THE HELIUM FILM

A THESIS

presented by

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I hereby declare that this thesis has been composed by me, is a record of work carried out by me, and that it has not been previously presented for a higher degree. The work was carried out in the Physical Laboratories of St. Andrews University under the supervision of Professor J. F. Allen, F.R.S.

I certify that Cecilia Mary Walker, B.Sc. has spent four terms as a research student in the Physical Laboratory of the United College of St. Salvator and St. Leonard in the University of St. Andrews, that she has fulfilled the conditions of Ordinance No. 51 of the University Court of St. Andrews and that she is qualified to submit the accompanying thesis in application for the Degree of Master of Science.

(Supervisor)

CAREER

I matriculated in October 1957 in the United College of St. Salvator and St. Leonard in the University of St. Andrews and followed a course leading to graduation in June 1960 with the Ordinary Degree of Bachelor of Science. The following year I followed a course leading to the Degree of Bachelor of Science with Honours in Natural Philosophy. In October 1961 I was admitted by the Senatus Academicus of St. Andrews University as a research student in the Department of Natural Philosophy in the same college and have since been engaged upon the work described in this thesis. In September 1961 I was awarded a British Oxygen Fellowship for the following year.

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CHAPTER I.

Introduction

1.1 The Phenomenon of the Helium Film

Although about twenty-five years have passed since its discovery there is still no satisfactory explanation of many properties of the helium film. A theory of the film based on van der Waals' forces has been developed, which accounts satisfactorily for some properties of the static film, but no complete hydrodynamic theory of the helium film has yet been put forward to explain the extremely unusual flow properties. The work presently reported was undertaken to aid development of a satisfactory hydrodynamic theory by providing further information on the film properties.

The possible existence of a film of liquid helium on the walls of a vessel was first mentioned by Rollin (Kurti, Rollin, and Simon, 1936), to explain the rapidity with which temperature equilibrium was established in a system containing liquid helium below the lambda-point. Rollin and Simon (1939) subsequently developed the idea and applied it to the results of their own experiments and to some earlier unexplained observations.

Kamerlingh Onnes (1922) made the first recorded observation of one of the properties of the film. He noted the quick disappearance of a difference in liquid levels introduced between the helium in a beaker and in a bath containing the partially immersed beaker. The phenomenon

was attributed to distillation, though the speed of this supposed vapour distillation was commented upon.

Rollin and Simon (1939) demonstrated the existence of the film in experiments which measured the rate of evaporation from vessels of known geometry. The rate of evaporation was found to vary with the radius of the tube connecting the vessel to the pumping system, as would be expected if the surface of the tube was covered with liquid helium. The material of the walls appeared to have no effect on the phenomenon.

Two theories were put forward to explain heat transport in the film: (1) that the film was stationary and had a very high thermal conductivity, similar to that of the bulk liquid (Keesom and Keesom, 1936), or, (2) that the film was in continual motion up the walls, evaporating in regions of higher temperature. The early experiments could be equally well explained by both theories. Rollin and Simon (1939) concluded that the film was "creeping" over the walls when they observed that a vessel emptied at the same rate when closed by a needle valve as when it was open, whereas if there had been distillation the transfer rates would have been very different in the two cases. Daunt and Mendelssohn (1939a) obtained further confirmation that the transfer was a surface phenomenon when they made the first comprehensive investigation of the film properties. They observed that the flow rate from the higher liquid level was enhanced when a wick of copper wires was introduced between two connected vessels containing helium at different levels, and all the helium removed from the higher level by the increase in flow appeared in the lower vessel. Further mention of the experiments

of Daunt and Mendelssohn will be made later.

The film is apparently a consequence of the superfluidity of liquid helium II and, therefore, any hydrodynamic theory of the film must take account of the theories of superfluidity which have been developed. Some of these theories and the properties of liquid helium leading to them will be briefly discussed below.

1.2 General Properties of Liquid Helium

The phase diagram of helium exhibits several features which indicate that the liquid may be expected to have some unusual properties. There is no triple point and it is impossible to solidify helium under its own saturated vapour pressure. The phase diagram has been compiled from magnetic cooling measurements down to very low temperatures, and, when the curves are extrapolated, it appears that helium is still liquid at absolute zero. The solidification pressure required at this temperature would be about twenty-five atmospheres. There is a phase change between two types of liquid (which are called helium I and helium II) at about 2.18°K.

Experimental observations show that $\frac{dp}{dT} \longrightarrow 0$ as $T \longrightarrow 0$. Using this and the Clausius-Clapeyron equation for the melting curve it can easily be seen that at 0°K. the entropy of the liquid and solid are equal. Thus from the third law of thermodynamics there exists at absolute zero both a solid and a liquid form with zero entropy or perfect ordering. The existence of a liquid at absolute zero implies that it might be expected to show quantum mechanical properties.

The specific heat curve for liquid helium shows the form of a co-operative phenomenon at the temperature of the liquid phase change. This indicates a second order transition where the first order derivatives of the Gibbs function (entropy and volume) are continuous, but the second order derivatives (specific heat at constant pressure and coefficient of expansion) show discontinuities at the transition temperature. Experimental determination of the coefficient of expansion has shown the expected discontinuity. The transition temperature is called the lambda-point because of the similarity of the specific heat curve to the Greek letter lambda.

Above the lambda-point the liquid (helium I) behaves very nearly classically, but below this temperature the liquid (helium II) exhibits some quite unusual properties. Under isothermal conditions helium II is able to flow through very narrow channels, and the flow shows complete divergence from classical properties (Allen and Misener, 1939). Also different values of the viscosity are obtained by different experimental techniques. The high mobility of liquid through the helium film is another phenomenon not encountered in other liquids.

The thermal properties of the bulk liquid are also very unusual. The extremely high thermal conductivity is found to be an inverse function of the temperature gradient (Allen, Peierls, and Uddin, 1937), and to increase with decrease in the size of a capillary along which there is a temperature gradient (Allen and Jones, 1938). Allen and Jones (1938) also observed the thermomechanical, or "fountain" effect, where a difference in levels of liquid helium is observed in two con-

nected vessels if a small quantity of heat is supplied to one of them.

1.3 Theories of Liquid Helium

The unusual properties cannot be explained by classical theory and it is therefore to be supposed that they may be connected with quantum phenomena (Landau, 1941).

In several ways He^4 may be likened to a gas rather than to a normal liquid. For example helium I has a gas like viscosity which increases with temperature, while in a normal liquid the viscosity decreases with increase in temperature. The atoms contain an even number of fundamental particles and can thus be considered to obey Bose-Einstein statistics. London (1938a,b) pointed out the similarity between the lambda-transition and the condensation which could occur in an ideal gas of Bose particles. This condensation had previously been neglected in theoretical consideration of the statistics as having no practical application. In helium the condensation is distorted from that of an ideal gas because the particles are subject to interatomic forces, also helium is in the liquid phase and is not a gas. Below the transition temperature a finite fraction of the atoms will "condense" into the lowest available quantum state, or ground state, the fraction condensed varying from unity at 0°K. to zero at the transition temperature. To describe the contribution of the atoms in the lowest state to the total density of the liquid, London considered the analogy to a condensation in momentum space, so that the atoms in the ground state have zero energy and momentum and hence also zero viscosity.

Tisza (1938) developed the "two fluid" theory on this picture. The atoms in the ground state are considered as the superfluid component of the liquid and have zero entropy, while the atoms in excited states are associated with the normal fluid.

Landau (1941; 1947) considered liquid helium as a continuum with its possible modes of excitation. He envisaged two types of excitation which he called phonons, corresponding to longitudinal Debye sound waves, and rotons, which he suggested might correspond to rotational modes of motion. In the energy spectrum envisaged by Landau, the excitations with small values of momentum (p) correspond to phonons and will be predominant below 1°K. At higher energy values (ϵ) he envisaged the roton spectrum to be of the form

$$\epsilon = \Delta + \frac{(p - p_o)^2}{2\mu}$$

where Δ is the minimum energy of a roton and μ its effective mass. The existence of the energy gap (Δ) implies that rotons will not be excited at low energies. The theory is based on temperatures close to the absolute zero, and hence fails to give a satisfactory explanation of the lambda phenomenon, but the presence of an energy gap and the resulting exponential factors in the calculated thermodynamic quantities make it relatively easy to relate theory and experiment. In this theory the superfluid is the "background" in which the normal fluid is embedded. The excitations themselves are considered as quasi-particles whose density at any temperature gives the normal fluid density (ρ_n) at that temperature.

The theory provides a semi-quantitative explanation of many of the observed phenomena in liquid helium. The non-classical flow through narrow channels may be described as flow of the superfluid with zero viscosity, while the excitations collide with the walls of the channel and are unable to get through. The helium film may be considered as an extremely narrow channel. Thus in the two fluid picture flow through the film will be of the superfluid fraction. It appears improbable that the normal fluid can take part in film flow, and thus the film offers a readily obtained medium for the study of pure superfluid flow.

A rigorous explanation of the properties of liquid helium can only be obtained from solution of the Schroedinger equation for all the atoms in a sample of liquid. This is prohibitively difficult and no complete solution can therefore be expected, but Feynman (1955) has developed Landau's picture using a phenomenological approach. The phonons in Landau's picture are longitudinal sound waves and involve changes in density as the value of the wave function changes in the Schroedinger equation. It is because only phonons exist as excited states close to the absolute zero that the specific heat varies as T^3 when T tends to zero. Any other types of excitations, not involving changes in density, have much higher energies, since the wave function changes very rapidly from positive to negative when there is a change of configuration between states corresponding to maximum and minimum possible values of the wave function. Feynman considers the rotons of Landau's excitation spectrum as the excitations produced by altering the configuration of the atoms without altering the density. The minimum energy is involved when the

gradient of the wave function is a minimum.

Feynman's picture indicates that there will be a minimum in the energy spectrum, and also predicts a linear spectrum for small values of the wave number, as envisaged by Landau. This quantum mechanical calculation gives a value of the minimum in the curve larger than the experimental value. Feynman also considers the implications of irrotational superfluid flow and this is discussed in the following chapter.

CHAPTER II.

The Concept of Vorticity in Helium II

2.1 Introduction

The irrotational flow of superfluid helium has been considered an essential to be accounted for in any development of a hydrodynamic theory explaining the unusual flow properties (Landau, 1941). The existence of a critical velocity ($\underline{v}_c$) was appreciated, and it was thought that flow must be frictionless and irrotational below this velocity. The restricting equation then becomes

$$\text{curl } \underline{v}_s = 0 \quad \text{for } \underline{v}_s < \underline{v}_c \ ,$$

where $\underline{v}_s$ is the superfluid velocity. Early experimental evidence tended to support the idea. The Andronikashvili pile of discs oscillating in helium II only influenced the normal part of the fluid.

Osborne (1950) found that the surface of a beaker of helium II set in rotation was described by the equation

$$y = \frac{\Omega^2}{2g} x^2 \ ,$$

where y is the height, x the distance from the axis of rotation and Ω is the angular velocity, not by the equation

$$y = \frac{\rho_n}{\rho} \frac{\Omega^2}{2g} x^2 \ ,$$

as would result if only the normal fraction was affected by the rotation

of the beaker. It can be argued that this might be the result if the critical velocity was exceeded, as the peripheral velocity was from 35 to 70 cm. per sec., but repetition of this type of experiment with velocities down to 4 cm. per sec. has yielded the same result (Andronikashvili and Kaverkin, 1955).

The superfluid has been regarded as the medium in which the normal fluid (or excitations) moves. It is thought to be frictionless itself, and thus the only way in which friction can be introduced is by the formation of excitations in the fluid. The superfluid might react with the walls to form these excitations. Two kinds of excitations have already been mentioned, phonons and rotons. The critical velocity required in the superfluid for creation of these excitations is given by

$$v_c = \left| \frac{\epsilon}{p} \right|_{\text{min.}},$$

which gives values for the creation of a phonon $v_c = 2.39 \times 10^4$ cm/sec., and for the creation of a roton $v_c = 6 \times 10^3$ cm/sec., values far in excess of any observed critical velocities. It seems very unlikely therefore, that creation of excitations like these can cause the onset of friction at low velocities, and the suggestion has been made (Ginsburg, 1949) that excitations of much lower values of $\left| \frac{\epsilon}{p} \right|$ must exist.

2.2 Quantised Vortex Lines

Feynman (1955) has developed the idea of quantised vortices in helium which was first mentioned by Onsager (1949). The theory will be briefly reviewed here.

Considering wave functions satisfying the Schroedinger equation for the system, flow can be represented by a function

$$\Psi_{\text{flow}} = \Psi \exp\left[i \sum_i S(\underline{r}_i) \right] ,$$

where Ψ is the wave function of the liquid at rest, S is a function varying with position and $S(\underline{r}_i)$ is its value at the position of the i^{th} atom. Taking a suitable value of S the velocity at any point is

$$\underline{v}_s = \frac{\hbar}{m} \text{ grad } S.$$

This implies irrotational flow, therefore,

$$\text{curl } \underline{v}_s = 0.$$

In a simply connected region this has only one solution

$$\underline{v}_s = 0.$$

However, in a multiply connected region this is not the case. Flow which is irrotational can exist, but, in any closed circuit surrounding a hole, there is the stipulation that the wave function must be single valued and change phase by an integral multiple of 2π along a complete path.

Therefore,

$$\oint \text{grad } S.\underline{ds} = 2\pi n$$

or

$$\oint \underline{v}.\underline{ds} = 2\pi n.\frac{\hbar}{m} .$$

Evaluating this in a simple case, that of a circular path of radius r, one obtains

$$v = \frac{n\hbar}{m}.\frac{1}{r} ,$$

which describes the velocity field of a simple vortex. This satisfies the stipulated condition that curl $\underline{v}_s = 0$ everywhere except at points on the axis of the vortex.

The energy of a vortex, bent to form a vortex ring of radius R, is

$$\epsilon \doteq 2\pi R.\pi\rho_s.\frac{\hbar^2}{m^2} \ln\left(\frac{R}{a}\right) ,$$

where a is the size of the vortex core. Its momentum perpendicular to the plane of the ring is

$$p \doteq 2\pi R.\pi R\rho_s.\frac{\hbar}{m} .$$

This gives a critical velocity for the formation of vortices in a narrow channel of

$$v_c = \frac{\epsilon}{p} \doteq \frac{2\hbar}{md} \ln\left(\frac{d}{2a}\right)$$

where 2d is the size of the channel, since 2d is the maximum value which R could attain in the channel. As can be seen the critical

velocity increases with decreasing channel size, and the product $v_c d$ increases slowly with channel size. This gives a nearer estimate for critical velocity than any other excitation postulated so far, but the theoretical values are still too large by an order of magnitude.

2.3 Rotating Helium

It has been seen that if there is a hole in the liquid, circulation can exist while maintaining irrotational flow. If the size of this hole is visualised to be just a line, then in a rotating beaker circulation might exist around a single vortex. However this gives a total energy of rotation greater than that of a rigid body. If more smaller vortices can be formed the energy would be reduced. This process may be continued until the smallest vortices satisfying the condition of quantisation of circulation are obtained. The circulation of these vortices is $2\pi\hbar m^{-1}$.

Considering a rotating cylindrical beaker (with height L, radius R and moment of inertia I) of angular velocity Ω filled with liquid of angular velocity $\omega(r)$ at a distance r from the axis, the energy and angular momentum of the system may be represented by

$$E = \tfrac{1}{2} I\Omega^2 + \pi L\rho \int_0^R \omega^2 r^3 dr$$

$$M = I\Omega + 2\pi L\rho \int_0^R \omega\, r^3 dr \ .$$

In equilibrium the function $(E - \lambda M)$ must be a constant, where λ is a

constant, and it can be shown that $\lambda = \Omega$. Choosing ω so that $(E - \Omega M)$ is a minimum

$$E - \Omega M = -\tfrac{1}{2} I\Omega^2 + \pi L\rho\int_0^R (\omega^2 - 2\omega\Omega)r^3 dr ,$$

then $\omega = \Omega$ for all values of r.

The velocity at any point is Ωr, therefore curl $\underline{v} = 2\Omega$. However in helium there is the condition that curl $\underline{v}_s = 0$. The helium could be considered as satisfying both of these conditions if there were embedded in it a large number of vortices parallel to the axis of rotation.

The circulation around N vortices of unit quantisation is

$$\oint \underline{v}.\underline{ds} = N.\frac{2\pi\hbar}{m}$$

or

$$\text{curl } \underline{v}_s = 2\Omega = n.\frac{2\pi\hbar}{m}$$

where n is the number of vortices per unit area. The free surface is then parabolic (as observed by Osborne (1950)) with small indentations where the vortices meet the surface. These vortices are however so small that the indentations, of 10^{-4} cm. in diameter, would be invisible by optical means.

The density of vortex lines is $\frac{m\Omega}{\pi\hbar}$ or $2.1 \times 10^3\Omega$ lines/cm^2.

2.4 Formation of Vortices in Flow from a Narrow Channel

It is well known that complicated patterns of turbulent motion are set up when a normal viscous fluid travels with a finite velocity into a reservoir of initially stationary fluid. The change in velocity between the two parts of the fluid implies circulation, and, having developed the idea that vortices could be formed in helium, Feynman (1955) applied an analogy between this classical type of turbulent flow and the flow of helium from an orifice.

The vortices are visualised to be formed as in Fig. 2.4.1. The circulation is

$$v = z.\frac{2\pi\hbar}{m}$$

where z is the number of vortices formed per centimetre.

Vortices can only be formed if there is sufficient kinetic energy in the moving fluid to form excitations of this type. Considering a slit of unit length, the energy required to create these vortices is

$$\frac{v^2}{2\pi\hbar}\, m \frac{\rho\pi\hbar^2}{m^2} \ln\left(\frac{d}{a}\right) \text{ per second},$$

and the kinetic energy available is

$$vd\,\frac{\rho v^2}{2} \text{ per second}.$$

Therefore, the velocity required to create vortices is

$$v = \frac{\hbar}{md} \ln\left(\frac{d}{a}\right),$$

which gives a value about a factor of ten larger than the experimental

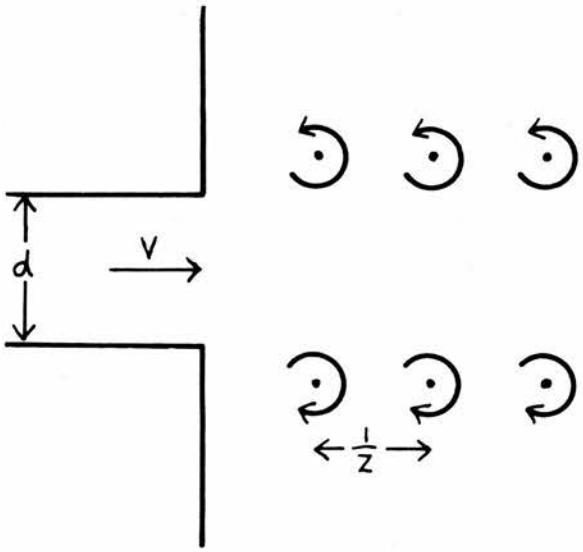

Fig. 2.4.1. Formation of Vortices in Flow from an Orifice.

values for critical velocity in a channel of size 10^{-5} cm.

This is a very idealised picture of what could be happening in liquid helium. If there are any irregularities on the walls of the slit, vortices may form in the slit due to disturbance of the flow pattern and not just at the ends. This could lead to a reduction in the observed critical velocity.

Peshkov (1960) calculated that the minimum velocity for formation of a vortex in a capillary would give a vortex of the largest available diameter, and deduced that vortices would be formed inside the channel. This idea has been developed (Peshkov and Tkachenko, 1962) to envisage vortex rings of small diameter moving down a capillary in a direction governed by the sign of their circulation. A vortex ring of diameter comparable to the size of the capillary is thought to interact with its image in the wall and move in the opposite direction. This has been used to explain the motion of the front of a temperature gradient along a capillary. Fineman and Chase (1962) think vortices are more likely to be formed at the ends of a channel. If vortex lines are the limiting factor in helium flow, they deduce that these vortices must be formed where the liquid is accelerating, and not in the centre of a tube where they presume uniform flow exists.

2.5 Experimental Evidence for the presence of Vortices in Bulk Helium II

Hall and Vinen (1956a,b) have investigated the attenuation of second sound in a rotating beaker of helium. Their results show an anisotropy in the amount of excess attenuation, according to the relative directions of the propagation of second sound and the axis of rotation. They have calculated in detail the dissipative forces which could be causing this attenuation, and obtain good agreement between the experimental results and their theory based on the picture of a uniform array of single vortex lines parallel to the axis of rotation. They have also calculated the effect which would be observed if the vortices were in concentric cylindrical sheets instead of a uniform array, and conclude that the uniform array gives the picture in best agreement with experimental results.

Vinen (1961) has detected single quanta of circulation in agreement with the Feynman picture. He has observed the rotation of the plane of vibration of a wire stretched along the axis of a cylindrical vessel when the whole apparatus was rotated. Circulation of $\frac{h}{m}$ has been shown to have much greater stability than any other value, but no higher values of stable circulation have been found.

2.6 Vorticity in the Helium Film

The unusual flow rate measurements which will be discussed in Chapter four led to the suggestion by Allen (1960) that in a film the "critical" velocity idea might be abandoned, and replaced by the idea of the film moving on vortex roller bearings. The theory of this con-

cept has been developed by Kuper (1960a,b).

Pure potential flow in a channel implies uniform velocity throughout the cross section of the channel, and a large velocity of slip at the walls. However van der Waals' forces effectively fix the first few layers of helium atoms in their positions above the walls in a film or channel. Therefore, there must be a velocity discontinuity, which implies circulation in the liquid. The velocity discontinuity could be explained by the presence of a vortex sheet, but this would mean a large energy per unit area resulting from the drastic change in the wave function over a very small distance.

The idea of a Karman row of unit quantised vortices (Fig. 2.6.1(i)) was postulated and the mean velocity on a plane parallel to the wall, in the x direction, calculated to be

$$-\frac{\pi h}{ma}\tanh\frac{\pi(y-b)}{a}\,.$$

The velocity at the wall can be made equal to zero by superimposing the velocity

$$-\frac{\pi h}{ma}\tanh\frac{\pi b}{a}$$

on the fluid.

Due to the instability of a single row of vortices, Kuper suggests that a Karman street of vortices (see Fig. 2.6.1.(ii)) is to be preferred inside a two walled channel. The effective velocity in such a system is of the order $\frac{\hbar}{mt}$, where 2t is the thickness of the channel. This is in order of magnitude agreement with measured critical flow rates.

