How to Kill 999 Flowers: In Defence of Logical Monism

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Abstract

How many correct logics are there? For much of logic's history it was widely assumed that there was *exactly one* correct logic, a position known as *logical monism*. However, the monist's hegemony has recently become increasingly precarious as she has simultaneously come under attack from two sides. On one side she faces *logical pluralists* who contend that there is *more than one* correct logic, and on the other she faces *logical nihilists* who contend that there are *no* correct logics. This thesis aims to defend monism against the twin threats of pluralism and nihilism.

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Contents

Notation

$A, B \ldots$	Natural language sentences
P	Set of natural language sentences
$\varphi, \psi \ldots$	Formal language sentences
Γ	Set of formal language sentences
$\ulcorner \ldots \urcorner$	Name of a sentence
$\neg$	Negation
$\wedge$	Conjunction
$\vee$	Disjunction
$\supset$	Material conditional
$\leftrightarrow$	Material equivalence
$\exists$	Existential quantifier
$\forall$	Universal quantifier
$=$	Identity
$\bot$	Absurdity
$\rightarrow$	Relevant conditional
$\models$	Model-theoretic consequence
$\vdash$	Proof-theoretic consequence
$\{\ldots\}$	Set
$\langle \ldots \rangle$	Ordered tuple
$\in$	Set membership
$\varnothing$	Empty set
$<$	Less than
$\leq$	Less than or equal to
$>$	Greater than
$\geq$	Greater than or equal to
$\mathscr{L}_i$	Logic i

Introduction

How many correct logics are there? For much of logic's history it was widely assumed that there was *exactly one* correct logic, a position known as *logical monism*. However, the monist's hegemony has recently become increasingly precarious as she has simultaneously come under attack from two sides. On one side she faces *logical pluralists* who contend that there is *more than one* correct logic, and on the other she faces *logical nihilists* who contend that there are *no* correct logics.

The aim of the present work is to defend monism against the twin threats of pluralism and nihilism. A number of papers defending pluralism have florally-themed titles, such as Andrea Sereni and Maria Paola Sforza Fogliani's *How to Water a Thousand Flowers: On the Logic of Logical Pluralism* and Roy Cook's *Let a Thousand Flowers Bloom: A Tour of Logical Pluralism*. In keeping with this theme, a defence of logical monism can therefore be depicted as killing off all but one of the pluralist's flowers, the remaining flower being the one true logic. From the outset it is important to note that I shall not be taking a stance on *which* of the pluralist's flowers remains – that is, on *which* logic is the one true logic – and will content myself with trying to justify the weaker thesis that there is but one true logic, whatever that may be. The rest of this introduction clarifies what it is for a logic to be correct, before mapping out the conceptual terrain and giving a preview of what is to come.

1 CORRECTNESS

The question which the nihilist, monist, and pluralist disagree over concerns the number of correct logics. But what is a logic, and what is it for one to be correct?

A logic, $\mathscr{L}_i$, is a mathematical structure comprised of a formal language and a consequence relation defined between sentences of that language. The formal language contains sets of logical and non-logical symbols that form the basic vocabulary, along with a set of rules that recursively define which strings of symbols qualify as grammatical sentences. $\mathscr{L}_i$'s consequence relation may be given model-theoretically as $\models_{\mathscr{L}_i}$, using cases in which formal sentences can be true along with a set of clauses that recursively assign truth-values to grammatical sentences within a case. More specifically, $\Gamma \models_{\mathscr{L}_i} \varphi$ iff φ is true in every case in which Γ is true. Alternatively, $\mathscr{L}_i$'s consequence relation can be given proof-theoretically as $\vdash_{\mathscr{L}_i}$ using a set of axioms and rules that determine which sentences can be proven from others. $\Gamma \vdash_{\mathscr{L}_i} \varphi$ iff there is a sequence of sentences in the formal language whose last line is φ, and in which every previous line is either an axiom, a member of Γ, or follows from previous lines via a rule.

So defined, there are uncountably many logics, ranging from propositional and first-order classical, intuitionistic, and relevant logics to higher-order, many-valued, and quantum logics. But what does it mean for one of these structures to be correct? The standard view is that, in much the same way that a theory of the atom is correct iff it correctly represents atoms, so too a logic is correct iff it correctly represents something, namely, the *logical consequence relation proper* that holds between *natural language* sentences.[1] To make sense of this, more must be said about the logical consequence relation and what it might be for a mathematical structure to correctly represent it. Before we do so, however, it is crucial to note that this view of correctness not only has historical precedent (e.g. Tarski, 1956a), but when the thorny issue of what it is for a logic to be correct is raised, nihilists (e.g. Cotnoir, 2018, pp. 301–302) and pluralists (e.g. Cook,

[1]For present purposes we can stay neutral on whether the relata of the logical consequence relation are interpreted sentences, propositions, or anything else, and I use 'sentences' merely as a placeholder. However, in §1.4 we will consider a view according to which there are different consequence relations depending on what the relata are.

2010, p. 495; Haack, 1978, p. 222; Shapiro, 2014, p. 7 fn. 1) alike have endorsed it. Thus, in adopting this definition, we are not beginning from a position that unfairly favours monism over its competitors.

The logical consequence relation is such that a sentence – the argument's conclusion – is a logical consequence of a (potentially empty) set of sentences – the argument's premisses – iff the argument is *necessarily truth-preserving solely in virtue of its logical form*.[2] For an argument to be necessarily truth-preserving, it must be *impossible* for the premisses to be true but the conclusion false. And for it to be necessarily truth-preserving in virtue of its logical form, its truth-preservingness must be due to the premisses' and conclusion's syntactic and semantic structure as well as the positions and meanings of the so-called *logical terms* (MacFarlane, 2017) – terms such as, 'and', 'or', 'not', 'if', 'some', and 'all'.[3]

Necessary truth-preservation in virtue of form is to be contrasted with necessary truth-preservation in virtue of *content*, where 'content' refers to the arguments' non-logical terms' content. For instance, the argument 'Platypuses lay eggs. Therefore: Some mammals lay eggs' is necessarily truth-preserving, not in virtue of its form but in virtue of the contents of the non-logical terms 'platypuses' and 'mammals'. This is illustrated by the fact that substituting 'Americans' for 'mammals' yields a non-truth-preserving argument of the same form as the original. By contrast, the argument, 'If Hypatia is a woman, then Hypatia is mortal. Hypatia is a woman. Therefore: Hypatia is mortal' is necessarily truth-preserving in virtue of its logical form because, irrespective of which terms we substitute for the non-logical terms 'Hypatia' and 'is a woman', the resulting argument is necessarily truth-preserving.[4]

So far we have said what a logic is, what the logical consequence relation is, and that a logic, $\mathscr{L}_i$, is correct iff it correctly represents the logical consequence relation. But for this correctness criterion to be informative something more must be said about what it is to correctly

[2]As per usual, an argument is valid iff its conclusion is a logical consequence of its premisses.

[3]Precisely which terms qualify as logical is a vexed matter, and one that we can set aside until we encounter pluralisms which claim different logics are correct depending on where the logical–non-logical distinction is drawn – see §1.3.

[4]That is, provided that they belong to the appropriate syntactic category.

represent the logical consequence relation. A natural suggestion is that, in this context, correct representation is a matter of correspondence between a logic's own consequence relation and the logical consequence relation proper. Suppose that we were able to translate natural language sentences into $\mathscr{L}_i$'s formal language. Then, for any natural language argument, we can obtain its 'formal counterpart' by translating its premisses and conclusion into the formal language. We can then say that $\mathscr{L}_i$ correctly represents the logical consequence relation proper iff *all* and *only* valid natural language arguments have formal counterparts that are $\mathscr{L}_i$-valid.

Following Cook (2010, p. 495), we can formally implement this idea by introducing a translation function, $\mathscr{T}$, which maps natural language sentences to their counterparts in the formal language. Letting *P* be a set of natural language sentences and *C* be a single natural language sentence, we can then define the following correctness criterion:

Correct: $\mathscr{L}_i$ is correct iff, for any *P*, *C*: *C* is a logical consequence of *P* iff $\mathscr{T}(C)$ is a $\mathscr{L}_i$-consequence of $\mathscr{T}(P)$.

Sometimes I will say that correct logics validate all and only valid natural language arguments, or that they correctly codify the logical consequence relation, but these slogans all express the same thought. Now that we know what it is for a logic to be correct, we are in a position to map out the possible positions one can take when it comes to questions concerning logics being correct.

2 MAPPING OUT THE TERRAIN

When mapping out the terrain, it is important to highlight that nihilists, monists, and pluralists all agree that logics are the kinds of things that can be correct, even if they disagree over how many are. As such, they are united in their opposition to *logical instrumentalism*, the thesis that logics are only more or less *useful* for accomplishing certain aims, and are not 'correctness-apt' (e.g. Rescher, 1969, Ch. 3). Whilst nihilists, monists, and pluralists can agree with the instrumentalist that logics can be more of less useful for certain purposes, it is the latter part of the instrumentalist's thesis that they take issue with.

We can therefore depict the terrain as follows:[5]

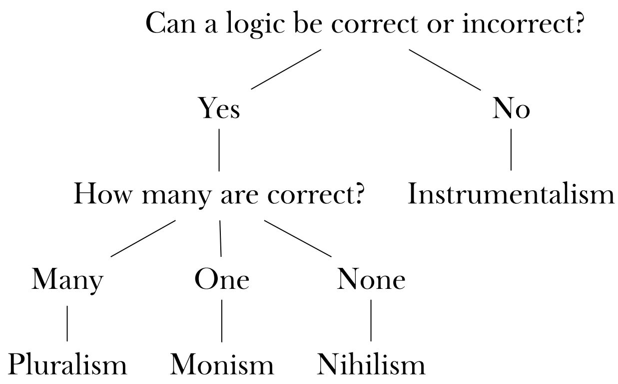

Figure 1

Insofar as the present work seeks to address the question of how many logics are correct, it is predicated upon logics being correctness-apt and instrumentalism being false. Although offering substantive arguments against instrumentalism would take us too far afield, some brief remarks are in order. The instrumentalist's thesis that logics are not correctness-apt commits them to two further claims: that the logical consequence relation proper is empty, and that logics are not in the business of trying to represent any such relation. After all, if the consequence relation were non-empty and some sentences really were logical consequences of others, then it is unclear what grounds there could be for thinking that logics do not aim to represent this relation. Conversely, if the consequence relation proper were empty but logics were trying to represent it then, in much the same way that all theories of phlogiston are incorrect, these logics are incorrect and nihilism follows, not instrumentalism.

