obscure feature, invariably observed and imperfectly understood, is the increased transfer associated with proximity of a bulk liquid level to the rim of a vessel. This was first noted by Daunt and Mendelssohn (1939a) and attributed by them to surface tension effects. Atkins (1950b), however, inclined to the view that the high rate is a consequence of the variation of film thickness with height. On the other hand, Webber, Fairbank and Lane (1949) suggested that the effect is accounted for by a change in evaporation rate as the inner level falls.

Eselson and Lasarev (1952) pointed out that none of these explanations can explain all the observed facts. In particular, they discussed film transfer from a vessel consisting of two parts of different diameters, the wider part being uppermost. If such a vessel is plunge-filled and raised, a large rate of transfer is observed when the inner level is near the rim. This rate smoothly decreases to the accepted value for enhanced transfer (σ_E). As the level drops into the narrow part, the rate immediately increases by a few multiples of itself to a value equal to that when the vessel was initially raised. Finally, when the inner level has dropped a further few mm within the narrow part, the rate decreases again to σ_E . This behaviour is contrasted with that resulting from filling the vessel through the film so that the inner level is

in the wide part initially. In this case, during subsequent efflux, the rate is given at first by $\sigma_{\varepsilon'}$. However, as the inner level falls to the narrow part the transfer rate behaves in a manner similar to that already described for this section of the beaker.

Eselson and Lasarev attribute the initial high rates of emptying when the inner level is near the rim to the large film thickness associated with small film heights. They further suggest that there is a prolonging of this effect throughout the remainder of the transfer.

A different interpretation of the change in rate occurring at the junction of two different diameters has been given by Miss Walker (1962). She considered that the observed transfer as the level drops to the narrow part appears to be anomalously high because the wrong governing perimeter has been used in deriving the flow rate over the transition period.

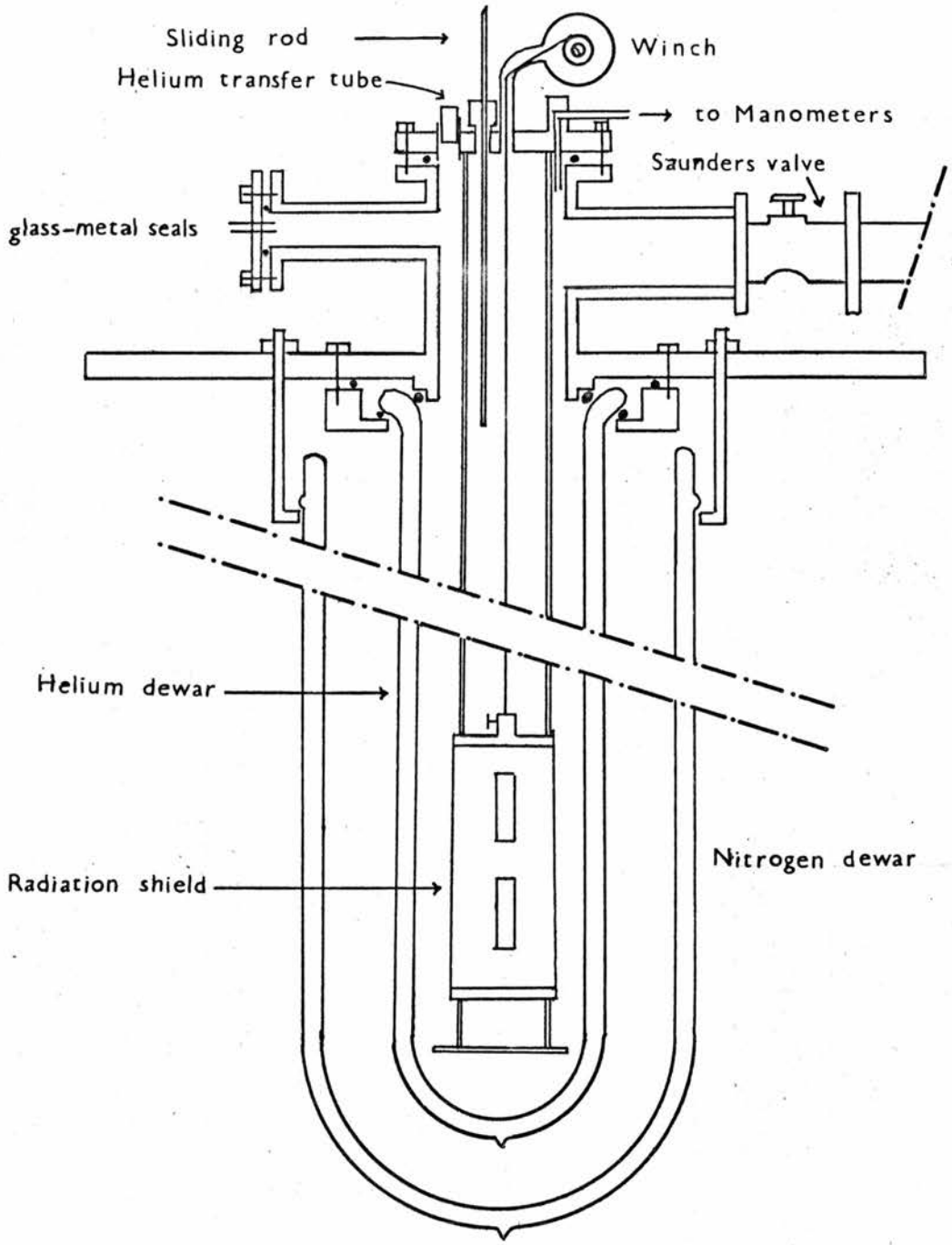


Figure 5.2.1.1 Schematic Representation of the Helium^4 Cryostat.

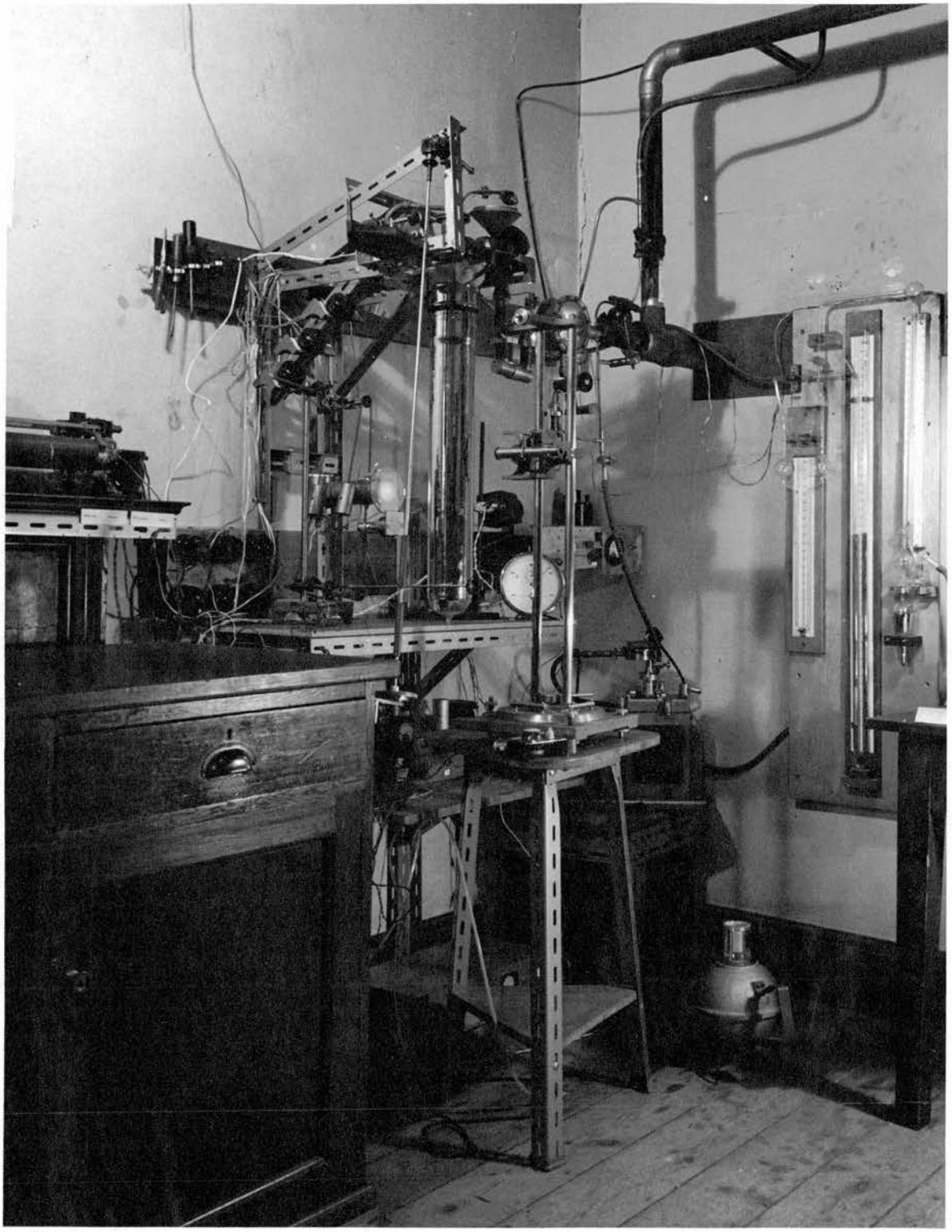


Figure 5.2.1.2 General Lay-out of Helium^4 Cryostat.

CHAPTER 5.

General Apparatus and Experimental Techniques.

5.1 Principle of the Method.

Film transfer rates were determined from observations of the rate of rise and fall of the helium meniscus in a small cylindrical glass beaker as it filled and emptied by film flow. Different types of beaker were used; these will be described in the appropriate sections. The meniscus was observed through a cathetometer, the magnification being chosen so that the meniscus traversed the length of a vertical eyepiece scale in one emptying or filling. Except for the experiments described in Chapter 7.7, the magnification was such that a single division of the 100 division eyepiece graticule measured 1.2×10^{-2} cm. Unless otherwise specified, the meniscus was always observed over the same region of the beaker, from 6 mm to 12 mm below the rim.

Transfer rates are given in units of cubic centimetres of helium transferred over 1 cm of the appropriate solid connecting surface in 1 second.

5.2 The Helium^4 Cryostat.

5.2.1 The Cryostat.

The cryostat and the various connections to it are shown schematically in Figure 5.2.1.1. Figure 5.2.1.2 illustrates the actual lay-out of apparatus. The cryostat was of the conventional type, consisting of two coaxial Monax glass dewars mounted on a brass cryostat head.

The inner, helium dewar was about 70 cm long, with an inner diameter of 4.5 cm. The top 12 cm was of single thickness glass. Except for narrow observation slits, the vacuum section was completely silvered on the inside. To facilitate pre-cooling to liquid nitrogen temperatures, air at a pressure of several torr was left within the vacuum interspace. During the pre-cooling this freezes onto the inner wall with the result that the dewar is hard at helium temperatures. Since Monax glass is practically impervious to helium, the dewar did not require re-evacuation during the entire course of the experiments.

The ~~fluted~~ flanged rim of the helium dewar was seated on a hard rubber ring below which was a brass ring. The brass ring was screwed to the top plate of the cryostat. A vacuum seal was provided by two rubber O rings on a suitably shaped cylindrical extension of the top plate.

The outer dewar, which was pumped hard and silvered on its inner surface, was kept filled with liquid nitrogen during an experiment. A ~~flange~~ bulge on the outer wall of this dewar allowed it to be supported by a padded brass ring suspended from the top plate. Diametrically opposite clear strips in the silvering were lined up with those of the helium dewar.

The cryostat head was made up of a square brass plate through which was mounted a brass T-piece with a 1 in. diameter horizontal side-arm terminating in a 1 in. Saunders valve. The helium bath was connected to the pumping line through this side-arm. Diametrically opposite the side-arm was a copper-nickel tube

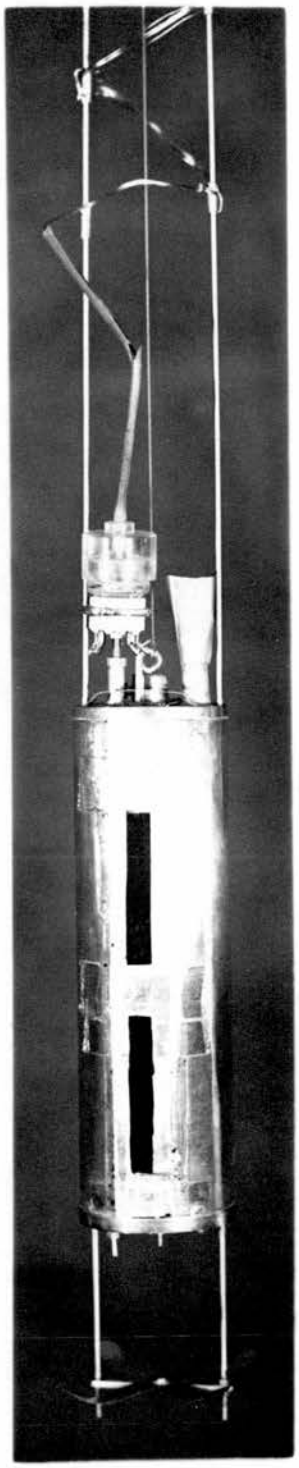


Figure 5.2.2.1 Radiation Shield.

through which all the electrical connections were led from a brass plate containing the requisite number of glass-to-metal seals. A circular brass cap was screwed onto a flange at the top of the cryostat head. A vacuum seal was obtained with a rubber O ring.

Apart from an entry which was only opened during filling with liquid helium, and a copper-nickel re-entrant tube to the pressure measuring system, there were two other connections to the cryostat cap.

The first of these was a rod passing through a rubber gland in the cryostat cap. Vertical motion of this rod was remotely controlled by a rack and pinion. This type of motion was only required in the displacer experiments to be described in Chapter 7.6.

The other connection to the cryostat cap was a tube through which passed a stranded terylene thread supporting the radiation shield. The thread was attached at its upper end to the drum of a small vacuum-tight winch.

5.2.2 The Radiation Shield.

The radiation shield inside which the observation beakers were mounted could be moved in a vertical direction on slides which extended the length of the helium dewar. This is shown in Figure 5.2.2.1. Remote control of the winch through a low geared coupling gave a smooth sliding motion of the radiation shield comparatively free of cryostat movement and vibration.

The walls of the cylindrical radiation shield were of 0.005 in. copper sheet polished on the outside and painted dull black on the inside. Viewing slits 3 mm wide in the shield were covered by

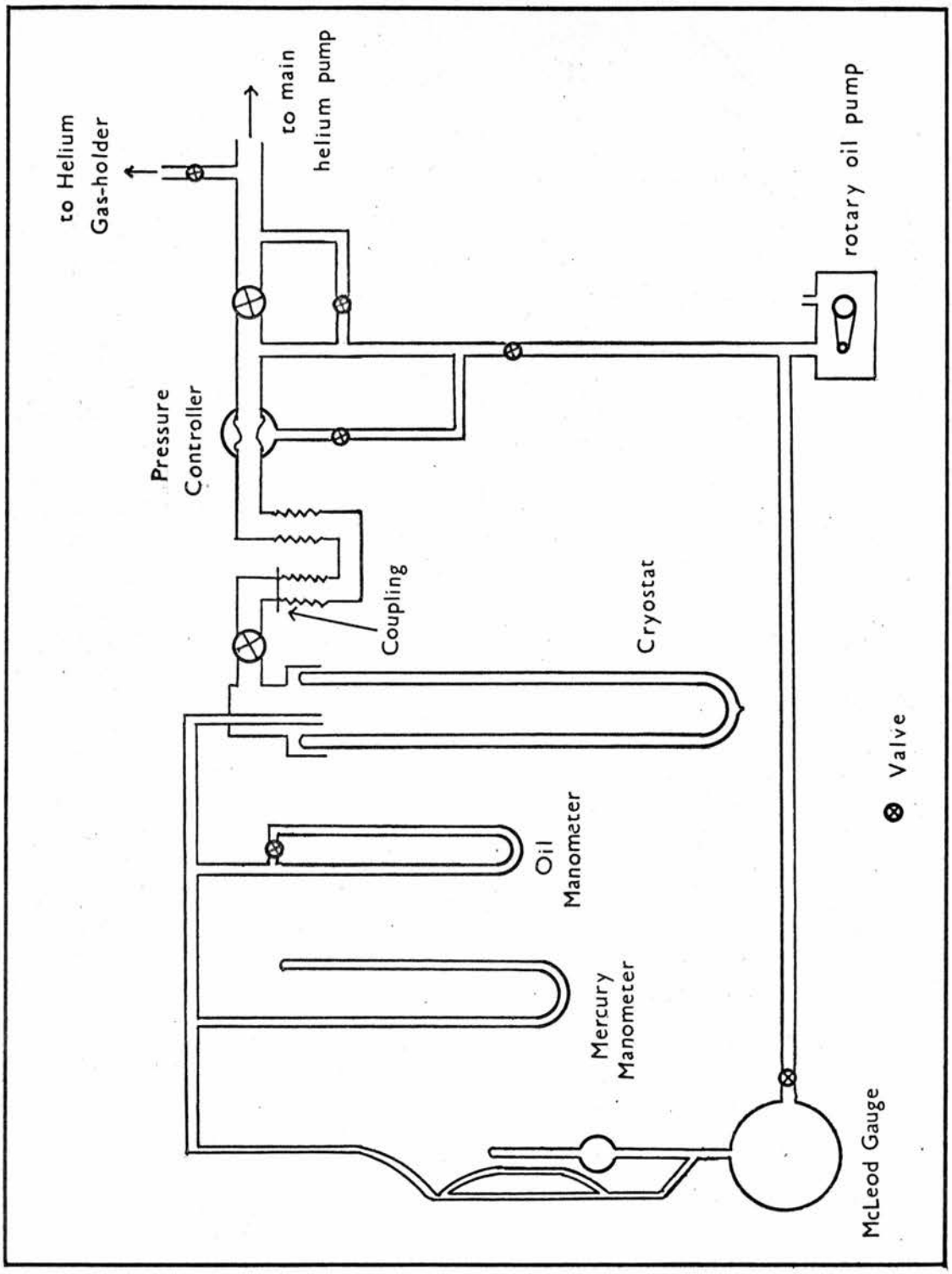


Figure 5.2.3.1 Helium^4 Gas-Handling System.

Chance ON22 heat absorbing glass.

Illumination was provided by a twelve volt incandescent bulb screened by a water cell and windows of Chance ON22 glass, and run at a low level of brightness. With these precautions no thermo-mechanical effects were detectable.

Electrical connections from within the radiation shield were made to the base of a valve holder securely attached to the top of the shield. The leads from the top of the cryostat were taken to the pins of a miniature valve, the glass envelope of which had been removed. This arrangement allowed the radiation shield to be conveniently dismantled for whatever changes were necessary between experiments.

5.2.3 Ancillary Equipment.

The gas handling system is shown in Figure 5.2.3.1. A rotary oil pump, producing an ultimate vacuum of the order of 1 torr, was used to evacuate the helium dewar for several days prior to filling with liquid helium. The cryostat was never diffusion pumped as it was unsuitable for this treatment.

During a run the pressure in the cryostat was reduced using an Edwards 'Speedivac High Vacuum Pump'. The speed of pumping was controlled by a 1 in. Saunders valve in conjunction with a needle valve connected in parallel. Steady pressures were maintained by means of a semi-automatic device (Walker, 1959). This consisted of an extremely flexible rubber tube which formed part of the main pumping line. This tube was surrounded by a closed chamber which was normally connected to the pumping line at a point nearer the

pump. When the desired pressure was reached, the chamber was sealed using a valve. The cross-sectional area of the pumping line then varied in such a way that the pressure above the helium bath remained equal to the reference pressure trapped in the closed chamber. With this device the variation of temperature was never greater than $\pm 10^{-2}$°K.

The main pumping line could be connected to storage tanks containing helium at atmospheric pressure.

Temperatures were calculated from the 1958 (van Dijk and Durieux) scale of vapour pressures. Pressure measuring devices included a mercury manometer, an oil manometer and a McLeod Gauge. The oil manometer was used to monitor small pressure changes in the cryostat, while the McLeod Gauge was used for accurate pressure measurement. Most of the experiments were carried out at the lowest helium bath temperature of about 1.2°K. In this region the pressure could be read to an accuracy of 0.001 torr.

5.2.4 General Procedure.

Helium gas at atmospheric pressure was introduced into the cryostat from the return line. Once the cryostat had been pre-cooled to liquid nitrogen temperatures, the manometer tube and pumping line were sealed off and disconnected. The cryostat was then taken to the liquifier.

After filling, the pumping line was re-connected and evacuated with the rotary oil pump. The helium return line valve was opened. The 1 in. Saunders valve on the cryostat head was opened at the same time as the manometer tube was re-connected; at this point,

the helium was boiling at atmospheric pressure and was therefore at a temperature of 4.2°K. Reduction in temperature was achieved by slowly opening the main pumping line to the helium bath.

A single fill of liquid helium lasted generally from 10 to 16 hours. The heat input to the cryostat, estimated from the rate of fall of the bath level, lay between 24 joules/hour and 36 joules/hour depending on the position of the helium level in the dewar.

5.3 The Helium^3 Cryostat.

5.3.1 The Cryostat.

The measurements of film transfer below 1°K to be reported in Chapter 8 were made in a cryostat in which a closed bath of helium^4 was cooled by a small quantity of helium^3 evaporating under its saturated vapour pressure. By reducing the vapour pressure of the helium^3 to about 0.138 torr, a temperature of 0.49°K was obtained. The helium^3 was pumped with an oil diffusion pump (Edwards 203B) backed by two hermetically sealed rotary oil pumps (Balzers DUO 1). Two litres of gas at N.T.P. provided about 3 cc of liquid.

The main difference between this cryostat and that already described lay in the cryostat cap and the apparatus it supported. The glass dewars and the means whereby they were supported were identical to those of the helium^4 cryostat. Moreover, the connections to the helium^3 cryostat head were the same as to the helium^4 cryostat head, except that a rotating plate valve was used in place of the 1 in. Saunders valve in the horizontal side-arm.

Within the helium dewar, and immersed in the main bath of helium^4, were three separate coaxial vessels. These were the

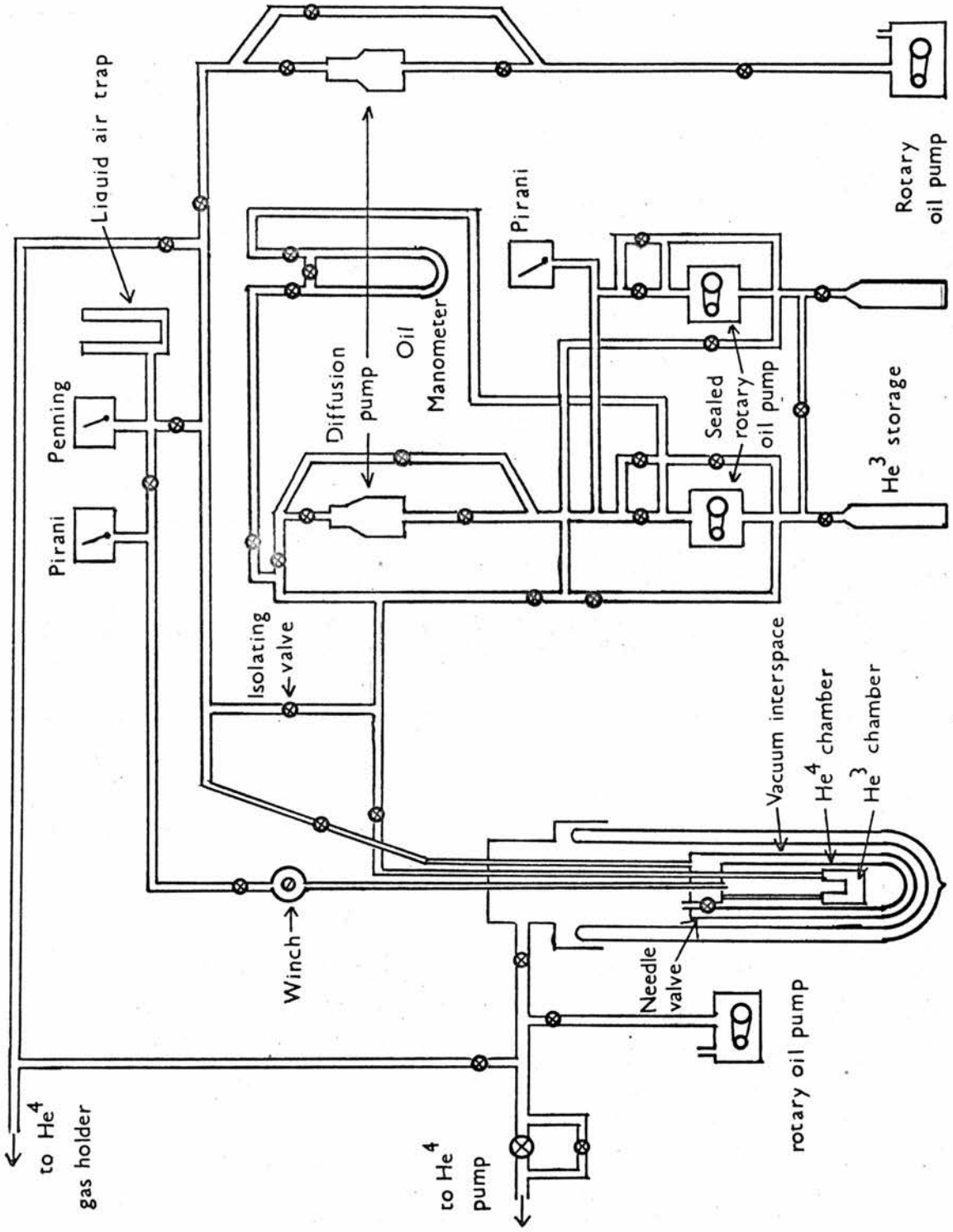


Figure 5.3.1.1 Schematic of Helium^3 Cryostat and Gas-handling System.

helium^3 chamber, the helium^4 experimental chamber and the high vacuum chamber. A diagrammatic representation of the cryostat and gas-handling system is shown in Figure 5.3.1.1.

The helium^3 chamber was an annular copper vessel supported from the top plate of the experimental chamber by the helium^3 pumping line (0.121 in. i.d. stainless steel tube) and a 0.125 in. diameter stainless steel rod. The outer diameter of the helium^3 container was 0.75 in. and the inner diameter was 0.375 in. The top of the container was 61 cm below the top of the inner dewar.

Surrounding the helium^3 chamber was a tight-fitting Kodial glass vessel to contain the helium^4 under investigation. This vessel was 2.3 cm in inner diameter and 12 cm long. Braised to the Nilo-K seal at the top of the glass was a flanged ring of stainless steel. The flange was screwed to a plate supported from the cryostat cap. The joint was made tight with an indium O ring. It was arranged that the vessel could be filled with helium^4 to the desired level by a needle valve operated from the cryostat cap.

The observation beaker was rigidly mounted so as to move up and down within the experimental chamber by means of tubes free to move on the supports of the helium^3 chamber.

The high vacuum chamber was a somewhat larger Kodial glass vessel surrounding the helium^4 vessel, and supported by a stainless steel flange screwed to a plate supported from the cryostat cap. This vessel was 21 cm long and 3.75 cm in inner diameter and just fitted inside the helium dewar. The plate on which it was mounted was 45 cm below the top of the dewar. The pressure in this

chamber (the vacuum interspace) could be reduced to, and maintained at, $< 10^{-5}$ torr.

Both the vacuum interspace and the helium^4 container were surrounded by 0.005 in. copper sheet with viewing slits 3 mm wide. Radiation influx was further avoided by windows of 3 mm thick Chance ON22 glass mounted on the outer radiation shield.

The cryostat cap had six connections through it. These were the filling entry, the tube to the manometer system monitoring the pressure in the main helium^4 bath, a control for the needle valve, a winch tube, the helium^3 pumping line and the vacuum interspace pumping line.

The rod controlling the needle valve, which allowed access of liquid to the experimental chamber, passed through a rubber gland in the cryostat cap.

The winch was identical with that described previously. The lower end of the winch tube, between the top plates of the vacuum interspace and the helium^4 chamber, was constricted to a diameter of 0.009 in. to reduce the heat leak by film flow. In addition, nylon fishing line of diameter 0.006 in., rather than stranded terylene thread, was used to minimize the perimeter available for film flow. A connection was made between the casing enclosing the winch drum and the high vacuum pumping system. This arrangement allowed alternative pumping access to the experimental chamber prior to a run.

The connection from the helium^3 chamber to the helium^3 gas-handling system was by a rotating plate valve. There was a join

in the pumping line beyond this valve to allow the cryostat to be disconnected and taken to the liquifier. A similar arrangement was adopted with the connection from the vacuum interspace.

Small mercury manometers attached to the helium^3 pumping line and the vacuum interspace pumping line enabled the pressure of exchange gas to be observed.

5.3.2 General Procedure.

During the procedure outlined below the vacuum interspace and helium^3 chamber were continuously checked for air leaks, He I leaks and He II leaks.

The vacuum interspace and helium^3 chamber were evacuated to $< 10^{-5}$ torr. The helium dewar and experimental space were evacuated to a rough vacuum. Following isolation of the helium^3 system from the high vacuum system, the vacuum interspace was isolated from the diffusion pump. Helium^4 from the return line was allowed into the vacuum interspace to a pressure of about 5 cm mercury. The high vacuum line was re-evacuated and opened to the helium^3 system. Helium^4 at atmospheric pressure was allowed into the cryostat and experimental chamber.

