

**CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS
WORKING PAPER SERIES**



CDMA12/01

A Producer Theory with Business Risks*

Seong-Hoon Kim†
University of St Andrews

Seongman Moon‡
Universidad Carlos III de Madrid

JANUARY 2012

ABSTRACT

In this paper, we consider a producer who faces uninsurable business risks due to incomplete spanning of asset markets over stochastic goods market outcomes, and examine how the presence of the uninsurable business risks affects the producer's optimal pricing and production behaviours. Three key (inter-related) results we find are: (1) optimal prices in goods markets comprise 'markup' to the extent of market power and 'premium' by shadow price of the risks; (2) price inertia as we observe in data can be explained by a joint work of risk neutralization motive and marginal cost equalization condition; (3) the relative responsiveness of risk neutralization motive and marginal cost equalization at optimum is central to the cyclical variation of markups, providing a consistent explanation for procyclical and countercyclical movements. By these results, the proposed theory of producer leaves important implications both micro and macro, and both empirical and theoretical.

JEL Classification: D21, D42, D81.

Keywords: Uninsurable Business Risks, Markup, Risk Premium, Hedge and Offer, Price Inertia, Stochastic Dominance, Conditional Sales Ratio.

*We thank Tatiana Damjanovic, Arup Daripa, John Driffill, Ron Smith, Alan Sutherland, and Arnold Zellner for their encouragement and constructive comments on the earlier drafts of the current work. All remaining errors are ours. Financial support from the Scottish Institute for Research in Economics (SIRE) and from the Spanish Plan Nacional de I+D+I (SEJ2007-62908) are gratefully acknowledged.

†s.kim@st-andrews.ac.uk; School of Economics and Finance, University of St Andrews, Fife, United Kingdom, KY16 9AL.

s.kim@ems.bbk.ac.uk; Department of Economics, Birkbeck College, Malet Street, London, United Kingdom, WC1E 7HX.

‡smoon@eco.uc3m.es; Department of Economics, Universidad Carlos III de Madrid, Calle Madrid 126, 28903 Getafe Madrid, Spain.

CASTLECLIFFE, SCHOOL OF ECONOMICS & FINANCE, UNIVERSITY OF ST ANDREWS, KY16 9AL

TEL: +44 (0)1334 462445 FAX: +44 (0)1334 462444 EMAIL: cdma@st-and.ac.uk

1 Introduction

The General Motors safeguards its business against losses from a variety of natural disasters but cannot do so against losses from normal business outcomes. The Gosch, a small Greek restaurant, holds a portfolio of insurance policies alike.—It loss-proofs the business against unprecedented flood, but exposes to unprecedented vapour of customer visits. In this paper, we model uninsurable business risks within the otherwise standard framework of a monopolistic producer, and examine how the presence of the uninsurable business risks affects the producer’s optimal pricing and production behaviours.

Business risks can be related to either firm-specific or economy-wide factors that would have negative impacts on profitability. In the present paper, business risks result in the form of market-clearing failure which are a natural outcome of realistic transaction process for many markets. Sellers do not adjust prices at every instant of time for various reasons. Furthermore, they do not know for sure how much of their product will be sold. Consequently, markets may not clear, and firms end up with “unwanted” inventory buildup or temporary line shutdowns to deal with excess supplies.¹ Empirical evidence supports this form of business risks. For example, [Hendricks and Singhal \(2009\)](#) sample the public announcements of excess inventory holdings that appeared in the Wall Street Journal and the Dow Jones News Service during 1990-2002, and find negative stock market reactions to the announcements. On the other hand, we do not have complete financial contracts that can be used to insure *a priori* this form of business risks. A rational producer, in the absence of financial derivatives market for his goods market outcomes, will optimally reflect the uninsurable business risks onto his pricing and production decisions.²

In our model, stochastic market-clearing failures arise as a consequence of the two following assumptions: (i) uncertain demand at the time of pricing and production decisions, and (ii) commitment to those decisions over some interval of time. By defining one unit period by the length of time for which the commitment assumption (ii) holds, we allow monopolistic producers to fully reset their prices and production plans in every

¹Inventory may have other roles such as promoting sales ([Bils and Kahn, 2000](#)), production-smoothing inventory creation ([Blinder, 1986](#)), and the use of collateral. This paper does not incorporate them into the model of business risks, and focuses on the role of “unwanted” part.

²[Eden \(1990\)](#) enlarges the commodity space, defining goods by the sales probability in addition to physical characteristics. [Carlton \(1979\)](#) considers a framework with long-term contracts where transaction costs of using organized (spot or resale) markets to clear markets are explicitly modelled.

period. In other words, one price and one production plan jointly define piecewise one period. As a result, the model environment is free from inter-temporal price stickiness, while leaves open for possible market-clearing failure within period. Mapping one period in the model to one period in the calendar time varies much across sectors and products, and relies on the particular scope of research as well. For example, a quarter length of a year corresponds to the common use of data frequency in the business cycle literature.³

This small modification with the two assumptions to a textbook model of monopolist renders the nature of producer decision problem largely different. In the textbook model, pricing and production decisions are just the other side of the coin, since the market-clearing assumption automatically matches price to quantity on one-to-one basis given demand curve. In contrast, with the two assumptions (i) and (ii), pricing and production become two distinct choice instruments and counterpoint in formation of profit function. *Ex ante*, pricing decision does condition demand distribution in contrast to the textbook model, while production decision does censor the distribution of demand schedules. Having them together, pricing decision determines the distribution of period profits at chosen level of production, and production decision becomes sunk and leads to concave period profit function by limiting the maximum attainable period profit at chosen price. It is their distinct roles in formation of profit function that make risks matter in our model even when we assume that the monopolist is risk-neutral.

Two mechanisms are identified in relation to their distinct roles. A rational producer who maximizes expected profit takes in account all contingent wedges that would lie between his pricing and production decisions *ex ante* and market outcomes *ex post*. In the model, each contingency is related to two contrasting states (excess demand *versus* excess supply). Profit-maximization of pricing decision neutralizes the uninsurable business risks, one against the other, and leads to an inverse relationship between prices and quantities. Profit-maximization of production decision equates marginal cost of production to the sum of expected marginal revenues conditional on each demand state, and leads to a positive relationship between prices and quantities. Suppose the monopolist wants to produce

³Bils and Klenow (2004)'s empirical work endorses this model environment reasonable and realistic: Using data from the Bureau of Labor Statistics during 1995-97, they document that among 350 categories of goods and services covering about 70 percent of consumer spending, half of prices last less than 4.3 months. Compared to prior studies about price stickiness, their result shows much more frequent price changes in consumer products and services. But at the same time, the price changes seem to be far less frequent than required to guarantee continuous market-clearing across these sectors.

more: the more he produces, the larger risk of excess supply relative to the risk of excess demand he faces. Rebalancing the two contrasting risks requires to lower price through the risk neutralization channel. At the same time, re-equating the two conditional expected marginal revenues with marginal cost of production raises price to compensate for taking a larger relative risk through the marginal cost equalization channel.⁴

Equipped with this new decision framework, we extend the market-power theory of the markup towards a direction of business-risk theory. In contrast to the textbook theory of the markup, we characterize a markup rule as a pricing kernel of business risks as well as market power, and show that optimal price includes ‘traditional markup’ to the extent of market power and ‘risk premium’ for compensation of potential losses related to uninsurable market-clearing failure. Our interpretation can be contrasted to the notion of effective cost in the literature. [Greenwald and Stiglitz \(1993\)](#) assume a bankruptcy cost in financing and production decision problem faced by a risk-averse firm. However, their interpretation of optimal price concerning the risk of business closure is confined within a notional extension of marginal cost to include marginal bankruptcy cost, because they assume perfect competition by which equilibrium market price must equal marginal cost of operation.⁵

In addition to the level effect of business risks on the composition of markups, we also examine their variation effect on prices and quantities. The two identified mechanisms (the risk neutralization and the marginal cost equalization) now jointly provide a unified framework for explanation of the well-known stylized facts over business cycles: prices vary little, while quantities vary much (price inertia);⁶ and markups fluctuate as output moves up and down (counter and procyclical variation of markups).⁷ The inertial behaviour

⁴Put it differently in a Bayesian decision context, if we want to induce a player to take a larger odd ratio, we have to compensate him by allowing to increase the magnitude of prize he would get if he succeeds. [Greenwald and Stiglitz \(1993\)](#) also captures this mechanism by assuming a scale effect: that is, the larger scale at operation, the more costly bankruptcy.

⁵Similarly, [Prescott \(1975\)](#), [Eden \(1990\)](#), and [Dana \(1998\)](#) extend the notion of marginal cost to include marginal cost of carrying underutilized capacity in addition to physical costs (covering marginal operating costs).

⁶A long series of literature finds that prices and wages appear to show little response to shifts in demand. See, for cardinal references, [Eckstein \(1972\)](#) and [Carlton \(1986\)](#). [Stiglitz \(1984\)](#) discusses about several alternative explanations for the inertial behaviour of prices. See also [Baudry et al. \(2007\)](#) and references therein for recent empirical reexaminations using large micro data sets. [Baudry et al. \(2007\)](#) documents long average duration of prices (around 8 months), although individual durations of prices significantly differ across sectors.

⁷[Bils \(1989\)](#), [Warner and Barsky \(1995\)](#), [Chevalier, Kashyap, and Rossi \(2003\)](#) and many others have long documented evidence on the countercyclicality of markups. To the contrary, some other studies find the degree of countercyclicality is significantly reduced or find even the opposite results. For example,

of prices relative to quantities is generated in response to changes in business risks. A higher demand (in terms of stochastic dominance relation between the distributions of demand schedules) implies that the probability of market-clearing failure becomes lower, other things being equal. As a market becomes more likely to clear with a higher demand for every given price and quantity, we find two opposite forces at work. With the risk neutralization channel on one hand, the monopolist becomes freer to use monopolistic market power and to increase price. With the marginal cost equalization channel on the other hand, the monopolist feels safer with every feasible combination of his pricing and production decisions, and tends to lower price to the extent of reduced business risks. The two forces counteract each other on determination of optimal prices, while optimal quantities increase through both channels. Consequently, the producer theory of uninsurable business risks predicts price inertia, less variation in prices relative to quantities.

This model also sheds light on the cyclical variation of markups and connects it to the cyclical nature of uninsurable business risks.⁸ And with changing business risks, the model provides a unified explanation for both countercyclical and procyclical behaviours of markups. A key condition determining whether markups are countercyclical or procyclical is related to the degree of the relative responsiveness between the risk neutralization channel and the marginal cost equalization channel, which is shown to be measured by the relative size of the expected volume of sales conditional on successful market-clearing to the one conditional on market-clearing failure. An increase in the relative size means that the monopolist expects to see relatively higher level of sales, and will be more satisfied about the goods market business condition. In other words, the monopolist expects that the overall risk level of the goods market reduces along his chosen optimum path, when the risk neutralization channel responds less than the cost equalization channel to changing business risks. Only then, the monopolist lowers the optimal markup rate.⁹

[Nekarda and Ramey \(2010\)](#) finds that markups are either procyclical or acyclical at both the aggregate level and the industry level in the US economy. [Beccarelli \(1997\)](#) finds procyclical markup movements across G-7 countries.

⁸Empirical evidence suggests that business risks may change over business cycles. For example, [Jaimovich and Floetotto \(2008\)](#) provide evidence that the number of business closures classified by industry in the US is larger during recessions. [Devereux, Head, and Lapham \(1996\)](#) measure the aggregate business failures by the number of bankruptcies reported in Dun and Bradstreet's Record of Business Closings, and find it countercyclical. [Van Nieuwerburgh and Veldkamp \(2006\)](#) also document from professional survey data, that forecast errors become smaller during booms.

⁹In contrast to our business risk theory, much of the existing theoretical explanation about cyclical

The present paper is related to two strands of the existing literature. With respect to theme and aim, our business-risk theory is close to [Greenwald and Stiglitz \(1989\)](#)'s portfolio choice theory of pricing and production.¹⁰ Both their work and the present paper aim to incorporate risk factors into monopolistic pricing decision. As a theory of price inertia, they carry out a mean-v

-averse (quadratic utility function); (2) the firms are uncertain about the consequences of their pricing and production choices; (3) the greater change from the state in which they are, the more the uncertainty; and (4) more uncertainty is associated with pricing decision than with production decision. However, these assumptions are built-in in that the combination of the assumptions (1) and (4) bears the conclusion of price inertia in itself. The present paper derives price inertia without relying on the assumptions. So theoretical and empirical implications that follow also differ.

With respect to baseline model environments, the present paper is closely related to [Leland \(1972\)](#). He classifies a firm's decision problem by looking at which choice variable between price and quantity is made before the realization of demand. He considers three cases; price, quantity, and both price and quantity. The baseline decision problem facing the monopolist in our model are close to the last case. However, the aim and the scope we have here differ from his. Also, [Prescott \(1975\)](#), [Eden \(1990\)](#), and [Dana \(1998, 1999\)](#) share common in the assumption of decision timing; commitment to pricing and production decisions with uncertain demand.¹¹ However, as an explanation of price dispersion, their

behaviour markups solely resort to wax and wane of monopolistic market power. For example, markups move up and down (1) through direct cyclical variation of demand elasticity (whose mechanism is mostly found from the literature of market share models based on switching costs, habit formation etc. See [Stiglitz \(1984\)](#); [Bils \(1989\)](#); [Klemperer \(1995\)](#); [Bagwell \(2004\)](#); [Ravn, Schmitt-Grohe, and Uribe \(2006\)](#) among others); or (2) through timely differentials of consumer valuations in separable markets (whose mechanism is mostly found in the literature of discriminative markup pricing including some recent works by [Nocke and Peitz \(2007\)](#); [Moller and Watanabe \(2010\)](#)); or (3) through cyclical variation of collusion among oligopolistic competitors (see [Green and Porter \(1984\)](#); [Neubecker \(2006\)](#) for reasons why collusion may be procyclical, and also [Rotemberg and Saloner \(1986\)](#); [Rotemberg and Woodford \(1992\)](#); [Jaimovich and Floetotto \(2008\)](#) for reasons why collusion may be countercyclical); or (4) due to the economies of scale in shopping technologies that relatively weaken seller's market power when shopping intensity is high ([Warner and Barsky, 1995](#)).

Some notable exception includes [Chevalier and Scharfstein \(1996\)](#) who connect markup pricing to hardship of external financing, and [Lal and Matutes \(1994\)](#); [Lal and Villas-Boas \(1998\)](#); [Chevalier, Kashyap, and Rossi \(2003\)](#) based on the loss-leader model. [Chevalier and Scharfstein \(1996\)](#) claim that monopolist raises prices to help cash flow during recessions, because then he suffers greater hardness in raising external funds. The loss-leader model is another markup pricing theory which does not explicitly resort to monopolistic market power.—Higher demands appear associated with lower markups because of strategic selection of high demand items at discounted prices to attract customers.

¹⁰See also [Greenwald and Stiglitz \(1993\)](#).

¹¹The recent literature with similar model environments also includes [Berk, Gurler, and Levine \(2007\)](#),

models concern optimal price discrimination under uncertain customer valuations. So they allow firms to present a menu of several pairs of prices and quantities using various rationing rules for each state of demand ordered from lowest to highest, and customers to arrive in a sequential manner. In contrast, the present model is interested in the nature (and the consequences) of business risks entailed in each pair of pricing and production decisions. So the decision problem agents face, and the implications we find both differ much from theirs.

The rest of the paper is organized as follows. Section 2 develops a model of monopolist with uninsurable business risks. Section 3 finds optimality conditions and discusses their economic meanings. Section 4 presents key results of the study. Section 5 makes some concluding remarks.

2 The Business Environments

In this section, we present the model environments of a monopolist market with business risks. To capture business risks present in goods market, we consider a simple form of possible market-clearing failure with stochastic demand as follows: A risk-neutral monopolist sets price p_t and produces output y_t at the beginning of each period t before the realization of demand. Period demand is unknown until the end of period. As a result of this timing differential, the market may fail to clear at the end of period. If the current period is the case, the producer carries forward unsold products to the next period. In what follows, we detail the risky business environments.