As a result, instrumentalists are susceptible to two lines of criticism. First, one may simply object that the logical consequence relation is non-empty because, for instance, it is impossible for a conjunction to be true but its conjuncts false in virtue of their logical form. This is certainly the orthodox view and those who accept it need not give instrumentalism any further thought. However, for those who are unorthodox, during the discussion of logical nihilism I provide an

[5]The following diagram is based on Haack (1978, p. 225).

extended argument that the logical consequence relation proper is non-empty. Accordingly, those tempted by instrumentalism can read this argument as also ruling out instrumentalism.

Second, as Susan Haack notes, the history of logic seems to indicate that logics *do* aim to represent the logical consequence relation:

> It is clear enough from the history of formal logic (consider Aristotle, for instance, or Frege) that the motivation for the construction of formal systems has been, on the basis of an initial conception of some arguments as good and others as bad, to sort out logical from other, e.g. rhetorical, features of good arguments, and to give rules which would permit only the logically good arguments and exclude the bad (1978, p. 227).

Thus, given that logics aim to represent the logical consequence relation proper, if this relation were empty, nihilism ensues not instrumentalism. Having gestured towards some reasons for setting instrumentalism aside, we can now focus on introducing the *dramatis personae* in further detail: the nihilists, the pluralists, and the monists.

At one extreme we encounter the nihilists, who claim that there are *no* correct logics. Given *Correct*, this is to say that there is no logic which validates every valid natural language argument. Accordingly, there are two straightforward ways to be a nihilist. One can either show that for any logic, $\mathscr{L}_i$, there are invalid natural language arguments that have $\mathscr{L}_i$-valid formal counterparts, or that there are valid natural language arguments lacking $\mathscr{L}_i$-valid formal counterparts. As we shall see later on, there are two prevalent strains of nihilism found in the literature. One takes the first route by arguing that there are no valid arguments and so all logics are incorrect as they validate invalid natural language arguments. The other takes the second route by claiming that there are valid arguments that cannot have formal counterparts due to the inherent limitations of formal languages.

At the other extreme we happen upon the pluralists, who maintain that there are multiple logics satisfying *Correct*. At first glance this may seem strange: how could there be multiple logics, all of which correctly codify the logical consequence relation, and yet they be distinct? If there were only a single unchanging logical consequence relation this worry would be well taken. However, the pluralists' contention is precisely that this is not so. Rather, following Shapiro (2014, Chs. 1–2), pluralists can be understood as claiming that the consequence relation's extension varies with some independent parameter, such as which cases the consequence relation quantifies over or where the boundary between logical and non-logical terms is drawn. Consequently, as the 'value' taken by the independent parameter changes, the logical consequence relation's extension varies and so too does which logic satisfies *Correct*. Consequently, there are multiple correct logics. As we shall see, different pluralisms can then be obtained depending on the parameter relativised to and which logics are correct relative to the different values that this parameter can take.

Finally, sandwiched inbetween the nihilists and the pluralists, we find the monists who claim that there is exactly one true logic, although they invariably disagree over which logic this is. Given *Correct*, this is to say that there is just one logic, $\mathscr{L}_i$, such that all and only valid natural language arguments have $\mathscr{L}_i$-valid formal counterparts. A slight qualification is required here. Monism, as it is usually understood, is not only the claim that there is one logic satisfying *Correct*, but also that this logic has a non-empty consequence relation. Otherwise we risk classifying as a monist a nihilist who claims that the logical consequence relation proper is empty and so the one true logic has an empty consequence relation. Accordingly, in keeping in current usage, I reserve the term 'logics' exclusively for those structures with non-empty consequence relations.

In virtue of being surrounded on both sides, monists face an unenviable challenge when defending their position. On the one hand, when defending themselves against pluralists, the justification that they give for rejecting the existence of multiple correct logics must not simultaneously undermine their claim that there is a correct logic. On the other hand, when defending themselves against nihilists, the justification that they give for there being at least one correct logic must not commit them to there being multiple correct logics.[6] Despite these difficulties, the present work endeavours to show that monism remains defensible.

[6]This way of framing the monist's predicament is inspired by Shelly Kagan's (1989) *The Limits of Morality*.

3 LOOKING AHEAD

Notice that monism is entailed by the negations of pluralism and nihilism, and so one way of defending monism is to defend the following two theses:

Upper Bound: There is at most one correct logic.

Lower Bound: There is at least one correct logic.

This observation informs the structure of the present work. Part I defends monism from pluralism by defending *Upper Bound*. Chapter 1 introduces pluralism more thoroughly as well as the plethora of pluralisms presently on offer, whilst chapters 2 and 3 construct the main argument in favour of *Upper Bound*. Chapter 2 argues that logic is *doubly* normative for reasoning. That is, logic not only constrains the combinations of beliefs that agents may have, as is commonly supposed, but it also constrains the methods by which agents may form them – a point that has hitherto gone unacknowledged. Chapter 3 then builds on this by developing the normative contradiction argument which demonstrates that, given logic is doubly normative for reasoning, a wide range of pluralisms are untenable because they entail logically contradictory claims about how agents ought to reason. Chapter 4 concludes Part I by arguing that the pluralisms which proved immune to the normative contradiction argument collapse into monism.

Part II completes the defence of monism by defending *Lower Bound* from the logical nihilists. For there to be at least one correct logic, two further claims must be true. First, the *non-emptiness claim* that the logical consequence relation proper is non-empty – that is, there must be valid arguments. Second, the *existence claim* that there actually is a logic which correctly codifies this non-empty relation. Recently, both claims have come under fire from nihilists. Chapter 5 defends the non-emptiness claim against Gillian Russell's nihilism, whilst Chapter 6 defends the existence claim from Aaron Cotnoir's nihilism. With *Lower Bound* in place the defence of monism is complete because, together with *Upper Bound*, it entails that there is one true logic.

Part I

Logical Pluralism

1

A Plethora of Pluralisms

Logical pluralists claim that there is more than one correct logic, and this chapter introduces logical pluralism more thoroughly before surveying the different varieties of pluralism on offer. Building on Shapiro's (2014) classification of logical pluralisms as different forms of logical relativism, §1.1 identifies two key components that lie at the heart of any logical pluralism and how different pluralisms can be obtained by varying them. The remainder of the chapter then puts this template to work as the existing pluralisms are introduced.

1 LOGICAL PLURALISM AND LOGICAL RELATIVISM

There are a plethora of species of logical pluralism and, following Shapiro (2014, Chs. 1–2), they can be classified as different forms of *logical relativism*. In general, and somewhat loosely, to be a relativist about some domain is to say that the things belonging to this domain – be they objects, events, actions, or anything else – do not instantiate certain properties *simpliciter* but only relative to some parameter, and therefore claims about this domain are only true or false relative to this parameter (Shapiro, 2014, p. 7). For instance, relativists about etiquette maintain that which actions are polite or rude is relative to parameters such as culture and time, so whilst it is neither true nor false *simpliciter* that it is rude to give someone an even number of flowers, it is true relative to present-day Russia but false relative to 18th century Britain.

According to the logical relativist, whether a natural language argument is valid is relative to some parameter, $\mathscr{P}$. Since a logic, $\mathscr{L}_i$, is correct just in case *all* and *only* valid natural language arguments have $\mathscr{L}_i$-valid formal counterparts, whether $\mathscr{L}_i$ is correct is also relative to $\mathscr{P}$. As a toy example, suppose that a natural language argument, A, is valid when $\mathscr{P}$ takes value p_1, invalid when it takes value p_2, and the validity of all other arguments remains unchanged between p_1 and p_2. If logics $\mathscr{L}_1$ and $\mathscr{L}_2$ are such that A's formal counterpart is $\mathscr{L}_1$-valid but $\mathscr{L}_2$-invalid, and both logics get the validity of all the other arguments right, then relative to p_1 $\mathscr{L}_1$ is correct but $\mathscr{L}_2$ is incorrect, and *vice versa* relative to p_2. Provided that at least two different logics are correct relative to at least two values of $\mathscr{P}$ – or else at least two different logics are both correct relative to one value of $\mathscr{P}$ – the resulting position is a logical pluralism.[1]

On this picture, the core of any logical pluralism is comprised of two parts, either of which can be varied to obtain different pluralisms. First, the parameter that logics are correct relative to, $\mathscr{P}$. As we will see, most of the pluralisms presently on offer relativise logical consequence to different parameters and can be classified accordingly. Second, a relation which maps each of the values that $\mathscr{P}$ can take to the logic(s) that are correct relative to them. Let us call this second component a pluralism's *specification relation*. Accordingly, to ascertain which is the correct logic for a particular argument being considered by an agent, one only needs to identify which value $\mathscr{P}$ takes in that context, and which logic is correct relative to that value.

For a pluralism to be plausible both components require support. In the first instance, we need to be told why arguments' validity should depend upon $\mathscr{P}$. And once we are told this, something must be said about why its specification relation is plausible – that is, why certain logics are correct relative to some values of $\mathscr{P}$ but incorrect relative to others. With this template in place, we are now in a position to introduce the different pluralisms in a systematic manner. For each pluralism, I begin by stating the parameter to which each relativises logical consequence and why one might think this to be so, before discussing the pluralism's specification relation. Since many pluralisms relativise logical consequence to parameters that can take

[1] Notice that logical pluralisms being understood species of logical relativism is quite compatible with the formality of logic. All that formality requires is that, given some value of $\mathscr{P}$, if some argument is valid then so too is every argument of the same form.

a large number of potential values, I only discuss a few of the values the parameter can take.

2 CASE-RELATIVE PLURALISM

Beall & Restall (2000, 2006) argue that logics are correct relative to *cases*, where cases are "'things' in which claims may be true" (2006, p. 89).[2] According to Beall & Restall, our concept of validity – or, equivalently, logical consequence – is vague but its core is given by the *Generalised Tarski Thesis* (*GTT*):

> *GTT*: "An argument is valid$_x$ if and only if, in every case$_x$ in which the premises are true, so is the conclusion" (2006, p. 29).

This core can then be precisified in different ways by varying which cases *GTT* quantifies over, thereby yielding different relations some of which are logical consequence relations. Since different logical consequence relations can be obtained by varying the cases that *GTT* quantifies over, it follows that logical consequence is relative cases.

