Mathematics for history's sake: a new approach to Ptolemy's *Geography*

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Abstract

Almost two thousand years ago, Claudius Ptolemy created a guide to drawing maps of the world, identifying the names and coordinates of over 8,000 settlements and geographical features. Using the coordinates of those cities and landmarks which have been identified with modern locations, a series of best-fit transformations has been applied to several of Ptolemy's regional maps, those of Britain, Spain, and Italy. The transformations relate Ptolemy's coordinates to their modern equivalents by rotation and skewed scaling. These reflect the types of error that appear in Ptolemy's data, namely those of distance and orientation.

The mathematical techniques involved in this process are all modern. However, these techniques have been altered in order to deal with the historical difficulties of Ptolemy's maps. To think of Ptolemy's data as similar to that collected from a modern random sampling of a population and to apply unbiased statistical methods to it would be erroneous. Ptolemy's data is biased, and the nature of that bias is going to be informed by the history of the data. Using such methods as cluster analysis, Procrustes analysis, and multidimensional scaling, we aimed to assess numerically the accuracy of Ptolemy's maps. We also investigated the nature of the errors in the data and whether or not these could be linked to historical developments in the areas mapped.

Declarations

Candidate

I, Daniel V. Mintz, hereby certify that this thesis, which is approximately 51,000 words in length, has been written by me, that it is the record of work carried out by me and that it has not been submitted in any previous application for a higher degree.

I was admitted as a research student in September 2006 and as a candidate for the degree of Doctor of Philosophy in September 2007; the higher study for which this is a record was carried out in the University of St Andrews between 2006 and 2010.

Signature of candidate: .... February 21, 2011

Supervisor

I hereby certify that the candidate has fulfilled the conditions of the Resolution and Regulations appropriate for the degree of Doctor of Philosophy in the University of St Andrews and that the candidate is qualified to submit this thesis in application for that degree.

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Contents

Introduction

0.1 The *Geography*

The Κλαυδίου Πτολεμαίου Γεωγραφικῆς Ὑφηγήσεως, literally 'Concerning a guide for drawing the world by Claudius Ptolemy' or, simply, the *Geography*, is an instruction manual for creating world maps. The work is divided into eight books. The first book introduces us to Ptolemy's purpose in writing: to produce maps, planar or spherical, of the known world. Because these maps are to represent such large areas, they are not to be overly detailed. The primary information to be conveyed is the location of the important settlements and geographical features (Ptolemy, *Geo.* 1.1). Ptolemy then turns to an analysis of the work of his predecessor in the field, Marinus of Tyre (also written as Marinos),[1] taking care to simultaneously praise the efforts of the past while emphasising his own improvements. At the end of Book 1, Ptolemy describes how to create both a spherical and a flat map of the known world. For this he gives two different projections (see the English translation and annotations to Book 1 in Ptolemy 2000).

Books 2 through 7 contain a gazetteer of all settlements and geographical landmarks Ptolemy thought appropriate to include on a map of the known world. He hints at the beginning of Book 2 that there are far too many locations listed to include on a single map and that smaller, regional maps would be more appropriate (Ptolemy, *Geo.* 2.1). Ptolemy describes the main bulk of his work as follows:

> . . . the number of degrees in longitude and latitude of well-trodden places are to be considered as quite close to the truth because more or less consistent accounts of them have been passed down without interruption; but [the coordinates] of the [places] that have not been so traveled, because of the sparseness and uncertainty of the research, have been estimated according to their

[1]There is little information available concerning Marinus beyond Ptolemy's own comments; however, it is generally accepted that he took some of his data from military sources of the Flavian period (AD 69-96) (Jones and Keillar 1996, 44).

> proximity to the more trustworthily determined positions or relative configurations, so that none of the [places] that are to be included to make the *oikoumenē* complete will lack a defined position. We have therefore put the degrees corresponding to each [place] at the outer edge of the columns in the manner of a table, setting the [degrees] of longitude before those of latitude, so that if anyone should come across corrections from fuller research, it will be possible to put them alongside in the remaining spaces of the columns (Ptolemy, *Geo.* 2.1).

Essentially, Ptolemy's catalogue is a table of three columns: name, longitude, and latitude, which was intended to be edited as data improved.

At the conclusion of the gazetteer portion of the *Geography* in Book 7, Ptolemy gives a caption for the world map. He then describes how to construct an armillary sphere with the known world on its surface. Ptolemy concludes Book 7 with an additional caption to be used with a flat map, whereas the former caption was for use with a globe. In Book 8, Ptolemy discusses how to construct regional maps of the known world and includes captions for each region. How many regions he divided his world map into and whether or not maps were included in the first manuscripts are discussed in Section 0.3.

0.2 Ptolemy and his work

Ptolemy is estimated to have been born in approximately AD 100. His name indicates that he had Greek ancestry as well as Roman citizenship (Toomer 1975, 186-187). '. . . the only place mentioned in any of Ptolemy's observations is Alexandria, and there is no reason to suppose that he ever lived anywhere else' (Toomer 1975, 186). Either way, he was in Alexandria by 127 as astronomical observations he made that year were recorded in his *Almagest*, though this was not published for over another twenty years (see Hamilton in Ptolemy 2000, 17). The latest observation contained in the *Almagest* dates to 141. As several works of his were published after the *Almagest*, it is probable that Ptolemy flourished under the emperor Hadrian and died during the reign of Marcus Aurelius (161-180) (Toomer 1975, 186).

The *Almagest* is Ptolemy's major work and possibly his earliest significant surviving publication. It contains theories on celestial mechanics but touches very briefly on the subject of geography. Ptolemy says this:

> . . . the only remaining topic in the foundations [of the rest of the treatise] is to determine the coordinates in latitude and longitude of the cities in each province which deserve note, in order to

> calculate the [astronomical] phenomena for those cities. However, the discussion of this subject belongs to a separate, geographical treatise, so we shall expose it to view by itself [in such a treatise], in which we shall use the accounts of those who have elaborated this field to the extent which is possible. We shall [there] list for each of the cities its distance in degrees from the equator, measured along its meridian, and the distance in degrees of that meridian from the meridian through Alexandria, to the east or west, measured along the equator (for that [Alexandria] is the meridian for which we establish the times of the positions [of the heavenly bodies]) (Ptolemy, *Alm.* 2.13).

By the time Ptolemy wrote the *Geography*, possibly his final work (Toomer 1975, 187), he included the coordinates for significantly more than the noteworthy cities and moved his prime meridian from Alexandria to the far west of the known world. In total, there are approximately 8,000 places listed, though in at least one manuscript a marginal notation seems to have marked the noteworthy cities he may have originally intended to be the sole occupants of the work (Ptolemy 2000, 19). These cities are further designated in Book 8 of the *Geography*, which contains the captions of twenty-six regional maps. In these captions, the important cities are given coordinates in terms of hours from the equator and Alexandria (Ptolemy, *Geo.* 8.3-8.28).

0.3 Manuscript tradition

The oldest extant manuscript of the *Geography* was created no earlier than the late thirteenth century (Ptolemy 2000, 42). Printed editions were not available until 1533 (Diller 1935, 534).[2] Manuscripts are grouped together into families based upon certain shared characteristics. For the *Geography*, these characteristics include handwriting, styles of lettering and numbering, errors, and, to a lesser extent, the included (or excluded) maps. For the most part, the maps of the *Geography* have their own tradition and have little or no relationship to the rest of the text.

The manuscripts that come down to us today belong to one of two different family trees or 'recensions'. The forty-six extant manuscripts are descendants of one or both of these lines (Hyde 1941, 244). The forebear of these two branches was not the Ptolemaic original, but an unknown younger manuscript.

[2] Latin manuscripts are not the focus of this discussion. However, as a point of reference, it should be noted that there was no Medieval Latin manuscript tradition of the *Geography*. Latin texts first appeared in the Renaissance, no doubt translated from the Greek manuscripts of the day (Diller 1935, 534).

Such a manuscript does not survive today, but must be assumed '... to explain the errors common to all branches of the tradition, which are too numerous and often too serious to be the author's own' (Ptolemy 2000, 42).

The nature of such errors points to a manuscript that utilized capital letters, which were used in manuscripts prior to the ninth century (Diller 1983, 88). Miniscule was not utilized until the tenth century, giving us a *terminus ante quem*. It is possible that this missing link could have been written as far back as late antiquity (Ptolemy 2000, 42).

This assumed manuscript naturally had errors of its own, as well as emendations differentiating it from the original text. Ptolemy had his own original errors, as can be seen from variations between coordinates given in the *Geography* and Ptolemy's *Handy Tables* which are not likely to have been produced from scribal copying errors. As more data became available, deliberate attempts at improving the text were also made. As time went on, however, distinguishing between original errors, copying errors, and improvements becomes more and more difficult (Ptolemy 2000, 42).

It is interesting to note that while such a work would seemingly have been a valuable tool throughout its history for the common traveler, pilgrim, merchant, military commander, and ruler, its use in the Middle Ages was minimal, with only a few copies in existence. Such was the case when the oldest of the surviving manuscripts was created at the turn of the fourteenth century. This was a major turning point in the history of the *Geography*, marking the beginning of a new proliferation of manuscripts (Ptolemy 2000, 43).

The probable explanation for this renewed interest in Ptolemy's *Geography* is the work of Byzantine scholar Maximos Planudes (*c*.1255-1305). Planudes claimed to have '... discovered through many toils the *geōgraphia* of Ptolemy, which had disappeared for many years' (Kugéas in Ptolemy 2000, 43).

It is unclear what exactly Planudes 'discovered'. The text? The maps? Both? What is clear, though, is that a family of manuscripts was created about 1300 from a single lost copy.

> Three of the most important are beautiful large-format parchment codices containing maps. The cost of materials and workmanship must have been enormous, suggesting that these were presentation copies for very wealthy (or imperial) patrons (Ptolemy 2000, 43).

This family can by identified by the common corrections of errors present in the archetype. Connections can also be drawn via the maps (or lack thereof) that accompany the texts.

While a certain amount of transmission errors is to be expected and was noted as early as the seventh century (Diller 1983, 90), emendations have also been made which are not errors, as can be seen by comparing this family to the other. Some of the Greek has been corrected or reworded, corruptions from previous traditions have been fixed, and errors in the data possibly dating all the way back to Ptolemy have been amended. These alterations seem to have been made due to discrepancies with the text and the maps drawn to the text's specifications (Ptolemy 2000, 44). Examples of these discrepancies include cities being plotted in the sea and points along a coastline being plotted in reversed order.

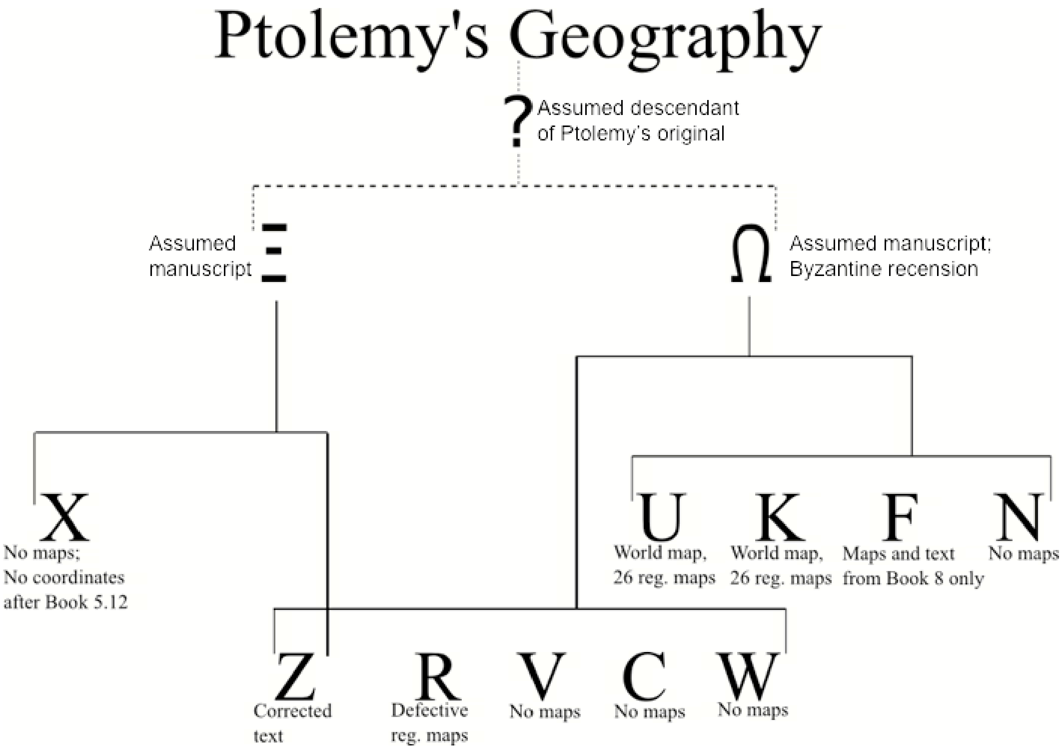

Figure 1: A tree of sources for Ptolemy's *Geography*

The text of the *Geography* and the accompanying maps have two disjoint traditions. It is not unusual to find a manuscript of the text with maps done in a different hand and style or with no maps at all. There are collections of Ptolemaic maps independent of any text save the captions of Book 8. While the rest of the *Geography* up to Book 8.2, excepting errors and corrections, can be attributed to Ptolemy with some certainty, Ptolemy's authorship of Book 8.29-30 and the world map is doubted, and even rejected by several scholars. The intervening section of Book 8 containing the regional maps and captions also has been a matter of some debate (Diller 1940, 335). Did Ptolemy have a map of the world in front of him, from which he wrote the text? Did Ptolemy create world and/or regional maps based on his text? Or did the maps come later, produced by a student or reader of Ptolemy's text?

Whatever the case may be, in the medieval manuscripts, the maps appear in several distinguishable groups. Firstly, there are the world maps. Very nearly all manuscripts containing any maps, have the world map drawn in the first projection that Ptolemy describes in Book 1. The great exception is manuscript K, which uses Ptolemy's second projection. Secondly, there are the regional maps. Adopting the scheme of Berggren and Jones, we shall denote two classes of manuscripts, A and B, based on the number of regional maps associated with the manuscripts (Ptolemy 2000, 45).

The A version of the manuscripts contains twenty-six regional maps, which corresponds to the number given in the text of the Book 8.2. The maps alternate with their captions. The B version contains sixty-four regional maps. These maps are interspersed within the main body of the text where the various areas are catalogued. In both versions, the regional maps follow the projection described for them by Ptolemy, namely a cylindrical projection in which distances along the central latitude and longitude are in the proper ratio (Ptolemy 2000, 45).

The maps of the A manuscripts, for the most part, derive from the maps in U. The key feature connecting the A group is found on the world maps. These contain an unknown country to the south of the Indian Ocean. Ptolemy does not give any data or coordinates for this country. The world maps of the A class, nonetheless, have the same, rough northern coastline for this unknown territory (Ptolemy 2000, 45). The maps of the B version are likely to have derived from those of the A version, the difference being that the B manuscripts have smaller page dimensions. To fit all of the maps into the manuscript, smaller maps were needed, and so the regional maps were divided into more manageable sizes (Ptolemy 2000, 45-46).

Did Ptolemy draw a map or maps to accompany his text? Berggren and Jones reason for the affirmative: 'As Ptolemy insists in 1.17, the way to detect and eliminate inconsistencies such as those he detects in Marinos' writing is to draw a map' (Ptolemy 2000, 46). However, such maps may only have been preliminary and intermediate sketches to guide the work and weed out errors. It does not necessarily follow that Ptolemy published any maps with the final version of the text. The way Ptolemy speaks of the construction of maps and globes in Books 1 and 2, leads us to believe that he is speaking through experience. Furthermore, if the point of writing the text is to instruct its reader on how to draw a world map, surely Ptolemy must have tried it himself (Ptolemy 2000, 46). Fischer takes this view a step further, concluding that Ptolemy, in fact, published a world map and several of the regional maps. Fischer claims that the notion that Ptolemy only used drafts of maps to aid in the writing process 'rests on a false rendering of a passage in the text' (Hyde 1941, 245).

Arguments against the publication of maps with the text rest on the sheer practicality of the matter. Neither manuscript version A nor B 'could have contained the maps, which must always have occupied large sheets' (Diller 1983, 92). For the world map to accommodate all of the cities listed in the catalogue, it would need to be, at best, one by two metres. One metre is over three times the average height of papyrus rolls in the second century AD. Maps of any substantial size were displayed publicly, painted or affixed to walls, not in manuscripts (Ptolemy 2000, 47). Besides this point, the copying of some thirty maps with each manuscript would have been a vast and expensive undertaking.

Because of the lack of practically of fitting the maps into the relatively small manuscripts before 1300,

> Planudes and his assistants therefore probably had no pictorial models, and the success of their enterprise is proof that Ptolemy succeeded in his attempt to encode the map in words and numbers. The copies of the maps in later manuscripts and printed editions of the *Geography* were reproduced from Planudes' reconstructions (Ptolemy 2000, 50).

Unfortunately, unless several lost manuscripts, predating those which are known to us, come to light, we shall never fully understand from where the extant manuscripts are derived, or, more importantly, how different today's texts are from Ptolemy's original work.

0.4 Current edition

Berggren and Jones published 'an annotated [English] translation of the theoretical chapters' of Ptolemy's *Geography* in 2000. In their preface, they lament that a complete Greek edition of the text has not appeared since 1845, good German and French translations exist only for parts of the work, and that the only near-complete English edition is 'very unsatisfactory' (Ptolemy 2000, xi). The first two complaints were remedied in 2006 by Stückelberger and Graßhoff when they published a complete Greek text with a side-by-side German translation.

In addition to compiling a complete Greek text based on the study and compilation of several extant manuscripts,[3] where the manuscript families di-

[3]Especially important to the 2006 edition is the examination of manuscript K, *Seraglien-*

verge, Stückelberger and Graßhoff have provided alternative readings.[4] For example, Paris is given the coordinates $(23°30', 48°10'[30'])$, where $48°30'$ is an alternative latitudinal coordinate found in the manuscript tradition. These divergent readings are found in the list of names as well as the coordinates. Another invaluable tool included in the 2006 edition is the identification of Ptolemy's place names with modern locations (should such identifications exist). These 'known' localities form the basis of our study.

0.5 Aims

Preliminary examination of Ptolemy's map of Great Britain prompted an investigation into the feasibility of using mathematics to find the as yet unidentified settlements on the map. Coordinates from the identified cities on Ptolemy's map were subjected to linear, affine, and quadratic transformations in attempts to minimise the sum of squared distances between Ptolemy's transformed coordinates and the modern or 'true' coordinates (see Mintz 2009, 4-6). This did not produce very satisfactory results as the errors remained quite large.

To reduce the errors, the map was divided into small regions, and each of these was subjected to minimising transformations. This approach produced very small errors, mostly due to the fact that the minimisation under a quadratic transformation could be solved exactly for zero if a region contained six or less points. This was the case for some of the regions. The others were not much larger.

While this nearly forced Ptolemy's known points exactly onto their modern counterparts, there were two basic problems with this approach. Firstly, the quadratic transformation did not reflect errors that Ptolemy or his data would have contained. This will be discussed below. Secondly, in pursuing a strict minimisation agenda and creating small groups of points to facilitate this, we did not learn much about Ptolemy's map. It was not possible to glean how the errors were spread across the map, nor the nature of the errors. Patterns could not be discerned in the data. The goal was to force the error to zero rather than to learn what we could from the data.

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sis gr. 57, of Istanbul. Until this publication, only 'unsatisfactory photographs dating from the 1930s' were accessible (Jones 2008, 129-130).

[4] The main readings presented are from the Ω, or Byzantine, manuscript recension. Alternative coordinates are from manuscript X, the only text free from revisions based on Ω (Jones 2008, 129-130; see also Ptolemy 2000, 44).

0.5.1 The errors we can and cannot fix

We knew that an error of zero could be achieved by a simple polynomial transformation with a sufficient number of variables, but, as this would in no way guarantee that unknown settlements would be moved to their correct positions, we had to rethink our process. Furthermore, even at the quadratic level, the transformation wreaked havoc on the geometry of the maps, causing, for example, Ireland's coastline to collapse and twist around itself (see Mintz 2009, 5). In order to transform Ptolemy's maps in such a way as to extract information concerning his accuracy, we needed to limit the transformations to minimise only those errors which would have been been inherent in his data.

Ptolemy's baseline was the work of Marinus (Ptolemy, *Geo.* 1.5). To this, he made corrections and additions based on the reports from travellers. With the exception of the few points that were given coordinates based on astronomical observations, new data would have taken the form of distance and orientation between one location and the next (Ptolemy, *Geo.* 1.2). That is, something along the lines of 'we marched north by north-west for seven hours from city A to get to city B'. Then Ptolemy would have to convert the length of the journey into stadia and establish the coordinates of city B from those of city A based on the distance and heading. If roads and milestones had been established, so much the better.

Therefore, the errors that the transformation needs to be able to detect are errors in rotation and scaling. The mechanics of the transformation will be discussed in Section 1.7. The rotations and scalings will be dealt with separately. The error will be minimised first by rotating the data, and then the rotated data will be scaled along the two main axes. This, as opposed to minimising the error by simultaneously rotating and scaling the data, is to emphasise that these two types of errors are distinct and need to be examined in turn. While this will not reduce the error as much as the simultaneous minimisation, it will help us to better understand from where the problems in Ptolemy's maps stem.

There are other and often greater errors inherent in Ptolemy's data beyond those of size and orientation. The data coming to Ptolemy certainly contained a level of misinformation and misinterpretation as it was relayed to Alexandria. Data, whether astronomical or travel-based, could simply be wrong for any number of reasons, from shoddy methodology to poor reporting. The thousands of kilometres the information needed to travel to get to Ptolemy from the borders of the Roman Empire could have had no small effect on the trustworthiness and accuracy of the raw data.

For our purposes, however, the greatest difficulty is time. Almost nineteen hundred years separates us from Ptolemy's raw data. Over a thousand years separates the oldest surviving manuscripts from Ptolemy's raw data. The *Geography* contains long columns of numerical data. Numerals in Ancient Greek are simply letters of the alphabet. The recopying of the data over the centuries without doubt created many transmission errors. It would have been impossible not to have done so. Add to this any deliberate changes, omissions, additions, 'improvements', and 'corrections' made by scribes and scholars over the years, and Ptolemy's original work and data are quite difficult to detect. We shall never know the *Geography* as Ptolemy wrote it. All we can do is work with the data and its errors that are presented to us today.

0.5.2 The case studies

To examine the accuracy of Ptolemy's work, we shall examine three of his regional maps, those of Great Britain, Iberia, and Italy (Figures 2 - 4). There are two considerations that led to the selection of these case studies. The first is statistical in nature. As an island and peninsulas, these areas are isolated from neighbouring regions. In studying the errors and accuracy of the three maps, we want the data to be internally independent from any other maps. We want to limit, as much as possible, any distance and orientation data that is based on points in adjacent provinces. For example, the western settlements on Ptolemy's map of Germany may have been placed based on their distance and bearing from settlements in France. The maps of Germany and France would then need to be examined together. Britain, Iberia, and Italy have the advantage of minimal contact with bordering provinces and the English Channel, Pyrenees, and Alps help to further isolate them.

The second consideration in choosing these three regions as our case studies is comparative. As well as determining the general accuracy of Ptolemy's maps, we would like to examine if this accuracy is in any way reflective of the history of these areas. Do the errors present in the data follow historical lines? Is there a difference between the accuracy of a map of a long established, peaceful region and one of an area that has known only constant warfare, for example? Britain, Iberia, and Italy have been chosen, not only because of their isolation, but also because they represent three distinct stages of development within the Roman Empire of the second century AD. Britain is an incomplete conquest. Spain is the first province and a fairly established part of the Empire. Italy is the centre of the Roman world. We hypothesise that Ptolemy's map of Italy will be the most accurate, Britain the least accurate, and Iberia falling between them.

In order to test this hypothesis, we need to familiarise ourselves with the histories of Britain, Spain, and Italy. Of necessity, these will be rather brief, paraphrastic, and superficial. In Britain and Spain, we shall begin with the

Roman world's first recorded (or speculated) contacts with these two future provinces. In Italy, we shall begin with some of Rome's earliest conquests and expansions. These potted histories are not meant to be in-depth historical criticism or analysis, but are merely guides to provide context for our numerical study. The accounts of our ancient authors should be taken with a grain of salt provided by our few secondary sources. For brevity's sake, many of the subtleties of these centuries must be glossed over or even omitted. The important thing to glean from these histories is the expansion of the Roman sphere of influence which would enable the transmission of geographical knowledge back to Rome and thence to the scientific community at Alexandria.

To illustrate our methodology, we first turn to Ptolemy's map of Ireland. Ireland and Great Britain are included together on Ptolemy's map of the British Isles, but Ireland has too few points to be a case study in its own right. However, because of its small number of identified locations, it is a convenient and transparent example with which to demonstrate our method.

A note on names

Place names proved to be a difficulty during this study. Ptolemy translated or transliterated the place names of Roman Britain, Spain, and Italy into Greek from their original Latin, Celtic, Iberian, etc. forms. The modern names were no less challenging as they appeared in a number of languages and regional dialectal forms. Compromises have been inevitable. On the maps accompanying the case studies, the modern names have been used. The local spellings have been utilised as far as has been possible. In the main body of text, though, better known cities and geographical features are referred to by their Anglicised names. At their first instance, at least, settlements mentioned by historical sources are given both their ancient and modern names (if known). For example, on Ptolemy's map we would have found Neapolis, but on our statistical maps,[5] Napoli. During the statistical discussion, we use Naples. In the historical narrative, depending on the sources, we find either Naples or Neapolis, but at its introduction, Neapolis (Naples). Appendix A contains the full list of place names, ancient and modern, as well as ID numbers, which were used on maps that proved too crowded to label with names.

[5]Our statistical maps follow the rectangular projection proposed by Ptolemy for his regional maps, namely that distances along the mean latitude and those along the mean longitude should be in the correct ratio (Ptolemy, *Geo.* 8.1).

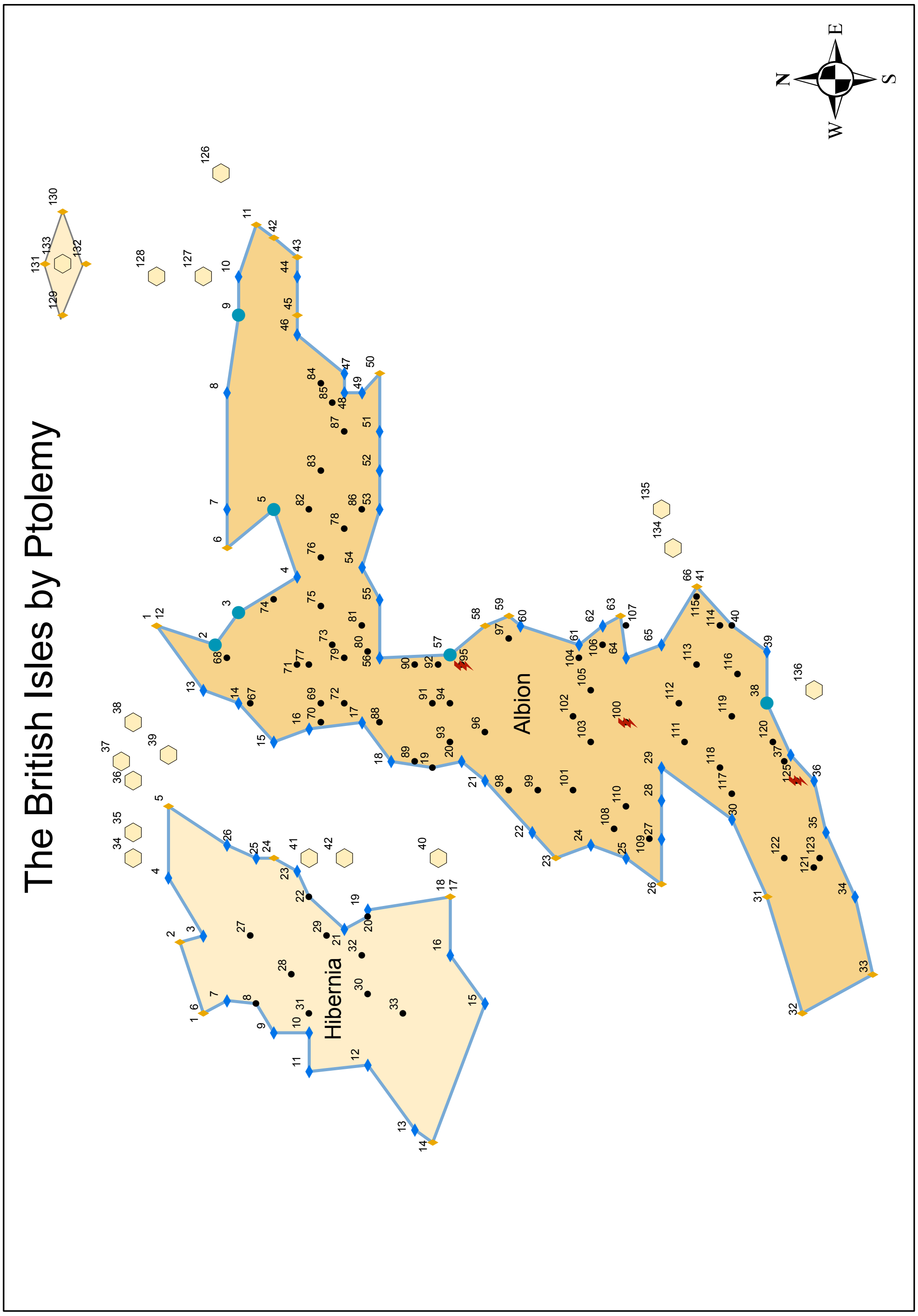

Figure 2: Ptolemy's map of the British Isles

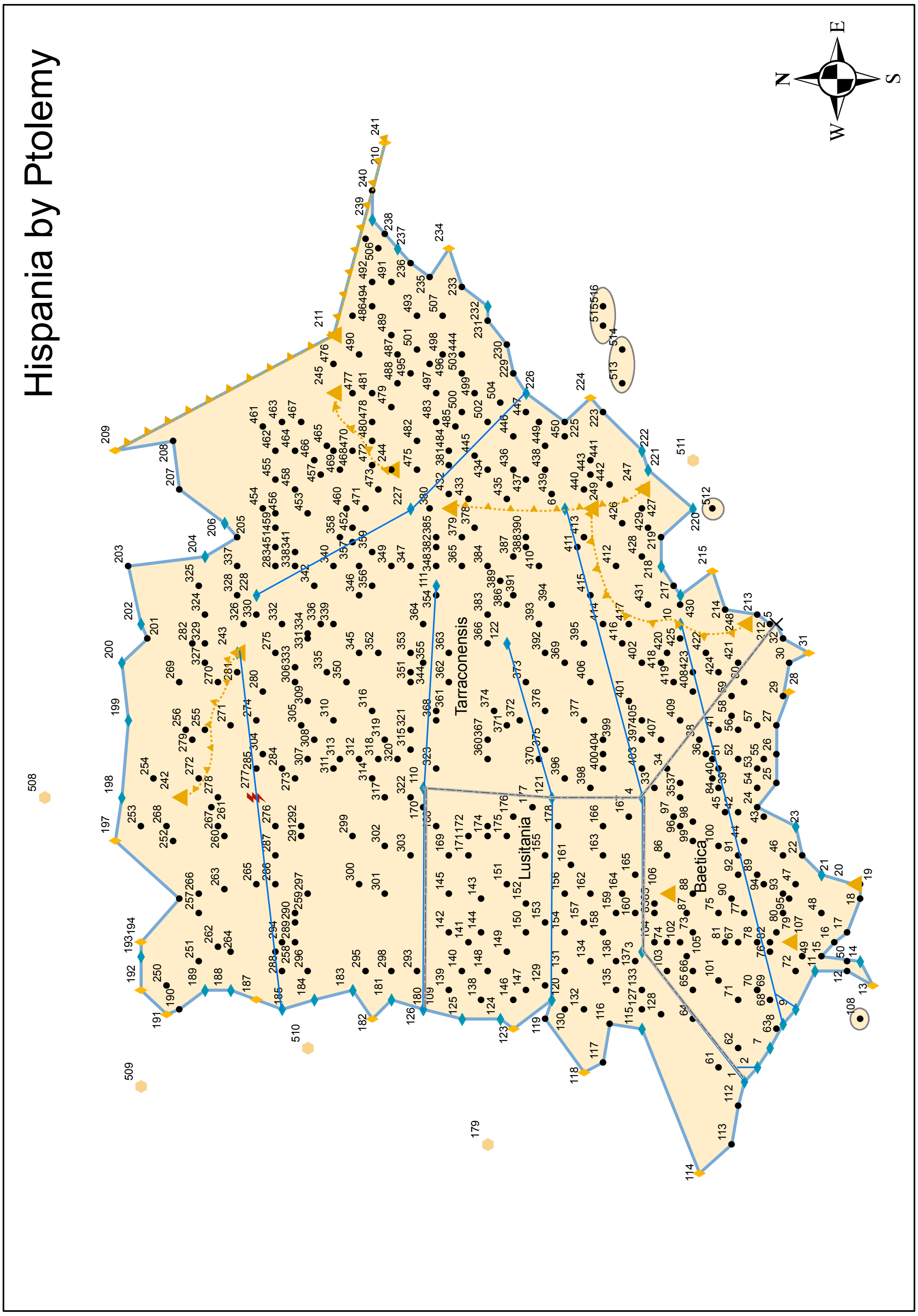

Figure 3: Ptolemy's map of Hispania

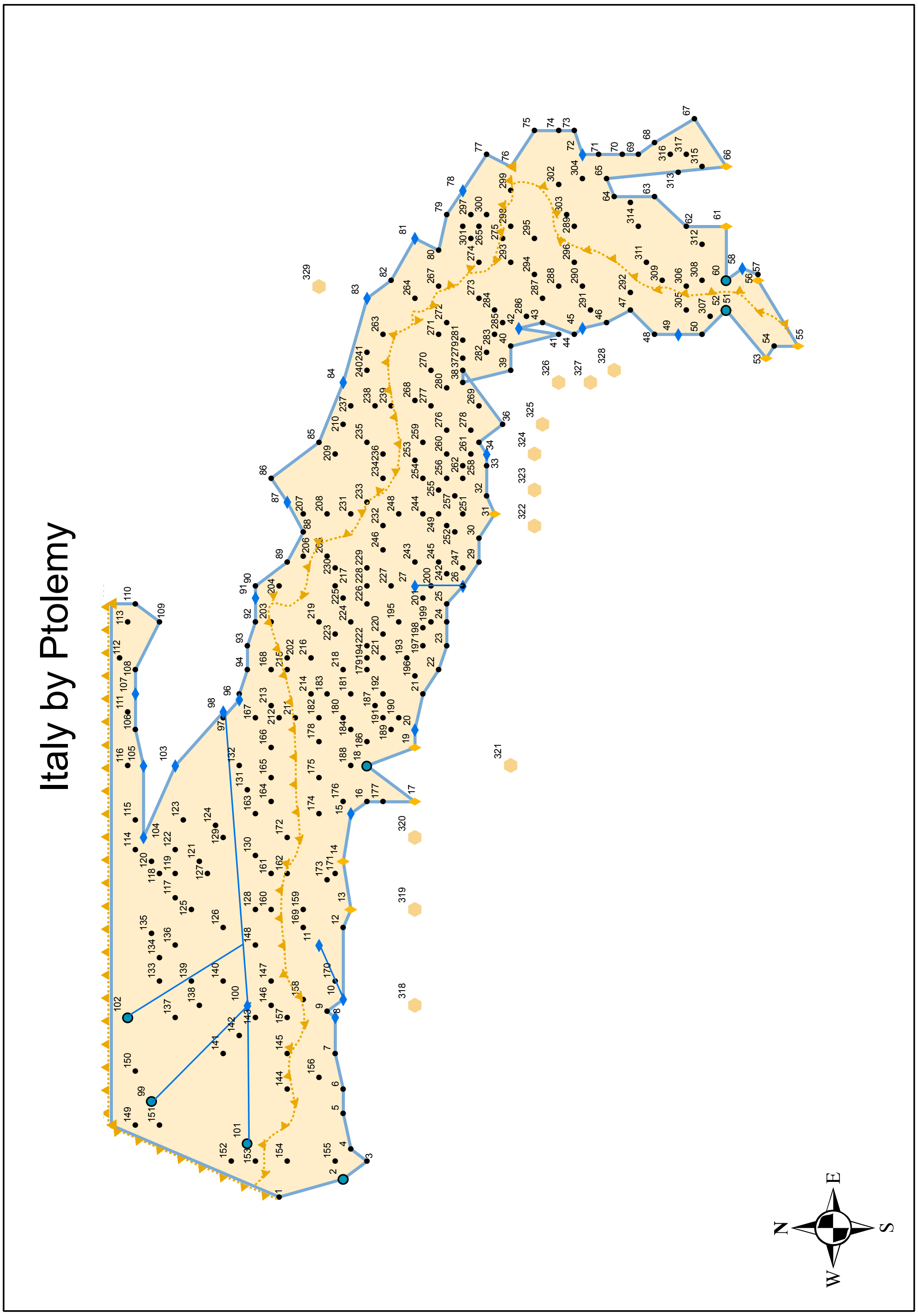

Figure 4: Ptolemy's map of Italy

A note on translations

All translations of ancient sources, save Ptolemy, are taken from the Loeb Classical Library editions. We use Toomer's translation and annotations of Ptolemy's *Almagest*. We are very much indebted to Berggren and Jones' English translation of Book 1 of the *Geography* as well as portions of Books 2, 7, and 8. All data for this study were taken from Stückelberger and Graßhoff's 2006 edition. We have translated the relevant portions of Books 2, 3, and 8, which discuss Ireland, Britain, Spain, and Italy from the Greek.

To distinguish between information provided by an ancient author and commentary by his translator, the citations of the work differ slightly. Ancient sources are referred to by titles, books, and sections, whereas modern scholars are referenced with years and page numbers. For example, a quote of Caesar from his *De Bello Gallico* will be referenced as (Caesar, *B.G.* 4.20), but a note from the translator as (Caesar 1998, 232).

Chapter 1

Methodology

1.1 Procrustes analysis

'Procrustes analysis' is named after a mythical Greek who would force strangers to sleep on his bed with the stipulation that they fit the bed exactly. His name means 'forth-beater', describing the action of a blacksmith who stretches a piece of metal by repeated hammering (Plutarch, *Theseus* 11; Diodorus, 4.59).

At its most basic, Procrustes analysis takes two shapes and alters one of them under given parameters to best fit the second according to least-squares regression (Peres-Neto and Jackson 2001, 170). Given two configurations of n points in t dimensions (the configurations are both descriptions of the same shape), what transformations can be performed on the first configuration to minimise the residual sum of squares with the second configuration? Finding the best transformations is 'Procrustes analysis', and the residual sum of squares is the 'Procrustes statistic' (Sibson 1978, 234). The optimised Procrustes statistic, that is the Procrustes statistic calculated after the best transformations have been found, can be used as a measure quantifying the association between the two configurations (Peres-Neto and Jackson 2001, 170).

If we assume that we are working in two dimensions, a configuration of n points will be a $2 \times n$ matrix, thus: $M = \begin{bmatrix} \bar{m}_1 & \bar{m}_2 & \cdots & \bar{m}_n \end{bmatrix}^{\mathrm{T}}$ and $P = \begin{bmatrix} \bar{p}_1 & \bar{p}_2 & \cdots & \bar{p}_n \end{bmatrix}^{\mathrm{T}}$, where M and P represent two different configurations of the same points with $\bar{m}_k = (x_k, y_k)$ and $\bar{p}_k = (x'_k, y'_k)$. That is, $\bar{m}_k$ and $\bar{p}_k$ represent the same point, k, but in different configurations with, most likely, different coordinates.

Let

$$G(M, P) = \sum_{n=1}^{N} (\bar{m}_n - \bar{p}_n)^{\mathrm{T}} (\bar{m}_n - \bar{p}_n) = \mathrm{trace}(M - P)^{\mathrm{T}} (M - P), \qquad (1.1)$$

which is none other than the sum of the squared distances between the coordinates of P and their equivalents in M. G is simply one manner of comparing the map produced by the coordinates in M to the one produced by those in P. However, until certain transformations are performed on P the initial result of such comparison is rather meaningless. In particular, a common centre, orientation, and scale must be chosen for the two configurations (Sibson 1978, 235).

Transformations need only be performed on one of the configurations, otherwise, depending on the transformations used, $G(M, P)$ may be completely unaffected or reduced trivially to zero if both P and M are subjected to the same transformation. We hold M constant and attempt to match P to M. We now define the Procrustes statistic:

$$G_\Gamma(M, P) = \min\{G(M, \phi P) : \phi \in \Gamma\}, \tag{1.2}$$

where Γ is a group of transformations in the given dimension. There are four transformation groups to consider in Procrustes analysis:

1. Euclidean: translations, rotations, reflections;
2. Similarity: uniform scale changes;
3. Affine: oblique scale changes; and
4. Special Euclidean: translations, rotations but NOT reflections (Sibson 1978, 235).

Which transformations are chosen depends on the nature of the data. For the time being, let us assume that we are using the generalised orthogonal Procrustes analysis which limits itself to rigid Euclidean transformations and uniform scaling. This will preserve the shapes of the configurations, as shape is unaffected by position, scale, and rotation (Stegmann and Gomez 2002, 2).

The first step in the Procrustes analysis is to align the two configurations. Simple translations are applied to M and P so that both have their centroids (the points having the mean longitude and latitude of the configurations) at the origin.[1] Having a common centroid reduces G_Γ as much as possible for a translation and has the bonus that subsequent algebra is simplified (Sibson 1978, 235).

The second step in the generalised orthogonal Procrustes analysis is to perform an orthogonal transformation on P, matching it to M. That is, we

[1] As stated above, only P needs to be transformed, but the mathematics required is simplified significantly if we take the origin as the centroid of both configurations instead of using the original centroid of M.

need to find a Euclidean transformation, G_E (minus translation which has already been performed) such that

$$G_E(M,P) = \min\{G(M,RP) : R^{\mathrm{T}}R = I\}, \tag{1.3}$$

where R is an orthogonal matrix which represents a rotation and/or reflection (Sibson 1978, 235).

Thirdly, a minimising dilation, d is performed. At this point, then, we are minimising $G(M, dRP)$, where R has already been optimised (Sibson 1978, 237). Lastly, a graph displaying M and dRP together is produced. Residual bars connecting the corresponding points between the two configuration may be useful to examine the configurations' difference point-by-point (Peres-Neto and Jackson 2001, 172). Of course, if we desire a less rigid approach, generalised orthogonal Procrustes analysis may be abandoned, and transformations from the affine transformation group may be performed as well as or instead of the dilation. Likewise, in the second step, a special Euclidean transformation may be used if reflections are unwanted. The order of the steps may also vary (see Stegmann and Gomez 2002, 4; Peres-Neto and Jackson 2001, 170-171).

1.2 The Mantel test

First published in 1967, the Mantel test examines the correlation, if any, between two distance matrices. Take two $n \times n$ matrices, M and P. Entries m_{ij} and p_{ij} represent the distance between points i and j. The distances do not need to be Euclidean (Peres-Neto and Jackson 2001, 169). M and P represent different configurations of the same data. Both matrices will be symmetric and have zeros down the main diagonal as $m_{ij} = m_{ji}$ and $m_{ii} = 0$ (Manly 1986, 53-54). The test begins with the null hypothesis that M and P are independent (Platt 2005, 2).

The test statistic

$$Z = \sum_{i=2}^{n}\sum_{j=1}^{i-1} m_{ij}p_{ij} \tag{1.4}$$

is computed both for M and P in their original form and when the entries of one of the matrices is randomised. (Note that only the entries in the lower half of the matrices are used to avoid unnecessary doubling of the distances as the entries of the upper half will be equal to the entries in the lower half.) By calculating Z for many randomised matrices, a random distribution of Z can be obtained and compared to the original (Manly 1986, 54).

If the two matrices, M and P are completely unrelated then their corresponding Z value will be similar to the randomised Z values. However, if their distance matrices are related, their 'observed Z will tend to be larger than values given by randomisation' (Manly 1986, 55). Should this be the case, we can conclude that the data does not support the null hypothesis (Platt 2005, 3).

1.3 Cluster analysis

1.3.1 Introduction

Given a set of data points, it is the purpose of cluster analysis to determine if there exist any 'natural groupings' (Chatfield and Collins 1980, 187). The groupings are based on the similarity of the objects examined, where the similarity is not necessarily Euclidean distance. Manly (1986, 100) notes that cluster analysis should be completely numerical and without a predetermined number of groups. The analysis can then reveal groups not apparent on visual inspection, as well as classify data points whose true group membership is unknown.

> More formally, cluster analysis aims to allocate a set of individuals to a set of mutually exclusive, exhaustive, groups such that individuals within a group are similar to one another while individuals in different groups are dissimilar (Chatfield and Collins 1980, 212).

1.3.2 Types of cluster analysis

Hierarchic cluster analysis results in a tree diagram or 'dendrogram'. Firstly, the distances between all points are determined. Cluster analysis then proceeds by starting with either one group with all the points or as many groups as are necessary for each point to be in its own group. Groups are then either spliced or merged until the opposite configuration is reached (Manly 1986, 100-101). There are three basic types of hierarchic cluster analysis: 'nearest neighbour', 'furthest neighbour', and 'group average'. With nearest neighbour, groups are joined when a point in one group is less than a threshold distance from the nearest point in a second group. Furthest neighbour works on the same principle, except that the points in question must be the furthest two points in the groups. Group average, as its name implies, links groups when the average of two groups' points are below the threshold distance (Manly 1986, 101-103).

A more partition-based approach uses fluid groups. Data points are not bound to remain in a single group once assigned. As a first step, arbitrary centres are chosen throughout a configuration. Points are then assigned to their closest centre. The actual centres of those groups are then determined based on the points in each group. If any individual point in a group is then closer to another group's centre, the point is moved to that group. Groups are then joined and/or split by an iterative process until the number of groups desired is reached (Manly 1986, 101).

A third method is based on a network rather than a hierarchy or partition. The minimum spanning tree (MST) is a series of straight lines joining points under a threshold distance. All points are connected to the network in such a way that the total length of all line segments is a minimum. The MST easily fits onto a two dimensional display of the data points much like roads connecting cities on map, giving visual clues to clusters and outliers (Chatfield and Collins 1980, 223-224).

Another method of cluster analysis applies specifically to maps and is similar to the partition-based approach discussed above. This technique is called Jenks optimisation or Jenks natural breaks after its primary developer, George Jenks. The development arose due to concerns of how to divide data in statistical maps into categories for the sake of colouring them. For example, on a map of grain output, should the boundary between what values are shaded blue versus what values are shaded green be based on arithmetic progression, quantiles, standard deviation, etc. (Jenks and Caspall 1971, 222)?

Jenks optimisation is built into the ArcGIS (©ESRI) software utilised to draw our maps. Given the statistic being mapped, the Jenks method divides the data into groups such that within-group variance is minimised while between-group variance is maximised. This is done through an iterative process (see Jenks and Caspall 1971, 232-236) which is completed once the sum of absolute deviations from the group means is minimised (Brewer and Pickle 2002, 670), which allows us to find 'groupings and patterns inherent in [the] data' (Murray and Shyy 2000, 654).

1.4 Multidimensional scaling

1.4.1 Introduction

If we know the distances between all points in a data set, however 'distance' is defined, can a graphical configuration of the points be found? This is the question that multidimensional scaling (MDS) seeks to answer. The configuration is in the form of a set of coordinates which, ideally, is in two or three dimensions to allow for a graphical solution, but this is not always possible. If the distances are the Euclidean distances between cities, for example, the MDS would produce a two-dimensional map of the cities (Chatfield and Collins 1980, 189; Manly 1986, 126).

Chatfield and Collins sum up the purpose of MDS nicely:

> Given the co-ordinates of a set of points (or individuals), it is easy to calculate the Euclidean distance between each pair of points. Scaling methods work the other way round. Given information about the dissimilarities [i.e. distances] between points, scaling methods try to find the co-ordinates of the points. In particular, the smaller the dissimilarity (or the larger the similarity) between two points, the closer we would like these points to be in the resulting spatial map (Chatfield and Collins 1980, 189).

The graphical result of MDS will not be unique. The result of the MDS may need to be translated, rotated, and/or reflected to match the true configuration of the points, should such a true configuration exist. The overall size may also need to be adjusted, but that would only be necessary if the computer program performing the MDS did not use the same scale for the MDS result and the true configuration. The angles and distances between the points will remain constant (Chatfield and Collins 1980, 198; Manly 1986, 133).

Accompanying the graphical result, there should also be what Kruskal and Wish call an 'objective function', which in other contexts may be known as an error or goodness-of-fit function. It is possible that the distances between the points as shown on the post-scaling configuration are not exactly the same as those given in the original data. 'For any given set of data and for any given configuration, the objective function yields a single number which shows how well (or how poorly) the data fit the configuration - i.e., it indicates how well the configuration represents the data' (Kruskal and Wish 1978, 24). One such objective function is called 'stress' and is discussed in Section 1.4.5.

1.4.2 Types of MDS

Multidimensional scaling can be 'classical' or 'ordinal'. Classic scaling is basically an algebraic procedure wherein dissimilarities are assumed to be Euclidean distances, and from these distances the coordinates of the points are found. Although Euclidean distances are assumed, the method is robust to deviations and error distortions. Ordinal scaling, on the other hand, is not so much concerned with the absolute dissimilarities, but rather the resulting rank ordering of the data points (Chatfield and Collins 1980, 190). Often a transformation will be applied to the dissimilarities before classical MDS is performed. The function used is a way of relating the original dissimilarities to those computed from the results of the scaling (Chatfield and Collins 1980, 204).

Chatfield and Collins make note of experiments conducted at Bath University comparing ordinal and classical scaling (OS and CS, respectively). A data set whose configuration was known was altered by transformations and error disturbances and then subjected to OS and CS.

> The resulting configurations were translated, rotated, scaled, and possibly reflected in order to make them as close as possible to the original configuration. Then the success of the reconstruction procedure was measured by
>
> $$\sum_{\text{all points}} \begin{pmatrix} \text{squared Euclidean distance from} \\ \text{actual position to achieved position} \end{pmatrix}$$
>
> after normalizing by dividing by the trace of $X^{\mathrm{T}}X$, where X is the original configuration matrix (Chatfield and Collins 1980, 209-210).

When dealing with Euclidean distances (or those closely resembling Euclidean distances), the two methods give the same basic results. However, when the distances are not Euclidean, only OS produces acceptable results (Chatfield and Collins 1980, 210). The statistic minimised in OS and CS is the aforementioned stress, to be discussed in Section 1.4.5.

1.4.3 An example

A typical example of MDS involves plotting a map of towns based on road distances (see Manly 1986, 129-133; Chatfield and Collins 1980, 202-203). Because road distances are usually not straight-line Euclidean distances, MDS will provide only an approximation of the true locations of the towns. This is especially true of points that are separated by large geographical features, such as bays and channels. If no bridges are present, the road distances will be much greater than the Euclidean distances. Such points may be placed well away from their true locations. Chatfield and Collins (1980, 203) cite the case of Carmarthen and Penzance, separated by the Bristol Channel, as such an example.

It is the large distances that will be most responsible for defining the configuration. To quote Kruskal and Wish:

> ... MDS does a much better job in representing larger distances (the global structure) than in representing small distances (the local structure). In fact, Graef and Spence (1976) have shown that discarding only the smallest third or the middle third of the dissimilarities does not disturb the reconstruction of the multidimensional space, but discarding the most dissimilar third of the values

> in the matrix (the smallest similarities) causes a severe degradation (Kruskal and Wish 1978, 46).

In other words, MDS is interested in the big picture. If we look at an English roadmap for example, the distances between Oxford, London, and Cambridge will not affect the scaling nearly as much as the distances between Exeter, London, and York. This is further seen in the test statistic, stress (Section 1.4.5): all other things being equal, stress will be lower when a large area is examined than when zooming in on a smaller region. That is to say, errors in a configuration become more apparent when a closer look is taken.

'All that is important with the configuration produced by multidimensional scaling is the relative positions of the objects being considered. This is unchanged by a rotation or a reflection' (Manly 1986, 133). Provided that the scale of the real configuration and that produced by the MDS are the same, uniform scaling to both will not have any effect on the relative positions of the data. The map resulting from MDS may not be aligned along the north-south, east-west axes. There is nothing particularly special about these axes, of course, beyond that of convention and their use with longitude and latitude. It may even be the case that axes other than these may reveal more information about the data than might be seen in the standard orientation. Determining the best set of axes, in fact, is one of the key goals of MDS (Chatfield and Collins 1980, 203).

1.4.4 Procedure

While each computer program will have its own method for carrying out MDS, the basic procedure is as follows:

1. An $n \times n$ matrix is constructed with each entry being the distance between two points. That is, in the ith row and jth column will be the distance between data points i and j, denoted δ_{ij}.

2. A dimension, t, is chosen. In the case of road maps discussed above, for instance, $t = 2$.

3. Based on the distance matrix, the computer calculates coordinates of each of the n points.

4. The computer then uses these coordinates to determine the Euclidean distance between each point, denoted d_{ij}. We now have two sets of distance data: the δ_{ij}'s (original distances from the input matrix) and the d_{ij}'s (new distances based on the configuration produced by the MDS).

5. A regression relating the d_{ij}'s and the δ_{ij}'s is then found. This can be linear, affine, polynomial, or monotonic, with some error term added. The distances produced from the regression, $\hat{d}_{ij}$ are known as 'disparities'. 'That is to say, the disparities $\hat{d}_{ij}$ are the data distances δ_{ij} scaled [or otherwise transformed] to match the configuration distance d_{ij} as closely as possible' (Manly 1986, 128).

6. A statistic, stress, is employed to measure the goodness of fit between the disparities and the original distances.

7. The computer alters the coordinates of each point repeatedly until the stress or similar statistic is minimised.

8. The final coordinates are the outcome of the scaling, usually accompanied with a plot if the dimension allows (Manly 1986, 128-129).

It may then be beneficial to repeat the MDS process for clusters of points within the data (see Section 1.3). A better understanding of the internal structure of the data may result (Kruskal and Wish 1978, 48).

1.4.5 Stress

Stress, in our particular context, is defined as

$$S = \sum_{i,j} \left(d_{ij} - \hat{d}_{ij} \right)^2 / \sum_{i,j} d_{ij}^2. \tag{1.5}$$

This is the normalised residual sum of squares.[2] The residuals themselves will not conform to any single scale, hence the normalisation. S will usually fall between 0 and 1 and is often expressed as a percentage. However, if the mean error is greater than the mean distance between cities, the stress will exceed 100%. The closer the regression is to the true relationship between the original distances and those of the configuration produced by the MDS, the closer the stress will be to zero. The stress depends on the dimension and the number of data points. As dimension, t, increases, stress usually decreases. As n increases, however, stress also tends to increase. The greater the number of dimensions and the fewer points, the easier it is for the scaled configuration

[2]There are several variations of the formula for stress. Chatfield and Collins take the summation for $i < j$. Manly and Kruskal and Wish take the square root of S to be their stress. With a lack of a standard definition and since the values of S that represent the thresholds of 'good fit' and 'bad fit' are highly subjective, Equation 1.5 represents a compromise.

to match the true configuration, thereby lowering the stress (Chatfield and Collins 1980, 207; Kruskal and Wish 1978, 24-25; Manly 1986, 129). However, if, by reducing the number of points, the overall range of the data is decreased significantly, the denominator of Equation 1.5 drops, causing stress to increase.

To minimise stress, the residual sum of squares, i.e. the numerator of Equation 1.5, must be minimised. The best possible regression relating the d_{ij}'s and the δ_{ij}'s must be found to accomplish this. A scatter diagram, plotting the d_{ij}'s against the δ_{ij}'s, should should show which type of regression (linear, etc.) best fits the data. A suitable regression technique can then be selected (Kruskal and Wish 1978, 25-26,28). In minimising stress, iterative procedures are used, taking advantage of the fact that S is explicitly differentiable. Unfortunately, local minima can cause problems. If a local minimum is found instead of a global minimum, then any changes made to the configuration will neither reduce the stress much at all nor will it provide a very good overall fit. Several different initial configurations may need to be tried to ensure a global minimum is reached. It is also possible that no minimum value exists (Chatfield and Collins 1980, 207).[3] Once minimised, a scatter diagram with the fitted distances can be used to see how well the regression fits the data. MDS is very robust to changes in the type of regression, provided n is large enough. There may be little difference between the scatter plot of a true monotonic relationship and one of the same data that is assumed to be linear. However,

> if the points clearly show some other curve than the one plotted, then the values of stress will be unduly inflated by the inappropriate assumption made about [the type of regression, and it] is best to reanalyse the data using a more appropriate assumption (Kruskal and Wish 1978, 29, see also 78).

The scatter diagram of the d_{ij}'s and the δ_{ij}'s may also be useful for determining whether or not cluster analysis (Section 1.3) should be performed. Instead of all the points lying along a curve (provided that there even is a relationship between the d_{ij}'s and the δ_{ij}'s), the points will be in a few different bunches. This may be the result of natural clusters in the data in which the dissimilarities between data points within each cluster is, on the whole, smaller than those between data points located in two different clusters. A separate analysis of each cluster should then be carried out, though one should be conscious of how much n is reduced by examining a single cluster instead of the whole (Kruskal and Wish 1978, 29-30).

Interpreting the value of S depends upon the number of data points and the size of the dimension, n and t, respectively. Kruskal and Wish advocate

[3] For a real-life analogy, see Kruskal and Wish 1978, 27-28.

$n > 4t$ as a rule of thumb. Above this threshold, S can be interpreted without much regard to n and t. If this is the case, an S of under 2% indicates a 'very good fit' (Kruskal and Wish 1978, 52), though Chatfield and Collins call 5% the threshold for a 'good' fit (see 1980, 207). Obviously, these are not hard and fast rules, especially as the sources use slightly different formulae for S. However, when $n < 4t$, randomness can account for a high percentage of the value of S. '...for 7 objects in 3 dimensions, a stress as small as 0.02 or smaller will occur for contentless random data about 50% of the time...' (Kruskal and Wish 1978, 52).

1.5 Choosing our statistics

Now that we have seen some of the statistical methods available for geographic and shape analysis, we must choose which methods are to be used, modified, or rejected. Procrustes analysis, as it stands, is perfect for our examination of Ptolemy's maps. It allows for the translations, scalings, and rotations, which are the very errors of Ptolemy that we aim to fix. The Mantel test, by comparison, provides relatively little that we can use. As Peres-Neto and Jackson (2001) observe, the Mantel test only provides a correlation statistic between distance matrices, whereas the Procrustes analysis provides not only this, but also a graphical result which can be used to analyse individual distance observations.

> In addition, Procrustes residuals can be also related to other sources of information, becoming useful in many designs of *post hoc* analysis such as regression or analysis of variance, cluster analysis, ordinations, and even additional Procrustes analysis (Peres-Neto and Jackson 2001, 176).

Cluster analysis will be very useful in determining whether Ptolemy was placing his cities and landmarks in geographic or historical blocs or if there is province-wide consistency or inconsistency. The minimum spanning tree method in particular could be used to model a system of roads connecting Ptolemy's cities. However, such a road network will almost certainly contain closed loops which the MST would leave out(Chatfield and Collins 1980, 223). Furthermore, the shortest roads may not be the main roads marked by milestones, and large bodies of water or mountain ranges may have prevented the connection of cities. Thus, the minimum spanning tree would be overly complicated at best.

The Jenks natural breaks method, on the other hand, is just right. Beyond having the advantage of being built into the mapping software, Jenks can divide the error data into appropriate clusters based solely on the relationships of the error figures. This information can then be colour-coded into the maps. How the colours are or are not regionalised can then be compared to the historical and geographical regions. We shall then be able to see if the errors are regionalised, if the errors are consistent throughout, or if the errors are almost random. This can lead us to speculate on whether or not Ptolemy's data came from different sources.

To the extent that it seeks to find a two-dimensional configuration given a distance matrix, multidimensional scaling is rather pointless for our purpose. Our data comes to us already in the form of a two-dimensional map based on a list of coordinates. MDS's initial power, then, is moot. However, MDS provides stress, which is a particularly good statistic for relating two configurations based on internal distances and consistencies. Whereas Procrustes analysis and the Procrustes statistic are based on the association of pairs of points *between* the two configurations, MDS and stress are based on the association of all points *within* the configurations. This association of points is that intercity distance and orientation data which Ptolemy used to place his settlements and determine their coordinates.

1.6 Adapting our statistics

Statistics should begin even before data is collected. Statistics should dictate how an experiment is set up, what data is collected, and how it is collected. Unfortunately, Ptolemy collected his data about nineteen hundred years ago, well before Procrustes analysis and MDS were formalised. Hence, we cannot control the way in which our data is collected. We can convert the data from coordinates into the more useful form of kilometre distances, but it is the statistical models that must be modified to work with our data.

1.6.1 Cluster analysis

Cluster analysis can take place at two stages: during the examination of the original configurations (Ptolemy's map and the modern map) and after the Procrustes analysis and MDS. At the first stage, the cluster analysis will essentially function as it would with any distance-data. Points may be put into groups based on their distances to the other points, the geography of the area, or the history of the area. There may even be predefined groups such as provincial boundaries. A visual inspection, rather than a raw computation, is preferable. A raw computation based entirely on straight-line distances will not take into account geographical features or historical developments. For example, two cities directly across the Italian peninsula may be closer in the Euclidean sense than two cities along the same coast, but there may be no road across the Apennines for some kilometres. Also, in Britain, certain cities in the south may have been established well ahead of cities elsewhere on the island. Though Colchester is nearer to Lincoln than it is to Exeter, it may need to be in Exeter's cluster because of the involvement of the two cities in earlier Roman campaigns than those which established Lincoln.

The second stage of cluster analysis comes after Procrustes analysis and MDS have been performed. New groups may emerge visually when the Ptolemaic points have been optimised to the modern points. More importantly, however, are the residual errors. Cluster analysis of the residuals may introduce groups not readily apparent on visual inspection of the configurations, whether optimised or not. Such analysis may support some historical hypotheses concerning the data. For example, the errors may follow patterns of settlement or conquest that can then be analysed in their own right.

1.6.2 Procrustes analysis

Procrustes analysis needs no altering to work with our data. The input is precisely what we have: two matrices describing the positions of points in two configurations. The output is exactly what we want: an optimisation of one of the configurations onto the other, presented graphically with a statistic describing the fit. The process allows us to choose the nature of the transformations used to optimise the configurations, and these transformations can be chosen to reflect the nature of Ptolemy's errors. As the Procrustes method is a type of shape analysis, it will be especially useful for determining the accuracy of coastlines and borders.

1.6.3 Multidimensional scaling

As stated above, we do not need the configuration-generating powers of MDS. We begin with a configuration and derive our distance matrix from it. What we want from MDS is its optimisation capabilities and its statistical output. Some modifications do need to be made to the process. Typically, the input is only a single distance matrix. A second distance matrix is produced after the configuration is computed. There is the implicit assumption that these matrices will be different. The original distances are not taken from a configuration and are not expected to fit with 100% accuracy into such a configuration. As our data is derived from a two-dimensional map, that exact same map will be reproduced by MDS (though possibly reflected, translated, scaled, and rotated uniformly).

We must provide both distance matrices in the form of the distances derived from the modern map and those derived from Ptolemy's map. As the procedure outlined in Section 1.4.4 dictates, the Ptolemaic distances can then be subjected to various optimisation transformations. It is thus that we obtain our d_{ij}'s, $\hat{d}_{ij}$'s, and δ_{ij}'s. We are then free to examine the stress between the two configurations and attempt its minimisation.

1.7 Applying our statistics - Ireland

Ireland has far too few identified locations for any conclusions to be drawn based on its statistics. But because of the small numbers of data points involved, it can serve as a convenient example to demonstrate our method of application.

1. Preparation of the data.

   From the text of the *Geography*, the coordinates of all identified locations need to be extracted. For computational purposes, the data should be set out in two $n \times 2$ matrices, one for Ptolemy's coordinates (matrix P) and one for the modern coordinates (matrix M). The coordinates should be in the same order in both matrices so that the coordinates describing the same locations are in the same positions in their respective matrices.[4] Coordinates should be converted into decimal format with longitudes west of the prime meridian given as negative numbers. The locations that have been identified on Ptolemy's map are Robogdium Promontory (Fair Head), R. Senus (Shannon), R. Dabrona (Lee), R. Birgus (Barrow), Sacred Promontory (Carnsore Point), Eblana (Dublin), R. Buvinda (Boyne), and R. Logia (Belfast Lough) as well as the Isles of Monaoeda (Man) and Mona (Anglesey). The coordinates,

[4]During pre- and post-processing, it will naturally be helpful to include ancient and modern names and possibly ID numbers for each location, along with any additional information deemed relevant, but here we are only discussing the data necessary for the statistical computations.

in that order, of these location are given in the matrices below:

$$P_{\text{raw}} = \begin{bmatrix} 16.33 & 61.50 \\ 9.50 & 59.50 \\ 11.25 & 57.00 \\ 12.50 & 57.50 \\ 14.00 & 57.50 \\ 14.00 & 59.50 \\ 14.67 & 59.67 \\ 15.33 & 60.67 \\ 17.67 & 61.50 \\ 15.00 & 57.67 \end{bmatrix} \qquad M_{\text{raw}} = \begin{bmatrix} -6.15 & 55.22 \\ -9.93 & 52.48 \\ -8.27 & 51.80 \\ -6.95 & 52.13 \\ -6.37 & 52.17 \\ -6.27 & 53.35 \\ -6.25 & 53.72 \\ -5.67 & 54.72 \\ -4.57 & 54.23 \\ -4.37 & 53.28 \end{bmatrix}.$$

2. Conversion of the coordinates into kilometre distances centred on the origin.

   Because the length of a degree longitude shrinks as one moves north from the equator, it is desirable to standardise the coordinates into a uniform measure. For comparison's sake, we would like the measure to be the same for Ptolemy's coordinates and the modern coordinates. This does not mean that we can directly convert the two sets of coordinates into kilometres in exactly the same way. Ptolemy reckoned the circumference of the earth to be roughly 18% smaller than the true measurement.[5] A simple conversion from degrees to kilometres would not be accurate. We must take into account how Ptolemy measured his degrees.

   The kilometre conversion and centring of the modern coordinates is as follows:

$$x_m = 111.320\left(\phi - \bar{\phi}\right)\cos\bar{\theta} \tag{1.6}$$
$$y_m = 111.133\left(\theta - \bar{\theta}\right). \tag{1.7}$$

   The longitude is converted to a rectangular x-coordinate and the latitude to y. Longitude, mean longitude, latitude, and mean latitude are represented, in degrees, by ϕ, $\bar{\phi}$, θ, and $\bar{\theta}$, respectively. The means were calculated over the 10 data points in M. The constants are the number of kilometres per degree along the equator and the meridians, respectively. The $\cos\bar{\theta}$ adjusts the distance per degree

[5]See Ptolemy 2000, 20-22, for a discussion Ptolemy's initial use of Eratosthenes' 700 stadia per degree and his later use of 500 stadia.

longitude as one moves away from the equator. The conversion and centring for the Ptolemaic coordinates is

$$x_p = 500\left(\phi - \bar{\phi}\right)(11/20)(0.185) \tag{1.8}$$

$$y_p = 500\left(\theta - \bar{\theta}\right)(0.185). \tag{1.9}$$

The 500 refers to the number of stadia per degree given by Ptolemy (Ptolemy 2000, 21). While Ptolemy knew the exact way to calculate the changing length of a degree longitude with his *chord* function,[6] for his regional maps he was content to use a simple approximation based on the ratio of the lengths of a degree longitude and latitude at the mean latitude of the map in question.[7] For the British Isles, this ratio is 11 : 20 (Ptolemy, *Geo.* 8.3.1), hence the replacement of $\cos\bar{\theta}$ in Equation 1.6 with 11/20 in Equation 1.8. Assuming that Ptolemy used the Attic stade, the 0.185 in Equations 1.8 and 1.9 is the number of kilometres per stade.[8]

Converting and centring the data yields

$$P = \begin{bmatrix} 117.44 & 212.75 \\ -230.21 & 27.75 \\ -141.18 & -203.50 \\ -77.58 & -157.25 \\ -1.27 & -157.25 \\ -1.27 & 27.75 \\ 32.64 & 43.17 \\ 66.56 & 135.67 \\ 185.27 & 212.75 \\ 49.60 & -141.83 \end{bmatrix} \quad M = \begin{bmatrix} 21.84 & 211.89 \\ -229.80 & -91.87 \\ -118.95 & -167.81 \\ -31.37 & -130.77 \\ 7.43 & -127.06 \\ 14.08 & 4.45 \\ 15.19 & 45.19 \\ 53.99 & 156.33 \\ 127.15 & 102.61 \\ 140.45 & -2.96 \end{bmatrix}.$$

3. Calculation of the Procrustes statistic and stress.

   The Procrustes statistic for the centred matrices is simply the sum of squared distances between the points in P and their correspond-

[6] Ptolemy's *chord* function is equal to 'the length of a chord subtended by a given angle in a circle of radius 60 [...or put another way,]

$$\cos\theta = chord\left(180^\circ - 2\theta\right)/120$$

(Ptolemy 2000, 16 footnote 11).

[7] We have followed Ptolemy's approximation when converting the modern coordinates into distances, hence $\cos\bar{\theta}$ instead of the individualised $\cos\theta$.

[8] See Ptolemy 2000, 14, for the various units of distance at use in the ancient world, Ptolemy's conversions of them, and their estimated values in kilometres.

ing points in M:

$$G(M,P) = \sum_{i=1}^{n} (x_{m_i} - x_{p_i})^2 + (y_{m_i} - y_{p_i})^2$$
$$= \text{trace}\left\{(M-P)^{\mathrm{T}}(M-P)\right\} \tag{1.10}$$

This is equivalent to Equation 1.1 but makes use of the standard (x, y) coordinates. To compute the stress, distance matrices, P_{D} and M_{D}, must be computed from P and M. Taking Equation 1.5 from Section 1.4.5, stress is

$$S = \sum_{i,j}\left(d_{ij} - \hat{d}_{ij}\right)^2 / \sum_{i,j} d_{ij}^2,$$

where the d_{ij}'s are the entries of M_{D} and the $\hat{d}_{ij}$'s of P_{D}. After centring, then, our results are thus:

$$G_{\mathrm{C}} = G(M,P) = 73{,}759.94\,\mathrm{km}^2 \qquad S_{\mathrm{C}} = 12.12\%.$$

The individual kilometre distance errors and individual stress errors are summarised in Tables 1.1 and 1.2, respectively. The individual Procrustes errors are simply the distances between a point in P and its corresponding point in M. Individual stress errors are computed as in Equation 1.5 but by holding i constant for each city, i. Figures 1.1 and 1.2 display Ptolemy's centred configuration, the former with individual stress errors, the latter with individual Procrustes errors. Because Procrustes analysis is concerned with shape comparison, Figure 1.2 has both the modern (dark blue with triangles) and Ptolemaic (light blue with circles) configurations centred on one another with their coastlines drawn (only identified points are included, hence the simple and misshapen configurations). Since stress is concerned with interpoint distances, only the configuration in question is displayed in Figure 1.1, the relevant connections between settlements being too numerous to display.

4. Rotation to minimise Procrustes statistic.[9]

[9]Stress remains constant under rotation as the inter-point distances in P remain unchanged. However, the rotation will determine where the principal axes fall. Scaling to minimise stress will then take place along these axes. The rotation, thus, will affect the minimisation of the stress.

	Centred	Rotated	Uniform	Skewed
Fair Head	95.60	45.95	38.26	64.77
R. Shannon	119.62	62.15	84.70	72.05
R. Lee	42.05	71.77	52.83	29.06
R. Barrow	53.26	41.06	4.50	13.69
Carnsore Point	31.42	39.50	23.13	38.14
Dublin	27.91	31.31	26.15	26.50
R. Boyne	17.58	7.48	6.27	12.61
Belfast Lough	24.19	24.35	50.17	57.92
Isle of Man	124.53	149.17	98.50	72.18
Isle of Anglesey	165.95	135.15	121.45	99.61

Table 1.1: Individual Procrustes results (km).

	Centred	Rotated	Uniform	Skewed
Fair Head	5.14%	5.14%	2.60%	2.30%
R. Shannon	3.17%	3.17%	5.49%	2.16%
R. Lee	9.51%	9.51%	4.78%	4.00%
R. Barrow	11.65%	11.65%	5.22%	4.55%
Carnsore Point	12.27%	12.27%	6.84%	8.56%
Dublin	10.08%	10.08%	5.62%	4.24%
R. Boyne	7.08%	7.08%	3.54%	2.87%
Belfast Lough	3.67%	3.67%	4.81%	4.89%
Isle of Man	36.12%	36.12%	15.12%	10.02%
Isle of Anglesey	30.83%	30.83%	24.39%	13.43%

Table 1.2: Individual stress results.

Figure 1.1: Ireland: Ptolemy's configuration with initial stress

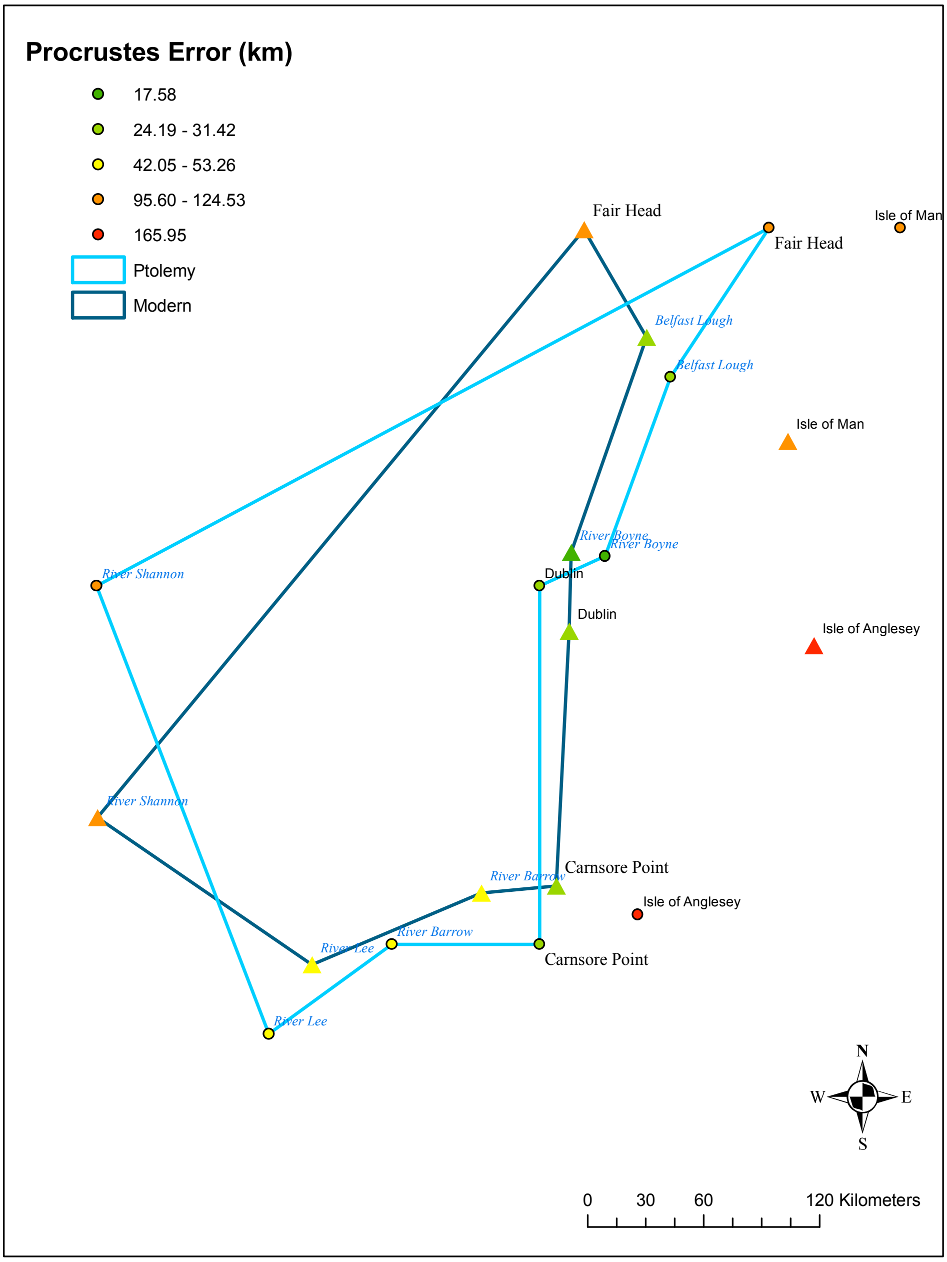

Figure 1.2: Ireland: modern and Ptolemy's configurations

To rotate P so that the Procrustes statistic is minimised requires multiplying P by the rotation matrix

$$R = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}. \tag{1.11}$$

We choose θ to minimise the error. Substituting PR for P in Equation 1.10 gives

$$G_{\mathrm{R}} = G(M, PR) = \mathrm{trace}\left\{(M - PR)^{\mathrm{T}}(M - PR)\right\}. \tag{1.12}$$

Computer minimisation[10] of G_{R} gives

$$\theta = -14.24^\circ \qquad PR = \begin{bmatrix} 61.50 & 235.10 \\ -229.96 & -29.72 \\ -86.79 & -231.97 \\ -36.52 & -171.50 \\ 37.44 & -152.73 \\ -8.06 & 26.58 \\ 21.02 & 49.87 \\ 31.15 & 147.87 \\ 127.25 & 251.78 \\ 82.96 & -125.28 \end{bmatrix}.$$

The angle of -14.27° refers to a anti-clockwise rotation about the origin bringing the points of P to those of PR. This reduces the sum of squares error to

$$G_{\mathrm{R}} = 56,516.45\,\mathrm{km}^2.$$

The individual errors are summarised in Table 1.1.

5. Scaling to minimise Procrustes statistic and stress.

Scaling occurs along the north-south and east-west axes. It can be applied either uniformly or independently along the two axes. We shall apply the scaling as a skew, treating each axis independently.[11]

[10] Minimisation was carried out by both the 'Solver' tool of Microsoft Excel and the 'procrustes' command of R found in the 'vegan' library. The sum of squares errors of the two programs were within $2\,km^2$ of each other. Likewise the various MDS programs in R use slightly different variations of the stress formula and are not at all transparent in their processes. Numbers quoted in the text refer to those produced by Excel.

[11] R's 'procrustes' command performs either no scaling or uniform scaling. Excel, being more manually operated, can perform both uniform and skewed scaling. Excel, therefore, is our tool of choice.

This will give us a better understanding of where Ptolemy's errors lie. Furthermore, his longitude and latitude (i.e. distances east-west versus north-south) are unrelated, as, for example, thinking that a city is 100 km farther north than it truly is has no bearing on how far east of its real location it is placed. The computer will compute a dilation matrix in the form of $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$. The minimised Procrustes statistic, G_{S}, is equal to $G(M, DPR)$, where D denotes the dilation matrix. Minimisation is achieved with the dilation matrix $\begin{bmatrix} 1.05 & 0 \\ 0 & 0.69 \end{bmatrix}$ with a result of

$$G_{\mathrm{S}} = 31,220.44\,\mathrm{km}^2,$$

and new coordinates of

$$\underset{\text{Procrustes}}{DPR} = \begin{bmatrix} 64.30 & 162.99 \\ -240.44 & -20.61 \\ -90.74 & -160.82 \\ -38.19 & -118.90 \\ 39.15 & -105.89 \\ -8.43 & 18.43 \\ 21.98 & 34.57 \\ 32.57 & 102.51 \\ 133.05 & 174.55 \\ 86.74 & -86.85 \end{bmatrix}.$$

Stress is minimised by the dilation matrix $\begin{bmatrix} 1.11 & 0 \\ 0 & 0.70 \end{bmatrix}$. Scaling the configuration gives the following stress:

$$S_{\mathrm{S}} = 5.36\%.$$

The scaled coordinates are

$$\underset{\text{stress}}{DPR} = \begin{bmatrix} 68.50 & 165.42 \\ -256.12 & -20.91 \\ -96.66 & -163.22 \\ -40.68 & -120.67 \\ 41.70 & -107.46 \\ -8.97 & 18.71 \\ 23.42 & 35.09 \\ 34.69 & 104.04 \\ 141.72 & 177.16 \\ 92.40 & -88.14 \end{bmatrix}.$$

The individual errors are in Tables 1.1 and 1.2. Figures for uniform scaling have been included for comparison's sake. We can readily see why skewed scaling is preferred as, with it, Ptolemy's configuration is brought much closer to its modern counterpart. Also, in the dilation matrices above, we have already seen that the inherent scaling errors of Ptolemy's Ireland are not uniform. How Ptolemy's map is affected by the various transformations can be seen in Figures 1.3 and 1.4. Ptolemy's identified locations are colour-coded by error.

6. Repetition of the above for any clusters that present themselves in the data.

   Much like trying to identify outliers, assigning clusters with only ten points is statistically dubious. But as an example, from the map we can see that the R. Shannon is off on its own on the west coast. The two islands, Anglesey and Man, naturally form a group. The three points along the southern coast could be their own cluster, likewise with the two northern points, as well as Dublin and the R. Boyne. This is, of course, taking things to the extreme. Let us examine Figure 1.2 to see how the computer can assist in cluster analysis. Again, trying to divide ten points into five clusters is fruitless, but we can see that the Procrustes error of Anglesey is much higher than the other locations and is set aside for its own group. While the Boyne is alone in the lowest error class, the points on either side of it are in the next class, and the entire east coast could be made into a single cluster. Looking at the range of values of the lowest three classes versus the highest two, it would be reasonable, in fact, to include the coast from Belfast Lough to the River Lee in one group. The actually physical geography of the island would make it absurd to include the most erroneous four points in a group together, but perhaps the 'west' coast of Fair Head and the Shannon could be one cluster, and the two islands another. In any case, the computer's Jenks natural breaks method for cluster analysis must be subjected to the reality of the map's geography.

7. Investigation of outliers.

   An outlier is defined here as any place having an error greater than the third quartile plus 1.5 times the interquartile range.[12] Ireland

[12]An outlier could also be a location that has an error less than the first quartile minus

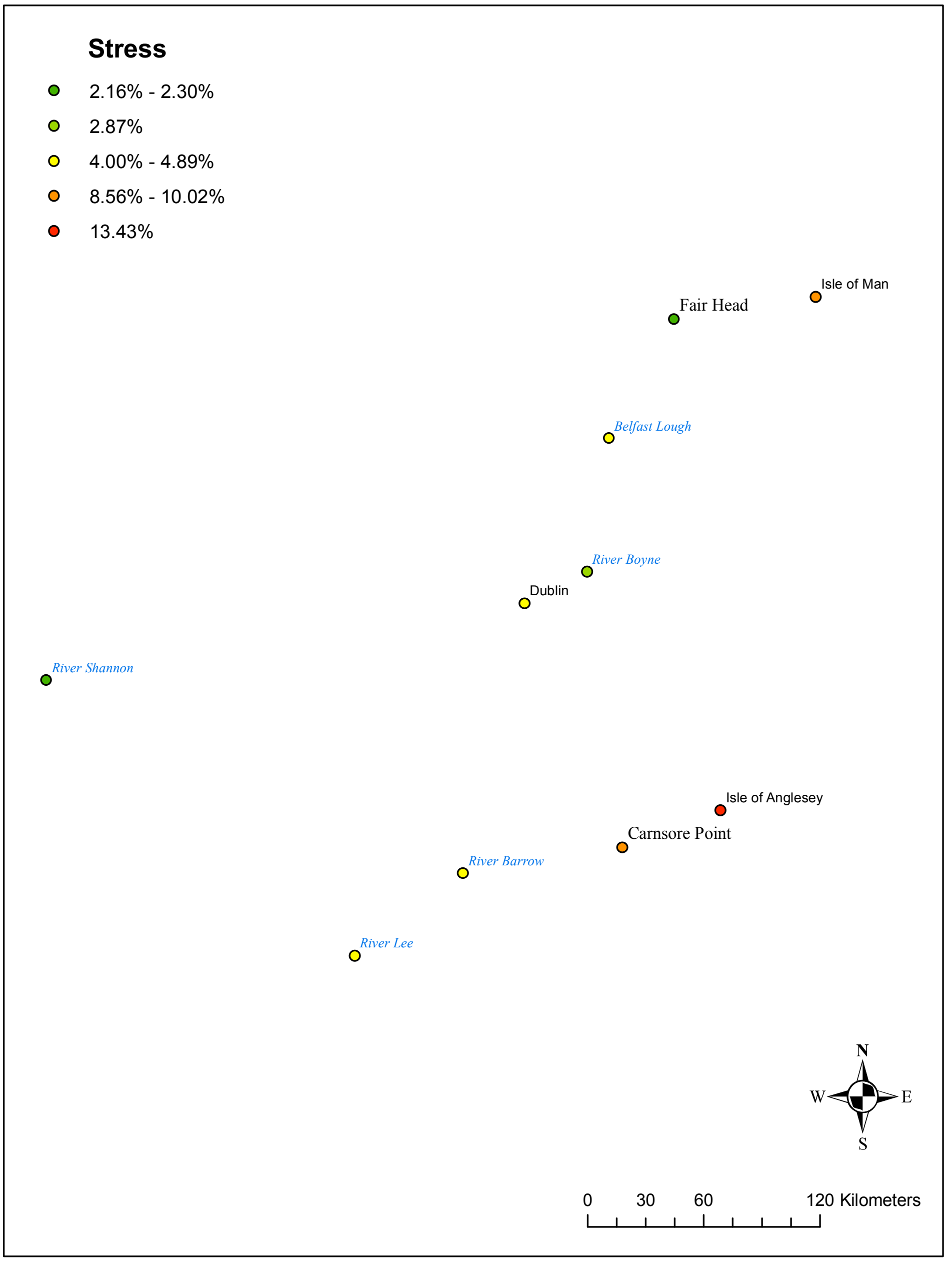

Figure 1.3: Ireland: transformed configuration with transformed stress

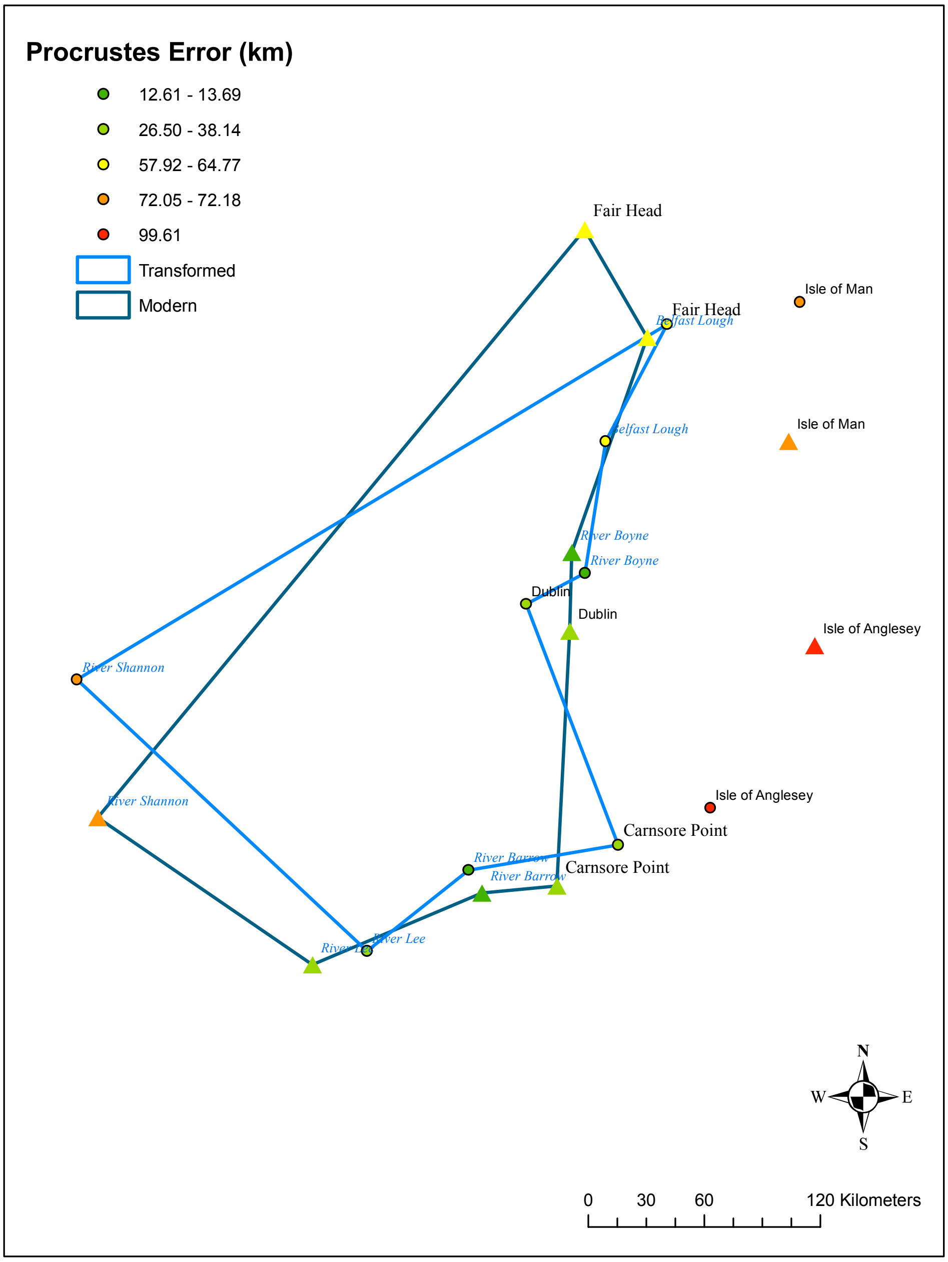

Figure 1.4: Ireland: modern and transformed configurations

has no points that are outliers in both their original errors and after their transformation, which is not surprising with so little data. The two isles, though, are both outliers when we focus on initial stress. For the sake of demonstration, though, let us suppose that Man and Anglesey remained outliers after transformation. We would then look back at the current edition of the *Geography* and see if alternative coordinates exist. If so, we would repeat the above steps to see if the overall map and statistics improve. If not, we would explore possible reasons why the errors are so large and, potentially, repeat the analysis with the outliers removed. In this case, the simple fact that the outliers are islands might compel us to exclude them from the study of the points of the mainland.

1.7.1 Interpreting the results

If we again suspend our statistical propriety and pretend that Ireland has sufficient data for analysis, we can be free to interpret the results as presented in this chapter's tables and maps. Examining Figure 1.2, we can see that the Fair Head-Shannon line is the source of greatest error as regards Ireland's shape. The two small isles, obviously, have their own large errors, and they should be excluded from analysis of Ireland proper. Looking at Figure 1.7, the two isles, along with the Shannon, shed the most error after the transformation, while the originally accurate east coast has no such gains. Indeed, Belfast Lough, stuck between the inaccurately placed Fair Head and Isle of Man more than doubles its error due to the transformation. Practically speaking, the difficulties of pinpointing the locations of islands are much greater, or at least significantly different, than charting points along a coast. Islands should be excluded from the shape analysis of the coast.

The same can be said with regards to the isles when looking at the initial stress and transformation-improvement results in Figures 1.1, 1.5, and 1.8. The stress errors beyond those of the two isles, though, are not consistent with the Procrustes error. Fair Head and Shannon are now towards the lower end of the error while places such as Dublin and Carnsore Point are towards the higher end. This phenomenon attests to the fundamental difference between the two error measurements and analyses. The River Shannon is approximately 120 km from its true location when Ptolemy's map is centred on the modern map. However, it is correctly placed to the west of all the other points, and, with the exception of Anglesey, is correctly placed north or south of all the other

1.5 times the interquartile range, but for the vast majority of regions studied, this would require a negative error.

