NECESSITY AND MODAL SYSTEMS:

AN ESSAY ON MODAL TERMS.

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M.Litt. Thesis.

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INTRODUCTION

The purpose of this paper is to determine if our ordinary-language notions of modal terms furnish a basis for the development of modal logics, and if so, how and to what extent. I shall begin by distinguishing the "use" of a modal term and the "ground" for it. Briefly, the logical relations between statements are due to the uses of their terms, and the grounds for a statement are the reasons we would produce to justify its assertion. In Chapter One I shall distinguish five uses of modal terms, and in Chapter Two I shall enumerate a number of grounds for sentences containing modal terms. In the next two chapters I shall employ the results of Chapter One and Chapter Two in connection with the two most important problems facing modal logic: iterated modalities and statements in which a quantifier binds a variable which is within the scope of a modal term.

CHAPTER ONE

THE USES OF MODAL TERMS

Section A: The Notions of Use and Ground.

In this section I shall introduce two notions which I will make use of throughout this essay: these notions are use and ground. I will establish these notions in a somewhat formal manner. The reason for doing this is to show that these notions can in fact be given a definite formulation. I shall, however, after this section tend to neglect the more formal considerations which are present in it. In particular, when I go on in the succeeding sections of this chapter, I shall follow ordinary language and will not directly apply many of the definitions on which the semi-formal formulations of "use" in this section depend.

To begin with "use", this needs to be said: the (To page 3) logical relations which a statement has to other statements are determined by, amongst other things, the words which are contained in the statement. We cannot say that they are determined solely by the words contained in the statement, because, e.g., the reference of the demonstratives and the definite article are dependent on the circumstances in which they are uttered; hence, the truth values of the two statements which contain the same words may be different.

I will even add to what I have said that there seems to be no reason why the results of this section could not be extended to accommodate an unrestricted class of statements.

Now for a given statement there is a class of statements which have a certain logical relation, e.g., "contradictory to", to this statement. Thus we will have, for any statement, a class of statements which are all those, and only those, which are contradictory to the statement; another which are all those, and only those, which are entailed by it; a third, which are all those, and only those, which are sub-contraries to it; and, in general, for any logical relation, we will have the class of all those, and only those, statements which have that logical relation to the given statement. At this point I wish to introduce a useful definition:

> Definition 1: A use-class of "p" (where "p" is any statement) $=_{df}$ the ordered class consisting of classes, each of which consists of those, and only those, statements which have a certain logical relation to "p".

Here it is obvious that the use-class of a statement is dependent upon the analysis used. If in one analysis one is concerned with logical relations R_1, R_2, and R_3, then the use-class of a statement "p" will be the ordered triplet consisting of the class of all, and only those, statements which have relation R_1 to "p"; the class of all those, and only those, statements which have relation R_2 to "p"; and the class of all those, and only those, statements which have relation R_3 to "p"---in that order. On the other hand, if in another analysis one is concerned with relations R_1, R_3, R_4, and R_5, then the use-class of a statement "p" will be the ordered quadruplet consisting of the class of all and only those statements which have relation R_1 to "p"; the class of all and only those statements which have relation R_3 to "p"; the class of all and only those statements which have relation R_4 to "p"; and the class of all and only those statements which have relation R_5 to "p"---in that order.

At this point it would be helpful to introduce a notation which explicitly mentions the logical relations investigated in a certain analysis.

<u>Definition 2</u>: $A\ (R_1, R_2, \ldots, R_n) =_{df}$ the analysis which is concerned with relations (logical relations) $R_1, R_2, \ldots, R_n$.

Making use of definitions one and two, we have:

<u>Definition 3</u>: The use-class of statement "p" relative to $A\ (R_1, R_2, \ldots, R_n)$, in symbols $(\emptyset_1, \emptyset_2, \ldots, \emptyset_n) =_{df}$ the ordered n-tuple consisting of the class of all and only those statements which

have relation R_1 to "p" (i.e., $\emptyset_1$); the class of all and only those statements which have relation R_2 to "p" (i.e., $\emptyset_2$); ...; and the class of all and only those statements which have relation R_n to "p" (i.e., $\emptyset_n$)---in that order.*

It should be pointed out here that what I mean by "logical relation" is to be taken in a very wide sense. Not only is it to include relations which are studied in formalized systems, but also such relations as "...is analyzable as..."

Now for certain statements, the use-class relative to A (R_1, R_2,..., R_n) may not be uniquely specifiable.** In these cases one or more of the classes $\emptyset_1$, $\emptyset_2$,..., or $\emptyset_n$ for "p" is not uniquely determined by A (R_1, R_2,..., R_n). I shall say here that a "determinant analysis" of statement "p" is an analysis which uniquely determines a use-class for statement "p". Thus it can be seen that A (R_1, R_2,..., R_n) can determine a class of any number of determinant analyses, their common characteristic being only that they are all concerned with the relations R_1, R_2,..., R_n.

Let $\emptyset_1^1$, $\emptyset_1^2$,..., $\emptyset_1^m$ be all the classes of statements

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* for "Ø" read the Greek letter "phi".

** Intuitively, we would say that at least some of the logical relations of the sentence are ambiguous; hence, perhaps, also that the sentence is ambiguous.

to which statement "p" has relation R_1; ϕ_1^1 being established by one determinant analysis; ϕ_1^2 being established by another determinant analysis; etc. I can now introduce another notation:

<u>Definition 4</u>: $(\phi_1^k, \phi_2^k, \ldots, \phi_n^k) =_{df}$ the use-class of statement "p" relative to the $k^{\underline{th}}$ determinant analysis of the form $A(R_1, R_2, \ldots, R_n)$.

Of course, for any integers k, l,&m, it might be the case that $\phi_m^k = \phi_m^l$.

The use-classes relative to $A(R_1, R_2, \ldots, R_n)$ of the statements we are concerned with are determined by the terms which these statements contain. Thus the distinction between one determinant use-class for a particular statement relative to $A(R_1, R_2, \ldots, R_n)$ is determined by the terms that statement contains. We can now begin to introduce the notion of the use of a <u>term</u>. If a statement has only one determinant use-class relative to $A(R_1, R_2, \ldots, R_n)$ then it has <u>not</u> been shown that any of the terms contained in it have more than one use. As <u>n</u> is increased, the evidence becomes greater that the terms, in a particular context, do not have more than one use.

Establishing that a term has a certain number of (logical) uses is not always a clear-cut process. For one thing, the number and kinds of logical relations which are involved in a particular analysis indicate what uses a statement has and thus also the uses of constituent terms. A unique list of use-classes of a statement could not be determined unless all the classes $\emptyset^1_1$, $\emptyset^1_2, \ldots, \emptyset^1_n$, $\emptyset^2_1$, $\emptyset^2_2, \ldots, \emptyset^2_n$, $\ldots$, $\emptyset^m_1$, $\emptyset^m_2$, $\ldots, \emptyset^m_n$ for that statement could be determined. But this would entail that all logical relations, R_{1-n}, which the statement could have to any other statement could be uniquely specified; and exactly what is to be accepted as a logical relation is not in all cases evident. Also, this would presuppose that all the statements of each class $\emptyset$ could be listed, or at least indicated. But this is obviously impossible, for these classes are infinite. And from the fact that the logical use-classes of a sentence cannot be uniquely specified it follows that the logical uses of a term cannot be uniquely specified. Also, there is the difficulty that, even when a statement has two distinct determinant analyses, the term or terms to which this multiplicity can be traced are not usually thereby determined from that statement alone.

In determining the (logical) uses of a term one must consider, usually, a group of statements in which that term occurs. Then it is a process of abstracting that which is common to the use-classes of a group of these statements and it would automatically change if the term in question were replaced by a different term. Notice, I do not say "...abstracting that which is common to the use-classes of <u>all</u> of these statements", but only "...to the use-classes of a <u>group</u> of these statements." For, given one term there may be a number of uses.

To put it more formally the uses of a term could be represented, at least in part, as a use-class of a statement-schema in which all but the particular term in question (but possibly a few other terms as well) is represented schematically. For example, to represent part of the use of the term "or", we could have a use-class for "p or q" as ($\emptyset_1^1$, $\emptyset_2^1$, $\emptyset_1^2$, $\emptyset_2^2$) where $\emptyset_1^1$ = ("p", "q",...), R_1 being the relation "...entails..."; $\emptyset_2^1$ = ("not p and not q", "not (p or q)", "neither p nor q", ...), R_2 being the relation "...is contradictory to..."; $\emptyset_1^2$ = (" p and not q", "q and not p", ...), where the second determinant analysis of "p or q" is as "either p or q (but not both)"; $\emptyset_2^2$ = ("not p and not q; or p and q", "not (p or q)", ...).

Of course, the use of a term is usually determined by the context in which it appears. In determining the use of a term from a context, one may have to take a relatively large linguistic unit into consideration, e.g., a paragraph; or perhaps the use can be determined from one or two words immediately preceding or following the word in question.

To illustrate how the use of a term can be determined from a context I take the example of the definite article, "the". There is one use of it in which "the A is B" entails, and is entailed by, " All A's are B's". Examples of this use are: "The whale is a mammal"; "The Triumph is a fast car"; and "The Fir tree is green all year." What is common to all the contexts here is that the definite article precedes a quantitative noun in the singular which, despite the juxtaposition of the definite article, refers to a number of things (in fact, to the whole class of things to which it is correctly applied). On the other hand, there is another use of "the" in which " The A is B" does not entail "All A's are B's". Examples of this are: "The car ran off the road"; "The factory has stopped operation"; and "The company is bankrupt"

Despite being in a clear context a term's use might not be definitely specifiable. But context is very often, if not usually, a good indication of use. As a consequence of this, a similarity in context very often, if not usually, indicates a similarity of use.

The second factor in my analysis is what I shall call the ground for applying a term. Just as two occurrences of the same term can be two different uses of it, and, in general, a term can have a number of uses, so can two occurrences of the same term have two different types of grounds for their application, and, in general, a term can have a number of types of grounds for its application. This is not to suggest that there is a one-to-one correlation between uses and types of grounds. On the contrary, some types of grounds are appropriate for a number of uses of a term, while some uses of a term are related to a number of types of grounds. For example, that p is true and q false is a ground for applying "or" in "p or q" in both the use expressed in Latin by "vel" and the use expressed in Latin by "aut". And both that p and that q are grounds for applying "or" in " p or q " in the use expressed in Latin by ~~"aut"~~. "vel".

In order to reach a fairly exact notion of ground it is necessary to begin by considering statements, as we did with the notion of use, since statements are what are true or false. Given a particular statement, "S", and a class of statements, ("E_1","E_2",...,"E_n"), the relationship is the support which ("E_1","E_2", ...,"E_n") gives to "S". This relationship can range from no support at all, i.e., where "S" and ("E_1", "E_2",..., "E_n") are independent, to the case where ("E_1", "E_2",...,"E_n") entails"S". Here it is desirable to abstract so that we talk of the support which a class of the kind ("E_1", "E_2",...,"E_n") gives to a statement of the <u>form</u> "S". Now, obviously, the terms which a statement contains are intimately connected with the form of that statement. So, we can talk about the amount of support which certain kinds of classes of statements give to statements of the form "... f ...",where "f" is a particular term. This, then, is what is meant by something being the ground for the application of a term: if a class of statements of the kind ("E_1", "E_2",...,"E_n") gives some support to a statement of the form "... f ...", then a class of statements of the kind ("E_1", "E_2",..., "E_n") is a ground for the application of the term "f".

I say " a class of sentences of the <u>kind</u>..." rather than "...of the <u>form</u>..." because there may be more than simply formal considerations involved in a class of sentences being a ground for the application of a term.

It is probably obvious by this time that there is an over-lap between the use-class of a statement of a certain form relative to a certain analysis and the grounds for applying the distinctive term in a statement of this certain form. The case in which this over-lap occurs is where a class of statements of the kind ("E_1", "E_2",...,"E_n") entails a sentence of the form "S". If we think of ("E_1", "E_2",...,"E_n") as containing only one member, "E_m" (which would be "E_1 & E_2 &...& E_n"), then, if the use-class of "S" includes the class of those statements which entail "S", then "E_m" will be a member of a class of the use-class of "S". And since "E_m" entails "S", a statement of the same kind as "E_m" is a ground for applying the distinctive term (i.e., the term whose occurrence makes "S" of the form that it is) since entailment is certainly a relation of support.

<u>Section B: Epistemic Uses.</u>

As mentioned at the beginning of <u>Section A</u>, I shall follow ordinary language in distinguishing uses of modal terms. But the results of the last section are still relevant here, even though I do not apply them to any great degree. One result which is of importance here is that the distinction between uses is somewhat indeterminate. We can often distinguish two uses by finding two different determinate analyses with regard to a single logical relation. But until we have gone through all logical relations we will not be certain of how many uses there are for the term in question.

However, the uses which I distinguish here I feel are not only naturally distinguished, but it is also important to distinguish them in that statements which contain the different uses differ in many and in particularly important logical relations to other statements. Also I feel that to distinguish them more specifically would evolve into nit-picking. There is one use which I present , the imperative use, which is certainly distinguished from the others, but which might be rather artificial in that it is distinguished from other uses, which I do not mention, which share many of its characteristics.

This brings up the question of the completeness of my list of uses. Certainly it is incomplete in that it does not include the uses akin to the imperative use. But the imperative use was included as representative of the group. (It was chosen in particular because it is easier to analyse than the rest.) With this exception, however, the list is practically complete in that it covers the most obvious cases. However this may be, the notion of a complete list of uses for a class of terms is a very foggy notion,,which one can see from <u>Section</u> <u>A</u>.

I shall begin by discussing three uses of "possibility" covered by M.R. Ayers in his book entitled, "<u>The</u> <u>Refutation</u> <u>of</u> <u>Determinism</u>". He writes as follows,

> "The first kind of possibility to be discussed is sometimes called 'epistemic possibility'...The sentences 'It is possible that it will rain tomorrow' and 'He might possibly call tomorrow', as normally used, would express relative possibility-statements.
> "The second kind of possibility is 'natural possibility'...An example of a natural possibility-statement would be 'It is not possible that life should exist on the sun.' I shall also talk of the related concepts of 'natural necessity', 'natural power' and 'natural potentiality'. An example of an attribution of a natural power would be a proposition expressed by 'This car can do 100 m.p.h.', taken in its normal sense.
> "Thirdly, I shall distinguish a general kind of possibility that I shall call 'possibility for choice'. An example of this kind of possibility-statement would be 'It is possible for him to come to dinner tonight', which might also be expressed 'He could come...' or 'It is in his power to come...' Consequently it may be described as an ascription of a power to a person, or, simply, an ascription of 'personal power'."[1]

To begin, then, with epistemic possibility, this can be said. This use of "possibility" is relative to knowledge, hence its close affiliation with the term "probable". This use is common in ordinary language; for example, when we see dark, low clouds rolling in from the West at evening, we say, "It's possible that it will rain tomorrow."

1. M.R. Ayers, The Refutation of Determinism, (Methuen, 1968) pp. 12-13.

However, if the clouds blow over and the barometer rises by the next morning, we then might say, "It's not possible that it will rain today." And even though both statements refer to the same day, they both might be correct since the evidence available on the one evening is different from the evidence available on the next morning.

The close connection between this sort of possibility and probability comes out in ordinary language. We often say that something is possible, but not very probable. Again, we often use the terms "possible" and "probable" interchangeably, but with this difference: it makes sense to ask how probable something is, but not how possible it is. That is, we recognize degrees of probability, but not degrees of possibility.

That epistemic possibility is not capable of degrees*

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* "Certain" is often used in a sense such that a person is certain that p iff he is utterly convinced that p. I shall use "certain" only in contexts which are, or could be paraphrased to be, of the form "It is certain that..." The truth or falsity of a statement of this from, unlike a statement of the form "X is certain that p", where "X" refers to a person, is dependent on the knowledge of the whole society of which the utterer is a member. Another distinction between these two uses of "certain" is that "It is certain that p" entails "p", but "X is certain that p" does not. For example, "Some ancients were certain that the earth was flat" is a true statement. But "It was certain that the earth was (is?) flat" is not true.

"It is certain that..." is identical with "It is known that..." except that certainty entails utter conviction (on the part of some member of the society).

That certainty is not capable of degrees will be questioned in Chapter Four. But this will depend on considerations which will be brought up later.

is a simple corollary of the fact that to say that something is possible in this use is equivalent to saying that it is not certain that ... not .... For example, to say "It is possible that it will rain tomorrow" is equivalent to saying "It is not certain that it will not rain tomorrow." There is also the converse relation; if something is certain, then it is not possible (in the epistemic use) that ... not ....

This relationship with certainty is a distinguishing characteristic of epistemic possibility. The other uses of "possible" which I shall consider have a similar relationship with "necessary", but not with "certain". And "certain" is distinguished from "necessary" in that a statement of the form "It is certain that ..." entails a statement of the form "It is known that...", whereas a statement of the form "It is necessary that..." entails no such statement.

Section C: Natural Uses.

Ayers[1.] makes the distinction between "It is possible *for* Smith to call" and "It is possible *that* Smith will call" as he puts it in his example. This particular example,

1. ibid, p. 14.

however, is relevant to possibility for choice, which I shall take up next. But the same distinction exists between natural and epistemic possibility. In general, epistemic possibility is expressed by a statement of the form "It is possible <u>that</u>...", and natural possibility is expressed by a statement of the form "It is possible <u>for</u>..."[1]

But, I think, it is generally felt that this verbal difference is not sufficient to establish that there are two different senses or uses of possible here. First of all, it should be pointed out that a natural possibility statement does not entail an epistemic possibility statement. This can be seen by considering that what is epistemically possible is relative to the knowledge we possess at the time; whereas what is naturally possible has only to do with the particular object or objects, and is independent of our knowledge. For the same reason, an epistemic possibility statement does not entail a natural

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1. An occurrence of the natural use of "necessity" can be followed by "that". For example, "It is necessary that the tides are higher in Spring." This suggests that an occurrence of the natural use of "possible" can be followed by "that". For example, "It is not possible that the tides are not higher in Spring." The point is that epistemic modal terms are always followed by "that". And, we might say, it is more fitting for natural modalities to be followed by a phrase containing "for".

possibility statement.

For example, a cetain car may be capable (i.e., it is naturally possible) of doing 100 m.p.h., but yet we may know for sure that it will not (i.e., it is epistemically impossible). Again, all the evidence may indicate that it is possible that a certain car does 100 m.p.h., but yet the car might not be capable of doing 100 m.p.h.

Perhaps the most compelling reason for treating these two kinds of possibility as separate senses is that epistemic possibility is associated with certainty, while natural possibility is associated with (natural) necessity. That is, "It is not (epistemically) possible ...not..." is equivalent to "It is certain...", and "It is not (naturally) possible ...not..." is equivalent to "It is necessary..."

Kneale, in his book entitled Induction and Probability, brings out the point that though science aims at "principles of necessitation" (i.e., propositions stating connexions of natural necessity), yet a scientist is never certain that he has arrived at such a principle. For a scientific law applies to an open-ended class, all the members of which obviously cannot be examined.

However, we need not go into the realms of science to show that the notions of necessity and certainty are distinct. Neither entails the other. For example, we may be certain that a particular building is forty feet high, but it certainly is not necessary that the building be forty feet high. Again, it may be necessary that a particular rock in a rock pile rolls down such-and-such a side of that pile (because of the forces acting upon the rock), and yet we may not know for certain that the rock will roll down the pile.

Section D: Possibility and Necessity for Choice.

The third use of "possible" which Ayers discusses is "possibility for choice". I shall here present in outline Ayers' argument for the distinction between this use and the natural use. But in the next section I will argue that "possibility for choice" is best considered as only a particular application of a wider use of "possible" which I shall call "conditional possibility".

With regard to "possibility for choice" this use usually occurs in a context containing "for" rather than containing "that". And the same points apply to the distinction between this use of "possible" and the epistemic use of "possible" which were mentioned in regard to the distinction between the epistemic and natural ~~uses of "possible". In particular, a statement of the form "It is not possible for...not...", where the possibility is possibility for choice, is equivalent to a~~

~~statement of the form "It is necessary for...". This is like the natural use of "possible" and distinct from its epistemic use. Ayers does not discuss "necessity for choice", but it is important to notice that such a notion can be introduced quite simply.~~

The distinction in question here is that between natural possibility and possibility for choice. It would only be begging the question to say that the distinction is a clear one since possibility for choice applies only to people qua agents, and natural possibility applies only to physical objects and to people qua physical objects. What has to be shown is that there are two separate senses of "possible" which are applicable to different sorts of things because of their different logical relations.

~~I see it in this way:~~ Ayers has two basic, although related, arguments for his thesis that possibility for choice is distinct from natural possibility. He presents his first argument in one paragraph:

> "Probably the most striking logical difference between the powers of a thing and a person's ability to do various particular actions is that the latter may depend very much on 'extrinsic' circumstances. A car that is locked in a garage does not thereby lose its powers, but a prisoner is deprived of his. When we ascribe a power to a thing we are characteristically saying something about its own nature but nothing about the circumstances in which it is placed; but if

> we assert that a man could have done an action we imply not only something about the man himself but also that the circumstances are favorable to the action, that there is nothing in them to prevent it."[1.]

Ayers goes on to add that the commonplace distinction between "general capacity" and "opportunity" is relevant here. Natural possibility has to do with general capacities, but neglects opportunity. For example, if a car can do 100 m.p.h., we say that it has the capacity regardless of whether it is being driven down the open road or is locked in a garage. But possibility of choice has to do with both general capacities and opportunities. A person locked in a cell cannot run a mile in five minutes simply because being locked in his cell he does not have the opportunity.

Ayers' second argument stems from his belief that a natural possibility statement can be analysed as follows: "It is possible for x to be k ~~can be taken to~~ means in some circumstances, ~~that~~ x would be k."[2.] ~~Now Ayers argues, not very convincingly, that the analysis of modal statements as hypotheticals is a real advance in an analysis of their~~

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1. ibid, p. 103.

2. ibid, p. 69.

~~meaning. However this may be, the only point of interest here is the view that statements expressing natural modalities can be paraphrased as hypotheticals.~~

As opposed to this analysis, statemnnts expressing possibility for choice can be paraphrased only as so-called " bogus hypotheticals". Now the antecedent of a hypothetical states a condition, the fulfilment of which is sufficient to bring about the state of affairs expressed in the consequent. Basically, there are two suggestions on how to analyse a statement attributing a power to a person, e.g.,"George can solve the problem", into hypotheticals. One is that the antecedent expresses a mental act and the consequent expresses a personal power, e.g., "If he wishes, George can solve the problem." The second is also that the antecedent expresses a mental act, but that the consequent expresses an exercise of the power expressed by the modal statement, e.g., "If he wishes, George would solve the problem."