Notice that not just any old precisification of *GTT* quantifying over any old set of cases yields a logical consequence relation. Rather, the relation is one of logical consequence just in case it is *necessary*, *formal*, and *normative* (2006, p. 35). That is, for any argument whose premisses and conclusion stand in this relation, it must be the case that: (i) it is impossible for the premisses to be true but the conclusion false (2006, pp. 14–16); (ii) one goes wrong if one believes the premisses but disbelieves the conclusion (2006, pp. 16–18); and (iii) the premisses and conclusion of every argument of the same logical form also stand in this relation (2006, pp. 18–23). For instance, take a precisification of *GTT* which only quantifies over cases in which it is true that 'Trump has solved Curry's Paradox'. The relationship so defined will hold between any sentence whatsoever and 'Trump has solved Curry's Paradox'. Thus, this relation cannot be logical as it is not normative: there is nothing amiss with one who believes that the Australians lost the Great Emu War but disbelieves that Trump has solved Curry's Paradox.

[2] Here we focus on the model-theoretic version of their pluralism. See Restall (2014) for the proof-theoretic form.

Turning to the specification relation, which kinds of cases yield logical consequence relations, and which logics are correct relative to these different relations? Beall & Restall argue that there are at least three precisifications of *GTT* that are necessary, normative, and formal, and therefore conclude that "there is more than one relation of logical consequence" (2006, p. 25). In particular, the precisifications of *GTT* quantifying over complete and consistent Tarskian models, over potentially incomplete but consistent constructions, and over potentially incomplete and inconsistent situations yield three different logical consequence relations. These three relations are correctly codified by classical, intuitionistic, and a relevant logic, respectively.

3 DISTINCTION-RELATIVE PLURALISM

Building on Tarski (1956a), Varzi (2002) contends that logics are correct relative to *where the distinction between logical and non-logical terms is drawn*. According to the standard model-theoretic account of logical consequence, an argument is valid iff it is necessarily truth-preserving in virtue of its logical form and the meanings of the logical terms. Typically, the natural language logical terms are taken to be 'and', 'or', 'all' and the like, but Varzi argues that there is no uniquely correct place to draw this distinction. In fact, "*every* term can in principle be treated as a logical term or a non-logical term" (2002, pp. 198–199). Consequently, one can obtain different logical consequence relations – or, as Varzi calls them, 'different senses of validity' – by varying where the distinction between logical and non-logical terms is drawn. In turn, different logics are correct depending on where the distinction is drawn.[3]

To get an idea of what Varzi's pluralism's specification relation might look like, consider the following example. We might draw the logical–non-logical distinction such that 'and', 'or', 'not', and 'if' are the only logical terms. Or we might draw it so that 'Everything' and 'Something' are also logical terms. Relative to the first distinction, 'Something is fluffy' is not a logical consequence of 'Guinness is fluffy', but relative to the second it is. Accordingly, a propositional

[3]It is unclear whether Varzi himself would agree with this characterisation as he talks only of *relativism* and makes no mention of correctness. However, I follow Shapiro (2014, pp. 49–57) in interpreting his and Tarski's position in this way. Moreover, Varzi's pluralism also has a Carnapian component which allows the meanings of logical terms to vary, but here we shall focus only on the 'Tarskian fragment' of his pluralism.

logic will be correct relative to the first distinction but a first-order logic relative to the second. Similar remarks apply for modal, epistemic, deontic logics and so on when expressions such as 'Possible', 'Knows that', and 'Ought to', are counted as logical terms.

4 TRUTHBEARER-RELATIVE PLURALISM

Russell (2008) maintains that logics are correct relative to the *kind of truthbearers* that arguments' premisses and conclusions are taken to be. Consider the argument, 'Water is water. Therefore: Water is H_2O'. Suppose that the premiss and conclusion are standard propositions – that is, sets of possible worlds – and are therefore identical since both are necessary. Given that the latter is not a logical consequence of the former, when the logical consequence relation's relata are standard propositions it cannot be *reflexive*.[4] But if the relata are taken to be hyperintensional propositions, the premiss and conclusion express different propositions and the resulting consequence relation may well be reflexive.[5]

Accordingly, this pluralism's specification relation will be such that when the truthbearers are standard propositions the correct logic will be one with a non-reflexive consequence relation, but when the truthbearers are hyperintensional propositions the correct logic's consequence relation might be reflexive. Russell also considers other truth-bearers – such as interpreted sentences, Kaplanian characters, and statements – and argues that each has their own logic:

> Is there room for, say, a logic of propositions? Or a logic of statements? If what is important for validity is that in all cases where the premises are true, the conclusion is true, then it makes sense to talk of validity for *any* kind of truth-bearer. Though it may be that we normally only call arguments composed of a certain kind of truth-bearer valid, the notion will be extendable to arguments composed of other things (2008, p. 607).

[4] That is, not every premiss entails itself.
[5] For an account of hyperintensional propositions employing impossible worlds, see Berto & Jago (2019).

5 FRAMEWORK-RELATIVE PLURALISMS

Kouri Kissel (2018) builds upon the pluralism espoused by Rudolf Carnap (1959) by arguing that logics are correct relative to *linguistic frameworks*. A linguistic framework is a vocabulary coupled with a syntax and a set of rules specifying which inferences are permissible. The framework itself is chosen on pragmatic grounds and, in particular, whether adopting the framework is conducive to achieving our theoretical aims, whatever those might be.[6] Naturally, the correct logic relative to a linguistic framework is determined by which inferences are permissible within it, and this is encoded by the specification relation in the usual way. For instance, if *ex falso quodlibet*, double negation elimination, disjunctive syllogism, and so on are permitted in a linguistic framework, then the correct logic relative to this framework is classical.

Although Kouri Kissel endorses this Carnapian picture, there is one point of difference concerning the meanings of logical terms. Carnap maintained that the logical terms have different meanings in different logics since they assign different truth-conditions and/or proof rules to said terms:

> [L]et any postulates and any rules of inference be chosen arbitrarily; then this choice, whatever it may be, will determine what meaning is to be assigned to the fundamental logical symbols (Carnap, 1959, p. xv).

By contrast, Kouri Kissel allows that "there are some contexts in which distinct logics have logical terms that are synonymous" (2018, p. 578). Consider a context where a proponent of a classical analysis system and a proponent of an intuitionistic analysis system are discussing that they can both prove the same theorem. Kouri Kissel argues that since both participants are talking about the same fundamental theorem, a negation featuring in the statement of the theorem must have the same meanings even though classical and intuitionistic negations have different truth-conditions.

[6]One might wonder whether this blurs the line between pluralism and instrumentalism insofar as the rules of inference specified in a framework are chosen on purely pragmatic grounds. Although I am sympathetic to this, I shall treat it as a pluralism in accordance with its proponent's intentions.

6 AIM-RELATIVE PLURALISMS

Blake-Turner & Russell (2018) and Field (2009a) argue that logics are correct relative to agents' *epistemic aims*. Focussing on Blake-Turner & Russell's account, from a set of premisses an agent may aim to draw true, true and demonstrable, or true and relevant conclusions. Whether a conclusion is a logical consequence of these premisses then depends on which of these aims the agent has. For instance, if their aim is to draw true and demonstrable conclusions, then '$\sqrt{2}$ is irrational' is not a logical consequence of 'It is not the case that $\sqrt{2}$ is rational'; but if their aim was merely to draw true conclusions, then it would be.

The specification relation for Blake-Turner & Russell's pluralism is as follows. When an agent's aim is to draw true conclusions, the correct logic is classical; when their aim is to draw true and demonstrable conclusions, the correct logic is intuitionistic; and when their aim is to draw true and relevant conclusions, the correct logic is a relevant logic (Blake-Turner & Russell, 2018, pp. 16–17). Something similar holds for Field's pluralism, although this is not to say that their pluralisms are identical – whereas Field thinks that logic is inherently normative for reasoning, Blake-Turner & Russell deny this.[7]

7 STRUCTURE-RELATIVE PLURALISM

Shapiro (2014) maintains that logics are correct relative to *mathematical structures* or *theories*. The rationale for this begins with a Hilbertian approach to mathematics on which "any consistent axiomatisation characterises a structure, something at least potentially worthy of mathematical study" (2014, p. 65). Of course, consistency is logic relative, and Shapiro argues that:

> [T]here are a number of interesting and important mathematical theories that employ a non-classical logic, and are rendered inconsistent if classical logic is imposed. This suggests logical consequence is relative to a theory or a structure (2014, p. 3).

[7] Insofar as the rationale for whether a logic is correct relative to an aim seems to turn on whether it is conducive to accomplishing said aim, one might wonder whether these pluralisms would be better described as instrumentalisms. Whilst I am inclined towards this line of thought, I shall continue to treat them as pluralisms.

That is, a logic is correct relative to a mathematical theory if, at the very least, it renders the theory consistent.[8]

One important implication of relativising logical consequence to mathematical structures is that it means abandoning the formality of logical consequence (Shapiro, 2014, p. 96). Roughly, logical consequence is said to be formal in the sense that, if an argument's conclusion follows logically from its premisses, then the same holds true of all arguments of the same logical form irrespective of their premisses' subject-matter. However, if different consequence relations hold between sentences concerning different theories, then one argument might be valid but another invalid despite sharing the same form.

To see how Shapiro's pluralism's specification relation works, let us consider a couple of mathematical theories, Peano arithmetic and smooth infinitesimal analysis (*SIA*). Peano's axioms of arithmetic define the natural numbers and their arithmetical properties. Following Gentzen's consistency proof, it is widely accepted that Peano's axioms are consistent in first-order classical logic, and therefore classical logic is correct relative to Peano arithmetic.[9] By contrast, *SIA* is a theory of infinitesimals – that is, non-zero numbers which are closer to zero than any real number – which is intuitionistically consistent but classically inconsistent. To see this, let a *nilsquare* be a number, *r*, such that $r^2 = 0$. Within classical systems zero is the only nilsquare, but in *SIA* zero is not the only nilsquare – that is:

$$\neg\forall r(r^2 = 0 \rightarrow r = 0)$$

Yet, at the same time, there are no nilsquares distinct from zero in *SIA*:

$$\forall r(r^2 = 0 \rightarrow \neg(r \neq 0))$$

[8]Given that a legitimate theory, T, being $\mathscr{L}_i$-inconsistent is *sufficient* for $\mathscr{L}_i$ to be incorrect relative to T, Shapiro is committed to $\mathscr{L}_i$-consistency being at least a *necessary* condition for $\mathscr{L}_i$ to be correct relative to T. Is $\mathscr{L}_i$-consistency also a *sufficient* condition? Shapiro is equivocal on this point, and we return to it in Chapter 3.