Figure 1.5: Ireland: modern configuration with initial stress

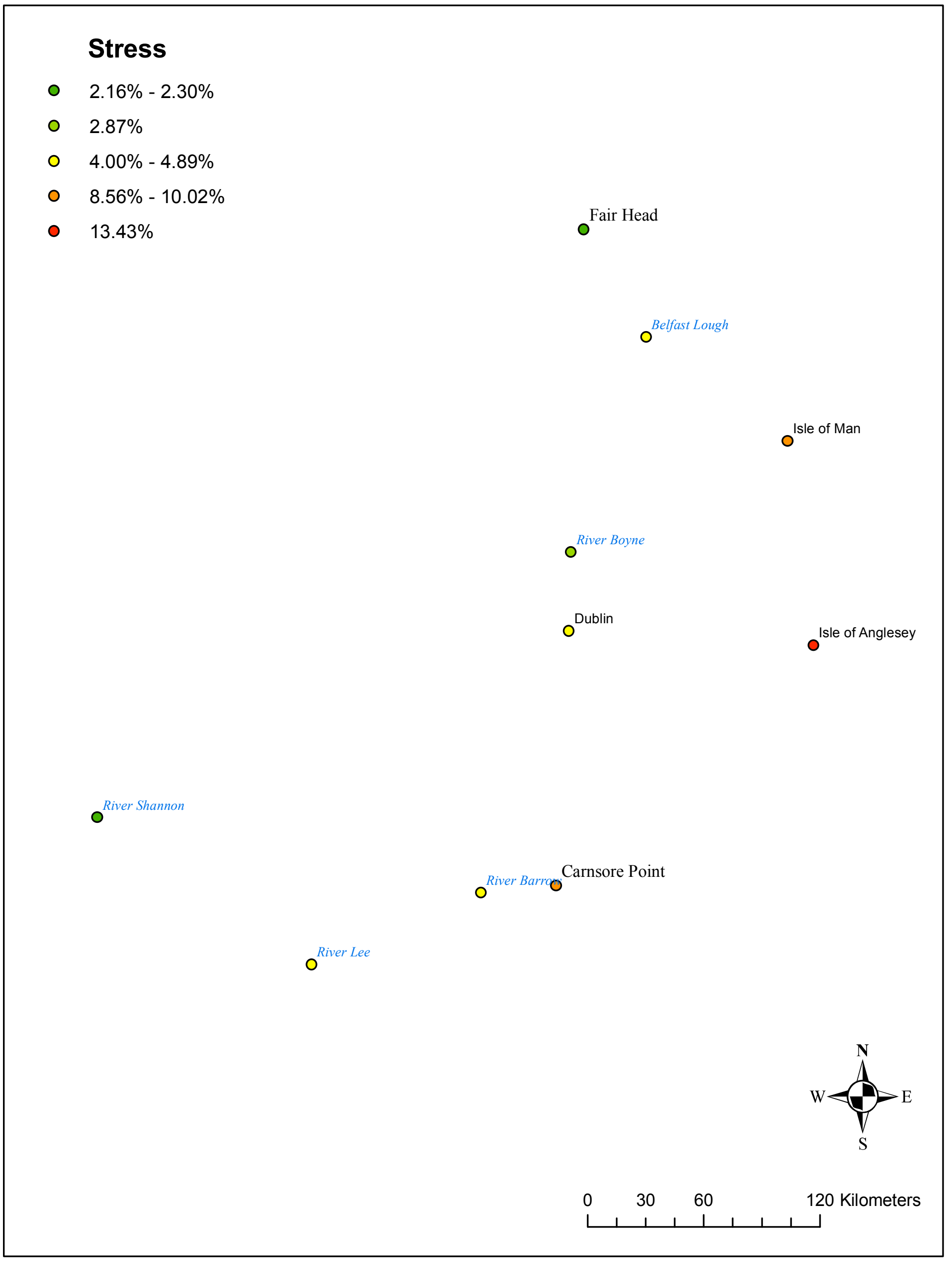

Figure 1.6: Ireland: modern configuration with transformed stress

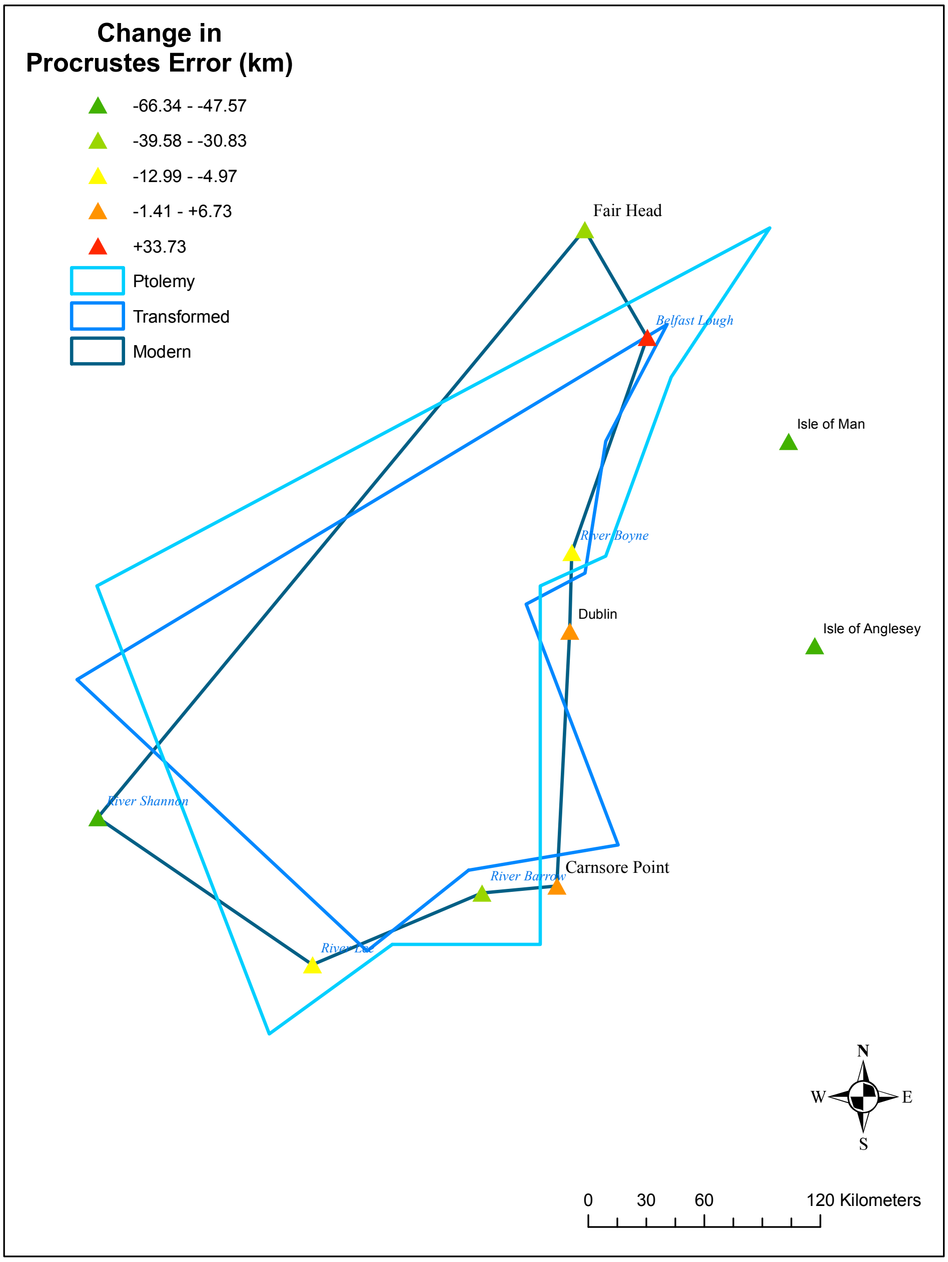

Figure 1.7: Ireland: all configurations with change in Procrustes error

points. Its location relative to the rest of the data is fairly accurate, hence its lower stress. Carnsore Point, on the other hand, is only 31 km from its rightful place, but because Anglesey is so close to it on Ptolemy's map and its other neighbours are farther away than they should be, its stress is high.

The transformations themselves and the resulting decreases in errors were presented above. A comparison between the two graphical outputs of the MDS and Procrustes analysis can be seen in Figure 1.9. Both have undergone the same rotation (as they always will since the rotation is based only on the Procrustes analysis), and their dilation matrices were very similar. Comparing transformed stress to original stress is quite difficult graphically, because one must be conscious of all the interpoint connections, which simply cannot be drawn without further confusing the map. In the subsequent case studies, there are also far too many points to have both the modern and Ptolemaic points on the same map.

Procrustes, as a shape analysis, will be left solely to the coastlines. This certainly has a graphical advantage as can be seen in Figures 1.4 and 1.7. There it is easy to see that the transformation has brought the Fair Head-Shannon line closer to what it should be, though at the cost of distorting the east coast between Dublin and Carnsore Point. The southern and eastern coast above Dublin have improved though.

What Ireland has allowed us to do is to take each step of the minimisation process and examine it transparently. With only ten data points, the numbers at each stage could be examined in easily managed lists, tables, and maps. This will not be the case for the case studies of Britain, Spain, and Italy. What data can be shown will be displayed as maps. Each group of cities analysed will have five accompanying maps:

1. the settlements in their modern locations colour-coded by their initial stress;

2. the settlements in their modern locations colour-coded by their transformed (i.e. minimised) stress;

3. the settlements in their modern locations colour-coded by the change in their stress due to the transformation;

4. the settlements in their Ptolemaic locations colour-coded by their initial stress; and

5. the settlements in their locations following the transformation colour-coded by their transformed stress.

Each stretch of coastline analysed will have three accompanying maps:

Figure 1.8: Ireland: modern configuration with change in stress

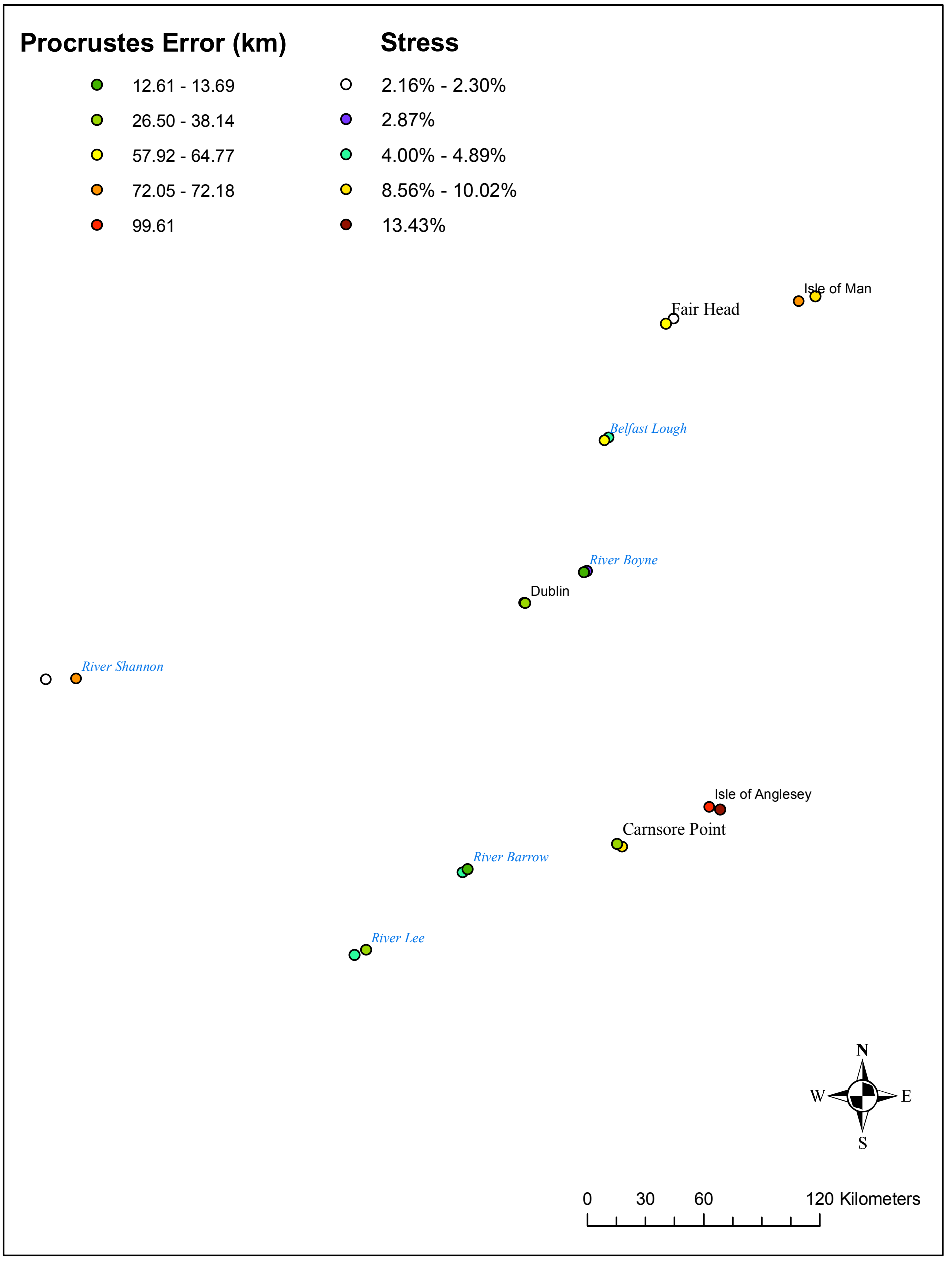

Figure 1.9: Ireland: transformation comparison between Procrustes analysis and MDS

1. the modern and Ptolemaic coastlines overlapped with points colour-coded by their initial Procrustes error;

2. the modern and transformed coastlines overlapped with points colour-coded by their transformed Procrustes error; and

3. the modern, Ptolemaic, and transformed coastlines overlapped with points colour-coded by the change in their Procrustes error due to the transformation.

Because coastal points are arranged essentially linearly with a single location having only one neighbour on each side, coastal maps are fairly uncluttered. This allows us to directly compare two or even three configurations on a single map. This is not the case with the cities. A single settlement can be surrounded by several other towns. Such a configuration can be very densely packed on its own and would be downright confusing if multiple configurations were displayed together on a single map. Thus, it is for purely visual reasons that there are two more inland maps than coastal maps.

Concerning our two error measurements, it is clear that Procrustes analysis is ideal for coastlines and borders. That we can 'connect the dots' of our data points and examine shapes that can easily and graphically be compared is more than helpful in determining the accuracy of Ptolemy's maps. Stress, unfortunately, does not afford this luxury, and accuracy based on stress will rely much more heavily on numeric rather than visual clues. Fortunately, the Jenks optimisation and colour-coding on our output maps will help in visualising this error. With Procrustes analysis being assigned to the coastal points, multidimensional scaling and stress must take on the role of analysing inland points. This makes perfect sense as the inland points in question are all settlements connected to one another by the vast road networks of the Roman Empire. There is little doubt that Ptolemy's data came from people measuring the distances and orientations between cities along these very roads. This interconnectivity is exactly what stress is designed to investigate.

As all roads lead to Rome, let us begin our investigation in Italy.

Chapter 2

Italy

2.1 A history of Rome's conquest of Italy

Most of the Roman conquest of Italy predates the drawing of Ptolemy's maps by several hundred years. After the Latin (ending 338 BC), Samnite (ending 290 BC), and Pyrrhic (ending 275 BC) Wars, most of the Italian peninsula was under Roman sway. How Rome related to its Italian allies was redefined during the Social War (ending 89 BC). The subjugation of northern Italy and the establishment of its border in the Alps, though, was not complete until 6 BC. The following is a brief sketch of Rome's growth and dominance during that time.

2.1.1 The Latin War

The Etruscan kings of Rome were overthrown, and the Republic supposedly established about 500 BC, though the evidence of our sources on that point is unclear. The neighbouring Etruscan city of Veii across the Tiber was forcibly joined to Rome in 396 BC, but what could have been a rapid expansion was checked by a Gallic sacking of the city ten years later (Livy, 5.41; Salmon 1982, 3).[1] By 350 BC, the peoples of Italy were divided into tribes of diverse languages, cultures, religions, alliances, and backgrounds. The Romans had strong ties to the Latini with whom they shared a common language, as well as the Sabini. For a hundred some years, a treaty existed between the Romans

[1]We must be suspicious of Livy's account prior to the First Punic War as barely any written sources survive. Livy began publishing his history in *c.*27 BC (Livy 1998, xii), and the earliest Roman historian, Fabius Pictor, had only written towards the end of the third century BC. Livy himself claims that all reliable historical works had been destroyed in the sack of 386 BC (Livy 1998, xvi-xvii). 'But in matters of so great antiquity I should be content if things probable were to be received as true' (Livy, 5.21).

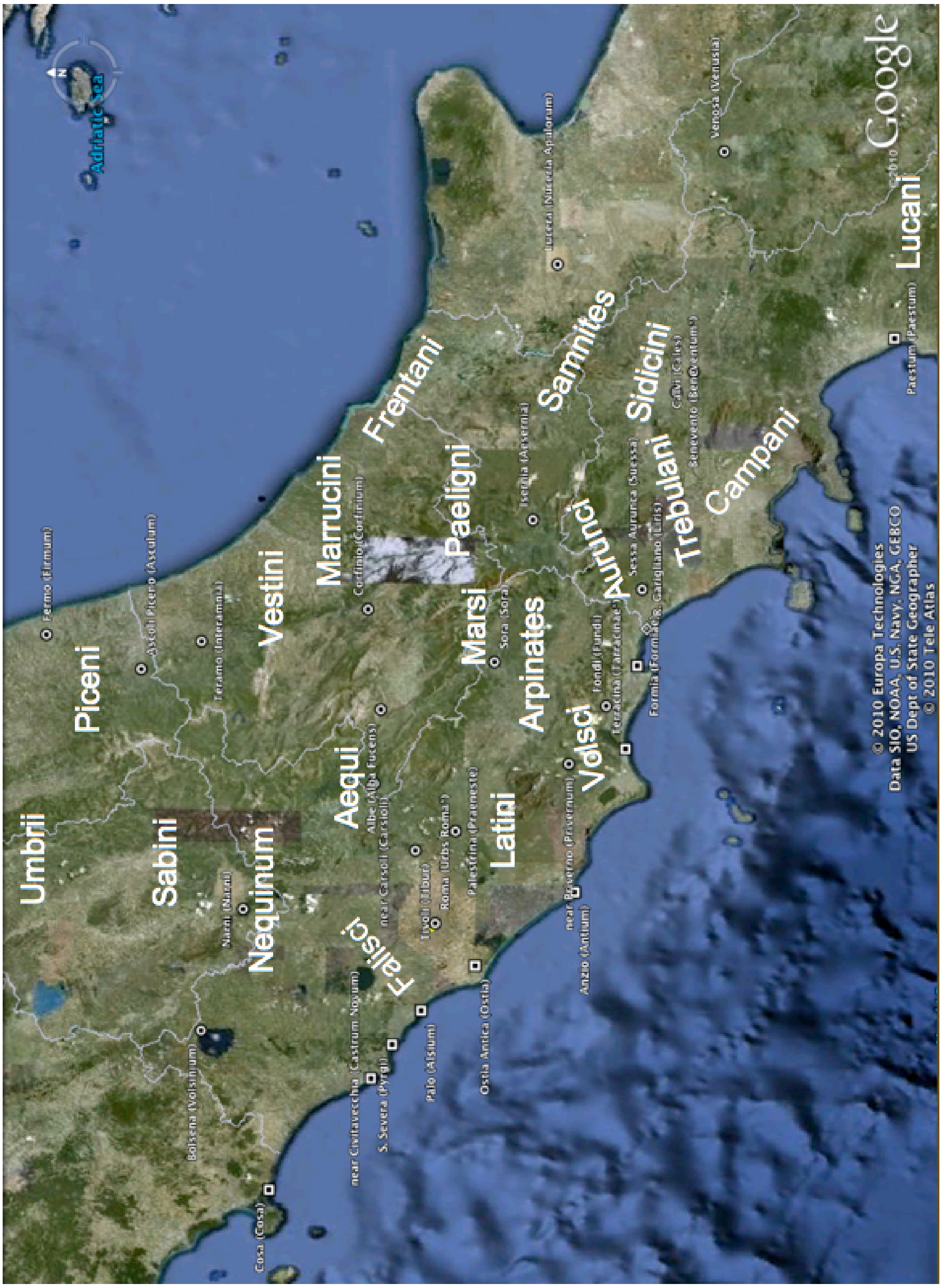

Figure 2.1: Central Italy (base map ©Google 2010)

and Latini, at the time united in the Latin League. This was for mutual defence against the neighbouring Aequi and Volsci. After Rome's taking of Veii and her recovery after the sacking by the Gauls, the other members of the Latin League challenged Rome's growing supremacy (Salmon 1982, 39-40).

Most of the Latini, along with the Aurunci, Campanians, Sidicini, and Volsci joined forces against Rome. For her part, Rome enlisted the help of the Samnites, a people of the Apennine Mountains. Rome won complete mastery of Latium and the surrounding area. This stretched from southern Etruria into northern Campania (see Pliny, *NH* 3.56-63 for a description of Latium and Campania). As well as territory being directly ceded to Rome, the Latin League was also forced to disband. The ceded land was given to Roman citizens, especially poorer ones. Additionally, two fully functioning *coloniae*,[2] consisting of Roman citizens, were planted at Ostia and Antium. The conquered cities each had separate alliances directly with Rome. These cities were given either the full or partial rights belonging to Roman citizens. These cities were now *municipia* of Rome. They were mostly free to carry out their own affairs except foreign policy (Livy, 8.11,14; see also Salmon 1982, 40-51 and Potter 1987, 45-46).

The Latin cities that did not fight against Rome were no longer held together by the Latin League. Rather, they became independent allies of Rome, but technically not allies of each other (Salmon 1982, 51-52). Other non-Latin cities, including some like Praeneste and Tibur who fought against Rome, were made into neither *coloniae* nor *municipia* but subjected allies or 'dependent satellites' (Salmon 1982, 54). After the end of the Latin War in 338 BC, Rome never lost control of Latium despite the numerous conflicts that marked its long history. Rome now had a platform from which to expand its holdings so that in less than a hundred years it would become one of the dominant powers in the Mediterranean (Salmon 1982, 55-56).

2.1.2 The Samnite Wars

The First Samnite War was a short affair concluded before the uprising of the Latini. In 354 BC, Rome and Samnium signed a treaty which most likely established the River Liris (Garigliano) as the border between the two 'nations' (Lomas 1996, 12; see also Potter 1987, 45). Both parties began to establish relations of various kinds with cities on either side of the river resulting in the First Samnite War (343-341 BC). This war is poorly attested to and, in any

[2] *Coloniae* were settlements of emigrant Roman citizens who retained all the rights thereof, as opposed to *municipia*, which were pre-existing towns whose citizens were granted certain rights and obligations belonging to Roman citizens.

Figure 2.2: Southern Italy (base map ©Google 2010)

case, the resolution seems to have been the reaffirmation of the treaty of 354 BC (Potter 1987, 34; Lomas 1996, 12).

Between the Latin War and the Second Samnite War, Rome sought to consolidate its position between southern Etruria and Campania. The Aurunci and most of the Volsci were brought under Roman control, solidifying Rome's dominance west of the Liris. A Latin *colonia* was planted at Cales (Calvi) in 334 BC to keep an eye on northern Campania (Salmon 1982, 57). The Volscian settlements of Privernum (Priverno), Fundi (Fondi), and Formiae took arms in 330 BC and were subjected the next year as *municipia*. A *colonia* of Roman citizens was established nearby at Tarracinae (Tarracina) to watch them and protect the coast (Salmon 1982, 50,57).

Troubles with the Samnites were renewed in 328 BC when the Romans established the *colonia* of Fregellae on the Samnite side of the Liris (Salmon 1982, 57; see also Potter 1987, 46). According to Livy, this and other matters led to failed negotiations:

> When the Roman legate invited them to discuss the question with the common allies and friends of both, the Samnite spokesman said, 'Why do we beat about the bush? Our differences, Romans, will be decided, not by the words of envoys nor by any man's arbitration, but by the Campanian plain — where we must meet in battle, — by the sword, and by the common chance of war. Let us encamp then face to face betwixt Suessula [Sessa Aurunca] and Capua [S. Maria Capua Vetere], and settle the question whether Samnite or Roman is to govern Italy' (Livy, 8.23).

Thus began the Second Samnite War in 326 BC. Various tribes in central and southern Italy were in and out of alliance with Rome and/or the Samnites throughout, and many cities fell in and out of the combatants' hands. Rome generally had the upper hand, according to the account Livy gives, with the notable exception of the disaster at the Caudine Forks in 321 BC. The entire army was ambushed by the Samnites between two ravines known as the Caudine Forks and forced to surrender without a fight. The Samnites spared the Romans under the conditions that they would disarm, remove their *coloniae* from Samnium, and ceremoniously pass under the yoke as a symbol of submission (Livy, 9.2,4-5). Unfortunately for the Samnites, this only led to renewed vigour on behalf of the Romans, but it was not until 304 BC that the Samnites sued for peace. A Roman army was sent to Samnium to investigate whether indeed the Samnite war machine was at rest and, being satisfied, the 'ancient treaty' was restored (Livy, 9.44-45; see footnote to chapter 45).

The Romans made substantial territorial gains following the conclusion of the Samnite war. The Romans turned to face the Aequi who had been supplying the Samnites. The Romans quickly took thirty-one of their cities, destroying most of them. Intimidated by the Romans' successes, the neighbouring tribes of the Marrucini, Marsi, Paeligni, and Frentani entered into treaties of alliance with Rome (Livy, 9.45). In the following years, *coloniae* were set up at Sora (Sora) and Alba (Albe), and the Arpinates and Trebulani obtained citizenship (Livy, 10.1). The Vestini entered into a treaty of friendship with Rome while the Etruscans were busy breaking theirs. Though at first ambushed by the Etruscans, the Romans raised reinforcements (Livy, 10.3-4) and 'the might of the Etruscans was broken for the second time' (Livy, 10.5).

The Etruscans and Samnites were quiet for a time (Livy, 10.6), and, in 299 BC, the Nequinum were subdued by Rome, and a *colonia*, Narnia (Narni), was founded in their lands as a forward post against the Umbrians. Rome also sought and obtained a treaty with the Picentes after the Etruscans came close to hiring the Gauls as mercenaries to engage the Romans (Livy, 10.10). In 298 BC, however, both the Etruscans and Samnites renewed their wars with Rome. The Etruscans invaded Lucania after the people there refused to join in their uprising. The Lucanians formed a league with Rome, who then declared war on the Samnite nation. Thus began the Third Samnite War (Livy, 10.11-12).

In 296 BC, realising that they could not stand alone, a united force of Etruscans, Samnites, some Umbrian tribes, and Gallic mercenaries mustered for an assault on Rome's territory (Livy, 10.18). After great slaughter on both sides, in 295 BC, Rome had the mastery, and the enemy force was scattered (Livy, 10.21,26-29). The following year, a deputation from three major cities in Etruria, Volsinii (Bolsena), Perusia (Perugia), and Arretium (Arezzo), sued for peace. A forty years' truce was made (Livy, 10.35-37). By 290 BC,

> the Samnites sought peace and the treaty with them was renewed for the fourth time. Curius Dentatus the consul having slaughtered the Samnites and conquered the Sabines, who had revolted, and received their submission, triumphed twice in the same year of office (Livy, *Periochae* 11).

Thus ended the Third Samnite War, during which time Rome gained control of the entirety of central Italy.

2.1.3 The War against Pyrrhus

In 281 BC, Rome found itself at war with the Tarentines, its first substantial, direct engagement with the Hellenistic World. The people of Tarentum (Tarent) invited an experienced Greek general, Pyrrhus, to lead their forces. Pyrrhus, formerly of Macedonia, was king of Epirus at the time. Pyrrhus and

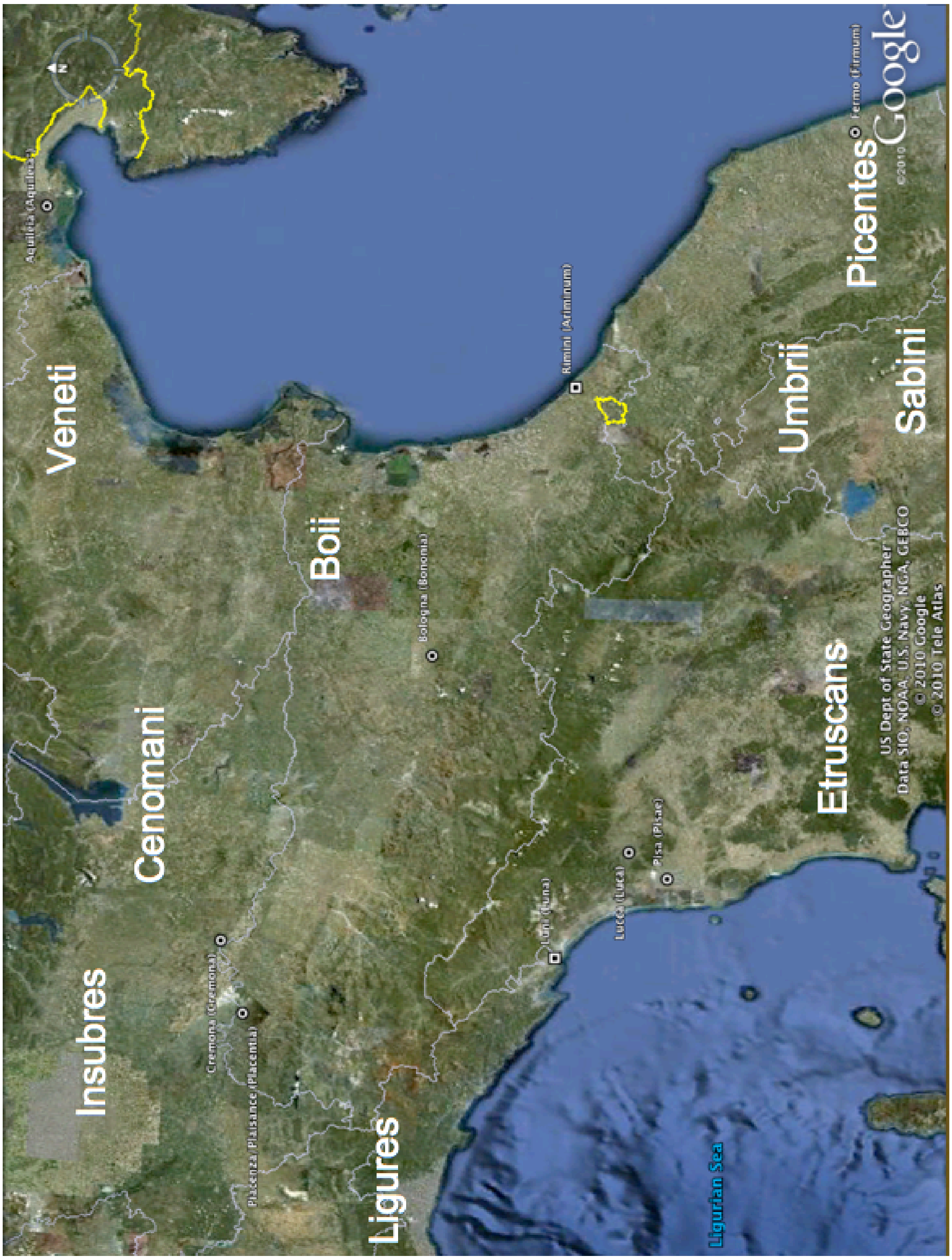

Figure 2.3: Northern Italy (base map ©Google 2010)

his Epirot followers accepted the invitation (Plutarch, *Pyrrhus* 13). Rome was also busy with the Gauls, who had destroyed a legion, and the Samnites, who had revolted yet again (Livy, *Periochae* 12). Successful campaigns were also fought against the Bruttii and Etruscans (Livy, *Periochae* 13).

In their first engagement, the Romans had the worst of it, the elephants of Pyrrhus causing panic in the Roman line. Pyrrhus then marched towards Rome with Lucanians and Samnites joining him. A two day battle was fought near Asculum (Ascoli Piceno), which was the source of the idiomatic 'pyrrhic victory'. (Livy, *Periochae* 13; Plutarch, *Pyrrhus* 16-17,21). Pyrrhus then left Italy for a campaign in Sicily, but returned in 275 BC. He met the Romans in battle near Beneventum (Benevento) and lost decisively. Pyrrhus left Italy in 275 BC (Livy, *Periochae* 14; Plutarch, *Pyrrhus* 23,25-26). The defeated Tarentines 'were granted peace and liberty' (Livy, *Periochae* 15). About this time, the Picentes, Umbrians, and Sallentines were also conquered by the Romans, and *coloniae* were established at Posidonia (Paestum), Cosa (Cosa), Ariminum (Rimini), and Beneventum (Livy, *Periochae* 14-15).

With due respect for the complex problems of precise dating, Salmon summarises Roman expansion to this point by listing chronologically the Latin *coloniae* established by Rome (all dates BC):

> Cales (334) countered Samnite-controlled Teanum Sidicinum [Teano]. . . . Fregellae (328), Luceria [Lucera] (314), Saticula, Suessa Aurunca [also written Suessula], Pontiae [Ponza] (313), Interamna Lirensas (312), Sora, Alba Fucens (303), Narnia (299), Carseoli [Carsoli] (298), Venusia [Venosa] (291), Beneventum (268) and Aesernia [Isernia] (263) encircled and even dismembered the Samnite League [and] penetrated the mid-Italic region. . . . Hadria and possibly Castrum Novum [Civitavecchia] (*c.*289), Cosa, Paestum (273) and Ariminum (268) helped to strengthen the Roman grip on Picenum, Etruria, Lucania and the Ager Gallicus respectively [and] protected the coasts of Italy. . . (Salmon 1982, 64-65).

By the founding of Ariminum in 268 BC, there was no truly politically independent settlement or tribe south of the Pisa-Ariminum line save Rome itself. All of this area was somehow tied to and dependent on Rome (Salmon 1982, 67). Rome was now a world player and immediately found herself in conflict with the other major powers of the Mediterranean.

2.1.4 Solidifying control with the help of Carthage

Most of the events of the First Punic War (264-241 BC) occurred outside of Italy proper, but Rome took the precaution of founding *coloniae* along her coasts for protection: Firmum (Fermo), Castrum Novum, Pyrgi (S. Severa), Brundisium (Brindisi), Alsium (Palo), and Fregenae. To prevent any inland uprisings, Aesernia was founded in 263 BC in Samnium while Etruscan Volsinii and Faliscan Falerii were destroyed (Salmon 1982, 73). The Carthaginian threat gone, the Romans looked to their northern border.

Following Carthage's defeat, Rome pushed north into Liguria in the north-west. To the east, the Gallic Boii put up stiff resistance. An alliance of Boii, Insubres, and transalpine mercenaries marched south-west over the Appennines but were beaten back in 225 BC. The Romans followed up their successful defence by attacking the Gallic tribes in their own territories, winning much of Cisalpine Gaul, and founding the *coloniae* of Cremona (Cremona) and Placentia (Piacenza) on the River Po in 218 BC. The Via Flaminia, begun in 220 BC, connected Rome to Ariminum to ensure swift access for troops to Rome's northern frontier. Rome had extended her control to the Po (Salmon 1982, 77-78; Livy, *Periochae* 16-20).

Rome's northern expansion was cut short by the sudden appearance of Hannibal crossing the Alps and the start of the Second Punic War in 218 BC. Despite several victories, Hannibal, with the exception of the newly subdued Insubres, Boii, and Ligures, was unable to turn the Italian tribes against Rome. It was not until he found himself in the south and victorious over the Roman army at Cannae in 216 BC that Rome's allies began to turn.

> Nevertheless, the universal revolt that Hannibal was hoping for did not materialize. In fact, no tribe or community went over to him in its entirety except in Campania and the far south [which were inhabited by Greeks]. Latium, central Italy, Picenum, most of the ports, and the Nomen Latinum everywhere made no move to support him; and in the upshot neither did Umbria or even Etruria (Salmon 1982, 79).

Paradoxically, Hannibal's invasion did not sunder Rome's alliances, but instead made them stronger (Salmon 1982, 83). Hannibal was recalled to Africa and, only a few years later, peace was declared in 201 BC.

2.1.5 The push to the Alps and the Social War

As soon as the Carthaginans were defeated, the Romans sought to re-establish their control of the north and extend it to the Alps. The Cenomani sought peace with Rome in 197 BC, and the Insubrues followed in 196 BC (Salmon 1982, 96). The Boii held out until the 190s BC. Once these Gallic tribes surrendered, Rome concentrated on the Ligures to the west, and it was not until the 150s BC that most, but not all, of the Ligurian tribes were subdued.

The people living in the Alps would take another hundred years before they submitted (Salmon 1982, 90).

A *colonia* was established in Cisalpine Gaul in 189 BC, Bononia (Bologna), and two in Liguria in 180 BC and 177 BC, Luca (Lucca) and Luna (Luni), respectively. The Veneti in the north-east had been allies of Rome since the previous century, and the *colonia* of Aquileia (Aquileia) was planted beyond their lands in the east in 181 BC (Salmon 1982, 96-97).

> Although largely pacified and organized, and although inhabited by many Roman citizens, Cisalpina remained distinct from the peninsula to the south. Officially it was Gallia, even though conceded to be geographically a part of Italy. As a war zone in which Roman armies were needed almost every year throughout the second century, Cisalpina is depicted as a sort of provincial area, and, in fact, early in the first century, it was officially pronounced a Roman province. But whatever its political vicissitudes, Romanisation made the same steady headway there as in the rest of Italy (Salmon 1982, 97-98).

Cisalpina and, indeed, all of Italy was further interconnected through a vast network of roads, most of which existed by the second century (see Salmon 1982, 99-100).

The physical unification of Italy under Rome soon brought demands from her Italian allies for equal benefits, namely Roman citizenship. The allies had supported Rome through manpower and resources but enjoyed little of the spoils of war. The Social War began when the Italians of Asculum in Picenum put to death the local Roman citizens in 91 BC. The Latin allies (*socii* in Latin, hence Social War), save Venusia, stuck with Rome along with a mix of the other Italian allies. Some stayed neutral. The tribes which revolted formed an Italian confederacy with a representative council and a capital, Italia, located at the settlement of Corfinium (Corfinio) in the centre of the peninsula. Italia even issued its own money (Salmon 1982, 129).

Rome had the worst of it to begin with and offered the Roman citizenship to the Italian allies who had not rebelled and to those who would surrender (Keaveney 1987, 171). With the help of now more willing allies, Rome was able to turn the tide, and, by 87 BC, the last of the rebels made peace. The full citizenship was extended to all the Italian allies, that is all Italy south of the Po. Cisalpine Gaul was very much excluded from the arrangement, excepting the Latin *coloniae* there (Salmon 1982, 129-130). It was not until the 40s BC that the Roman citizenship was extended into Cisalpina and the area fully incorporated into Italy (Salmon 1982, 139; Pliny, *NH* 3.133-137).

In 32 BC, all of Italy swore an oath of allegiance to Octavian (Le Glay 1996, 166). Italians began to see themselves as sort of dual citizens, belonging both to their birthplace and to Rome (Le Glay 1996, 168). By 6 BC, the last of the Alpine tribes were brought to heel, and the boundary of Italy firmly established (Salmon 1982, 149). About AD 6, Augustus divided Italy, including Cisalpine Gaul, into eleven regions (Pliny, *NH* 3.46) which 'gave a definite political organization and administrative reality to all Italy' (Salmon 1982, 153).[3] It also meant that non-Roman Italians were able to participate more actively in politics as they could now vote in their regions instead of having to travel to Rome (Le Glay 1996, 219). Despite the civil wars that would trouble Italy throughout the first century BC and the difficulties in the Alps, it was after the conclusion of the Social War that Italy, with Rome, became truly unified, both politically and socially (see Keaveney 1987, 189-192).

2.2 The cities

2.2.1 Initial results

Much of Ptolemy's Italian coastline is comprised of cities. Therefore, in the analysis, the coastal data and the inland data will not necessarily be treated as separate groups. The Procrustes analysis of the coast will include both cities and terrain points that Ptolemy used to outline the peninsula. The stress analysis will include all cities, whether Ptolemy included them in the coast or the inland data.

The 234 identified cities on Ptolemy's map of Italy have an average longitude of 36.66° and an average latitude of 42.18°. The modern centroid is $(12.94°, 42.65°)$. Ptolemy's latitude is less than half of a degree from its modern equivalent. If we simply move his longitude 20° west following his assignment of 20° to London, Italy is over 3.7° too far east. The conversion is not as simple as that, of course, but it is interesting to see how quickly Ptolemy's longitudinal errors grow.

Initial Procrustes error[4] for Italy is 3.75 million km^2, reduced to 2.09 million

[3] See Laurence 1999, 162-176 for discussion on the historical, ethnic, and geographical context of these divisions.

[4] As a reminder, Procrustes error is

$$G(M,P) = \sum_{i=1}^{n} (x_{m_i} - x_{p_i})^2 + (y_{m_i} - y_{p_i})^2$$

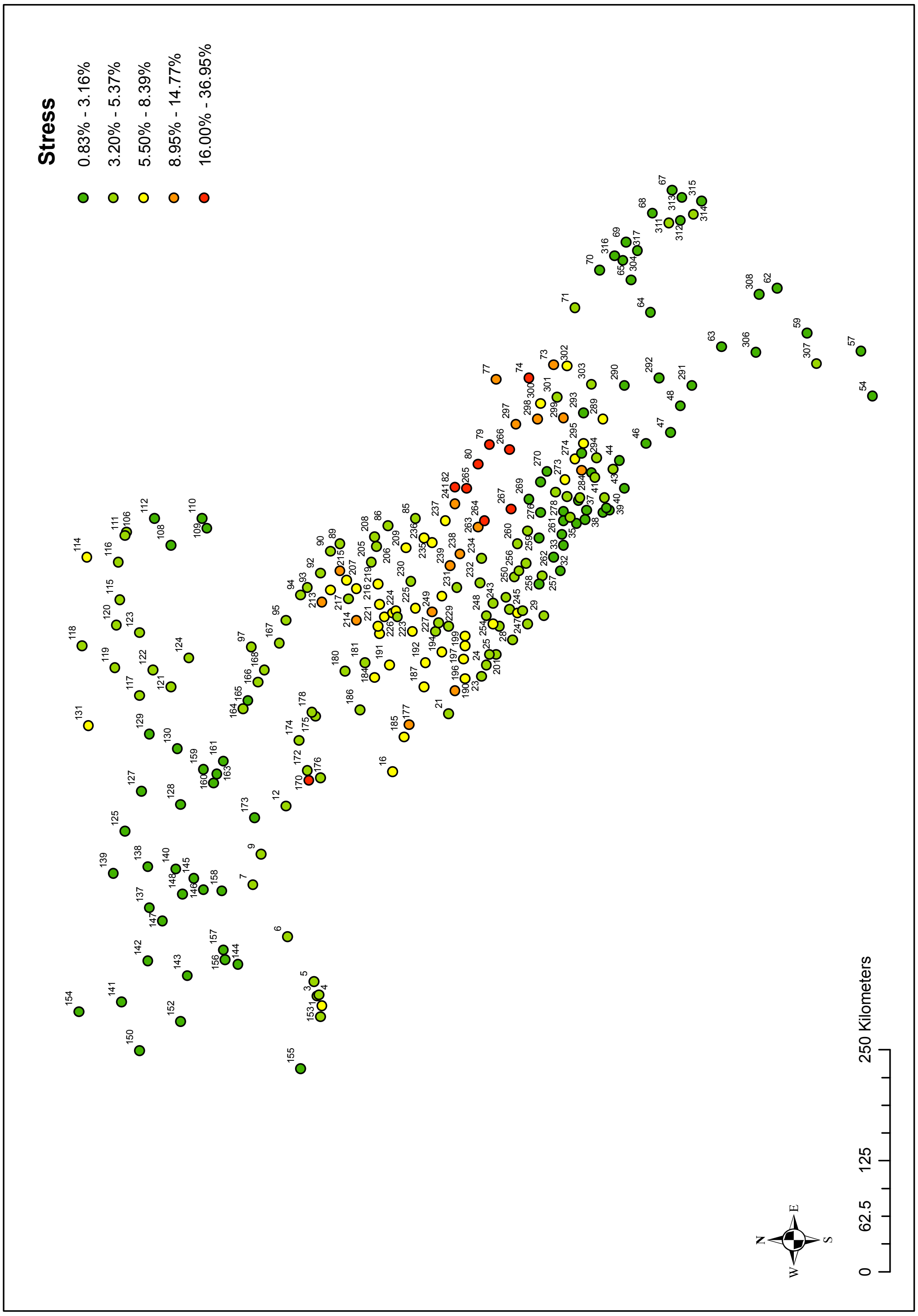

Figure 2.4: Italy: modern configuration with initial stress

after a 17° clockwise rotation. Stress[5] begins at 4.14%. The desired scaling calls for a 22% reduction in longitude but a 13% elongation in latitude. This produces a final stress of 3.13%. Mean individual stress is 4.96%, reduced to 3.48% after skewing.

The modern map of Italy (Figure 2.4) has a densely packed centre becoming sparser towards the peninsula's extremities. Ptolemy's configuration (Figure 2.5) still has a cluttered centre, but on the whole the cities are more equally distributed throughout the map. As the modern map's points spread out as they move away from the centre, they give a rough approximation of the familiar boot shape of the peninsula. On Ptolemy's map, though, the foot of the boot is not well defined. In the north, the points do fan out to the sides, but do not spread as far to the north as they should. After the transformation (Figure 2.7), Ptolemy's map is almost a straight band of evenly spaced points.

No clear groupings emerged from clustering the cities according to whether the above transformations either improved or reduced their values of stress. There are 55 cities whose stress increased as a result of the overall city transformations. Some came from the north, some the centre, others towards the west coast above the foot of the boot, and a fourth group from the heel of the boot. These groups are not exclusive. The areas from which they come contain many points that did improve. Of these areas, the heel and the north-east, where modern Italy meets Slovenia and Croatia, are the more distinct regions, with a substantial number of settlements taking on additional stress but few experiencing reductions. Stress overall for these 55 cities, though, starts quite low at 2.13% improving to 1.61% after the longitude is expanded by 18%, and the latitude is shrunk by 14%. Mean individual stress drops from 2.38% to 1.83%. Given that these points are spread across the entire length of the peninsula, with large empty spaces between groups, this stress is artificially low (as the total intercity distance increases enormously without a comparable increase in error) and is no cause for celebration.

Likewise, the 179 cities that improved do not form any distinct groups. Indeed this reduced map is barely distinguishable from Ptolemy's original. The north-west corner is missing, and the bottom is a bit more streamlined, but otherwise not much different. The modern map shows more obviously that the north has been thinned out. The transformed map, with the exception of the extreme south, is just a straight band of points that flares out ever so slightly at the top. At best, looking at the modern map, we can maybe say that a cohesive group exists in the centre and that further examination is

[5]Stress is

$$S = \sum_{i,j} \left(d_{ij} - \hat{d}_{ij} \right)^2 / \sum_{i,j} d_{ij}^2 .$$

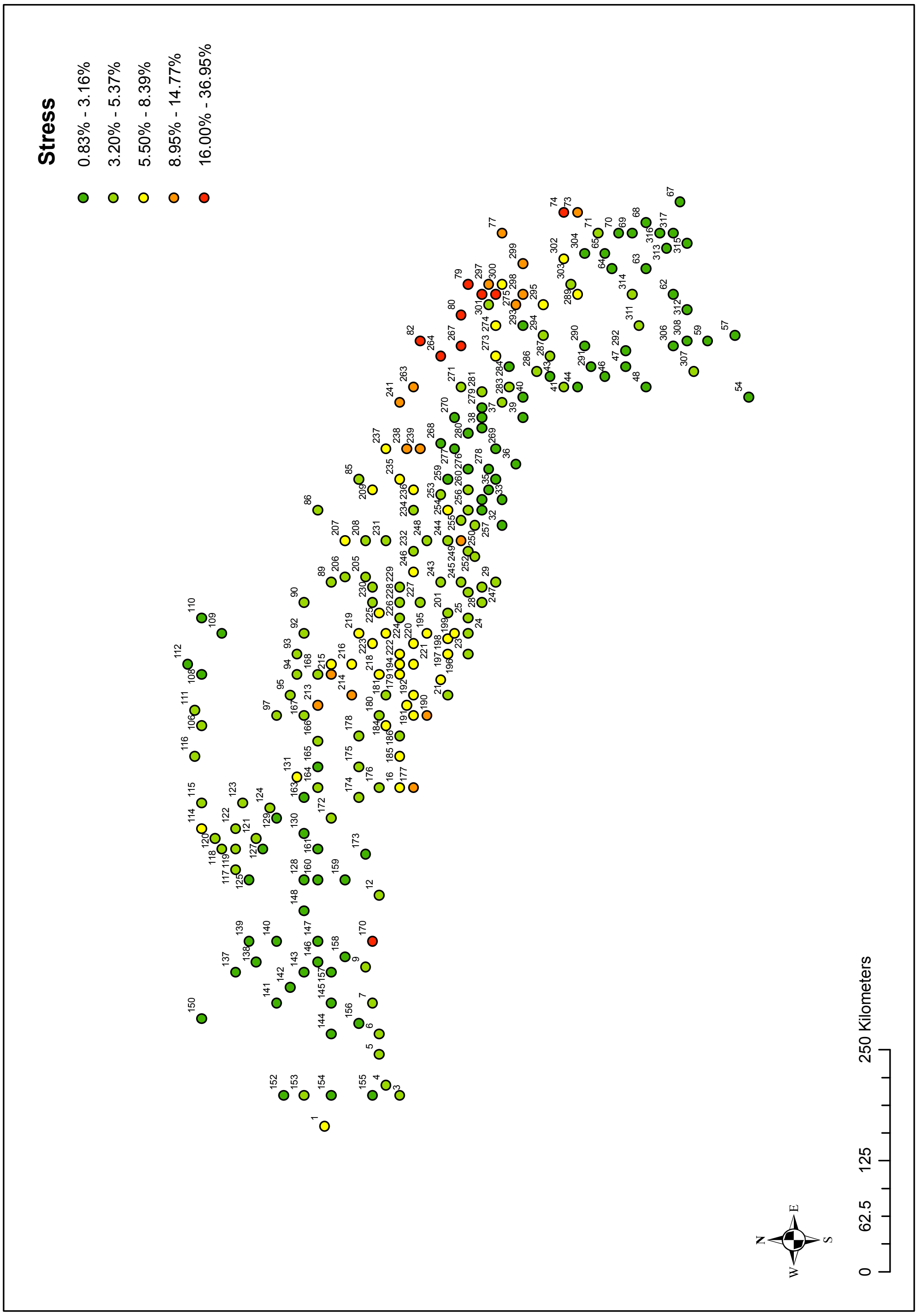

Figure 2.5: Italy: Ptolemy's configuration with initial stress

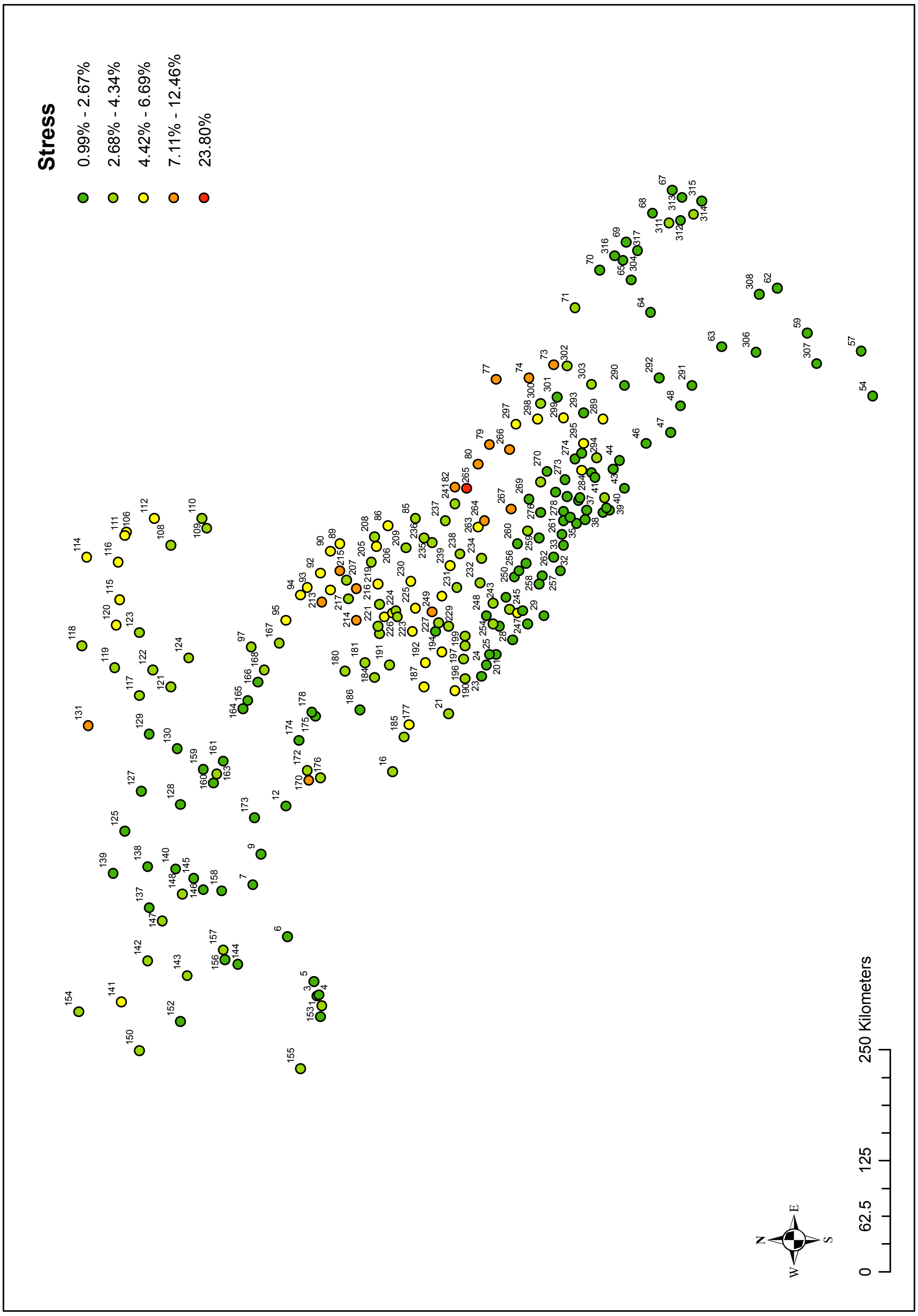

Figure 2.6: Italy: modern configuration with transformed stress

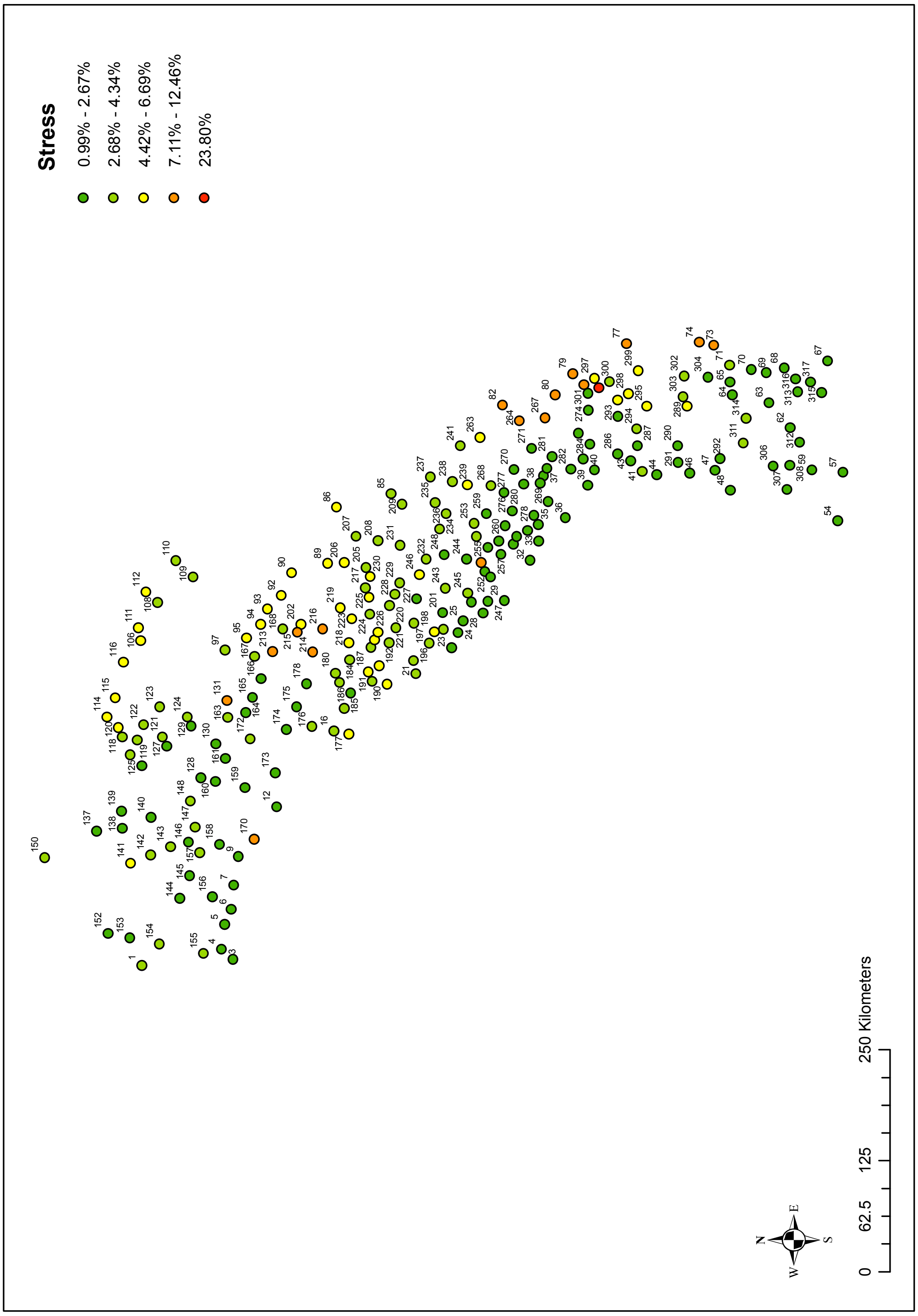

Figure 2.7: Italy: transformed configuration with transformed stress

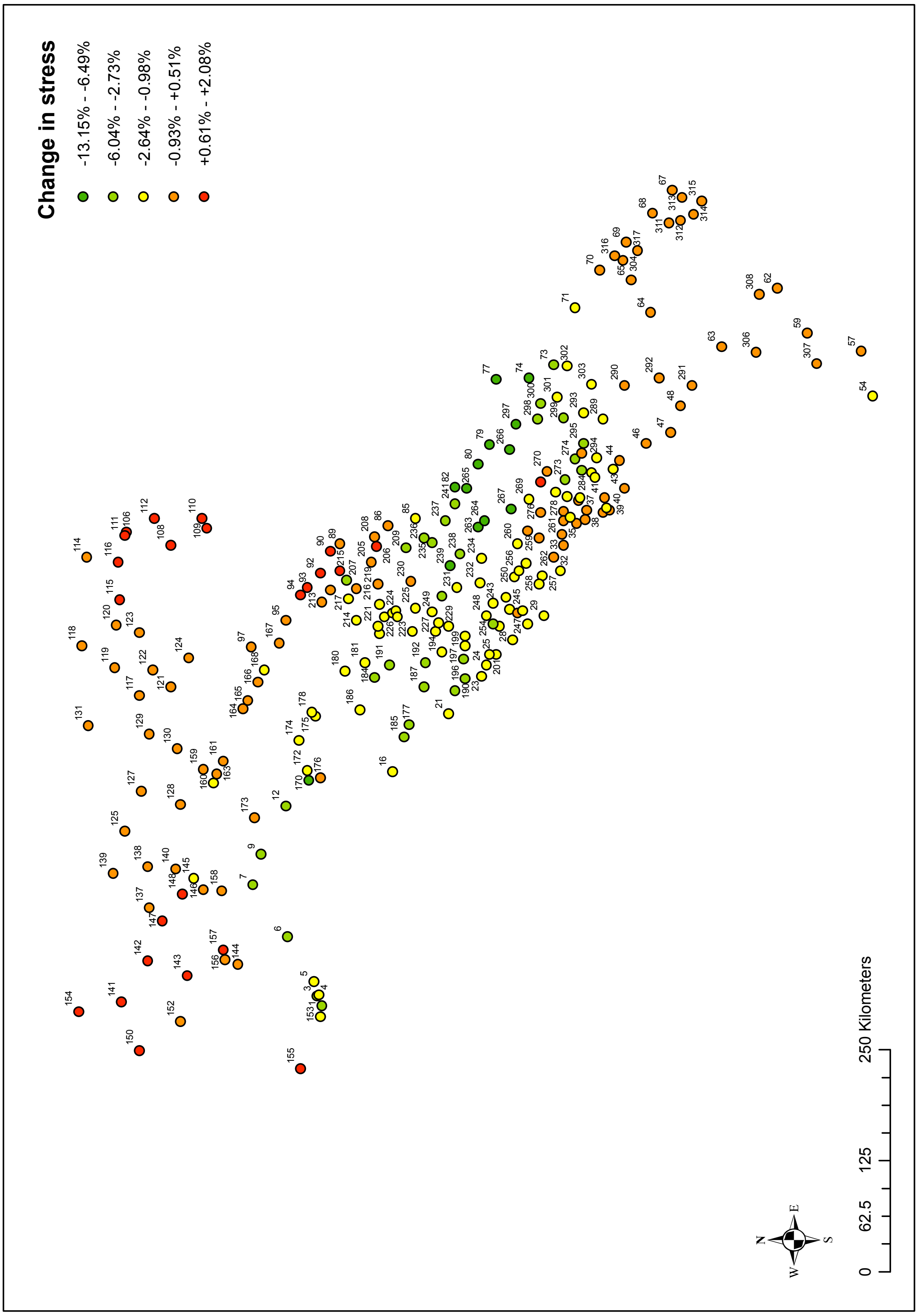

Figure 2.8: Italy: modern configuration with change in stress

needed on either end of that. Initial stress is 5.95%. Skewed scaling calls for a 37% reduction in longitude and an 18% increase in latitude, more than halving the stress to 2.64%. Individual mean stress also improved significantly, being reduced from 7.17% to 3.03%. In contrast with the previous group, this stress can be taken as a true measurement of the accuracy of Ptolemy's placements because of the even distribution of cities throughout the map.

Returning to the full map, we examine the distribution of the cities, and three obvious sections present themselves: the sparse north and south and the densely packed centre. These sections are no accident, but correspond to geographical and historical divisions of Italy. The northern area is the mainland portion of Italy down to the border of peninsular Italy formed of the Arno and Rubicon rivers (the Pisa-Ariminum line). This area was the last to fall to Rome, parts of which resisted into the reign of Augustus. The southern area was under Greek influence until the conclusion of the Pyrrhic War. The centre is the area of Rome's early conquests. In Figure 2.4 of initial stress, three areas of central Italy stand out. They conform roughly to Etruria and Umbria, Latium and Campania, and Samnium and the east coast. These may need to be examined in their own rights, but judgement will be postponed until central Italy is isolated from the north and south (see Figure 2.9).

2.2.2 Northern Italy

The north of Italy consists of 69 identified cities. Its southern boundary corresponds roughly to the lines of the Arno and Rubicon rivers, the traditional border between Cisalpine Gaul and Italy proper. The region requires a small rotation of 10° in the clockwise direction reducing the Procrustes error 12% from 472 thousand km^2 to 415 thousand km^2. Initial stress is 8.01%. Skewing reduces stress to 6.33% with a longitudinal increase of 5% and a latitudinal increase of 49%. Mean individual stress falls from 8.28% to 6.67%.

Immediately upon detachment from the rest of Italy, the stress of the northern cities shoots up. This is not solely due to cutting the total modern distances by reducing the number of points in question (234 to 69), but also to Italy's slender shape. At first, these cities simply needed to be more northerly than the cities in central and southern Italy. After their isolation, they have lost this advantage, and, in and amongst themselves, the errors in Ptolemy's placement results in a rather high stress.

On the modern map (Figure 2.10), there is a wide corridor extending up the middle, which is missing on Ptolemy's map (Figure 2.11). This corridor corresponds to the Apennines which then turn south-west to meet the Alps at the bottom corner of the map. Surprisingly the cities in the mountains from Senez to Luni are on the lower end of the spectrum in terms of stress. Pisa

Figure 2.9: Italy's divisions (base map ©Google 2010)

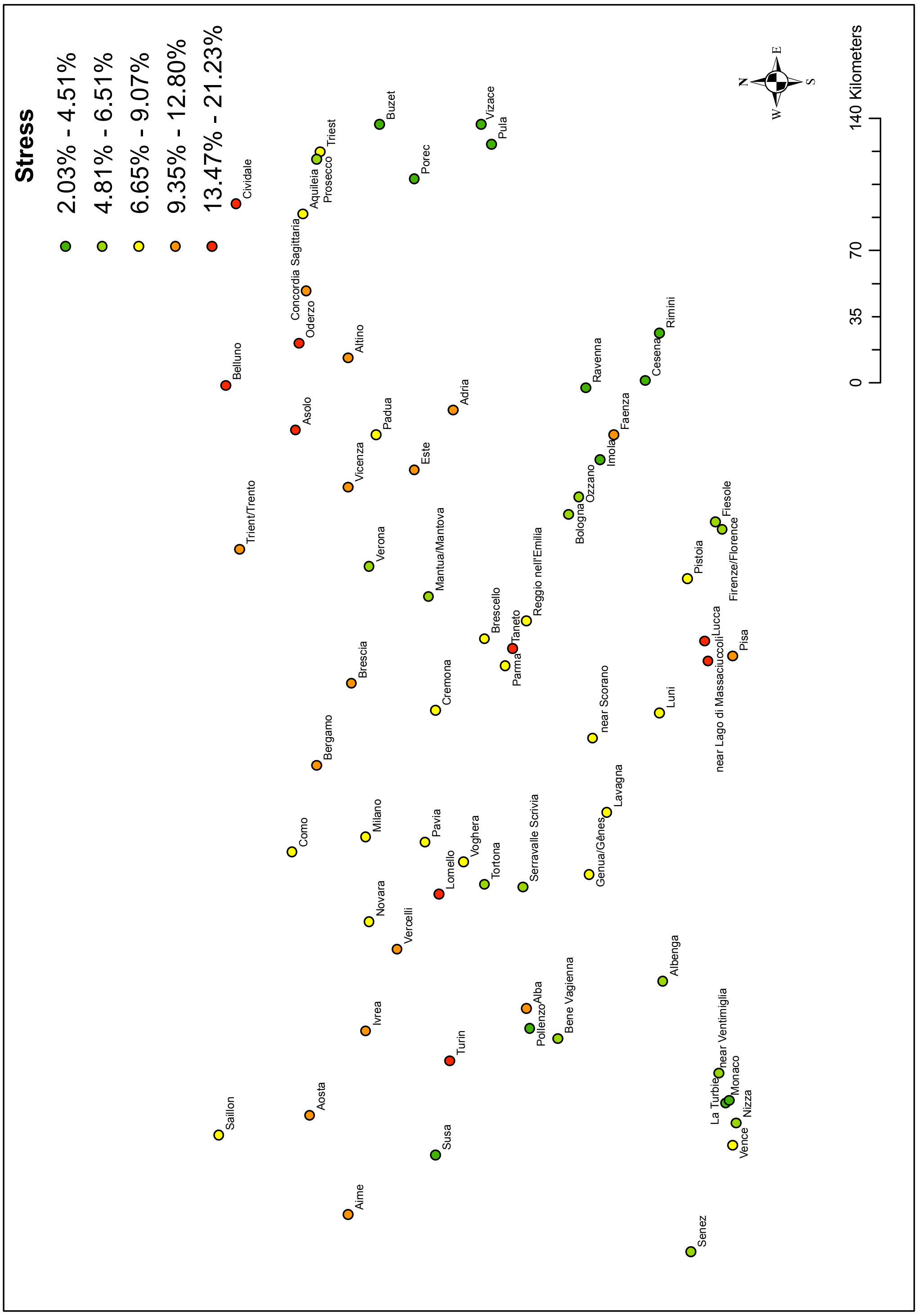

Figure 2.10: Northern Italy: modern configuration with initial stress

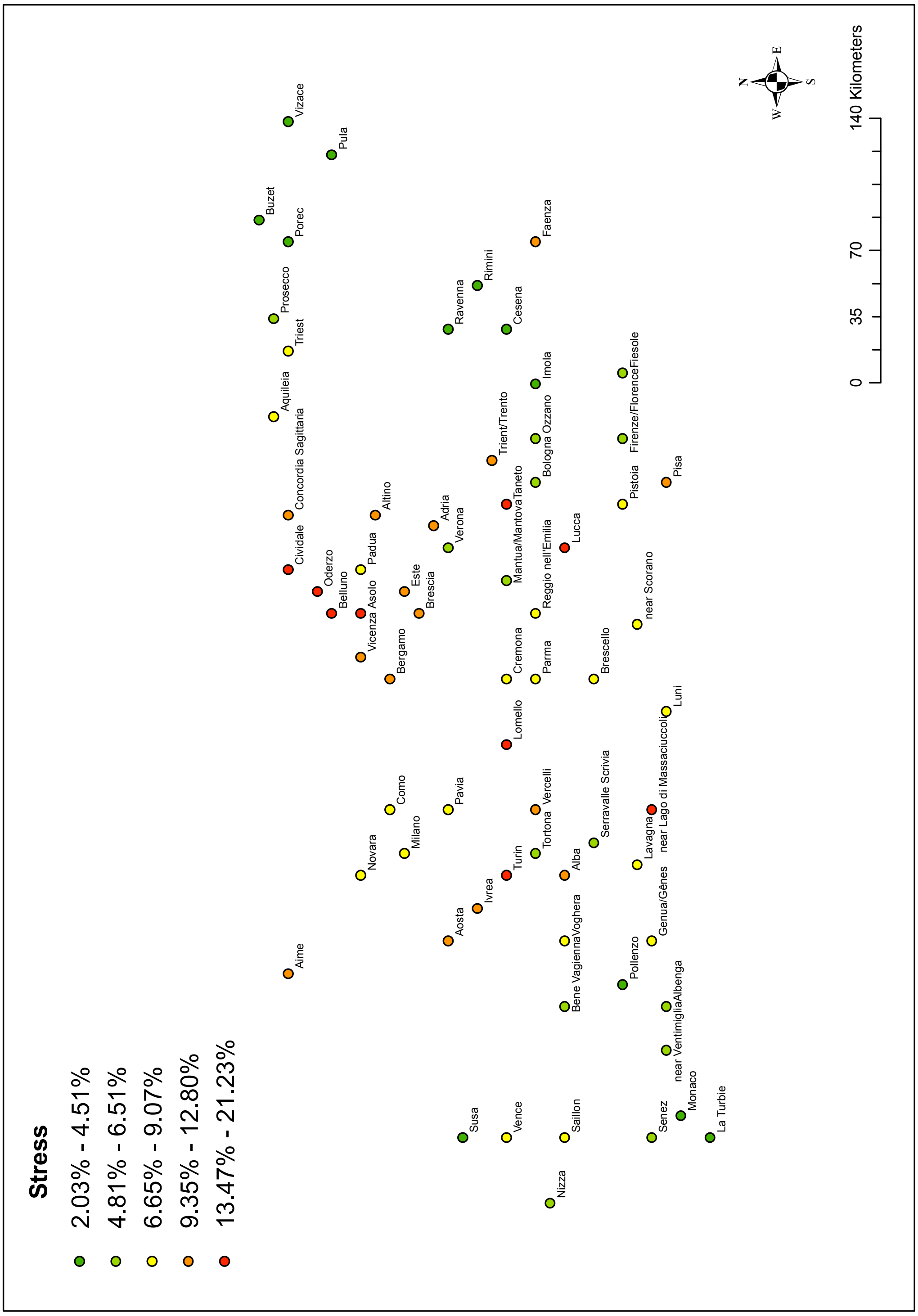

Figure 2.11: Northern Italy: Ptolemy's configuration with initial stress

and its two nearest neighbours are broken up significantly by Ptolemy, raising their error. The lowest stresses are observed on the north-east spur whose four cities are in modern Croatia. No readily apparent divisions appear to exist based on the initial stress, with varying stress levels spread throughout the area.

The Apennine corridor can be found more clearly opening between Fiesole and Faenza after Ptolemy's map is transformed (Figure 2.13). The Gulf of Venice has widened with the cities of Croatia maintaining their low stresses with marked improvements along the coastline leading to those cities. Stress gains and losses appear across the map (Figure 2.14). There is a small group between Asolo and Triest in the north-east that is consistent in its stress reduction, and the centre is dominated by cities falling in the middle tier of stress-change.

2.2.3 Southern Italy

Southern Italy corresponds approximately to the Greek territories gained as a result of the Pyrrhic War, measured roughly by the Paestum-Bari line. Included are 29 identified cities. Initial Procrustes error of 168 thousand km^2 is reduced to 125 thousand km^2 by an anti-clockwise rotation of 24°. Initial stress stands at 18.49%. A 31% longitudinal increase and a 53% latitudinal increase bring the error to 10.59%. Mean individual stress is reduced from 18.83% to 11.74%.

When we zoom in on southern Italy, the familiar foot shape is rather lacking in Ptolemy's map (Figure 2.16). There are quite large stress errors distributed throughout the region. Major errors seem to stem from a gross widening of the heel (Salento) and putting Alezio, Nardò, and Terranova da Sibari on the wrong sides of the Gulf of Taranto. The east coast of the heel contains the most consistently accurately placed cities from Otranto north to Bari.

After transformation, while the stresses do drop, the same problems remain (Figures 2.17 and 2.18). Salento is too wide and confusion remains as to which side certain cities on the Gulf of Taranto belong. The Gulf of Taranto itself, and, thereby, the foot shape of the south is still lacking its definition, though if Crotone were moved, the two distinct peninsulas of southern Italy would be readily visible. Still, there are great improvements across the board on individual stress errors.

On the modern map (Figure 2.19), one can clearly see a gap between the cities to the east and west. This corresponds to the southern Apennines, which occupy the western half of the region which in turn corresponds to the ancient regions of Lucania and Bruttium. A case can be made for dividing the south along these lines, and, indeed there are vast differences between the two

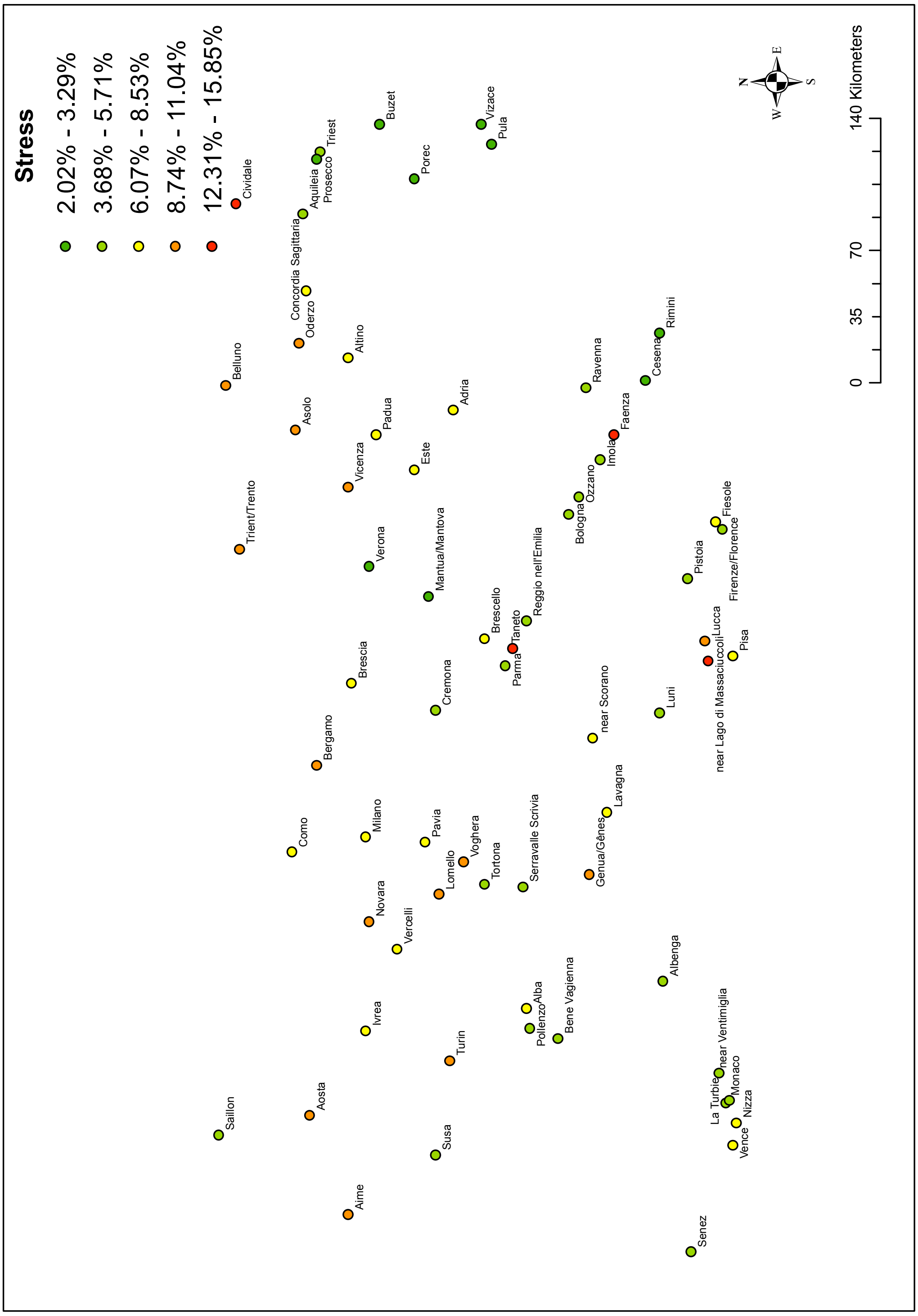

Figure 2.12: Northern Italy: modern configuration with transformed stress

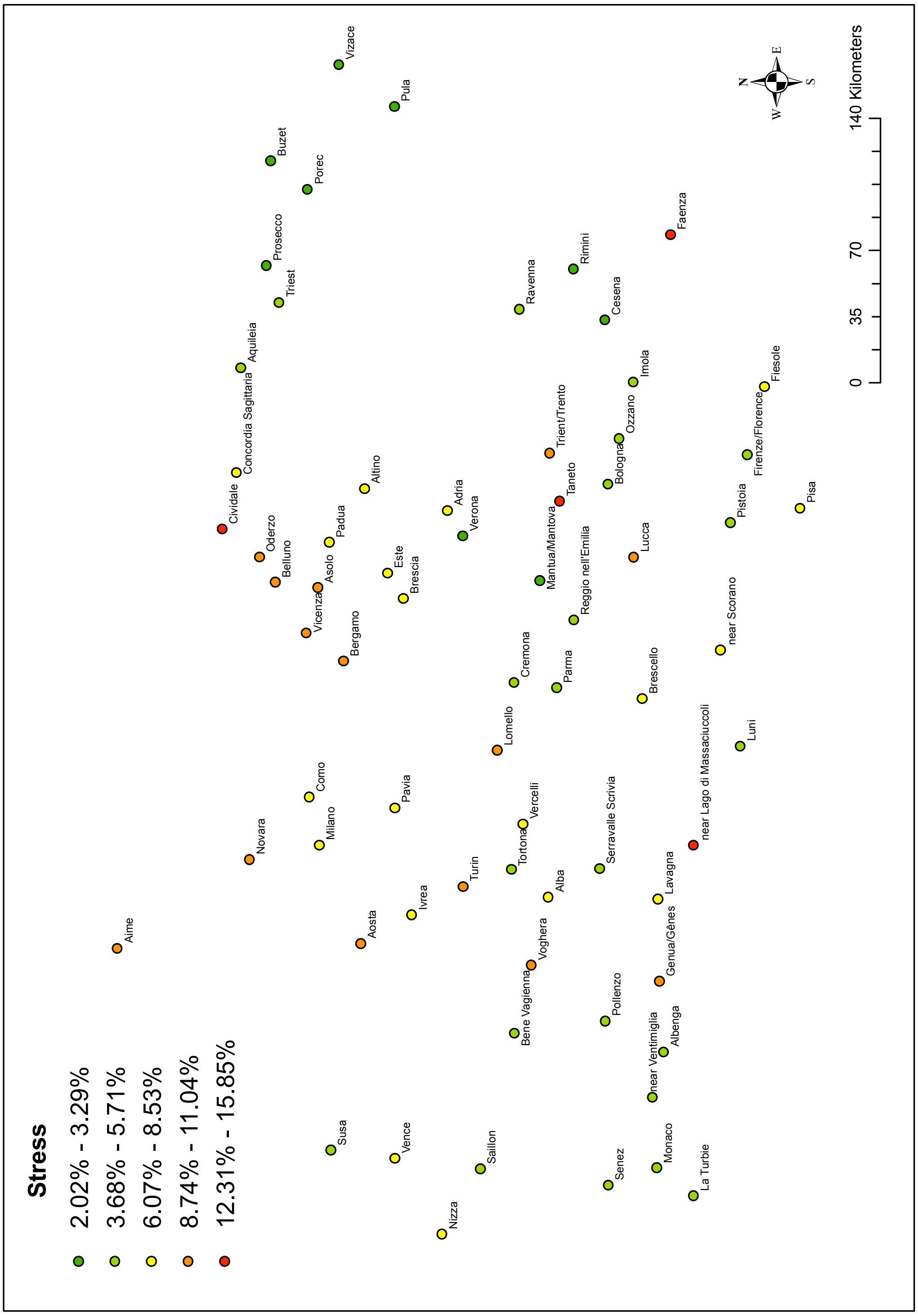

Figure 2.13: Northern Italy: transformed configuration with transformed stress

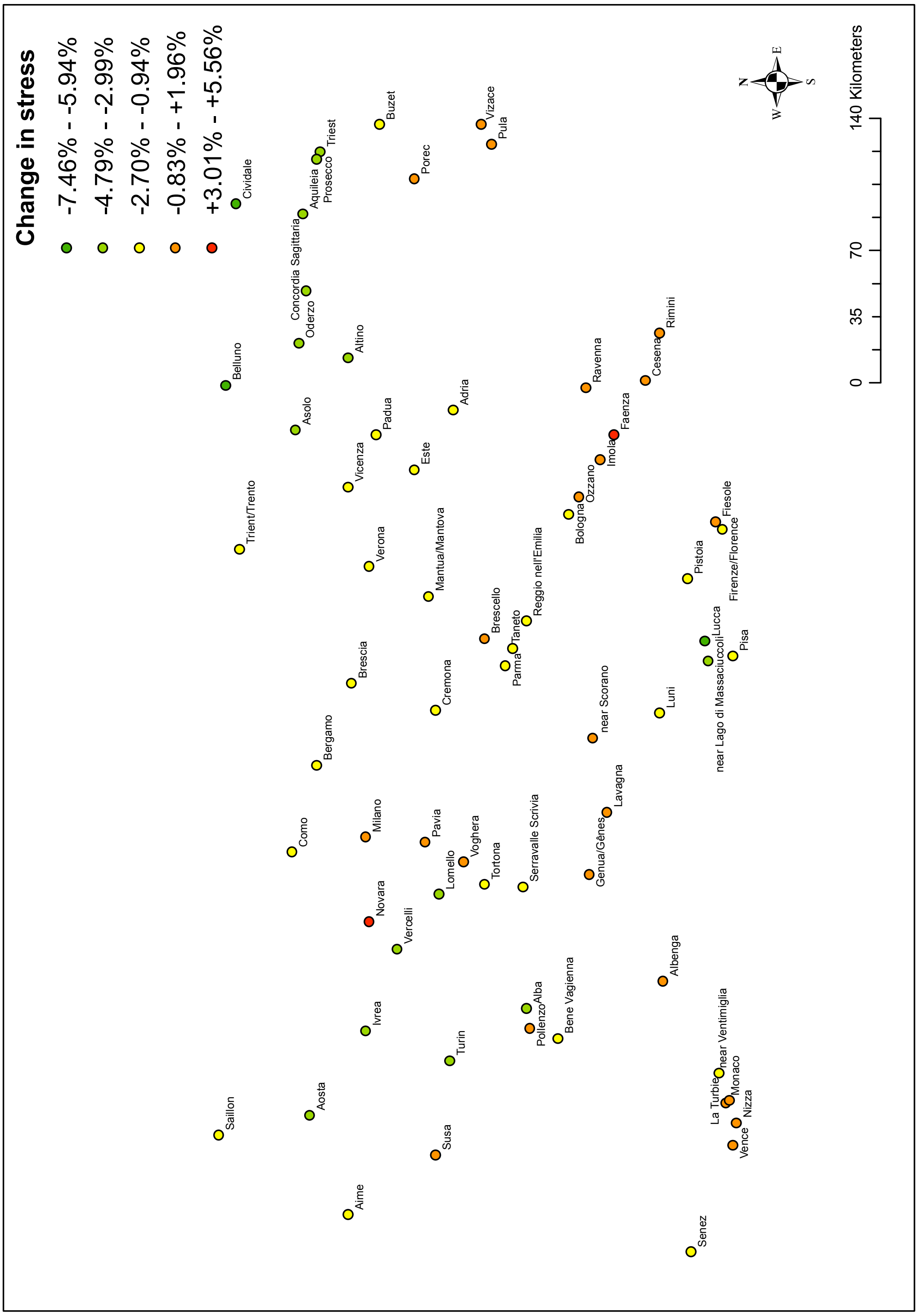

Figure 2.14: Northern Italy: modern configuration with change in stress

Figure 2.15: Southern Italy: modern configuration with initial stress

Figure 2.16: Southern Italy: Ptolemy's configuration with initial stress

Figure 2.17: Southern Italy: modern configuration with transformed stress

Figure 2.18: Southern Italy: transformed configuration with transformed stress

Figure 2.19: Southern Italy: modern configuration with change in stress

halves. The south-east, or heel, requires a 64° anti-clockwise rotation, a 2% longitudinal expansion, and a 14% latitudinal reduction to bring stress down from 30.36% to 29.86%. The south-west or toe, on the other hand, needs an 11° anti-clockwise rotation, a 9% longitudinal expansion, and a 59% latitudinal expansion to reduce its stress from 18.34% to 8.88%. The 15 cities of the heel have a final mean stress of 31.86%, while the 14 cities of the toe stand at 10.40% after transformation. The oddity is that the south-west is covered in mountains, and it is far more accurate than the relatively flat south-east. This could be indicative of source problems.

2.2.4 Central Italy

Central Italy includes Rome and its earliest neighbours and conquests. Included in central Italy are 136 known cities. Initial Procrustes error is 996 thousand km^2 which receives a 28% reduction after an 18° clockwise rotation. This brings the error to 715 thousand km^2. Stress begins at 16.94%. Minimum stress of 5.17% is achieved with a 40% reduction in longitude and a 7% reduction in latitude. Mean individual city stress drops from 18.22% to 5.73%.

Central Italy is roughly rhomboid and divided in two by the Apennine mountains. Strictly speaking, of course, the Apennine range is not a straight line neatly dividing cities into west- and east-central groups. Many of the central Italian cities are situated in the mountains themselves, so any division along the mountains will be greatly aided by what the numbers dictate.

There is little to give shape to this section of the peninsula. The modern cities (Figure 2.20) appear to be less densely packed than Ptolemy's (Figure 2.21). There is a wide corridor on the modern map through which part of the mountain chain passes. This is less pronounced on Ptolemy's map but still present. Ptolemy has most of his cities with high errors strictly to the east of this corridor when, in actuality, they line the corridor on both sides.

The lowest stresses are found in Campania and Latium. To the northern end, in Etruria, Umbria, Sabina, and Picenum, we find a large group of low- to mid-range stresses. The higher stresses, as noted above, occur in and around that mountain corridor and to the east. This corresponds roughly to Samnium and other areas brought under Rome's dominion during the Samnite Wars.

Stress is greatly reduced after the transformation, though the transformed map appears too narrow and stretched compared to the modern (Figures 2.22 and 2.23). The troubled area along the Apennine corridor is home to the greatest improvements, while Campania, previously home to the lowest stress, showed little to no improvement, naturally enough (Figure 2.24). The area to the north experienced moderate to low improvement. The errors are more spread throughout central Italy than in Ptolemy's original. Campania is now

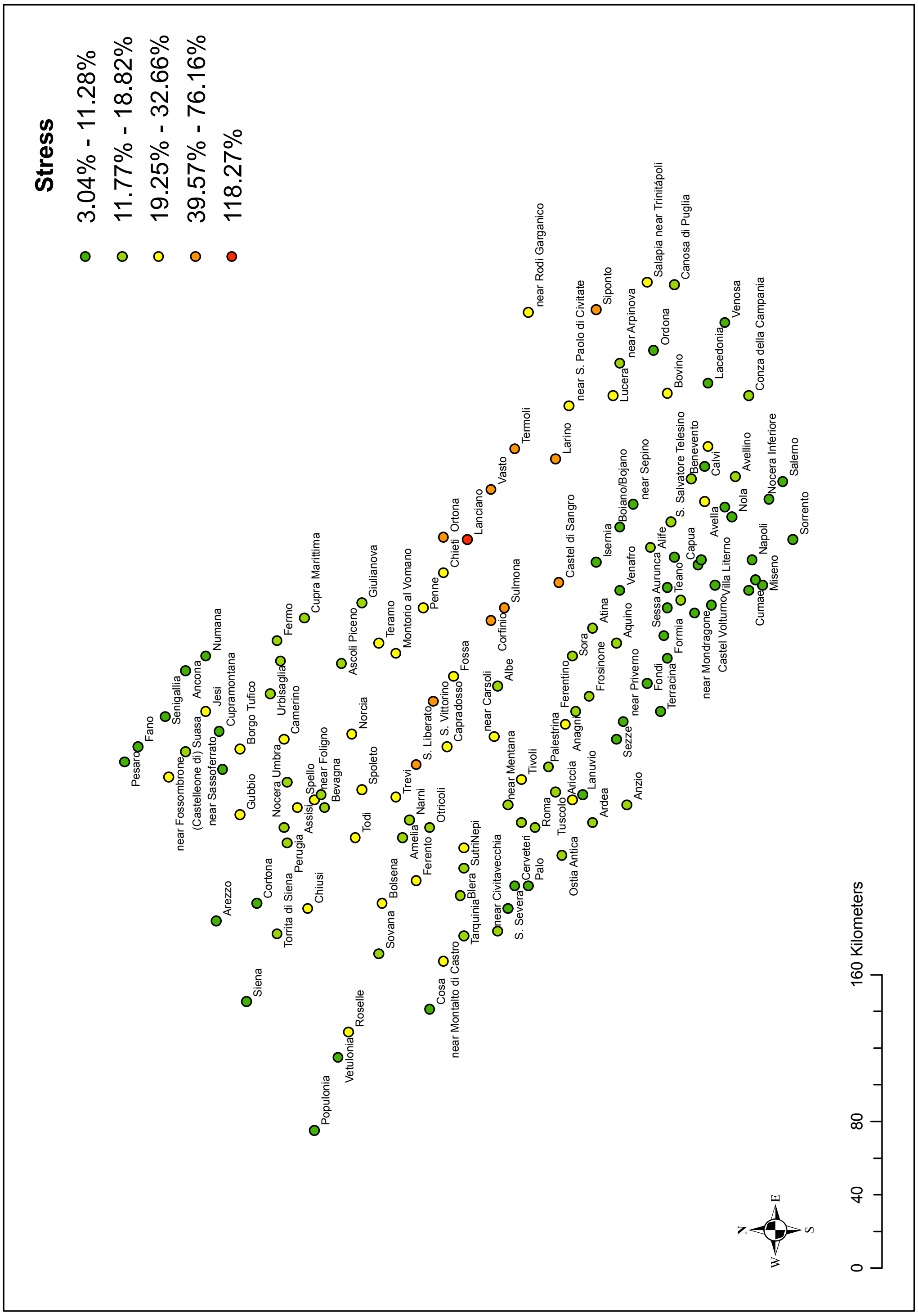

Figure 2.20: Central Italy: modern configuration with initial stress

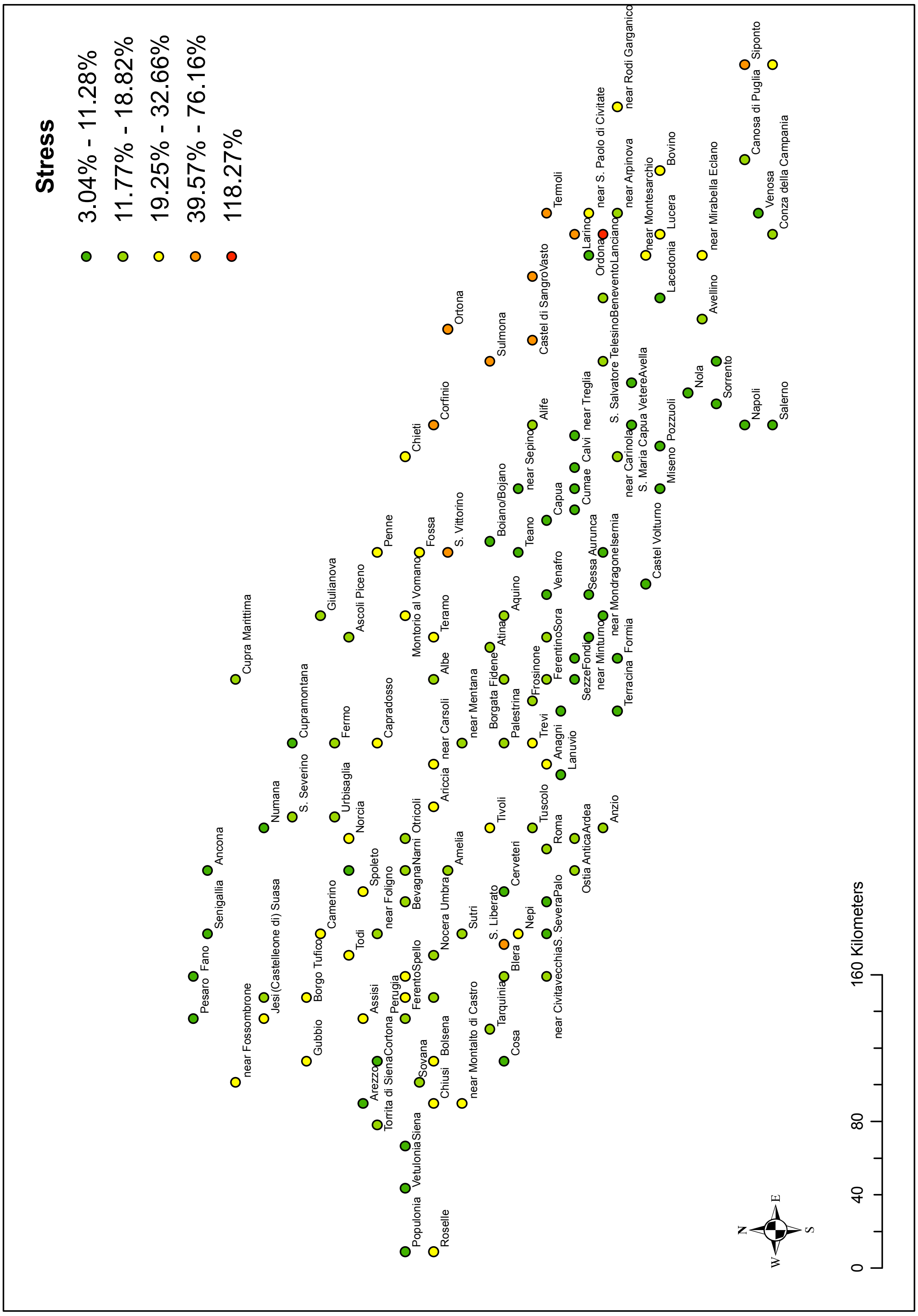

Figure 2.21: Central Italy: Ptolemy's configuration with initial stress

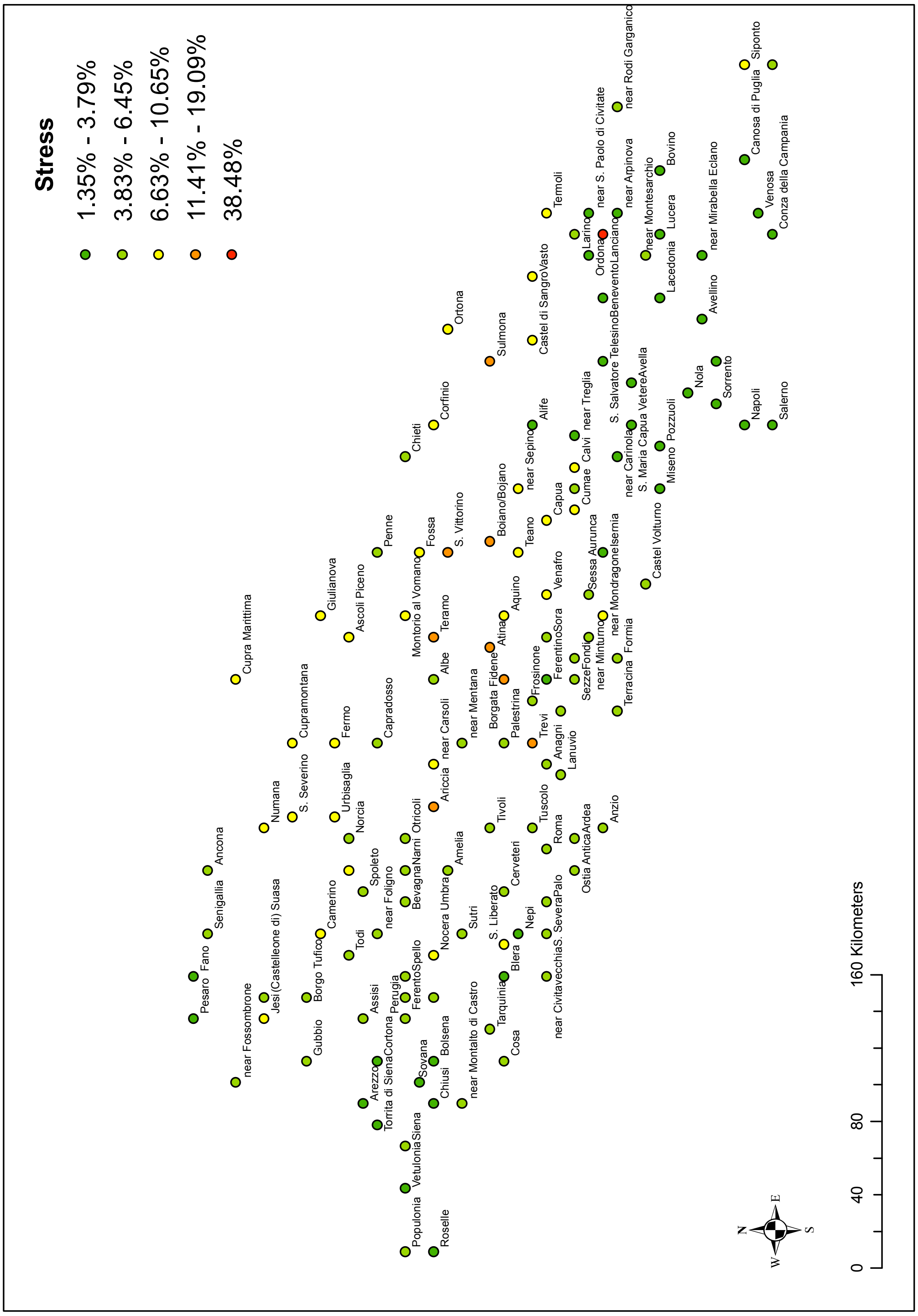

Figure 2.22: Central Italy: modern configuration with transformed stress

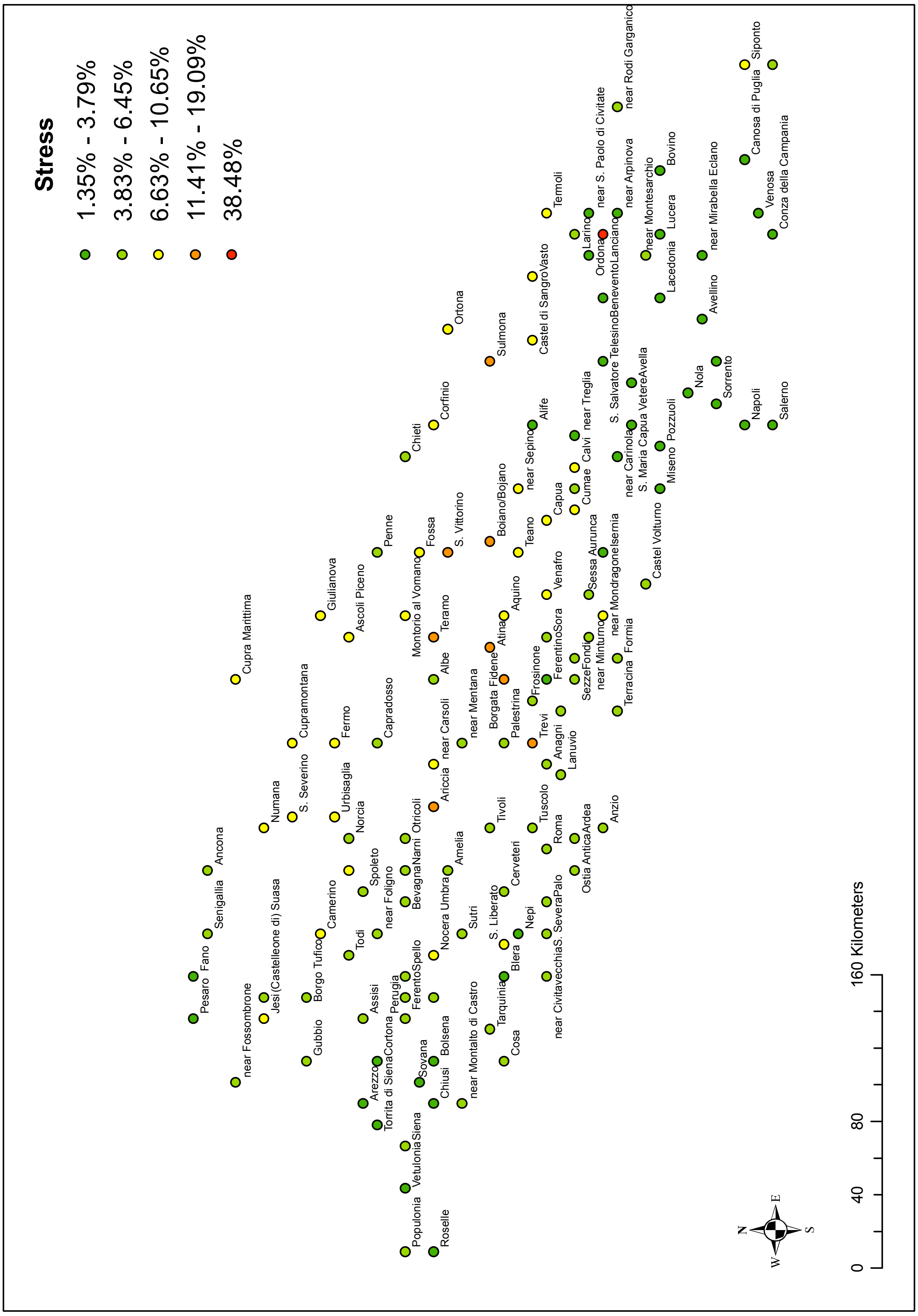

Figure 2.23: Central Italy: transformed configuration with transformed stress

dotted with moderate levels of stress (on the modern map) whereas Etruria and Umbria are quite low. The mountain area now has a range of stresses, but most points are middleweights.