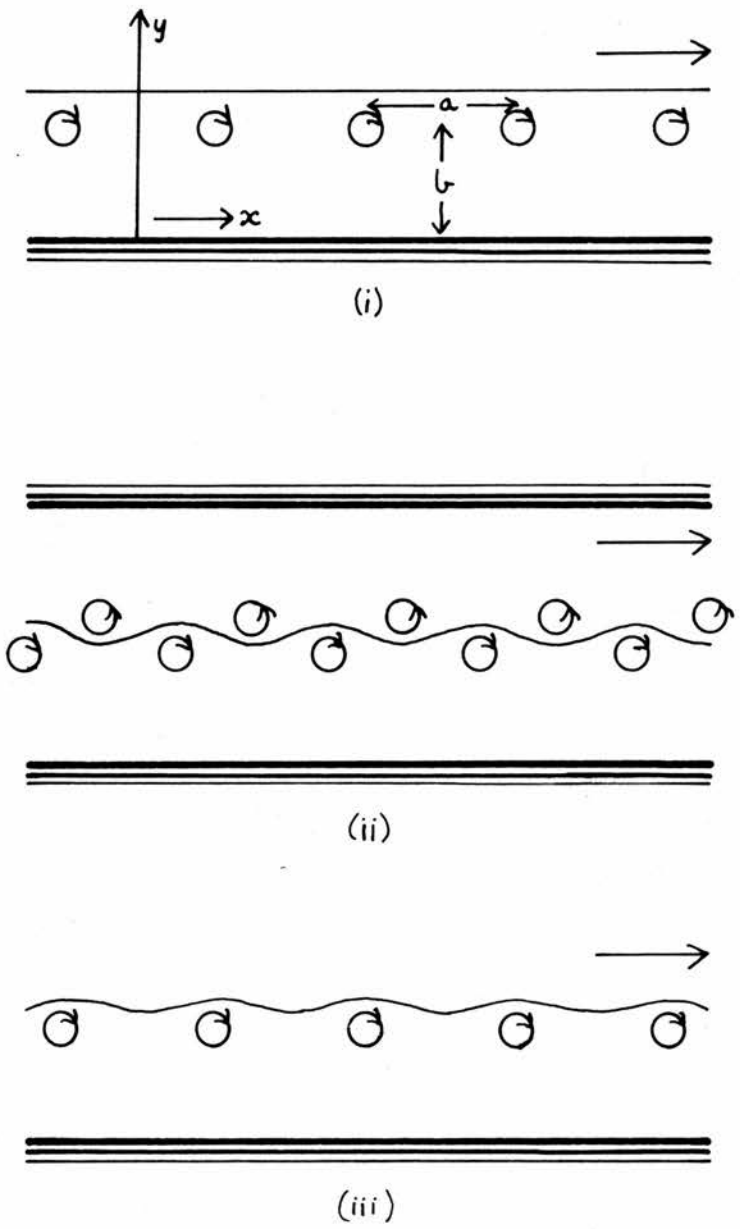

Fig. 2.6.1 Vortices in a Channel
(i) Karman row of vortices in a film
(ii) Karman street of vortices in a channel
(iii) Possible vortex configuration in a film

In a film where there is only one wall exerting any influence on the moving fluid he suggests the surface of the film may be corrugated due to the presence of a single row of vortices near the surface (see Fig. 2.6.1.(iii)). For stability the second row could be considered as the image of the row of vortices in the corrugated surface.

Evidence in favour of the picture of flow of the film on vortex roller bearings is obtained from observations of the decay of isothermal film oscillations. It might be assumed that the damping, if it occurs in the film and not at the ends, is due to scattering of rotons by the vortex lines. Allen (1961) used the damping force proportional to $e^{-\Delta/kT}$ where Δ is the energy of roton formation. A linear plot of the logarithmic decrement against $\frac{1}{T}$ was obtained. The slope gave $\Delta = 7.5 \pm 0.4°K.$ which is a satisfactory result as it is between the values of $\Delta = 8.5°K.$ for a free roton in the bulk liquid, and $\Delta \doteq 7°K$ for a roton held by its image force at 10 Å from the wall. A similar calculation based on the results of Picus (1954) gives a value of $\Delta \doteq 7.8°K.$ No data is available for the variation of the damping force with path length, and this would indicate whether damping occured in the film or at the ends.

CHAPTER III

Properties of the Helium Film

3.1 Thickness of the Helium II Film

The earliest measurements of film thickness (Daunt and Mendelssohn, 1939b; Kikoin and Lasarev, 1938) were performed with very simple apparatus and only gave average values. The thickness of the film below the lambda-point was found to be approximately 3×10^{-6}cm. but there was a considerable scatter in the values obtained. Above the lambda-point they found the film to be less than 10^{-7}cm. thick.

The theories for film formation lead one to expect the film thickness (d) to vary with the height above the bulk liquid (H) according to the relationship

$$d = \frac{k}{H^{1/z}} .$$

Different theories and experimental techniques give different values for the constants k and z. Individual theories probably only give a first approximation to the true picture. There is departure from the general formula when the value of H is very small. Three main methods have been used to measure the film thickness, all employing different experimental techniques.

An optical method has been used by Jackson and several co-workers (Burge and Jackson, 1951; Ham and Jackson, 1957). When plane polarised

light is reflected by a polished metal surface it becomes elliptically polarised, and the ellipticity of the reflected light is changed when there is a thin layer of transparent material on the surface. The thickness of the covering layer may be calculated if its refractive index is known. A metal mirror is used covered with monomolecular layers of barium stearate, with a horizontal line dividing areas with a different thickness of stearate covering. The two parts of the field of view (above and below the line) appear equally bright when a Nicol prism and a compensating plate are suitably adjusted. When a helium film is present the ellipticity of the reflected light is changed. The thickness of the film may be calculated from the angle through which the Nicol prism must be rotated to restore uniform illumination in the field of view. The method assumes that the refractive index of helium in the film is equal to that of the bulk liquid. This method has the advantage over other methods that local variations in thickness can be seen as light and dark regions in the field of view. In the other methods any local variations are smoothed out in the final results. This means that several properties of the helium film, otherwise undected, have been observed by this method.

Above the lambda-point, when the mirror was lifted quickly out of the helium bath it was covered by a thick film which was seen to drain off slowly. No such thick film was observed below the lambda-point, presumably due to the high mobility of the superfluid. If the temperature is quickly reduced from just above to just below the lambda-point, while the mirror is partially immersed in the bath, a

film can be seen creeping up over the surface at a rate of a few centimetres per second (Burge and Jackson, 1951). With a modified form of the apparatus an attempt has been made to measure the rate of extension of the film. Some experimental difficulties remain to be overcome before any conclusive results can be obtained (Ham and Jackson, 1953b). The most recent results from the optical method giving the film profile in the form mentioned earlier give k varying from 2.96×10^{-6} at 1.32°K. to 3.15×10^{-6} at 2.05°K., and z almost constant at 2.27. The variation of thickness with temperature is very slight and it has been found that the film obeys this sort of relationship to within 0.005° of the lambda-point.

Atkins (1950a) measured the film thickness by observing the period of oscillation of a moving film when the levels were approaching equilibrium. The formula connecting the period of oscillation (T) with the height of the film was estimated to be

$$T = 2\pi \left[\frac{\rho}{\rho_s} \cdot \frac{r}{2g} \left(1 + \frac{r}{R} \right) \int_0^L \frac{dH}{d} \right]^{\frac{1}{2}}$$

where r and R are the inner and outer radii of the vessel and L is the height of the rim above the bath level. The values obtained by this method gave $k = 1.5 \times 10^{-6}$ at 1.1°K. to $k = 2.2 \times 10^{-6}$ at 2.0°K. and z approximately 7.1, which is significantly different from the results of optical measurements and from theoretical estimates. Rather different results were found with a second beaker, so no real conclusions can be drawn yet, particularly as some very broad assumptions, such as presumed constant velocity throughout the thickness of the film, were made.

Only the thickness of the moving part of the film can be measured by this method.

The third method is a gravimetric one used by Bowers (1953). A carefully constructed sheet of metal foil with a thin wire attached to its lower edge is suspended above a bath, with the wire dipping into the liquid. The weight of the foil covered by a helium film is measured as conditions vary. The height of the foil above the bath and its surface area may be accurately known. Results from this method give $k = 11.8 \times 10^{-6}$ and z approximately equal to two which is in agreement with the theory of Bijl, de Boer, and Michels (1941), though some of their assumptions are inapplicable to the conditions in a film (Atkins, 1959a). No significant variation of thickness with temperature has been observed.

Thermal radiation was found to affect results in several of the above experiments at the higher temperatures near the lambda-point. It appeared that helium could not flow fast enough in the film to replenish the liquid lost by evaporation. When the heat source causing the radiation was removed the film gradually reformed.

The non-existence of a thick film in equilibrium with bulk liquid above the lambda-point can probably be attributed to evaporation. According to the theories of film formation on the basis of van der Waals' forces, there should be no difference in film thickness above and below the lambda-point, a fact which is definitely not supported by experimental observations. However, above the lambda-point there is no superfluid present, and therefore no flow can occur to replace the film once

it has evaporated.

Impurities (i.e. a layer of solid air) were found by Ham and Jackson (1957) to increase the thickness to about 20×10^{-6} cm., which was too thick for any variations of thickness to be detected by the apparatus. This increase may be due to the rough microscopic finish on such a surface, of the form described by Bowers as a "roughness factor" in his calculations.

Possible variations of thickness with movement of the film have still not been satisfactorily investigated. Burge and Jackson (1951) placed a heater at the top of their mirror and observed a 20% increase in thickness at a given height, but this could be the result of some thermal effects. Jackson and Henshaw (1950) attempted simultaneous measurements of thickness and flow rate. The thickness was found to increase with temperature, while the volume flow rate and velocity of flow decreased. No record of the thickness of the static film at the different temperatures has been published.

Atkins (1959b) mentioned the possibility of obtaining a wave motion in the form of a variation in film thickness. He called this postulated wave motion "third sound" and envisaged it as an oscillation of the superfluid component parallel to the wall, while the normal component was virtually stationary. In effect this would involve areas of increased evaporation next to areas of increased condensation. The velocity which has been estimated for this wave motion is 50 cm. per second, which is much less than the velocities of first and second sound in liquid helium.

Detection of a travelling wave of this type has been reported

(Everitt, Atkins, and Denenstein, 1962) using a modified version of the optical method of Jackson et al. (Burge and Jackson, 1951; Ham and Jackson, 1957). The wave was excited by periodic evaporation of the film from a narrow strip on the wall.

3.2 Flow of the Film

The basic properties of the film were well demonstrated in the experiments of Daunt and Mendelssohn (1939a). They showed that a beaker partially immersed in a bath of liquid helium slowly filled by film transfer, until the liquid level inside the beaker was equal to that of the bath. When the beaker was then raised, the level in the beaker fell until equilibrium was again established. If the beaker was lifted clear of the bath, drops could be seen forming on the bottom and dripping into the bath, this process continuing until the beaker was empty. The flow rate out of the beaker remained approximately constant throughout the whole emptying process, except when the inside level was close to the rim. They did, however, state that there was a slight dependence of flow rate on the difference in the heights of the levels. This dependence was very small and they concluded that gravitational potential could only play the part of a higher order correction in the controlling factors. An adjustment to reach equilibrium flow conditions was at first thought to cause the fast initial flow rate when the inside level was close to the rim, but no similar adjustment in flow rate was observed when a half filled beaker was lifted from a bath of helium.

Atkins (1950b) gives examples of the curves obtained for filling

and emptying of a beaker by film flow, plotting flow rate against pressure head for values of the pressure head up to 4 cm. of helium. There is a continual change of slope in the curve for emptying of the beaker, the change being greatest near to the beaker rim and when the levels are approaching equilibrium. The curve for filling of the beaker does not show any flow rate variation for pressure heads from 3.5 cm. to below 1 cm., but again there is a rapid decrease in flow rate as the levels approach equilibrium. Atkins concludes that flow rate is controlled by the height of the rim of a beaker above the upper level of bulk liquid. The flow rate will vary as the thickness of the film at the rim varies. When a beaker is being filled no significant change in the height of the rim with respect to bath level occurs since the ratio of bath area to beaker area is normally so great, and dependence of the flow rate on pressure head is only noted when the difference in levels becomes small. However, when a beaker is emptying the level of liquid in the beaker (the upper level) is changing continuously with respect to the rim, consequently the thickness of the film at the rim is also changing and the rate of transfer is variable throughout. The experiments of Eselson and Lasarev (1952) support this interpretation.

The small influence of gravitational potential on flow rate indicates that the mechanism of flow must differ considerably from the classical case. The analogous effect with a normal liquid is a siphon, but the flow in this case is proportional to the difference in the two levels of liquid. Various theories have been formed to resolve this problem. A normal viscous fluid experiences a frictional retarding

force proportional to its velocity, which means a steady build up of resistive force. The idea of a critical velocity in liquid helium arises from the observation that under normal conditions the velocity does not rise above a certain value, and this value appears to be attainable very rapidly. This could imply zero resistance up to a certain velocity and above this prohibitively large frictional resistance preventing further acceleration. The Gorter and Mellink theory (1949) predicts a mutual friction force between the superfluid and normal fractions in helium, proportional to the cube of the relative velocity of the two components. There are also other suggested resistive mechanisms as mentioned earlier. It is possible that helium transport is a mixture of two flows of liquid, a frictionless part, and a part experiencing a mutual friction force of some type similar to that in the Gorter-Mellink theory.

Further information on the frictional forces acting in film flow should be obtained from study of sub-critical flow. In a steady state of flow the pressure head between opposite ends of the film should be a direct measure of the frictional forces opposing flow. If flow is indeed frictionless it should be capable of existing under zero pressure head. This is demonstrated by the double beaker experiment of Daunt and Mendelssohn (1946). Once the level of liquid between the two beakers becomes coincident with either of the other levels these two remain coincident until complete equilibrium is established. Calculation shows that flow over the wall of the outer beaker must be sub-critical if the velocity of flow over the wall of the inner beaker is critical, therefore

flow is shown to exist under zero pressure head.

Picus (1954) obtained a steady pressure head by gradually lowering a plunger into a beaker of helium and observed flow out of the beaker. Seki (1962) has calculated from Picus' results that no pressure head is required to maintain flow at up to half the critical flow rate, but above this speed a small pressure head is required, which increases as the velocity further increases. When the plunger was stopped the level difference decayed and damped oscillations about the equilibrium position were observed. Atkins (1950b) concluded from his observations on the decay of oscillations that the frictional forces must be very small, otherwise any oscillations would be damped immediately.

In the film the flow rate is equal to the product of the critical velocity and the thickness of the film.

Daunt and Mendelssohn (1939a) and Brown and Mendelssohn (1950) have concluded that the length of the flow path has no effect on flow rate.

The experiment of Daunt and Mendelssohn (1939a) with the wick of copper wires connecting two vessels and causing increased transfer between them, had shown that the film was carried on the connecting surfaces of the apparatus. They also showed that the flow rate was apparently governed by the least perimeter of the film path above the upper liquid level. A constriction in the flow path below the upper level has no control over the flow rate. In this case the excess helium, which cannot be carried in the film, forms bulk liquid at the constriction and runs over the surface of the constriction and down on

the film until it can once more be absorbed into the film.

It is difficult to see why bulk liquid cannot always form on a constriction which is on the downstream flow side of a vessel wall. Presumably any bulk liquid formed above the upper level would have to flow back through the film because it would be at a higher gravitational potential than the original upper level, and, in spite of the evidence that film transfer rate is independent of gravitational potential, flow in isothermal conditions is only started by the existence of a level difference, therefore gravitational potential does have some effect.

As mentioned Atkins (1950b) showed that it is probably the thickness of the film above the upper level which controls the flow rate from a vessel. Let us consider first a difference in levels of bulk helium inside and outside a beaker which has infinitely thin walls and rim. If the two sides of the wall could be considered independently under static conditions, there would be a difference in film thickness on the two sides of the wall at any horizontal section through the beaker. At the infinitely thin rim, these two films would meet and there would be a discontinuity in the profile if the films both maintained their static shape. Therefore, presumably some helium from the thicker film flows into the thinner one at the rim, making the latter unstable. In attempting to remove this instability the helium would drain down to the lower level. This may be the way in which film flow is initiated, an indirect result of the gravitational potential of the bulk liquid levels.

It would be helpful to know more about the thickness of a moving

film as compared with a static film. In the above picture one might expect the film on the downstream side to be thicker than in the static case or upstream side. Some of the theories mentioned in a later section predict a change of thickness throughout a moving film when compared to a static film.

The finite thickness of the wall in any real experiment would not alter the picture. Assuming the rim is flat and horizontal, the thickness of the film over it will be equal to the thickness of a film at the same height above the upper liquid level on a vertical wall. Flow will then be governed by the thickness above the upper level, and by the inner perimeter of the rim of the beaker. The type of "overflow" discussed could still occur at the downstream side of the rim. On this picture the flow rates in and out of the beaker should be equal, as is intuitively obvious but rarely observed experimentally due to the critical effects of thermal radiation.

The apparent inability of helium to form bulk liquid above the upper of two levels between which flow occurs could also be explained. It is reasonable to assume that bulk helium might only form on the downstream side of flow if the film already has a tendency to be thicker than the stable situation here. If bulk helium formed the thickness of the film at any point above the bulk helium would be altered, and the process outlined above would then occur in the opposite direction. The observations of Jackson and Henshaw (1953) with a beaker of the type shown in Fig. 3.2.1 (page 35) do not support this supposition. They found the film thickness on the outside of part A when the beaker was

emptying to be the same whether the level inside was in part A or part B, although when the inner level was in part A bulk liquid formed on the outside of part B. The non-existence of a change in film thickness could perhaps be due to irregularities in the bulk liquid formed. The irregularities were observed as large local variations in thickness, and the total change of film thickness might then be small, due to conflicting influences of bulk helium at different loci. There is also the possibility that film thickness above moving bulk liquid is different from that above static bulk liquid.

A further hitherto unexplained effect of flow rate is obtained when the surface area of the higher bulk liquid level suddenly decreases (Daunt and Mendelssohn, 1939a; Eselson and Lasarev, 1952). Calculations have been made on the basis of Daunt and Mendelssohn's (1939a) statement that "the transfer between levels of liquid helium is limited by the narrowest place in the connecting surface above the higher level". The flow rate is effectively constant while the surface area remains constant in the larger portion of the beaker, but when the change is reached there is a sudden increase in the flow rate, which then decreases again until the flow rate regains its earlier constant value.

Eselson and Lasarev attributed this increase in flow rate at the change in perimeter to a large rate of transport, because at this point the film is thicker. They attribute any continued fast rate to a "prolonging effect" of a fast initial rate, and say the film will continue to take away the same quantity of liquid as it did in the larger part of the tube. They discount the idea of surface tension playing

any part in influencing the change of mass flow.

A possible simple explanation is that the results arise from a slight misinterpretation of the experimental results of Daunt and Mendelssohn (1939a). The control of flow rate by a constriction has generally been described in the following way: "The transfer rate is determined by the smallest perimeter lying above the higher of the two liquid levels". (Atkins, 1959a). The use of the word "perimeter" implies the geometrical surface of the substratum over which the film is flowing. It is probably more correct to say the flow rate is determined by the minimum "channel" above the higher of the two liquid levels, where "channel" is defined as the product of the geometrical perimeter and the thickness of the film at the perimeter considered.

Applying this to the particular experiment under consideration (Fig. 3.2.1), the exact position of the minimum channel available for flow is indeterminate when the inner liquid level is passing the decrease in beaker diameter. When the level of bulk liquid is just inside the narrow part of the tube, the available channel sizes are:

(1) at the rim of the vessel $2\pi R.\dfrac{k}{(h_1 + h_2)^{1/z}}$

(2) at the rim of the narrow portion $2\pi r.\dfrac{k}{h_2^{1/z}}$

Clearly there will be a period of time over which the value of (1) is less than the value of (2), and during this interval flow will be controlled by the channel at the rim. To calculate the height at which

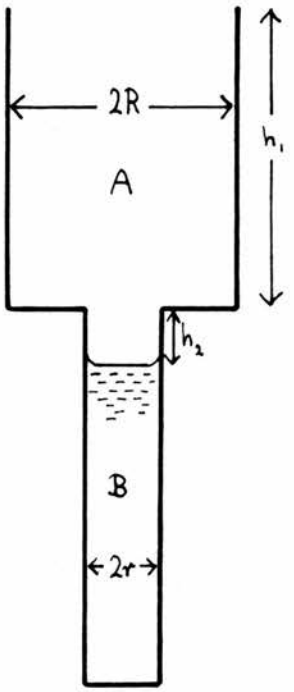

Fig. 3.2.1 Form of beaker used by Daunt and Mendelssohn (1939a) and Eselson and Lasarev (1952).

the change in controlling channel would occur the two expressions must be equated. From the paper of Eselson and Lasarev the value of $\frac{R}{r}$ is given as 2.08 and h_1 is approximately 2.6 cm., the value for $\frac{1}{z}$ is taken from the results of Ham and Jackson (1957) to be 0.44. Equating (1) and (2)

$$\frac{R}{r} = \left[\frac{h_1 + h_2}{h_2}\right]^{0.44}$$

which gives h_2 approximately 0.6 cm., in very good agreement with the height at which the flow rate regained the steady value in the earlier calculations. The results of Daunt and Mendelssohn give similar support to the idea. The transfer is not in fact any greater than usual, but the wrong perimeter has previously been used in calculating the flow rate over the transition period.

In the experiments used to test the earlier statement the positions of the minimum channel and minimum perimeter are probably coincident. It would be advantageous, if it is possible, to construct other geometries whereby the difference may be clearly shown.

The structure of the moving film must be sufficiently well defined throughout to give complete control of the whole film by the narrowest channel available in the flow path, no matter where in the path the constriction occurs. The conditions below the bulk liquid level supplying liquid for transfer have no effect on the flow.

3.3 Formation of Bulk Liquid from the Film

In all experiments where bulk liquid has been observed to form from a moving film, no report mentions the formation of bulk liquid at any point above the upper liquid level. Jackson and Henshaw (1953) have observed that bulk liquid forms if at any point below the upper level the film passes over a perimeter less than that controlling the flow. The bulk liquid appears to move down independent of the velocity of the film, until a point is reached where the perimeter is again larger than that controlling the flow and the helium is reabsorbed into the film. The conical beaker used by Ham and Jackson (1953a) shows this formation of bulk liquid well, and has been successfully photographed.

Chandrasekhar and Mendelssohn (1955) have given an extremely elegant demonstration of the formation of bulk liquid using the apparatus shown in Fig. 3.3.1. The inner vessel A can be filled by lowering the apparatus until the upper rim of tube C is below the bath level. The time taken to fill A is short enough for the film flow of helium into B to be insignificant. The vessel A slowly empties when the apparatus is raised, and bulk liquid forms where the perimeter of the outside of vessel A decreases. The bulk helium collects in the bottom of the conical vessel B. The level of liquid in B is only observed to rise until the perimeter of the level in the cone is equal to the inside perimeter of A (the perimeter controlling flow). Although bulk liquid continues to enter vessel B from the base of A, the level does not rise further and an equivalent amount of helium creeps out by film flow over the walls of

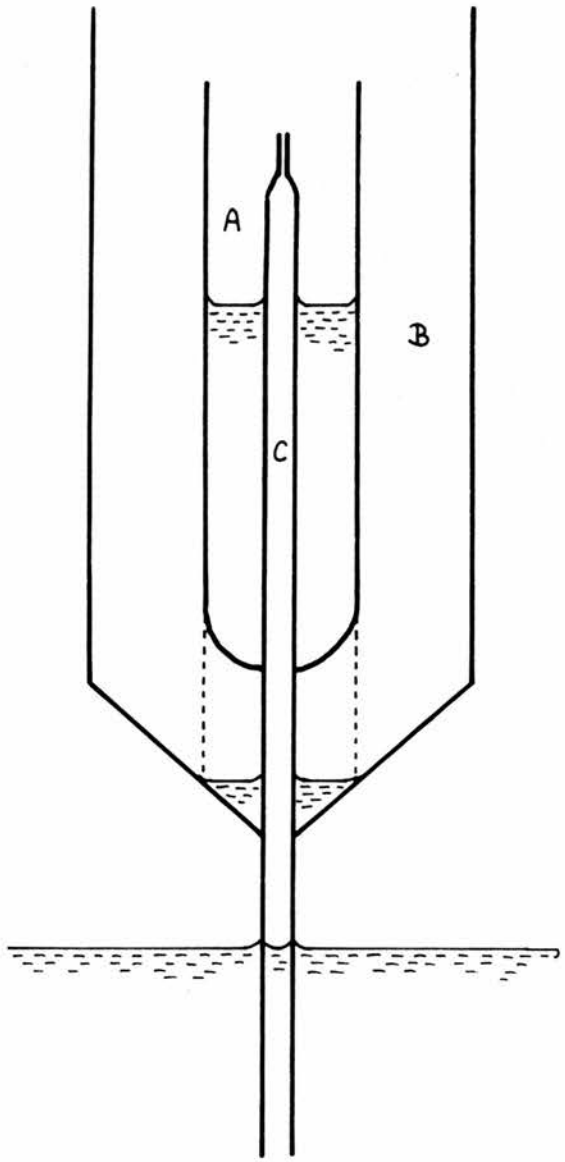

Fig. 3.3.1 The apparatus of Chandrasekhar and Mendelssohn (1955).

vessel B.