[9]Of course, Gödel's second incompleteness theorem demonstrates that the consistency of Peano arithmetic cannot be proved in Peano arithmetic itself (or any weaker system). However, Gentzen proves the consistency of Peano arithmetic using a system which is weaker than Peano arithmetic in some aspects but stronger in others. See Smith (2013, Ch. 32) for a brief overview.

These two claims are not intuitionistically inconsistent since $\neg(r \neq 0) \nvdash_I r = 0$. However, as soon as double-negation elimination is added contradiction quickly ensues, and so *SIA* is classically inconsistent. Within the Hilbertian tradition, if *SIA* is a legitimate mathematical theory as Shapiro (2014, pp. 74–75) maintains, then it must be consistent. Accordingly, Shapiro's pluralism's specification relation is such that intuitionistic logic is correct relative to *SIA* but classical logic is not, though the latter is correct relative to Peano arithmetic.

8 DOMAIN-RELATIVE PLURALISMS

Domain-relative pluralists such as Bueno & Shalkowski (2009), Lynch (2009), and Pedersen (2014) contend that logics are correct relative to *domains of inquiry*. Domains are individuated according to their subject-matters, and common examples include the domains of mathematics, macro-physical objects, and morality. Although different domain-relative pluralists carve up the world into different domains, the basic idea is that these domains differ radically and so the consequence relations holding between sentences about these domains differ. Since domain-relative pluralists maintain that there are argument forms which are valid when their premisses belong to one domain but invalid when they belong to another, domain-relative pluralists are united in their rejection of the formality of logical consequence.

Lynch and Pedersen's domain-relative logical pluralism is built atop their domain-relative alethic pluralism, according to which different properties realise truth in different domains.[10] Let us say that truth in a domain is *epistemically constrained* just in case a sentence belonging to that domain is true iff it is *superwarranted*. A sentence, *s*, is superwarranted iff believing *s* is warranted and remains warranted irrespective what additional information is acquired in the future (Lynch, 2009, p. 38; Pedersen, 2014, pp. 263–264).

According to Lynch and Pedersen, in domains concerned with mind-independent entities, such as electrons and stars, truth is not epistemically constrained but a matter of correspond-

[10] Although their pluralisms are not identical, I will talk as though they are because the differences are immaterial for present purposes.

ing to reality.[11] Suppose, as Pedersen (2014, pp. 273–274) does, that the mind-independent world is complete: for any mind-independent object o and mind-independent property F, it is determinate whether o is F or not. Resultantly, sentences about this domain like 'o is F' satisfy the law of excluded middle, and so the consequence relation holding between them is such that the law of excluded middle and double-negation elimination are valid. By contrast, truth is epistemically constrained in domains concerned with mind-dependent entities, and Pedersen cites the mathematical domain as a prime example. Since there are some sentences, such as Goldbach's Conjecture, such that belief in neither them nor their negations is presently super-warranted, for these sentences the law of excluded middle fails and double-negation elimination is invalid (Pedersen, 2014, p. 266). Thus, Lynch and Pedersen's pluralism's specification relation maps mind-independent domains to classical logic but mind-dependent domains to intuitionistic logic.

What consequence relation holds between sentences about multiple domains? Take the argument, 'If the Crown Jewels are locked in your safe, then what you did was illegal. The Crown Jewels are locked in your safe. Therefore: What you did was illegal'. Since it concerns both the arrangement of mind-independent physical objects and the legal status of certain actions, it straddles the mind-independent and mind-dependent domains. This is the so-called *problem of mixed inferences*, and Lynch's (2009, pp. 99–102) general solution is as follows. The logical consequence relation holding between sentences about different domains is the weakest of the consequence relations holding between sentences in the individual domains concerned. And if there is no uniquely weakest consequence relation, then the consequence relation is the intersection of the consequence relations that hold in the individual domains – that is, the argument forms that are valid when reasoning across multiple domains are those which are valid in all the domains in question. Thus, if an argument's premisses belong to domains D_1 and D_2 and logics $\mathscr{L}_1$ and $\mathscr{L}_2$ are correct relative to these domains, then the specification relation maps it to whichever of $\mathscr{L}_1$ and $\mathscr{L}_2$ is weakest, or their intersection.

[11] An entity exists mind-independently iff it would continue to exist even if there were no minds, and mind-dependently otherwise (Lynch, 2009, p. 33; Pedersen, 2014, pp. 272–273).

By contrast, Bueno & Shalkowski (2009) carve the world up into three domains whose subject-matters are the *character of the world*, *claims made about the world*, and *epistemic states*. Sentences about this first domain make claims about how the world is or could be. Given Bueno & Shalkowski's assumption that "the world is both consistent and complete" (2009, p. 309), the consequence relation that obtains between sentences about this domain satisfies the laws of excluded middle and non-contradiction (in addition to all the usual suspects), and so the correct logic in this domain is classical.

But sometimes we reason not about the world itself but about claims made about the world. And, unlike the world, claims made about the world need not be either consistent or complete. For instance, an outdated database of Australian fauna might include the following inconsistent entries: 'Duck-billed platypuses are mammals', 'Duck-billed platypuses lay eggs', and 'No mammals lay eggs'. To avoid triviality, the logical consequence relation holding between sentences about this domain cannot be explosive, and so the correct logic must be paraconsistent:

> [T]o reason about these inconsistent claims. . . it needs to be possible for the inconsistent claims not to trivialize our whole database. In other words, it makes perfect sense, in this case, to adopt a logic in which the inconsistency of the claims under consideration does not entail everything. It makes sense to use a paraconsistent logic (2009, p. 312).

Lastly, in the domain whose subject-matter is *agents' epistemic states*, the consequence relation will be one in which double-negation elimination and the law of excluded middle fail:

> At most stages of the baseball season one might, in quite ordinary contexts, say: 'I am not certain that the Yankees will not win the World Series'. Quite clearly, suppose that a hearer used a simple-minded version of the classically sanctioned double negation elimination to infer that the speaker has, effectively, claimed to be certain that the Yankees would, indeed, win the World Series. The hearer would be guilty of not only conversational but also inferential error (2009, p. 315).

Thus, whilst Bueno & Shalkowski's pluralism's specification relation maps the domains of the character of the world and claims made about the character of the world to classical and paraconsistent logics, respectively, it maps the domain of epistemic states to intuitionistic logics. Finally, although Bueno & Shalkowski do not address the issue of reasoning across domains, nothing precludes them from adopting Lynch's approach.

9 MEANING-RELATIVE PLURALISM

Haack (1978) maintains that logics are correct relative to the *meanings of logical terms*. Logical terms are formal representations of their natural language counterparts, and focus on some of their counterparts' features whilst abstracting from others. This abstraction makes room for there to be multiple correct logics. Although different logics give logical terms different meanings by giving them different truth-conditions and/or proof rules, they may provide equally correct formalisations by perspicuously representing different aspects of their natural language counterparts:

> Formalisation involves a certain abstraction from what are taken to be irrelevant or unimportant features of informal discourse... And this leaves scope for, so to speak, alternative formal projections of the same informal discourse; i.e. scope for the idea that, for instance, material implication, strict implication, relevant implication, and other formal conditionals might all have some claim to represent some aspect of 'if' (1978, p. 230)

Naturally, focussing on different aspects of natural language logical terms' meanings yields different consequence relations relative to which different logics are correct. For instance, on Haack's account, both the material conditional and relevant implication represent different aspects of the meaning of 'If', and relative to the former classical logic is correct but relative to the latter a relevant logic is correct.

CONCLUSION

This chapter argued that at the core of any pluralism lies two components: the independent parameter to which logical consequence is relativised, and the specification relation which relates each value the parameter can take to the logics which are correct relative to that value. This template was then used to systematically introduce the plethora of pluralism found in the literature.

2

A Double Dose of Normativity

A time-honoured tradition has it that logic is normative for reasoning. In Kant's words, "In Logic we do not want to know how the understanding is and thinks, and how it has hitherto proceeded in thinking, but how it ought to proceed in thinking" (Kant, 1885, p. 4). This chapter argues that logic is *doubly* normative for reasoning as it constrains both the *combinations of beliefs* that we may have *and* the *methods* by which we may form them. In doing so, it lays the foundation for my central argument in favour of the upper bound thesis that there is at most one correct logic – namely, that most pluralisms are inconsistent given that logic is doubly normative for reasoning.

We begin in §2.1 by arguing for the following *Incompleteness Thesis*:

Incompleteness Thesis: The prevailing conception of the normativity of logic is incomplete because it overlooks the fact that, in addition to constraining the *combinations of beliefs* that we may have, logic also constrains the *methods* by which we may form them.

I then going on to explain why, following the seminal work of Gilbert Harman (1984, 1986), the normativity of logic for reasoning in either sense turns on whether there are true *bridge principles*. §2.2 recapitulates and builds upon the contributions of John MacFarlane (2004) and Florian Steinberger (2019a, 2019c) to articulate *validity bridge principles* capturing the normativity of logic for belief combinations. Finally, in §2.3 we enter unchartered waters and begin the task of formulating *invalidity bridge principles* that capture the normativity of logic for the methods by which we form our beliefs.

1 THE INCOMPLETENESS THESIS

To motivate the claim that logic is doubly normative for reasoning, we begin with two examples of reasoning gone awry:[1]

Suky: Suky correctly believes that there is a cat on the mat and that there is a dog on the log, but fails to believe their conjunction.

Max: Max is curious about how the Greek tragedian Aeschylus died. He reasons: if an eagle dropped a tortoise on Aeschylus's head then Aeschylus is dead; Aeschylus is dead, so an eagle dropped a tortoise on his head.

Both Suky's and Max's reasoning contain logical errors, albeit of different kinds. Suky's reasoning is problematic because of the combination of beliefs that she has: she believes two conjuncts but fails to believe the conjunction that they entail. By contrast, Max's fault lies not in his combination of beliefs – indeed, they are all true – but in the method by which he formed his belief, namely, via the invalid deductive inference of affirming the consequent.