Given the natural divisions in the geography of the area, the historical divisions, and the fact that the statistics indicate several regions with diverse error ranges reacting differently to the transformation, central Italy will now be split into three groups for further analysis (Figure 2.9). The regions are carved out of central Italy roughly along historical divisions such as those put in place by Augustus, but there is of necessity compromise with geographical features, physical proximity of cities, and trends in the error. The three areas to be examined are Etruria and points east, Latium and Campania, and Samnium and its surroundings to the south and east.

Etruria and east

The 56 cities that are roughly located in Etruria, Umbria, Sabina, and Picenum are included in this group. Initial Procrustes error of 165 thousand km^2 is reduced to 119 thousand km^2 by a clockwise rotation of 20°. Initial stress stands at 16.07%. Reduction of the longitude by 26% and expansion of the latitude by 2% minimise the stress at 10.86%. Mean individual stress is brought down to 12.10% from 18.18%.

With the exception of Trevi, there are no outstanding errors that catch the eye. It is clear that a rotation is needed to bring the Pesaro-Numana line into agreement with the modern map, but the hook shape along the southern border is present in both the Ptolemaic and modern maps (Figures 2.25 and 2.26). The cities between Populania and Cortona are bunched up on Ptolemy's map, though still retain relatively low stresses, though we notice that even the lowest stress bracket includes stresses over 11%. The main errors are centred around the misplacement of cities around Assisi (or the misplacement of Assisi), and in Ptolemy's easternmost points.

Etruria and its neighbours do indeed get their rotation, putting the Pesaro-Numana line about where it should be, except Numana itself, which is too far south (Figure 2.27). The Populania-Cortona area is spread out a bit, but not enough, and the southern boundary, even excepting Trevi, is far too much of a horseshoe shape than the hook or cane shape seen on the modern map. There are significant reductions in stress (Figure 2.29), but all on the upper end, with the bottom stress threshold dropping very little.

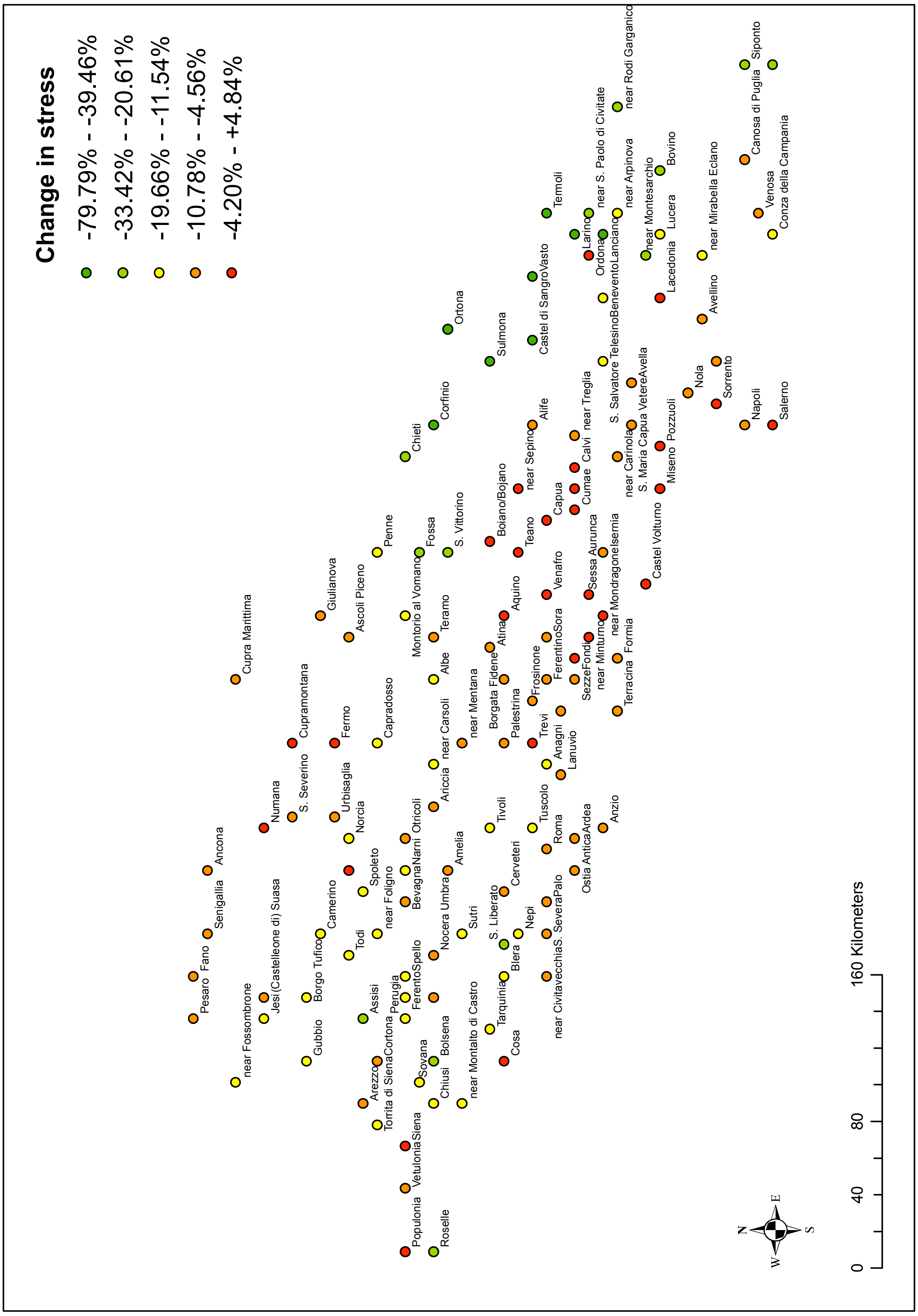

Figure 2.24: Central Italy: modern configuration with change in stress

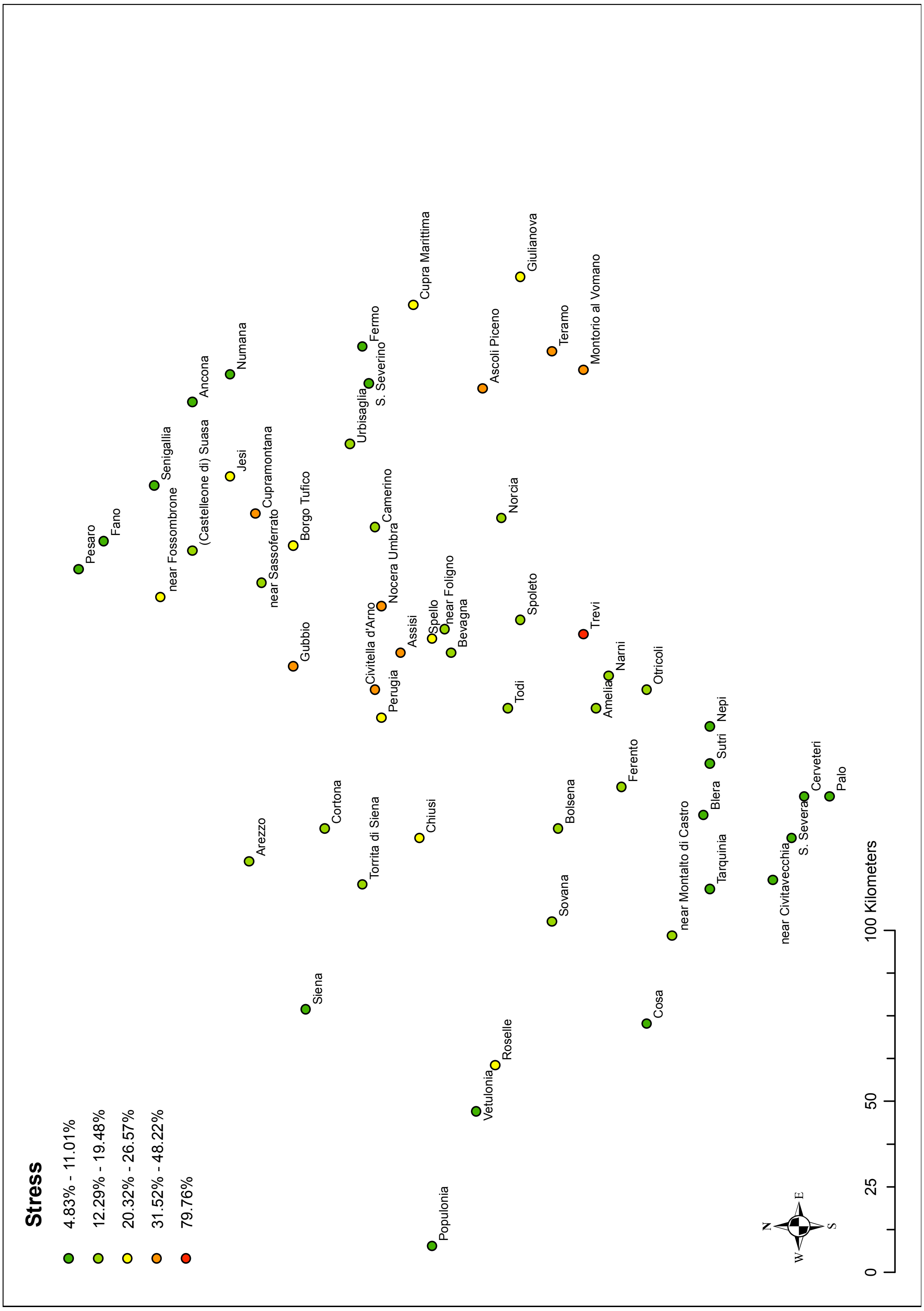

Figure 2.25: Etruria: modern configuration with initial stress

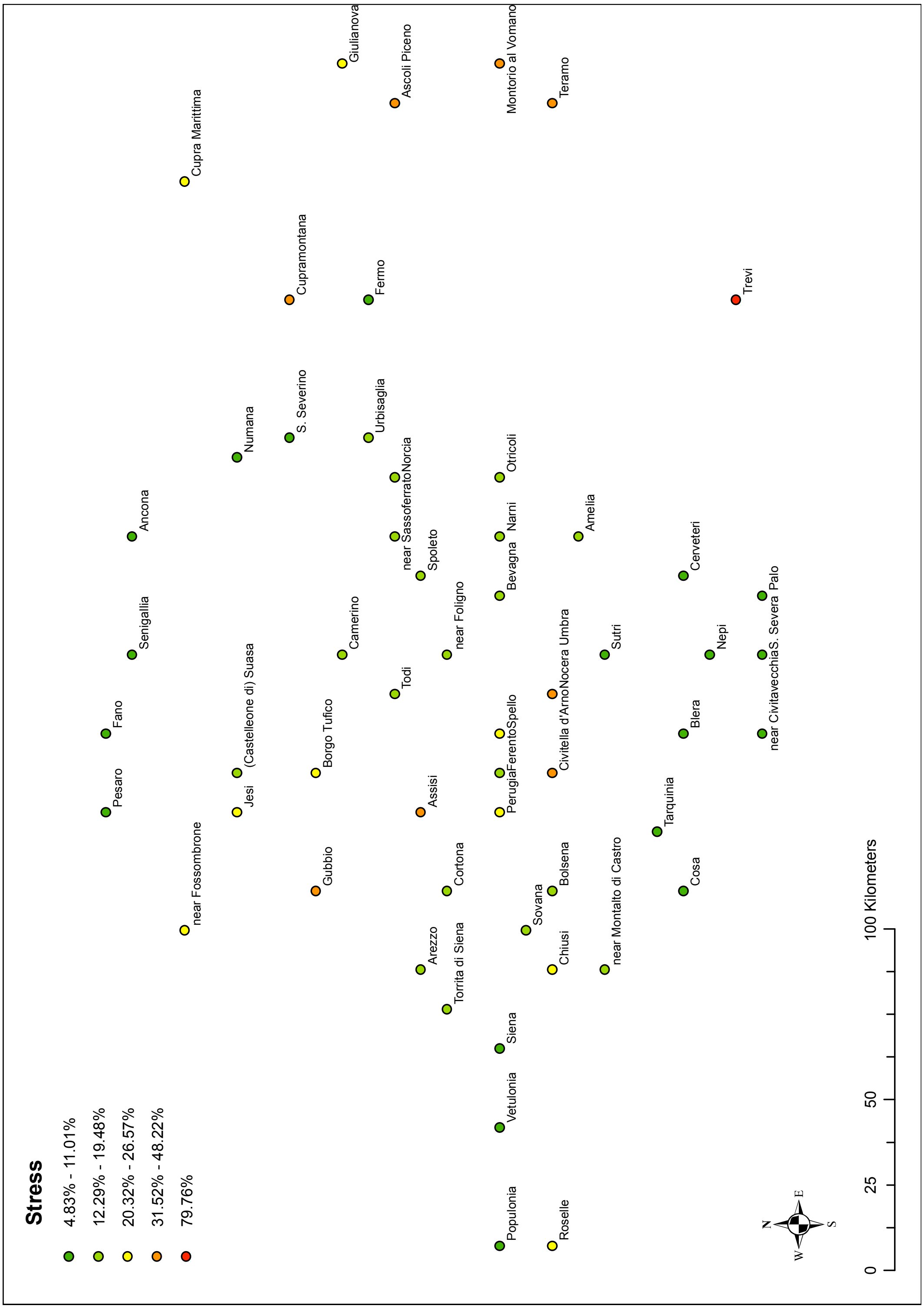

Figure 2.26: Etruria: Ptolemy's configuration with initial stress

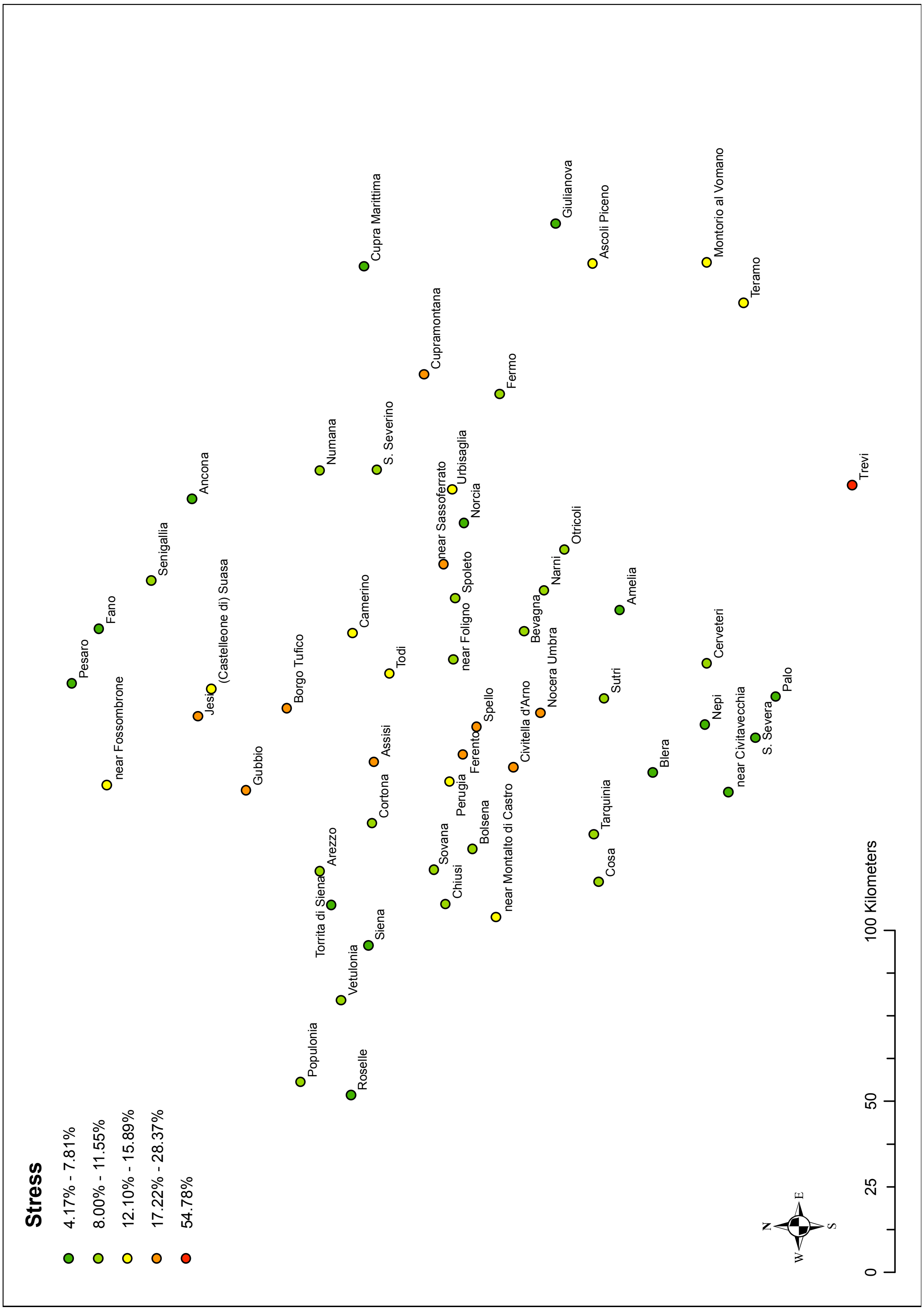

Figure 2.27: Etruria: Ptolemy's configuration with transformed stress

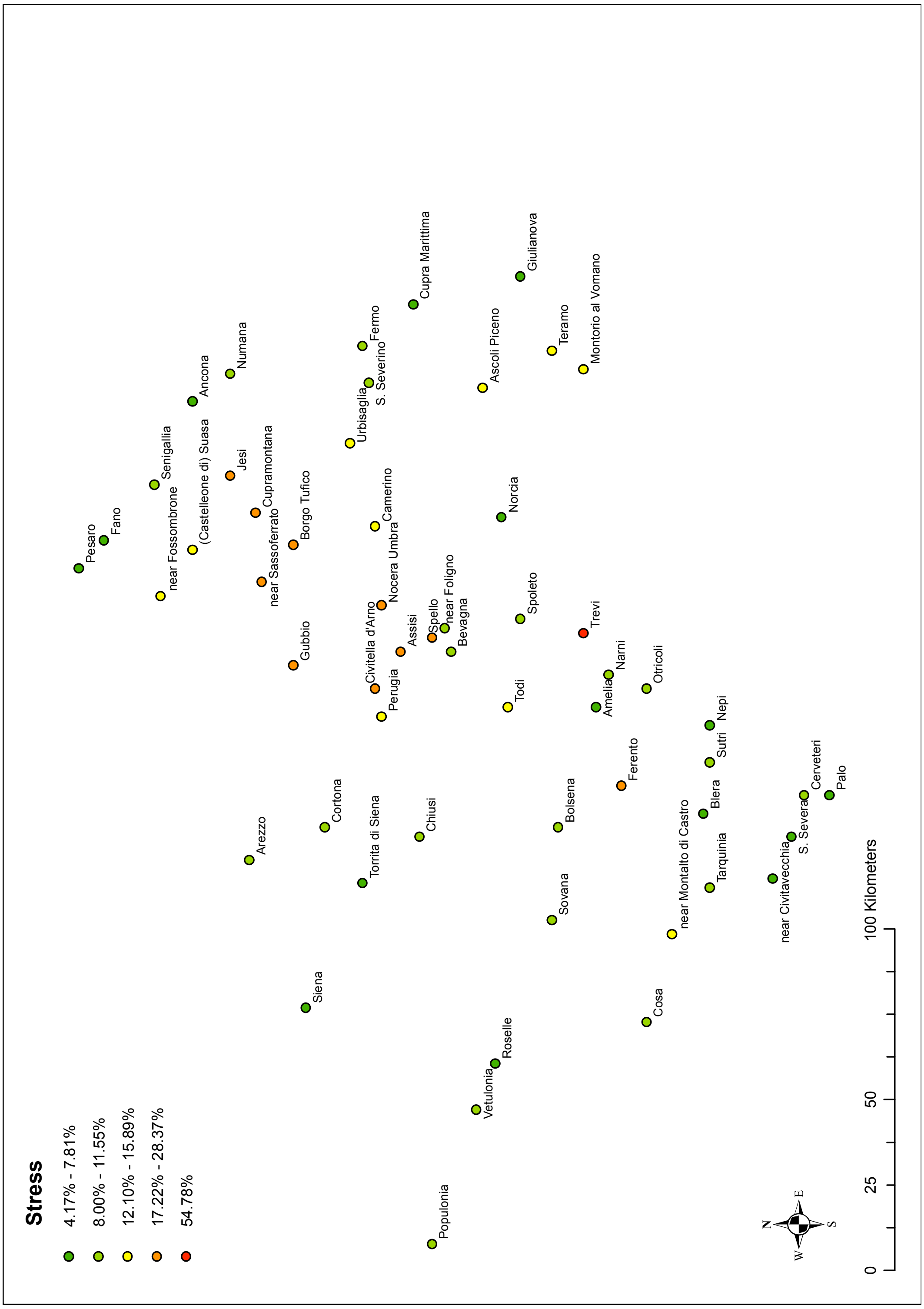

Figure 2.28: Etruria: modern configuration with transformed stress

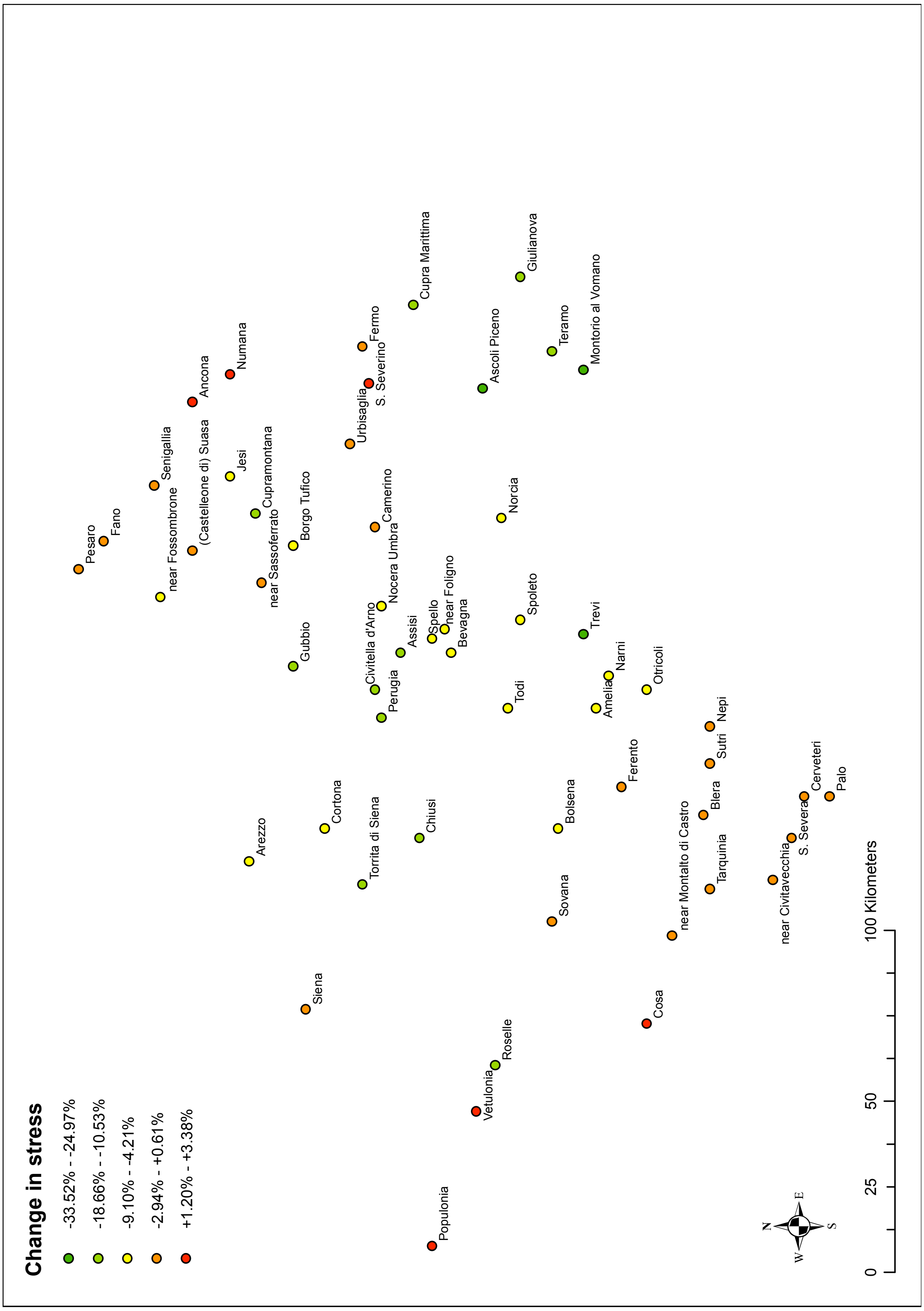

Figure 2.29: Etruria: modern configuration with change in stress

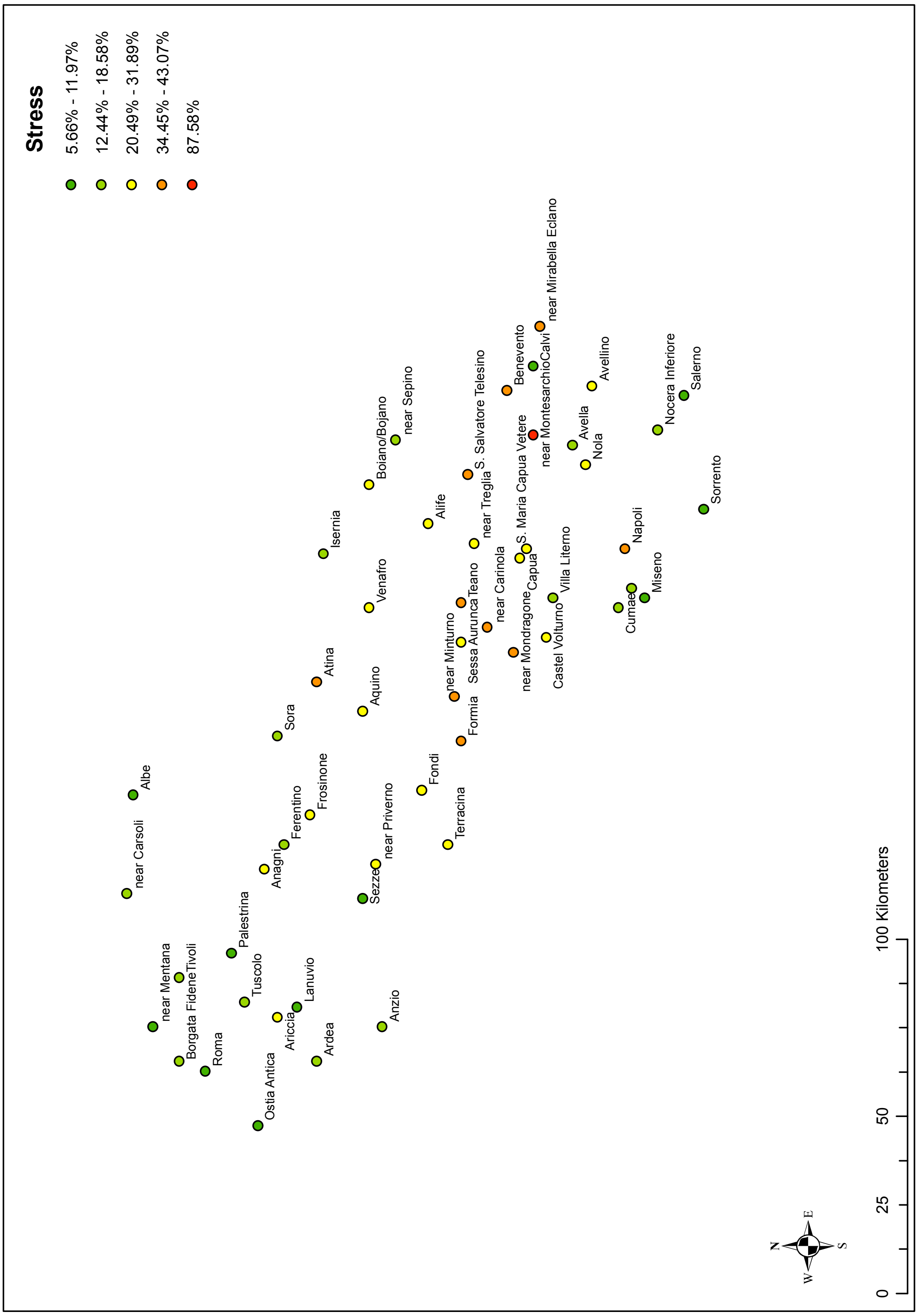

Figure 2.30: Latium and Campania: modern configuration with initial stress

Latium and Campania

Latium and Campania occupy the narrow plain between the Apennines and the west coast of Italy. Included in this group are some settlements of south-west Samnium because of their proximity to the cities of Campania as well as the long intertwined history of the two areas. There are 54 cities in all. An initial Procrustes error of 112 thousand km^2 is reduced by a clockwise rotation of 13° to 92 thousand km^2. Initial stress is 20.87%. For all practical purposes, the skewed scaling is uniform with a longitudinal reduction of 28% and a latitudinal reduction of 26%, which results in a drop of the stress to 8.05%. Individual stress as well plunges significantly upon transformation, from 22.98% to 8.95%.

Looking at the modern map (Figure 2.30), the heavy errors are grouped south-east of centre, in northern Campania and south-western Samnium. In Ptolemy's map, however, the error appears more evenly spread throughout the centre (Figure 2.31). On both maps, the area around Rome is the area of lowest stress. The area surrounding Naples seems most confused with the relative positions of Salerno, Sorrento, and Nocera Inferiore jumbled and Naples being nowhere near Cumae, Pozzuoli, and Miseno. Of course, those last three cities are also placed quite far apart from each other by Ptolemy. Yet, with the exception of Naples, those six cities are in the lowest two stress brackets. And with the lowest two brackets extending to over 18%, the inter-distance data of Ptolemy's cities is clearly quite erroneous.

After the transformation has been made, there is a great reduction in individual stresses across the board. With the exception of Montesarchio, all of the highest stresses are in the centre of the transformed map, which is essentially northern Campania (Figure 2.33). The Naples-Sorrento areas still have many out of place settlements, but, again, have comparatively low stress. Indeed, on the transformed map (Figure 2.32), southern Campania almost rivals northern Latium for low stress. Looking at how the individual stresses change in each city after the transformation (Figure 2.34), we can really appreciate how inconsistent the data in this area must be. Some cities shed over 60% while others gained almost 10%. This is not a region where simple rotations and scalings can account for the misplacements.

Samnium and its surroundings

The Samnium group consists of 26 known cities contained in the historic district of Samnium and the northern part of Apulia. Initial Procrustes error is reduced from 98.7 thousand km^2 to 98.0 thousand km^2 by a 3° anti-clockwise rotation. Initial stress of 28.78%. Scaling calls for a 48% reduction of the lon-

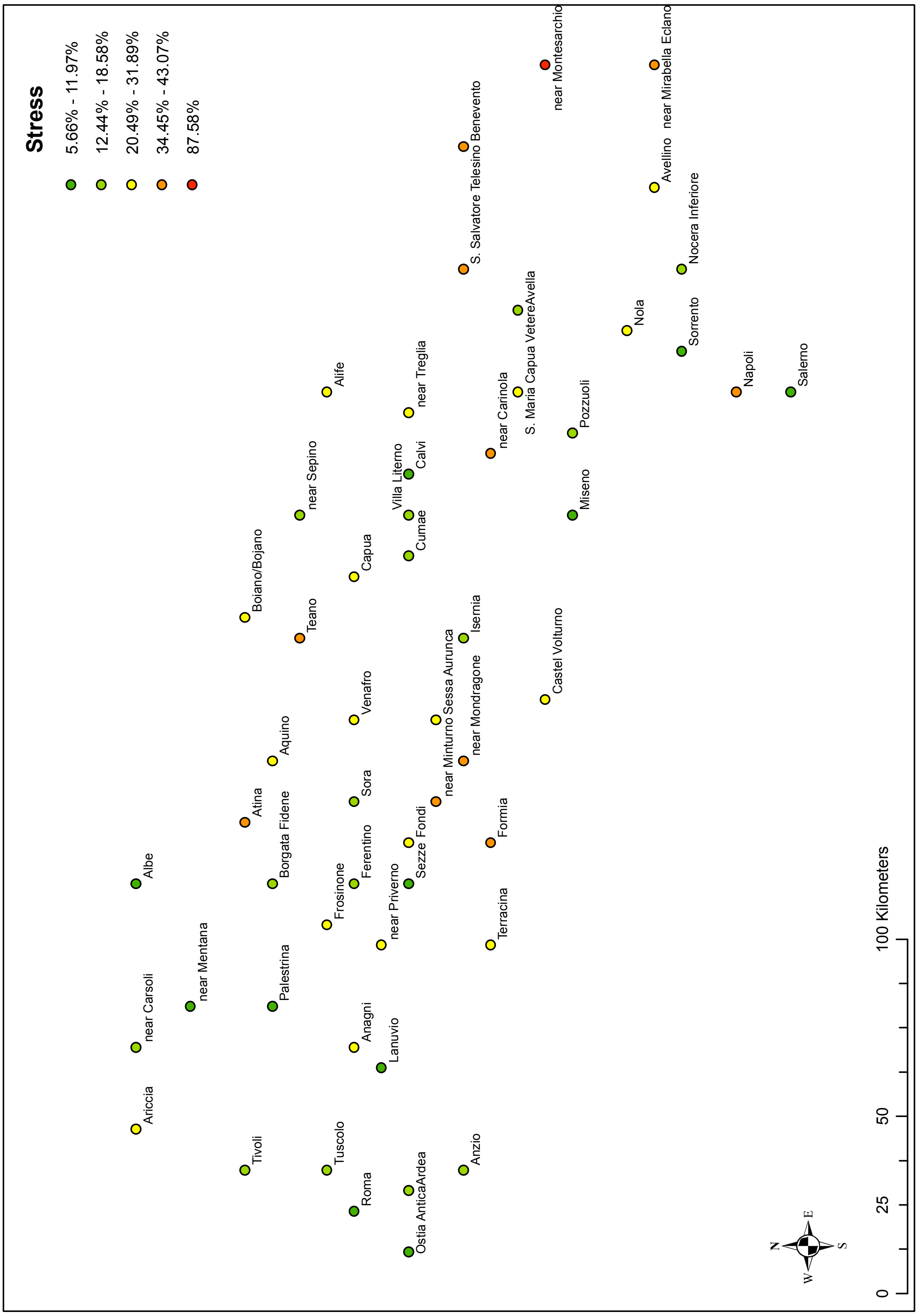

Figure 2.31: Latium and Campania: Ptolemy's configuration with initial stress

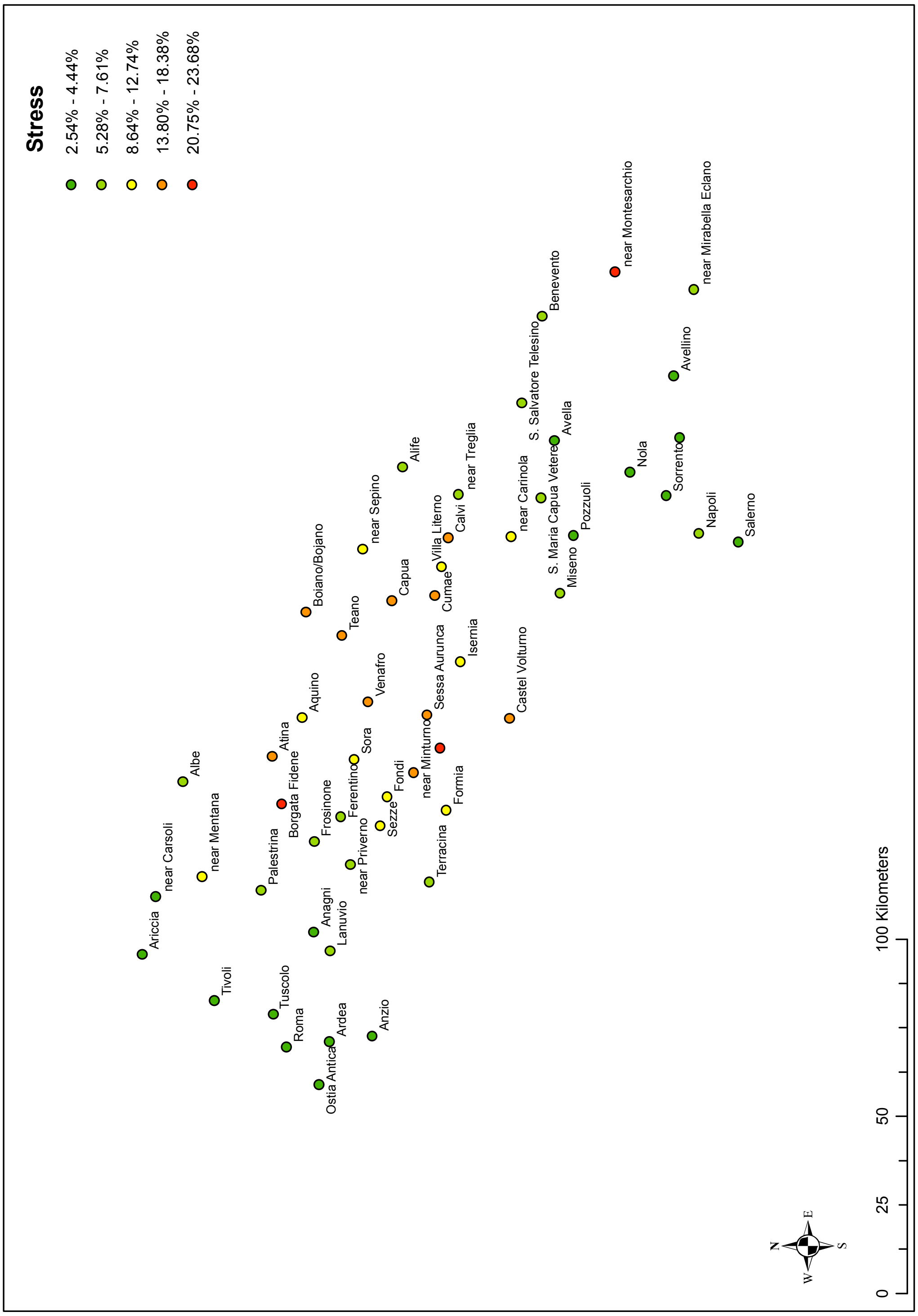

Figure 2.32: Latium and Campania: transformed configuration with transformed stress

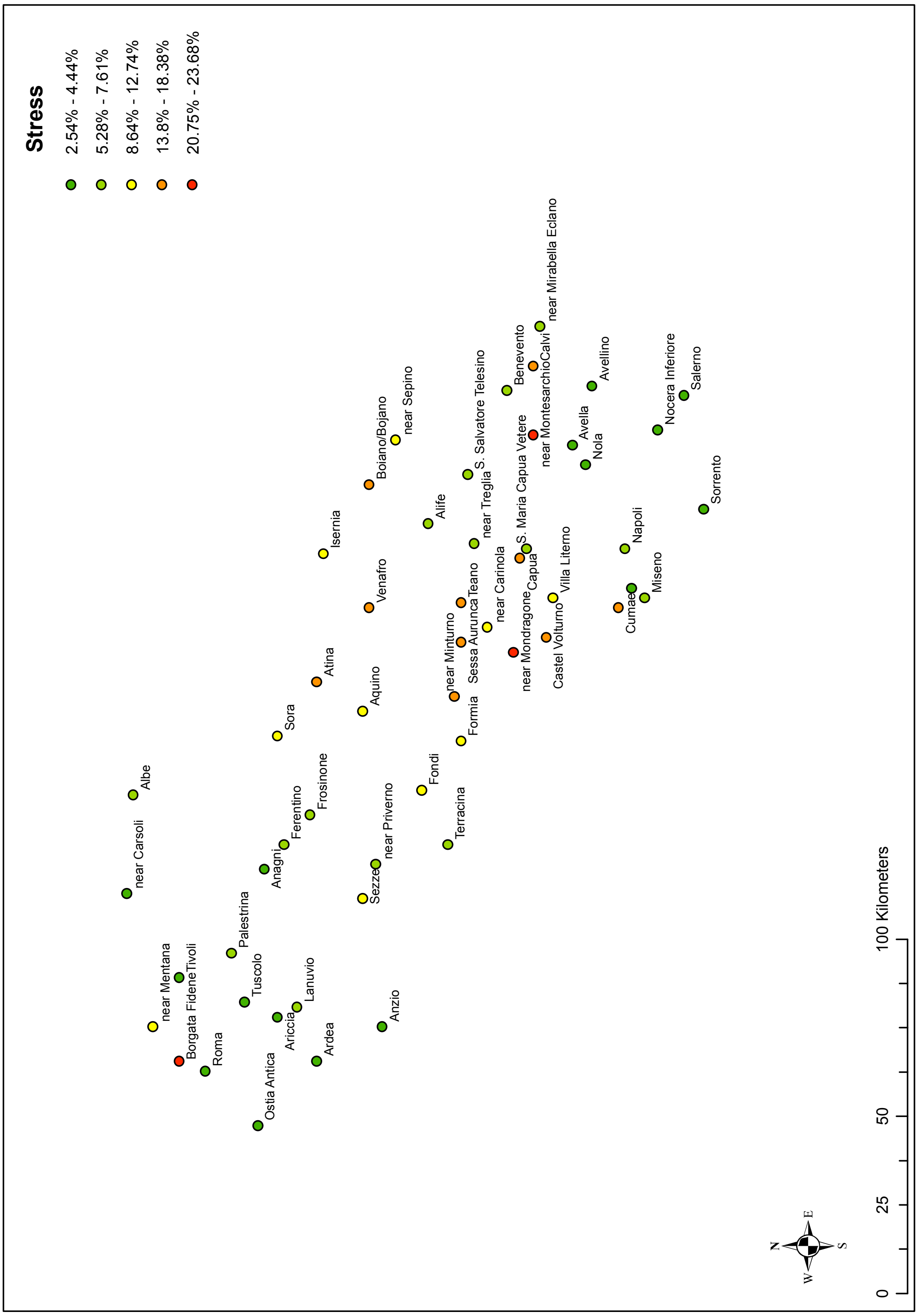

Figure 2.33: Latium and Campania: modern configuration with transformed stress

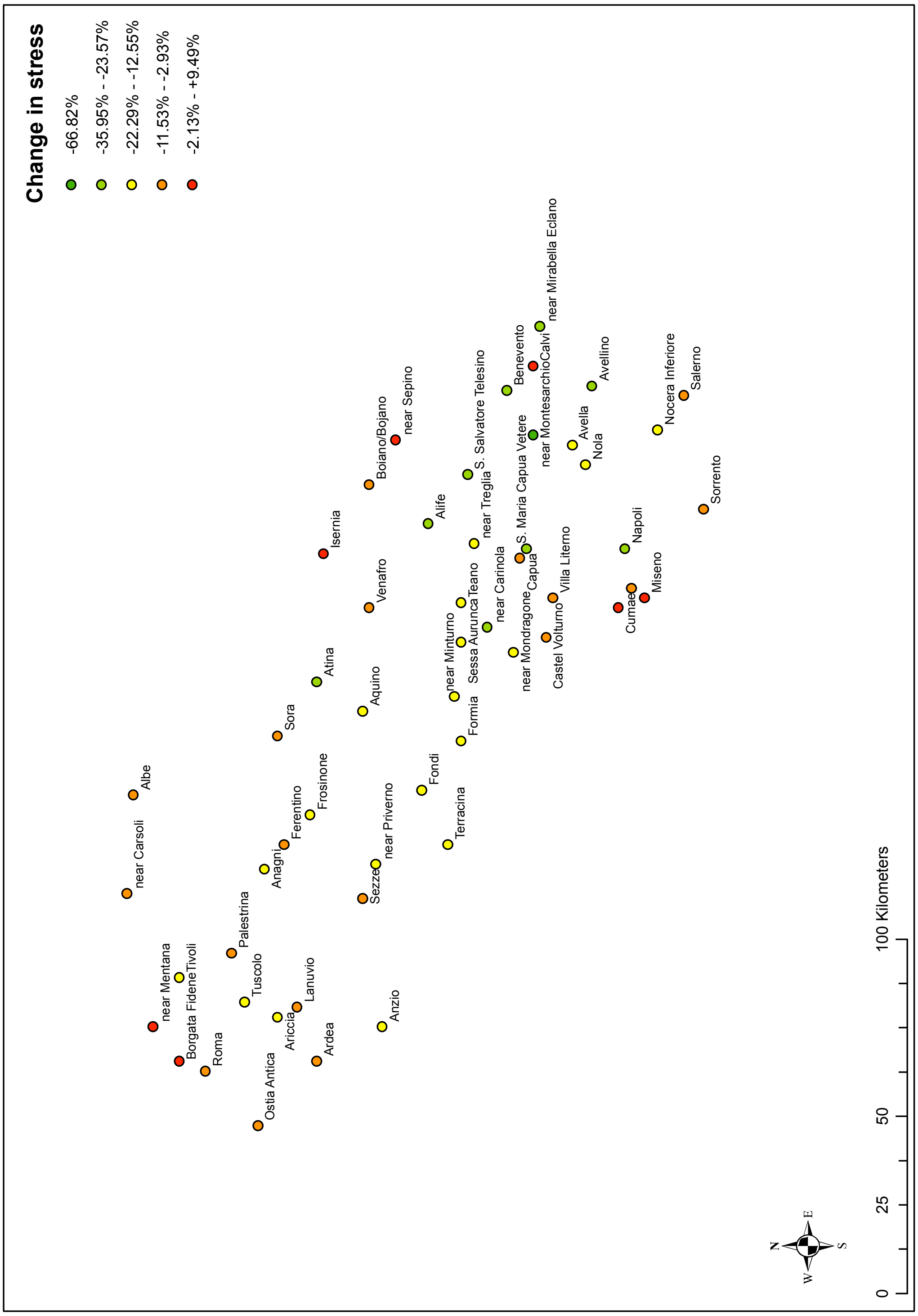

Figure 2.34: Latium and Campania: modern configuration with change in stress

gitude and a 13% expansion of the latitude to minimise the stress at 8.33%. The individual cities also greatly benefit from this transformation with their mean error dropping from 28.44% to 9.36%.

Individual errors are very high in Samnium and its surrounding neighbours. Ptolemy densely packs his cities around Lanciano in the south-east (Figure 2.36). Unfortunately Lanciano should be north of centre and the cities spread out fairly evenly throughout the area (Figure 2.35). S. Liberato is also far south of where it should be. The errors are rampant and distributed throughout. Even the cities in the lowest stress bracket seem to be placed precariously. The need for a longitudinal scaling can be seen along the whole length of the region.

The transformation does much to lower individual city stresses (Figures 2.37 and 2.38). There is great improvement across the board as both the upper and lower bounds of error drop significantly. S. Liberato's faulty placement was corrected to the extend that its stress dropped over 75 percentage points. Further correction was not possible without the rest of the cities contracting too much. Most improvement was seen in the centre with less gains made in the southeast (Figure 2.39). As in Etruria, though, it seems that the mountains have made it very difficult to judge relative placements and distances between cities. Even mile marked roads are little avail in determining straight-line distances through the passes of the Apennines.

2.3 The coast

2.3.1 Initial results

A modern Italian coast, reduced to the 90 points identified with Ptolemy's map, has an average longitude of 13.77° and latitude of 41.95°. Ptolemy's centre is $(37.73°, 41.56°)$. His latitude, being within $4/10^{\text{ths}}$, of a degree is excellent. To give an idea of longitude, Rome is approximately 12.61° east of Greenwich (≈London), whereas Ptolemy measures 16.67°. It appears that his longitude is expanding as he moves across the globe.[6]

After mutual centring, Italy's coastal error is 1.4 million km^2. A clockwise rotation of 12.8° makes the error drop to 946 thousand km^2. In the scaling, the longitude is reduced by 18% and the latitude remains unchanged. The skewing results in a final Procrustes error of 783 thousand km^2, a reduction of 42% from the original error. The original mean error of each coastal point is 107.50 km, reduced to 80.97 km after transformation.

[6]See graph in Ptolemy 2006, 47, for examples of how Ptolemy's longitudinal distances increase as we move east across his world map.

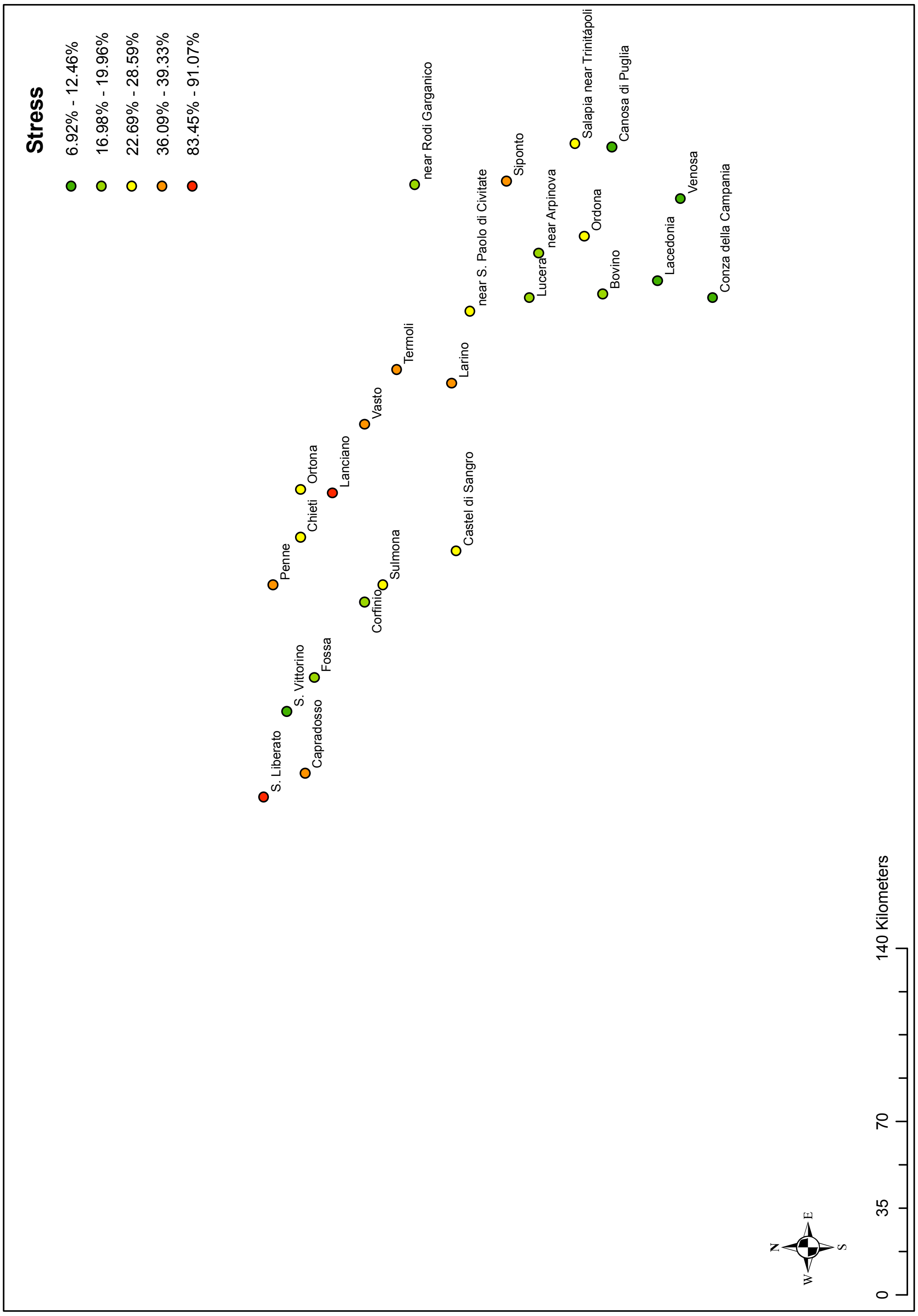

Figure 2.35: Samnium: modern configuration with initial stress

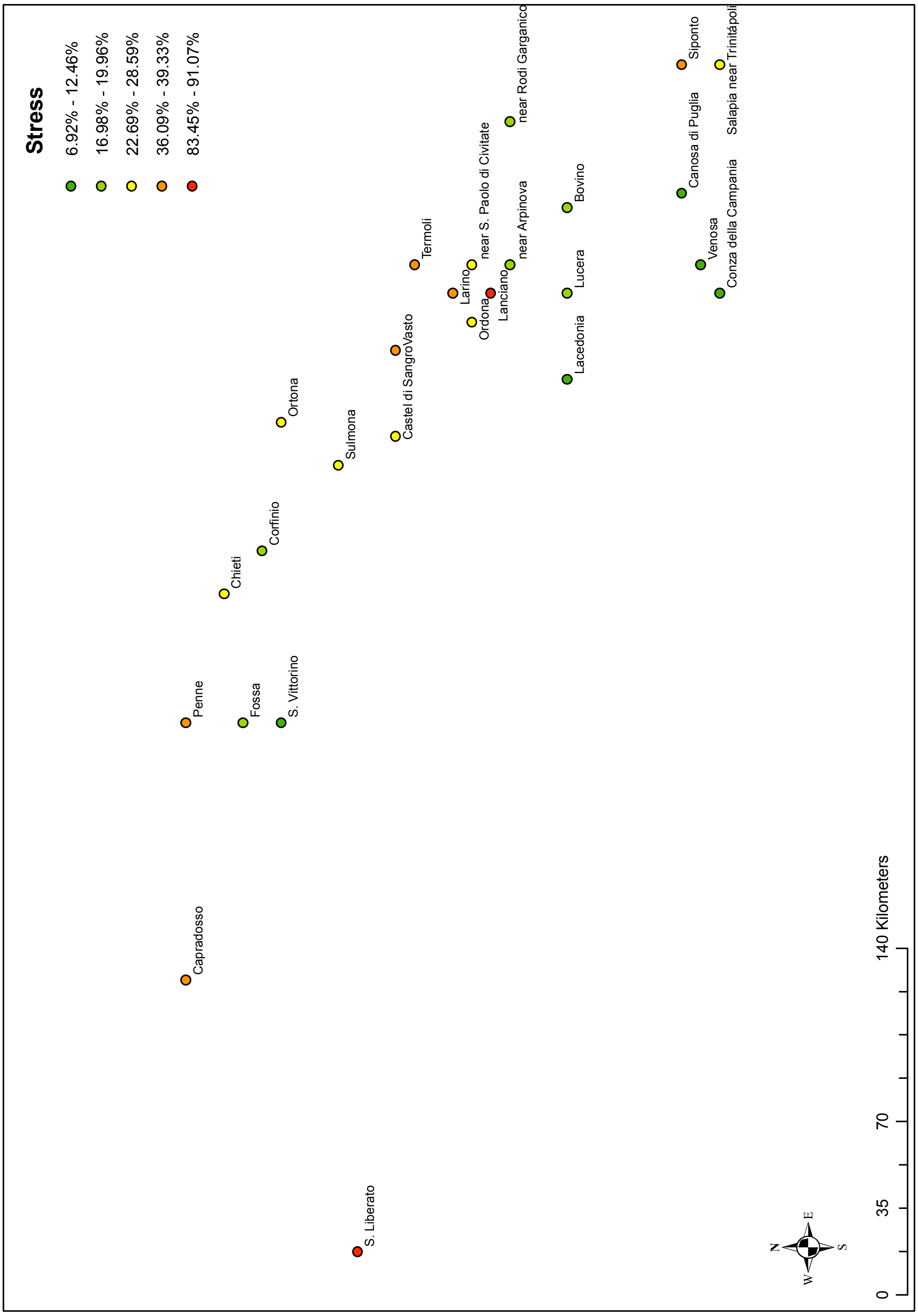

Figure 2.36: Samnium: Ptolemy's configuration with initial stress

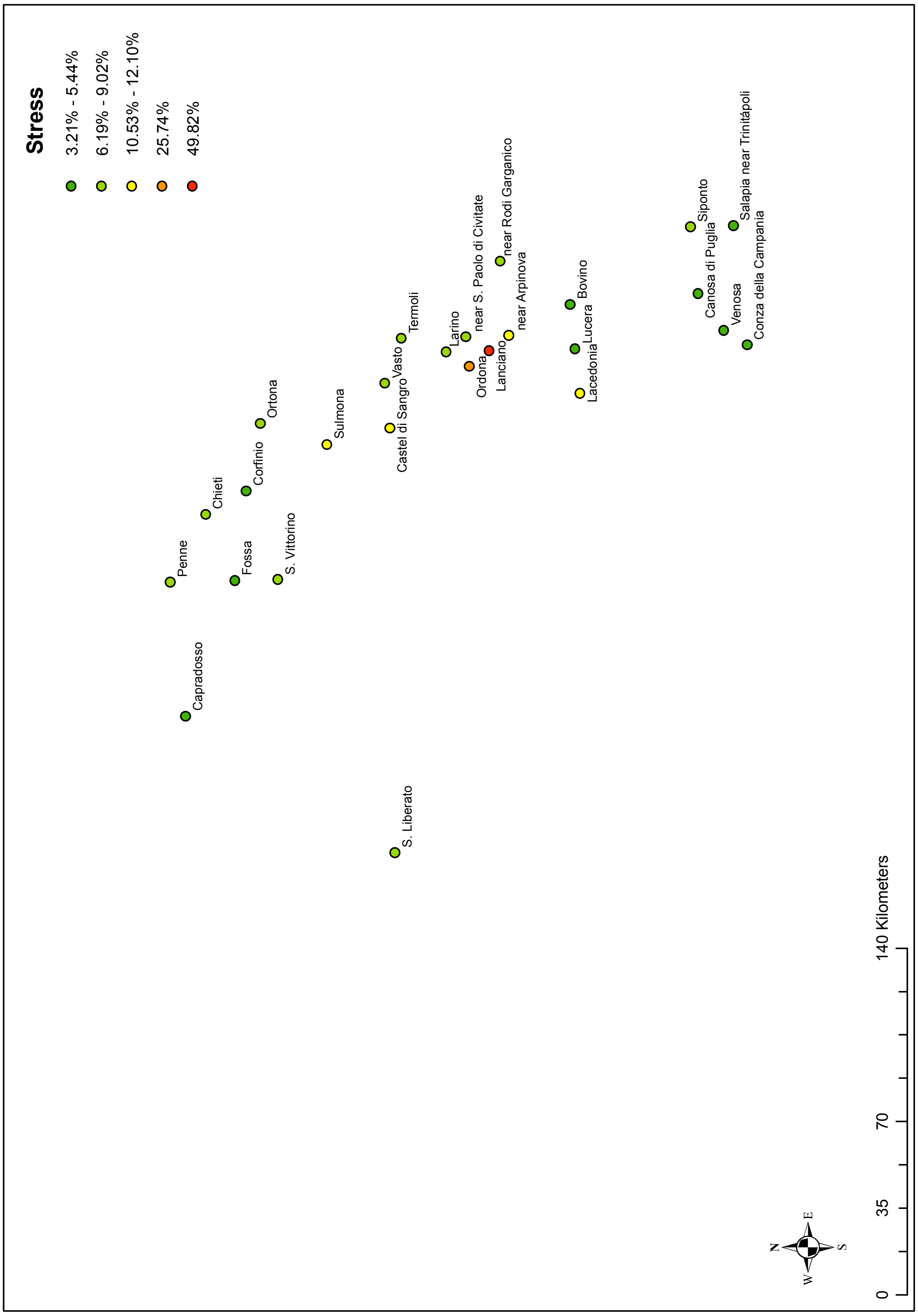

Figure 2.37: Samnium: transformed configuration with transformed stress

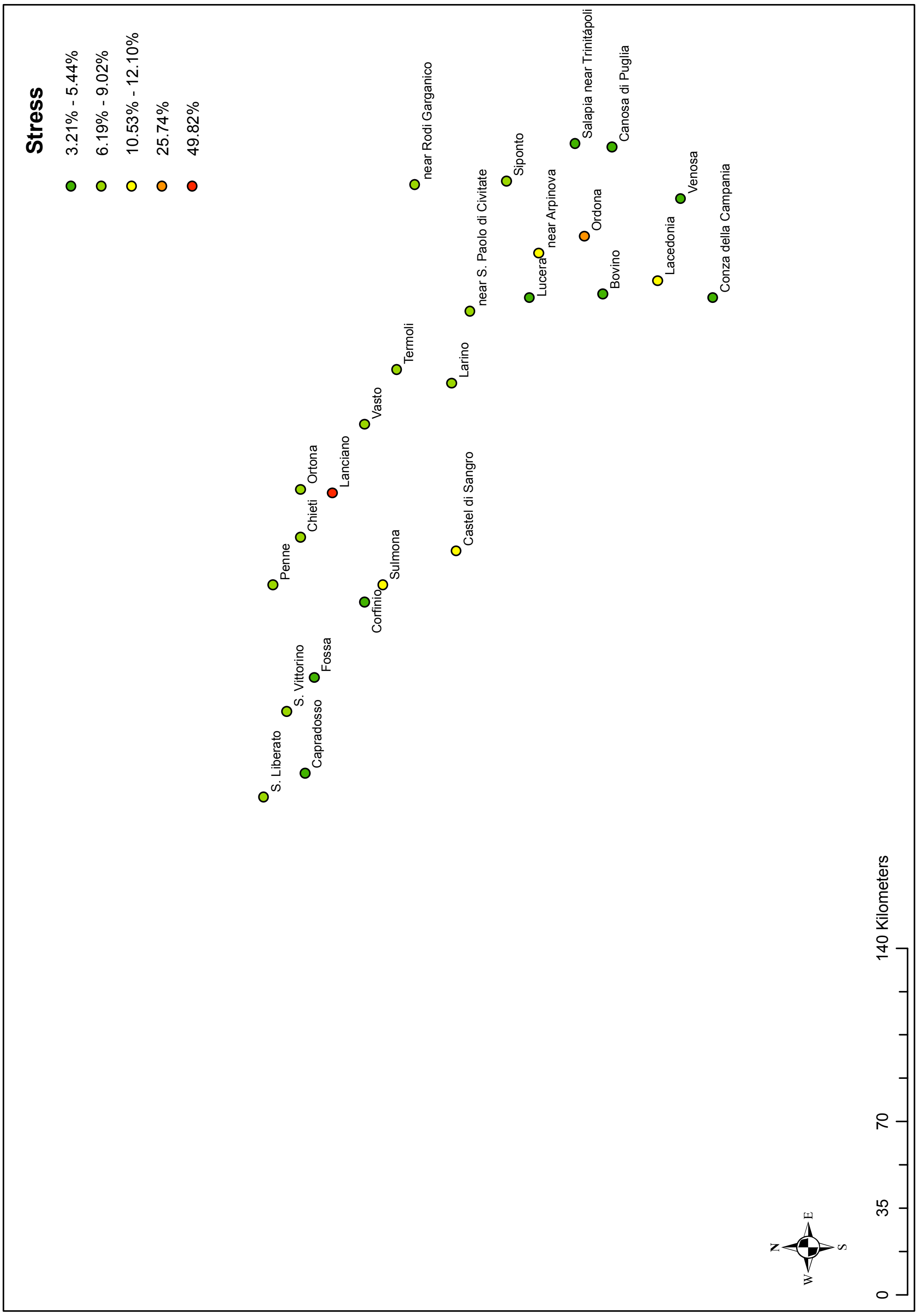

Figure 2.38: Samnium: modern configuration with transformed stress

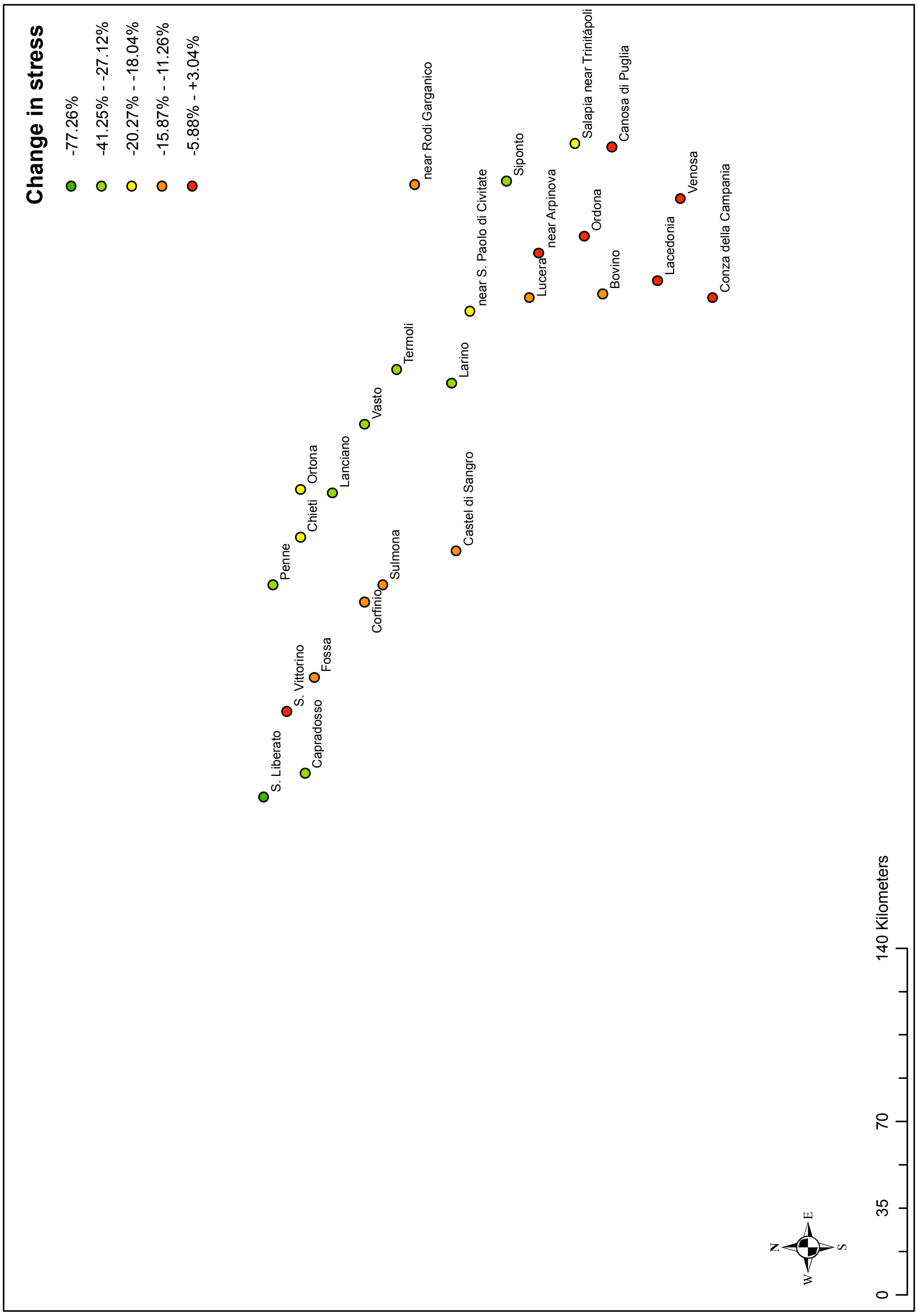

Figure 2.39: Samnium: modern configuration with change in stress

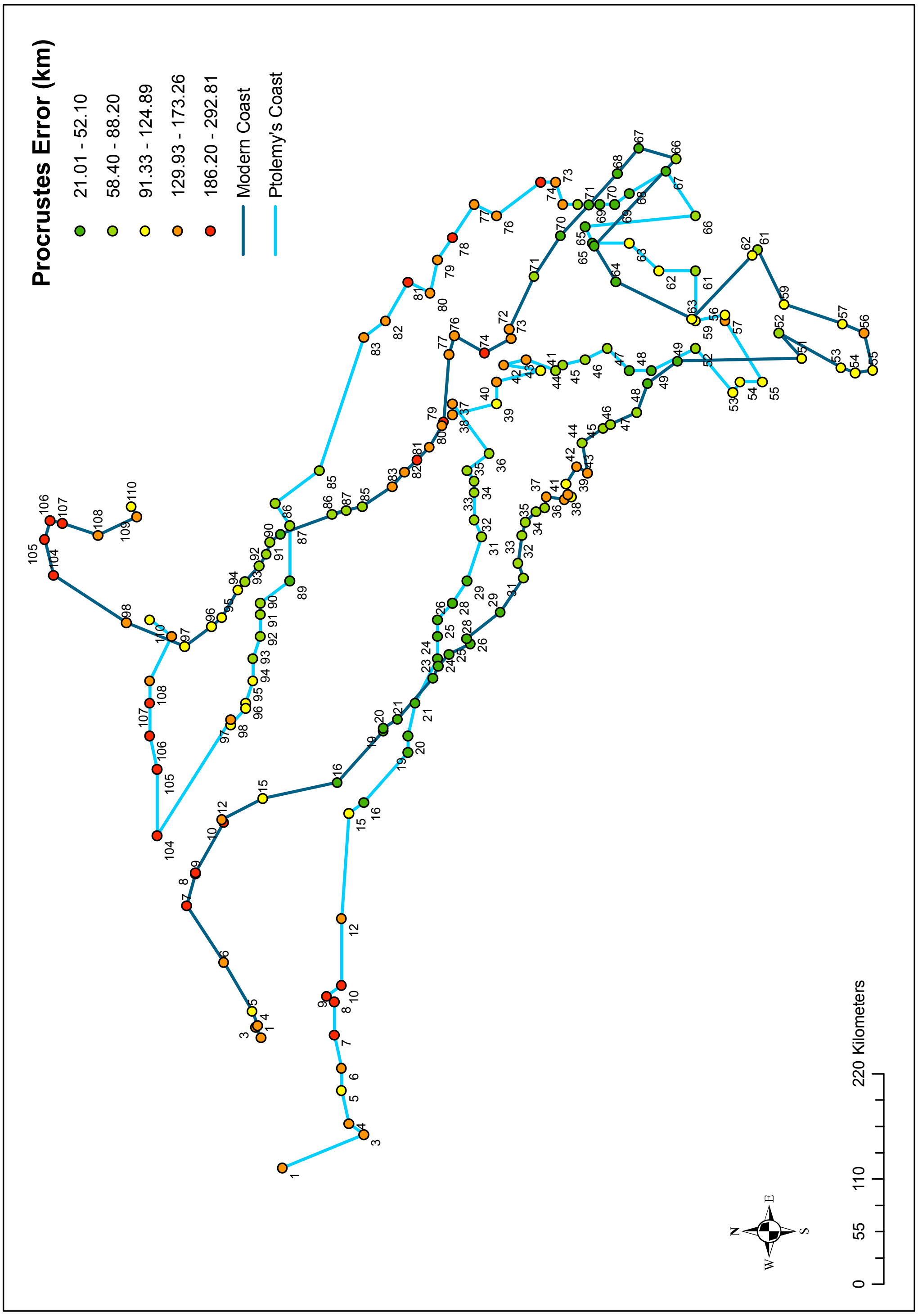

Figure 2.40: Italy's Coast: Initial Procrustes error

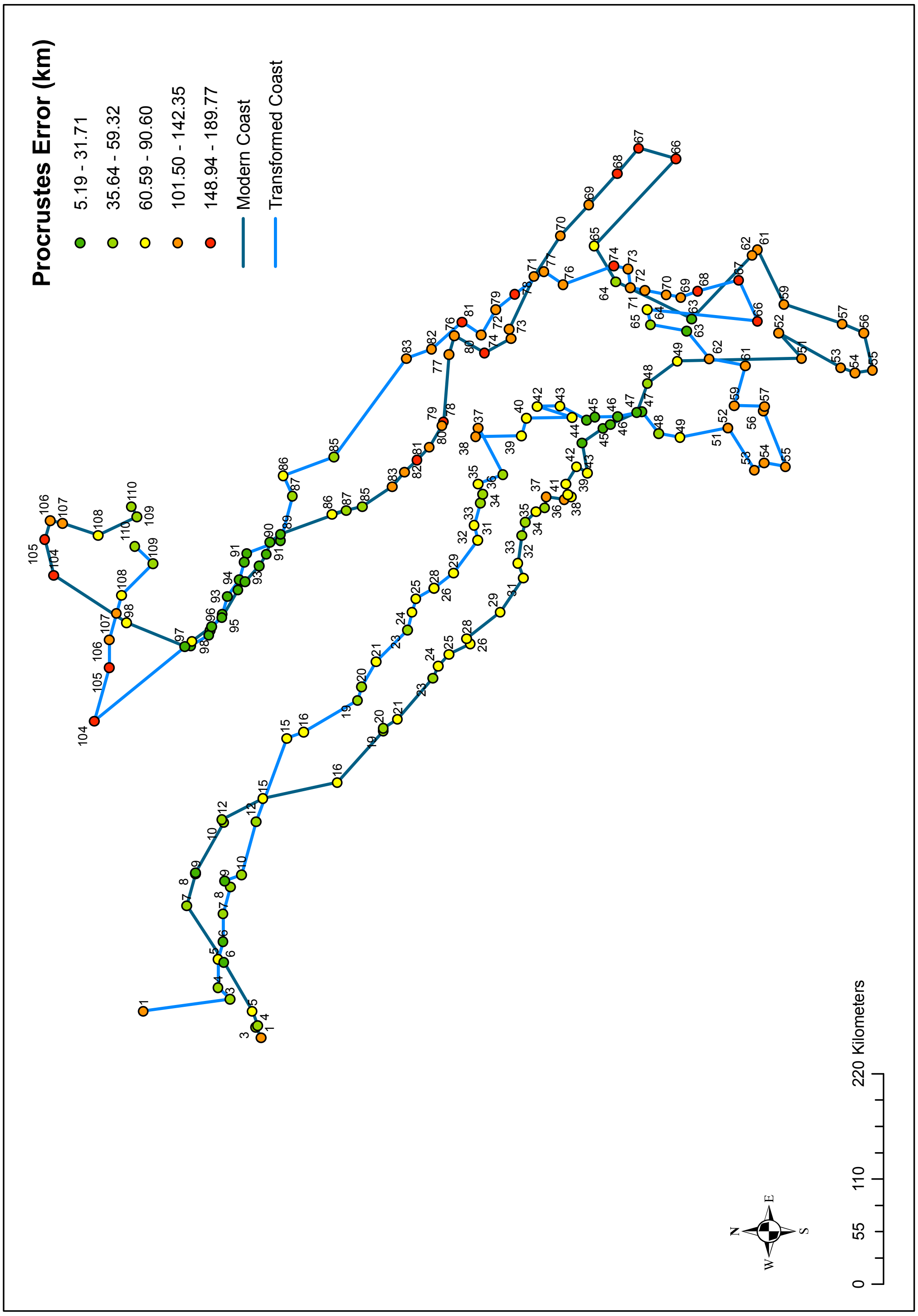

Figure 2.41: Italy's Coast: Transformed Procrustes error

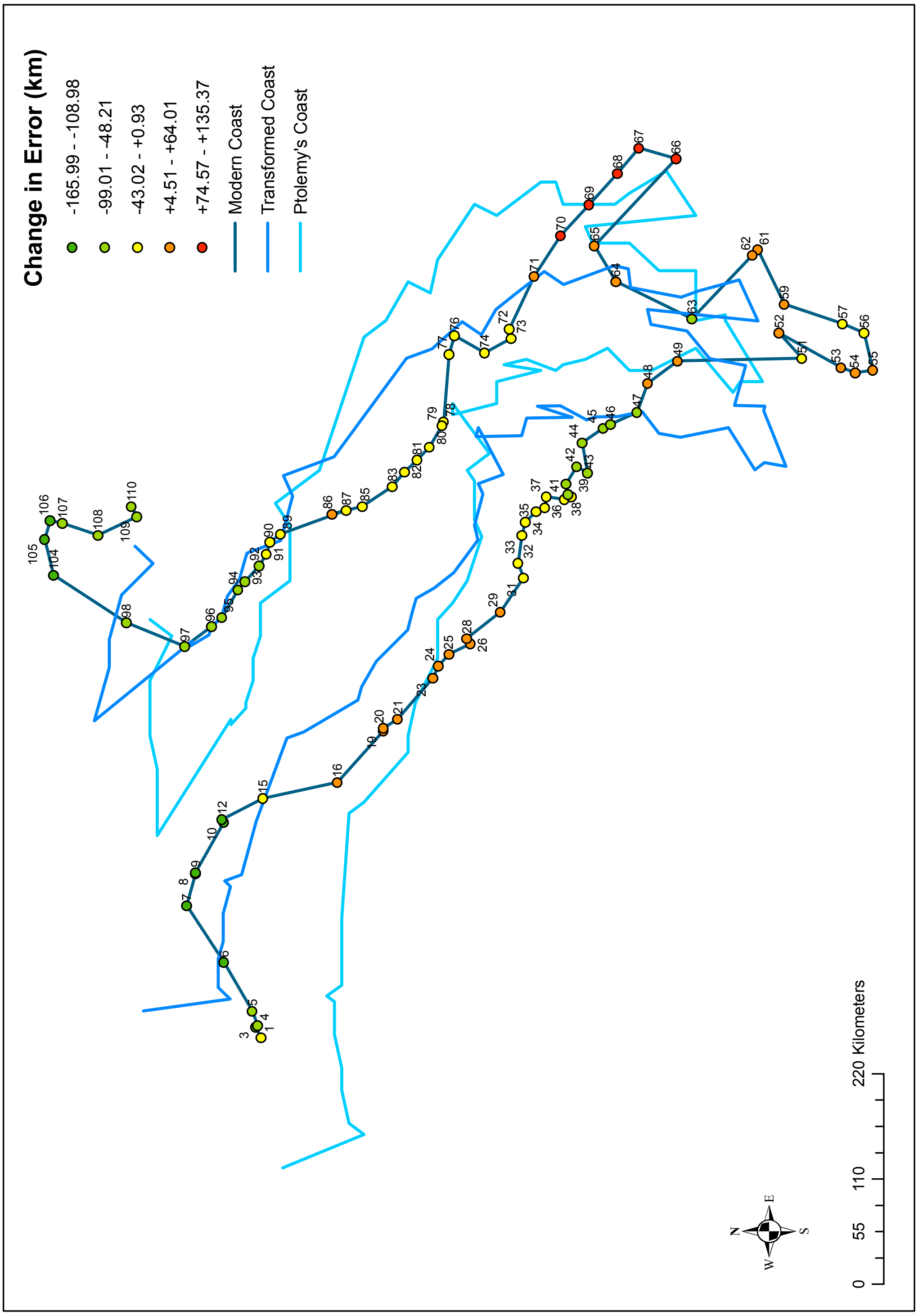

Figure 2.42: Italy's Coast: Change in Procrustes error

Looking at the maps, we can see that the peninsula is contorted compared to its modern shape (Figure 2.40). The north coasts are far from alignment. Moving south along the east coast, there is a very large gap between #85 and #83 (Giulianova and R. Pescara), after which begins a long string of a high error locations. Shape-wise, the two configurations appear as if they could line up in the region of #76 (Promontorio del Gargano) but for the fact that Ptolemy's coast by this point is so far to the east-south-east of where it should be. The two configurations catch up to each other at the heel of the boot of Italy. The toe of the boot has mostly mid-range errors, while most of the west coast until #15 (R. Arno) has relatively low errors, the one area of exception being about #40 (Pozzuoli).

After transformation, great improvements have been made (Figure 2.41). The north-west certainly lines up much more accurately. The north-east is closer to where it needs to be, especially the two end points, #109 and #110 (Pula and Vizače), but clearly needs a translation and rotation all its own. There is near perfect alignment down the coast from #97 to #89 (Ravenna and Numana), but then the troubles remain along the rest of the east coast. The majority of the boot, as well now, is out of sorts - the cost of improvements elsewhere. The west coasts of the two configurations run about parallel but are farther apart than they are initially.

In both the initial Ptolemaic configuration and the transformed configuration, there is a significant bend about the foot of the boot. The modern configuration, on the other hand, runs fairly straight along its entire length. On the west coast, this bend occurs at #37 and #38 (Villa Literno and Cumae) where the coastline twists and intersects itself, suggesting that Ptolemy ordered his cities wrongly. These cities are also the focal point of some higher range errors on both the Ptolemaic and transformed maps. Figure 2.42 compares the three configurations and gives the change in Procrustes error following the transformation. The areas immediately around the bend improve, but the vast majority of points that make up the foot, as well as a healthy portion of the west coast, worsen as a result of the transformation. It will be interesting to see how this turn in Ptolemy's map is holding back a correcting transformation by separating the regions on either side of it.

2.3.2 Coastal divisions

The first obvious place to draw a dividing line is about Italy's foot, close to the southward turning of Ptolemy's map. On the west coast, this line falls between Sorrento and Salerno (#43 and #44), very close to the inland divisional boundary. In Figure 2.40 these points fall on either side of series of high and low error. The other candidate for the division is between Castel

Volturno and Villa Literno (#36 and #37), but this brings us further from the historical division. Furthermore, after transformation (Figure 2.41), the error boundary remains distinct between Sorrento and Salerno, but not so much anymore between the others. On the east coast, the division is between Bari and R. Ofanto (#71 and #72), matching the inland divisions, as well as having a natural gap and distinct error divisions in Figures 2.40 and 2.42.

The other coastal divisions take account of the peninsula's shape and terrain as opposed to being informed by the history of the area. Naturally, measurements taken up and down the east coast have nothing to do with measurements up and down the west coast. Furthermore, as we have seen, the Apennine Mountains make cross-peninsula travel difficult and measurements unreliable. Therefore, the eastern and western coasts will be examined as their own groups.

The southern coast

The southern coast consists 25 points from Salerno to Bari. Initial Procrustes error is 121 thousand km^2. An anti-clockwise rotation of 20° reduces the error to 81 thousand km^2. A longitudinal and latitudinal expansion of 29% and 49%, respectively, minimise the error at under 42 thousand km^2, making a total improvement of 66%. Initial mean error for the points is 66.0 km, which is reduced to 34.7 km after the transformation.

Examining Figure 2.43, we can see that the errors are fairly scattered about the coast. The north-western stretch at first glance appears to be the most consistently accurate. Looking at the error groupings, though, we see that the difference between the first two tiers is more distinct than between any of the three middle ranges, so we should be cautious about treating the two shades of green as a single low error group. The most distinct tier is the highest error, but the three points in this category are spread far apart.

The transformation certainly brings the two peninsulas into line with the modern configuration (Figure 2.44). The error levels are still spread throughout the map, but they appear now in series of two or three locales at a time. The errors have certainly dropped, and this is readily apparent when noting the tiers. The lower three have tightened up in terms of range, and the lowest category is now very distinct from its neighbour. The main problem seems to be that the western peninsula is too fat. The west coast is convex when it needs to be concave, and the Gulf of Taranto needs to expand, while at the same time thinning the western peninsula. The range of error reduction is from −70 to +40 km (Figure 2.45). Those locations that take on more error are the same trouble spots as in Figure 2.45, namely the falsely convex area of the west coast and along the Gulf of Taranto.

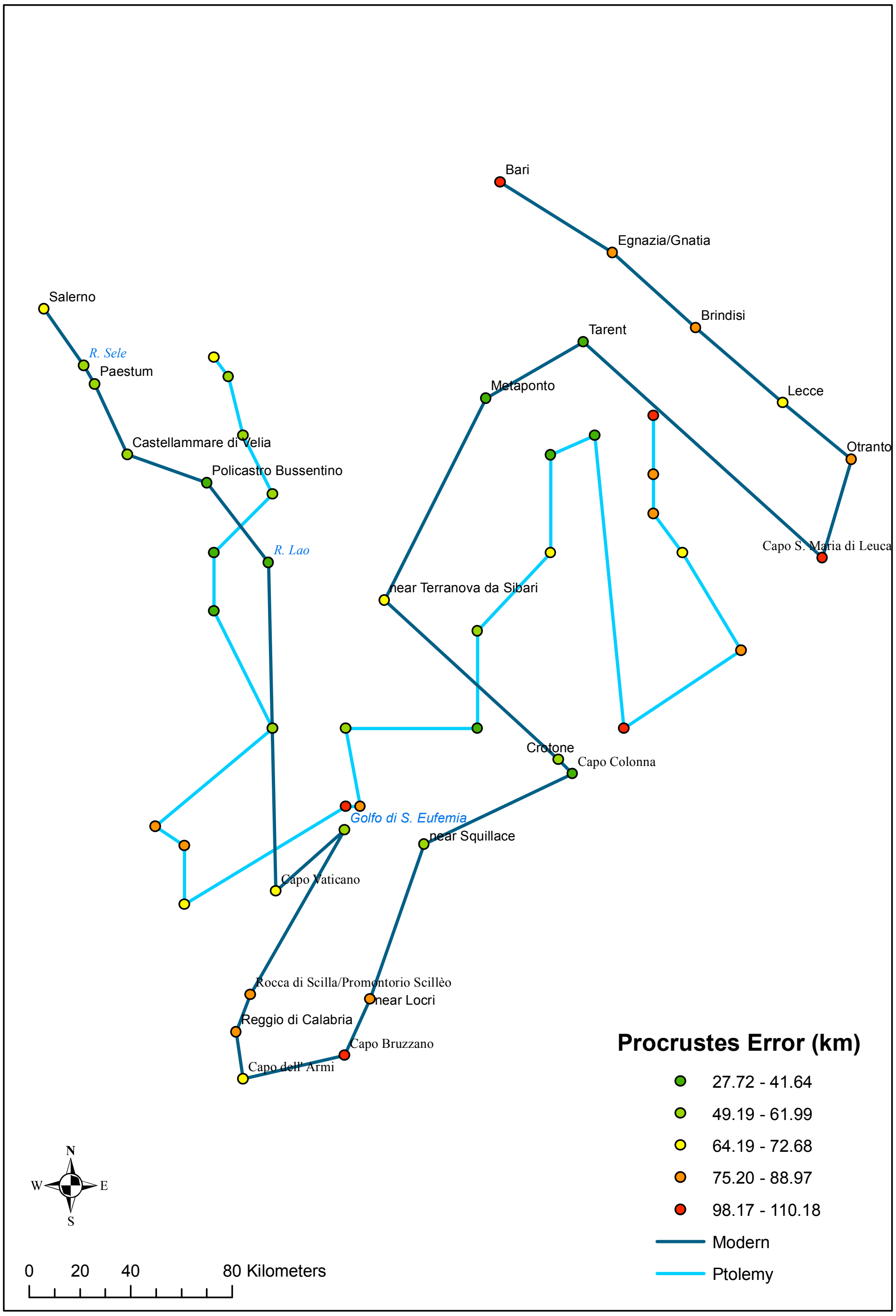

Figure 2.43: Italy's Southern Coast: Initial Procrustes error

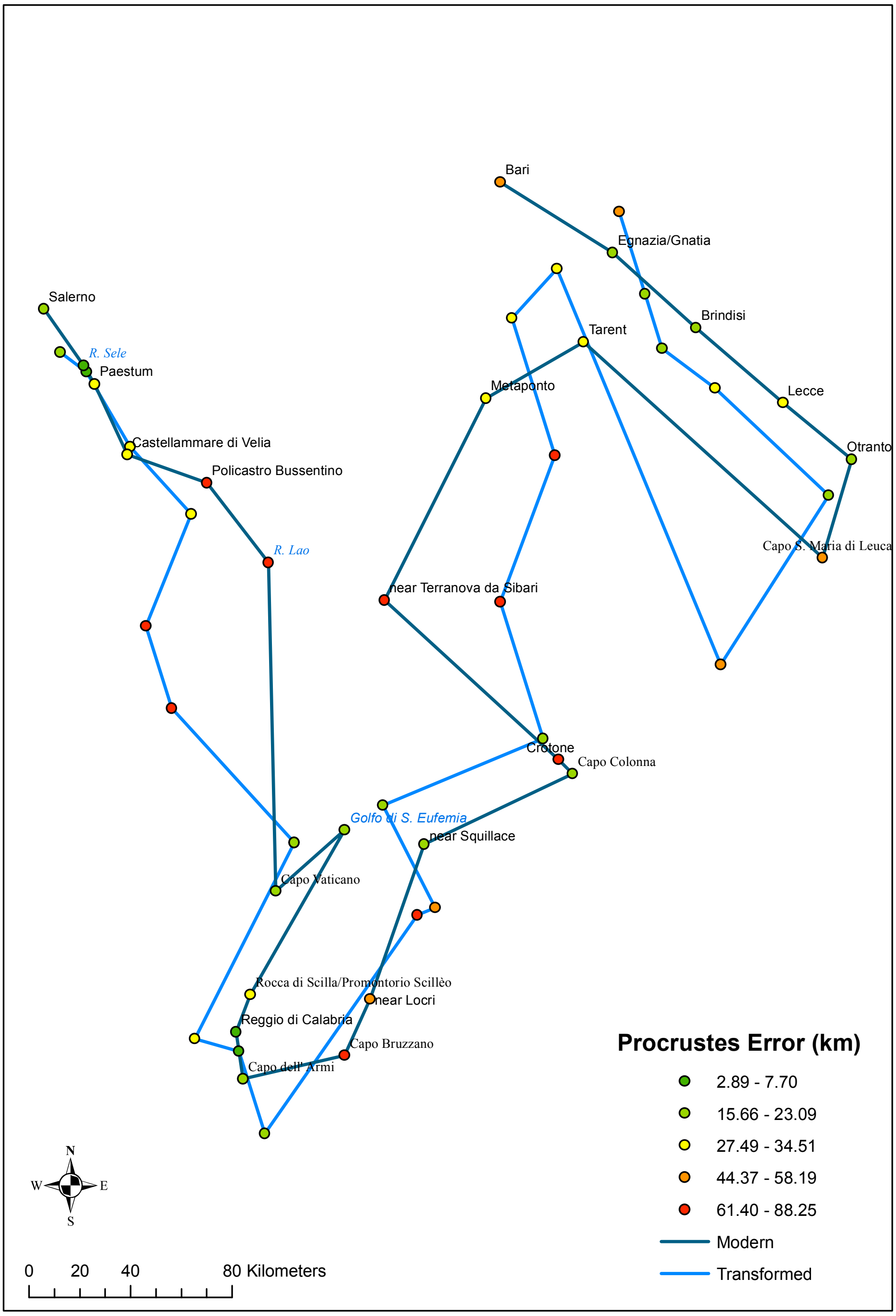

Figure 2.44: Italy's Southern Coast: Transformed Procrustes error

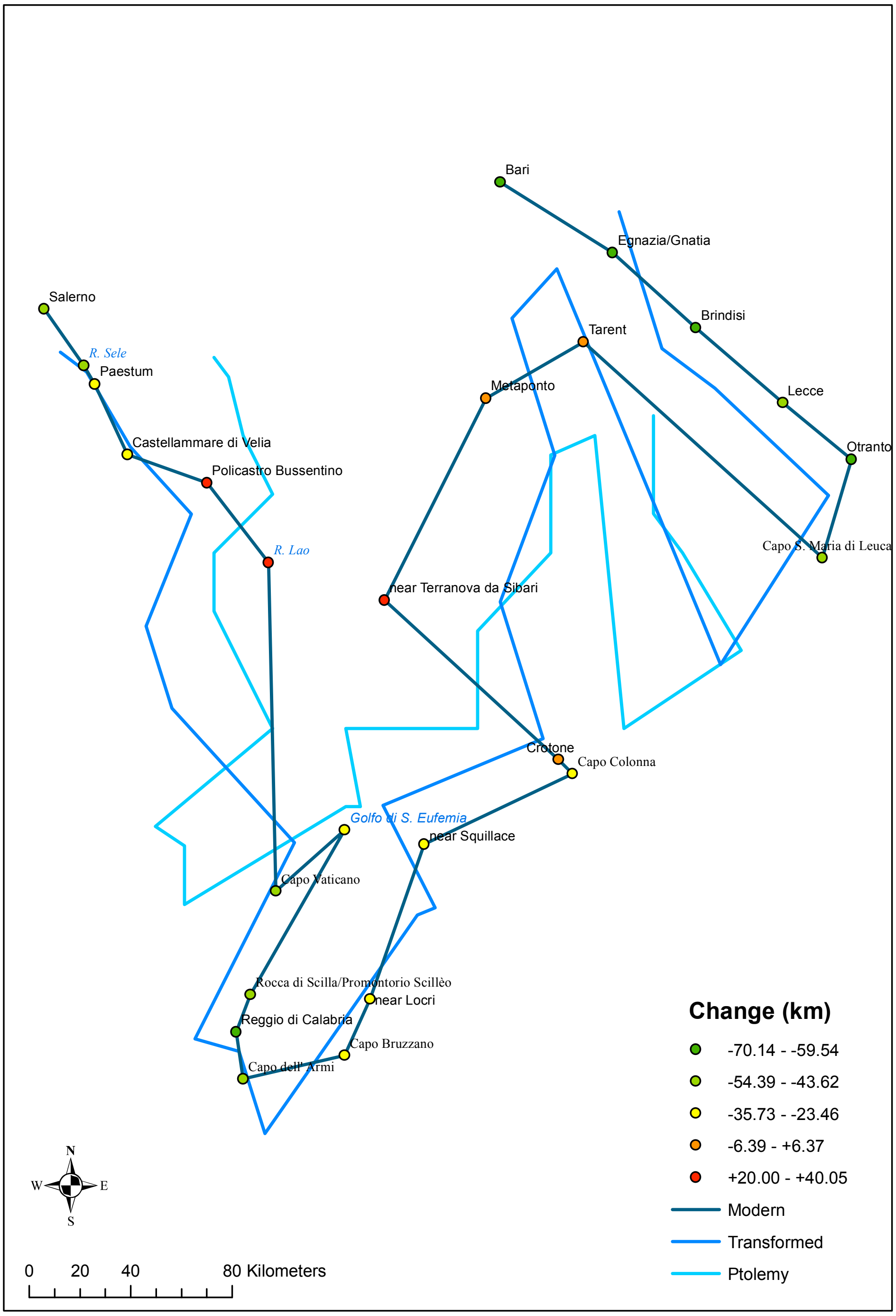

Figure 2.45: Italy's Southern Coast: Change in Procrustes error

The western coast

The western coast of Roman Italy runs from Nizza to Sorrento and contains 34 points. Initial Procrustes error is 472 thousand km^2, reduced to 129 thousand by a 22° clockwise rotation. Minimisation is achieved at 55 thousand km^2 by reductions in longitude and latitude by 17% and 16%, respectively. This represents a shedding of 66% of initial Procrustes error. Mean individual displacement is 103.8 km, dropping to 31.4 km after transformation.