2.1 Fundamentals

Demand Function Period demand d_t is subject to exogenous random demand factor x_t as follows;

$$d_t = D(x_t, p_t). \tag{1}$$

and [Bensoussan et al. \(2008\)](#), who follow a long series of operational research on the classical theme of the newsboy inventory problem pioneered since [Arrow, Harris, and Marschak \(1951\)](#) and [Scarf \(1959\)](#). Their studies also share common in their assumptions of demand uncertainty and commitment to production (capacity investment) before the realization of demand. Particularly, [Berk, Gurler, and Levine \(2007\)](#) develop numerical approximation technique for Bayesian updating of demand distribution when realized demand data are censored by production (stock order) decision. [Bensoussan et al. \(2008\)](#) focus on the theoretical existence of dynamic optimal feedback rules of stock orders and sales with the observation structure of censored demand.

Random demand factor x_t realizes at the end of period t , independent of the history of past realizations, $x^{t-1} = \{x_1, x_2, \dots, x_{t-1}\}$. Demand function $D(\cdot)$ is twice-continuously differentiable, with standard properties $D_x > 0$, $D_p < 0$, $D_{xp} \leq 0$, and $D_{pp} \geq 0$.

Examining pricing and production behaviour with business risks, our focus of analysis will be drawn to intertemporal stochastic dominance relation of demand distributions. To capture stochastic shifts in demand distribution over periods, we introduce a finite number of distribution parameters that characterize the distribution function for each period. Let $F(x_t; \theta_t)$ denote the probability distribution function of random demand factor x_t for each period t . Here θ_t is a set of time-varying parameters that characterize the distribution function $F(\cdot)$ for each period. Assume that θ_t realizes at the beginning of each period t and follows an *i.i.d.* process given by the probability distribution function $G(\theta_t)$. Let us consider a non-negative support $[\underline{x}_t, \bar{x}_t]$ with $0 \leq \underline{x}_t < \bar{x}_t \leq \infty$. As G is stationary, the distribution of the supports (and thus the distribution of distribution function F 's) is stationary.

Stock Transition Equation A risk-neutral monopolist sets price p_t and produces output y_t at the beginning of each period t . Making pricing and production decisions, the producer knows θ_t in $F(x_t; \theta_t)$ and thus the distribution of period demand d_t . Upon the realization of random demand factor x_t at the end of period, however, the market may end in either excess demand or excess supply. Unsold products from the current period can be carried forward to the next period.¹² Consequently, the level of product stock available at the beginning of the current period is the sum of output produced at the beginning of the period plus unsold goods brought forward from the previous period.

Let s_t denote the stock available at the beginning of period t . Also let n_t denote inventory holdings brought forward from period $t - 1$;

$$n_t = (s_{t-1} - d_{t-1})^+,$$

where $(s_{t-1} - d_{t-1})^+ = s_{t-1} - d_{t-1}$ for $s_{t-1} - d_{t-1} > 0$; and $(s_{t-1} - d_{t-1})^+ = 0$ otherwise. Then, the stock transition equation is expressed as follows;

¹²As is discussed in Section 2.2, we assume that unsatisfied demand does not roll over to the next period.

$$s_t = y_t + n_t. \quad (2)$$

Making production decision is equivalent to choosing the level of stock s_t , for $n_t = (s_{t-1} - d_{t-1})^+$ is known.

Profit Function Period profit is the total sales revenue minus the total cost. First, the total sales revenue in period t is the product of price and sales volume; $p_t\{s_t - (s_t - d_t)^+\}$. Next, let $C(\cdot)$ denote the total production cost, and $H(\cdot)$ the total storage cost. Both the cost functions are twice-continuously differentiable and strictly increasing in the level of output and of stock, respectively, with $C(0) \geq 0$ and $H(0) \geq 0$. Then the total cost in period t is the sum of the total production cost $C(y_t)$ and the total inventory maintenance cost $H(n_t)$. Using the stock transition equation (2), we have the total cost in period t as $C(s_t - n_t) + H(n_t)$. So period profit π_t is given by

$$\pi_t = p_t\{s_t - (s_t - d_t)^+\} - C(s_t - n_t) - H(n_t). \quad (3)$$

At the beginning of period t , the monopolist makes pricing and production decisions to maximize the expected present value of its profit stream,

$$E[\Pi_t | \mathcal{I}_t] = E \left[\left\{ \sum_{j=0}^{\infty} q_{(t,t+j)} \pi_{t+j} \right\} | \mathcal{I}_t \right], \quad (4)$$

where $q_{(t,t+j)}$ denotes a cumulative discount factor composed of short-term rates between periods t and $t+j$. $E[\cdot | \mathcal{I}_t]$ is the expectations operator conditional on information set $\mathcal{I}_t = \{x^{t-1}, d^{t-1}, n^t, p^t, s^t, q^t, \theta^t\}$ available at the beginning of period t , where time superscript t denotes the history of concerning variable up to period t . We shorthand $E[\cdot | \mathcal{I}_t] = E_t[\cdot]$.

Market-Clearing (Failure) Condition Given the monopolist's pricing and production decisions (p_t, s_t) , the state of market-clearing and market-clearing failure will follow realization of x_t . Let \hat{x}_t define the *lowest* admissible value of realized x_t in order to clear the market, for given pricing and production decisions. Conversely, since the market demand is monotonically increasing in x_t (i.e., $D_x > 0$), the market will fail to clear for x_t

such that

$$\underbrace{D(x_t, p_t)}_{\substack{\text{period-}t\text{-demand} \\ \text{realization at } x_t \in [\underline{x}_t, \hat{x}_t]}} < s_t,$$

for given (p_t, s_t) .

So we have the threshold value \hat{x}_t which divides the continuum of demand states into market-clearing and market-clearing failure;

$$\left\{ \begin{array}{ll} \text{excess supply} & \text{for } x_t \in [\underline{x}_t, \hat{x}_t), \\ \text{exact clearing} & \text{for } x_t = \hat{x}_t, \\ \text{excess demand} & \text{for } x_t \in (\hat{x}_t, \bar{x}_t]. \end{array} \right.$$

In effect, bearing $D_x > 0$ in mind, we can summarize these distinct states in the line with the standard market-clearing condition;

$$D(\hat{x}_t, p_t) = s_t. \tag{5}$$

It is important to understand that, for known $n_t = (s_{t-1} - d_{t-1})^+$, the threshold \hat{x}_t will be *implied* by the firm's pricing and production decisions. So, *ex ante*, it works like a "point estimator" imparted by the Bayesian decision theory (for example, the minimum expected loss estimator). In what follows, we discuss about the nature of "point decision instruments" (pricing and production decisions) under the business environments we have modelled so far.

2.2 The Nature of Decision Problem

Figure 1 loosely sets out a baseline decision problem facing a producer with the business risks, and illustrates how the producer's pricing and production decisions are related with ex post market outcome: Pricing decision *does condition* demand distribution, while production decision *does censor* it at chosen price. To be clear, the upper panel pictures their distinct roles in forming expected profit under the probability distribution of demand function. The bottom panel pictures their distinct contributions in connection with total revenue and total cost.¹³ Further, it shows the two decision instruments jointly induce

¹³In this illustration, initial level of inventory holdings is set to zero; $n_0 = 0$.

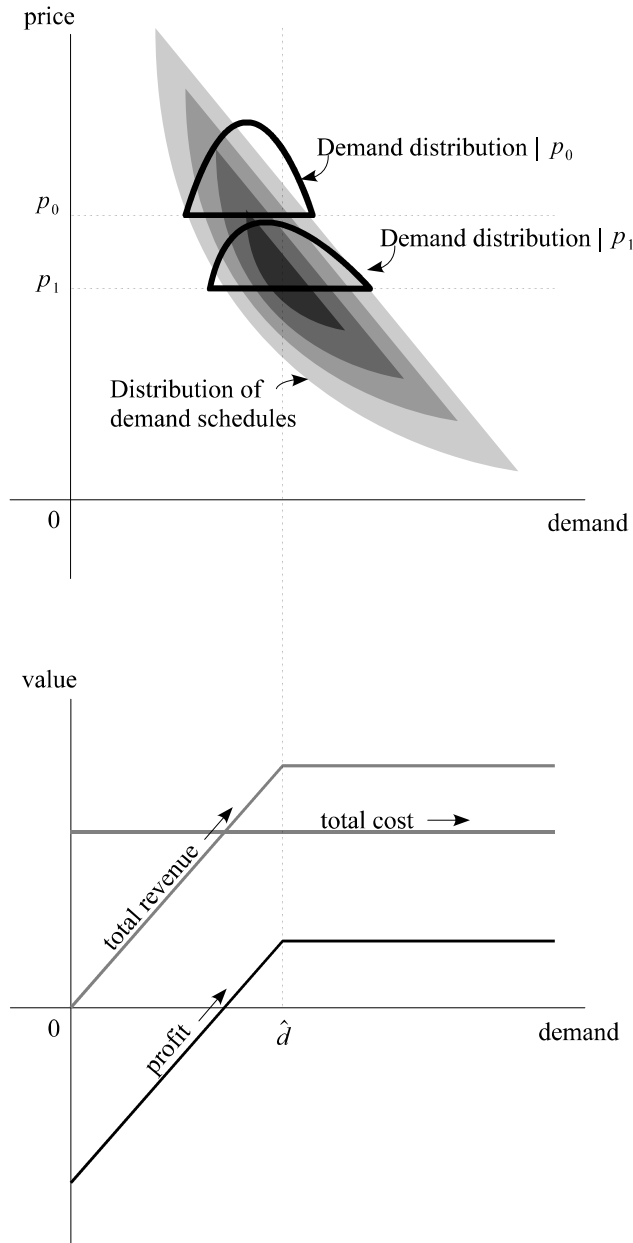


Figure 1: A baseline decision problem facing producer with business risks

the period profit function to be concave in the level of demand. Once pricing decision is made, the distribution of relevant demand level is pinned down. Once production decision is made, the total cost function turns to a fixed level of sunk cost. It is this concave profit function that makes risk matter in our model even when the firm is risk-neutral.¹⁴

One aspect of financial market implicitly assumed in the business environments is incompleteness of traded assets. As seen in the bottom panel. \hat{d} denotes the level of demand that corresponds to the lowest admissible level of x for temporal market-clearing; $\hat{d} = D(\hat{x}, p)$. The shown kink at \hat{d} reflects missing asset markets. Suppose that the economy hold a complete range of assets which offers contingent claim against every state of goods market failure. With complete asset markets, a producer would insure his main business against excess demand (excess supply) states, and recover the market value of lost sales opportunity (loss from unsold goods) at a premium known in advance. If this were the case, the producer's profit profile would turn flat over the whole range of demand (or claimable range of demand realization, if he held partial insurance). So the exact functional form of profit over the states of goods market relies on how complete asset markets are. However, such financial contracts have been rarely implemented for goods and service markets in general. A few exceptions are some futures contracts for wholesales of utilities and commodities.

The profit profile in the bottom panel of Figure 1 has been drawn under some technical assumption that would differ across industries: No value attaches to unwanted inventory buildup (unsold products) and lost sales (stock-out). As we relax this assumption, the profit function becomes less concave in its shape and so less sensitive to the risk of stochastic failure of market-clearing. Unsold products in manufacturing sectors are usually carried forward for the next period sales at certain storage costs. So the market value of inventory stock equals the expected present value of future production costs minus storage costs, and will decrease in the realized level of demand in the current period. Figure 2a incorporates this value into the profit function, by assuming a linear relationship between these costs and the level of stock carried forward. On the other side, for the case of excess demand, sellers lose sales and thus concern about the potential impact of stock-out on their future business. If they are able to organize such lost sales in the form of advanced

¹⁴In analogy but not by chance, the concavity of the period profit function takes after profit function in put option trades.—The choice of production level against random demand corresponds to the choice of strike price in writing a put option against random movement of underlying asset price.

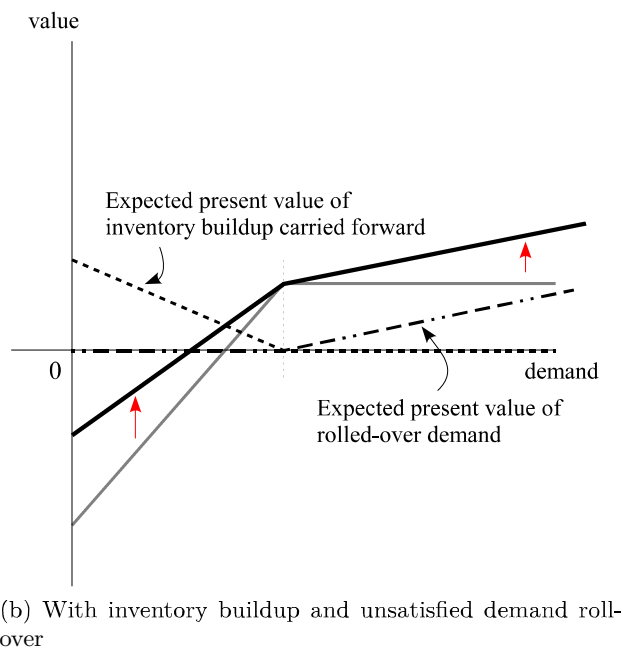
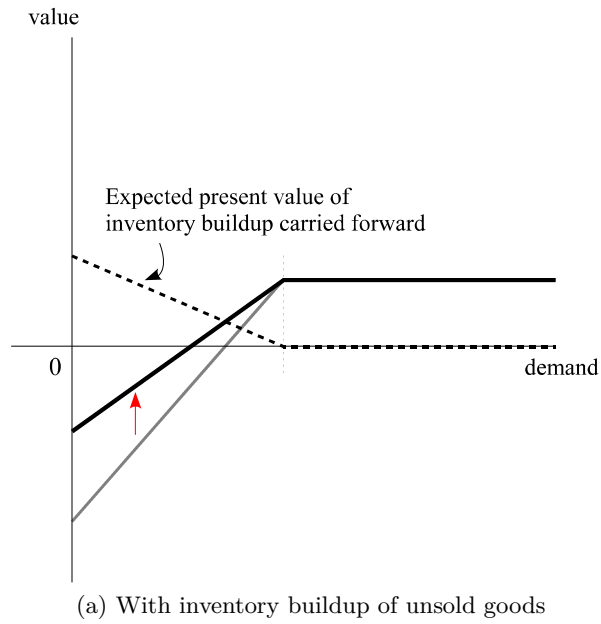


Figure 2: Profit function following pricing and production decisions

orders to be delivered in a later date, the certainty of so portioned future demand will have information value possibly translated in terms of pecuniary unit. Figure 2b describes the profit function modified to fit into such situation. Nevertheless, lost sales often cut the seller's reputation, and may undermine potential customer base and future market share through, for example, deep habit formation mechanism. The consequences of lost sales are as obscure as that one may find the two opposite impacts canceling out each other. In this respect and for the virtue of simplicity, we assume zero future value of intertemporal transfer of lost sales, while allowing intertemporal inventory buildup. Regardless of what specific assumptions to be made about lost demand and stock roll-over, however, the concavity property of profit function induced by the business risks will hold as far as market discount factor is less than one and inventory maintenance is costly.

3 The Decision Rule

This section searches optimal pricing and production decisions under the risky business environments. To start, we reformulate the maximization problem in a recursive way. Given any two consecutive periods, we drop time subscript for the current period variables and use prime ($'$) to denote the next period variables. In particular, we will shorthand $q = q_{(t,t+1)}$ and $q' = q_{(t+1,t+2)}$ at the current period t .

3.1 The Optimality Conditions

Restating the Maximization Problem A producer prices and produces to maximize

$$\int_x \Pi(x, n, \theta) dF(x; \theta) = \max_{p, s} \left\{ \int_x \pi(x, n) dF(x; \theta) + qE \left[\int_x \Pi(x', n', \theta') dF(x; \theta) \right] \right\} \quad (6)$$

with

$$\pi(x, n) = \begin{cases} pD(x, p) - C(s - n) - H(n) & \text{for } x \in [\underline{x}, \hat{x}], \\ ps - C(s - n) - H(n) & \text{otherwise,} \end{cases} \quad (7)$$

$$n' = (s - D(x, p))^+, \quad (8)$$

$$s = D(\hat{x}, p), \quad (9)$$

and given n . As the selection of a price-production pair is equivalent to the selection of price-stock pair, we define a pricing-production decision rule as follows:

Definition 1 (Optimal Pricing-Production Rule). A *pricing-production* decision rule (p, s) is a function of the predetermined inventory level n and the distribution parameter θ , apart from the parameters for demand and technologies. An optimal pricing-pricing decision rule (p, s) is the set of $p = p(n, \theta)$ and $s = s(n, \theta)$ that solves the maximization problem (6) with (7) to (9), for given n and θ .