Now it is difficult to draw a line between George's wishing, trying, or choosing to do something, and his actual execution of it. "Wishing", "choosing", and "trying" are often used simply to express that the agent in question does something intentionally. It is apparent that these words mention actions ~~words~~ and do not express conditions in the same sense that, say, rain is a condition for the grass to be wet.

Ayers sums up his argument against the view that statements of possibility of choice are analysable as hypotheticals as follows:

> "...not every action requires another as means to performing it; not every action has to be, or could be, 'brought about', some must simply be done. It is a related truism ... that the experiment or trial verifying the existence of an ability may simply be the action in question. Trials need not always have some element of action in common, independent of their result in the way in which a successful and an unsuccessful attempt at hitting a golf ball may sometimes have in common an identical swing. Thus the possibility of any hypothetical analysis of personal power is ruled out. In fact, that testing one's ability to do something is not the same kind of thing as testing the truth of a hypothetical, simply follows from the truism that in order to do something it is not always necessary to do something else first."[1].

Section E: Conditional Uses and "Possibility for Choice" Reconsidered.

In what follows I shall present a use of the modal

1. ibid, p. 145.

terms "necessary" and "possible" which occurs in connection with both physical objects and people <u>qua</u> agents. I shall try to show that Ayers' "possibility for choice" is an instance of this sort of possibility which I shall call "conditional possibility"; the correlative term, of course, is "conditional necessity". I will also have to show that these modalities are distinct from natural modalities; in doing so I shall rely in part on the discussion of Ayers' "possibility for choice" discussed in the last section.

Ayers' first argument that "possibility for choice" is distinct from natural possibility is that the former depends on "extrinsic" circumstances and the latter does not:

> "...if we assert that a man could have done an action we imply not only something about the man himself but also that the circumstances are favourable to the action, that there is nothing to prevent it."[1]

But the possibilities and necessities for physical objects often depend on the "extrinsic" circumstances, or as I shall call them, or often call them, the conditions. For example, if a car is parked with its bumper against a

1. ibid, p. 103.

stone wall, we say that it is not possible for the car to move forward. And this is not because of the state of the car itself (we can assume it is in working order) but because of a condition on the car, i.e., it is against a stone wall. Again, if a ship is sailing into a strong wind, we say that it is necessary for it to go at a speed of less than so-many miles per hours, even though it is (naturally) possible for it to go at a faster speed when it is not heading into the wind.

Certain Stoics made the distinction between modal statements which are true because of the nature of the thing which the statement is about, and modal statements which are true because of extrinsic conditions.[1] The first sort are those which I call natural modal statements, and the second sort are those which I call conditional modal statements. To consider these two sorts of modalities it is convenient to have them arranged in the traditional square of opposition; ~~besides, that they do fit the square of opposition is usually taken as a condition for terms being modal terms.~~ We can take

1. cf, William and Martha Kneale, The Development of Logic, (Oxford, Clarendon Press, 1962), pp. 117-128, Chpt 111, " 2. Megarian and Stoic Theories of Modality".

"It is (naturally) possible for x to (be) k" as:

(1) "x's nature is such that x is not prevented from being k"

The contradictory of "It is (naturally) possible for x to be k" is "It is (naturally) impossible for x to (be) k" And the contradictory of (1), which we take for impossibility, is:

(2) "x's nature is such that x is prevented from being k."

The contrary of "It is (naturally) impossible for x to k" is "It is (naturally) necessary for x to (be) k" The contrary of (2), which we take for necessity, is:

(3) "x's nature is such that x is prevented from being not k."

The contradictory of "It is (naturally) necessary for x to (be) k" The contradictory of (3), which we take for non-necessity, is:

(4) "x's nature is such that x is not prevented from being not k."

From the formulation of (1), (2), (3), and (4), we have straight off that (1) and (2) are contradictories, and (3) and (4) are contradictories. Also, (2) and (3) are contraries. We must now check whether (1) and (4) are subcontraries. For (1) and (4) not to be subcontraries it must be possible for their contradictories to both be true. But, as we saw, (2) is the contradictory of (1); and (3) is the contradictory of (4); and (2) and (4) are contraries. So (1) and (4) cannot both be false, thus they are subcontraries. We have yet to show that (3) entails (1); and that (2) entails (4). If (3) did not entail (1), then it must be possible for (3) and the contradictory of (1) to be true. But the contradictory of (1) is (2), which is the contrary of (3). So (3) cannot be true and (1) false, i.e., (3) entails (1). Likewise, if (2) did not entail (4), then it must be possible for (2) to be true and (4) false. But the contradictory of (4) is (3) which is the contrary of (2). So (2) cannot be true and (4) false, i.e., (2) entails (4). It should be added that these relations between (1), (2), (3), and (4) would meet any reasonable criterion of relevance. (Many logicians feel that for "p" to entail "q", or to be the contradictory, contrary, or subcontrary of "q", "p" and "q" must be relevant, in some way, in subject matter.)

For conditional modalities we start out by taking "It is possible for _x_ to (be) _k_" as:

(5) "There are no extrinsic conditions which prevent _x_ from being _k_."

For impossibility we have:

(6) "There are extrinsic conditions which prevent _x_ being _k_."

For necessity we have:

(7) "There are extrinsic conditions which prevent _x_ from not being _k_."

And for non-necessity we have:

(8) "There are no extrinsic conditions which prevent _x_ from not being _k_."

It should be evident that, as the conditional modalities are presented here, they are not used in ordinary language except in the cases of necessity and impossibility. For example, we do not say that it is possible for a broken-down car to be driven ten feet forward even if there is nothing blocking its way. Neither do we say of a person in the open air that it is possible for him to jump 100 feet in the air. What we do use in ordinary language are modalities which can be derived from (5) - (8) by substituting "extrinsic' or 'intrinsic' conditions" for "extrinsic conditions." (In the jargon I am using here, an intrinsic condition would be a facet of something's nature.) Making this substitution, and applying de Morgan"s law in the first and fourth cases, we get for possibility, impossibility, necessity, and non-necessity, respectively:

(9) "There are no extrinsic and no intrinsic conditions which prevent _x_ from being _k_."

(10) "There are extrinsic or intrinsic conditions which prevent _x_ from being _k_."

(11) "There are extrinsic or intrinsic conditions which prevent _x_ from not being _k_."

(12) "There are no extrinsic and no intrinsic conditions which prevent _x_ from not being _k_."

It is evident now why we can say that conditional necessity and impossibility are used in ordinary language, but that conditional possibility and non-necessity are not. This is because in (10) and (11) we say "extrinsic _or_ intrinsic conditions"; thus it is enough for establishing modal statements of these sorts that there are only extrinsic conditons which prevent _x_ from being _k_ or not _k_. But in (9) and (12) we say "no extrinsic _and_ no intrinsic conditions"; thus it is not enough for establishing modal statements of the form of (9) and (12) that there are only no extrinsic conditions which prevent _x_ from being _k_ or not _k_. Another result which is brought out in the above formulation and which accords with ordinary usage is that a statement of natural necessity or impossibility implies a statement of this hybrid form of necessity or impossibility, but not vice verse; and a statement of this hybrid form of possibility or non-necessity implies a statement of natural possibility or non-necessity. An example might bring out how this accords with ordinary usage. We might say on a very windy day of a plane that it is not possible for the plane to take off. But, then again, it is possible for the plane to take off, because aerodynamically it satisfies the right specifications. Also, we might say of a system comprising two bodies that it is necessary for the one to attract the other with a force equal to $G\frac{m_1 m_2}{r^2}$. Then no matter what the extrinsic conditions are, the two bodies attract each other accordingly.

From what has been said, it follows that conditional modalities, in relation to ordinary language, are abstractions. However, I have shown how a set of modalities concerned with physical objects and people, and yet distinct from natural modalities, can be analyzed as a synthesis of natural and conditional modalities. In particular, Ayers' "possibility for choice" is an instance of these hybrid modalities.

~~One question which was left unanswered in setting out the modalities above is: What does the variable "x" range over in formulations (1) - (12)? We could say that it ranges over things in a very wide sense, but this is not~~

~~very helpful, except that it rules out predicates, propositions, functions, etc. It certainly ranges over spatio-temporal congregations of matter---cars, planets, boats, etc.---what are usually known as physical objects. But it also ranges over spatio-temporal configurations of matter which would hardly be called physical objects. What I have in mind here are what are usually referred to as systems---two or more bodies which mutually attract or repel each other, two or more particles which collide, etc. We need not restrict the range of "x" to material things, though. We can include such things as waves and images, and even force fields. There is quite a bit of debate as to whether to include things like fields in the ontology of the physical sciences. But this is not very relevant here, for whatever ontology suffices for the physical science suffices for the expression of natural and conditional modalities. The important thing is that "x" ranges over spatio-temporal units which are convenient for empirical analysis.~~

A [redacted] problem which arises from the above discussion is the explication of the terms "nature", "intrinsic condition" and "extrinsic condition". Now any condition which is not an extrinsic condition is an intrinsic condition; and the sum of a thing's intrinsic conditions is its nature; so if we explicate the notion of nature, we shall be able to offer an explication for the other two notions as well.

To begin with a pragmatic consideration: I use the word "nature" because, ~~for one thing,~~ of its generality. For instance, we can say that it is necessary for a certain generator to produce an alternating current because of the way the generator is constructed, i.e., because of its nature. Again, we can say that it is necessary that two bodies are attracted towards each other in accordance with the equation $F = G\frac{m_1 m_2}{r^2}$, not because of their construction, but simply because of their nature, and here we leave "nature" unspecific.

There is some danger of this explication becoming circular. We must not say that the nature of a thing is what is necessarily or possibly true of it. What is the case is that it is the nature of the thing which <u>determines</u> what is necessarily or possibly true of it. However, this still could be circular in an epistemic sort of way: the nature of a thing determines what is necessarily or possibly true of it, and we find out the nature of a thing by finding out what is necessarily or possibily true of it. But the truth is that there are other ways of determining the nature of a thing, ways which do not rely directly on establishing certain modal statements about it. An obvious case is simply observing the construction of a thing. For example, we need not have driven a car 70 m.p.h., to be quite sure that it is (naturally) possible for it to go 70 m.p.h. We could simply open the bonnet and have a look around. Of course, we would not be completely sure that it is possible for the car to go 70 m.p.h., but then we would not be completely sure about its construction (i.e., nature) either. In fact, we could be sure that it is possible for *x* to *k*, and yet know next to nothing about *x*'s nature; but yet, that it is possible for *x* to *k* is true because of *x*'s nature.

Perhaps it sounds as if I am getting close to the Aristotelian notions of essence and accident in my division of conditions into intrinsic (the sum of which is the thing's nature) and extrinsic. Actually I am not. The nature of a thing may include what are accidents as well as what are essences. In fact, it may be misleading to say even that the nature of a thing includes its essence, for the nature of a thing is found by empirical investigation, and not by *a priori* considerations, except obliquely, in that in referring to a thing or type of thing we usually use a word or phrase which gives a partial description of it, and hence attribute to it certain properties, relations, etc. In addition, the nature of a thing changes in time.

A thing's nature exists whether we know about it or not; and we rarely know every facet of something's nature. But the intrinsic conditions which are taken as important for a certain modal statement about a thing are usually quite few in number, and often it is just one facet of a thing's nature which is relevant to a certain natural modal statement. It is this abstraction of certain facets of a thing's nature (i.e., certain intrinsic conditions) which allows us to form generalizations about an infinite or open set of things.

Intrinsic conditions are general. And it is this generality which makes natural modalities basically general in the sense that it is theoretically possible to derive all natural modal statements from universal natural modal statements.

On the other hand, natural modal statements are specific in a sense. This is most apparent with regard to universal natural modal statements, which, following the general form we have used for singular statements,, we can typify as: "It is (possible, impossible,necessary, or non-necessary) for all <u>x</u> such that $\emptyset(\underline{x})$ to be <u>k</u>." In this case the intrinsic condition which is important is ϕ, and it also indicates in which particular cases the modal statement applies - viz., whenever we have something which is ϕ. In this sense, then it is specific: The statement indicates all the factors which are involved in the truth or falsity of it. This is obviously also true in the case of many definite descriptions, although here we often have more specified than is necessary. For instance, we might have " It is necessary for the thing which is ϕ and ψ* to be k;" and perhaps that it is necessary for it to be k is due only to it being ϕ. The situation is somewhat different with regard to definite descriptions for which the intrinsic condition(s) on which the modality of the statement in which they occur depends is (are) not mentioned, and also for individual constants. Here the intrinsic conditions are not specified; but they are indicated in the modal statement in so far as the thing upon whose nature the modal statement depends is specified. In this sense, then, all natural modal statements are specific; they refer to the thing upon whose nature their truth depends.

In contrast to this, the extrinsic condition on which the truth of conditional modal statements depends is not referred to at all. When we say,, in the conditional use,

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* for "ψ" read the Greek letter "psi".

"It is possible for x to (be) k" we make no allusion at all to any extrinsic conditions other than the implication that there are no external conditions which prevent x from being k. Again, when we say, in the conditional use, "It is necessary for x to k", there is only a non-specific existential implication to the effect that there is some extrinsic condition which prevents x from not being k. Of course, as regards statements of conditional necessity and possibility, it is usually clear from the context what the particular external condition is which prevents x from being k or not being k. But the validity of these statements does not depend on any particular extrinsic condition existing. In fact, a person could utter one of these statements on the mistaken ground that such-and-such extrinsic conditions prevent x from being k or from not being k, and yet the conditional modal statement could be true because some other extrinsic condition prevents x from being k or from not being k. It is important here to distinguish between the grounds for a statement and what is actually asserted.

It is a result of this non-specificity of conditional modal statements that they are not general. This lack of generality can perhaps best be appreciated in an attempt to formulate a universal conditional modal statement.

This would be of the form: "It is (possible, impossible, necessary, or non-necessary) for any x such that $\emptyset(x)$ to be k." Or to rewrite it in terms of extrinsic conditions: " There are (no) extrinsic conditions which prevent any x such that $\emptyset(x)$ from (not) being k." A statement like this could quite easily be true of a finite number of things. All we would have to do to verify it would be to go through each of the things which are $\emptyset$ and determine whether there are (no) extrinsic conditions which prevent each of them from (not) being k. But the situation is quite different when those things which are $\emptyset$ are an infinite or open set. If there were in every possible instance of $\emptyset(x)$ some extrinsic condition which prevented x from (not) being k, the existence of the extrinsic conditions would be correlated with the common facet of the x's natures --- their being $\emptyset$; and these conditions would then not be extrinsic conditions, since they are connected with the things' natures. Likewise, if there were in every possible instance of $\emptyset(x)$ no extrinsic condition which prevented x from (not) being k, the lack of extrinsic conditions would then be connected with a facet of the thing's nature --- their being $\emptyset$. The point is, a natural modal statement gives us something, although perhaps only vaguely referred to, to generalize over --- a thing's nature or a particular facet of a thing's nature. But a conditional modal statement gives us nothing to generalize over; at the most conditional modal statements have completely non-specific existential implications.

The affinity between natural and conditional modalities, especially in subject matter, is apparent. And, ~~returning to~~ considering the generality of things which the variable "x" can range over in "It is naturally (possible, impossible, necessary, or non-necessary) for x to be k", this affinity is seen to be all the more close. It often happens that where a conditional modal statement is in order, the corresponding natural modal statement is also in order, but with this difference: the designator "x" is replaced by another expression which designates not only the thing designated by "x" but also some of the circumstances which are extrinsic to x. In this way, although, obviously, not all the circumstances extrinsic to x can be included, we can safely assimilate some conditional modal statements to natural modal statements (especially where the modality is necessity or impossibility). The increase in specificity accomplished by this assimilation makes possible the sort of generality which is characteristic of natural modal statements and completely lacking for conditional ones. And it is this drive for generality in empirical statements which is perhaps the most characteristic feature of modern science. But in non-scientific life we tend to have a generally accepted, though admittedly vague, domain of thing-hood. It is only to be expected that in this area conditional necessity statements are not usually assimilated to natural ones.

Ayers' second argument for a distinct set of modalities for human agents is that natural possibility statements are analyzable as conditional (not modal conditional) statements; whereas statements of "possibility for choice" are not.

Ayers' holds that "It is (naturally) possible for *x* to be *k*" can be analyzed as: "In some circumstances *x* would be *k*." But I hope to show that this is incorrect, and to make it correct would be to make it circular. The trouble with it is that even though it is possible for *x* to be *k*, there may be no circumstance in which *x* was *k*, is *k*, or will be *k*; that is, there may be no *actual* circumstances in which *x* is *k*. (That Ayers has to say "*x would* be *k*" ---i.e., in the subjunctive ---is an implicit admission of this.) To make this analysis correct, one would have to speak about a *possible* circumstance. Ayers' analysis thus becomes: "It is possible for *x* to *k*" is analyzed as "In some possible circumstances, *x* would be *k*." All this analysis accomplishes is to switch grammatically the modal term from modifying the clause " for *x* to be *k*" to modifying the noun "circumstances." It seems to me extremely doubtful whether it is a helpful analysis at all, for the possible circumstance which would satisfy *x* being *k* most directly is simply the circumstance in which *x* is *k*. Anyway, it obviously fails at giving anything like a definition of "(naturally) possible" since it is so patently circular.

This result certainly makes a hole in Ayers' argument. But we have yet to consider conditional statements in connection with the rest of the natural and all of the conditional modalities. In what follows, I shall restrict the discussion of conditionals to hypotheticals (i.e., statements of the form "If..., then..."). It is not difficult to see that natural necessity statements entail hypotheticals. I start with universal natural necessity statements, i.e., statements of the form: " It is necessary for all *x*'s such that Ø(*x*) to be *k*." This is true (when it is true) because of a facet of the *x*'s nature - viz., their being Ø. And it is not just that the things which are Ø which are *k*, but anything which would be Ø would also be *k*. Thus a universal natural necessity statement entails a subjunctive hypothetical of the form: "For anything at all, if it were Ø, then it would be *k*." A singular natural necessity statement refers to the thing upon whose nature its truth depends. Usually, if not always, it is just particular facets of its nature which are relevant; and if any other thing were like the thing mentioned in the necessity statement in the relevant respects, then it too would be _k_. So singular natural necessity statements entail subjunctive hypotheticals of the following form:

(13) "If something were like _x_ (in the appropriate respects), then it would be _k_."

For impossibility we have:

(14) "If something were like _x_ (in the appropriate respects), then it would not be _k_."

For possibility we have:

(15) "It is not the case that (if something were like _x_ [in the appropriate respects], then it would not be _k_)."

And for non-necessity we have:

(16) "It is not the case that (if something were like _x_ [in the appropriate respects], then it would be _k_)."

I intend to leave it an open question as to whether the natural modal statements of the forms "It is necessary for _x_ to be _k_" and "It is impossible for _x_ to be _k_" are entailed by any hypotheticals. "If something were like _x_ (in the appropriate respects), then it would (not) be _k_" entails "All things like x (in the appropriate respects) are (not) k." Something stronger which (13) [(14)] entails is: "All things like x (in the appropriate respects) are (not) k, and there is a reason for them (not) being k." This last statement entails: "x is (not) k, and there is a reason for it (not) being k." Whether this in turn entails (3) [(2)] I shall leave an open question here. If there is a reason for x (not) being k, in some sense, then, it is prevented from not being (being) k. But whether it is prevented from not being (being) k because of its nature is another question. "Nature", as I have presented it, is rather indeterminate in some respects; and its indeterminacy appears to be such that it bars a simple solution to the present problem.

Natural possibility and non-necessity statements, on the other hand, are obviously entailed by a host of hypotheticals. For example, "If something is like x (in the appropriate respects), then it is (not) k" entails "It is (naturally) possible for x to (not) be k." This is also entailed by "If x = y, then y is (not) k." But both of these also entail the non-modal statement " x is (not) k." It is obvious that any hypothetical which entails a non-modal statement also entails the corresponding possibility statement since, for natural modalities at least, we have "p" entails "Mp". The question is: Are there any hypotheticals which entail natural possibility statements but not the corresponding non-modal statements? The answer seems to be: no. As we have seen, a subjunctive hypothetical entails statements stronger than non-modal statements, and it is evident that indicative hypotheticals entail statements at least as strong as non-modal statements.* Apparently, then, the only sorts of hypotheticals which entail natural possibility statements and no non-modal statements are those which explicitly or implicitly make use of the notion of possibility. But the relation between the negation of subjunctive hypotheticals and natural possibility and non-necessity statements is another aspect of the same problem which we came up against in the discussion about the relation between subjunctive hypotheticals and natural necessity and impossibility statements. The negation of a subjunctive hypothetical does not entail a statement as strong as a simple non-modal statement; but whether it entails only statements as weak and weaker than a natural possibility statement is difficult to decide.

The relation between conditional modal statements and hypotheticals is significantly different. This is best seen with regard to conditional necessity statements. Here no reference is made to any sort of extrinsic condition; we just assert that there is some extrinsic condition(s) which prevent x from not being k. Thus

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*By '"p" is stronger than "q"' I mean that "p" entails all those statements entailed by "q", and more as well. "Weaker there is no particular facet of the situation of which we could say that if it held in another situation, than a situation which shares certain characteristics with the original situation would also hold.

This is just a result of the lack of generality of conditional necessity statements which was mentioned before. ~~mentioned.~~ It follows that conditional necessity statements do not entail hypotheticals, nor do they have the close relationship to them which was discussed earlier in connection with natural necessity statements --- that they might not be exactly entailed by hypotheticals, but they "almost" are. By similar lines of argument, one can arrive at the conclusions that conditional impossibility statements do not have these relations to hypotheticals, nor do conditional possibility and non-necessity statements have these relations to the negations of hypotheticals. It is true, however, that conditional possibility statements are entailed by some hypotheticals. But this is simply because a conditional possibility statement, like most other possibility statements, is entailed by the corresponding non-modal statement. To conclude: it is this close affinity between natural modal statements and hypotheticals and the lack of it with conditional modal statements which is perhaps the most important difference between the natural and con-

than" is the converse relation.

ditional uses of "necessary" and "possible".