Looking at Figure 2.46, we see that ‘west’ really only applies to the modern map. Ptolemy’s west coast mostly faces the south. The errors of the individual locations are meaningless at this stage. The errors naturally decrease towards the centre where the two coastlines intersect and increase towards the ends, but Ptolemy’s coastline is so misaligned that a discussion of placement errors is pointless. Shape-wise, it is clear the the ‘northern’ end is lacking the curve of the Ligurian Sea and that the end point, Nizza, is completely out of place. Ptolemy’s coast, on the whole is too long and there is a bizarre overlapping between Villa Literno and Cumae. There is an alternative longitudinal coordinate for Villa Literno which would put it west of Cumae and resolve the overlap, however, Villa Literno is supposed to be east of Cumae. The problem is that Cumae is too far north (or Literno too far south).

After the transformation, the two coastlines are now worthy of comparison (Figure 2.47). The three lowest tiers of error are not very distinct, and their total range is smaller than that of the single lowest tier of initial error. The southern half of the coast lines up very closely. Again, there is some confusion in the area of Cumae with Ptolemy’s coast being decidedly more jagged than its modern equivalent. The northern half of the coastline still does not admit an indentation for the sea. Either that or Ptolemy thought that the Ligurian Sea did not reach that far east, and what looks like the misplacement of Nizza is actually Ptolemy’s idea of its eastern shore. For the most part, improvement follows the same pattern as the initial error (Figure 2.48): the change is greatest at either end and becomes less impressive towards the centre.

The eastern coast

The eastern coast of Italy consists of 31 points running from Vizače to R. Ofanto. Initial Procrustes error is 707 thousand km^2. This error is reduced 85% by the transformations. A clockwise rotation of 30° drops the error down to 352 thousand km^2. A 37% reduction in longitude and a 34% reduction in latitude bring the error to its minimum at 105 thousand km^2. Mean initial individual error is 138.3 km, which is minimised at 55.1 km by the transformations.

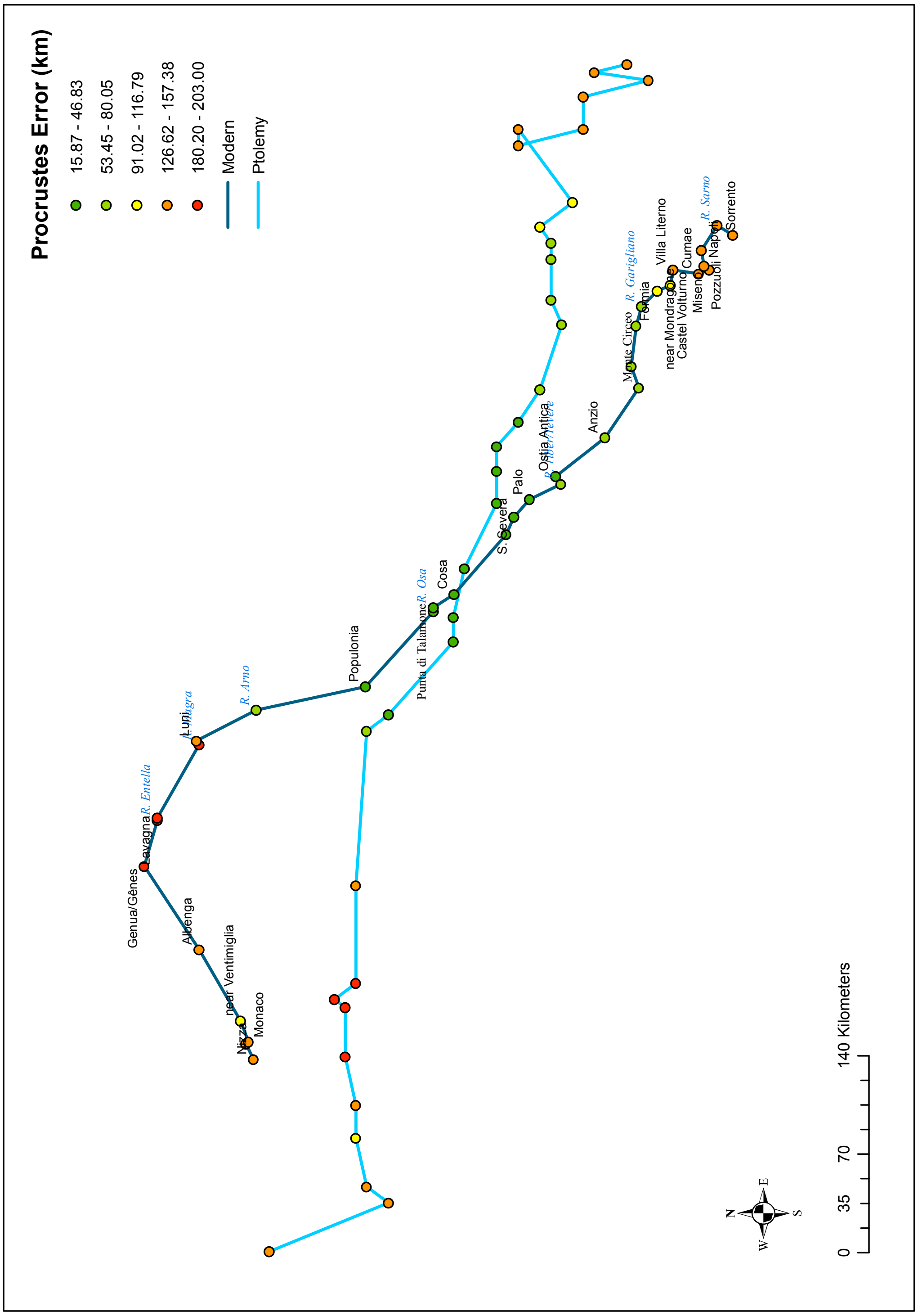

Figure 2.46: Italy's Western Coast: Initial Procrustes error

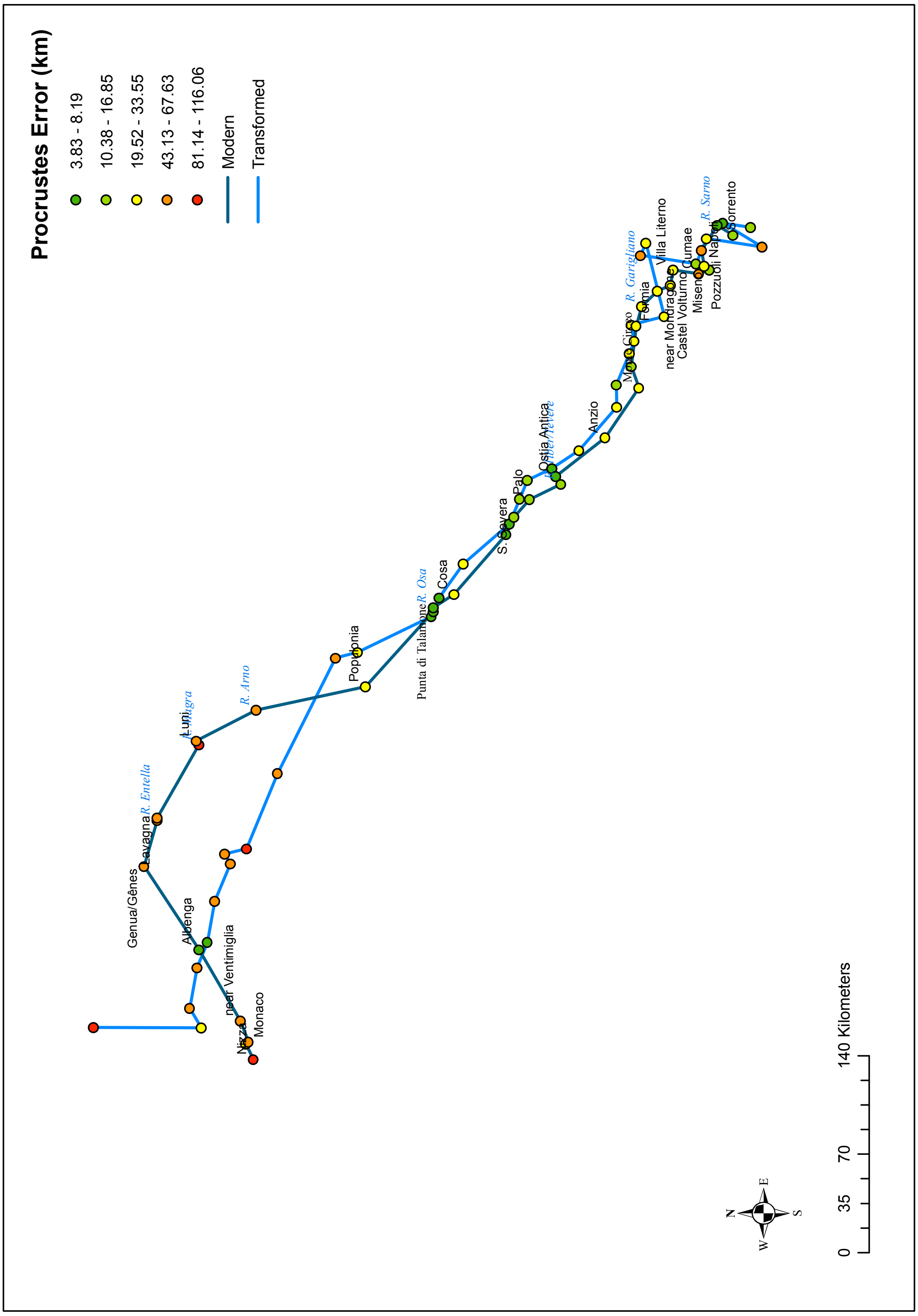

Figure 2.47: Italy's Western Coast: Transformed Procrustes error

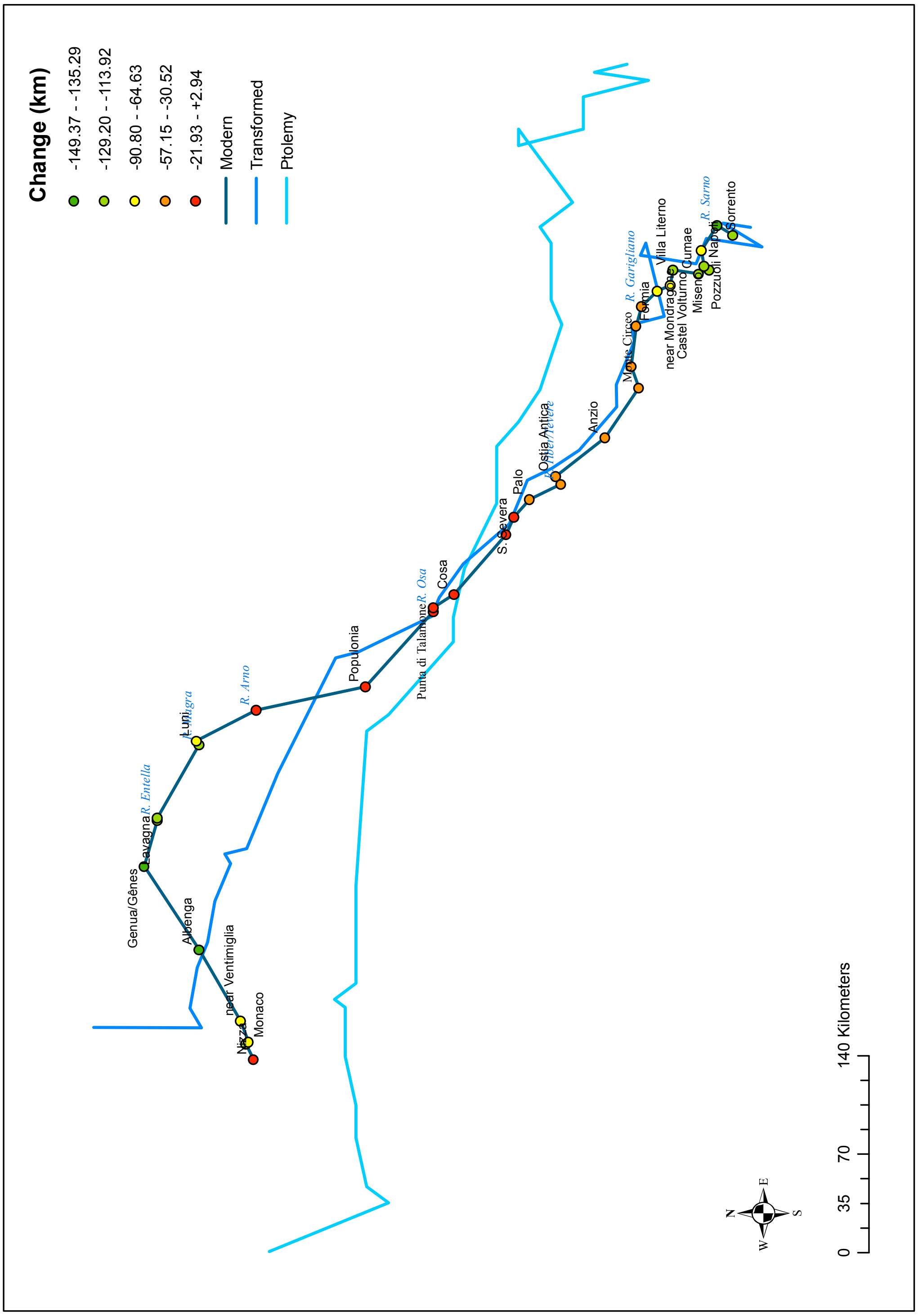

Figure 2.48: Italy’s Western Coast: Change in Procrustes error

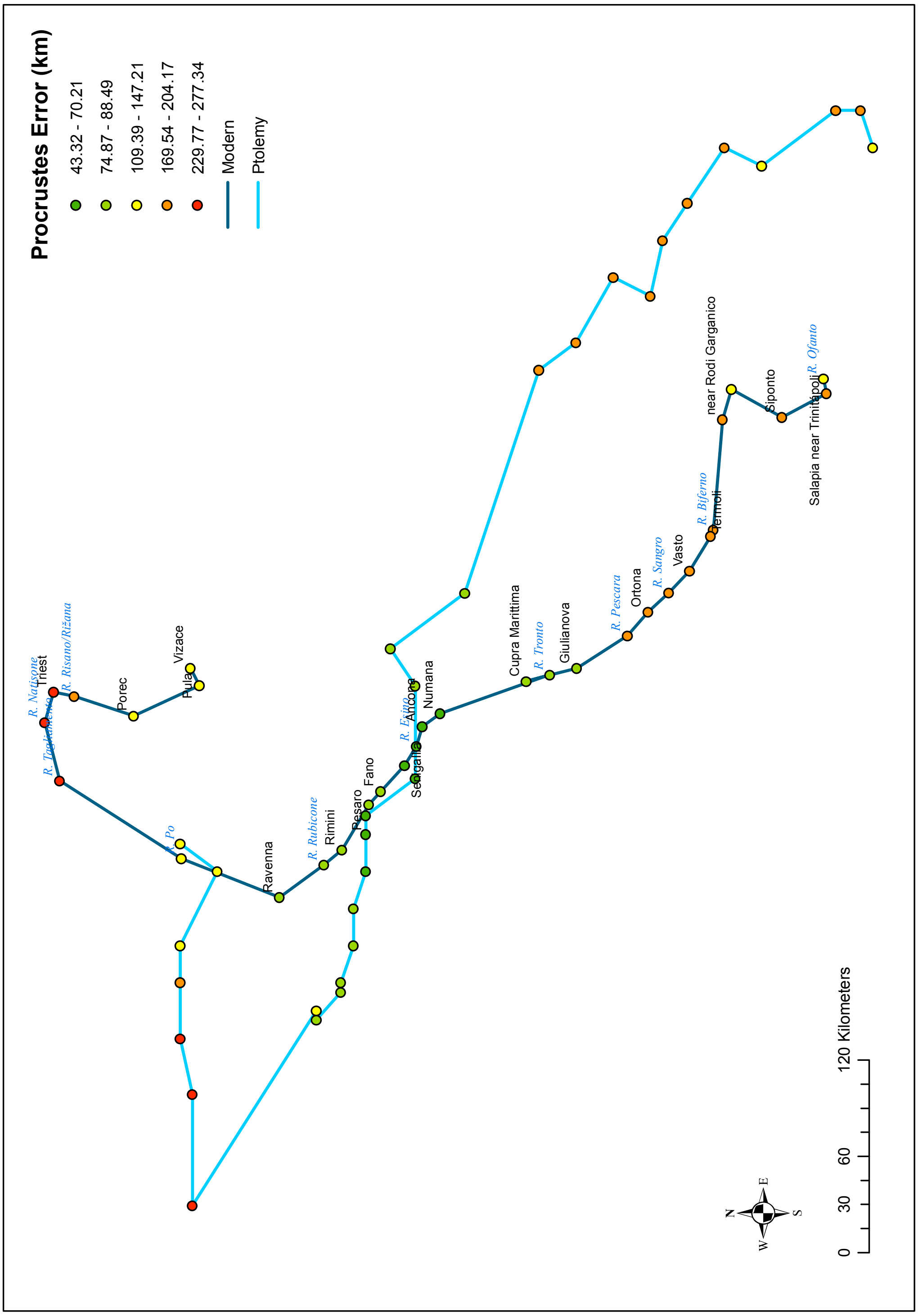

Figure 2.49: Italy's Eastern Coast: Initial Procrustes error

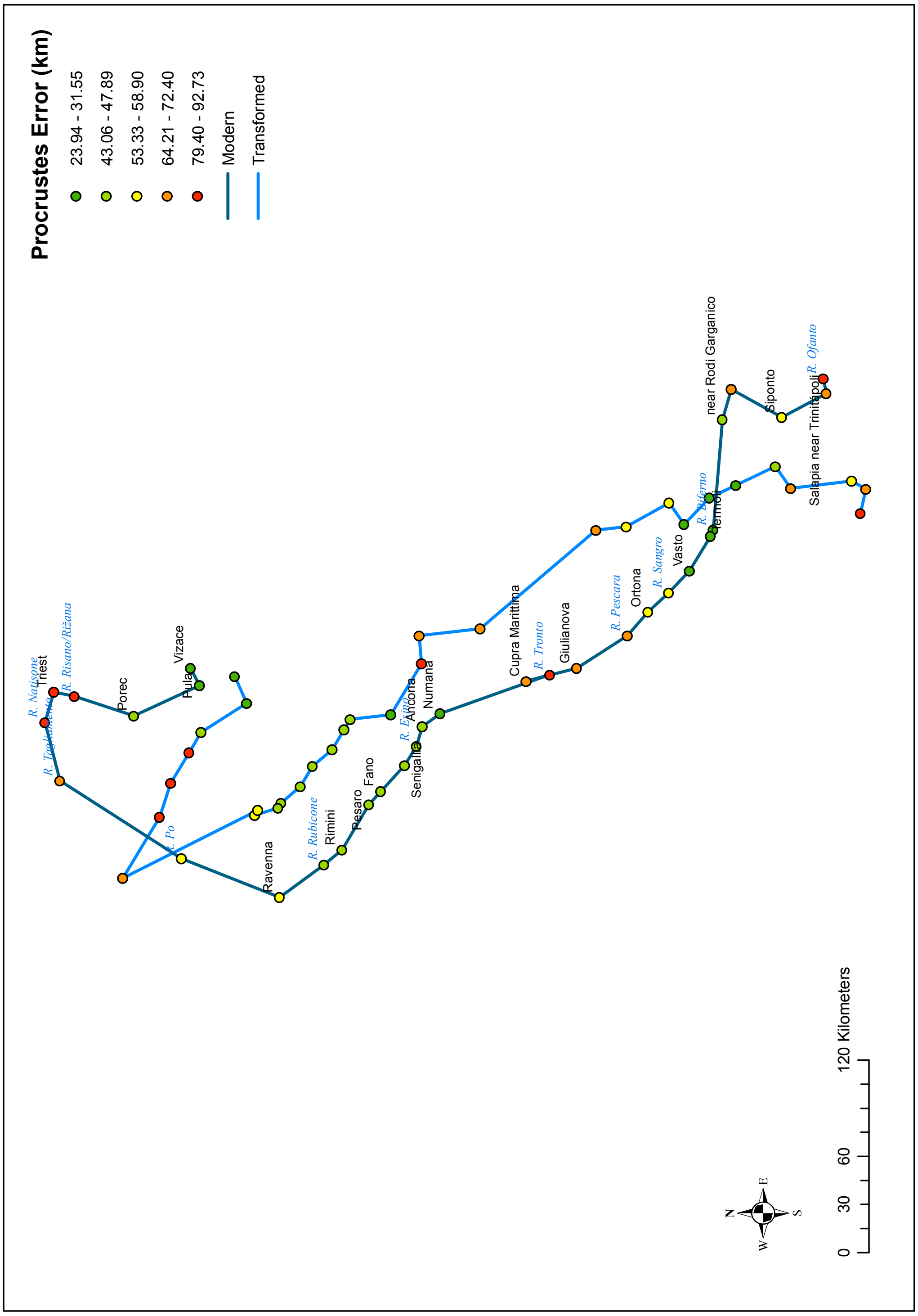

Figure 2.50: Italy's Eastern Coast: Transformed Procrustes error

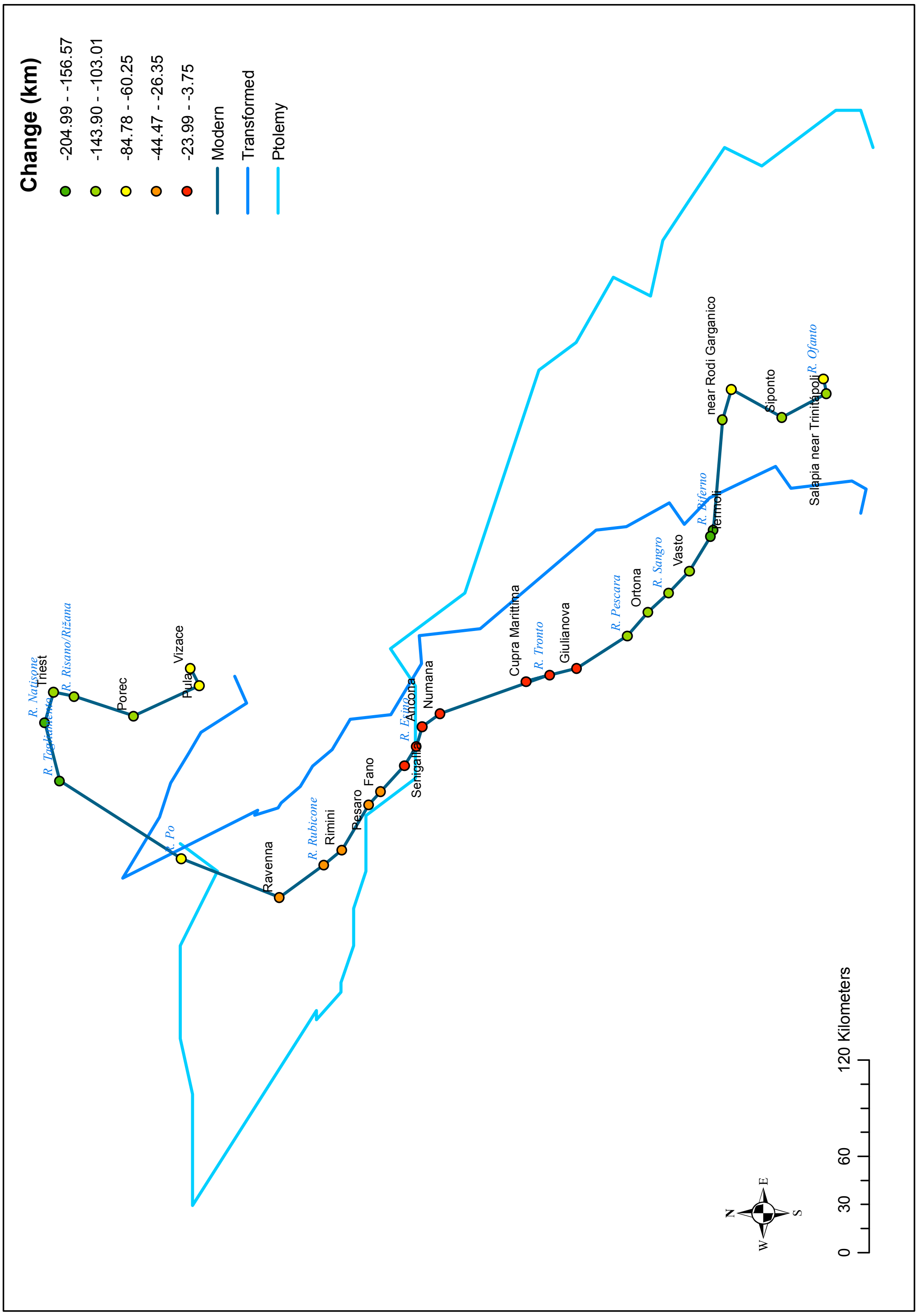

Figure 2.51: Italy's Eastern Coast: Change in Procrustes error

As with his west coast, Ptolemy's east coast suffers from being too horizontal and the errors radiate out from the centre (Figure 2.50). Of the few things that can be noted, the Gulf of Venice is too sharp, and the Po is much too close to Ravenna. Towards the southern end, the modern map has a smooth concave shape from Numana to the neighbourhood of Rodi Garganico. Ptolemy's coast, though, is jagged so that, overall, it does not have a definite curve to it. Again, it is only after transformation that we can really discuss the individual errors.

Unfortunately, it seems the correcting demands of the east coast are not internally consistent, resulting in a rather poor fit of the two configurations (Figure 2.50). The centre of Ptolemy's coastline runs fairly parallel to the modern coast and is kept from proper alignment by the far north and south. Moving the centre westward would result in errors on the extremities that would counter the minimisation of all. The trouble spot in the centre is mostly due to the lack of concavity of Ptolemy's coast, but there is also an ordering problem. Ptolemy places R. Tronto north of Cupra Marittima and in his catalogue lists them in order of R. Tronto, then Cupra Marittima, then Giulianova (from north to south). The modern ordering of points follows Ptolemy's and so, though difficult to see without zooming in, there is an overlap between Cupra Marittima (which is farther north, but listed second) and R. Tronto. Regardless, those two locations, along with Giulianova are too far north on Ptolemy's map, further adding to their errors.

In the north, Ptolemy's Gulf of Venice remains too sharp, too narrow, and too far south. A separate expansion could do wonders for the area. The south of Ptolemy's configuration is too vertically minded and pulls west, whereas the modern coastline comes out of its smooth curve before turning abruptly south-south-west, forming a small concavity, and ending by pointing out to the east. The maps do not correspond as to which points are the promontories jutting out into the sea and which are the interiors of bays. With the exception of the Gulf of Venice, which is mostly a matter of size, the eastern coast of Ptolemy's Italy suffers from a confusion of curvature.

In terms of improvement, Ptolemy's points had no choice but reduce their errors as they were so far out initially (Figure 2.51). Not a single location took on additional error from the transformation, but like the west coast, the improvements almost exactly match the initial error: the greater the distance from the centre, the greater the error and the greater the improvement.

	#	Initial stress	Transformed stress	Initial city stress	Transformed city stress
Italy	234	4.14%	3.13%	4.96%	3.48%
North	69	8.01%	6.33%	8.28%	6.67%
Central	136	16.94%	5.17%	18.22%	5.73%
South	29	18.49%	10.59%	18.83%	11.74%
Etruria, etc.	56	16.07%	10.86%	18.18%	12.10%
Latium, etc.	54	20.87%	8.05%	22.98%	8.95%
Samnium, etc.	26	28.78%	8.33%	28.44%	9.36%
South-west	14	18.34%	8.88%	18.52%	10.40%
South-east	15	30.36%	29.86%	33.05%	31.86%

Table 2.1: Italian cities stress summary

	Rotation	Longitude skew	Latitude skew
Italy	+17°	-22%	+13%
North	+10°	+5%	+49%
Central	+18°	-40%	-7%
South	-24°	+31%	+53%
Etruria, etc.	+20°	-26%	+2%
Latium, etc.	+13°	-28%	-26%
Samnium, etc.	-3°	-48%	+13%
South-west	-11°	+9%	+59%
South-east	-64°	+2%	-14%

Table 2.2: Italian cities transformation summary

2.4 Concluding thoughts

Let us examine Table 2.1. Italy as whole has the lowest stresses across all categories. Italy's long, narrow shape naturally aids it in keeping its stress down. Over such a configuration, the points at the extremities of the country will be relatively accurately placed so long as they are farther north or south of the bulk of the other points, which we can see are located in the centre. The points in the foot, for instance, will be accurate in terms of stress regardless of their east-west error, so long as they are farther south than the 205 points of central and northern Italy. A country that is squarish in shape, loses this 'advantage', and the points at its extremities will need to be accurate in all directions to keep stress at a minimum.

The tripartite divisions of north, central, and south, we expect to have higher stresses due to the shortening of the overall distances. The central region, having over twice the number of cities as the other two regions, is densely packed and has a very high stress, especially when compared to the northern region. This is remedied after transformation, however, suggesting that while the central group's initial configuration is riddled with errors, these errors are consistent throughout. Because of this, the transformation was able to have a significant effect.

The errors of the central region, however, are not consistent with those of its neighbours on either side. Looking at Table 2.2, the north and central groups need a similar rotation, but the south needs to turn about 40° from them in the opposite direction. The longitudinal skews are all over the place. The north is fine, the centre is entirely too wide, and the south entirely too narrow. The north and south are in agreement that substantial latitudinal lengthening is required, while the central region is ever so slightly too long. Even had history not dictated that these regions be treated separately, we can see clearly that the statistics demand it.

Reflecting the high initial stress of the central region as a whole, its three subdivisions also begin with large errors. Additionally, like the region as a whole, each subregion's stress drops off considerably after their transformations. This is especially the case in Samnium, which almost claims the minimum stress of the three despite it initially having the highest error.

The consistency of error of the central group that we theorised above due to its effective transformation does not appear to be the case as surmised. While the Etruria and Latium subgroups behave very similarly on the rotation and longitudinal skewing, they diverge on their latitudinal scaling (Table 2.2). The Samnium subgroup is all together different with its very slight rotation in the opposite direction and much larger longitudinal reduction. The central region has large initial errors which are corrected, on the one hand, by a single

	Etruria, etc.	Latium, etc.	Samnium, etc.
Initial stress	14.64%	18.16%	14.93%
Transformed stress	9.71%	7.47%	5.87%
Initial city stress	16.12%	19.92%	15.18%
Transformed city stress	10.50%	8.29%	6.47%
Rotation	+20°	+13°	-8°
Longitudinal skew	-27%	-25%	-37%
Latitudinal skew	+9%	-26%	+13%

Table 2.3: Italy inland results without outliers

transformation applied to the whole group, but on the other hand, by three different transformations applied to each of its subgroups. Is it that a few outliers are causing inflated initial stresses and masking the real errors?

Let us begin in Etruria. We noted that Trevi is just plain out of place. Ptolemy places it in the middle of nowhere to the south-east of Etruria. Its initial error is 79.76% compared to the mean city error of 18.18%. In the Latium group, Montesarchio holds the record with a stress of 87.85% compared to the mean of 22.98% initially. Samnium has two culprits: S. Liberato and Lanciano. The former has initial error of 83.45% and the latter, 91.07%. Mean city stress for Samnium is 28.44%. Results of the removal of these cities is shown in Table 2.3.

By removing the outliers, the central group as a whole reduces its initial and transformed stress to 15.42% and 4.65%, respectively. The transformation, though, only changed negligibly. Thus, the reduction of stress is simply due to the removal of the high error cities, but their removal did not significantly alter the group's configuration. Likewise, Etruria and Latium experience small reductions in their stresses, but even smaller changes to their transformations. At most, Etruria's latitude is expanded an extra 7% which Trevi had limited by its extreme southern placement by Ptolemy. Samnium experiences a much sharper reduction in stress as a result of S. Liberato and Lanciano' removal. Without S. Liberato's easterly misplacement, Samnium's required longitudinal reduction is eased by 11 percentage points. Otherwise, the transformation is negligibly distinct from its original form.

Without the outliers, the dynamics of the central group and its three subgroups have changed very little. How then can the group as a whole experience such a huge reduction in stress under its transformation when its three divisions undergo diverse transformations (also with substantial minimisations of their own stresses)? The latitudinal scaling of −7% is a compromise between the three competing needs of the groups. The rotation of 18° leans heavily

	#	Initial	Rotated	Scaled	Improvement
Italy	90	1,362K	946K	783K	42%
South	25	120.6K	81.0K	41.5K	66%
West	34	472.2K	128.8K	54.8K	88%
East	31	707.2K	352.3K	105.4K	85%

Table 2.4: Italy coastal error summary (in km^2)

towards the needs of Etruria. Etruria, which spans the breadth of the group as compared to the narrow Latium and Samnium, is more dependent on the rotation for the minimisation of its Procrustes errors. The 39% reduction in overall longitude is the problem. That reduction is greater than any of the reductions of the subgroups. What may be happening here is that the whole is greater than the sum of its parts. While each subgroup has its own errors requiring to be fixed, once points from Latium are related to points far away in Umbria, for instance, the greater errors borne over the greater distances become the primary concern for the minimisation process. Just as the stress errors become more apparent as we zoom in on Ptolemy's maps, they are smoothed away as we zoom out. The transformation that so incredibly reduced the central group's stress signalled a consistency seen only from a distance, and this was lost when we got too close to the map while looking at the subgroups.

As mentioned previously, the south of Italy is not internally consistent. The source of that inconsistency is the south-east or heel of the boot. The toe in the west needs an incredible amount of north-south stretching, after which, its stress drops to levels comparable to the other subgroups. The heel, on the hand, simply does not improve. A huge rotation certainly helps to amend initial Procrustes issues (by about 50% in fact), but that does nothing for its stress. There is basically no longitudinal scaling and a small condensing of the latitude. The stress, effectively, does not change. The placement errors in this area are so egregious and irregular that a transformation cannot be found to significantly reduce the stress. Yet, this is not an area that was unfamiliar in any way to the Romans and to the Greeks before them. This area is Italy's maritime connection to the eastern Mediterranean, and one of the relatively flattest areas of Italy. Even if its distance from Rome was erroneous because of the Apennine barrier, the distance-relationships of the cities of the south-east amongst themselves should be very accurate. It is fair to assume that there should have been abundant sources for Ptolemy for this area. Manuscript corruption is an easy way out of this difficulty, but, more than likely, these errors will remain a mystery.

	Initial	Transformed	Improvement
Italy	107.5	81.0	25%
South	66.0	34.7	47%
West	103.8	31.4	70%
East	138.3	55.1	60%

Table 2.5: Italy coastal location error summary (in km)

	Rotation	Long. Scale	Lat. Scale
Italy	+13°	-18%	+0%
South	-20°	+29%	+49%
West	+22°	-17%	-16%
East	+30°	-37%	-34%

Table 2.6: Italy coastal transformation summary

As with the case of the cities and their stress-errors, the south coast requires a transformation essentially opposite to that of the other divisions (Table 2.6). Its rotation is between 40° and 50° in the opposite direction from the two side coasts and requires huge expansions in scaling, versus the significant shrinking of the others. The south coast's transformation is very similar to that being utilised by the southern cities. The higher stresses though, do not correspond to the higher Procrustes error, giving testimony to the difference between the two types of error. The Gulf of Taranto, though, is still a cause of concern under both species of error.

The southern and western coasts come out fairly evenly in terms of Procrustes error after transformation (Tables 2.4 and 2.5). The south certainly begins in a much better position, but both finish with about the same average individual error and total error (considering the extra number of points on the west coast). The eastern coast, on the other hand, does not fare so well. It is far more out of place initially and, despite massive improvement, fails to line up accurately with its modern counterpart. Historically, though, this is understandable. Rome (via Ostia) is on the west coast. Coastal trade routes took ships north to Massalia (Marseille), south to Sicily, around the boot to Brundisium (Brindisi) and from there to the east. The western and southern coasts of Italy were well travelled by the Romans and Greeks. The eastern coast however, was for a long time enemy territory. The northern portions of it, which we saw were significantly out of place, were some of the last areas to be taken by Rome. Unfamiliar waters make for uncertain maps.

The east and west coast need a larger rotation than any of the stress divisions. Both coasts needed essentially uniform scaling which is only the case in the Latium stress group. We can see the large effect the Apennines have on the numerical analysis. Latium is the only stress group north of the foot of the boot that is almost entirely to one side of the mountain chain. Like the coasts, it requires a clockwise rotation and significant uniform negative scaling. The other groups north of the foot span the width of the peninsula and/or include large swathes of mountainous territory. While they all, except Samnium (which includes the area of the southern turning of the peninsula), need a clockwise rotation about the order of 15°, their scalings are far from uniformity and consistency. Somehow, despite Rome's constant engagement in and beyond the Apennines, the mountain range distorted distances and orientations between cities. Or was it rather the case that reasonably accurate knowledge of the geography of the Apennines did exist but was not sought by Ptolemy? Would ancient geographers seek updated information on the position of cities and coastlines if such positions were already thought to be correct?

Chapter 3

Spain

3.1 A history of Rome's involvement in Iberia

3.1.1 Colonisation from across the sea

Roman involvement in the Iberian Peninsula[1] was predated by Greek and Phoenician, and subsequently Carthaginian, colonisation in the ninth to sixth centuries BC (Richardson 1996, 14). Herodotus notes that several Greek peoples had sailed as far as the mouth of the River Baetis (Guadalquivir). An indigenous kingdom was there which the Greeks called Tartessus (Herodotus, 1.163, 4.152). The Romans later named it Turdetania, and it was noted for its rich mineral wealth (Richardson 1996, 12-13).

About the time of the founding of Carthage, the Phoenicians established *coloniae* at Gadir (Roman Gades, modern Cádiz), Malaca (modern Malaga), Sexi (Almuñécar), and Abdera (Adra) along the southern coast of Spain. During the sixth century, Carthage seems to have taken over from its Phoenician founders and began planting its own settlements as well as assuming control of those previously established by Phoenicia (Richardson 1996, 14).

Greek *coloniae* also began appearing in the sixth century BC, though mostly on the northern end of the east coast: Rhode (Roses) and Emporion (Empúries). Emporion was founded by colonists from Massilia (Marseilles), which in its turn, was founded by the Greeks of Phocaea in *c.*546 BC. The settlers of Rhode may have been Massiliotes or 'first generation' Greeks (Richardson 1996, 14). Strabo says that there are three Massiliote cities south of the River Sucro (Júcar), of which he only names Hemeroscopeium (Strabo, 3.4.6).

[1] 'Spain', unless modified by the adjective 'modern' refers to the whole of the Iberian Peninsula, Portugal being an entity postdating this study. 'Hispania' is used to denote the area of the Iberian Peninsula officially under Roman control (whether such control was a reality or not).

A Greek city in ruins, Maenaca, was located between Malaca and Sexi in the south (Strabo, 3.4.2). These settlements were not fundamentally military or political in nature. While these cities surely influenced the native culture of the Iberians, the tribes remained independent (Richardson 1996, 16).

3.1.2 Rome gets worried

By 237 BC, Carthage had lost most of its Mediterranean holdings. The Carthaginian general Hamilcar Barca was sent to Spain in 237 BC where he 'applied himself to recovering Spain for the Carthaginians' (Polybius, 2.1). Within nine years, all of Turdetania (i.e. the Guadalquivir river valley) was brought under Carthaginian control, as well as the Mediterranean coast to a point just south of the River Júcar. Here was founded Acra Leucê (Alicante). Hamilcar was killed in battle outside the city of Helicê (Diodorus, 25.10; Polybius, 21.5-9; Livy, 24.41; see Richardson 1996, 16-17).

Hamilcar's son-in-law, Hasdrubal, assumed command upon his death. Hasdrubal continued the expansion of Carthaginian holdings. He formed an alliance with several Iberian tribes through the marriage of his daughter to a local king and was himself acknowledged as general of the Iberians (Diodorus, 25.12; Richardson 1996, 17). He then built Carthago Nova (Cartagena). In 221 BC, Hasdrubal was murdered by a slave, and Hannibal, son of Hamilcar, was elevated to the command of the Carthaginian forces. Hannibal began his conquest of Italy in 218 BC, leaving behind him his brothers Hasdrubal (Hamilcar had both a son and son-in-law named Hasdrubal) and Mago to oversee further operations in Spain. The Carthaginians were driven out of Spain entirely by the Romans by 206 BC following their decisive victory at Ilipa (Alcála del Río) (Richardson 1996, 18).

Rome's earliest known involvement in Spain came in the mid-third century BC. It was at this time that Rome became concerned at the rapid Carthaginian expansion in the peninsula, not necessarily because they desired to expand into Iberia themselves, but simply because Rome did not want Carthage to regain any of its former power. Not wanting to risk open war as one with the Celts was imminent, Rome concluded a treaty with Hasdrubal in 226 or 225 BC 'in which no mention was made of the rest of Spain, but the Carthaginians engaged not to cross the Ebro in arms' (Polybius, 2.13; see Richardson 1996, 19-20).

This treaty was not to last long after the command of Hannibal began. The treaty was complicated by the fact that the city of Saguntum (Sagunto) was an ally of Rome and located south of the River Iberus (Ebro). Previous treaties with Carthage forbade attacks on one another's allies, but Saguntum was only allied with Rome after these treaties were put in place. In any case,

Figure 3.1: Spain (base map ©Google 2010)

Roman delegates arrived first at Carthago Nova to remind Hannibal to attack neither Saguntum nor to cross the Ebro. The same delegation then went to Carthage to argue the same points (Polybius, 3.15). Whatever the specifics of the treaties and the motives, real or imagined, war was declared.

Second Punic War

In 219 BC, Hannibal took his army from Carthago Nova and proceeded to besiege Saguntum for eight months before it fell to him (Polybius, 3.17). The following year he marched north, crossing the Ebro and subduing the Spanish tribes in his path. He left a large part of his army under the command of Hanno to guard the rear from native revolts and Roman landings in northern Spain. (He previously left the southern areas under the command of his brother, Hasdrubal (Polybius, 3.33).) Hannibal himself left Spain to begin his famous march that would take him through the Alps and into Italy (Polybius, 3.35).

While Hannibal was crossing the Alps, the brothers Publius and Gnaeus Scipio landed in Spain with the mission of disrupting Hannibal's supply chain. Striking from their base at Tarraco (Tarragona), the Scipios were able to secure the area north of the Ebro before crossing the river and cutting off Hannibal's reinforcements. With the help of native revolts in the south, Rome was able to take back Saguntum by about 212 BC. From there, the brothers divided their forces and pushed into Carthaginian territory, but both were killed and their armies defeated. The leaderless survivors, though, were able to regroup and maintain the stranglehold on Hannibal's supply route.[2]

Publius Scipio's son, also named Publius, was sent to Spain in 210 BC to take command. The next year he took Carthago Nova, gaining control of Spain's east coast. Pushing further into Carthaginian territory, Scipio routed Hasdrubal, who fled to his ultimate defeat in Italy. In 208 BC, the Romans, now allied with several Iberian tribes, won the decisive battle of the Spanish theatre at Ilipa, outside modern Seville. In full retreat, the Carthaginians made a failed last attempt to retake Carthago Nova by sea before completely withdrawing from the peninsula.[3]

3.1.3 Rome decides to stay

Scipio, having expelled the Carthaginians, turned to pacifying any opposing tribes and towns in his immediate vicinity. Hannibal was still fighting in Italy,

[2]See Richardson 1996, 27-30; Livy, 25.32-39; Polybius, 9.11; Appian, *Iberike* 16.

[3]See Richardson 1996, 30-35; Livy, 26.17-19, 27.18-20, 28.2; Polybius, 10.7-9, 10.38-40, 11.20-24.

and Scipio's mission to prevent his resupplying meant holding on to the former Carthaginian territory in Spain. To this end, Scipio founded Italica (Santiponce) just north of Seville for his wounded soldiers (Richardson 1996, 35-36; Appian, *Ib.* 38). For the time being, Spain was viewed as a military *provincia* where all administrative procedures were undertaken by the commanders for the purpose of maintaining their armies, preventing or subduing native uprisings, and preventing Carthaginian resurgence (see Richardson 1996, 36-38). Romanisation, despite the lack of official sanction, began in the wake of Rome's military successes in Spain. This was especially true in areas with large Roman populations, whether military or civilian. Great numbers of Romans and Italians settled in Emporion, Tarraco, and Carthago Nova looking for commercial gain in supplying the war effort. Thus trade between Spain and Italy was already beginning to flourish before the Roman Senate took political control of the peninsula (see Richardson 1996, 39).

Initial state of the provinces

Until the final defeat of Hannibal in Africa in 201 BC, decisions regarding Spain were made by the commanders there on a case-by-case basis. Once the war had ended, the Senate decreed that the number of troops in Spain should be significantly reduced (Livy, 30.41; Richardson 1996, 45-46). However, this was not acted upon by the commanders, who continued battling in the peninsula and collecting booty. The Senate, therefore, altered its plan and began sending official magistrates,[4] and in 198 BC two newly established praetorships were first employed for the overseeing of Spain. The new praetors were to establish the borders of two new provinces into which Spain was divided, Hispania Citerior (Nearer Spain) and Hispania Ulterior (Further Spain) (Richardson 1996, 47-48; Livy, 32.27-28). These names did not refer to the whole of Spain, but only the areas which were nominally under Roman control.

At that time, the designation of *provincae* meant that Citerior and Ulterior were areas of active military engagement. While the Carthaginians were no longer present and there was no real threat that they would appear again in southern Spain, holding on to the coastal areas gained during the war would take significant manpower. There was also the threat of Iberian-Gallic alliances that might threaten Rome (Richardson 1996, 50-51; Livy, 29.1). Fighting broke out near Emporion in *c.*195 BC as well as in the south of Ulterior (Livy, 34.9-16; Richardson 1996, 53). Quite often during the next twenty years, the two praetors and their respective armies conducted operations together and pursued their enemies beyond the boundaries of the provinces assigned

[4]Irregularly, many of the previous commanders had not held very high office in Rome before being appointed to their command.

to them. Indeed, there were instances when the Citerior army was fighting in Ulterior at the same time that the Ulterior army was fighting in Citerior (Richardson 1996, 54-55; Livy, 34.17,19; 35.7). The commanders even took it upon themselves to found settlements, as is the case of the praetor of Citerior, Gracchus, who founded the town of Gracchurris (Alfaro) in the Ebro valley (Richardson 1996, 58; Livy, *ep.* 41).

Pacification and conquest

After 178 BC, very little is known for some twenty years. Warfare decreased, and Appian and Livy indicate little else. All that really comes down to us is the list of the commanders sent to the provinces (Richardson 1996, 59). It is not until the mid-150s BC that Appian resumes his tale of the peninsula with accounts of two military operations (Appian, *Ib.* 44). One is the revolt of the Lusitani which lasted four or five years until it was finally put down in 150 BC through the deceit of Ser. Sulpicius Galba, who had the vast majority of the Lusitanians slaughtered after he agreed to a truce (Richardson 1996, 60-61; Appian, *Ib.* 56-60).

It was in 153 BC that a change of policy, or at least of view, from Rome began to have serious consequences for the peninsula. It seems that a lack of major fighting in other parts of the Roman world left the two consuls with little purpose. 'Generals, not interpreters of words, are chosen at consular elections' (Cicero, *Pro Murena* 38). Because other theatres of war were now closed, the Senate began to send one of the consuls to Spain, beginning with Fulvius Nobilior in 153 BC, though renewed warfare in Africa and Greece temporarily saw the consuls fighting elsewhere soon after.

Both Carthage and Corinth were destroyed in 147 BC, and in 146 BC the consul Q. Fabius Maximus Aemilianus was sent to Spain where a revolt of the Lusitanian survivors threatened Ulterior. The Lusitanians were led by Viriathus, who gathered a large coalition of natives to his banner and led them in guerrilla warfare for eight years (Richardson 1996, 64-65; Appian, *Ib.* 61-62). Viriathus wreaked havoc on the Roman forces under several different consuls and praetors. Things came to an end in 140 BC, when after a false peace, Q. Servilius Caepio, consul assigned to Ulterior, was able to have Viriathus murdered by his own friends. Caepio's successor D. Iunius Brutus gave those who surrendered land to settle while pacifying the remaining fighters over several years throughout the peninsula. Eventually, his campaigns took him to the land of the Callaeci who lived in the uttermost north-west of Spain (Richardson 1996, 65; Appian, *Ib.* 63-74).

Before his demise, Viriathus made alliances with the Celtiberian tribes in Citerior. The consul Q. Metellus Macedonicus was sent against them in 143

BC, pushing the natives back to Termantia (Tiermes) and Numantia (Muela de Garray). In 141 BC, Q. Pompeius took over the command (Richardson 1996, 66; Appian, *Ib.* 76). After failing to make significant headway, Pompeius attempted to settle with the Numantines through formal surrender. Unfortunately, the next consul, M. Popillius Laenas, arrived during the negotiations in 139 BC, whereat Pompeius denied the whole episode. Pompeius and a Numantine delegation had to take the dispute before the Senate, where it was voted that the fighting should continue (Richardson 1996, 66-67; Appian, *Ib.* 79).

The consul of 137 BC, C. Hostilius Mancinus, was defeated by the Numantines and forced to surrender. The Senate again rejected the terms that were arrived at after the surrender (Richardson 1996, 67; Appian, *Ib.* 80). Meanwhile, M. Aemilius Lepidus, Mancinus' fellow consul, was sent out to Spain in his stead while the negotiations at Rome were underway. Joining forces with D. Iunius Brutus, who still held the command in Ulterior, they attacked Pallantia (Palencia), under the possibly false pretext that the Vaccaei, whose city it was, were supplying the Numantines. The Senate ordered Lepidus to desist in his assault and, after he failed to obey and suffered defeat, stripped him of his command and fined him. His successor, the consul of 135 BC, Q. Calpurnius Piso, acted no better and continued fighting about Pallantia before wintering near modern Toledo (Richardson 1996, 68; Appian, *Ib.* 80-83).

In 134 BC, the Senate chose P. Scipio Aemilianus to bring about the end of the Numantine war. Scipio had won great fame by sacking Carthage the decade before. Scipio first subdued the nearby tribes sympathetic to the Numantines, including the Vaccaei. Then, surrounding the city with a string of seven forts, Scipio forced Numantia to capitulate unconditionally (Richardson 1996, 69; Appian, *Ib.* 84, 87, 89-90, 96).

3.1.4 Assimilation

The first true signs of provincial structure and management date back to the 170s BC and the peace treaties of Ti. Sempronius Gracchus. These treaties, concluded with the Celtiberians, contained precise stipulations regarding taxation. Other tribes seem to have had similar arrangements. It should be noted that grain was not the only form of tax levied. The mineral wealth of Spain was also documented as flowing into Rome as tax as early as Cato's command in the 190s BC (Richardson 1996, 72-73; Livy, 34.21). Polybius gives quite impressive figures for the output of mines near Carthago Nova in the mid-second century (fragment of Polybius, 34.9, quoted by Strabo, 3.2.10). By 171 BC, delegations from subdued tribes were before the Senate accusing Roman officials of various misdeeds. Though the offenders escaped conviction, the Senate passed several measures ensuring the rights of these communities (Richardson 1996, 70-71; Livy, 43.2).

Besides establishing taxation through treaties, Gracchus is also credited with founding Gracchurris on the Ebro in the 170s BC (Richardson 1996, 75; Livy, *ep.* 41), possibly as well as a town in the upper valley of the Guadalquivir, Iliturgi (near Mengíbar) (Richardson 1996, 75). Ten years earlier, the proconsul of Ulterior, L. Aemilius Paullus, established an independent town of natives at Turris Lascutana (Alcalá de los Gazules) (Richardson 1996, 76). These towns were not Roman, but places for conquered or allied tribes to settle. Scipio had founded Italica in 206 BC, and it is the only Roman establishment founded before 171 BC for which records exist. It was in this year that a *colonia* with Latin (as opposed to Roman) rights was established for the illegitimate children of Roman soldiers and native women at Carteia (El Rocadillo), on the south coast, west of Gibraltar (Richardson 1996, 77; Livy, 43.3).

Carteia was an anomaly in the Roman system. However, it was the only settlement in Spain at this time with any official legal status. Italica was, practically speaking, a *colonia*, but not officially and was more or less an independent entity. Likewise, two other settlements established for veterans lacked official recognition: Corduba (Córdoba), founded by M. Claudius Marcellus in *c.*152 BC, and Valentia (Valencia) founded by Brutus (Richardson 1996, 77-78). Possibly, this is because the creation of Carteia was petitioned for in front of the Senate, while the others were founded on the authority of the commanders only. Spain was still very much under the control of the commanders up until the Numantian war in 153 BC when it became the norm that consuls were assigned to the provinces. (Richardson 1996, 78-79).

Little is noted in the ancient sources between the Numantian war and the first of several civil wars beginning with that of Q. Sertorius, who arrived in Citerior in 83 BC. Campaigns were waged against the pirates off the east coast about the Balearic Islands where the consul Q. Caecilius Metellus founded the Roman settlements of Palma (Palma) and Pollentia (Alcúdia) on the island of Mallorca in 123 BC (Richardson 1996, 83; Livy, *ep.* 60). On the mainland, the Lusitani were in and out of revolt at the end of the second century. Results were mixed, though Rome did gain, in 104 BC, the town of Villavieja (near Alcántara) (see Richardson 1996, 84). Otherwise, little is known about the affairs of Spain during the fifty some years following the Numantian war.

There were a few signs, though, that a permanent civilian system of administration was being established. Cicero notes that at least as early as 113 BC there was a regular judicial tribunal set up in Corduba (Richardson 1996, 89; Cicero, *2 Verr.* 4.56). Further evidence of conversion from military state to civilian government appear at this time at Emporion, now called Emporiae, and Tarraco. Excavations have revealed that the military base at Emporiae was destroyed at the end of the second century BC and replaced by a Roman style town. Likewise, the walls of Tarraco were expanded to hold not only the stronghold, but also the civilian town (Richardson 1996, 90-91).

At the turn of the century, Roman villas and farmsteads began appearing in the vicinity of Baetulo (Badalona), founded about this time, and near modern-day Barcelona. Of greater import, perhaps, was the construction of roads with milestones. Stones were found marking out a road from Ausa (Vic), in the territory of the Ausetani, north-west of Barcelona, down to the coast at Tarraco. Another road from the same period runs inland through the Ebro valley from the coast. Whether these improvements were the work of the Senate or the field commanders is open for debate. Obviously, road building serves a basic military function, and was a primary reason for the construction of the Via Domitia from Italy to Spain towards the end of the second century BC, '... but this in itself suggests a greater stabilisation of the Roman military presence....' (Richardson 1996, 92).

3.1.5 Civil strife

In 83 BC Q. Sertorius was sent to govern Hispania Citerior. He became very popular with the Spanish natives by relieving tax and billeting requirements. In 81 BC, Sertorius was replaced but was called back to the province by the Lusitanians to command them and their allies in a campaign against the Roman establishment. Many disaffected tribes as well as Romans rallied to him. In 77 BC, the young Cn. Pompeius (better known to us as Pompey) was chosen by the Senate in addition to the consul already in place in Ulterior, Q. Caecilius Metellus Pius, to destroy Sertorius' forces. Sertorius shifted to guerrilla warfare. Eventually cut off from his allies, he was assassinated at Osca (Huesca), after which most of the natives surrendered to Pompey. It was during this campaign that Pompey founded a native settlement at Pompaelo (Pamplona) and Metellus, the Roman settlement at Caecilia Metellinum (most likely Medellín) (see Richardson 1996, 95-100).

In 68 BC, C. Iulius Caesar was sent to Hispania Ulterior as quaestor under C. Antistius Vetus, where his task was to travel about the province hearing cases (Suetonius, *Div. Caes.* 7; Richardson 1996, 105). Caesar returned in 61 BC as proconsul and was active in both military and civilian matters. He counterattacked the Lusitanians after they had made several raids into the Baetis river valley and used this as an excuse to push north into modern Galicia.[5]

[5]See Appian, *Bell. Civ* 2.8; Plutarch, *Caesar* 12; Livy, *ep.* 103; Cassius Dio, 37.52-53;

In 55 BC, Pompey, as consul, was given all of Spain as his command as part of the political arrangement of the First Triumvirate, during which time he captured the hill stronghold of Clunia (Peñalba de Castro) during a revolt the Vaccaei (Cassius Dio, 39.33,54). When the Triumvirate broke down and the civil war between Pompey and Caesar began, Spain was a source of great concern to Caesar. Though Pompey abandoned Rome for Greece, there were three commanders and seven legions in Spain loyal to him (Caesar, *De Ballo Civili* 1.29-30,38; Richardson 1996, 108). Caesar himself went to Spain and engaged his enemies at Ilerda (Lleida). After hard going and failed manoeuvring in the territory of sympathetic tribes, the Pompeian forces surrendered (Caesar, *B.C.* 1.37,41,61-84; Richardson 1996, 110-111). The final Pompeian commander, Varro, had stationed himself in the south at Gades. As news of Caesar's northern victory spread, a great multitude of the populace went over to him with the towns of Corduba, Carmo (Carmona), and eventually Gades expelling the Pompeian forces. Varro surrendered, and Caesar returned to Rome via Tarraco and Massilia, where he learned that he had been proclaimed dictator (Richardson 1996, 111-112; Caesar, *B.C.* 2.20-21).

Unfortunately for Caesar, the deputy he left behind in Spain, Q. Cassius Longinus, was extremely unpopular with the inhabitants and the army. The unrest led to a resurgence of the Pompeian cause. In 46 BC, Cnaeus Pompeius, son of the now dead Pompey, besieged Carthago Nova. By the end of the year, Caesar returned to Spain to engage Cnaeus and his brother Sextus. The brothers consolidated their forces in the Baetis valley, besieging Ulia and Corduba. Cnaeus did little to ingratiate himself with the population, slaughtering everyone in the town of Ucubi (Espejo) for the belief that they were sympathetic to Caesar. Caesar crushed the Pompeians, and eventually Cnaeus' head was displayed at Hispalis (Sevilla). Despite Sextus' eluding capture, Caesar left Spain for Rome at the end of the following spring (see Richardson 1996, 112-116).

In these last stages of the civil war, several municipal and Romanising developments arose. Contrebia (Botorrita) and Azaila, though destroyed during the conflict, had Roman-style buildings, as did Hispalis and Corduba (Richardson 1996, 118). Caesar gave Roman citizenship to the peoples of Gades in 49 BC, gaining official recognition as a Roman *municipium* at some point in the following six years (Livy, *ep.* 110; Cassius Dio, 41.24; Richardson 1996, 119). Olisipo (Lisbon) may also have been honoured with such a status, though it is possible it took this honour upon itself (Richardson 1996, 119). Caesar also established several official *coloniae* of Roman citizens as a policy of resettlement: Tarraco (Tarragona), Carthago Nova (Cartagena), Hasta (Mesa de Asta), Hispalis (Sevilla), Urso (Osuna), Ucubi (Espejo), and the unidenti-

Richardson 1996, 106.

fied Itucci. With the exception of Tarraco and Carthago Nova (and possibly Itucci), all of these settlements were in the Baetis river valley (Richardson 1996, 120). These *coloniae* must have significantly contributed to the Romanisation of the Mediterranean coast as well as the Baetis and Ebro river valleys (Richardson 1996, 124-125).

In 41 BC, Octavian (C. Iulius Caesar), took control of Spain following M. Aemilius Lepidus (Cassius Dio, 48.1). From the beginning of Octavian's command in Spain to his acceptance of the title 'Augustus', details of the military activity in Spain is far from detailed (Richardson 1996, 130). The Cerretani tribe of northern Spain near the Pyrenees was defeated in 39 BC (Cassius Dio, 48.42). In 29 BC, the proconsul T. Statilius Taurus was engaged with the Vaccaei, Cantabri, and Astures in north-central Spain near the Durius (Douro) river. At least two settlements are thought to have been founded during this period: Celsa (Velilla de Ebro) and Norba Caesarina (Cáceres). Celsa was originally founded as Victrix Iulia Lepida, though its name was shortened and eventually changed after the fall of Lepidus. It is located in the Ebro valley to the east of modern Zaragoza (see Richardson 1996, 132-133).

3.1.6 Spain in the early Empire

The Senate bestowed the unprecedented title of 'Augustus' on Octavian in 27 BC. At once, he mounted a military expedition and marched north into Gaul. There was an expectation that he would continue north and invade Britain, but was turned aside and spent 26 and 25 BC in Spain (Cassius Dio, 53.22). The campaigns were conducted primarily in the north and north-west, meeting some tribes of these areas for the first time, or, at least, the first time in their own lands. The Astures and the Cantabri were sufficiently subdued that Augustus officially declared peace throughout the Roman world by closing the doors of the temple of Janus in Rome. However, Spain was not entirely in Roman hands and fighting continued, even with the Astures and Cantabri, in 22 BC. In 19 BC, Augustus' right hand, Agrippa, dealt yet again with the Cantabri by completely annihilating the fighting men and relocating the rest of the population into less defendable areas (Cassius Dio, 53.26, 54.5,11; see Richardson 1996, 133-134).

It is about this time (Cassius Dio gives 27 BC though this is far from certain) that Spain was further divided into three provinces: Lusitania, Baetica, and Tarraconensis. Lusitania and Baetica were formed out of Hispania Ulterior, while the third province was either referred to as Tarraconensis (taken from its chief city Tarraco) or its original name, Hispania Citerior. Under the arrangement made between the Senate and Augustus, Baetica was to be administered through the Senate, whilst Augustus would have direct command over Lusitania and Tarraconensis (Cassius Dio, 53.12; see Richardson 1996, 135-136).

Several important settlements were established during the reign of Augustus. The Emperor is said to have established Emerita Augusta (Mérida) as a *colonia* on the River Anas (Guadiana) after the campaigns he supervised in 26 and 25 BC for his retiring veteran soldiers (Cassius Dio, 53.26). Emerita Augusta was made the capital of the newly formed Lusitania (Corduba and Tarraco being the capitals of Baetica and Tarraconensis, respectively). Other *coloniae* established in this period include Barcino (Barcelona) and Caesaraugusta (Zaragoza). Development was widespread throughout the whole of Spain from new settlements to the refashioning of native towns (Richardson 1996, 142-143). Despite the foundation of Roman towns at Lucus Augusti (Lugo), Bracara Augusta (Braga), and Asturica Augusta (Astorga) in the newly pacified areas of Tarraconensis, urbanisation was essentially limited to the older parts of Roman Spain (Richardson 1996, 145).

3.1.7 After Augustus

Between civil wars, founding of towns and *coloniae*, and the new division of the provinces, Spain had undergone many abrupt changes. Under the remaining Julio-Claudian emperors, a period of calm and consolidation was ushered in (Richardson 1996, 149). Clunia may have been granted *municipium* status by Augustus as it seems to be the case with Baelo Claudia (Bolonia) on the Atlantic coast by Claudius (Richardson 1996, 158). The increased status of some newer settlements even overshadowed the importance of long established towns and cities, forcing them into decline. For example, Emporiae and Baetulo were overtaken by Gerunda (Girona) and Barcino. The *colonia* of Celsa was altogether abandoned, and its population was integrated into Caesaraugusta (Richardson 1996, 190).

3.1.8 Rise of the Flavians

After the death of Nero in AD 68, the Roman world was thrown into political and military turmoil. Peace was restored in AD 70 when Vespasian defeated his opponents and established the Flavian dynasty. After the turmoil of the previous few years, Vespasian reduced the legionary complement of Spain to one, the VII Gemina. This was part of a restructuring of the military in which the legions were sent to more permanent locations along the frontier of the Empire as opposed to the previous system of raising and sending legions to problem areas as events warranted. Consequently, there was a much greater mingling of the military and local populations in the areas to which legions were stationed. Following the wars which saw Vespasian rise to power, the forces which had previously been in Spain were not returned but sent to these frontier posts (Richardson 1996, 189).

The Iberian peninsula, with this great reduction of military presence, was no longer the frontier area it had once been, but a core part of the Empire. In fact, Pliny the Elder writes that Vespasian granted Latin rights to all of Spain (Pliny, *NH* 3.30; see Richardson 1996, 189-190). Many Flavian-era *municipia* made their first appearances in Baetica and eastern Tarraconensis. The north-west of the latter province, which was the only military area left on the peninsula, received a single *municipium* at Aquae Flaviae (Chaves) in the north of modern Portugal (Richardson 1996, 191).

Pliny, writing in the AD 70s, records that within each of the three Spanish provinces there were judicial regions, known as *conventus*, to aid the governor in his duties.[6] It is true that during the reigns of Augustus and Tiberius, the governors of Tarraconensis would judge cases throughout their province at fixed settlements, but it was not until the Flavian dynasty was established that the *conventus* system arose with its clearly defined geographical boundaries arose. It was certainly not until then, that these areas began to be used for more than simply judicial purposes (Richardson 1996, 211). By the ascension of Trajan in AD 98, it is clear, at least in the minds of the Romans living there, that Spain was indeed a significant and substantial part of the Empire.

3.2 The cities

The development of towns and cities of which the Romans had knowledge (and eventually direct control) began in the south with the Carthaginians and the north-east with the Greeks. These two spheres of influence grew up and down the Mediterranean coast. When they met, Rome, which had begun to exert influence on the Greek settlements, was irreversibly drawn into Spanish affairs. After the Carthaginians were expelled, Rome found itself in nominal control of the coast from the Pyrenees to the mouth of the Guadalquivir beyond the Pillars of Hercules. Development, or, more properly, conquest and subsequent development, grew inland along the major rivers. The province of Hispania Ulterior expanded northward along the Atlantic coast and inwards along the rivers Guadalquivir and Guadiana. Hispania Citerior expanded north-west along the River Ebro and eventually westward to the mouths of the rivers Tajo and Douro. After the division into three provinces, despite continued

[6] For a list of all *conventus*, see Pliny, *NH* 3.7 (Baetica), 3.18 (Tarraconensis), and 4.17 (Lusitania).

expansion of controlled territory in the north-west, most new settlements were in the province of Baetica and along the Ebro.

Given this system of development, we shall examine Spain in several configurations. First, we shall look at the peninsula as a whole. Next, we shall divide Spain into its three constituent provinces as Ptolemy himself did. Finally, we shall look at divisions along this historic development, namely configurations involving the north-east and Ebro river valley and the eastern and southern coasts (see Figure 3.2).

3.2.1 The peninsula as a whole

Initial analysis shows that the mean coordinates of Ptolemy's known cities are $(10.72°, 40.45°)$. The corresponding modern coordinates are $(-3.70°, 39.85°)$. The Ptolemaic latitude is $6/10^{\text{ths}}$ of a degree from its modern equivalent. Naturally, the longitude is substantially different, though with Ptolemy's London (and so, Greenwich), at a longitude of 20°, we can see that his longitudes in general become inaccurate very quickly.

Looking at the 155 identified cities, the initial sum of squares Procrustes error is approximately 2.3 million km^2. An anti-clockwise rotation of a little over 13° reduces the error to approximately 1.3 million km^2. Stress comes in at 3.32%. A skewed scaling with a longitudinal increase of 7.0% and a latitudinal decrease of 2.9% reduces the stress to 3.20%.

Mean city stress begins at 3.66% and improves to 3.59%. Looking at Figure 3.3, we can see that the highest errors are located in the sparse centre of the map, with mid-range errors scattered about. The highest concentration of low stress cities is in the south, with strings of low error in the north and north-east along the coasts. Ptolemy's configuration (Figure 3.4) is more densely packed, but the relative positions of the higher and lower stresses are very similar to those of the modern configuration. Perhaps an exception to this is the north-west, which excludes any cities outside the lowest tier of stress.

The range of stresses is not significantly altered by the transformation. Looking at Figures 3.5-3.7, we can see that a large number of cities took on more stress during the minimisation. These are heavily concentrated in the south with a nontrivial presence in the centre and centre-north. With the exception of cities #113 and #112 (Faro and Tavira), though, the cities of the south were not lifted out of the two bottom stress brackets. The cities that improved the most appear to be those in the north-east, cities that did not need improving in the first place.

The city map of Hispania is rather too crowded to get a good idea of how the cities relate to one another. Two areas do stand out for comment, though. On Ptolemy's map the northern half of the west side has a gap, where on the

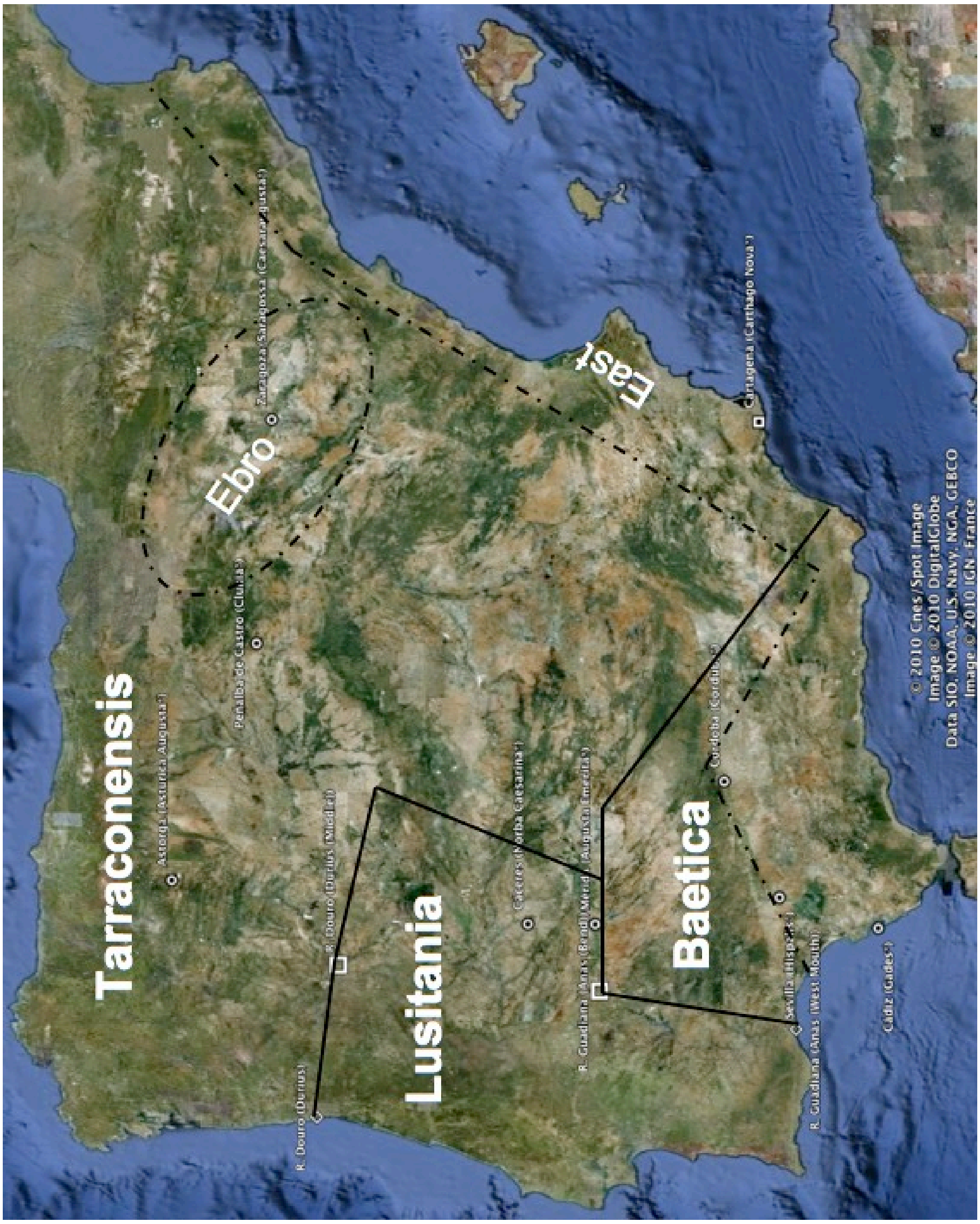

Figure 3.2: Hispania's divisions (base map ©Google 2010)

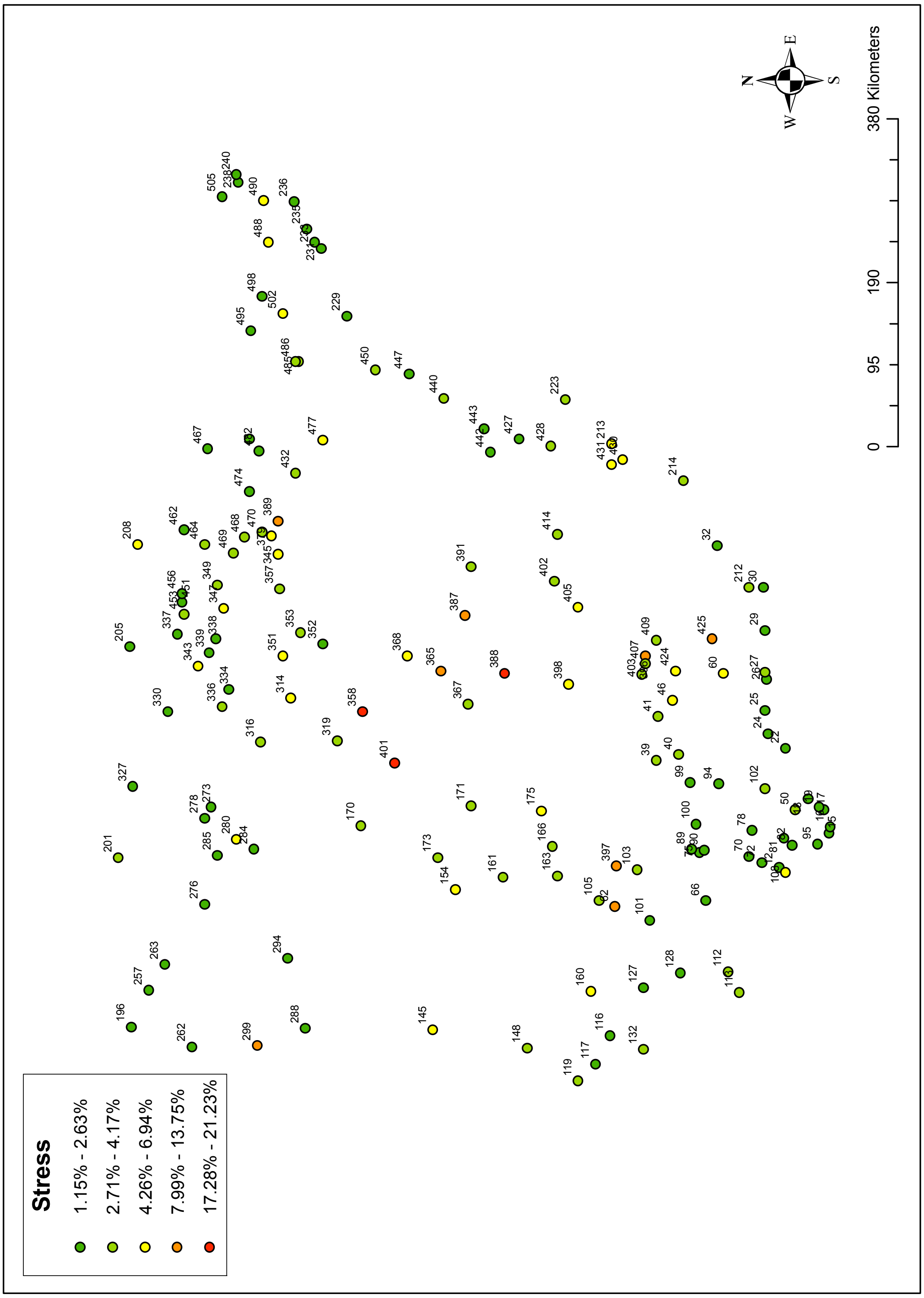

Figure 3.3: Hispania: modern configuration with initial stress

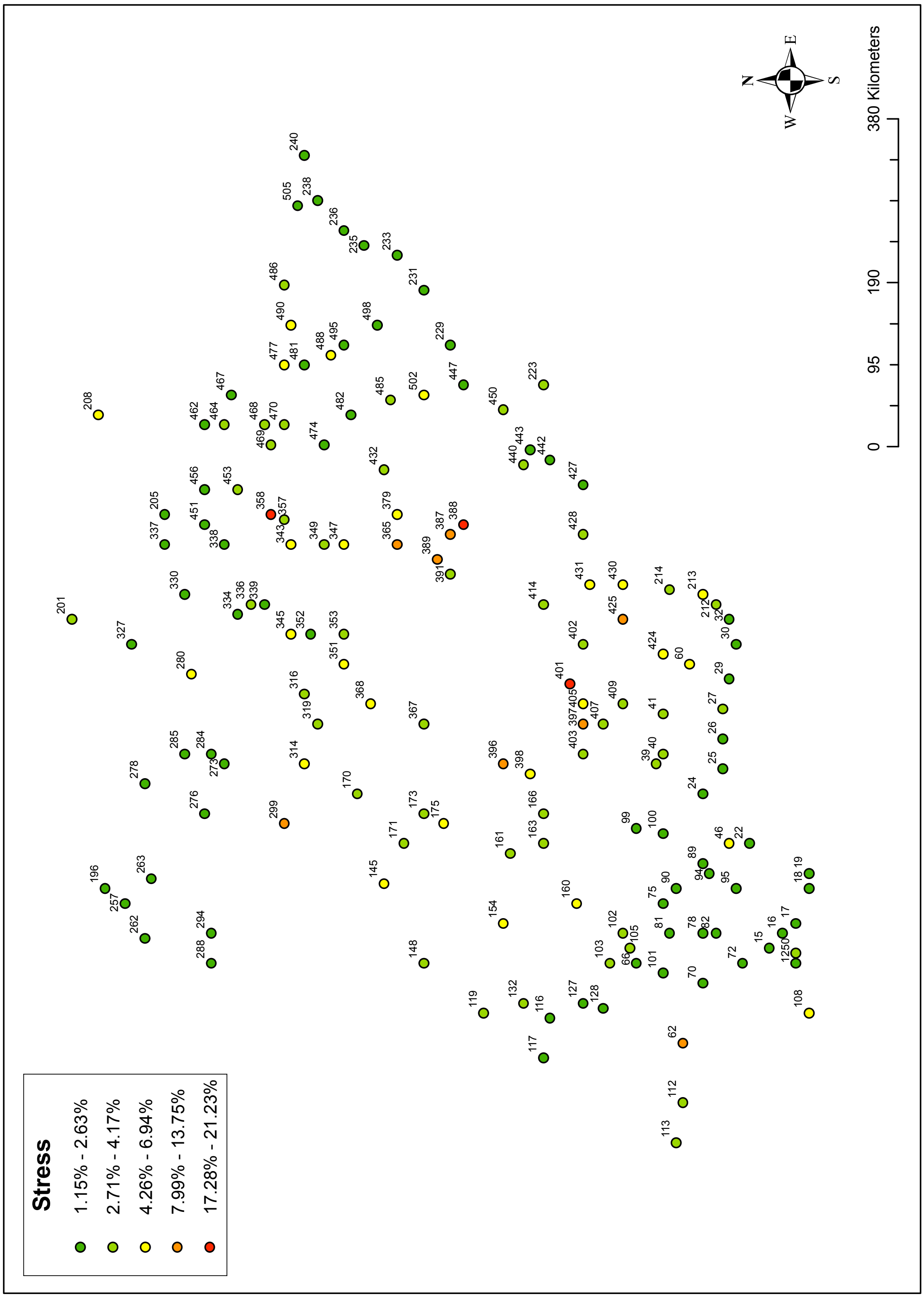

Figure 3.4: Hispania: Ptolemy's configuration with initial stress

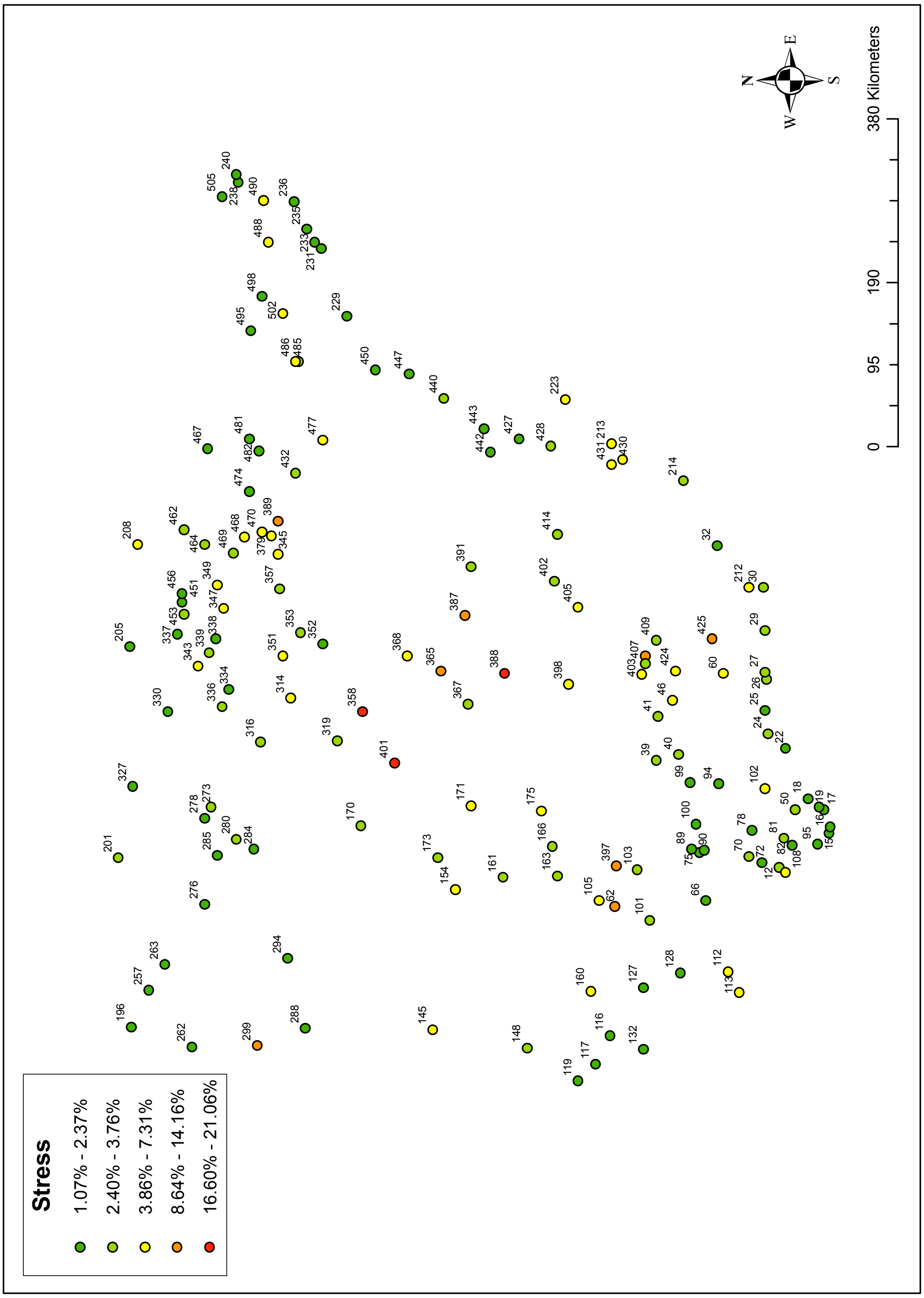

Figure 3.5: Hispania: modern configuration with transformed stress

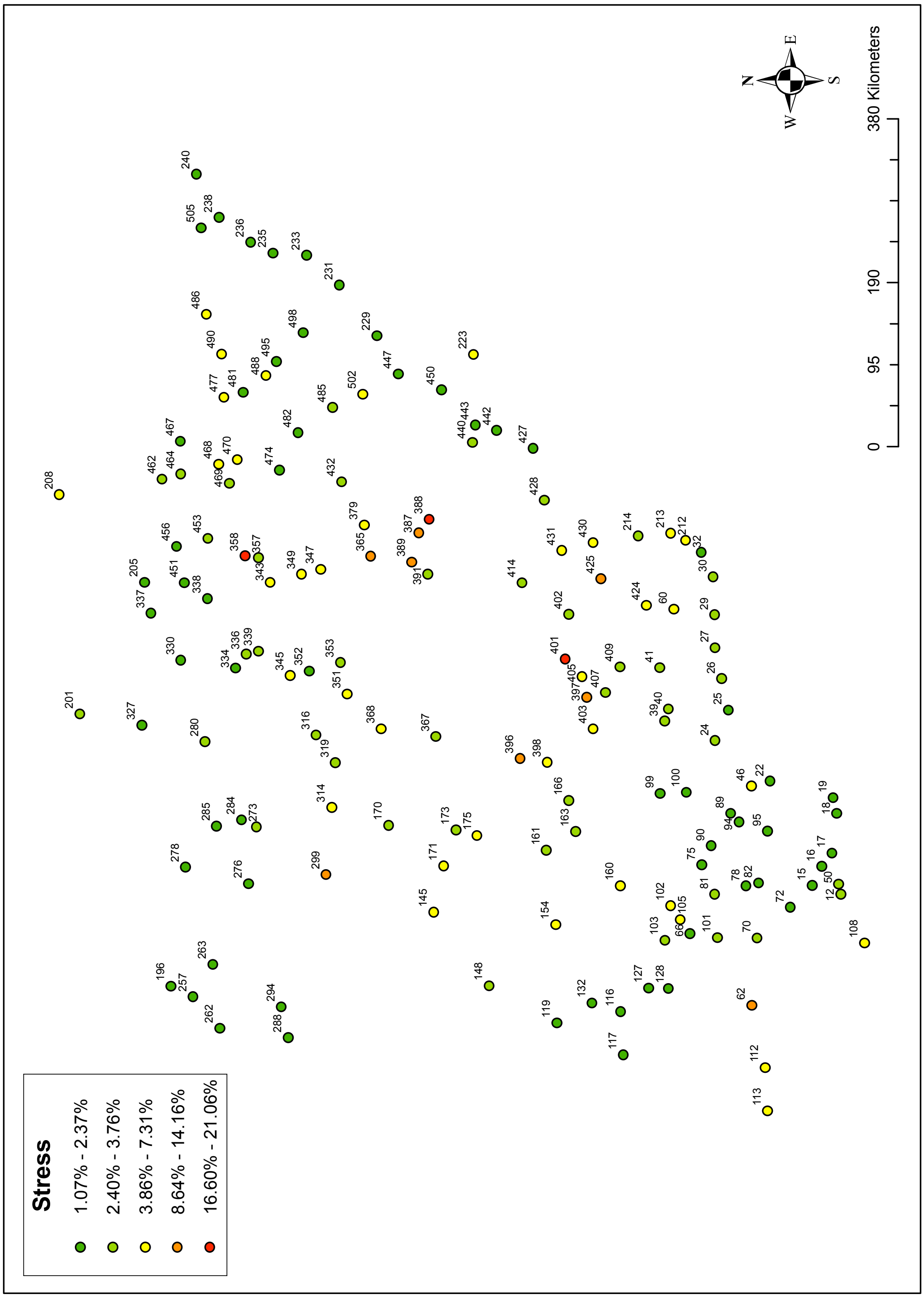

Figure 3.6: Hispania: transformed configuration with transformed stress

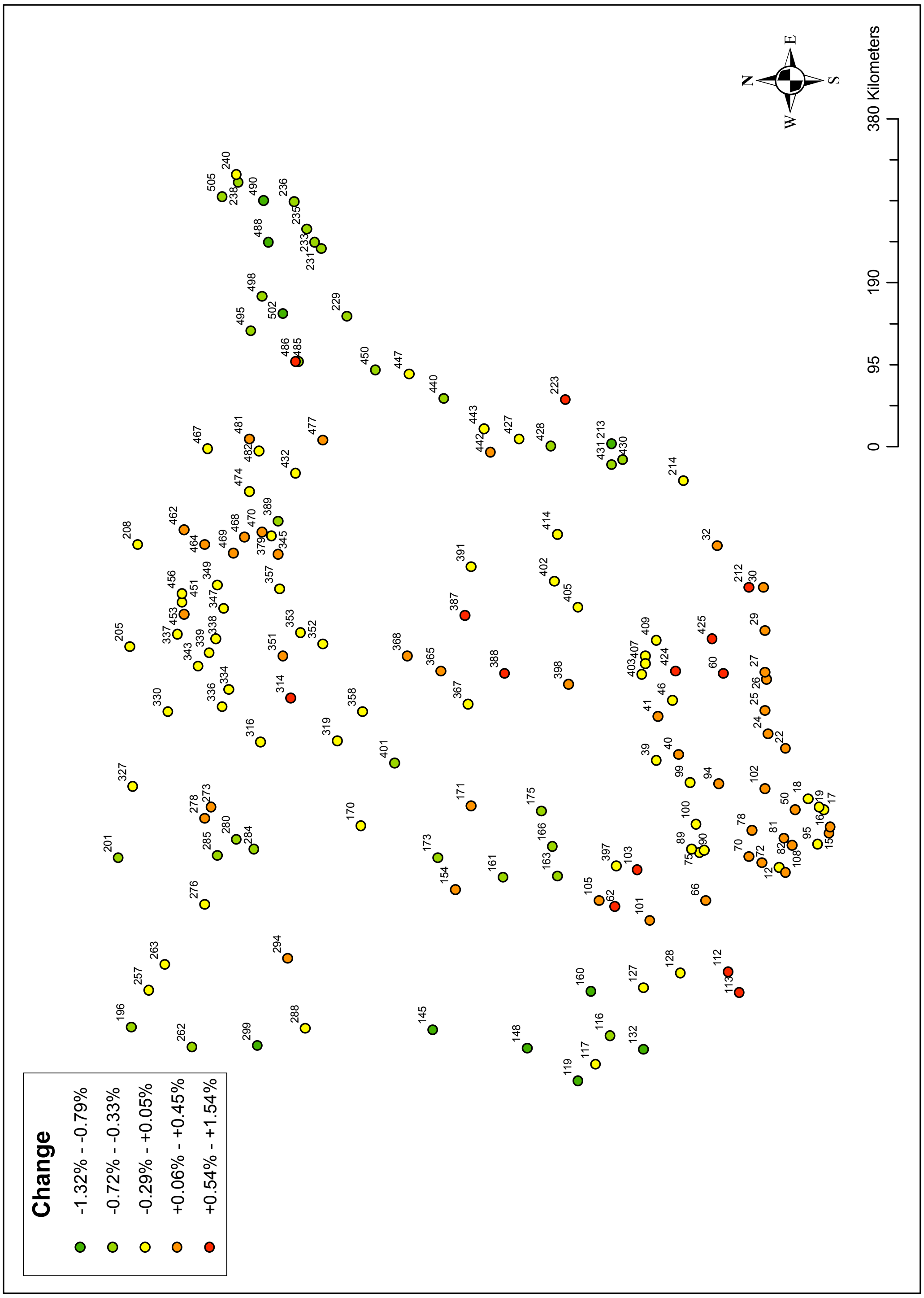

Figure 3.7: Hispania: modern configuration with change in stress after transformation

modern map there is a north-south row of cities. Also, on the modern map to the right and above the origin is a large ovoid gap which is not present on Ptolemy's map, unless it is the slender diagonal area surrounding the origin.

3.2.2 Baetica

There are 39 identified cities in Baetica, including Cádiz (ancient Gades). Cádiz is technically on an island, but, as it played such an important role in the history of Roman Spain, was very close to the mainland, and had major roadways leading to it, it has been included with the other known cities. The initial Procrustes error for Baetica is 183 thousand km^2, reduced to 180 thousand km^2 after an anti-clockwise rotation of 4.3°. The initial stress is 12.55%. The needed scaling is a uniform reduction. Scaling resulted in a 14% reduction in both latitude and longitude, bringing the stress down to 10.20%.