The existence of these two kinds of errors indicate that logic is doubly normative for reasoning. First, logic constrains the *combinations of beliefs* that agents may have. At the very least, one should not disbelieve the logical consequences of one's belief set and, in cases like Suky's, one should either believe said consequences or else revise one's belief set. Second, logic constrains the *methods* by which agents may form beliefs: one should not, as Max does, form beliefs via invalid deductive inferences. Whereas the former has been widely recognised by the literature on the normativity of logic, the latter has been ignored entirely.

In light of the work of Harman (1984, 1986), the claim that logic is normative for reasoning in either sense faces a significant obstacle. Harman's central insight was that deductive logic and

[1] As usual, reasoning is the process of forming and revising our doxastic attitudes via inference (Harman, 1984, p. 107).

a *theory of reasoning* are not one and the same, for they have entirely distinct subject-matters. Whereas deductive logic concerns logical facts delineating static and non-psychological relations between sentences, a theory of reasoning consists of normative constraints governing how agents ought to form and revise their beliefs (Harman, 1984, p. 107; 1986, pp. 3–4). For instance, *modus ponens* – the logical fact that, 'If A then B' and 'A' entail 'B' makes no mention of beliefs: it does not say that if one believes 'If A then B' and 'A' then one ought to believe 'B'. Thus, logic cannot be normative for reasoning simply because logical facts are *themselves* constraints on reasoning.

Disabused of the illusion that logical facts and normative constraints on reasoning are identical, a gap opens up between the two. For logic to be normative for reasoning in either way, there must be true *bridge principles* – conditionals whose antecedents are logical facts and whose consequents are the normative constraints on belief combinations or methods that these facts induce (MacFarlane, 2004). Crucially, insofar as the existing literature has been exclusively preoccupied with bridge principles linking logical facts to constraints on belief combinations (e.g. Field, 2009b; Harman, 1986; MacFarlane, 2004; Milne, 2009; Steinberger, 2019a, 2019c; Streumer, 2007), the prevailing conception of the normativity of logic neglects its normativity for methods of belief formation. Hence the *Incompleteness Thesis*. In the next two sections we explore the bridge principles that together capture both dimensions of logic's normativity.

2 VALIDITY BRIDGE PRINCIPLES

We begin with the bridge principles that relate logical facts to constraints on belief combinations. Broadly speaking, there are two kinds of logical facts: *validity* facts of the form $\Gamma \models \varphi$, and *invalidity* facts of the form $\Gamma \not\models \psi$. As such, our first task is to establish which kind of logical fact induces constraints on the combinations of beliefs agents may have.

The constraint that Suky violates by believing both conjuncts but failing to believe their conjunction is grounded in the fact that the conjuncts *necessitate* the truth of their conjunction. As such, this constraint and other constraints on belief combinations must be induced by a validity fact since, given $\Gamma \not\models \psi$ alone, Γ necessitates the truth of neither ψ nor $\neg\psi$. Let us call bridge principles relating validity facts to constraints on belief combinations *validity bridge principles*. Thus, to say that logic is normative for belief combinations is to say that there is a true validity bridge principle. The task of this section is to formulate such a principle. §§2.2.1–2.2.2 recapitulate the existing literature on validity bridge principles, focussing in particular upon the contributions of John MacFarlane and Florian Steinberger. §§2.2.3–2.2.4 then proceed to build upon these contributions to articulate validity bridge principles capturing the normativity of logic for the combinations of beliefs we may have.

2.1 *The MacFarlane Three-Step*

We begin with the seminal work of John MacFarlane (2004). The importance of MacFarlane's contribution resides in his methodology for addressing the question of whether there are any true validity bridge principles. Broadly speaking, MacFarlane proceeds in three stages. First, he formulates a blueprint for validity bridge principles which renders explicit the 'dimensions' across which they can vary, thereby enabling the space of all the possible permutations to be mapped out. Second, MacFarlane identifies the set of criteria that any true validity bridge principle must satisfy. Finally, these criteria are applied to the various validity bridge principles and conclusions are drawn about whether any are true.

Following Steinberger (2019a, p. 312), MacFarlane's blueprint for validity bridge principles can be generalised as:

VBP$_{BP}$: If $\delta(\Gamma \models \varphi)$, then $D(\alpha(\Gamma), \beta(\varphi))$.

The antecedent of *VBP* states a validity fact or an agent's doxastic attitude towards such a fact, where δ is a variable denoting said attitude that is empty when the antecedent is factual. *VBP*'s consequent is a normative constraint on the agent's doxastic attitudes, where D is a deontic operator, and α and β are the potentially distinct attitudes being constrained. The relation between the two is that the antecedent is a triggering condition specifying when agents are bound by the normative constraint given in the consequent.

Different validity bridge principles can then be generated by varying the values taken by *VBP*'s parameters:

1. δ: Does the validity bridge principle have a factual antecedent in which δ is empty, or an attitudinal antecedent where δ denotes a doxastic attitude? That is, are agents subject to the normative constraints induced by the validity facts which actually obtain, or those that they think obtain?[2]
2. α, β: Does the normative constraint govern agents' beliefs or degrees of belief?
3. *D*: What kind of deontic operator does the normative constraint employ? Is it the indefeasible 'ought to' operator, the defeasible 'has reason to' operator, or the even weaker 'is permitted to' operator?
4. *D's Scope*: The normative constraint in *VBP*'s consequent is typically a conditional of the form $\alpha(\Gamma) \supset \beta(\varphi)$. What scope does the deontic operator take with respect to this conditional? Does it embed into both the antecedent and consequent, just the consequent, or does it take wide scope over the entire conditional?
5. *Polarity*: Does the normative constraint in the validity bridge principle's consequent positively require that agents hold certain doxastic attitudes, or does it negatively preclude them from holding specific doxastic attitudes?

All in all, the variation of these parameters generates a space of seventy-two distinct validity bridge principles. With the first step of MacFarlane's method complete, we move to the second stage of articulating the adequacy criteria. MacFarlane (2004, pp. 9–12) endorses the following seven criteria.[3]

(1) *Belief Revision*. A true validity bridge principle must allow agents to revise their belief set when it entails an absurdity rather than requiring that they believe the absurdity. It thereby precludes a true validity bridge principle's deontic operator from embedding into the normative

[2] I use 'think' as a placeholder for any of the doxastic attitudes to which δ may refer.

[3] Criteria (1)–(4) are found in Harman (1986, Ch. 2) and MacFarlane (2004, p. 12) introduces (5)–(7).

constraint's consequent, as the constraint would then require that subject *s* believe an absurd logical consequence of their belief set rather than allowing them to revise their initial beliefs.

(2) *Excessive Demands*. A true validity bridge principle cannot require cognitively limited agents to believe all the logical consequences of their beliefs, since the deduction of many of these consequences will be extremely complex and lies beyond their cognitive reach. Thus, *Excessive Demands* requires validity bridge principles' antecedents to be attitudinal since this restricts the normative constraints to which an agent is subject to those induced by the validity facts that they think obtain.

(3) *Clutter Avoidance*. The content of any given belief has infinitely many consequences, many of which are infinitely long disjunctions that the agent has no reason to consider. Thus, a true validity bridge principle cannot require that agents believe *all* the logical consequences of their beliefs as this would clutter their finite cognitive capacity. This can be avoided by supplementing *VBP*'s antecedent with the clause 'and *s* has reason to consider φ' so that agents are only subject to the normative constraints induced by relevant validity facts.

(4) *Priority Question*. A true validity bridge principle must respect the fact that the normative constraints induced by validity facts constrain our reasoning irrespective of whether we are aware of them. According to MacFarlane, saying that we are only subject to the validity facts that we think obtain get matters the wrong way round and has the unwanted consequence that:

> The more ignorant we are of what follows logically from what, the freer we are to believe whatever we please – however logically incoherent it is. But this looks backwards. We seek logical knowledge so that we will know how we ought to revise our beliefs: not just how we *will* be obligated to revise them when we acquire this logical knowledge, but how we are obligated to revise them even now, in our state of ignorance (MacFarlane, 2004, p. 12).

Since bridge principles' antecedents delineate when agents are subject to the normative constraints on reasoning given in their consequents, *Priority Question* requires that their antecedents be factual.

(5) *Preface Paradox*. In D. C. Makinson's (1965) Preface Paradox, an author has meticulously researched every sentence in a non-fiction book she has written and therefore, given her evidence, is rationally required to believe that each sentence is true. However, since she recognises that there is strong inductive evidence for her fallibility, she is also rationally required to believe that at least one sentence is false. Given the author is rationally required to believe both that each sentence in her book is true and that at least one is false, a true validity bridge principle should not *require* that she forgo either belief, despite their inconsistency. Thus, to satisfy *Preface Paradox*, a validity bridge principle cannot employ the indefeasible 'ought to' operator.

(6) *Strictness*. A true validity bridge principle must indefeasibly require that agents either believe the straightforward consequences of their belief sets or else forgo at least one of their existing beliefs. For instance, if s believes φ and ψ, s is indefeasibly required to either believe $\varphi \wedge \psi$, or else jettison their belief in at least one of the conjuncts. Therefore, *Strictness* demands that validity bridge principles employ the indefeasible 'ought to' operator.

(7) *Obtuseness*. The normative constraint on reasoning given in a true validity bridge principle's consequent must not merely forbid an agent who believes φ and ψ from disbelieving $\varphi \wedge \psi$, but must positively require that they believe $\varphi \wedge \psi$.

With the adequacy criteria formulated, we turn to step three of MacFarlane's method: ascertaining whether there are any validity bridge principles satisfying all the adequacy criteria. Each criterion precludes at least one of a validity bridge principle's parameters from taking certain values – for instance, *Strictness* precludes a validity bridge principle's deontic operator from taking any value other than 'ought to'. As such, each criterion partitions the space of validity bridge principles into two cells comprised of those validity bridge principles which satisfy it and those which do not. We may think of the application of a criterion to the space of validity bridge principles as eliminating the portion of the space coinciding with the latter cell. The question of whether there are any true validity bridge principles can therefore be understood as follows: once all the adequacy criteria have been applied to the space of validity bridge principles, which, if any, remain uneliminated?