In the standard model of monopolist, pricing and production decisions are just the other side of the coin because the assumption of market-clearing automatically matches price to quantity one-to-one for a given demand curve. In contrast, when market can stochastically clear or fail, pricing and production become two distinct instruments. A profit-maximizing producer will not waste either side of the decision space. As discussed in Section 2.2, he takes into account the nature of each of pricing and production decisions and optimally combine one with the other.

The First Order Conditions Applying the Leibniz rule to the maximization problem (6) with (7) to (9), we have the following first order conditions w.r.t. (p, s) ;

$$\underbrace{s \int_{\hat{x}}^{\bar{x}} dF(x)}_{\text{MR.P.XD}} = \underbrace{- \int_{\underline{x}}^{\hat{x}} [D(x, p) + pD_p(x, p)] dF(x)}_{\text{MR.P.XS(cont'd)}} + qE \left[\underbrace{\int_{\underline{x}}^{\hat{x}} \{C_{y'}(s' - s + D(x, p)) - H_{n'}(s - D(x, p))\} D_p(x, p) dF(x)}_{\text{MR.P.XS}} \right] \quad (10)$$

and

$$\underbrace{C_y(s - n)}_{\text{MC.Y}} = \underbrace{p \int_{\hat{x}}^{\bar{x}} dF(x)}_{\text{MR.Y.XD}} + qE \left[\underbrace{\int_{\underline{x}}^{\hat{x}} \{C_{y'}(s' - s + D(x, p)) - H_{n'}(s - D(x, p))\} dF(x)}_{\text{MR.Y.XS}} \right], \quad (11)$$

where D_p , C_y , and H_n are the partial derivatives. Just to simplify notation, we drop the distribution parameter θ where no confusion arises (until we examine the impact of the distributional changes on the optimal pricing-production decision rule). Here instead, we attach a set of acronyms to the first order conditions to help to interpret the mathematical terms in the language of economics;

- MR.P.XD: marginal revenue associated with excess demand state w.r.t. pricing decision
- MR.P.XS: marginal revenue associated with excess supply state w.r.t. pricing decision
- MC.Y: marginal cost of production
- MR.Y.XD: marginal revenue associated with excess demand state w.r.t. production decision
- MR.Y.XS: marginal revenue associated with excess supply state w.r.t. production decision.

All the acronymic terms are non-negative.

The Second Order Conditions Let $\Pi^e = \int_x \Pi(x, n, \theta) dF(x; \theta)$, the maximum value function in (6); and the notation of its partial derivatives follows the subscript-convention. It should also hold that for a decision rule (p, s) to be optimal,

$$\Pi_{pp}^e < 0, \tag{12}$$

and the determinant of the Hessian matrix

$$\begin{vmatrix} \Pi_{pp}^e & \Pi_{ps}^e \\ \Pi_{sp}^e & \Pi_{ss}^e \end{vmatrix} > 0, \tag{13}$$

where

$$\left\{ \begin{array}{l}
\Pi_{pp}^e = -\frac{\{D_p(\hat{x}, p)\}^2}{D_{\hat{x}}(\hat{x}, p)} f(\hat{x}) \{p - qB(\hat{x})\} + \int_{\underline{x}}^{\hat{x}} [2D_p(x, p) + pD_{pp}(x, p)] dF(x) \\
\quad - q \int_{\underline{x}}^{\hat{x}} \{B(x)D_{pp}(x, p) + B_p(x)D_p(x, p)\} dF(x), \\
\Pi_{ss}^e = -\frac{1}{D_{\hat{x}}(\hat{x}, p)} f(\hat{x}) \{p - qB(\hat{x})\} - C_{yy}(s - n) + q \int_{\underline{x}}^{\hat{x}} B_s(x) dF(x) \\
\Pi_{ps}^e = \int_{\hat{x}}^{\bar{x}} dF(x) - \frac{D_p(\hat{x}, p)}{D_{\hat{x}}(\hat{x}, p)} f(\hat{x}) \{p - qB(\hat{x})\} + q \int_{\underline{x}}^{\hat{x}} B_p(x) dF(x) \\
\Pi_{sp}^e = \Pi_{ps}^e;
\end{array} \right. \quad (14)$$

and $f(x) = dF/dx$ is the probability density function; and

$$B(x) = C_{y'}(s' - s + D(x, p)) - H_{n'}(s - D(x, p)). \quad (15)$$

$B_p(x)$ and $B_s(x)$ are the partial derivatives of w.r.t. p and s , respectively. That is, $B_p(x) = \{C_{y'y'}(s' - s + D(x, p)) + H_{n'n'}(s - D(x, p))\}D_p(x, p)$, and $B_s(x) = -\{C_{y'y'}(s' - s + D(x, p)) + H_{n'n'}(s - D(x, p))\}$.

The conditions (12) and (13) together with (10) and (11) are necessary and sufficient for a local maximum. If Π^e is a strictly concave, these conditions constitute a unique global maximum for Π^e . In what follows, we characterize these conditions and proceed to specifications for demand function, technologies, and probability distribution.

3.2 Characterization of the Optimal Decision Rule

The Business Risks In the present model, the business risks take a form of stochastic market-clearing and market-clearing failure. As commonly appeared in the two optimality conditions (10) and (11), the risks are basically captured by the probability distribution function $F(x)$. In particular, $\int_{\underline{x}}^{\hat{x}} dF(x) = F(\hat{x})$ is the probability that a market may fail to clear (excess supply), and $\int_{\hat{x}}^{\bar{x}} dF(x) = 1 - F(\hat{x})$ the probability that a market clears (excess demand). The two contrasting risks play important roles in forming an optimal

pricing-production rule, as reflected in the above acronymic terms followed by either XD or XS.

To help to read the first order conditions (10) and (11) in the language of economics, let us facilitate the acronymic terms introduced above, and restate (10) to MR.P.XD = MR.P.XS and (11) to MC.Y = MR.Y.XD + MR.Y.XS. As briefed in this expression, Equation (10) states that, for a chosen level of production, the monopolist should set price at which the two conditional responses of revenue do offset in expectations. In other words, the pricing rule seeks to neutralize production risks by hedging one in the form of excess supply against the other in the form of excess demand. Let us call this risk neutralization channel a “*hedge rule*” for short. The hedge rule optimally spots how much a producer should adjust prices to counteract additional risk arising when he increases production.

Equation (11) is the standard equalization condition between marginal revenue and marginal cost of production. It states that for a chosen price, the level of ex ante optimal production will be found where marginal production cost equals the sum of expected marginal revenues conditional on each state of market-clearing and market-clearing failure. Let us call this marginal cost equalization channel a “*offer rule*” for short. The offer rule optimally spots how high prices a producer will ask if he is asked to produce more and thus to take additional risk. For a limiting case in which unsold goods are fully perishable, the expected marginal revenue associated with excess supply becomes zero and thus the offer rule reduces to MC.Y = MR.Y.XD.¹⁵

Forward-looking Reservation Price The two terms MR.P.XS and MP.Y.XS in (10) and (11) and the four Hessian elements in (13) with (14) share one common factor, $B(x) = C_{y'}(s' - s + D(x, p)) - H_{n'}(s - D(x, p))$ in (15). $B(x)$ is the difference between future marginal costs of production and of inventory maintenance for each level of x . In effect, it amounts how much the producer could save tomorrow if he carries forward one additional unit of output produced today. So we can think of the expected present value of $B(x)$, i.e., $qE[B(x)] = qE[C_{y'}(s' - s + D(x, p)) - H_{n'}(s - D(x, p))]$, as a “forward-looking reservation price” at which price the producer is indifferent between if he sells additional unit now

¹⁵This reduced form expression appears similar to the optimality condition found in the models of Prescott (1975), Eden (1990), and Dana (1998). In particular, they express $p = C_y(s - n)/\{1 - F(\hat{x})\}$, and call the r.h.s. “marginal capacity cost”, regardless of market structure.

and if he turns down sales now and carries forward it tomorrow.

In general, the relative size of the forward-looking reservation price to the marginal cost depends on a full description of cost structure, existence of externalities, and so on.¹⁶ Consider a producer who currently produces at $qE[B(x)] > C_y(s - n)$. His production motive must be two-fold; to sell and to hoard.—A fraction of output is produced for sales today, and the rest of output is produced to hoard in expectations of higher production cost tomorrow (e.g., cost-push inflation). If no business risks exist, production by the latter motive (for speculative inventory accumulation) will end where the forward-looking reservation price equals the marginal cost of production, $qE[B(x)] = C_y(s - n)$, assuming a textbook U -shaped marginal cost structure. At optimum, the marginal cost sets the upper bound of the forward-looking reservation price. In contrast, in the presence of business risks, the forward-looking reservation price is strictly less than the marginal cost of production at optimum; $qE[B(x)] < C_y(s - n)$.

One can also easily reason the lower bound of the forward-looking reservation price at optimum path. At a first glance, it seems plausible that the next period storage cost exceeds the next period production cost in expectations, and thus $E[B(x)] < 0$. However, when maintenance of inventories is more expensive than producing new units in the next period, no rational producers will carry forward any additional inventories and rather will scrap them at some costs. So the forward-looking reservation price at optimum path must be greater than (and at least as great as) the writing-off cost of inventories. Without loss of generality, we assume zero scrap cost. In effect, the zero lower bound is seen as equivalent as the case of completely perishable goods. The following result summarizes our discussion about the forward-looking reservation price.

Lemma 1 (Forward-looking Reservation Price). *Assume there are no externalities with which inventories are more productive or more destructive. Assume zero cost of scrapping inventories. In the present model of business risks where markets stochastically succeed or fail to clear, the following relationship holds at any interior optimum regardless of cost*

¹⁶Notable examples of externalities that make inventory holdings desirable include Ramey (1989) and Bils and Kahn (2000). In Ramey (1989), inventories are productive because inventories are assumed to be a production factor. Also in Bils and Kahn (2000), inventories are productive because inventories contribute to generation of greater sales.

structure (i.e., for any economically sensible total cost function C);

$$0 \leq qE[B(x)] < C_y(s - n). \quad (16)$$

Proof. The zero lower bound is obvious from the assumption of zero scrap cost. As per the upper bound, first look at the r.h.s. of (11). Note in order for (11) to hold, $0 < \text{MR.Y.XD} < C_y(s - n)$ and $0 < \text{MR.Y.XS} < C_y(s - n)$ at any interior optimum. However, a rational producer currently active will set price and produce where $p > C_y(s - n)$ at optimum ex ante. So we have that $p \int_{\hat{x}}^{\bar{x}} dF(x) < C_y(s - n) < p$. Finally, note that $F(x)$ bases on a space of measure one. Consequently, $qE \left[\int_{\underline{x}}^{\hat{x}} B(x) dF(x) \right] < qE[B(x)] < C_y(s - n) < p$. \square

Links to the Standard Assumptions Next, to help to read the second order conditions (12) and (13) in the language of economics, let us express the first order conditions (10) and (11) in terms of acronymic terms as follows;

$$\begin{cases} \Pi_p^e = \text{MR.P.XD} - \text{MR.P.XS} & = 0 \\ \Pi_s^e = -\text{MC.Y} + \text{MR.Y.XD} + \text{MR.Y.XS} & = 0. \end{cases}$$

Using the expressions, we can equivalently state the second order condition (12) to

$$\text{MR.P.XD}_p - \text{MR.P.XS}_p < 0$$

and (13) to

$$\left| \begin{array}{cc} \text{MR.P.XD}_p - \text{MR.P.XS}_p & \text{MR.P.XD}_s - \text{MR.P.XS}_s \\ -\text{MC.Y}_p + \text{MR.Y.XD}_p + \text{MR.Y.XS}_p & -\text{MC.Y}_s + \text{MR.Y.XD}_s + \text{MR.Y.XS}_s \end{array} \right| > 0,$$

where the notation again follows the subscript-convention for partial derivatives.

Bearing in mind the acronymic expressions, we can read the baseline economic meanings related to these second order conditions. First, regarding the properties for the cross-derivatives, we already know that a producer facing stochastic market-clearing and market-clearing failure uses pricing and production as two distinct instruments, although p and s are related through the definition (9) of the market-clearing level of random demand

factor. So we have $MR.Y.XD_p + MR.Y.XS_p = MR.P.XD_s - MR.P.XS_s$, following Young's theorem and the fact that $MC.Y_p = 0$. This is as indeed shown in the third equation from (14). It says that the change in the marginal revenue of production (conditional on over- and under-production) following an increase in price should be the same as the change in the marginal revenue of pricing (conditional on over- and under-pricing) following an increase in production.

Regarding the requirement for the leading principal minor of order 1, we can understand (12) or $MR.P.XD_p - MR.P.XS_p < 0$ as a statement that, as a pricing rule deviates from the optimum and leads to higher prices than optimal level (while holding s), the expected marginal revenue conditional on market-clearing states decreases more than the expected marginal revenue conditional on market-clearing failure. So this statement is basically about the functional form of marginal revenue, and thus ultimately about the specification of demand function. Also, we know that for (13) to hold, $\Pi_{ss}^e < 0$, or $-MC.Y_s + MR.Y.XD_s + MR.Y.XS_s < 0$. It says that, if a production rule s (while holding p) leads to a higher output level than optimum, the marginal cost of production lies above the marginal revenue (the sum of the two conditional expected marginal revenues). So this statement corresponds to the standard requirement about the position of marginal cost relative to marginal revenue. The following result links the requirement for the leading principal minors of order 1 to the standard assumptions for demand function and production-storage technologies.

Lemma 2 (Links to the Standard Assumptions). *Assume that $2D_p(x, p) + pD_{pp}(x, p) \leq 0$ for any given x . Assume also that a pricing-production decision rule (p, s) leads the marginal cost of production for the present period to be weakly increasing, and the forward-looking reservation price to be weakly decreasing at optimum. Then, for any non-degenerate and continuous distribution function $F(x)$, we have that $\Pi_{pp}^e < 0$ and $\Pi_{ss}^e < 0$. That is,*

$$MR.P.XD_p < MR.P.XS_p, \text{ and}$$

$$MR.Y.XD_s + MR.Y.XS_s < MC.Y_s.$$

Proof. It is obvious that $\Pi_{pp}^e < 0$ and $\Pi_{ss}^e < 0$ under the following standard assumptions

$$\begin{cases} 2D_p + pD_{pp} \leq 0, \\ C_{yy} \geq 0, \\ B_s \leq 0. \end{cases}$$

□

All the three assumptions for the above result are standard: The first assumption that $2D_p + pD_{pp} \leq 0$ requires the schedule of marginal revenue (for any fixed level of x) derived from period demand function to spot a downward-sloping curve over the coordinate of price and quantity. The second assumption looks at the cost side, and requires that the marginal cost of production for the present period is also weakly increasing at optimum. To see economic meaning of the third assumption, recall that $B_s(x) = -\{C_{y'y'}(s' - s + D(x, p)) + H_{n'n'}(s - D(x, p))\}$. So the assumption $B_s \leq 0$ is equivalent to $C_{y'y'} + H_{n'n'} \geq 0$, which means a rational producer produces where the marginal costs of production and inventory maintenance next period weakly increase.

Although these standard assumptions made in Lemma 2 are alone neither necessary nor sufficient for the second order conditions (12) and (13) to hold, we have seen them to be useful to illuminate stability properties for the present model of a producer with business risks, as such referring back to our existing knowledge on the traditional market-clearing producer theory. Toward further analysis of the model, we consider analytically tractable functional specifications for preferences (demand function) and technologies (production and inventory maintenance), with which Lemma 1 and 2 hold.

3.3 Specification of the Fundamentals

Change of Variables As the required properties are mostly about the specifications of the demand function $D(x, p)$ and the probability distribution $F(x; \theta)$, it is clearer and easier to characterize the existence requirement in terms of (p, \hat{x}) rather than (p, s) . In effect, with the identity (9), the selection of an optimal pricing-production rule (p, s) can be transformed to the selection of an optimal pricing-estimation pair (p, \hat{x}) because s and \hat{x} play the same decision role of censoring the probability distribution that is conditioned

by pricing rule p . Henceforth, we focus on (p, \hat{x}) as defined as follows:

Definition 2 (Change of Variables). An optimal pair of pricing p and estimator \hat{x} is the set of functions $p = p(n, \theta)$ and $\hat{x} = \hat{x}(n, \theta)$ that solves the maximization problem (6) with (7) to (9), for given n and θ . As (p, s) and (p, \hat{x}) are one-to-one interchangeable (injective mapping), we call a *pricing-estimation* rule a *pricing-production* rule interchangeably depending on the context.