Looking at Figure 3.8, we can see that the centre of the map contains the cities of higher stress, with the cities of lower stress off to the north and east. Ptolemy (Figure 3.9) indeed fairly accurately portrays the cities heading eastward along the coast, but his northern cities are intermixed with the main body. It is not difficult to see why Ronda has a high stress. Cádiz is much too far away from the mainland. In general, Ptolemy's configuration is much too dense, lacking the gaps of the modern configuration between Aroche and Niebla, Sevilla and Lebrija, and Osuna and Granada.

After its transformation (Figures 3.10 and 3.11), the individual mean stress for the cities of Baetica drops from 14.87% to 11.43%. The maximum stress drops from 65.29% to 44.44%, but the minimum actually rises from 2.21% to 3.44%. In fact, the city with the initially minimum stress, Vera, has its stress increased to 4.24% by the transformation. Thus, the range of each stress bracket changes quite a bit after the transformation. The east spur of cities remains at the bottom of the stack, but the northern group lost second place and has become rather mixed. The south improves across the board with the big exception of Barbate, which worsens by over 3 percentage points. The centre has many scattered, but no means universal, improvements (see Figure 3.12). The lack of those major gaps seen in the modern map is not fixed, and, indeed, the configuration becomes more condensed after scaling. This reminds us that stress is not a measure of shape analysis but of inter-point distances, and Ptolemy's extreme northern and southern points (Monesterio and El Rocadillo, respectively), for example, need to be brought closer together to minimise stress. These two are, in point of fact, not the modern configuration's extreme points, which further explains why a contraction and not an expansion was needed for the minimisation of the stress in Baetica.

On the modern map of Baetica, it is quite evident where the road and river

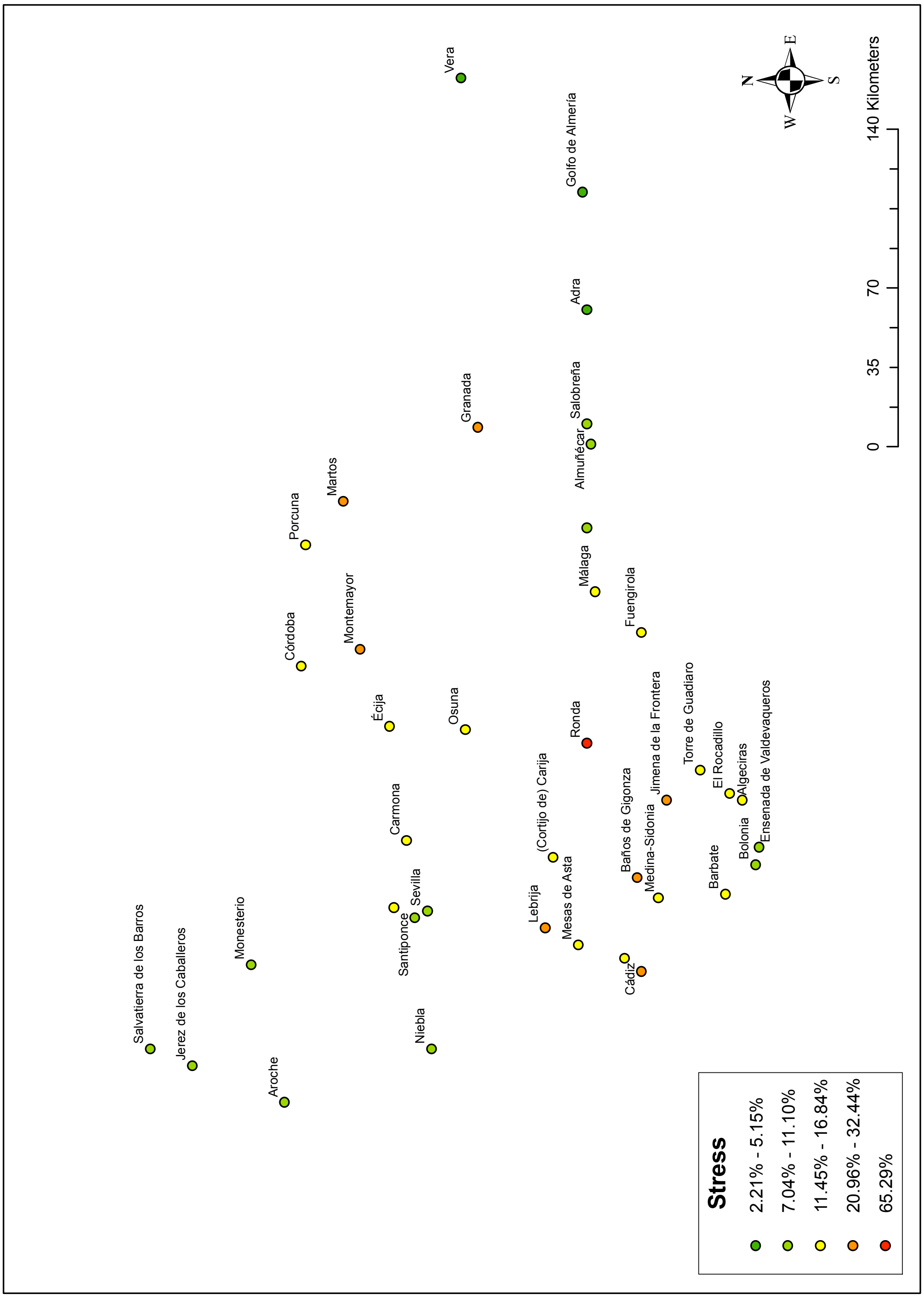

Figure 3.8: Baetica: modern configuration with initial stress

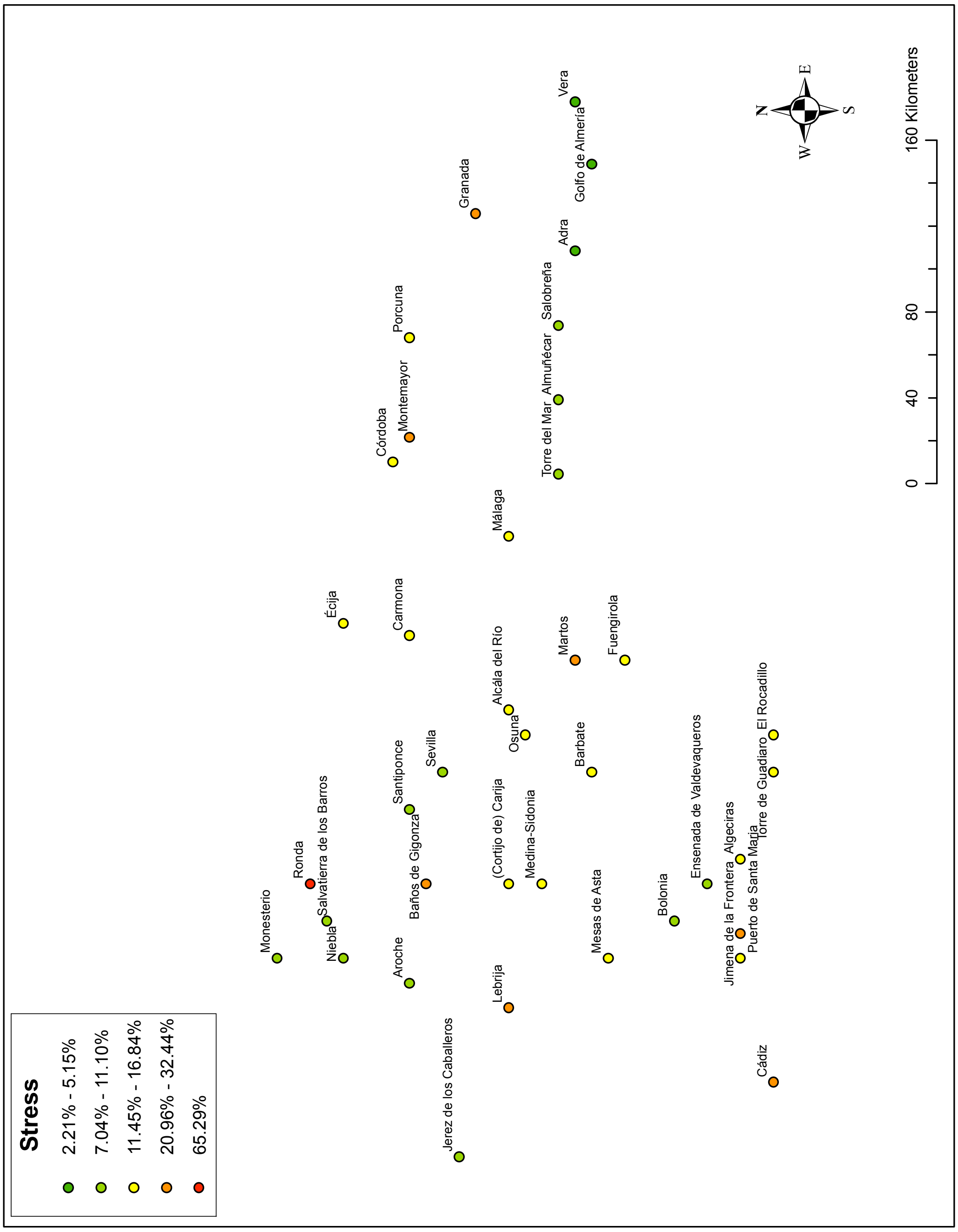

Figure 3.9: Baetica: Ptolemy's configuration with initial stress

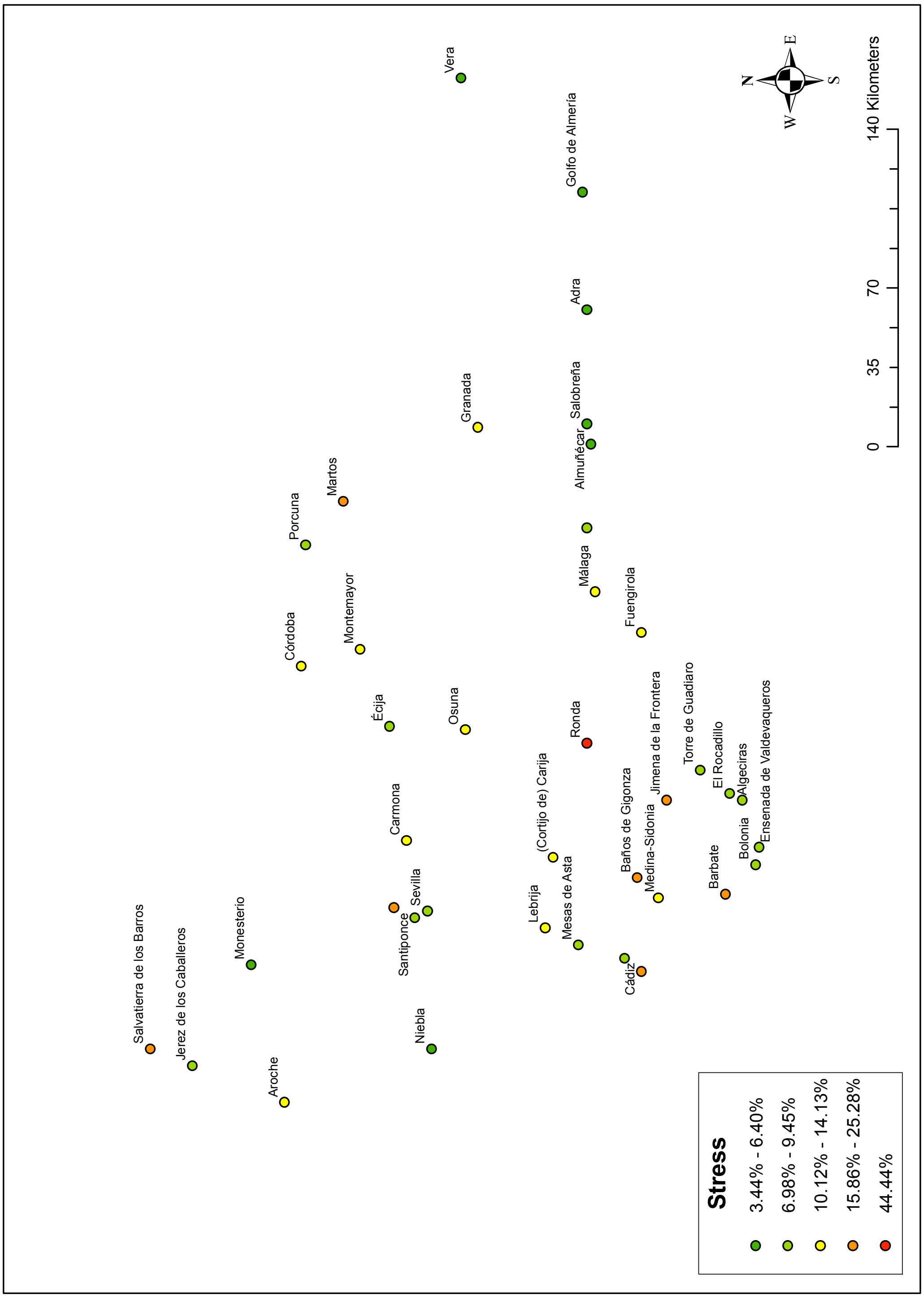

Figure 3.10: Baetica: modern configuration with transformed stress

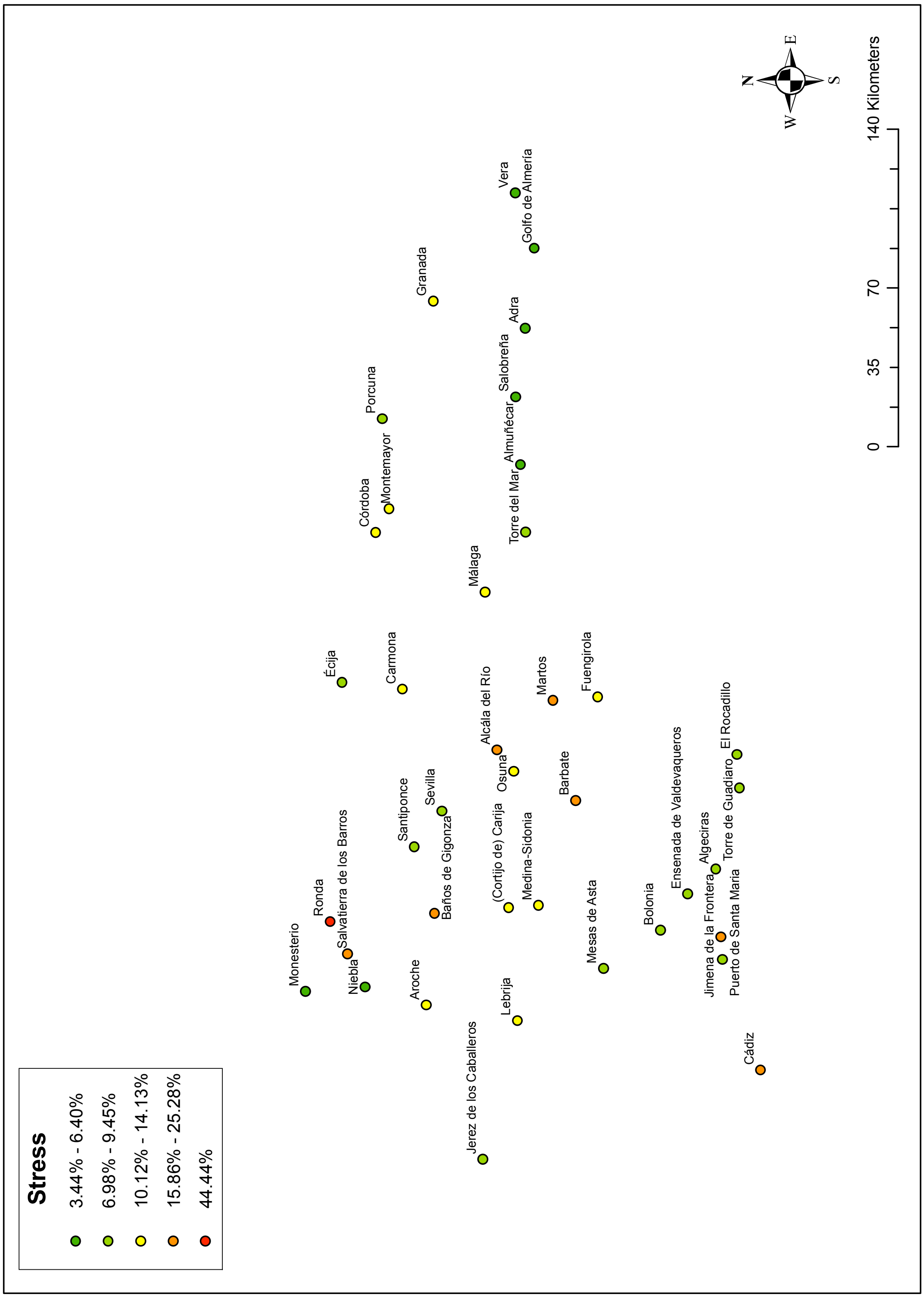

Figure 3.11: Baetica: transformed configuration with transformed stress

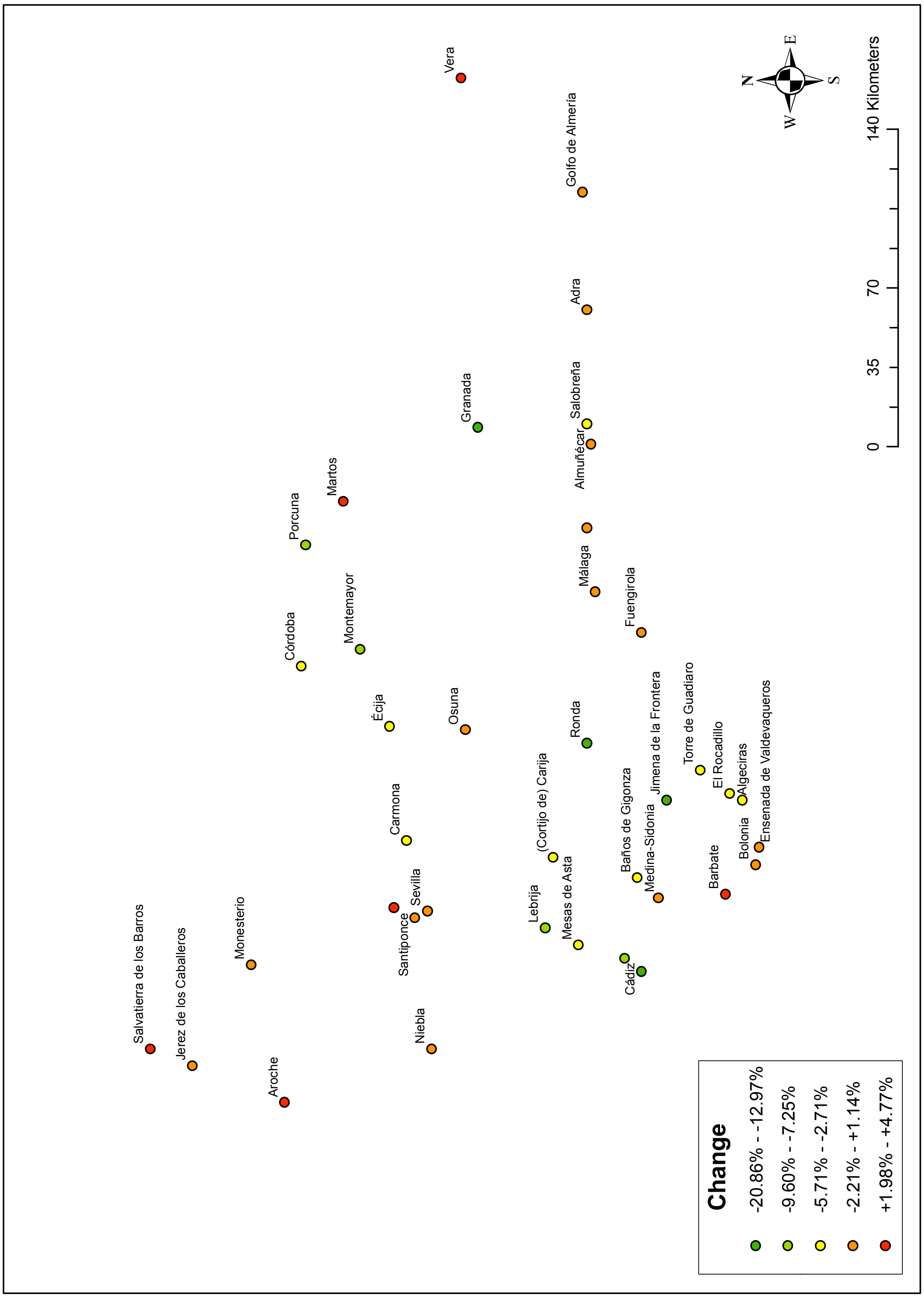

Figure 3.12: Baetica: modern configuration with change in stress after transformation

networks of the Guadalquivir diverge from the coast, creating a loop of points to the east of the origin. This is much less clearly defined in Ptolemy's map, and begins with Granada, which is much too far east. Besides this, the ring of cities of the modern map is missing on Ptolemy's.

3.2.3 Lusitania

Lusitania has 19 known cities with an initial stress of 9.63%. The sum of squares error is 168 thousand km^2. An anti-clockwise rotation of 20.5° reduces that error to 116 thousand km^2. Skewed scaling provides a longitudinal expansion of 11% combined with a latitudinal reduction of 14%. This scaling minimises the stress at 8.92%.

Figures 3.13 and 3.14 show the modern and Ptolemy's configurations, respectively, of Lusitania with initial city stress. We can see clearly that the modern map has two main groups of cities, essentially a coastal group and an inland group. Ptolemy's map, by contrast, has four groups: Faro and Tavira, Lisbon south to Mértola, Santarém south to Évora, and Coimbra and Salamanca south to Mérida and Medellín. Each group in the modern configuration has its good and bad points. Ptolemy has his worst points almost straight down the middle of his map. In fact, with the exception of the north-eastern corner, Ptolemy's map can almost be sectioned by the color-coding of Jenks stress brackets.

Following its transformation, Lusitania's mean city stress falls from 9.77% to 9.23%. The range, however, of stress values increases as the minimum is reduced by 0.50% compared to the rise of 1.52% of the maximum. The rotation and scalings certainly help distinguish the western, coastal group from the eastern, inland group (see Figures 3.15 and 3.16). Faro and Tavira are still quite far detached from the main body of cities, but further reductions in scaling would condense the rest of the configuration too much. Coria and Évora can be seen moving in the wrong direction from their modern counterparts and together forming a centre line in the province, but each should be separated by quite a distance. Coimbra also finds itself on the wrong side of the divide. On either side of the centre line, the error is fairly evenly mixed, though the biggest losses of stress are all found on the west side: Tavira, Faro, and Santarém all shed over 3 percentage points (see Figure 3.17).

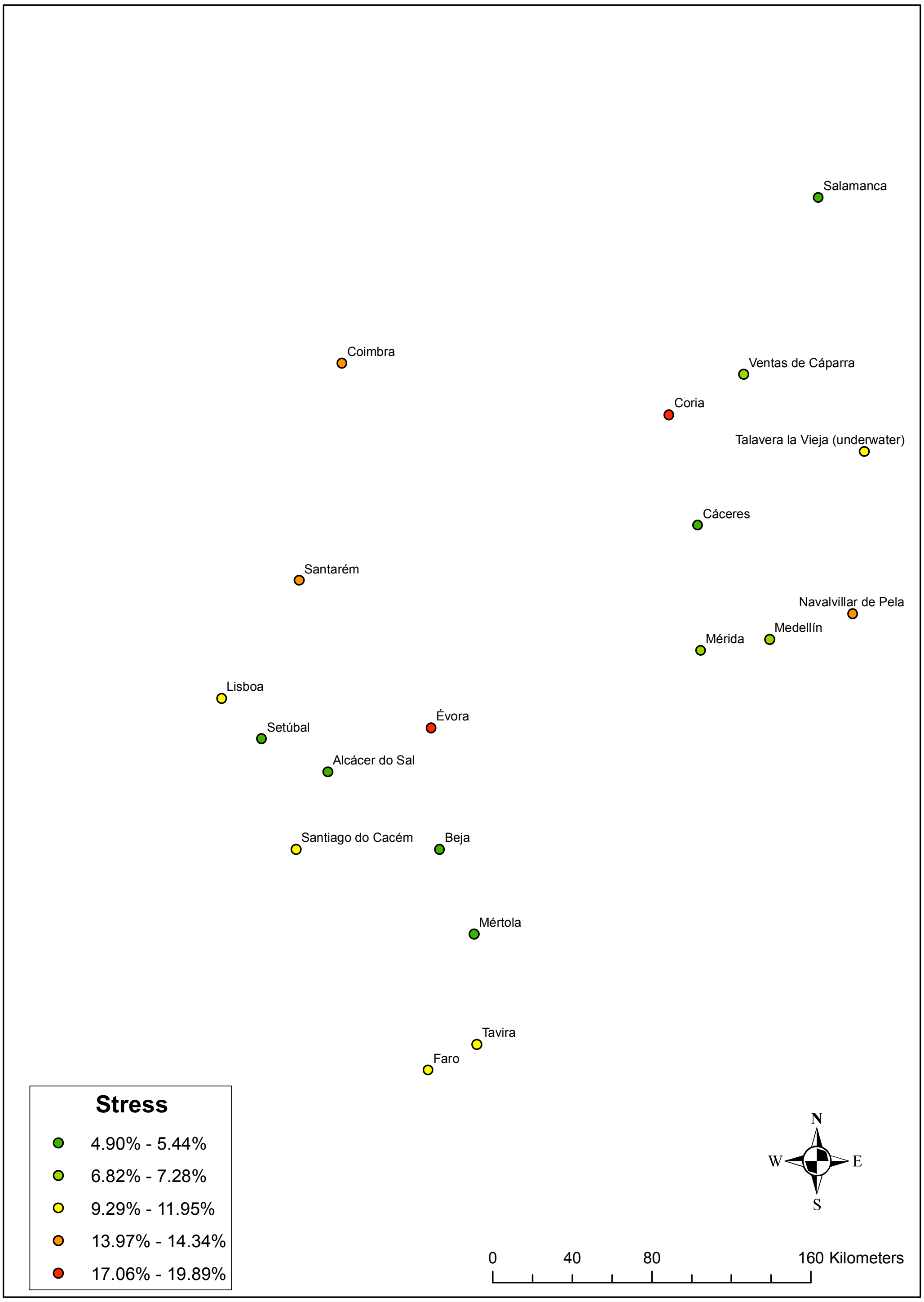

Figure 3.13: Lusitania: modern configuration with initial stress

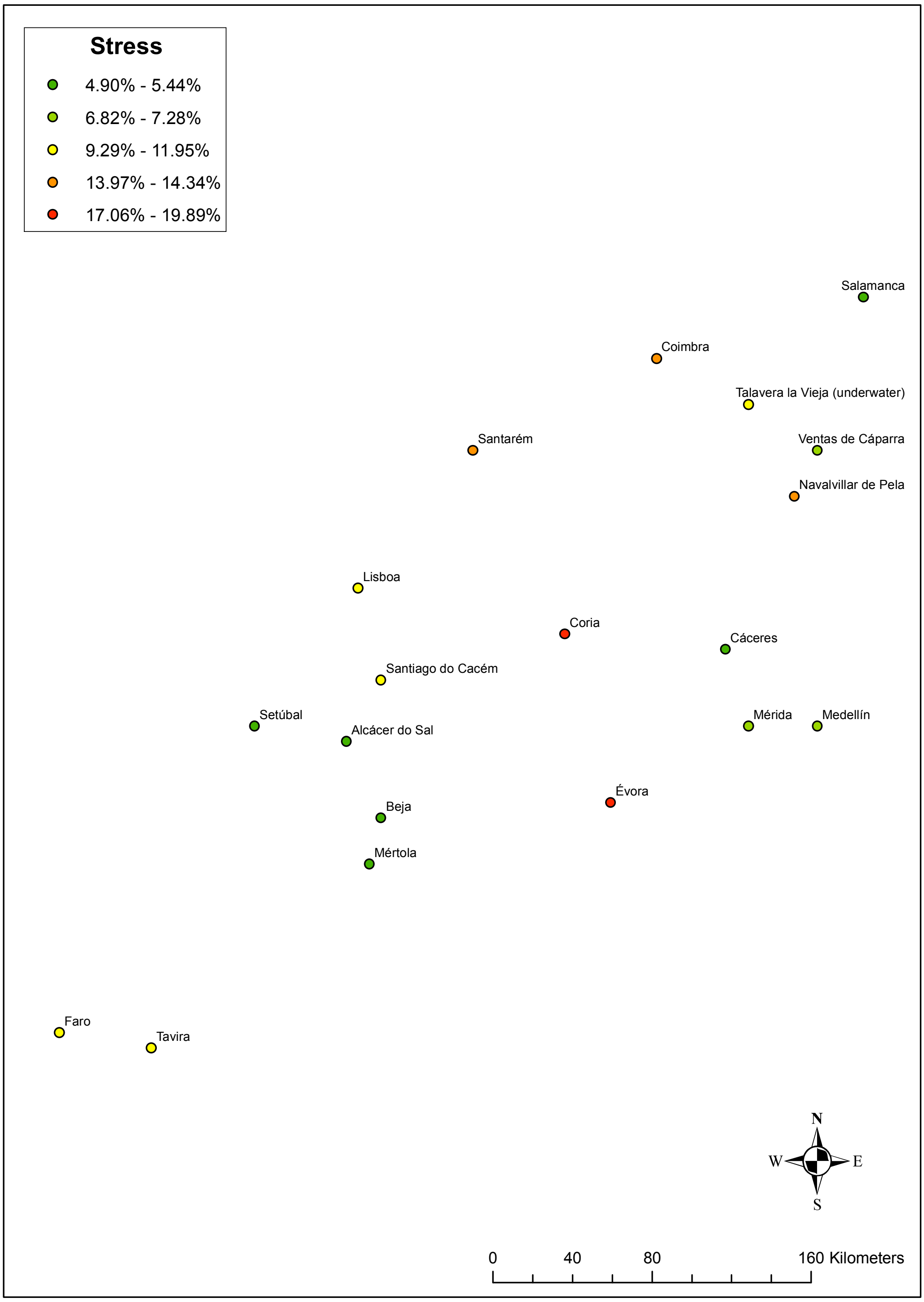

Figure 3.14: Lusitania: Ptolemy's configuration with initial stress

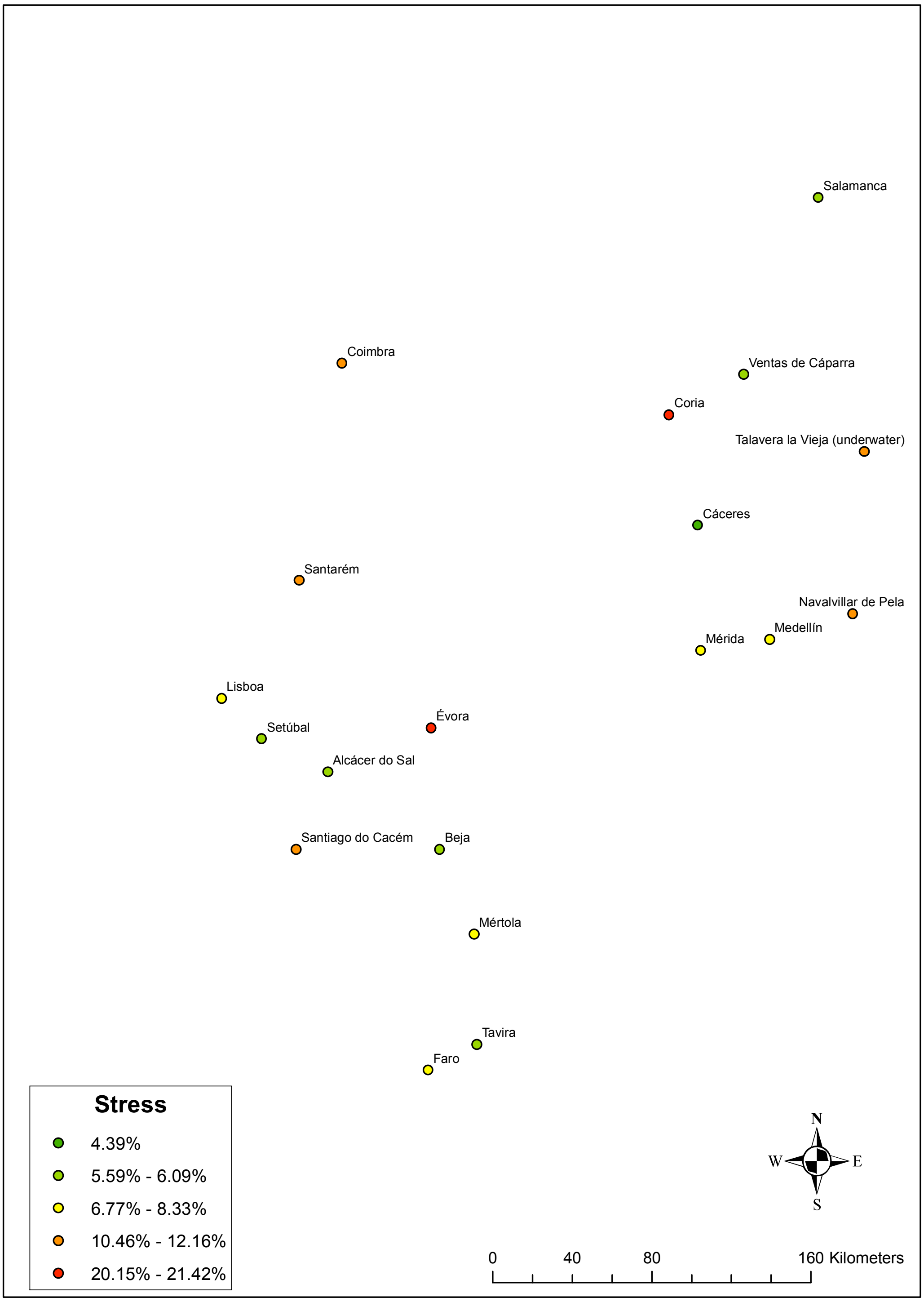

Figure 3.15: Lusitania: modern configuration with transformed stress

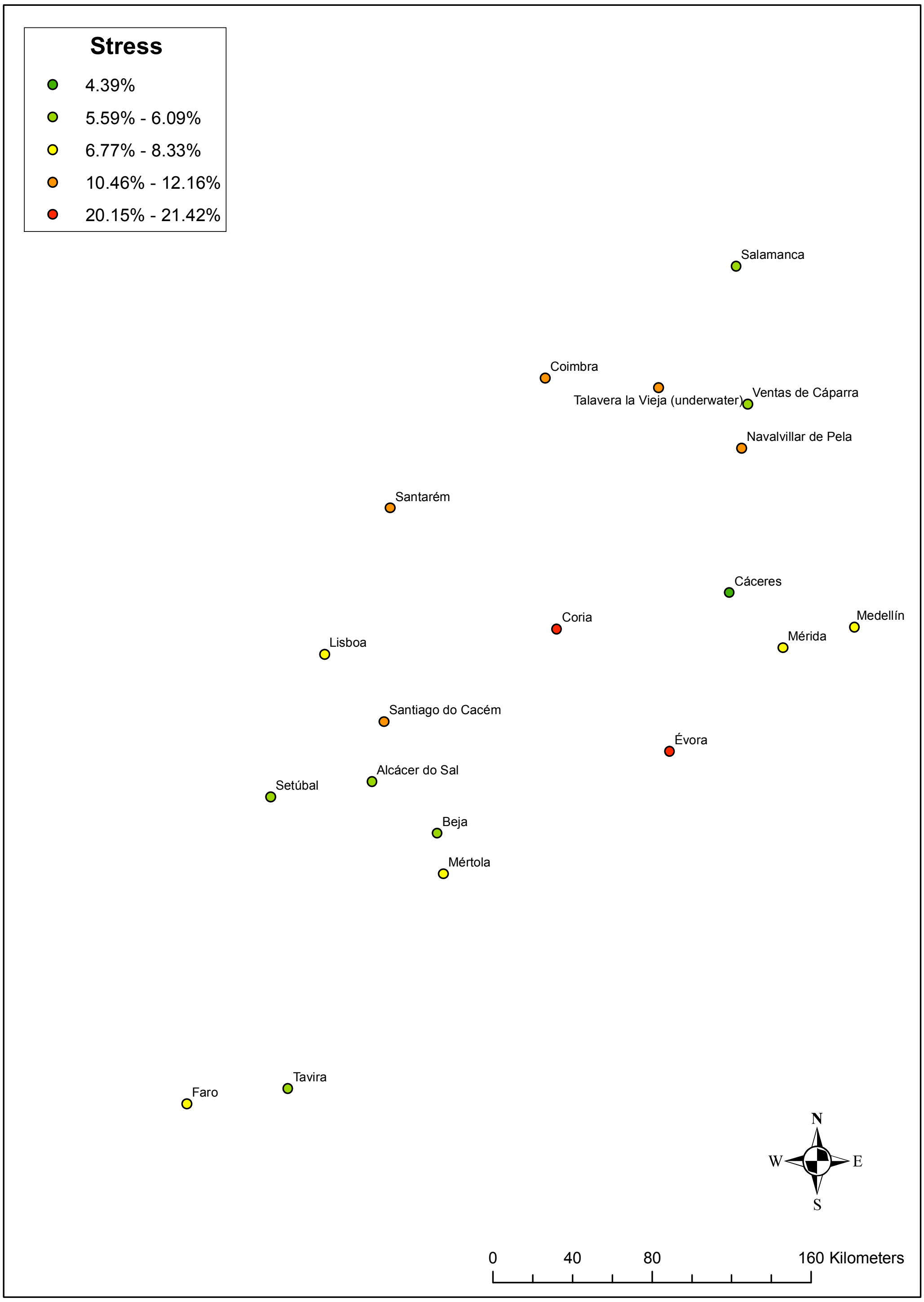

Figure 3.16: Lusitania: transformed configuration with transformed stress

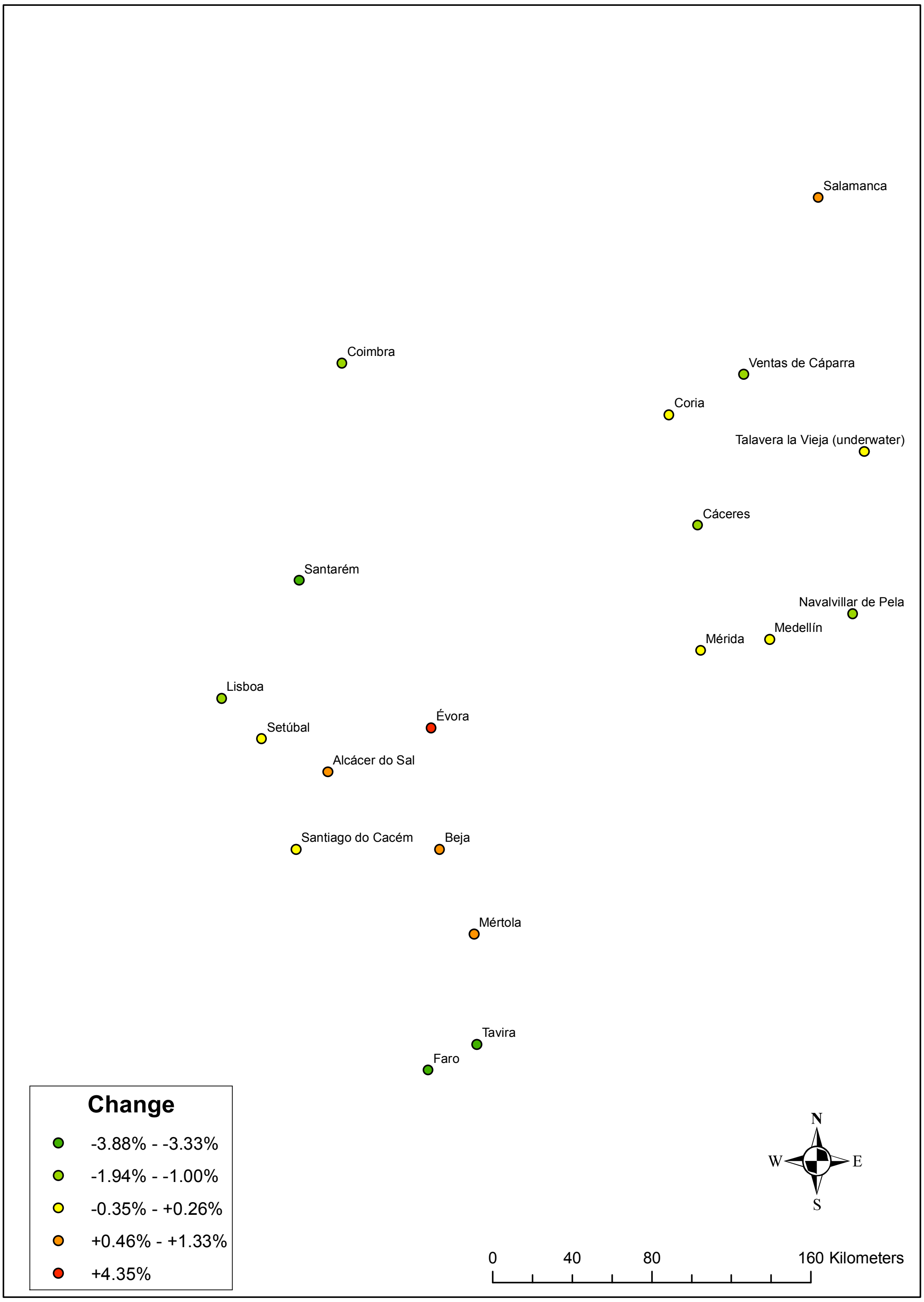

Figure 3.17: Lusitania: modern configuration with change in stress after transformation

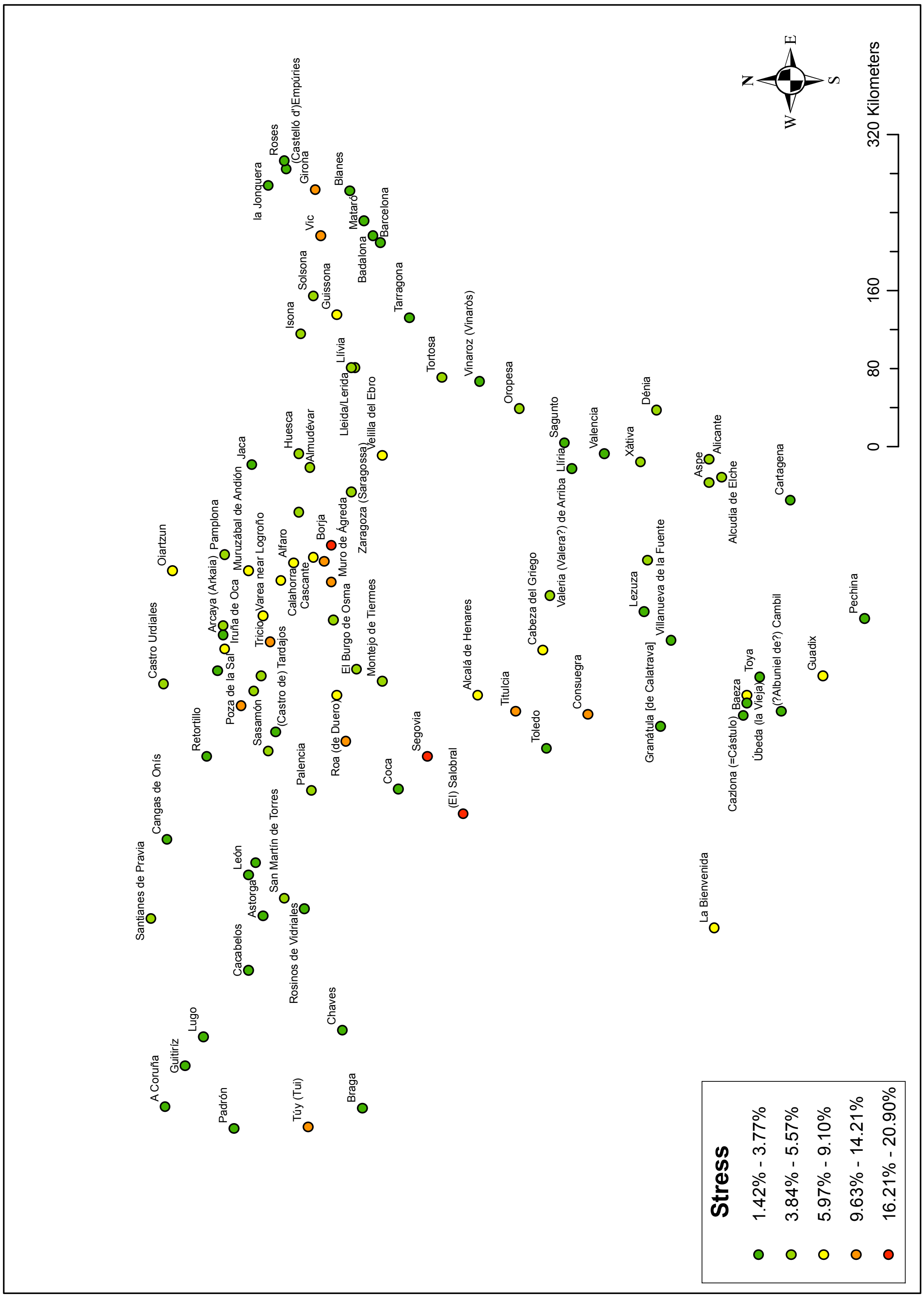

Figure 3.18: Tarraconensis: modern configuration with initial stress

3.2.4 Tarraconensis

The whole of the province

The 97 cities of Tarraconensis that can be identified begin with a stress of 4.98%. Initial Procrustes error is 1.4 million km^2, down to 890 thousand after a 14.2° anti-clockwise rotation. Scaling calls for a 9% increase in longitude and a 5% increase in latitude. This leaves the stress at 4.54%.

If we examine Figure 3.18, we can see that with the exception of Vic and Girona, the east coast of Tarraconensis is a bastion of low stress. The north-west, likewise, with the exception of Túy, contains mostly below average stresses. The cities of higher stresses are concentrated in the centre and centre-north. These are the headwaters of the rivers Douro and Ebro, respectively. Looking at Ptolemy's configuration (Figure 3.19), we note indeed that Vic, Girona, and Túy are placed at some distance from the east coast and north-west, respectively, and that otherwise these areas are very low in stress. The cities with the higher stresses are more clustered around the centre of Ptolemy's map than in the modern configuration. The whole centre of gravity of Ptolemy's map has migrated south, filling in what is empty space on the modern map.

Mean city stress is reduced from 5.46% to 5.20% by the transformation. The maximum stress of Salobral is increased by 2.01 percentage points, expanding the range of stress values. Figure 3.20 shows a substantial increase in the number of cities classified in the highest two stress brackets at the cost of the mid-range cities. Figure 3.22 displays the improvements, or lack thereof, of the cities. Most either worsen or experience only mediocre improvement, confirming what we saw in Figure 3.20. The scalings make little impression visually (Figure 3.21), but the rotation is clearly visible. Instead of the error being clustered north of centre as in Ptolemy's original, the cities of highest stress make a 'T' shape east from Túy to Llívia and south from Segovia to Consuegra. One portion of this 'T', at least, will be examined further: the Ebro river valley.

The Ebro valley

Though getting a later start than Baetica, the Ebro valley was the centre of urbanisation in the north-east of Spain. The 18 known cities along the Ebro and its tributaries have been analysed. Far from being a model of internal cohesion, though, the errors are very high. Stress begins at 41.66% with a corresponding Procrustes error of 153 thousand km^2, reduced to 148 thousand km^2 after a 10° anti-clockwise rotation. Skewed scaling reduces the longitude by 27% and the latitude by 39%. Final stress is an upsettingly large 22.98%.

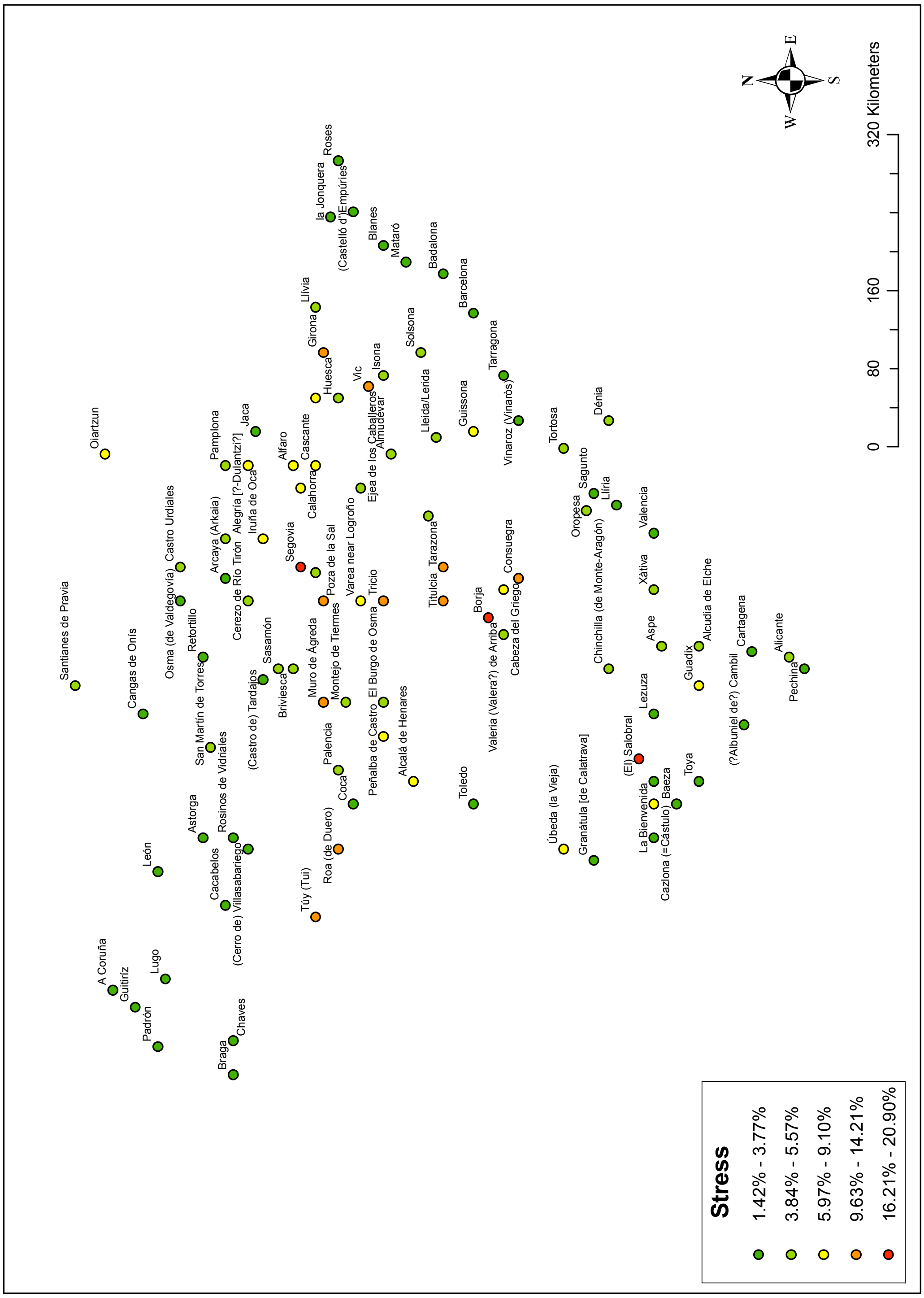

Figure 3.19: Tarraconensis: Ptolemy's configuration with initial stress

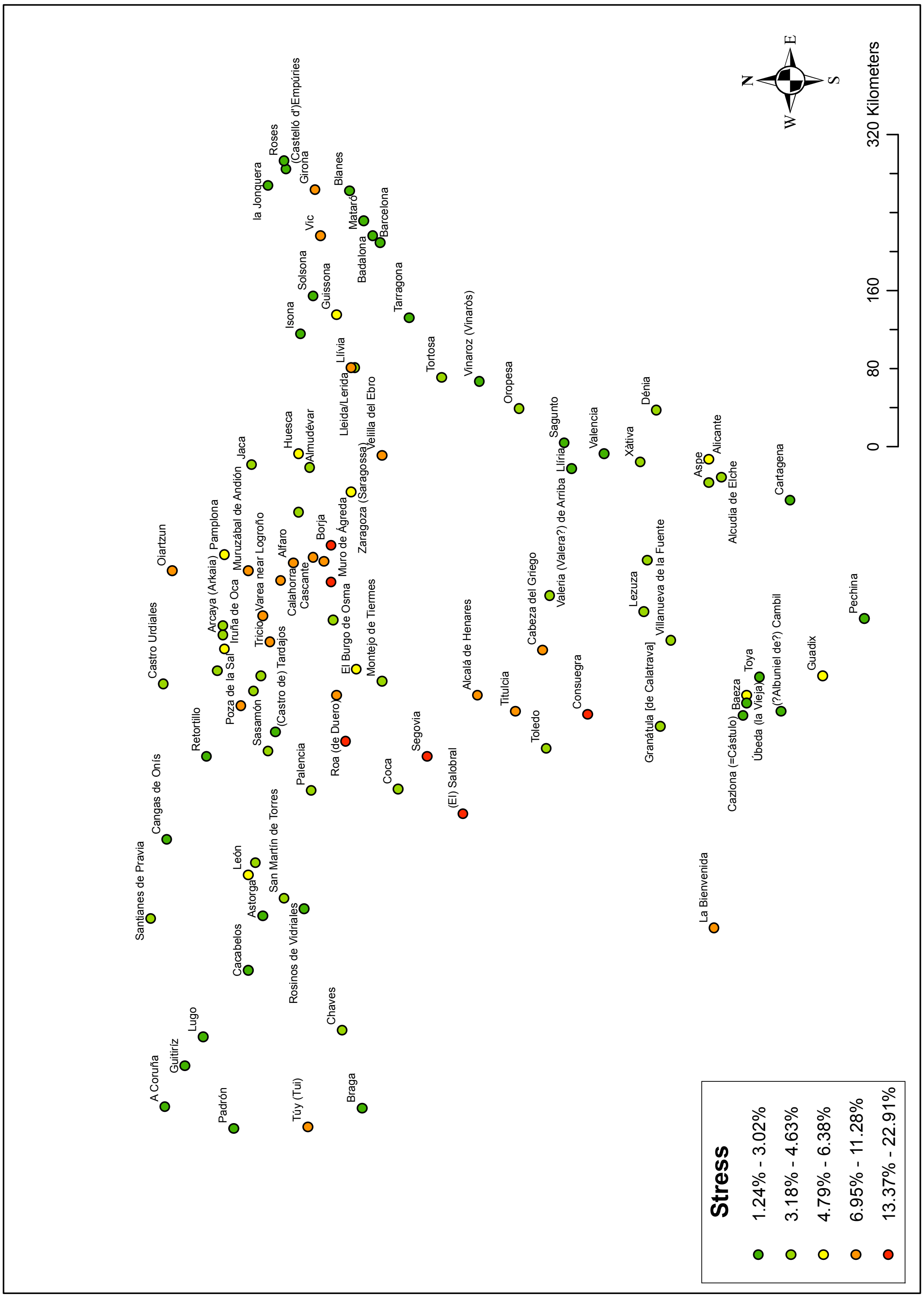

Figure 3.20: Tarraconensis: modern configuration with transformed stress

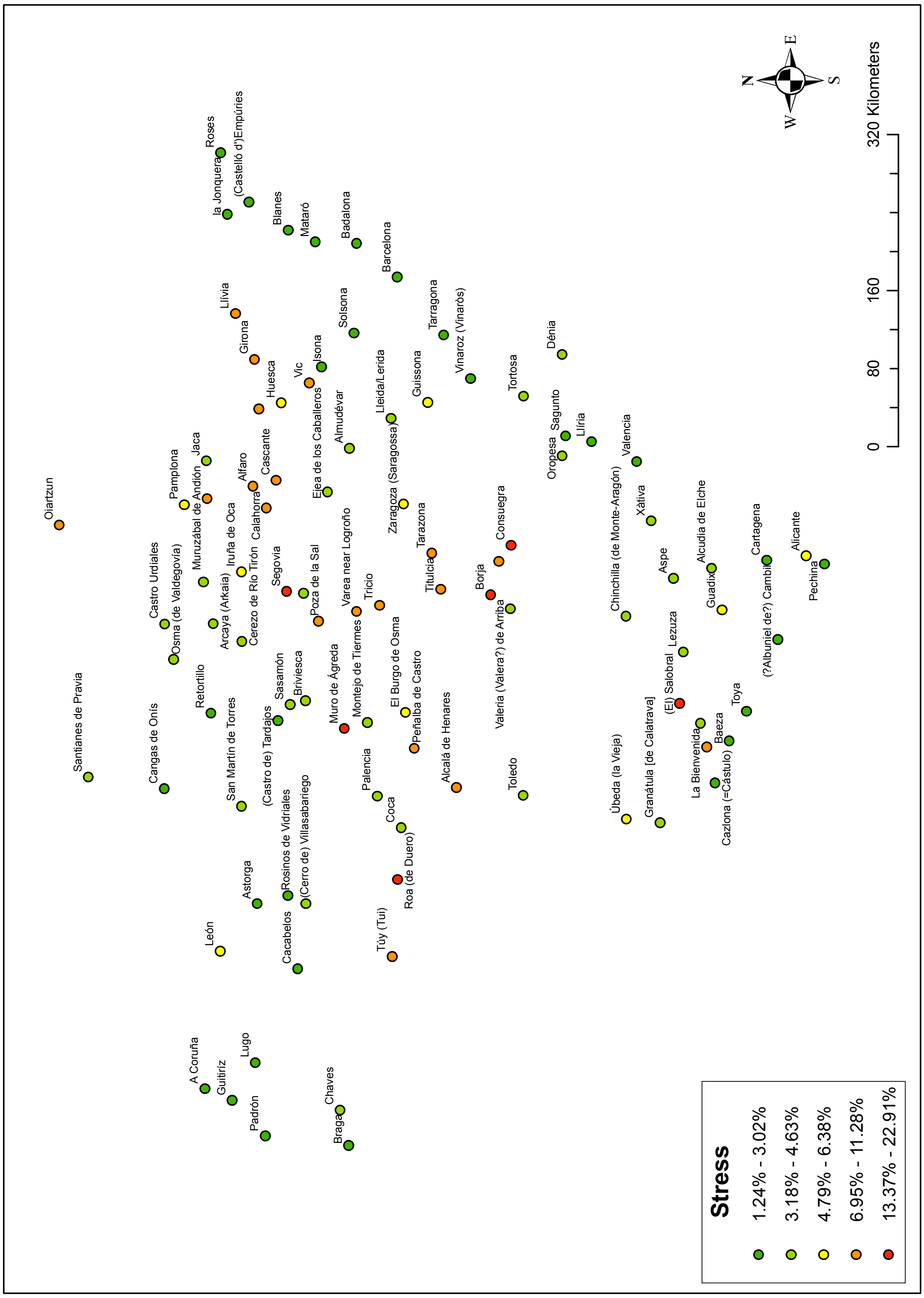

Figure 3.21: Tarraconensis: transformed configuration with transformed stress

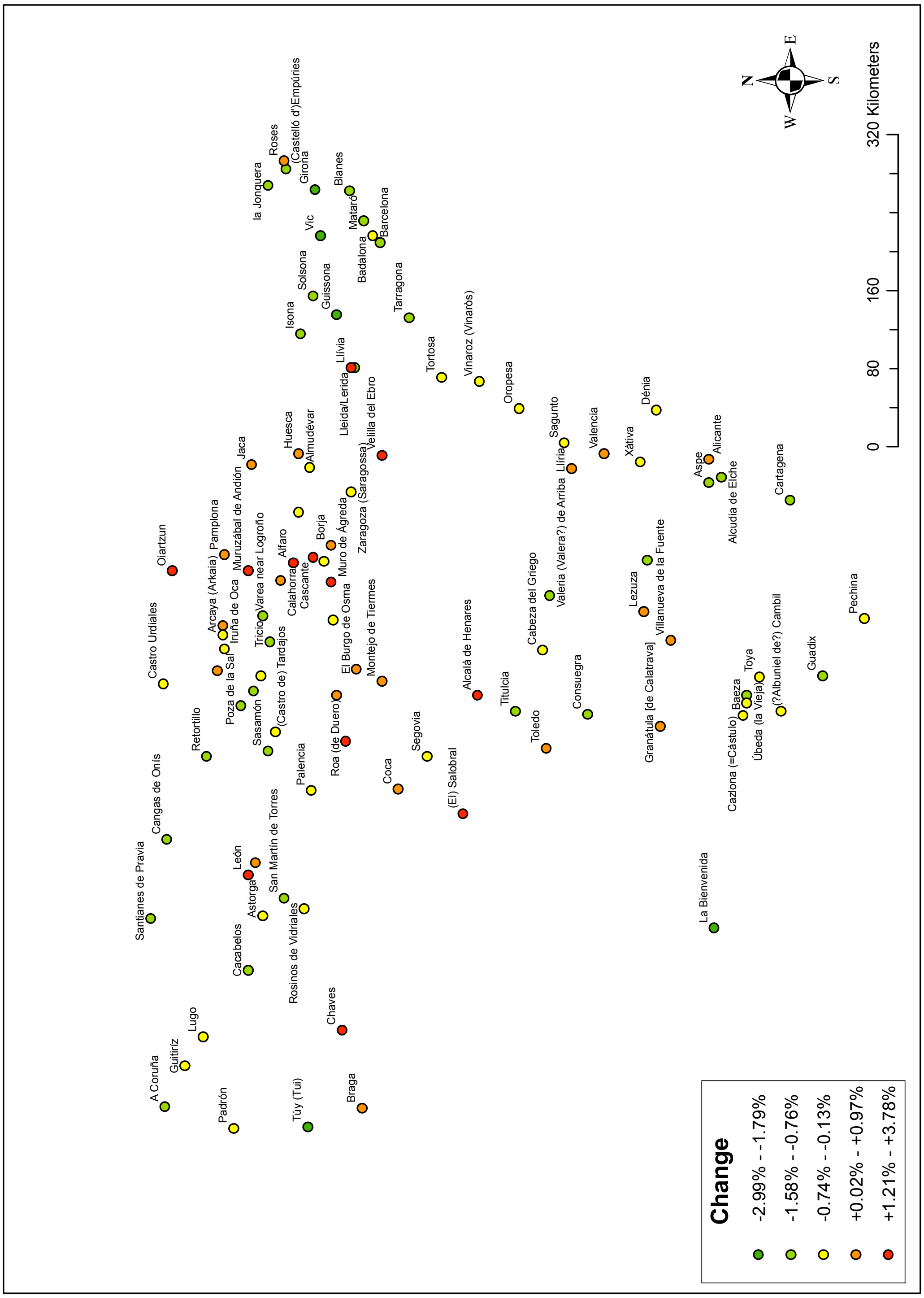

Figure 3.22: Tarraconensis: modern configuration with change in stress after transformation

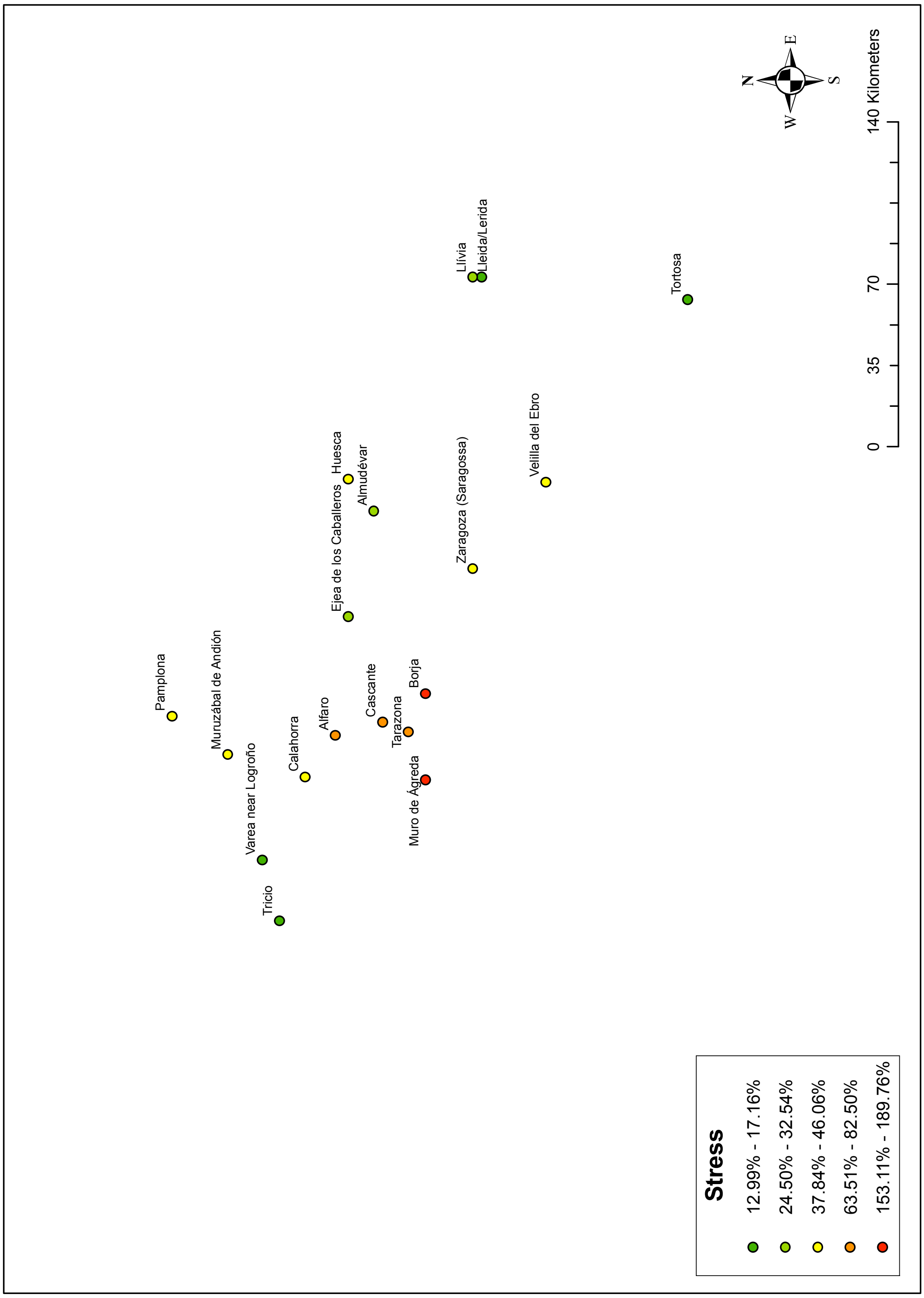

Figure 3.23: Ebro: modern configuration with initial stress

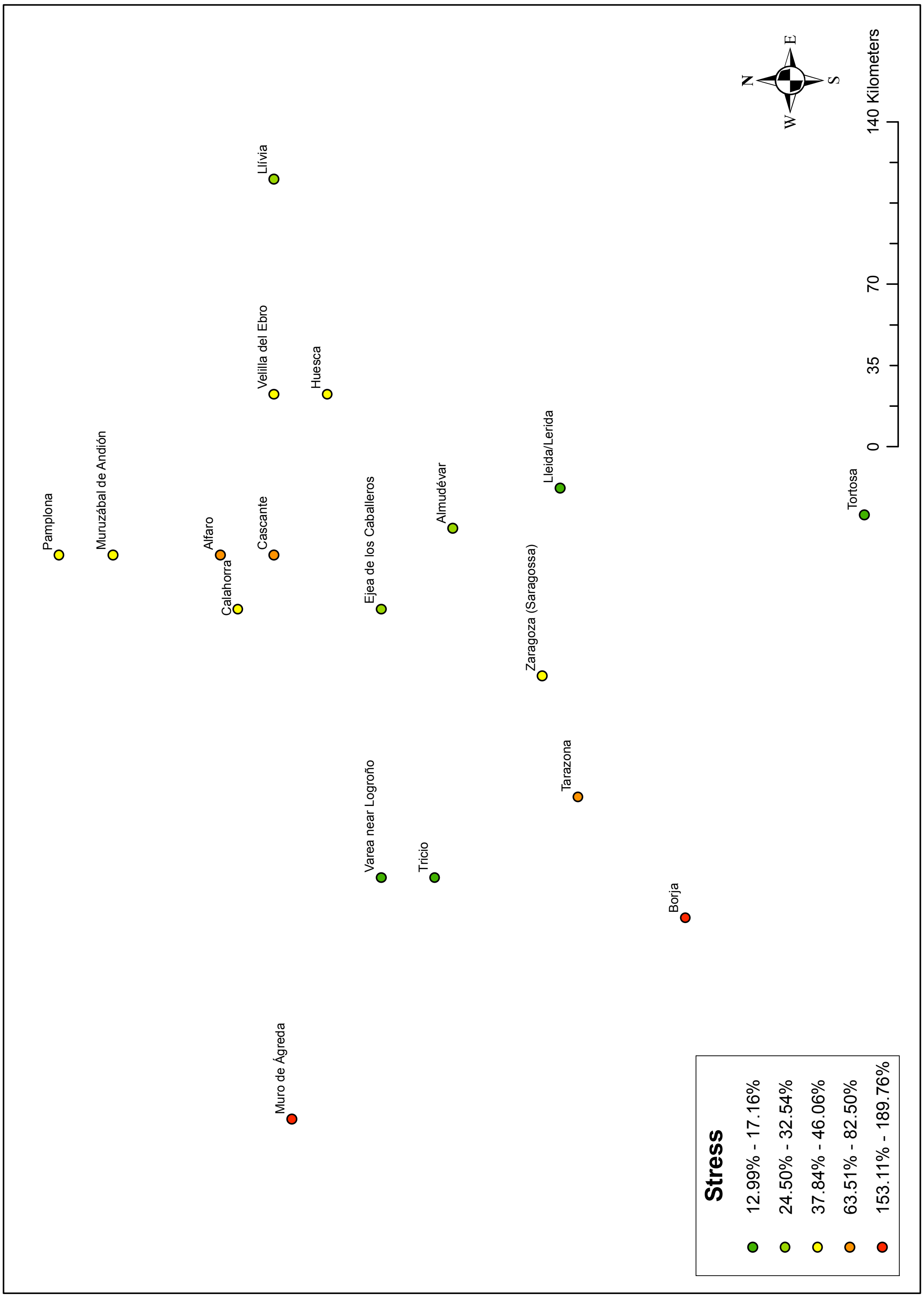

Figure 3.24: Ebro: Ptolemy's configuration with initial stress

The modern configuration of the Ebro valley is rather simple (Figure 3.23). There is a straight diagonal from Calahorra to Tortosa approximately representing the river itself, with several branches of tributaries. This is not the picture presented by Ptolemy (Figure 3.24). The cities are scattered everywhere. Llívia and Lleida are 3.7 km from each other on the modern map. Ptolemy has them 181.4 km apart, and they are on the lower end of the stress spectrum! The numbers speak for themselves - the lowest stress is 12.99%. This is Tortosa which is correctly placed south of all the other cities. Two cities, Muro de Ágreda and Borja, are so far out of place as to have stresses over 100%, 153.11% and 189.76% respectively. Orientations and distances are simply wrong on this map.

The transformation drops the mean city stress from 52.78% to 25.74%. Tortosa's minimum of 12.99% is only brought down to 12.93% and remains the minimum. Borja's maximum of 189.76% falls precipitously to the new maximum of 52.57%. The post-transformation modern configuration (Figure 3.25) shows that most problems are upriver. Figure 3.26 shows the severe contraction of Ptolemy's map in the computer's attempt to reduce the configuration to the mostly straight line that is the modern map. Like the cites themselves, the higher errors are scattered across the map. Lleida's vast erroneous distance from Llívia is now recognised by its expulsion from the lowest stress bracket and inclusion in the second highest.

The east

If we look at the settlements that make up the eastern coast of Spain, we find 62 identified cities. These represent the most urbanised area of Spain and make up a large portion of Baetica, which had an extensive road network connecting the cities of the Baetis valley with those of the coast. The cities of the river valley have been included in this group both because of their short distance to the coast and the fact that the routes down the coast to the mouth of the Baetis included one which turned inland and went down the river itself. Initially, the stress for this area is 2.44% with a sum of squares error of 469 thousand km^2. The Procrustes error drops to 326 thousand km^2 after a 8.4° anti-clockwise rotation. The skewing minimisation demands a 1% shrinking of the longitude and a 9% expansion of the latitude to finalise the stress at 2.35%.

Examining Figure 3.28, we can see the initial stress displayed on the modern configuration. We have very low stresses across the board, the greatest concentration of which is in the far north. In the far south-west, we encounter mainly lower stress errors, but mid- and high-range stresses enter in as we move eastward. Ptolemy's map (Figure 3.29) does not differ much in its error dis-

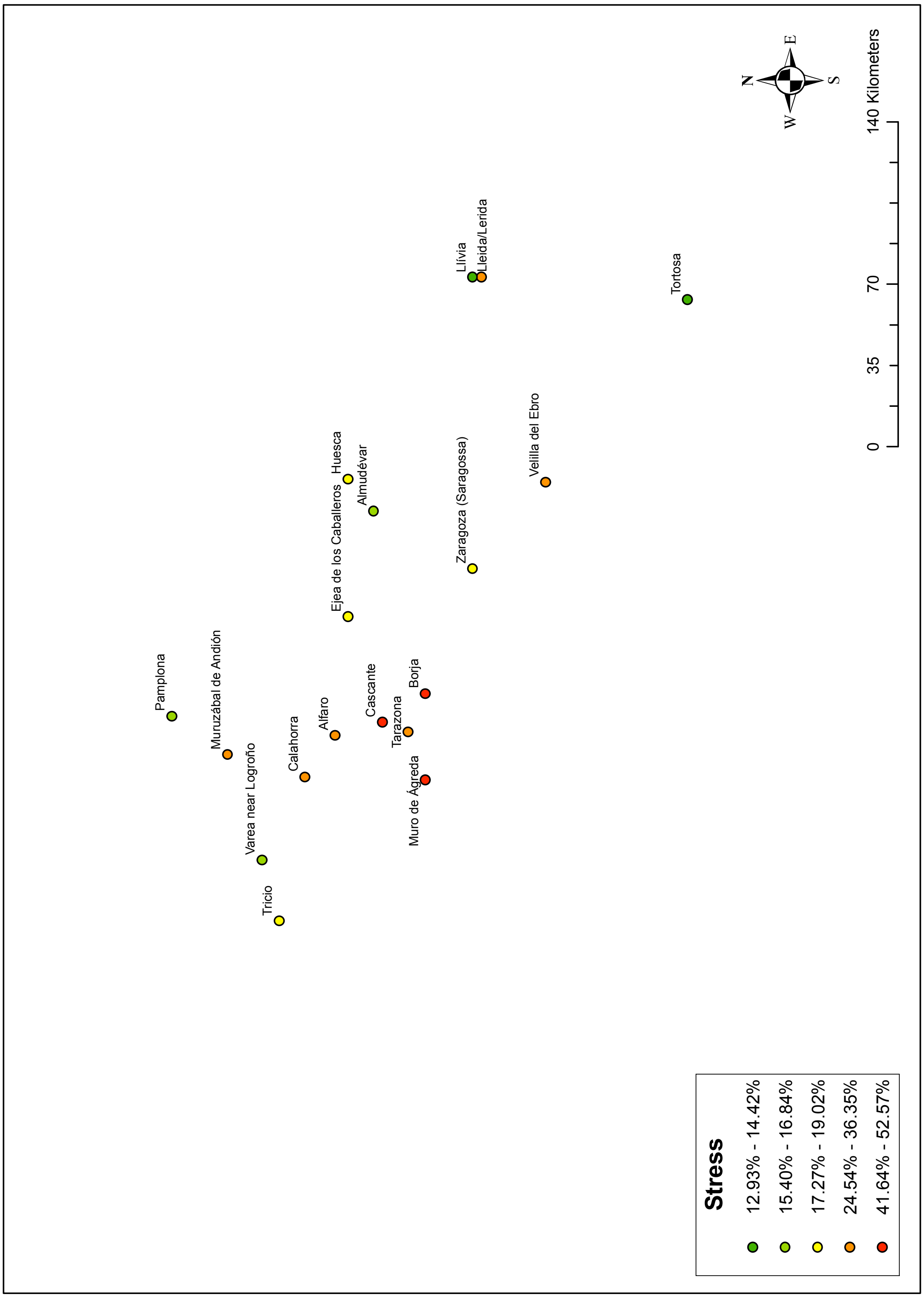

Figure 3.25: Ebro: modern configuration with transformed stress

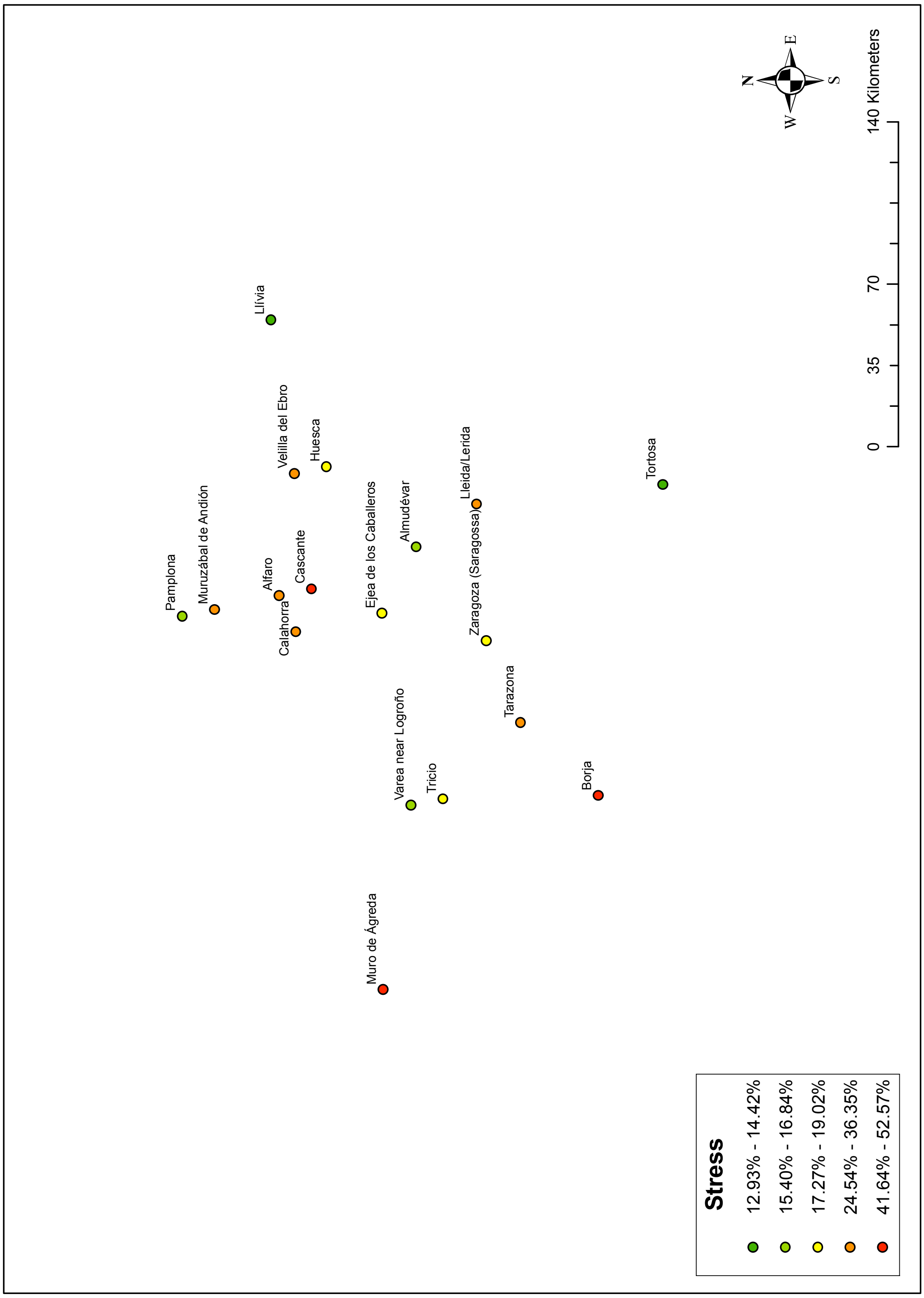

Figure 3.26: Ebro: transformed configuration with transformed stress

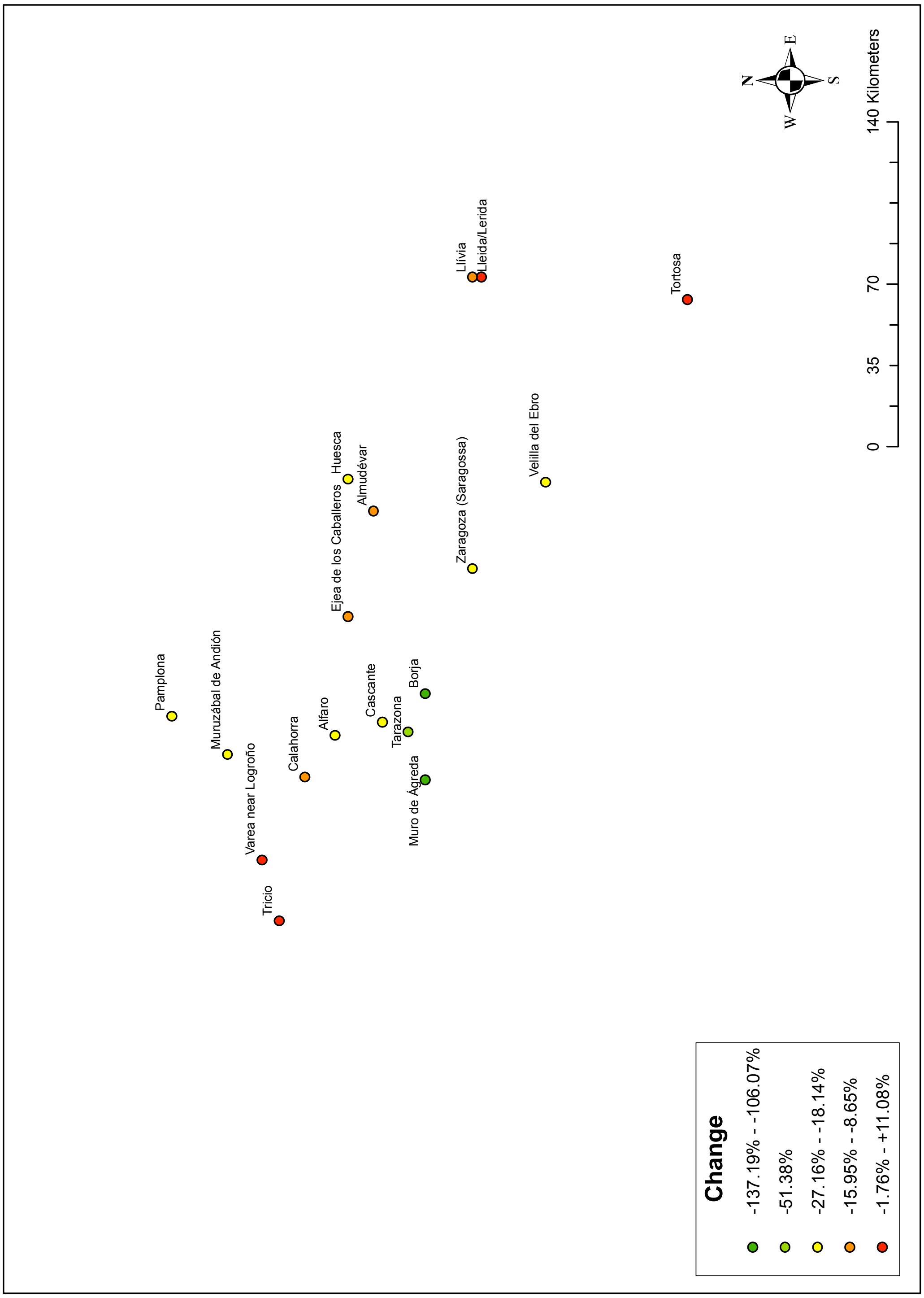

Figure 3.27: Ebro: modern configuration with change in stress after transformation

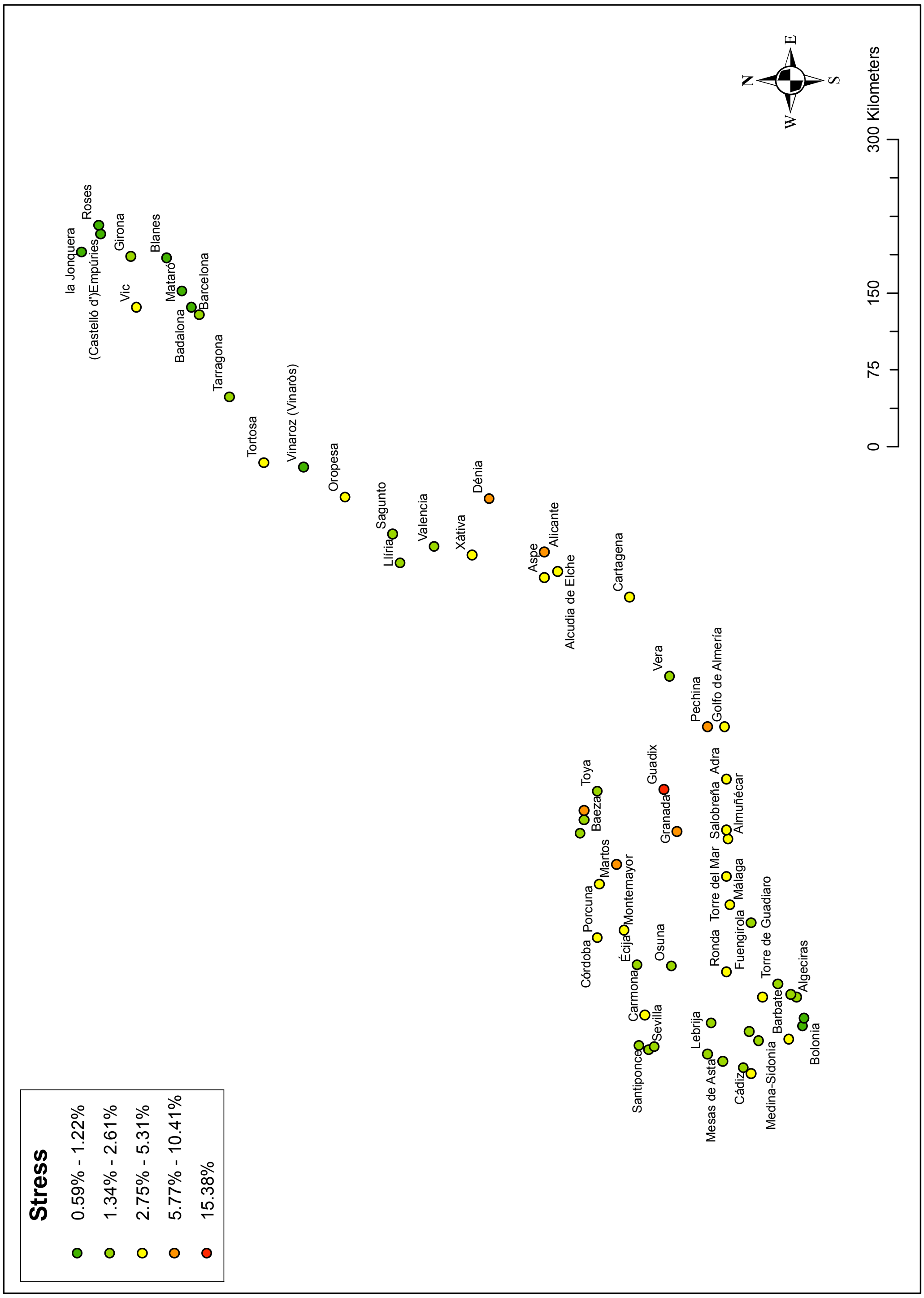

Figure 3.28: East: modern configuration with initial stress

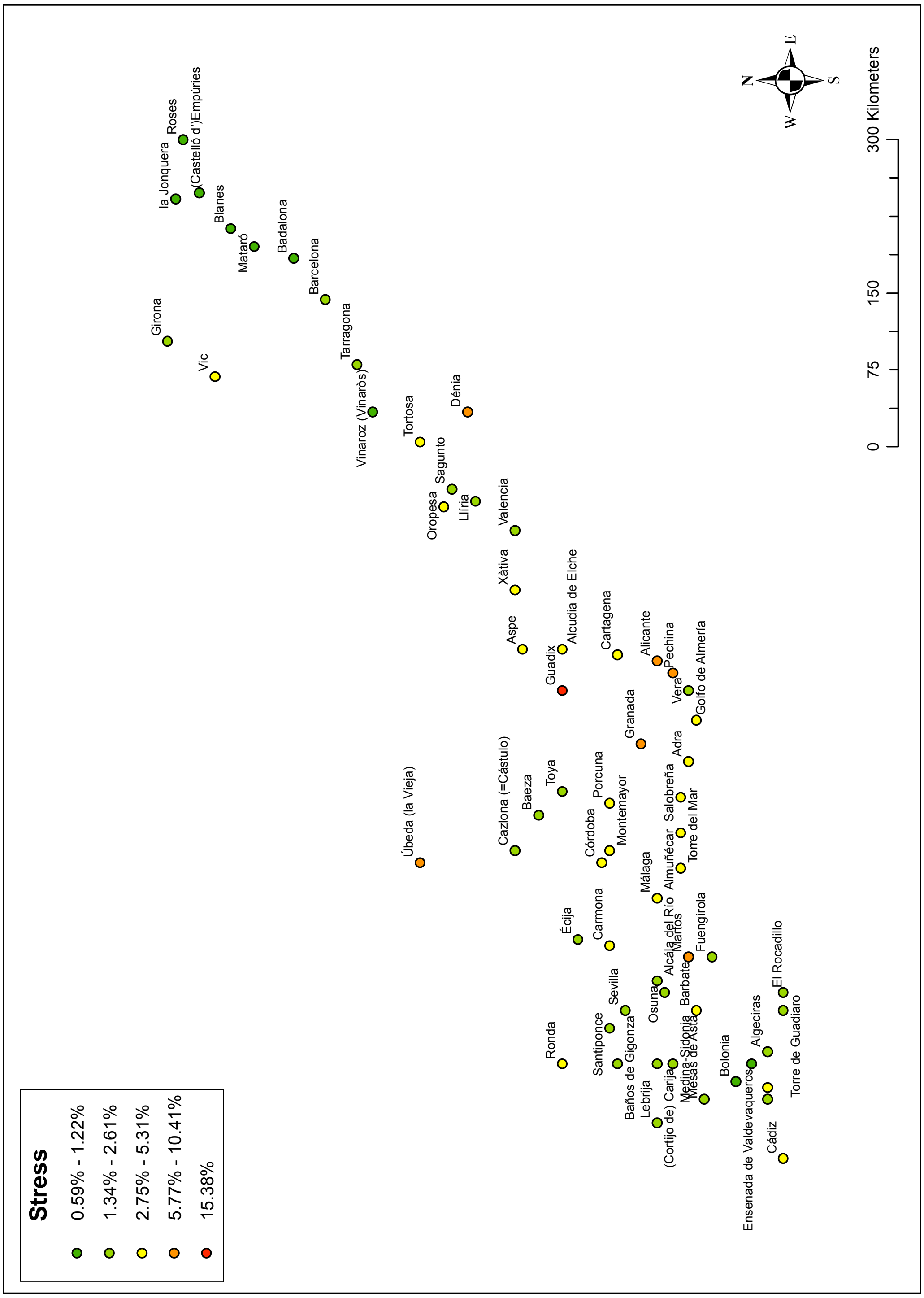

Figure 3.29: East: Ptolemy's configuration with initial stress

tribution. The errors that arise in the south are moved farther north-eastward and, in general, the cities in the south are more spread out than in the modern configuration.

Mean city stress begins at 3.06% and is brought down to 2.99% by the transformation. Both the minimum and maximum values decrease, from 0.59% and 15.38% to 0.51% and 14.87%, respectively. La Jonquera holds the minimum both before and after the transformation, as Guadix the maximum. The realigning of the stress tiers makes it seem that the cities have become worse. Figure 3.30 certainly has more cities in the two highest brackets than Figure 3.28. The expansion of the latitude post-transformation is quite evident in Figure 3.31 both in the north-south extent of the map, but also in the further spread of the cities of the south. Again, especially in the south, it appears that most cities have moved to higher stress brackets. Examining Figure 3.32, though, we see that while several cities have indeed had their stresses increased, most remain very close to 'no change' at all.

3.3 The coast

3.3.1 The coast of Hispania

The centroid of Ptolemy's coast of Hispania is $(9.73°, 40.22°)$. The modern counterpart is $(-4.50°, 39.52°)$. His latitude, as it is for his cities, is very accurate. It should be noted that the longitudes of the coast are farther west than those of the cities. This is simply because the Pyrenees greatly reduces the number of coastal points in the eastern half of the peninsula, pushing the mean westward.

The coastline has an initial Procrustes error of 1.4 million km^2, which is reduced to 585 thousand km^2 after a 13° anti-clockwise rotation. A 6% expansion of longitude coupled with a 6% reduction of latitude improves the error to 532 thousand km^2. There are 77 identified points on the coast, some of which are cities included in the stress analysis above. The initial mean error is 119.27 km, reduced to 73.39 km after transformations.