A single report (Jackson and Henshaw, 1950) mentions the formation of drops of liquid on the surface of the moving film when the upper level was near to the rim of the vessel. The number of drops decreased as the upper level fell, and they had all disappeared when the level was 3 mm. from the rim. They concluded that the fast flow rate when the upper liquid level was near the rim of a beaker was caused by transport of liquid drops in addition to normal film transfer. No further mention of this phenomenon has been made and it is possibly a spurious observation, perhaps produced by jogging the full beaker causing some of the helium to spill out on top of the film.

3.4 Thermal Effects of Film Flow

The flow rate of the film is affected by the temperature of the cryostat. The general form of this variation was first studied by Daunt and Mendelssohn (1939b), who found the volume flow rate at temperatures below about 1.5°K. to be approximately 7.5×10^{-5} cm.^{3}/sec. over each centimetre of the perimeter controlling flow. At higher temperatures the rate steadily decreased as the temperature increased until it was zero at the lambda-point. They observed a slight maximum in the curve at 1.5°K., but this has not generally been reproduced in later investigations. Smith and Boorse (1955d) have calculated curves based on a seventh power dependence of flow rate on temperature, and these curves have been found to fit quite well to their experimental values normalised at 1.1°K. The factor controlling the temperature dependence of flow rate is estimated

to be of the form

$$\left[1 - \left(\frac{T}{T_\lambda}\right)^\sigma\right],$$

where T_λ is the temperature of the lambda-point and σ is a constant. A similar dependence was observed by Mendelssohn and White (1950), but they found the value six for σ in most cases, though for one specimen beaker they preferred a value of eight. They did not normalise their results. The general form of this dependence agrees moderately well with the dependence of superfluid fraction on temperature, calculated from the results of Andronikashvili's oscillating disc experiment (Andronikashvili, 1946), where the expression for the variation of the superfluid fraction is

$$\left[1 - \left(\frac{T}{T_\lambda}\right)^{5.6}\right] \qquad \text{(see London, 1954).}$$

This tends to confirm the idea that flow through the film may be considered as pure superflow, with only the fraction ρ_s/ρ taking part, and flow rates varying with the amount of superfluid available.

Daunt and Mendelssohn (1939b) showed that there is no appreciable heat conduction through the film itself, by applying heat to the film emptying out of a Dewar vessel. In subsequent work (1950) they looked for the thermomechanical effect observed by Allen and Jones (1938) producing a difference in liquid levels when a little heat is supplied to one of two vessels connected by a capillary. The film forms a very narrow channel connecting two vessels, and can be considered to transmit the superfluid up the temperature gradient without allowing any diffusion

of the normal fluid in the opposite direction. The effect was observed when heat was supplied inside a beaker which was partially immersed in the bath, and the variation of level difference produced with different heat inputs was noted at several temperatures. With small heat inputs they found the level inside rose until a steady difference was maintained. This difference will be maintained when the quantity of helium evaporating due to the heat input is equal to the volume of helium flowing into the beaker through the film.

The heat input ($\dot{Q}$) to the beaker is used to evaporate the helium in the vessel and to change the superfluid entering the beaker from the film into normal fluid, that is raising the atoms to an excited state to maintain the balance of ρ_n to ρ_s required at the temperature of the cryostat. When an equilibrium state has been reached it may be expressed in the form

$$\dot{Q} = \sigma \rho n \, (L + T.\Delta S) \, ,$$

where σ is the volume flow rate, ρ the density of the helium, n the perimeter over which the film flows, L the latent heat of evaporation of the bulk liquid and $T.\Delta S$ is the energy required to change the superfluid to normal fluid. The curves showing variation of the equilibrium level difference with heat input at different temperatures exhibit maxima. At higher heat inputs a critical value is reached, where the film flowing in is unable to supply as much helium as is evaporated, and the level in the beaker steadily falls. If $T.\Delta S$ is ignored with respect to L as being very small the formula implies that the flow rate is directly proportional to the heat input at a given temperature up to a critical value

of the heat input. This critical value of the heat input is that above which the flow rate cannot compensate for helium evaporation. Estimation of the values of the critical heat inputs from the above formula using this approximation, and using the value of critical velocity found in isothermal flow meausrements, give values in good agreement with the observations. This implies that the critical velocity of film flow under a temperature gradient is the same as that under isothermal flow. The maxima in the curves indicate that the term $(L + T.\Delta S)$ cannot be constant, although the method by which this variation takes place is not clear.

Brown and Mendelssohn (1950) have investigated possible dependence of flow rate on temperature gradient and conclude that none exists.

Under isothermal conditions the opposite effect to the thermomechanical effect, that is the mechanocaloric effect, should be observed, because the superfluid flow will cause an entropy deficiency at the downstream end of the flow, and lead to a lowering of temperature. The resulting temperature gradient between the ends of the film should tend to force the liquid back into the film. Instead of this temperature difference building up, it is dissipated by helium evaporating from the upstream end and recondensing at the downstream end of the film, giving up its latent heat on condensation. Atkins (1950b) has calculated the temperature difference necessary to produce this form of distillation in the gas phase to be 10^{-6}°K., and this would be capable of producing a thermomechanical pressure difference of 2×10^{-3} cm. of helium, which is very small by comparison with the gravitational heads under con-

sideration.

Eselson and Lasarev (1952) have applied this to explain the decrease of flow rate as two levels approach equilibrium. The effect is very small and would not be noticeable while the gravitational level difference is much greater than the effective value of the mechanocaloric effect, but it should begin to influence the film transfer and cause a decrease in flow rate when the level difference becomes comparable with the mechanocaloric effect. The level difference at which the flow rate falls off steeply is given by Atkins (1950b) and Eselson and Lasarev (1952) as 0.5 cm. Eselson and Lasarev started their calculations with the level difference at which the decrease in flow rate commences and estimated the temperature difference to be 10^{-4}°K. They did not observe the decrease when thermal contact between the inside of the beaker and the bath was improved by means of a copper base on the beaker.

In view of the effects that small variations in temperature can have on the film, it is to be expected that incident radiation might have pronounced effects. There is considerable disagreement on this topic. De Haas and van den Berg (1949) observed high flow rates and attributed them to the complete absence of incident radiation, but their observations have now been shown to be due to the presence of impurities. Bowers and Mendelssohn (1950) did an experiment in which the apparatus was shielded from, or exposed to radiation, and observed no difference in flow rate. Atkins (1950b) found that incident radiation appeared to affect flow rates from a beaker, causing a decrease in filling rate and

an increase in emptying rate. These changes could be attributed to increased evaporation of the film inside the beaker by incident radiation. It is possible that there was not sufficient incident radiation to interfere on the occasions when no effect has been observed. It is generally considered advisable to shield the apparatus from possible radiation influx in all film experiments.

3.5 Impurities and the Effect of the Substrate Material

Extremely high reproducible flow rates were reported by Atkins (1948) and de Haas and van den Berg (1949) which were not obtained by other investigators. It has now been established that these results were almost certainly caused by the presence of a small quantity of solid air impurity, which had entered the cryostat during the filling process. Bowers and Mendelssohn (1950) have shown how successive increases in the amount of air present in the cryostat produce increases in the observed flow rates. When the amount of contamination is very small nothing is observed visually, but when the amount of air present is increased a fogging of the apparatus can be seen, which implies a granular deposit on the walls. This may be the explanation for the increased flow rates observed on contaminated surfaces, as a granular deposit might considerably increase the effective perimeter available for film flow. There is a second possible explanation for the increased flow rates. Ham and Jackson (1957) have reported that the thickness of the helium film increases to 20×10^{-6} cm. when a small quantity of air is deliberately introduced into the cryostat, which would mean a greater

channel available for film flow.

Chandrasekhar and Mendelssohn (1952) investigated the flow of helium over a well polished steel beaker and found the flow rate to be almost identical with the flow rate over glass. On baking the beaker to a red heat and then cooling, thus destroying the smooth finish and introducing a crazed surface, they found the flow rate over the surface was three times as high. Mendelssohn and White (1950) have also reported a higher rate of flow on metals than on glass. Smith and Boorse in a series of papers (1955a,b,c,d) conclude that the substrate material has no significant effect, but that surface finish does play a part in determining the flow rate. A popular view supposes that very small cracks on the surface provide narrow channels through which the film can flow and thus increase the total flow rate. Smith and Boorse (1955b) prefer the explanation that it is the "total microscopic perimeter seen by the film" which governs the transport rate.

3.6 Theories of the Helium Film

There are two basic approaches to the theory of the helium film. In one the intermolecular forces holding the atoms in position and balancing the gravitational forces are considered, and in the other possible wave functions suiting the unusual boundary conditions pertaining to a liquid film are considered.

Schiff (1941) and Frenkel (1940) attributed the film to the existence of van der Waals' forces between the atoms of helium and the atoms of the substrate material forming the walls. In later con-

siderations (Temperley, 1949) the forces between the helium atoms were also considered. The potential energy of the atom in such a field of force can be added to the gravitational potential energy of an atom raised from the bulk liquid to give a total potential energy of the form

$$\Psi = mgH + \frac{m\alpha}{z^3},$$

where m is the mass of the helium atom, z is the distance from the wall, H is the height of an atom above the bulk liquid and α is a constant determined by the strength of the interatomic forces. As there can be no difference between the energy of the helium in a film and in the bulk liquid in contact with the film, this total potential energy must be zero. Hence, considering the atoms at the outside of the film (distance d from the wall) and at height H, a profile of the form

$$d = \frac{k}{H^{1/3}}$$

is obtained. Schiff calculated values of the constant k for different materials and found the values for metals to be between 4.3×10^{-6} and 4.7×10^{-6}, for glass 4×10^{-6} and for rock salt 2.2×10^{-6}. The absence of the film above the lambda-point is attributed to the inability of the helium to flow and replenish the film as it is evaporated at these temperatures.

Atkins (1954) extended this theory and considered the variation of pressure at different depths in the film. He found that the pressure (P) at any point in the film is given by

$$P = P_o + \rho\left(\frac{\alpha}{z^3} - gH\right)$$

where P_o is the saturated vapour pressure at the surface of the bulk liquid. Clearly in the surface condition considered by Schiff and Frenkel $P = P_o$, and their original result is again obtained. In addition this theory shows that, because the pressure varies throughout the film, the density must vary. Substituting for H in the equation one obtains

$$P = P_o + \rho\left(\frac{\alpha}{z^3} - \frac{\alpha}{d^3}\right).$$

At points very close to the wall the pressure will become very high and at such pressures helium is in the solid phase. It has been shown that there are two atomic layers of solid helium adjacent to the walls supporting a film. From consideration of the phase diagram there will be pressures existing in the film which indicate the presence of helium I at temperatures between about 1.75°K. and the lambda-point. One might then expect three layers above the wall, solid helium, helium I and helium II. Helium I is thought unable to flow in the film, and the channel profile available for flow might vary considerably from the measured film profile even to the extent of making a uniform channel throughout. However, calculation has shown that the correction which must be applied to the channel size is negligible.

Bijl, de Boer, and Michels (1941) treated the liquid as a condensed Bose-Einstein gas. The ground state wave function was considered to have nodes at the surface of the film and at the wall. When the total energy of the film was minimised, the relationship obtained was

$$d = \frac{h}{2m}\left(\frac{1}{2gH}\right)^{\frac{1}{2}} = \frac{k}{H^{\frac{1}{2}}}$$

The total energy is considered as the sum of the gravitational potential energy and the zero point energy of an atom in a box with dimensions the thickness of the film. The difficulties in this theory have been discussed by Mott (1949), who has shown that the variation in density, implied by the wave function for the ground state, gives a minimum at the walls and surface and a maximum at the centre. This is implausible when a similar concept is applied to the bulk liquid, as all the atoms would be expected to congregate at the centre of a vessel away from the walls, though Dingle (1952) points out that this may be a suitable wave function for a thin film, but not for application to bulk liquid. Dingle notes that the formation of the film is unexplained by this theory, though the film may be formed by other effects and this theory could then apply to the film once formed.

Atkins (1954) considers the zero point energy and obtains the following formula from initial non-rigorous calculations for the dependence of the film thickness on height

$$H = \left(\frac{4 \times 10^{-6}}{d}\right)^3 + \left(\frac{4 \times 10^{-6}}{d}\right)^2 .$$

In thin films the term in $\frac{1}{d^3}$ due to the van der Waals' forces predominates, while for thicker films the zero point energy must be considered, giving a value of $\frac{1}{z}$ between two and three in the standard expression for the dependence of thickness on height.

McCrum and Eisenstein (1955) have considered the possibility of

polar molecules adhering to the surface of a wall covered by a film, and have shown that the accumulated electrostatic field from a configuration of such impurities could exert considerable influence on the film thickness.

Temperley (1949) investigated the variation of thickness with temperature. As the atoms become thermally agitated when temperature increases an energy defect would be required to counteract this effect. The energy states of the film were considered as states in a Bose-Einstein system, and it was assumed that at any height in the film there is only one atom in any one state. This condition is necessary because of the interatomic forces. The ground state energy of the liquid was assumed to have some value in the range of possible energies for an atom in the film. At absolute zero all atoms are in the ground state. At higher temperatures some atoms go into higher energy states of the film and an increase in thickness will then be expected, decreasing again above the transition temperature corresponding to the energy of the ground state atoms.

Franchetti (1957) has also considered this problem, calculating the free energy of the film arising from van der Waals' forces, gravitational energy and zero point energy. In the general form of the profile equation, $d = k/H^{1/z}$, he found the term $\frac{1}{z}$ to vary with temperature. A steady increase of thickness with rising temperature is expected from this theory.

The theory of the moving film has not yet received much thorough investigation. Picus (1954) suggested that the shape of the moving film

might differ from the shape of the static film to explain some of his experimental results on film flow at low level differences. Meyer (1955) enlarged on this by considering the kinetic energy of the moving film with the free energy. His results gave an increase in the thickness of the moving film over the static film, the variation in thickness with height being less than in the static case.

Franchetti (1958) and Kontorovich (1956) both consider the free energy of the moving film and obtain conflicting results. Franchetti calculates an increase in the thickness of a moving film over a static film, while Kontorovich predicts a decrease in the thickness. Arkhipov (1958) also predicts a decrease in the thickness of a moving film.

CHAPTER IV

The Effect of Turbulence on the Helium Film

4.1 Observations of Enhanced Flow Rates

The previously discussed ideas of factors governing the rate of film transfer in and out of a beaker can in no way be applied to explain some of the observed enhanced rates. Care has been taken to prevent impurities being present in all the experiments mentioned in this section.

The earliest report of this "enhanced" flow rate was by Eselson and Lasarev (1951), who mentioned the possibility that transfer might depend on the pre-history of formation of the film. A fuller account of their work was subsequently published (Eselson and Lasarev, 1952). The experimental procedure was as follows:

(1) The flask was filled by complete immersion in the helium bath, then it was raised and the rate of emptying by film flow determined.

(2) When the levels had equalised the flask was lowered and the rate of filling by film flow measured.

(3) After further equalisation of the levels, the flask was raised and the rate of emptying by film flow again determined.

They found the rate of flow in case (1) was always greater than in case (3), and the flow rate into the flask in case (2) was near to, but slightly less than, that in case (3). They found the flow rate in case

(3) was independent of the time at which it was noted after the levels had equalised for the second time, and found similar rates after case (1) if some of the helium had spilled out of the flask while it was being raised. They also noted that the appearance of this variation of flow rate on the method of filling the flask was a temperature dependent phenomenon, the effect decreasing with increasing temperature. The experiment was performed at three different temperatures and the effect was no longer detectable at 1.88°K.

The dependence of the flow rate on the history of the helium in the beaker has also been noted by Snyder and Donnelly (1959), Allen (1960; 1961) and Seki (1962). Experiments similar to those of Eselson and Lasarev were carried out, and similarly it was found that the flow rate out of a beaker was greater when the beaker had been filled by complete immersion, than if it had just been filled by film flow. Allen used narrow beakers of diameters between one and three millimetres, and the others used wider beakers of between six and ten millimetres.

Allen (1961) found enhanced flow rates were obtained in several cases:

(1) After plunge filling the beaker.

(2) After filling when the rim of the vessel was close to the bath level, so that the film would be thicker than in the normal state and the volume flow rate considerably greater.

The last case was further enhanced when the rim of the beaker had been ground with emery powder. Allen deduced that the enhanced rates were obtained at times when the helium in the beaker might have been more

turbulent than usual as a result of the method of filling. If this turbulence is considered as an increase in the vorticity in the bulk liquid, and increased flow rates are observed, it could imply that the film moves on vortex "bearings" (Allen, 1960; Kuper, 1960a,b). If more vorticity is available the flow rate might be enhanced.

The next extension of these experiments was an attempt to stir the helium mechanically. Stirring was accomplished with a solenoid actuated ferrite rod in the beaker. Flow from the beaker was increased and flow into the beaker diminished when the helium was continually stirred. The increase and decrease obtained were both about the same order of magnitude. It is plausible that the disturbance produced could be in the form of vortices. The magnitude of the effect seemed to depend on the rate of stirring, and a dependence of the flow rate on the vorticity at both ends of the film could be implied from the observation of both an increase in emptying and a decrease in filling rate.

The observations imply an appreciable entropy due to turbulence, and one would expect an equilibrium effect in the form of a difference in levels between two vessels when the specific amount of turbulence in one was greater than in the other. An equilibrium level difference of 0.5 mm. was observed. All the experiments were carried out at the lowest bath temperature of 1.15°K.

Seki used the same procedure as Eselson and Lasarev, and occasionally noted that there was no change in the flow rates between cases (1) and (3). His work was done at temperatures between 1.2 and 1.3°K.

4.2 Experiments with a Rotating Beaker

If the flow rate is influenced by the quantity of turbulence at both ends of the film, one would at first expect the helium in a rotating beaker to flow out by film flow more quickly than out of a stationary beaker. The theory of Feynman (1955) gives a density of vortices in a rotating beaker equal to $\frac{2m\omega}{h} = 2.1 \times 10^3 \omega$ lines/cm.^{2}, where ω is the angular velocity. The vortices are parallel to the axis of rotation, and Hall and Vinen (1956a,b) have demonstrated the probable existence of these lines in their measurements on the attenuation of second sound in a rotating bucket of helium. Thus, since there is vorticity at one end of the film, one might expect the flow rate to be affected.

In some early work Donnelly (1956)* thought an increase in flow rate of up to 35% had indeed been detected, but these high rates were explained in a subsequent investigation (Snyder and Donnelly, 1959). When a beaker of helium II is rotated the surface meniscus is parabolic as for a normal liquid and the shape of the parabola is not dependent on the normal fluid density. In the reported experiments the variation in the level of the tip of the meniscus was noted. No change in flow rate was found for angular velocities between 4 and 220 radians per second when corrections were made for the height of the rim of the beaker above the liquid level inside the beaker, including the correction for the height of the limb of the parabolic meniscus. The change in thickness

*Not available for consultation by the present author.

of the film on the inside and outside of the beaker, due to rotation, was negligible.

From the observations on flow rates Snyder and Donnelly expected helium to flow out of a rotating beaker until the limb of the parabolic meniscus was adjacent to the bath level. Instead they found that the limb of the meniscus was below the bath level in the equilibrium situation. The differences in height were 0.860 cm. at 1400 rpm. and 0.325 cm. at 800 rpm.

Some similar experiments were carried out using carbon tetrachloride to see if there was an analogous effect with a normal liquid. The experimental arrangement is shown in Fig. 4.2.1. The perspex beaker of internal diameter 0.812 in., outer diameter 1.288 in. and height 2 in., was partially immersed in a bath of carbon tetrachloride. The rotation speed could be adjusted by varying the current in the driving motor. In the first experiment a 1 mm. diameter hole was drilled in the base of the rotating beaker. When the beaker was rotated the limb of the parabola was observed to rise slightly higher than the theoretical value for the height in a beaker isolated from the surrounding bath. In a second experiment the hole in the bottom of the beaker was closed and a similar hole was drilled in the side wall of the beaker, near to the base. When the beaker was now rotated the liquid level fell below the expected height. When the hole is in the side liquid appears to be forced out by centrifugal action, while a hole in the base appears to have little effect. The levels of the limb of the parabolic meniscus relative to the bath level are given for three possible cases against the

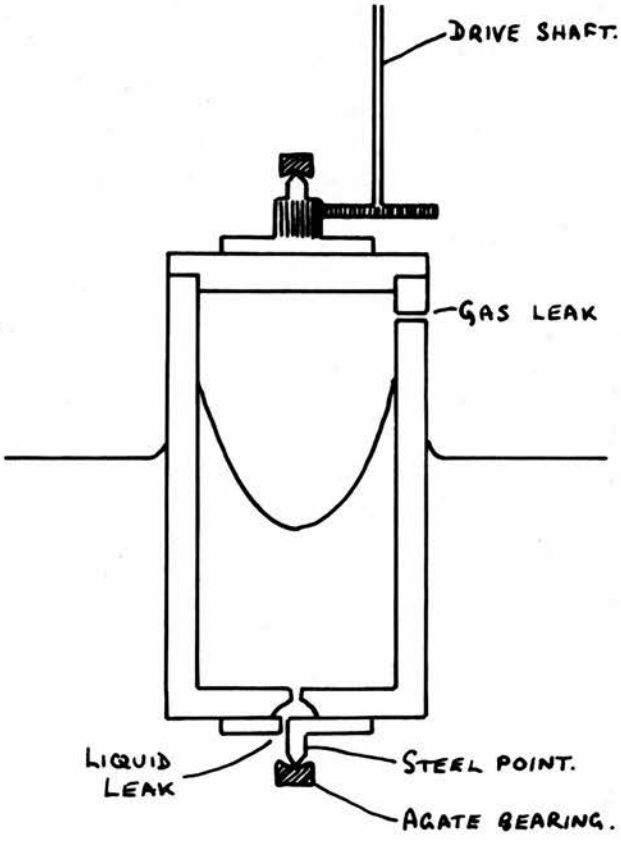

Fig. 4.2.1 The apparatus used in the rotation experiments.

angular velocity in Table 4.2.1 and Fig. 4.2.2. The three cases shown are the theoretical value for liquid in a beaker isolated from the bath, the experimental value for carbon tetrachloride in a beaker with a hole in the base, and the experimental value for carbon tetrachloride in a beaker with a hole in the side.

Table 4.2.1

Position of Hole	Rotation Speed Ω rad./sec.	Ω^2	Depth of Parabola (exptl.) cm.	Depth of Parabola (theor.) cm.	Limb to Bath Level (exptl.) cm.	Limb to Bath Level in Isolated Beaker cm.
Side	24.46	598	0.396	0.321	-0.400	+0.161
	34.66	1201	0.652	0.642	-0.828	+0.321
	43.2	1866	0.975	0.998	-1.348	+0.499
	47.8	2285	1.066	1.222	-1.715	+0.611
Base	21.76	473	0.293	0.253	+0.25	+0.127
	31.66	1002	0.653	0.537	+0.53	+0.269
	41.45	1718	0.938	0.919	+0.733	+0.460
	51.7	2673	1.320	1.430	+0.995	+0.715

There are considerable errors in the readings as the optical system for viewing the levels was rather simple. The angular velocity was calculated by timing the rotation speed of the driving shaft with a stop watch and multiplying by the gear ratio.