Technologies and Demand Function As mostly found in the literature, let us consider the case of constant marginal cost in both production and storage technologies. This specification enhances analytical tractability, by simplifying the functional form of the forward-looking reservation price, $qE[B(x)]$. Henceforth, let $C_y(\cdot) = c$ and $H_n(\cdot) = h$, where c and h are constant within period. However, we allow them to vary over time as technological shocks hit and/or factor prices move up and down. Given the specification for technologies, the forward-looking reservation price turns free from realization of x , and only subject to market discount rate q and relative technological advances between production and storage sectors: $qE[B(x)] = qE[c' - h']$. For conciseness, let $B = E[c' - h']$, and thus $qE[c' - h'] = qB$.

Also we consider a period demand function that is subject to a random factor x in a multiplicative form, $D(x, p) = xD(p)$. Here we slightly abuse the demand notation D , to save letters, while we maintain the same functional properties for demand as previously assumed in Section 2.1. With the demand function, the price elasticity of demand $\epsilon(p)$ turns free from random factor x . That is, $\epsilon(p) = |D_p(x, p)/\{D(x, p)/p\}| = |D_p(p)/\{D(p)/p\}|$. To be consistent with the specifications for technologies and demand, we assume $0 \leq qB < c$ and $2D_p + pD_{pp} \leq 0$, and thus Lemma 1 and 2 hold.

The Hedge Rule and the Offer Rule Given the specification for technologies and demand function, we can rewrite the two optimality conditions (10) and (11) as follows;

$$0 = \underbrace{\hat{x}D(p) \{1 - F(\hat{x})\}}_{\text{MR.P.XD}} + \underbrace{\{D(p) + pD_p(p) - qBD_p(p)\} T(\hat{x})F(\hat{x})}_{\text{MR.P.XS}},$$

and

$$\underbrace{c}_{\text{MC.Y}} = p \underbrace{\{1 - F(\hat{x})\}}_{\text{MR.Y.XD}} + \underbrace{qBF(\hat{x})}_{\text{MR.Y.XS}}.$$

where $T(\hat{x})$ the conditional mean of x truncated from above \hat{x} ,

$$\begin{aligned} T(\hat{x}) &= \int_{\underline{x}}^{\hat{x}} x dF(x | x < \hat{x}) = \frac{\int_{\underline{x}}^{\hat{x}} x dF(x)}{F(\hat{x})} \\ &= E(x | x < \hat{x}). \end{aligned}$$

In this new expression, we have substituted (9) and removed s out of the system of equations to fully focus on the pricing-estimation rule (p, \hat{x}) . A few further steps of rearrangement lead to

$$\text{the hedge rule: } p \left\{ 1 - \frac{1}{\epsilon(p)} \left(1 + \frac{\hat{x} \{1 - F(\hat{x})\}}{T(\hat{x})F(\hat{x})} \right) \right\} = qB \quad (17)$$

and

$$\text{the offer rule: } p = \frac{1}{1 - F(\hat{x})} c - \frac{F(\hat{x})}{1 - F(\hat{x})} qB. \quad (18)$$

It holds that $T(\hat{x})F(\hat{x})\{1 - F(\hat{x})\} \neq 0$ at an interior optimum. As discussed before, the probability of market-clearing failure $F(\hat{x})$ basically captures the excess supply risk (XS-risk), and the probability of market-clearing $\{1 - F(\hat{x})\}$ basically captures the excess demand risk (XD-risk). Another distributional term $T(\hat{x})$ can be read as the expected sales volume conditional on market-clearing failure. As will be seen in the next section, the three distributional terms are central to the analysis of cyclical variation of prices and quantities.

From this new look, we can see that the standard mechanism of demand and supply basically underlies the optimal decision rules (17) and (18), respectively. Apart from the distributional terms, the price elasticity of demand $\epsilon(p)$ is an essential component in the hedge rule (17), and the marginal cost of production c is in the offer rule (18). Holding everything else constant, one can see a clear association between higher markup rates and lower price elasticities of demand, and association between higher prices and higher marginal costs of production.

This characterization is useful for inspection of the neighbourhood around where the hedge rule and the offer rule hold at the same time. Assuming the forward-looking reser-

vation price qB within the optimal boundary established in Lemma 1, we can visualize each of the hedge rule and the offer rule over the decision coordinate (p, \hat{x}) .

Recall first that in the present model, a producer faces the two contrasting types of business risks, excess demand (XD) and excess supply (XS). As discussed before, the hedge rule basically seeks to neutralize the risks, one against the other, whereas the offer rule equates marginal cost of production to the sum of expected marginal revenues conditional on each state of XD and XS. By formulating a producer's decision space in terms of choice of prices and probabilities, we can intuitively understand how the hedge rule gives rise to a tradeoff relationship between pricing p and optimal choice of probability $F(\hat{x})$, and how the offer rule relates them in the opposite direction. Suppose that the producer is about to produce more. It is equivalent to choose a higher odd ratio $F(\hat{x})/\{1 - F(\hat{x})\}$ for every given price. Put it differently, the more he produces, the larger risk of market-clearing failure (XS-risk) relative to the risk of market-clearing (XD-risk). Rebalancing the two contrasting risks requires to lower price. At the same time, the offer rule allows him to utilize "means of pricing" to compensate for taking a larger XS-risk relative to the XD-risk. If we want to induce the producer to produce more and confront a higher odd ratio $F(\hat{x})/\{1 - F(\hat{x})\}$, we have to compensate him. Consequently, the producer will not offer the same price even when the marginal cost of production is given constant, and will ask a higher price along the offer rule.

We close this section with an quantitative example that demonstrates how this simplified system helps us to establish an optimal pricing-estimation (p, \hat{x}) and then obtain the corresponding optimal pricing-production choice (p, s) .

Example 1. Let $d = x(1 - p)$ where x is uniformly distributed over $[0, u]$. To have market active, we also assume that $0 < mc < 1$. Under the assumptions for demand and distribution, (17) and (18) become

$$p = 1 - \frac{(1 - qB)}{2u} \hat{x}, \quad (19)$$

$$p = qB + \frac{(c - qB)u}{u - \hat{x}}, \quad (20)$$

respectively. Solving the system of the simultaneous equations, one can obtain a unique optimal pair (p, \hat{x}) that satisfies the condition (16) for no speculative inventories at op-

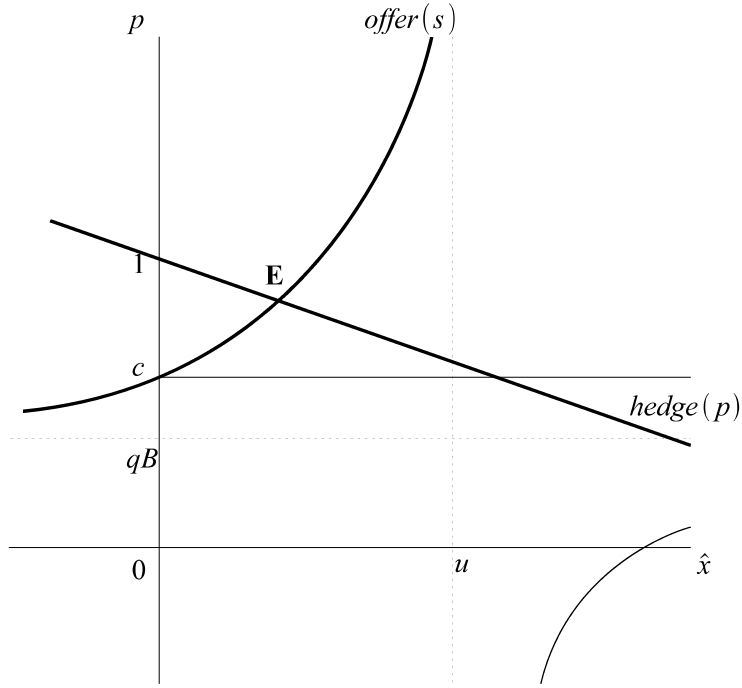


Figure 3: Example 1

timum. Then, the optimal stock for sales in the present period directly follows from the definition (5). Using the stock transition equation (2), one can straightforwardly work out the optimal production level for given the level of stock $n = (s_{-1} - d_{-1})^+$ brought forward from the previous period.

Graphically, Equation (19) is linear and intercepted at $\hat{x} = 2u/(1 - qB)$ and $p = 1$. Equation (20) is a rectangular hyperbola with asymptotes $\hat{x} = u$ and $p = qB$. See Figure 3. We label each equation by $hedge(p)$ and $offer(s)$, reminding of where each condition has been originated from. The unique optimal decision (p, \hat{x}) is represented by the point **E** at which the hedge curve meets the offer curve.

4 The Analysis of the Model

In this section, we establish three main results of the model. First, we derive an optimal markup rule and contrast it with the standard markup rule that is derived under the assumption of complete market-clearing. We highlight the role of uninsurable business risks in determination of the markup rate. Second, we conduct comparative statics to examine the optimal responses of pricing and production decisions to changing business

risks. We show that quantities are more volatile than prices. So the present model provides an alternative explanation of price inertia. Third, we inspect a mechanism through which shocks to uninsurable business risks transmit to the cyclical variation of markups. We provide conditions for procyclical or countercyclical markups.

4.1 Markup with the Business Risks

Let μ denote the markup rate (the ratio of price to marginal cost) in the presence of the business risks. Plugging (17) into (18) w.r.t. the forward-looking reservation price qB , we obtain the markup formula as follows;

$$\mu(p, \hat{x}) = \frac{1}{1 - \frac{R(\hat{x})}{\epsilon(p)}}, \quad (21)$$

where

$$R(\hat{x}) = F(\hat{x}) + \frac{\hat{x}}{T(\hat{x})}\{1 - F(\hat{x})\}. \quad (22)$$

It is evident from (21) that the optimal markup rate depends critically on the distributional terms as well as the inverse of the demand elasticity (the Lerner index), $1/\epsilon(p)$. If we assume the demand elasticity is constant, we can separate the measure of monopolistic market power from risk factors represented by $R(\hat{x})$, and easily confirm a standard result that markups increase in the market power. If the demand elasticity varies with the price level, the Lerner index which measures market power will also be affected by the business risks.

Let us look at the composition of the term $R(\hat{x})$. It makes a balance between the probability of market-clearing failure $F(\hat{x})$ and the probability of market-clearing $1 - F(\hat{x})$ times the ratio $\hat{x}/T(\hat{x})$. In economics terms, $T(\hat{x})$ can be read as the expected volume of sales conditional on market-clearing failure, and \hat{x} as the expected volume of sales conditional on successful market-clearing.¹⁷ So $R(\hat{x})$ balances the two contrasting risks over the ratio $\hat{x}/T(\hat{x})$. In this sense, we call it a “risk-balancing factor” although

¹⁷Strictly to say, the actual unit of sales volume is not the same with the numeraire of $T(\hat{x})$ or \hat{x} . It is measured at the unit of market demand. However, given the multiplicative specification for demand function, $T(\hat{x})$ and \hat{x} are proportional to the expected volumes of sales conditional on market-clearing failure and market-clearing, respectively, at the common rate of $1/D(p)$. So we measure them in the unit of sales within economics context, which has no harm to our discussion since our concern is their relative size $\hat{x}/T(\hat{x})$.

the producer’s risk attitude is assumed neutral. The following result connects the risk-balancing factor to the notion of “risk premium” in the markup rate.

Proposition 1 (Risk Premium in Markup Rate). *Consider a benchmark model that is identical to the present model in all aspects other than that the producer in the benchmark model is safeguarded by the assumption of complete market-clearing. Let μ_o denote the markup rate derived from the benchmark model. That is, $\mu_o(p) = 1/\{1 - 1/\epsilon(p)\}$. Let μ^* and μ_o^* denote the markup rates at optimum for the present and the benchmark model, respectively. Then, μ^* is greater than μ_o^* for any interior optimum. That is, $\mu^* - \mu_o^* > 0$.*

Proof. See Appendix. Essentially, we show that $R(\hat{x}) > 1$ and $\epsilon(p)$ is weakly increasing in price p . □

By definition, uninsurable risks have no spot market prices. However, the result implies that an optimal pricing of business risks prevalent in goods markets has been already reflected in the observed market prices of goods. Let us call μ_o the risk-free rate of the markup in the sense that it is determined in the absence of business risks and solely geared by market power. Then the difference $\mu - \mu_o$ amounts a “risk premium” that compensates for the uninsurable business risks. In terms of the size of the markup (rather than the rate), we have a more straightforward decomposition expression as follows: $p - c = (p - p_o) + (p_o - c)$, where p_o denotes the price corresponding to the risk-free rate of the markup μ_o . Here, the second factor $p_o - c$ accounts for the size of the markup solely geared by market power. And the first factor $p - p_o$ contributes to compensation for potential losses due to possible market-clearing failure. So it is implied that everything else equals, a producer sets higher markups and thus higher prices when he faces the uninsurable business risks.

By implication, the business risk theory of markups indicates that the traditional view that “the markup ratio is a good measure of market power” (Hall, 1988, pp.924-925) may be misleading.¹⁸ As the traditional proposition tends to overestimate the true effect of market power on determination of the markup rate, those empirical studies that are based on the framework forwarded by Bils (1987), Hall (1988), or Rotemberg and Woodford (1991) would contain estimation biases due to a model misspecification problem.

¹⁸Eden (1990) also provides a similar argument that the traditional measurement of market power may be misleading.

One most popular form of demand function used in empirical studies is the one with constant elasticity. With the special case of a constant price elasticity of demand, $\epsilon > 1$, one can immediately pin down the shadow prices of the uninsurable risks from the current theoretical framework. That is, $\mu - \mu_o = \epsilon / \{\epsilon - R(\hat{x})\} - \epsilon / \{\epsilon - 1\}$. In addition, for a case in which unsold goods are fully perishable, we have $qB = 0$ and therefore $R(\hat{x}) = \epsilon F(\hat{x})$ following (17) and (20). At the optimum, $\mu = \epsilon / \{\epsilon - \epsilon F(\hat{x})\} = 1 / \{1 - F(\hat{x})\}$, and thus the risk premium is given by $\mu - \mu_o = 1 / \{1 - F(\hat{x})\} - 1 / \{1 - 1/\epsilon\}$.

The following example looks at a simple model with a perishable good, while allowing the price elasticity of demand to vary along the linear demand function used in the previous example.

Example 2. *Continuing from Example 1:* Let us assume that $B = 0$ and $(s_{-1} - d_{-1})^+ = 0$. In this case, a rational producer in the present model chooses price and output given by

$$p = \frac{1 + \sqrt{1 + 8c}}{4}, \text{ and } y = s = \frac{\{3 - \sqrt{1 + 8c}\}^2}{8}u.$$

Assuming all functional forms to be identical between the present model and the traditional textbook market-clearing model, a producer in the benchmark model faces the following specific demand function; $d_o = \hat{x}_o(1 - p_o)$ with known (and exogenously given) \hat{x}_o . Under the assumption of complete market-clearing, $d_o = y_o$, the benchmark producer chooses price and output as follows;

$$p_o = \frac{1 + c}{2}, \text{ and } y_o = \frac{(1 - c)}{2}\hat{x}_o.$$

One can easily verify that $p > p_o$, as far as $0 < c < 1$ the assumption for the market to be active for the present period. So the risk premium in terms of the markup rate is shown as

$$\mu - \mu_o = \frac{\sqrt{1 + 8c} - 2c - 1}{4c} > 0,$$

for $0 < c < 1$.

Although the welfare analysis is not under the scope of our current study, it would interest readers to compare the levels of output. Given the example, let us set \hat{x}_o to the mean of random variable x ; i.e., $\hat{x}_o = u/2$. It is not hard to find some welfare loss in the sense that $y < y_o$. In brief, the rational producer prices higher and produces less when he

boldly faces uninsurable business risks. This inefficiency is not surprising.—The producer has no access to complete asset markets for risks prevalent in his goods market business.