After pre-cooling to liquid nitrogen temperatures, the dewar and experimental chamber were roughly evacuated. The needle valve to the experimental chamber was closed and helium^4 at atmospheric pressure was allowed into the dewar. All connections at the cryostat head and cap were shut off. The cryostat was disconnected at the screw couplings adjacent to all valves on the cryostat head.

After the main bath had been filled with helium^4, the helium^4 pumping line was re-connected and the usual procedure followed to connect the helium^4 bath to the helium^4 gas handling system. The lines from the vacuum interspace and the helium^3 chamber were re-coupled to the high vacuum system. The vacuum interspace and helium^3 bath were evacuated for about an hour to $< 10^{-5}$ torr. The helium^3 system was isolated from the high vacuum system. The temperature of the helium^4 bath in the cryostat was reduced to the lowest obtainable by pumping with an Edwards 'Speedivac High Vacuum Pump'. The experimental chamber was filled to the desired level with helium^4 by opening the needle valve to the main bath.

Helium^3 from the storage cylinder was allowed to condense into the helium^3 bath. The temperature in the experimental chamber was lowered by pumping the helium^3 back into storage with a diffusion pump backed by sealed rotary oil pumps. The temperature was maintained constant by manual regulation of the pumping speed. Temperatures were determined from vapour pressure measurements according to the 1962 (Roberts, Sherman and Syndoriak, 1964) helium^3 scale of temperatures.

CHAPTER 6.

Transfer by the Normal Film at 1.2°K.

6.1 Introduction.

As a preliminary to investigating enhanced transfer rates in the helium film, a study was made of normal rates and in particular the reason for the variability of measured rates in successive transfers.

A normal rate is defined as that observed when the film is a normal one. In all measurements of normal rates care was taken to ensure that the levels of liquid in both beaker and bath were never closer to the rim than 5 mm. Thus, at the start of a sequence of normal transfers the film on the walls and rim of the beaker can only be formed by superfluid creep.

6.2 Procedure and Apparatus.

To obtain a normal rate of filling, an empty beaker was partially immersed in the helium bath. Lifting the beaker after the levels had come to equilibrium allowed the normal rate of emptying to be determined. The observations were always arranged to cover the same region of the beaker, from 6 mm to 12 mm below the rim.

The beakers were usually of between 0.9mm and 1.4mm in inner diameter, and of wall thickness about 1mm. In the initial experiments, the length of the beakers was 3cm but this was later reduced to 2cm to facilitate complete emptying of the beaker between runs. The finish of the rim, whether flame-polished or ground with

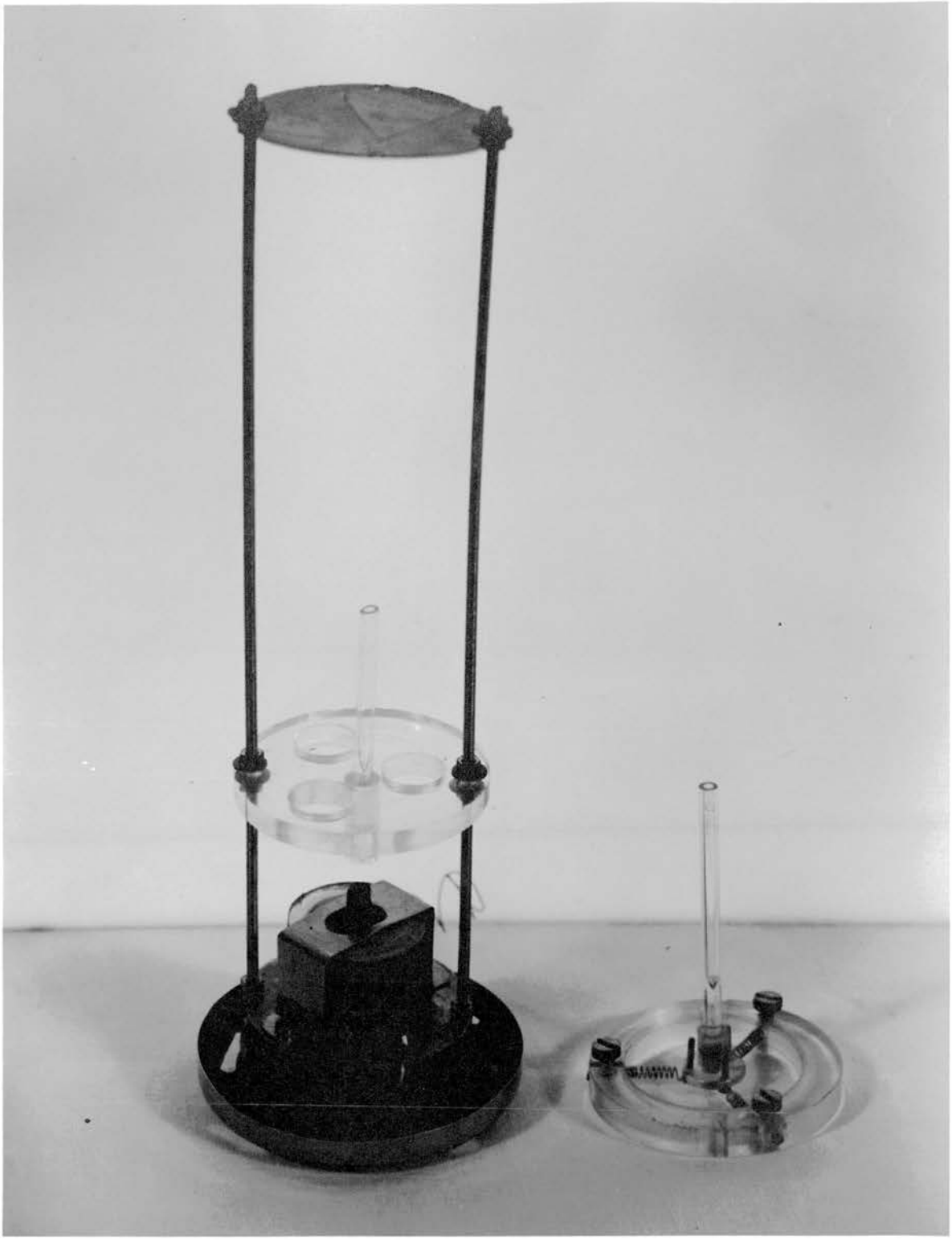


Figure 6.2.1 Apparatus for Measurement of Film Transfer.

Table 6.3.1

Rate of Transfer ($\times 10^{-5}$ cm^2 sec^{-1})	
normal filling	normal emptying
9.47	9.43
9.60	9.05
9.52	9.60
9.69	9.43
9.60	9.39
9.52	9.30
9.25	9.48
9.48	9.27
9.68	9.35
9.73	9.27

Run 1V : Temperature 1.27°K.

Figure 6.3.1 Normal Film Transfer Rates in Different Types of Beaker

Table 6.3.2

Run Number	normal emptying rate		normal filling rate	
	average rate ($\times 10^{-5} cm^2 sec^{-1}$)	coefficient of variation (%)	average rate ($\times 10^{-5} cm^2 sec^{-1}$)	coefficient of variation (%)
I	8.54	1.69	8.60	0.58
II	9.61	0.73	10.07	0.79
III	9.14	1.75	8.95	0.78
IV	9.36	0.53	9.56	0.52
V	9.77	0.61	10.15	0.39
VI	9.83	0.41	9.95	0.60
VII	8.77	0.34	8.83	0.23
VIII	8.70	0.46	8.74	0.23
	8.81	0.45	8.81	0.56
X	9.68	1.03	9.94	0.90
XI	9.49	0.84	9.68	0.72

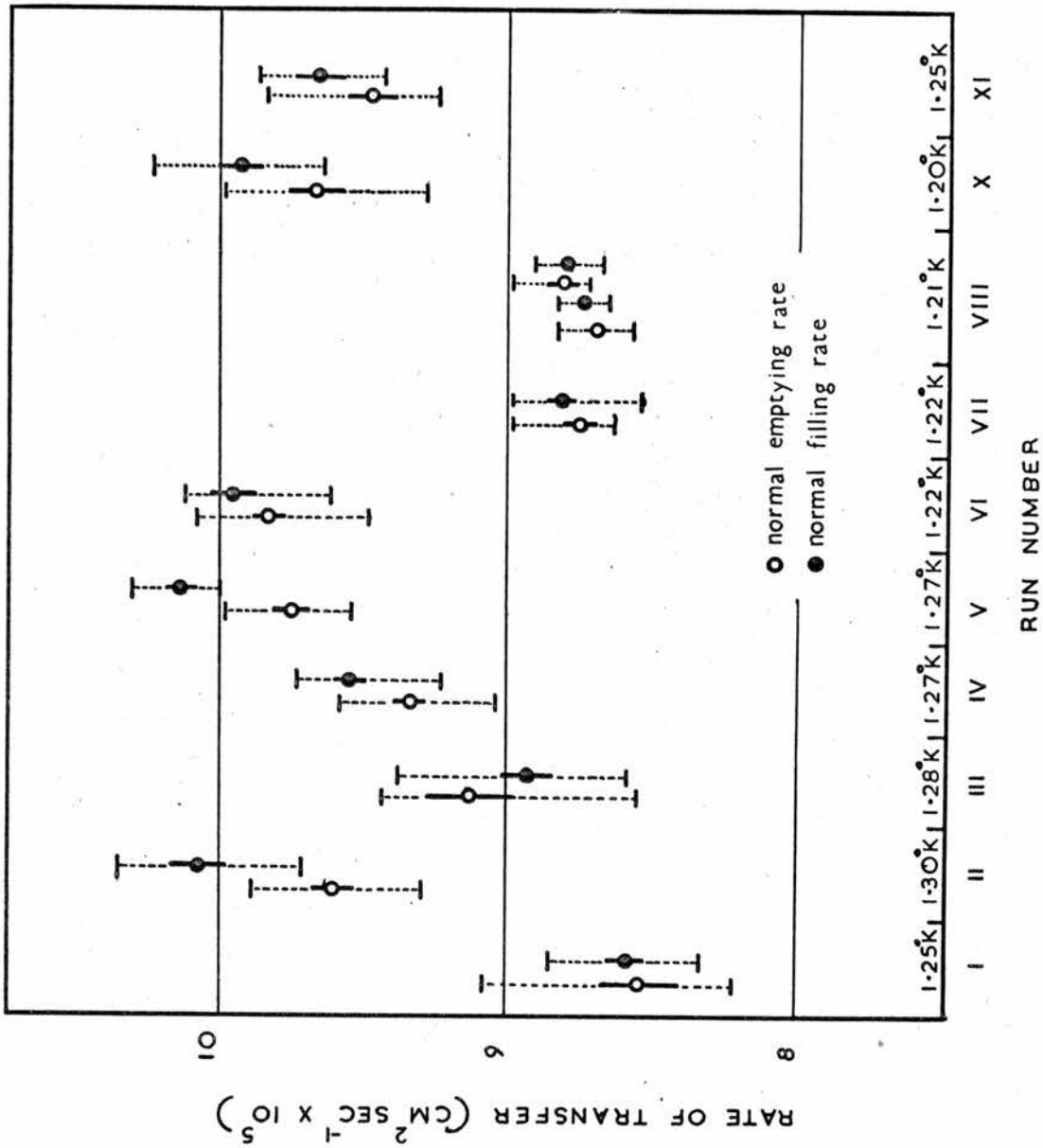


Figure 6.3.2 Magnitude and Scatter of Normal Film Transfer Rates in Open Beakers.

320 grade emergy, appeared to have little effect on the flow rates. Most of the experiments were conducted with ground rims since flame-polishing tended to round off the edges of the rims, thus making exact determination of the governing perimeter difficult.

The beaker chosen for a particular experiment was rigidly supported in a vertical position between screwed wire struts attached to the base plate of the copper radiation shield. An example of the method of mounting is shown in Figure 6.2.1 (a).

6.3 Normal Transfer in Open Beakers.

Many published results reporting film transfer have been obtained using open cylindrical beakers. There remained a need, however, to establish the degree of reproducibility of a sequence of normal transfers in such a beaker. Moreover, improvement on the existing paucity of measurements within a run allows statistical comparison of the variability of normal transfer with that of enhanced transfer.

The scatter in measured values of a typical sequence of normal film transfers is shown in Table 6.3.1 and Figure 6.3.1A. The estimated uncertainty (possible error) arising from observational inaccurracy and plotting error is of the order of 2%. It is clear that the spread of observed values is somewhat larger than this.

Altogether ten runs were carried out using open beakers to investigate the run-to-run variation of the average normal rate and the scatter of normal rates within a run. The results are shown in Table 6.3.2 and Figure 6.3.2. The average value of the measured emptying rate together with that of the filling rate was calculated

Table 6.3.3

Run number	Significant difference between the coefficients of variability of filling and emptying rates?
11, 111, 1V V, V111, X, X1	No
1, V1, V11	Yes

for each run. These averages are the plotted points on the figure. The solid lines parallel to the abscissa extend above and below the average values by an amount equal to the standard deviation (s). In Table 6.3.2 **s** is expressed in terms of the coefficient of variation which is given by $\frac{100\,s}{\bar{\sigma}}$, where $\bar{\sigma}$ is the average transfer rate. The least and greatest values of flow rate contributing to a given sequence are shown in Figure 6.3.2 by horizontal bars above and below each average.

The apparently random range of values of normal rates led us to examine the observations statistically. In particular, we were interested in finding out whether the scatter of filling rates was statistically different from that of emptying rates within a run. Significance tests were applied on the basis of a 4% confidence level, or in other words, a 96% probability that two populations were significantly different. Confidence limits were determined from the coefficient of variation for each sequence of rates. The test then consisted in seeing where the coefficient of variation of one sequence lay relative to the confidence limits of the other sequence in the run. If the coefficient of variation lay outside the limits, there was considered to be a significant difference between the two scatters. On the other hand, if the confidence limits included the coefficient of variation, the difference between the scatters was not significant, at least on the 4% level. The results of the tests are shown in Table 6.3.3. It is seen that only in three cases was there any significant difference between the variability of filling and emptying rates.

Table 6.3.4

Run number	Significant difference between average values of filling and emptying rates?
1, 111, V1, V111	No
11, 1V, V, V11, X, X1	Yes

The difference between the average values of filling and emptying within a run has also been examined statistically. Tabulation of results is given in Table 6.3.4. In all cases where a significant difference exists, the filling rate is greater than the emptying rate.

As can be seen from Figure 6.3.2 there is considerable variation of the average value of normal transfer rate from run-to-run. This variation was observed either with the same beaker or with different beakers. It has been pointed out in Chapter 3 that surface contamination by solid air or other impurity can cause such effects and it is possible that this is the cause of the observed variation of the average rates. The simple beakers were carefully cleaned with Chromic acid before mounting, but this was not possible with the more complicated beakers to be described later. None of the beakers were baked-out before mounting, but prior to runs the cryostat was given prolonged evacuation by mechanical pumps. No systematic variation of transport rate with the length of the initial pumping period was detected. We must therefore concur with the statement made by Smith and Boorse (1955b) that " complete cleanliness appears highly incompatible with the study of surfaces of known microstructure at helium temperatures." It should perhaps be emphasised, however, that this variation of transfer rate with contamination is of secondary importance as far as this thesis is concerned. The data presented in this section are still of use as control measurements for later work. Moreover, the large variation which we have found to exist from run-to-run in no way invalidates

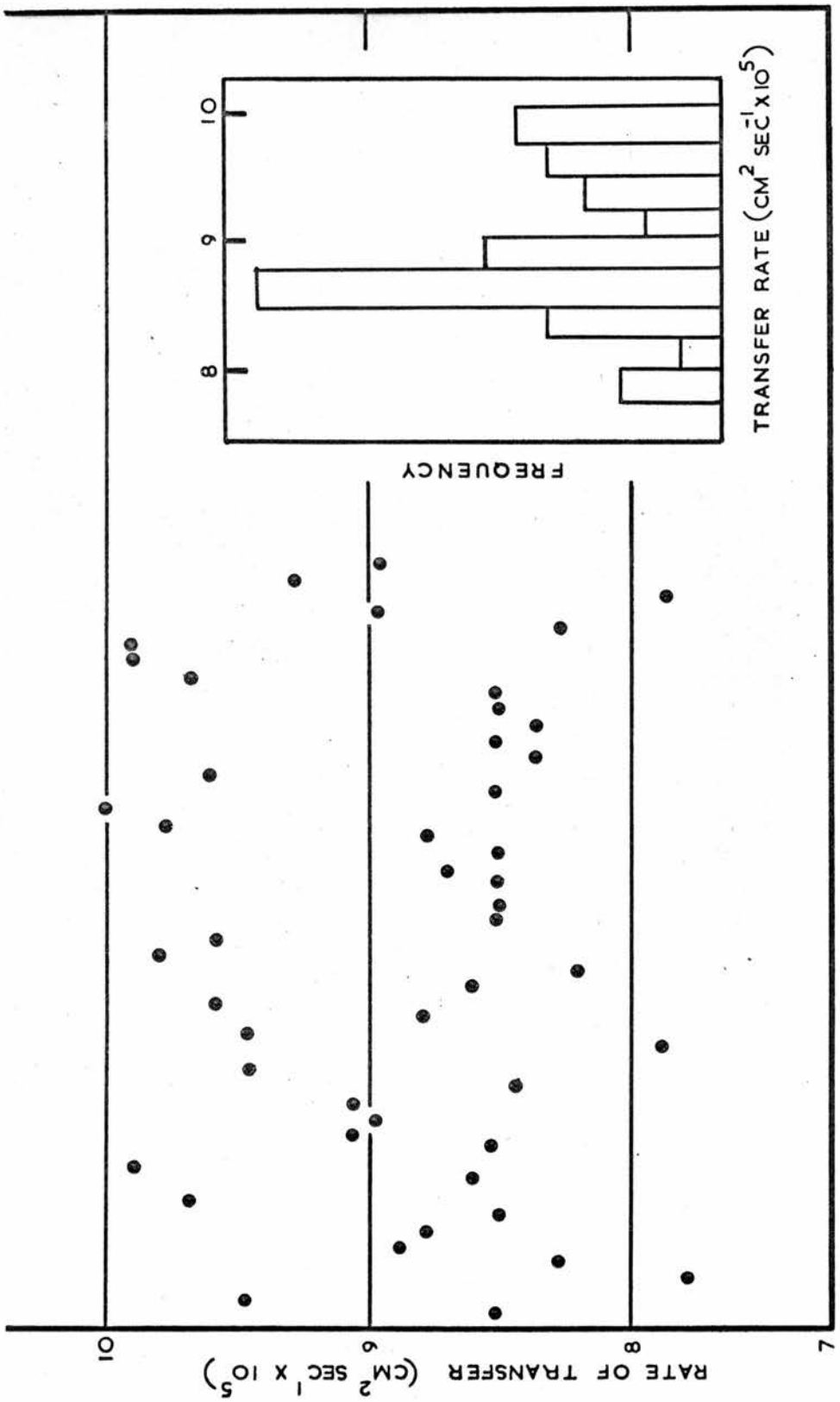


Figure 6.4.1 Variation of Filling Rate within a Single Transfer.

any investigation of scatter within a single run.

6.4 Closer Examination of Normal Transfer in an Open Beaker.

As has been described, the usual method of determining the rate of meniscus motion consequent upon film transfer was to note the successive time intervals as the inner level passed fiducial marks on the eyepiece of the cathetometer. Using the cathetometer with the 100 division graticule spanning a field of view of 1.2 cm, the position of the meniscus was generally marked every five divisions as the beaker filled or emptied. During normal transfer the variation of meniscus level with time (for depths below the beaker rim of between 0.6 and 1.2 cm) was linear within the accuracy of the plotted points.

A few measurements, however, were carried out in which the time was noted as the meniscus passed every division of the graticule (i.e. every 0.12 mm). In these cases, successive time intervals were too short to allow accurate measurement by simple 'clock-watching'. If, however, the pen of a constant speed recorder was triggered manually every time the level passed a division the time intervals were reduced to a series of lengths on chart paper. Calibration with respect to time was checked using a scalar to inject pulses onto the paper at one second intervals.

Film transfer in an open beaker was studied in this way with a Kipp Micrograph BD2 recorder running at a chart speed of 7200 mm/hour. The manner in which the film-filling rate varied as the inner liquid level rose between successive divisions of the graticule is shown in Figure 6.4.1. The frequency with which a flow rate

occurred throughout the observed range of values is indicated by the histogram. It is clear that the range over which this sequential variation of flow rate occurred is greater than any single range depicted in Figure 6.3.2. It is, in fact, of the order of the total range covered by all the normal rates examined in open beakers. To be more specific, the mean deviation within one film-filling is 5.4%, while for a sequence of rates (such as given in Table 6.3.1) it is of the order of 1.5%. We must therefore conclude that the transfer rate measured over single divisions of the eyepiece scale varied in a random way with a scatter larger than that among a group of successive transfers.

In this connection a further observation is of interest. When film transfer was observed under high magnification, the beaker level was seen to move in an uneven manner. Indeed, at times the motion took the form of a series of jumps rather than a steady rise or fall.

6.5 Reduction of Scatter of Normal Transfer Rates.

The presence of a certain amount of bath surface agitation was noticed during all the experiments reported in this dissertation. This is presumably caused by unavoidable vibration of the cryostat which is necessarily mechanically linked to its surroundings*. Since it was suspected that agitation of the bath surface might have influenced the transfer rate, two other types of beaker were constructed so as to excluded or diminish this kind of disturbance. These are shown in Figure 6.3.1B, C_1, C_2 together

Footnote*. Unfortunately, the laboratory is housed in an unstable Victorian semi-detached house.

Table 6.5.1

Type of beaker	average transfer rate ($cm^2\ sec^{-1}$)	mean deviation (%)	temperature (°K)
A	9.46×10^{-5}	1.47	1.27
B	7.35×10^{-5}	0.7	1.22
C_1	8.17×10^{-5}	0.9	1.20
C_2	7.65×10^{-5}	1.9	1.22

with representative groups of transfers made with them. B was an inverted U-tube of which one arm was extended by a standpipe about 0.4mm in diameter. The rim at the top of the standpipe was further reduced to a diameter of 40 microns. Brief complete submersion of the beaker partially filled one arm of the U-tube after which normal film transfer took place into the other arm. This device completely prevented the bath surface from influencing film transfer, but the U-tube was difficult to empty, so making accumulation of data tedious.

In both C_1 and C_2 an open beaker was enclosed within a cylindrical hood; C_1 had an annular gap of 0.2mm while in C_2 the annular gap was 2.0mm. This type of beaker was the most successful since the bath surface influence could be greatly diminished, and both normal and enhanced transfer could be studied with the same ease as in a simple open beaker. Unfortunately the magnitude of the surface tension rise of liquid helium within an annular gap imposed a practical limitation on the size of the gap. Thus the film could not be completely isolated from the main bath in beakers of this design.

Table 6.5.1 shows the results of normal transfer rates measured in various beakers. The scatter of transfer rates in a sequence of successive transfers is given in terms of mean deviation. It can be seen that transfer in the inverted U-tube exhibits the lowest scatter. It is of interest to compare this scatter with the scatter in observations of helium flow through very narrow channels where bath surface conditions play no significant role.

Watmough* has quoted a value of 0.5% for the mean deviation of flow through a 50 mμ wire-filled channel.

The results from beakers C_1 and C_2 show that the scatter depended strongly on the size of the annular gap; it required a gap as small as 0.2mm in width to reduce the scatter to that comparable with the U-tube. Since bath surface waves could not penetrate the annuli in type C beakers whatever the annular width, it must be presumed that the wider annuli, which are equivalent to tubes of 4 to 5 mm diameter, are themselves acting as small baths capable of exhibiting a surface wave mode.

The role of random disturbance of the bath surface is further demonstrated if the results of film flow in an inverted U-tube with a capillary opening at the top of the tube are compared with those in type B beakers. The mean deviation of transfer rate is increased roughly three-fold if the standpipe which constitutes a liquid lock between the film and the bath is removed and replaced by such an opening.

It is clear from Figure 6.3.1 that there is extensive variation of the average value of the transfer rate measured in different beakers. Earlier in this chapter such variation was attributed to surface contamination. Of all the beakers used, the cleanest was undoubtedly B since the operative surface is completely internal; and indeed, this beaker showed the lowest normal transfer rate of $7.35 \pm 0.05 \times 10^{-5}$ cm^2 sec^{-1}, which is in the neighbourhood of the generally accepted value for transfer over a clean surface at 1.2°K.

Footnote*. D. J. Watmough, private communication.

6.6 Discussion.

It is apparent that bath surface agitation in the form of surface waves impinging on the base of the film can influence the normal rate of transfer in the film. In simple open beakers, surface waves on the main bath will cause the outer meniscus to move up and down at the base of the film. Well away from the beaker this oscillatory motion will be of the whole fluid and therefore isothermal. Close to the beaker, however, where the meniscus merges with the film the motion of the normal fluid will be negligible and any remaining perturbation will be of the superfluid alone. Such a motion would constitute a source of third sound.

Hitherto, third sound has only been produced by periodic heat input to the film. There is, however, no reason to suppose that it cannot also be generated by liquid movement of the type described above.

The oscillatory motion of the meniscus due to bath waves was normally less than 10 cps. Since the velocity of third sound at 1.2°K is about 50 cm sec^{-1} (Everitt, Atkins and Denenstein, 1964), a wavelength of several cm is to be expected.

The presence of third sound in the film would allow a significant increase of mass transfer similar to that observed in classical liquid films (Portalski, 1960), or in other words to wave motion in a shallow liquid. Tilley and Kuper (1965) have calculated that extra mass transfer by a travelling wave of third sound could occur to the extent of 10% of the normal film transfer rate, assuming a

wave amplitude of one quarter the film thickness. This appears to be the most likely mechanism giving rise to the frequent quick changes of rate observed within a single transfer and to the scatter of rates found in a group of successive transfers. Mass transfer by wave action would increase the rate of filling of a beaker and decrease the rate of emptying. It is possible that the difference between the average values of filling and emptying rates which we have sometimes observed in open beakers is due to third sound, sporadically generated by bath surface agitation.

Table 7.1.1

Enhanced rate of transfer ($\times 10^{-5}$ cm^2 sec^{-1})
11.96
11.54
11.75
11.16
11.92
11.41
11.41
11.50
12.00
12.14
11.41
11.20
11.58
11.80
11.88
11.58
11.92
11.84
11.84
11.37
11.62
11.33

Run IV : Temperature 1.27°K.