It is here that MacFarlane comes unstuck. Once all the criteria have been applied, none remain: there are no validity bridge principles satisfying all the criteria (MacFarlane, 2004, pp. 12–13). Why is this? If two adequacy criteria conflict – that is, they demand that a single parameter takes two different values – then the intersection of the sets of validity bridge principles satisfying them both is empty. Consequently, any conflicting adequacy criteria will together eliminate the entirety of the space of validity bridge principles as inadequate. Thus, we have the following necessary condition for a space of bridge principles, *S*, to contain an adequate member:

*Conflict-Free**: None of the adequacy criteria pertaining to *S* conflict.

Yet, despite MacFarlane's adequacy criteria being fairly weak, there are two separate conflicts amongst them. First, between *Excessive Demands* and *Priority Question*: whereas *Excessive Demands* requires validity bridge principles to have attitudinal antecedents, *Priority Question* demands that their antecedents be factual. Second, between *Strictness* and *Preface Paradox*: *Strictness* favours principles featuring the indefeasible 'ought to' operator but *Preface Paradox* pulls in the opposite direction. What to do?

2.2 *Steinberger's Tripartite Distinction*

This is where the contributions made by Florian Steinberger (2019a, 2019c) enter the fray. Steinberger aims to progress the debate by showing that bridge principles themselves are norms, and there are three functionally distinct kinds of norm they may be:

(I) *Evaluations*. An evaluative norm states an *objective* and *ideal* standard of correctness. It is *objective* since it is the standard of correctness that actually obtains as opposed to that which is thought to obtain by a particular agent. And it is *ideal* because it abstracts away from various limitations that those subject to it may have.

(II) *Directives*. A directive norm is a principle that guides ordinary agents' actions.

(III) *Appraisals*. An appraisal norm is a standard determining when ordinary agents' conduct is worthy of praise or criticism (Steinberger, 2019a, p. 316; Steinberger, 2019c, p. 16).

To see this distinction in action and why a given principle may be plausible when construed as one kind of norm but not another, consider the following falsity norm for belief:

Falsity: If p is false, then s ought not to believe that p.

Falsity is a plausible *evaluative* norm – after all, if beliefs aim at truth then, from an objective and ideal standpoint, it is incorrect to believe falsehoods. However, it is a deeply implausible *directive* or *appraisal* norm. In the first instance, *Falsity* cannot serve as a directive because it does not provide any guidance in scenarios where there is no way of knowing whether p is true or false, and may direct agents to not believe p even when all their evidence points in the opposite direction. Nor is *Falsity* a plausible appraisal norm because, supposing p is false, it would be unfair to criticise an agent for believing p if they have no way of knowing that p is false or if all the evidence available to them points towards p being true.

This distinction can be equally fruitfully applied to bridge principles. Bridge principles *qua* evaluative norms determine whether our reasoning is correct or incorrect from an objective and ideal standpoint; bridge principles *qua* directive norms provide us with guidance on how to reason; and bridge principles *qua* appraisals determine when our reasoning is reprehensible or praiseworthy as far as logic is concerned. Accordingly, there is not one space of validity bridge principles but three, each comprised of the same set of principles but differentiated by the normative functions these principles perform. Due to space limitations, the remainder of this chapter focusses solely upon evaluative and directive bridge principles.

Crucially, different sets of adequacy criteria apply to each of these three spaces. As Steinberger (2019c, p. 23) puts it, "*Different normative roles invite different criteria of adequacy*". The hope, then, is that disambiguating the three spaces will enable *Conflict-Free** to be met for each of them, since some of the criteria that were previously thought to conflict do not apply to the same spaces. And this is exactly what happens.

Belief Revision is relevant to both spaces of validity bridge principles (Steinberger, 2019c, p. 24). If one's belief set entails an absurdity then, from an objective and ideal standpoint, it is incorrect to believe said absurdity rather than revising one's initial belief set. Moreover, agents should not be directed to believe the absurdity. Both *Excessive Demands* and *Clutter Avoidance* are irrelevant to evaluative bridge principles but relevant to their directive counterparts (Steinberger, 2019c, pp. 23–24). Insofar as evaluative validity bridge principles are in the business of stating *ideal* standards for correct reasoning, they abstract from our cognitive capacities and therefore worries concerning their demandingness gain no purchase. By contrast, for a norm to function as a directive and guide ordinary agents' reasoning, it cannot direct agents to do that of which they are incapable, such as believing *all* their belief sets' logical consequences.

Priority Question only applies to the space of evaluative validity bridge principles (Steinberger, 2019c, p. 23). The reason for this is that *Priority Question* requires validity bridge principles to capture what logic demands of us as opposed to what we think it demands of us. And whilst this is appropriate for evaluative validity bridge principles which state objective standards for reasoning, it is inappropriate for directive validity bridge principles which, to guide agents' reasoning, must be sensitive to which validity facts they think obtain.

Preface Paradox applies to the space of directive validity bridge principles but not their evaluative counterparts (Steinberger, 2019c, pp. 24–27). Given that the author has fastidiously researched each sentence in her book, she is rationally required to believe that each individual sentence is true. Moreover, given the author has strong inductive evidence for her own fallibility, she is rationally required to believe that she has made at least one error. Accordingly, a true directive validity bridge principle should not direct her to forgo either belief given that she is rationally required by other non-logical doxastic norms to believe both. However, *Preface Paradox* is far less compelling as adequacy criterion for evaluative validity bridge principles. The Preface Paradox only arises because we are non-ideal reasoners who lack the cognitive resources to ascertain which of these beliefs is misplaced. However, an evaluative validity bridge principle is in the business of stating an objective and ideal standard for correct reasoning and, from an objective and ideal standpoint, it surely is incorrect for the author to have inconsistent beliefs, even if other norms require that she does so.

Strictness and *Obtuseness* both pertain to the spaces of evaluative and directive validity bridge principles (Steinberger, 2019b, pp. 25–26). Recall Suky who believes that there is a cat on the mat and that there is a dog on the log. From an objective and ideal standpoint, there is something wrong with Suky's reasoning if, whilst retaining her wholehearted beliefs in the conjuncts, she were to either disbelieve their conjunction due to countervailing reasons or fail to believe their conjunction. Similarly, if Suky recognises that her beliefs entail that there is a cat on the mat and a dog on the log, a directive validity bridge principle should direct her to either believe the conjunction or else forgo her belief in at least one of the conjuncts – even if she has a countervailing reasons against believing the conjunction.

Steinberger's findings for evaluative and directive validity bridge principles are summarised in the table below. Recall that MacFarlane came unstuck because there were two conflicts amongst his adequacy criteria – between *Excessive Demands* and *Priority Question*, and between *Preface Paradox* and *Strictness* – which eliminated the entirety of the space of validity bridge principles as inadequate. Distinguishing between the spaces of evaluative and directive validity bridge principles relieves much of this tension as it transpires that, for the most part, these conflicting adequacy criteria do not apply to the same spaces.

Criterion	Evaluations	Directives
Belief Revision	✓	✓
Excessive Demands	✗	✓
Clutter Avoidance	✗	✓
Priority Question	✓	✗
Preface Paradox	✗	✓
Strictness	✓	✓
Obtuseness	✓	✓

In the next two subsections these findings are used to identify true evaluative and directive validity bridge principles; we begin with evaluations.

2.3 *Evaluative Validity Bridge Principles*

Although Steinberger never applies his findings in search of a true evaluative validity bridge principle, it is straightforward to see that his distinction clears the way for us to do so because the space of evaluative validity bridge principles now satisfies *Conflict-Free**. Both the conflict between *Excessive Demands* and *Priority Question* and that between *Preface Paradox* and *Strictness* have been resolved since neither *Excessive Demands* nor *Preface Paradox* pertain to this space. Thus, all that remains is to identify which permutations of MacFarlane's blueprint satisfy the criteria to which the space of evaluative validity bridge principles is subject – namely, *Priority Question*, *Strictness*, *Belief Revision*, and *Obtuseness*.

In §2.2.1 we saw that different validity bridge principles can be obtained by varying the five parameters in MacFarlane's blueprint: whether their antecedent is attitudinal or factual, the kind deontic operator employed and its scope, the polarity of the normative constraint, and whether the normative constraint governs agents' beliefs or degrees of belief. We also saw how each of the adequacy criteria constrains which values these parameters could take in a true validity bridge principle. Now we know which adequacy criteria pertain to the space of evaluative validity bridge principles, we can straightforwardly identify which permutations of MacFarlane's blueprint can serve as evaluative validity bridge principles.

By *Priority Question*, a true evaluative validity bridge principle's antecedent must be *factual* because objective standards are those induced by the logical facts that actually obtain as opposed to those that we think obtain. *Strictness* requires the deontic operator to be the *indefeasible* 'ought to' operator, whilst *Belief Revision* dictates that this operator must take *wide scope* over the bridge principle's consequent so as to allow agents to revise their beliefs when they entail absurdities. Finally, *Obtuseness* ensures that the normative constraint has a *positive polarity* – that is, it must not merely forbid agents from disbelieving their belief sets' logical consequences, but must positively require that they believe these consequences (or else revise their initial belief set).

The value of one parameter is not settled by these adequacy criteria: whether the normative constraint governs agents' outright beliefs or their credences. Supposing that logic induces constraints on our beliefs, the foregoing observations yield the following evaluative validity bridge principle:

$VBP_{E,B}$: If $\Gamma \models \varphi$ then $O(Bs\Gamma \supset Bs\varphi)$.

That is, from an objective and ideal standpoint, if $\Gamma \models \varphi$ then *s* ought to see to it that if they believe Γ then they believe φ.[4] However, credence-based validity bridge principles have also been proposed. For instance, letting $\Gamma = \{\gamma_1, \ldots, \gamma_n\}$ and $Cr(\gamma_1)$ be an agent's credence in γ_1, Field (2009b, p. 255) proposes:

$VBP_{E,C}$: If $\Gamma \models \varphi$ then $Cr(\varphi) \geq Cr(\gamma_1 \wedge \ldots \wedge \gamma_n)$.[5]

This asserts that, from an objective and ideal standpoint, if $\Gamma \models \varphi$ then one's credence in the conclusion, φ, should be at least as great as one's credence in the conjunction of the premisses, $\gamma_1 \wedge \ldots \wedge \gamma_n$.