4.2 Price Inertia relative to Quantity Movement

So far we have uncovered the basic link between riskiness of business and markup pricing. We now examine whether or not the model generates price inertia relative to quantity movements as we observe in data using the analysis of comparative statics. Again, the hedge rule (17) and the offer rule (18) are central to our analysis of comparative statics. The most natural form of experiments of comparative risk takes the first-order stochastic dominance (FSD) shock. Draw θ and θ' , over two consecutive periods, from the distribution of distribution parameters $G(\theta)$. If $\{1 - F'(x; \theta')\} \geq \{1 - F(x; \theta)\}$ for every x , it is said that F' FSDs F , or r.v. x' FSDs r.v. x . Let us brief it by an “FSD shock”. This shock is a favourable demand-side shock in stochastic terms. An FSD-deteriorating shock is a negative demand shock. The following proposition presents an important result of comparative analysis of business risks, and shows that the presence of uninsurable business risks can explain the observed price inertia in data.

Proposition 2 (Price Inertia). *Suppose F' FSDs F . Everything else equal (i.e., $B' = B$ and $c' = c$ and $q' = q$), the hedge rule (17) associates every given \hat{x} with a higher price (or at least equal price) following the FSD shock, whereas the offer condition (18) associates every given \hat{x} with a lower price (or at most equal price).*

Proof. See Appendix. □

This proposition states that the hedge rule pushes up prices in response to an FSD shock, and in contrast, the offer rule reacts to pull down. The result is price inertia. New optimum is associated with a large movement along the dimension of \hat{x} with little variation in p , and therefore associated with a large movement in s . For schematic appreciation, refer to Figure 4, a typical high-school exposition for the resultant force between vertical and horizontal forces. Section 4.3 will further examine economic conditions by which we can tell about the ultimate direction of the resultant force in the dimension of price.

To see the key economic mechanism behind Proposition 2, suppose that an FSD shock hits the market. By definition, the probability of market-clearing $1 - F(\hat{x})$ becomes larger

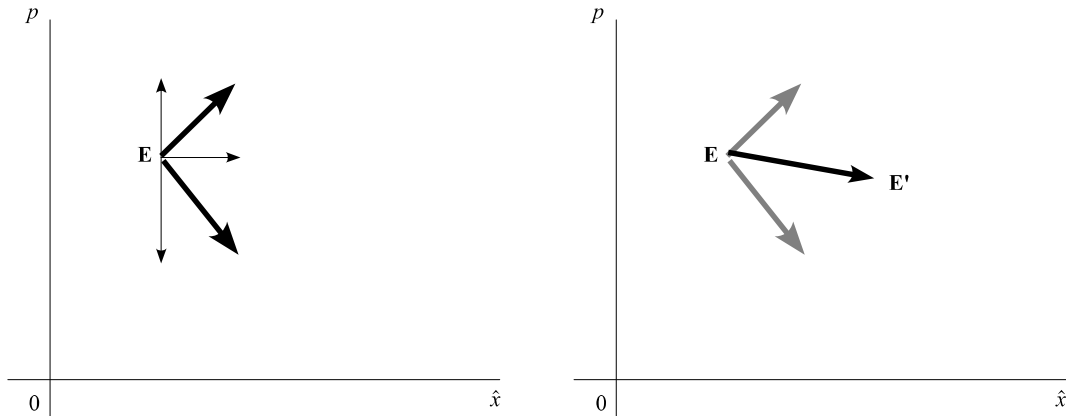


Figure 4: Price Inertia relative to Quantity Movement

for every \hat{x} , and the probability of market-clearing failure $F(\hat{x})$ becomes smaller. Put differently, the market becomes more likely to clear for every given production level and for every given price, as market demand is higher on average (as implied by the FSD relation). So the producer feels *safer* than before with every feasible combination of his pricing and production decisions.¹⁹ In a sequel, as he feels safer, the producer would set a higher price for every given production level. So reduction in the risk of market-clearing failure tends to free up the producer in exploitation of his monopolistic market power. This is what the first part of Proposition 2 captures through the reaction of the hedge rule (17) following an FSD shock.

At the same time, the offer rule (18) also actively reacts to the demand shock. Clearly, when market demand is higher, the market becomes more likely to clear for every given output level. As a result, the producer will feel safer with his production decision, and produce more for every given price. While the offer rule is basically identified as the optimal production schedule, we find that the supply side actively reacts to even non-technological demand-side shocks. Graphically, the offer curve will shift down as if there were a positive supply shock. This is what the second part of Proposition 2 captures. The next example takes both parts of the proposition and illustrates a case of perfect price

¹⁹Notice that we use the term “safer” to describe the rational agent’s response to the comparative experiment with “higher mean”. In a traditional usage of terms, we do not usually associate riskiness of events with the first moment of the event’s distribution but with the second moment. However, in the present paper, such convention has no much relevance because the monopolist is risk-neutral in the Arrow-Pratt measure as appeared in his objective function (4), and the importance of distribution actually arises from the fact of asymmetric profit structure between excess supply state and excess demand state. So we describe a “more likely to clear” market environment for every given price as a “less risky” situation even without taking into account the monopolist’s risk attitude.

inertia.

Example 3. *Continuing from Example 1:* Consider the same shock environments as in Proposition 2. That is, F' FSDs F , and $B = B'$ and $c = c'$ and $q = q'$. Specifically, let x' be uniformly distributed over $[0, u']$ where $u' > u$. Obviously, $1 - F'(\hat{x}) = 1 - (\hat{x}/u) > 1 - (\hat{x}/u') = 1 - F(\hat{x})$ for every \hat{x} . Figure 5 illustrates the impact of the FSD shock on pricing and production.

On the hedge side, Equation (19) from Example 1 turns to

$$p' = 1 - \frac{(1 - qB)\hat{x}'}{2u'}$$

following the FSD shock. One can easily see that with $u' > u$, the schedule associates every $\hat{x}' = \hat{x}$ with a higher price $p' > p$. In other words, the hedge curve shifts up from $hedge(p)$ to $hedge'(p)$ as illustrated by Figure 5a, where the relation $p' > p$ is represented in terms of the markup size (denoted by \mathbf{mu} , price minus marginal cost at a given production level): $\mathbf{mu}' > \mathbf{mu}$.

On the offer side, Equation (20) turns to

$$p' = qB + \frac{(c - qB)u'}{u' - \hat{x}'},$$

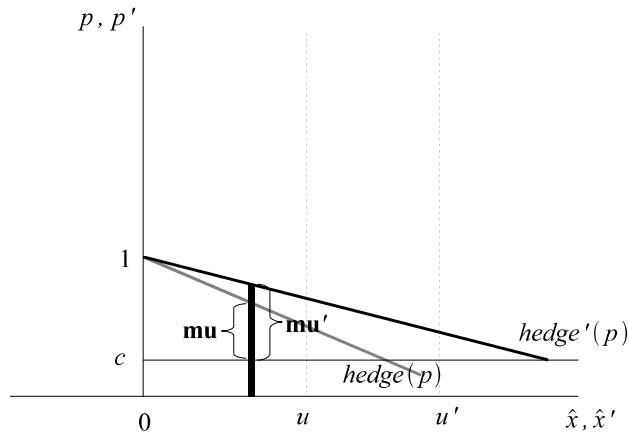
following the FSD shock. As shown in Figure 5b, the impact of the demand shock on markups reverses. As the offer curve shifts out from $offer(s)$ to $offer'(s)$, every $\hat{x}' = \hat{x}$ is associated with a lower price and so reduces the markup size; $\mathbf{mu}' < \mathbf{mu}$.

Figure 5c summarizes the overall impact of the demand shock on pricing and production. The shock increases production, but neither increase nor decrease price. Although the experiment is made with simple functional forms, the economic implication is clear: At optimum, quantities are more sensitive than prices in response to demand shocks.²⁰

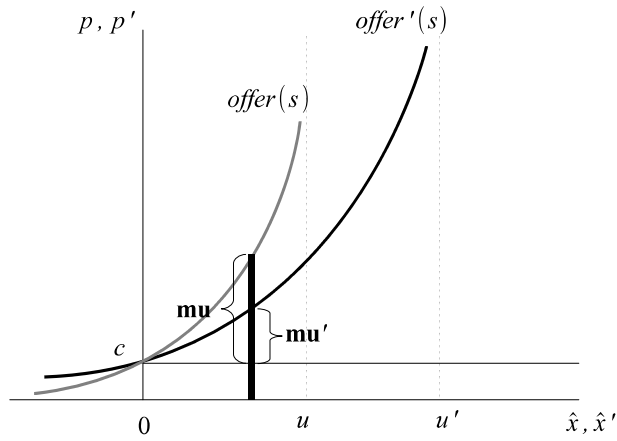
4.3 Cyclical Variation of Markup

Our previous comparative statics of business risks implies that the ultimate direction of optimal price depends on the relative responsiveness of the hedge and the offer, while the optimal level of production increases in response to a favourable demand shock. But,

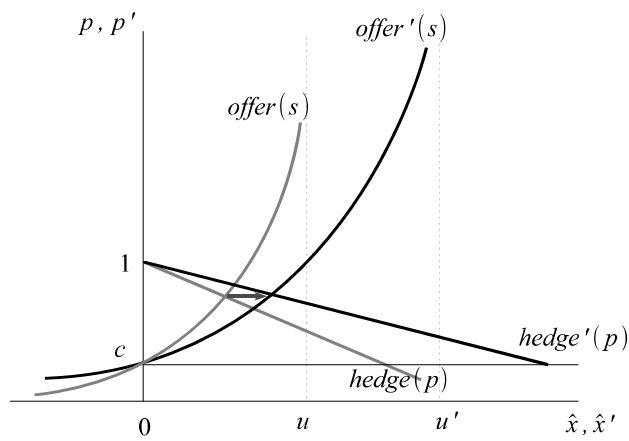
²⁰Note that when replacing the constant marginal cost with an increasing schedule in production level, the same FSD shock may lead to reduction in the markup rate.



(a) The hedge curves and markups



(b) The offer curves and markups



(c) The overall impact of demand shock

Figure 5: Example 3

what determines their relative responsiveness? To answer this question, refer back to the markup rule (21) with the risk-balancing factor (22). Apparently, the risk-balancing factor $R(\hat{x})$ has a predictive power about the relative responsiveness of the two decision rules, as it is present only in the hedge side.²¹ Among the distributional terms that constitute the risk-balancing factor $R(\hat{x})$, we can see the probability of market-clearing failure $F(\hat{x})$ and the probability of market-clearing $1 - F(\hat{x})$ move in the opposite direction for every \hat{x} following an FSD shock. So the predictive power of $R(\hat{x})$ concerning the ultimate response of markups (and thus optimal pricing) is summarized in the conditional sales ratio $T(\hat{x})/\hat{x}$. We first look at the expected volume of sales conditional on market-clearing failure alone, and then combine it with the expected volume of sales conditional on market-clearing.

Lemma 3 (Expected Volume of Sales). *Suppose F' FSDs F . Then, the expected volume of sales conditional on market-clearing failure becomes larger,*

$$T'(\hat{x}) \geq T(\hat{x}),$$

for every \hat{x} .

Proof. See Appendix. □

Lemma 4 (The Conditional Sales Ratio). *Suppose F' FSDs F . Assume that both distributions have finite expected values: $E(x) < \infty$, and $E(x') < \infty$. Then, the set $\{(\hat{x}', \hat{x}) \mid \hat{x}' > \hat{x} \text{ and } T'(\hat{x}')/\hat{x}' \geq T(\hat{x})/\hat{x}\}$ is non-empty for any fixed $\hat{x} \in (\underline{x}, \bar{x})$.*

Proof. [**Step 1**]: We already know that $0 \leq T(\hat{x})/\hat{x} < 1$, or equivalently $0 \leq T(\hat{x}) < \hat{x}$ ($\hat{x} \neq 0$) for any non-degenerate probability distribution of a non-negative random variable.

Combining the fact with Lemma 3, one can easily assure that $T(\hat{x}) \leq T'(\hat{x}) < \hat{x}$ for all \hat{x} 's.

[**Step 2**]: We then use the fact that T is monotonically increasing in \hat{x} but bounded from above by a finite value; i.e., $T(\hat{x}) \rightarrow 0$ as $\hat{x} \rightarrow \underline{x}$ (the lower bound) and $T(\hat{x}) \rightarrow E(x) < \infty$

²¹Notice that using the definition of the risk-balancing factor (22), one can rewrite the hedge rule (17) as follows;

$$p \left\{ 1 - \frac{1}{\epsilon(p)} \frac{R(\hat{x})}{F(\hat{x})} \right\} = qB.$$

This expression also clues how the risk-balancing factor is related to the price elasticity of demand in their relative size. That is,

$$\epsilon(p) \geq \frac{R(\hat{x})}{F(\hat{x})} > 1,$$

at any interior optimum.

as $\hat{x} \rightarrow \bar{x}$ (the upper bound). Due to the monotonicity of T and the existence of finite supremum, there must exist at least one $\hat{x}_1 > \hat{x}_0$ (for fixed \hat{x}_0) such that the following condition hold around non-empty neighbourhood $(-\Delta, \Delta)$,

$$\frac{T(\hat{x}_0)}{\hat{x}_0} \leq \frac{T'(\hat{x}_1 - \Delta)}{\hat{x}_1 - \Delta}, \text{ and } \frac{T(\hat{x}_0)}{\hat{x}_0} > \frac{T'(\hat{x}_1 + \Delta)}{\hat{x}_1 + \Delta}.$$

□

Now we tailor the usual definition of cyclicity into the current context of optimal pricing-estimation decision, and proceed to the third main result of this research.

Definition 3 (Cyclicity). Let $\mu(p, \hat{x})$ and $\mu'(p', \hat{x}')$ denote pre- and post-shock schedules of markup rates, respectively. Let also $\mu^* = \mu(p^*, \hat{x}^*)$ and $\mu'^* = \mu'(p'^*, \hat{x}'^*)$ denote the optimal markup rates before and after shock, respectively. If a shock leads to $\{\mu'^* < \mu^* \text{ and } \hat{x}'^* \geq \hat{x}^*\}$ or $\{\mu'^* > \mu^* \text{ and } \hat{x}'^* \leq \hat{x}^*\}$, then the shock will be said to cause a countercyclical markup behaviour. If a shock leads to $\{\mu'^* < \mu^* \text{ and } \hat{x}'^* \leq \hat{x}^*\}$ or $\{\mu'^* > \mu^* \text{ and } \hat{x}'^* \geq \hat{x}^*\}$, then the shock will be said to cause a procyclical markup behaviour.

Noteworthy is that the statement $\{\mu'^* > \mu^* \text{ and } \hat{x}'^* \leq \hat{x}^*\}$ concerns the times when a shock occurs in the reverse direction of stochastic dominance (such as like FSD-deteriorating or SSD-deteriorating distributional changes). Alternating dominance of a shock (like FSD and FSD-deteriorating) in the line with the alternating statements in Definition 3 is a *sufficient* condition for countercyclical markups. However, this is not a necessary condition for the usual definition of countercyclical markups, since there might exist such occasions where output y (or stock for sales s) decreases even when both \hat{x} and $\mu(\hat{x}, p)$ increase.

Proposition 3 (Countercyclicity). Consider the same shock environments as in Proposition 2. That is, F' FSDs F , while $B = B'$ and $c = c'$ and $q = q'$. Then, every element (\hat{x}, \hat{x}') pairwise taken from the set $\{(p, \hat{x}), (p', \hat{x}') \mid \mu'(p', \hat{x}') < \mu(p, \hat{x})\}$ is also a member of the set

$$\left\{ (\hat{x}, \hat{x}') \mid \frac{T'(\hat{x}')}{\hat{x}'} > \frac{T(\hat{x})}{\hat{x}} \right\}, \quad (23)$$

but not vice versa.