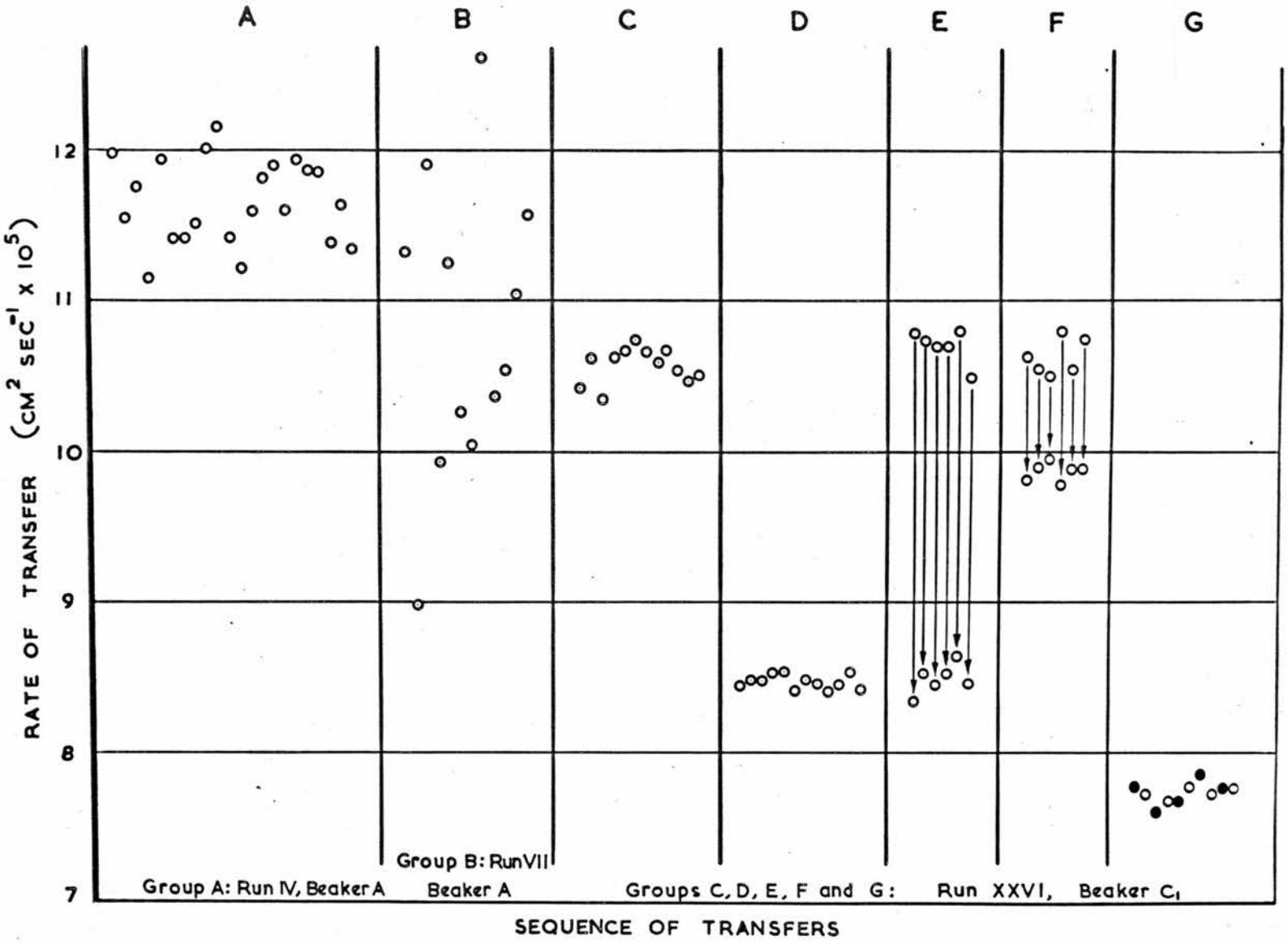


Figure 7.1.1 Enhanced Film Transfer at 1.2°K

Table 7.1.2

Run number	average enhanced emptying rate ($\times 10^{-5}$ cm^2 sec^{-1})	coefficient of variation (%)
1	10.80	4.53
11	10.58	0.85
1V	11.64	0.51
V	11.59	1.81
V1	12.92	1.62
V111	11.49	1.49
X1	13.61	0.82

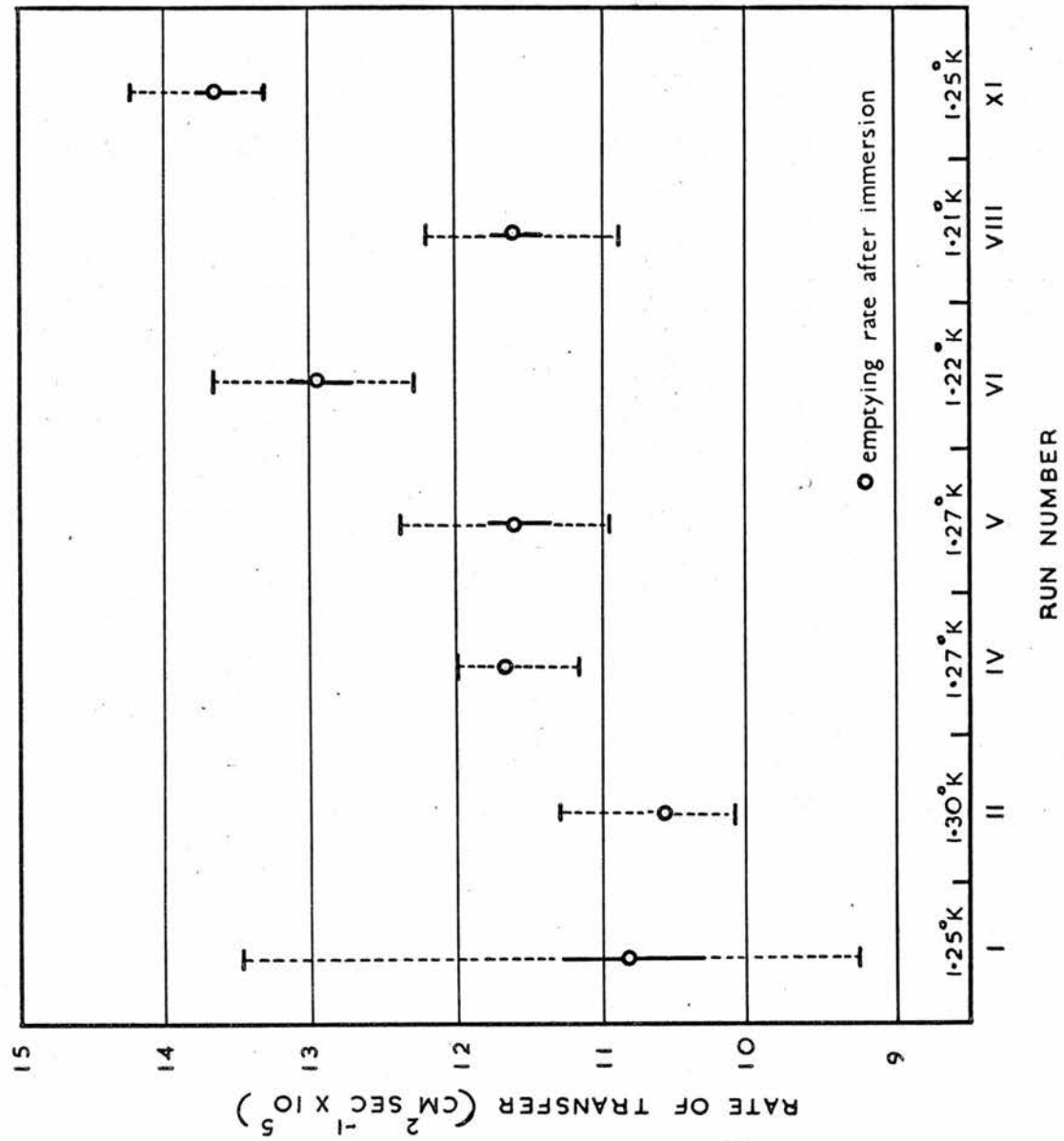


Figure 7.1.2 Magnitude and Scatter of Enhanced Transfer Rates in Open Beakers.

CHAPTER 7.

Enhanced Transfer Rates at 1.2°K.

7.1 Enhanced Transfer in Open Beakers.

We now consider the enhanced transfer rates out of a beaker filled by plunging it beneath the bath and raised so that the rim was 1.5 cm above the bath level. Flow rates were calculated from data taken from the same range of the beaker as that over which normal film transfer was determined, i.e. from 6mm to 12mm below the rim. The procedure adopted is shown by stages 1 and 2 of Figure 4.2.1.

Typical results are shown in Table 7.1.1 and Figure 7.1.1. Group A shows a succession of enhanced transfer rates, each following a plunge-fill, out of a simple beaker of type A, Compared with the normal transfer rates of group A, Figure 6.3.1, they show that the enhanced rate was about 20% greater than normal and much more scattered.

In seven of the ten runs in which normal rates in open beakers were determined (see Figure 6.3.2) enhanced rates were also measured. The results are shown in Table 7.1.2 and Figure 7.1.2. As in Figure 6.3.2 the average value, standard deviation and maximum and minimum value of a sequence of transfers is shown in the figure. A comparison was made between the standard deviation of a sequence of enhanced transfer rates and that of normal rates in the same run. In all cases where there was enough data to be statistically significant, the difference between the standard deviations was significant; enhanced transfer appears to be less reproducible than

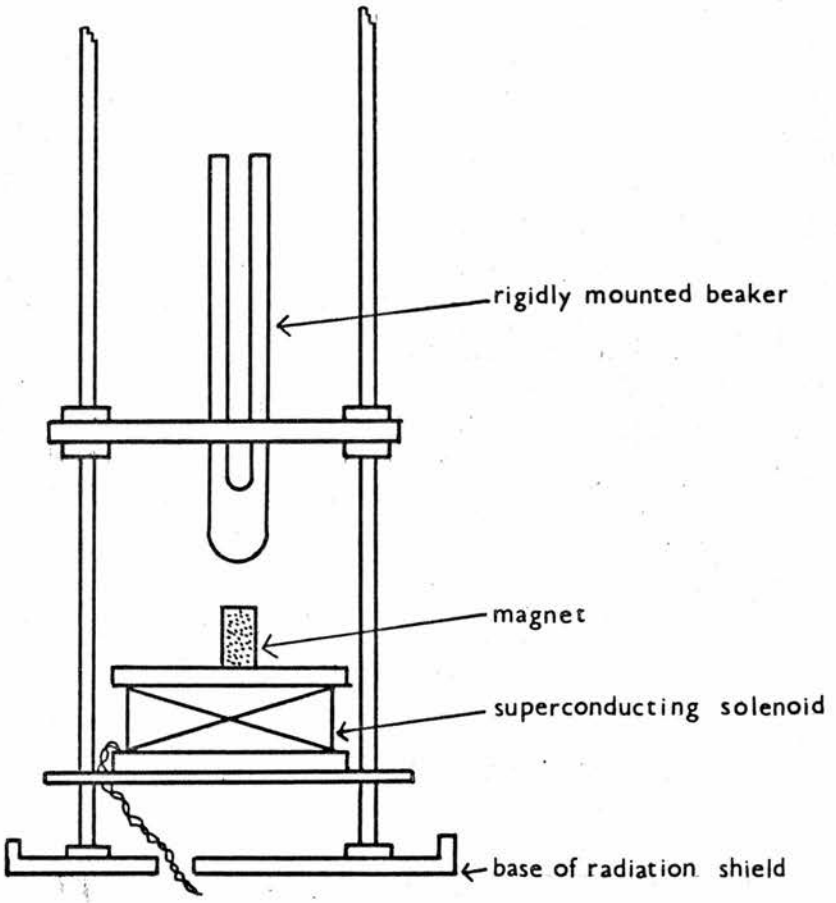


Figure 7.1.3 Apparatus for Local Vibration Experiments.

normal transfer in an open beaker.

An anomalously large scatter of efflux rates from a plunge-filled beaker has also been reported by Allen (1960b) and Webber, Fairbank and Lane (1949).

This irreproducibility was at first very puzzling until it was noted by chance that vibration of the cryostat seemed to influence enhanced transfer. Group B of Figure 7.1.1 was made with an open beaker, but in this group there was appreciable casual cryostat disturbance coming from background vibration of the building. In several of the transfers the cryostat was deliberately shaken using an electric buzzer mounted on the cryostat head. The scatter is very pronounced in this group, indicating a close connection between cryostat disturbance and reduction of enhanced rate. Indeed, the transfer rate was frequently seen to alter abruptly to a lower value in the middle of an observation, the change coinciding with an imposed vibration of the cryostat.

To obtain a clearer idea of the behaviour of flow rates under vibration, a small magnetically-operated hammer was used to vibrate the beaker only and not the whole cryostat. The apparatus was a modification of that used by Miss Walker (1962), and is shown in Figures 7.1.3 and 6.2.1 (a). The hammer was a cylindrical barium ferrite magnet (Mullard Magnadur, type FD 288) which moved in and out of the magnetic field of a superconducting solenoid mounted axially with and directly under the beaker. To lessen friction, the magnet was tightly enclosed in a perspex sheath. The solenoid, constructed of niobium wire, was of the type to be described in

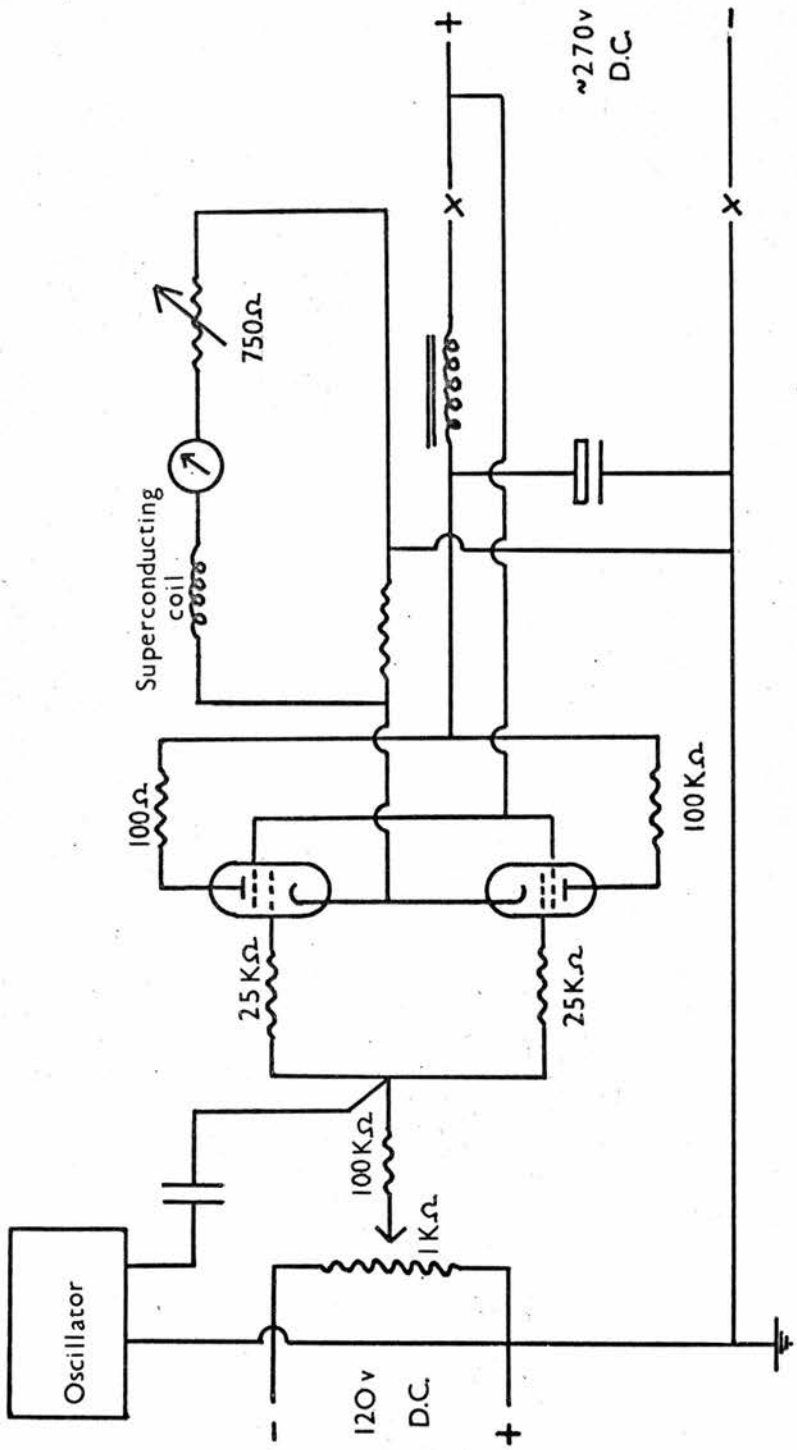


Figure 7.1.4 Electrical Circuit for Magnetically-operated Hammer.

Figure 7.1.5 Influence of Local Vibration on Enhanced Transfer in an Open Beaker.

Chapter 10. The frequency and magnitude of the taps given to the base of the beaker by the magnet at the top of its travel were varied using a low frequency oscillator and the circuit of Figure 7.1.4 which produced an alternating current superposed on a steady direct current. The tapping was of undetermined magnitude, but could have given momentary accelerations of the order of 10 g to the beaker.

The influence of local vibration on film transfer rates in an open beaker of type A is shown in Figure 7.1.5. The first two groups of flow rates are (A) normal film transfer and (B) enhanced transfer after plunge-filling. No deliberate vibration was present during these measurements. The flow rates in group C were determined after plunge-filling followed by magnetic tapping of the beaker prior to measurement. The figures above each symbol refer to the number of taps given to the base of the beaker at a rate of 1 per second. In view of the large spread of values of enhanced transfer after successive immersions followed by an identical number of taps, one is not justified in drawing quantitative conclusions from this experiment. Nevertheless, it can be inferred that there is a strong tendency for repeated tapping to produce a cumulative decrease in enhanced transfer rate towards the normal value.

Further attempts to establish a less tenuous connection between vibration and diminution of enhanced transfer in an open beaker have failed, the results being typified by their irreproducibility rather than their usefulness. It was realized, however, that the irreproducibility stemmed in part from the inherently large scatter

which is present in all transfer rate measurements in open beakers.

7.2 Enhanced Transfer in Hooded Beakers.

Following the successful introduction of hooded beakers into the investigation of normal film transfer, a series of measurements of enhanced transfer was undertaken in this type of beaker. For ease of comparison with the results obtained using open beakers, the results are shown in Figure 7.1.1.

Groups C, D, E, F and G were all made on the same day with the hooded beaker C_1. Care was taken to ensure that cryostat disturbance was as small as possible. Group G was a control group of normal transfers. The undisturbed enhanced rates, group C, were about 30% higher than the control group but the mean deviation was about the same. Groups D, E and F involved the use of the magnetic hammer with which the beaker could be quite vigorously tapped. In group D, after the beaker had been immersed and raised and while the beaker level was falling towards the observing range, the beaker was repeatedly tapped for about a minute. The subsequent enhanced transfer rate was found to have been reduced consistently to about 10% above normal. In E the undisturbed enhanced transfer was observed for two minutes, the beaker was then given 20 taps in quick succession, after which the transfer rate was found to have been diminished by the same amount as in D. The vertical arrows joining the upper and lower points indicate that they belong to the same transfer. In F the procedure was similar to E but the number of taps given was 5, in quick succession; the drop in rate was less but again uniform.

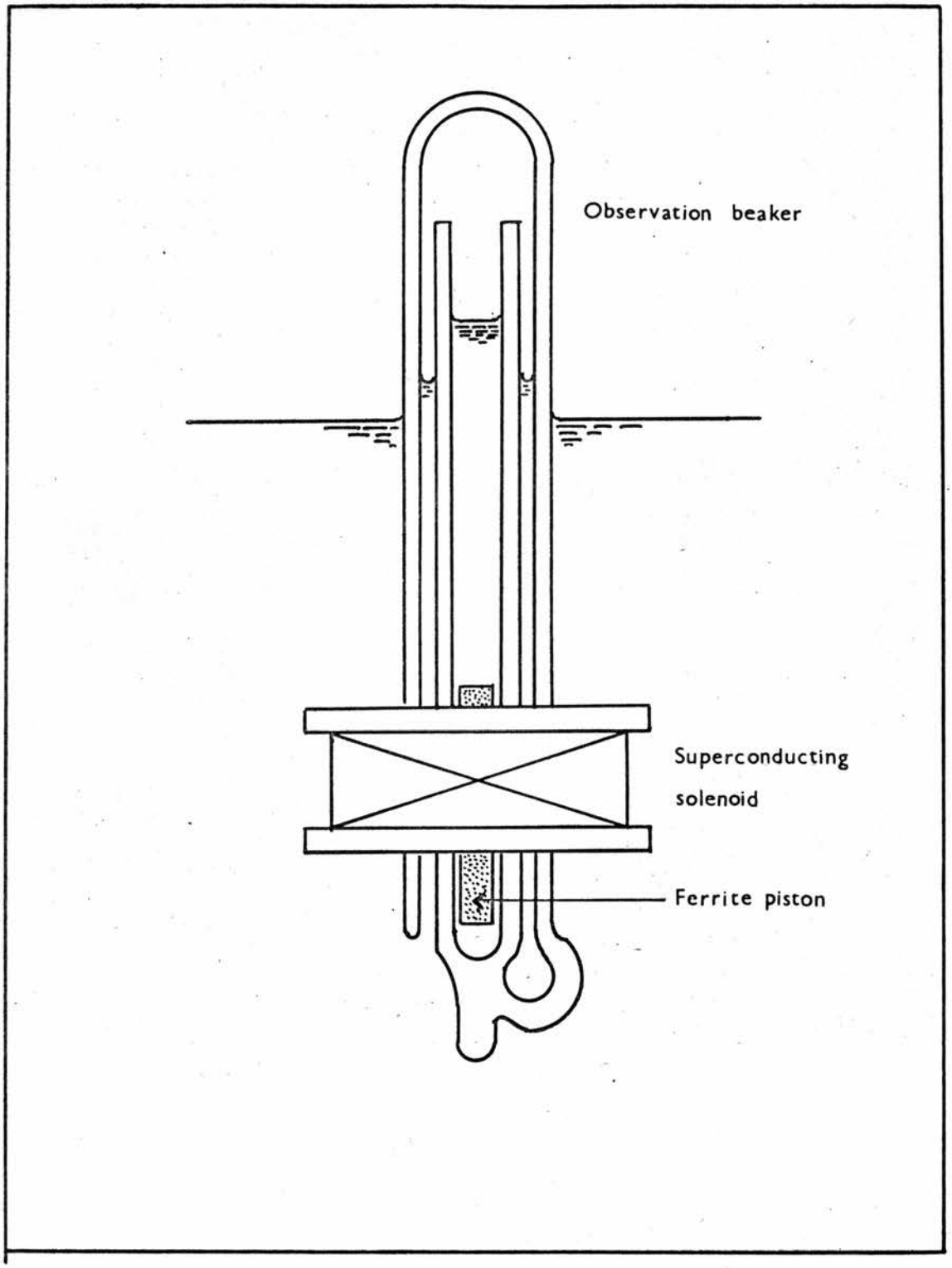


Figure 7.3.1 Apparatus for Agitating the Beaker Level.

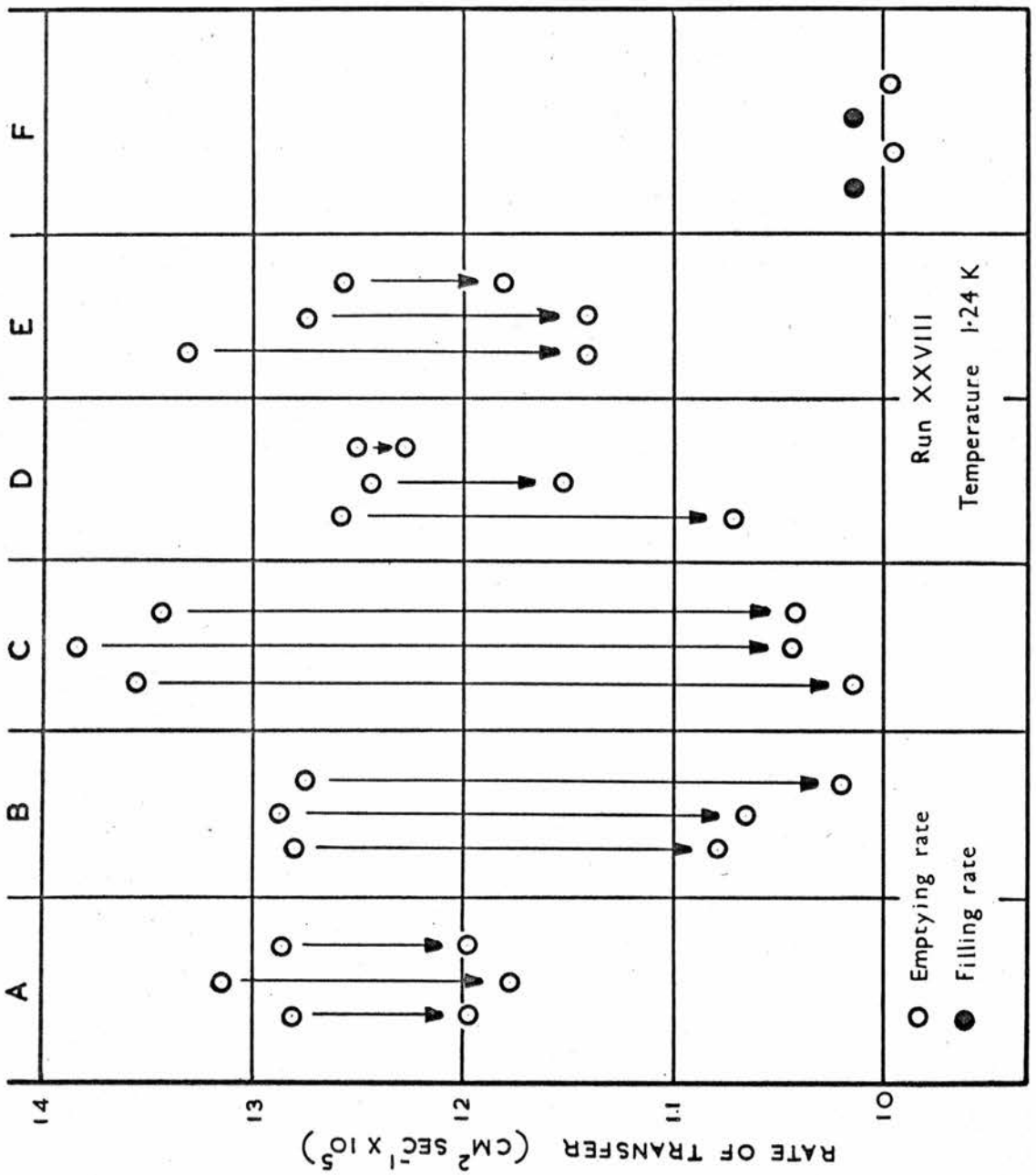


Figure 7.3.2 Enhanced Transfer and Surface Wave Disturbance.

It is difficult to say whether perturbation of the transfer rate was caused by agitation of the beaker or agitation of the liquid meniscus, but it is probably the former since in the beaker C_1 bath surface waves were inoperative and the surface wave disturbance in group B of Figure 7.1.1 produced a much more scattered transfer rate.

7.3 Effect of Surface Wave Disturbance on Enhanced Transfer.

It was inferred from the experiments reported in the previous section that the cause of diminution of enhanced transfer is bodily disturbance of the beaker rather than agitation of the liquid meniscus. It is, however, practically impossible to investigate the effect of the two causes independently. Nevertheless, enhanced transfer after the beaker level alone had been subjected to severe disturbance was studied. The apparatus is shown in Figure 7.3.1. A beaker of slightly greater length than usual was firmly mounted in a superconducting solenoid so as to lie along the magnetic axis. The up-and-down movement of a ferrite piston (Mullard Ferroxcube, type FX 1147) situated within the beaker was controlled by means of the circuit shown in Figure 7.1.4.

Figure 7.3.2 shows a series of transfers all observed in a hooded beaker of type C_1 fitted with a ferrite piston. At all times the top of the piston was never nearer to the beaker level than 2 cm. Group F was a control series of normal transfers. In groups A, B, C, D and E the beaker was totally immersed and lifted until the rim was 1.5 cm above the bath level. The enhanced rate was determined as soon as the inner level fell to the range of measurement. The

piston was then operated for 30 seconds, its motion being of different amplitude and/or frequency for each of the groups. The enhanced transfer rate was again determined. Vertical arrows indicate that the upper and lower points belong to the same transfer. In A the piston moved up and down over a range of 1 cm at a frequency of 3 c/s. Very little disturbance of the beaker level was observed to be associated with this motion. In B and C the frequency was increased to 6 c/s and 10 c/s respectively. Meniscus movement within the beaker was appreciable at 6 c/s and increased still further at 10 c/s. In groups D and E the amplitude of the piston was reduced to 5 mm and the observations were repeated at 6 c/s and 10 c/s. The meniscus was still visibly perturbed, although not to such a large extent as in the previous two groups.

It is clear that least reduction of enhanced transfer rate was obtained at low piston frequencies. Since the inner level was not noticeably disturbed by the action of the piston at these frequencies, it is suggested that the slight reduction observed is due to vibration of the beaker. This is bound to occur when the piston drops to the base of the beaker on switching off the activating current. From the results at higher frequencies it is inferred that agitation of the beaker level results in diminution of the enhanced transfer rate towards the normal value. Qualitatively, the amount of reduction appears to be correlated with the magnitude of the disturbance. Reduction to near normal rates requires a severe surface perturbation. This is borne out by an observation made during the local vibration experiments mentioned in a previous

Figure 7.4.1 Transfer in Spring-mounted and Wide Diameter Beakers.

section. It was noticed that reduction of enhanced transfer rate was increased if the beaker level jerked visibly in phase with the magnetic hammer.

7.4 Transfer in Spring-mounted and Wide Diameter Beakers.

Film transfer rates were measured in an open beaker of the usual diameter which was held in place by a spring-mounting. The beaker assembly is shown in Figure 6.2.1 (b). The springs were wound of 0.0075 in. diameter phosphor bronze wire.

Typical transfers made in this beaker are shown in Figure 7.4.1. Group A consisted of a sequence of normal transfers. In B the beaker was plunge-filled, raised and the subsequent outflow rate observed. Only in three cases was this rate significantly enhanced. It was concluded that the tendency of enhanced transfer to decrease towards the normal value under disturbed conditions was accentuated by the flexible mounting which allowed the beaker to resonate with considerable amplitude.

In another experiment, a simple open beaker of 8 mm internal diameter was used. Normal transfer rates and rates after immersion are shown in groups C and D of Figure 7.4.1. It is clear that this beaker failed to show any evidence of enhanced transfer. It is possible that surface waves which can exist both inside and outside such a large beaker are responsible for the almost complete reduction of enhanced transfer to normal.