Assuming centrifugal forces are pushing the liquid out, then one can calculate the pressure difference between the two levels when the equilibrium situation is reached. The change in pressure through the hole in the side is $\rho\Omega^2\frac{(R^2 - r^2)}{2}$ where ρ is the density, Ω the angular

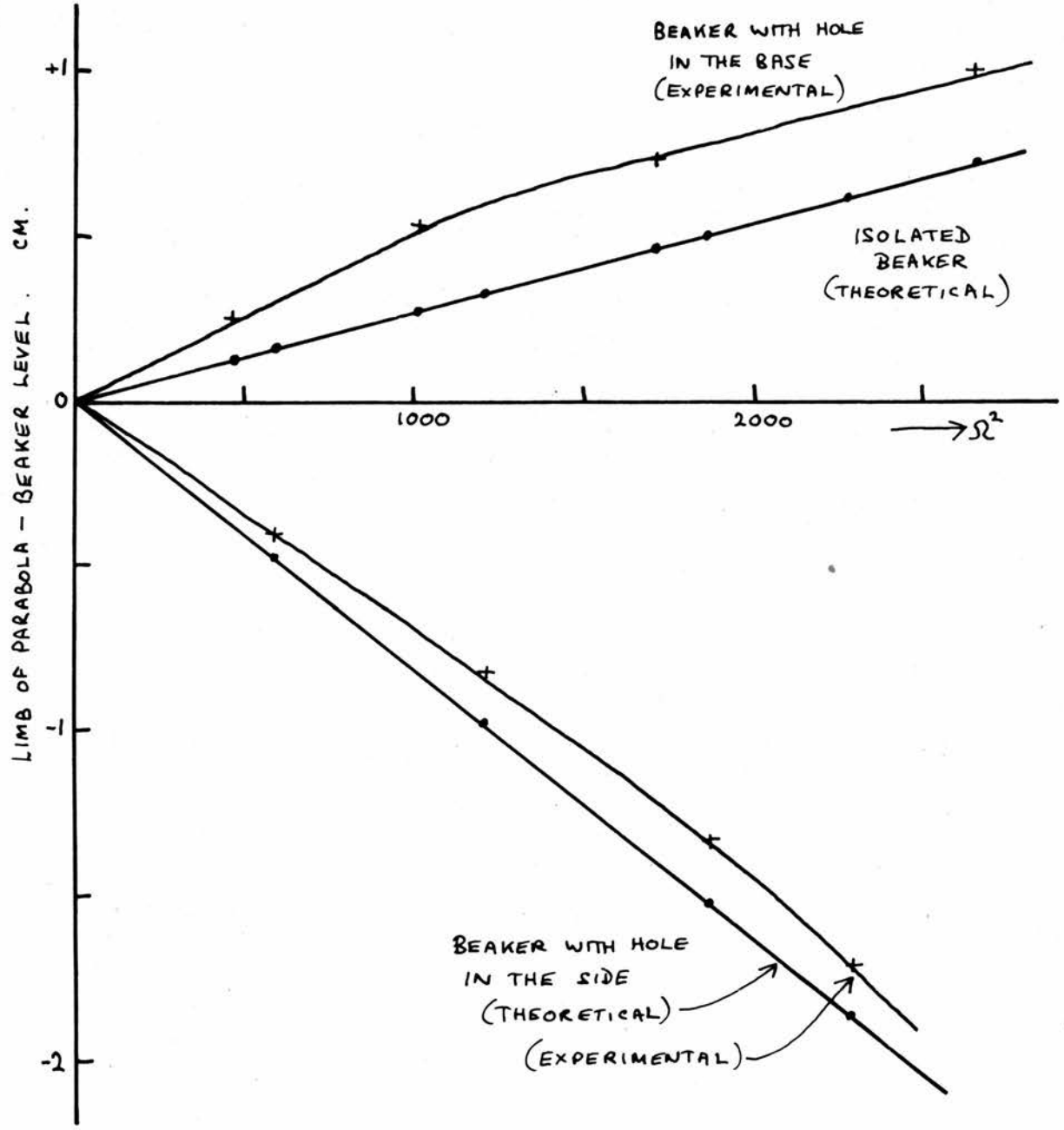

Fig. 4.2.2 The distance between the limb of the parabola and the bath level as a function of the square of the angular velocity.

velocity and R, r the outer and inner radii of the vessel. This should be equal to the difference in hydrostatic pressure on the two sides of the hole, $H\rho g$, where H is the difference in the heights of the levels. Therefore

$$\rho\Omega^2 \cdot \frac{(R^2 - r^2)}{2} = H\rho g$$

$$\frac{\Omega^2}{H} = \frac{2g}{(R^2 - r^2)}$$

In the present experiment $\frac{2g}{(R^2 - r^2)}$ is 1224. The corresponding values of $\frac{\Omega^2}{H}$ are given in Table 4.2.2

Table 4.2.2

Ω rad./sec.	H cm. (experimental)	$\frac{\Omega^2}{H}$	H cm. (theoretical)
24.46	-0.400	1496	-0.488
34.66	-0.828	1452	-0.981
43.2	-1.348	1383	-1.523
47.8	-1.715	1332	-1.868

In this calculation it has been incorrectly assumed that the fluid outside the beaker is stationary. However exact solutions would be very involved because the situation outside is hydrodynamically unstable and extremely complicated.

It appears plausible that Snyder and Donnelly's results may also be explained in this way. The equilibrium level difference observed by them could be a result of centrifugal forces acting through the film.

Snyder and Donnelly explained the results by suggesting that the liquid in the beaker must be cooler than that in the bath, thus causing a fountain effect.

4.3 Discussion

Assuming rotation does have no effect on film properties one can visualise the different effects observed in the following way. In a rotating beaker the density of vortices is just that required to satisfy the condition curl $v_s = 0$ throughout most of the liquid and give the minimum energy of rotation. This number of vortices must be maintained to satisfy the conditions. In the other experiments the beaker is not in rotation, and in its steady state there will only be the vorticity present which it is sometimes thought is inherent in any quantity of bulk liquid helium. On stirring or filling very quickly an excess of vorticity over the "background" may be produced, and this will cause the increased flow rate. In the stirring experiments more vorticity is continually being created and the flow rate is found to increase when more turbulence is produced. Thus in all cases the vorticity only appears to affect the film in so far as the bulk helium at the ends of the film is attempting to reach its state of lowest energy.

Alternatively, in a rotating beaker the vortices are thought to be formed parallel to the axis of rotation, and possibly the lower ends of the lines attach themselves to the base of the beaker and form a rather stable configuration. In the experiments with non-rotating beakers the vorticity is most probably formed in lines round the walls of the beakers.

If the Allen-Kuper picture (Kuper, 1960) of the film moving on vortex rollers is correct, then vortices so formed might go into the film more readily than any which might come away from the base in a rotation experiment.

An attempt is made in the following work to help clarify the situation in some of the non-rotating experiments, since only a small amount of experimental data is so far available. Experiments of two types have been performed. In the first the dependence of the flow rate on the method of filling and its variation with temperature have been noted. In the second a beaker containing a solenoid activated stirrer was used, and the main investigation has been into the static level difference produced.

CHAPTER V

General Apparatus and Procedure

5.1 The Cryostat

The general lay out of the apparatus may be seen in Fig. 5.1.1 and the form of the cryostat may be seen in Fig. 5.1.2. The cryostat consists of two Dewars, the inner one to contain the helium and the outer one to be filled with liquid air, mounted on the brass cryostat head. The head is built up on a 7 in. square brass plate. Above the plate there is a brass T junction with a 1 in. diameter side arm. This arm was connected to a 1 in. Saunders type valve, and from there to the pumping system. A copper-nickel tube, terminating in a flanged brass tube, was inserted in the other side of the main arm of the T junction. Attached to this was a brass plate with a number of glass metal seals, through which all electrical connections to the inside of the cryostat were made. The joint was made vacuum tight with a rubber O ring. The electrical connections inside the Dewar were of 34 S.W.G. silk covered copper wires.

A brass flange was fixed to the top of the main arm of the T junction and the different cryostat caps used were attached to this. The vacuum seal was again obtained with a rubber O ring.

A cylinder of 1.6 in. internal diameter was fixed below the plate. The outside of this cylinder was "stepped" in shape, and the whole section

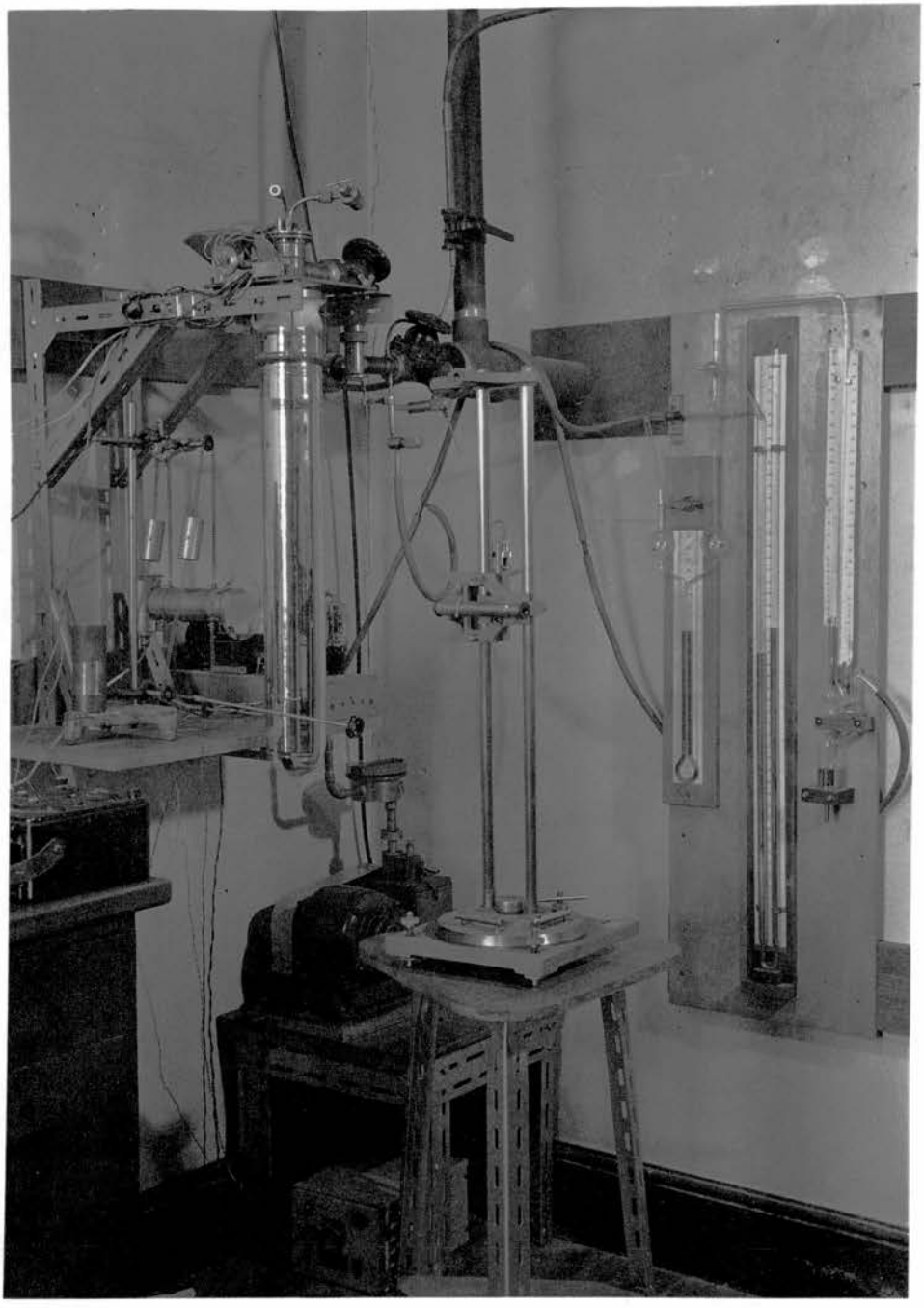

Fig. 5.1.1 General view of the apparatus.

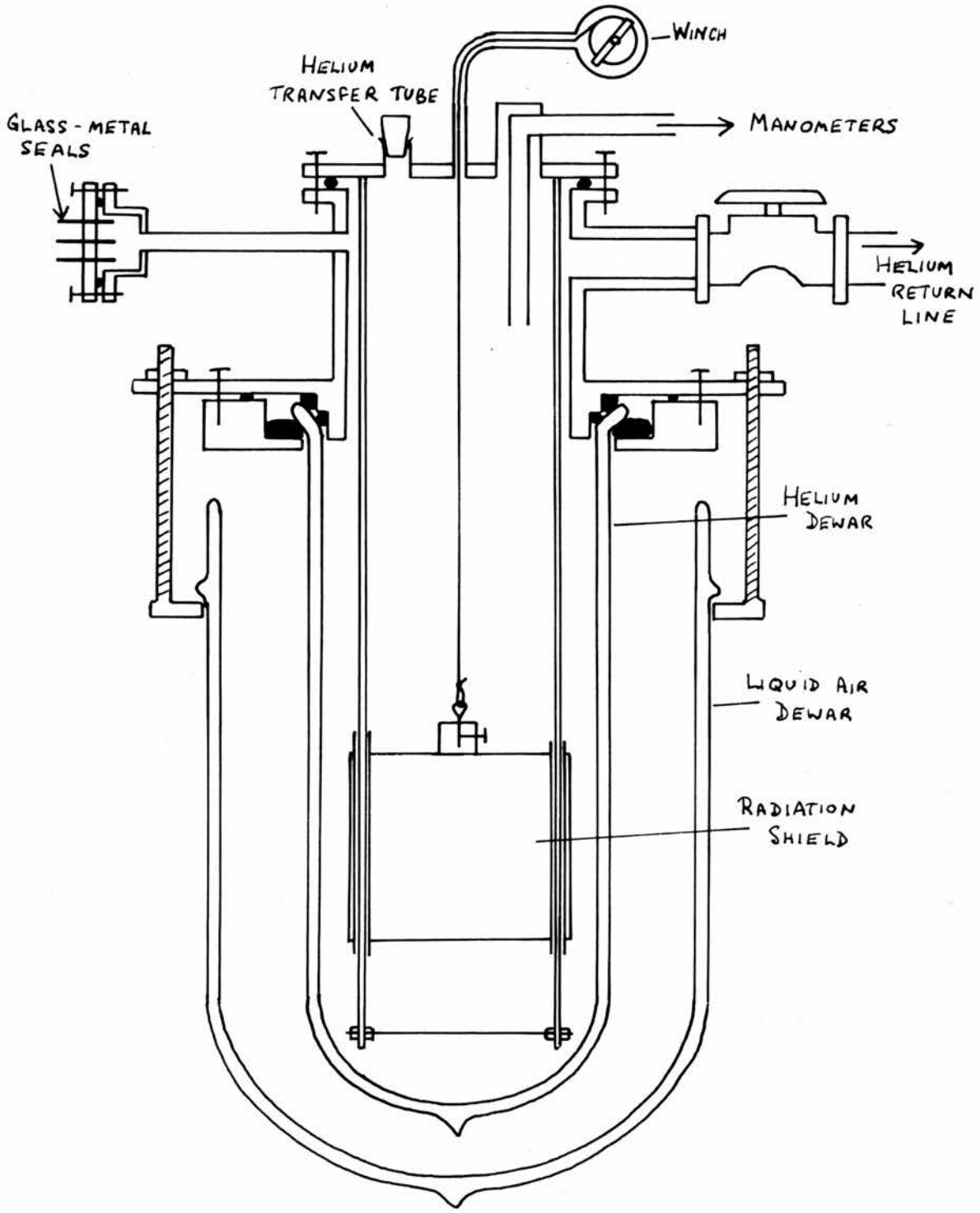

Fig. 5.1.2 Schematic diagram of the cryostat.

was screwed to the plate with a Gaco ring between the brass pieces.

The inner Dewar was held in place by a brass ring shaped to hold the rim of the Dewar, with a ring of Gaco sheet between the Dewar and the brass. The brass ring was screwed to the plate above. As the screws were tightened a vacuum tight seal was made between the Dewar and the cryostat head by pressure on the Gaco sheet in the support ring, and on two Gaco rings on the shaped part of the cylinder protruding below the plate. The screws on all these vacuum seals where Gaco rings were used had to be tightened initially when the pressure inside the cryostat was atmospheric and again when the cryostat was evacuated. If the seals are not very tight the Gaco rings do not always provide a reliable seal when there is a slight over pressure on one side.

The air Dewar was held in place by a brass ring covered in felt, which fitted round a flange in the Dewar some 5 cm. from the rim. Four brass stud-bolts suspended the ring from the brass plate of the cryostat head.

5.2 The Cryostat Caps

The first cryostat cap used during the work had four connections through it. These were the filling entry for the liquid helium, a rod to support the apparatus for individual experiments, a winch with a nylon line for further control of the apparatus when required and a connection to the manometer system.

The filling entry consisted of a short tube through the cap, which was sealed with a rubber bung except during filling at the liquefier.

The connection to the manometers was made of copper-nickel re-entrant tube. The end of this tube was connected to the manometers by a length of rubber pressure tubing.

The rod controlling the depth of the apparatus in the cryostat was made of steel tube, closed at both ends, and the apparatus was fixed to the bottom by a tube terminal connection. The rod went through a rubber gland in the cryostat cap, which held it in place while maintaining a vacuum seal at the joint.

The winch consisted of a small drum in a vacuum sealed outer casing, on which was wound 0.017 in. nylon fishing lead.

The second cryostat cap required only three connections through it. The filling entry and connection to the manometer system were exactly as before. The third connection was another winch with a smaller drum, fixed so that the 0.017 in. nylon line came directly down the centre of the cryostat. To the under side of the cap were fixed two 0.1 in. diameter rods, extending to the bottom of the Dewar (see Fig. 5.2.1). The radiation shield, in which all apparatus was mounted, moved up and down by means of tubes fixed through the shield and free to move on the rods. The height was controlled by altering the length of the nylon line in the Dewar. The weight of the shield and its contents was sufficient to keep the nylon line taut, and the height of the apparatus in the Dewar could be controlled very easily using the winch.

The use of a winch as described was particularly advantageous when readings had to be taken as soon as possible after the apparatus was in the correct position, as the tendency for the apparatus to turn slightly

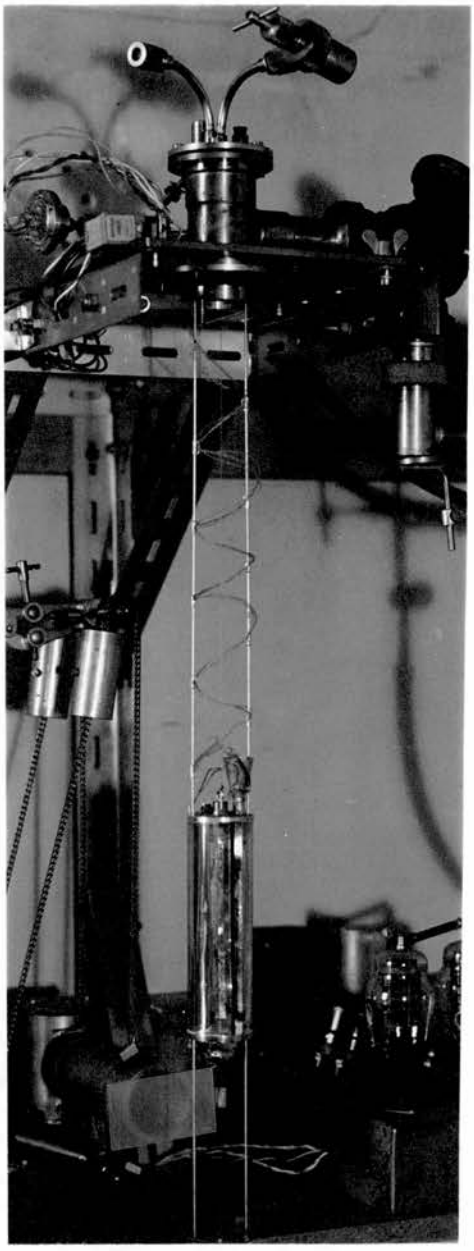

Fig. 5.2.1 The interior of the cryostat with the second cryostat cap in position and showing the radiation shield.

in the Dewar was eliminated, and the cathometer setting therefore remained constant.

5.3 The Dewars

The Dewars of Monax glass were made in the department. The approximate dimensions of the two Dewars were as follows:

Helium Dewar	Length	69.0 cm.
	Inner diameter	4.5 cm.
	Outer diameter	6.0 cm.
Air Dewar	Length	66.0 cm.
	Inner diameter	7.0 cm.
	Outer diameter	8.8 cm.

The air Dewar was formed entirely of a vacuum flask, and was silvered inside except for a strip 14 mm. wide down each side. The helium Dewar was of single thickness glass for the top 11 cm. and the lower portion was a vacuum flask silvered on the inside except for a 7 mm. wide clear strip down each side.

The air Dewar was pumped hard, but the helium Dewar contained air at a pressure of approximately 0.1 mm. of mercury at normal temperatures between the walls. This facilitated precooling of the cryostat before filling, and when the helium was introduced into the inner Dewar the air immediately solidified so that the Dewar became hard and worked efficiently.

5.4 The Radiation Shield

In all experiments the apparatus was mounted inside a radiation shield to prevent any incoming heat radiation affecting the results.

The radiation shield consisted of a cylindrical box 15 cm. long, with walls of 0.005" copper sheet and top and bottom plates of 0.01" copper-nickel. The shield just fitted inside the helium Dewar. There were two windows of 3 mm. thick Chance ON22 glass in the sides.

Precautions were taken that background light and the necessary viewing light were kept to a minimum. The light source was a twelve volt incancescent bulb, and the light passed through a 15 cm. water cell and a plate of ON22 glass before entering the cryostat.

Radiation still appeared to be affecting results, and further precautions were taken in later runs. The inside of the metal shield was painted matt black and the outside of the chamber was given a high polish to reflect any radiation falling on it. The amount of radiation able to get through the windows was restricted by covering much of the glass with a thin sheet of aluminium, polished on the outside and blackened on the inside, so that only a strip approximately 3 mm. wide was left clear for viewing. With these added precautions the scatter of values in measurement of a single quantity was greatly reduced.

5.5 The Pumping System

As mentioned the connection from the cryostat to the pumping system was via a 1" Saunders type valve. The whole cryostat could be disconnected close to the pump side of this valve for transfer to the liquefier by a

join beside a flexible part of the pumping line (see Fig. 5.5.1). Beyond this point a rotary oil pump was connected via a needle control valve. This pump was used to evacuate the cryostat prior to filling with helium.

The main pump, a "Speedivac High Vacuum Pump", was connected via a 1" Saunders type valve with a needle valve connected in parallel to give fine control. At the very low working pressures best control was afforded by the Saunders valve alone. The needle valve was used when the cryostat was being pumped to lower the temperature to below the lambda-point initially. The pump was connected to the working point by a 3" line.

There was a tap connection between the cryostat pumping line and the helium gas storage containing helium at atmospheric pressure.

With the small rotary oil pump a lowest pressure of about 1 mm. of mercury could be reached after pumping for several hours. This was low enough to be effective in removing most of the air absorbed on the apparatus and inside surface of the cryostat while it was open to the air.

Using the large helium pump a lowest pressure of approximately 0.45 mm. of mercury was obtained, giving a lowest helium bath temperature of 1.15°K.

5.6 Pressure Measurement

Three gauges were connected to the cryostat for pressure measurement:

(1) A mercury manometer.

(2) An oil manometer.

(3) A McLeod gauge.

The mercury manometer was used to give a rough indication of pressure.