Proof. See Appendix. Essentially, to become a member of the set $\{((p, \hat{x}), (p', \hat{x}')) \mid \mu'(p', \hat{x}') < \mu(p, \hat{x})\}$, it should meet the membership requirement implied by both decision rules (17) and (18) (or equivalently, both the markup rule (21) and either of the two decision rules). So the proof involves joint examination of the two key equations. First, we derive two separate necessary conditions, each of which satisfies (17) and (18), respectively. Next, we show that any $((p, \hat{x}), (p', \hat{x}'))$ that satisfies both necessary conditions should also satisfy the claimed ordering $T'/\hat{x}' \geq T/\hat{x}$. Finally, to show that the reversed relationship does not necessarily hold, we use the markup rule (21) and show that some paired element (\hat{x}, \hat{x}') belonging to the set (23) may not be found from the set $\{((p, \hat{x}), (p', \hat{x}')) \mid \mu'(p', \hat{x}') < \mu(p, \hat{x})\}$. \square

To appreciate the claimed ordering within the economics context, recall that $T(\hat{x})/\hat{x}$ measures the relative size of the expected volume of sales conditional on successful market-clearing to the sales volume conditional on market-clearing failure. So the proposition basically states that the producer would lower the optimal markup rate only when the expected volume of sales conditional on market-clearing failure $T(\hat{x})$ increases more in proportion than the expected volume of sales conditional on market-clearing \hat{x} . An increase in the conditional sales ratio $T(\hat{x})/\hat{x}$ means that the producer can expect to see relatively higher level of sales than before. Consequently, the producer feels more satisfied with his business condition, and expects the overall risk level of the goods market to decrease. Only then, he lowers the markup.

One can associate this kind of risk assessment directly to the risk-balancing factor $R(\hat{x})$ in (22), and will find it a sensible risk measure indeed. As seen before, $R(\hat{x})$ uses a normalized weight vector, 1 *versus* $\hat{x}/T(\hat{x})$, to balance the two contrasting types of risks associated with each of the two mutually exclusive events (market to clear *or* not) for given choice \hat{x} . According to the formula (22), an increase in $T(\hat{x})/\hat{x}$ and thus a fall in $\hat{x}/T(\hat{x})$ along the optimum path leads to a fall in $R(\hat{x})$, holding the other terms constant. And then, according to the formula of the markup rate (21), a fall in $R(\hat{x})$ will lead to a fall in the markup rate. As such, through the risk measure $R(\hat{x})$, the conditional sales ratio $T(\hat{x})/\hat{x}$ is tightly linked to pricing decision.

Another way of appreciating the proposition's necessary condition is to notice the expressional closeness between the two definitions of $T(\cdot)$ and the second-order stochastic

dominance (SSD).²² Along the line of the literature of stochastic dominance, we define a comparative risk relationship that can precisely capture the necessary condition in Proposition 3.

Definition 4 (Normalized Conditional SSD). If (\hat{x}, \hat{x}') is a member of the set

$$\left\{ (\hat{x}, \hat{x}') \mid \frac{T'(\hat{x}')}{\hat{x}'} > \frac{T(\hat{x})}{\hat{x}} \right\},$$

we say that it lies in “normalized conditional SSD” (NC-SSD) relation.

Starting from the familiar notion of the SSD, we first think about ‘conditional SSD’ and define it by the relation $T'(\hat{x}') \geq T(\hat{x})$. As the SSD relation normally captures stochastically less risky relationship, the notion of ‘conditional SSD’ can be employed to capture the subset of events conditional on economic agent’s choice when r.v. x' becomes less risky than r.v. x . Using the notion back to Lemma 3 in particular, we can equivalently state that if r.v. x' FSDs r.v. x then every estimate $\hat{x}' = \hat{x}$ lies in ‘conditional SSD’ relation. In the same spirit, Definition 4 refines the notion further, and introduces a notion of ‘normalized conditional SSD’ by indicating the relation $T'(\hat{x}')/\hat{x}' \geq T(\hat{x})/\hat{x}$. Using this notion, we can rephrase Proposition 3 in an intuitive statement as follows: *The monopolist lowers the markup only if a distributional shock leads to a ‘normalized conditional SSD’ (NC-SSD) relation at his optimum path.* Remind of which risk environments the SSD relation usually means in the literature, and then combine this rephrasing of Proposition 3 with the risk premium result of Proposition 1. Now, readers may easily grasp the intrinsic link between business risks and markups. This finalizes our theorization of business risks for the goods markets. The following auxiliary result employs the notion of NC-SSD to state a sufficient condition for the procyclical markup behaviour.

Corollary (Procyclicality). *Any (\hat{x}, \hat{x}') that does not lie in an NC-SSD relation can be paired from a member of the set $\{((\hat{x}, p), (\hat{x}', p')) \mid \mu'(\hat{x}', p') \geq \mu(\hat{x}, p)\}$.*

Proof. Obvious by contrapositioning of Proposition 3. □

Remark. No particular order of stochastic dominance involves in the proof of Proposition 3 and Corollary. So the result holds for any form of distributional shocks (and holds in

²²It is said that random variable x' second-order stochastically dominates (SSDs) x if $\int_0^{\hat{x}} \{1 - F'(x')\} dx' \geq \int_0^{\hat{x}} \{1 - F(x)\} dx$ for all \hat{x} .

parallel for the reverse cases like FSD-deteriorating, and so on).

The following example serves to demonstrate Proposition 3 and Corollary, and shows how one can use the NC-SSD relation to capture the economic consequences of changing risk environments.

Example 4. Assume that $d = xp^{-\epsilon}$, where $\epsilon > 1$. The demand function with constant price elasticity has been often used in the applied literature. Let also $B = 0$. This assumption saves algebraic work without loss of generality. But this time, we replace the zero lower bound with a positive real variable l ; i.e., $x \sim U[l, u]$, where $0 < l < u$. Then, we obtain

$$\hat{x}^* = \frac{u + \sqrt{u^2 + (\epsilon^2 - 1)l^2}}{\epsilon + 1},$$

and

$$p^* = \frac{(u - l)(\epsilon + 1)}{\epsilon u - \sqrt{u^2 - l^2 + \epsilon^2 l^2}} \epsilon,$$

where $l < \hat{x}^* < u$. We conduct comparative statics of markup pricing by changing the parameters u and l . Notice first that any increase in either the upper bound or the lower bound (or both) can be qualified as an FSD shock; and second that the optimal estimate \hat{x}^* increases in response to either case of the distributional shock (i.e., $\hat{x}'^* > \hat{x}^*$). How about the optimal responses of the markup rate? Using Proposition 3 and Corollary, we examine the impact of the FSD shock on the optimal markup rate for each of the following two cases: *Case 1.* an increase in u , *Case 2.* an increase in l .

For this, we first take a direct approach to the comparative statics of the markup rate. Assuming each case of the distributional shock is infinitesimally small, we take simple partial differentiation to the optimal markup rate.

Case 1. An infinitesimal increase in u : Let $u = \alpha l$, where $\alpha > 1$. Then, the current optimal markup rate is expressible as

$$\mu^* = \frac{\alpha l - l}{\alpha \epsilon - \sqrt{\alpha^2 l^2 - l^2 + \epsilon^2 l^2}} = \frac{\alpha - 1}{\alpha \epsilon - \sqrt{\alpha^2 - 1 + \epsilon^2}}.$$

Verifying the sign of $\partial \mu^* / \partial u$ is equivalent to verifying the sign of $\partial \mu^* / \partial \alpha$. One can show that $\partial \mu^* / \partial \alpha > 0$.

Case 2. An infinitesimal increase in l : Let $l = \alpha u$, where $0 < \alpha < 1$. Then,

$$\mu^* = \frac{u - \alpha u}{u\epsilon - \sqrt{u^2 - \alpha^2 u^2 + \epsilon^2 \alpha^2 u^2}} = \frac{1 - \alpha}{\epsilon - \sqrt{1 - \alpha^2 + \epsilon^2 \alpha^2}}.$$

Verifying the sign of $\partial\mu^*/\partial l$ is equivalent to verifying the sign of $\partial\mu^*/\partial\alpha$. One can show that $\partial\mu^*/\partial\alpha < 0$.

In other words, for the given uniform distribution, a demand shock that increases the upper-bound/lower-bound leads to an increase/decrease in the markup rate. So the results contrast each other in the movement of markups, although the market demand increases on average in both cases. Proposition 3 confirms the result:

Case 1. $x' \sim U[l, u']$ with $u' > u$: Check whether the optimum path (\hat{x}^*, \hat{x}'^*) is a member of the set $\{(\hat{x}, \hat{x}') \mid T'(\hat{x}')/\hat{x}' > T(\hat{x})/\hat{x}\}$. We have

$$T(\hat{x}) = \frac{\hat{x} + l}{2}.$$

Taking difference between the conditional sales ratio,

$$\frac{T'(\hat{x}')}{\hat{x}'} - \frac{T(\hat{x})}{\hat{x}} = \frac{\hat{x}' + l}{2\hat{x}'} - \frac{\hat{x} + l}{2\hat{x}} = \frac{l(\hat{x} - \hat{x}')}{2\hat{x}'\hat{x}} < 0$$

at optimum. So (\hat{x}, \hat{x}') of our interest does not belong to the set $\{(\hat{x}, \hat{x}') \mid T'(\hat{x}')/\hat{x}' > T(\hat{x})/\hat{x}\}$. From Corollary above, we know that the monopolist sets a higher (at least as equal as) rate of the markup if he cannot find an NC-SSD relation along the optimum path. So the shock to the upper bound generates *procyclical* markup behaviour.

Case 2. $x' \sim U[l', u]$, with $l' > l$: Similarly,

$$\frac{T'(\hat{x}')}{\hat{x}'} - \frac{T(\hat{x})}{\hat{x}} = \frac{\hat{x}' + l'}{2\hat{x}'} - \frac{\hat{x} + l}{2\hat{x}} = \frac{l'\hat{x} - l\hat{x}'}{2\hat{x}'\hat{x}} > 0$$

at optimum. So an NC-SSD relation is found, and the shock to the lower bound leads to *countercyclical* markup behaviour.

To intuitively understand why the NC-SSD relation works well in identification of the cyclicity of markups, let us think about what distributional characters can be captured by the notion of NC-SSD relation. Assume that $l < l' < u < u'$. Both cases are with FSD

shocks. However, *Case 2* shock will locate less probability mass on both tails, relative to *Case 1* shock. With *Case 2* shock, the monopolist can expect a higher level of sales volume even when market fails to clear (i.e., “conditional”), and even in the ratio term relative to the expected volume of sales conditional on market-clearing (i.e., “normalized”).

5 Conclusion

We introduce uninsurable business risks to the goods markets and study how their presence affects the producer’s optimal pricing and production behaviours. Business risks are modelled in the form of stochastic market-clearing and market-clearing failure within the otherwise standard framework of monopolist. This small modification to a textbook model renders the nature of producer decision problem largely different, and leads pricing and production decisions to counterpoint in formation of profit function.

We identify two mechanisms in relation to their distinct roles. Profit-maximization with respect to pricing decision works to neutralize the uninsurable business risks. Profit-maximization with respect to production decision works to equate marginal cost of production to the sum of expected marginal revenues conditional on each demand state. Equipped with this new decision framework for a monopolistic producer, we characterize a markup rule as a pricing kernel of business risks as well as market power, and show that optimal price includes ‘traditional markup’ to the extent of market power and ‘risk premium’ by the shadow price of the business risks. In addition, we take the new decision framework to provide a unified explanation for price inertia, countercyclical markups, and procyclical markups. Central to understanding these empirical regularities is the extent of business risks that varies across sectors and moves over business cycles.

Grounded on these results, the producer theory of business risks leaves important implications both micro and macro, and both empirical and theoretical. First, our theory provides clues to the nature of business cycles, consistently with the empirical evidence that suggests the extent of business risks varies over business cycles. Although the results we find have not been derived from a general equilibrium model by which one could establish certain causal links between business risks and business cycles, we can still address some important issues in macroeconomics. If the times of economic recessions are coincident with the times when unwanted inventories build up faster and the bankruptcy rate rises

as we observe in data, individual firms will consider the times of recessions as the times of high business risks, and react through the two key mechanisms; the risk neutralization and the marginal cost equalization. Our theory predicts that under the harder macroeconomic conditions, the firms reduce output and stock but adjust little price. Consequently, we find price inertia over business cycles as optimal choices, but not intertemporal nominal price stickiness. It suggests that monetary policy (nominal aggregate demand shocks) may have real effects if nominal shocks have some risk consequences to individual firm's business conditions.

Second, whether markups are countercyclical or procyclical at aggregate level is still controversial, although many pioneering empirical studies have found them countercyclical. Above all, the countercyclical movement of markups (either in a sector or at the aggregate level) implies that exercise of market power is far more limited than predicted under Chamberlinian monopolistic competition. Indeed, it is this implication of countercyclical markups that has long attracted research interest in both theoretical explanation for it and empirical reexamination against it. Our theory has constructive suggestions for both directions: At the bottom line, exercise of market power is limited in the presence of uninsurable business risks. As a natural consequence, markups *tend* to move countercyclically, if business risks are larger in recessions and smaller in booms.—But, not necessarily so. It is shown that precise cyclical behaviour of markups depends on the relative responsiveness of the two channels to changing business risks. However, most of the empirical studies have maintained one common theoretical premise that markup pricing is an exercise of market power in imperfectly competitive markets. As this traditional view neglects the shadow price (firm-internal pricing) of uninsurable business risks in determination of markups, the existing empirical models may suffer from a model misspecification problem, and thus estimation biases.—Markup data we access may overestimate the magnitude of market power actually at work.

Third, in micro data, we find prices rise during periods of demand peaks in some sectors and fall in other sectors. According to the theory proposed here, the mixed evidence across sectors is consistent with the fact that industries differ in the nature of business risks influenced by turnover of new products, time taken from service/production planning to completion, and industrial relations, and so on. Our theory also sheds light on the contradicting pricing behaviours we often observe even within an industry; for example,

advanced purchase discount (the earlier the cheaper) *versus* last minute sales (the later the cheaper). Most of the existing literature focuses on the role of market power that is endowed to a monopolist to conduct price discrimination over heterogeneous consumer valuations when transactions are made in a sequential manner. Complementary to the existing explanation, our theory of business risks suggests that such pricing pattern can arise as a means of “active risk management” against market-clearing failure.

Appendix

Proof of Proposition 1

It is sufficient to show that $R(\hat{x}) > 1$ and $\epsilon(p)$ is weakly increasing in price p . From (22), note that $R(\hat{x})$ is a linear combination of 1 and $\hat{x}/T(\hat{x})$. And from the definition of “truncation from above”, we also know that $0 < T(\hat{x})/\hat{x} < 1$ (and thus $\hat{x}/T(\hat{x}) > 1$) for any non-degenerate distribution of positive random variable x . As a result,

$$R(\hat{x}) > 1$$

for all \hat{x} 's other than where $F(\hat{x}) = 1$.

Next, we show that $\epsilon(p)$ is weakly increasing in p , which is a basic property of demand function that satisfies the standard assumption $2D_p(p) + pD_{pp}(p) \leq 0$ as we have discussed in Lemma 2. Differentiating $\epsilon(p) = |D_p(p)/\{D(p)/p\}|$ w.r.t. p , we obtain

$$\begin{aligned} \epsilon_p(p) &= -\frac{D(p)\{D_p(p) + pD_{pp}(p)\} - p\{D_p(p)\}^2}{\{D(p)\}^2} \\ &= -\frac{D_p(p)}{D(p)} \left\{ 1 - \frac{pD_p(p)}{D(p)} + \frac{pD_{pp}(p)}{D_p(p)} \right\}. \end{aligned}$$

By definition $\epsilon(p)$, we can rewrite it to

$$\epsilon_p(p) = -\frac{D_p(p)}{D(p)} \left\{ 1 + \epsilon(p) + \frac{pD_{pp}(p)}{D_p(p)} \right\}.$$

Clearly it holds that $\epsilon_p(p) \geq 0$ for all p , since $D_p(p) < 0$ and

$$1 + \epsilon(p) + \frac{pD_{pp}(p)}{D_p(p)} \geq 0.$$

The latter follows from the fact that $\epsilon(p) > 1$ (indeed also implied by that $\epsilon(p) \geq R(\hat{x})/F(\hat{x})$ as shown in Footnote 21) and from the standard assumption that $2D_p(p) + pD_{pp}(p) \leq 0$ discussed in Lemma 2. The equality holds when demand function has a constant elasticity. Consequently, $\epsilon(p)$ is weakly increasing in p .

Combine the facts; $R(\hat{x}) > 1$ and $\epsilon_p(p) \geq 0$ for all p . It then follows that at optimum, the markup rate with business risks strictly larger than the markup rate without business risks. That is, $\mu^* > \mu_o^*$.