7.5 Delay Time Measurements.

Experiments were made to test the persistence of the enhanced rate of emptying after a delay time τ . The results obtained

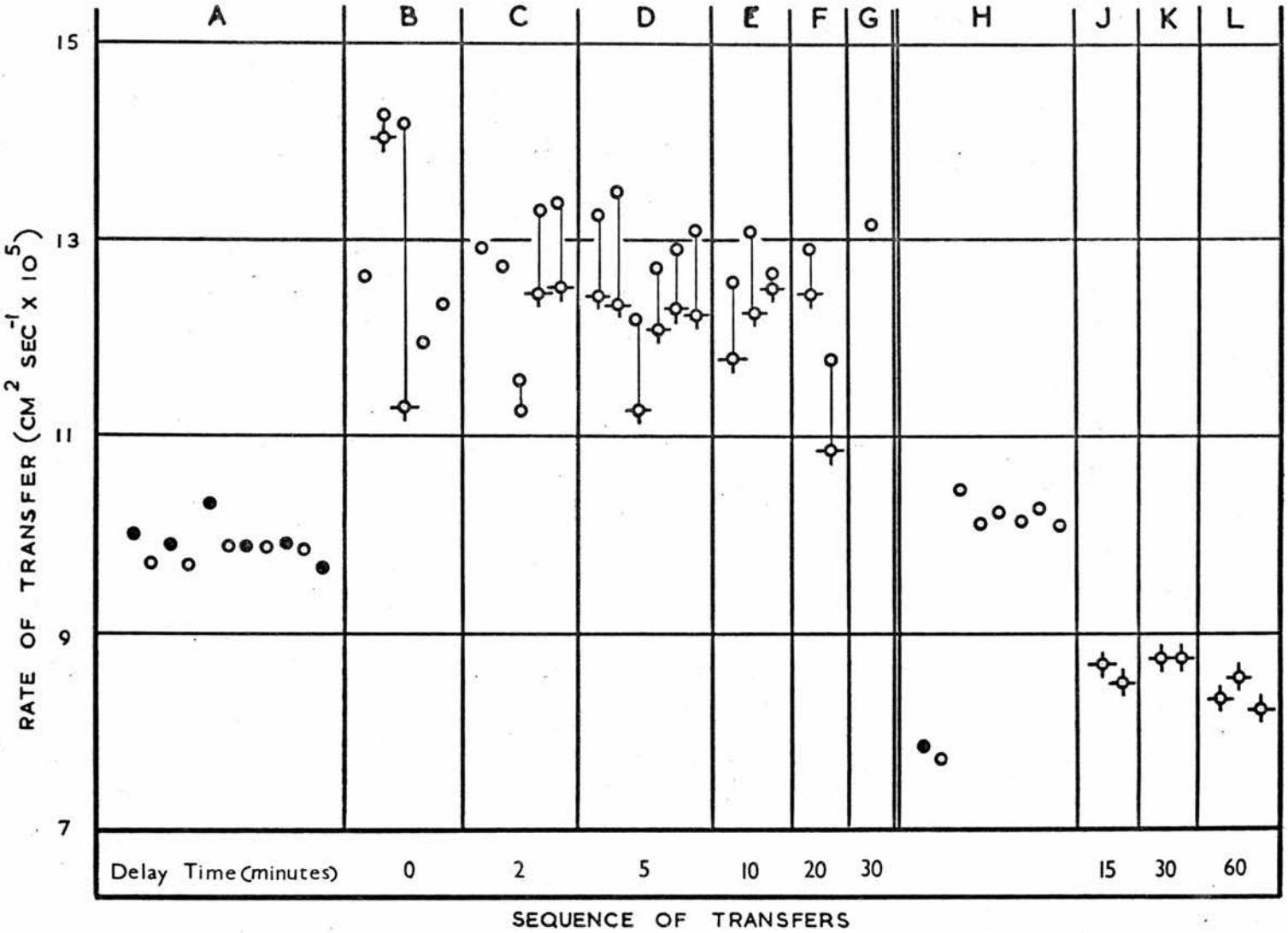


Figure 7.5.1 Delay-time Measurements in Open and Hooded Beakers.

with a beaker of type A are shown in groups B to G of Figure 7.5.1. Group A is a control sequence of normal transfers. After a plunge-fill the beaker was raised and the enhanced rate observed as the inner level fell from 6 mm to 10 mm below the rim. The bath and beaker levels were then equalized. This state was maintained for a time τ after which the beaker was again raised and the enhanced rate determined over the range 10 mm to 16 mm below the rim. Plotted points representing the rate of transfer before and after the delay are connected by a vertical line. In cases where the delay time produced no significant change of rate, only the initial rate is plotted. The delay times are given below each group.

As in the investigation of local vibration in an open beaker, interpretation of results is hindered by the inherent spread of values of transfer rate due to bath surface waves. Hence, this experiment serves merely to emphasize once again the futility of studying enhanced transfer in open beakers.

In an effort to obtain more conclusive evidence on the influence of delay times on enhanced transfer, the measurements were repeated using the hooded beaker C_1. Representative sequences of transfers in this beaker are shown in groups H to L of Figure 7.5.1; normal transfers and undisturbed enhanced transfers are shown in group H. The procedure whereby the delay was achieved was slightly different from that used in the open beaker experiments. The hooded beaker was raised after plunge-filling so that its rim was 5 mm above the bath level. It was left untouched for a time τ after which it was raised a further 10 mm to determine the

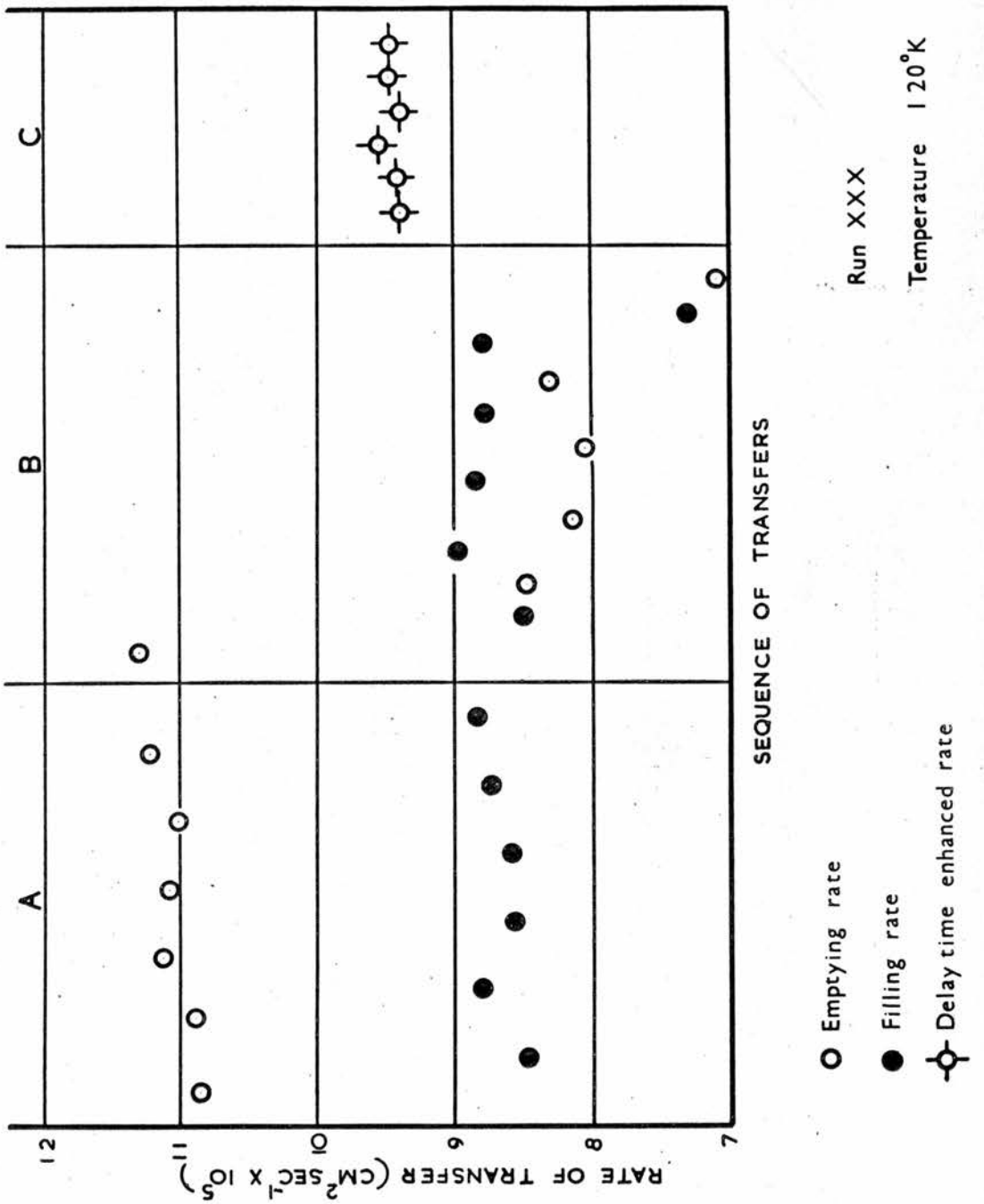


Figure 7.6.1 Transfer by Film-filling and Subsequent Efflux after Enhanced Emptying.

enhanced rate.

In each case varying degrees of enhanced transfer were present, with a slight tendency for the longer rest periods to show the smaller rates. Although this investigation was carried out at night, the chance of casual cryostat disturbance increasing with time was considered sufficient to account for this tendency. It was therefore concluded that there was no clear evidence that the enhanced rate was seriously diminished within the space of an hour.

The results of Seki (1962) confirm this conclusion, while it is inferred that the delay time measurements reported by Eselson and Lasarev (1952) can be accounted for by cryostat disturbance.

7.6 Enhancement of Film-filling Rates.

Since enhanced transfer appeared to have considerable stability as far as emptying the beaker after a plunge-fill was concerned, the question of an enhanced rate of filling arises. We are concerned with the filling rate immediately following an enhanced emptying. Groups A and B of Figure 7.6.1 show attempts to observe this in beaker C_1. In group A each enhanced emptying transfer after a plunge-fill was followed by carefully depressing the beaker, with as little disturbance as possible, so that the transfer was into the beaker. The filling rate is still invariably enhanced but with values fairly uniformly between the enhanced emptying rate and the normal filling or emptying rate; the latter are shown by a pair of control observations at the end of group B. Group B shows first an enhanced emptying following a plunge-fill, followed by a sequence of fillings and emptyings initiated by carefully lowering or raising

the beaker. The partially enhanced rate in each transfer was found to persist. Prior to taking the last pair of control normal transfers, the beaker was well raised above the bath so that it was emptied and warmed sufficiently to evaporate any film.

It is obvious that the enhanced rate persisted during both emptying and filling, but that the reduction in rate was pronounced. It must be remembered that even if the beaker was moved up or down very smoothly the accompanying shift of the annular meniscus up or down the film itself constitutes an inevitable disturbance to the film so that a reduction of enhanced rate is probably to be expected.

The latter is also true of delay time measurements of enhanced transfer. In group C of Figure 7.6.1 the procedure for delay time measurements in hooded beakers (section 5 of this chapter) was followed, but with a zero waiting period. The similarity in magnitude between the reduced enhanced rates in the three groups lends support to the conclusion that neither reversal of the flow direction nor a rest period in the measurements affects enhanced transfer; rather is the diminution in rate a consequence of unavoidable perturbation of the film.

These measurements clarify the discrepancy which exists between the magnitude of the rates reported by Eselson and Lasarev (1952), which we have called σ_F and $\sigma_{E'}$, and normal transfer rates. Clearly film transfer following enhanced emptying does not take place at the normal rate.

It must be pointed out that, in contrast to Eselson and Lasarev, we find the filling rate (σ_F) following an enhanced emptying to

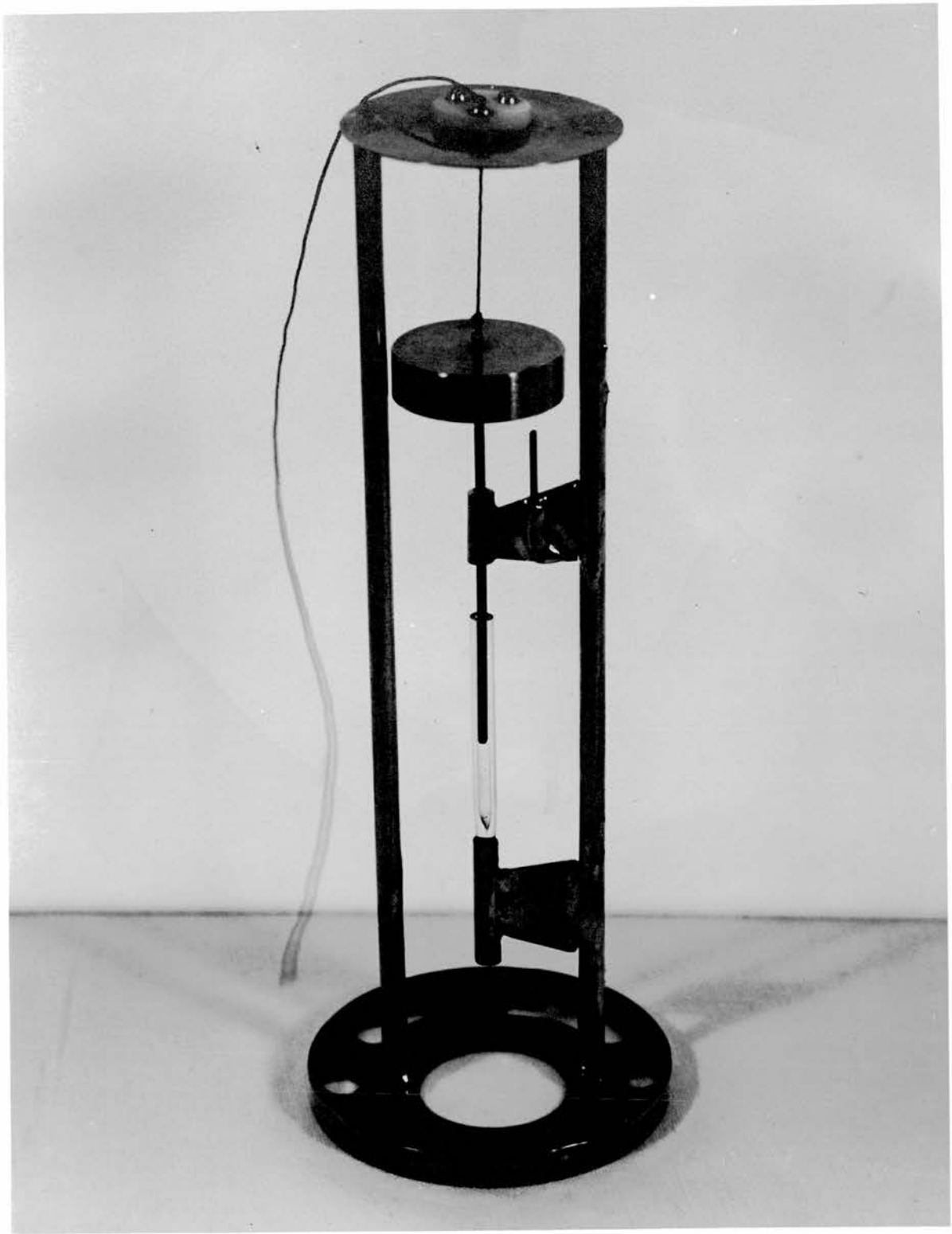


Figure 7.6.2 Apparatus Used to Obtain Filling Rates After Immersion.

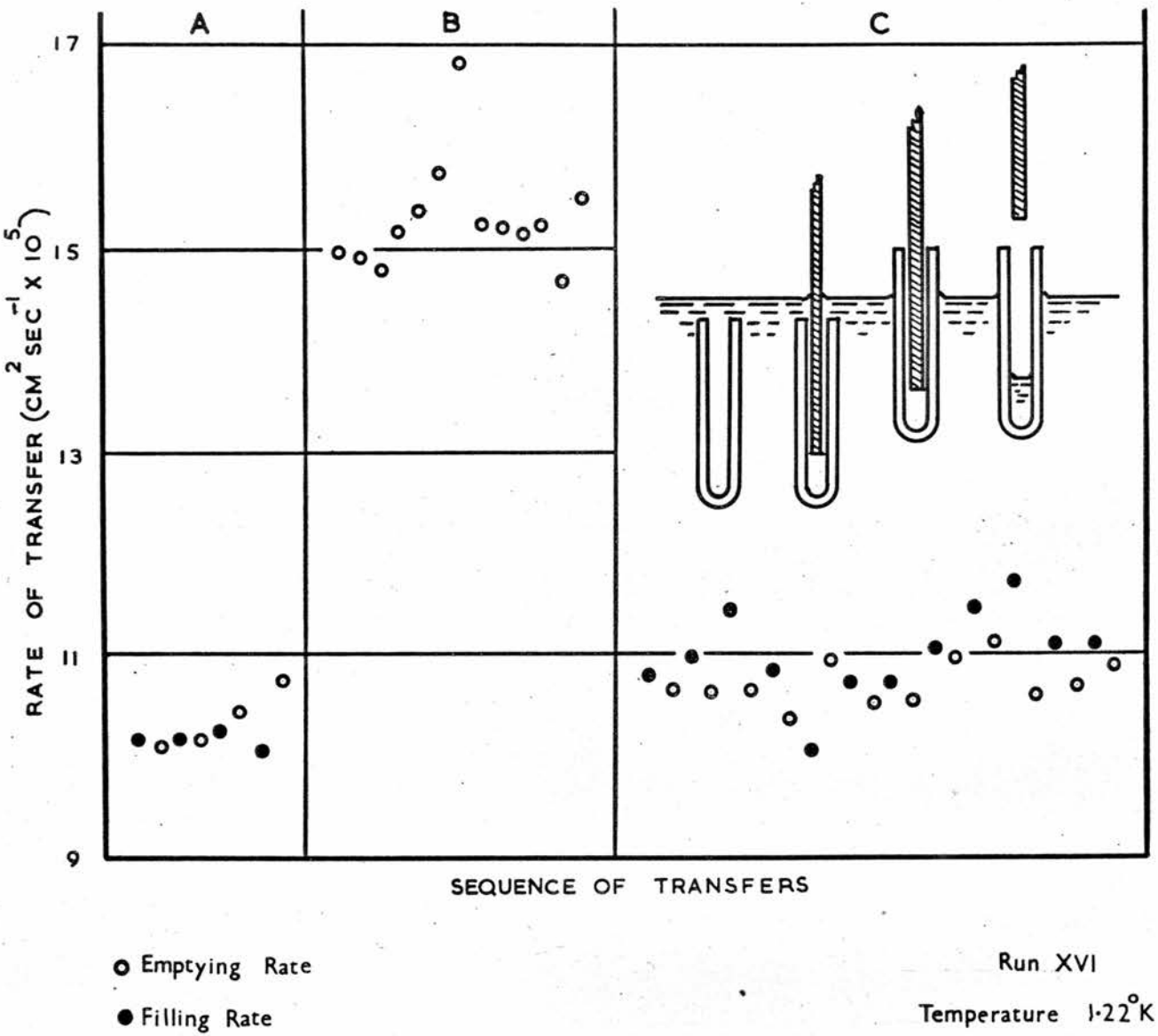


Figure 7.6.3 Film Transfer Rates After Use of Displacer.

be slightly greater than the subsequent emptying rate ($\sigma_{E'}$). The reason for this discrepancy is not understood, but it is not thought to be of prime importance. Moreover, we have never found, as did Seki (1962), that the filling rate after an enhanced emptying sometimes retained the full enhanced value. This difference is presumably because conditions of vibration are never the same in two cryostats. The influence of changes in environment on the stability of enhanced transfer will be further commented on in the following section.

One further experiment in which enhanced filling rates are involved deserves mention. A loose-fitting displacer was suspended above a beaker of type A. The displacer could be inserted into the beaker by means of a nylon line and remotely controlled rack and pinion. The apparatus is shown in Figure 7.6.2. The procedure adopted in the experiment is shown schematically in Figure 7.6.3 (inset). The beaker was plunge-filled and the displacer inserted while the beaker rim was still below bath level. The beaker was then raised with the displacer still in place until the rim stood the usual 1.5 cm above the bath level. The filling rate after the displacer had been quickly and completely withdrawn was determined.

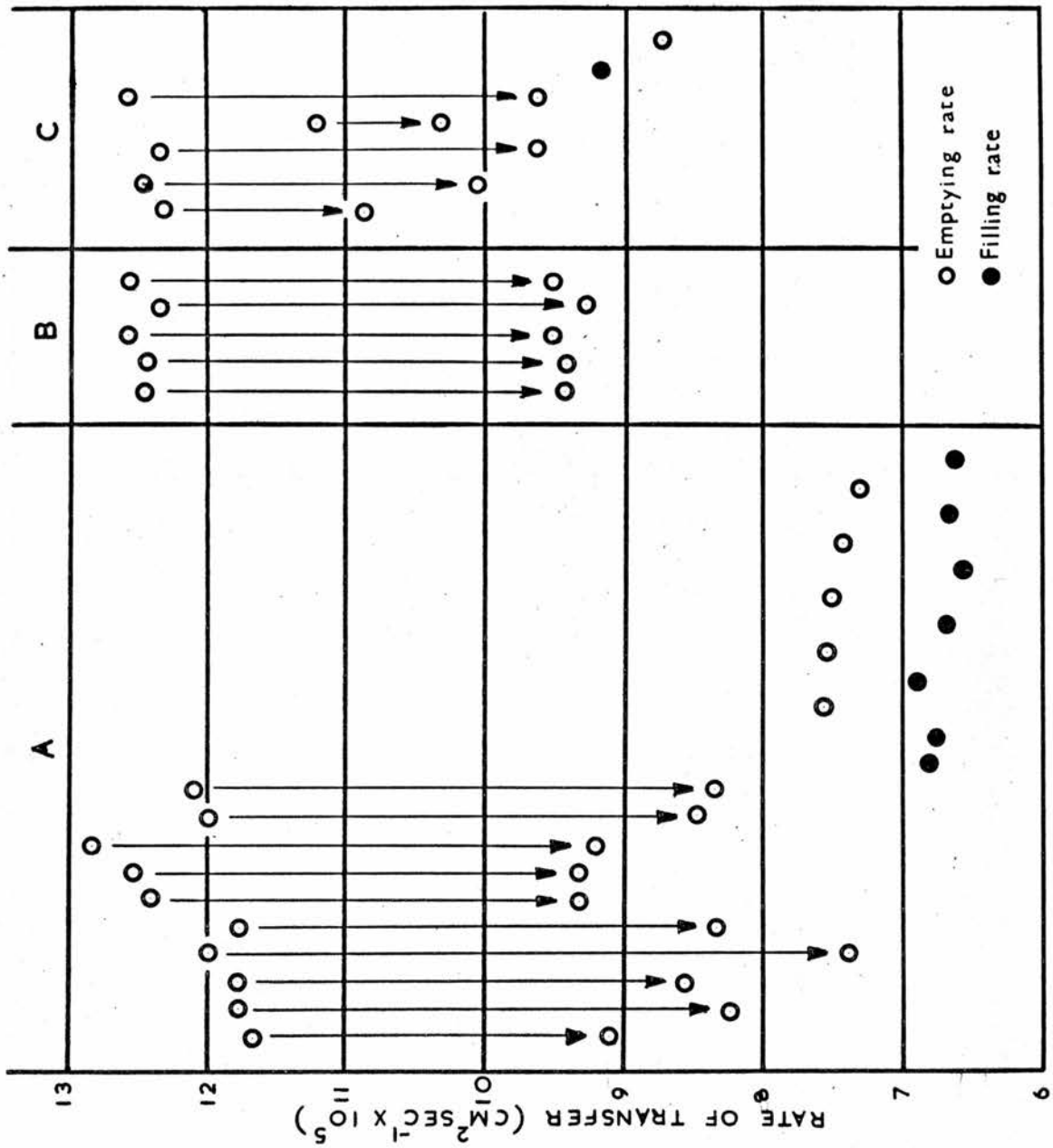
The results of a run employing this technique are shown in Figure 7.6.3. Groups A and B are control groups of normal transfer and enhanced transfer respectively. In group C the filling rate and subsequent emptying rate after successive withdrawals of the displacer are recorded. A slight enhancement of film transfer following removal of the displacer is evident. As before, the

filling rate in successive transfers is generally greater than the subsequent emptying rate. It was impossible to lift the displacer without its rubbing on the inner wall of the beaker. Moreover the displacer was not perfectly smooth and could not be withdrawn from the liquid without creating a disturbance.

In pursuance of this effect, the displacer was used to disturb the free surface of the film formed after plunge-filling. The displacer was moved up and down inside the top of the beaker before the falling meniscus reached the range of measurement. Such a coarse perturbation of the film at the governing section by the tip of the displacer always produced a reduction in the emptying rate from its enhanced value.

7.7 The Stability of Enhanced Transfer in Long Films.

In the short 2 cm long beakers, an abrupt diminution of enhanced rate of transfer was sometimes observed towards the extreme end of the transfer. This appeared to be a spontaneous drop in flow rate not associated with the onset of deliberate or accidental disturbance. In order to study this effect, three hooded beakers, each of type C_1 and just over 5 cm in length were made. The rim treatments of the three were different; one had a rim ground flat with 320 grade emery, one rounded by flame-polishing, and one in which the outer wall of the beaker had been ground off at an angle of 30° to the vertical to leave a narrow flat rim at the inside edge about 0.1 mm wide. A lower magnification in the cathetometer enabled the beaker level to be observed continuously over beaker lengths in excess of the usual 12 mm; several enhanced transfers



Group A : Run XXXI , Beaker C_1 with conical-ground rim. $T = 1.19^{\circ}K$

Group B,C : Run XXXII, Beaker C_1 with flame-polished rim. $T = 1.20^{\circ}K$

Figure 7.7.1 Stability of Enhanced Transfer in Long Films.

were followed until the inner meniscus was 3.75 cm below the beaker rim.

After a plunge-fill the beaker was raised until the rim stood 4.5 cm above the bath. The enhanced rate which has already been reported was observed until the beaker level had dropped to a certain depth below the rim. At this depth the rate of transfer dropped sharply to a lower value but still higher than that for normal transfer. This change in rate during the transfer was typical of all three rim shapes. Results obtained with two of the beakers are shown in Figure 7.7.1. A group of enhanced transfers, each transfer exhibiting a spontaneous change in rate, is shown in the first part of group A. The arrows indicate that the upper and lower values belong to the same transfer. This group was made with the beaker having the conical rim. The last twelve plotted rates in group A constitute a control sequence of normal fillings and emptyings. In all the considerable number of normal transfers in various beakers this is the only group in which a significant difference between filling and emptying is resolvable without recourse to statistical analysis. It will be recalled from Chapter 6 that significance tests, at a level of 4%, were used to compare the average values of filling and emptying rates in open beakers. In contrast to the behaviour of normal rates made in the hooded beaker with the conical rim, it was found that where a significant difference existed, the filling rate was greater than the emptying rate. To ascribe this difference in behaviour to the geometry of the rim is perhaps unwise, especially in view of the

Table 7.7.1

Type of beaker rim	depth x in cm in successive transfers	remarks
flat ground rim	1.72, 1.57, 1.59, 1.59	night run ; quiet lab.
conical ground rim	1.04, 1.02, 1.01, 1.02, 1.05, 0.99, 1.02, 0.99, 0.95, 0.95	day run: usual building vibration present
round, flame-polished rim	1.82, 1.81, 1.66, 1.74, 1.61	might run : quiet lab.
	1.38, 1.46, 1.60, 1.85, 1.40	small mechanical vacuum pump running near cryostat

fact that the hood enclosing the beaker with the conical rim was of a thicker glass than usual. The reason for the difference in normal transfer rates is, therefore, unknown, but is not thought to be of significance as far as this dissertation is concerned.

Groups B and C of Figure 7.7.1 were made with the beaker with the flame-polished rim. The last pair of rates in group C are normal transfers. The partially collapsed enhanced rates in group B exhibited a lower scatter than those in group A. Since the latter transfers were carried out at night, this is possibly connected with a reduction in cryostat disturbance. During measurement of enhanced transfer in group C a small mechanical vacuum pump was left running on the floor near the cryostat; the much greater spread in values of the partially collapsed enhanced rate is apparent.

The depth χ of the beaker level below the rim at which the break in enhanced transfer occurred in the three differently rimmed beakers is shown in Table 7.7.1. For a given run, the depth χ was very constant, and most remarkably so for the conical rim. The last five transfers in the table were those during which the small vacuum pump was left running. Compared with the previous five transfers made in the same beaker under quiet conditions, the shallower depth χ at which the break occurred is marked as is also the much greater spread in value.

It is apparent that a very small disturbance indeed is sufficient to affect both the time of onset of instability and the amount by which the enhanced rate is diminished towards normal.

The depth χ is the film path length <u>within</u> the beaker at

TAble 7.7.2

Type of beaker rim	depth x in successive transfers		maximum film path length l in cm
	average value (cm)	mean deviation (%)	
flat ground rim	1.62	4.7	6.4
conical ground rim	1.00	2.7	5.7
round, flame-	1.73	4.3	6.5
polished rim	1.53	9.7	6.3

which the instability set in. The average depth in successive transfers is given in Table 7.7.2, together with the mean deviation of this depth. Also given in the table is an approximate value for the maximum film path length $\underline{l}$ at which the break in rate occurred. This includes the film on the outer wall of the beaker as well as that on the rim.

These experiments show that under undisturbed conditions film transfer can only maintain its full enhanced value provided the film path length is less than about 6.5 cm. For lengths in excess of this, the rate diminishes spontaneously, although not to normal. It is also apparent that the instability in enhanced transfer rate sets in at a significantly lower value of film path length if the transfer is observed under the usual disturbed conditions present in the laboratory during the day.

Although maximum film path lengths of up to 8.5 cm have been investigated, we have never observed more than one spontaneous change in enhanced transfer rate within a single emptying of the beaker. There is no evidence that the diminution of enhanced transfer can occur in several stages as the film path lengthens, as has been reported by Mate *.

7.8 Interpretation of Experimental Results.

The investigation of film transfer described in the last two chapters has firmly established the existence of two types of transfer rate; normal rates are obtained when the beaker has been partially

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Footnote *. Mate, C. F., private communication.

filled by film flow alone, and enhanced rates when the beaker has been completely immersed.

The behaviour of normal transfer rates can be explained in terms of the mobile helium film which apreads over a dry surface held in contact with the bulk liquid. This film we have called the normal film; it has a concave profile given, in general by $d \propto H^{-1/2}$. On the other hand, the presence of enhanced transfer by the helium film is both embarrassing and completely foreign to helium film theory. The vorticity hypothesis of film transfer will be examined in Chapter 10 and will be shown to be incompatible with the observed behaviour of transfer rate.

The following general characteristics of enhanced transfer remain to be explained. The irreproducibility of the enhanced rate is most apparent in open beakers when up to 40% enhancement above normal is observed at 1.2°K. Reproducibility is markedly improved if the transfer takes place in the absence of any disturbance. Perturbation of the meniscus at either end of the film, or at the governing section for flow, can reduce the enhanced rate, as can vibration of the beaker itself. Enhanced transfer persists if the flow is stopped and re-initiated after a waiting period of the order of a kilosecond. Enhanced transfer also persists after reversal of the flow direction. A spontaneous drop in enhanced transfer occurs when the total film path length exceeds a reproducible maximum value of the order of several cm. This value is diminished if the level of vibration is increased.