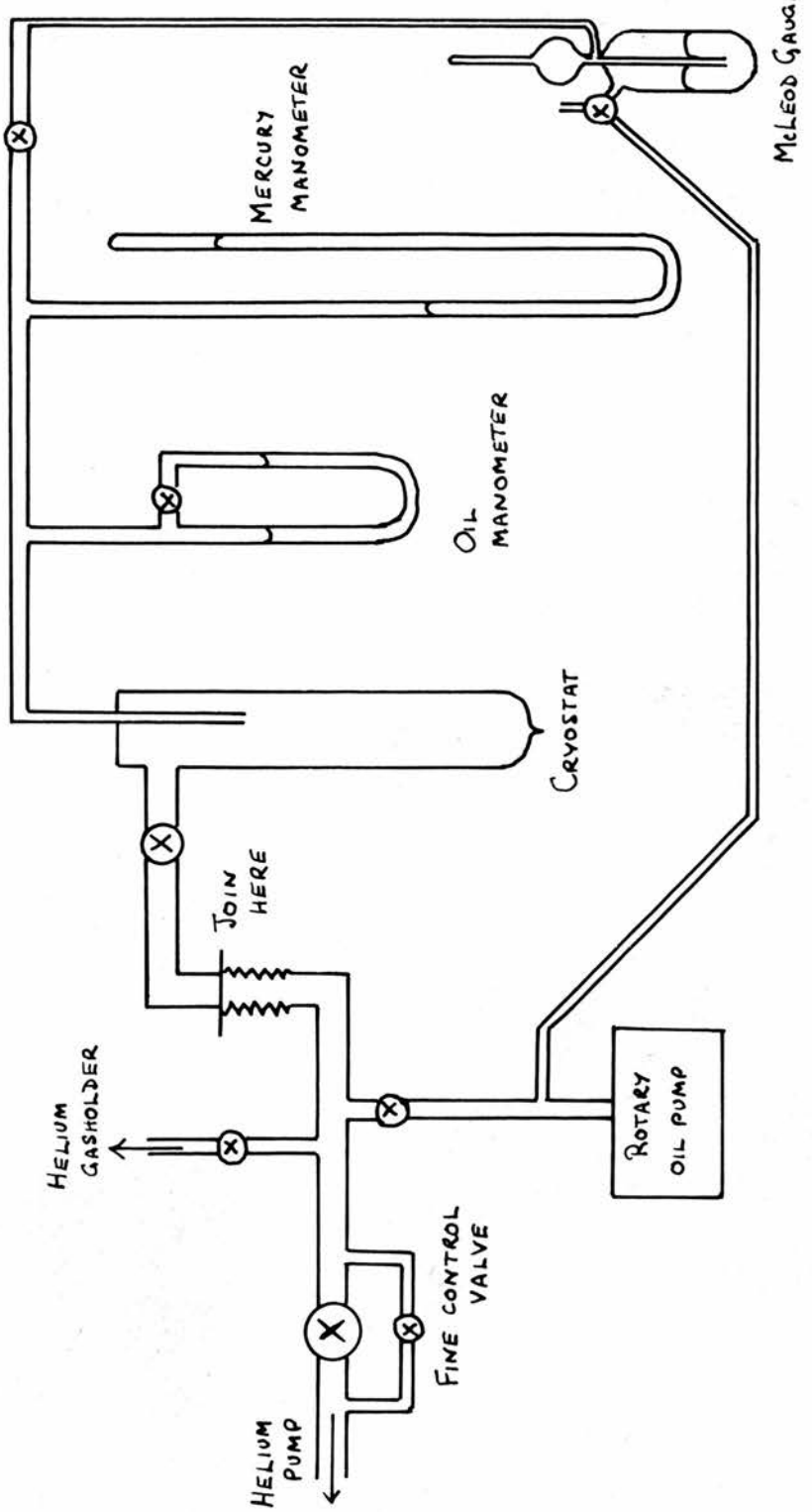

Fig. 5.5.1 Schematic diagram of the gas handling system

The McLeod gauge could be used to read to 0.01 mm. of mercury when the level was set to the lower of the two fixed marks in the closed tube of the gauge. When the level was set to the upper mark the pressure could be read to 0.001 mm. of mercury.

The oil manometer consisted of a U tube filled with butyl phthalate, with a tap in the top connection between the tubes. When this tap was closed the drift of the levels in the two arms of the tube indicated small pressure changes, and thus also temperature changes in the cryostat when it was filled with helium. A drift of 1 cm. difference in the levels indicated a change of pressure in the cryostat equivalent to approximately 1 mm. of mercury.

5.7 Filling the Cryostat

The cryostat was usually pumped for several hours when first set up to remove any gas adsorbed on the surface of the apparatus, as impurities of this sort have a pronounced effect on the results of film flow. About an hour before filling the valve to the pump was closed and the tap to the helium return line opened, to allow helium at atmospheric pressure into the cryostat. The outer Dewar was filled with liquid air and this was replenished as required throughout the run, so that the level of liquid air was always about two inches from the top of the Dewar. The helium gas inside the cryostat enabled the whole apparatus to be precooled to the liquid air temperature. Shortly before the cryostat was taken to the liquefier for filling, the helium return line was again opened to bring the helium in the Dewar up to atmospheric pressure again.

The return line and valves all now being shut and the electrical connections removed, the manometer tube was sealed off by a screw clip, and the cryostat disconnected at the join in the pump line and carried to the liquefier.

After filling, the cryostat was reconnected and the space in the line which had been open to the air quickly evacuated. The pump used for this purpose was then shut off. The helium return line was opened. The manometer tube was reconnected and at the same time the 1" Saunders valve was opened. The helium in the cryostat was now under vapour at atmospheric pressure, and therefore at a temperature of 4.2°K. Lower temperatures were then obtained by opening the main pumping line slowly and lowering the pressure until the desired temperature was attained. The bath temperature was calculated from the pressure in the cryostat using the tables prepared in 1958 (van Dijk and Durieux, 1958). The temperature could be raised if desired later in the experiment by slightly closing the valves and allowing pumping to continue until a steady pressure was again obtained.

5.8 Taking of Readings

In all the experiments observations were made of rise and fall of the meniscus of the helium level inside a narrow beaker. The meniscus was in all cases viewed through a cathetometer with a hundred division graticule. The magnification was such that each division measured 1.2×10^{-2} cm. When it was considered advantageous to take readings over a distance greater than 1.2 cm. conveniently spaced marks were made

on the outside of the beakers and cathetometer readings taken relative to these fixed marks. When flow rates were being measured careful positioning of the clock with respect to the cathetometer enabled almost simultaneous measurement of the time and the helium level at a fixed reference point on the scale.

CHAPTER VI

Dependence of Transfer Rates on the Method of Filling the Flask

6.1 Apparatus and Method

In the experiments described here the beakers were all between 1.2 and 1.5 mm. diameter and about 3 cm. long. The rims were all lightly flame polished.

In the early experiments the beaker was mounted inside the copper radiation shield and the height of the apparatus controlled by the steel rod through the cryostat cap. Flow out of the beaker after three different methods of filling was investigated. The methods were:

(1) Filling by film flow with the rim of the beaker at least 2 mm. above the bath.

(2) Filling by plunging the beaker below the bath level, so that it was completely immersed in one quick movement.

(3) Filling by lowering the beaker until the rim was below the bath level, but surface tension prevented the helium from pouring into the beaker instantaneously. The beaker could be immersed to a depth of about 0.6 mm. without breaking the surface tension meniscus and filling was by film flow at a much enhanced rate, due to the increased film thickness. The profile of the helium at the rim of the beaker is shown in Fig. 6.1.1.

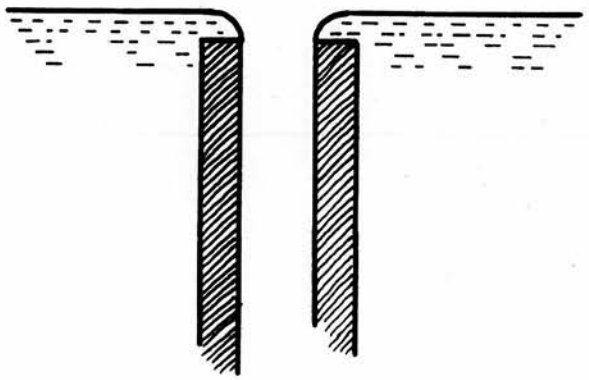

Fig. 6.1.1 Profile of the helium at the rim of a carefully immersed beaker.

The results were similar to those previously reported. The rates of emptying after procedures (2) and (3) were greater than the rate after procedure (1) by as much as 60%. The first results were however unsatisfactory because the rates of flow after (2) and (3) were not reproducible to within 15%. The values of the ordinary film flow rates for filling and emptying also showed a large scatter and were higher than the normally accepted values, possibly due to the presence of some impurities. The irreproducibility was thought to be a consequence of different depths and rates of immersion of the beaker in successive runs. It was not very convenient to use the rod for depth control when fine adjustment was desired, as considerable force had to be exerted to overcome resistance in the gland, and this led to jerky motion of the apparatus.

The modified apparatus (Fig. 6.1.2) was fixed inside the radiation shield. The beaker was mounted on a frame which was free to move on a pair of parallel rods. The position of the frame was controlled by a nylon thread (0.017 in. diameter) which was fixed to the frame and then passed through small holes in the top plates of the apparatus and radiation shield to the drum of a small winch attached to the cryostat cap. The height of the radiation shield was controlled by the rod as before.

A piece of wire, flattened and made into a pointer, was fixed with the point about 0.5 mm. above the rim of the beaker. When the beaker was immersed to fill by fast film flow (procedure (3)), the depth of immersion could be controlled by adjusting the height until the pointer

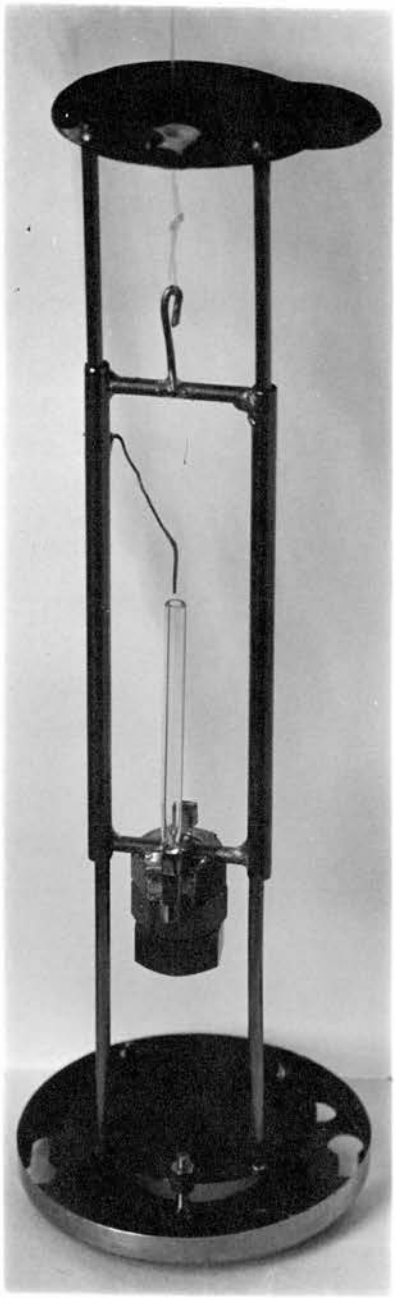

Fig. 6.1.2 Apparatus for film flow measurements.

touched the bath surface. To observe this the pointer had to be set a little to one side of the beaker. Control of the depth of immersion was much improved in this way.

The three brass nuts fixed to the bottom of the frame were required to keep the thread taut. The whole apparatus was mounted in the radiation shield on its own base plate and held in position by a bayonet fitting.

The scatter of results was probably due to thermal radiation effects. Care was taken to prevent any direct path for radiation to get into the shield. The radiation shield nearly filled the Dewar and to facilitate filling several large spaces had to be left in the top and bottom. These spaces were backed by another sheet of metal some 0.1 inch behind the gaps, to give protection from radiation while allowing free passage of helium.

Before each experiment the beakers were cleaned with carbon tetrachloride and carefully dried. The position of the pointer varied from day to day, but the height was constant to within 0.1 mm.

6.2 Variations in Flow Rate arising from the Method of Filling the Beaker

The procedure of previous experiments (e.g. Eselson and Lasarev, 1952) was repeated several times. The results are shown in Table 6.2.1 and Fig. 6.2.1, with some readings for normal filling and emptying by film flow measured prior to filling by fast film flow.

Table 6.2.1

Flow Rates x 10^{-5} $cm^3/cm.sec.$

Emptying after fast film flow filling		Filling by film flow	Emptying after filling by film flow
			9.48
		9.28	9.34
		9.28	9.50
		9.33	9.48
16.52	10.92	8.43	9.26
20.00	10.55	8.65	9.38
17.78	10.63	9.00	9.38

Temperature 1.177°K.

The filling rate when the beaker has emptied after a quick filling is below the filling rate when the beaker has been filled and emptied by ordinary film flow. The results of Allen (1961) with stirred helium imply that the flow rate might depend on the vorticity at both ends of the film. This would explain the decrease in the filling rate after a quick filling and emptying. Assuming the flow rate out of the beaker is still enhanced when the beaker is lowered for refilling, there will be some excess vorticity remaining at what becomes the "downstream" end of the film and this could cause the decreased flow rate. When the beaker is then emptied the quantity of turbulence remaining is presumably negligible and the emptying rate is once again normal.

Much conflicting evidence has been put forward on differences in the filling and emptying rates of beakers by film flow. There is no

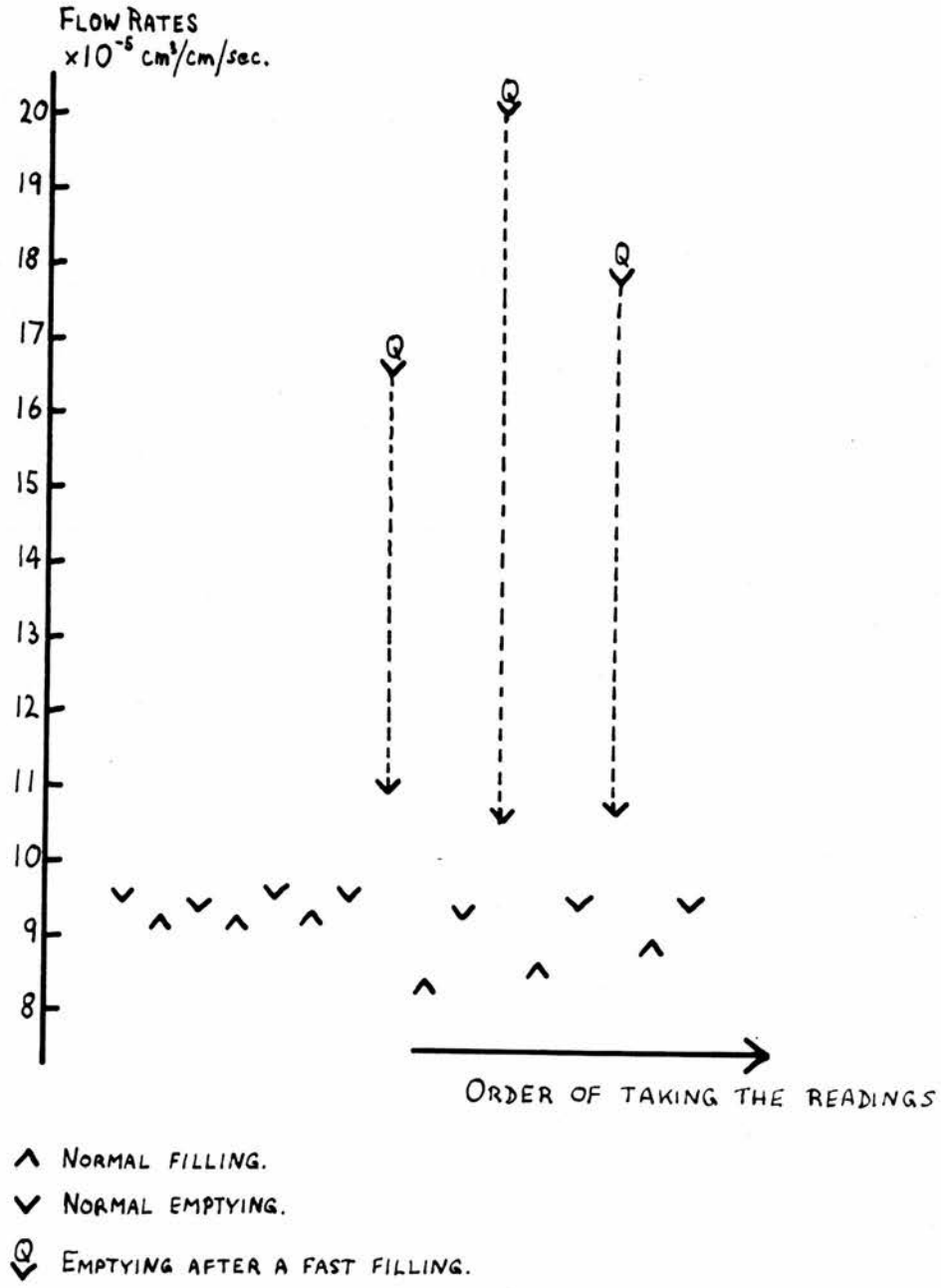

Fig. 6.2.1 Flow rates in and out of the beaker.

logical reason why the two rates should be different and it is reasonable to assume that any constant variations in the normal rates for filling and emptying (as used for the standards here) can be attributed to radiation and thermal effects. However, the decrease in filling rate noted here, as in previously published work, is a definite decrease below the standard rate and can be explained by the turbulence theory.

In the results given here a change of slope was observed in the graphs of the level of the meniscus against time after a quick filling of the beaker. This was observed quite frequently in the first experiments but was noticed less often as the technique was modified. In this group of runs the beaker was not emptied completely each time before quick filling. If the turbulent fluid going in and the liquid already present do not mix very quickly there might be a higher "specific turbulence" nearer the top of the tube than at a lower part, where the turbulence is beginning to mix with the bulk liquid previously present. In later work the beaker was generally completely emptied before filling, or at least emptied to far below the region over which the level of the meniscus was noted. With this precaution only one rate was ever observed once edge effects had disappeared.

6.3 Temperature Variation of the Effect

Eselson and Lasarev (1952) found the magnitude of the difference in flow rates resulting from the two methods of filling varied with temperature. They worked at three temperatures and did not observe the effect when the temperature was raised to 1.88°K. The other temperatures

investigated were 1.52 and 1.69°K.

A further investigation into this variation of the effect with temperature was carried out. At each temperature a number of measurements of standard film flow rates for filling and emptying were made. Then the beaker was filled by careful immersion and fast film flow (procedure (3)) and the emptying flow rate determined. This rate was also noted several times.

Two examples of the results obtained are given. Figs. 6.3.1 and 6.3.2 show the results in Table 6.3.1, and Figs. 6.3.3 and 6.3.4 show the results in Table 6.3.2. In Figs. 6.3.1 and 6.3.3 all the flow rates are given in the order in which they were measured. In Figs. 6.3.2 and 6.3.4 the average values of each flow rate are given, together with the total scatter of values if this is appreciably greater than the part of the scale covered by the symbol used. In Fig. 6.3.2 there appears to be a slight maximum at 1.55°K. in the standard film flow rates. This maximum was also observed by Daunt and Mendelssohn (1939b) but has not been found in subsequent rigorous studies of the effect of temperature on film flow (e.g. Smith and Boorse, 1955d). There are possibly insufficient readings to justify the use of mean values in this experiment. In a subsequent experiment this was investigated more carefully and mean values of twenty measurements of flow rates for filling and emptying at each temperature showed a negligible difference in flow rates between 1.77 and 1.448°K. In Fig. 6.3.4 no maximum is observed.

The standard filling and emptying rates in both experiments are not ideally reproducible, but, as explained previously this is probably

Table 6.3.1

Run on 7th March, 1962.
Flow Rates x 10^{-5} $cms^3/cm.\ sec.$

Pressure mm. of Hg.	Temperature °K.	Flow Rates			
		Normal Filling	Normal Emptying	Emptying after quick filling	
0.489 mm	1.166	8.505 8.64 8.5 8.87 9.02	8.64 8.64 8.87 8.73 8.67	13.08 13.94 15.40	10.91 13.17 13.88 13.91
1.718 to 1.829 Average 1.773	1.365	9.02 9.04 8.88 8.98	8.66 8.48 8.40 8.55	12.90 12.72 13.09 13.36	11.15 11.85 11.38 11.59
3.280 to 3.171 Average 3.225	1.474	9.09 9.17 9.38	8.66 8.60 8.78	12.89 12.53 12.73	11.83 11.67 11.68
5.12 to 4.48 Average 4.95	1.568	9.18 9.59 8.97	8.96 9.81 8.68	12.60 12.42 13.49	11.38 11.20 11.25
8.39 to 8.43 Average 8.41	1.695	8.725 8.95 8.88	8.75 8.82 8.68	10.55 10.69 11.11	10.00 10.03 10.08

Table 6.3.1 (continued)

Pressure mm. of Hg.	Temperature °K.	Flow Rates			
		Normal Filling	Normal Emptying	Emptying after quick filling	
13.28 to 14.35 Average 13.82	1.830	8.4 8.5 8.36	8.17 8.05 7.66	9.32 9.17 9.53	8.43 8.60 8.77
23.71 to 23.92 Average 23.82	2.000	7.21 6.91 6.66	6.04 6.09 5.93	8.00 7.51	7.05 6.54 6.13
32.99 to 33.96 Average 33.48	2.124	4.430 4.065 4.010	2.843 2.965 2.888	4.765 4.900	3.520 3.323
16.27 to 13.57 Average 14.92	1.852	8.34 8.47	7.86 8.14	10.00 9.06	8.81