Proof of Proposition 2

The first part of the following proof shows that the hedge rule (17) associates every given \hat{x} with a higher price (or at least equal price) in response to an FSD shock, and the second part shows that the offer condition (18) associates every given \hat{x} with a lower price (or at most equal price).

1. The Hedge Rule's Reaction: First, rewrite the hedge rule (17) as follows;

$$p = \frac{qB}{1 - \frac{1}{\epsilon(p)} \left[1 + \frac{\hat{x}\{1 - F(\hat{x})\}}{T(\hat{x})F(\hat{x})} \right]}.$$

Suppose the contrary were true: i.e., $p' < p$ following the FSD shock. Then,

$$\frac{1}{1 - \frac{1}{\epsilon(p')} \left[1 + \frac{\hat{x}\{1 - F'(\hat{x})\}}{T'(\hat{x})F'(\hat{x})} \right]} < \frac{1}{1 - \frac{1}{\epsilon(p)} \left[1 + \frac{\hat{x}\{1 - F(\hat{x})\}}{T(\hat{x})F(\hat{x})} \right]},$$

or

$$\frac{1}{\epsilon(p')} \left[1 + \frac{\hat{x}\{1 - F'(\hat{x})\}}{T'(\hat{x})F'(\hat{x})} \right] < \frac{1}{\epsilon(p)} \left[1 + \frac{\hat{x}\{1 - F(\hat{x})\}}{T(\hat{x})F(\hat{x})} \right].$$

Rearranging both sides, we obtain

$$\frac{1 + \frac{\hat{x}\{1 - F'(\hat{x})\}}{T'(\hat{x})F'(\hat{x})}}{1 + \frac{\hat{x}\{1 - F(\hat{x})\}}{T(\hat{x})F(\hat{x})}} < \frac{\epsilon(p')}{\epsilon(p)}. \quad (24)$$

Next, we will derive a contradiction by showing that the l.h.s. is greater than (or at least equal to) one, and the r.h.s. less than one.

[Step 1] (l.h.s. ≥ 1): Consider a measure \mathbb{F} over the space of non-negative x , given by $\mathbb{F}(x) = F(x) - F(x')$. By definition (FSD), $\mathbb{F}(x) \geq 0$ for every x . Consequently,

$$\int_{\underline{x}}^{\hat{x}} x d\mathbb{F}(x) \geq 0.$$

By the method of integration by parts,

$$\hat{x}\mathbb{F}(\hat{x}) - \int_{\underline{x}}^{\hat{x}} \mathbb{F}(x) dx \geq 0,$$

or

$$\hat{x}\{F(\hat{x}) - F'(\hat{x})\} - \int_{\underline{x}}^{\hat{x}} \{F(x) - F'(x)\}dx \geq 0.$$

Rearranging it,

$$\hat{x}F(\hat{x}) - \int_{\underline{x}}^{\hat{x}} F(x)dx \geq \hat{x}F'(\hat{x}) - \int_{\underline{x}}^{\hat{x}} F'(x)dx.$$

Reversing the method of integration by parts, we obtain

$$\int_{\underline{x}}^{\hat{x}} x dF(x) \geq \int_{\underline{x}}^{\hat{x}} x dF'(x),$$

which is equivalent to by definition $T(\cdot)$,

$$T(\hat{x})F(\hat{x}) \geq T'(\hat{x})F'(\hat{x}).$$

Consequently, it holds that

$$\frac{1 + \frac{\hat{x}\{1 - F'(\hat{x})\}}{T'(\hat{x})F'(\hat{x})}}{1 + \frac{\hat{x}\{1 - F(\hat{x})\}}{T(\hat{x})F(\hat{x})}} \geq 1,$$

since $1 - F'(\hat{x}) \geq 1 - F(\hat{x})$ by definition (FSD).

[Step 2] (r.h.s. ≤ 1): Under the supposition that $p' < p$, we would see that $\epsilon(p')/\epsilon(p) \leq 1$ since $\epsilon(p)$ is weakly increasing in price p . Combining the two steps contradicts (24) everywhere. So it contradicts the previous supposition, and completes proof.

2. The Offer Rule's Reaction: Rewrite the offer rule (18) as follows;

$$p(\hat{x}) = \frac{1}{1 - F(\hat{x})}c - \frac{F(\hat{x})}{1 - F(\hat{x})}qB = \frac{c - qBF(\hat{x})}{1 - F(\hat{x})}.$$

Everything else constant, we have that for every given \hat{x} ,

$$\begin{aligned}
\{p' \leq p\} &\iff \left\{ \frac{c - qBF'(\hat{x})}{1 - F'(\hat{x})} \leq \frac{c - qBF(\hat{x})}{1 - F(\hat{x})} \right\} \\
&\iff \left\{ \frac{1 - F'(\hat{x})}{1 - F(\hat{x})} \geq \frac{c - qBF'(\hat{x})}{c - BF(\hat{x})} \right\} \\
&\iff \left\{ \frac{1 - F'(\hat{x})}{1 - F(\hat{x})} \geq \frac{1 - (qB/c)F'(\hat{x})}{1 - (qB/c)F(\hat{x})} \right\} \\
&\iff \{ \{1 - F'(\hat{x})\} \{1 - (qB/c)F(\hat{x})\} \geq \{1 - F(\hat{x})\} \{1 - (qB/c)F'(\hat{x})\} \} \\
&\iff \{F(\hat{x}) - F'(\hat{x})\} \{1 - (qB/c)\} \geq 0
\end{aligned}$$

The last line holds jointly by definition (FSD) and by the condition of no-speculative inventory holdings $0 \leq qB < c$ as discussed in Lemma 1.

Proof of Lemma 3

We know that for non-negative random variable z , the expected value equals the area under the complementary c.d.f., $1 - F(z)$. That is,

$$E(z) = \int \{1 - F(z)\} dz.$$

Refer to [Wolfstetter \(1999, p.341, Rule 5 and 6\)](#) for the statement. See also [Ross \(2007, p.92, Exercise 46\)](#) for the proof of a discrete case. Figure 6a depicts the statement in the current context. We also know that a first-order stochastic dominance shock between two periods (like the one shown in Figure 6b) implies that the following inter-temporal relationship holds in terms of expected values, ([Wolfstetter, 1999, p.137, Lemma 4.1](#));

$$E(x') = \int \{1 - F'(x)\} dx \geq \int \{1 - F(x)\} dx = E(x).$$

Since $\{1 - F'(\hat{x})\} \geq \{1 - F(\hat{x})\}$ for *all* \hat{x} 's, it should also hold that

$$1 - F'(x | x < \hat{x}) \geq 1 - F(x | x < \hat{x}),$$

for any given \hat{x} . An example of the FSD relation for a given common truncation point is illustrated by Figure 6c. Then, by the definition of T ,

$$\begin{aligned} T'(\hat{x}) = E(x' | x' < \hat{x}) &= \int_0^{\hat{x}} \{1 - F'(x | x < \hat{x})\} dx \\ &\geq \int_0^{\hat{x}} \{1 - F(x | x < \hat{x})\} dx = E(x | x < \hat{x}) = T(\hat{x}). \end{aligned}$$

Figure 6d is a graphical representation of this final statement.

Proof of Proposition 3.

[Step 1]: Select a paired element (\hat{x}, \hat{x}') from the set $\{(p, \hat{x}), (p', \hat{x}') \mid \mu'(p', \hat{x}') < \mu(p, \hat{x})\}$. Each of \hat{x} and \hat{x}' must satisfy the offer condition (18). Let us tailor (18) to fit into the definition of the markup rate by dividing both sides of (18) by c ;

$$\mu(p, \hat{x}) = \frac{p}{c} = \frac{1 - (qB/c)F(\hat{x})}{1 - F(\hat{x})}.$$

Since (\hat{x}, \hat{x}') has been paired from the set $\{(p, \hat{x}), (p', \hat{x}') \mid \mu'(p', \hat{x}') < \mu(p, \hat{x})\}$, it must satisfy the following strict inequality

$$\frac{1 - (qB/c)F'(\hat{x}')}{1 - F'(\hat{x}')} < \frac{1 - (qB/c)F(\hat{x})}{1 - F(\hat{x})},$$

or equivalently

$$\frac{\{1 - (qB/c)\} \{F'(\hat{x}') - F(\hat{x})\}}{\{1 - F'(\hat{x}')\} \{1 - F(\hat{x})\}} < 0.$$

Under the no-speculation condition derived from Lemma 1, this requires that

$$F'(\hat{x}') < F(\hat{x}).$$

So

$$\{\mu'(p', \hat{x}') < \mu(p, \hat{x})\} \implies \{F'(\hat{x}') < F(\hat{x})\}.$$

[Step 2]: Let us rewrite the hedge rule (17) as follows;

$$p = \frac{qB}{1 - \frac{1}{|\epsilon(p)|} \left[1 + \frac{\hat{x}}{T(\hat{x})} \frac{1 - F(\hat{x})}{F(\hat{x})} \right]},$$

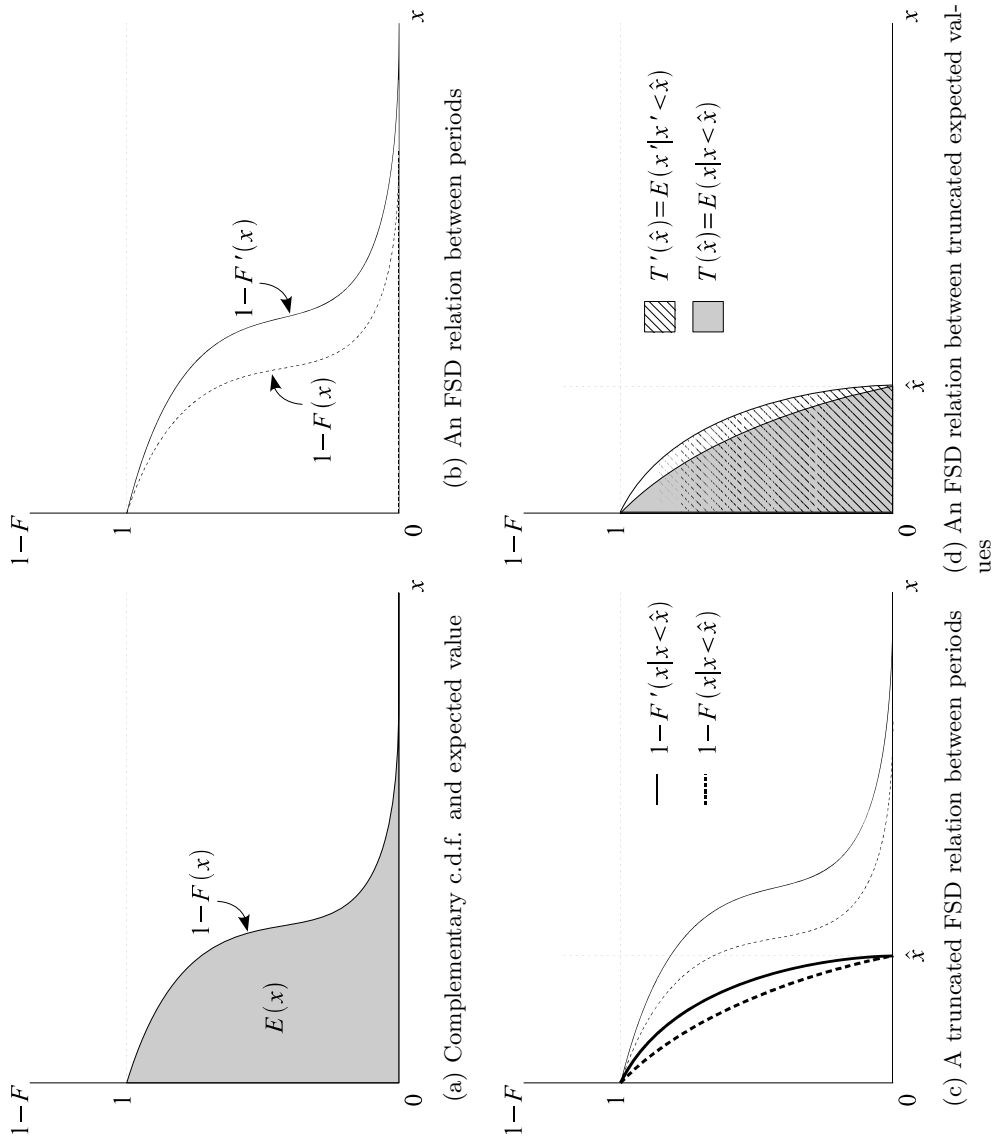


Figure 6: A graphical assistance to the proof of Lemma 3

and then tailor it to fit into the definition of the markup rate by dividing both sides by mc . Select a paired element (\hat{x}, \hat{x}') from the set $\{((p, \hat{x}), (p', \hat{x}')) \mid \mu'(p', \hat{x}') < \mu(p, \hat{x})\}$. It should hold that by the same token with Step 1,

$$\frac{qB/c}{1 - \frac{1}{\epsilon(p')} \left[1 + \frac{\hat{x}'}{T'(\hat{x}')} \frac{1 - F'(\hat{x}')}{F'(\hat{x}')} \right]} < \frac{qB/c}{1 - \frac{1}{\epsilon(p)} \left[1 + \frac{\hat{x}}{T(\hat{x})} \frac{1 - F(\hat{x})}{F(\hat{x})} \right]}.$$

After rearranging both sides, we obtain

$$\frac{1 + \frac{\hat{x}'}{T'(\hat{x}')} \frac{1 - F'(\hat{x}')}{F'(\hat{x}')}}{1 + \frac{\hat{x}}{T(\hat{x})} \frac{1 - F(\hat{x})}{F(\hat{x})}} < \frac{\epsilon(p')}{\epsilon(p)}.$$

However, since c is constant, that any paired element (p, p') from the set $\{((p, \hat{x}), (p', \hat{x}')) \mid \mu'(p', \hat{x}') < \mu(p, \hat{x})\}$ implies that $p' < p$. For such price pair (p, p') , it holds that

$$\frac{\epsilon(p')}{\epsilon(p)} \leq 1,$$

since $\epsilon(p)$ weakly increases in p as shown during the proof of Proposition 1. So clearly,

$$\frac{1 + \frac{\hat{x}'}{T'(\hat{x}')} \frac{1 - F'(\hat{x}')}{F'(\hat{x}')}}{1 + \frac{\hat{x}}{T(\hat{x})} \frac{1 - F(\hat{x})}{F(\hat{x})}} < 1.$$

Consequently,

$$\{\mu'(p', \hat{x}') < \mu(p, \hat{x})\} \implies \left\{ \frac{\hat{x}'}{T'(\hat{x}')} < \frac{\hat{x}}{T(\hat{x})} \frac{1 - F(\hat{x})}{1 - F'(\hat{x}')} \frac{F'(\hat{x}')}{F(\hat{x})} \right\}.$$

[Step 3]: Suppose the contrary that the claimed ordering (23) does not hold;

$$\frac{T'(\hat{x}')}{\hat{x}'} \leq \frac{T(\hat{x})}{\hat{x}},$$

or,

$$\frac{\hat{x}'}{T'(\hat{x}')} \geq \frac{\hat{x}}{T(\hat{x})}.$$

According to the necessary condition obtained in Step 2, this implies that

$$\frac{\hat{x}}{T(\hat{x})} \leq \frac{\hat{x}'}{T'(\hat{x}')} < \frac{\hat{x}}{T(\hat{x})} \frac{1 - F(\hat{x})}{1 - F'(\hat{x}')} \frac{F'(\hat{x}')}{F(\hat{x})},$$

which in return requires

$$1 < \frac{1 - F(\hat{x})}{1 - F'(\hat{x}')} \frac{F'(\hat{x}')}{F(\hat{x})},$$

and therefore

$$F'(\hat{x}') > F(\hat{x}).$$

But, this final statement *contradicts* the necessary condition obtained in Step 1. Consequently, Step 1 and 2 show together that

$$\{\mu'(p', \hat{x}') < \mu(p, \hat{x})\} \implies \left\{ \frac{T'(\hat{x}')}{\hat{x}'} > \frac{T(\hat{x})}{\hat{x}} \right\}.$$

[Step 4]: Using the markup rule (21) and the risk-balancing factor (22), we can see that some (\hat{x}, \hat{x}') belonging to the set (23) may not be extracted from the set $\{(p, \hat{x}), (p', \hat{x}') \mid \mu'(p', \hat{x}') < \mu(p, \hat{x})\}$. First, let us rewrite (22) as follows by subtracting 1 from both sides;

$$R(\hat{x}) - 1 = \left\{ \frac{\hat{x}}{T(\hat{x})} - 1 \right\} \{1 - F(\hat{x})\}.$$

Then

$$\begin{aligned} \{R'(\hat{x}') \geq R(\hat{x})\} &\iff \{R'(\hat{x}') - 1 \geq R(\hat{x}) - 1\} \\ &\iff \left\{ \frac{\frac{\hat{x}'}{T'(\hat{x}')} - 1}{\frac{\hat{x}}{T(\hat{x})} - 1} \right\} \left\{ \frac{1 - F(\hat{x}')}{1 - F(\hat{x})} \right\} \geq 1. \end{aligned}$$

From this statement, it is clear that one cannot select an element (\hat{x}, \hat{x}') that belongs to the set $\{(\hat{x}, \hat{x}') \mid F'(\hat{x}') < F(\hat{x}) \text{ and } T'(\hat{x}')/\hat{x}' > T(\hat{x})/\hat{x} \text{ and } R'(\hat{x}') > R(\hat{x})\}$. Second, by looking at the relationship (21) and (22), it is also clear that one cannot select an element (\hat{x}, \hat{x}') that satisfies $\mu'(p', \hat{x}') \geq \mu(p, \hat{x})$ and at the same time, belongs to the set $\{(\hat{x}, \hat{x}') \mid F'(\hat{x}') < F(\hat{x}) \text{ and } T'(\hat{x}')/\hat{x}' > T(\hat{x})/\hat{x}\}$.