This experimental study together with the theoretical work of

Tilley * led Matheson and Tilley (1965) to the belief that there are two kinds of helium film. In addition to the normal film formed on a dry substrate, there is the film which is left on the same substrate after it has been in contact with bulk liquid. The thickness of the latter film, formed as a draining layer of liquid, is probably greater than that of the normal film. Bearing in mind the experimental evidence (e.g. Atkins, 1959, Ch.6) that the critical velocity in channels of the order of 10^{-5} cm in width is almost insensitive to variations in channel width, it is clear that enhanced transfer can be associated with a thick film.

To see how drainage of bulk liquid becomes involved in film flow experiments it is necessary to consider the procedure which we have used to obtain enhanced transfer. In other words, we must examine the film which remains on a beaker surface after it has been raised from beneath the bath. Three distinct surfaces must be considered: the outer wall, the inner wall and the rim.

On the outer wall of the beaker, a film is created immediately the beaker is lifted after complete immersion. Although superfluid flows through this film, the rate of transfer is controlled by the thickness of the film at the inner rim of the beaker. The profile of the film on the outer wall is, therefore, unimportant.

Inside the beaker, the level falls as a result of superfluid efflux through the outer film. It is assumed that the shape of the

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Footnote *. Tilley, J., presently engaged in writing a thesis on the theoretical aspect of superfluid film transfer.

film left on the wall by the retreating meniscus is determined only by the normal component. The entire fluid is acted on by gravity, but only the normal fluid experiences a viscous drag. Thus the normal fluid provides a channel through which the superfluid can pass, consistent with keeping the whole film isothermal. At a specified depth below the beaker rim, the initial thickness of the film on the inner wall is governed merely by the kinematic viscosity of the normal fluid and the velocity of recession of the bulk liquid. However, as soon as this film has been 'deposited' by the falling liquid level, drainage of the normal fluid will ensue. The behaviour of the film on the inner wall, therefore, resembles that of a slab of classical liquid lying on a vertical wall, acted on by gravity and retarded by viscosity.

It will be shown in Appendix A that the thickness d at depth z below the vertex of the film at time t from the start of drainage is given by

$$d = \left[\frac{\eta z}{\rho_n g} \cdot \frac{1}{t}\right]^{1/2}, \qquad \text{(A.1.11)}$$

where η is the viscosity and ρ_n the density. Thus, except at the rim where the film merges with the surface of liquid on the rim, and at the base of the film where it merges with the beaker meniscus, the film on the inner wall will posses a parabolic profile. The vertex of the parabola will be situated at the junction of the free surface of the film and that of liquid on the rim. The profile of the draining film is quite different from the concave profile of the normal film.

Table 7.8.1

Film thickness (cm)	time in sec to reach thickness d at depth z (cm) given by				
	3×10^{-6}	10^{-5}	10^{-3}	10^{-1}	1
3×10^{-5}	9×10^{-3}	3×10^{-2}	3	3×10^{2}	3×10^{3}
3×10^{-6}	9×10^{-1}	3	3×10^{2}	3×10^{4}	3×10^{5}
3×10^{-7}	90	3×10^{2}	3×10^{4}	3×10^{6}	3×10^{7}

An idea of the time scale of the drainage process can be gained from Table 7.8.1 in which equation (A.1.11) has been used to calculate the time taken for the film to reduce to certain thicknesses at various depths. z is taken as the depth below the horizontal surface of liquid on the rim. Viscosity and normal fluid density have been taken from the measurements made by Tough, McCormick and Dash (1963). At a temperature of 1.27°K, η = 1.56×10^{-5} poise and ρ_n = 5.98×10^{-3} gm cm^{-3}. It is clear that at the governing section for flow drainage occurs at such a rate that the thickness would reach normal film thickness in a very few seconds, assuming no mean free path effects or quantum effects. Moreover, no limit is set by equation (A.1.11) on the ultimate thickness to which a draining film may diminish. These predictions are obviously not compatible with observations of enhanced transfer rate which can persist essentially unchanged for times of the order of a kilosecond.

The difficulty is resolved when we take into consideration the liquid trapped on the beaker rim when it is raised from beneath the bath. Immediately the beaker rim appears above the bath surface, a roughly hemi-spherical drop of liquid is seen on top of the completely full beaker. The profile of this drop rapidly becomes horizontal; presumably the large volume of liquid held within the drop by surface tension drains away in the form of bulk liquid rivulets or even individual drops. Some 5 sec to 7 sec after lifting the beaker, a liquid level appears within the beaker as true film transfer commences. Thereafter, drainage from the annular puddle

Table 7.8.2

Temperature (°K)	film thickness d (cm)
2.1	4.1×10^{-6}
2.0	4.2×10^{-6}
1.9	4.4×10^{-6}
1.8	4.7×10^{-6}
1.7	5.3×10^{-6}
1.6	5.9×10^{-6}
1.5	6.8×10^{-6}
1.4	7.9×10^{-6}
1.3	9.5×10^{-6}
1.2	11.9×10^{-6}

of liquid lying on the beaker rim is restricted by the very small hydrostatic pressure head available to drive the viscous normal fluid over the edge of the rim.

Qualitatively, it is clear that as soon as the rate of leakage from the rim reservoir becomes equal to that of drainage in the vertical film, the thickness of the film near the rim will attain a constant value. It must be remembered that even the narrowest rim used in our experiments (conical ground rim of 0.1 mm width) can contain a reservoir 3000 normal film thicknesses wide and possibly 100 film thicknesses deep. It must also be borne in mind that attainment of the final depth of liquid at the governing section will cease to be controlled by normal fluid viscosity but will become dominated by mean free path and quantum effects.

The dramatic change in the time scale of the drainage process arising from the inclusion of a slowly leaking rim reservoir has been discussed by Tilley and Kuper (1965). Using a suitable model to approximate the physical situation they find the thickness of the vertical film near the rim of the beaker to be given by

$$d = 7{\cdot}28 \times 10^{-5} (\nu_n)^{1/3}, \qquad \text{(A.2.10)}$$

where ν_n is the kinematic viscosity of the normal fluid. Derivation of this equation will be indicated in Appendix A.

Using equation (A.2.10) an estimate can be made of the steady thickness of the film at various temperatures. This is shown in Table 7.8.2. It is inferred that within a short time following the commencement of transfer after a plunge-fill the opening of the

governing section will be controlled by the rim reservoir and that this opening will stay fairly constant for long laboratory times. Moreover, the thickness of a draining film of liquid helium is likely to be greater than that of the normal film. Direct confirmation of the presence of a thick film should be possible from thickness measurements.

Although the theory assumes a horizontal flat rim with sharp edges, these edges are not sharp to the order of film thickness and it is probable that the above considerations will hold for any rim shape. This is borne out by the results presented in section 7 of this chapter in which there is little difference between the enhanced rate of transfer in beakers with rounded and conical rims. If enhanced transfer is indeed linked with the presence of a rim reservoir, then only in the case of a rim sharp to the order of film thickness may we expect to find no enhanced transfer. Such a rim is likely to be difficult to produce.

The tendency of enhanced transfer rates to decrease towards the normal value when the film is disturbed by severe wave motion indicates that the thick film is somewhat unstable. Since the theory of the draining film takes no account of the van der Waals attraction of the wall, the normal film must be regarded as the final condition of the draining film. The comparative ease of destruction of the thick film and its consequent replacement by the normal film is presumably why enhanced transfer has acquired the reputation of being somewhat elusive to observe.

The partial destruction of the thick film by meniscus wave

action may be due to third sound resonance effects. The mechanical hammer is presumably an indirect means of setting up third sound in the film by the imposition of impulsive forces to the liquid in various parts of the film. Third sound waves in the normal film produce only second order temperature fluctuations. Since the thick film is a meta-stable state, however, evaporation effects during an imposed wave motion may become increasingly important.

Third sound resonance probably also accounts for the instability of long thick films which is found to exist when the film exceeds a certain critical length. The frequency of forced vibration of the apparatus is likely to be within or near the laboratory vibration spectrum, say 10 c/s to 15 c/s. Since the velocity of third sound is of the order of 50 cm sec^{-1} at 1.2°K, the corresponding wavelength under these conditions would be between 3.5 cm and 5 cm. This confirms experimental observations in which the maximum stable length over which enhanced transfer was observed is of the order of a few cm.

Everitt, Atkins and Denenstein (1964) found that third sound was heavily attenuated at an inside corner of the film formed by a step in the substrate. It remains for further research to elucidate the importance of an outside corner, such as the beaker rim, on the attenuation of third sound.

In view of the incomplete understanding of the mechanism of evaporation and condensation in helium films, one is not able to put forward a complete explanation of the effects suggested above.

Nevertheless, it is thought (Osborne, 1962) that helium vapour exchanges momentum with the normal fluid only, although the process whereby single atoms are organized into the collective motion arising from the boson nature of He II is not clear. Of the two helium films considered in this thesis, only the normal film is in thermodynamic equilibrium; an unusual variation of chemical potential would be required to allow the helium film to be stable over two well-defined ranges of thickness (Dzyaloshinskii, Lifshitz and Pitaevskii, 1960). Accordingly, a large coefficient of evaporation may result in rapid destruction of the thick film. Unfortunately the coefficient of evaporation under the conditions of our experiment is not known. However, the energy of a roton is uncertain to the order of $\hbar/\tau$, where τ is the average time over which a roton is undisturbed by collision processes. Allowing for the latent heat, therefore, the energy of a free particle in the vapour is equal to the quasi-particle energy only over a small range of energies lying between the intersection of the two energy spectra. Below 1°K this restriction limits the quasi-particles which can evaporate to high energy phonons and low momentum rotons. The occupancy of both these thermal states is low. It is expected, therefore, that only a fraction of the thermal excitations impinging on the free surface of the film will evaporate *. If one tentatively infers from this that evaporation cannot destroy a thick film

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Footnote *. C, G. Kuper, private communication.

below 1°K, it is likely that a thick film will exist over the entire temperature range investigated. This is only true, of course, if one assumes that the mechanism whereby enhanced transfer is sustained does not change radically between 0.5°K and 1.8°K.

CHAPTER 8.

Temperature Dependence of Normal and Enhanced Transfer Rates.

8.1 Introduction.

It will be recalled that Eselson and Lasarev (1952) observed that the difference in magnitude between normal and enhanced transfer rates is strongly dependent on the temperature and disappears above 1.88°K. The general trend of the variation of enhancement of film transfer with temperature was further investigated by Miss Walker (1962). Her measurements, however, were conducted using simple open beakers and were not ideally reproducible.

It is of interest to note that results similar to those of Miss Walker lie latent in the experiments of Webber, Fairbank and Lane (1949). In addition to determining the filling rate of a beaker between 1.4°K and 2.1°K, they apparently measured the emptying rate following complete immersion of the beaker. Unfortunately, they remained unaware of the significance of the growing difference between the two rates as the temperature fell. Much swayed by the data of Daunt and Mendelssohn (1939b), and undaunted by a 30% difference between the rates of filling and emptying at 1.4°K, they averaged the two rates at each temperature. Although the resulting transfer rate was still increasing with decreasing temperature at 1.4°K, the agreement between their data and that of Daunt and Mendelssohn was stated to be " reasonably satisfactory". These experiments are probably the most glaring example of the disruptive effect of unrecognized enhanced transfer on studies of the helium

Table 8.2.1

Temperature (°K)	rate of transfer ($\times 10^{-5}$ cm^2 sec^{-1})		
	normal filling	normal emptying	enhanced emptying
1.99	4.42	4.26	4.46
1.91	5.75	5.31	5.83
1.83	6.75	6.87	7.07
1.73	7.68	7.52	8.00
1.63	7.99	7.96	9.21
1.57	8.08	7.92	9.41
1.49	8.12	8.04	9.97
1.36	8.00	8.04	10.73
1.23	7.80	7.88	11.52

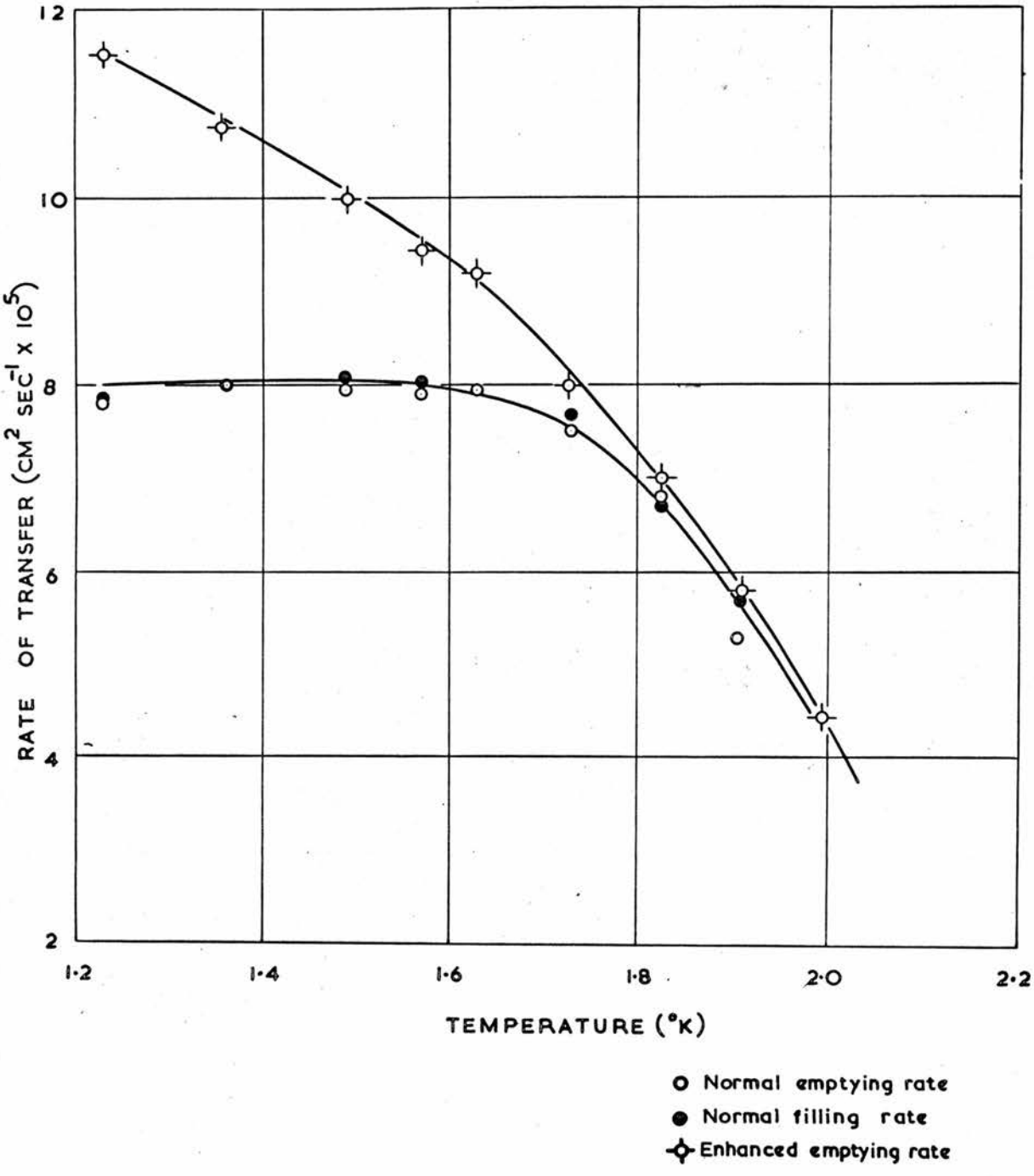


Figure 8.2.1 Temperature Variation of Normal and Enhanced Transfer Between 1.2°K and the Lambda Point.

film.

8.2 Variation of the Rates of Transfer Above 1.2°K.

The ability to exclude most of the random element of bath disturbance by the use of the hooded beaker C_1 enabled us to obtain a fairly accurate determination of both normal and enhanced transfer rates as a function of temperature down to 1.2°K. All measurements of normal film transfer were made after the beaker had been emptied completely. Enhanced transfer was observed after the beaker had been plunge-filled and then raised.

Typical results are given in Table 8.2.1 and shown graphically in Figure 8.2.1. At 1.2°K the enhanced rate is about 40% greater than the normal rate and is still steeply increasing. Above about 1.9°K, however, any difference between the two rates of transfer dependent on the method of filling becomes indistinguishable.

8.3 Normal and Enhanced Transfer Below 1.2°K.

Measurements of normal and enhanced transfer have been extended down to 0.49°K using the helium^3 cryostat described in Chapter 5. Transfer rates were determined in the usual way in a hooded beaker of type C_1. Although the beaker meniscus was viewed through eight layers of assorted glass, visibility was as good as that in the helium^4 cryostat. The temperature was controlled manually by adjusting the speed with which the helium^3 was pumped. At temperatures below 0.7°K the pressure within the helium^3 chamber was measured with a Pirani gauge which was subsequently calibrated against an oil manometer.

Since the vertical movement of the beaker assembly was

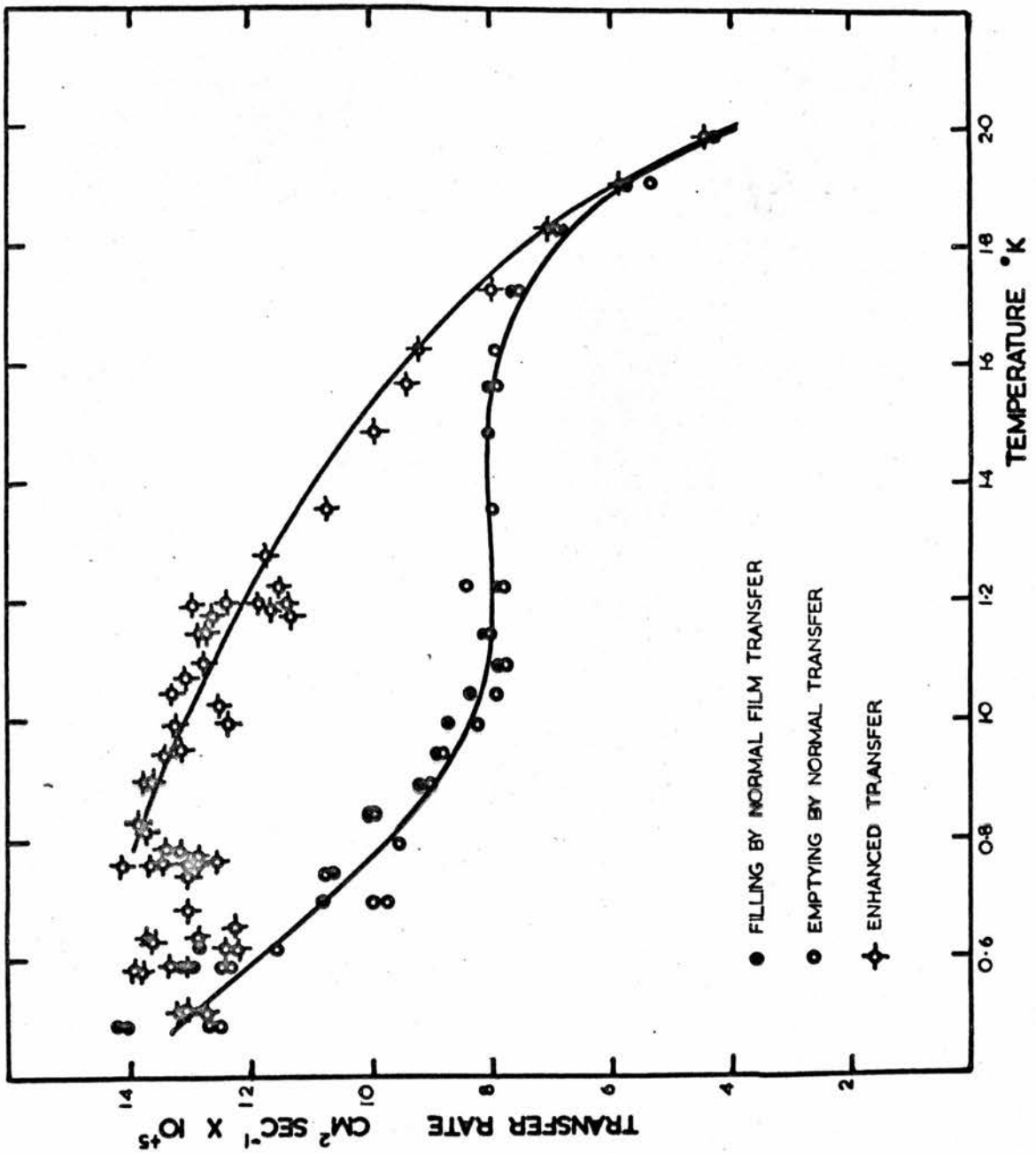


Figure 8.3.1 Normal and Enhanced Transfer Rates Below 1.2°K

restricted to a few cm, it was not possible to lift the beaker completely clear of the bath. Accordingly, measurements of normal film transfer had to be made before a thick film was formed by plunge-filling.

Following some indication that the enhanced rate below 1°K became progressively more difficult to observe, all mechanical pumps were switched off during measurements. Incorporation of a ballast volume between the diffusion and mechanical pump stages of the helium^3 pumping system enabled this to be done. When this procedure was adopted, the slight rise in temperature was recorded and an average value was used. The rise in temperature was never greater than 0.05°K and generally much less.

Control measurements of normal and enhanced transfer rates at 1.2°K in the helium^3 cryostat revealed equality of transfer rates with those measured in the helium^4 cryostat at the same temperature. A composite picture of the temperature variation of normal and enhanced film transfer was therefore constructed from observations made in both cryostats. This is shown in Figure 8.3.1. Above 1.2°K transfer rates were determined in the helium^4 cryostat; their values have been given in Table 8.2.1. Transfer rates below 1.2°K were taken from data obtained during five separate runs with the helium^3 cryostat. In this cryostat both the normal and enhanced rate of transfer were strictly reproducible, presumably because of the inherent cleanliness of beakers of type C_1.

As found by other observers, there is a resumed rise in the normal rate of transfer below about 1.05°K. The average percentage

increase in normal rate between 1.2°K and 0.49°K is about 60%. The scatter of data points is greater at lower temperatures. There is no indication of a lessening of the rate of rise of normal transfer towards the lowest temperatures investigated.

On the other hand, it is apparent that the steep rise in enhanced transfer is abruptly halted below temperatures of about 0.8°K. At 0.52°K it is impossible to distinguish any enhancement of film transfer following complete immersion of the beaker. This observation cannot be attributed to a change in the level of vibration since the level was maintained at a constant value over the entire temperature range investigated in the helium^3 cryostat.

8.4 Discussion.

The thickness of the normal film does not change appreciably with temperature above 1.2°K. This experimental observation is in agreement with equation (3.2.1). Thus the variation of normal film transfer rate with temperature above 1.2°K is probably a consequence of the temperature variation of the superfluid fraction ρ_s/ρ.

On the other hand, the thickness of the film formed after a beaker has been raised from beneath the surface of the bath depends on the kinematic viscosity of the normal fluid (Chapter 7.8). In the temperature range from 1.3°K to 1.9°K, the normal fluid can be regarded on the atomic scale as a gas of excitations, mainly rotons. At 1.9°K the theory of Landau and Khalatnikov (1949a,b) predicts that the mean free path of a roton is about 10^{-7} cm. As the temperature decreases to 1.3°K, however, the roton mean free path increases by an order of magnitude. Accordingly, between 1.9°K and

1.3°K the kinematic viscosity of He II increases by roughly a factor of ten. Thus the rapid rise in the enhanced transfer rate below 1.8°K can be explained qualitatively by the rapid increase in kinematic viscosity and in particular by the rapid increase in roton mean free path. Both these quantities become small above 1.8°K, permitting rapid drainage and absence of enhanced transfer.

Tilley and Kuper (1965) have put forward a prediction of the functional dependence of enhanced rate on temperature in the range $1.2° < T < 1.8°K$. Since it is useful to compare this prediction with our measured values of enhanced rates, the theory will be discussed at this point.

For completeness, Tilley and Kuper assume that the critical velocity is a function of both temperature and film thickness. The normal film transfer rate can be written

$$\sigma_n(T) = \frac{\rho_s}{\rho} v_c(d_n, T)\, d_n, \qquad (8.4.1)$$

where d_n is the thickness of the normal film at its governing section. This thickness is taken to be 3×10^{-6} over the temperature range in question.

Similarly, the rate at which superfluid is conveyed through the thick film is defined to be

$$\sigma_t(T) = \frac{\rho_s}{\rho} v_c(d_t, T)\, d_t(T). \qquad (8.4.2)$$

And by equation (A.2.10) we have

$$d_t(T) \propto (\gamma_n)^{1/3}. \qquad (8.4.3)$$

Table 8.4.1

Temperature (°K)	Rate of Transfer ($\times 10^{-5}$ cm^2 sec^{-1})			
	measured		theoretical enhanced	
	normal	enhanced	ν_c proportional to $d^{-1/2}$	ν_c independent of d
	σ_n	σ_t	σ_{t_2}	σ_{t_1}
1.9	5.85	5.95	5.85	5.85
1.8	7.05	7.35	7.35	7.60
1.7	7.70	8.35	8.50	9.30
1.6	7.95	9.20	9.30	10.8
1.5	8.00	10.05	10.0	12.5
1.4	8.05	10.80	10.8	14.6
1.3	8.05	11.45	11.8	17.5
1.2	8.05	12.05	13.30	22.0

Film transfer rates, other than normal, have not been observed above 1.9°K. This is taken as evidence that the draining film cannot maintain a thickness greater than normal above 1.9°K, that is

$$d_n = d_t(1.9^\circ). \tag{8.4.4}$$

Hence,

$$\sigma_t(T) = \frac{v_c(d_t,T)}{v_c(d_n,T)} \cdot \frac{d_t(T)}{d_t(1.9^\circ)} \cdot \sigma_n(T). \tag{8.4.5}$$

According as v_c is independent of d or proportional to $d^{-1/2}$, equation (8.4.5) becomes

$$\sigma_{t_1}(T) = \left[\frac{\nu_n(T)}{\nu_n(1.9^\circ)}\right]^{1/3} \cdot \sigma_n(T) \tag{8.4.6}$$

or

$$\sigma_{t_2}(T) = \left[\frac{\nu_n(T)}{\nu_n(1.9^\circ)}\right]^{1/6} \cdot \sigma_n(T) \tag{8.4.7}$$

respectively.

Experimental values of normal transfer rate, taken from the smoothed data of Figure 8.3.1, are given in Table 8.4.1. Also given are the calculated values of σ_{t_1} and σ_{t_2} which are to be compared with the experimental values of enhanced transfer rate. It appears that there is remarkable agreement between the measured values of enhanced transfer rate and those predicted on the assumption that v_c is proportional to $d^{-1/2}$.

At 1.2°K the roton mean free path calculated from the Landau-

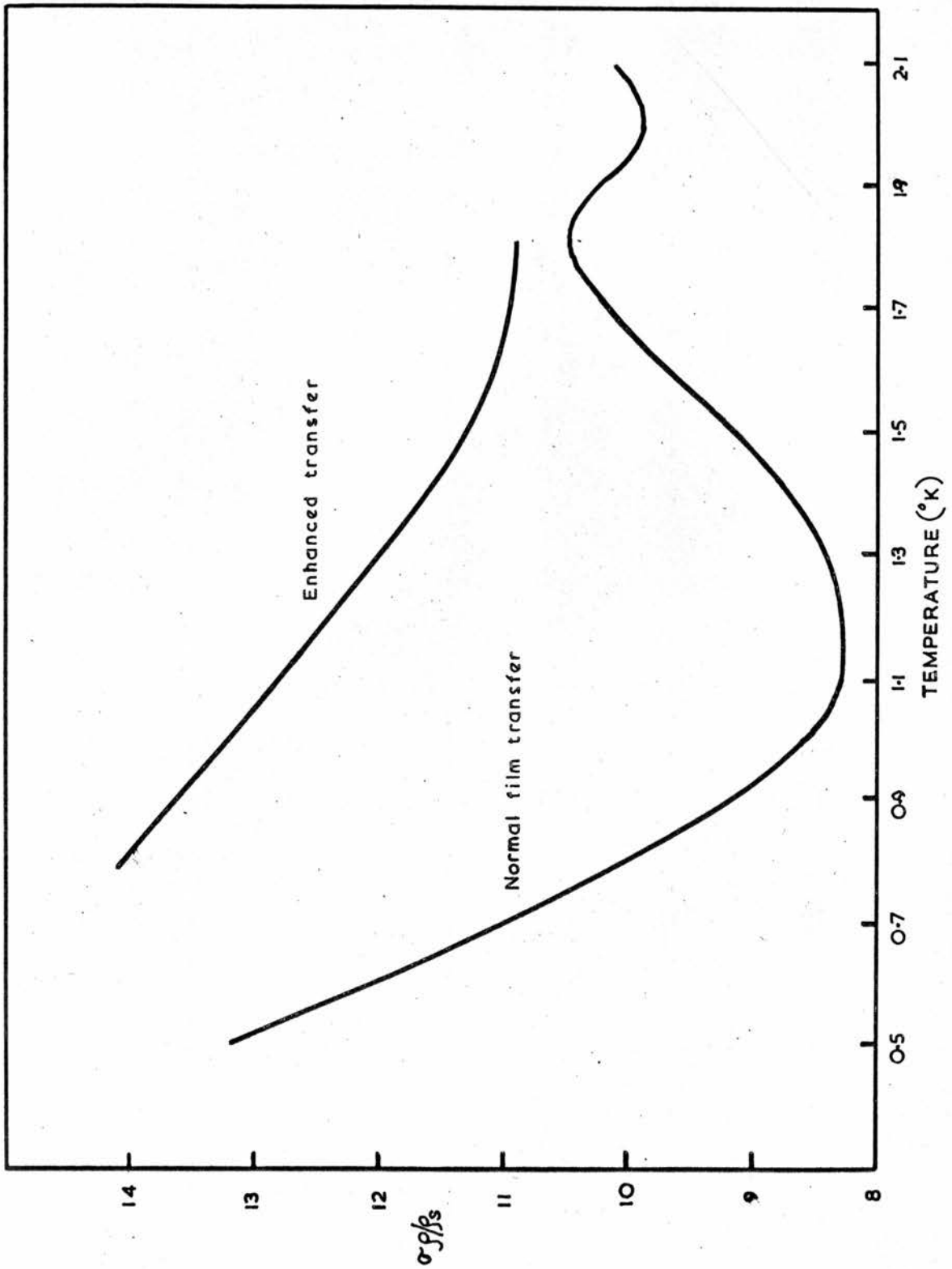


Figure 8.4.1 Dependence of Normalized Film Transfer Rates on Temperature.