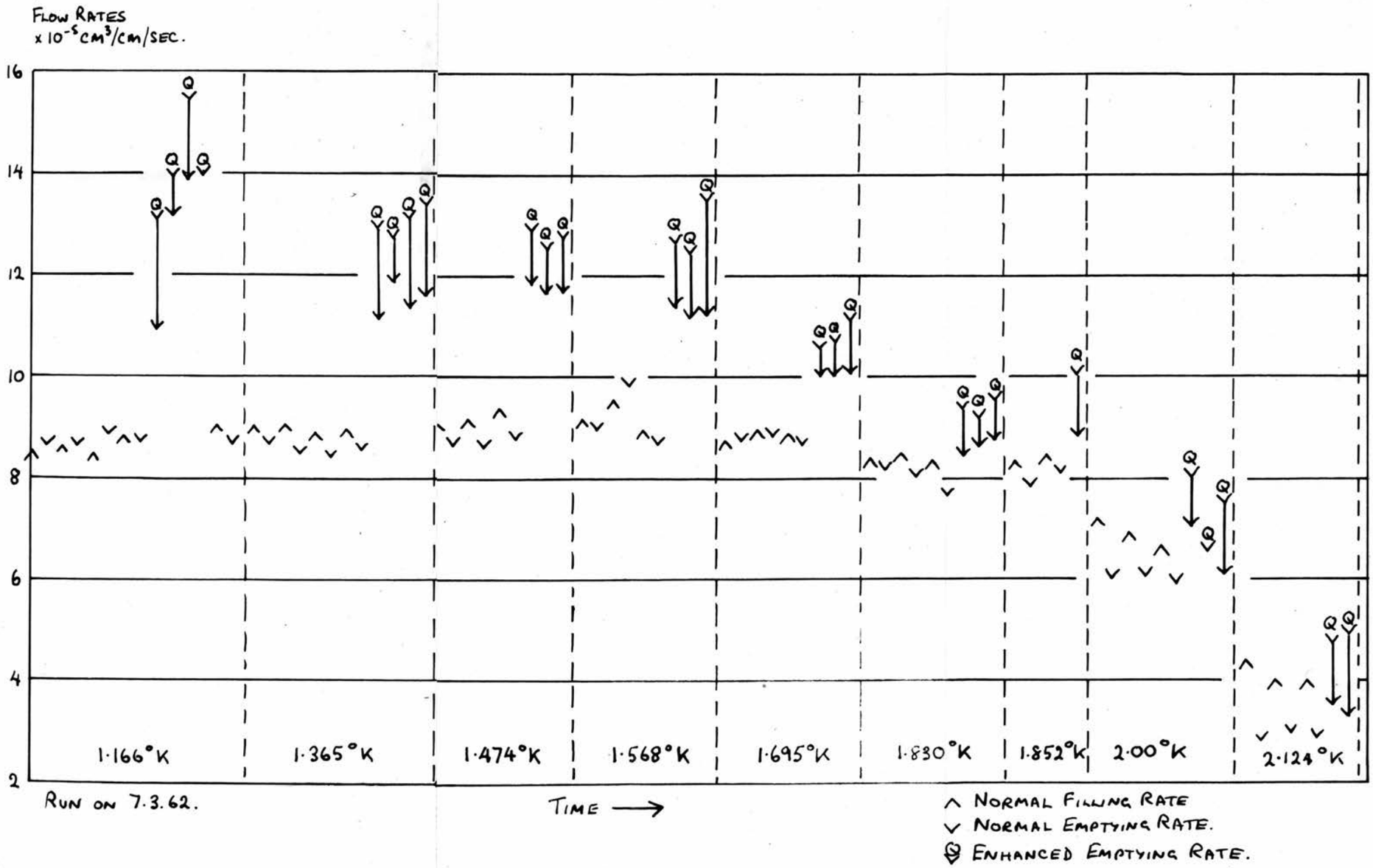

Fig. 6.3.1 Flow rates at different temperatures

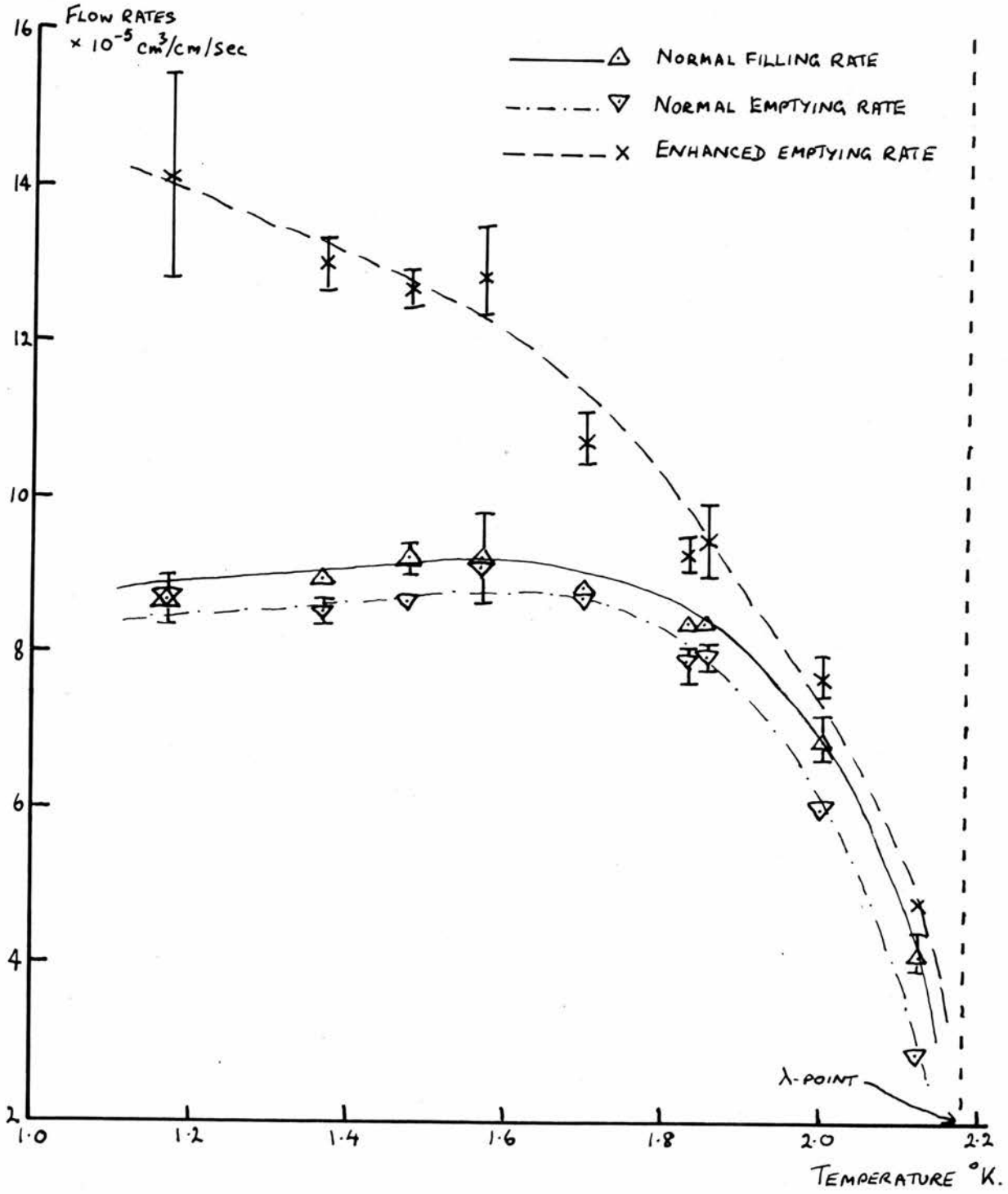

Fig. 6.3.2 Variation of flow rates with temperature.

Table 6.3.2

Run on 29th March, 1962.
Flow Rates x 10^{-5} $cms^3/cm.\ sec.$

Pressure mm. of Hg.	Temperature °K.	Flow Rates		
		Normal Filling	Normal Emptying	Emptying after quick filling
0.565 to 0.542 Average 0.553	1.183	7.9 8.1 8.01 7.745 8.025 8.125 8.18 8.05	8.07 8.36 8.4 8.28 8.18 8.295 8.125 8.49	13.26 13.16 12.84 12.95 12.92
1.465 1.326 1.298 Average 1.363	1.32	7.98 7.98 7.96 8.815	8.02 8.135 8.135 8.62	12.48 12.46 12.62 12.92
8.48 8.15 8.07 Average 8.23	1.689	7.88 8.09 8.17 7.845	7.975 7.975 7.91 7.88	9.08 8.85 8.85 8.85

Table 6.3.2 (continued)

Pressure mm. of Hg.	Temperature °K.	Flow Rates			
		Normal Filling	Normal Emptying	Emptying after quick filling	
		7.3	7.28		
13.93		7.245	7.245		
13.15		7.36	7.23		
13.17	1.821	7.295	7.32	7.92	
Average				7.955	
13.42				7.97	
				7.86	
		6.415	6.45		
		6.15	6.27		
19.55 to		6.32	6.17		
19.79	1.938	6.375	6.25	6.42	
Average				6.54	
19.67				6.28	
				6.455	
		6.18	6.145		
25.13 to		4.92	4.6		
25.23	2.02	4.88	4.65		
Average		4.965	4.58	5.055	4.92
25.18				5.0	4.78
		8.48	7.88		
1.635 to		7.855	7.88		
1.644		7.92	7.75		
Average	1.351	7.815	7.935	9.735	
1.639				9.63	
				12.22	
				9.9	

Table 6.3.2 (continued)

Pressure mm. of Hg.	Temperature °K.	Flow Rates			
		Normal Filling	Normal Emptying	Emptying after quick filling	
2.538 to 2.601 Average 2.569	1.433	8.06 8.325 8.3	8.13 8.13 8.005	11.92 9.96 11.76	
8.89 to 9.02 Average 8.95	1.711	8.22 7.89 8.07	7.9 7.85 7.8	8.72 8.805 8.855	8.56

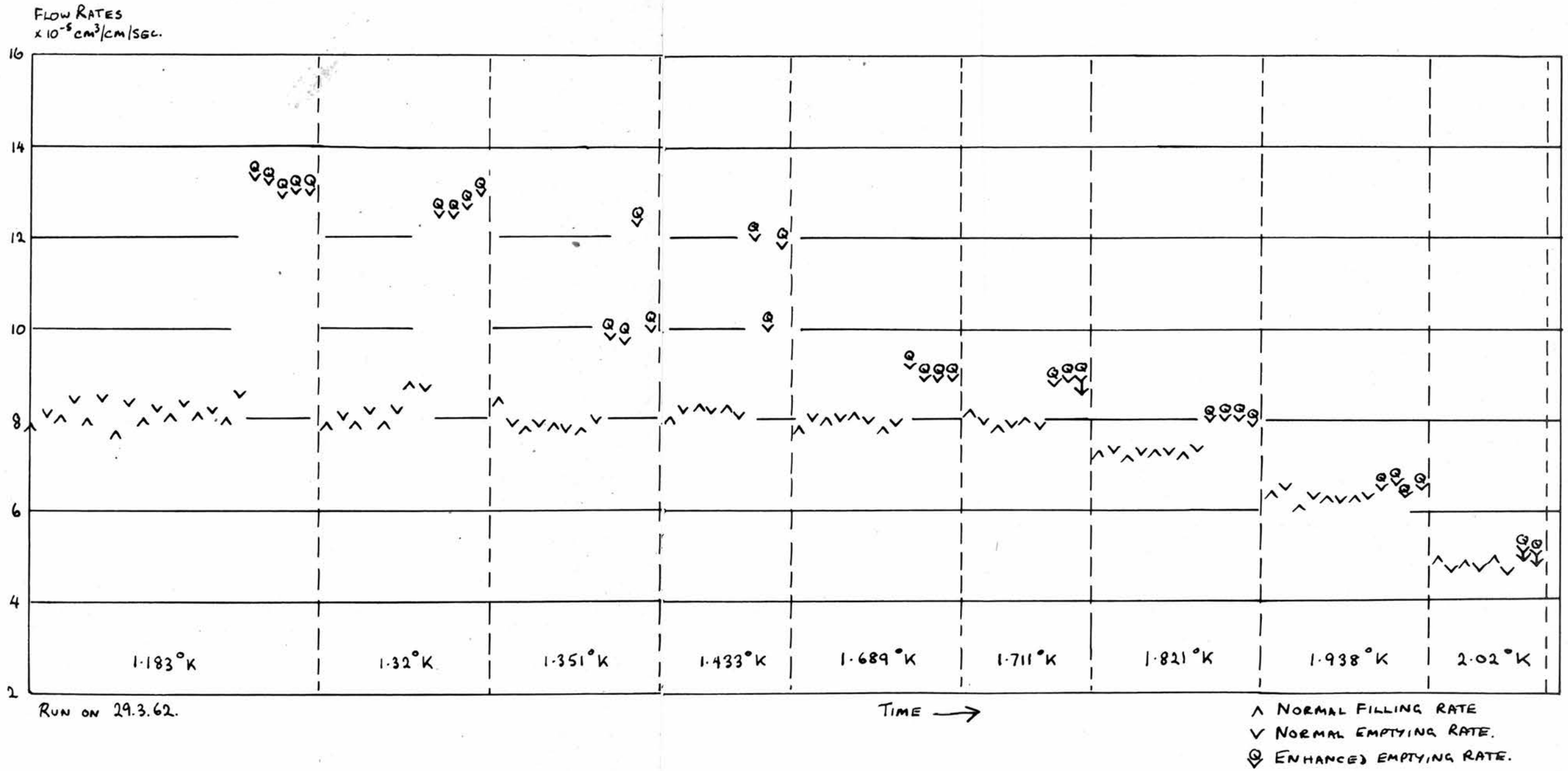

Fig. 6.3.3 Flow rates at different temperatures

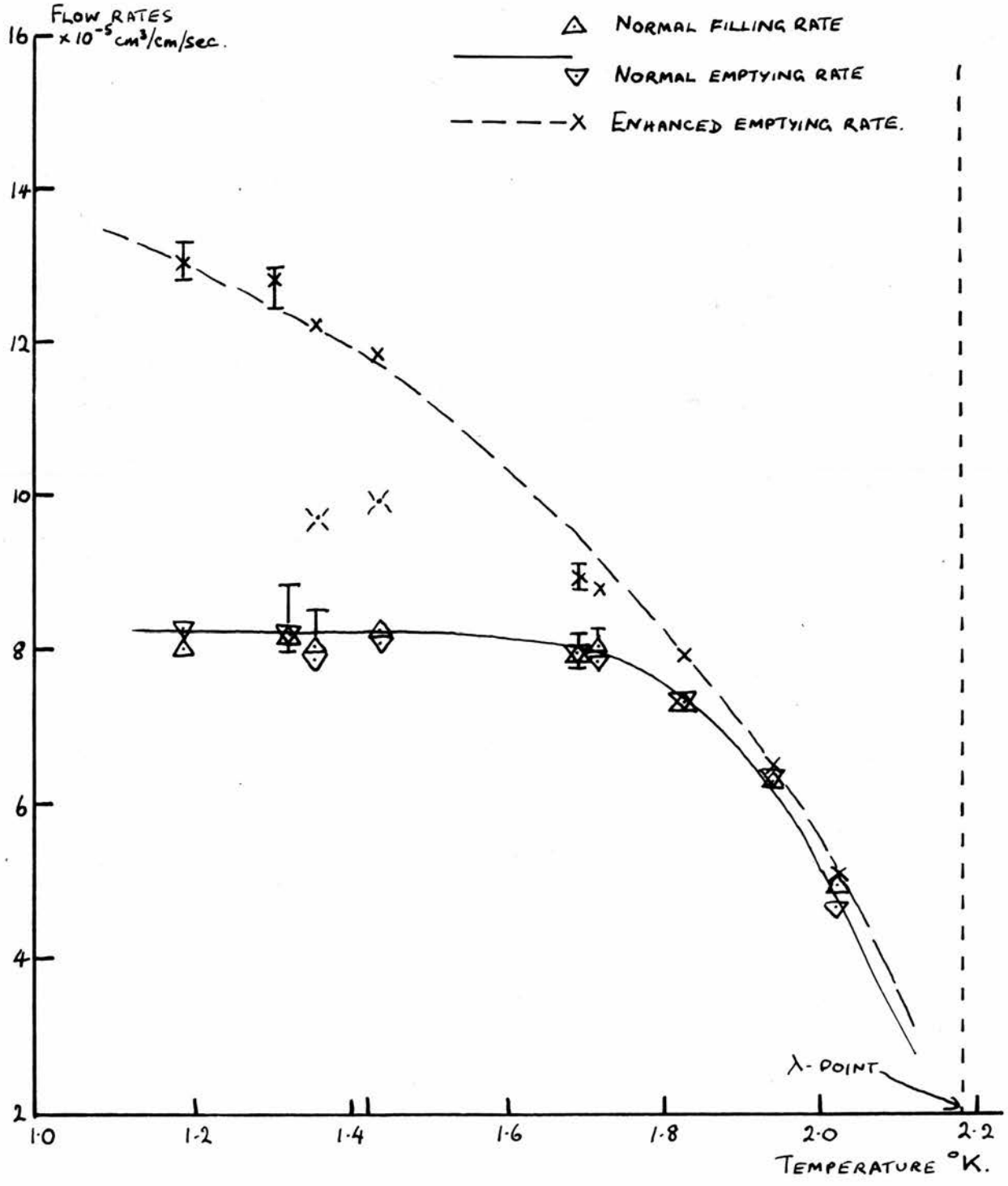

Fig. 6.3.4 Variation of flow rates with temperature.

due to temporary fluctuations in the radiation entering the shield. The temperature was also not always steady, particularly if the helium pump was being used for a second working point in the building, and this can cause fluctuations in the flow rate. In the experiment giving the results shown in Fig. 6.3.3 the radiation shielding precautions mentioned on page 69 were taken, and comparison with Fig. 6.3.1 shows how the reproducibility is improved.

The lower values for the enhanced emptying rates at temperatures 1.351 and 1.433°K. in Table 6.3.2 are thought to be spurious results as no similar observations have been made in other runs.

A search for possible relationships between the enhanced flow rate and superfluid fraction or temperature has been carried out. So far no relationship has been found which is obeyed by either the relative enhancement in flow rate, $\frac{Q - R}{R}$, where Q is the enhanced flow rate and R is the normal flow rate, or by the enhanced flow rate.

CHAPTER VII

The Effect of Stirring Helium in a Small Beaker

7.1 Introduction

A copper-nickel stirrer, connected as shown in Fig. 7.1.1 to a solenoid activated piece of ferrite (Mullard Ferroxcube, type FX.1147), was used in the first experiments. This system stirred the helium satisfactorily and flow rate measurements showed the form previously reported (Allen, 1961) with similar changes in flow rates with stirring frequency and a very small equilibrium level difference.

In all experiments the stirrer only appeared to stir the helium effectively if the level of the meniscus in the beaker was noticeably agitated.

Several times in different experiments the helium in the beaker suddenly ceased to be stirred, although motion of the stirrer continued. Once the helium had exhibited this strange effect it appeared unable to regain its earlier properties during the same day for more than a minute or two. Normally it ceased to be stirred for the rest of the run. It was thought that this might be due in some way to thermal effects inside the radiation shield, possibly a result of the considerable heat dissipation from the solenoid. The solenoid was made of copper wire and the bath level fell noticeably when current was passed in the coil. Superconducting solenoids were made to overcome this large source of heat input.

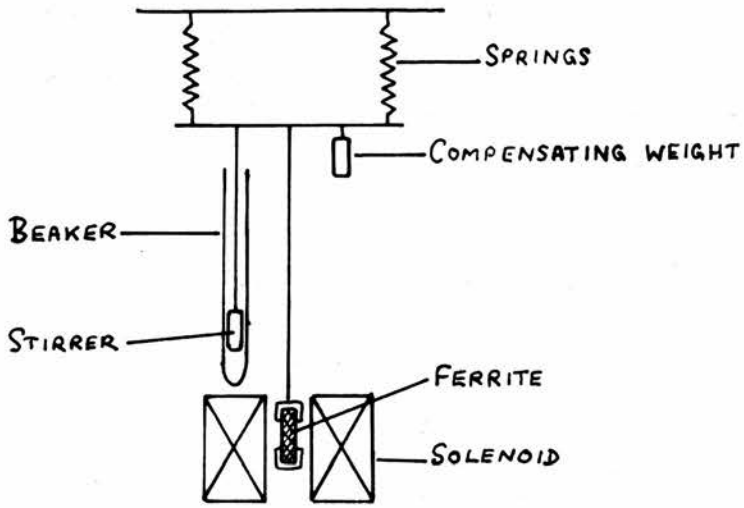

Fig. 7.1.1 Apparatus for stirring helium with a copper-nickel stirrer.

7.2 The Superconducting Solenoids

The coils eventually used were of 0.008 in. niobium wire and had 690 turns of wire. The anodised film on the metal surface provided sufficient insulation for the wire and each layer of turns was separated by a layer of oiled silk. The wire was too brittle to endure much handling and leads of niobium from the ends of the coil could not be taken to the top of the cryostat before joining to another metal. Instead, they were cut off an inch or two from the coil and joined to 34 S.W.G. Lewmex copper wires.

The junctions were made by exerting pressure on the wires in contact between plates of indium. Small plates of nickel and indium were prepared, with a hole through the centre of each one. Four of the plates were connected in the order nickel, indium, indium, nickel, and fixed with a 10 BA screw through the holes. Pressure was maintained by the screw in a tapping in one of the 0.08 in. thick nickel plates. The ends of the niobium and copper wires were cleaned and laid round the screw between the indium plates. Contact was readily made between the copper and indium, but greater care was necessary in making contact between the indium and niobium because of the tendency for niobium to form a covering layer of oxide.

7.3 Apparatus

A superconducting solenoid was incorporated in apparatus similar to that used by Allen (1961). A piece of ferrite was used as the stirrer and the whole beaker placed through the superconducting coil. The beaker was

narrow as before but much longer, being about 7.5 cm. long, and had marks made on it at 5 mm. spacing. The whole apparatus (Fig. 7.3.1) was secured on its own base, which then fitted inside the radiation shield with a bayonet fitting.

For easy positioning of the apparatus with respect to the bath the second cryostat cap (see page 66) was used. The junctions for the superconducting solenoid were kept below the coil and outside the shield, so that if any heating occurred at the junctions the effect would be quickly dissipated throughout the bath.

7.4 Taking of Readings

When the helium was stirred the meniscus in the beaker was visibly agitated at all frequencies investigated and a large equilibrium static level difference was obtained. The graphs in Figs. 7.5.1 and 7.5.3 show the variation in static level difference with frequency on two different runs. It was necessary to wait a considerable time for the steady level differences to develop, and during these times the bath level was continually falling. It was, therefore, impossible to keep the same helium level inside the beaker throughout. Readings were only taken when this level was between two of the fixed marks on the scale in case there was a variation in level difference with the amount of helium in the beaker. In this experiment the rim was sufficiently far from the bulk liquid levels for any rim effects normally observed to be neglected completely.

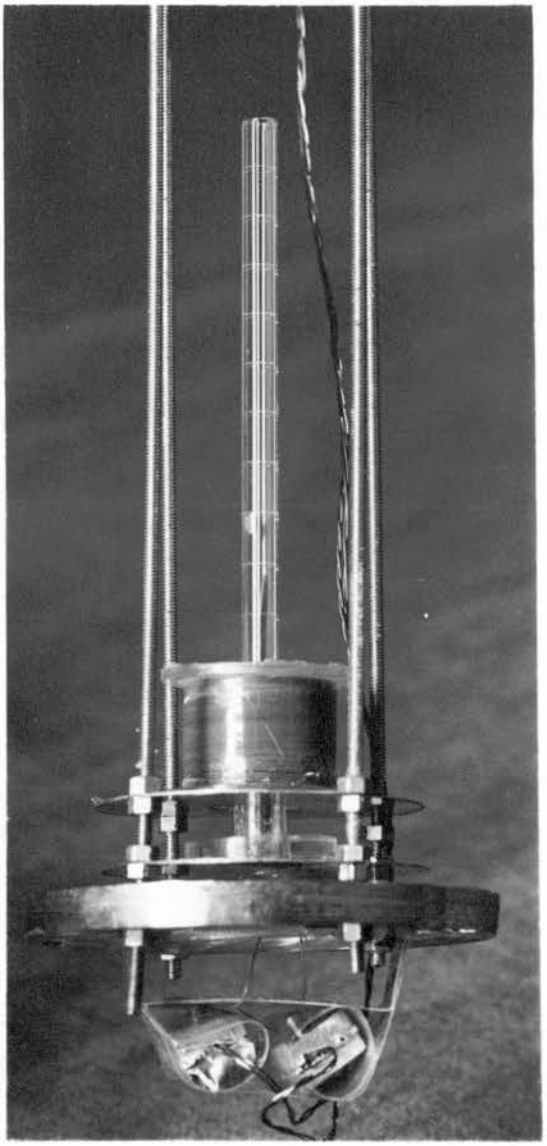

Fig. 7.3.1 Apparatus for stirring helium with a solenoid activated piece of ferrite rod. The superconducting coil and junctions can be seen.

7.5 Interpretation of the Equilibrium Level Difference Readings

In Fig. 7.5.1 the points in the graph are noted in the order in which they were taken, as it is felt this may help in the explanation of the results. Starting at 1.5 c/s no level difference was noted until 4 c/s on the oscillator scale. (For convenience the frequencies are all given here as on the oscillator scale, for any claculations a correcting factor of 0.678 must be applied). Readings of the equilibrium level difference were taken going up in steps of 2 c/s to 14 c/s, then 15 c/s and down again in steps of 2 c/s filling in the gaps in the frequencies on the table (Table 7.5.1). It was noted that the motion of the stirrer decreased in amplitude at about 12 c/s when no adjustments, apart from frequency change, were made to the electrical circuit (Fig. 7.5.2), the ferrite apparently being unable to follow the magnetic field exactly at the higher frequencies. The amplitude decreased from about 24 divisions on the cathetometer scale (approximately 3 mm.) to between 12 to 15 divisions.

A marked effect is observed when the liquid is stirred. The first four positive readings indicate a linear dependence of equilibrium level difference on frequency at constant amplitude, no difference being observed below 4 c/s. The amplitude was then observed to fall and a different linear dependence was observed in the range 13, 14, 15 c/s. Decreasing the frequency again there was a sudden increase in level difference when the amplitude of the stirrer increased again between 13 and 11 c/s, then a further linear dependence of level difference on frequency as the frequency decreased. The slopes of lines I and III (see

Fig. 7.5.1) are approximately the same but they have different intercepts on the frequency axis. If both the stirrer and the tube were sufficiently asymetrical the velocity of helium flow past the stirrer could be changed if the stirrer was turned round while being violently agitated. This change could cause the apparent shift of the line connecting the readings if one assumes the amount of turbulence depends on the velocity of helium past the stirrer. Close examination of the part of the tube containing the stirrer later revealed considerable asymetry in the tube.

The decrease in amplitude would also give a different velocity of helium past the stirrer. The velocity depends on the size of the gap, frequency and amplitude. Assuming the gap size is constant, for equal velocities of flow the product of frequency and amplitude should be constant. If the static level difference is also a measure of the velocity, points on the graph corresponding to the same level difference should show the same product of frequency and amplitude. Starting with values on the line through the points at 13, 14, 15 c/s, corresponding frequencies for the larger amplitude of oscillation are calculated in Table 7.5.2. The level differences and amplitudes are given as a number of divisions on the cathetometer scale.