The Iberian peninsula is connected to mainland Europe at the Pyrenees along a roughly east-west line, but Ptolemy has them running obliquely from the north-west to the south-east (Figure 3.33). This, along with the inaccuracies inherent in Ptolemy's data, results in a seriously erroneous depiction of the north coast, for he places his cities over 240 km from their modern equivalents. The west coast appears much more accurate in shape, and this is reflected in the relatively smaller error figures. As a whole, though, it is too far east when the modern and Ptolemaic maps are centred on one another. The south-west

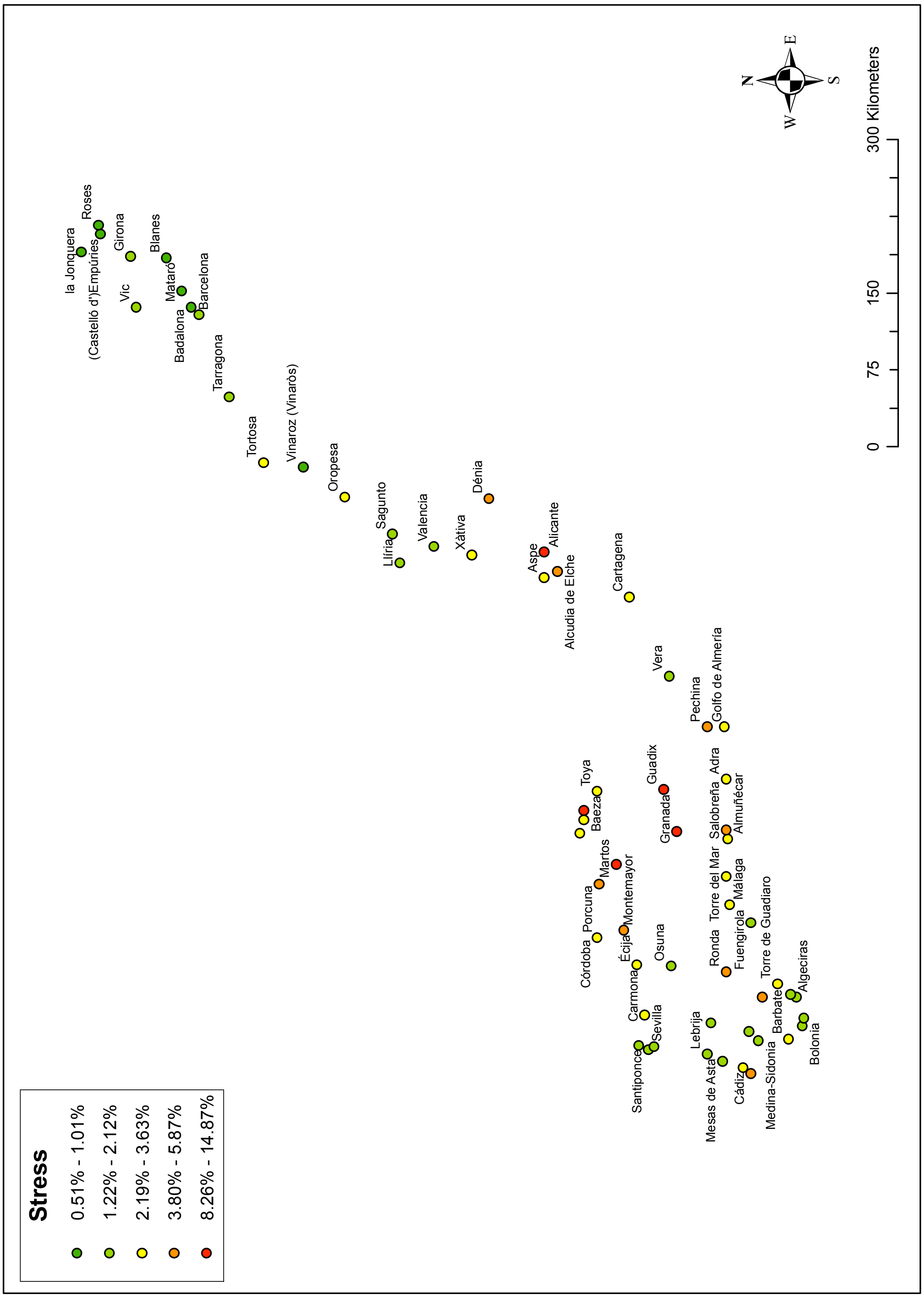

Figure 3.30: East: modern configuration with transformed stress

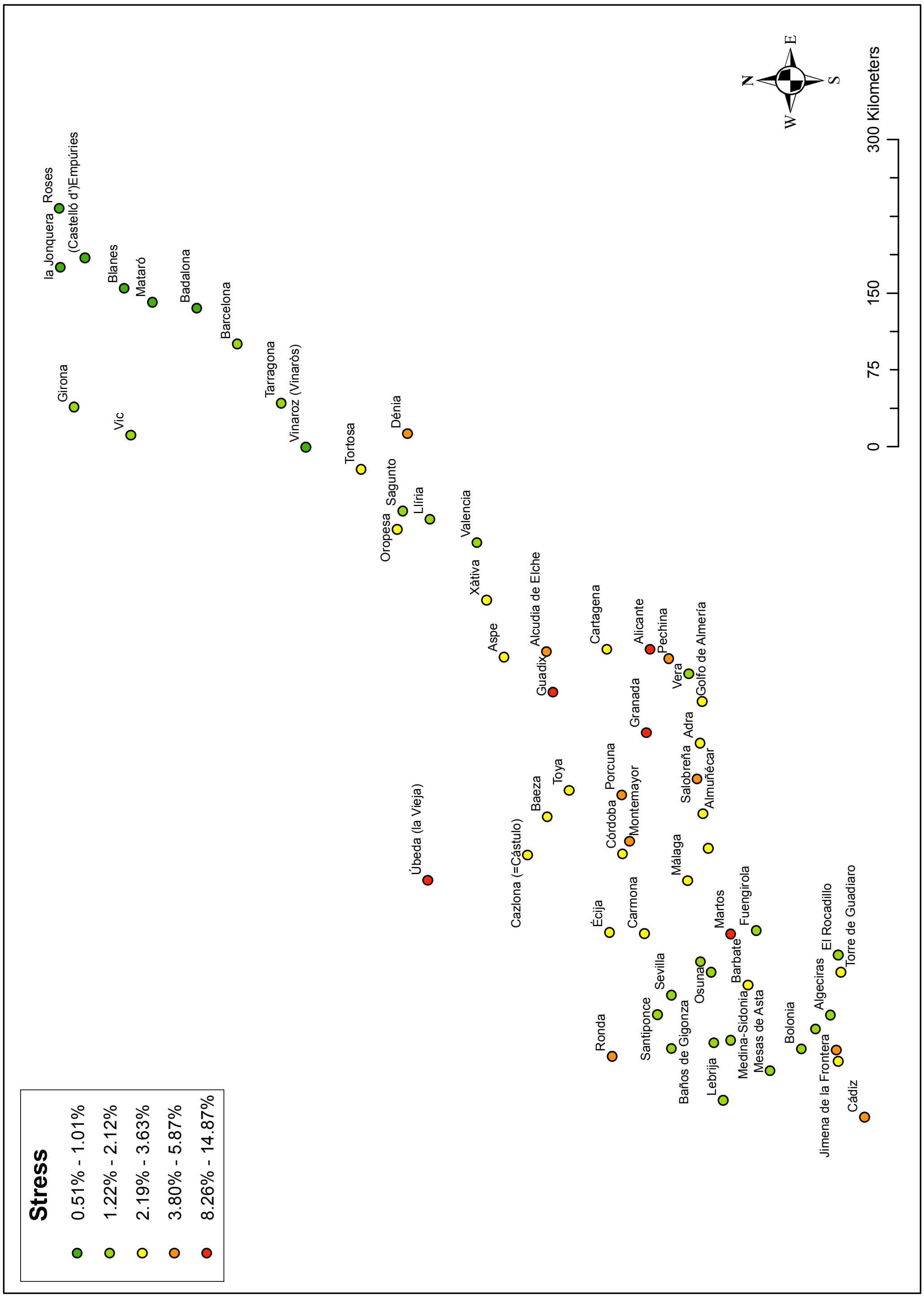

Figure 3.31: East: transformed configuration with transformed stress

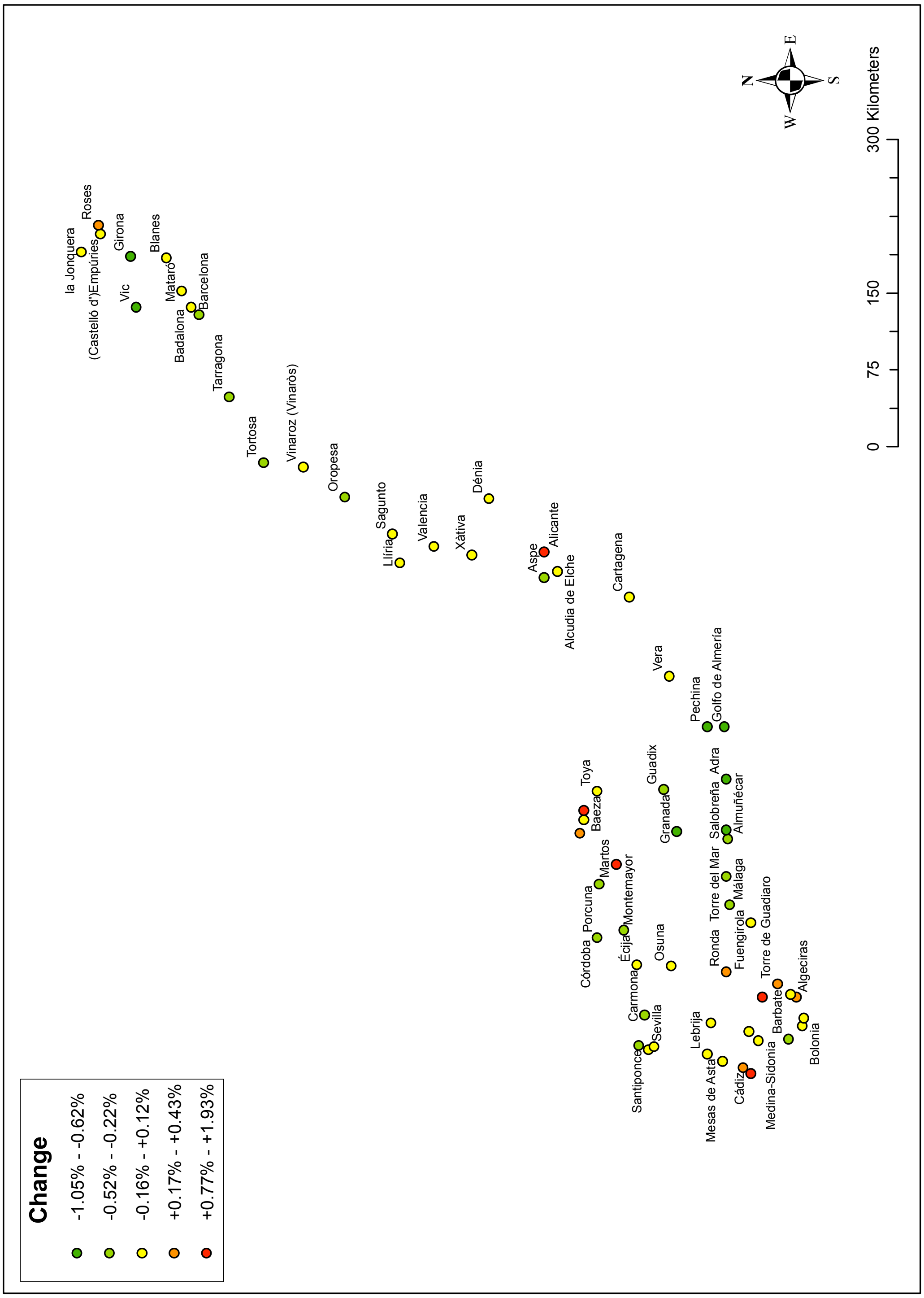

Figure 3.32: East: modern configuration with change in stress after transformation

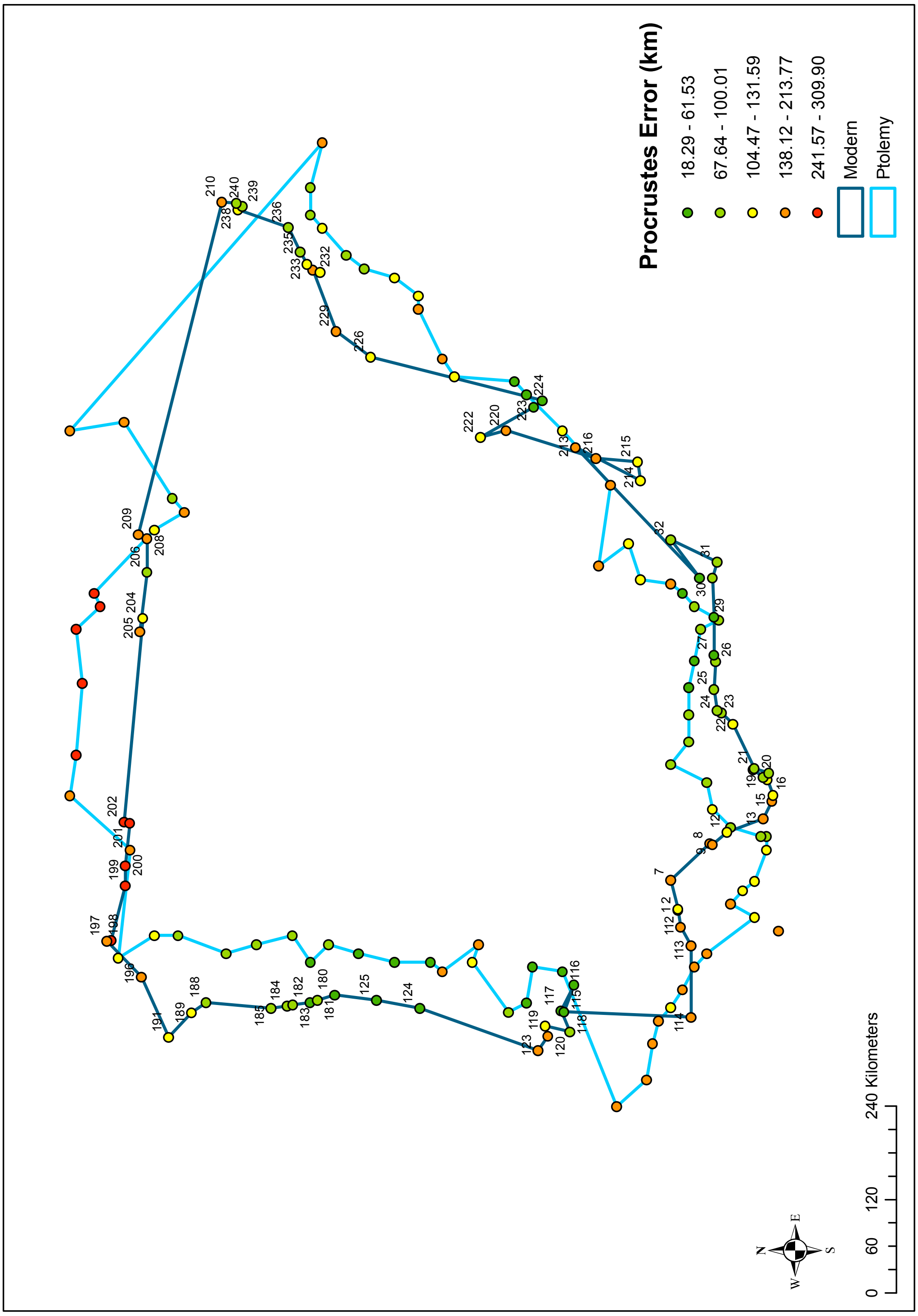

Figure 3.33: Hispania's Coast: Initial Procrustes error

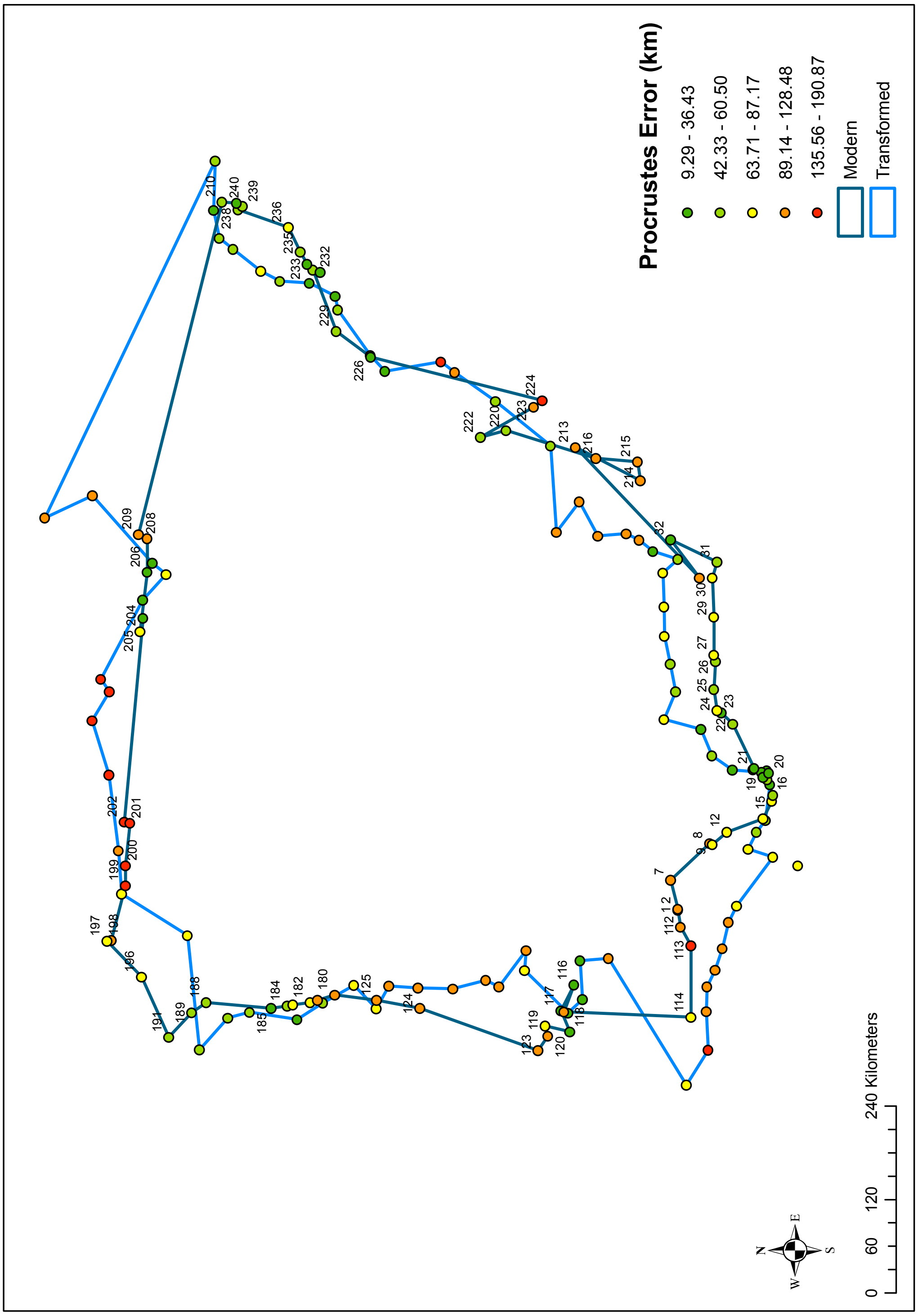

Figure 3.34: Hispania's Coast: Transformed Procrustes error

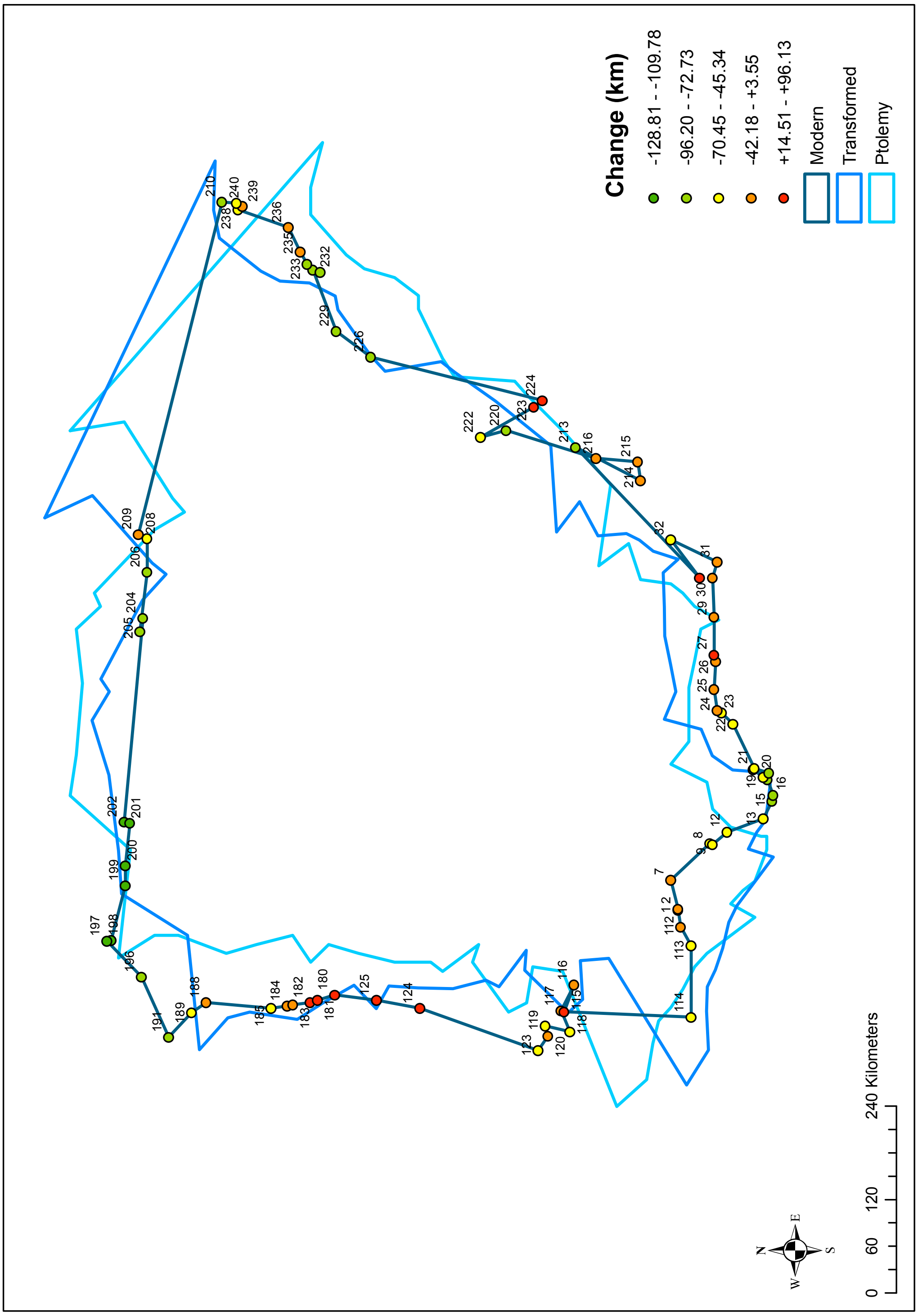

Figure 3.35: Hispania's Coast: Change in Procrustes error

coast comes next after the north coast in sustained inaccuracy. It needs both a re-centring and a rotation. It is unclear whether this rotation results from the slant given to the Pyrenees, but to judge from Figure 3.33, it appears that the rotations needed are similar. The southern portion of the Mediterranean coast is fairly accurate, but the severity of the errors becomes more mixed the further north we move.

After the transformation, the Mediterranean coast follows its modern counterpart more closely, now with the northern and southern ends both having good runs of consistently low errors (Figure 3.34). The northern end of the west coast matches its modern counterpart very closely now, but at the cost of the central and southern portions. The north and south-west coasts certainly improve, but still remain the areas of highest error. This is despite the fact that the north coast saw the most improvement (Figure 3.35). The transformation of the south-west only provided a modest reduction in the size of the error, while the more accurate areas of the west and Mediterranean coasts took on additional error.

As with the city stress analysis, we shall now examine the coasts of the three Roman provinces embedded in the Iberian peninsula.

3.3.2 The coast of Baetica

The coast of Baetica has 25 identified points. The initial Procrustes error is 34,109 km^2 which is only reduced to 34,066 km^2 after a clockwise rotation of 0.5°. Scaling results in a longitudinal reduction of 14% and a latitudinal reduction of 21%. The Procrustes error is left at 19,815 km^2. Mean individual error drops from 35.51 km to 25.28 km.

Despite initial errors appearing on the map of Hispania in this region, the coast of Baetica is the most accurate piece of coastline once it is freed from the errors of the rest of the peninsula. These are the lowest errors, by far, of any division of the coastline. Ptolemy's coast zigzags quite a bit compared to the modern coast, crossing it several times (Figure 3.36). There are several clumps of high-, mid-, and low-range errors (centred about Cabo Trafalgar, El Rocadillo, and the Guadalhorce, respectively) but barely any transitions of medium-low or medium-high errors. The most worrying error is Ptolemy's placement of Cabo Trafalgar which juts out to the south-west below Puerto de Santa Maria. This misplacement is so severe that on the maps of the whole of Hispania, the map making software could not incorporate Trafalgar into the polygon of the peninsula.

We can see that virtually no rotation is needed, and scaling alone is relied upon (Figure 3.37). While the mean and minimum errors drop, Vera takes an additional 18 km of error to become a new and higher maximum. In fact, Vera

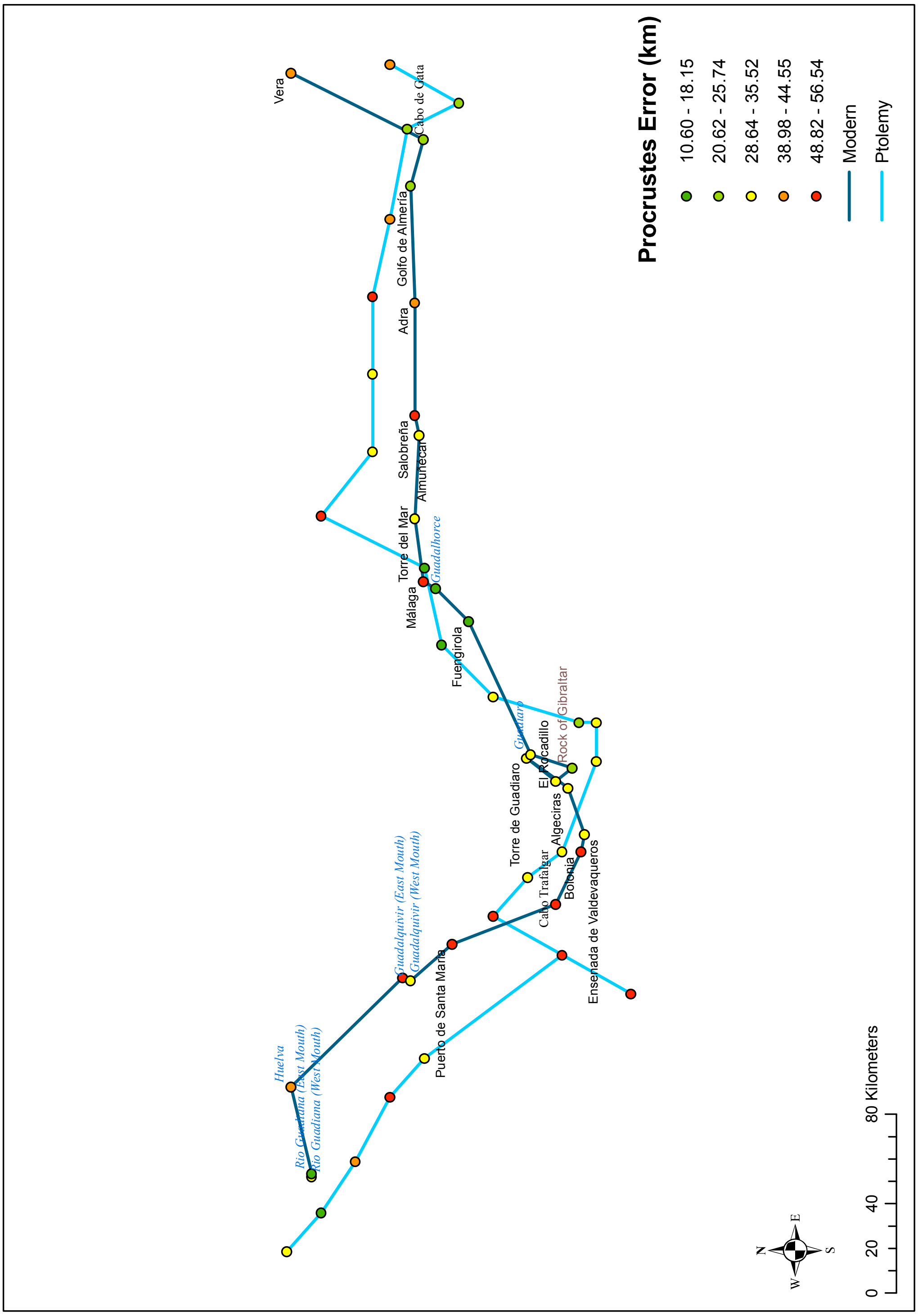

Figure 3.36: Baetica's Coast: Initial Procrustes error

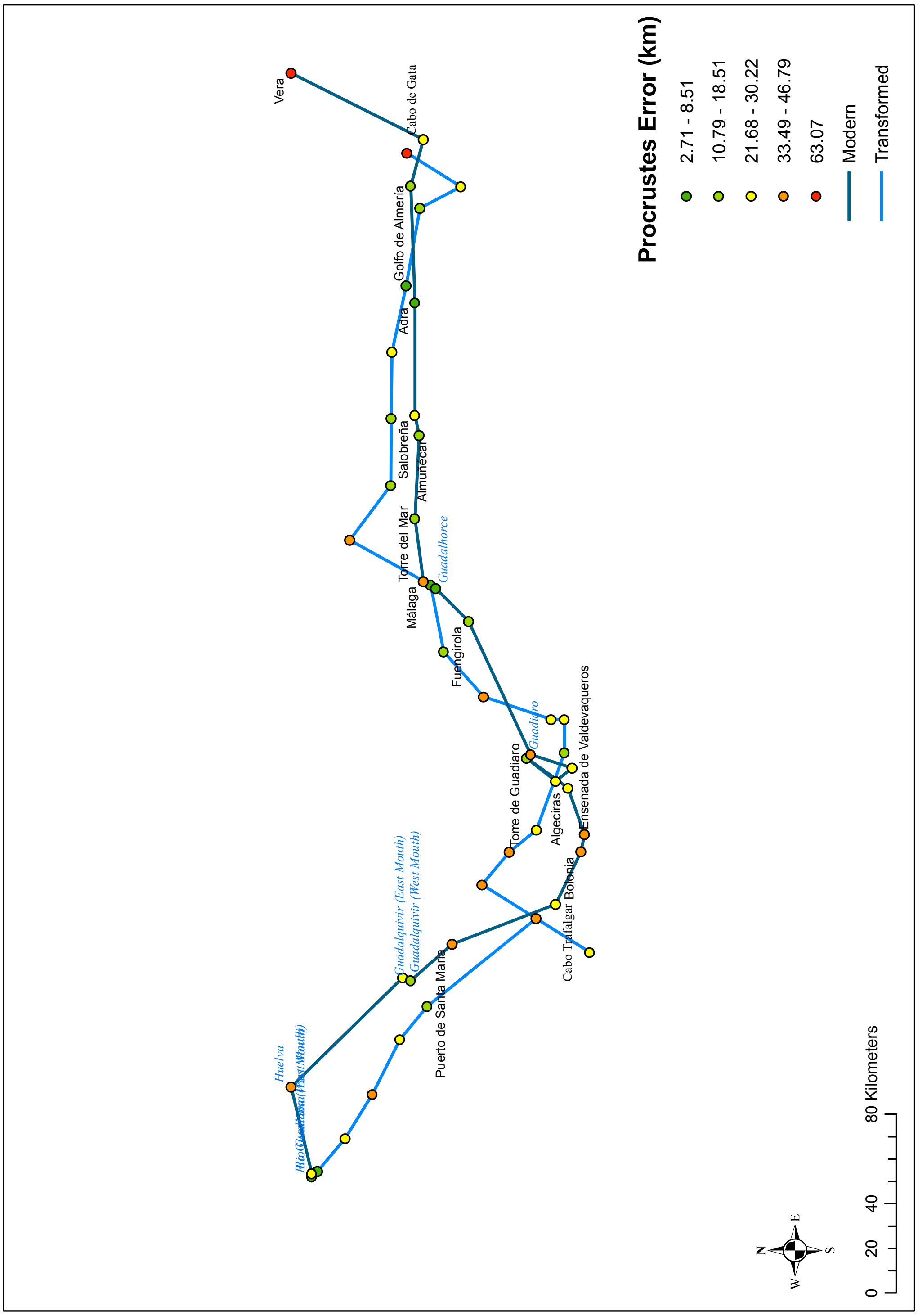

Figure 3.37: Baetica's Coast: Transformed Procrustes error

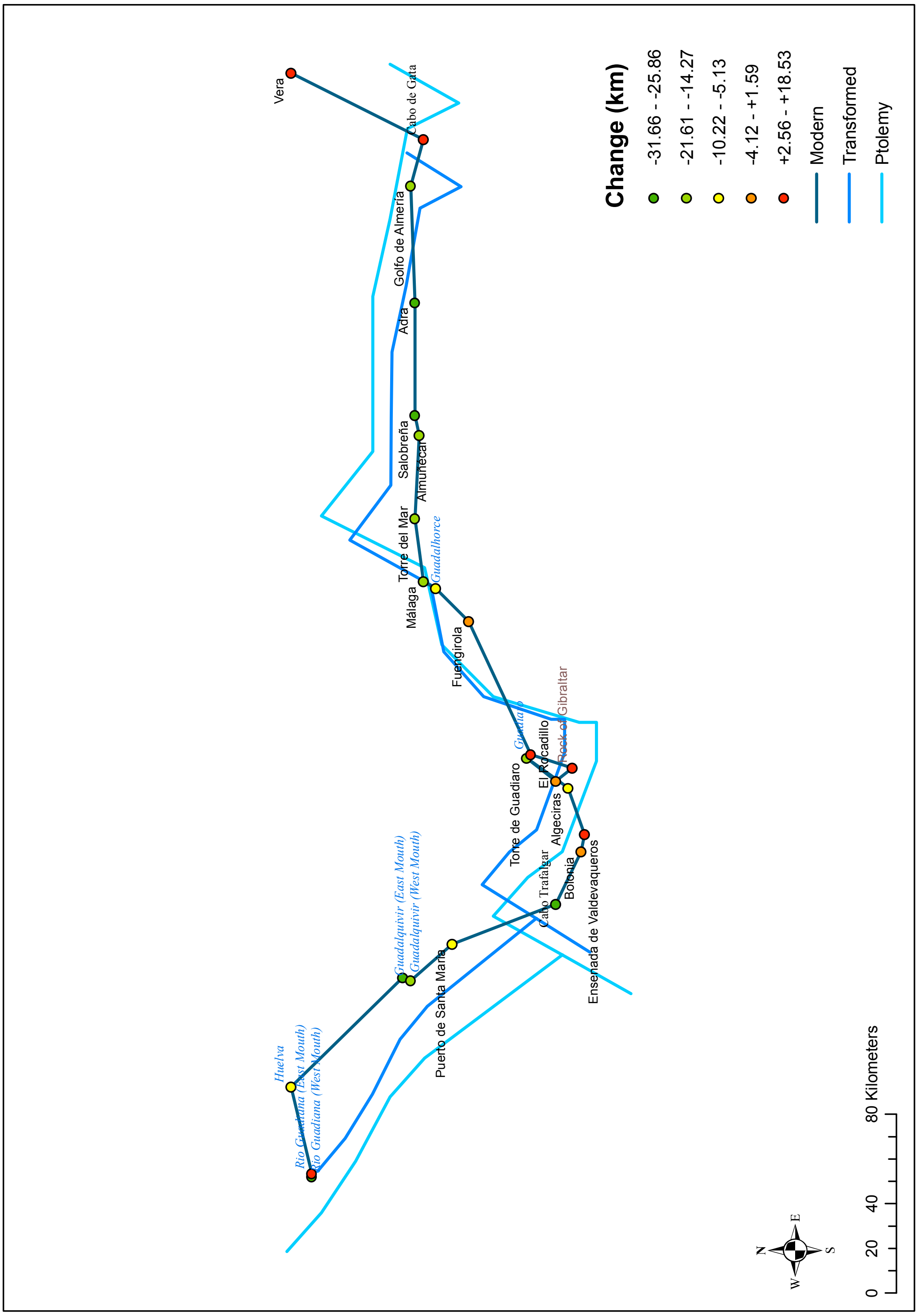

Figure 3.38: Baetica's Coast: Change in Procrustes error

sits in the highest error group by itself with a very substantial gap before the medium-high group is reached. Thanks to Vera's 'sacrifice', however, the rest of the Baetica coastline lines up relatively well atop its modern counterpart. The primary area of difficulty is the southern curve between the Atlantic and the Mediterranean. The main length of the Mediterranean coast is mostly contained in the two lowest Jenks divisions. Ptolemy's misplacement of the Huelva is holding back the Atlantic coast proper, though we can not necessarily fault him for his long separation of the east and west mouths of the other rivers. River deltas are highly changeable and after almost two thousand years, nature will take its course. The improvements from the transformation (Figure 3.38) occur along with two main coasts except for their end points and their meeting at the Pillars of Hercules.

3.3.3 The coast of Lusitania

Known points on the coast of Lusitania number 14. An anti-clockwise rotation of 34.2° brings the Procrustes error to 62,799 km^2, down from the original 137,974 km^2. The problem appears to be that Ptolemy's longitude is spread far too much. A reduction of 65%, as compared to a 4% expansion of the latitude, results in a sum of squares error of 54,117 km^2. The average coastal error for Lusitania is 89.05 km, reduced to 58.11 km after transformation.

There appear to be three distinct areas of interest in Lusitania (Figure 3.39). The northern- and southernmost three points appear to be close to their proper places in relation to themselves, but are quite far removed from where they should be compared to the rest of the coastline. In the south this looks as though it can be fixed by a re-centring followed by an anti-clockwise rotation, but only a rotation of that bit of the coast. The three northernmost points, the rivers Douro, Vouga, and Mondego just need a simple translation, again, independent of the rest of the coast. The third area, the points from Cabo da Roca to the River Sado, is the real source of error. This area happens to have the lowest numerical errors, though that is due to the centring of the two configurations and not their shapes. It looks as though an effort has been made to portray the headlands and inlets near Lisbon accurately, but the order of the features is wrong with Lisbon being far to the north of this area. The relative positions of Setúbal, the River Sado (the city is located at the river's mouth), and Alcácer do Sal (which is quite far inland), all contribute to this error.

The transformation mostly consists of flattening the coast into a straight north-south line to minimise the error (Figure 3.40). Minimisation is achieved, but the coastlines fail to match at all. Again, the lowest errors are in the centre simply because the configurations have been centred on one another. Certainly

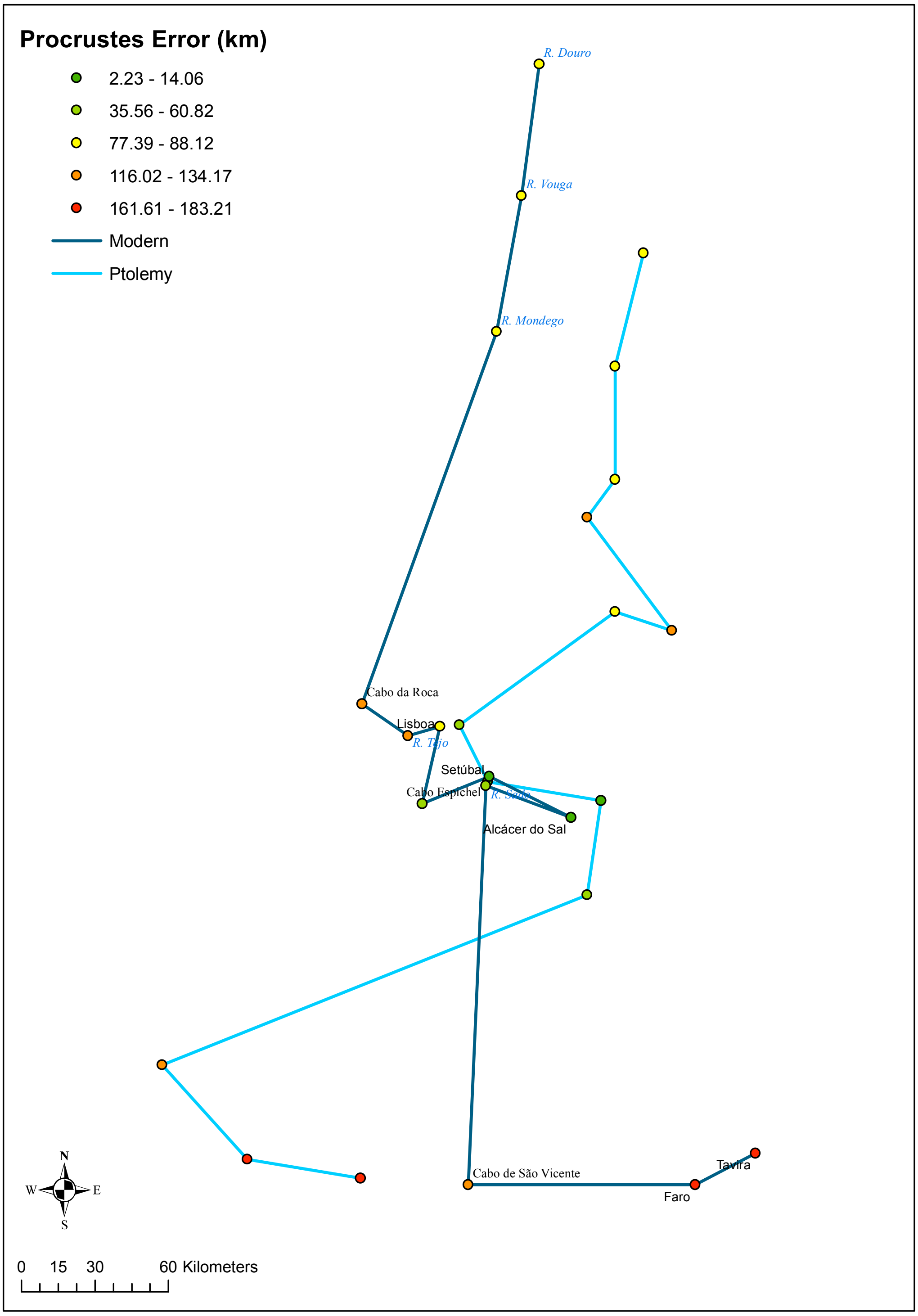

Figure 3.39: Lusitania's Coast: Initial Procrustes error

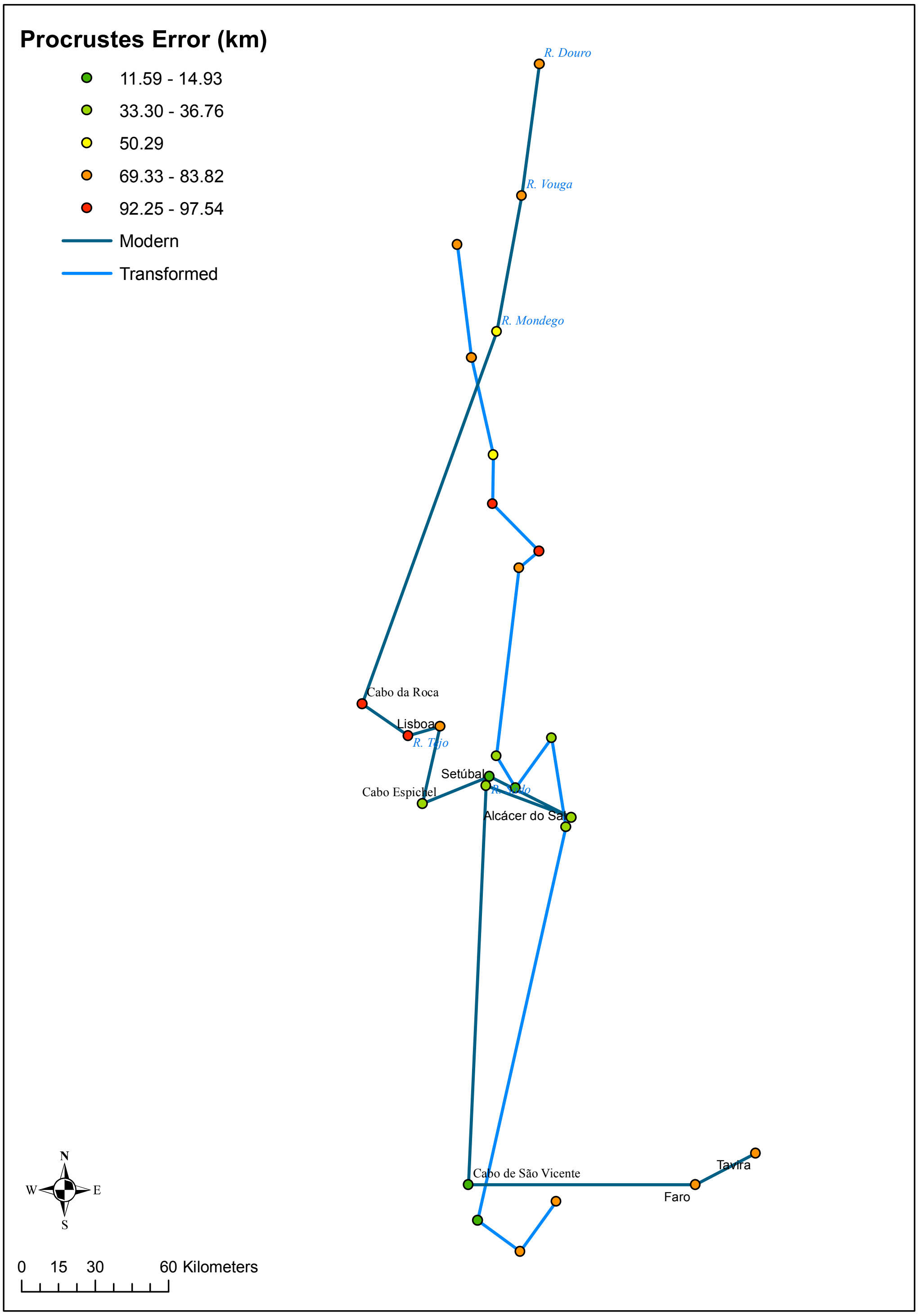

Figure 3.40: Lusitania's Coast: Transformed Procrustes error

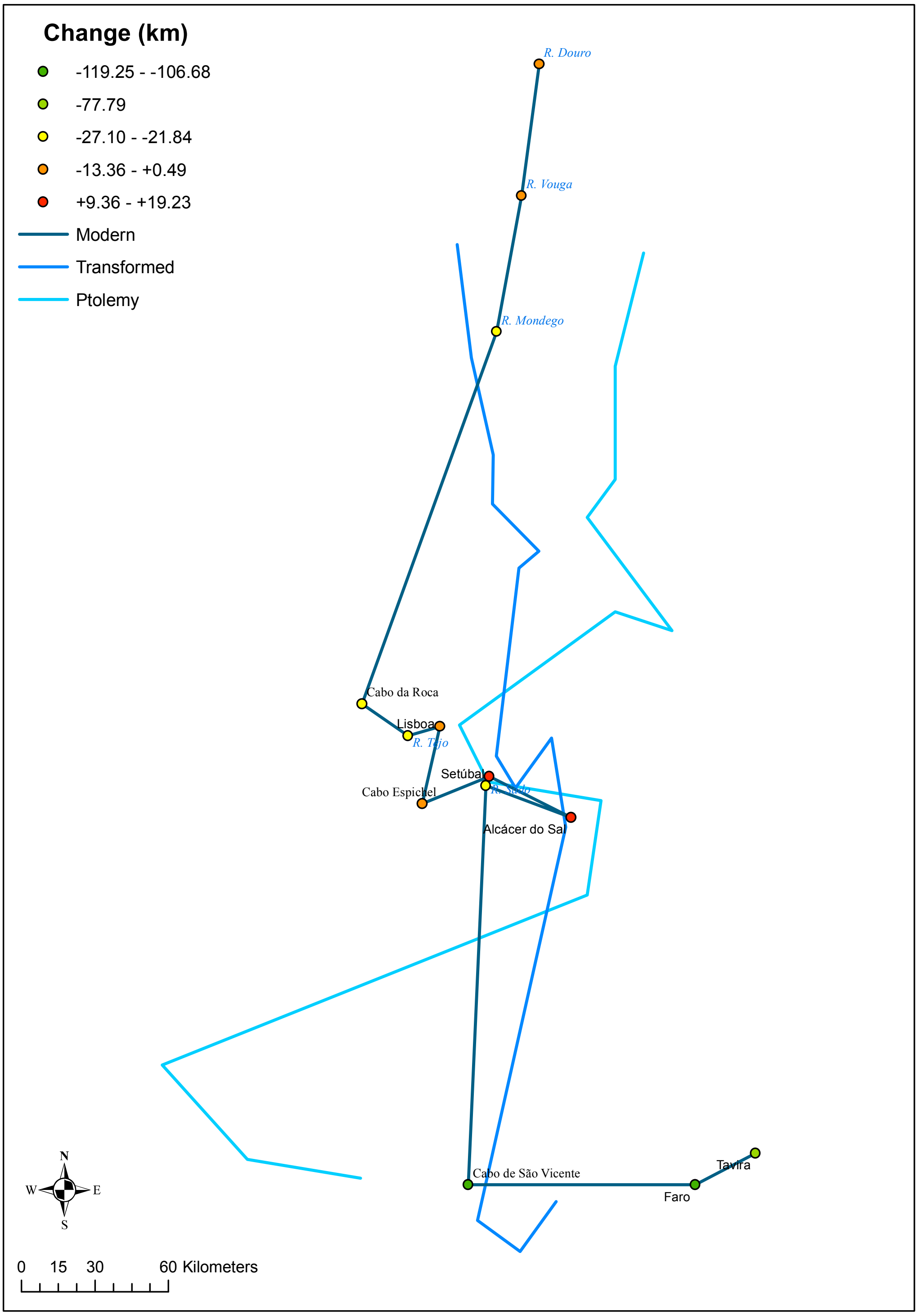

Figure 3.41: Lusitania's Coast: Change in Procrustes error

the mean error has fallen (Figure 3.41), but the shape of the coastline is far from improved. It is far too short and far too narrow. For such a small stretch of coastline with only 14 points, the minimisation achieved rather poor results. Ptolemy's Lusitania, simply put, is inaccurate.

3.3.4 The coast of Tarraconensis

Tarraconensis in its entirety

The 41 known points of the coast of Tarraconensis begin with an error of 688 thousand km^2. A 12.8° anti-clockwise rotation more than halves this down to 340 thousand km^2. Expanding the longitude by 8% and shrinking the latitude by 7% brings the Procrustes error down to 300 thousand km^2. Initial mean error is 113.73 km, shrinking to 75.77 km after rotation and scaling.

The coast of Tarraconensis, along with the line of the Pyrenees, has much the same errors as when it was examined as part of Hispania (Figure 3.42). The portion of the Atlantic coast left to Tarraconensis is very well aligned, but the northern coast continues to be the source of the largest errors. Ptolemy's coast erroneously bulges out into the Bay of Biscay. The Mediterranean coast looks as though slight rotations could produce some accurate results, most of its error being at the southern end. However, the Mediterranean coastline presents not only the numerical error but also another problem. Ptolemy's order is confused in several places. Because the overlapping of the coastline is visible only on the modern configuration, it is possible that Ptolemy reordered his points to avoid these overlaps or that he moved his points to erroneous locations to avoid them. Either way, there are serious problems from R. Turia to Pechina.

The transformation aligns the Pyrenees more accurately but they are too far north (Figure 3.43). The west coast is pulled out of place to facilitate improvements elsewhere. The northern coast no longer protrudes into the Bay of Biscay, but there is the problem that the points around the Navia are pulled too far eastward. The southern portion of the Mediterranean coast is still causing trouble, but the northern half is much closer to alignment. Improvements, with few exceptions, were across the board except for the west coast (Figure 3.44), but despite reductions of more than 100 km in places, more than a few locations remain over 100 km from their true places.

The east coast

The east coast is measured from Cap Béar down through the Strait of Gibraltar to the mouths of the River Guadalquivir (Baetis). There are 41 identified

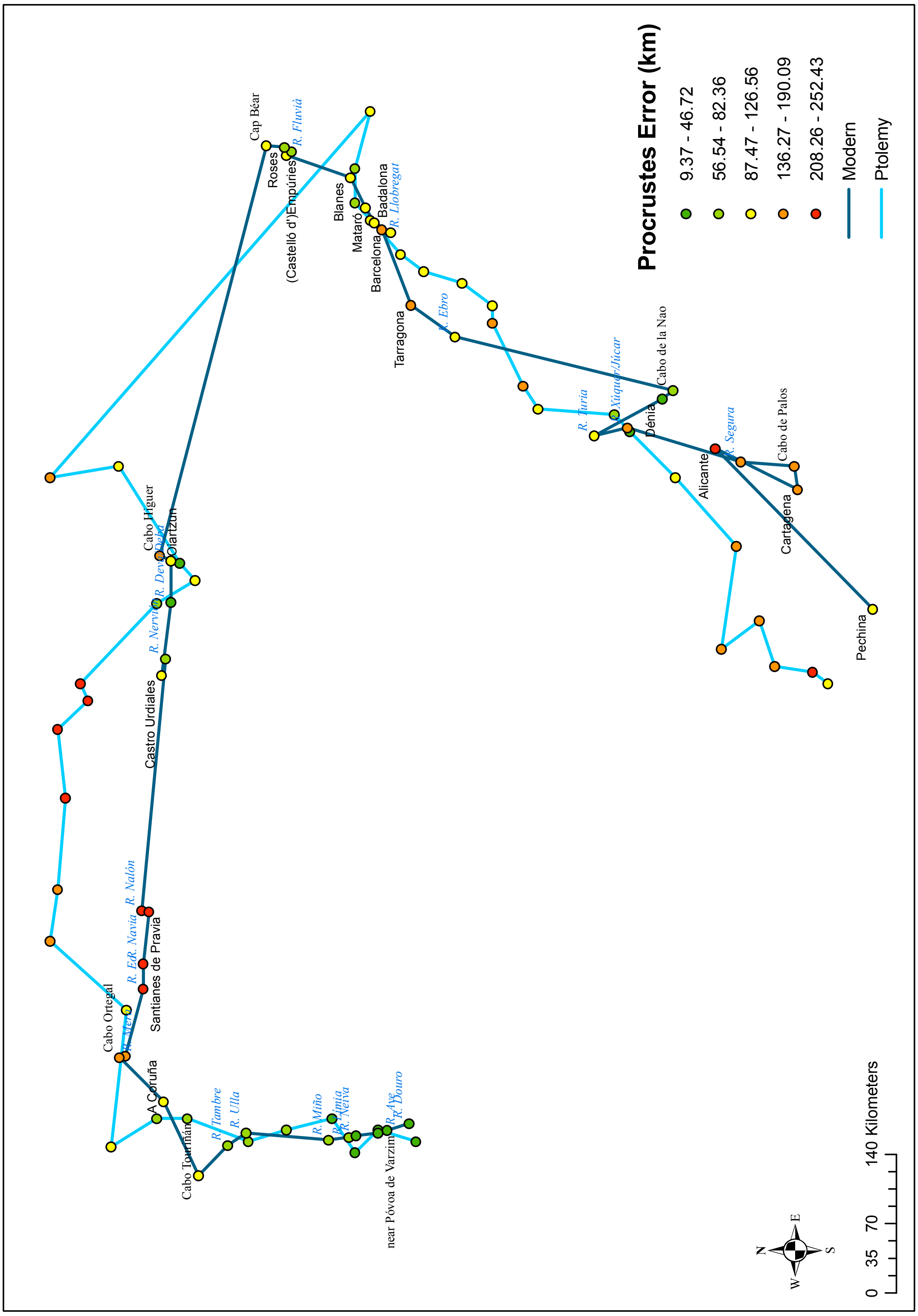

Figure 3.42: Tarraconensis' Coast: Initial Procrustes error

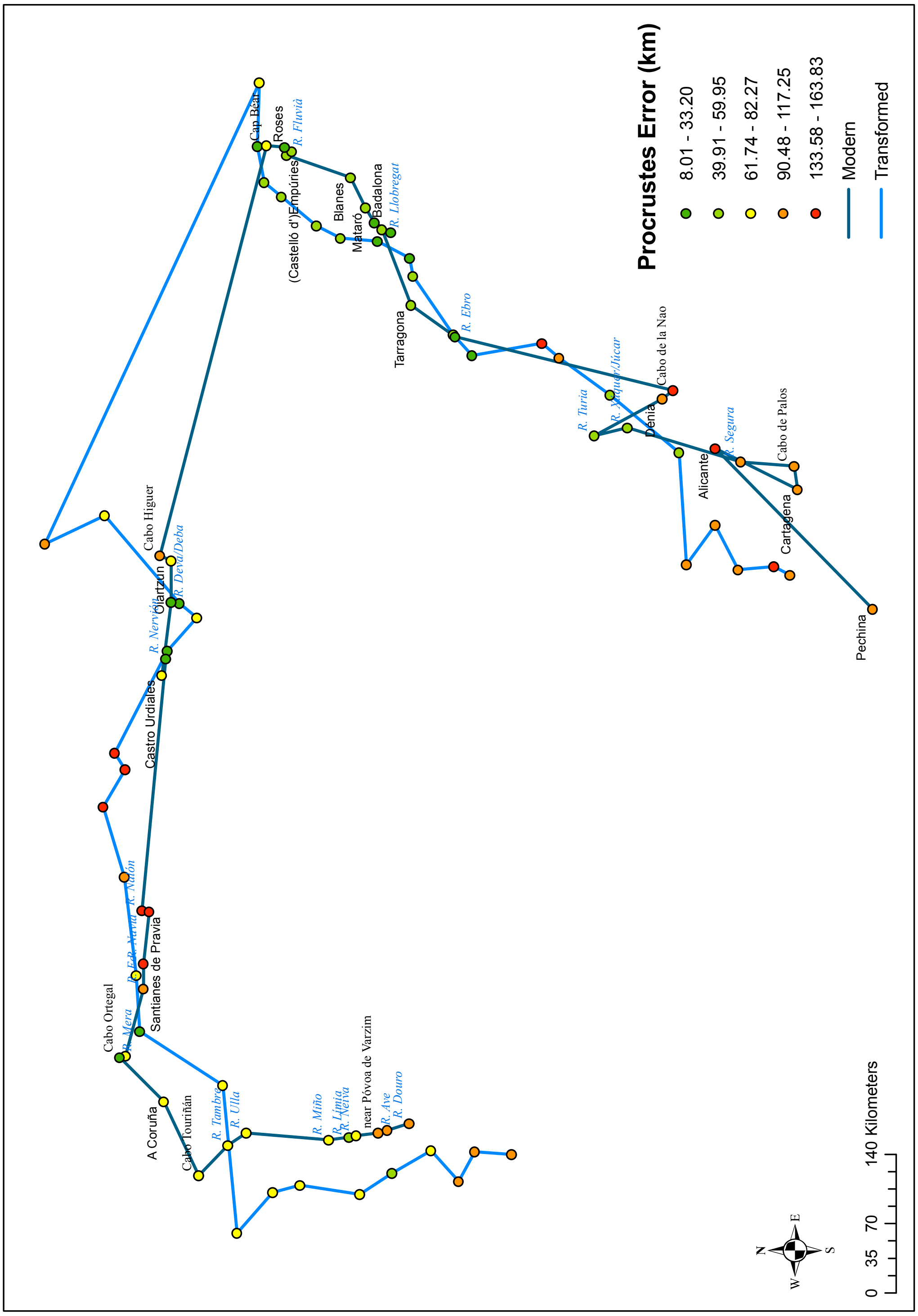

Figure 3.43: Tarraconensis' Coast: Transformed Procrustes error

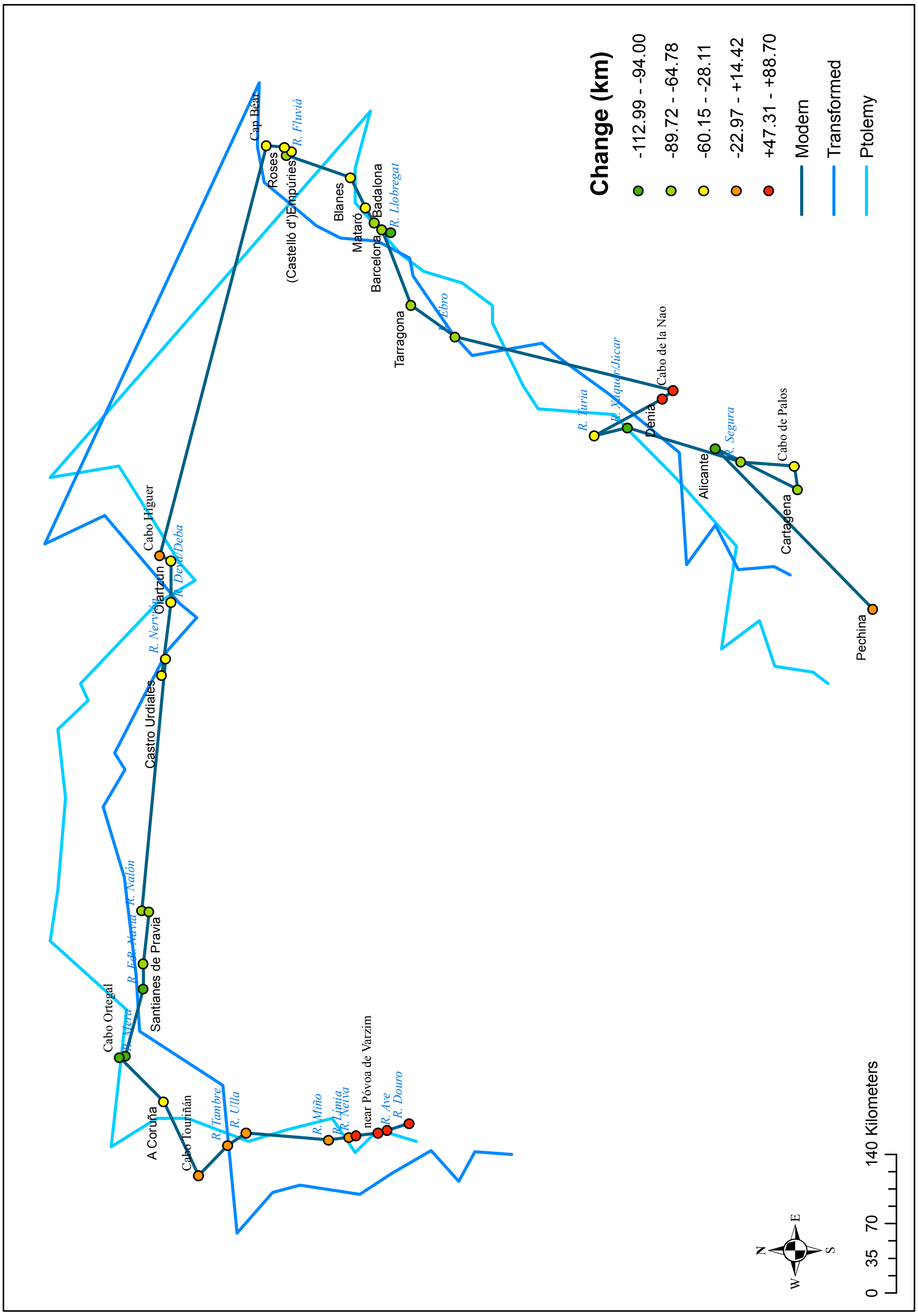

Figure 3.44: Tarraconensis' Coast: Change in Procrustes error

points that make up this coastal strip, which was well developed and travelled over hundreds of years before Ptolemy. The initial Procrustes error is 276 thousand km^2. A 9.0° anti-clockwise rotation cuts the error by over a half to 137 thousand km^2. Little scaling is needed. A longitudinal reduction of 2% with less than a 1% expansion of the latitude brings the error to 135 thousand km^2. Beginning with a Procrustes error of 76.11 km, the mean individual error is brought to 50.19 km by the transformation.

Ptolemy's coast matches up fairly well with the modern, and it is clear that a rotation will bring them closer in line (Figure 3.45). The main problem area appears to be the places between Cartagena and the Ebro, approximately halfway up the coast. The error seems to stem from Ptolemy's ordering of the coast. Between the Ebro and Adra, he has no more than 3 points in the correct order at any time. The modern coast looks so odd and twisted because it is crossing over itself following Ptolemy's ordering of the points. The problem is rampant. It also occurs twice north of the Ebro, but the points involved are very close together and the error much less severe. Had this been a one-off occurrence between two consecutive points, the fault reasonably could be assigned to transmission error. Unfortunately, the data here is either corrupt or just plain inaccurate, but because the individual distance errors are so low compared to the other parts of Hispania, the east coast still comes in as the most accurate section after Baetica.

After the transformation, the areas around the Rock of Gibraltar and Roses are the most accurate (Figure 3.46). The locations centred around Adra are in the middle error bracket, but the shape of the coast, i.e. flat, is very accurate. Unfortunately, Ptolemy's points are pulled north by the rest of the coast. The centre remains jagged, confused, and inaccurate. There are a few substantial improvements, mostly in the far north thanks to the rotation (Figure 3.47). The successes become mediocre rather quickly after that, though, as little can be done to fix the erratically placed central points.

3.4 Compare, contrast, conclude

3.4.1 Outliers

In the various groupings above, 23 settlements meet the definition of outlier both in their initial individual stress errors and their errors after transformation. In actuality, these are not 23 distinct cities as 5 'qualify' in 2 divisions of the cities (e.g. in Hispania and Baetica) and 1 in 3 divisions. Going back to the text, 9 of these 16 cities have alternative coordinates according to differing manuscript traditions. Of those 9, changing to the alternative coordinates

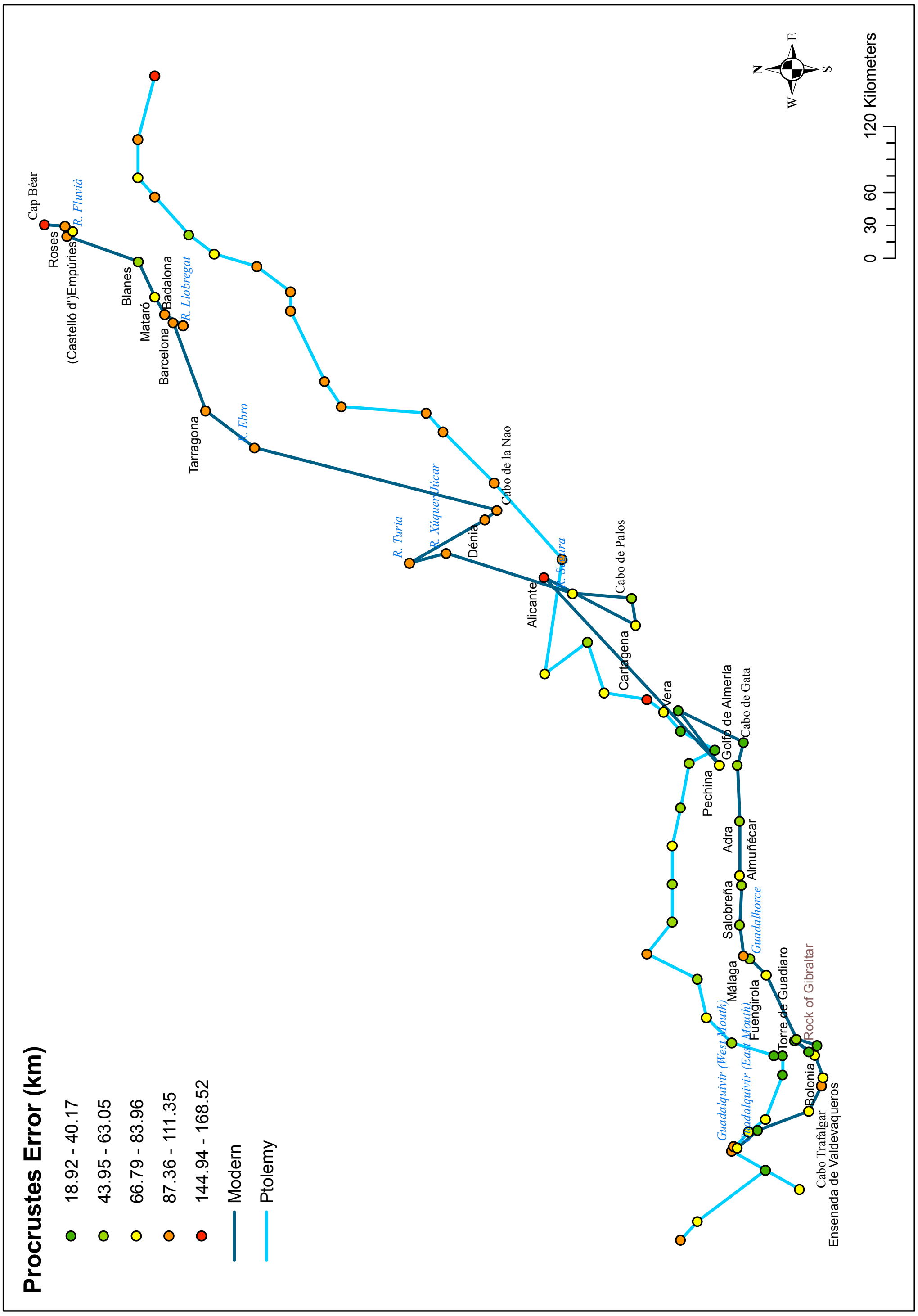

Figure 3.45: East Coast: Initial Procrustes error

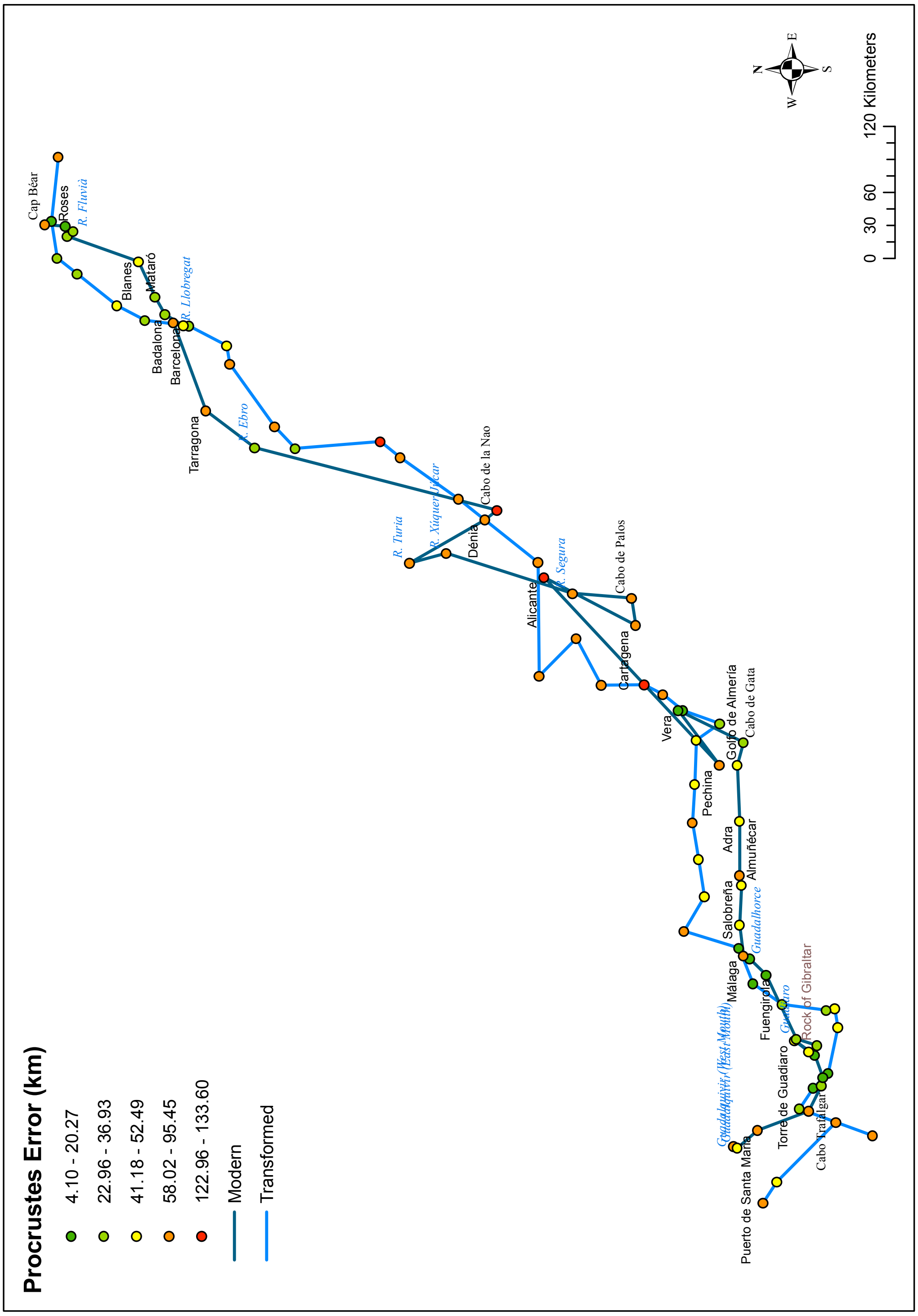

Figure 3.46: East Coast: Transformed Procrustes error

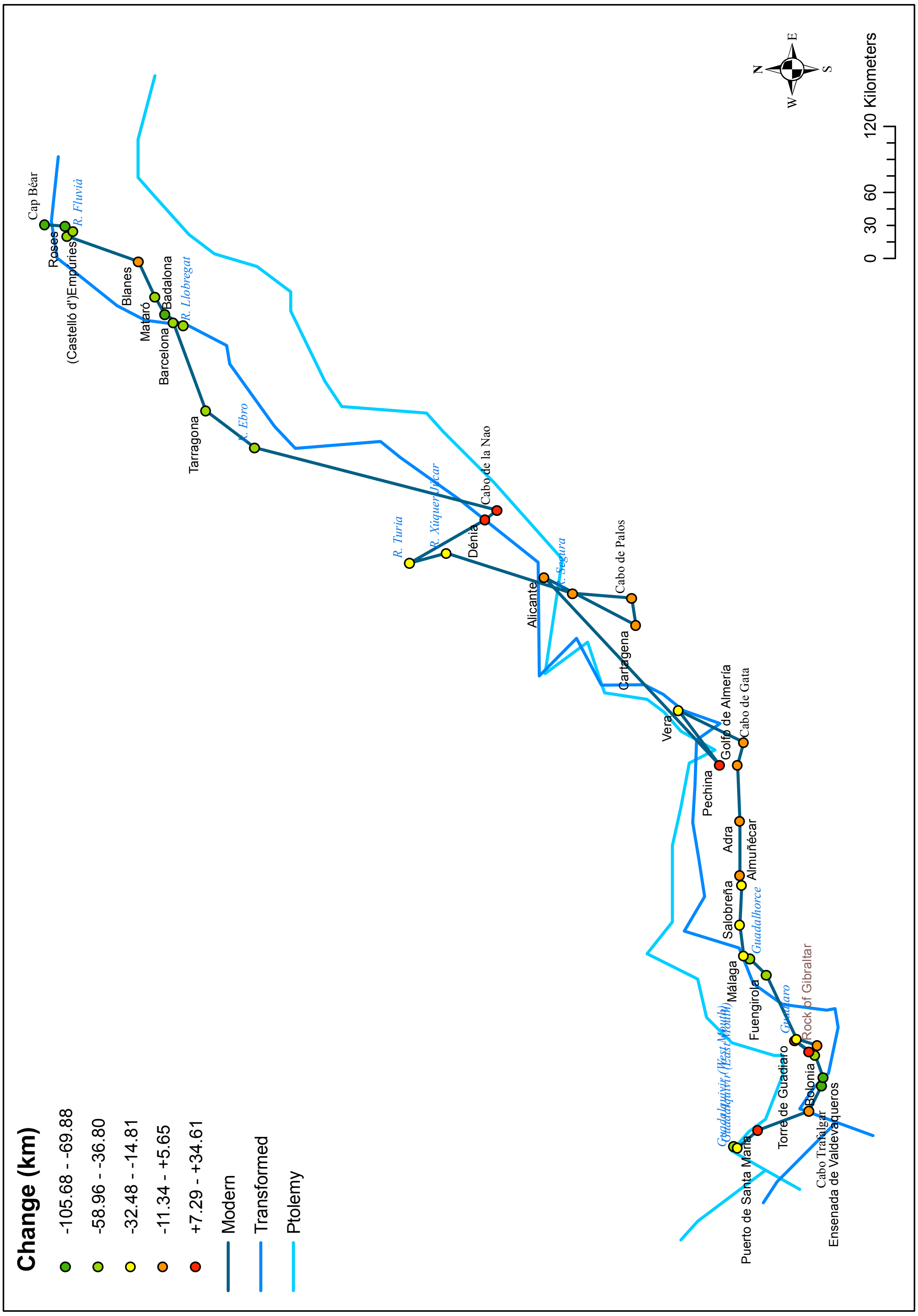

Figure 3.47: East Coast: Change in Procrustes error

improved 4 of the cities' stress while also moving them closer to their true modern locations. Those 4 cities are Granada (Iliberri), Guadix (Acci), Segovia (Segovia), and El Salobral (Salica). As well as being in the Hispania stress analysis, Granada is also in the analysis of Baetica and the East, Guadix is in Tarraconensis and the East, and both Segovia and El Salobral are in Tarraconensis.

The strange case of Granada

The analysis of Hispania, Baetica, Tarraconensis, and the East is repeated with these alternative coordinates in place. In Hispania at large, the alternative coordinates drop initial stress from 3.32% down to 3.23% and skewed stress from 3.20% to 3.12%. Interestingly, Baetica is worsened by Granada's change. Initial stress rises from 12.55% to 12.82%, and skewed stress is 0.14% higher with the alternatives. How can putting a city closer to where it belongs improve the overall map, but worsen regional maps? Let us examine the individual statistics for Granada.

In the original configuration of Hispania, Granada is 40.97 km from its modern location with an individual stress of 4.28%. For comparison, the mean Procrustes and stress errors per city are 110.20 km and 3.66%, respectively. (There are 155 cities.) Granada, therefore, is much closer to its true location than the average city, but still has above average stress. When we examine the East group of 62 cities, Granada is now out by 92.28 km and its stress jumps to 9.16%. The means, however, drop down to 79.10 km and 3.06%. When we further zoom into Baetica (39 total cities), Granada is out by 90.10 km with a stress of 24.44%. Average Procrustes error is 59.88 km, and average stress is 14.87%. (It is only the 9.16% stress in the East group that is statistically an outlier.)

What happens when the alternative coordinates are used?[7] Granada's initial Procrustes and stress errors drop significantly in the Hispania analysis after the coordinates are switched. Procrustes drops to 24.29 km and stress down to 3.92%. Mean values are 109.38 km and 3.56%, respectively. The errors relating to Granada then rise dramatically in the East and Baetica groups, higher than they were under the original coordinates. In the East group, Granada's Procrustes error is 97.23 km with a stress of 9.40%, an outlier. The means are 78.70 km and 2.95%. In Baetica, Granada has errors of 103.51 km and 29.11%, also an outlier. Average errors for Baetica are 60.26 km and 15.15%.

[7] We remember that Hispania and the East will be affected by the other alternative coordinates of Guadix, Segovia, and/or El Salobral.

Granada, in the foothills of the Sierra Nevada, is neither on the Mediterranean coast nor alongside the Guadalquivir. It is at the confluence of several rivers, one being the Genil, a tributary of the Guadalquivir, but quite some distance from it. Sources disagree exactly where the major road networks led in this particular area,[8] but it is very conceivable that Granada was not on the major routes through Cartagena and Guadix to Córdoba, Seville, and/or Cádiz. Because of its potential lack of networking with the rest of Baetica and the main east coast routes, it is possible that Granada's placement by Ptolemy is unrelated to the other cities of the region, but rather related to the general placement of cities in the peninsula. Relative to the 115 cities in Spain outside of Baetica, Granada may be well placed, but compared to the other 38 cities of Baetica it is quite far off. If so, this means that Baetica's data and potentially that for the well-travelled east coast are separate from the data for the rest of the peninsula. Because the statistics for Granada classed along with Baetica work out lower when drawn from the original coordinates, we shall revert to those coordinates both for the whole of Hispania and of Baetica. It is appropriate to remove Granada from the eastern group, in order to assist the detection of errors in the data for the major routes through the peninsula.

Outliers after Granada's restoration

Stress for Hispania with Granada back to its original coordinates and alternatives in place for Guadix, Segovia, and El Salobral starts at 3.24% with skewing bringing that number down to 3.13%. Both of these are 0.01% higher than when Grenada's alternative coordinates were used but 0.08% and 0.07% lower than with no alternatives, respectively. With Granada unchanged, Baetica and Lusitania do not change as a result of the alternative coordinates. Tarraconensis, on the other hand, drops from its stresses of 4.98% (initial) and 4.54% (skewed) to 4.81% and 4.39%, respectively, after the alternative coordinates are applied. The east group has initial stress of 2.44% and skewed stress of 2.35%. With alternative coordinates given to both Granada and Guadix the stress drops to 2.35% and 2.25%. With Granada removed from the group and Guadix still using its alternative coordinates the stress becomes 2.22% and 2.14%.

Looking now for outliers, in Hispania there are 10 outliers in terms of their original stress, but every single one of them falls into line after skewing. Unfortunately, 10 different cities become outliers because of skewing. Looking at the Procrustes analysis, only 2 cities are outliers throughout the transformations: Santianes de Pravia and Túy. Santianes de Pravia is never an outlier

[8] See Map 1 in Fear 1996 and Map 4 in Curchin 1991.

in terms of stress, but Túy is only after scaling.

When we zoom in on Baetica, we find 3 cities with a starting stress at outlier level, only 1 of which remains an outlier throughout the transformations: Ronda. Ronda is also a Procrustes outlier after rotation and scaling. (There are no original Procrustes outliers, though Ronda was an original Procrustes outlier when Granada's alternative coordinates were employed.) Lusitania has no original stress outliers and only 2 post-skewing outliers. Faro is the only initial Procrustes outlier, but it is corrected significantly after rotation. Tarraconensis has 10 initial stress outliers, reduced to 6 after skewing. Initial Procrustes outliers number 2, with post-transformation outliers at 5. The 2 original Procrustes outliers were fixed after the rotation and do not correspond with any of the stress outliers. Of the 6 final stress outliers, 3 are Procrustes outliers under rotation and skewed scaling: Segovia, Consuegra, and El Salobral. (Segovia and El Salobral at this point have alternative coordinates in place, but they are still outliers under their original coordinates.)

The East group begins and ends with 5 stress outliers, though only 4 of them are the same cities. There are 2 initial Procrustes outliers, reduced to 1 after skewing. Of the 4 cities that remain stress outliers despite transformation, one, Úbeda, is the final Procrustes outlier. The Ebro Valley has 2 initial stress outliers, 1 of which, Borja, remains an outlier throughout transformation. While there are no initial or final Procrustes outliers, there are 4 as a result of rotation. These are then fixed again by the scaling. Borja is one of those four, as is Muro de Ágreda, the second initial stress outlier.

Coastal outliers

Looking at the Procrustes analysis of the entire coast of Hispania, 5 of the 77 points are identified as outliers initially. The mean error is 119.27 km, whereas the errors for these outliers range from 241.57 km to 309.90 km. After rotation, only 2 remain outliers: the rivers Nalón and Navia. They are still outliers after scaling. Santianes de Pravia is an initial outlier, but ceases to be under rotation, returning to outlier status after skewing despite a constant improvement throughout the transformations. The final mean error for the entire coastline is 73.39 km with the 3 outliers ranging from 170.70 km to 190.87 km.

Baetica has no initial Procrustes outliers. It is only after scaling that Vera becomes an outlier with an error of 63.07 km. The final mean error of the 24 points of the Baetica coast is 25.28 km. The coast of Lusitania has no outliers. The coast of Tarraconensis has no initial or post-scaling outliers. The rivers Nálon and Navia become outliers only after rotation. The coastal points from Cap Béar down to the mouths of the Guadalquivir include 1 initial outlier:

	#	Initial stress	Transformed stress	Initial mean stress	Transformed mean stress
Hispania	155	3.32%	3.20%	3.66%	3.59%
Baetica	39	12.55%	10.20%	14.87%	11.43%
Lusitania	19	9.63%	8.92%	9.77%	9.23%
Tarraconensis	97	4.98%	4.54%	5.46%	5.20%
Ebro	18	41.66%	22.98%	52.78%	25.74%
East	62	2.44%	2.35%	3.06%	2.99%

Table 3.1: Hispania inland stress summary

Cap Béar itself. This is fixed immediately by rotation. Alicante and Cabo de la Nao become outliers as a result of rotation and remain so throughout their scaling. Their final Procrustes errors are 133.60 km and 122.96 km, respectively, compared to the mean error of 50.19 km.

Repeat offenders

The most egregious outliers are Segovia, Consuegra, and El Salobral. Removing them causes initial stress in Hispania to drop to 2.94% and in Tarraconensis to 4.28%. Skewing further brings Hispania down to 2.81% and Tarraconensis to 3.84%. The mean city stress post-skewing in Hispania before the outliers are removed is 3.50%. Outlier removal brings this figure down to 3.05%. In Tarraconensis, this mean decreases from 5.03% to 4.37%. None of the outliers seen in the coastal data persist through several groupings as these 3 cities (i.e. they are not outliers in both the east group and Tarraconensis, for example), and, as such, we have not recalculated any of the coastal statistics with them removed.

3.4.2 Comparisons

The various divisions of the peninsula are difficult to compare because stress varies dramatically when the size of the regions analysed differ. Tarraconensis, the largest by far, has both the lowest overall and mean individual stresses. Lusitania takes second place. Is Lusitania, then, more accurate than Baetica? Let us examine the provinces in turn.

Tarraconensis differs little from the whole of the Iberian peninsula (Table 3.1). Its stress is slightly higher as the loss of points in Lusitania and Baetica reduce the total modern distances, increasing the stress. Its transformation is very similar to that of Hispania as a whole, the only significant difference being the latitudinal skew, but even then it is only a difference of 8% (Table

	Rotation	Longitude skew	Latitude skew
Hispania	-13°	+7%	-3%
Baetica	-4°	-14%	-14%
Lusitania	-20°	+11%	-14%
Tarraconensis	-14°	+9%	+5%
Ebro	-10°	-27%	-39%
East	-8°	-1%	+9%

Table 3.2: Hispania inland transformation summary

3.2). The discrepancy can be blamed on Baetica. Baetica is the southern portion of Hispania which is missing from Tarraconensis. Baetica's need of a 14% reduction of latitude affects the whole of the peninsula, but is not what Tarraconensis on its own needs. The longitudinal skewing however, is largely unchanged as Tarraconensis still stretches across the entire width of Hispania.

Baetica's stress is the highest of the three Roman provinces. This is particularly troubling as Baetica was the most developed and well established of the three. However, it is also the most densely packed province. With 39 cities together in such a relatively small swathe of land, even the tiniest errors in misplacement can result in large errors in stress. Lusitania, on the other hand, which has slightly more area than Baetica, but far fewer cities, has a lower stress despite having larger absolute errors. (Baetica's initial configuration, on average, has cities misplaced by 60 km from their modern counterparts, while Lusitania averages 81 km.) The fewer cities of Lusitania are not tightly grouped, and so their erroneous placements do not so readily affect their relative positions. Thus, Lusitania is able to keep its stress lower than the urban sprawl of Baetica.

The east group of cities has the lowest stress, and while we might be quick to say that this is due to the eastern coastal region being well travelled with a superb road network linking it to southern France and northern Italy, we would be neglecting an important numerical consideration. The east group consists of a large portion of the high stress of Baetica. This included portion of Baetica is not an area of any particularly low stresses. It is the long, narrow shape of the east group rather than native accuracy that keeps the stress low. Because of the great distance spanned by the cluster, localised misplacement errors have very little affect on the stress of the region. Even though Roses is placed over 100 km from its true location, it is still north of most of the other 61 cities, and, consequently, it has an initial stress of only 0.6%. The low stress of the east group is a result of its shape, not of Ptolemy's arrangement, unlike the Ebro group which is simply inaccurate.

	#	Initial error	Transformed error	Initial mean error	Transformed mean error
Hispania	77	1,352K	532K	119.27	73.39
Baetica	24	34.1K	19.8K	35.51	25.28
Lusitania	13	138.0K	54.1K	89.05	58.11
Tarraconensis	41	688.1K	300.2K	113.73	75.77
East	41	276.5K	135.4K	76.11	50.19

Table 3.3: Hispania coastal error summary (in km and km^2)

	Rotation	Longitude skew	Latitude skew
Hispania	-13°	+6%	-6%
Baetica	+1°	-14%	-21%
Lusitania	-34°	-65%	+4%
Tarraconensis	-13°	+8%	-7%
East	-9°	-2%	+1%

Table 3.4: Hispania coastal transformation summary

As for the coastal error analysis, we see that Baetica has the lowest Procrustes error across the board (Table 3.3).[9] This is in juxtaposition to it having the highest stress of the three Roman provinces of Iberia. It is not the case that Baetica's coastal points are more accurately placed than its inland cities, but rather, due to its compact size and high density, the small misplacement errors of the cities produce large stress errors.

The inaccuracies in Lusitania are hidden in the stress analysis, but they become apparent in its coastal analysis. With about half the number of points of Baetica, Lusitania has more than twice the Procrustes error in all categories of measurement. With so few points for which to account, the transformation has much more freedom to correct the errors as compared to the transformation of Baetica's coast, yet Lusitania's points could not be brought in line. The misplacement of points along Lusitania's coast are therefore far from consistent.

Tarraconensis, as with its stress analysis, has similar Procrustes figures to those of Hispania as a whole. Obviously the total error is far reduced when the 37 points of Baetica and Lusitania are removed, but its individual errors are about the same. In fact, the individual numbers are better for the whole peninsula post-transformation than for Tarraconensis. Certainly the small errors of Baetica aided the mean of Hispania at large. We can also see

[9]Tarraconensis and Lusitania both contain the River Douro. This is why the total number of points in Hispania is one less than the summation of the three provinces.

that the east coast group, having the same number of points as Tarraconensis' entire coastline, has much smaller errors. The eastern parts of Tarraconensis and Baetica seem to have a relationship that may well factor in to the results of Hispania, but Tarraconensis on its own is left in an unresolved situation.

The coastal transformations for Hispania and Tarraconensis are essentially equal (Figure 3.4), yet as we have seen, this transformation works slightly better for the whole peninsula than just Tarraconensis thanks to the hoarding of accurately placed points by Baetica. Lusitania's coastal transformation may be viewed dismissively. As we have observed above, its longitudinal skew serves to shrink the coast into a straight line to resolve its displacement. This is not reflective in anyway of what has happened to Ptolemy's coast, serving only as an indication that it is grossly inaccurate. Baetica's coastal transformation differs substantially from the rest of the peninsula. It barely rotates and needs to be reduced along both axes. All others require an anticlockwise rotation and contrary scalings. The misalignment of the Pyrenees does not seem to carry through to Baetica in the opposite corner of the peninsula, while still affecting the rotations of all other sides of the coast. Is Baetica properly aligned to the globe or improperly aligned to the rest of Iberia?

Hispania's inland and coastal transformations are practically identical. Tarraconensis' transformations only differ in the latitudinal scaling. The northern coast's diversion into the Bay of Biscay accounts for some of the need of the reduction in the coastal transformation that is lacking in the inland analysis. Similarly, Baetica's two transformations really only differ with regards to latitude. The coast of Baetica, though, is so narrow in the north-south direction that the additional 7% reduction over the inland transformation amounts to very little actual movement of points.

Lusitania's two transformations are quite distinct from one another. The problem, again, appears to be the flattening of Ptolemy's coast during error minimisation. The inland area has a more substantial width with which to work and simply reducing the region to a narrow band is not a viable option for its minimisation. Like Baetica and Tarraconensis, the two eastern transformations differ only in the latitudinal scaling. With the inclusion of the inland points of south-eastern Tarraconensis and eastern Baetica, the east group's inland configuration has more of a stake in the latitudinal scaling than the narrow coastline alone, and a more substantial expansion is required. The Ebro valley has no coastline for comparison's sake.

3.4.3 Putting it all together

As predicted, the coast along the Mediterranean and around the Strait of Gibraltar represent the most accurate section of coastline, while the northern and eastern coasts, far from Roman authority during most of Spain's history, are ill placed by Ptolemy. This is especially true along the coast of the Lusitania. The Mediterranean coast is not without its troubles, though, as we have seen with regard to the ordering of points. It may be that the errors here sprung from Ptolemy's integration of new data into older maps, such as those drawn by Marinus.

As a whole the interior stress is quite low, and the inaccuracies only become apparent when smaller sections of the peninsula are viewed in turn. Lusitania, traditionally a difficult province for the Romans to rule, is at a level of accuracy that we would expect given its history. Tarraconensis, due to its large area and spread of points, glosses over many of its troubles and reflects the relatively low stress of the entire peninsula. The historical development of the province, however, is not reflected in Baetica or in the Ebro valley. Apologies have been made for Baetica because of its small area and high density, however, the Guadalquivir river valley is wide, flat, and home to a major transportation artery. To be sure, there are mountainous areas as we saw in the discussion of Granada, but on the whole better results were expected in Baetica. The Ebro river valley, on the other hand, is surrounded by mountains up to its banks, with each tributary behind another set of foothills and spurs. The topography of the region certainly contributes to the inaccuracy of the region. However, this is not an excuse for an initial stress of over 41%. The data is highly suspect. We hope that the error lies with competing information flowing to Ptolemy and not with a single misguided source.

Chapter 4

Britain

4.1 The Romans in Britain

4.1.1 Caesar makes contact

While campaigning in northern Gaul, Julius Caesar turned his attention to Britain. In 55 BC, he writes that the Gauls, though often receiving reinforcements from and having trade relations with Britain, did not know anything

> . . . except the sea-coast and the districts opposite Gaul. Therefore, although he summoned to his quarters traders from all parts, he could discover neither the size of the island, nor the number or the strength of the tribes inhabiting it, nor their manner of warfare, nor the ordinances they observed, nor the harbours suitable for a number of large ships (Caesar, *De Bello Gallico* 4.20).

Crossing from Gaul, Caesar made for Dover, describing the steep cliffs abutting the sea. However, due to the mass of enemy troops waiting on said cliffs, Caesar had to sail some seven miles further on until he found a shore that 'was even and open' (Caesar, *B.G.* 4.23). Caesar left Britain at the time of the autumnal equinox, about one month later. In his account, it seems that his camp was very close to the initial beachhead and that no exploration of the surrounding territory was made save to find grain to feed the troops (Caesar, *B.G.* 4.32,36).

The following spring, Caesar again made ready to invade Britain. Landing at an unspecified location 'on a sandy, open shore' (Caesar, *B.G.* 5.9), Caesar headed inland to attack a unified force of various British tribes lead by Cassivellaunus who, according to Caesar, controlled the lands on the further side of 'the river called Thames, about eighty miles from the sea' (Caesar, *B.G.* 5.11).

After giving battle, Caesar forded the Thames at an unknown location. During the march, he was continually hounded by the chariots of Cassivellaunus, but at the same time several tribes surrendered to Caesar. These included five tribes of uncertain location in south-eastern Britain and the Trinovantes of Essex with their capital at Camulodunum (Colchester). From these tribes Caesar learned the location of Cassivellaunus' headquarters, which some scholars identify with Verulamium (St Albans), though Caesar describes the 'stronghold' as simply a fortified wooded area. (Caesar, *B.G.* 5.20-21; Caesar 1998, 232) Whatever the nature of Cassivellaunus's stronghold and whether or not it was Verulamium, Caesar notes that it was not far from his position on the north side of the Thames. He quickly marched on the place and captured it with minimal effort (Caesar, *B.G.* 5.21).