Which should we opt for? Does logic induce constraints governing our beliefs or our credences? This is a false dichotomy: nothing precludes logic from inducing constraints on *both*. Indeed, as we have seen above, the adequacy criteria pertaining to the space of evaluative validity bridge principles leave this option open. That said, it is *$VBP_{E,B}$* that will form an integral part of the argument against pluralism constructed in the next chapter.[6]

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[4] My use of the locution 'see to it that' bears no relation to STIT logic, and is merely being used to capture the wide scope that the 'ought to' operator has over the conditional.

[5] Actually, Field ends up endorsing a slightly different principle, but I have used this one for ease of exposition as it makes no difference to the present discussion.

[6] Might *$VBP_{E,B}$* and *$VBP_{E,C}$* contradict one another in some cases, thereby forcing us to choose between them? I think not. Given that credences and beliefs are distinct attitudes (even if one is more fundamental than the other), for this worry to materialise there would have to be a connexion between credences and beliefs such that having credences satisfying *$VBP_{E,C}$* commits one to having a belief set inconsistent with that mandated by *$VBP_{E,B}$*. Such a connexion is articulated by the descriptive and normative variants of the *Lockean Thesis* (*LT*) – that *s* (ought to) believe *p* iff their credence in *p* exceeds some threshold, τ. However, there are compelling arguments against both variants. According to Douven (2016), "The vast majority of philosophers take the [Lottery and Preface Paradoxes] to show that the [normative] Lockean Thesis is to be given up" (e.g. Douven & Williamson (2006), Kelp (2017), Lehrer (1990), Maher (1993), Smith (2010a), and Staffel (2017)). For objections to both variants centring on naked statistical evidence, see Buchak (2014), Jackson (2018), Smith (2010b), and Staffel (2016, 2019b). For arguments that the descriptive *LT* fails due to the differing functional roles of beliefs and credences, see Buchak (2014), Fantl & McGrath (2009, Ch. 5), Friedman (2019), Jackson (2019), Ross & Schroeder (2014), and Staffel (2019a). Finally, Weisberg (2020) presents evidence from cognitive psychology indicating that both beliefs and credences are fundamental, and therefore that the descriptive *LT* is false.

Thus, since it satisfies all the relevant adequacy criteria, $VBP_{E,B}$ – henceforth, 'VBP_E' – captures the normativity of logic for the combinations of beliefs we may have from an objective and ideal standpoint. Returning to Suky, VBP_E asserts that, from an objective and ideal standpoint, she has gone wrong because she ought to either believe the conjunction or else rescind her belief in at least one of the conjuncts. We now turn to the logical norms which guide our reasoning.

2.4 *Directive Validity Bridge Principles*

Steinberger (2019a) is exclusively concerned with articulating a true directive validity bridge principle, and therefore this subsection closely follows much of his argument.

As argued in §2.2.2, the space of directive validity bridge principles is subject to the *Belief Revision*, *Excessive Demands*, *Clutter Avoidance*, *Preface Paradox*, *Strictness*, and *Obtuseness* adequacy criteria. Unlike the space of evaluative validity bridge principles, then, the space of directive validity bridge principles does not satisfy *Conflict-Free** since both *Preface Paradox* and *Strictness* pertain to it. Whereas *Strictness* requires the deontic operator to be the indefeasible 'ought to' operator, *Preface Paradox* demands that it be the defeasible 'has reason to' operator so as to avoid the author being required to forgo either her belief that there is a mistake in the book or that each individual sentence is correct.

The way forward here is, again, due to Steinberger (2019a, pp. 323–324). The intuition behind *Strictness*, recall, is that when it comes to obvious consequences of an agent's belief set that they have reason to consider, agents should either modify their belief set or believe the consequences. Rather than jettisoning *Strictness* altogether, Steinberger proposes that we can capture the *Strictness* intuition using the defeasible 'has reason to' operator because it is *quasi-strict* – in some contexts it behaves strictly and in others defeasibly – thereby allowing *Strictness* and *Preface Paradox* to be simultaneously satisfied.

The basic observation with which this account begins is that how we weight reasons for and against doing certain things varies across contexts. For instance, if your Chihuahua-loving boss asks if you like Chihuahuas, tactfulness takes priority over honesty and so the appropriate answer is 'Of course – they're adorable! Do you have any photos?'. However, during a family discussion about what breed of dog to get, honesty takes priority and so the appropriate answer is 'No, they're small and yappy! How about a German Shepherd?'.

Likewise, the weightiness of the reason provided by a reasons-based directive validity bridge principle for either modifying one's initial belief set or believing its consequences is context-variant. This enables such a principle to capture the intuition behind *Strictness* whilst satisfying *Preface Paradox*. In *most* contexts, the reason that this principle provides for either modifying one's initial belief set or believing its consequences is sufficiently weighty to outweigh any reasons one might have for retaining one's belief set but disbelieving one of its straightforward consequences, thereby giving the appearance of indefeasibility and capturing the *Strictness* intuition. However, there are *some* contexts in which the reason provided by the principle for either modifying one's initial belief set or believing its consequences can be outweighed by countervailing reasons. One such context is provided by the Preface Paradox, where the reasons provided by the author's evidence and the demand for epistemic humility outweigh the reason for consistency. Of course, ascertaining precisely when the reason for consistency is outweighed by other reasons is a tricky matter. However, an account of why and when competing reasons outweigh one another lies well beyond the question of whether logic is normative, and therefore need not be settled here.[7]

With *Preface Paradox* and *Strictness* out the way, we are free to apply the remaining adequacy

[7] Steinberger (2019a, pp. 323–324) himself distinguishes between many-premiss and few-premiss contexts, arguing that in few-premiss contexts deductive reasons have lexical priority over other reasons, but may be outweighed in many-premiss contexts. My worry with this is that there seem to be many-premiss contexts in which our obligations are strict. For instance, suppose that, for all n such that $1 \leq n \leq 10,000$, I believe $n < (n+1)$, but I lack beliefs about whether $n < (n+i)$ when $i \geq 2$. One day I discover the 'greater than' relation is transitive which, together with my 9999 '$n < (n+1)$' beliefs, entails $1 < 10,000$. Despite this being a many-premiss context, *Strictness* applies: I am indefeasibly required to believe $1 < 10,000$, rather than merely having a reason to do so. Moreover, there are few-premiss contexts in which *Strictness* does not apply. The classical sceptical paradox only has two premisses, an instance of the closure principle and another stating that one does not know one is not in a sceptical scenario. Yet, as with the Preface Paradox, I am rationally permitted – if not required – to believe both premisses and the negation of the paradoxical conclusion. And therefore, as with the Preface Paradox, a true validity bridge principle should not *require* that I believe the paradoxical conclusion or else forgo my belief in either premiss, though it may plausibly assert that I have reason to do so.

criteria and ascertain which permutations of MacFarlane's blueprint can serve as directive validity bridge principles. As was the case for evaluative validity bridge principles, *Belief Revision* and *Obtuseness* require that the deontic operator takes wide scope over the normative constraint whose polarity is positive. This leaves *Clutter Avoidance* and *Overdemandingness*. A true directive validity bridge principle should not direct us to believe all of the infinitely many consequences of our belief sets – even those which are obvious – since many of these consequences are pointless disjunctions that we have no reason to consider. Thus, its antecedent should be supplemented with the clause, 'and *s* has reason to consider or considers φ'.

Overdemandingness requires that a directive bridge principle does not direct agents to do that of which they are incapable, such as abiding by the normative constraints induced by validity facts lying beyond their cognitive reach. Such worries are avoided by principles that have an attitudinal antecedent, so that agents are only subject to the constraints induced by validity facts which they bear some attitude towards. We must, however, be careful about *which* attitude features in the antecedent. As Steinberger (2019a, pp. 324–325) points out, were the attitude belief then those agents who lack beliefs about which arguments are valid – perhaps because they lack the requisite concepts – would not be directed by logical norms. To avoid this implausible result, the attitude should be one that does not presuppose explicit belief or possession of the requisite concepts. For convenience I follow Steinberger in opting for 'In *s*'s best estimation…'.

Putting all this together, the following directive validity bridge principle satisfies all the relevant adequacy criteria:

VBP$_D$: If in *s*'s best estimation $\Gamma \models \varphi$ and *s* has reasons to consider or considers φ, then $R(Bs\Gamma \supset Bs\varphi)$.

That is, if in *s*'s best estimation $\Gamma \models \varphi$, and *s* has reason to consider φ, or does consider φ, then *s* has reason to see to it that if they believe Γ then they believe φ. Returning to Suky, *VBP*$_D$ directs her to either believe the conjunction of forgo her belief in at least one conjunct unless she has Preface Paradox-like reasons against doing so, which she does not.

Thus, VBP_E and VBP_D together capture the normativity of logic for belief combinations, with the former laying down objective and ideal standards and the latter providing guidance to ordinary agents on how to manage their beliefs. With these principles in place, we now turn to the bridge principles which capture the normativity of logic for the methods by which we may form our beliefs.

3 INVALIDITY BRIDGE PRINCIPLES

We began in §2.1 by highlighting that, in addition to the combinations of beliefs that agents may have, logic also constrains the methods by which they may form them. This was illustrated by Max, whose reasoning was logically erroneous because he formed his belief that an eagle dropped a tortoise on Aeschylus' head by affirming the consequent. Since the problem with Max's reasoning is that it is *invalid*, the constraint he violates must be induced by an invalidity fact of the form $\Gamma \not\models \varphi$. Let us call bridge principles relating invalidity facts to the constraints on methods of belief formation that they induce *invalidity bridge principles*.

This section enters unchartered waters and sets about articulating invalidity bridge principles which capture the normativity of logic for methods of belief formation. To do this, we will employ MacFarlane's method and Steinberger's tripartite distinction. §2.3.1 enacts the first step of MacFarlane's method by articulating a blueprint for invalidity bridge principles. §2.3.2 completes the second step by ascertaining the relevant adequacy criteria and whether they apply to the space of evaluative or directive invalidity bridge principles, or both. §2.3.3 begins the third step by identifying the method of belief formation governed by invalidity bridge principles of any kind. Finally, §§2.3.4–2.3.5 complete the third step by articulating the evaluative and directive invalidity bridge principles satisfying the relevant adequacy criteria, respectively.