Table 7.5.2

	Line II			Line I or III	
Level Difference	Amplitude	Frequency c/s	Product	Amplitude	Expected Frequency c/s
15	13	13	169	24	7
22	13	14	182	24	7.6
29	13	15	195	24	8.1

Table 7.5.1

No. of reading	Frequency c/s on B.F.O. Scale	Amplitude of Oscillation Divs. on Cathetometer (approx.)	Static Level Difference Divs. on Cathetometer
1	4	24	7.5
2	6	24	49.5
3	8	24	72
4	10	24	107
5	12	24	62
6	14	15	21.5
7	15	12	31
8	13	15	15
9	11	25	95
10	9	24	63
11	7	24	8.5
12	6	24	$\div 2$
13	5	24	not detectable

Run on 1st June, 1962.

Lowest available bath temperature.

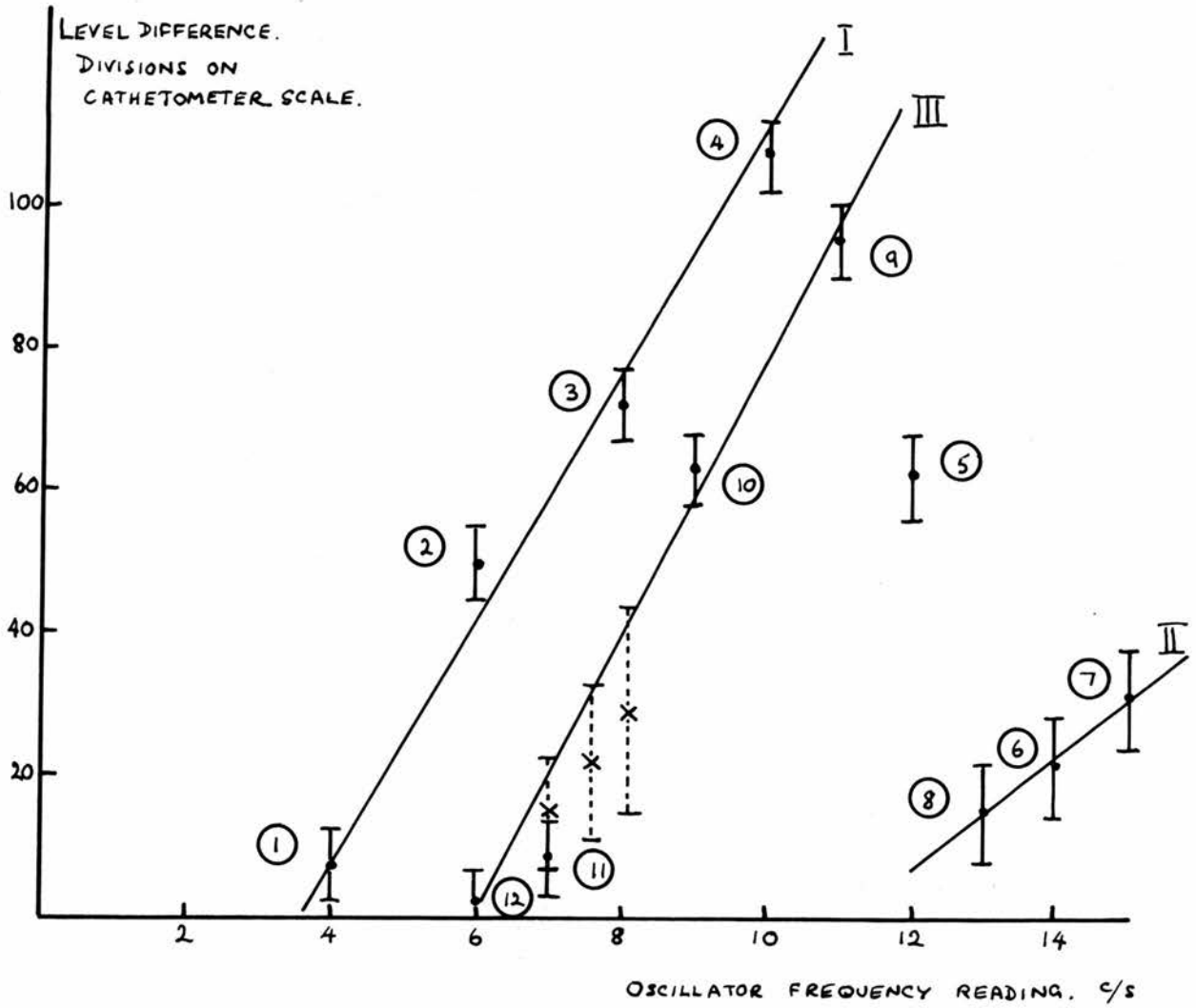

Fig. 7.5.1 Variation of equilibrium level difference with stirring frequency. (Run on 1st June, 1962). 100 divisions = 1.2 cm.

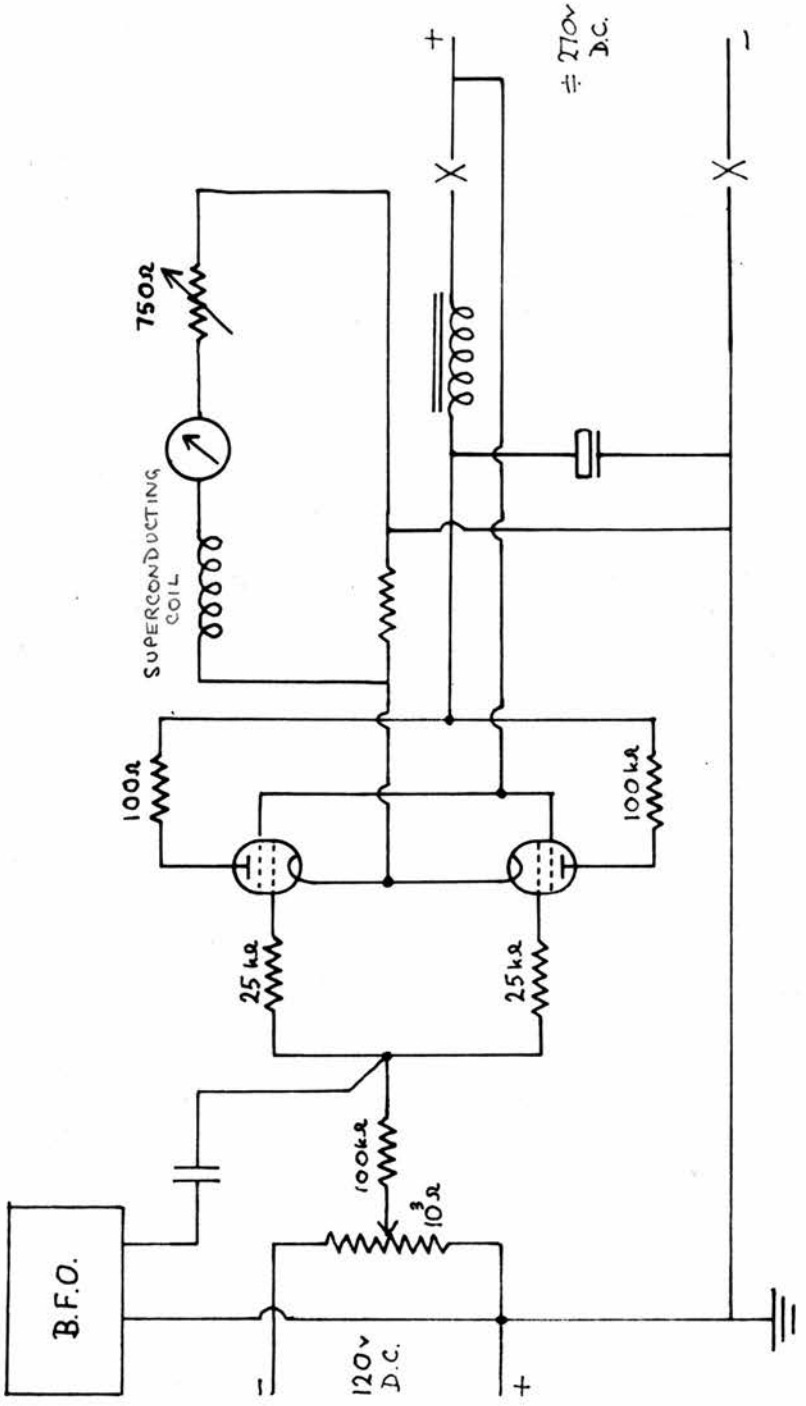

Fig. 7.5.2 Electrical circuit for operating the stirrer

There is quite good agreement between these calculated points and line III (see Fig. 7.5.1).

On Feynman's (1955) picture there should be a critical velocity of helium flow through the gap between the stirrer and the tube, below which vorticity cannot be created. Thus one expects to find the observed sharp intercept with the frequency axis on the graph. The sharp intercept also indicates that the observations cannot be attributed to thermal effects. No fountain effect is observed when D.C. only is supplied to the superconducting solenoid, thus eliminating thermal effects from this source. If there was any heating inside the beaker, due to eddy currents in the ferrite or frictional heating, it should be proportional to the frequency of stirring. The presence of such heating might be expected to show first as a fountain in the beaker and then by evaporating away the helium, producing a continuous steady fall of the liquid levels as in the observations of Daunt and Mendelssohn (1950) on the thermomechanical effect. However, in these experiments no change of level in either direction was observed at frequencies below the cut off values mentioned. Also, evaporation would result in a steady decrease in the quantity of helium present and an equilibrium level difference would not be obtained.

In the experiment giving the results in Table 7.5.3 and Fig. 7.5.3 the whole apparatus had been dismantled and reassembled, though the same tube and stirrer were used. The settings of the uncalibrated potentiometers in the electrical circuit were also slightly different. The amplitude of stirring on this occasion remained constant throughout the whole

Table 7.5.3

Frequency c/s on B.F.O. scale	Amplitude of oscillations Divs. on cathetometer	Static Level Difference Divs. on cathetometer
2	31	0
3	30	0
4	30	0
4.6	30	0
5	30	0
6	30	2?
6.5	30	8
7	30	8
8	30	17 ± 4
9	30	21.5
10	30	14
11	30	27
12	30	84
13	30	112
14	30	137
15	30	150

Run on 18th July, 1962.

Temperature 1.186°K.

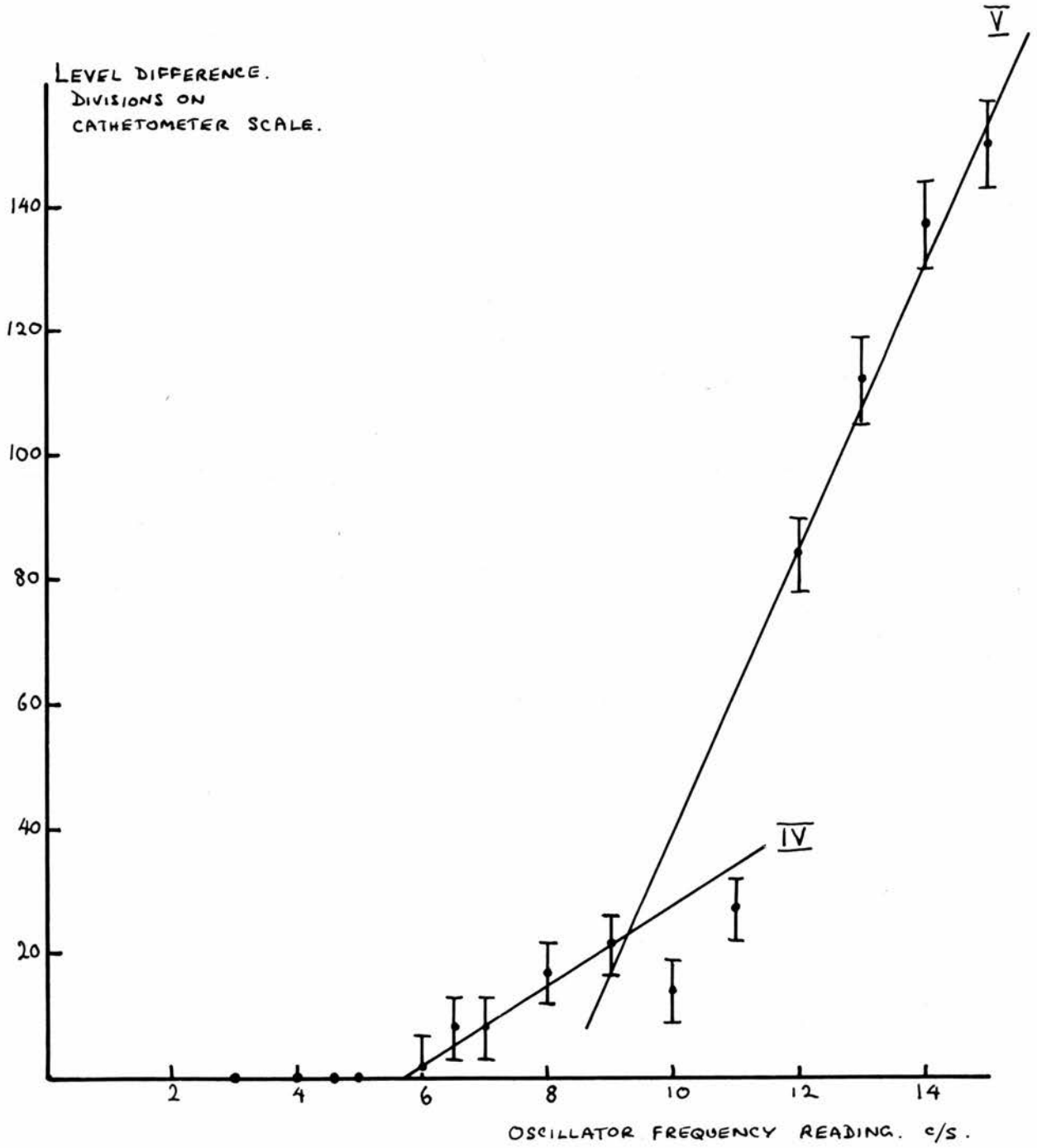

Fig. 7.5.3 Variation of equilibrium level difference with frequency of stirring. (Run on 18th July, 1962). 100 divisions = 1.2 cm.

frequency range, and the readings were taken in ascending frequency order only.

The change in slope of the graph at about 9.5 c/s is a feature not previously observed and the slopes are not the same as before. The line connecting the points from 6 to 9 c/s (line IV) coincides well in slope with line II of the previous set of results, but line V is steeper than any lines obtained before. The possibility that the stirrer gets into different modes of vibration cannot be neglected. This could explain the different slopes, correlation between lines obtained only being possible if the stirrer happened to strike the same mode each time, possibly for example in lines I and III, or lines II and IV.

Exact calculation of the critical parameters for the creation of turbulence is not possible because of the scatter of cut off values of frequency. Orders of magnitude of the different quantities have been estimated.

For the purpose of these calculations the motion of the stirrer has been assumed simple harmonic, which is only an approximation to the proper mode of motion.

The geometrical constants of the apparatus are given below:

Internal diameter of the tube = 1.715 mm.

Diameter of the ferrite stirrer = 1.582 mm.

Size of the gap (assumed regular) = 0.066 mm.

$$\frac{\text{Velocity of helium past the stirrer}}{\text{Velocity of the stirrer}} = \frac{5.66}{1}$$

The critical velocity of flow past the stirrer is calculated in Table 7.5.4.

Table 7.5.4

Cut off frequency (B.F.O.)	Cut off frequency corrected c/s	Amplitude of oscillation (cathetometer reading)	cm.	Maximum velocity of stirrer cm./sec.	Velocity of helium cm./sec.
5.7	3.86	30	0.36	4.37	24.75
3.6	2.44	24	0.288	2.21	12.52
6.0	4.07	24	0.288	3.685	20.85
11.2	7.59	13	0.156	3.72	21.05

The critical velocity of helium past the stirrer for creation of turbulence appears to be approximately 20 cm./sec., about a factor of ten greater than the values quoted by Atkins (1959a) for critical velocity of helium flow in glass capillaries of dimensions similar to the gap size.

Feynman's (1955) formula for the critical velocity for the creation of turbulence,

$$v = \frac{\hbar}{md} \ln \frac{d}{a} ,$$

is calculated for the idealised picture where helium is flowing into a bath of initially stationary fluid. The situation in the present experiments is very complicated and this formula cannot therefore be expected to apply exactly. The calculated critical velocity is 0.32 cm./sec., two orders of magnitude less than the observed critical velocity. The formula is critically dependent on the size of the vortex core, which can only be estimated theoretically.

Staas et al. have found a critical value of Reynolds number above which turbulent flow in capillaries occurs. They found the expression for Reynolds number (R)

$$R = \frac{d \rho v_n}{\eta_n}$$

where d is the channel size, ρ the density of helium, v_n the velocity of the normal component and η_n the viscosity of the normal component, fitted their results better than other possible expressions for the Reynolds number in liquid helium. The critical Reynolds number in their experiments appeared to be about 1200. Critical values for the Reynolds number using this formula are calculated in Table 7.5.5 for the present experiments. ρ is taken to be 0.145 gm./cm.^{3} and η_n as 19 x 10^{-6} poise (Atkins, 1959a).

Table 7.5.5

Critical velocity (experimental) cm./sec.	Reynolds number
24.75	1261
12.52	638
20.85	1068
21.05	1072

Thus the observed critical velocities give values of Reynolds number in order of magnitude agreement with the values found for turbulence in a glass capillary.

7.6 Flow Rate Measurements

Measurements of the flow rate when the helium in the beaker was being continually stirred were attempted, but the corresponding static level differences which could be obtained were so large that it became difficult to measure the very slow filling rates. The readings shown only give emptying rates for the beaker. Readings were taken over the same region of the beaker each time. Only a few values have yet been obtained (Table 7.6.1 and Fig. 7.6.1).

A linear plot is obtained for flow rate against frequency of stirring. The apparent frequency, obtained by extrapolation of the graph, at which any increase in emptying rate might commence is much lower than that at which an equilibrium level difference has been found. One might expect the two cut off values to be the same, but, as discussed previously, the stirrer may just have been in a different mode of oscillation.

7.7 Development and Decay of the Equilibrium Level Difference

Readings of the variation of both bath and beaker levels with time were taken to see how the static level difference was developed. The results are shown in Table 7.7.1 and Fig. 7.7.1. The solenoid was switched on when both beaker and bath levels were 5 mm. from the rim of the beaker.

A rapid splitting of the levels occurred at the start and then the beaker level settled to a steady rate of fall. The difference between the two levels increased gradually as the beaker level fell over a

Table 7.6.1

Flow Rates x 10^{-5} $cms^3/cm.sec.$

Current in the coil	Filling Rate	Emptying Rate
None	9.09	8.77
	9.29	9.17
D.C.	9.045	9.65
	9.01	9.13
D.C. and 4 c/s A.C.	6.03	11.32
	5.67	11.08
5 c/s		11.31
		11.69
6 c/s		12.37
		13.41
8 c/s		13.55
3 c/s		10.77
2 c/s		9.32
4 c/s		12.06

Run on 18th July, 1962.

Temperature 1.186°K.

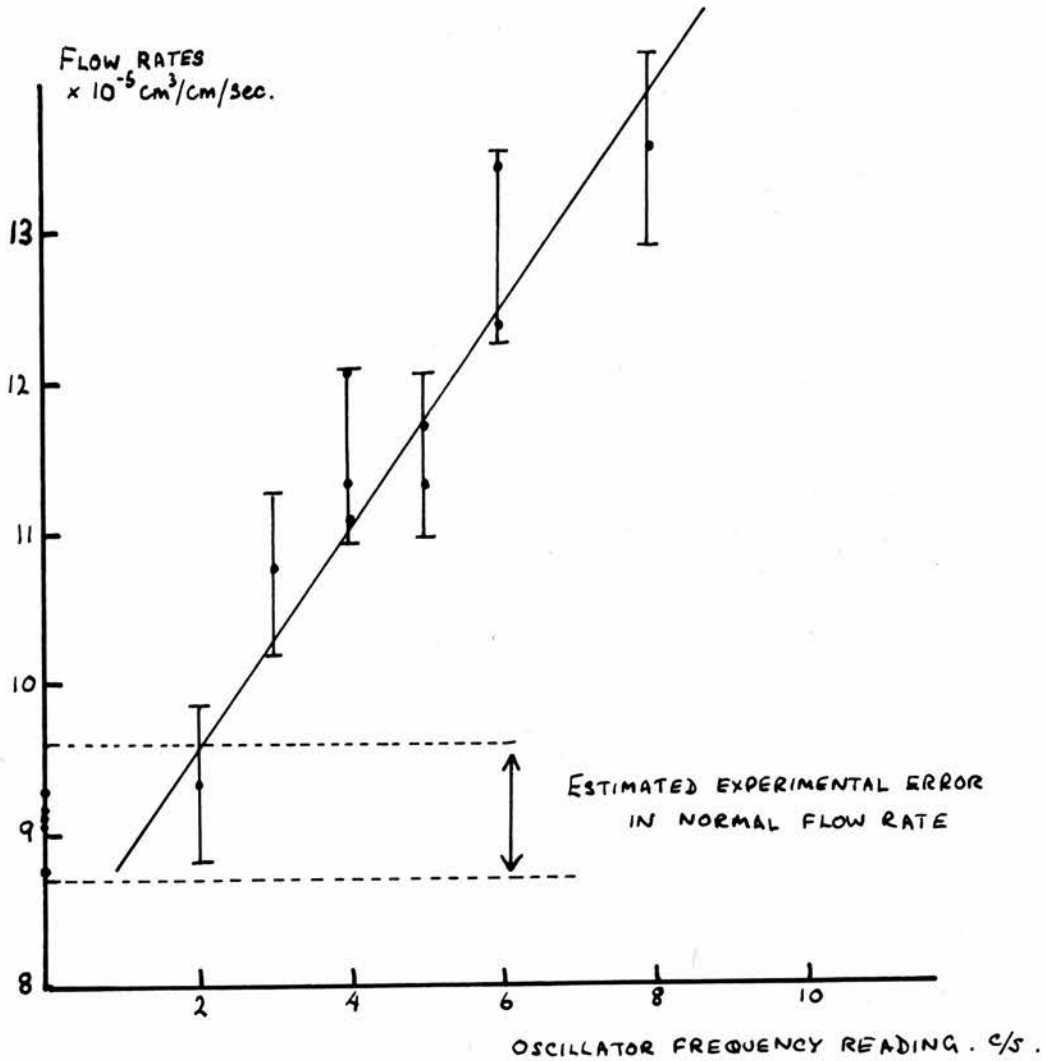

Fig. 7.6.1 Variation of the flow rate out of a beaker with frequency of stirring. (Run on 18th July, 1962).

Table 7.7.1

Stirring at 5 c/s, amplitude of stirring 30 divisions.

Cathetometer scale readings

Bath level	Beaker level	Time Min.	Time Sec.
	10	0	19
	15	0	32
	20	0	49
	25	1	5
	30	1	34
	35	2	4
	40	2	30
	45	3	10
5	50	3	37.5
6	55	4	26
9	60	5	15
12	65	6	10.5
14	70	6	59.5
16	75	7	47
17.5	80	8	12.5
22	85	9	26.5

Run on 18th July, 1962.

Temperature 1.186°K.