Khalatnikov theory is of the same order of magnitude as the film thickness. Thus, below this temperature mean free path effects might be expected to control the drainage process. The roton contribution to the effective viscosity of normal fluid in the film will be slightly reduced as the flow regime acquires the characteristics of a Knudsen type of flow. This reduction, however, is not sufficient to curb the rapid increase of viscosity which occurs below 1.2°K. For these reasons the macroscopic theory of drainage in its present form is not expected to be applicable at temperatures below 1.2°K.

The resumed rise in the normal rate of transfer below 1.05°K may be connected with an increase of film thickness (Mate, Harris-Lowe and Daunt, 1964 ; Peshkov, 1964), although there is little experimental evidence to support this. In this connection it is of interest to plot* the variation with temperature of the product of critical velocity and controlling film thickness for both the normal film and the thick film. It is clear from equations (8.4.1) and (8.4.2) that this product is simply obtained by division of the measured rate of transfer by ρ_s/ρ . The lower curve of Figure 8.4.1 shows the temperature dependence of such a normalized transfer rate for the normal film. In the range $1.2° < T < 2.1°K$ where the thickness of the normal film is essentially independent of temperature, the curve characterizes the variation of v_c with temperature.

Footnote*. This plot was made in conjunction with J. Tilley.

The minimum in the curve at a temperature of about 1.1°K followed by a sharp increase in the normalized rate towards the lower temperatures is presumably due to an increase in film thickness. This is borne out by the dependence of the normalized transfer rate of the thick film on temperature, shown in the upper curve of Figure 8.4.1. In this case it is known that, at least above 1.2°K, the film thickness is increasing with decreasing temperature as the one third power of the kinematic viscosity.

Despite many attempts, no theory of critical velocity in the film has met with complete success. Nevertheless, it is apparent that our measurements of enhanced transfer rate give tentative support to theories in which $v_c \propto d^{-1/2}$. It should be borne in mind, however, that a temperature dependent critical velocity for superflow, as evident from Figure 8.4.1, is not be expected from theories based on the Landau criterion. There would seem to be a need, therefore, for a reappraisal of the idea of critical velocity in the helium film.

The trend of normal film transfer below 1°K resembles that of the velocity of second sound. Noting this, Allen and Matheson (1965) tentatively suggested that not only is third sound involved with variations in the transfer rate but that third sound of zero frequency may be the actual mechanism of film transfer itself.

The increased scatter of normal film transfer rates below 1°K is presumably a result of the declining influence of normal fluid. At temperatures below, say, 0.5°K any wave motion in the film will effectively be of pure superfluid. One might expect this

motion to be almost unattenuated with a restoring force provided only by the van der Waals attraction of the substrate.

Clearly much experimental and theoretical work remains to be done before the mechanism of normal film transfer below 1°K is fully understood.

The full value of enhanced transfer following plunge-filling was never observed below 0.75°K and any difference between normal and enhanced transfer became indistinguishable at 0.52°K. This is presumably because of the extreme sensitivity of the thick film to disturbance which arises from the negligible amount of normal fluid present and its consequent lack of ability to support the thick film. Put very crudely, the disappearance of enhanced transfer is a direct manifestation of the inadequacy of the thermo-mechanical restoring force of the normal fluid to cope with the ever-increasing fraction of superfluid.

The increasing tendency, as the temperature is lowered, for enhanced transfer to break down may well be partly due to cryostat stability, and in a very quiet cryostat the onset of such instability may be delayed.

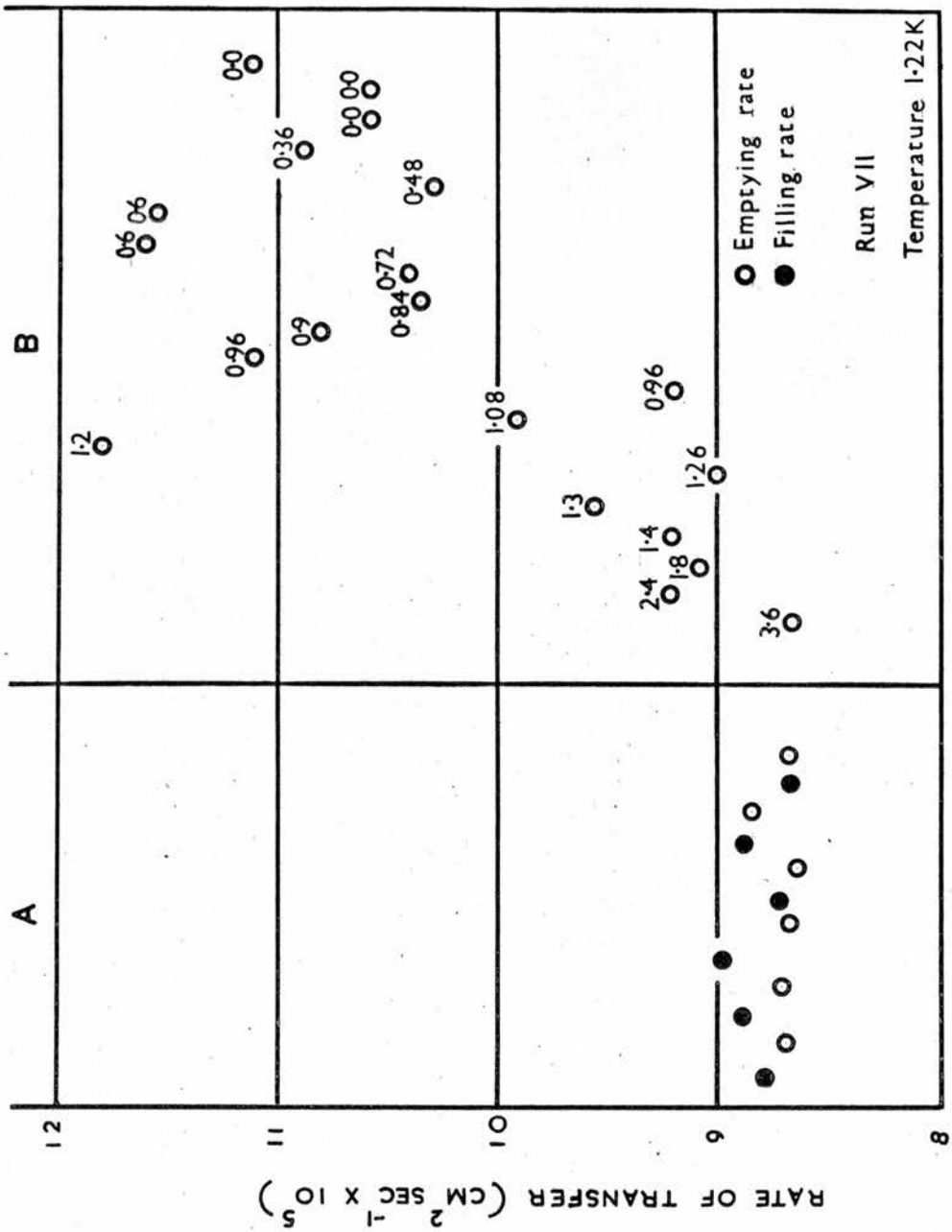


Figure 9.1.1 Enhanced Transfer after Fast Film-filling.

CHAPTER 9.

Variations in Film Transfer Rate Associated with Proximity of the Liquid Level to the Beaker Rim.

9.1 Fast Film-Filling and Enhanced Transfer.

Fast rates of transfer into a beaker occur when the beaker is filled with the bath level near the rim (Allen, 1963). The effect of fast filling on subsequent superfluid efflux from a beaker was studied using an open beaker of type A. Starting with an empty beaker, the normal filling procedure was followed, the bath level being 6 mm from the beaker rim. Following the equalization of levels, the beaker was carefully depressed until the rim was a specified height above the bath level. The levels were again allowed to equalize, filling occurring through the film at a fast rate. The beaker was then quickly raised until the rim stood 1.5cm above the bath level and the rate of emptying was determined over the usual range of the beaker.

The results are shown in Figure 9.1.1. Group A is a control sequence of normal rates of filling and emptying. Group B shows a sequence of emptying rates, each following a fast filling. The figures above each symbol refer to the height of the rim above the bath level in mm at the end of the fast filling procedure. The last three rates of the sequence are rates following simple plunge-filling.

It is evident that if the beaker is filled until the inner level is nearer than 0.84 mm to the rim, the subsequent emptying

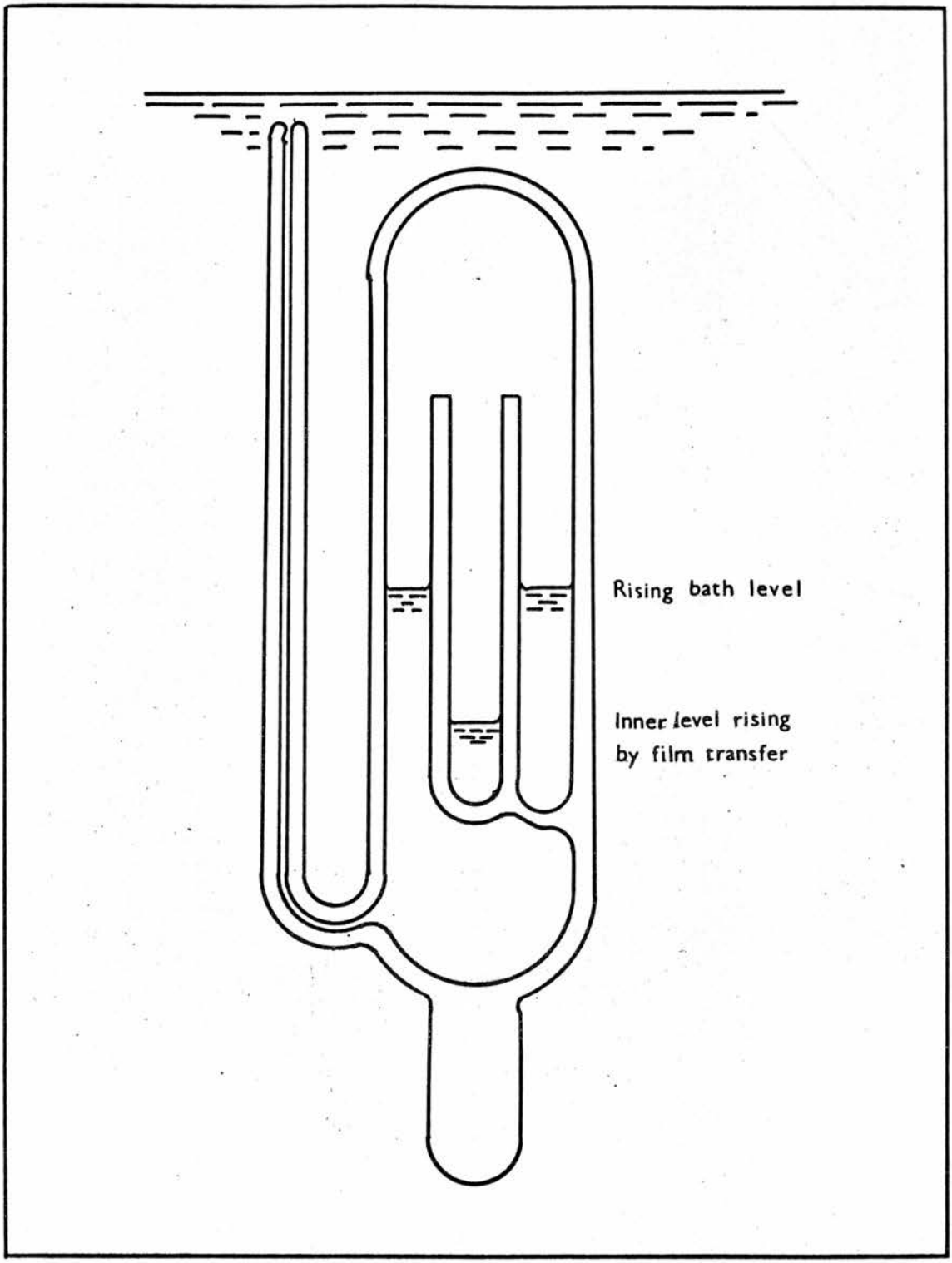


Figure 9.2.1 Hooded Beaker with Stand-pipe.

rate is enhanced. No enhancement of transfer is produced, however, if the bath level is always further from the rim than 3.6 mm. Fast film-filling to intermediate depths below the rim results in a partially enhanced rate; the amount of enhancement is dependent upon the depth to which the beaker was filled.

9.2 The Onset of Fast Film-filling Rates.

It is obviously of interest to determine the exact height of the beaker rim above the bath level at which the film-filling rate starts to exceed the normal value. To do this by carrying out a sequence of transfers, each involving a different distance between the rim and the bath level, is both tedious and inaccurate. Accordingly, film transfer into a beaker was studied by a dynamic method in which the bath level was made to rise towards the beaker rim at a constant speed. The rate of filling was constantly observed during this process; the onset of fast film-filling was apparent as a departure from linearity in the rate of rise of the inner level. The change of filling rate could subsequently be correlated with the height of the beaker rim above the bath level.

Figure 9.2.1 shows the apparatus in which the measurements were made. A simple open beaker was completely surrounded by a cylindrical glass envelope. The only way in which liquid could enter the envelope was by a stand-pipe about 0.4 mm in diameter. Complete submersion of the stand-pipe allowed the envelope to fill slowly with liquid. Film transfer then took place from the rising annular bath into the beaker. A convenient rate of rise of bath level occurred when the stand-pipe was reduced to a diameter of

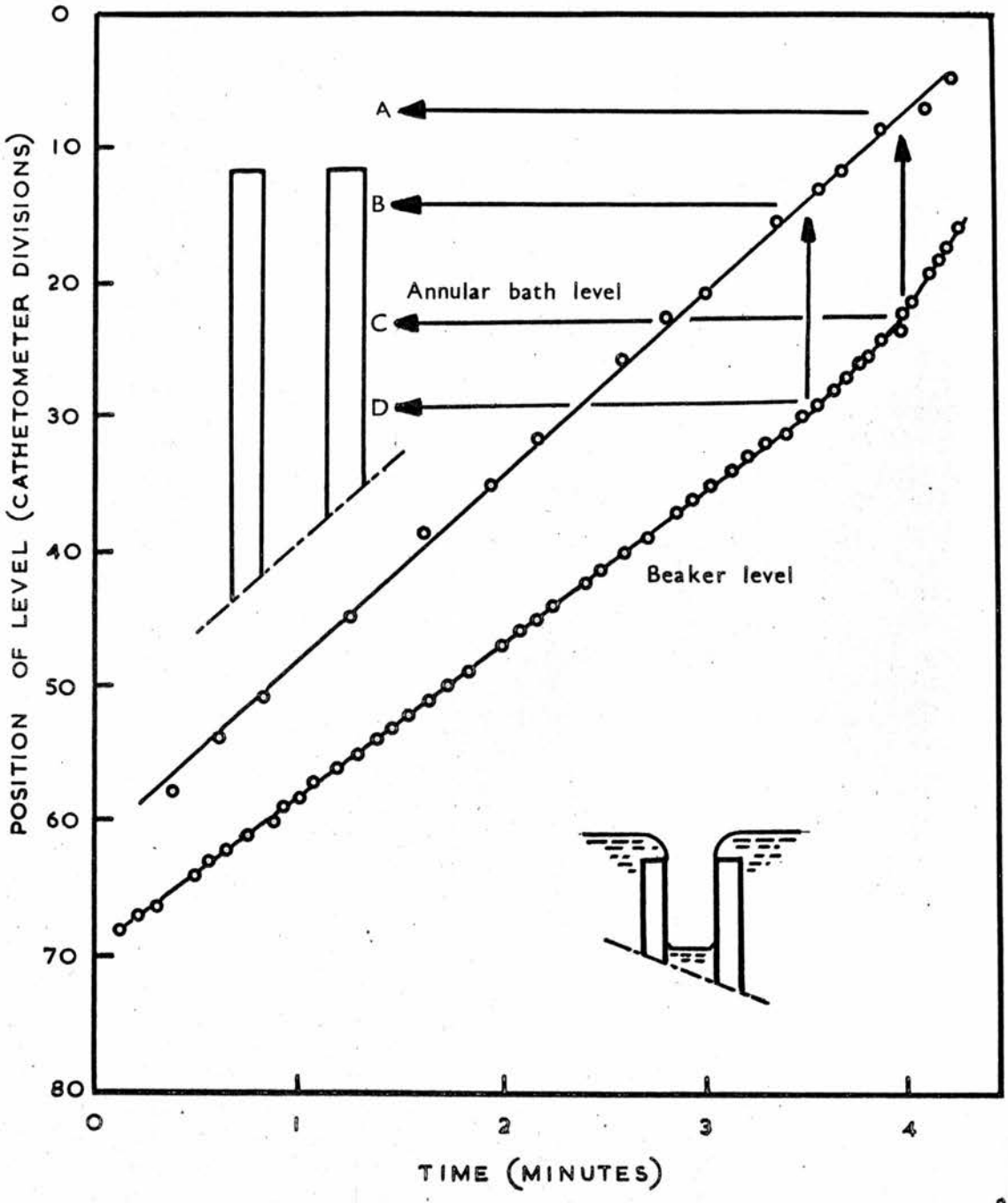


Figure 9.2.2 Stages of Fast Film-filling.

47 microns at the rim, and the annular gap between beaker and envelope was 2 mm.

A typical example of the behaviour of the inner level as the bath level approached the rim during filling is given in Figure 9.2.2. To simplify subsequent discussion, the beaker has been included in the figure beside the time variation of the menisci positions. By means of the construction shown, the position of the bath level relative to the rim can be determined at the time of an observed alteration of film-filling rate. For example, as the inner level was observed over the range DC, the bath level rose from B to A.

When the bath level rose above the rim of the beaker, filling still occurred through the film, the surface tension meniscus preventing bulk liquid gushing into the beaker. The profile of bulk liquid surrounding the beaker in this condition is shown in Figure 9.2.2 (inset). Allen (1963) reported that it was possible to preserve film flow into the beaker in this way, provided the depth of immersion of the rim was not greater than 0.6 mm. Using the apparatus described above, however, the onset of plunge-filling was frequently delayed until the bath level was 0.9 mm above the rim. The difference is presumably because there was less disturbance in the moving bath level experiments to break the surface tension meniscus.

It is clear that the filling rate shown in Figure 9.2.2 appears to increase in two distinct stages; a slight increase while the rim was still above bath level was followed by a larger increase as the

Table 9.2.1

height of rim above bath level (mm) at initial change of rate	depth of rim below bath level (mm) at final change of rate
0.18	0.54
0.66	0.24
0.78	0.12
0.48	0.0
0.36	0,12
0.30	0.24
0.12	0.18
0.66	0.0
1.10	0.0

Table 9.3.1

Depth of inner level below rim when rate of fall becomes linear (mm)	time at which linearity sets in (sec)
4.14	94
4.02	88
3.78	84
3.54	126
4.61	108
4.56	96
3.96	78
3.45	94
4.14	100
4.14	112
4.62	102
4.26	92
3.96	102
4.32	94
3.96	90

bath level passed the rim. This behaviour was characteristic of all the transfers investigated. The position of the beaker meniscus when the filling rate changes is unimportant since the thickness of the film above the upper liquid level controls the flow. Table 9.2.1 shows the position of the bath level relative to the rim at the beginning of each increase in rate in a sequence of similar transfers. The final change in film-filling rate is of secondary importance to this investigation; it is merely due to greatly increased film thickness at the governing section for flow. This increase is primarily a surface tension effect.

The initial change of rate, however, must be considered more fully. The height of the rim above the bath level at which this change occurred, from 0.12 mm to 1.10 mm, was not very reproducible. Nevertheless, there is a close correspondence between this range of heights and that to which it was necessary to fill the beaker to obtain enhanced efflux from the beaker. It is therefore inferred that enhancement of emptying rate when the beaker has been almost completely filled is due to a thick film formed on the walls and rim of the beaker during the filling process. Since film thickness at the governing section is determined by the height of the film, the magnitude of enhancement after fast film-filling depends on the proximity of the upper liquid level to the rim during filling. It must be borne in mind that as the level approaches the rim the increasing influence of surface tension must be taken into account; eventually the film profile (3.2.1) will cease to be valid.

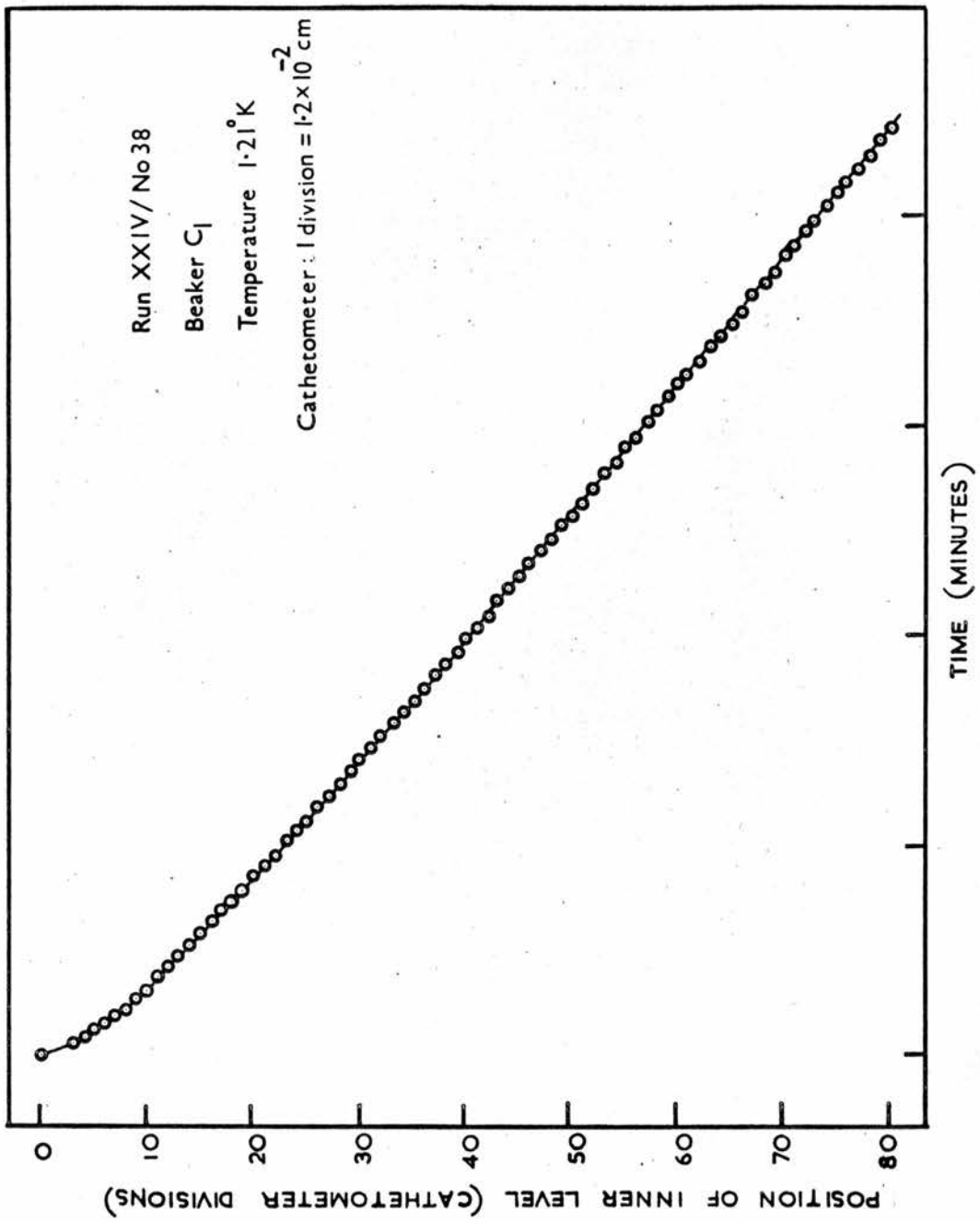


Figure 9.3.1 Enhanced Transfer Near the Beaker Rim.

9.3 Observations of Enhanced Transfer near the Beaker Rim.

In an effort to dissociate the phenomenon of enhanced emptying after plunge-filling from a simple thickening of the film due to small film heights, the emptying of a beaker was observed from the instant the inner meniscus became visible after raising the beaker from beneath the surface of the bath. The position of the falling meniscus is plotted against time in Figure 9.3.1. It is evident that the rate of fall of the liquid level varies in a different way from the rate of rise observed during filling and shown in Figure 9.2.2.

To establish the reproducibility of the high rate of emptying present at the commencement of enhanced transfer from the beaker, a sequence of such measurements was carried out, each after an identical plunge-fill. The depth of the beaker level below the rim at which the rate of fall of the level became constant is shown in Table 9.3.1. It is also of interest to tabulate the time from the start of emptying to the attainment of this depth.

Apparently enhanced transfer can assume a constant magnitude only when the inner level has fallen at least 3.78 mm from the beaker rim. On the other hand, the filling rate does not become fast until the upper liquid level is nearer to the beaker rim than 1.10 mm. Moreover, the full value of enhanced transfer from a beaker filled by the film was not obtained unless the beaker had been filled to within 0.84 mm of the rim.

It is clear, therefore, that enhanced transfer after plunge-filling is not due simply to large film thicknesses associated with

small film heights.

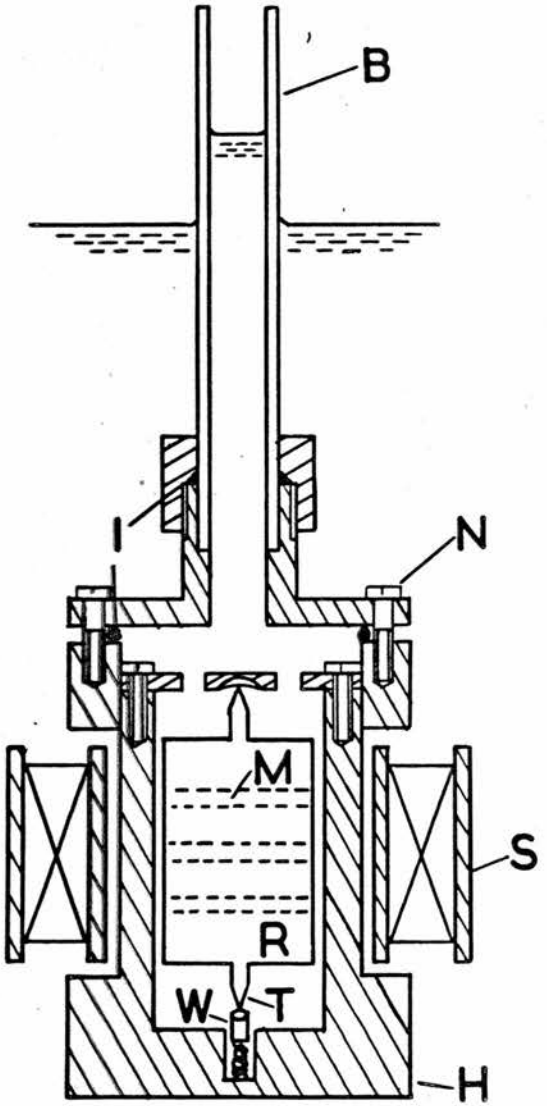
The large rate of transfer while the inner level is near the rim of the vessel followed by a steady decrease to a constant enhanced emptying rate is better understood if enhanced transfer out of the beaker is linked with a thick film formed by drainage, and in particular with the presence of a rim reservoir. Studying viscous flow in a thin layer round the corner of the rim, Tilley* has shown that drainage from a horizontal rim reservoir is rapid for times of the order of a minute, but becomes slower when the depth of the reservoir is about 10^{-5} cm. It is therefore suggested that the initial emptying rate, ~~depicted in~~ inferred from Figure 9.3.1 and several orders of magnitude higher than expected enhanced rates, is due to such fast drainage of the rim reservoir.

It is apparent from Table 9.3.1 that it takes from 88 sec to 126 sec to establish a constant enhanced rate. Prior to this, the thick film thickness is still slowly varying because leakage from the rim reservoir is occurring at a rate greater than that at which the vertical film can transport liquid.