We have seen that Ayers' attempt to differentiate natural possibility and "possibility for choice" on the grounds that only a natural possibility statement can be analyzed as a statement of the form "In some circumstances, _x_ would be _k_" fails because a statement of natural possibility cannot be so analyzed without becoming circular. We have seen, however, that natural modalities are related to hypotheticals in a way in which conditional ones are not. And, to my mind ~~however~~, any attempt to distinguish uses of "possible" and "necessary" within the conditional uses (or, rather, within the hybrid uses conbining natural and conditional uses as described above) into those which apply to physical objects and those which apply to people _qua_ agents by appeal to "bogus conditionals" ("If he wishes, he will...", and "If he tries, he will...," etc.) would be unsuccessful.

The peculiarity of these hypotheticals is due to what might be called the bogus action verbs ("wish", "try", etc.), and not to the possibility statement which entails them. Ryle, in _The Concept of Mind_[1](especially chapter 1, sections (1), (2), and (3); and chapter V, sections (1), (2), (3), (4) and (5)) argues that certain

1. Gilbert Ryle, _The Concept of Mind_, 1949.

verbs, especially verbs which purportedly refer to mental actions, are not action verbs at all. They are better considered as such things as dispositionals or adverbs. The problems connected with these verbs are obviously not restricted to their occurrences in hypotheticals. And obviously these verbs are applicable only to human agents (and in some circumstances to animals). Thus that there are certain possibility statements which are analyzable into statements containing these terms shows a similarity in subject-matter (they are about people) and not necessarily of logic, which we are concerned with here. The crucial point is that these bogus action verbs allow us to form bogus hypotheticals from a sort of possibility statement which does not entail "proper" hypothetical statements or their negations.

The hybrid use of modal terms discussed before probably occurs much more commonly with regard to people than the natural uses. This is understandable, since, for one thing, we have difficulty in finding characteristics of people which we would attribute to something as hard and fast as a nature. This is especially true for general statements about all people. This can be seen in the rather ethereal aspect of discussions about human nature. But we also find the same difficulty with regard to individuals or sub-groups of the human race.

Another point which makes the hybrid uses of modal terms more useful with regard to people than the natural ones is that we usually consider people as agents, and if the extrinsic circumstances are such that it is necessary, impossible, possible, or non-necessary for an agent to do or be something, this often is an important factor in our appraisal of the agent. This is particularly evident in moral or legal areas.

Section F: Logical Uses.

The fourth use of modal terms which I shall consider is logical and mathematical necessity, and logical and mathematical possibility. For brevity I shall simply refer to these as logical necessity and possibility. I shall first consider the distinctions between logical and epistemic modalities. To begin with, although in both uses the term "possible" is employed, the correlative term differs between the two cases: for logical modal uses it is "necessary", but for epistemic modal uses it is "certain". However, this still does not show that "necessary" has a logical use distinct from "certain".

Perhaps the most important and obvious distinction between the logical use of "necessary" and "certain" is that "It is certain that p" implies "It is known that p"; whereas "It is (logically) necessary that p" does not.*

*See footnote on page I5

For the sense noted before, certainty about p implies knowing that p. It is also quite obvious that something can be logically necessary without being known. For example, if Fermat's last theorem is true, then it is logically necessary; but it is not known whether Fermat's last theorem is true or false.

It is a consequence of what was said in the last paragraph that "It is certain that p" does not imply "It is (logically) necessary that p" nor vice versa. But we can go even further than this in that we can cite whole families of sentences which are logically necessary, but not certain, or are certain but not logically necessary. Now there is an infinite number of logically necessary sentences which have not been proved. One example was given in the last paragraph. And all but a finite number of these have not even been formulated. These are all examples of sentences which are logically necessary but are not certain. Again, we are certain about some ~~se~~ empirical ~~s~~ statements, but these, it is evident, are not logically necessary.

It remains to show that the logical use of "necessary" and "possible" is different from their natural and conditional uses. This can be done by considering the substitution of individual constants in modal contexts. For natural necessity the following is a valid inference schema:

$$(1).\quad \frac{\begin{array}{c} L\ \emptyset(a) \\ a = b \end{array}}{L\ \emptyset(b)}$$

where "L" is "It is necessary that...", " $\emptyset$ " is a predicate, and "a" and "b" are individual constants. For example, from "It is necessary that the morning star travels in an elliptical orbit about the sun" and "The morning star is the evening star" one can infer "It is necessary that the evening star travels in an elliptical orbit about the sun." It can be stated that (1) is also a valid inference schema for conditional necessity. For example, from "It is necessary that my car did not start (because the battery was disconnected)" and "My car is the car parked across the street" one can infer "It is necessary that the car parked across the street did not start."

But (1) is not a valid inference schema for logical necessity. For example, from "It is necessary that nine is greater than eight" and "Nine is the number of planets" one obviously cannot infer "It is necessary that the number of planets is greater than eight", since this is false. The nearest thing to (1) which is valid for logical necessity is:

$$(2).\quad \frac{\begin{array}{c} L\ \emptyset(a) \\ L\ (a=b) \end{array}}{L\ \emptyset(b)}$$

An instance of (2) is: "It is necessary that nine is greater than eight" and "It is necessary that three squared equals nine"; therefore, "It is necessary that three squared is greater than eight."

<u>Section G</u>: <u>Imperative Uses</u>.

The fifth, and last, use of modal terms which I shall consider is what I call the "imperative use". I call it this because of the close connection which it has with imperatives, which will come out later on in this section.

It is often clumsy to phrase a modal sentence of this type in terms of "necessary". This is sometimes the case with the other uses of "necessary" which I have considered; hence, the tendency to formulate statements of the form "<u>x</u> must <u>k</u>", rather than of the form "It is necessary for <u>x</u> to <u>k</u>". This tendency is even greater in connection with the imperative use of "necessary". So the examples of this use which I shall now give are first formulated in terms of "must":

(1) "<u>x</u> must hand in the paper by Tuesday."

(2) "<u>x</u> must finish his meal."

(3) "In Britain one must not drive over 30 m.p.h. in a built up zone."

(4) "In Britain one must pay a certain amount of his income to the government in tax."

These statements can be rephrased in terms of "necessary" which makes for a much more clumsy formulation. But "necessary" is the term in which we are basically interested. Rephrased, the statements become:

(1') "It is necessary for x to hand in the paper by Tuesday."

(2') "It is necessary for x to finish his meal."

(3') "In Britain it is necessary not to drive over 30 m.p.h. in a built-up zone."

(4') "In Britain it is necessary to pay a certain amount of one's income to the government in tax."

An important distinguishing feature of this use of "necessary" is the relation between imperative necessity statements and commands uttered with authority. Suppose a teacher says to a pupil: "Hand in the paper by Tuesday!" This entails (1') because the teacher is in a position of authority over the pupil in academic matters. Likewise, (2') is entailed by a parent saying to his five-year-old child: "Finish your meal!" In general, an imperative addressed to a person(s) by a person(s) who has authority over that person(s) in the subject matter of which that imperative is an instance entails an imperative necessity statement with a reference to the person(s) to whom the imperative is addressed and containing a subordinate phrase which is a grammatical adaptation of the imperative.

However some imperatives are not addressed to particular people, but to anyone who satisfies a certain condition. For example, we can imagine a host at a party standing on a chair and shouting: "Don't put your glasses on the piano!" In this case, although the imperative was addressed to a particular group of people (the people in the room), it applies to anyone who meets a particular condition (being in the room now or later). This suggests that the necessity statement entailed by an imperative should not be characterized as containing a reference to the person(s) to which the imperative is addressed, but a condition which determines whether the statement of necessity applies to any person or does not apply to that person.

An analogous relation exists between imperative necessity statements and laws which are issued by people with authority in the subject-matter of which the imperative is an instance.* For example, that Parliament has passed a law forbidding people to drive over 30 m.p.h. in built-up zones entails (3'). And that Parliament has passed a law that everyone is to pay a certain amount of his income to the government as tax entails (4'). Further considerations on the relationship between laws and imperative necessity statements are exactly parallel to those discussed in the

*I would add that whether something could be a law, and yet not be issued with authority seems doubtful. I shall not discuss this problem for it is not relevant to my essay.

last two paragraphs in connection with the relationship between imperatives and imperative necessity statements. There may be other things besides laws and imperatives issued with authority which entail imperative necessity statements because of their dependence on some authority. It should be obvious from the discussion what conditions these further things must meet. If we want a general name for them, perhaps we could call them commands. Imperatives and laws issued with authority would then be instances of commands.

~~The peculiarity of the use of "entailment" in the preceding paragraphs is such that the word needs to be explained. The notion of entailment, as it is usually used, involves the notion of the preservation of truth: If the premises are true, then the conclusion must be true. But commands are never true (nor are they false), so how could a command entail anything? Perhaps it would be best to introduce a looser notion than entailment; one could call it the relation of "basing on"* If "p" is based on "q", then if "q" is uttered, then, (logically) necessarily, "p" may be uttered as well. Asserting is an instance of uttering, and entailing is an instance of basing upon.** If we have the rule that if "q" is not only uttered, but asserted, then, necessarily, "p" may be not only uttered, but asserted, then we have the rule that~~

~~*I am using "basing on" in an artificial sense. Usually, '"q" is based on "p"' means something like '"p" is a ground for~~ ~~(continued p.56~~

~~"p" entails "q".~~

~~I will take "imperative necessity" in such a way that for any imperative necessity statement, there is a command based on it. This will simplify the discussion in this essay, especially when I come to consider grounds. As mentioned on page 13, imperative uses are artificially distinguished from certain other uses of modal terms. The source of this artificial distinction is the limiting of imperative necessity statements to only those statements for which there is a command which is based on them. This is artificial in that it uses a ground (that there is a command to the effect that...) in determining what is purportedly a distinct use. For now we have both that a command is based on a necessity statement and a necessity statement is based on a ground. If we defined a looser notion of equivalence which is related to the usual notion in the same way that "basing on " is related to "entailing", then we have the result that a command and an imperative necessity statement are equivalent. Thus it is seen that "necessary" in its imperative use comprises just all those occurrences of "necessary" for which the ground could be that there is a command such that....~~

One distinction between the imperative use of "necessary" and the other uses which have been covered is that "Lp" does not entail "p".* It might be necessary

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~~*(See footnote on page 56)~~

in Britain to pay a certain amount of one's income to the government as tax. Yet there may be people (tax evaders) who do not pay the specified amount. Also, it may be necessary for me to hand my paper in by Tuesday, yet I might be lazy and hand it in (perhaps in vain) on Wednesday.

Another point which distinguishes this use of "necessary" from the other uses mentioned is that its correlative term is "permissible". For example, if it is not necessary not to drive more than 30 m.p.h. on the motorway, then it is permissible to drive more than 30 m.p.h. on the motorway. And if it is not necessary not to finish the meal in more than thirty minutes, then it is permissible to finish the meal in more than thirty minutes.

But both of these features which distinguish imperative necessity from natural, conditional, and logical necessity are also features of occurrences of "necessary" which have not got as grounds that there is a command such that.... For example, that it is necessary not to kill a person does not entail that someone has never killed another person. And it does entail that it is not permissible to kill a person. However, only a theist could point to an authority over all people who commanded: " Do not kill!" In moral areas we can find many such examples of necessity or permissibility statements. And to hold that they are all based on commands, which involve an authority, would require an extreme form of theism, or something similar. To say that all uses of modal terms of this sort are imperative uses would be begging a philosophical point. However, the imperative use was chosen as representative of this wider use.

(from p.53)

"q"*. However, the notion I wish to introduce here is a <u>logical</u> notion. It is a looser notion than entailment only in so far as it applies to cases in which one or both of the grammatically complete sequences of words so related are neither true nor false. Of course, if "p" is based on "q", then "q" is a ground for "p"; but the converse is not true. For the remainder of this section I shall use "basing on" in the artificial sense defined here.

**Only statements which are true or false can be assented; in particular, commands are not assented, but only uttered. And entailment is a relation only between statements which are true or false.

(from page 54)

*"p" in this case is a statement which is true or false. A command (which need not contain a modal term) is based on an imperative necessary statement, but a command is not true or false; so a command is not <u>entailed</u> by an imperative necessity statement.

CHAPTER TWO

GROUNDS FOR MODAL TERMS

Section A: Grounds Afforded by Entailment.

In what follows, I shall present various different sorts of grounds for applying modal terms.* In each case the ground will be presented as a determination of whether a non-modal statement (or, rather, what the non-modal sentence expresses) is necessary, impossible, possible, or non-necessary. For uses of modal terms which are usually followed by "that" (epistemic and logical uses), this construal of a ground raises no problems since the clause following "that" is in the grammatical form of a sentence.

But many of the uses of modal terms which I have considered (natural, conditional, and imperative uses) are usually followed by "for", and "for" is generally followed by a clause which is not in the grammatical form of a sentence. However, it was said that the following rules of inference are valid for natural and conditional uses: "Lp" entails "p"; and "p" entails "Mp". This suggests that the grammatical form of "p" within the scope of a natural or conditional modal term is irrelevant to our purposes here. Also, if

* It is important to bear in mind that I am for the most part discussing sorts of grounds, not particular grounds.

we consider imperative modal statements, it is the action or state of a person which is necessary, impermissible, permissible, or non-necessary. And this person's action or state can be expressed by a sentence; and we can say that it is this state of affairs which is described by the sentence whose non-occurrence is ruled out, occurrence is ruled out, occurrence is not ruled out, or non-occurrence is ruled out by an imperative modality statement. In general, then, for these three uses the grounds for them are determinations of the necessity, impossibility (impermissibility), possibility (permissibility), or non-necessity of a non-modal statement. The non-modal statement will be of the form "x k's", but the corresponding modal statement will be of the form "Y(for an x to k)", where "Y" is one of these modalities. For our purposes here, we can consider it a grammatical peculiarity of modal terms of these three uses that they are followed by a clause introduced by "for".*

In the discussion of each particular sort of ground, I shall indicate for which logical uses that sort of ground is appropriate and whether it is a strict ground (i.e.,

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~~* Onemight be able to reformulate the medieval distinction between modalities de dicto and de re in terms of the distinction between use of modal terms followed by "for" and those followed by "that". But I shall ignor this distinction for the remainder of this chapter.~~

entails the modal statement) or not.

The first sort of ground which I shall mention is what I call "the grounds afforded by entailment." In presenting this ground one must cover each of the four modalities separately. Given that "p" entails "q", and "Lp", one has a ground for "Lq"; given that "p" entails "q" and "L -q", one has a ground for "L -p"; given that "p" entails "q" and "Mp", one has a ground for "Mq"; and given that "p" entails "q" and "M -q", one has a ground for "M -p".

The grounds afforded by entailment compose a strict sort of ground since all of the four divisions of it are adaptions of logically valid inference schemata. Also, this sort of ground is appropriate for all five uses of modal terms which I have discussed except for the epistemic use. But in regard to imperative uses we must replace "M" with "P" (for "permissible") in two of the above formulations. ~~And in regard to epistemic uses we must replace "L" with "C" (for "certain") in the other two formulations~~. Entailment is a logical relation. If "p" entails "q", then it is because of the meanings of "p" and "q" that this relation holds. This is the reason that the grounds afforded by entailment are so important for establishing statements of logical uses of modal terms, and for other uses are often almost incidental. For the remainder of the discussion I will be concerned with the grounds afforded by entailement only in so far as they apply to logical uses. It is convenient to narrow the scope further, and consider these grounds in connection with only logical necessity since logically necessary statements, and not logically possible ones, are what one usually wishes to estblish in logic and mathematics.

In logic and mathematics the notion of deduction is very important. But we cannot assimilate deduction to entailment without qualification. The difficulty lies with deductive systems in which theorems are derived by means of rules of inference (and not entailment) before the axioms are interpreted. But in systems of this type the rules are normally such that if a theorem is derivable from the other theorems and/or axioms, then, when the system is given its normal interpretation, the theorems and/or axioms from which the theorem is derived entail it. Now only interpreted formulae are necessary; so, for our purposes here, we can treat the deduction relationship between the axioms and theorems of this type of system as entailment relationships.

We can safely state, with this result, the restricted ground afforded by entailment, with which we are presently concerned, in this way. In a deductive system with logically necessary premises, that "p" is deducible in this system is a strict ground for "Np", where "N" is the logical use of "necessary". This ground could be made more general by omitting the qualification "logically" and adding "or certain" in parentheses. However, it is only with regard to logical necessity that such a ground would be of particular interest at all.

The question arises: What is meant by "deductive systems"? Cetainly it includes formalized systems with a specified set of axioms or axiom schemata, and specified rules of inference. But, as I use it here, it will include systems which do not have a specified set of axioms or axiom schemata,and/or do not have specified rules of interence. The only requirements are that certain of the statements are known to be necessary not simply because they can be derived from the others, and that the others have been derived by means of entailment from the former. We could call this first group of statements the premises of the system---this is why I used the word "premises" rather than "axioms" in the formulation of the restricted ground afforded by entailment.

Typifying "deductive systems" as I do is admittedly extending its reference further than is normal. Such things as informally developed branches of mathematics are included under this notion of deductive system. But the restricted ground given two paragraphs ago is valid for all these systems.

Section B: Grounds Afforded by Meaning.

The second sort of ground which I shall consider are the grounds afforded by meaning. Sometimes the truth or falsity of a statement can be determined by considering only the meanings of its constituent terms and without reference to extra-linguistic facts. In such cases we might say that the truth or falsity of these statements is "independent of the world." So there is no (logically) possible circumstance which would falsify a true statement of this type, nor is there a (logically) possible circumstance which would verify a false statement of this type. But if the meaning of a statement is such that there could not be a circumstance described by its negation or by it, then that statement is logically necessary or logically impossible.

The formulations of this sort of ground for the four different modalities are as follows. Given that "p" has been shown to be true by virtue of the meaning of its constituent terms, we have a ground for "Lp". Given that "p" has been shown to be false by virtue of the meanings of its constituent terms, we have a ground for "L -p". Given that "p" has been shown not to-be-false-by-virtue-of-the-meanings-of-its-constituent-terms, we have a ground for "Mp". And given that "p" has been shown not to-be-true-by-virtue-of-the-meanings-of-its-constituent-terms, we have a ground for "M -p". We should add the restriction that "p" is well-formed to the formulation of the sorts of grounds for possibility and non-necessity since "p" might not be true or false because it is not well-formed and hence meaningless. For the remainder of this essay this restriction will be assumed.

It should be obvious that the grounds afforded by meaning are applicable only for logical modalities. Also this sort of ground is a strict ground. It could not be the case that something is true by virtue of the meanings of its constituent terms and yet not be logically necessary. After all, "true by virtue of it constituent terms" is one way of explaining logically necessity. This comment can be adapted to the other formulations of the sort of ground afforded by meaning with regard to the other modalities.

Here one must be careful not to conflate a sort of ground for a use of a modal term and the explanation we give of that use itself. In particular, we must distinguish the sort of ground afforded by meaning for the application of logical uses of modal terms and an explanation which can be given of the logical uses of modal terms. This explanation of the logical use of "necessary" is specific in that the clause, which is grammatically of the form of a sentence, which follows "It is necessary that..." is true because of its meaning. Of course, if it is true because of its meaning, it is also true because of the meanings of its constituent terms. Similar comments can be made about the other three modalities.

There is a difference between something being true and _showing_ that something is true. _A fortiori_, there is a difference between a statement being true for such-and-such reasons, and showing by direct appeal to those reasons that it is true for those reasons. We have already an example of a sort of ground for showing that a statement is true for a certain reason which is not a direct recourse to that reason. This is that we have a ground for "Lp" (where "L" is the logical use of "necessary")-that is, that "p" is true because of the meanings of its constituent terms- given that "p" is derivable in a deductive system with logically necessary premises.

In the case of the grounds afforded by meaning, the reason for "Lp" being true, as well as "p", is the same as the ground for "Lp", with the exception that when we use this reason as a ground it must be _shown_ that the reason does in fact hold. For the grounds afforded by entailment, on the other hand, we are interested in the entailment relationships between statements and not directly in the meaning of the statement itself. But this ground is dependent on the meanings of the statements in so far as a relation of entailment holds, if it does hold, because of the meanings of the statements so related.

Now there are statements for which the grounds afforded by meaning are straightforwardly applicable and for which the grounds afforded by entailment are, if not inapplicable, at least superflous. "All brown cows are brown" can quite easily be seen to be true by virtue of the meanings of its terms. But what sort of logically necessary premises would entail this? Obviously quite a few, but it is doubtful whether any of them would be more certain than "All brown cows are brown." In other words, it would be pointless to use the grounds afforded by entailment to establish "It is necessary that all brown cows are brown." In general, in ordinary discourse, or in any area which is not deductively systematized (in the loose sense which I am using here) the grounds afforded by meaning are the most practical grounds for establishing the (logical) modality of a statement.

Logical statements (or statement schemata) often present no practical problems to the employment of the grounds afforded by meaning. In fact, we would have good reason to suspect that a person did not understand it very well, if he failed to recognize the necessary character of an uncomplicated logical statement uttered in that language.

For more complicated logical statements, the employment of the grounds, afforded by meaning, appears to be rather impractical. The difficulty is apparently overcome in some cases (especially where there is a decision procedure) by explicating the pre-scientific (as Carnap would call it) notion of true by virtue of the meanings of its constituent terms. Such an explication involves the explication of a number of other terms and the formulation of rules of truth, rules of ranges, etc., and ends up with "semantically valid" replacing "true by virtue of the meanings of its constituent terms." But it is not obvious at all that all formalized semantics which have been used in connection with interpreting axiomatic systems are based soley on explications of pre-scientific notions connected with meaning.

The grounds afforded by meaning are in general impractical to employ in connection with statements which both occur as theorems in deductive systems and are rarely used in ordinary discourse. Exceptions to this are the cases where the statement occurs in a system for which there is a formalized semantics; and these are only exceptions if one assumes that determining the validity or non-validity of a statement or its negation by means of a particular semantics is an extension of the sort of ground afforded by meaning.

The difficulty of applying the grounds afforded by meaning to statements which are theorems in a deductive system but are rarely used in ordinary discourse is perhaps best appreciated by considering mathematical examples. Now the modality of certain elementary statements in mathematics, e.g. " 2 + 2 = 4 " or " 1 + 2 = 5 ", is apparently determinable by means of the grounds afforded by meaning. We can also apparently determine the modalities of certain statements in more advanced fields of mathematics. But this is generally due to the importance of definition in mathematics since a definition equates the meanings of two symbols or groups of symbols. For example, given the definition "$e^{a+b\underline{i}} =_{df} \cos a + \underline{i} \ \sin \ b$" and knowing that $\cos \frac{pi}{2} = 0$ and $\sin 0=1$, we might say that we have a ground afforded by meaning for "$L(e\frac{pi}{2} = \underline{i})$."