References

- Arrow, Kenneth J., Theodore Harris, and Jacob Marschak. 1951. "Optimal Inventory Policy." *Econometrica* 19 (3):250–272.
- Bagwell, Kyle. 2004. "Countercyclical Pricing in Customer Markets." *Economica* 71 (284):519–542.
- Baudry, Laurent, Herve Le Bihan, Patrick Sevestre, and Sylvie Tarrieu. 2007. "What do Thirteen Million Price Records have to Say about Consumer Price Rigidity?" *Oxford Bulletin of Economics and Statistics* 69 (2):139–183.
- Beccarello, Massimo. 1997. "Time Series Analysis of Market Power: Evidence from G-7 Manufacturing." *International Journal of Industrial Organization* 15 (1):123–136.
- Bensoussan, A., M. Cakanyildirim, J. Minjarez-Sosa, A. Royal, and S. Sethi. 2008. "Inventory Problems with Partially Observed Demands and Lost Sales." *Journal of Optimization Theory and Applications* 136 (3):321–340.
- Berk, Emre, Ulku Gurler, and Richard A. Levine. 2007. "Bayesian Demand Updating in the Lost Sales Newsvendor Problem: A Two-Moment Approximation." *European Journal of Operational Research* 182 (1):256–281.
- Bils, Mark. 1987. "The Cyclical Behavior of Marginal Cost and Price." *American Economic Review* 77 (5):838–855.
- . 1989. "Pricing in a Customer Market." *The Quarterly Journal of Economics* 104 (4):699–718.
- Bils, Mark and James A. Kahn. 2000. "What Inventory Behavior Tells Us about Business Cycles." *The American Economic Review* 90 (3):458–481.
- Bils, Mark and Peter J. Klenow. 2004. "Some Evidence on the Importance of Sticky Prices." *The Journal of Political Economy* 112 (5):947–985.
- Blinder, Alan S. 1986. "More on the Speed of Adjustment in Inventory Models." *Journal of Money, Credit and Banking* 18 (3):355–365.

- Carlton, Dennis W. 1979. "Contracts, Price Rigidity, and Market Equilibrium." *Journal of Political Economy* 87 (5):1034–1062.
- . 1986. "The Rigidity of Prices." *The American Economic Review* 76 (4):637–658.
- Chevalier, Judith A., Anil K. Kashyap, and Peter E. Rossi. 2003. "Why Don't Prices Rise during Periods of Peak Demand? Evidence from Scanner Data." *The American Economic Review* 93 (1):15–37.
- Chevalier, Judith A. and David S. Scharfstein. 1996. "Capital-Market Imperfections and Countercyclical Markups: Theory and Evidence." *The American Economic Review* 86 (4):703–725.
- Dana, James D. 1998. "Advance-Purchase Discounts and Price Discrimination in Competitive Markets." *Journal of Political Economy* 106 (2):395–422.
- . 1999. "Equilibrium Price Dispersion under Demand Uncertainty: The Roles of Costly Capacity and Market Structure." *The RAND Journal of Economics* 30 (4):632–660.
- Devereux, Michael B., Allen C. Head, and Beverly J. Lapham. 1996. "Aggregate fluctuations with increasing returns to specialization and scale." *Journal of Economic Dynamics and Control* 20 (4):627–656.
- Eckstein, Otto. 1972. *The Econometrics of Price Determination*. Board of Governors of the Federal Reserve System.
- Eden, Benjamin. 1990. "Marginal Cost Pricing When Spot Markets Are Complete." *Journal of Political Economy* 98 (6):1293–1306.
- Green, Edward J. and Robert H. Porter. 1984. "Noncooperative Collusion under Imperfect Price Information." *Econometrica* 52 (1):87–100.
- Greenwald, Bruce and Joseph E. Stiglitz. 1989. "Toward a Theory of Rigidities." *The American Economic Review* 79 (2):364–369.
- Greenwald, Bruce C. and Joseph E. Stiglitz. 1993. "Financial Market Imperfections and Business Cycles." *The Quarterly Journal of Economics* 108 (1):77–114.

- Hall, Robert E. 1988. "The Relation between Price and Marginal Cost in U.S. Industry." *The Journal of Political Economy* 96 (5):921–947.
- Hendricks, Kevin B. and Vinod R. Singhal. 2009. "Demand-Supply Mismatches and Stock Market Reaction: Evidence from Excess Inventory Announcements." *Manufacturing and Service Operations Management* 11 (3):509–524.
- Jaimovich, Nir and Max Floetotto. 2008. "Firm dynamics, markup variations, and the business cycle." *Journal of Monetary Economics* 55 (7):1238–1252.
- Klemperer, Paul. 1995. "Competition When Consumers Have Switching Costs: An Overview with Applications to Industrial Organization, Macroeconomics, and International Trade." *Review of Economic Studies* 62:515–539.
- Lal, Rajiv and Carmen Matutes. 1994. "Retail Pricing and Advertising Strategies." *The Journal of Business* 67 (3):345–370.
- Lal, Rajiv and J. Miguel Villas-Boas. 1998. "Price Promotions and Trade Deals with Multiproduct Retailers." *Management Science* 44 (7):935–949.
- Leland, Hayne E. 1972. "Theory of the Firm Facing Uncertain Demand." *The American Economic Review* 62 (3):278–291.
- Moller, Marc and Makoto Watanabe. 2010. "Advance Purchase Discounts Versus Clearance Sales." *The Economic Journal* 120 (547):1125–1148.
- Nekarda, Christopher J. and Valerie A. Ramey. 2010. "The Cyclical Behavior of the Price-Cost Markup." *mimeo* .
- Neubecker, Leslie. 2006. *Strategic Competition in Oligopolies with Fluctuating Demand*. Birkhauser.
- Nocke, Volker and Martin Peitz. 2007. "A Theory of Clearance Sales." *The Economic Journal* 117 (522):964–990.
- Prescott, Edward C. 1975. "Efficiency of the Natural Rate." *Journal of Political Economy* 83 (6):1229–1236.

- Ramey, Valerie A. 1989. "Inventories as Factors of Production and Economic Fluctuations." *The American Economic Review* 79 (3):338–354.
- Ravn, Morten, Stephanie Schmitt-Grohe, and Martin Uribe. 2006. "Deep Habits." *Review of Economic Studies* 73:195–218.
- Ross, Sheldon. 2007. *Introduction to Probability Models*. Burlington, MA, USA: Elsevier, 9th ed.
- Rotemberg, Julio and Garth Saloner. 1986. "A Supergame-Theoretic Model of Price Wars during Booms." *The American Economic Review* 76 (3):390–407.
- Rotemberg, Julio and Michael Woodford. 1991. "Markups and the Business Cycle." *Macroeconomics Annual* 6:63–129.
- . 1992. "Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity." *The Journal of Political Economy* 100 (6):1153–1207.
- Scarf, Herbert. 1959. "Bayes Solutions of the Statistical Inventory Problem." *The Annals of Mathematical Statistics* 30 (2):490–508.
- Stiglitz, Joseph E. 1984. "Price Rigidities and Market Structure." *The American Economic Review* 74 (2):350–355.
- Van Nieuwerburgh, Stijn and Laura Veldkamp. 2006. "Learning Asymmetries in Real Business Cycles." *Journal of Monetary Economics* 53 (4):753–772.
- Warner, Elizabeth J. and Robert B. Barsky. 1995. "The Timing and Magnitude of Retail Store Markdowns: Evidence from Weekends and Holidays." *The Quarterly Journal of Economics* 110 (2):321–352.
- Wolfstetter, Elmar. 1999. *Topics in Microeconomics: Industrial Organization, Auctions, and Incentives*. Cambridge, UK: Cambridge University Press.

ABOUT THE CDMA

The **Centre for Dynamic Macroeconomic Analysis** was established by a direct grant from the University of St Andrews in 2003. The Centre facilitates a programme of research centred on macroeconomic theory and policy. The Centre is interested in the broad area of dynamic macroeconomics but has particular research expertise in areas such as the role of learning and expectations formation in macroeconomic theory and policy, the macroeconomics of financial globalization, open economy macroeconomics, exchange rates, economic growth and development, finance and growth, and governance and corruption. Its affiliated members are Faculty members at St Andrews and elsewhere with interests in the broad area of dynamic macroeconomics. Its international Advisory Board comprises a group of leading macroeconomists and, ex officio, the University's Principal.

Affiliated Members of the School

Dr Fabio Aricò.
Prof George Evans (Co-Director).
Dr Gonzalo Forgeue-Puccio.
Dr Laurence Lasselle.
Dr Seong-Hoon Kim.
Dr Peter Macmillan.
Prof Rod McCrorie.
Prof Kaushik Mitra (Director).
Dr Geetha Selvaretnam.
Dr Ozge Senay.
Dr Gary Shea.
Dr Gang Sun.
Prof Alan Sutherland.
Dr Alex Trew.

Senior Research Fellow

Prof Andrew Hughes Hallett, Professor of Economics,
Vanderbilt University.

Research Affiliates

Prof Keith Blackburn, Manchester University.
Prof David Cobham, Heriot-Watt University.
Dr Luisa Corrado, Università degli Studi di Roma.
Dr Tatiana Damjanovic, University of Exeter.
Dr Vladislav Damjanovic, University of Exeter.
Prof Huw Dixon, Cardiff University.
Dr Anthony Garratt, Birkbeck College London.
Dr Sugata Ghosh, Brunel University.
Dr Aditya Goenka, Essex University.
Dr Michal Horvath, University of Oxford.
Prof Campbell Leith, Glasgow University.
Prof Paul Levine, University of Surrey.
Dr Richard Mash, New College, Oxford.
Prof Patrick Minford, Cardiff Business School.
Dr Elisa Newby, University of Cambridge.
Prof Charles Nolan, University of Glasgow.
Dr Gulcin Ozkan, York University.

Prof Joe Pearlman, London Metropolitan University.
Prof Neil Rankin, Warwick University.
Prof Lucio Sarno, Warwick University.
Prof Eric Schaling, South African Reserve Bank and
Tilburg University.
Prof Peter N. Smith, York University.
Dr Frank Smets, European Central Bank.
Prof Robert Sollis, Newcastle University.
Dr Christoph Thoenissen, Victoria University of
Wellington and CAMA
Prof Peter Tinsley, Birkbeck College, London.
Dr Mark Weder, University of Adelaide.

Research Associates

Miss Jinyu Chen.
Mr Johannes Geissler.
Miss Erven Lauw
Mr Min-Ho Nam.

Advisory Board

Prof Sumru Altug, Koç University.
Prof V V Chari, Minnesota University.
Prof John Driffill, Birkbeck College London.
Dr Sean Holly, Director of the Department of Applied
Economics, Cambridge University.
Prof Seppo Honkapohja, Bank of Finland and
Cambridge University.
Dr Brian Lang, Principal of St Andrews University.
Prof Anton Muscatelli, Heriot-Watt University.
Prof Charles Nolan, St Andrews University.
Prof Peter Sinclair, Birmingham University and Bank of
England.
Prof Stephen J Turnovsky, Washington University.
Dr Martin Weale, CBE, Director of the National
Institute of Economic and Social Research.
Prof Michael Wickens, York University.
Prof Simon Wren-Lewis, Oxford University.

**RECENT WORKING PAPERS FROM THE
CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS**

Number	Title	Author(s)
CDMA10/12	Finance and Balanced Growth	Alex Trew (St Andrews)
CDMA10/13	Economic Crisis and Economic Theory	Mark Weder (Adelaide, CDMA and CEPR)
CDMA10/14	A DSGE Model from the Old Keynesian Economics: An Empirical Investigation	Paolo Gelain (St Andrews) and Marco Guerrazzi (Pisa)
CDMA10/15	Delay and Haircuts in Sovereign Debt: Recovery and Sustainability	Sayantan Ghosal (Warwick), Marcus Miller (Warwick and CEPR) and Kannika Thampanishvong (St Andrews)
CDMA11/01	The Stagnation Regime of the New Keynesian Model and Current US Policy	George W. Evans (Oregon and St Andrews)
CDMA11/02	Notes on Agents' Behavioral Rules Under Adaptive Learning and Studies of Monetary Policy	Seppo Honkapohja (Bank of England), Kaushik Mitra (St Andrews) and George W. Evans (Oregon and St Andrews)
CDMA11/03	Transaction Costs and Institutions	Charles Nolan (Glasgow) and Alex Trew (St Andrews)
CDMA11/04	Ordering Policy Rules with an Unconditional	Tatjana Damjanovic (St Andrews), Vladislav Damjanovic (St Andrews) and Charles Nolan (Glasgow)
CDMA11/05	Solving Models with Incomplete Markets and Aggregate Uncertainty Using the Krusell-Smith Algorithm: A Note on the Number and the Placement of Grid Points	Michal Horvath (Oxford and CDMA)
CDMA11/06	Variety Matters	Oscar Pavlov (Adelaide) and Mark Weder (Adelaide, CDMA and CEPR)
CDMA11/07	Foreign Aid-a Fillip for Development or a Fuel for Corruption?	Keith Blackburn (Manchester) and Gonzalo F. Forgues-Puccio (St Andrews)

CDMA11/08	Financial intermediation and the international business cycle: The case of small countries with big banks	Gunes Kamber (Reserve Bank of New Zealand) and Christoph Thoenissen (Victoria University of Wellington and CDMA)
CDMA11/09	East India Company and Bank of England Shareholders during the South Sea Bubble: Partitions, Components and Connectivity in a Dynamic Trading Network	Andrew Mays and Gary S. Shea
CDMA11/10	A Social Network for Trade and Inventories of Stock during the South Sea Bubble	Gary S. Shea (St Andrews)
CDMA11/11	Policy Change and Learning in the RBC Model	Kaushik Mitra (St Andrews and CDMA), George W. Evans (Oregon and St Andrews) and Seppo Honkapohja (Bank of Finland)
CDMA11/12	Individual rationality, model-consistent expectations and learning	Liam Graham (University College London)
CDMA11/13	Learning, information and heterogeneity	Liam Graham (University College London)
CDMA11/14	(Re)financing the Slave Trade with the Royal African Company in the Boom Markets of 1720	Gary S. Shea (St Andrews)
CDMA11/15	The financial accelerator and monetary policy rules	Güneş Kamber and Christoph Thoenissen
CDMA11/16	Sequential Action and Beliefs under Partially Observable DSGE Environments	Seong-Hoon Kim

For information or copies of working papers in this series, or to subscribe to email notification, contact:

Kaushik Mitra
Castlecliffe, School of Economics and Finance
University of St Andrews
Fife, UK, KY16 9AL
Email: km91@at-andrews.ac.uk; Phone: +44 (0)1334 462443; Fax: +44 (0)1334 462444.