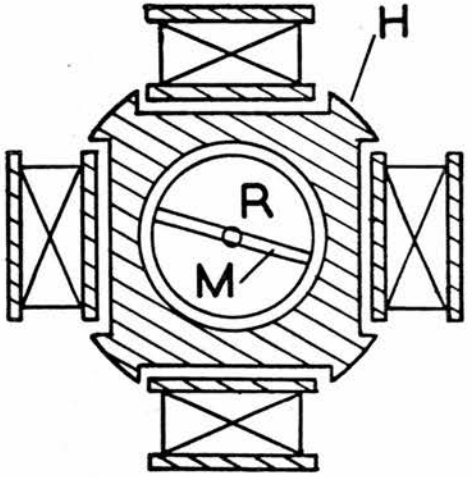
Footnote*. Tilley, J., Private communication.

Figure 10.2.1 Rotary Stirrer

B : Observation beaker
H : Perspex motor housing
I : Indium seals
M : Barium ferrite magnets
N : Nylon screws
R : Perspex rotor
S : Superconducting solenoids
T : Teflon pivots
W : Watch jewel bearings



VERTICAL SECTION



HORIZONTAL SECTION

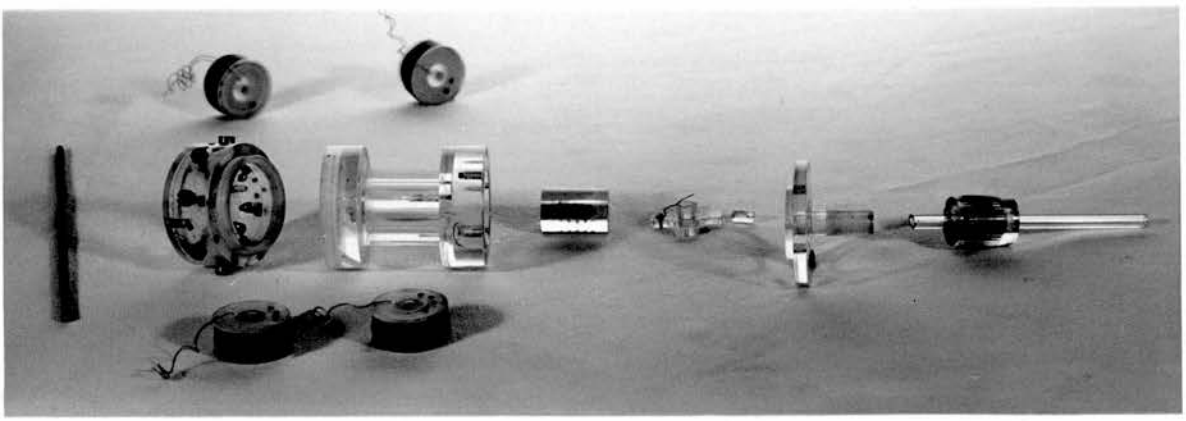


Figure 10.2.3 Components of the Rotary Stirrer.

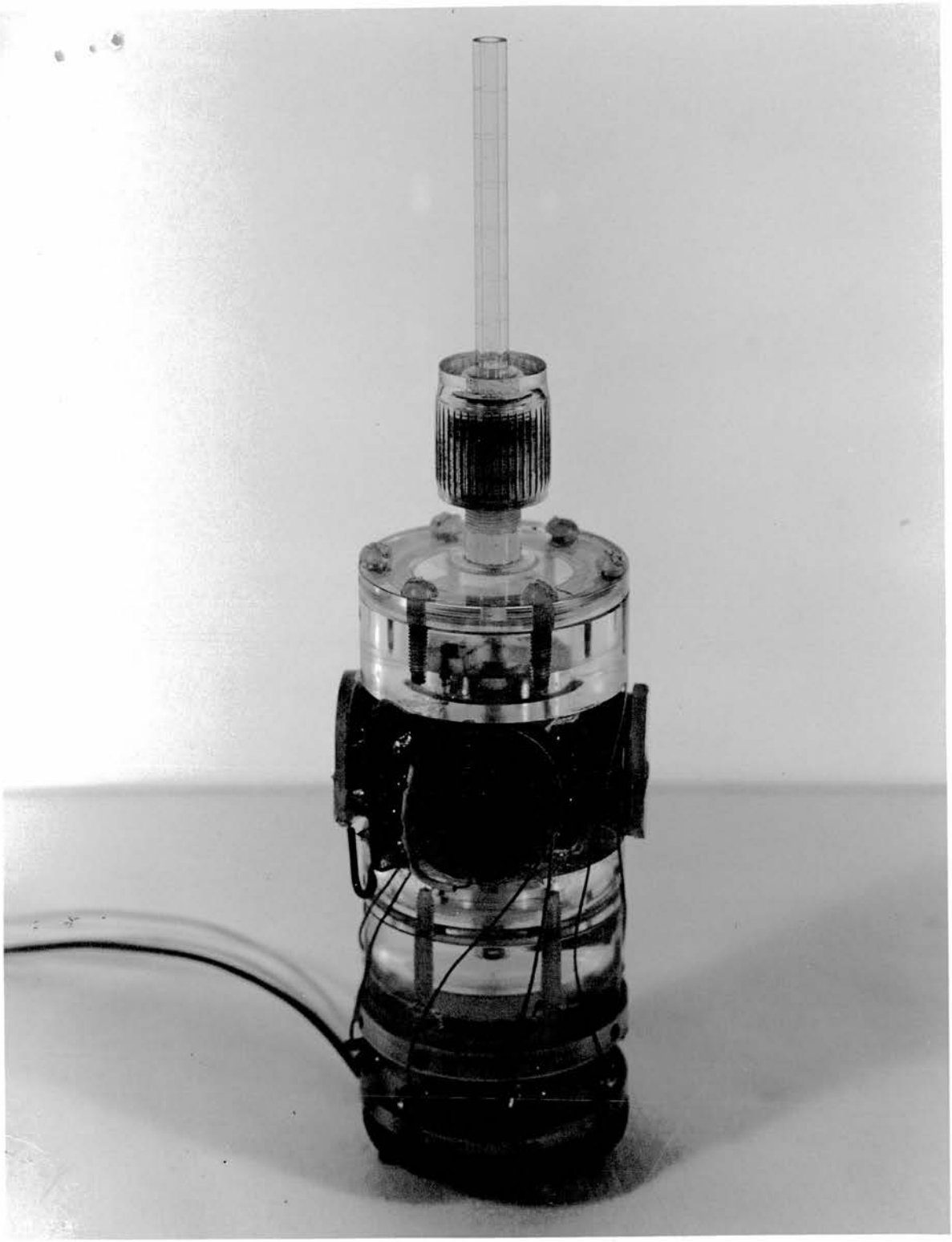


Figure 10.2.2 Assembled Rotary Stirrer.

CHAPTER 10.

Examination of the Vorticity Hypothesis of Film Transfer.

10.1 Introduction.

The enhanced transfer rate out of a beaker which had been filled by complete immersion was considered by Allen (1960a,b: 1963) to arise from turbulence introduced into the beaker by the violent filling process. Support was given to this by the effect of stirring the liquid in a narrow beaker by means of the up-and-down motion of a ferrite piston. Since the piston was limited in its action to a frequency of about 12 c/s and the resulting liquid flow to and fro past the stirrer had an uncertain Reynold's number, a more elaborate stirring system was devised.

10.2 The Rotary Stirrer.

The salient points of the design of the new stirrer are shown in Figure 10.2.1. The assembled stirrer (Figure 10.2.2) was a tight fit inside the radiation shield. In Figure 10.2.3 the various parts of the device can be seen.

Five cylindrical barium ferrite magnets (Mullard Magnadur, type FD 497) were cemented into transverse holes in a perspex rotor of diameter 1.082 cm. The sintered nature of the magnetic material made it possible to machine the rotor with the magnets in place. The surface of the rotor was smooth and highly polished. The rotor was set into motion by the rotating magnetic field produced by two pairs of superconducting solenoids set in quadrature. The superconducting solenoids were embedded in a perspex stator giving a

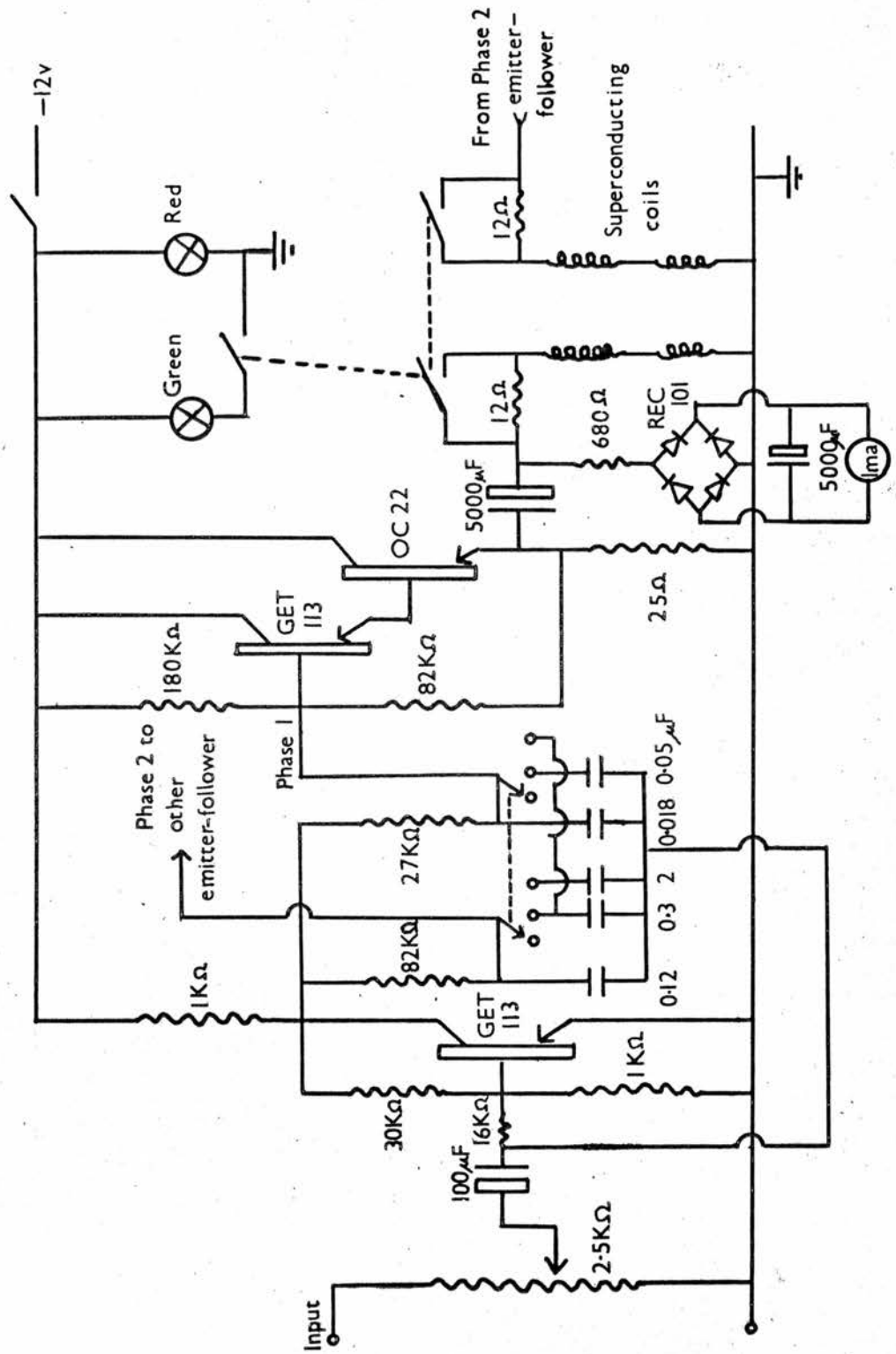


Figure 10.2.4 Two Phase Supply.

rotor clearance of about 0.001 mm. Rotor pivots of teflon (poly-tetrafluoroethylene) points resting in watchmaker's jewel bearings were found to be practically free of heat generation. The lower bearing was spring-loaded with a phosphor bronze spring. Attached to the top of the stator and giving access to the otherwise enclosed rotor space was a 1.795 mm bore glass tube which acted as the transfer beaker. Glass-to-perspex and perspex-to-perspex seals were made with indium O rings. The brass screws used to compress the rings were replaced by nylon ones prior to a run.

The four superconducting solenoids were constructed of 0.008 in. niobium wire wound on perspex formers. On average, the coils had 465 turns of wire wound in 17 layers separated by strips of oiled silk. Insulation between adjacent turns was provided by the anodized film on the niobium. After scraping off the somewhat tenacious oxide layer, the ends of the wire were encased within platinum sheaths and spot-welded. The platinum was soldered to brass terminal tags on the base of the rotor from which copper wires (Lewmex 38 swg) were taken to the top of the cryostat.

The two phase supply required to produce a rotating magnetic field was obtained by means of the circuit shown in Figure 10.2.4. The power supply was a Levell transistor RC oscillator (type TG 150M). Fairly low frequencies ($\sim$ 3 c/s) were required to start the rotor from rest. Provided smooth changes in the frequency of the controlling field were made, the rotor remained locked to the field for frequencies between 1.5 c/s and 73 c/s.

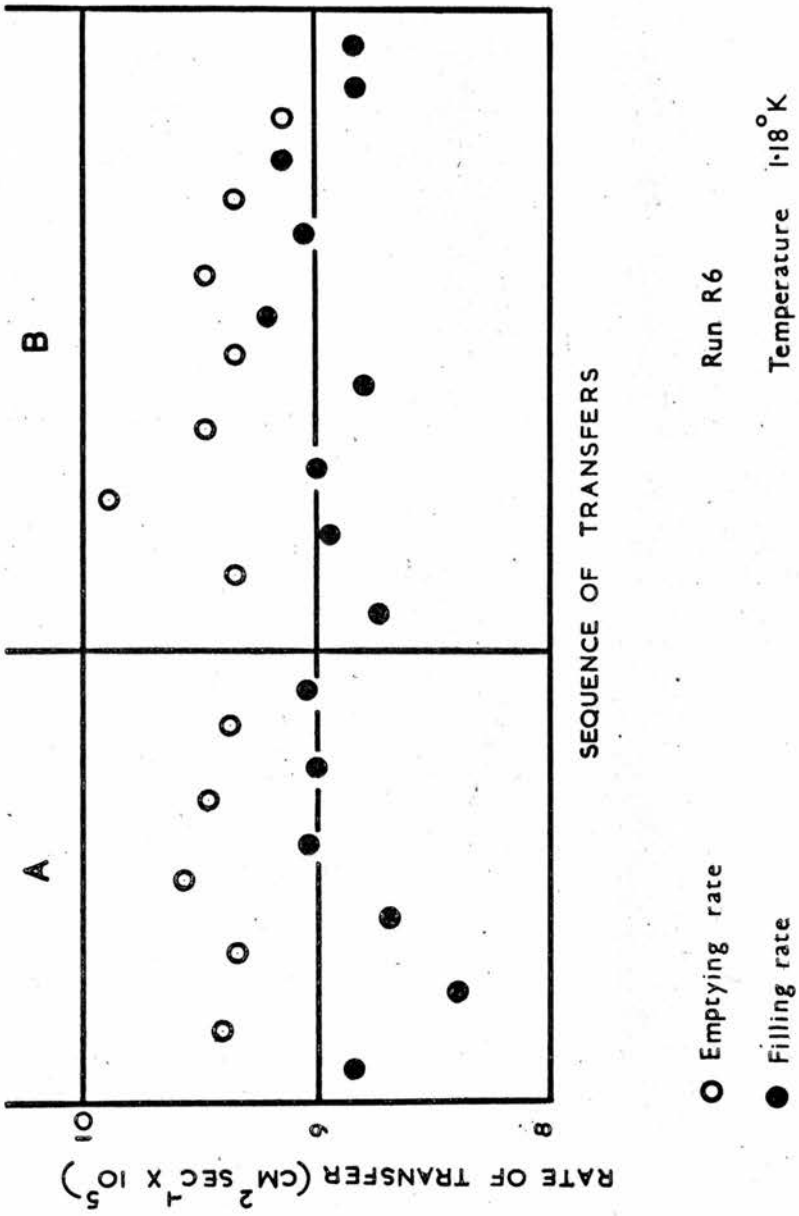


Figure 10.2.5 Film Transfer Rates and Rotary Stirring.

10.3 Flow Rate Measurements.

The effect of rotary stirring on normal film transfer rates is shown in Figure 10.2.5. The meniscus was observed over the usual range of the beaker, from 0.6 cm to 1.2 cm below the rim. Group A is a sequence of normal transfers made before the stirrer was set in motion. In B the rotor was operated at frequencies from 6 c/s to 73 c/s and for times from 1 minute to 5 minutes prior to observing transfer into and from the beaker.

It is clear that stirring in this manner had no effect on the film transfer rate which maintained its normal value at all times.

10.4 Discussion.

By use of the rotary stirrer, a very high rate of stirring was achieved with resulting Reynold's numbers well in excess of that required to produce turbulence in helium II (Staas, Taconis and van Alphen, 1961). It is therefore inferred that any vorticity introduced by mechanical stirring has no effect on the film transfer rate. Thus, the smaller amounts of turbulence resulting from plunge-filling the beaker can be considered to be irrelevant to the problem of enhanced transfer in the helium film.

It remains to discuss the enhancement and inhibition of film transfer observed in the beaker with the piston stirrer (Allen, 1963). As was mentioned in Chapter 4, the transfer rate was only affected when the action of the piston was violent enough to agitate the beaker level. The most likely explanation, therefore, of the results obtained by Allen is that the great agitation of the

meniscus during stirring generates continuous waves in the film with consequent mass transfer. A similar but more sporadic type of wave motion was described earlier in connection with the scatter of normal film transfer rates. Since mass transfer by wave action can be greater in magnitude and opposite in direction to normal film transfer, the liquid level in a piston-stirred beaker can be made to fall below the bath level by this means. Thus the static level difference, attributed by Allen to a difference in specific turbulence between the ends of the film, can easily be comprehended.

In the case of the rotary stirrer described above, the design was such that the meniscus in the observation beaker never suffered a disturbance of its surface. Hence the transfer rate was never affected by the action of the rotor.

The recently published experiments by Selden and Dillinger (1965) lend support to the ideas outlined above. During film flow under a gravitational potential, they found that mechanical agitation of the higher liquid level increased the transfer rate, while agitation of the lower liquid level decreased the rate. It was also noticed that changes in the flow rate were accompanied by increased scatter in the data, such as might be accounted for by variations in mass transfer.

One further experiment deserves mention at this point. It was found that prolonged submersion of a simple beaker for periods of up to half an hour showed the same enhanced efflux rate as did a momentary plunge-fill. Since it was unlikely that turbulence could remain trapped within a small open immersed beaker for any

length of time, it is considered that there can be no connection of any kind between vorticity and film transfer.

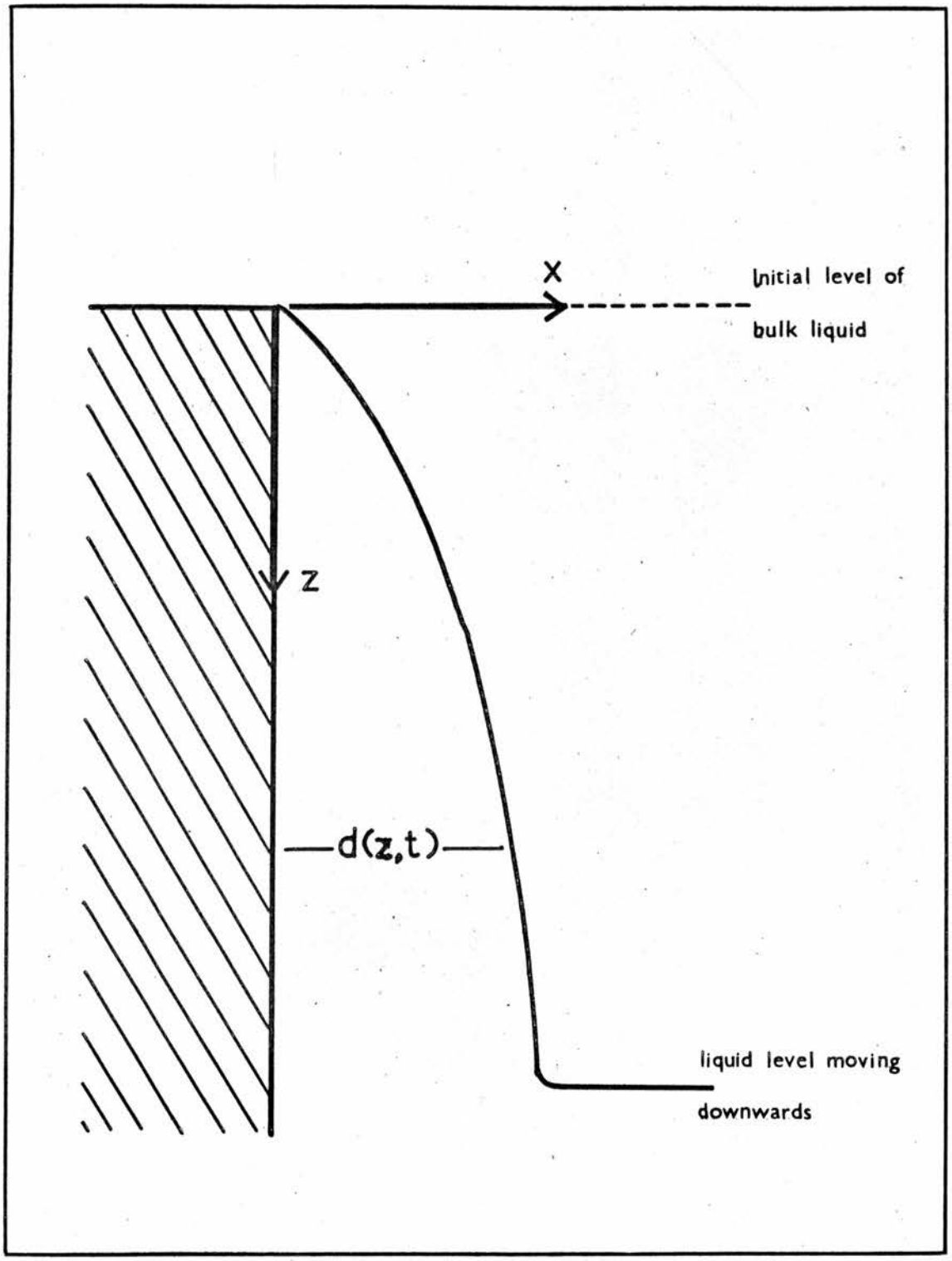


Figure A.1.1 Clinging of Viscous Fluid to Vertical Wall During Drainag

APPENDIX A.

Theory of Film Drainage.

A.1 Drainage on an Infinite Vertical Surface.

The clinging of viscous fluid to a vertical surface which had been raised from beneath the bulk liquid was considered by Jeffreys (1930) and at greater length by van Rossum (1958). Their analysis of the draining of a slab of classical liquid was considered (Matheson and Tilley, 1965: Tilley and Kuper, 1965: Allen and Matheson, 1965) to be applicable to the motion of the normal fluid within a helium film formed by recession of the bulk liquid. The assumptions which have to be made before Jeffrey's theory is applicable to the helium film have already been outlined in Chapter 7. We are concerned here merely with a theoretical examination of normal fluid flow during the drainage process. In particular, we are interested in discovering the differential equation which governs the motion; solution of this equation will indicate the rate at which the liquid drains, or in other words, the profile of the clinging film.

The situation analysed by Jeffreys is shown in Figure A.1.1. The x -axis is perpendicular to the stationary wall, with the z -axis directed downwards along the wall. At time $t = 0$, the bulk liquid level coincides with the x -axis, whence it falls at a constant speed. The liquid remaining on the vertical wall is free to fall under gravity, retarded only by viscosity. The ensuing motion closely approximates to that of laminar flow. Hence the velocity of the fluid V_z will be a function of x only.

The local film thickness, denoted by d, is expected to be a function of both x and t.

The Navier-Stokes equation for laminar flow in the z direction in a gravitational field (see, for example, Levich, 1962, Ch.1 and Ch.12) is

$$\frac{\partial v_z}{\partial t} = g + \frac{\eta}{\rho_n}\frac{\partial^2 v_z}{\partial x^2} - \frac{1}{\rho_n}\frac{\partial p}{\partial z}, \qquad \text{(A.1.1)}$$

where η is the viscosity and p the pressure. At $x = d$, $p = p_\sigma$, the surface tension pressure. Since the pressure gradient in the horizontal direction is zero, the pressure throughout the entire film is p_σ. If surface tension effects are neglected, however, the pressure term in (A.1.1) can be omitted. Moreover, except in the initial stages of the process, the acceleration $\frac{\partial v_z}{\partial t}$ is negligible in comparison with g.

The boundary conditions applicable at the free surface and at the solid surface respectively are

$$\eta \frac{\partial v_z}{\partial x} = 0, \quad \text{at} \quad x = d \qquad \text{(A.1.2)}$$

and

$$v_z = 0, \quad \text{at} \quad x = 0. \qquad \text{(A.1.3)}$$

Integrating (A.1.1) with respect to z and considering the conditions (A.1.2) and (A.1.3), the velocity profile is found to be

$$v_z = \frac{g\rho_n}{\eta}\left(dx - \frac{x^2}{2}\right). \qquad \text{(A.1.4)}$$

The Nusselt equation immediately follows, that is,

$$q_n = \int_0^d v_z \, dx = \frac{1}{3} \frac{\rho_n g}{\eta} d^3, \tag{A.1.5}$$

where q_n is the flux of fluid in the film. This equation, together with the continuity equation

$$\frac{\partial q_n}{\partial z} = - \frac{\partial d}{\partial t}, \tag{A.1.6}$$

leads to a variation of thickness given by

$$\frac{\partial d}{\partial t} + \frac{g \rho_n}{\eta} d^2 \frac{\partial d}{\partial z} = 0. \tag{A.1.7}$$

Dividing by $\frac{\partial d}{\partial z}$ and making use of the identity

$$\left(\frac{\partial d}{\partial t}\right)_z \Big/ \left(\frac{\partial d}{\partial z}\right)_t = - \left(\frac{\partial z}{\partial t}\right)_d, \tag{A.1.8}$$

we find

$$\left(\frac{\partial z}{\partial t}\right)_d = \frac{g \rho_n}{\eta} d^2. \tag{A.1.9}$$

The general solution of this equation is

$$z = \frac{g \rho_n}{\eta} d^2 t + \phi(d), \tag{A.1.10}$$

where $\phi(d)$ depends on the initial conditions. Since $z = 0$ at $t = 0$, $\phi(d) = 0$. Hence the shape of the free surface of the draining film is given by

$$d = \left[\frac{\eta}{g \rho_n} \cdot \frac{z}{t}\right]^{1/2}. \tag{A.1.11}$$

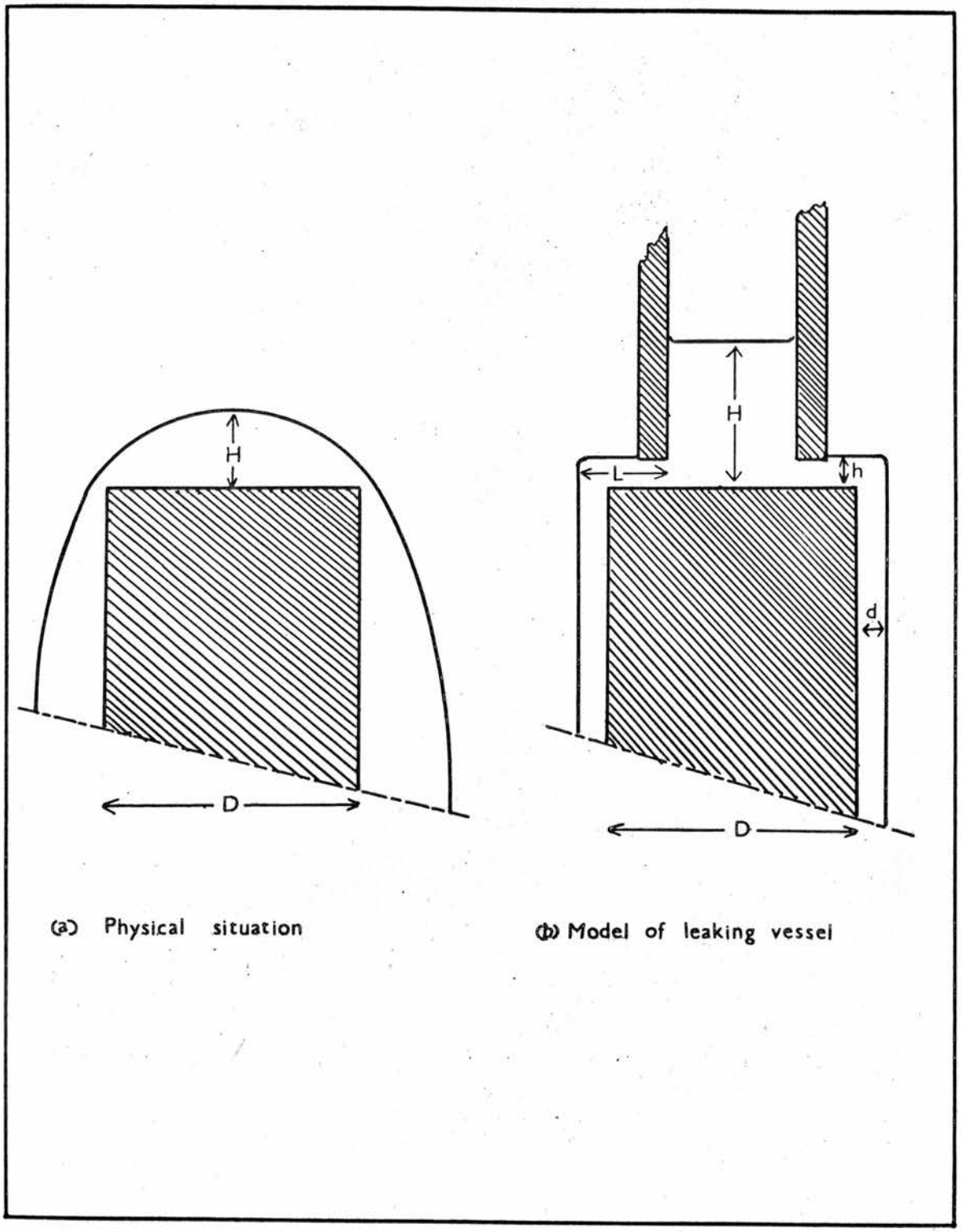


Figure A.2.1 Drainage at the Rim of a Beaker.