But there are many theorems in mathematics which, even given all the definitions, apparently cannot be shown to be necessary by considering just the meanings of their constituent terms. We can take this as a consequence of, for one thing, the impossibility of formulating a semantics which validates all mathematical formulae and none of their negations.

At this point I shall introduce a notion which is connected with, although contrasted with, the notion of a ground being a strict ground for the application of a term. This notion is that of the reliability of a ground for determining whether or not a term applies. This is not an absolute term in that we do not say that a particular sort of ground is either reliable or not. Rather, there are degrees of reliability. Now one ground is more reliable than another ground for ddtermining the application of a particular term if, when using the first sort of ground to determine whether the term applies, we have better reason to believe that the term does in fact apply in those cases than we have when using the second sort of ground. It is seen, then, that "reliable" is being used here as it is commonly used.

An obvious difference between "reliable" and "strict", as used here, is that a ground employed in a particular occasion is either strict or not; whereas, the reliability of a ground is a matter of degree. Again, that a ground is strict does not involve that it is very reliable. We shall see later that there are strict grounds which are quite unreliable in some cases, and non-strict grounds which are quite reliable in some cases. The reason for this is that a particular strict ground entails the statement in which the term is applied; but sometimes it is difficult to establish the statement which is being used as a ground in the first place.

Now the sort of ground afforded by meaning for logical uses of modal terms varies quite a bit in its reliability. This sort of ground is most reliable in the case where there is a formalized semantics by which one can determine the validity or non-validity of a statement or its negation.

(But the determination of this may not even be classifiable with the sort of ground afforded by meaning in the case of some formalized semantics.) Yet the sort of ground afforded by meaning is nearly as reliable, if not as reliable, for many simple statements for which the above method is not reliable. But with more complicated statements, as has been noted, this sort of ground in its "pre-scientific" form is less reliable. In fact, with many mathematical statements this ground is very unreliable. And there is good reason to suspect that this sort of ground is of no use at all with regard to many non-elementary mathematical statements.

The reliability of the sort of ground for the application of "possible" and "necessary" afforded by meaning can be contrasted with the reliability of the sort of ground afforded by entailment. Now if "p" is said to be necessary on the ground that it is entailed by "q", and "q" is necessary, then there are two reasons why the necessity of <u>p</u> might be doubted: (a) "q" might not be necessary, and (b) it might be doubted that "q" in fact entails "p". So we can see that the ground afforded by entailment might not be very reliable in some cases. However, the reliability of this ground can be improved immensely by relying on premises for which there is little reason to question their necessity and by having rigorous criteria for entailment. This is in fact what is done in logic and mathematics where we have either formalized systems or a very rigorous proof procedure.

Section C: Grounds Afforded by Imagining.

The third sort of ground for the application of modal terms that I shall consider is the sort of ground afforded by imagining. The determination of the truth or falsity of a modal sentence by appealing to what can and cannot be imagined played a much larger role in early modern philosophy of logic than it has in twentieth-century philosophy of logic.

One reason for this change is that the imprecision of the word "imagine" has become more appreciated. "To imagine" can be used as a synonym for "to assume" and for "to believe". But these senses are obviously not what we want for a notion affording a sort of ground for the application of modal terms. What we want is more on the order of the sense of "to imagine" where it is taken as something like "to form a mental picture of". But to construe imagining as just this will not do.

For one thing, another reason why twentieth-century philosophy of logic has tended to ignore the concept of imagining is that this concept has very close connection with psychologism which is particularly obvious when " to imagine" is construed as "to form a mental picture of". One thing which makes psychologism particularly repugnant for the philosophy of logic is that what is thought, imagined, etc., varies from person to person. In particular, what one person imagines when considering a particular sentence is quite likely different from what another person imagines when considering the same sentence. For example, when X hears the statement "The sea is blue" he may form a mental picture of the last time he flew over the sea and looked down; when Y hears the same statement, on the other hand, he may form a mental picture of the last time he was looking over the side of a ship. Thus it can be seen that there is no one-to-one correlation between a statement and what is imagined in connection with it as there is between a statement and (ambiguities and multiplicity of uses aside), its meaning.

One way out of this difficulty is to associate with the statement "p" the class of all those states-of-affairs which one could imagine and would be described by "p", in an analogous fashion to the association of a meaning with a statement. In the case of statement variables, a class of the classes formed as above for each instance of the variable would be associated with the variable.

This liberates "to imagine" to some extent from "to form a mental picture of" since, for example, we could imagine in this sense that all A's are B's---a state of affairs---where the number of A's is infinite; but we could not form a mental picture which would contain some sort of representation of all A's. However, even widening what can be imagined to include all possible states of affairs still leaves problematic cases. Perhaps the most noticeable are mathematical statements. What states-of-affairs would be described by " $2 + 2 = 4$ "? Perhaps we could explicate "two" as "the class of all imaginable pair-classes" and give an analogous explication of "four". In this case we would have the states-of-affairs which are imaginable and are described by " $2 + 2 = 4$" as those states-of-affairs in which the union of two pair-classes is a class containing four members. I present this as an example of how some of the problems might be solved when one applies an explication of "to imagine" to particular sorts of statements. I do not intend to present a completely sufficient explication of the notion, much less solve the problems which arise in its application.

There is one more problem in connection with the formulation given two paragraphs ago which is worth mentioning. This formulation in itself is still open to the objection that different people can imagine different states-of-affairs.

But perhaps it does furnish a way out of this difficulty. We might be able to give an objective explication for the notion of imagining. This might be done by taking that which is imaginable as what is imaginable by anyone and by producing a criterion for determining when an imaginable state of affairs is described (in a very wide sense) by a statement.*

Assuming that an adequate explication of "to imagine" has been given, and that a wide enough sense of "a statement describes a state-of-affairs" has been found, we could state the sorts of ground afforded by imagining for each of the four modalities as follows: given that there are no imaginable states-of-affairs described by "-p" (and "p" is well-formed), we have a ground for "Lp". Given that there are no imaginable states-of-affairs described by "p", we have a ground for "L -p". Given that there is at least one imaginable state-of-affairs described by "p", we have a ground for "Mp". And given that there is at least one imaginable state-of-affairs described by "-p", we have a ground for "M -p".

~~* The notion of meaning is not free from subjective factors either. Often we speak about one person meaning one thing by a statement, and another person meaning something else by the same statement. Quine (esp., Word and Object, chapters 1, 11, and V1) presents more philosophical objections to the objectivity of the notion of meaning.~~

These formulations make no reference to extra-linguistic facts, other than the reference to what is imaginable. But one of the criteria for an adequate application of " to imagine" is that it affords a way of determining what is imaginable by considering a particular statement and nothing else. It is apparent, then, that imagining affords grounds for the application of only logical uses of modal terms.

~~That we want a notion of imagining which makes reference to only one statement in determining what is imaginable shows that the grounds afforded by imagining are less reliable in relation to the grounds afforded by entailment for the same reasons that the grounds afforded by meaning are less reliable. For the grounds afforded by meaning are employed in connection with individual statements; whereas those afforded by entailment are always employed in connection with more than one statement. The consequences of this were discussed in the last section.~~

Also, it seems apparent that the grounds afforded by imagining are in general less reliable than those afforded by meaning. ~~I mentioned some of the problems involved with the unexplicated notion of imagining. Although the unexplicated notion of meaning does have its problems as well, they are not nearly as crucial as those encountered with imagining. So generally the unexplicated notion of imagining affords less reliable grounds than the unexplicated~~

~~notion of meaning. When "meaning" is explicated so as to furnish a basis for a formalized semantics, it affords very reliable grounds. But it seems doubtful that "imagining" could be explicated so as to furnish a basis for an analogous formalized discipline. It is conceivable, though, that an explication for "imagining" could be given which would afford grounds just as reliable as those afforded by meaning for statements which commonly occur in ordinary language and are of little interest as theorems in deductive systems. The problem, of course, is how such an explication could be given.~~

Nevertheless, the grounds afforded by imagining are strict grounds for logical possibility and non-necessity. For we cannot imagine what is logically impossible. We might _think_ we imagine something which is logically impossible, but then we are simply wrong. That we might think we imagine something which is logically impossible shows that this sort of ground is often relatively unreliable, not that it is not strict with regard to the logical uses of "possible" and "non-necessary". But this sort of ground is not strict with regard to the logical uses of "necessary" and "impossible" since we cannot imagine every state-of-affairs, and it does not even appear to be true that every possible state of affairs is imaginable.

Section D: Empirical Evidence as a Ground for Modal Terms.

A fourth sort of ground for applying modal terms is empirical evidence of almost any type.. For each of the four modalities I offer the following formulations. Given that there is empirical evidence that we should rule out "-p", we have a ground for "Lp". Given that there is empirical evidence that we should rule out "p", we have a ground for "L.-p". Given that there is empirical evidence that we should not rule out "p", we have a ground for "Mp". And given that there is empirical evidence that we should not rule out " -p", we have a ground for "M -p".

Empirical evidence obviously affords grounds for applying natural and conditional uses of modal terms since these uses occur only in statements which are empirical in subject matter. But it does not afford grounds for logical uses of modal terms, with one exception, since the actual state of the physical world is irrelevant to the truth or falsity of statements which contain occurrences of these uses. Nor does empirical evidence afford grounds for imperative uses, except in two oblique ways: (1), Commands, it might be said, are sometimes discovered by empirical investigation, e.g., when one discovers some obscure law which was passed in the last century, (2) If we have "L -p", where "L" is the natural or conditional use of "necessary", then, where "L" and "P" are imperative uses, "Lp", "L -p", "Pp" and "P -p" are rendered, if not meaningless, then superfluous.

It is evident that this sort of ground is generally not a strict ground. No matter how many observations we make, we can never arrive at a class of statements which entails a statement of natural or conditional necessity. The situation is generally the same with regard to natural and conditional possibility statements, as one would expect from the definability of (natural and conditional) "necessity" and "possibility". There is one exception however. If we actually observe the state-of-affairs described by "p", then we have a strict ground for "Mp" for natural, conditional, logical, and epistemic uses of "possible". That this is a strict ground for all these uses follows from the rule that "p" entails "Mp" for these uses of "possible". This ground, by the way, is the exception referred to in the last paragraph to the generalization that empirical evidence does not afford grounds for logical uses of modal terms.

Section E: Grounds for Epistemic Uses.

All of the sorts of grounds mentioned so far for the application of "possible" are also grounds for the application of the epistemic use of "possible" since this use is relative to evidence, and a ground for something being possible is also evidence that it is not necessarily false. For the same reason empirical evidence is generally a strict ground for the epistemic use of "possible".

But not all the grounds mentioned above for applying "necessary" are also grounds for applying "certain". Only those grounds which for "necessary" are both strict and reliable are grounds for "certain". "It is possible that...", where this is the epistemic use of "possible", is equivalent to "It is not certain that...not...". We have another sort of ground for the epistemic use of "possible". Given that there are no strict or reliable ground for "-p", we have a ground for "Mp".

Of course, from the fact that "certain" is equivalent to no use of "necessary", it follows that there are grounds for applying "certain" which are grounds for no use of "necessary".

There is an interesting result which follows from the point that for something to be a ground for "certain" it must be reliable. We have seen that "reliable" is a relative term---grounds are more or less reliable, not either reliable or not reliable. From these two points we can conclude that the grounds for applying "certain" are strict in relatively few cases in connection with necessity statements---only where we might say that the ground for the necessity statement was "completely reliable". And these cases are restricted to logical and imperative necessity statements since these are the only uses of "necessary" for which there are strict grounds.

Section F: Grounds for Imperative Uses.

For imperative uses of modal terms we have the sort of ground afforded by command.* For the four modalities this sort of ground can be construed as follows. Given that there is a command which expresses the same thing as "p" (itself a command; hence neither true nor false), we have a ground for "Lp" (with the appropriate grammatical changes in "p"). Given that there is a command which expresses the same thing as "-p", we have a ground for "L -p". Given that there is no command which expresses the same thing as "-p", we have a ground for "Pp". And given that there is no command which

*"Command" was taken at page 53 as an imperative, law, etc., uttered with authrity.

expresses the same thing as "p", we have a ground for "P -p".

This sort of ground is a strict ground. This is a consequence of what was stated on page 54, i.e., that a statement of the form "Lp", where "L" is the imperative use of necessary, is true, if and only if there is a command which expresses the same thing as "p".

This sort of ground is not always very reliable. There is often the question whether the authority which issues a purported command actually has authrity over the particular individuals in the particular field. When a law has been judged unconstitutional it is because it has been decided that the authority which issued the law did not have authority in the particular field over the individuals involved.

That we cannot always determine whether a particular command expresses the same thing as "p" is another factor which sometimes makes the sort of ground formulated above relatively unreliable. Perhaps the most obvious examples of this are in jurisprudence with regard to the interpretation of the law. Another type of example would be children maintaining that what their parents told them to do did not actually cover a cetain case which they failed to do.

That commands afford grounds for only imperative uses of modal terms should be obvious. Commands are neither true or false, and the other uses of modal terms have a close connection with the notion of truth and falsity in that for these other uses "L (or C) p" entails "p", "L (or C) -p" entails " -p", "p" entails "Mp" and " -p" entails "M -p".

CHAPTER THREE

ITERATED MODALITIES

Section A: Iterated-Modality Statements Containing Only Logical Uses of Modal Terms.

The formulations of the various sorts of grounds in the last chapter all contained reference to a statement or command "p" and a modal statement of the form "L(or C)p", "L(or C) -p", "M (or P)p", or "M(or P)-p". The question now arises whether "p" itself can be a modal statement. This is obviously a question which must be answered individually for each sort of ground. Also, which uses of modal terms occur in "p" is important. And, since we found it essential to mention in the last chapter for which uses of modal terms each sort of ground was appropriate, it will likewise be important here to distinguish the uses of "L" ("M", "C",or "P") in "L (M,C, or P)p", where "p" in this case is itself a modal statement, even when we are discussing the same sort of ground for "L(M, C, or P)p".

~~This approach to the problem of iterated modalities might be a *prima facie* confusion between the notions of use and ground. For the relation between iterated modality statements which we are looking for are logical relations, and hence have to do with the use of modal terms. But first of all, the last two of the three considerations mentioned in the last paragraph were.~~

~~about use.~~

~~Secondly,~~ In ordinary discourse we very rarely use iterated modality statements, with the possible exception of iterated modality statements of the second degree in which the first modal term occurs as an epistemic use, and the second as another use. So we have not got the precedent of ordinary discourse to follow in this case in determining the logical relations which hold for modal statements. I here take an alternative to ordinary discourse to determine whether certain logical relations hold: investigating the relations between the grounds for a modal statement of one form and use and a modal statement of another form, containing perhaps some modal terms of a different use. This evidently is the best (and perhaps only acceptable) alternative to following ordinary discourse in establishing the characteristics of different uses. ~~For the relations between the truth and falsity of different statements is of crucial importance to the logical relations of statements, and grounds are what we use in determining (although often not unequivocally and with certainty) the truth or falsity of a statement.~~ So to some extent in this chapter I am using results about the sorts of grounds for the application of modal terms to establish results about the uses of modal terms. But this could hardly be called a confusion, since it is being overtly done because of the inability to employ the precedent which was used earlier in establishing the characteristics of the different uses - viz., ordinary discourse. In most cases the principles and relationships will be explicable by appeal to the points made in Chapter One about the various uses. It is only in connection with iterated-modality statements which contain only logical uses of modal terms where an unavoidable appeal is made to grounds which are not reflected at all in the prior discussion of use.

The iterated modal statements which I shall consider will all be of the form "Xp", where "X" is a sequence of two or more modal terms. Thus I shall not be considering all forms of iterated modal statements. For example, "L (p $\supset$ Lq)" is an iterated modal statement, but not of the form "Xp". However, iterated modal statements not of the form "Xp" are usually equivalent to statements of this form, or conjunctions or disjunctions of this form, especially in a system in which all the connectives are truth-functional or definable in terms of truth-functional connectives and modal terms. Also, the results obtained with regard to iterated modal statements of the form "Xp" would probably form a prolegomena for considerations of other forms of iterated modalities.

In this chapter I shall consider almost exclusively iterated modal statements which contain no negation signs before modal terms. This does not affect the generality of these results at all. For by means of the equivalences "L(or C)p ≡ -M(or P)-p" and "M (or P)p ≡ -L (or C) -p" (where "p" occurs for imperative uses and "C" occurs for epistemic uses), any negation sign before a modal term can be eliminated.

I shall discuss only iterated modality statements which contain only logical uses of modal terms in this section. In the next section I shall discuss the other uses. In the first paragraph of this section it was said, in effect, that, where "X" is a sequence of one or more modal terms, there are three important factors to consider in determing whether "L(M,C, or P) Xp" is meaningful andhow it is related to other iterated-modality statements: (a) the grounds for "L" ("M", "C", or "P"), (b) the uses of the modal terms which comprise "X", and (c) the use of "L" ("M", "C" or "P"). For iterated-modality statements which contain only logical uses of modal terms, the sort of <u>ground</u> for the modal terms which comprise "X" is also an important consideration. For there are at least three sorts of grounds which are particularly important for logical uses of modal terms, and since ordinary discourse furnishes us with no precedent here, I rely on consideration of the grounds for these terms.

I shall begin by discussing iterated-modality statements in which the ground for each occurrence of a logical use of a modal term is the same. At the end of this section I shall comment on statements containing iterated logical modalities in which there are different grounds for different modal terms.

To begin with iterated-modality statements in which the ground for each term is afforded by imagining. In Chapter Two, section C, page 13, the following formulations were given for the grounds afforded by imagining: Given that there are no imaginable states-of-affairs which are described by "-p" (and "p" is well-formed) we have a ground for "Lp". And given that there is at least one imaginable state-of-affairs described by "p", we have a ground for "Mp". But does this make sense when "p" is either of the form "Lp" or "Mp", where the ground for "L" or "M" is also afforded by imagining? To determine this, let us write out "Lp" and "Mp" by using adaptions of the formulations given above for "L" and "M" obtained by eliminating the quotation marks around p. "Lp" becomes: "The state-of affairs -p is not imaginable". "Mp" becomes: "The state-of-affairs p is imaginable."* We can now substitute these

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* "The state-of-affairs p..." could be made more explicit by saying "The state-of-affairs, that p,..." but I shall hold to my first formulation.

construals of "Lp" and "Mp" for "p" in the formulations of the grounds afforded by imagining. The grounds for "MMp" is then: Given that there is at least one imaginable state-of-affairs described by "The state-of-affairs p is imaginable", we have a ground for "MMp".

But what sort of state-of-affairs would we be imagining here? Evidently, someone (perhaps ourselves) being able to imagine something (a state-of-affairs). But it seems evident that what we can imagine someone else being able to imagine is dependent on what we can imagine, although, perhaps, we can imagine them imagining it "better" or "worse", "more clearly" or "less clearly". And evidently, if we have an explication of "imaginable" in accordance with which what is imaginable is not dependent on particular people, then if it is imaginable that a state-of-affairs can be imagined, this, if it means anything, is true if and only if the state-of-affairs is imaginable. So we have the result that iterated-modality statements in which the grounds for each modal term is afforded by imagining, if they are meaningful, are redundant.

The grounds afforded for "necessary" and "possible" by meaning were stated in Chapter Two, section B, page 62, as: Given that "p" has been shown to be true by virtue of the meanings of its constituent terms, we have a ground for "Lp". And given that "p" has been shown not to-be-false-by-virtue-of-the-meanings-of-its-constituent-terms, we have a ground for "Mp". We can give construals of "Lp" and "Mp" by replacing the modal term with adaptions of the ground for that term, but omitting the reference to showing that.... "Lp" then becomes: " 'p' is true by virtue of the meanings of its constituent terms." And "Mp" becomes: " 'p' is not false-by-virtue-of-meanings-of its constituent terms."

Substituting these construals of "Lp" and "Mp" for "p" in our formulations of the grounds afforded by meaning for "necessary" and "possible" results in the following. Given that " 'p' is true by virtue of the meanings of its constituent terms" has been shown to be true by virtue of the meanings of its constituent terms, we have a ground for "LLp". Given that " 'p' is not false-by-virtue-of-the-meanings-of-its-constituent-terms" has been shown to be true by virtue of the meanings of its constituent terms, we have a ground for "LMp". Given that " 'p' is true by virtue of the meanings of its consituent terms" has been shown not to-be-false-by-virtue-of-the-meanings-of-its-constituent-terms, we have a ground for "MLp". And given that " 'p' is not-false-by-virtue-of-the-meanings-of-its-constituent-terms" has been shown not to-be-false-by-virtue-of-the-meanings-of-its-constituent-terms, we have a ground for "MMp".

Now all of these cases of grounds for second degree modal statements are meaningful and unproblematic. For the semantical predicates "...is true by virtue of the meanings of its constituent terms" and "...is not false-by-virtue-of-the-meanings-of-its-constituent-terms" can be shown to apply or not to apply to a statement by considering only the meanings of the terms of that statement. Hence, not only is a statement of the form " 'p' is true by virtue of the meanings of its constituent terms" and " 'p' is not false-by-virtue-of-the-meanings-of-its-constituent-terms" true because of the meanings of its terms, or not false-because-of-the-meanings-of-its-terms, or the negation of it is one of these two, but it can often be shown to be so. The problems of showing whether or not a statement of one of the above two forms, or its negation, is true because of the meanings of its terms or not false-because-of-the-meanings-of-its-terms is almost always, if not always, the same problem encountered in showing that "p" or "-p" is true because of the meaning of its terms or not false-because-of-the-meaning-of-its-terms in the first place. For the two semantical predicates mentioned above are not in the least complicated and offer no problems for an analysis of their meaning.

It is obvious that we can arrive at formulations of grounds for iterated modalities of any degree where the grounds for the modal terms are afforded by meaning. All we need do is to go through the above procedure for formulating the grounds for second degree modal statements, but using a second degree modality instead of a first to formulate the grounds for third degree modal statements; we could take these formulations, but substitute third degree modal statements for second degree modal statements, to formulate the grounds for fourth degree modal statements, etc.