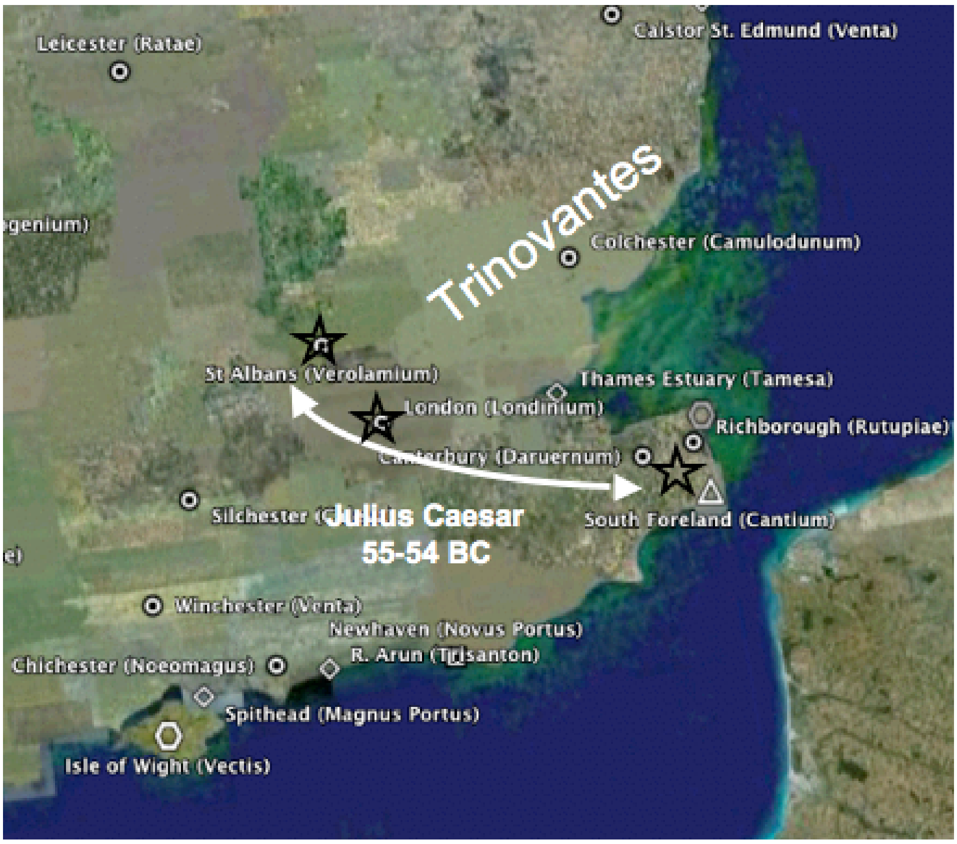

Figure 4.1: Julius Caesar's Invasion of Britain BC 55-54 (base map ©Google 2010)

After this and a failed attempt by the allies of Cassivellaunus to destroy Caesar's beached fleet, Caesar sailed back to Gaul never to return. He makes no other mention of Britain in *De Bello Gallico* save for a brief discussion on the origin of the Druids (Caesar, *B.G.* 6.13).

4.1.2 Britain on the back burner

After Caesar's withdrawal from Britain, Rome soon became engulfed in civil war and the various upheavals in the aftermath of Caesar's assassination. Trade between Britain and Rome seems to have continued, and diplomatic relations with some tribes were restored once Augustus took the helm of state (Ireland 1996, 37). Strabo indicates that such ties were rather strong, noting that some chieftains 'have also managed to make the whole of the island virtually Roman property' (Strabo, 4.5.3; Ireland 1996, 37). No knowledge of northern Britain had been gained, and the Romans, realising that there was no military or financial gain to be had, showed no real interest in invading the island (Strabo, 2.5.8; Ireland 1996, 38). Dio Cassius (second to third century AD) writes that Augustus had set out against Britain but did not get far into Gaul before he needed to quell a revolt in Spain. Possibly Augustus wanted to invade Britain as a symbolic gesture of following in the footsteps of his adopted father, Julius Caesar; possibly he wanted to force certain tribes into paying a tribute. Whatever the reasons, they were not significant enough to trump maintaining order in previously conquered areas of the new Empire (Cassius Dio, 49.38.2, 53.22.5, 53.25.2; Ireland 1996, 38).

The reign of Tiberius (AD 14-37) brought nothing new to British-Roman relations. Caligula brought an army to the shores of the Channel but did not cross it. Instead, it seems he positioned his men to attack the water before demanding that they collect seashells. 'Was it simply a manifestation of imperial insanity, or a shrewd attempt to humiliate troops who refused to embark for action in a land still regarded with awe, suspicion and fear' (Ireland 1996, 41)?

4.1.3 The invasion of Claudius

Dio Cassius writes that Aulus Plautius was given command of an invasion in AD 43 under the Emperor Claudius. Even at that time 'the soldiers were indignant at the thought of carrying on a campaign outside the limits of the known world' (Cassius Dio, 60.19; Ireland 1996, 45). A battle was fought at the crossing of an unnamed river and again at a crossing of the Thames Estuary near a pre-existing bridge. The Romans were victorious in both, but lost some men in nearby marshes in the pursuit of the retreating Britons (Cassius Dio, 60.20; Ireland 1996, 46).

The Romans held their position on the south side of the Thames while awaiting the Emperor's arrival from Rome. Claudius joined the army there, crossed the river, and lead the army into Essex. He defeated the natives and captured Camulodunum. 'He deprived the conquered of their arms and handed them over to Plautius, bidding him also subjugate the remaining districts' (Cassius Dio, 60.21; Ireland 1996, 47).

4.1.4 Expansion

Four legions were left in Britain to carry out Claudius' orders of subjugation.

> While the XX Valeria remained to hold what had already been gained, the IX Hispana advanced northwards, the XIV Gemina north-west over the Midlands, and the II Augusta, the best documented of them all since its Legate was the future Emperor Vespasian, into the west country (Ireland 1996, 51).

According to Suetonius, writing in the second century AD, more than twenty towns and the Isle of Vectis (Wight) surrendered to Vespasian (Suetonius 1989, *Vespasian* 4). Funerary inscriptions tell us that the IX Hispana reached Lindum (Lincoln), and the XIV Gemina reached Viroconium (Wroxeter) (Ireland 1996, 52). This was all in the governorship of Aulus Plautius (43-47). He was succeeded in 47 by Ostorius Scapula.

Ostorius' first campaign was of securing those areas newly gained against native uprisings. Tacitus states that Ostorius wanted to hold the area between the rivers Sabrina (Severn) and Trisantona (sometimes identified with the Trent) (Tacitus, *Annals* 12.31; Ireland 1996, 52-53). This is the area then approximately south-east of the angle made by Lincoln, Wroxeter, and Wight. Tacitus says that the Iceni were the first tribe to fall to Rome in this campaign. No location is given. Afterwards, Ostorius attacked the Decangi (Deceangli) which puts the army close to the Irish Sea (Tacitus, *Annals* 12.31-32; Ireland 1996, 53). The difficulty here is that Ptolemy puts the Iceni in East Anglia, quite the distance from the Irish Sea where the Decangi are located. Tacitus also claims that Ostorius was not using the legions at this time, but if the XX Valeria was in Colchester, surely it would have dealt with the Iceni, and the XIV Gemina in Wroxeter would have subdued the Decangi. However, Tacitus notes that both tribes were met in battle by auxiliaries under the command of the governor Ostorius (Tacitus, *Annals* 12.31).

After establishing a *colonia* at Colchester, Ostorius moved against the Silures (located in southeast Wales by Ptolemy), presumably with XX Valeria as it was no longer needed at Colchester. The campaign eventually moved into the territory of the Ordovices (located in northern Wales by Ptolemy). Tacitus is mute on the location of these battles, mentioning neither settlements nor rivers by name. The Romans had the victory, though with significant casualties (Tacitus, *Annals* 12.32-35; Ireland 1996, 53-54).

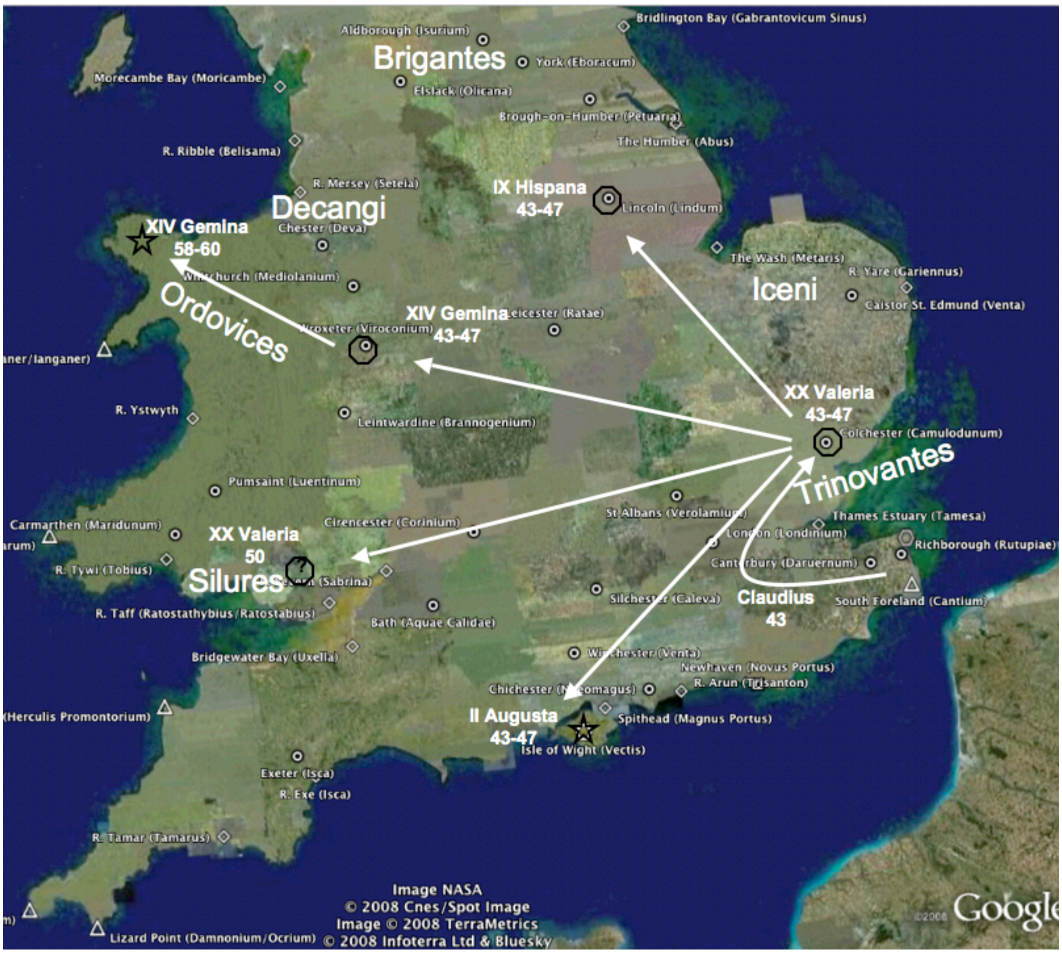

Figure 4.2: The invasion of Claudius and beyond AD 43-60 (base map ©Google 2010)

Taking over the governorship of Britain after Ostorius' death in 52 was Aulus Didius. Little territory was gained for Rome under his leadership. Most of his work consisted of securing the areas fought for under Ostorius and sending aid to the allied Brigantes to the north (Tacitus, *Annals* 12.40; Ireland 1996, 57). With the death of Claudius and the ascension of Nero, there was a flagging interest in Britain as it was viewed as more trouble than it was worth. The only thing stopping Nero from recalling the legions from Britain was his imperial honour (Suetonius, *Nero* 18; Ireland 1996, 57-58). In 57, the new governor, Quintus Veranius, renewed action against the Silures in southern Wales but died soon after taking command. He was succeeded in 58 by Suetonius Paulinus who planned to subdue the island of Mona (Anglesey). Despite his success here with, presumably, the XIV Gemina, the Iceni and Trinovantes in the east revolted. Colchester was sacked and the IX Hispana, coming to the rescue, was routed after sustaining heavy casualties (Tacitus, *Annals* 14.29-32; Ireland 1996, 58-60).

> Suetonius, on the other hand, with remarkable firmness, marched straight through the midst of the enemy upon London; which, though not distinguished by the title of colony, was none the less a busy centre, chiefly through its crowd of merchants and stores (Tacitus, *Annals* 14.33).

This is the first mention of Londinium (London) by Tacitus. Presumably, London had existed before the Roman invasions of this period, but lacked much importance until the Romans put it to good use as a trading depot. Suetonius decided to abandon London and wait for more reinforcements before making a victorious stand against the rebels. Both London and St Albans were sacked (Tacitus, *Annals* 14.33; Ireland 1996, 60-61). Following this was a period of rebuilding and recovery under a new governor, Petronius Turpilianus. London seems to have taken over as the commercial and political capital of the province at this time (Ireland 1996, 71).

4.1.5 Rebuilding

In 63 Petronius was replaced by Trebellius Maximus, who carried on the work of internal pacification and stability. No expansion of the province was attempted as the XIV Gemina was withdrawn from Britain, leaving only three legions. These legions had been restored to their former numbers with soldiers from Germany (Tacitus, *Annals* 14.38; Ireland 1996, 70). The XX Valeria took over at Wroxeter, leaving its unnamed base in southern Wales (Ireland 1996, 73). In 68, the Emperor Nero committed suicide, and Rome once again plunged into civil war. While there was some intrigue within the legions of Britain, very little was noted during this time. There were no campaigns or recorded revolts. Trebellius was expelled by his own army and replaced by Vettius Bolanus, who was appointed by the leading imperial candidate, Vitellius. Also, the XIV Gemina was sent back to Britain, not so much to campaign but as to keep them away from the main body of Vitellius' supporters as that legion had been opposed to his rule (Tacitus, *Histories* 2.65-66; Ireland 1996, 75). It seems the only major action under Bolanus was a rescue operation of Cartimandua, queen of the Brigantes. Her former husband took over the kingship of the tribe and led a revolt against her. Cartimandua found asylum amongst the Romans (Tacitus, *Histories* 3.45; Ireland 1996, 77-78). In 70, XIV Gemina was again withdrawn from Britain, and, in 71, Bolanus, whose 'rule was milder than a warlike province requires' (Tacitus, *Agricola* 8), was recalled and replaced by Petillius Cerialis (Ireland 1996, 78).

Over a decade had now passed since the rebellion which saw the sack of Colchester, London, and St Albans. In that time there had been no campaigns or major revolts. Tacitus gives little indication of what did happen in Britain at that time beyond the politics of the Civil War. He mentions the inactivity of the governors of the period in a rather contemptuous tone. Assuredly, though, the military inaction of the governors must have been compensated for by their civil activities. Could this not have been a decade of construction? Rebuilding the sacked towns? Paving old tribal paths and building new roads? Setting up a working civil government? Surveying landscapes? Dedicating temples? Integrating Britons into Roman society and culture? It is very difficult to believe that the bulk of three or four Roman legions sat completely idle for ten years. Much more likely is that this is the time when south-eastern England truly established itself as a functioning Roman province.

4.1.6 Renewed conquest

Petillius Cerialis brought with him to Britain the II Adiutrix legion and had them stationed at Lincoln. The IX Hispana was marched north and eventually were quartered at Eboracum (York) (Ireland 1996, 78). Petillius immediately set to work with a campaign against the Brigantes in northern England much of whose territory was gained for Rome (Tacitus, *Agricola* 17; Ireland 1996, 78). Julius Frontinus succeeded Petillius and renewed Rome's campaign against the Silures in Wales. Though lacking in primary source materials beyond a mention in Ptolemy's *Geography*, the II Augusta apparently had been stationed at Isca (Exeter) before moving to Gloucester to replace the XX Valeria when it moved to Wroxeter. Frontinus used the II Augusta in his campaign against the Silures and stationed them at a new fortress at Caerleon. During this time, a new fortress was also build for the II Adiutrix who moved to Chester (Ireland 1996, 79).

4.1.7 Tacitus' account - *Agricola*

In 77, Julius Agricola succeeded Frontinus as governor. His son-in-law, Tacitus, wrote his biography, and though glossed with flattery, it is the most contem-

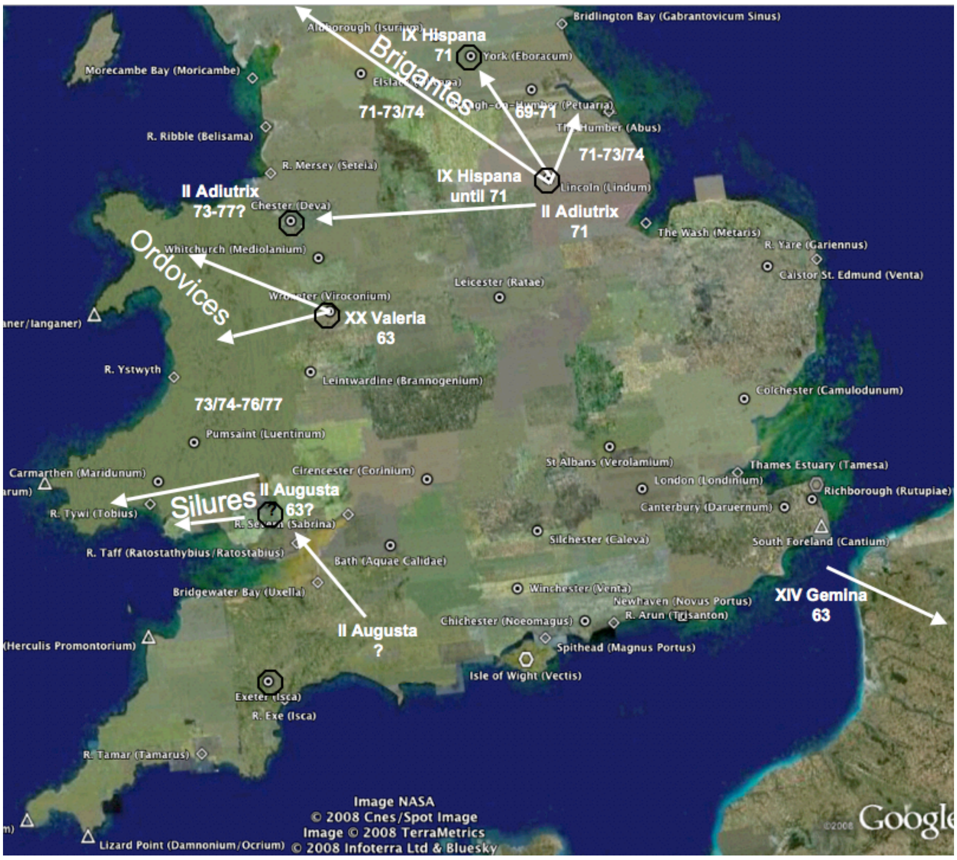

Figure 4.3: Expansion AD 61-77 (base map ©Google 2010)

porary and thorough account of Roman involvement in Britain. Immediately upon his arrival in Britain, Agricola met the unruly Ordovices in battle and almost annihilated them. Ptolemy puts this tribe in northern Wales. Agricola then proceeded to finish the subjugation of Mona (Anglesey) that began until Paulinus. According to Tacitus, as he had no boats for the crossing, Agricola sent unarmed soldiers swimming across the strait and 'launched them upon the enemy so suddenly that the astonished islanders ... promptly came to the conclusion that nothing was hard and nothing invincible to men who fought in this fashion' (Tacitus, *Agricola* 18). The islanders surrendered without a fight.

Agricola spent the winter reforming the provincial government. The following summer, he was out on campaign, though Tacitus does not inform us where or against what tribes. The text indicates that this was merely a set of raids in order to instil fear into the tribes of Britain. Were it not for the following remark, one would assume these raids were restricted to those tribes previously subdued: '... they were then so carefully and skilfully surrounded with Roman garrisons and forts that no newly acquired district ever before passed over to Rome without interference from the neighbours' (Tacitus, *Agricola* 20). Once this fear of the Roman war machine was established, Agricola spent the next winter sponsoring civic building campaigns. '... he would exhort individuals, assist communities, to erect temples, market-places, houses... ' (Tacitus, *Agricola* 21). Here is the first clear indication that Romanisation had begun.

The third summer saw Agricola push north up to the Taus (Tay) Estuary. There was very little in the way of fighting, but a string of forts was established. The next year he made sure this new area was secured. The Forth-Clyde isthmus was now garrisoned, and all lands to the south were in Roman hands (Tacitus, *Agricola* 22-23). Tacitus does not have Agricola march north the next year. '... he crossed in the leading ship and in repeated and successful battles reduced tribes up to that time unknown: he also manned with troops that part of the British coast which faces Ireland... ' (Tacitus, *Agricola* 24). Does this mean that Agricola was conducting a naval campaign along the coast of Ayrshire and Dumfries and Galloway despite Tacitus claiming that all of Great Britain south of the Forth-Clyde isthmus was subdued? A campaign against Ireland was never undertaken, even with the perceived advantage it would give as Ireland was thought to have been halfway between Britain and Spain. This belief persisted despite the fact that the Romans, through trade, knew 'the approaches to the island and its harbours. [Furthermore,] Agricola had given shelter to one of [Ireland's] petty chieftains... ' (Tacitus, *Agricola* 24).

The summer of Agricola's sixth year, 'he embraced in his operations the tribes beyond the Forth' (Tacitus, *Agricola* 25). The army and the fleet moved

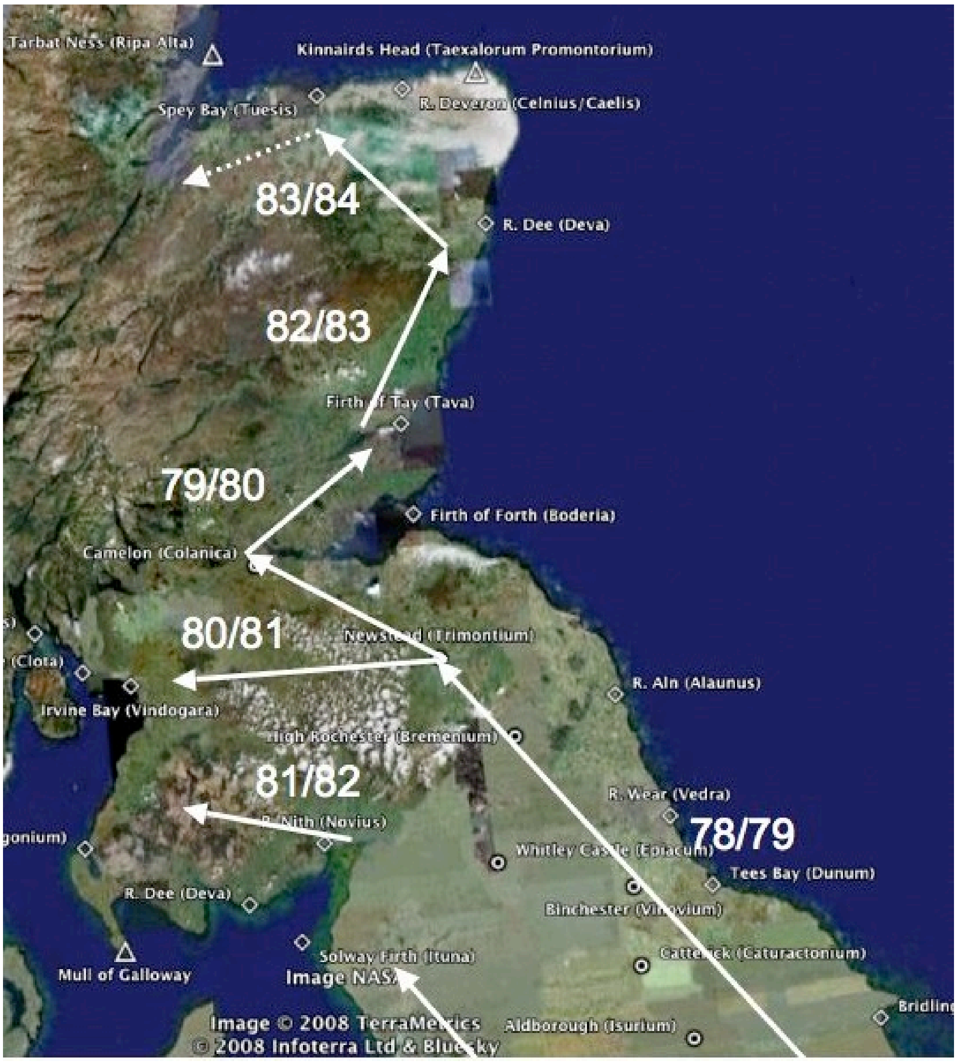

Figure 4.4: Expansion under Agricola AD 77-84 (base map ©Google 2010)

in tandem up the east coast. The tribes in the area banded together and attacked the IX Hispana. Reinforcements arrived, and the Britons were routed, but not defeated. Both sides prepared for a major engagement the following summer (Tacitus, *Agricola* 25-27). This battle took place at the Graupian Mountain, the whereabouts of which are the subject of debate. The Roman victory was decisive, and the surviving Britons fled in all directions. It was at this time that Agricola ordered the fleet to circumnavigate the island. The fleet discovered and 'conquered' the Orcades (Orkneys) as well as sighting Thule (Shetland) from afar (Tacitus, *Agricola* 10). Agricola was then recalled from Britain by the Emperor Domitian the next spring in 84 (Tacitus, *Agricola* 29-38).

4.1.8 From Agricola to the walls

'Britain [was] subdued and immediately let go' (Tacitus, *Histories* 1.2; Ireland 1996, 84). When Agricola was recalled from Britain in 84, four legions were in Britain. It seems three or four of these had been used in the Scottish campaigns. The II Adiutrix was removed about 87 to serve in Dacia. A legionary fortress had been built in Inchtuthil and abandoned a few years later. Which legion was stationed there is unclear. Lincoln was converted from a legionary fortress into a *colonia* at some point in the 90s. During the reign of Nerva (96-98), Gloucester was also made into a *colonia*. The three legions were now stationed in Caerleon (II Augusta), Chester (XX Valeria), and York (IX Hispana) (Ireland 1996, 84-86).

The Emperor Hadrian visited Britain in 122, and, soon after, the VI Victrix arrived. As the IX Hispana is not mentioned again in the context of Britain, perhaps the VI Victrix was a replacement for it. In Britain, Hadrian 'corrected many faults and was the first to build a wall, 80 miles long, to separate the Romans and barbarians' (*Scriptores Historiae Augustae*, *Hadrian* 11.2; Ireland 1996, 87). Several inscriptions indicate that the VI Victrix, II Augusta, and XX Valeria all worked on the wall's construction (Ireland 1996, 90, 99 note 3).

In 138, Antoninus Pius ascended to the imperial throne and had the Roman frontier pushed north. 'Through the governor Lollius Urbicus he defeated the Britons, and having driven back the barbarians, he built another wall, this time of turf' (*Scriptores Historiae Augustae*, *Antoninus Pius*, 5.4; Ireland 1996, 91). Corbridge was improved at this time by the II Augusta before the legion began construction of the Antonine Wall. The VI Victrix and XX Valeria also left inscriptions detailing their involvement in the building of the Wall (Ireland 1996, 91-92). An inscription from Newcastle from the 150s notes that reinforcements arrived for all three legions under the governor Julius Verus. Various other inscriptions from the south of Scotland attest to movement of the Roman frontier back and forth between the two walls (Ireland 1996, 94-95). Such was the state of Britain at the approximate time of the publication of Ptolemy's *Geography*.

4.2 The cities

4.2.1 A province united

Ptolemy's identified cities have a mean longitude of 18.89° and mean latitude of 55.82°. The modern centroid is at $(-1.59°, 52.83°)$. Longitude cannot be commented on as Ptolemy's prime meridian through the Blessed (Canary) Isles is rather difficult to pinpoint. (Also, for comparison's sake, the longitudes of Italy and Spain were discussed in the context of Ptolemy's London, which would be moot here.) Ptolemy's latitude, however, is a full 3° higher than it should be. That is a distance of over 330 km.

The inland cities of Britain include 30 identified settlements. An anti-clockwise rotation of 12° reduces the initial Procrustes error of 205 thousand km^2 to 159 thousand km^2. Initial stress is 4.58%. Scaling does little to improve this error. A 12% expansion of the longitude accompanied with a 3% reduction in latitude minimises the stress at 4.42%. Mean individual city stress drops from 5.21% to 5.02%.

Figure 4.5 shows those 30 cities in their modern locations with their initial stress errors. Most cities are located in modern England, with only 2 identified settlements each in Wales and Scotland. The highest errors are in the centre of the map, especially along the Welsh border. Chester has such a high stress that it is in its own category with an error more than twice that of any other location. The lowest errors are found in the south-east of England. A smaller string of low stress cities is just south of the very top of the map, on either side of the Scottish border.

Looking now at Ptolemy's configuration (Figure 4.6), we get a completely different picture. Central England on the modern map contains only Leicester, but Ptolemy has a much more crowded centre. Lincoln and Leicester are brought together as are Caistor-by-Norwich and Colchester. These are relatively minor errors, though, compared to the positioning of Chester and Saint Albans. Saint Albans should only be 31 kilometres from London, yet Ptolemy, or at least the manuscripts, place it over 142 kilometres away from the capital. Chester should be on the northern Welsh border and not in south-central England.

The north maintains low to moderate stresses despite Ptolemy's divergence from the relatively linear arrangement of the cities in the modern configuration.

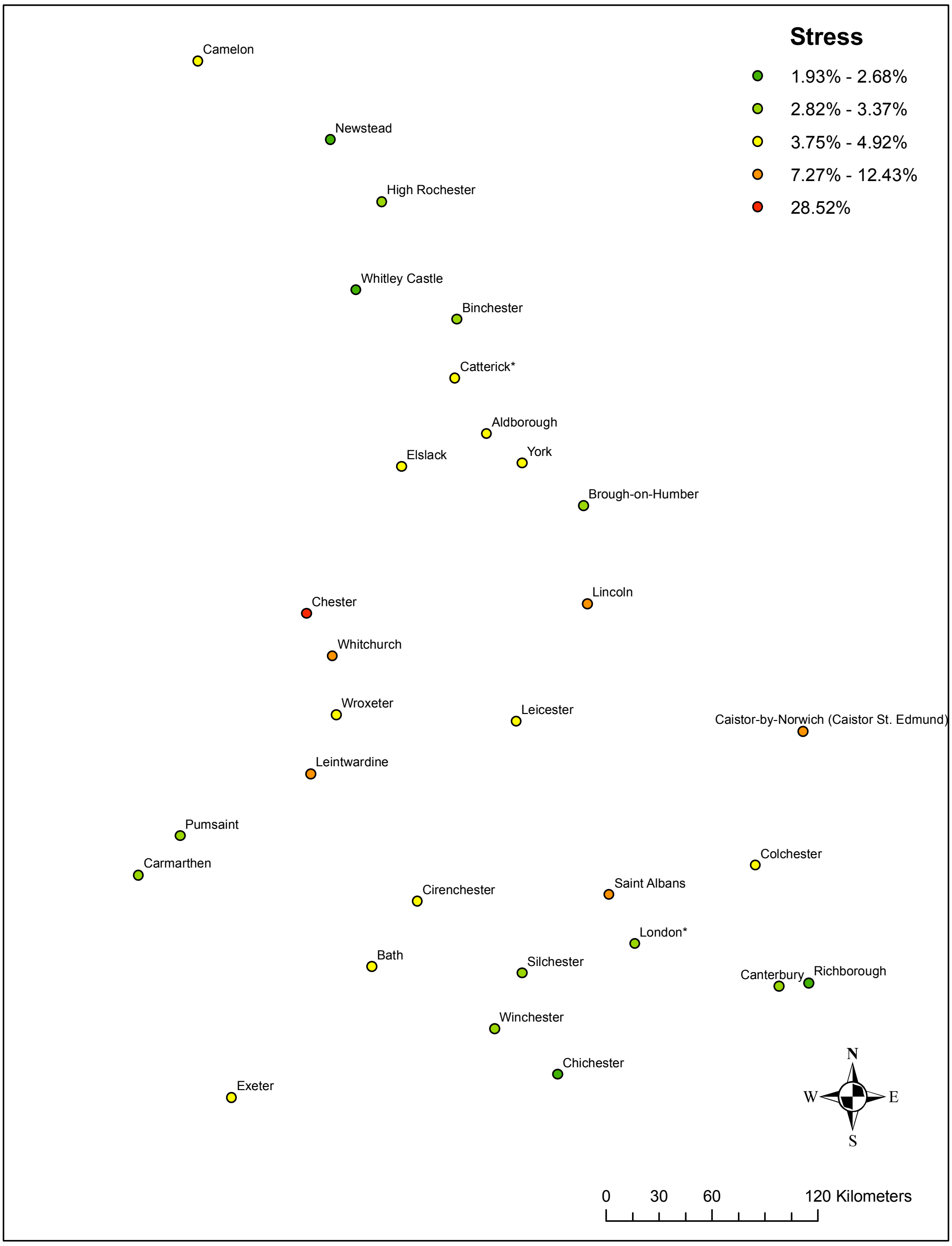

Figure 4.5: Albion: modern configuration with initial stress

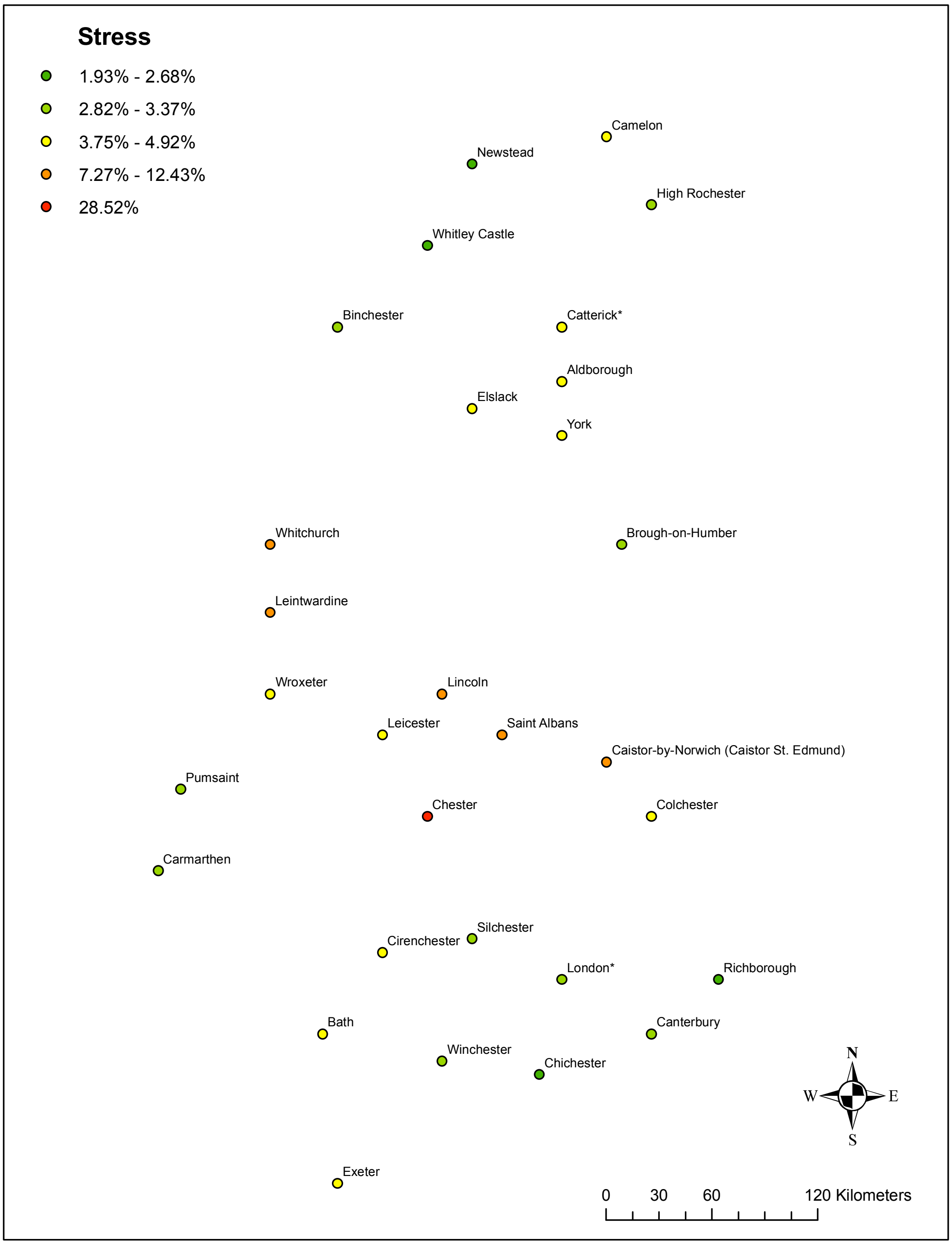

Figure 4.6: Albion: Ptolemy's configuration with initial stress

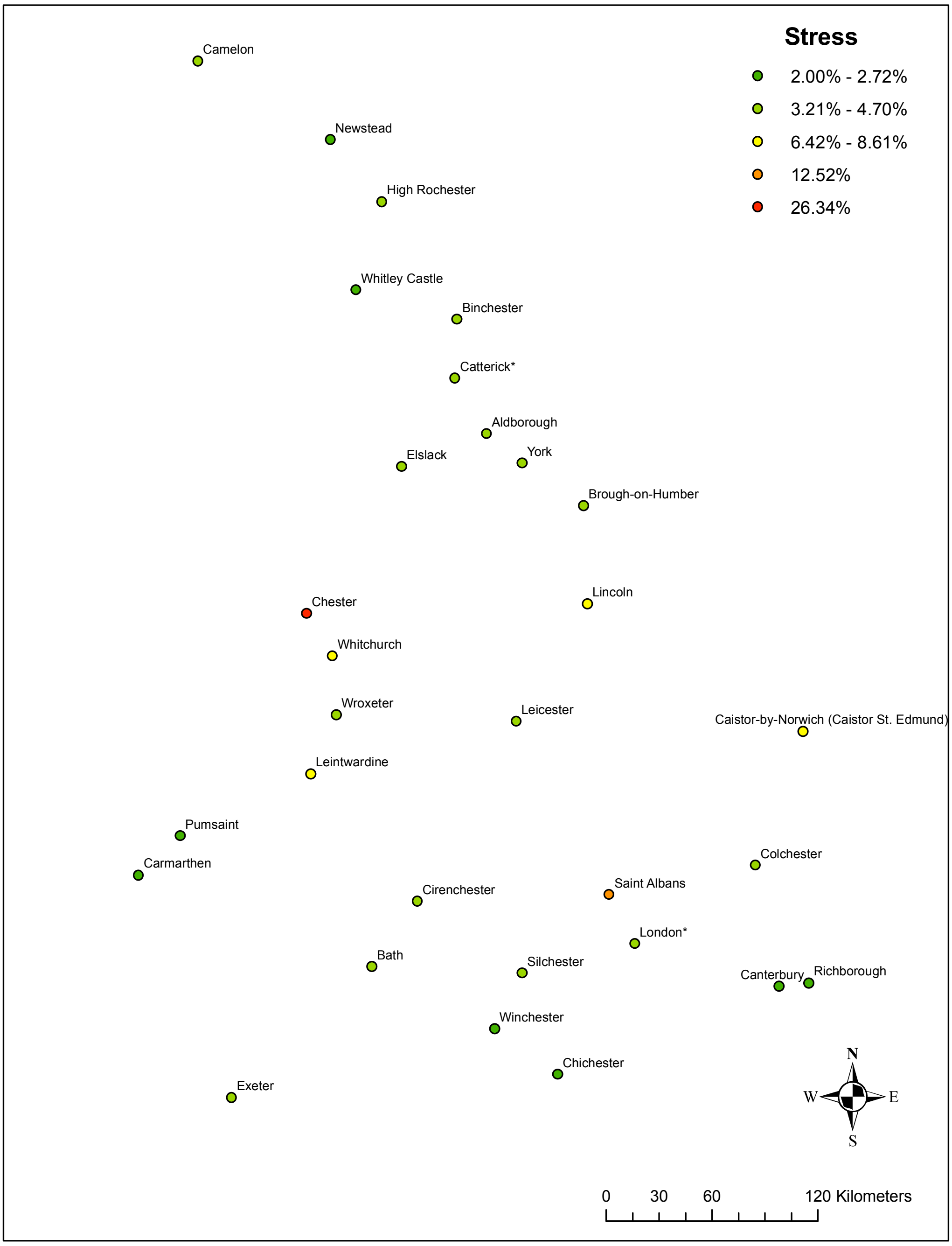

Figure 4.7: Albion: modern configuration with transformed stress

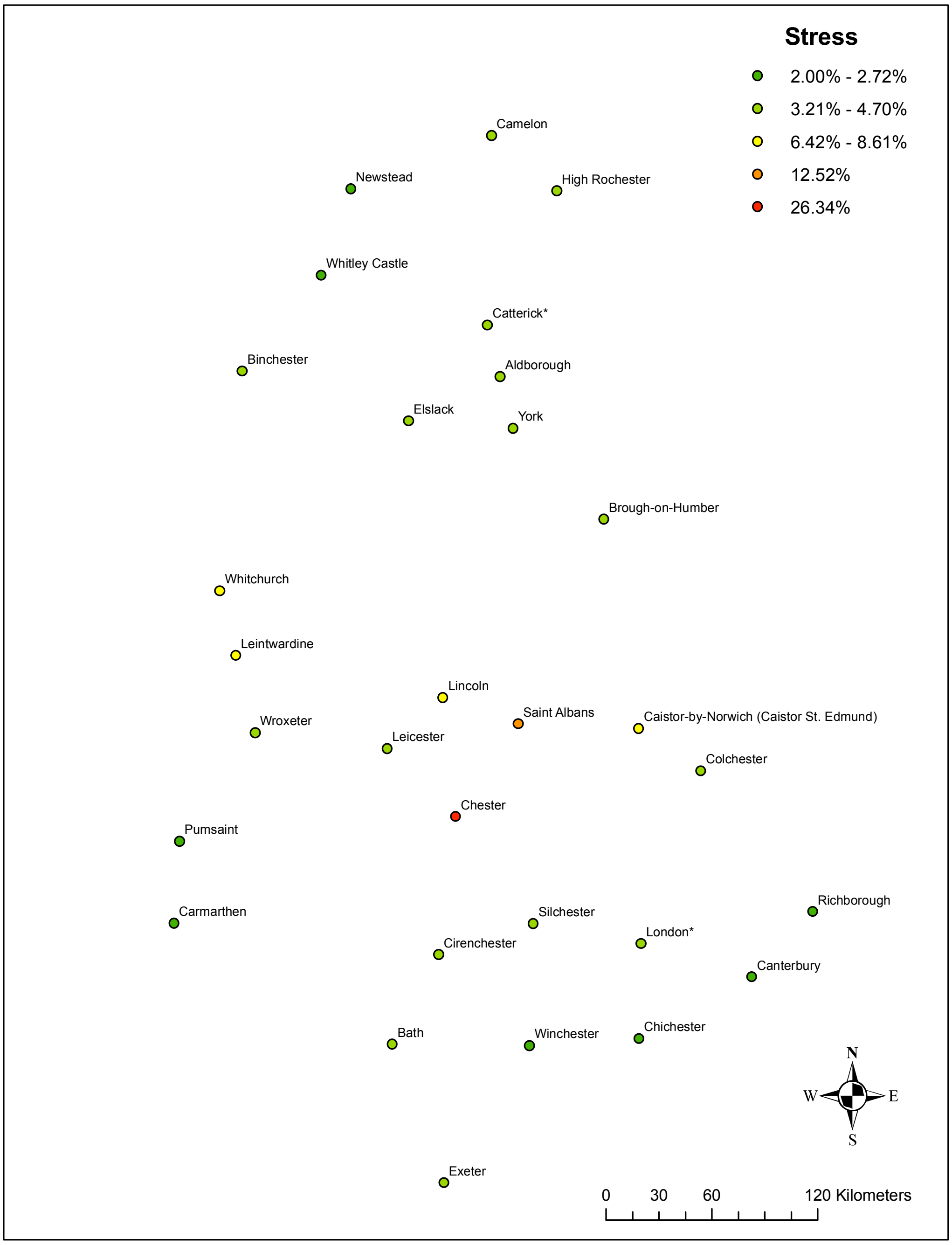

Figure 4.8: Albion: transformed configuration with transformed stress

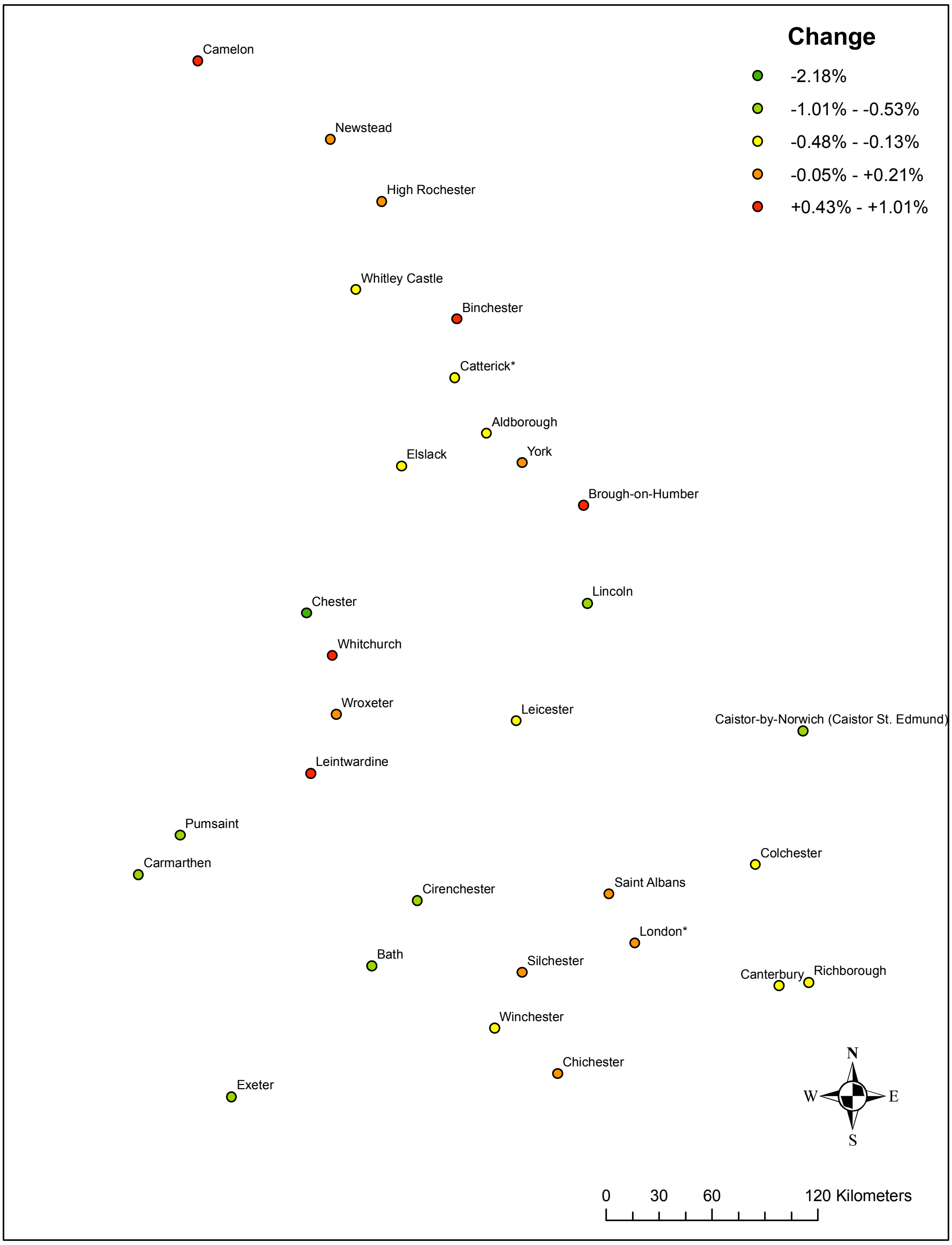

Figure 4.9: Albion: modern configuration with change in stress after transformation

The error due to the misplacements will have been relieved somewhat from the northern position of these cities in the fairly narrow configuration. As we see, though, even this does not bring Camelon, the northernmost point, into the lowest Jenks bracket of stress. Quickly glancing through the rest of the map, we notice that Leintwardine and Wroxeter are in the wrong north-south positions relative to one another, Exeter is too far east, Silchester is erroneously north of London, and Canterbury is too far removed from Richborough.

Once the transformation is made (Figures 4.7 and 4.8) all but 6 points are in the two lowest stress brackets. However, the brackets have changed substantially. There is a much larger separation between all but the two top tiers, and all but the top tier have higher values assigned to them. Very little change is visible to the eye. The problem area is still the centre of the map. Chester and Saint Albans, each in their own stress bracket, are misplaced and are affecting the surrounding areas. The improvements were minimal (Figure 4.9), with a very small range of values from most stress lost to most stress gained. Chester somehow lost over 2 percentage points of stress, though the city itself moved very little during the transformation being so near the centre. The movement of the other cities relative to Chester must have brought its stress down. Unfortunately, two of the three other Welsh border cities worsened (Wroxeter neither gained nor lost any stress) despite being amongst the cities with the highest stress initially.

To further analyse Ptolemy's Britain it is necessary to divide it into parts. While several possible clusters of cities present themselves, we must remember that we need at least 9 cities in each group for the statistics to remain valid (see Section 1.4.5). Historically, it is best to divide the country into a northern and southern group. We take our dividing line as that drawn horizontally across the island just north of Anglesey. Lincoln, Chester, and all points south are in the first group. Brough-on-Humber, Elslack, and points north are in the second. This reflects the various periods of conquest and contention of Roman involvement in Britain. By AD 60, the Romans had moved into Wales, Anglesey, and Lincoln. (Whether they held these areas in peace is another story.) There was then a decade-plus lull before the Romans pushed north and into the second group (see Figures 4.2 - 4.4).

4.2.2 Southern cities

The southern cluster consists of 20 identified points. Initial Procrustes error is 84,587 km^2, reduced to 84,051 km^2 after a 2° clockwise rotation. Initial stress is 9.68%. The scaling results in a 32% extension of the longitude and a 20% diminution of the latitude. Stress is minimised at 6.85%. Mean individual error begins at 9.88% and is reduced to 7.13% by the transformation.

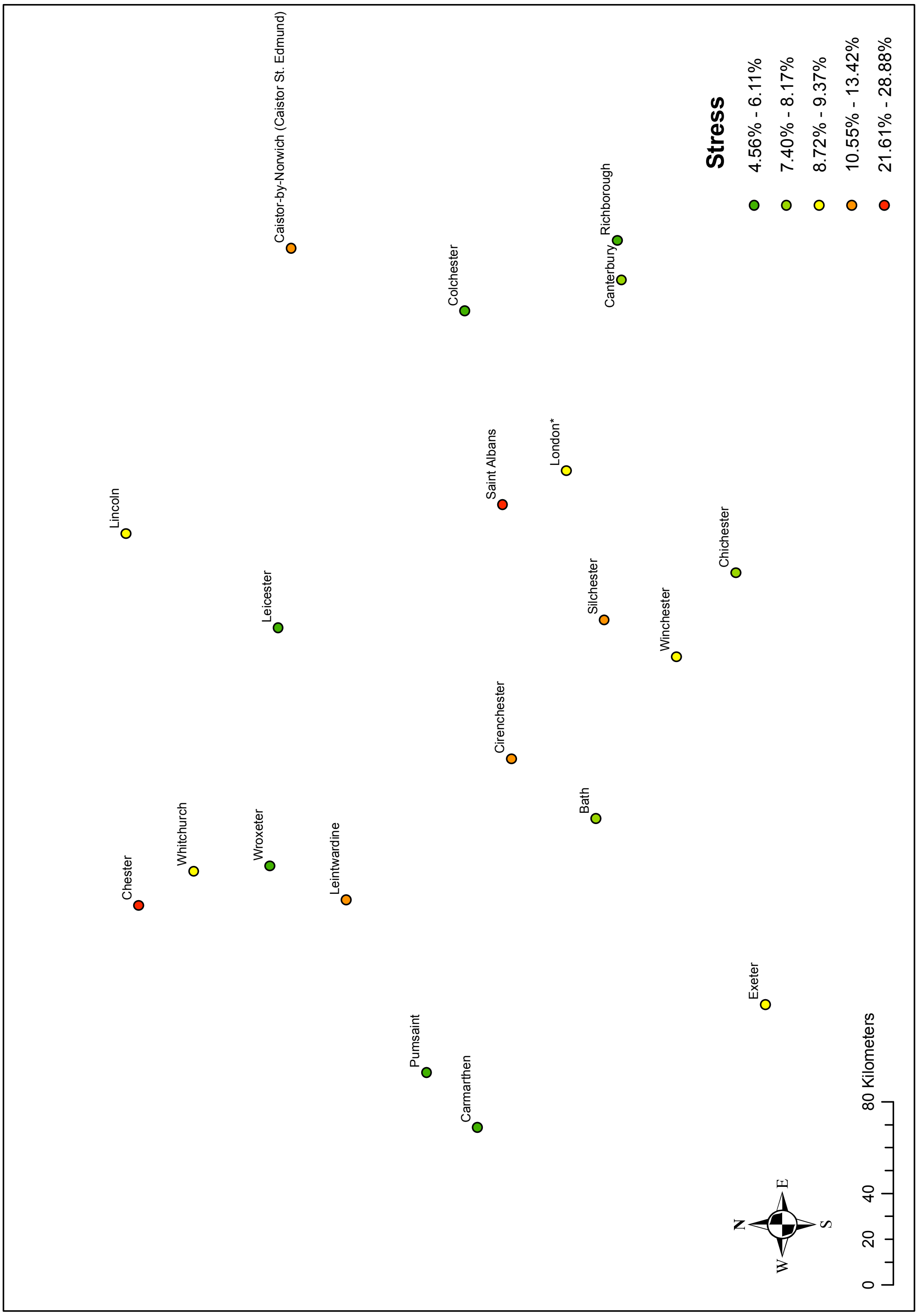

Figure 4.10: Southern Albion: modern configuration with initial stress

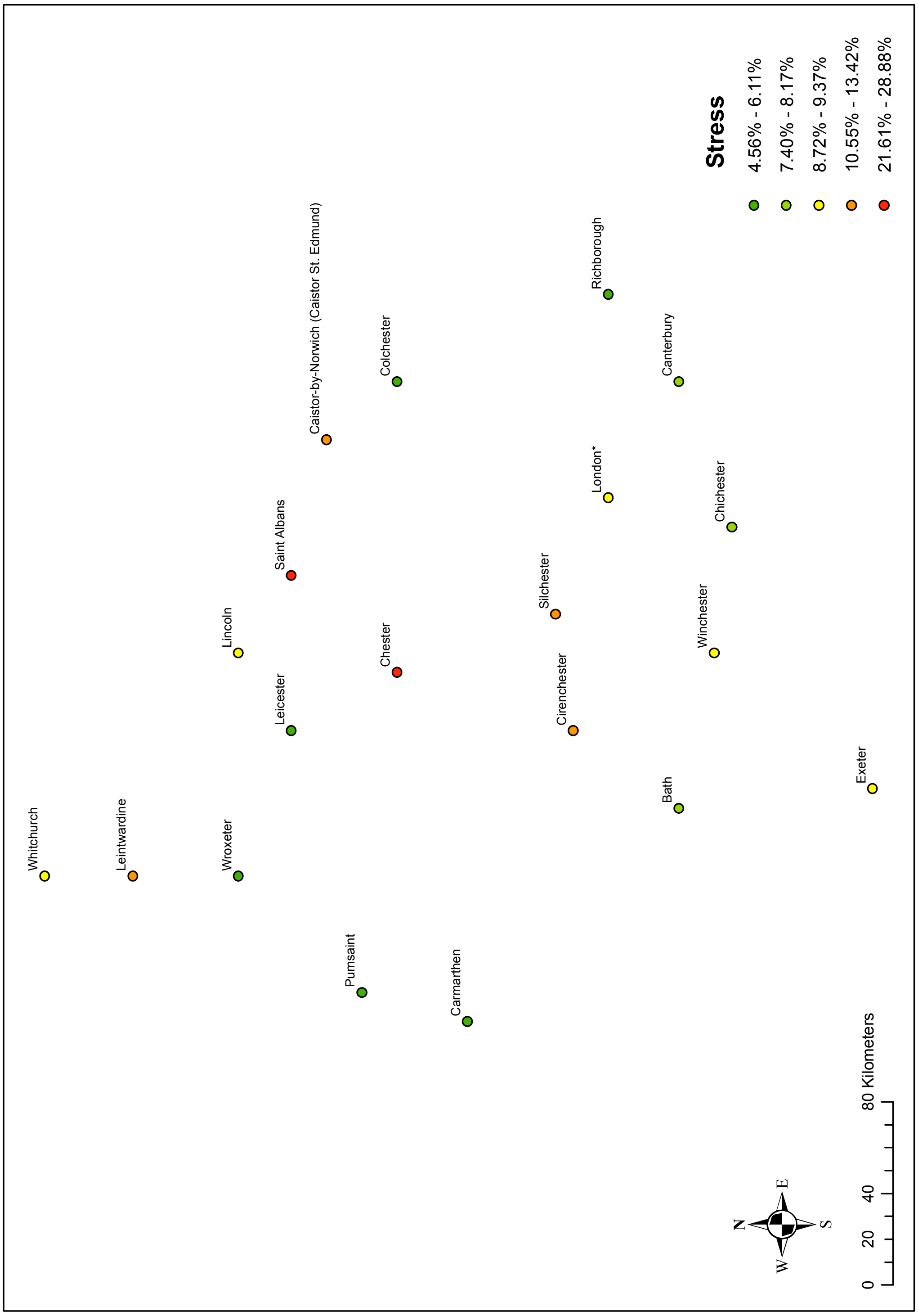

Figure 4.11: Southern Albion: Ptolemy's configuration with initial stress

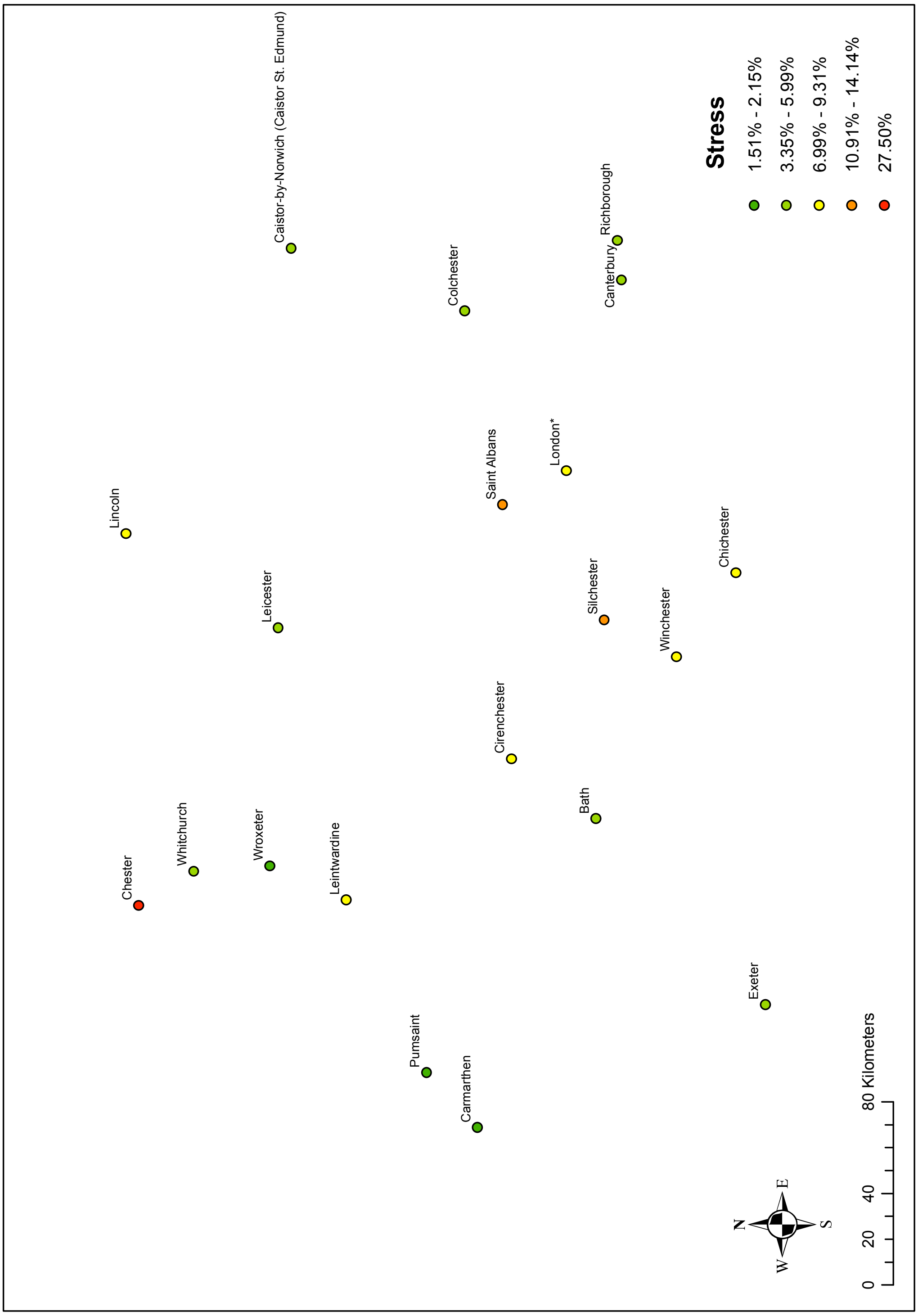

Figure 4.12: Southern Albion: modern configuration with transformed stress

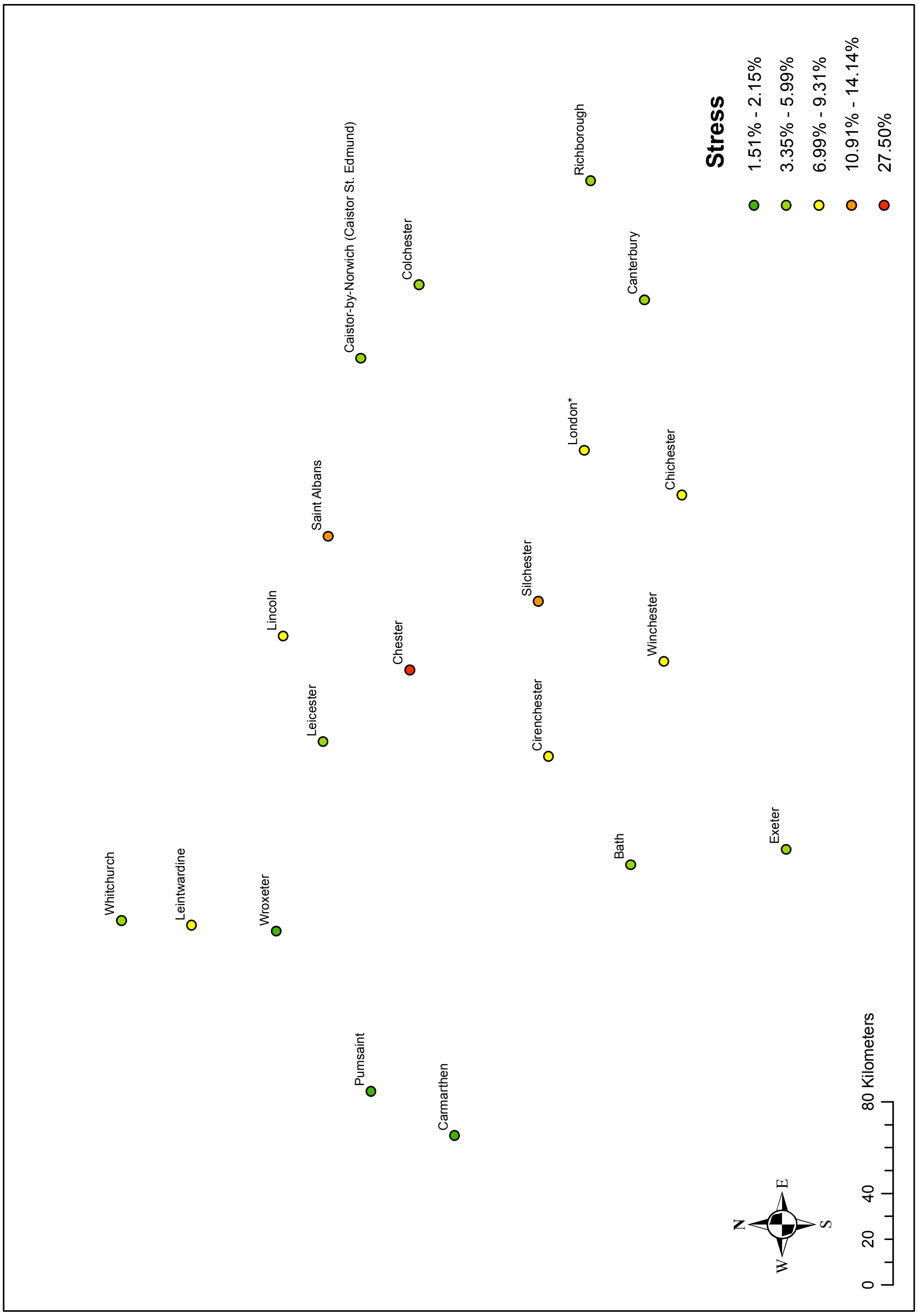

Figure 4.13: Southern Albion: transformed configuration with transformed stress

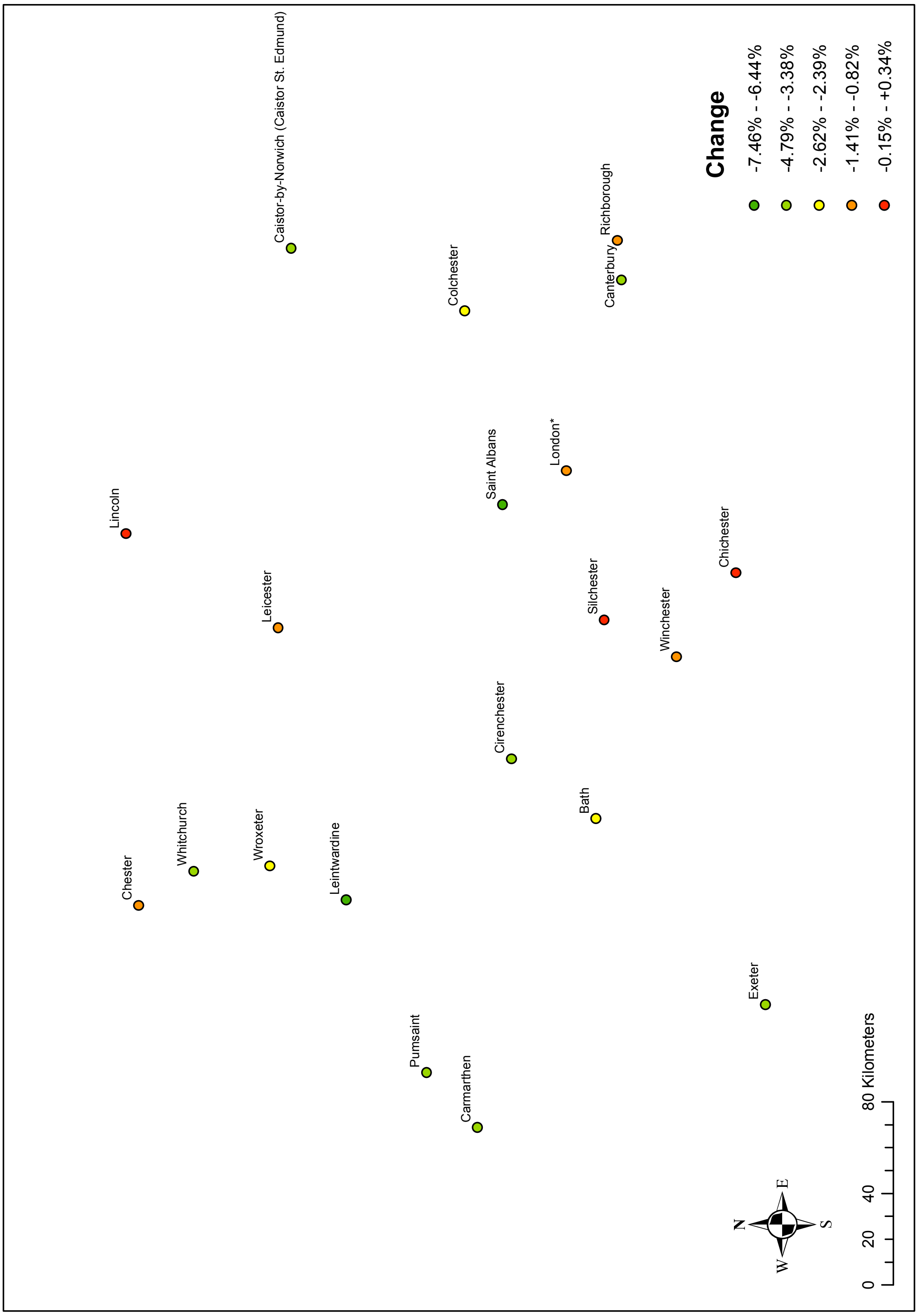

Figure 4.14: Southern Albion: modern configuration with change in stress after transformation

The errors are evenly distributed about the configurations (Figures 4.10 and 4.11). Most of the problems we see in Britain as a whole are present in the southern group: Chester and Saint Albans' cluttering of the centre, Leintwardine and Wroxeter's relative positions, Exeter's misplacement, Silchester's move north of London, and Canterbury and Richborough's separation. Chester and Saint Albans are certainly the worst, and their errors reflect this. As to the Welsh border, the blame seems to rest entirely on Leintwardine. Wroxeter has the smallest stress in the configuration after Pumsaint and Carmarthen, which sit quite accurately in southern Wales. Exeter is simply too far east but this may be due to a more general error of the south-west peninsula. Silchester has no readily available excuses for its misplacement, and the difference in stress between Richborough and Canterbury suggests that it is Canterbury that is the more out of place.

The transformation makes quite a difference to the lower end of the stress spectrum (Figures 4.12 and 4.13). The lowest tier especially is small and distinct and contains much smaller values than it did before being rotated and scaled. Pumsaint, Carmarthen, and Wroxeter hold the lowest errors, all in the west of the map. Saint Albans loses almost 7.5 percentage points of error and leaves Chester in its own stress bracket. The vast majority of errors fall in the medium-low tier, with the second and third lowest tiers expanding their ranges and number of included points. Chichester and Lincoln are the only cities to take on additional stress, though only a very minimal amount (Figure 4.14). Overall, improvements were made across the board.

4.2.3 Northern cities

The cities of northern Roman Britain primarily lie on a fairly straight line along the ancient road, Dere Street, from York to the Antonine Wall. Brough-on-Humber is on the southern road, Ermine Street, out of York, while Whitley Castle and Elslack are on the opposite side of the Pennine Mountains from Dere Street. Camelon is off by itself just north of the Antonine Wall. These 10 cities have an initial Procrustes error of 62,022 km^2, reduced to 40,902 km^2 by a 33° anti-clockwise rotation. Stress begins at 13.07% and is brought down to 9.67%. The required transformation is a 28% reduction in longitude with a 32% expansion of the latitude. Mean individual city stress is initially 14.67% and minimised at 12.00%.

The modern and Ptolemaic configurations do not resemble one another very much (Figures 4.15 and 4.16). Only York, Aldborough, and Catterick maintain any semblance of the linear configuration of the modern map, but they take the concept too far and form a perfectly straight vertical line with the cities evenly spaced upon it. Elslack leads the group with its minimum stress

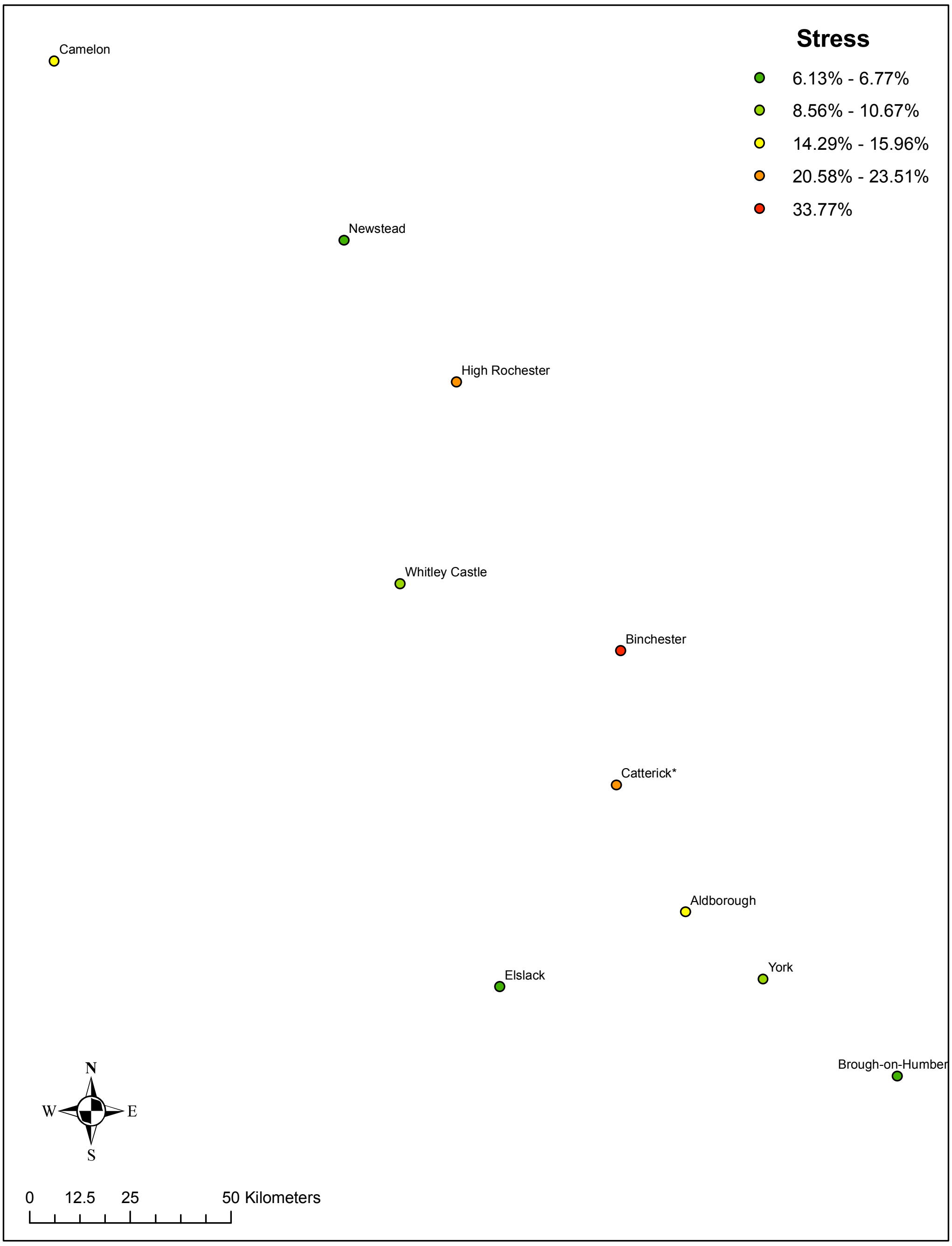

Figure 4.15: Northern Albion: modern configuration with initial stress

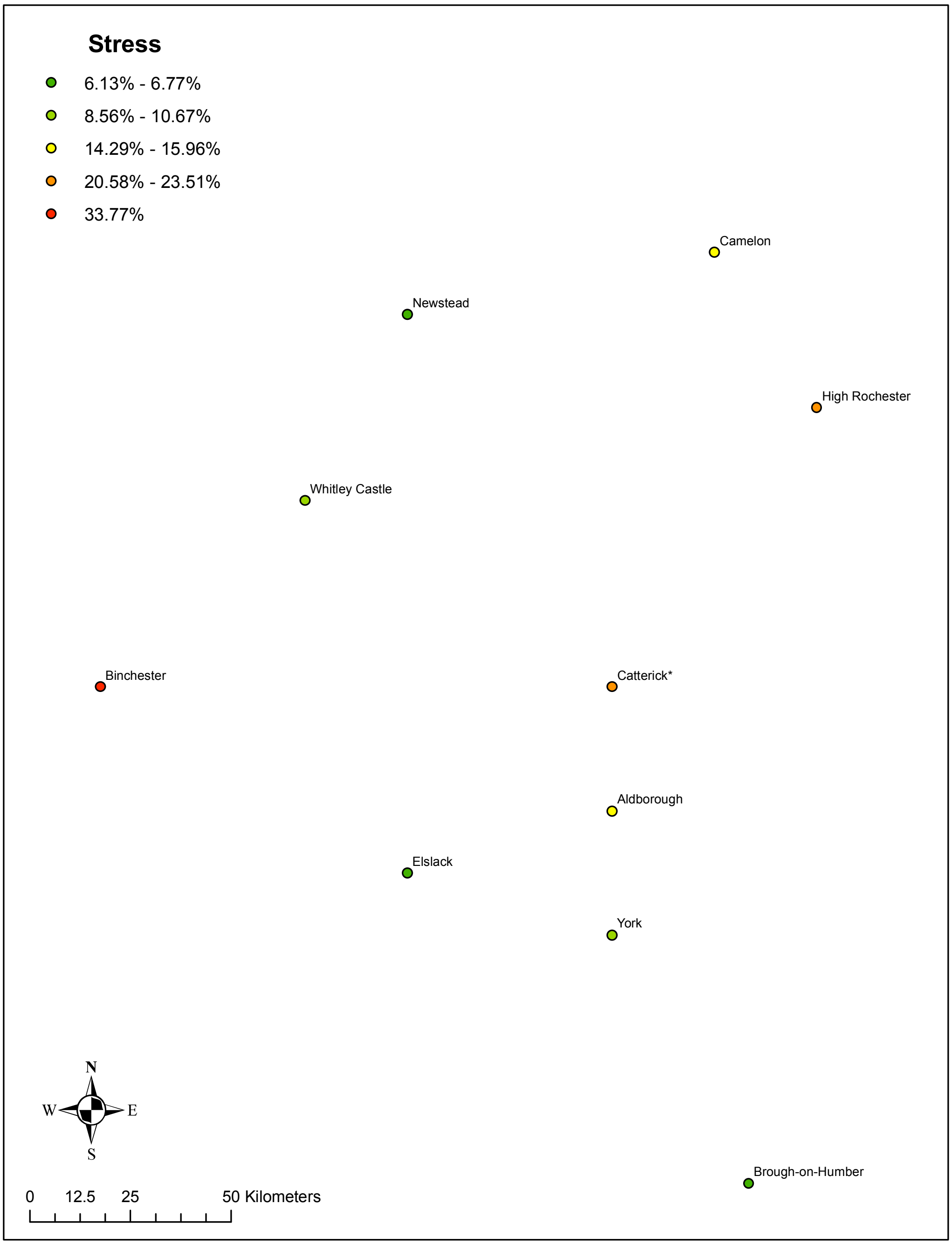

Figure 4.16: Northern Albion: Ptolemy's configuration with initial stress

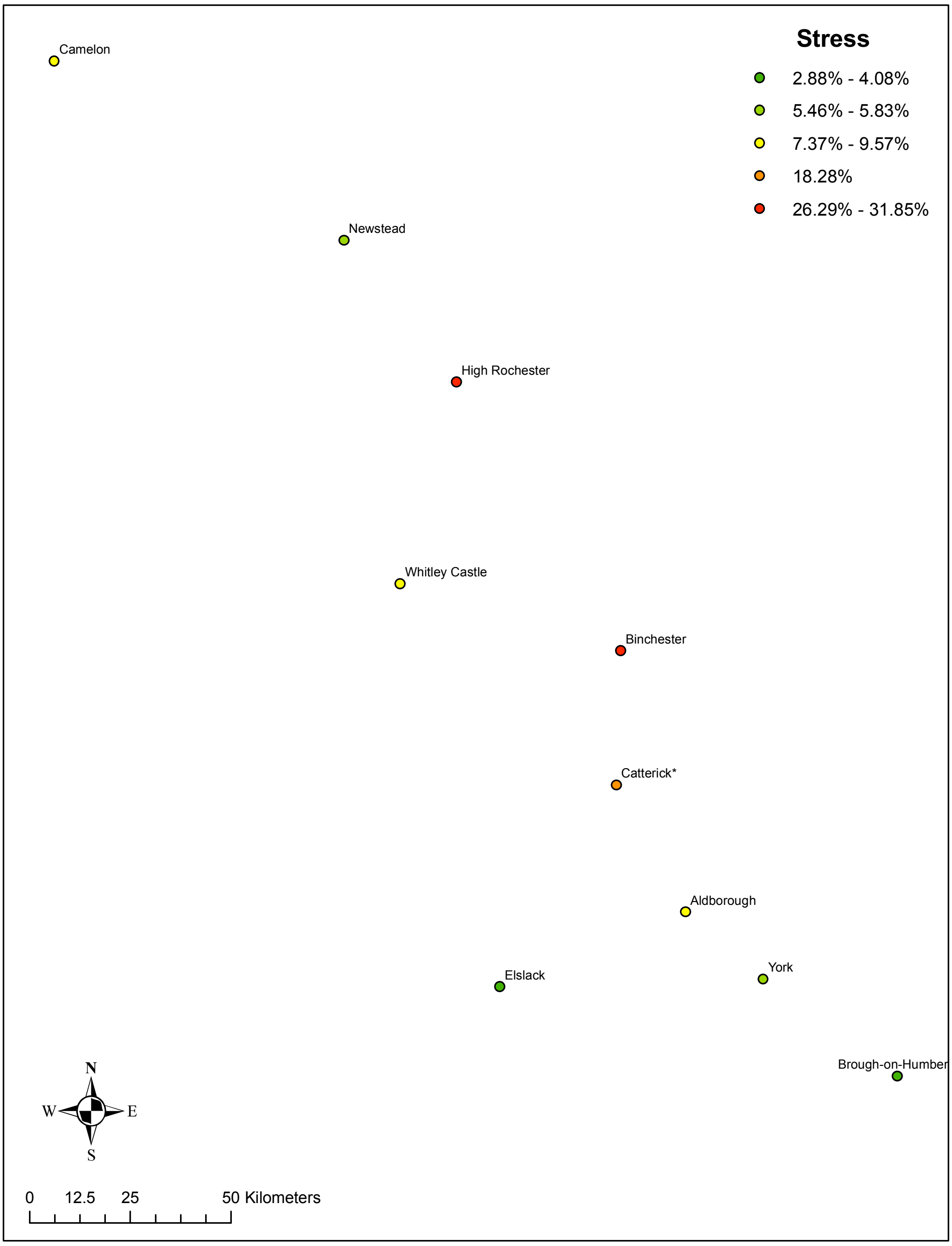

Figure 4.17: Northern Albion: modern configuration with transformed stress

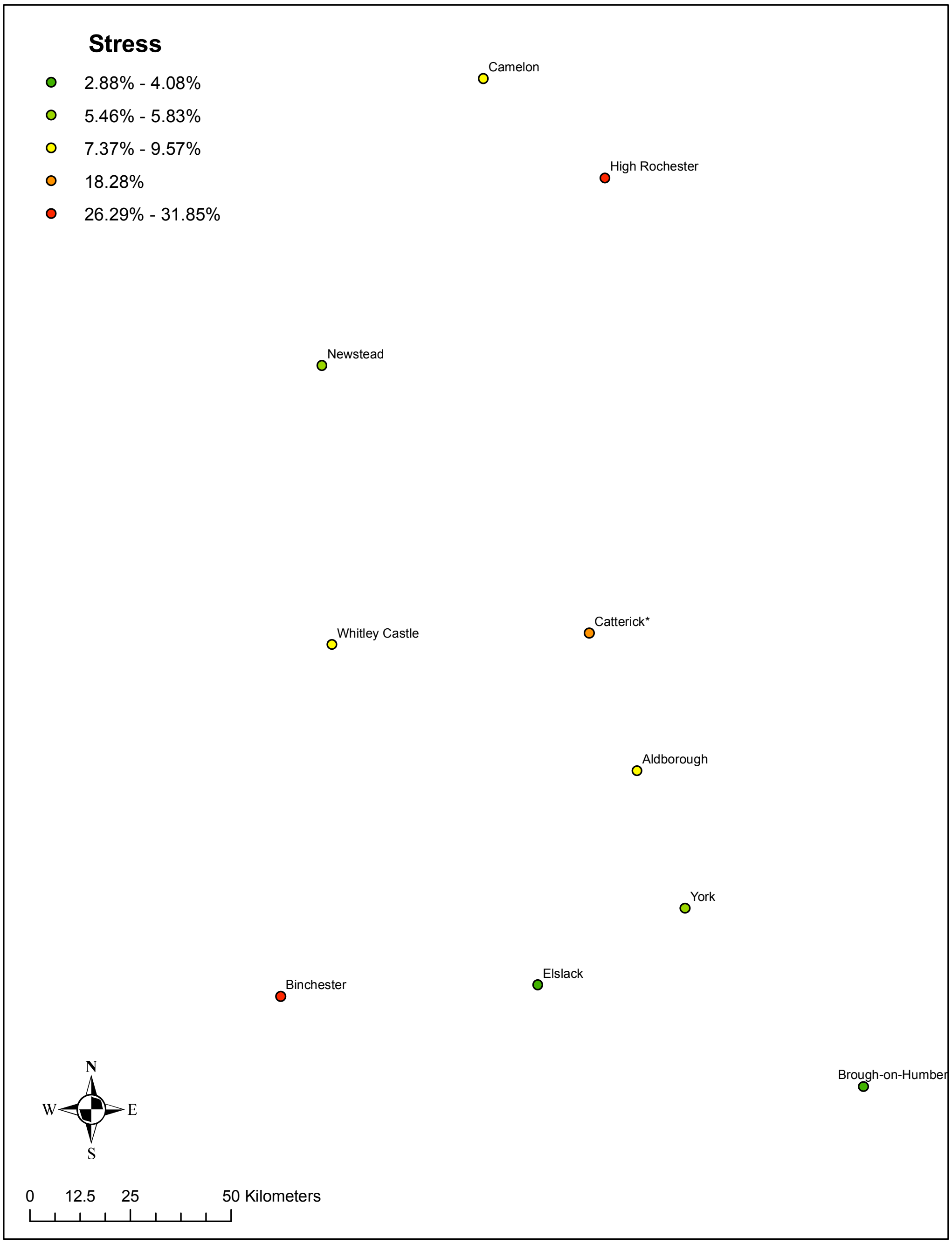

Figure 4.18: Northern Albion: transformed configuration with transformed stress

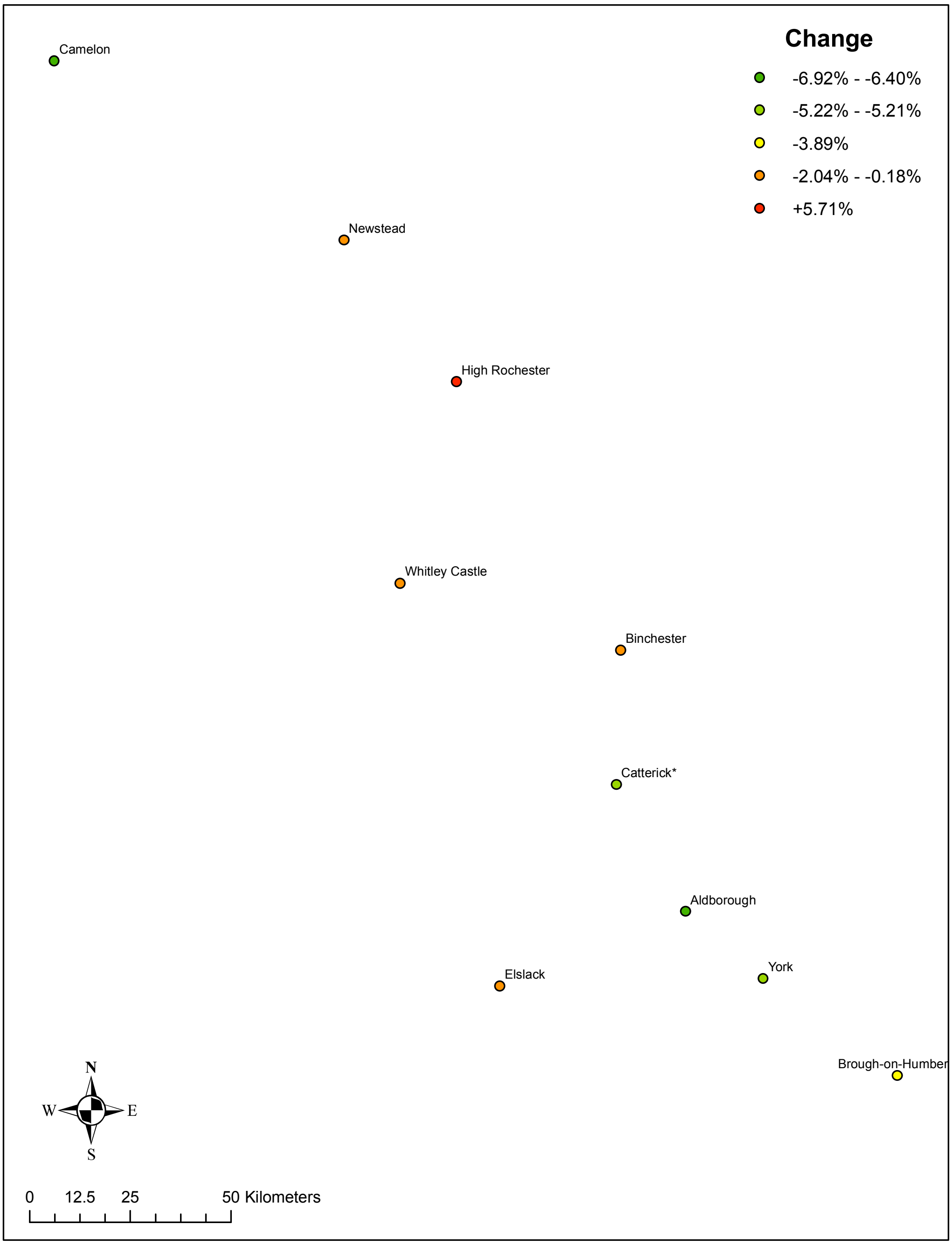

Figure 4.19: Northern Albion: modern configuration with change in stress after transformation

off to the south-west. High Rochester and Binchester are the most obvious errors. Both are at roughly the correct latitudes but have been banished to the edges of the map far from their places on the main road. Catterick also demands a mention. Ptolemy lists Catterick as one of the chief cities of Roman Britain, possibly even one that was placed using scientific means about which the other cities would have been positioned based on orientation and distance. Catterick does indeed have the lowest initial Procrustes error, yet it has the second highest stress. We are again reminded that absolute placement and relative placement produce different errors and that the misplacement of a city's neighbours will be reflected in its own stress.

With the exception of High Rochester, which took on quite an additional 5.71% of stress, there were improvements across the northern cluster (Figures 4.7 - 4.8). The rotation of the group is easily discernible in the better alignment of the York, Aldborough, and Catterick line, which almost can be said to include Brough-on-Humber now, though it is still too distant from York. Camelon is much closer to the north-west corner of the map where it belongs, but is still a long way east of its true position. Binchester seems without hope, and, though it lost almost 2 percentage points, appears even farther from its modern position. With High Rochester and Binchester on opposites ends of the map instead of in the centre as neighbours on the same road combined with the fact that there are so few cities in total, it is difficult to see how accurately placed the other settlements are. There is little doubt that all cities in the northern cluster have had error thrust upon them by the misplacement of these two.

4.3 The coast

4.3.1 The island

The coastline of Ptolemy's map of Britain has average coordinates of (20.73°, 56.97°), compared to the modern point of $(-2.84°, 54.05°)$. Ptolemy's coastal centroid is much farther north-east than his inland centroid due to the inclusion of Scotland, which is turned due east. His inland data lacks both this turning and the inclusion of any identified settlements, save two, in Scotland. The modern inclusion of Scotland likewise pulls the centre northward, but, due to Scotland's true orientation, it also brings the centre westward. Ptolemy's Britain, therefore, must be off on the longitudinal axis no matter where the prime meridian is drawn. Ptolemy's latitude is consistent, though, and, like his inland map, his coastal data is approximately 3° too far north.

There are 48 identified points making up the coastline of Great Britain.

Initial sum of squares error is 4.04 million km^2. An anti-clockwise rotation of 44° greatly reduces this error to 1.33 million km^2. Further reduction is accomplished by a 55% decrease in longitude and a 16% decrease in latitude. The minimised error is 883 thousand km^2, an improvement of over 78%. Mean individual Procrustes error is 257.44 km which is lowered to 123.58 km by the transformations.

The extremely high initial error is readily explained with a passing glance at Figure 4.20. Scotland is sideways, plain and simple. The northern coastline shares many of the same features shape-wise with its modern equivalent, but they are all sideways. The Welsh coastline looks fairly accurate. The south-west peninsula is much larger on Ptolemy's map than on the modern one. East Anglia does not push out into the North Sea as it should. Otherwise, there is little to discuss at this point. Individual errors and alignment cannot be properly discussed with Scotland's orientation throwing the rest of Ptolemy's configuration to the west to compensate. Unfortunately, fixing the coast as a single unit does not allow us to discuss individual errors and alignment either (Figure 4.21). The rotation aids Scotland but throws off England and Wales. Furthermore, the scaling shrinks Ptolemy's island to a wraith of its former self, collapsing the island to a narrow north-south band to minimise the error. The numerical improvements are great (Figure 4.22), but the shape is lost.

4.3.2 The Roman south coast

The coastal division is not quite the same as that for the inland cities, but farther north, at the line of the Solway Firth and River Wear. As it was Agricola who ordered the circumnavigation of the island, this line reflects the frontier line in the early years of his governorship (see Figure 4.4). Having jettisoned Scotland, Roman Britain proper can now be examined and its true accuracy judged. The 28 identified points of the southern coast have an initial Procrustes error of 127.6 thousand km^2. An anti-clockwise rotation of 5° lowers the error to 115.0 km^2. A 1% reduction in longitude and a 17% reduction in latitude minimise the error at 84.4 thousand km^2, a 34% improvement over the initial summation. Individual error has an initial mean of 56.95 km and a transformed mean of 47.23 km.