A.2 The Influence of a Liquid Reservoir on Drainage.

Tilley and Kuper (1965) also discussed the relevance to the drainage process of the puddle of bulk liquid which is trapped on the rim of a beaker lifted from beneath the bath. As a first approximation, leakage from the rim puddle (Figure A.2.1a) is considered to be similar to the flow of viscous liquid through two horizontal films situated at the base of a rectangular vessel filled to a depth H (Figure A.2.1b). The length of each horizontal film is

$$L = \tfrac{1}{2} K D, \qquad (A.2.1)$$

where D is the width of the rim and $0 < K < 1$.

The pressure gradient available to drive fluid through the horizontal film is $\frac{\rho_n g H}{L}$. Steady flow will prevail when

$$\frac{\rho_n g H}{L} = \eta \frac{d^2 v}{dy^2}, \qquad (A.2.2)$$

where v is the horizontal fluid velocity at a perpendicular distance y above the rim. Integrating (A.2.2) and taking account of the usual boundary conditions for laminar viscous flow, the velocity profile is found to be

$$v = \frac{\rho_n g H}{2\eta L}\left(yh - \frac{y^2}{2}\right), \qquad (A.2.3)$$

where h is the film thickness. The flux of fluid through each horizontal film is, therefore,

$$Q_n = \frac{g H h^3}{3 \nu_n L}, \qquad (A.2.4)$$

or, from (A.2.1)

$$Q_n = \frac{2gHh^3}{3\nu_n K D}, \qquad (A.2.5)$$

where ν_n is the kinematic viscosity of the fluid.

By (A.1.5), the flux of fluid through the inside vertical film is

$$q_n = \frac{1}{3}\frac{g}{\nu_n} d^3, \qquad (A.2.6)$$

where d is the inner film thickness assumed to be uniform. In the steady state the flow rate Q_n will be equal to the flux q_n. Hence

$$d^3 = \frac{2Hh^3}{KD}. \qquad (A.2.7)$$

Steady efflux of fluid from the leaking vessel on top of the rim will result in a decrease of H by some fraction ε in a time τ. During this time the volume of liquid leaving the vessel will be εHD. Hence

$$\tau = \frac{\varepsilon HD}{2Q_n}. \qquad (A.2.8)$$

From (A.2.5), (A.2.7) and (A.2.8)

$$d = \left(\frac{3D}{2g}\right)^{1/3}\left(\frac{\varepsilon H}{\tau}\right)^{1/3}(\nu_n)^{1/3}. \qquad (A.2.9)$$

To further relate this analysis to the problem of the draining helium film requires a sensible choice of the parameters in (A.2.9). The wall thickness of the beaker D was fixed at 1 mm. Taking the liquid reservoir depth H as 5×10^{-5} cm, Tilley et al

calculated the film thickness which would allow a 5% change in H in a time of 1 kilosecond. Thus

$$d = 7{\cdot}28\times10^{-5}(\nu_n)^{1/3}. \qquad \text{(A.2.10)}$$

A 5% change in H would cause, say, a 2% change in d over a period of 1 kilosecond. From an experimental point of view, such a change is not detectable. To all intents and purposes, therefore, the theory indicates a constant film thickness. Nevertheless, it is important to note that the film thickness given by (A.2.10) is not an absolute value. In particular, it depends on apparently arbitrary values of ε, H and τ. These parameters, however, appear to the one third power in (A.2.10) and hence d is not particularly sensitive to changes in them.

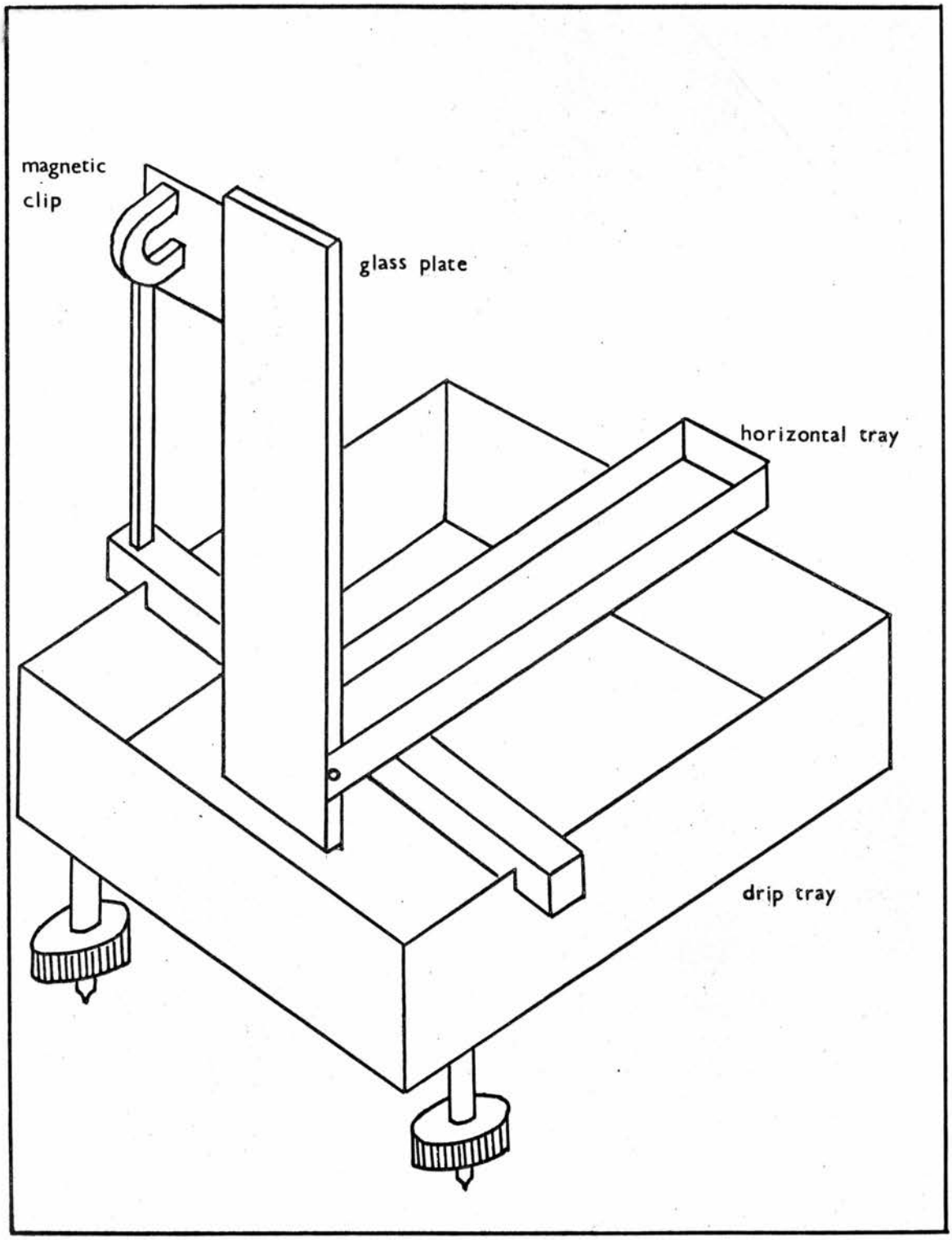


Figure B.2.1 Apparatus used to Obtain a Draining Layer of Syrup.

APPENDIX B.

Experiments on Drainage of a Classical Fluid.

B.1 Introduction.

When the possibility of draining helium films was first mooted, it was felt desirable to investigate the thickness of a classical liquid film draining on a vertical wetted plate. In particular, it was hoped to gain an idea of the validity of equation (A.1.11) proposed by Jeffreys.

A conveniently observable rate of drainage dictated the choice of fluid used in the experiments reported here. For this reason, and the sustaining nature of the fluid, Lyle's Golden Syrup was chosen. There are indications that Syrup is Newtonian in that the shearing stress is proportional to the rate of shear.

B.2 Apparatus and Procedure.

A flat glass plate, 5 cm wide and 32cm long, was rigidly pivoted at one end of a shallow metal tray (Figure B.2.1) in such a way that it could lie horizontally within the tray, and also be swung into a vertical position where it was held by a magnetic clip.

With the plate in the horizontal position a uniform layer of Syrup was deposited on the plate. This layer was generally about 4 mm in thickness. The plate was quickly elevated to the vertical position. Talcum powder was lightly puffed onto the film to enable a travelling microscope to be focussed on the free surface. The position of the surface could be determined to within 0.005 mm in this way. The rate at which the free surface approached the

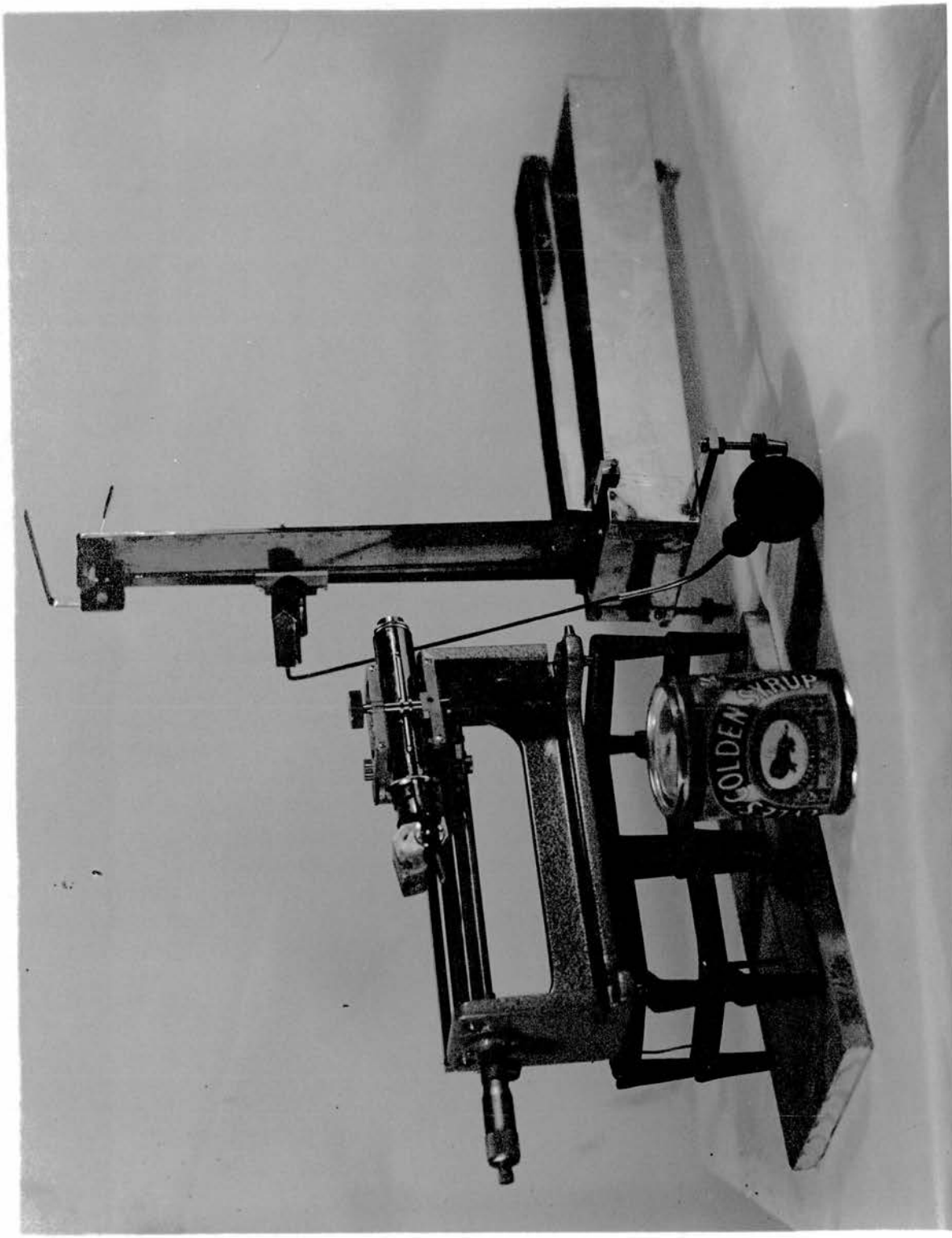


Figure B.2.2 Assembled Apparatus for Draining Film Experiments.

plate was observed for periods of the order of half an hour. Following this, any residual film was washed off the plate and the microscope focussed on the bare glass surface. Excess liquid was collected in a large tray beneath the plate.

In this way the variation with time of the thickness of a draining layer of Syrup was determined. All measurements were made at a constant depth below the top of the plate of 18 cm. The complete apparatus is shown in Figure B.2.2.

The temperature at which the experiment is conducted must be known in order that the viscosity of the fluid be determinate. Accordingly, the whole apparatus was enclosed within an insulated box. A series of thermostatically-controlled heaters and an electric fan enabled the temperature within the box to be maintained constant to within 0.5°C. The layer of syrup was given adequate time in a horizontal position to reach a steady temperature before measurements were begun.

B.3 The Viscosity and Density of Lyle's Golden Syrup.

In order that the drainage experiments constitute a useful test of Jeffreys' parabolic formula for film thickness, it is necessary to know the viscosity and density of Syrup.

A lamentable lack of knowledge regarding the temperature variation of the viscosity of Syrup is apparent from the literature. Accordingly, viscosity measurements were made over the temperature range $18.1°C \lesssim T < 31.1°C$ using a falling sphere viscometer of diameter 8.4 cm. The temperature of the liquid column was controlled thermostatically. Observations were made of the time taken

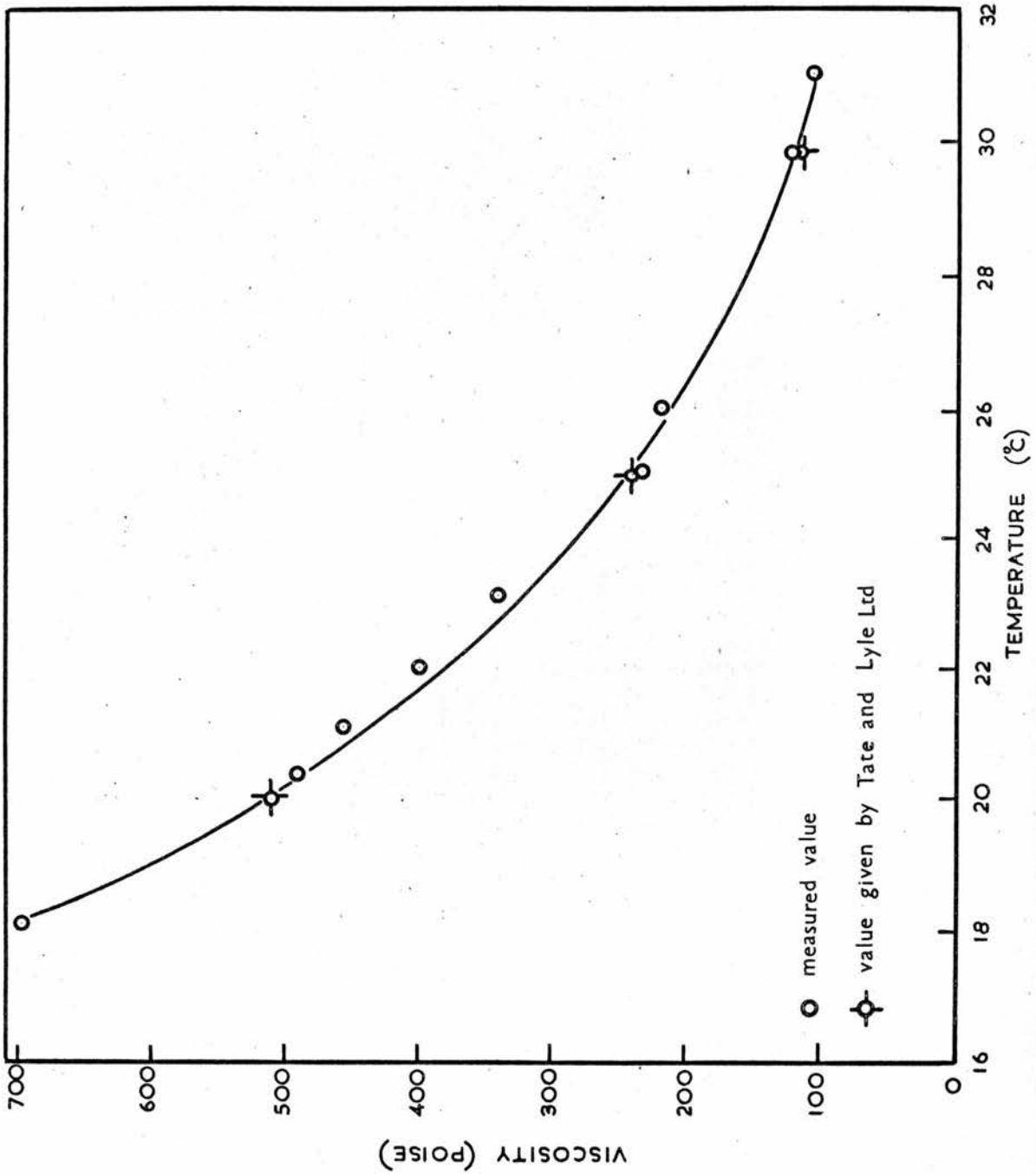


Figure B.3.1 Temperature Variation of Viscosity of Syrup.

Table B.3.1

Run number	temperature (°C)	viscosity (poise)	density (gm cm^{-3})
D2	23.5	325	1.437
D3	26.1	223	1.429
D4	25.9	229	1.429
D6	18.5	628	1.440
D7	19.5	570	1.439
D8	27.6	179	1.435
D9	30.8	113	1.432

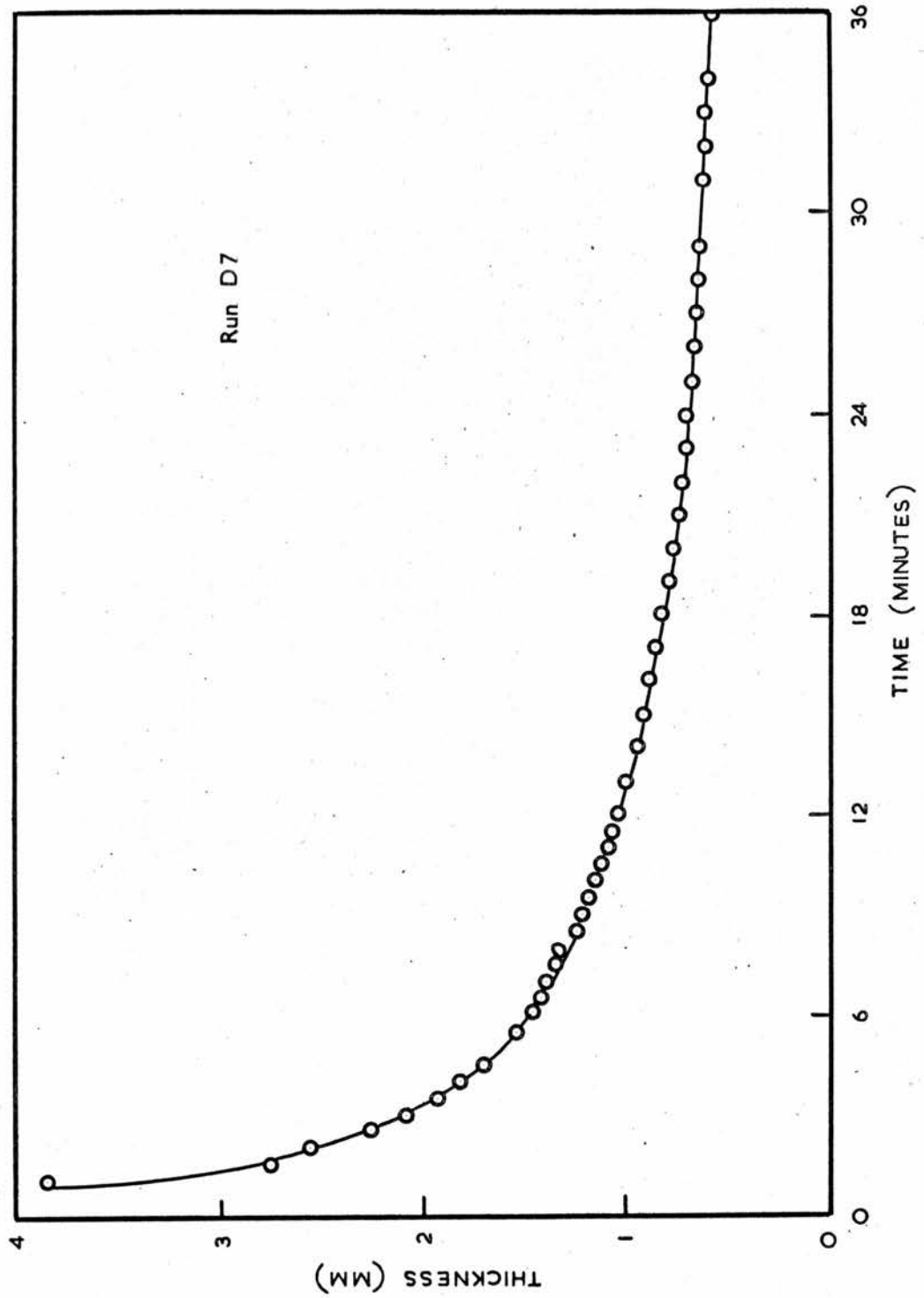


Figure B.4.1 Variation of Syrup Film Thickness with Time.

for steel ball bearings (diameter 3.175 mm) to fall through a distance of 20 cm. The measured viscosity was corrected for the effect of the walls of the viscometer.

The results are shown in Figure B.3.1. It can be seen that the variation of viscosity with temperature agrees well with that given by Tate and Lyle, Ltd*. Their measurements were made in a large number of experiments using a Hoppler viscometer, U-tube viscometer, falling sphere viscometer and a rotating cylinder viscometer. A logarithmic plot of viscosity against temperature allowed the value of viscosity at any intermediate temperature to be interpolated.

The density of Syrup is given by Tate and Lyle, Ltd. as

$$\rho = 1{\cdot}451 - 0{\cdot}006T \qquad \text{(B.3.1)}$$

over the temperature range $15°C < T < 60°C$.

Table B.3.1 gives the appropriate values of viscosity and density of syrup for the seven runs to be described in the following section.

B.4 Observations and Analysis of Film Drainage.

A typical observed variation of syrup film thickness with time is shown in Figure B.4.1. Drainage was initiated at time $t = 0$ by raising the uniform layer of syrup into a vertical position. As might be expected, drainage is most rapid at the commencement of

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Footnote*. Nicol, W. B., of Tate and Lyle, Ltd., private communication.

Table B.4.1

Run number	measured value of n	value of C (cm^2 sec) measured	theoretical
D2	0.49	2.44	4.15
D3	0.53	0.98	2.89
D4	0.51	1.61	2.95
D6	0.54	9.91	7.99
D7	0.52	9.40	7.27
D8	0.48	1.10	2.29
D9	0.49	0.89	1.45

of the process.

A graph of the data in which $\log d$ was plotted against $\log t$ revealed a linear relationship between the thickness d and the reciprocal of the time t to some power less than unity. It may therefore be deduced that the observed film thickness obeys the relationship

$$d = \left(\frac{C}{t}\right)^m, \qquad \text{(B.4.1)}$$

where m is the absolute value of the gradient of the logarithmic plot and $\log C = \frac{\kappa}{m}$, where κ is the intercept on the $\log d$-axis. Table B.4.1 shows the values of m and C obtained from analysis of the data of seven runs at different temperatures.

The theoretical drainage equation according to Jeffreys is

$$d = \left(\frac{\eta}{\rho g} \cdot \frac{z}{t}\right)^{1/2}, \qquad \text{(A.1.11)}$$

where d is the film thickness at depth z below the vertex of the parabola. In our experiments z was 18 cm. The theoretical value of m in equation (B.4.1) is 0.5. The theoretical value of C is $\eta z/\rho g$. This quantity was calculated for each run from Table B.3.1. Values are shown in the last column of Table B.4.1.

B.5 Discussion.

It appears that the power of the relationship between film thickness and time predicted by Jeffreys is strongly supported by experiment. Moreover, there is order of magnitude agreement between the value of C in (B.4.1) and that of $\eta z/\rho g$ in Jeffreys' equation. The slight discrepancy which is apparent between the

latter quantities is possibly a result of edge effects arising from the small width of the glass plate.

It is hoped that further experiments on the drainage of classical fluids will reveal further information about the factors governing drainage. In particular, the influence of surface tension and that of a bulk liquid reservoir feeding the film remain to be experimentally established.

Since this work was completed, our attention has been drawn to the experiments by Goucher and Ward (1922), Satterly and Stuckey (1932), Morey (1940) and van Rossum (1958). The parabolic profile of a draining film was studied photographically and by interference fringes. For a simple draining film the Jeffreys solution was found to be fulfilled almost exactly.

SUMMARY

From this experimental investigation of superfluid film transfer in narrow beakers in helium II, it has emerged that there are probably two kinds of helium film. A normal film is formed by superfluid creep over a dry substrate. A thick film remains when bulk liquid has drained from a substrate that has previously been immersed in the liquid. A comparison has been made of the superfluid flow between the two types of film.

The transfer rate exhibited by the normal film has been identified with the rate commonly accepted as standard. The thick film, however, showed an enhanced rate of transfer. The enhanced rate was as much as 60% greater than the normal rate at 1°K, but the difference between the two rates disappeared above 1.8°K.

Statistical significance tests on the standard deviation of normal and enhanced transfer suggested that there was a basic difference between the two types of flow. Comparing the scatter of film flow rates with the scatter of flow rate measurements in narrow channels emphasized the importance, in film transfer, of the free surface of the film and the consequent possibility of variable film thickness.

The scatter of values of transfer rate associated with a normal film has been attributed to third sound generated by bath waves impinging on the meniscus at the base of the film. This scatter was greatly reduced by the use of beakers in which bath surface disturbance was almost entirely eliminated. The increased spread of values of normal transfer rates below 1°K was thought to

be a consequence of the declining influence of normal fluid.

The enhanced transfer rate was found to persist for long periods of time in quiet conditions, but was abruptly diminished by disturbances such as agitation of the meniscus at either end of the film, or perturbation of the film itself at the governing section for flow. The sensitivity of enhanced transfer to disturbance is evidence of the fact that the thick film is a meta-stable state; the equilibrium state of the helium film is the normal film. Enhanced transfer also persisted after reversal of the flow direction. The latter observation shows that a true normal rate is only observed if the transfer takes place into an empty beaker. There was a maximum stable length of the order of a few cm for a thick film exhibiting the full enhanced transfer rate. The cause of the spontaneous decrease of enhanced transfer rate which occurred in long, thick films was thought to be third sound resonance. To substantiate this suggestion requires more detailed knowledge of the processes of evaporation and condensation in the film than is at present available.

The temperature dependence of the two types of transfer was compared between 0.49°K and the lambda point. The reason for the resumed rise in the normal rate of transfer below 1°K is not known; it may possibly be connected with a temperature dependent critical velocity or an increase in film thickness. In view of the inadequacy of the present theory of critical velocity in the film, the mechanism of normal film transfer below 1°K must remain obscure.

Unlike the normal transfer rate, the enhanced transfer rate rose steadily with decreasing temperature. However, this rise was halted below about 0.8°K and at 0.52°K it was impossible to distinguish any enhancement of film transfer following plunge-filling.

The theory of thick draining films put forward by Tilley has been examined. The thickness of the film formed by a receding bulk liquid level is found to be proportional to the one third power of the kinematic viscosity of the normal fluid. The theory takes account of the leaking puddle of liquid formed on the beaker rim when it is lifted from beneath the bath surface. The presence of this reservoir completely alters the time-scale of the drainage process which would otherwise be that occurring on, effectively, an infinite vertical wall.

The parabolic profile predicted for a classical liquid film draining on a vertical wall in the absence of a rim reservoir was verified using Golden Syrup.

The temperature dependence of enhanced transfer between 1.2°K and 1.9°K was satisfactorily explained by the drainage theory, assuming the critical velocity to be inversely proportional to the inverse square root of the controlling film thickness. The disappearance of the thick film below 0.8°K was taken as evidence of the final breakdown of the classical drainage theory.

The influence of the rim on both filling and emptying rates of a beaker was studied. Fast film-filling rates, which occurred when the upper liquid level approached within a mm or so of the rim, were attributed to increased film thickness associated with

short normal films. On the other hand, rates of emptying several orders of magnitude greater than enhanced rates persisted until the inner level was several mm below the rim. The observed decline of this fast initial rate towards the expected enhanced rate is believed to be a result of the declining thickness of the rim reservoir.

Observations made while the liquid in the beaker was continuously stirred failed to reveal any enhancement or inhibition of film transfer rate, and thus do not support the vortex model for film flow.

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