It is also apparent why I omitted reference to showing that...in the construals of "Lp" and "Mp". The reason is that " 'p' has been shown to be true by virtue of the meanings of its constituent terms" and " 'p' has been shown not to-be-false-by-virtue-of-the-meanings-of-its-constituent-terms" are never themselves true because of the meanings of their terms. For what is shown about a statement is not determined solely by the meanings of its terms. In fact, nothing at all may be shown about it.

But that the reference to showing that...is omitted does not usually affect these formulations. We saw above that the semantical predicates "...is true by virtue of the meanigs of its constituent terms " and " isnot false-by-virtue-of-the-meanings of its consitutent terms" offer no problems as far as grounds for them are concerned when they are applied to a statement which already contains one of them in the only phrase not contained in quotations marks. In fact, the only problematic cases of their application is to statements to which they are not already applied. So there are basically the same problems for applying these semantical predicates to "p" as there are for applying them to "Xp", where "X" is a sequence of any number of modal terms, the grounds for which are afforded by meaning. As long as a reference to showing that... is included in the ground for the last term, these problems are still allowed for. And the formulations which I gave satisfy this condition.

Assume that it has been shown that "p" is true by virtue of the meanings of its constituent terms. Then it is true that " 'p' is true by virtue of the meanings of its constituent terms." But this is not only true, but true by virtue of the meanings of its constituent ters. For "...is true by virtue of the manings of it s constituent terms" applies by virtue of the meanings of its constituent terms to those and only those statements which meet this description and it makes no reference to anything but terms. From what was said earlier about the lack of problems in applying this semantical predicate to statements which contain it as the only phrase not inside quotation marks it follows that there is a ground for " ' 'p' is true by virtue of the meanings of its constituent terms ' is true by virtue of the meanings of the meanings of its constituent terms". And this is a strict ground, since all grounds afforded by meaning are strict. But "Lp" was construed as " 'p' is true by virtue of the meanings of its constituent terms". So, when the ground for "L" is afforded by meaning, we have the result that when we have a ground for "Lp", we necessarily have a ground for "LLp". Thus, in this case, "Lp" entails "LLp". We can substitute "$L^{n-1}p$" where "n - 1" is any integer greater than zero, for "p" and arrive at the result that "$L^{n-1}p$" entails "$L^n p$". From the transitivity of entailment it follows that "Lp" entails "$L^n p$", where the ground for all occurrences of "L" are afforded by meaning.

Now it is a valid rule for logical necessity that "Lp" entails "p". Where the ground for "L" is afforded by meaning, we can substitute "$L^{n-1}p$" for "p" with the result that "$L^n p$" entails "$L^{n-1}p$". By transitivity of entailment it follows that "$L^n p$" entails "Lp". Since "$L^n p$" entails and is entailed by "Lp", "$L^n p$" is reducible to "Lp", where n is any integer greater than zero and the grounds for all occurrences of "L" are afforded by meaning.

By a similar line of argument we can conclude that

"$M^n p$" is reducible to "Mp" with the same conditions imposed on "M" and "n". But here we have as a result of a valid rule for logical possibility that "Mp" entails "$M^n p$", and we would then have to show by appeal to the sort of ground afforded by meaning that "Mp" entails "$M^n p$".

Likewise we can show that "LMp" is reducible to "Mp" and that "MLp" is reducible to "Lp". From these four results about the reducibility of iterated modalities we can conclude that, where the grounds for all modal terms are afforded by meaning, all iterated modality statements are reducible to statements of the form "Lp" and "Mp" - i.e., iterated modalities can be eliminated where the grounds for all the modal terms are afforded by meaning.

The sorts of grounds afforded by entailment for "necessary" and "possible" which were given in Chapter Two, section A, page 59 , are:

(1) Given "Lp" and that "p" entails "q", we have a ground for "Lq".

(2) Given "Mp" and that "p" entails "q", we have a ground for "Mq".

Since "p" can be any statement whatsoever, as long as it satisfies the conditions "Lp" and that "p" entails "q", we can replace "p" by a variable which ranges over names of statements as long as it satisfies these two conditions. I shall use "A", "B", etc., for these types of variables.

However, it is not the name "p" which occurs in "Lp", nor is it the name of a variable. It was said before that "p" is necessary because of its meaning. This suggests that we could use other variables, "a", "b", etc., which are correlated with the variables "A", "B", etc., in that "A" ranges over the names of statements and "a" ranges over the statements themselves, and where a name of a statement is given as the value of "A", the statement itself is taken as the value of "a".* (1) and (2) can be reformulated in accordance with this suggestion and replacing "q" with "p" as:

(1') Given that there is an A such that "La" and "A" entails "p", we have a ground for "Lp".

(2') Given that there is an A such that "Ma" and "A" entails "p", we have a ground for "Mp".

Construals of "Lp" and "Mp" can be given which are based on (1') and (2'). "Lp" becomes:

(3) "There is an A such that 'La' and A entails 'p' ".

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* Cf. Chapter Four, section B, for the analogous problem for individual variables. In that section many of the results are applicable to statements and statement variables as well.

"Mp" becomes:

(4) "There is an A such that 'Ma' and A entails 'p'".

Substituting (3) for the first occurrence of "p" in (1') and "Lp" for the second occurrence of "p" and replacing "a" and "a" with "B" and "b" in (1') gives:

(5) Given that there is a B such that "Lb" and B entails "There is an A such that 'La' and A entails 'p'", we have a ground for "LLp".

According to (5) we have a ground for "LLp" if there is a necessary statement which entails the statement, "There is an A such that 'La' and A entails 'p'". We could easily formulate analogous grounds for "LMp", "MLp", and "LLp". To have such a situation as this , we would have to have a deductive system (in the loose sense) in which at least some of the theorems state entailment relationships between statements. Such a system, of course, would be metalinguistic in that it would contain names of statements (the variables over which "A" ranges). And some of these names would be names of modal statements.

For certain inference schemata which are valid for logical modalities to hold in a system of the kind envisaged here, it would have to contain theorems which mention other theorems. Also, "B" in (5) ranges over statements which mention statements over which "A" ranges.

If we could justify this breach of the object language-metalanguage distinction here, we could presumably justify a breach of the metalanguage-<u>meta</u>metalanguage distinction, etc. Thus iterated modalities of any degree would be meaningful, and non-redundant. I shall later make reference to the problems of finding reduction relations for these iterated modalities.

Besides the objection that such construals of iterated modalities as are furnished by (5) involve a breach of the object language - metalanguage distinction, there is also the objection that such systems as were envisaged above are impractical. Both of these objections are avoided if we (a) eliminate <u>all</u> variables which range over names of statements in favor of variables which range over statements, and (b) eliminate all names of statements in favor of the statements themselves. We can accomplish (b) by introducing a dummy operator "That" which precedes the names of the statement stripped of its quotation marks (i.e., precedes the statement). The scope of this operator can be indicated by parentheses. Reformulating (1') and (2') along these lines gives:

(1") Given that there is an <u>a</u> such that That (L<u>a</u>) and <u>a</u> entails That (p), we have a ground for That (Lp).

(2") Given that there is an _a_ such that That (M_a_) and _a_ entails That (p), we have a ground for That (Mp).

(1") and (2") also give us grounds for "Lp" and "Mp" since "Lp" is true if and only if That (Lp) and "Mp" is true if and only if That (Mp).

(3) and (4) can be reformulated as:

(3') "There is an _a_ such that That (L_a_) and _a_ entails That (p)."

(4') "There is an _a_ such that That (M_a_) and _a_ entails That (p)."

And (5) becomes:

(5') Given that there is a _b_ such that That (L_b_) and _b_ entails That (there is an _a_ such that That (L_a_) and _a_ entails That (p)), we have a ground for That (LLp).

The analogous grounds for That (LMp), That (MLp) and That (MMp) are:

(6) Given that there is a _b_ such that That (L_b_) and _b_ entails That (there is an _a_ such that that That (M_a_) and _a_ entails That (p)), we have a ground for That (LMp).

(7) Given that there is a _b_ such that That (M_b_) and _b_ entails That (there is an _a_ such That

(La) and a entails That (p)), we have a ground for That (MLp).

(8) Given that there is a b such that That (Mb) and b entails That (there is an a such that That (Ma) and a entails That (p)), we have a ground for That (MMp).

(6), (7), and (8) are grounds for "LMp", "MLp" and "MMp", respectively, as well since "LMp" is true if and only if That (LMp), "MLp" is true if and only if That (MLp), and "MMp" is true if and only if That (MMp).

Whereas those formulations of the grounds for iterated modalities which contain names of statements and variables for names of statements are applicable only when there is a fairly rigorously developed deductive system which has theorems in which others of its theorems are mentioned, those formulations which contain no such names or variables are applicable in many other cases as well. It is convenient to include under the term "statement" not only those linguistic entities which are true or false and have been formulated, but also those which have not been formulated. Then (5'), (6), (7) and (8) can express relations between statements within the same deductive system (in the loose sense used here) even when this deductive system is not at all rigorously presented.

To take an example from informal number theory: It is possible that n is an integer between 9 and 15. This entails that That (n is not even, which is possible, entails That (n is prime)). Existential generalization gives us: There is a b - namely, That n is an integer between 9 and 15 - such that That (Mb) and b entails That(there is an a - namely, That n is even - such that That (Ma) and a entails That n is prime. So we have a ground for That it is possible that it is possible that n is prime, and hence also a ground for "It is possible that it is possible that n is prime."

What sort of reduction rules are these, then, between the iterated modalities furnished by the formulations of the grounds afforded by meaning which contain no names of statements or variables which range over names of statements? I maintain that there is not a single answer to this question. Entailment affords a sort of ground which might be called specific as opposed to the sort of ground afforded by meaning which is general.*

The grounds afforded by meaning are considerations of only one statement - the ralationships which hold between

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~~* The sort of ground afforded by imagining is also general, but I will not discuss it here.~~

statements are incidental to this sort of ground. Thus if it has been shown, for example, that a statement is true by virtue of the meanings of its constituent terms, then it has been shown to be true by virtue of the meanings of its constituent terms: full stop - no further specification can be added.

But the grounds afforded by entailment are considerations of the relationship between statements. Thus if it has been shown that a statement is entailed by a necessary statement, for example, then we can specify what particular statement this necessary statement is. Or if it is shown that a statement is deducible in a deductive system with necessary premises, then this deductive system can be specified. That is is possible to specify what statement entails the statement or in which system it is deducible is what is meant by the specificity of the grounds afforded by entailment.

Different deductive systems, even if they contain some of the same statements as theorems or premises, will usually have different deductive relationships between their theorems and premises. So for any two deductive systems it might well be the case that there are different general characteristics of the deductive relationships between their axioms and premises which determine which reduction rules hold between iterated modalities based on the second set of formulations of the grounds afforded by entailment. The question of what reduction rules are valid for iterated modalities based on the formulations of the grounds afforded by entailment which do not contain names of statements or variables which range over names of statements, then, is to be answered for individual deductive systems, or perhaps different sorts of deductive systems.

The points made in the last paragraph are also true for iterated modalities furnished by the formulations which include names of statements and variables which range over names of statements. So in this case, too, the problem of reduction rules must be answered for individual deductive systems or sorts of deductive systems.

I have considered how the three sorts of grounds for logical uses of modal terms individually furnish construals of iterated-modality statements. I shall now consider whether they furnish any additional construals when taken together.

Consider a second degree modality statement in which the ground for the first modal term is afforded by imagining, and the ground for the second is afforded by meaning. If we construe these modal terms in terms of their grounds, as has been done throughout this section, then iterated modality statements of this form would be true if and only if it were the case that something were imaginable or unimaginable in connection with the statement, or its negation, that some statement is true because of its meaning or not false-because-of-its-meaning. In this case, then, we would be concerned with the imaginability or unimaginability that a statement's meaning determines its truth, falsity, or neither. But then we would be concerned with the imaginability or unimaginability of a typographical entity or series of noises possessing a particular attribute. And this would just furnish a ground afforded by imagining for a first degree modal statement.

We might try to avoid this conclusion by taking entities which are expressed by statements- "propositions" in Carnap's sense. Then we might construe an iterated-modality statement of the type discussed here as a statement asserting that it is imaginable or unimaginable that a proposition or its negation, always or never holds. But this amounts only to a reformulation of the sort of ground afforded by imagining for first degree modal statements.

Second degree modality statements in which the ground for the first term is afforded by meaning, and the ground for the second term is afforded by imagining can be treated similarly. We can construe these such that they are true if and only if it were the case that "There is an (no) imaginable state-of-affairs described by 'p' (or '-p')" is true because of its meaning or is not-false-because-of-its-meaning. It is evident that "There is an (no) imaginable state-of-affairs described by 'p' (or '-p')" is never true or false because of the meanings of its terms. For imagining is not a logical or semantical notion.* But this does not rule out having grounds for "MLp", "MLp", "M-Lp" and "M~Lp". However, such modality statements would be pointless without the corresponding statements of possibility and ~~non-necessity~~ impossibility.

That the grounds afforded by meaning and the grounds afforded by imagining when taken together do not furnish a way of formulating iterated modalities (other than the pointless ones mentioned in the last paragraph) can be viewed as a consequence of the fact that these two sorts of grounds are checks on each other. If by means of the grounds afforded by imagining we conclude that a certain modal statement is true, then we could check this by means of the grounds afforded by meaning and vice versa. Of course, the grounds afforded by meaning are generally more suitable and reliable than those afforded by imagining; so there is more reason to check the results of applying the grounds afforded by imagining with the grounds afforded by

* It might be the case that no one can imagine anything. It might also be the case that there are no people at all, and again nothing would be imaginable.

meaning than there is for the converse situation.

We arrive at a different conclusion for grounds afforded by meaning and those afforded by entailment taken together. A sort of ground afforded by entailment for "LLp" is that "Lp" occurs as a theorem in a deductive system with necessarily necessary premises. Assume that the ground for asserting that these premises are necessary is afforded by meaning. Then, in "LLp" the ground for the second occurrence of "L" is afforded by meaning and the ground for the first occurrence is afforded by entailment.

Of course, to avoid an infinite regress of entailment relationships, somewhere along the line the ground for certain premises being necessarily necessary must be afforded by meaning or imagining. Objections have already been presented to having grounds afforded by imagining for second degree modalities when the ground for the second modal term is afforded by meaning. So the ground for the necessary necessity of the premises must be afforded by meaning as well. The same conclusion can be arrived at for necessarily possible, possibly necessary, and possibly possible premises for which the ground for the second modal term is afforded by meaning.

But we have seen that, where the grounds for all occurrences of modal terms in a statement are afforded by meaning, all modalities are reducible to first-degree modalities. Thus we may conclude that all premises in the sort of system envisaged have, if they are modal premises, are reducible to first-degree modal statements. And the theorems which are deduced in a system are of the same modal degree as the premises. Thus an iterated modality which has a term, the grounds for which is afforded by meaning, preceded by a term, the ground for which is afforded by entailment, is reducible to a modality which is identical to it except that the term for which the ground is afforded by meaning is deleted.

In considering second-degree modality statements in which the ground for the first modal term is afforded by meaning, and the ground for the second is afforded by entailment, it is convenient to consider the four combinations of "L" and "M" individually. "LLp", given the above condition, can be construed as:

(9) " 'q' entails 'p' and 'Lq'" is true because of its meaning.

A conjunction is true because of its meaning if and only if each of its conjuncts is true because of their meanings. So (9) is equivalent to the conjunction of

(10) " 'q' entails 'p'" is true because of its meaning,

and

(11) "Lq" is true because of its meaning.

It follows from the discussion of two paragraphs back that we can safely assume that the ground for "Lq" is afforded by meaning. It follows from the reduction rules given for iterated modalities based on the grounds afforded by meaning, then, that (11) is equivalent to:

(12) "q" is true by virtue of its meaning.

This is a construal of,

(13) Lq.

Now entailment is a relation which holds or does not hold because of the meanings of the statements so related. So (10) is true if and only if,

(14) "q" entails "p".

Since (13) is equivalent to (11), and (14) is equivalent to (10), the conjunction of (10) and (11), which is equivalent to (9), is equivalent to the conjunction of (14) and (13), which is:

(15) "q" entails "p" and Lq.

But this is a construal of "Lp". So we have the reduction rule, in this case, that "LLp" can be replaced by "Lp".

Construing "LMp" along these lines gives:

(16) " 'q' entails 'p' and 'Mq' " is true because of its meaning.

An argument analogous to the one given in the last paragraph could be given here to justify the reduction rule that "LMp" can be replaced by "Mp".

Analogous construals of "MLp" and "MMp" give:

(17) " 'q' entails 'p' and 'Lq'" is not false-because-of-its-meaning.

and

(18) " 'q' entails 'p' and Mq'" is not false-because-of-its-meaning.

But it is not true that a conjunction is not false-because-of-its-meaning if and only if its conjuncts are not false-because-of-their-meanings. There is no way of relating the semantical predicate "...not false-because-of-its-meaning" individually to the conjuncts of (17) and (18). So the reduction rules for replacing "MLp" with "Mp" or "Lp" and "MMp" with "Mp" are invalid.

For iterated modality statements based on both grounds of meaning and of entailment in general, then, we have the following result: Any iterated modality statement satisfying this condition can be reduced to one containing only modal terms for which the grounds are afforded by entailment unless the initial term is "M" and the grounds for it are afforded by meaning; in that case it is reducible to a statement in which the initial "M" is the only modal term for which the grounds are afforded by meaning.

Grounds afforded by imagining, on the other hand, together with grounds afforded by entailment do not offer a basis for any iterated modalities which are not based solely upon grounds afforded by entailment. In the first place, imagining does not furnish any iterated modalities on its own, so, for the ground "q" entails "p" and "Lq" (or "Mp"), "Lq" must be only a first degree statement if its ground is afforded by imagining. In this case the ground for "Lp" (or "Mp") would be afforded by entailment but would only be a first degree statement; in particular "L" (or "M") would not contain within its scope a modal operator construed in terms of imagining.

We can construe a second degree modal statement in which the ground for the first modal term is afforded by imagining and the ground for the second is afforded by entailment, as: A state-of-affairs is (not) imaginable which is described by "(It is not the case that) 'q' entails 'p' and 'Lq' or 'Mq' ." It is quite obvious that objections parallel to the objections to modal terms construed in terms of imagining preceding modal terms construed in terms of meaning can be raised here. I will not elaborate on them though.

In summary: The sort of ground afforded by imagining furnishes no basis for iterated modalities, on its own, or together with the other two sorts of grounds. The sort of ground afforded by meaning furnishes the basis for an infinite number of iterated modalities and all of these are reducible to first degree modalities. And the sort of ground afforded by entailment also furnish a basis for an infinite number of iterated modalities, but the reduction rules for these must be determined for individual deductive systems or sorts of deductive systems. But for the infinite number of iterated modalities furnished by the sort of ground afforded by entailment, together with the sort of ground afforded by meaning, there are certain reduction rules which are valid no matter what deductive system is being considered.

<u>Section B</u>: <u>Other Types of Iterated-Modality Statements</u>.

In this section I shall consider iterated modality statements which contain non-logical uses of modal terms; some, however, will contain a logical modality.

To begin with iterated modality statements containing logical and non-logical uses of modal terms. Statements which contain an occurrence of one of the four non-logical uses of modal terms which I have presented, and in which the scope of this occurrence includes the remainder of the statement, can be treated as non-modal statements in connection with the grounds for logical uses of modal terms. For all such sentences are <u>logically</u> possible, and none are <u>logically</u> necessary.

On the other hand, a second-degree modal statement in which the first modal terms is a natural, conditional, or imperative modality and the second modal term is a logical modality is meaningless. For empirical evidence and grounds afforded by imperatives are irrelevant to the truth or falsity of a statement which is true because of its meaning.

Statements which contain an epistemic modality followed by any other modality are meaningful. We can have knowledge or no knowledge relevant to the truth or falsity or any statement whatsoever - this includes any sort of modal statement. It is doubtful, though, that any natural or conditional necessity statement is certain; but in this case such are false, and not meaningless.

But a second-degree modal statement in which both modal terms are epistemic, although it is meaningful, is redundant. Such a statement would be relative to the state of our knowledge about the state of our knowledge about the truth of "p", where "p" is any statement except an epistemic-modality one. And the state of our knowledge about the state of our knowledge is, if anything, part of the state of our knowledge.

Similarly, a second-degree modal statement in which both modal terms are natural uses would be true or false according to the nature of the nature of whatever is mentioned in it. And a second-degree modal statement in which both modal terms are conditional uses would be true or false according to the extrinsic conditions on the extrinsic conditions on what is mentioned in it. The nature of the nature of a thing is, if anything, the nature of that thing; and the extrinsic conditions on a thing are extrinsic conditions on it. So both of these alleged cases of second-degree modalities are redundant.

Natural, conditional, or imperative modalities preceding an epistemic modality present problems. Presumably, a second-degree modal statement containing a natural modality followed by an epistemic modality would be true or false according to the nature of the state of our knowledge. But what is the nature of the state of our knowledge? Evidently this is either redundant or meaningless.

A second-degree modal statement containing a conditional modality followed by an epistemic modality would be true or false accoring to the extrinsic conditions on the state of our knowledge. But it does not appear that we can make sense out of talk about extrinsic conditions on knowledge.

Again, a second-degree modal statement containing an imperative modality followed by an epistemic modality is dependent on there being, or not being, a command(uttered by a person with authority in the field to which it is relevant over the people to which it applies) concerned with the state of our knowledge. But it is very doubtful indeed whether any person or group of people has authority over another person, much less over all people, with regard to the state of their knowledge.

Second-degree modal statements in which an imperative modality is followed by a natural or conditional modality are objectionable for similar reasons. For no one has authority to issue commands over the natures of things or their extrinsic conditions, but just over people.

Even though a natural modal term following a conditional modality is redundant since extrinsic conditions on the nature of a thing would be, if anything, extrinsic conditions on that thing itself, good sense can be made of a second-degree modal statement in which the first term is a natural modality and the second a conditional modality. For, even though the extrinsic conditions on X are not specified in a statement of the form "It is naturally necessary (impossible, possible, or non-necessary) for X to K", we can still sometimes determine, given X, what sort of conditions are naturally necessary (impossible, possible, or non-necessary). For example, it is conditionally possible for me to be attracted to the center of the earth, since there are no extrinsic conditions which prevent it. But, because of the nature of me and the earth (we both have mass) there could not be any extrinsic conditions which prevent me from being attracted toward the center of the earth. Thus it is naturally necessary that it is conditionally possible for me to be attracted toward the center of the earth.