Instantly, the two configurations find themselves in a profoundly more accurate alignment than we saw previously (Figure 4.23). There are three trouble spots: the north, the south-west, and East Anglia. In the north, we can see the beginnings of the turning of Scotland to the east instead of the curving of the modern configuration towards the west. Ptolemy's Cornwall and Devon are quite exaggerated in both length and width. Ptolemy's peninsula also curves in the opposite direction to the modern peninsula resulting in a wider Bristol

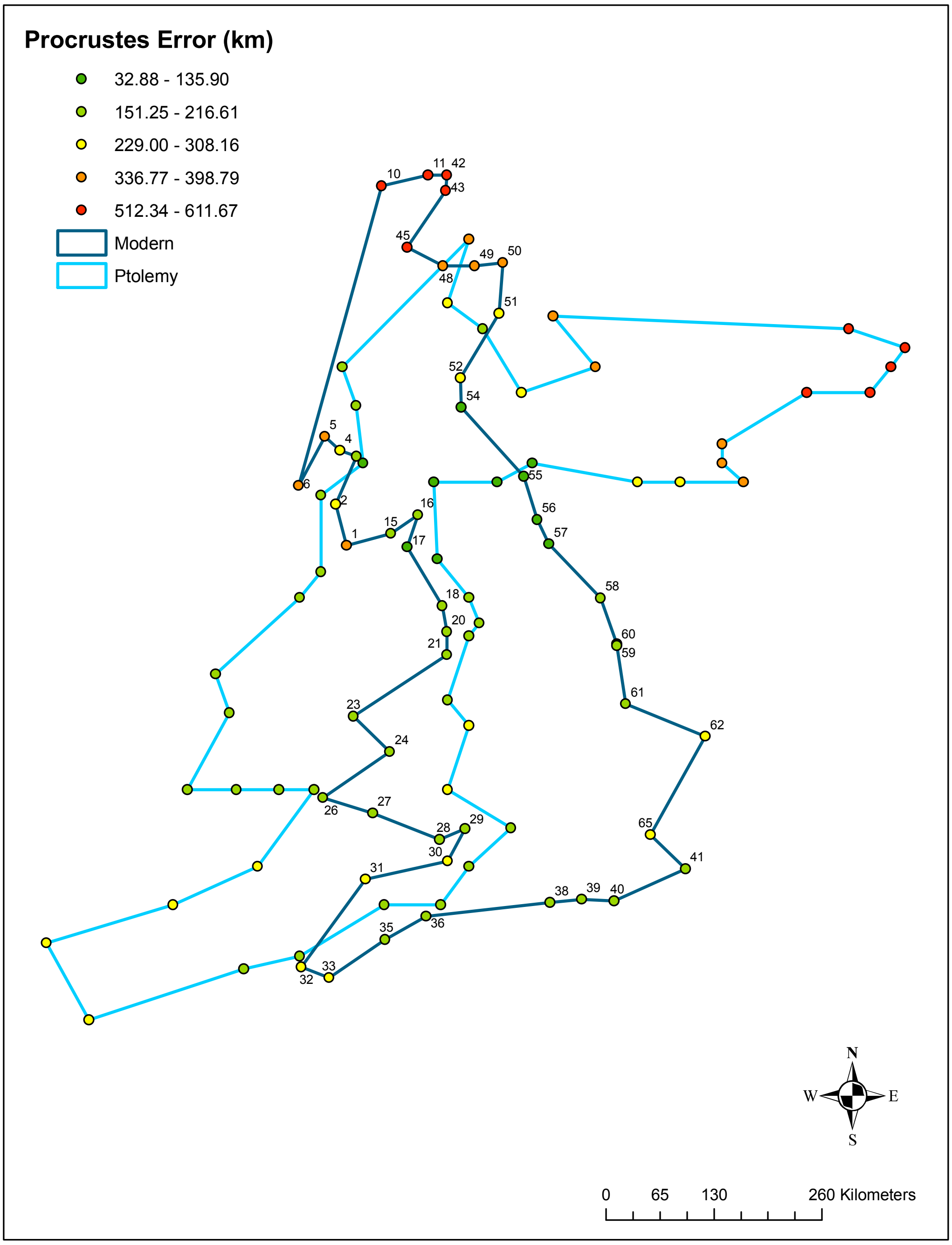

Figure 4.20: Albion's Coast: Initial Procrustes error

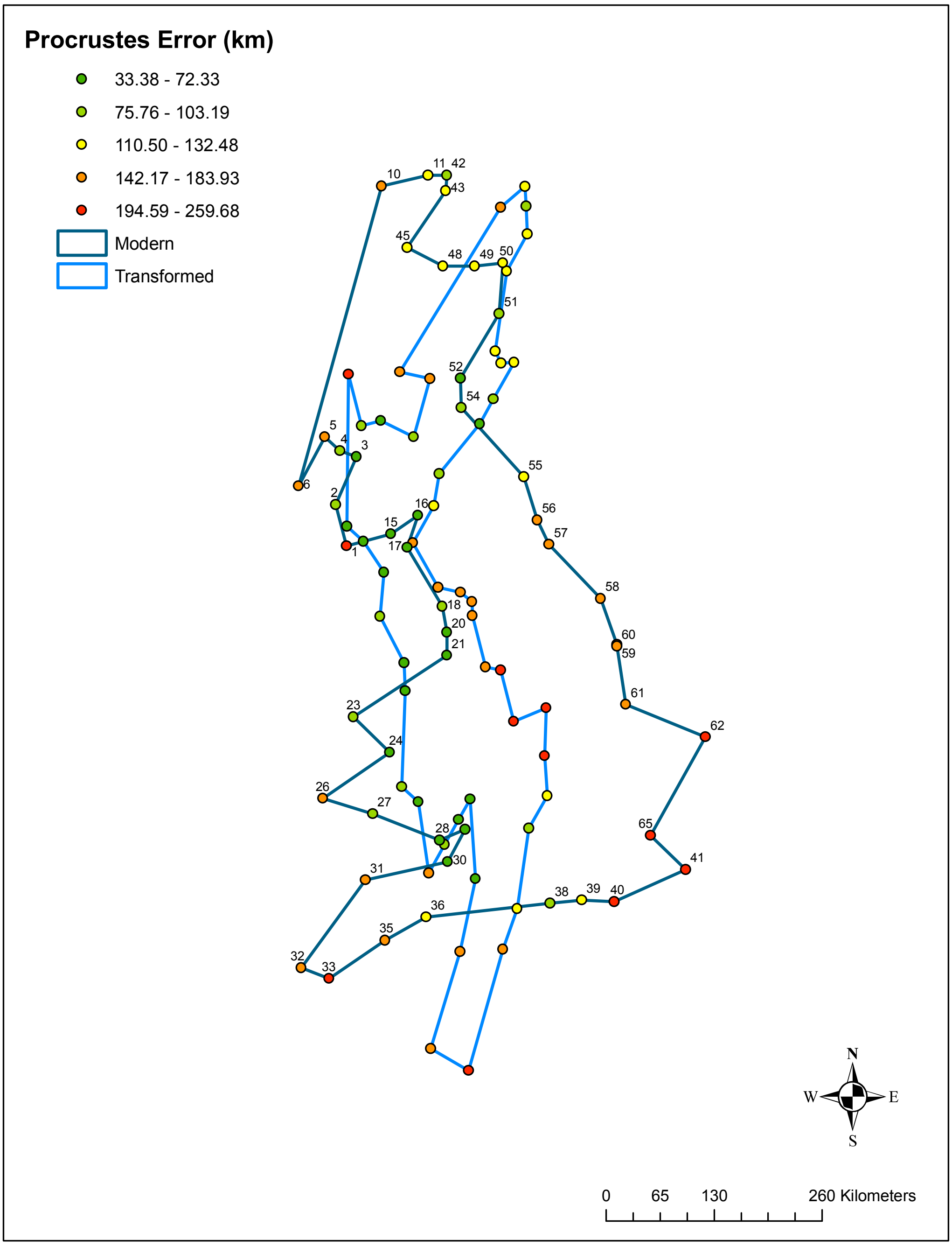

Figure 4.21: Albion's Coast: Transformed Procrustes error

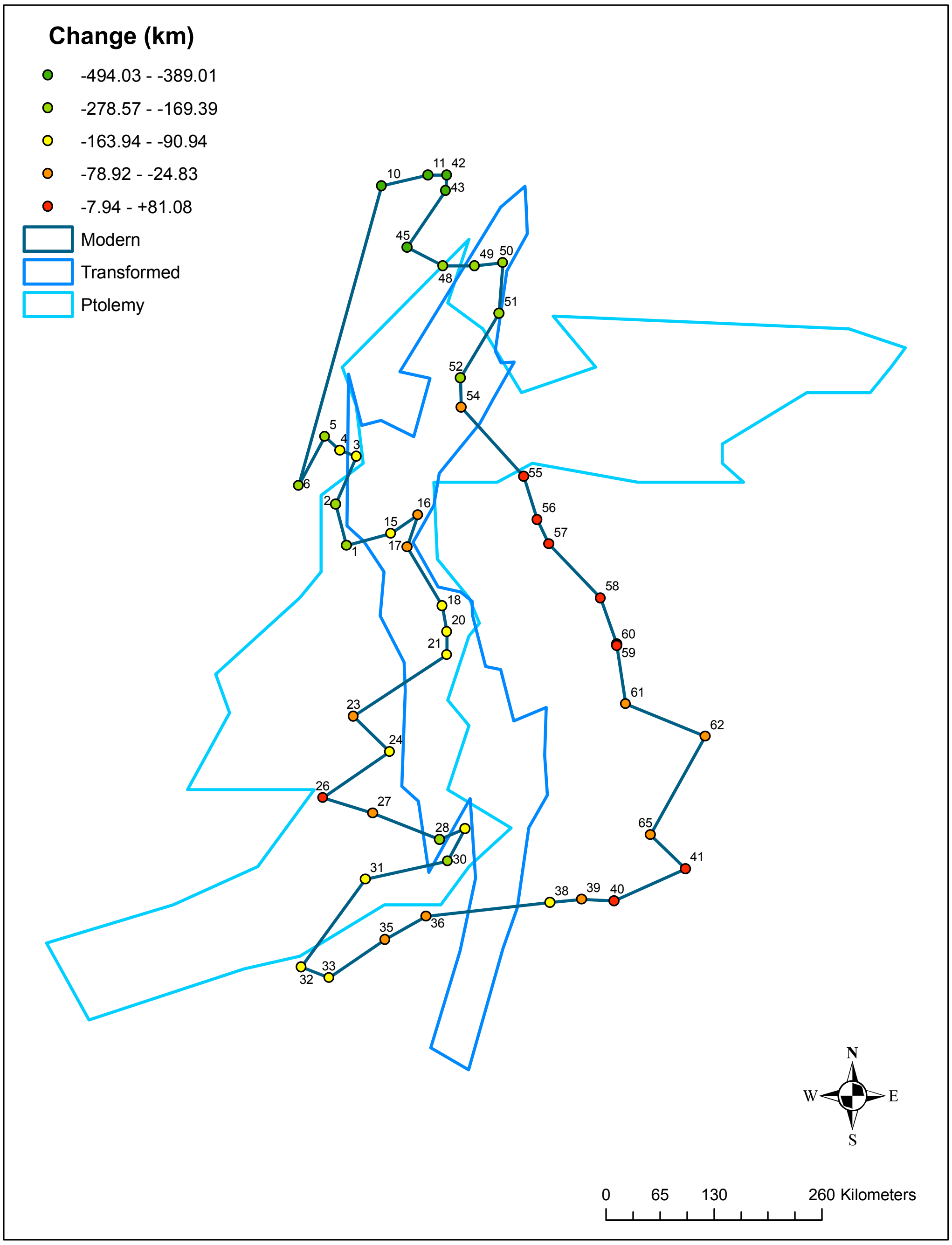

Figure 4.22: Albion's Coast: Change in Procrustes error

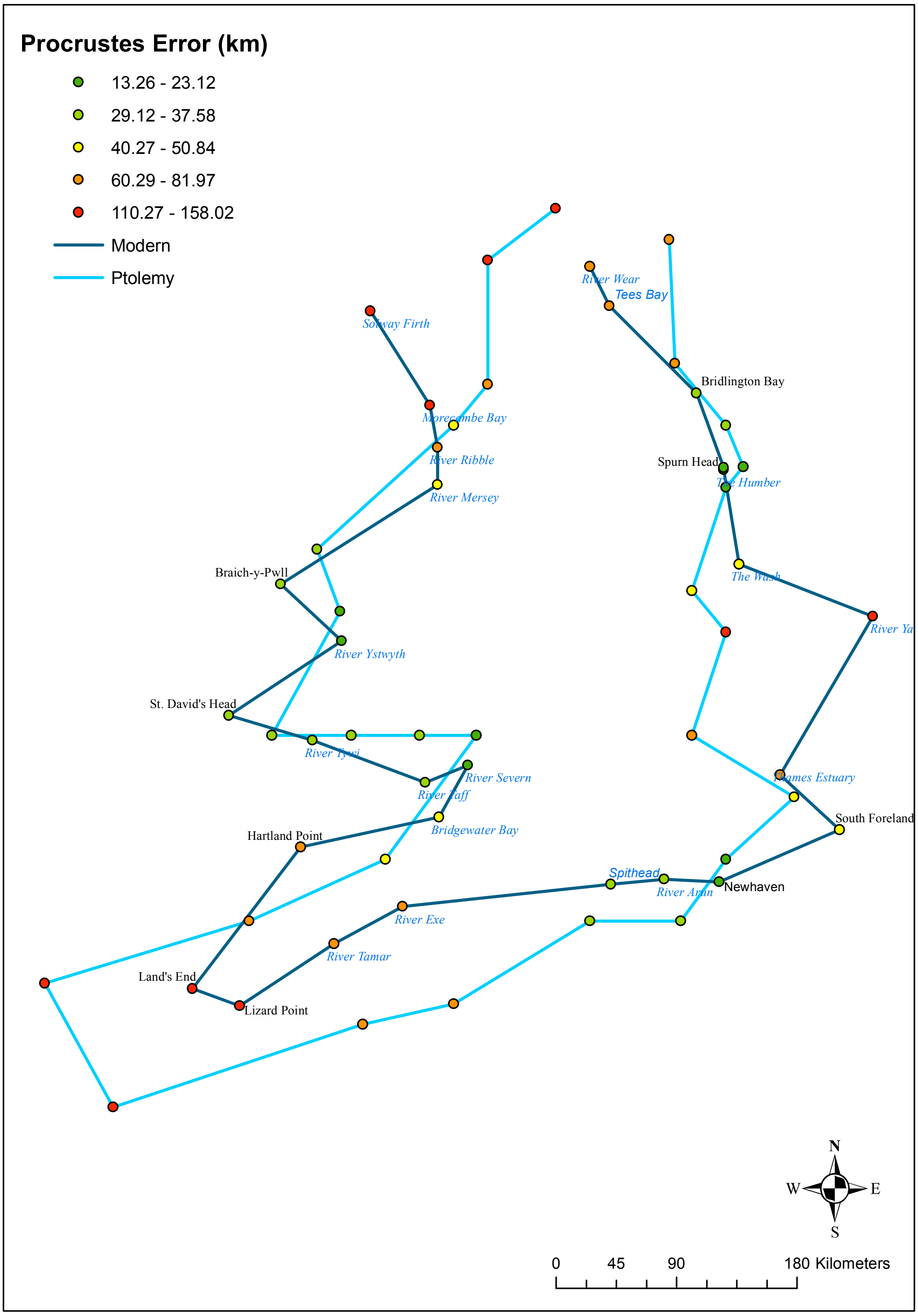

Figure 4.23: Albion's Southern Coast: Initial Procrustes error

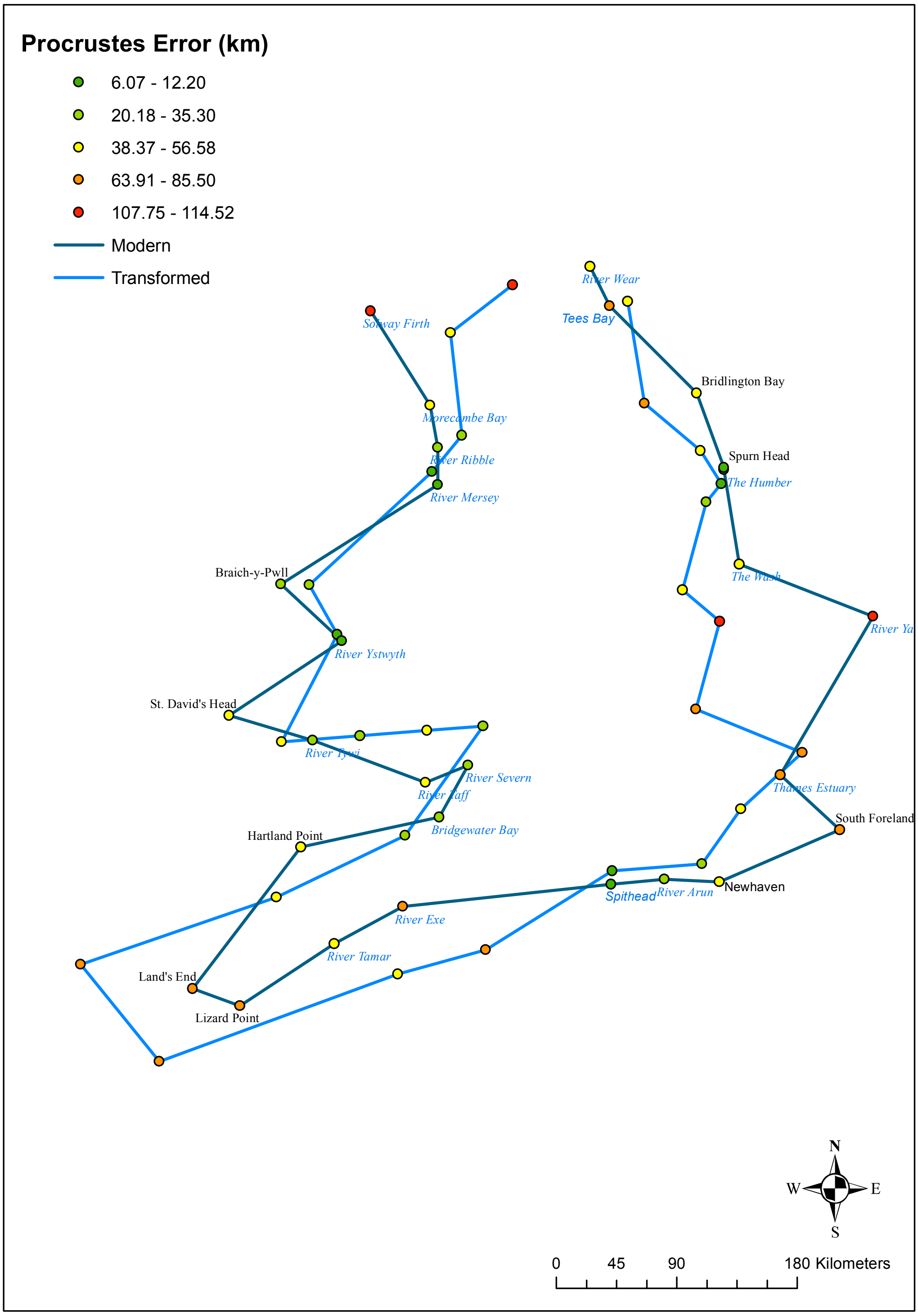

Figure 4.24: Albion's Southern Coast: Transformed Procrustes error

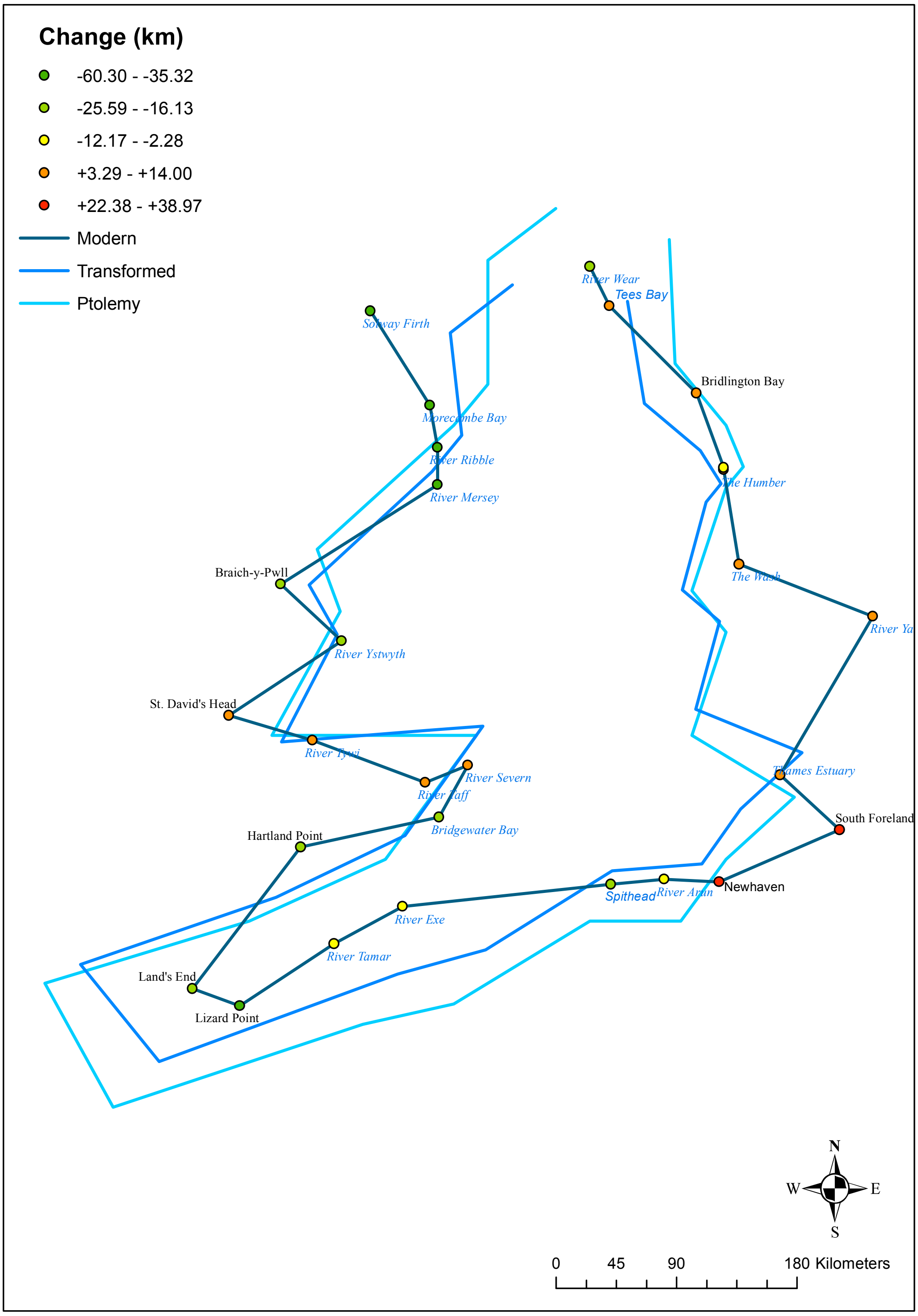

Figure 4.25: Albion's Southern Coast: Change in Procrustes error

Channel and potentially narrower English Channel (depending on his coast of northern France). East Anglia seems to be missing from Ptolemy's map with the mouths of the Thames, Yare, and Wash all pulled westward. In between these three trouble spots are rather accurately placed points, especially the 6 points that make up the coast of Wales.

The effects of the transformation are fairly regionalised (Figures 4.24 and 4.25). The largest improvements were along the west coast and the area around Land's End. This included the already accurately placed River Ystwyth, whose mouth is now a mere 6.07 km from its true location. The southern coast of Wales and most of the east coast down to Newhaven were forced further from their modern counterparts. In spite of this, East Anglia is not terribly worse off than it was before the transformation. New Haven suffers the most as the transformed coastline moves north from its Ptolemaic position. However, this brings the River Arun and especially Spithead much closer to their true locations.

4.3.3 The barbarian north coast

The northern coast, approximately that of Scotland, consists of 20 points. Initial Procrustes error is 1.52 million square kilometres. Over 90% of this error is eliminated by an 87° anti-clockwise rotation. The error now stands at 136.1 thousand km^2. A reduction of the longitude by 28% and the latitude by 27% minimises the error at 59.9 thousand km^2. This represents a total loss of error of over 96%. Mean individual error, likewise, drops abundantly: 259.79 km to 47.42 km.

As we have noted several times now, Scotland is sideways (Figure 4.26). With this enormous exception, the shapes of the two configurations look very similar. The only areas that stand out as being inaccurate are the Mull of Galloway, which Ptolemy has jutting out of the mainland at a more extreme angle than a modern cartographer would, and the line between Tarbat Ness and Spey Bay, which needs to be much shorter. Otherwise, Ptolemy's Scotland is very similar to the modern map.

The transformation does wonders (Figures 4.27 and 4.28). Most improvements range in the hundreds of kilometres, and not a single point moves farther away from its true location. Most of the error is grouped in the south-west on either side of the cluster of the best placed points. The problem stems from Ptolemy's placement of the Mull of Kintyre. If he had not put it so near to due south of the River Naver (technically, he put it near to due west, but let us assume the correcting rotation) and put it a bit farther away, that could have pulled the whole Firth of Clyde area into place. As predicted, the Mull of Galloway is also out of alignment and is at entirely the wrong angle to the

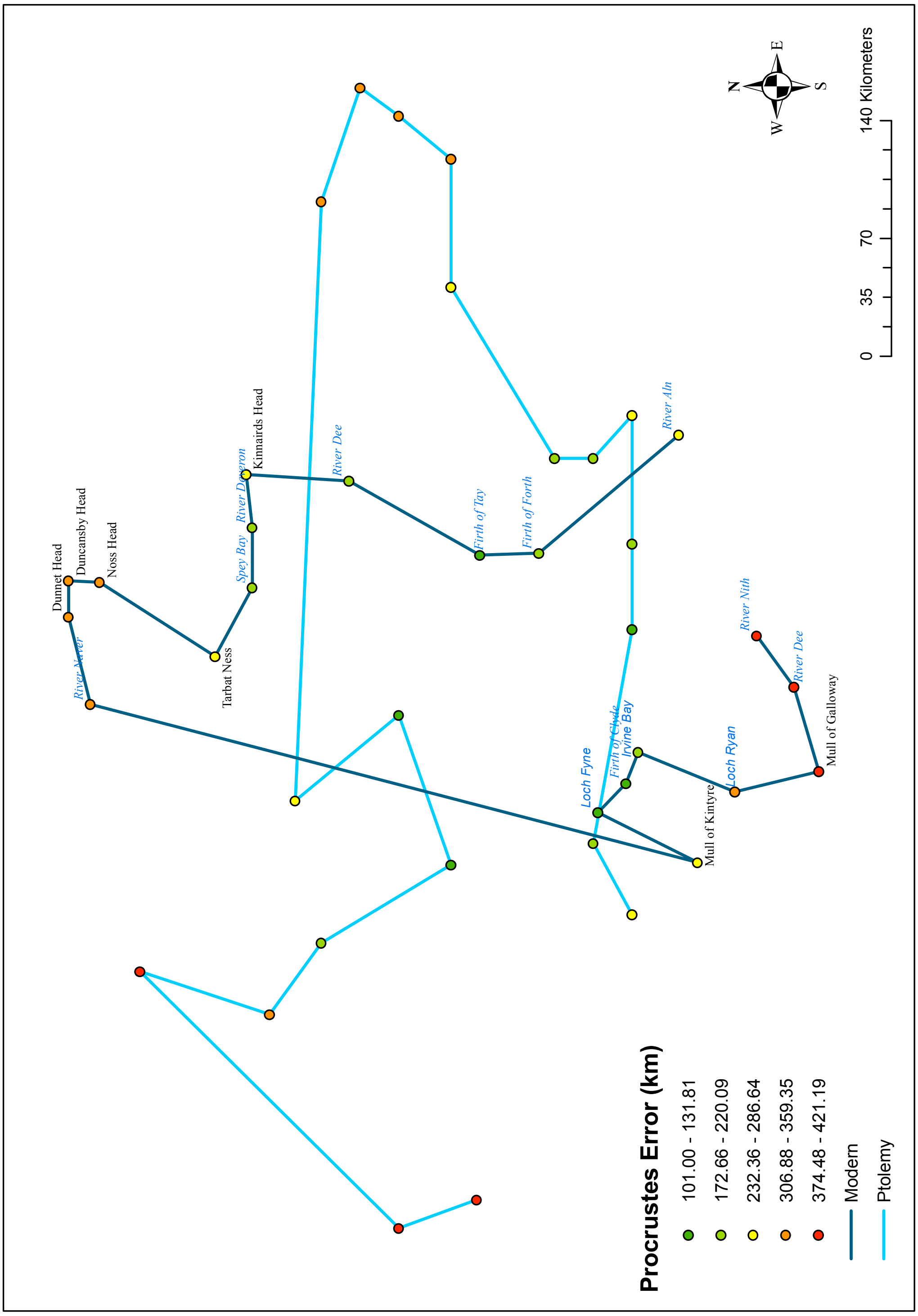

Figure 4.26: Albion's Northern Coast: Initial Procrustes error

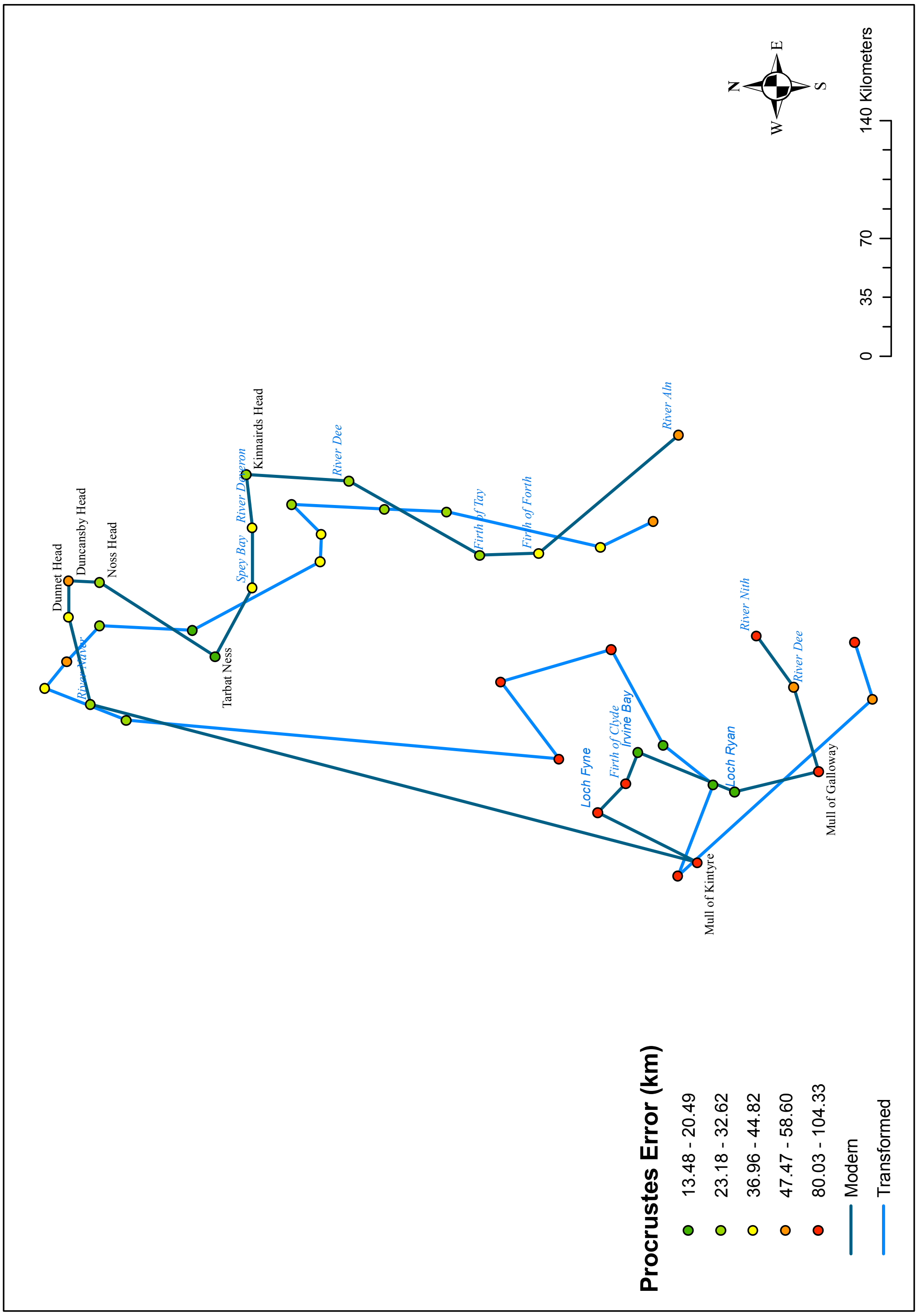

Figure 4.27: Albion's Northern Coast: Transformed Procrustes error

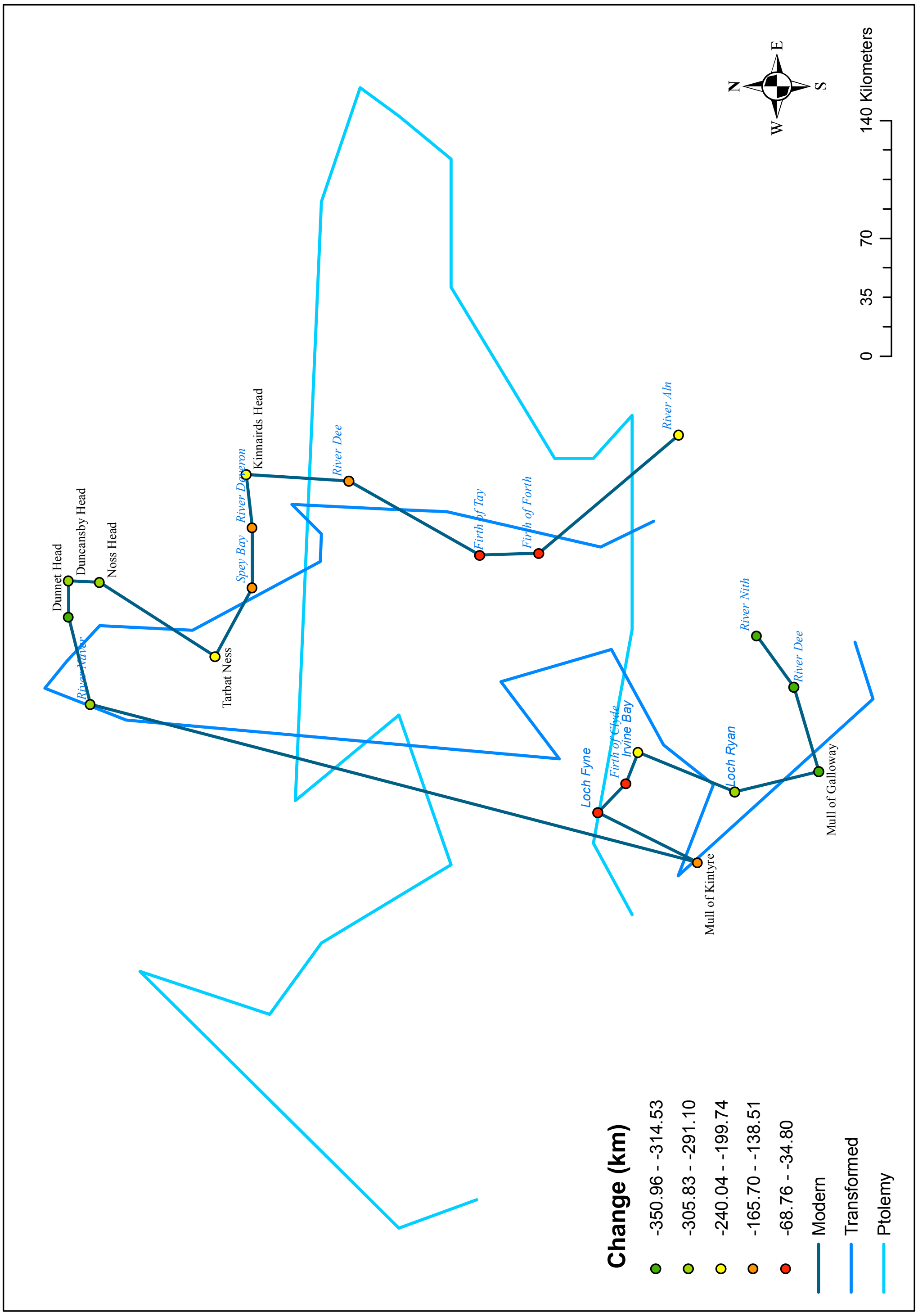

Figure 4.28: Albion's Northern Coast: Change in Procrustes error

mainland. Also as we saw, if the Spey Bay area were to be pulled north, the north-east region would be nearly aligned with the modern map. Still, we have seen an amazing turn around on Ptolemy's map of Scotland.

4.4 Concluding notes on Great Britain

4.4.1 Outliers

In the analysis of the inland cities of the whole island, 5 are both initial and post-transformation outliers: Whitchurch, Leintwardine, Chester, Lincoln, and Saint Albans. All of these cities are in the southern group. When this area is examined on its own only Chester and Saint Albans are at the outlier level. There are no coastal outliers.

Chester has alternative coordinates. The Ω manuscript tradition has Chester at $(18.50°, 55.00°)$ while the alternative manuscript, X, has (17.50°, 56.75°), a location north-northwest of the modern city. This second reading puts the city 45.43 km from its true location with a stress of 4.26%, while the original reading puts it 134.07 km away with a stress of 28.52%. This has a huge windfall for the configuration as a whole. Initial stress for all inland cities is now 3.57%, reduced to 3.47% after transformation. Mean city stress is now 3.86%, dropping to 3.74%. The transformation changes slightly as well, now requiring a 13° anti-clockwise rotation, 9% longitudinal expansion, and 2% latitudinal reduction. The benefits also extend to the southern group. Initial stress drops to 6.73%, transformed stress to 3.86%. Mean individual stress is now 7.05% and is reduced to 4.14% by the transformation. The necessary clockwise rotation is now under 1°. Longitudinal expansion is 30%, and latitudinal reduction is 22%. A further benefit is that Saint Albans ceases to be an outlier in the southern group after the transformation. It still has the maximum stress error, 9.75%, but as it has no alternative coordinates, it will have to remain as it is.

4.4.2 A united kingdom?

Table 4.1 summarises the stress figures for Britain and its two inland divisions. (Chester has been given its alternative coordinates when computing these numbers.) Both the visual displays above and the stress totals make it quite clear that the northern group is far less accurate than the southern group. The island as a unit has the lowest stress, but this is merely reflective of the greater total modern distances.

More important, perhaps, than the initial stress, is the transformed stress.

	#	Initial	Transformed	Initial Individual	Transformed Individual
Albion	30	3.57%	3.47%	3.86%	3.74%
North	10	13.07%	9.67%	14.67%	12.00%
South	20	6.73%	3.86%	7.05%	4.14%

Table 4.1: Britain inland stress summary (with alternative Chester)

The whole of Albion improves very little. The two divisions, however, improve dramatically, both in total and individual errors. This means that no effective transformation was found for the cities when taken together, but very effective ones were found for the cities when taken in groups. The errors, therefore, are not constant across Roman Britain. This is verified in Table 4.2, where we see vastly different transformations at work.

	Rotation	Longitudinal skew	Latitudinal skew
Albion (inland)	$-13°$	+9%	-2%
North (inland)	$-33°$	-28%	+32%
South (inland)	$+1°$	+30%	-22%
Albion (coast)	$-44°$	-55%	-16%
North (coast)	$-87°$	-28%	-27%
South (coast)	$-5°$	-1%	-17%

Table 4.2: Britain transformation summary (with alternative Chester)

We see that the south cluster needs almost no rotation and that the north cluster is to blame for the misalignment. This is not surprising when we recall that Scotland is almost entirely sideways. It must be that this phenomenon is also contorting northern England. Indeed, Ptolemy's turning can already be seen in his positioning Catterick, Aldborough, and York on a straight north-south line instead of the correct diagonal. We further see that the two areas differ almost oppositely in the way they need to be scaled. The North needs a 28% reduction in longitude and a 32% elongation in latitude while the South needs a 30% expansion and 22% diminution, respectively. Essentially, Ptolemy's north is short and fat, and his south tall and thin. Such drastic differences clearly point to a problem with the data. It seems highly likely that the data available to Ptolemy was gathered at different times and/or by different sources. The southern data may have been compiled during the relative peace and stability in the decades following the conquest of the area, while the northern data was hastily gathered in the turmoil of constant warfare.

In examining the coast, far more northern points have been identified than in the inland analysis, and so we anticipate the turning of Scotland to be an even more burdensome source of error here (see Table 4.3). Indeed, the north coast has a Procrustes statistic almost 12 times larger than the south coast with individual errors over 4 times as great. Surprisingly, though, after a rotation and scaling, the north actually has a lower total statistic and nearly identical individual error. Yes, there are 8 less points in Scotland, but this still represents an astonishing turnaround. This points to Scotland having the correct basic shape despite its east-west alignment.

	#	Initial	Transformed	Initial Individual	Transformed Individual
Albion	48	4.04M	883K	257.44	123.58
North	20	1.52M	59.9K	259.79	47.42
South	28	127.6K	84.4K	56.95	47.23

Table 4.3: Britain coastal summary (in km)

When we examine the transformations, we see that, again, the north is responsible for the rotation of the island. The north and south coastal groups do not follow the same pattern as the inland cities, though, in terms of scaling. However, because of Scotland's inclusion or exclusion, the two southern groups are really the only ones that can be compared across the stress-Procrustes divide. Especially intriguing is the difference in longitudinal scaling between the southern cities and the southern coast. The cities need to expand outwards, while the coast remains essentially as it is. This does not necessarily mean, however, that Ptolemy put his cities too far inland. If we look again at Figure 4.23, we see that the longitude is not capable of being expanded or reduced because of the south-west peninsula and East Anglia. An expansion would take Land's End and Lizard Point farther away from the modern configuration, whereas any attempt to bring them closer would doom East Anglia. As it is, Ptolemy's East Anglia may not even be as inaccurate as the configurations make it out to be. East Anglia even today is quite marshy, and it is not inconceivable that some two thousand years ago, the area was underwater. The river mouths forming the coastline would then have been much further to the west as on Ptolemy's map. It is also not difficult to imagine how early Roman navigators could have exaggerated the size of the south-west peninsula when they were used to the tranquil waters of the Mediterranean and not the rough, tidal waters of the English Channel and Atlantic Ocean.

That still leaves the mystery of the turning of Scotland. Surely the sailors sent by Agricola to circumnavigate the island could not have mistaken east for north so consistently. That being said, in the height of summer, the sun does rise relatively far north, especially compared to its rising in the Mediterranean regions. This may have been an unexpected phenomenon to the Romans. However, the sun also sets to the north and not to the south, and so the directions of south and west should not have been confused.

Looking again at Scotland's transformation, we see a rotation very close to 90° and a nearly uniform scaling of between a third and a quarter. Is it possible that the shape of Ptolemy's Scotland is slightly more inaccurate than our transformation has made it and that its size and orientation are a mathematical alteration induced deliberately by Ptolemy? He has three competing sets of data sitting before him: a tradition that says Great Britain is a triangle with a long side facing north and a corner pointing east,[1] a number of locations of tribal settlements (none of which have been identified) from an area never under Roman control, and a more recent set of coastal data from competent mariners specifically tasked with determining the shape of the island. If the inaccurate coordinates of the native settlements fit reasonably well with the tradition of an east-west Scotland, could it be possible that Ptolemy intentionally rotated the coastline by a right angle and expanded it by 25% or 33% so that it fit around the settlements and appeased tradition? In the south, Roman Britain had established and relatively accurate coastal and inland data that fit together. There he was able to break with tradition and scientifically render a truer map. No such inland data was coming to him from Scotland, though. Scotland, at the edge of the known world, could very well represent Ptolemy's compromise between tradition and scientific enquiry.

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[1]Caesar describes the island as follows:

> The natural shape of the island is triangular, and one side lies opposite to Gaul. Of this side one angle, which is in Kent (where almost all the ships from Gaul come in to land), faces the east, the lower angle faces south. This side stretches about five hundred miles. The second side bears towards Spain and the west, in which direction lies Ireland, smaller by one half, as it is thought, than Britain; . . . The length of this side, according to the belief of the natives, is seven hundred miles. The third side bears northwards, and has no land confronting it; the angle, however, of that side faces on the whole towards Germany. The side is supposed to be eight hundred miles long(Caesar, *B.G.* 5.13).

Diodorus Siculus, writing within fifty years of Caesar, maintains the triangular shape of the island (Diodorus, 5.21.3-6; Ireland 1996, 15) as does Strabo (Strabo, 4.5.2; Ireland 1996, 20).

Chapter 5

Conclusion

5.1 Advancing the field

We have seen the internal accuracies (or inaccuracies) now of each of our case studies. It is left to us to discuss how the three provincial maps compare to one another. First though, it will be beneficial to see how far Ptolemy had advanced the geographical knowledge of the ancient world. For this purpose, we shall compare Ptolemy's maps with the descriptions of the three countries given by Strabo. Strabo was a contemporary of Augustus, writing his *Geographica* approximately 150 years before Ptolemy. The two geographers should give us a glimpse of how Greco-Roman knowledge of the world changed from the beginnings of the Roman Empire to the days of its greatest extent.

5.1.1 Strabo's descriptions

Strabo describes Italy thus:

> . . . as for the Alps, their base is curved and gulf-like, with the cavities turned toward Italy; the central gulf are [*sic*] near the Salassi, while the extremities [of the gulf] take a turn, the one as far as [Mount] Ocra and the recess of the Adriatic, the other to the Ligurian seaboard as far as Genua (the emporium of the Ligures), where the Apennine Mountains join the Alps. . . . The remainder of Italy, however, is narrow and elongated, terminating in two heads, one at the Sicilian Strait and the other at Iapygia [that is, Apulia]; and it is pinched in on both sides, on one by the Adriatic and on the other by the Tyrrhenian Sea (Strabo, 5.1.3).

Cisalpine Gaul is bounded on the southern side 'by the seaboard of the Heneti [that is, the Veneti] and by those Apennine Mountains which reach down to the neighbourhood of Ariminum and Ancona' (Strabo, 5.1.3). The Veneti occupied the north-western coast of the Adriatic Sea. The sea must cut into Italy (the 'recess' Strabo mentions above), creating an east-west shoreline which connects to an east-west line of the Apennines, the line of the mountains meeting the line of the coast at Ariminum. This southern border of Cisalpine Gaul is 6,300 stadia, but the breadth of the area is less than 1,000. Northern Italy, then, seems to run mostly parallel to mainland Europe instead of jutting south-eastward into the Mediterranean.

To get an idea of the orientation, we turn to Strabo's account of earlier depictions of the peninsula:

> Now it is not easy geometrically to outline what is now Italy, as a whole, by means of a single figure, and yet they say it is a triangular promontory extending towards the south and the winter-risings of the sun [south-east], with its vertex at the Strait of Sicily and with the Alps as its base (Strabo, 5.1.2).

Strabo does go on to complain about the use of the term 'triangle' by his predecessors and remedies this with his description. The Tyrrhenian coast is curved and points to the winter risings. Strabo then turns to the Adriatic coast and its recess, which turns into the peninsula at Ariminum. The Adriatic coast runs parallel to the Tyrrhenian coast from Ariminum to Apulia. The bend in the coastline at Ariminum forms 'an angle, or, if not an angle, at least a considerable curve' (Strabo, 5.1.2). The southern coast from Apulia to the Strait of Sicily, i.e. the bottom of the foot, Strabo does not describe as any particular shape other than admitting that it is not a straight line. He also considers the angle made at Apulia to be as considerable as that made at Ariminum (Strabo 1988, 5.1.2). He describes Apulia and Calabria as peninsulas. Calabria, the 'toe' of Italy, and Apulia, the 'heel', are both 1,300 stadia at their maximum breadth and slightly less than 6,000 in length. He does not quibble with Polybius' estimate of between 2,500 and 3,000 stadia for the distance between the two (the former by sea, the latter by land) (Strabo, 5.1.3). Unfortunately, there is no estimate of the length of the two long coastlines.

Strabo describes Spain as follows:

> Iberia is like an ox-hide extending in length from west to east, its fore-parts toward the east, and in breadth from north to south. It is six thousand stadia in length all told, and five thousand stadia in its greatest breadth; though in some places it is much less than three thousand in breadth, particularly near the Pyrenees, which form its eastern side. That is, an unbroken chain of mountains, stretching from south to north, forms the boundary line between

> Celtica and Iberia; ... So the eastern side of Iberia is formed by the Pyrenees; the southern side is formed in part by Our Sea [i.e. the Mediterranean], from the Pyrenees to the Pillars, and from that point on by the ocean, up to what is called the Sacred Cape; the third is the western side, which is approximately parallel to the Pyrenees and extends from the Sacred Cape to that Cape of the Artabrians which is called Nerium; and the fourth side extends from Cape Nerium up to the northern headlands of the Pyrenees (Strabo, 3.1.3).

An ox-hide is like a rectangle the sides of which curve inward. Iberia is a fairly simple shape, according the Strabo, having four sides each facing a cardinal direction.

Strabo is not as forthcoming in his description of Britain. He has a triangular Britain with its longest side 4,300 - 5,000 stadia in length and parallel to Celtica's (France's) northern coastline. Its eastern most point is Cantium (Kent), directly across the Channel from the mouth of the Rhine. The westernmost point is an island opposite the 'Aquitanian Pyrenees' (Strabo, 2.5.28, 4.5.1). Ireland is said to run parallel to Britain's north side (Strabo, 4.5.4). This is not much of a revision of Caesar's description of the British Isles not one hundred years earlier (see footnote in Section 4.4.2).

5.1.2 Improvements made by Ptolemy's day

It is difficult to say whether or not Ptolemy's map of Italy is an improvement on Strabo's description of the Italian peninsula (ignoring the precision that a point-by-point map provides). There are certainly several differences. The Tyrrhenian coast is no longer a straight line towards the south-east 'pinched in' by the sea. Rather it consists of two essentially straight lines: one running east-south-east from Nice to Naples, the second running due south to the Strait of Sicily. On the Adriatic side, the 'pinch' is gone as is the angle at Ariminum. The recess of the Adriatic is angled north-west and no longer meets the Apennines in a straight east-west line. The coast now runs from the mainland in a fairly straight east-south-east line until Mount Garganus (Gargano), before turning due south to the end of Apulia. Apulia and Calabria are no longer equally sized peninsulas. Calabria appears to be just an extension of the main Italian peninsula.

Ptolemy's Spain is not the ox-hide shape that Strabo imagined. Ptolemy's map, with a lot of simplification, is five sided: west coast, north coast, Pyrenees, Mediterranean coast, and south-west coast. Spain has moved considerably with respect to the rest of Europe by Ptolemy's time. The Pyrenees no longer separate Spain from Gallia Aquitania (south-western France) in a straight north-south line, nor do they make up Spain's east side. Strabo must have had Spain side-by-side with France, thereby allowing the 'Aquitanian Pyrenees' to be opposite the island off Britain's Land's End. Ptolemy moves Spain so that it is hanging correctly off the south-west corner of France, allowing for the existence of the Bay of Biscay. The Pyrenees are still at a relatively steep angle, but certainly are more accurately aligned under Ptolemy than Strabo. With Ptolemy's re-angling of the Pyrenees, the Mediterranean coast is no longer the southern coast, but slants obliquely as it should. The direction of the coast changes at the Pillars, forming the fifth side we mentioned above. Ptolemy's western and northern coasts remain as Strabo's.

Britain sees the most change in the some 150 years between Strabo and Ptolemy. Britain is certainly no triangle in Ptolemy's view. Kent is no longer opposite the Rhine, and Spain is far away. The southern coast is not the longest side of Britain (but even Caesar knew that), and Ireland is now off the west coast. Even with Scotland on its side, the Roman vision of Britain has moved forward by leaps and bounds.

It seems that improvement from Strabo to Ptolemy is inversely related to how incorporated each province was within the Roman Empire. Augustus was himself engaged in the subjugation of parts of Spain, and Britain had not been touched by any Roman leader save Julius Caesar when Strabo was writing. New information clearly was being generated for these areas, especially Britain. With new information, improvements to geographical knowledge and the maps could be made. Knowledge of parts of Spain and all of Italy, though, may have been taken for granted already in Strabo's time. There simply may not have been an interest in trying to improve knowledge of these areas because it may have been thought accurate already. Indeed, the changes Ptolemy makes to Italy do not in some instances improve his map's accuracy over Strabo's description. Britain, on the other hand, was almost entirely unknown by Strabo. What Britain looked like may have been a question at the forefront of geography, fuelling investigation and enquiry. It is small wonder that the newest acquisitions of Rome saw the greatest improvements in their conception and description.

5.2 Comparison

5.2.1 By the numbers

From an historical standpoint, we would expect the general accuracy of the three provinces to be proportional to the length of time that the Greco-Roman scientific community had to gather the respective data. That is, Italy should be the most accurate as it was the most established and developed area, followed by Spain, and lastly the half-conquered Britain.

	Total Stress	City Stress	Total Proc.	Individ. Proc.
Italy	4.14%	4.96%	1,362K km^2	107.5 km
Spain	3.32%	3.66%	1,352K km^2	119.27 km
Britain	4.58%	5.21%	4.04M km^2	257.44 km

Table 5.1: Initial error summary

We see that this is not quite the case (see Table 5.1). Italy has a higher stress than Spain. Spain is rather squarish, but Italy is more of a narrow rhomboid. The 234 known cities of Italy are much more densely packed than the 155 known cities of Spain. While certainly the density plays a role in increasing Italy's stress error, the misplacement of the cities cannot be completely dismissed. Looking at Figure 2.4, we can see that there are densely packed areas, especially Campania, which have very low stress, while the loosely packed Samnium has very high stress. Likewise in Figure 3.3, the higher errors of Spain are all in the centre which is mostly open space. The higher stress of Italy as compared to Spain is a result of the Spanish cities having been placed with more relative accuracy.

Britain has the highest stress, and thereby the lowest accuracy, as we expected. This is not simply due to the small total distances as compared to Spain and Italy. Thanks to the fact that Britain only has 30 identified settlements, it was easy to comment on the placement of almost all the cities individually. There were quite a few out of the 30 that had been misplaced. However, if Britain were examined on the same scale as Spain and Italy, the misplacements would then seem very small indeed. We should not, therefore, be so quick to simply label Britain as 'inaccurate'. When compared to subsections of Spain and Italy much closer to its own size, Britain has a much lower stress than all save Tarraconensis and the eastern seaboard of Spain. Furthermore, the replacement of Chester's coordinates brought Britain's stress down to 3.57%. This one change puts Britain ahead of Italy. More than anything, this emphasises how far Italy is from meeting our expectation of being the most accurate of our case studies.

Breaking down the three maps into their major subgroups, a more scattered picture is presented (Table 5.2). Italy and Spain each have their highs and lows. The largest areas, Tarraconensis and central Italy, have the lowest stresses, post-transformation. Again, we remark how much the transformation

	#	Initial Error	Transformed Error
Southern Italy	29	18.49%	10.59%
Central Italy	136	16.94%	5.17%
Northern Italy	69	8.01%	6.33%
Baetica	39	12.55%	10.20%
Lusitania	19	9.63%	8.92%
Tarraconensis	97	4.98%	4.54%
Southern Britain	20	9.68%	6.85%
Northern Britain	10	13.07%	9.67%

Table 5.2: Overall stress errors of the major subdivisions (in km)

improved central Italy. Spain and Italy, as well as having the two divisions with the minimum stress errors, also have the two maximum stress errors (after transformation). Britain's two divisions fall in the middle. The truly amazing thing is the range of errors across the subgroups in each map. Each section has its own inaccuracies not readily seen when the maps are taken as wholes. The most profound examples of this are the Ebro valley in Spain and Chester in southern Britain. The Ebro had an initial stress of 41.66% compared to the mere 4.98% of Tarraconensis as a whole. Alternative coordinates for Chester brought southern Britain's initial error down to 6.73%, with a consequent transformed error of 3.86%, the lowest of any subdivision, save the east coast of Spain. Error is far from uniform across Ptolemy's maps, and it is only through breaking the maps down that the source of the error, sometimes only a single city, can be found. With highly regionalised errors, it is clear the Ptolemy's data was not from a single, unified source.

	Cities			Coastal points		
	Known	Total	%	Known	Total	%
Italy	234	272	86%	90	104	87%
Spain	155	429	36%	77	99	78%
Britain	30	61	49%	48	66	73%

Table 5.3: Identified cities

Comparing Procrustes errors very much depends on the number of identified coastal points. These are given in Table 5.3. Spain and Italy have similar Procrustes figures (Table 5.1). Based on the numbers, Italy has the most accurately mapped coastline with the lowest per-point error. Britain, with the turning of Scotland, has by far the highest error. Once Scotland and Roman

Britain are transformed independently, though, both have mean individual errors of 47 km. By the numbers, the two transformed subdivisions of Britain have an accuracy that is middle-of-the-road compared to those of the other provinces (see Table 5.4). Once each region is allowed to be transformed independently and all the results displayed together, a pattern emerges. The areas having the lowest error are all in the western Mediterranean: southern and western Italy and Baetica. The regions of highest error are on the further sides of Spain and Italy. (Tarraconensis is a bit of a mixed bag, it must be said. The east coast of Spain taken on its own has a transformed error of 50.19 km, which is lower than Lusitania and eastern Italy.) Britain, which we know was deliberately circumnavigated, has an error that falls in the middle, just as it does with stress. It seems that the coastlines of the western Mediterranean, which enjoyed the greatest urbanisation and Romanisation, not to mention maritime trade, were those most accurately mapped.

	#	Initial Error	Transformed Error
Southern Italy	25	66.0	34.7
Western Italy	34	103.8	31.4
Eastern Italy	31	138.3	55.1
Baetica	24	35.51	25.28
Lusitania	13	89.05	58.11
Tarraconensis	41	113.73	75.77
Southern Britain	28	56.95	47.23
Northern Britain	20	259.79	47.42

Table 5.4: Individual Procrustes error of the major subdivisions (in km)

5.3 Final thoughts

Ptolemy's maps represent a huge step forward from previous Greco-Roman conceptions of at least three regions of Europe: Italy, Spain, and Britain. While the several generations of cartographical knowledge before Ptolemy have not survived, it is safe to assume that these representations of the known world were overshadowed by the *Geography*, otherwise it would have been Ptolemy's work that was lost to obscurity. His mathematical approach to cartography, mapping the world with the use of coordinates, was revolutionary. Ptolemy's maps have such a high level of detail which is simply not possible to convey through the written descriptions of earlier geographers. Ptolemy was precise, systematic, and, all things considered, accurate.

In the current age of Google Earth and GPS satellites, it is easy to dismiss Ptolemy's work. 'How could anyone have thought Scotland pointed east!?' Remembering that Ptolemy's only tools were tradition, word-of-mouth descriptions, and the very occasional astronomical observations, we can truly appreciate how good his maps really are. Before we get carried away in the direction of over praising him, however, let us pause to consider how much of his maps of Britain, Spain, and Italy we do not know.

Table 5.3 shows just how much is left to discover about these three maps. Many of the coastal points have been identified, but a great majority of the settlements of Spain and Britain have yet to be linked to modern locations. Dozens may yet remain buried. When this study began, it was the intention to be able to correct Ptolemy's maps to such a degree, that the location of these unknown cities could be speculated upon with reasonable accuracy. Unfortunately, the non-uniform nature of the errors in the maps prevented this. It is even entirely possible that the stress errors of the maps may jump significantly if more cities are found. Perhaps the known cities represent those most accurately placed by Ptolemy.

The future of the statistical and historical analysis of the maps of Ptolemy, for which this study is merely a small beginning, relies on a concerted effort of many fields. Mathematicians, geographers, historians, linguists, statisticians, cartographers, geologists, and archaeologists need to come together to give this work its due. Detailed analyses of roadways and milestones, coastal shifts, changing names, manuscript transmissions, and regional histories and interactions all need to be collated in connection with the *Geography*. It is only with such an interdisciplinary approach that antiquity's masterpiece of cartography can be truly analysed and appreciated.

Appendix A

Table of Names

Place names on Ptolemy's map of Ireland.

#	Latin name	Modern name
1	Northern Promontory	N/A
2	Vennicnium Promontory	N/A
3	R. Vidua	N/A
4	R. Argita	N/A
5	Robogdium Promontory	Fair Head
6	Northern Promontory	N/A
7	R. Ravius	N/A
8	Nagnata	N/A
9	R. Libnius	N/A
10	R. Ausoba	N/A
11	R. Senus	River Shannon
12	R. Dur	N/A
13	R. Iernus	N/A
14	Southern Promontory	N/A
15	R. Dabrona	River Lee
16	R. Birgus	River Barrow
17	Sacred Promontory	Carnsore Point
18	Sacred Promontory	Carnsore Point
19	R. Modonus	N/A
20	Manapia	N/A
21	R. Oboca	N/A
22	Eblana	Dublin
23	R. Buvinda	River Boyne

Continued on next page

Ireland – continued from previous page

#	Latin name	Modern name
24	Isamnium Promontory	N/A
25	R. Vinderis	N/A
26	R. Logia	Belfast Lough
27	Regia I	N/A
28	Raeba*	N/A
29	Laberus	N/A
30	Macolicum	N/A
31	Regia II	N/A
32	Dunum	N/A
33	Hibernis*	N/A
34	Ebuda I	N/A
35	Ebuda II	N/A
36	Ricina	N/A
37	Maleus	N/A
38	Epidium	N/A
39	Monaoeda	Isle of Man
40	Mona	Isle of Anglesey
41	Edrus	N/A
42	Limnus	N/A

Place names on Ptolemy's map of Italy.

#	Latin name	Modern name
1	Nicaea	Nizza
2	Harbour of Hercules	N/A
3	Tropaea Augusti	La Turbie
4	Monoeci Portus	Monaco
5	Albintimilium	near Ventimiglia
6	Albingaunum	Albenga
7	Genua	Genua/Gnes
8	Entella	R. Entella
9	Tigullia	Lavagna
10	Macra	R. Magra
11	Boacias	R. Vara
12	Luna	Luni

Continued on next page

Italy – continued from previous page

#	**Latin name**	**Modern name**
13	Cape Luna	N/A
14	Temple of Hercules	N/A
15	Arnus	R. Arno
16	Populonium	Populonia
17	Cape Populonium	N/A
18	Trajan's Harbour	N/A
19	Cape Telamo	Punta di Talamone
20	Osa	R. Osa
21	Cosa	Cosa
22	Graviscae	Porto Clementino
23	Castrum Novum	near Civitavecchia
24	Pyrgi	S. Severa
25	Alsium	Palo
26	Tiberis	R. Tiber/Tevere
27	Tiberis	R. Tiber
28	Ostia	Ostia Antica
29	Antium	Anzio
30	Clostra	Archi di S. Donato
31	Cape Circeii	Monte Circeo
32	Tarracinae*	Terracina
33	Formiae	Formia
34	Liris	R. Garigliano
35	Sinuessa	near Mondragone
36	Volturnum	Castel Volturno
37	Liternum	Villa Literno
38	Cumae	Cumae
39	(Misenum)	Miseno
40	Puteoli	Pozzuoli
41	Neapolis*	Napoli
42	Sarnus	R. Sarno
43	Surrentum	Sorrento
44	Salernum	Salerno
45	Silerus	R. Sele
46	Paestum	Paestum
47	Velia	Castellammare di Velia
48	Buxentum	Policastro Bussentino

Continued on next page

Italy – continued from previous page

#	Latin name	Modern name
49	Laus	R. Lao
50	Tempsa	Torre del Casale
51	Taurianum	Capo Vaticano
52	Gulf of Hippo	Golfo di S. Eufemia
53	Cape Scylla	Rocca di Scilla/Promontorio Scillo
54	Rhegium Iulium	Reggio di Calabria
55	Cape Leucopetra	Capo dell' Armi
56	Cape Zephyrus	Capo Bruzzano
57	Locri	near Locri
58	Locanus	N/A
59	Scolacium	near Squillace
60	Scolacium	N/A
61	Cape Lacinium	Capo Colonna
62	Croto	Crotone
63	Thurii	near Terranova da Sibari
64	Metapontum	Metaponto
65	Tarentum*	Tarent
66	Cape Iapygia, Sallentinum	Capo S. Maria di Leuca
67	Hydruntum	Otranto
68	Lupiae	Lecce
69	Brundisium*	Brindisi
70	Egnatia	Egnazia/Gnatia
71	Barium	Bari
72	Aufidus	R. Ofanto
73	Salpia	Salapia near Trinitápoli
74	Sipontum	Siponto
75	Apeneste	N/A
76	Garganus	Monte/Promontorio del Gargano
77	Uria	near Rodi Garganico
78	Tifernus	R. Biferno
79	Buca	Termoli
80	Histonium	Vasto
81	Sagrus	R. Sangro
82	Ortona	Ortona
83	Aternus	R. Pescara
84	Matrinus	N/A

Continued on next page

Italy – continued from previous page

#	**Latin name**	**Modern name**
85	Castrum	Giulianova
86	Cupra Maritima	Cupra Marittima
87	Truentus	R. Tronto
88	Potentia	S. Maria a Potenza
89	Numana	Numana
90	Ancona*	Ancona
91	Aesis	R. Esino
92	Sena Gallica	Senigallia
93	Fanum Fortunae	Fano
94	Pisaurum	Pesaro
95	Ariminum	Rimini
96	Rubico	R. Rubicone
97	Ravenna*	Ravenna
98	Padus	R. Po
99	Lacus Larius	Lago di Como/Lake Como
100	Doria	Dora Baltea
101	Lacus Poeninus	Lake at the Great St. Bernard Pass
102	Lacus Benacus	Lago di Garda/Lake Garda
103	Atrianus	N/A
104	Tiliaventum	R. Tagliamento
105	Natiso	R. Natisone
106	Tergeste	Triest
107	Formio	R. Risano/Rižana
108	Parentium	Poreč
109	Pola	Pula
110	Nesactum	Vizače
111	Pucinum	Prosecco
112	Piquentum	Buzet
113	Alvum	N/A
114	Forum Iulii	Cividale
115	Concordia	Concordia Sagittaria
116	Aquileia*	Aquileia
117	Vicetia	Vicenza
118	Belunum	Belluno
119	Acelum	Asolo
120	Opitergium	Oderzo

Continued on next page

Italy – continued from previous page

#	Latin name	Modern name
121	Ateste	Este
122	Patavium	Padua
123	Altinum	Altino
124	Atria	Adria
125	Bergomum	Bergamo
126	Forum Iutuntorum	N/A
127	Brixia	Brescia
128	Cremona	Cremona
129	Verona	Verona
130	Mantua	Mantua/Mantova
131	Tridentium	Trient/Trento
132	Butrium	N/A
133	Vaunia	N/A
134	Carraca	N/A
135	Bretina	N/A
136	Anaunium	N/A
137	Novaria	Novara
138	Mediolanium	Milano
139	Comum	Como
140	Ticinum	Pavia
141	Augusta Praetoria	Aosta
142	Eporedia	Ivrea
143	Augusta Taurinorum	Turin
144	Augusta Bagiennorum	Bene Vagienna
145	Iria	Voghera
146	Dertona	Tortona
147	Vercellae	Vercelli
148	Laumellum	Lomello
149	Forum Claudii	N/A
150	Axima	Aime
151	Oscela	N/A
152	Segusio	Susa
153	Vintium	Vence
154	Salinae	Saillon
155	Sanitium	Senez
156	Pollentia	Pollenzo

Continued on next page

Italy – continued from previous page

#	**Latin name**	**Modern name**
157	Alba Pompeia	Alba
158	Libarna	Serravalle Scrivia
159	Brixellum	Brescello
160	Parma	Parma
161	Rhegium Lepidum	Reggio nell'Emilia
162	Nuceria	N/A
163	Tannetum	Taneto
164	Bononia	Bologna
165	Claterna	Ozzano
166	Forum Cornelii	Imola
167	Caesena	Cesena
168	Faventia	Faenza
169	Biracellum	N/A
170	Fossae Papirianae	near Lago di Massaciuccoli
171	Bondelia	N/A
172	Luca	Lucca
173	Lucus Feroniae	near Scorano
174	Pistoria	Pistoia
175	Florentia	Firenze/Florence
176	Pisae	Pisa
177	Rusellae	Roselle
178	Faesulae	Fiesole
179	Perusia	Perugia
180	Arretium	Arezzo
181	Cortona	Cortona
182	Aculca	N/A
183	Biturigia	N/A
184	Manliana	Torrita di Siena
185	Vetulonia	Vetulonia
186	Saena	Siena
187	Suana	Sovana
188	Saturnia	N/A
189	Eba	N/A
190	Volci	near Montalto di Castro
191	Clusium	Chiusi
192	Volsinium	Bolsena

Continued on next page

Italy – continued from previous page

#	Latin name	Modern name
193	Sudernum	N/A
194	Ferentia	Ferento
195	Sutrium	Sutri
196	Tarquinii	Tarquinia
197	Blera	Blera
198	Forum Clodii	S. Liberato
199	Nepeta	Nepi
200	Falerium	N/A
201	Caere	Cerveteri
202	Suasa	(Castelleone di) Suasa
203	Ostra	Le Muracce
204	Traiana	N/A
205	Urbs Salvia	Urbisaglia
206	Septempeda	S. Severino
207	Cupra Montana	Cupramontana
208	Firmum	Fermo
209	Asculum	Ascoli Piceno
210	Adria	N/A
211	Pitinum	N/A
212	Tifernum	N/A
213	Forum Sempronii	near Fossombrone
214	Iguvium	Gubbio
215	Aesis	Jesi
216	Tuficum	Borgo Tufico
217	Sentinum	near Sassoferrato
218	Asisium	Assisi
219	Camerinum	Camerino
220	Nuceria	Nocera Umbra
221	Arna	Civitella d'Arno
222	Hispellum	Spello
223	Tuder	Todi
224	Forum Flaminii	near Foligno
225	Spoletium	Spoleto
226	Mevania	Bevagna
227	Ameria	Amelia
228	Narni	Narni

Continued on next page

Italy – continued from previous page

#	**Latin name**	**Modern name**
229	Ocriculum	Otricoli
230	Nursia	Norcia
231	Cliternia	Capradosso
232	Carsioli	near Carsoli
233	Aex	N/A
234	Alba Fucens	Albe
235	Beregra	Montorio al Vomano
236	Interamnia	Teramo
237	Pinna	Penne
238	Aveia	Fossa
239	Amiternum	S. Vittorino
240	Angulus	N/A
241	Teate	Chieti
242	Urbs Roma*	Roma
243	Tibur	Tivoli
244	Praeneste	Palestrina
245	Tusculum	Tuscolo
246	Aricia	Ariccia
247	Ardea	Ardea
248	Nomentum	near Mentana
249	Treba	Trevi
250	Anagnia	Anagni
251	Vempsum	N/A
252	Lanuvium	Lanuvio
253	Atina	Atina
254	Fidenae	Borgata Fidene
255	Frusino	Frosinone
256	Ferentinum	Ferentino
257	Privernum	near Priverno
258	Setia	Sezze
259	Aquinum	Aquino
260	Sora	Sora
261	Minturnae	near Minturno
262	Fundi	Fondi
263	Corfinium	Corfinio
264	Sulmo	Sulmona

Continued on next page

Italy – continued from previous page

#	Latin name	Modern name
265	Anxanum	Lanciano
266	Larinum	Larino
267	Aufidena	Castel di Sangro
268	Bovianum	Boiano/Bojano
269	Aesernia	Isernia
270	Saepinum	near Sepino
271	Allifa	Alife
272	Tuticum	N/A
273	Telesia	S. Salvatore Telesino
274	Beneventum*	Benevento
275	Caudium	near Montesarchio
276	Venafrum	Venafro
277	Teanum	Teano
278	Suessa	Sessa Aurunca
279	Cales	Calvi
280	Casilinum	Capua
281	Trebula	near Treglia
282	Forum Popillii	near Carinola
283	Capua*	S. Maria Capua Vetere
284	Abella	Avella
285	Atella	S. Maria d'Atella
286	Nola	Nola
287	Nuceria [Alfaterna]	Nocera Inferiore
288	Ulci	N/A
289	Compsa	Conza della Campania
290	Potentia	Potenza
291	Blanda	Palecastro di Tortora
292	Grumentum	Grumentum
293	Aquilonia	Lacedonia
294	Abellinum	Avellino
295	Aeculanum	near Mirabella Eclano
296	Fratuolum	N/A
297	Teanum	near S. Paolo di Civitate
298	Nuceria Apulorum	Lucera
299	Vibarna	Bovino
300	Arpi	near Arpinova

Continued on next page

Italy – continued from previous page

#	Latin name	Modern name
301	Herdonia	Ordona
302	Canusium	Canosa di Puglia
303	Venusia	Venosa
304	Caelia	Ceglie Messapico
305	Numistro	Raia S. Basile
306	Consentia	Cosenza
307	Vibo Valentia	Vibo Valentia
308	Petelia	Strongoli
309	Abystrum	N/A
310	Rudiae	Rugge
311	Neretum	Nard
312	Aletium	Alezio
313	Bausta	Vaste
314	Uzentum	Ugento
315	Veretum	near Pat
316	Sturni	Ostuni
317	Uria	Oria
318	Aethalia	Elba
319	Capraria	Capraia
320	Ilva	Elba
321	Planasia	Pianosa
322	Pontia	Ponza
323	Pandateria	Ventotene
324	Parthenope	N/A
325	Prochyta	Procida
326	Pithecusae	Ischia
327	Capreae	Capri
328	Sirenusae	Li Galli
329	Diomedeiae	Isole di Tremiti

Place names on Ptolemy's map of Spain.

#	Latin name	Modern name
1	R. Anas (West Mouth)	Rio Guadiana
2	R. Anas (East Mouth)	Rio Guadiana

Continued on next page

Spain – continued from previous page

#	Latin name	Modern name
3	R. Anas (Bend)	Rio Guadiana
4	R. Anas (Boundary)	Rio Guadiana
5	Border	N/A
6	R. Anas (Source)	Rio Guadiana
7	Onoba Aestuaria	Huelva
8	R. Baetis (West Mouth)	Guadalquivir
9	R. Baetis (East Mouth)	Guadalquivir
10	R. Baetis (Source)	Guadalquivir
11	R. Asta	N/A
12	Menestheus Harbour	Puerto de Santa Maria
13	Temple of Hera	Cabo Trafalgar
14	R. Baelo	Rio del Valle
15	Baelo	Bolonia
16	Mellaria	Ensenada de Valdevaqueros
17	Traducta	Algeciras
18	Barbesula	Torre de Guadiaro
19	Carteia	El Rocadillo
20	Mount Calpe	Rock of Gibraltar
21	R. Barbesula	Guadiaro
22	Suel	Fuengirola
23	R. Salduba	Guadalhorce
24	Malaca	Málaga
25	Maenoba	Torre del Mar
26	Sexi	Almuñécar
27	Selambina	Salobreña
28	Prominence	N/A
29	Abdara	Adra
30	Portus Magnus	Golfo de Almería
31	Cape of Charidemus	Cabo de Gata
32	Baria	Vera
33	Segida	near Palma del Río
34	Iliturgi	near Mengíbar
35	Vogia	N/A
36	Calpurniana	N/A
37	Caecila	N/A
38	Biniana	N/A

Continued on next page

Spain – continued from previous page

#	Latin name	Modern name
39	Corduba*	Córdoba
40	Ulia	Montemayor
41	Obulco	Porcuna
42	Arcilacis	N/A
43	Detumo	near Posadas
44	Murgi	near Dalías
45	Salduba	N/A
46	Tucci	Martos
47	Sala	N/A
48	Barba	El Castillón
49	Ebora	near Sanlúcar de Barrameda
50	Onoba	Jimena de la Frontera
51	Illipula Magna	N/A
52	Selia	N/A
53	Vescis	N/A
54	Osqua	near Villanueva de Concepción
55	Artigis	N/A
56	Calecula	near Pinos Puente
57	Lacibis	N/A
58	Sacili	Alcorrucén
59	Lacippo	Alechipe
60	Iliberri	Granada
61	Canaca	N/A
62	Seria	Jerez de los Caballeros
63	Osca	N/A
64	Caeriana	N/A
65	Urium	N/A
66	Illipula	Niebla
67	Segida	N/A
68	Iptuci	Cabeza de Hortales
69	(Sala)	N/A
70	Nabrissa	Lebrija
71	Ugia	N/A
72	Asta	Mesas de Asta
73	Corticata	N/A
74	Laelia	near Albaida del Aljarafe

Continued on next page

Spain – continued from previous page

#	Latin name	Modern name
75	Italica	Santiponce
76	Maxilua	N/A
77	Ugia	near Las Cabezas de San Juan
78	Carissa	(Cortijo de) Carija
79	Calduba	N/A
80	Paesula	N/A
81	Saguntia	Baños de Gigonza
82	Asido	Medina-Sidonia
83	Nertobriga	near Fregenal de la Sierra
84	Contributa	near Medina de los Torres
85	Regina	near Casas de Reina
86	Cursu	N/A
87	Mirobriga	Capilla
88	Spoletinum	N/A
89	Illipa Magna	Alcála del Río
90	Hispalis*	Sevilla
91	Obulcula	La Monclova
92	Calecula	N/A
93	Oleastrum	N/A
94	Urso	Osuna
95	Baesippo	Barbate
96	Fornacis	N/A
97	Arsa	N/A
98	Asyla	N/A
99	Astigi	Écija
100	Carmo	Carmona
101	Arucci	Aroche
102	Arunda	Ronda
103	Curiga	Monesterio
104	Acinippo	Ronda la Vieja
105	Vama	Salvatierra de los Barros
106	Marianum Mountain	Sierra Morena
107	Illipula Mountain	Sierra de Ronda
108	Gades*	Cádiz
109	R. Durius	R. Douro, Atlantic Ocean
110	R. Durius (Border)	R. Douro

Continued on next page

Spain – continued from previous page

#	Latin name	Modern name
111	R. Durius (Source)	R. Douro
112	Balsa	Tavira
113	Ossonoba	Faro
114	Sacred Cape	Cabo de So Vicente
115	R. Callipus	R. Sado
116	Salacia	Alcácer do Sal
117	Caetobriga	Setúbal
118	Cape Barbarium	Cabo Espichel
119	Olisipo	Lisboa
120	R. Tagus	R. Tejo
121	R. Tagus	N/A
122	R. Tagus	N/A
123	Mountain of the Moon	Cabo da Roca
124	R. Munda	R. Mondego
125	R. Vacua	R. Vouga
126	R. Durius	R. Douro
127	Pax Iulia	Beja
128	Iulia Myrtilis	Mértola
129	Lancobriga	N/A
130	Caepiana	N/A
131	Braetolaeum	N/A
132	Mirobriga	Santiago do Cacém
133	Arcobriga	N/A
134	Meribriga	N/A
135	Catraleucus	N/A
136	White Towers	N/A
137	Arandis	N/A
138	Lauare	N/A
139	Aritium	near Abrantes
140	Selium	N/A
141	Elbocoris	N/A
142	Araducta	N/A
143	Verurium	N/A
144	(Velladis)	N/A
145	Aeminium	Coimbra
146	Chretina	N/A

Continued on next page

Spain – continued from previous page

#	Latin name	Modern name
147	Arabriga	N/A
148	Scalabis	Santarém
149	Tacubis	N/A
150	Concordia	N/A
151	Talabriga	N/A
152	Rusticana	near Galisteo
153	Mendeculia	N/A
154	Caurium	Coria
155	Turmogum	near Garrovillas
156	Budua	near Ermita de Nuestra Señora de Bótoa
157	Colarnum	N/A
158	Sallaecus	N/A
159	Ammaea	near Aramenha
160	Ebora	Évora
161	Norba Caesarina*	Cáceres
162	Licinniana	N/A
163	Augusta Emerita*	Mérida
164	Evandria	near Arroyo de San Serván
165	Geraea	N/A
166	Caecilia Metellina	Medellín
167	Capasa	N/A
168	Lancia Oppidana	N/A
169	Cottaeobriga	N/A
170	Salmantica	Salamanca
171	Augustobriga	Talavera la Vieja
172	Ocelum	N/A
173	Capara	Ventas de Cáparra
174	Manliana	N/A
175	Lacimurga	Navalvillar de Pela
176	Deobriga	N/A
177	Obila	N/A
178	Lama	N/A
179	Londobris	Berlenga
180	R. Durius	R. Douro
181	R. Avus	R. Ave
182	Cape Avarum	near Póvoa de Varzim

Continued on next page

Spain – continued from previous page

#	Latin name	Modern name
183	R. Nebis	R. Neiva
184	R. Limia	R. Límia
185	R. Minius	R. Miño
186	R. Minius (Source)	R. Miño
187	Cape Orvium	N/A
188	R. Ulla	R. Ulla
189	R. Tamaris	R. Tambre
190	Harbour of the Artabroi	N/A
191	Cape Nerium	Cabo Touriñán
192	Altars of Sestius	N/A
193	R. Vir	N/A
194	Cape	N/A
196	Flavium Brigantium	A Coruña
197	Cape Lapatia	Cabo Ortegal
198	R. Mearus	R. Mera
199	R. Nabius	R. Eo
200	R. Navia	R. Navia
201	Flavionavia	Santianes de Pravia
202	R. Naelo	R. Nalón
203	Noega Ucesia	N/A
204	R. Nerva	R. Nervión
205	Flaviobriga	Castro Urdiales
206	R. Deva	R. Deva/Deba
207	Menosca	N/A
208	Oiarso	Oiartzun
209	Cape Oiarso	Cabo Higuer
210	Temple of Aphrodite	Cap Béar
211	Pyrenees	N/A
212	Urci	Pechina
213	Lucentum	Alicante
214	Carthago Nova*	Cartagena
215	Cape Scombraria	Cabo de Palos
216	R. Tader	R. Segura
217	Alonis	N/A
218	R. Saetabis	R. Albaida
219	Ilicitanus Harbour	N/A

Continued on next page

Spain – continued from previous page

#	Latin name	Modern name
220	R. Sucro	R. Xúquer/Júcar
221	R. Pallantia	R. Palantia
222	R. Turia	R. Turia
223	Dianium	Dénia
224	Cape Tenebrium	Cabo de la Nao
225	Tenebrius Harbour	N/A
226	R. Iberus	R. Ebro
227	R. Iberus (Middle)	R. Ebro
228	R. Iberus (Source)	R. Ebro
229	Tarraco*	Tarragona
230	Subur	N/A
231	Barcino	Barcelona
232	R. Rubricatus	R. Llobregat
233	Baetulo	Badalona
234	Cape Lunarium	Cap Begur
235	Iluro	Mataró
236	Blanda	Blanes
237	R. Sambroca	N/A
238	Emporiae	(Castelló d')Empúries
239	R. Clodianus	R. Fluvi
240	Rhode	Roses
241	Temple of Aphrodite	Cap Béar
242	Vindium Mountains	N/A
243	Vindium Mountains	N/A
244	Edulium Mountains	N/A
245	Edulium Mountains	N/A
246	Idubeda Mountains	Sistema Ibérico
247	Idubeda Mountains	Sistema Ibérico
248	Orospeda Mountains	Sistemas Béticos
249	Orospeda Mountains	Sistemas Béticos
250	Claudionerium	N/A
251	Novium	N/A
252	Burum	N/A
253	Olina	N/A
254	Vica	N/A
255	Libunca	N/A

Continued on next page

Spain – continued from previous page

#	Latin name	Modern name
256	Pintia	N/A
257	Caronium	Guitiríz
258	Turuptiana	N/A
259	Glandomirum	N/A
260	Ocelum	N/A
261	Turriga	N/A
262	Iria Flavia	Padrón
263	Lucus Augusti	Lugo
264	Aquae Calidae	N/A
265	Dactonium	N/A
266	Flavia Lambris	N/A
267	Talamine	N/A
268	Aquae Quintinae	Baños de Guntín
269	Lucus Asturum	La Castañera
270	Laberris	N/A
271	Interamnium	N/A
272	Argenteola	N/A
273	Lancia	(Cerro de) Villasabariego
274	Maliaca	N/A
275	Gigia	N/A
276	Bergidum Flavium	Cacabelos
277	Interamnium Flavium	Xano de Arriba
278	Legio VII Gemina	León
279	Brigaecium	Dehesa de Morales
280	Baedunia	San Martín de Torres
281	Intercatia	N/A
282	Paelontium	N/A
283	Nardinium	N/A
284	Petavonium	Rosinos de Vidriales
285	Asturica Augusta*	Astorga
286	Nemetobriga	Trives Viejo
287	Forum Gigurrorum	near A Rúa
288	Bracara Augusta	Braga
289	Caladunum	N/A
290	Pinetum	N/A
291	Complutica	N/A

Continued on next page

Spain – continued from previous page

#	Latin name	Modern name
292	Tuntobriga	N/A
293	Araducca	N/A
294	Aquae Flaviae	Chaves
295	Volobriga	N/A
296	Koelerner	N/A
297	Forum Bibalorum	N/A
298	Forum Limicorum	near Xinzo de Limia
299	Tude	Túy (Tui)
300	Merua	N/A
301	Aquae	Baños de Bande
302	Cambaetum	N/A
303	Forum	N/A
304	Bargiacis	N/A
305	Intercatia	near Villanueva del Campo
306	Viminacium	near Calzadilla de la Cueza
307	Porta Augusta	N/A
308	Autraca	N/A
309	Lacobriga	near Carrión de los Condes
310	Avia	N/A
311	Segontia Paramica	N/A
312	Gella	N/A
313	Albocela	N/A
314	Rauda	Roa (de Duero)
315	Segisama Iulia	N/A
316	Palantia	Palencia
317	Eldana	N/A
318	Cougium	N/A
319	Cauca	Coca
320	Ectodurum	N/A
321	Pintia	N/A
322	Sentice	N/A
323	Sarabris	N/A
324	Concana	N/A
325	Octaviolca	N/A
326	Argenomescum	N/A
327	Vadinia	Cangas de Onís

Continued on next page

Spain – continued from previous page

#	Latin name	Modern name
328	Vellica	N/A
329	Camarica	N/A
330	Iuliobriga	Retortillo
331	Morica	N/A
332	Bravum	N/A
333	Sisaraca	N/A
334	Deobrigula	(Castro de) Tardajos
335	Ambisna	N/A
336	Segisamo	Sasamón
337	Uxama Barca	Osma (de Valdegovía)
338	Segisamonculum	Cerezo de Río Tirón
339	Virovesca	Briviesca
340	Antequia	N/A
341	Deobriga	N/A
342	Vindeleia	Cerro de la Cruz
343	Salionca	Poza de la Sal
344	Visontium	N/A
345	Augustobriga	Muro de Ágreda
346	Savia	N/A
347	Tritium Magallum	Tricio
348	Oliba	N/A
349	Vareia	Varea near Logroño
350	Confluenta	N/A
351	Clunia*	Peñalba de Castro
352	Termes	Montejo de Tiermes
353	Uxama Argaela	El Burgo de Osma
354	Segortia Langa	N/A
355	Veluca	N/A
356	Tucris	N/A
357	Numantia	(Muela de) Garray
358	Segovia	Segovia
359	Nova Augusta	N/A
360	Ilurbida	N/A
361	Etelesta	N/A
362	Ilarcuris	N/A
363	Varada	N/A

Continued on next page

Spain – continued from previous page

#	Latin name	Modern name
364	Thermida	N/A
365	Titulcia	Titulcia
366	Mantua	N/A
367	Toletum	Toledo
368	Complutum	Alcalá de Henares
369	Caracca	N/A
370	Libora	N/A
371	Ispinum	N/A
372	Mentercosa	N/A
373	Barnacis	N/A
374	Alternia	N/A
375	Paterniana	N/A
376	Rigusa	N/A
377	Laminium	N/A
378	Belsinum	N/A
379	Turiasso	Tarazona
380	Nertobriga	near Calatorao
381	Bilbilis	Cerro de la Bámbola
382	Arcobriga	near Arcos de Jalón
383	Caesada	near Espinosa de Henares
384	Mediolum	N/A
385	Attacum	N/A
386	Ercavica	near Cañaveruelas
387	Segobriga	Cabeza del Griego
388	Consabura	Consuegra
389	Bursao	Borja
390	Laxeta	N/A
391	Valeria	Valeria de Arriba
392	Istonium	N/A
393	Alaba	N/A
394	Libana	N/A
395	Urcaesa	N/A
396	Salaria	Úbeda (la Vieja)
397	Sisapo	La Bienvenida
398	Oretum	Granátula (de Calatrava)
399	Aemiliana	N/A

Continued on next page

Spain – continued from previous page

#	**Latin name**	**Modern name**
400	Mirobriga	Capilla
401	Salica	(El) Salobral
402	Libisosa	Lezuza
403	Castulo	Cazlona (Cástulo)
404	Lupparia	N/A
405	Mentesa	Villanueva de la Fuente
406	Cervaria	N/A
407	Vivatia	Baeza
408	Laccuris	N/A
409	Tugia	Toya
410	Lobetum	N/A
411	Pucialia	N/A
412	Salaria	N/A
413	Turbula	N/A
414	Saltigi	Chinchilla (de Monte-Aragón)
415	Bigerra	N/A
416	Abula	N/A
417	Asso	near Caravaca de la Cruz
418	Bergula	N/A
419	Carca	N/A
420	Ilunum	N/A
421	Arcilacis	N/A
422	Segisa	N/A
423	Orcelis	N/A
424	Vergilia	Cambil
425	Acci	Guadix
426	Menlaria	N/A
427	Valentia	Valencia
428	Saetabis	Xtiva
429	Saetabicula	N/A
430	Illici	Alcudia de Elche
431	Iaspis	Aspe
432	Caesaraugusta*	Zaragoza (Saragossa)
433	Bernaba	N/A
434	Ebora	N/A
435	Belia	N/A

Continued on next page

Spain – continued from previous page

#	Latin name	Modern name
436	Arsi	N/A
437	Damania	N/A
438	Leonica	N/A
439	Osicerda	N/A
440	Etobesa	Oropesa
441	Lassira	N/A
442	Edeta and Liria	Llíria
443	Saguntum	Sagunto
444	Carthago Vetus	N/A
445	Biscargis	N/A
446	Theava	N/A
447	Adeba	Vinaroz (Vinars)
448	Tiariulia	N/A
449	Sigarra	N/A
450	Dertosa	Tortosa
451	Suestatium	Arcaya (Arkaia)
452	Tullica	N/A
453	Veleia	Iruña de Oca
454	Gebala	N/A
455	Gabalaeca	N/A
456	Tullonium	Alegría
457	Alba	N/A
458	Segontia Paramica	N/A
459	Tritium Tuboricum	N/A
460	Thabuca	N/A
461	Iturissa	Ateabalsa
462	Pompaelo	Pamplona
463	Bituris	N/A
464	Andelus	Muruzábal de Andión
465	Nemanturista	N/A
466	Curnonium	N/A
467	Iacca	Jaca
468	Gracurris	Alfaro
469	Calagurris	Calahorra
470	Cascantum	Cascante
471	Ergavica	N/A

Continued on next page

Spain – continued from previous page

#	Latin name	Modern name
472	Tarraga	N/A
473	Muscaria	N/A
474	Segia	Ejea de los Caballeros
475	Alavona	N/A
476	Bergusia	N/A
477	Celsa	Velilla del Ebro
478	Bergidum	N/A
479	Erga	N/A
480	Succosa	N/A
481	Osca	Huesca
482	Burtina	Almudévar
483	Gallica Flavia	N/A
484	Orcia	N/A
485	Ilerda	Lleida/Lerida
486	Iulia Libica	Llívia
487	Aquae Calidae	N/A
488	Ausa	Vic
489	Baecula	N/A
490	Gerunda	Girona
491	Sebendunum	N/A
492	Bassi	N/A
493	Egosa	N/A
494	Beseda	N/A
495	Aeso	Isona
496	Udura	N/A
497	Ascerris	N/A
498	Setelsis	Solsona
499	Telobis	N/A
500	Ceresus	N/A
501	Bacasis	N/A
502	Iessus	Guissona
503	Anabis	N/A
504	Cinna	N/A
505	Deciana	la Jonquera
506	Iuncaria	near Figueres
507	Rubricata	N/A

Continued on next page

Spain – continued from previous page

#	Latin name	Modern name
508	Trileuci Scopuli	N/A
509	Cassiterides	N/A
510	Islands of the Gods	N/A
511	Ophiussa	Formentera
512	Ebusus (City)	Ibiza
513	Palma	Palma (de Mallorca)
514	Pollentia	Alcúdia (de Polença)
515	Iamo	Ciutadella (de Menorca)
516	Mago	Mahón/Maó

Place names on Ptolemy's map of Britain.

#	Latin name	Modern name
1	Novantum Peninsula	Mull of Galloway
2	Rerigonium Bay	Loch Ryan
3	Vindogara Bay	Irvine Bay
4	Clota Estuary	Firth of Clyde
5	Lemannonium Bay	Loch Fyne
6	Cape Edidium	Mull of Kintyre
7	R. Longus	N/A
8	R. Itys	N/A
9	Bay of Volsas	N/A
10	R. Nabarus	River Naver
11	Cape Tarvedum (Orcas)	Dunnet Head
12	Novantum Peninsula	Mull of Galloway
13	R. Abravannus	N/A
14	Iena Estuary	N/A
15	R. Deva	River Dee
16	R. Novius	River Nith
17	Ituna Estuary	Solway Firth
18	Moricambe Estuary	Morecambe Bay
19	Setantium Harbour	N/A
20	R. Belisama	River Ribble
21	R. Seteia	River Mersey
22	R. Toesobis	N/A

Continued on next page

Britain – continued from previous page

#	Latin name	Modern name
23	Cape of the Ganganes	Braich-y-Pwll
24	R. Stuccia	River Ystwyth
25	R. Tuerobis	N/A
26	Cape Octapitarum	St David's Head
27	R. Tobius	River Tywi
28	R. Ratostathybius	River Taff
29	Sabrina Estuary	River Severn
30	Uxella Estuary	Bridgewater Bay
31	Cape of Herakles	Hartland Point
32	Cape Antivestaeum (Bolerium)	Land's End
33	Cape Damnonium (Ocrium)	Lizard Point
34	R. Cenion	N/A
35	R. Tamarus	River Tamar
36	R. Isca	River Exe
37	R. Alaunus	N/A
38	Great Harbour	Spithead
39	R. Trisanton	River Arun
40	New Harbour	Newhaven
41	Cape Cantium	South Foreland
42	Cape Virvedrum	Duncansby Head
43	Cape Verubium	Noss Head
44	R. Ila	N/A
45	Lofty Height	Tarbat Ness
46	R. Loxa	N/A
47	Varar Estuary	N/A
48	Tuesis Estuary	Spey Bay
49	R. Celnius	River Deveron
50	Cape of the Taizales	Kinnairds Head
51	R. Deva	River Dee
52	Tava Estuary	Firth of Tay
53	R. Tina	N/A
54	Boderia Estuary	Firth of Forth
55	R. Alaunus	River Aln
56	R. Vedra	River Wear
57	Dunum Bay	Tees Bay
58	Cape with Good Harbours	Bridlington Bay

Continued on next page

Britain – continued from previous page

#	Latin name	Modern name
59	Cape of Ocelus	Spurn Head
60	R. Abus	The Humber
61	Metaris Estuary	The Wash
62	R. Gariennus	River Yare
63	Promontory	N/A
64	R. Sidumanis	N/A
65	Tamesa Estuary	Thames Estuary
66	Cape Cantium	South Foreland
67	Lucopibia	N/A
68	Rerigonium	N/A
69	Carbantorigum	N/A
70	Uxellum	N/A
71	Corda	N/A
72	Trimontium	Newstead
73	Colania	Camelon
74	Vindogara	N/A
75	Coria	N/A
76	Alauna	N/A
77	Lindum	N/A
78	Victoria	N/A
79	Curia	N/A
80	(Alauna)	N/A
81	Bremenium	High Rochester
82	Banatia	N/A
83	Tamia	N/A
84	Pinnata Castra*	N/A
85	Tuesis	N/A
86	Orrea	N/A
87	Devana	N/A
88	Epiacum	Whitley Castle
89	Vinovia	Binchester
90	Cataractonium*	Catterick*
91	Calatum	N/A
92	Isurium	Aldborough
93	Rigodunum	N/A
94	Olicana	Elslack

Continued on next page

Britain – continued from previous page

#	Latin name	Modern name
95	Eboracum	York
96	Camulodunum	N/A
97	Petuaria	Brough-on-Humber
98	Mediolanum	Whitchurch
99	Brannogenium	Leintwardine
100	Deva	Chester
101	Viroconium	Wroxeter
102	Lindum	Lincoln
103	Ratae	Leicester
104	Salinae	N/A
105	Verulamium	St Albans
106	Venta	Caistor-by-Norwich (St Edmund)
107	Camulodunum	Colchester
108	Luentinum	Pumsaint
109	Maridunum	Carmarthen
110	Bullaeum	N/A
111	Corinium	Cirenchester
112	Calleva	Silchester
113	Londinium*	London*
114	Durovernum	Canterbury
115	Rutupiae	Richborough
116	Noviomagus	Chichester
117	Iscalis	N/A
118	Aquae Calidae	Bath
119	Venta	Winchester
120	Dunium	N/A
121	Voliba	N/A
122	Uxella	N/A
123	Tamare	N/A
124	Isca	Exeter
125	Legio II Augusta	N/A
126	Scitis	Isle of Skye
127	Dumna	Isle of Lewis*
128	Orkades	Orkney Islands
129	W. Thule	N/A
130	E. Thule	N/A

Continued on next page

Britain – continued from previous page

#	**Latin name**	**Modern name**
131	N. Thule	N/A
132	S. Thule	N/A
133	Thule*	N/A
134	Tanatis	Isle of Thanet
135	Counnus	N/A
136	Vectis*	Isle Wight*

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