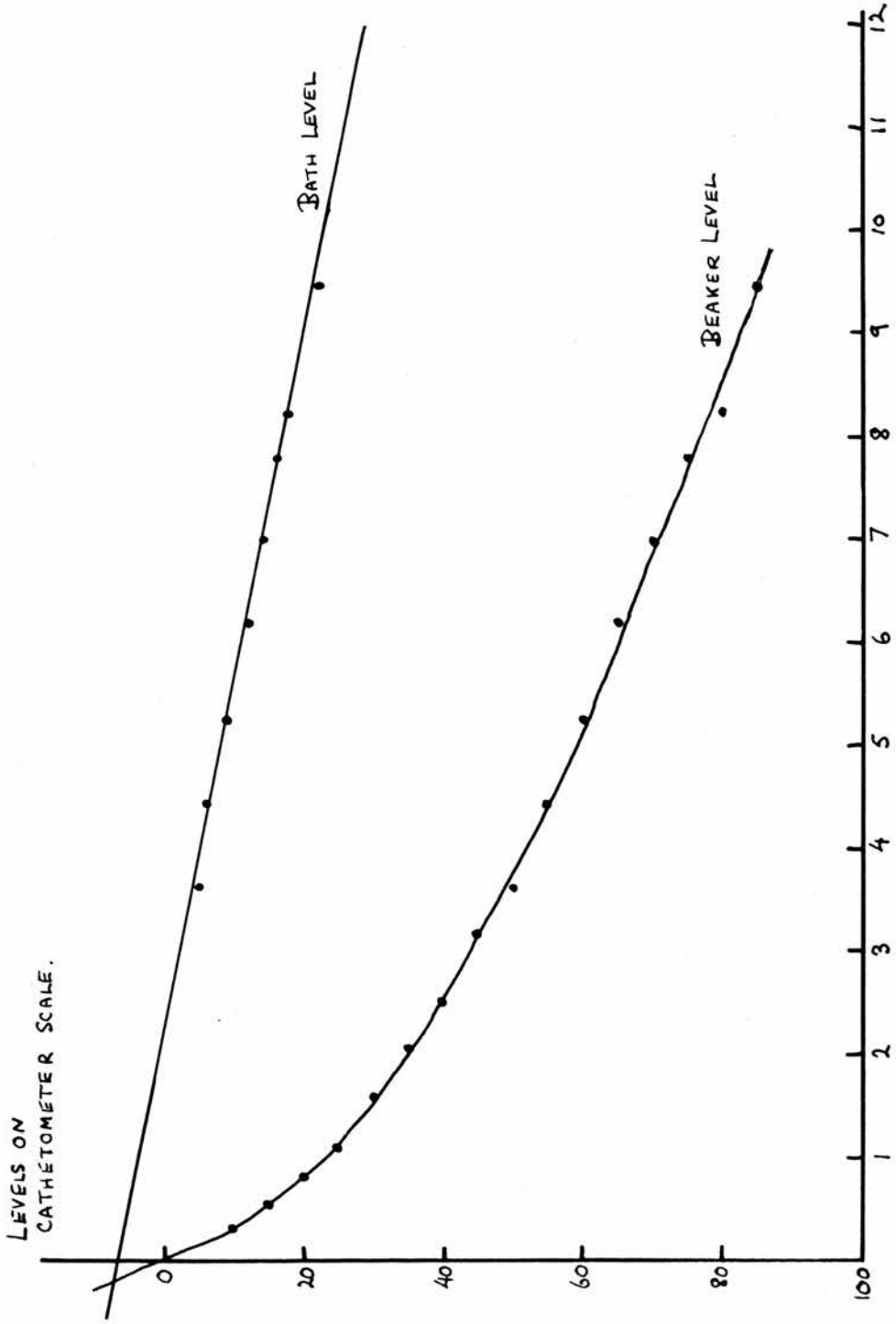

Fig. 7.7.1 Development of the Equilibrium Level Difference

period of forty minutes.

It was suspected that the quantity of helium in the vessel had some bearing on the results. The steady increase in the level difference as the quantity of helium in the beaker was reduced supports this view. The bath level fell by about 1.25 cm. during the forty minutes while the two levels were observed. Even after applying a correction for the volume of helium present in the beaker the setting up of the level difference does not seem to obey a simple saturation formula of the type $D = D_o(1 - e^{-\lambda t})$ where D is the level difference and D_o its value at saturation, t is the time after switching on and λ is a constant.

An unsuccessful attempt was made to measure the delay time between commencement of stirring and any influence on the flow rate. The time was too short to be measured by the techniques employed here. It was thought that this time might vary with the amount of helium present.

An attempt was also made to note the variation of static level difference with the volume of helium present in the beaker, by noting the level difference when the bath was at different marked levels on the beaker. A conclusive set of results was never obtained, because the effect of "tiredness" as observed in some of the early experiments was again noted.

The stirrer was switched off after an equilibrium level difference had developed and the decay of the level difference observed. From the results mentioned in Chapter VI it might be expected that the flow rate into the beaker would vary, being slower than the standard flow rate but gradually tending to the normal flow rate as the "specific turbulence"

at the downstream end of the film diminished. The results can be seen in Fig. 7.7.2. The expected variation in flow rate was not observed and subsequent measurements of the standard flow rate showed that the level difference decayed at a rate consistent with normal filling rates.

7.8 "Tired" Helium

This "tiredness" appeared when the helium ceased to be stirred and the meniscus in the beaker was no longer agitated, although motion of the stirrer appeared to remain the same. Sometimes the beaker meniscus still moved occasionally, while at other times the meniscus immediately became steady and easy to see. Any static level difference which had been set up immediately died away. In view of the precautions which have now been taken it is unlikely that this effect can be attributed to spurious thermal effects.

Possibly the effect could be caused by the stirrer creating so much turbulence that a sort of saturation position is reached, where instead of non-turbulent helium passing the sides of the stirrer, the stirrer just carries some of the turbulence (assuming this is in the form of vortices) up and down with it. This view is supported by the observation that in any run the effect always first appeared at one of the higher frequencies (not always at the same frequency) and stirring at lower frequencies after this was only sometimes accomplished. It was suspected that the effect developed when the beaker was being emptied fairly rapidly while being continually stirred, when no helium was getting into the beaker until after the effect had developed.

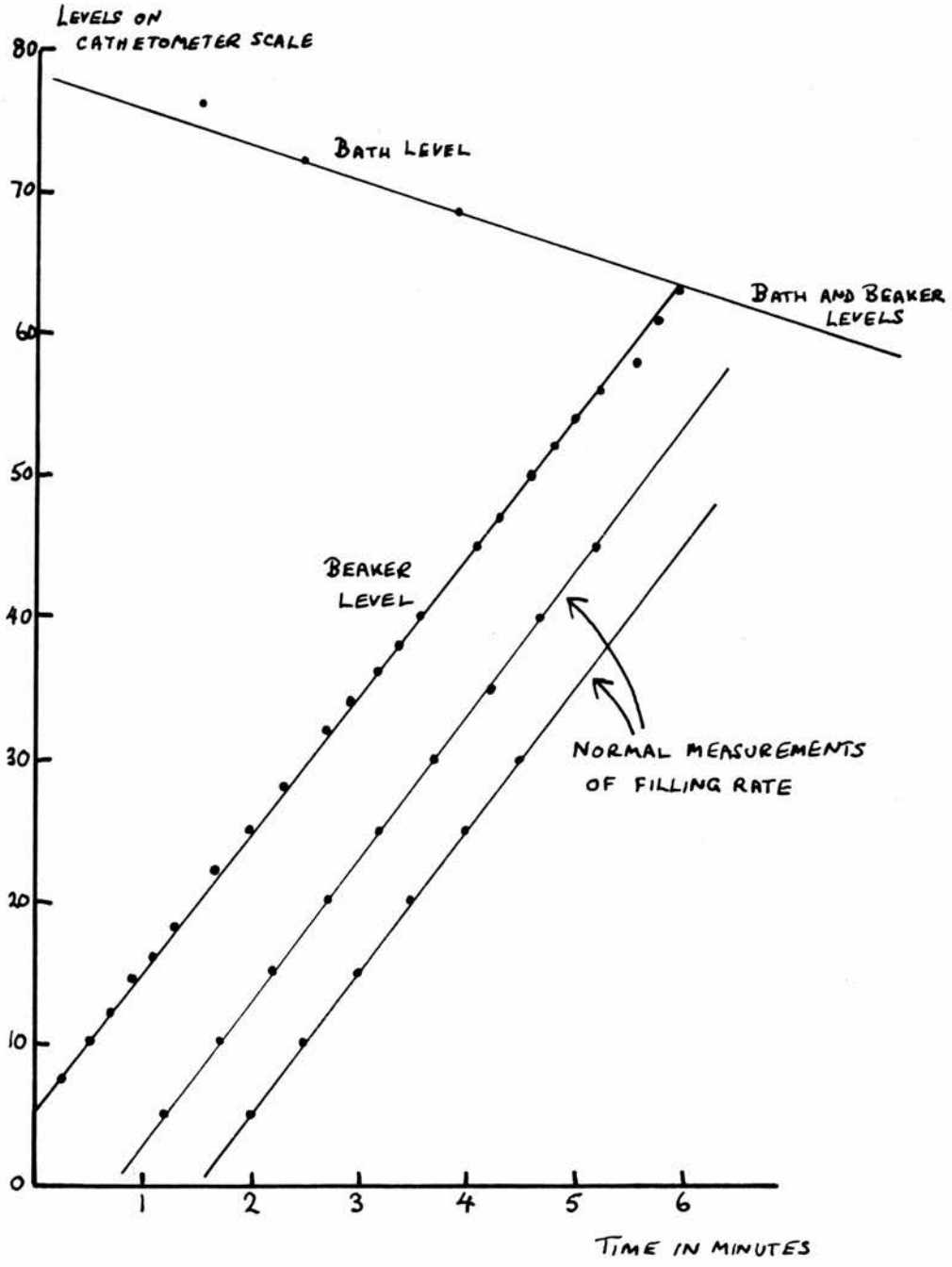

Fig. 7.7.2 Decay of the equilibrium level difference compared with normal filling rates for the beaker.

Three ways in which "tiredness" has been observed are shown in Figs. 7.8.1 and 7.8.2. In Fig. 7.8.1 the meniscus was not agitated during the intervals ab, cd and ef, but during intervals bc and de the meniscus was agitated and in the time interval after f the meniscus was agitated intermittently and the level difference gradually increased again. In Fig. 7.8.2 the straight portions of the plots of beaker level were observed when the meniscus was not agitated. In the lower graph the meniscus was agitated intermittently during the intervals when the beaker level does not follow a linear plot.

The possibility that this may be an experimental effect cannot be eliminated completely. It is however difficult to envisage the reasons for any mechanical inefficiency of the stirrer at times when the effect was observed. It was never observed at the start of a run, although the working time before it appeared varied from day to day and it was sometimes not observed at all. Stopping the stirrer for up to half-an-hour did not affect the state of the helium once it was "tired". Emptying the helium from the beaker and then filling by plunge filling did not completely remove any tiredness which had developed, although some stirring could be accomplished after this procedure.

7.9 Comparison of the Equilibrium Level Difference with Fountain Pressure

A fountain effect was set up in a small beaker for comparison with the pressure head of helium produced when the beaker was stirred. The apparatus consisted of a narrow beaker of 1.93 mm. internal diameter, containing a small coil of Eureka wire. Two pairs of copper leads were

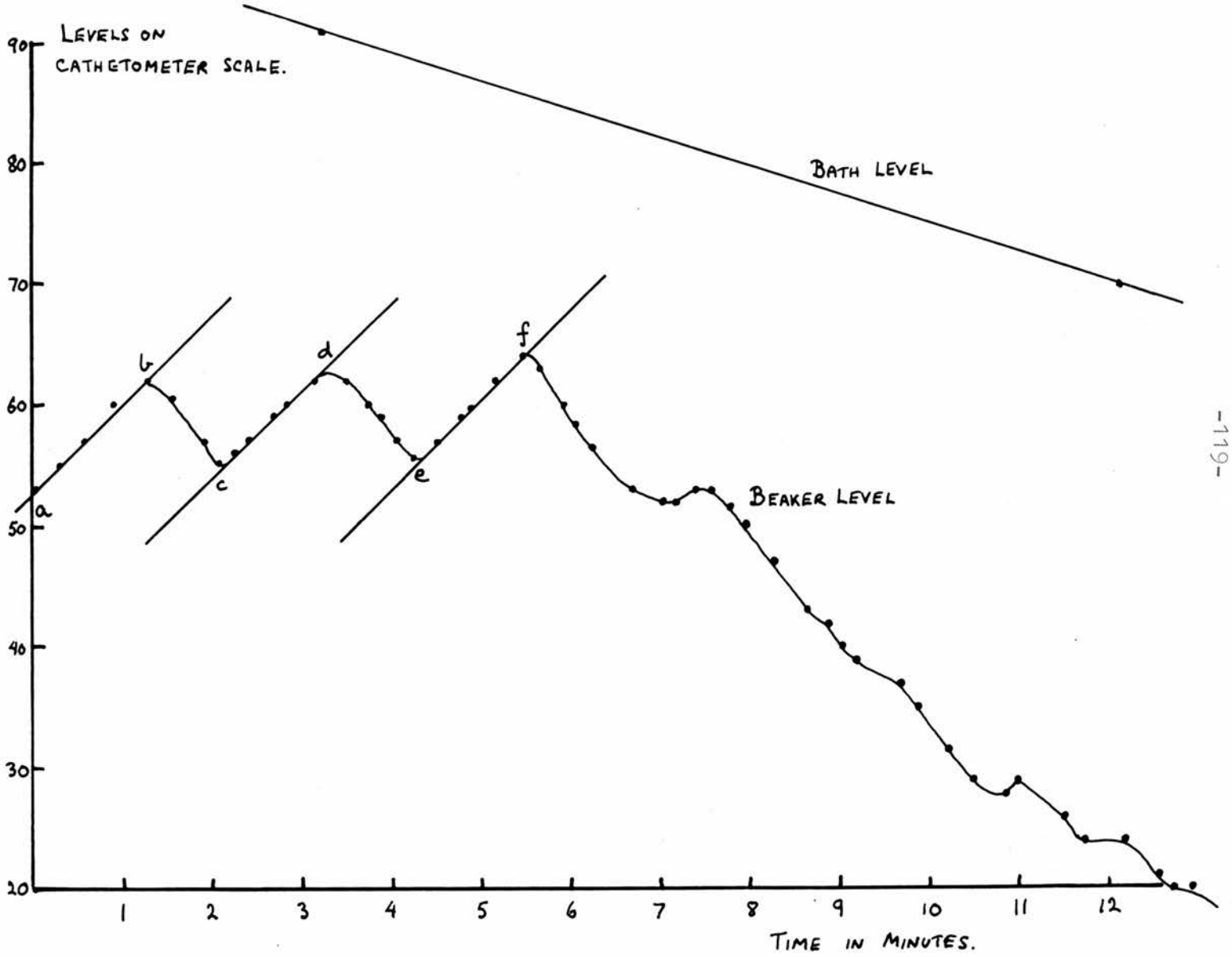

Fig. 7.8.1 A way in which "tiredness" develops

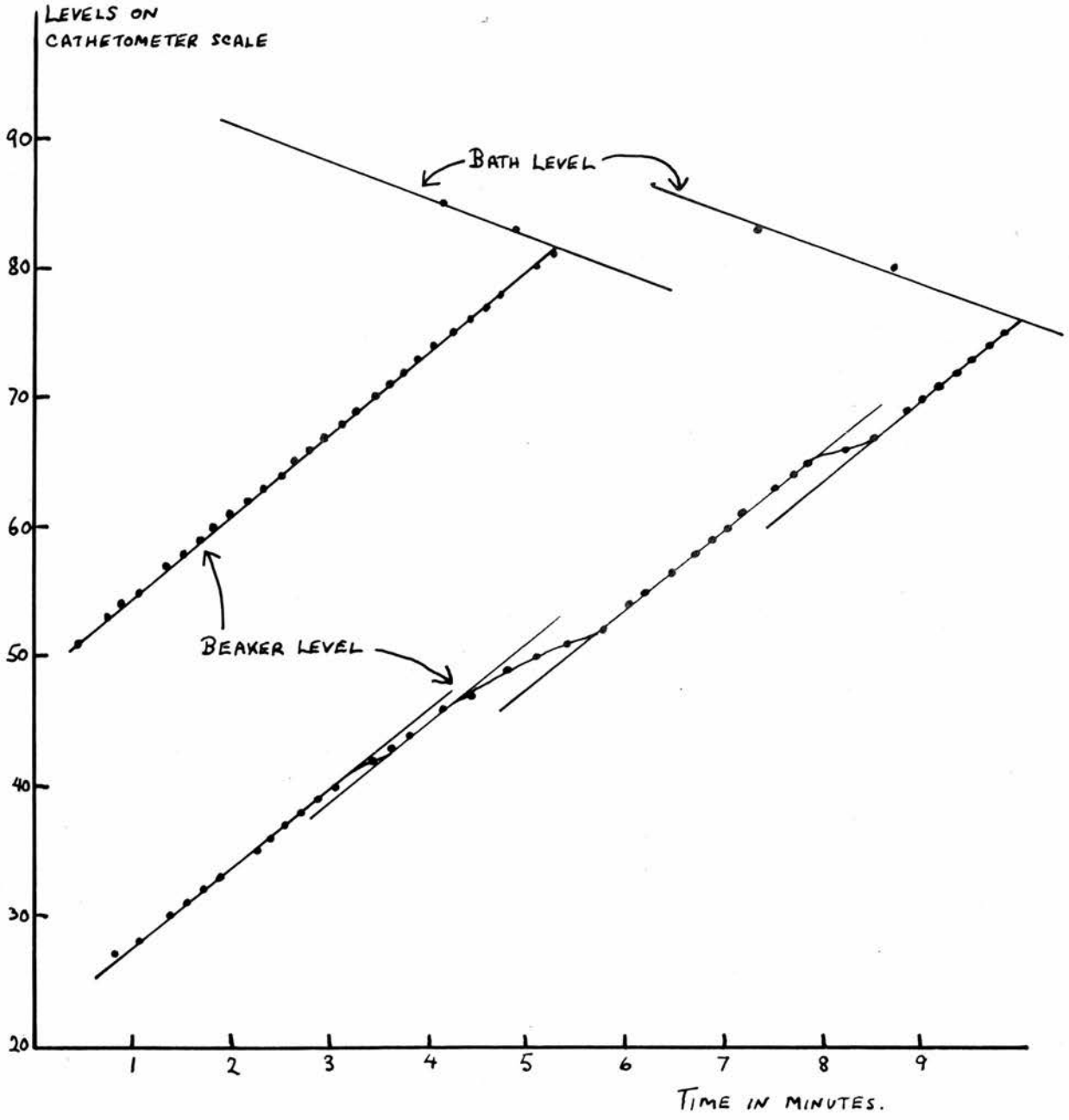

Fig. 7.8.2 Two ways in which "tiredness" develops.

provided so that both current and voltage in the small coil could be measured.

The fountain height in the beaker was noted for different values of the power input and a linear graph of the fountain pressure against heat input was obtained (Table 7.9.1 and Fig. 7.9.1). Above a certain critical heat input the helium in the beaker boiled away more quickly than it could be replaced by film flow from the bath and thus only a limited value of the fountain pressure could be reached. In Fig. 7.9.1 it can be seen that to obtain a fountain pressure of 30 divisions (0.36 cm.) an energy input of 0.72×10^{-3} joules/sec. is required.

The equilibrium level difference produced by stirring introduces a negative pressure head at the end of the film where the turbulence is created. This implies a further term due to the turbulence in the expression for pressure in liquid helium (Allen, 1961):

$$P = gh + sT - f(\text{vorticity}) \ .$$

In the equilibrium situation there appears to be a balance between the potential energy of the helium outside the beaker and the kinetic energy of turbulence inside the beaker.

Table 7.9.1

Current I ma.	Voltage V volts	VI x 10^{-3}	Fountain pressure Divs. on scale
2	0.001	0.0020	0
4	0.0044	0.0176	0
6	0.0065	0.0390	1.2
8	0.0088	0.0704	2.5
10	0.0111	0.1110	5.5
12	0.0133	0.1599	8
14	0.0157	0.2199	11.2
16	0.0179	0.2861	12.8
18	0.0199	0.3582	14
20	0.0216	0.4320	17.5
22	0.0249	0.5480	22.5
24	0.0267	0.6430	26
26	0.0299	0.7770	8.5
28	0.0316		evaporating

Run on 14th June, 1962.

Temperature not very steady but approximately 1.247°K.

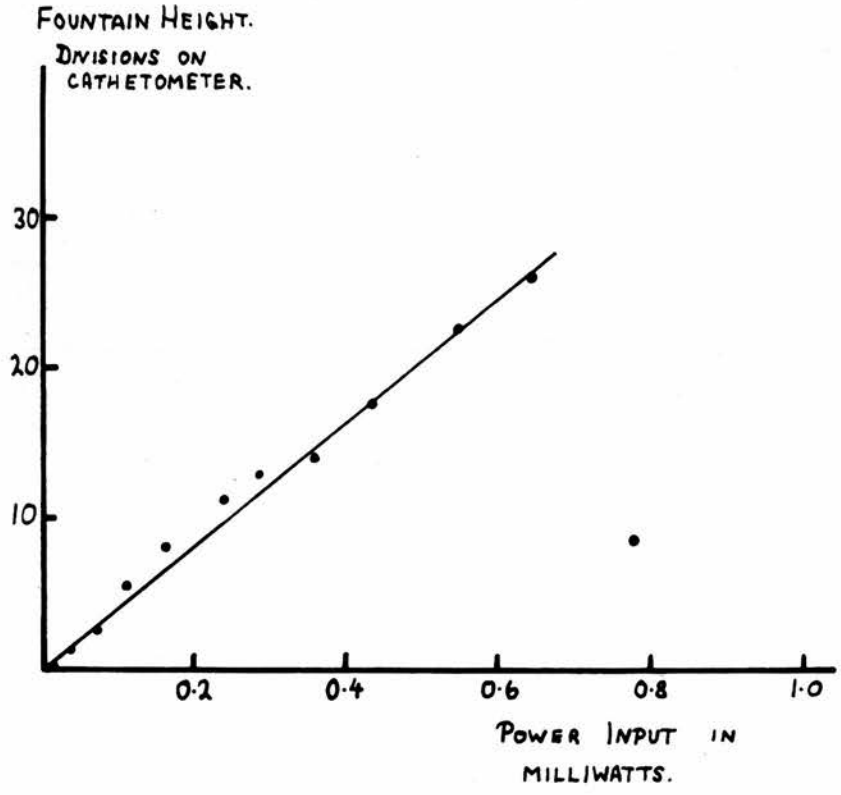

Fig. 7.9.1 Fountain pressure as a function of the power input to the beaker.

CHAPTER VIII

Summary

A qualitative survey of the experimental results on flow of the helium film has led to several observations which may have some bearing on the mechanism of the flow. The initial cause of flow between helium at two different gravitational levels has been estimated to result from an instability in the film profile at the rim of the wall separating the two levels. Further consideration has led to the belief that the statement which implies that flow is controlled by the smallest perimeter in the flow path should be amended to imply that it is the smallest "channel" which controls flow, although in the majority of cases the two will be coincident. If bulk helium formed above the upper of two levels between which flow occurred, it would be at a higher gravitational potential than the level initially supplying helium for the film flow and should then tend to flow in the opposite direction. This is thought to be the reason for the control exerted by a constriction on the flow of the film.

The experiments with the rotating beakers filled with carbon tetrachloride have shown that, in any rotating beaker experiments where the fluid inside is connected by a leak to the external bath, one must be careful to distinguish the effects of centrifugal forces from any other observed phenomena.

The experiments on the effect produced by turbulence in the bulk

helium on the helium film in contact with it require further development before good quantitative results may be obtained. Turbulence does appear to influence the helium film, although it is difficult to reconcile some of the recorded observations. Enhanced flow rates out of a beaker have always been observed when the beaker has been filled by quick immersion, followed by slower filling rates, but no corresponding slow filling rate has been observed when the stirrer in a beaker was switched off and the equilibrium level difference allowed to decay. The need for good shielding of the apparatus from thermal radiation to get reproducible results from film experiments has been demonstrated.

Considerable equilibrium level differences have been produced by stirring the helium in a beaker. The differences are not however reproducible from day to day and only rough estimates can be made of the critical velocity and Reynolds number. The values of Reynolds number, calculated using the measured critical helium velocity as v_n in the formula of Staas et al., agree moderately well with the critical values of Reynolds number for flow in capillaries. The equilibrium level difference indicates a balance between the kinetic energy of turbulence inside the beaker and the potential energy of the helium at the other end of the film.

It is unlikely that the present form of the apparatus can yield any more quantitative results, as the instabilities in the motion of the stirrer are such as to be inherent in any similar system. Development of another method of producing turbulence in the bulk liquid should help to clarify the picture, and it is to be hoped that the results will

provide a more quantitative picture consistent with the observations recorded here. It might also be instructive to investigate the effects on the film of more readily calculable forms of kinetic energy in the bulk liquid.

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