Finally, second-degree modality consisting of two imperative modalities is redundant. A statement containing this alleged type of modality would be dependent on there (not) being a command that there (not) be a command that something-or-other. This would involve there being a primary authority over a secondary authority which is over the people to which the command applies. But in this case we would say that the primary authority was over the same people (and perhaps more as well) and in the same fields (and perhaps in more as well) as the secondary authority.

In summary: Second-degree modality statements in which the first modality is a logical modality, and the second is any non-logical modality, are meaningful and non- redundant; but the only statements of this type which are true are those in which the logical modality is "M" or "M-". Second-degree modality statements which contain an epistemic modality followed by a non-epistemic one are meaningful and non-redundant; but it is doubtful whether any of these statements in which the first modality is "C" or "-C" and the second is natural or conditional necessity or possibility is ever true. Finally, a second-degree modality statement in which the first term is a natural modality and the second term is a conditional modality are meaningful and non-redundant. All other second-degree modalities, with the exception of those in which both terms are logical modalities, are meaningless, objectionable, or redundant. In particular, second-degree modalities in which both terms are either epistemic, natural, conditional, or imperative modalities, are meaningless, objectionable, or redundant.

CHAPTER FOUR

MODALITY AND QUANTIFICATION

Section A: Quantification and Non-logical Modalities.

In this chapter I shall discuss the consequences of combining quantifiers and modal operators. It will be seen that this presents no special problems for natural, conditional, and imperative modalities. I shall concentrate on these modalities and quantification in this section and justify certain entailment relationships between statements which contain both quantifiers and modal terms of these uses. These results, however, will vary according to the use of the modal terms which the statement contains. In the next section I shall concentrate on statements which contain quantifiers and logical uses of modal terms. It will be shown that basic objections can be raised in connection with these sorts of statements especially when a quantifier precedes a modal term. A possible resolution of these difficulties will also be offered. Epistemic modalities, because of a basic difficulty, will only be briefly mentioned in this chapter (in section A).

I have previously discussed a distinction between the natural and the logical uses of "necessary" and "possible", viz., the following inference schema is valid for natural modalities, but not for logical ones:

$$\text{(a)} \qquad \frac{\begin{array}{c} L\text{ (or M) } \emptyset\text{ (a)} \\ a = b \end{array}}{L\text{ (or M) } \emptyset\text{ (b)}}$$

What I propose to do now is to determine for which other uses of modalities this schema is valid, and to give an explanation of why it is or is not valid in each of the different uses.

An explanation of why schema (a) does not hold for logical modalities, at least in what might be called its standard interpretation, is in order here. As mentioned before, the truth of a modal statement in which the modality is a logical modality is due solely to the meanings of the constituent terms of that statement, although in determining whether the statement is true or false we often use some other grounds than simply semantical analysis - e.g., grounds afforded by imagining or the entailment relation.

If "Ø(a)" is true, false, or neither by virtue of the meanings of its terms, then it is true, false, or neither by virtue of the meanings of, for one thing, "a". Now "a" and "b" may refer to the same thing, that is, a, otherwise known as b, not because of the meanings of "a" and "b", but simply because it happens that "a" and "b" refer to the same thing. In this case, "Ø(a)" is also true, since the object, no matter what name is give to it, either has or does not have the property $\emptyset$; in this case it has. But "$\emptyset(b)$" is not true by virtue of the meaning of its terms because it just so happens that the object referred to by "a" is the same as is referred to by "b", so it just so happens that "$\emptyset(b)$" is true; that is, "it just so happens" in the sense that "$\emptyset(b)$" is not true by virtue of the meanings of its terms.

But the situation is different with regard to natural modalities. A natural modal statement, say "$L\emptyset(a)$", is true, not because of the meanings of its terms, but because of the nature of the object or objects which it is about. Consider the case where we have two names for an object, say "a" and "b". No matter what name we give to an object, its nature is the same. This is a result of the truism that how we speak about something does not affect how or what a thing is. So, if we know that "a" and "b" refer to the same thing, that is, if we know that $a = b$, "L (or M) $\emptyset(a)$" could not be true and "L(or M) $\emptyset(b)$" false. In other words, "L (or M) $\emptyset(a)$" together with "$a = b$" entail "L(orM) $\emptyset(b)$. So schema (a) is valid for natural modalities.

(a) is likewise valid for conditional modalities. I could give the same argument as given in the last paragraph for this, only changing "nature" to "extrinsic conditions".

However, this argument cannot be applied in the case of imperative modalities. An imperative modality is not true or false because of the way the world is. It is dependent upon what imperatives, laws, etc. have been uttered with authrity. On the other, hand, they are not solely dependent on the meanings of their constituent terms either. One way of determining whether the inference schema in question is valid for these modalities is to determine whether substitutibility of identicals holds for imperatives and laws. Now if someone is ordered or bound by law to do or not to do something, then it would be futile on his part, if he wanted to ignore the imperative or law, to argue that he has another name than the one by which he was referred to in the particular imperative or law. The point is, if a person is ordered or bound by law to do or not do something, then this holds of him independently of how he is referred to.

It is not so clear that what a person or people are ordered or bound by law to do or not to do is independent of how it is referred to.

For example, imagine that there is a law prohibiting the use of cars on a street called "B" after certain hours, Imagine, also, that one end of this street is also known as "A", and that most people, when they say "B", are referring to the other end of the street, although the official name of the whole street is "B". If someone were arrested for driving on the end of the street known as "A" after the specified time, he might argue that he was not on 'B' street at all, but was on A street. The prosecutor might argue that "A" is simply [redacted] name for [redacted] B. Such quibbles are not uncommon in law courts, but this does not show that i~~n~~denticals are not substitutable within the scope of imperative modal terms. What it does show, rather, is that it is sometimes difficult to determine whether two terms do in fact refer to the same thing. Apparently, in a case such as the one considered above, once it is determined that two terms refer to the same thing, the two terms can be used interchangeably in imperative modal contexts. This, after all, is only sensible - What one may, must, or must not do is communicated by language; hence meaning is important; but, as far as reference goes, it makes no difference how we refer to the particular objects involved other than as far as clarity is concerned. The conclusion, then, is that (a) is valid for imperative modalities.

The analogue of (a) obtained by substituting "C" for "L" is not valid. For a conclusion is certain only if <u>all</u> the premises are certain. In general, identicals cannot be substituted within the scope of any epistemic modal term.

Not only is the epistemic analogue of (a) not valid, but

(b)
$$\begin{array}{c} C\ \emptyset(a) \\ \underline{C\ (a = b)} \\ C\ \emptyset(b) \end{array}$$

is not valid either. If a conclusion is certain, not only must the premises be certain, but it must also be certain that the inference schema is valid. This eliminates any resolution to the problem of quantifying into epistemic modal contexts analogous to the resolution of the problem of quantifying into logical modal contexts suggested in the next section.

The problem of the substitutibility of identicals is closely related to the problem of quantifying into modal contexts. In particular, the problem of reference arises in connection with both. For the substitutibility of identicals to be valid, the individual terms must occur in a context which Quine calls "purely referential". That is, the occurrence of the term is of importance only in so far as it refers to an object. How it refers to an object, or if it is known to refer to a particular object, are irrelevant, in so far as simple truth is concerned, in these contexts.

Now in a statement containg a quantifier which binds a variable in that statement, the variable ranges over objects. And these variables are of importance solely because they range over objects. The meanings of the terms which are substitution instances for them are irrelevant as is the consideration whether these terms are known to refer to particular objects. So where the context is such that substitutability of identicals is valid, we have a "purely referential context" and hence quantification into these contexts presents no problems.

Of the five modal uses which I have considered, three of them have scopes which are purely referential contexts; viz., natural, conditional, and imperative uses. Thus quantification into the scopes of modal operators of these uses is allowable.

There are some interesting questions about the relations between operators of these uses and quantifiers though. I shall call any inference schema of the form " '$XQ\emptyset(x)$' $\longrightarrow$ '$QX\emptyset(x)$' " or " '$QX\emptyset(x)$' $\longrightarrow$ '$XQ\emptyset(x)$' " where "X" is a variable bound by "Q", a <u>commutation relationship</u>. I shall assume, and this is not an outrageous assumption, that there are two modal operators, viz., "L" and "M", although there are different uses of them, and in one use "M" is replaced by "P", and two quantifiers, viz., "(x)" and "$(\exists x)$". There are then eight possible commutation relationships:

(1) "L(x) Ø(x)" entails "(x) LØ(x)"

(2) "(x) LØ(x)" entails "L(x) Ø(x)"

(3) "L(∃ x) Ø(x)" entails "(∃ x) LØ(x)"

(4) "(∃ x) LØ(x)" entails "L(∃ x) Ø(x)"

(5) "M(x) Ø(x)" entails "(x) MØ(x)"

(6) "(x) MØ(x)" entails "M(x) Ø(x)"

(7) "M (∃ x) Ø(x)" entails "(∃ x) MØ(x)"

(8) "(∃ x) MØ(x)" entails "M(∃ x) Ø(x)"

But not all of these stand or fall together. Consider, for example, (8). Using the definition "Mp $=_{df}$ -L -p", we have:

"(∃ x) -L -Ø(x)" entails "-L -(∃ x) Ø(x)"

Since "p" entails "q" if and only if "-p" entails "-q":

"L -(∃ x) Ø(x)" entails "-(∃ x) -L -Ø(x)",

And

"-(∃ x) -Ø(x) $=_{df}$ (x) Ø(x)"; so,

"L(x) -Ø(x)" entails "(x) L -Ø(x)"

But the negation of a predicate is another predicate; so we can write the formula directly above as:

"L(x) Ψ(x)" entails "(x) L Ψ(x)"

and this is the same for as (1), and we are not interested in the predicate any way, but only in the commuting of the modal operator and the quantifier. So (1) and (8) stand or fall together. The same interdependence holds between (2) and (7), (3) and (6), and (4) and (5).

It is very likely that different commutation relationships are valid for the different uses considered here. So I shall consider these three uses individually.

To begin with natural uses of "necessary" and "possible". (1) and (8) are valid. For if there actually exists at least one object for which it is possible to Ø, then it is possible for there to exist an object which Ø's --- at least the one which already exists and could Ø. That is, any object which satisfies the condition "M Ø(x)" supplies a verifying instance for "M ($\exists$x) Ø(x)". Again if it is necessary for it to be the case that every object that there would be (not just actual objects) Ø's, then every object there actually is necessarily Ø's.

But (2) and (7) are not valid. It could be possible for there to exist an object which Ø's, and yet of all the objects which actually exist there may be none for which it is possible [redacted] to Ø. For example, it may be naturally possible that there exists a horse which flies; but it is very doubtful that there exists a horse for which it is naturally possible to fly. Again, of all the objects which actually exist, it might be necessary for each of them to Ø, and yet it might not be necessary that all objects Ø; for it might be possible for some object to exist, although it does not, which does not Ø. For example, for all the planets it might be naturally necessary for each of them to attract a body with a certain mass and a certain change, but it is not naturally necessary for all planets to attract a body of these specifications; for it is naturally possible that there would be a planet with the same charge as the body and would repel the body.

(3) and (6) are not valid. For each object, it might be possible for it to Ø; and yet it may not be possible for all objects to Ø. For example, it is naturally possible for any person to be in a certain room, but it is not naturally possible for all people to be in that room. Again, it might be necessary that something exists which Ø's and yet there might not exist something which necessarily Ø's. For example, it is naturally necessary for there to exist a man such that he is the father of a particular child; but there is no particular man for whom it is necessary that he is the father of that child.

Finally, (4) and (5) are not valid either. It might be possible for all things to Ø, but there might happen to be something for which it is impossible to Ø. For example, it is naturally possible for it to be the case that all cows have horns (in fact it was once the case that all cows had horns); but for all cows which there actually are, there are some ("polled" ones) for which it is naturally impossible to have horns. Again, there might exist something for which it is necessary to Ø, and yet it might not be necessary for this thing, or anything which $\emptyset$'s, to exist. To continue the example of the polled cattle: There are cattle for which it necessary that they not grow horns, and yet it is not necessary that such cattle exist (in fact, they are a recent introduction).

So, the only commutation relations which are valid for natural modalities are (1) and (8).

It is important to notice that (2) and (4) and (5) and (7) were shown to be invalid by appealing to objects which do not actually exist, but which could possibly exist; or which do exist, but it is possible for them not to exist. This is in accordance with the appropriate way for interpreting statments which include both quantifiers and natural modal operators. For, as we have seen, natural modal statements are intimately connected with subjunctive hypotheticals*. These include statements of the type: "If there were an object such-and-such, then it would be so-and-so."

But conditional modal statements have no important relationship with subjunctive conditionals.@ And conditional modal statements are basically particular, not

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* Cf. pp. 40-43.

@ Cf. pp. 43-45.

general like natural ones.* Also, when we say that it is conditionally necessary or possible for all A to be B, this is a general statement only in so far as it is an enumeration; "A", in this context, never applies to an open or infinite set of objects.@ And obviously only objects which actually exist can be enumerated. So we cannot appeal to objects which could possibly exist, or objects which actually exist but possibly could not-exist, in determining the validity or invalidity of commutation relationships (1)-(8).

But counter-instances to (2), (4), (5) and (7) depend on such objects. So we may conclude that, besides (1) and (8), (2), (4), (5) and (7) are valid commutation relationships for conditional uses of modal terms; and (3) and (6) are invalid. Also, "L(x) Ø(x)" is equivalent to "(x) LØ(x)" and "M(∃x) Ø(x)" is equivalent to "(∃x) M Ø(x)" for conditional uses of modal terms.&

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* Cf. pp. 35-36.

@ Cf. pp. 36-38.

& These conclusions are supported by results from the semantics of formal model systems. If the domain for each world is the same -- all possible objects are actual -- then (1), (2), (4), (5), (7) and (8) are usually valid. (3) and (6) are invalid in most systems. Cf. pp 141-148 of <u>An Introduction to Modal Logic</u> by C.E.Hughes & M.J.Cresswell, (Methuen, 1968).

We have already seen (p. 52) that imperative modal statements are similar to natural ones, and unlike conditional modal statements, in that they are basically general. And it is evident that, like natural modal statements, imperative modal statements have a close connection with subjunctive hypotheticals. For example, if it is necessary that all the people in a certain town pay their taxes on a certain day, then if X were a resident of the town, then he must pay his taxes on that day. Apparantly, imperative modal statements entail only subjunctive hypotheticals (or their negations) which contain imperative modal terms in their consequents. That this is so is not surprising since " 'Lp' entails 'p'" is not valid for imperative necessity, nor is " 'p' entails 'Pp'" valid for permissibility.

The important point is that some of the subjunctive hypotheticals entailed by imperative modal statements will be of the form (or its negation):"If there existed an X (or did not exist) such that...,then..." It is obvious, then, that counter-instances to the possible commutation relationships are allowable which appeal to possible objects which do not exist, or the possible non-existence of actual objects. Now (5)-(8) contain "M". For the analogous relationships for permissible we need only replace "ME with "P":

(5') "$P(x)\ \phi(x)$" entails "$(x)\ P\ \phi(x)$"

(6') "$(x)\ P\ \phi(x)$" entails "$P(x)\ \phi(x)$"

(7') "$P\ (\exists x)\ \phi(x)$" entails "$(\exists x)\ P\ \phi(x)$"

(8') "$(\exists x)\ P\ \phi(x)$" entails "$P\ (\exists x)\ \phi(x)$"

We may conclude, then, that analogous to the case of natural modalities and quantifiers, (1) and (8') are the only commutation relationships which are valid for imperative modalities.

Section B: Quantification and Logical Modalities.

We have seen earlier that

$$\frac{\begin{array}{c} N\phi(a) \\ a=b \end{array}}{N\phi(b)}$$

is not a valid inference for logical necessity. In connection with this, we have seen that quantification into a logical modal context is a very dubious operation indeed. However, if there were some justification for quantifying into a logical modal context, this would certainly be important, even if it would, as it probably would, require a departure from the usual view of quantification, modalities, individual terms and variables, or predicates. There have been a host of attempts at making sense of quantification into logical modal contexts and the substitutability of identicals. And any essay on modalities would hardly be complete without considering this problem. What I hope to do in the sequel is to shed some light on these problems with the help of what has been said so far in this essay.

I shall not consider any resolutions to the problem of quantification and logical modalities which would involve a reinterpretation of these modalities since the purpose of this essay is to investigate these modalities as they are actually used. The most fruitful sort of approach in my opinion is to consider alterations in our notion of what is an object. An alteration in our notion of an object would also involve an alteration in our notion of quantification, since we quantify over objects, and in our notions of individual terms and variables, since these refer, or range over, objects. Again, it would involve a reinterpretation of the notion of a predicate, although perhaps less directly in so far as predicates apply to objects. ~~Relations, of course, including identity, would like~~

~~wise be misinterpreted.~~

Despite the unclarity in the scope of the ordinary notion of an object, it is still obvious when "object" is being used in a novel sense. As Quine has pointed out, our ontology is at least reflected (he wants to say determined) by what values our variables of quantification take. If someone is using "object" in its ordinary sense, then he need not put specifications on the values which the variables are to take. On the other hand, if he is using "object" in a restricted or anovel sense, then he must specify certain restrictions and/or extensions to the values which these variables are to take.

Now Carnap offers a solution to the problem of quantification and logical modalities by introducing an extended sense of object - viz., intensional objects, or as he calls them, "individual concepts". To understand what is meant by "individual concept" requires a certain amount of background which Carnap presents in Meaning and Necessity. First of all, he introduces the notion of L-true as follows;

"A sentence $\mathfrak{S}_i$ is L-true in a semantical system S if and and only if $\mathfrak{S}_i$ is true in S in such a way that its truth can be established on the basis of the semantical rules of the system S alone, without any reference to (extra-linguistic) facts". (pg. 10)

The similarity between this account of L-truth and my account of logical necessity - i.e. true by virtue of the meanings of the constituent terms - is obvious. However there is an important distinction : Carnap's notion of "L-true" is defined in terms of a sort of ground for determining the truth of a sentence. ("..its truth can be established on the basis of the semantical rules"..). In fact, Carnap's account of "L-true" is practically identical with, although more developed than, my account of being shown to be necessary by means of the grounds afforded by meaning. There is, admittedly, a closer affinity between the grounds afforded by meaning and the reason why statements of logical necessity are true (viz. because of the meanings of constituent terms). This affinity aside the distinction between Carnap's notion of "L-true", which is dependent on a particular ground, and my notion of logical necessity, which is dependent on no ground, is important especially in so far as it obviously admits alternatives to Carnap's construal of logical modalities and objects over which the variables of quantification range in a logical modal context. I shall discuss these alternatives later

Carnap goes on to define other concepts in terms of "L-truth":

"a. $\mathfrak{S}_i$ is L-false in (S_I) $=_{Df}$ $\sim\mathfrak{S}_i$ is L-true

b. $\mathfrak{S}_i$ L-implies $\mathfrak{S}_i$ (in S_I) $=_{Df}$ the sentence $\mathfrak{S}_i \supset \mathfrak{S}_j$ is L-true.

c. $\mathfrak{S}_i$ is L-equivalent to $\mathfrak{S}_i$ (in S_I) $=_{Df}$ the sentence $\mathfrak{S}_i \equiv \mathfrak{S}_j$ is L-true.

d. $\mathfrak{S}_i$ is L-determinate (in S_I) $=_{Df}$ $\mathfrak{S}$ is either L-true or L-false." (pg. II)

If a sentence is not L-determinate, then it is L-indeterminate, or factual. Opposed to the L-concepts defined above are concepts which Carnap prefixes with "F", this being an evident abbreviation for "factual". Thus that a sentence is F-true is defined as its being true but not L-true. Analogous definitions are given for F-false, F-implies and F-equivalent.

Carnap finds it convenient to extend the notion of equivalence from a connective between statements to ~~a~~ connective between predicates, relations, individual terms and functions. Two relations of degree n (taking predicates as relations of degree one), "R" and "R^I", are equivalent "iff" $(x_I)\ldots(x_n)$ [$Rx_I\ldots x_N \equiv R^I x\ldots x_n$]" is true. And two individual terms are equivalent iff if they refer to the same thing, i.e., iff the identity relation works between them. Besides the notion of equivalence, we have of course the notion of L-equivalencein an extended use as well. Collectively, Carnap refers to statement terms, predicates and relations, individual terms and functions as designators. So for all types of designators he has defined equivalences and L-equivalences.

By means of these notions, Carnap arrives at two results which are generalisations to all designators of what has been said about individual terms:

"If two designator signs are equivalent, then any two sentences of simplest form (in S_I: atomic form) which are alike except for the occurrence of the two designator signs are likewise equivalent.

"If two designators (which may be compound expressions) are L-equivalent, then any two sentences (of any form whatever) which are alike except for the occurrence of the two designators are likewise L-equivalent." (pg. I6)

Carnap then gives the following definitions:

"Two designators <u>have the same extension</u> (in S_I) $=_{Df}$ they are equivalent (in S_I)

"Two designators <u>have the same intension</u> (in S_I) $=_{Df}$ they are L-equivalent (in S_I)." (pg.23)

Obviously these definitions do not define "intension" and "extension", but only the phrases "have the same extension" and "have the same intension". As Carnap puts it, it is now a matter of looking for the appropriate entities to be the extensions or intensions for the various sorts of designators.

For sentences (or statements) the obvious sort of "entities" to choose as extensions are truth values, since two sentences are equivalent iff they have the same truth value. The intensions of sentences are propositions in the sense of what are expressed by sentences. The extensions of predicates are classes, and their intensions are properties, and the extension of an individual term (or individual expression, as Carnap calls it) is an individual, and its intension is what Carnap calls an individual concept.

The same results can be extended to variables. A variable of the same type as some designator will have <u>value extensions</u> and <u>value intensions</u> of the same sort as the designators. Thus a sentential variable ranges over truth values as its value extension and propositions as its value intensions. Predicate variables range over classes as their value extensions and properties as their value intensions. And individual variables range over individuals as their value extensions and individual concepts as their value intensions.

I might quickly mention two more results which Carnap arrives at before discussing his solution to quantification and logical modalities. First, this multiplication of entities (two types of entities - extensions and intensions- for each type of designator) need not [redacted] be repeated in the metalanguage. To begin with, the object language has only one expression for both extensions and intensions. And in the metalanguage itself one can replace any talk about extensions or intensions by talk about equivalent or L-equivalent expressions. In such a way we arrive at what Carnap calls a neutral metalanguage.

Secondly, a sentence, it was mentioned earlier, is L-determinate iff its truth value can be determined by semantical values alone. But the truth value of a sentence is its extension. This suggests the generalisation of L-determinacy which Carnap accepts:

"A designator is L-determinate in S if and only if the semantical rules of S alone, without addition of factual knowledge, give its extension." (pg.72)

In particular, we can talk about L-determinate individual terms - individual terms whose referents can be determined by means of the semantical rules alone.

There is also the notion of an L-determinate intension, viz., the intension of an L-determinate designator. We have the result that any designators which are L-equivalent have the same intention because of the definition given earlier of "have the same intension". So to each extension ther e corresponds at most one L-determinate intension. In the case of individual terms, if we take a restricted class of individuals, it is possible to have a one-to-one correspondence between L-determinate intensions and the individuals in this restricted class. The example which Carnap takes is that of a coordinate language in which each individual is associated with a number, or an ordered pair, triple or quadruple of numbers. In general, this reduction of individuals to individual concepts is possible when some mathematical entity is associated with each individual and these entities are of a sort that they can be serially ordered.

Now Carnap finds it necessary to take intensions as the values of the variables in a formula in which the scope of a quantifier includes a variable within the scope of a modal operator. That he cannot take them as extensions is evident from what we said earlier. But there are only a finite number of individual terms that are actually ever expressed in any language, thus there are only a finite number of individual concepts that are ever actually expressed by a language. So Carnap extends the notion of individual concept to include an infinite number of things, any of which might be expressible, but all of which are not in fact expressed because they are infinite in number.

Taken in such a sense, there could not be a possible individual concept which is not also actual. So those and only those commutation relations are valid for formulae containing quantifiers and logical modal operators as are valid for formulae containing quantifiers and conditional modal operators.

Whether all sorts of individual terms express individual concepts is not at all clear. It is fairly obvious that definite descriptions do. For with a definite description the individual is referred to, one might say, by means of certain predicates which, taken together, distinguish it. But it is not clear whether proper names can be associated in such a way with predicates. At any rate, these predicates do not usually occur in the proper name as they do in definite descriptions; and if a proper name contains a predicate or is derived from a predicate, there is no reason why the individual to which this name applies should necessarily have the property in question. (A girl called "Prudence" may be very imprudent.) One class of names, if they can be called that, from which some of the properties of their referents can be determined, are numerals. (Not all of the properties of numbers are determined by numerals; for example, that nine equals the number of planets is an astronomical, not an arithmatical, fact) That there are individual terms for which one is hard put to find a corresponding individual concept is important to Carnap's project in that it reflects on the generality, or rather lack of generality, of it.

The difficulty of identifying individual concepts for names is implicitly admitted by Carnap in that he considers "a=b", where "a" and "b" are names, as always L-indeterminate -- that is "L(a=b)" is always false. It follows that "a" and "b" never express the same individual concept. But "a=b" is not L-false either. Now if names expressed individual concepts and there is no logical or linguistic reason why they should not express the same individual concept, one would expect that it would be possible for there to be two names which express the same individual concept. But Carnap rules out this possibility.

It is certainly applicable in the case of definite descriptions. But its applicability to most sorts of names is very dubious, and, if it were used here, it would certainly be very strained. One consequence of this is with regard to quantification. It is valid, in extensional logics, to infer $\emptyset(a)$ from $(x)\ \emptyset(x)$, where "a" refers to any individual at all. Although there is no reason to rule out descriptions as instances of individual constants, the most natural, if only because the most general, interpretation of individual constants is as names.* For here we are talking about the extension and simply want some sort of tag for the various objects without saying anything at all about ~~there~~ their properties. However, if there are no individual concepts expressed by these constants, there is no intensional object for "a" in "Øa" to refer to, and hence in an intensional logic, "Øa", in its general use, becomes meaningless. The only way out of this is to restrict the class of expressions themselves which can be used as instances of a variable which is bound by a universal quantifier. The same result applies to variables bound by existential quantifiers as well, as one can see by considering existential generalization.

Another difficulty arises with regard to the semantics for an intensional logic. Carnap takes as the range of variables objects in the usual sense. But the only way he can do this is to have something like his coordinate language discussed earlier in which there is a one-to-one correspondence between extensions of individual terms and L-determinate intensions. This is true of any language system which maps objects of any sort onto numbers. And

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~~* This is of course if one does not follow Russell and introduce definite descriptions by definition. We can define "ψ (ix) Øx" as "(∃'x) Øx · (y)(Øy ⊃ ψy)" where "(ix) can be read as "the x such that..." and (∃'x) can be read as "there is only one x such that..." "(x) ψx ψ(ix) Øx" is then short for "(x) ψx ⊃ ((∃'x)Ø x · (y) (Øy ⊃ ψy))" which is invalid.~~

the intensions of numerals are L-determinate. But in the general case there is no such one-to-one correspondence between extensions and L-determinate intensions, unless one carries physics to absurdity and has every object reduced to a spatio-temporal extension in some all-encompassing coordinate system. So in gneral one must have a semantics in which the domain contains intensional objects for the variables within the scope of a modal operator but bound by a quantifier outside this scope and objects in the usual sense for the range of the other variables. But we can also consider individual concepts as the range of variables which are bound by quantifiers which are within the scope of a modal operator but do not "quantify into" the scope of an additional modal operator.

Thus we can consider the range of "x" in "L(x) Ø(x)" as consisting of individual concepts, even though the range of "x" in "(x) Ø(x)" consists of extensional objects. This is the point of Carnap's [redacted] neutral metalanguage. For any object there is, it is possible to give a definite description for it, even though it is not possible to give a definite description to all objects. Thus, although we do not have a one-to-one correspondence between intensions and extension (we often have two or more definite descriptions which are not L-equivalent and describe the same object), we have the result that if a formula is valid in extensional logic, then when prefixed with "L" it is valid in an intensional logic as considered here. For in the wide sense of individual concept used here --- it is expressible, but need not be expressed ---we will have every possible object described; thus every domain which is formable will be a subclass of the domain of intensional objects.

So "L" and "M" can be taken as intensional operators in the sense that any variable, whether free or bound, which occurs in their scope ranges over intensional objects. But this means that we shall have two [redacted] sorts of domains: one containing objects in the usual sense; one containing intensional objects. However, we could consider the domain of intensional objects as the union of all domains.

There is a difficulty, however, with certain definite descriptions in that they refer to different objects at different times, and, possibly, places (e.g."the top card in the deck"). The way out of this difficulty is furnished by Quine, although in a different context. The difficulty which Quine faces is that often sentences change their truth-values depending on when and where they are uttered. His solution here is to introduce the notion of "eternal sentences" ---that is, a sentence which is identical with one of the problematic sentences, with the exception that it includes a reference to when and where it was uttered.

Thus an eternal sentence does not change its truth value. Now the same thing can be done for definite descriptions; viz, include in them a reference to where and when they were uttered. Thus these "eternal definite descriptions" always refer to the same object.

~~I shall briefly mention two undesirable, but not completely unacceptable, results of making all definite descriptions eternal. First, it is rather akin to, though not necessarily the same as, the coordinate language proposed by Carnap. Secondly, it certainly deviates from ordinary usage.~~

It was mentioned earlier that Carnap defines "L-true" in terms of grounds; in particular, in terms of grounds based on meaning. The result is that intension, and in particular individual concepts, are defined in terms of the grounds afforded by meaning. Thus two individual terms have the same intension if they can be shown by the analysis of their meanings alone to refer to the same object. But we have seen that there are other grounds for applying logical modal terms besides those afforded by meaning. In particular, there are those afforded by imagining and by entailment. I shall here briefly indicate some of the results which follow from defining "have the same intension" in terms of these other grounds.

We can characterize a statement as L-true according to the grounds afforded by imagining if one cannot imagine the statement being false. We can then derive all the definitions for imagining which Carnap derives for meaning. In particular, we have: Two individual terms, "a" and "b" have the same intension iff one cannot imagine the statement "a=b" being false.

Besides the fact that imagining does not afford strict grounds for logical necessity and that the grounds which it affords are often very unreliable, there is also the difficulty of finding appropriate entities to be intensional objects in connection with imagining. What we imagine are ships, cars, trees, flowers, etc., i.e., objects in the usually sense; so intensional objects would apparently be concerned with <u>how</u> we imagine something just as individual concepts are concerned with how we refer to something.

But the question how we imagine something is a psychological question as it stands. Perhaps we could explicate this question so as to make it objective and less psychological. Perhaps we could consider how we <u>must</u> imagine something described by an individual term. Thus one might arrive at a notion of an intentional object in connection with imagining as those facets of how we imagine something which are both sufficient and necessary to justify one in saying that he is imagining an object described by the particular individual term in question. It is hard to see what meaning, though, can be given to the phrase "those facets of how we imagine something" unless one reverts to psychologism.

In conclusion, the difficulties for giving a construal of "intensional object" in terms of imagining appear to be immense, if not insurmountable.

Likewise, we can characterize a statement as L-true according to the grounds afforded by entailment <u>iff</u> that statement is entailed by a logically necessary statement. And we can derive all the definitions in terms of grounds afforded by entailment which Carnap derives in terms of grounds afforded by meaning. In particular, we could derive: Two individual terms, "a" and "b", have the same intensions <u>iff</u> "a=b" is entailed by a logically necessary statement.

But there is a difficulty in finding appropriate entities for intensional objects defined in terms of entailment which were not encountered in finding entities for intensional objects defined in terms of ~~entailment~~ meaning. This difficulty is that, although individual terms have meaning, only statements, and not individual terms, are entailed by other statements.

However, we can investigate the dependence of entailment on individual terms in so far, at least, as there are entailment relationships between statements which differ only in that one has a certain individual term in a position(s) where the other statement has a different individual term. Also, there are valid and invalid entailment relations between statements which differ only in that one has an individual term where the other has a variable (and perhaps also is preceded by a quantifier absent in the other), or one has one individual variable where the other has a different individual variable. The dependence of entailment relations on individual terms and variables, then, would apparently furnish a notion of intensional object.

To show how the notion of intensional object might be used with regard to a deductive system (in the loose sense), I shall take an example of an informal proof in geometry. Let us take a triangle having a certain property, say having sides of lengths three, four, and five units. Let us form a definite description by means of this property, say "the geometric figure with sides of three, four, and five units." We can prove that this geometrical figure must be a right triangle. If we symbolize the definite description given above by "a" and the predicate "is a right triangle" by $\emptyset$, then what we have proved is "L $\emptyset$(a)". But we have also proved that there is something which is necessarily a right triangle; namely, the geometrical figure with sides of three, four, and five units. So we have a proof for "($\exists$ x)L $\emptyset$(x)".

This example shows that it is possible to quantify into a logical modal context when the grounds for "L" afforded by entailment.

The specificity of the sort of ground afforded by entailment was discussed in Chapter Three, section A. It is only to be expected that this specifity should be a factor in the formulation of a notion of intensional object in terms of entailment. A formulation of this sort would be a result of the investigation of the dependence of entailment relationship on individual terms and variables. And different sorts of deductive systemts have different sorts of theorems, rules of inference or acceptable methods of proof, etc.

CHAPTER FIVE

CONCLUSION

Natural, conditional, and imperative uses of modal terms present no logical problems. I have considered these three in connection with the two most important purely logical problems for modal statements: iterated modalities and quantification into modal contexts. We found that the attempt to formulate iterated-modality statements containing modal terms of only one of these uses resulted in redundancies. The attempt to formulate iterated modalities by taking these uses together resulted in either meaningless or redundant statements. The problem, then, of iterated modalities does not arise for these three uses. We also saw that quantification into the scope of a modal term of any of these uses is unproblematic. We were able to determine unambiguously commutation rules for statements containing initial occurrences of quantifiers and modal terms for each of these uses.

These uses present no logical problems; yet, natural and conditional modalities in other ways are philosophically problematic. In Chapter One, sections C, D, and E, it was shown that the natural use of modal terms is well established in ordinary discourse. But problems with these uses are apparent even here. Someone might say "It is (naturally) necessary for X to K." A common response to this is: "Yes, but is it really necessary?" Evidently, then, in ordinary discourse degrees of natural necessity are distinguished. This can be intelligently interpreted in accordance with the explanation of natural necessity presented in Chapter One, section E. There "It is naturally necessary for X to K" was rewritten as: "X's nature is such that X is prevented from not K-ing." For "weaker" natural necessity statements we could say "almost prevented". And one natural necessity would be "stronger" than another if the object(s) is more strictly prevented from not-K-ing.

It is apparently universally held by philosophers that modal terms (other than, perhaps, epistemic uses of them) are absolute; namely, that there are not degrees of necessity, possibility, or permissibility. For logical modalities a statement is either true because of its meaning or not, to cite one example. But with regard to natural uses of modal terms, only two alternatives seem to be the case: (a), The view that modal terms are absolute is an incorrect dogma, or (b) There is a grave philosophical difficulty here for the notion of natural necessity.

To find examples of natural necessity is a problem. Every-day experience presents little which is necessarily such-and-such. To argue that such-and-such must be the case the saffest course is to appeal to scientific laws. But many scientific laws are hardly presentable as natural necessity statements. For example, what sort of a statement of natural necessity would be furnished by "The speed of light _in vacuo_ = c"? What would prevent the speed of light from not being c?

Conditional modalities are not used in ordinary discourse, but are abstractions from a use which does occur in ordinary discourse. I called this use, which occurs in ordinary discourse, a hybrid use because it is analyzable as a combination of natural and conditional uses.

Conditional modal terms evidently are not absolute either. This can be interpreted in the same way as the non-absolute character of natural modalities. For the formulation of conditional modal statements in Chapter One, section E, was in terms of preventing as well. There is also the difficulty of finding unambiguous examples of the hybrid modalities, except, of course, when it is the case that X K's, hence possible for X to K. Finally, at pages 38 and 39, it was mentioned that conditional modality statements are not of much interest for scientific investigation since they lack the generality of natural ones. This certainly reflects on the usefulness of conditional modalities.

The difficulties I have mentioned with regard to natural and conditional modalities do not arise in connection with imperative modalities. A simple explanation for this is that imperative modality statements are so closely connected with human creations; namely, commands (but to be taken in the wide sense here). We could similarly say that imperative modalities (and not just statements of them) are human creations. I did mention, however, at page 13, and then again on pages 55 and 56, that imperative modalities have been rather artificially distinguished from other uses of modal terms. We might call this wider use, from which the imperative use has been distinguished, the <u>deontic</u> use of modal terms. It should be obvious, from what I said at pages 53 and 54, that not all deontic modal statements are based on commands. So we talk about many deontic modalities which, if they exist, are not human creations. For these modalities we would encounter some of the same problems which are en-

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~~* (from page 147): I might mention, though, an explanation of why natural and conditional modalities are often dismissed unreflectively. (This explanation, however, is more *ad hominem* in nature rather than philosophical.) With a slight knowledge about modalities, especially as they were treated by early modern writers, one often treats the construals of modal statements as appropriate for *all* modal statements. And it is usually, if not always, possible to imagine exceptions to statements of natural or conditional necessity or impossibility. Imagining affords grounds appropriate only for logical modalities. To take natural and conditional modal statements as explicable in terms of imagininging is an error.~~

countered in connection with natural and conditional modalities. For example, someone might say, "It is necessary not to lie." Someone might retort, "But is it *really* necessary not to lie?" And we also have the problem of all the circumstances which would exempt a person from a deontic necessity.

Problems were found, however, for the logic of epistemic uses of modal terms. The most crucial problem mentioned was that one cannot always deduce a conclusion, even given premises which are certain and a valid inference schema, which is certain. For one must be certain also that the inference schema is valid. As a consequence of this, it can be shown that one cannot quantify into the scope of an epistemic modal term in the usual way; further, there is no apparent way of reinterpreting quantification to accomplish this.

Epistemic uses of modal terms present philosophical problems of other sorts as well. For example, it was noted in Chapter Two, section E, that a ground for a certainty statement must satisfy the condition that it is a reliable ground for the statement derived by deleting the phrase "It is certain that". But reliability is not an absolute notion either; namely, there are degrees of certainty. On page 15, the footnote, I said that "certain" would be taken in a sense such that "It is certain that p" entails "p"; consequently, "It is certain that p " entails "It is known that p". The converse too may hold. But knowledge is not capable of degrees. In retrospect, then, the condition that "certain" be taken so that "It is certain that p" entails "p" appears to be too strong."It is certain that p" could be better interpreted as "All indications are that it is known that p". However, all the distinctions between certainty and necessity which were made in Chapter One are nevertheless valid on this interpretation.

On the other hand,, the logical use of modal terms is philosophically problematic mostly in so far as it is logically problematic. For example, logical necessity is an absolute notion; namely, what a statement expresses is either logically necessary or not. Logical necessity was explained as truth by virtue of meaning. Truth is an absolute notion (in the sense used here). Most of the statements with which we are concerned in logical and mathematical disciplines have a definite meaning.*

~~* In other areas which furnish examples of logical necessity and impossibility, e.g., philosophical analysis, this might not always be the case. In this event, we ~~would~~ might introduce at least one more truth value. The consequences of introducing a third truth value would be quite apparent for iterated modalities. There might also be consequences in connection with intensional objects.~~

Examples of true logical necessity statements are quite easy to come by: e.g., "It is necessary that 2 + 2 = 4."

There are, however, serious logical problems with logical modalities. In Chapter Four, section B, it was shown that quantification, as it is usually interpreted, into the scope of a logical modal operator leads to contradictions. The remedy is to reinterpret the notion of an object. Here we encounter a problem which appeared with regard to iterated modalities as well: we have no precedent in this problem; so the best we can do is to formulate notions in terms of the grounds afforded for logical modalities. This leads to a multiplicity of notions of intensional object with no way for determining which notion is "the correct one".

In the formulation of a notion of an intensional object in terms of the grounds afforded by meaning, not only must we reinterpret what is meant by "object", but the substitution instances of variables must be restricted to certain types of terms. And these terms must be formulated in a particular way. Presumably the substitution instances of variables which range over intensional objects, the notion for which is furnished by entailment, would be similarly restricted. So quantification into logical modal contexts also suffers from a lack of generality.

A multiplicity of results, dependent on which sorts of grounds logical modal terms are construed in terms of, is the best that can be arrived at in connection with the problem of iterated logical modalities.

Here it is tempting to say that the reduction rules arrived at by considering one of the sorts of ground are the correct reduction rules. That a statement is logically necessary has been explained as: that it is true by virtue of the meanings of its constituent terms. And a sort of ground for "Lp" was given as that "p" has been shown to be true by virtue of the meanings of its constituent terms. We construed, for example, "LLp" when considering the sort of ground afforded by meaning as "'p' is true because of the meaning of its constituent terms' is true because of the meanings of its constituent terms." And an explanation of "LLp", one would suspect, since the explanation of "Lp" is similar to the ground for it, would be similar to this.

But here it is essential to make the distinction between the use of a term and the ground for its application. " 'p' is true by virtue of the meanings of its constituent terms" is about the typographical entities which constitute "p". But "Lp" is <u>not</u> about the typographical entities; for "p" does not occur within quotation marks in "Lp". Nor is "LLp" about the typographical entities which constitute "Lp". This point is further brought out by the fact that the following is a valid rule of inference for logical necessity:

$$\frac{\begin{array}{l}Lp \\ \text{"p" entails and is entailed by "q"}\end{array}}{Lq}$$

That is, what might be called "strictly equivalent statements" are substitutable in logical modal contexts. We can now see that " & 'p' is true by virtue of the meanings of its constituent terms ' is true by virtue of the meanings of its consituent terms", the construal of "LLp" in terms of the grounds afforded by meaning (with some adaptions mentioned in Chapter Three, section A), is not the only possible construal, nor is it "the" correct construal of "LLp"; for this statement is about the typographical entities which constitute " 'p' " and "LLp" is not.

"Lp", where "L" is the logical use of "necessary", is explained as " 'p' is true by virtue of the meanings of its terms" because "p" gives us a way of referring to its occurrence in "Lp". The point made above is that it is the <u>meanings</u> of the terms constituting "p", not the typographical entities, which are essential for the truth or falsity of "Lp". But, of course, we have no way of specifying meanings except by using typographical entities, or series of noises, etc.*

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* The discussion in the last paragraph is closely related to the discussion in Chapter Four, section B, about "L" and "M" as intensional operators.

For a deductive system, it is often held that formulae -- typographical entities -- are related by entailment. We say that "p" is derivable in the system, not That p. But in Chapter Three, section A, we saw that the grounds afforded by entailment furnish the most comprehensive basis for iterated modalities when we do not restrict the entities related by the entailment relationship to typographical entities. This is closely connected to the fact that "Lp" is not about the typographical entities which constitute "p" and shows what difficulties may arise when the grounds for the application of a term are conflated with its use.

The conflation of the grounds for the logical use of modal terms and that use itself is very apparent in certain articles by Quine.* He interprets "L" either as "It is analytic that..." or as a semantical predicate based on formal semantical systems. In either of these cases, "p", the subordinate clause in "Lp", occurs within quotation marks within the scope of the semantical predicate. Quine is then able to treat logical modal contexts in the same way as quotational contexts. In particular, he concludes that iterated logical modalities are usually, if not always, pointless and that quantification into a logical modal context is even more problematic than was suggested in Chapter Two,

*Cf., <u>From a Logical Point of View</u>, Chpt viii; <u>The Ways of Paradox</u>, Chpt 13 and 14.

section B.

But it is not the case that a logical modal context is a quotational context, or even that logical modal contexts are in all ways similar to quotational contexts. There are important differences between the two. For example, if "p" and "q" are strictly equivalent, then they are interchangeable in logical modal contexts, but not necessarily in quotational contexts.

~~The relation between the grounds for the logical use of modal terms and that use itself does present a serious objection to modal logic however. We have seen that the best indications we have for accepting formulation and reduction rules for iterated logical modalities are furnished by the grounds for these modalities. Likewise, the best indications for formulations of an appropriate sense of "object" for interpreting quantification into logical modal contexts are furnished by the grounds, for logical modalities. All of these grounds with the possible exception of those afforded by imagining, which are very dubious anyway, are supplied by fields which are studied intensely in connection with non-modal logic. So the development of logical modal logic is dependent on the results established by investigations of non-modal logic, but not vice versa. And if the logic of logical modalities is an after-the-fact formulation of results established in non-modal logic, what is the point in formulating logical modal systems in the first place? --- especially since very difficult problems arise in modal~~

~~logic, e.g., with regard to iterated modalities and quantification, which are not encountered in non-modal logic.~~

Throughout the discussion of logical modalities, I referred to three sorts of ground: those afforded by meaning, entailment, and imagining. The inclusion of the grounds afforded by imagining was more-or-less by courtesy. Before this century logical modalities were often construed in terms of what is or is not imaginable. And even as a recent a text book on modal logic as An Introduction to Modal Logic of Hughes and Cresswell, to which I referred, gives interpretations of the semantics for logical modal systems mostly in terms of imaginability. And perhaps the most intuitive or every-day notion of logical modalities is in terms of imaginability.

But we say that the grounds afforded by imagining furnish no basis for the investigation of iterated modalities. And the problems of defining a notion of intensional object in terms of imaginability are apparently insurmountable. Even more basic than this, though, is the fact that in Chapter Two, section C, grave difficulties were encountered in the attempt to give formulations of the sort of ground afforded by imagining. In the end, appeal had to be made to possible explications of "imagining" in a last-ditch attempt to provide such formulations.

Appendix One

Ayer on Necessity.

In his book, Language, Truth and Logic, A.J. Ayer presents the view that the only proper use of "necessary" is its logical use. In arguing for this view he makes a confusion which will become obvious to one if one will keep in mind the five distinct uses of modal terms I have presented. Ayer argues thusly:

> "Having admitted that we are empiricists, we must now deal with the objection that is commonly brought against all forms of empiricism; the objection, namely that it is impossible on empiricist principles to account for our knowledge of necessary truths. For, as Hume conclusively showed, no general proposition whose validity is subject to the test of actual experience can ever be logically (sic.) certain. No matter how often it is verified in practice, there still remains the possibility that it will be confuted on some future occasion....
> "Where the empiricist does encounter difficulty is in connection with the truths of formal logic and mathematics. For whereas a scientific generalisation is readily admitted to be fallible, the truths of mathematics and logic appear to everyone to be *necessary* and *certain*. But if empiricism is correct no proposition which has a factual content can be *necessary* *or* *certain*. (p. 72, my italics)

Although Ayer does not explicitly state that "It is necessary that..." is equivalent to "It is certain that...", he argues as if this is the case. For where he does have an argument that something is (not) necessary, he takes this as an argument that it is also (not) certain, and vice versa.

In particular, Ayer wants to identify "certain" with the logical use of "necessary". I have earlier argued in chapter one, section E, that this identification is erroneous. For there are statements which, if they are true, are logically necessary, but are not certain (e.g., Fermat's last Theorem). And there are statements which are certain but are not logically necessary, or in any other use necessary.*

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* The late J.L.Austin gave a convincing criticism of restricting "certain" to particular forms of statements. (Cf. esp. Chpt. 10 of Sense and Sensibilia.)

Even some positivists usually admit a type of statement which is certain but not logically necessary in statements about sense data.

Admittedly, it is doubtful whether a statement of natural or conditional necessity is at all times, or ever, certain. But this is no objection to these uses of "necessary" since (a) not all statements of logical necessity are certain either, and (b) "It is certain that..." is not equivalent to "It is necessary that...", and (c) there are a large number of empirical statements (e.g. generalisations about an infinite or open class) which we find essential to employ in investigations of the physical world and about which we are not, nor could we be, certain.

Positivists rarely, if ever, discuss imperative necessity. An imperative necessity statement is often known with certainty. For example, if Parliament has passed a law which forbids anyone to drive more than 30 m.p.h. in a built-up zone, then it is certain that it is necessary that one drive not more than 30 m.p.h. in a built-up zome. Thus this use of "necessary" has in common with the logical use, and is distinct from the natural and conditional uses, that necessity statements of this use can be known with certainty.

I hope to have shown in this appendix not that all of the positivists' objections to non-logical uses of "necessity" are wrong, but only that one particular objection to these uses is based on a confusion. The objection that logical necessity statements are the only necessity statements which can be known with certainty is probably one of the more important of the positivists' objections to non-logical uses of necessity, and for one author, Ayer, it is the only objection he presents.

Appendix Two

Toulmin on Modalities and Elliptical Occurrences of Modal Terms.

Toulmin, in his book, The Uses of Argument, presents the view that the validity of modal statements is dependent on the argument we have for the truth of the sentence which is qualified by the modal term:

> "It is the quality of the evidence or argument at the speaker's disposal which determines what sort of qualifier he is entitled to include in his statements; whether he ought to say, 'This must be the case', 'This may be the case', or 'This cannot be the case'; whether to say, 'Certainly so-and-so', 'Probably so-and-so', or 'Possibly so-and-so'." (p. 90)

Here again we are confronted with a conflation of non-epistemic uses of modal terms to epistemic uses. Toulmin's construal of qualifiers is correct for epistemic uses, but is incorrect for non-epistemic uses of modal terms. However, non-epistemic uses of modal terms are often used elliptically in every-day discourse in such a way that the truth or falsity of the statements in which they occur is dependent on the state of our knowledge.

It is not difficult to see that Toulmin is correct with regard to epistemic modalities. Epistemic modal statements are relative to knowledge, and what one argues from is part of one's knowledge. So "...the quality of the evidence or argument at the speaker's disposal..." is just what the validity of epistemic modal statements depends on.

But we found that the other uses of modal terms do not depend for their applicability on the knowledge of the utterer. And it is a direct consequence of this that they do not depend for their applicability on the arguments which one has at his disposal. Modal propositions which assert a non-epistemic use of modal terms are discovered. Were there no people at all, it still would be necessary that all brown cows are brown, and probably necessary that all bodies are attracted to each other in accordance with the equation: $F=G\frac{m_1\ m_2}{r^2}$.

In the quotation give above, Toulmin mentions only modal terms which have an epistemic use ("must", "may", "cannot", "certainly", "probably", and "possibly"). But in the sequel he goes on, in particular, to use the term "necessary" in the same sort of context. To see exactly what this context is, it would be best to present his general schema for arguments:

$$\begin{tikzcd} D \arrow[rr] & {} \arrow[d, dash] & \text{So, } Q\,C \arrow[d, dash] \\ & \text{Since } W \arrow[d, dash] & \text{Unless } R \\ & \text{On account of } B & \end{tikzcd}$$

(Given at p. 104)

To put it briefly, "C" is the claim which is being put forward, and for which the argument is constructed to give support. Basically, it is the conclusion of the argument. "D" is the <u>data</u>, "...the facts we appeal to as a foundation for the claim..."(p. 97) According to Toulmin, we produce the data when challenged with: "What have we got to go on?" "W" is the <u>warrant</u> which justifies us in taking the step from the data to the claim. Briefly, the warrant is what is produced when, after the data and claim are presented, one is challenged with:"How do you get there (i.e.,from the data to the claim)?". "B" is the <u>backing</u> for the warrant. This is the evidence for the warrant itself. "R" are the conditions of exception or rebuttal "...indicating circumstances in which the general authority of the warrant would have to be set aside."(p. 101) "Q" is for the <u>qualifier</u> of the claim, which is what is important to my essay. Toulmin says of qualifiers:

> "Warrants are of different kinds, and may confer different degrees of force on the conclusion they justify. Some warrants authorise us to accept a claim unequivocally, given the appropriate data these warrants entitle us in suitable cases to qualify our conclusion with the adverb 'necessarily'(sic); others authorise us to make the step from data to conclusion either tentatively, or else subject to conditions, exceptions, or qualifications --- in these cases other modal qualifiers, such as 'probably' and 'presumably' are in place. It may not be sufficient, therefore, simply to specify our data warrant and claim: we may need to add some explicit reference to the degree of force which our data confer on our claim in virtue of our warrant. In a word, we may have to put in a <u>qualifier</u>."
> (p. 101)

It would be absurd to deny that we do not have arguments in which the conclusion contains a non-epistemic modal term. But this is explainable in terms of Toulmin's schema without restricting these terms to epistemic modalities. To concentrate just on "necessity"(the non-epistemic uses of "possible" would be parallel cases), I shall mention three sorts of arguments which fit into Toulmin's schema and from which a sentence qualified by the term "necessarily" can be inferred and hence in which the qualifier (if indeed we should retain this term) is not epistemic:

(a) when D is of the form "Lp" and W is of the form ""p" entails "q"," then we can infer "Lq".

(b) When W is of the form "If p, then Lq" and D is "p", then we can infer "Lq".

(c) Toulmin mentions ~~a type of~~ arguments which he calls warrant establishing and are to be contrasted with so-called warrant using arguments, which are of the form set out earlier in this appendix. A warrant establishing argument is basically one concerned with the relationship between the backing for a warrant and the warrant itself. Now under this class of arguments we can include the whole range of grounds for applying modal terms discussed earlier. Then in this case we have a modal conclusion and non-modal premises. This is perhaps best seen in the case of imperative necessity, where the backing is a law and the warrant is a statement that it is necessary for all persons (who satisfy a certain condition) that so-and-so. This warrant can then occur in a warrant using argument where, perhaps, D states that a certain person is such-and-such, and C states that it is necessary for this person to so-and-so.

But this hardly explains why Toulmin conflates non-epistemic with epistemic modalities. Now there is a common fallacy involving "necessary" and "If...,then..." A sentence of the form "If p, then necessarily q',as used in ordinary discourse, can be taken in two ways: First, as the same as "Necessarily (If p, then q)"; secondly, as simply, "If p, then necessarily q". In the first case we can, given "p", derive only "q"; but in the second case we can derive "necessarily q". The following example may instance how the ambiguity may lead to problems: If I know that "p", where "p" is any proposition whatsoever, then it is necessarily true that p. Taking this in the second sense, we have the result that everything which I know is a necessary truth. This is absurd. The mistake, obviously, is in taking the conditional in the second sense, with which it is grammatically identical. But in fact the adverb "necessarily" modifies the whole conditional, and not just the consequent. The distinction between "Necessarily (if p, then q)" and "If p, then necessarily-q" was made by medieval logicians.

The former was known as "necessitas consequentiae", and the latter as "necessitas consequentis".

Now it is my view that Toulmin is guilty of the same fallacy, but in terms of mention rather than use, which was discussed above in connection with" if...,then..." We can say that an argument schema is necessary, i.e., that given D and W, the conclusion necessarily follows (and this necessity need not be logical necessity --- I am not restricting this discussion to entailment). Or we can say that a certain argument in which a proposition of the form Lp occurs as the conclusion is valid, i.e., that given D and W, the conclusion that necessarily p follows. Both these forms of argument can be expressed as: "Given D and W, we have necessarily p".

The conflation of the two cases mentioned in the last paragraph does indeed lead to fallacies. It is a valid argument to conclude that Anne has red hair from the warrant that all of Mr. Smith's daughters have red hair and the datum that Anne is one of Mr. Smith's daughters. We can indeed say that given that all of Mr. Smith's daughters have red hair and Anne is one of Mr. Smith's daughters, necessarily Anne has red hair. But it is not a necessary truth that Anne has red hair.

Toulmin fails to make this distinction. We can agree that in ordinary language the following argument schema often appears with the "necessarily" substituted for Q:

$$D \xrightarrow[W]{\quad\quad} Q\ C$$

(We can safely ignore "B" and "R" for the remainder of this section) To make things more difficult, the warrant is often expressed in the form "If D, then necessarily C". And this leads straight to the fallacy involving "if...,then..."

The point is that it is vital to distinguish between:

$$(1)\quad D \xrightarrow[W]{\quad\quad} Q\ C$$

and:

$$(2)\quad Q(\ D \xrightarrow[W]{\quad\quad} C)$$

Indeed, Q in either case can be an epistemic or non-epistemic modality. But, maintaining Toulmin's distinction between a warrant and its backing, we can put a condition on the occurrence of a non-epistemic modality for Q in (1): A modality of the same use must occur in either D or W. Epistemic modalities, on the other hand, do not come under this condition. A non-epistemic modality may be established in a warrant establishing argument as Toulmin calls it, in so far as this warrant establishing argument coincides in what I have termed the using of particular sorts of grounds in determining whether or not a particular use of a modal term is applicable in a certain case.

In many cases where we would be able to offer an argument for a certain proposition, the data and warrants are common knowledge. And for many of these arguments the conclusion would follow necessarily from the premises. (Here again it need not be logical necessity.) But if the data and warrant are not only true but common knowledge, it is rather a waste of breath to state them. Nonetheless, we still speak as though an argument were being given. We would express this situation schematically as:

$$\left[L\ (D \xrightarrow[W]{\quad\quad} \right] C\,)$$

where what is bracketed is understood, and not presented. But we have seen that, grammatically, the qualifier is often stated with the claim, or conclusion. In this case, we would have, at least grammatically, the following:

$$\left[D \xrightarrow[W]{\quad\quad} \right] LC$$

But this is logically usually of the form,

$$L\ \left(\left[D \xrightarrow[W]{\quad\quad} \right] C \right)$$

This elliptical use of "necessary" has in common with "certain" the feature that it depends on our knowledge. For when we assert "Necessarily p" elliptically, we must know some reason for "p" being true and that this reason necessarily implies that p. But we need not admit a new and different use of "necessary" (the "elliptical" use) which is an epistemic use. It is explainable in terms of the other uses of "necessary", and, for any occurrence of this elliptical use, when the ellipsis is eliminated, the resulting modality is just an instance of one of the other uses of "necessary".

That "necessary" is often used elliptically explains many rather confusing instances of necessity statements. Somebody might say"Mr. Smith necessarily has a garage." Here the ellipsis can be eliminated as follows, "Mr. Smith has a garage" follows with (logical) necessity from "All people who live on Bash Street have garages" and "Mr. Smith lives on Bash Street": and we know that all people who live on Bash Street have garages and that Mr. Smith lives on Bash Street.

Appendix Three

"Can", "Must" and Modal Terms.

Ayers uses a terminology in The Refutation of Determinism which is somewhat misleading. This is his use of "x has the power to k" as interchangeable with "It is possible for x to k." The word "power" is fairly general and fairly vague, and I hope to show that it can be used in a way which is not analysable in terms of possibility.

When we say of something that it has the power to k, we can also say that it can k. But the converse of this does not hold. An obvious use where we would say "x can k" without attributing a power to x is with regard to social conventions or properties. For example, if someone says to me "You can go to the party in your scruffy clothes", he is not saying anything about my powers. Using "can" where one could use "permissible" in the imperative sense does not attribute a power to someone either. In general, where "can" is used in a way such that the statement or clause in which it occurs is supported by a statement describing an actual state of affairs, there "can" is used in a way which attributes a power to a thing or person. That is, where we have p is evidence for Cp (Cp being a statement of the form "x can k"), Cp attributes a power to a person or thing. So what I want to do amounts to showing that statements of the form "x can k" which satisfy the above condition are not always analysable as "It is possible for x to k".

The cases I shall consider in this regard are concerned with knowing how. If someone knows how to k, then he can k. For example, if I know how to swim, then I can swim; if I know how to speak German, then I can speak German. But there is a stronger relationship between knowing how and can. We often say of a person that he can do such-and-such when we mean precisely that he knows how to do such-and-such. When somebody says he can swim we would not think it particularly dangerous if we pushed him off a boat reasonably close to shore; after all, he knows how to swim. Again, if somebody says he can speak German, we might take him with us on a trip to Germany only because he is the only one who knows how to speak German. Now if someone actually does do K, then when K is the sort of thing which one can be said to know how to do, then we have evidence that he knows how to do K. For example, if I am swimming, then this is evidence that I know how to swim, and if I am speaking German, then this is evidence that Iknow how to speak German. So cases of knowing how furnish statements containing "can" which satisfy the condition given in the last paragraph, and this can be said to attribute powers to people.

But is "It is possible for x to k" ever taken as equivalent to "x knows how to k"? Cetainly, ~~it shows~~ if x knows how to k, then it is possible for x to k, for we cannot know how to do the impossible. But to begin with, if we say that it is possible for someone to swim or to speak German and we mean that he knows how to (or can) swim or speak German, then we are at least being misleading. If someone were paralyzed from the waist down, I would certainly be entitled to say that it is not (conditionally) possible for him to swim. But it is not impossible for a man in normal physical condition and not prevented by extrinsic conditions from swimming to swim, even though he might not know how to swim.

But perhaps this use of "can" is a modal use which we never, in ordinary speech, equate with possibility; and perhaps there would be no philosophical objections to extending the term "possible" to cover these cases. But I feel there are philosophical objection to such an extension of the term "possible". To begin with, for all the uses of "possible" which I have considered, the following rule is valid: "p" entails "Mp". But that x actually does k does not entail entail that x knows how to k; it is only evidence that x know how to k. For example, someone may have a book of German phrases which are written phonetically. We could imagine him saying the right German phrases in the appropriate circumstances, looking it up in his book each time. But in each case he has to look thephrase up in the book. In this case he would certainly be speaking German, andyet not know how to speak German. If a friend hears him uttering one of the phrases and says " I never knew you could speak German", the appropriate reply would be, "I can't; I just look the phrase up in my book." But if we hear a person speaking German, this is certainly evidence that he can speak German. If we have any doubts, we could get a German speaker to hold a conversation with him just to be sure that he was not a "special case" like the person with the phrase book.

But there are modal sentences which neither entail nor are entailed by the corresponding non-modal sentences. The particular use of modal terms which I have discussed and which have this characteristic are the imperative modalities. Perhaps "can", as used in the context of knowing how, is related to, or be assimilated to, the weaker of these modalities- viz., permissibility. But here again there is a difference. If x does k, this is no evidence at all that it is permissible for x to k.

Perhaps the most convincing reason for not admitting "can", in the context of knowing how, as a modal term is that there is no correlative term. Consider"It is not the case that x can not k." In this context, we can just as well consider:" It is not the case that x knows how to not k." What would be another way of saying this with no negation at all? One problem is how we are to construe a statement to the effect that someone knows how to not do something. What would it mean to say of someone that he knows how to not swim? Knowing how has the implication of achievement. This is what makes the example of some one knowing how not to swim so absurd, since swimming is something which is learned or achieved. We do not say that we know how to breathe, we just do breathe. On the other hand, we do not say of an adult that he knows how to walk, but we say of a two year old child that he does. Perhaps we cannot analyze this notion of learning, acquiring, or achieving any further. For the present purposes it is enough to note that knowing how is an achievement verb. What it is for which it is an achievement not to do is hard to say. If it is an achievement to do something, then it certainly is not an achievement not to do it, in the general sense in which this is meant in connection with knowing how. Of course, we can often derive a fairly artificial statement of the form "x knows how to k". For example, it follows from "x knows how to swim" that x knows how not to sink. Sometimes it is even natural to form a knowing-how statement with an internal negation- e.g.,"x knows how not to look guilty." Usually, though, what we have achieved is expressed by a positive verb.

Thus it is that when we can say "x knows how to k" we can rarely, if ever, say " x knows how to not k" for any x and the the same k as in the positive statement. Even in the few cases, if any, where we can, the two statements, for fixed x and k, are not sub-contraries, for x might know how to k and to not-k, and when he wants to k, he k's, and when wants to not-k, he not-k's. This is indicative of the impossibility of setting up the traditional square of opposition for "can" as used in the context of knowing how. The only part of it which still holds is "It is not the case that x can k"("It is not the case that x knows how to k") and "x can k" (" x knows how to k") are contradictories, as are statements which are formed by replacing "k" with "not-k". It follows from this, and is also obvious, that "It is not the case that x can (knows how to) not-k" does not entail "x can (knows how to) k".

The conclusion is that "can", in the context of knowing how, is an achievement, not a modal term. This shows the danger of Ayers' term "power" when used in discussion about possibility. "Power" and "can", it appears, are interdefinable if "can" is restricted to those uses in which "p" is evidence for "Cp". What Ayers' does, then, amounts to using a term which is determined by more than modal considerations to act as a sort of arbitrator in a discussion about modal terms.

It is common in discussions about modalities to find the terms "can" and "must" analyzed as much, if not more, than terms like "necessary", "Possible", "certain", and "permissible". We have already seen the danger of this in one respect, for "can" is non-modal in contexts of knowing how; but it is more than just this context that "can" and "must" are not good choices as "the" modal terms.

Now for all the modal uses which I have considered, the strong term (the one which entails the other) can be replaced, perhaps with some grammatical changes, by "must", and the weak term can be replaced, perhaps again, with some grammatical changes, by "can". Of course, there are some noticable differences in the way in which "can" and "must" occur in contexts of the different modal uses. Perhaps the most noticable is that in epistemic uses, "can" usually occurs in the subjunctive at the beginning of the sentence, i.e., in the form "It could be that,...". Admittedly, "could" can occur for other modal uses as well, but anything except the subjunctive couod be misleading in epistemic contexts.

But "can" and "must" are used in many more contexts than those in which they are eliminable in favor of the modal terms discussed here. And I think that it is fairly obvious that in many of these contexts they are not modal at all. To list some examples: " I can't think of any more cases", "You can see for yourself", "He can't have said that!" "I must have been tired to write that!", "You must come home tomorrow!", "You must see that film". The third and fifth examples have something in common: "can" or "must" are used here for what might be called emotive force. That is, they are not used to express any sort of modality, but are used to emphasize what the speaker believes, feels, desires, etc. In the fourth example, "must" is used in a tongue-in-cheek fashion. In the first example, "can" occurs in an idiomatic phrase, viz., "can think of" or "can't think of". This phrase is more akin to "know of" than to any simple modal term or phrase. In the second example, "can" is used to soften a command. Instead of saying "Look at this if you don't believe me", it would often times be more polite to say "You can see for yourself." In the last example, "must" is used simply to emphasize the advice which the speaker is giving, for it shows that he feels that the advice is very good.

Admittedly, "necessity" and "possibility" are sometimes used in extended senses where they are hardly modal. But most of these are jargon terms in which they occur in only the noun form and cannot be paraphrased into an adverbial form. We often speak of "The necessities of...", or "The necessity for...", or "The possibilities of...", or "The possibility for..." in obvious jargon uses. But "necessity" and "possibility" are not such versatile terms as are "must" and "can". It is this versatility of "must" and of "can" which makes them what one might call dangerous terms for an analysis of modal notions. It is apparent, then, why I have avoided analyzing statements containing "can" and "must" in the foregoing discussions.

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