3.1 *A Blueprint for Invalidity Bridge Principles*

To articulate a blueprint for invalidity bridge principles we must first say something more precise about the nature of the normative constraints induced by invalidity facts. In particular, *which* methods of belief formation do they govern? A natural answer is that the invalidity fact $\Gamma \not\models \psi$ forbids one from forming the belief that ψ via *deduction* from Γ.[8] Letting M be an appropriate specification of deduction *qua* method of belief formation, this thought yields the following blueprint:

IBP$_{BP}$: If $\delta(\Gamma \not\models \psi)$, then $D\neg(M)$.

That is, if $\Gamma \not\models \psi$, or s thinks $\Gamma \not\models \psi$, then s is forbidden from or has a reason against forming the belief that ψ via method M. Invalidity bridge principles therefore admit of four dimensions of variation:

1. δ: Does the invalidity bridge principle have a factual antecedent in which δ is empty, or an attitudinal antecedent? That is, are agents subject to the normative constraints induced by the invalidity facts which actually obtain, or those that they think obtain?
2. D: Is the deontic operator 'ought to', 'has reason to', or 'is permitted to'?
3. $\neg$ *Scope*: Should $\neg$ take wide or narrow scope with respect to the deontic operator? That is, should an invalidity bridge principle's consequent be $\neg D(M)$, and assert it is not the case that s ought to or has a reason to form the belief that ψ by deduction from Γ? Or should it be $D\neg(M)$, and assert s is forbidden from or has a reason against forming the belief that ψ by deduction from Γ?
4. M: How is the proscribed method of belief formation, deduction, to be specified?

By varying the values that these four parameters take, we can generate the spaces of evaluative and directive invalidity bridge principles – albeit indeterminately insofar as I have not catalogued the number of possible ways of specifying deduction. However, this will suffice for now and we turn to the adequacy criteria relevant to invalidity bridge principles.

[8]The question of whether incorrect deduction should qualify as deduction in the first place is addressed in §2.3.3. For now I use 'deduction' as a placeholder.

3.2 *Adequacy Criteria for Invalidity Bridge Principles*

This subsection identifies the adequacy criteria relevant to invalidity bridge principles and ascertains which of these criteria are relevant to the spaces of evaluative and directive invalidity bridge principles. A natural place at which to begin this task is with the adequacy criteria pertaining to validity bridge principles – though given that validity and invalidity bridge principles induce different kinds of constraints on reasoning, we should expect different sets of adequacy criteria to pertain to them.

Invalidity bridge principles forbid agents from forming beliefs in particular ways and, crucially, do not require agents to have any particular belief combinations. Thus, there is no risk of an invalidity bridge principle committing one to believing absurdities, having a cluttered belief set, contravening the norms giving rise to the Preface Paradox, or to being logically obtuse. Accordingly, none of *Belief Revision*, *Clutter Avoidance*, *Preface Paradox*, and *Obtuseness* are relevant to invalidity bridge principles. This leaves *Excessive Demands*, *Priority Question*, and *Strictness*.

Beginning with *Excessive Demands*, can it be overly demanding to forbid agents from forming beliefs in a particular way? Validity bridge principles could be overly demanding because, to believe the logical consequences of one's belief set one must first ascertain what they are, and many of the requisite deductions are beyond our cognitive abilities. Although deductive proofs of invalidity facts may be as laborious as proofs of validity facts, one need not have ascertained all the propositions not entailed by Γ to not invalidly deduce them from Γ. Nonetheless, invalidity bridge principles might be overly demanding in another way. Depending on how liable humans are to forming beliefs via invalid deduction, it may be overly demanding to require that agents not form the belief that ψ via deduction from Γ when they do not think $\Gamma \not\models \psi$ obtains. I will assume humans are prone to reasoning invalidly, and therefore *Excessive Demands* pertains to invalidity bridge principles.[9]

Priority Question asserts that the normative constraints on reasoning induced by logical facts

[9] I invite those who are more sanguine about our capacity to reason validly to jettison this criterion as it makes no difference to which invalidity bridge principles are endorsed in §§2.3.4–2.3.5.

bind us irrespective of whether we think said facts obtain. Accordingly, this criterion applies equally to invalidity bridge principles: the constraints on methods of belief formation induced by invalidity facts constrain our reasoning irrespective of whether we think these facts obtain. Finally, *Strictness*, which asserts that the normative constraints on reasoning induced by logic are indefeasible, also applies to invalidity bridge principles. One is forbidden from deducing ψ from Γ if one knows that $\Gamma \not\models \psi$ irrespective of any countervailing reasons that one may have. So much for the criteria which applied to validity bridge principles. Are there additional criteria that a true invalidity bridge principle must satisfy?

On the face of it, there is at least one more. Although $\Gamma \not\models \psi$ should induce a normative constraint that forbids agents from forming the belief that ψ via deduction from Γ, this constraint should not forbid agents from forming the belief that ψ via alternative methods – in particular, via non-deductive inference from Γ. For instance, 'The sun will rise tomorrow morning' is not entailed by 'The sun has risen every previous morning', but a true invalidity bridge principle cannot forbid agents from forming a belief in the former via inductive inference from the latter. Let us call this criterion *Alternative Methods*.

Thus, we have four adequacy criteria: *Priority Question*, *Excessive Demands*, *Strictness*, and *Alternative Methods*. Our next task is to clarify which of these criteria apply to the spaces of evaluative and directive invalidity bridge principles. *Priority Question*, *Excessive Demands*, and *Strictness* apply to evaluative and directive invalidity bridge principles just as they did to their validity counterparts, and do so for the same reasons (see §2.2.2). *Priority Question* applies to evaluative but not directive invalidity bridge principles because, although objective and ideal standards of reasoning obtain independently of what we think they are, directive norms must be sensitive to agents' beliefs about which arguments are invalid if they are to provide guidance. The reverse is true of *Excessive Demands* as directives should be sensitive to agents' limitations but evaluative norms should not. *Strictness*, meanwhile, applies to both. From an objective and ideal standpoint, it is wrong to form beliefs via invalid reasoning irrespective of any countervailing reasons that one may have for doing so. Similarly, at least in cases where the agent thinks that an argument is invalid, they should be directed to not form a belief in its conclusion via an invalid deductive inference from its premisses regardless of any reasons that they may have for doing otherwise. *Alternative Methods* also applies to evaluative and directive invalidity bridge principles alike. For supposing that $\Gamma \not\models \psi$ but Γ provides non-deductive support for ψ, it is neither incorrect for *s* to form the belief that ψ via non-deductive inference from Γ, nor should they be directed not to do so.

These findings are summarised in the table below:

Criterion	Evaluations	Directives
Priority Question	✓	✗
Excessive Demands	✗	✓
Strictness	✓	✓
Alternative Methods	✓	✓

This subsection and the one preceding it have identified a blueprint for invalidity bridge principles and the criteria for assessing the adequacy of its permutations, thereby completing the first and second steps of MacFarlane's method. We now embark upon the final step of determining whether there are any permutations of IBP_{BP} satisfying all the relevant adequacy criteria.

3.3 *The Prohibited Method*

Although it is clear that invalidity bridge principles constrain the method of forming beliefs via deduction, IBP_{BP} leaves open the question of how this method should be specified. Since evaluative and directive invalidity bridge principles both constrain deduction, this subsection settles the 'value' taken by M in invalidity bridge principles of both kinds.

A natural way of trying to cash out the idea that we are forbidden from or have reason not to form beliefs via invalid inferences is that agents are forbidden from or have reason not to believe ψ *because* they believe that Γ and $\Gamma \models \psi$. Let $\sqsupset$ denote the 'in virtue of' relation such that $p \sqsupset q$ reads 'In virtue of *p*, *q*', and let B be the binary predicate '…believes that…'. We may then capture this suggestion as follows:

IBP$_\sqsupset$: If $\delta(\Gamma \not\models \psi)$, then $D\neg(Bs(\Gamma \wedge (\Gamma \models \psi)) \sqsupset Bs(\psi))$.

That is, if $\Gamma \not\models \psi$, or *s* thinks $\Gamma \not\models \psi$, then *s* ought not (or has reason not to) believe ψ because they believe Γ and $\Gamma \models \psi$. Permutations of this blueprint run little risk of violating *Alternative Methods* since an agent who comes to believe that the sun will rise tomorrow morning via induction does not do so because they believe that this is entailed by the fact that the sun has risen every previous morning.

This blueprint must, however, be rejected because it cannot form the basis of a true directive norm, as it should be able to. To be a directive norm, its antecedent must be attitudinal, yielding something along the lines of: If $Bs(\Gamma \not\models \psi)$, then $D\neg(Bs(\Gamma \wedge (\Gamma \models \psi)) \sqsupset Bs(\psi))$. However, unless *s* believes or bears some other attitude towards both $\Gamma \not\models \psi$ and $\Gamma \models \psi$, this constraint is vacuous. Since a directive norm should guide our reasoning even when we lack these contradictory beliefs, an alternative specification of deduction must be found.

I endorse the following proposal. Let *F* be the ternary predicate '. . . forms the belief that. . . via method. . . ' and m_Γ be the method of forming a belief via deduction from Γ. We can then update *IBP*$_{BP}$ as follows:

*IBP**$_{BP}$: If $\delta(\Gamma \not\models \psi)$, then $D\neg(Fs\psi m_\Gamma)$.

That is, if $\Gamma \not\models \psi$, or *s* thinks $\Gamma \not\models \psi$, then *s* is forbidden from or has a reason against forming the belief that ψ via deduction from Γ.

The central issue here is how *m*, the method of forming beliefs via deduction, is individuated. Many distinctions between different ways of individuating methods have been drawn in the epistemology literature, but the one of relevance here is between *fallibilist* and *infallibilist* individuations. A method of belief formation is individuated fallibly just in case it is possible to form false beliefs via that method. By contrast, when individuated infallibly, methods are said to be *perfectly reliable* or *success-entailing* since forming beliefs via that method always yields, at the very least, true belief (Brown, 2018, p. 37; Lasonen-Aarnio, 2010, p. 8; Williamson, 2000, p. 182). For instance, perception may be fallibly individuated as having the perceptual experience that p, or infallibly as seeing that p, where seeing that p requires that p causes the perceptual experience. The former individuation is fallibilist because it is possible to form the false belief that p even if one has the perceptual experience that p if, say, one is hallucinating. By contrast, the latter individuation is infallibilist because it is impossible to form the false belief that p by seeing that p because seeing that p requires p to be the case.

Given that valid deduction is necessarily truth-preserving, an individuation of deduction on which *only* valid deductive inferences qualify as tokens of deduction is an infallibilist individuation, since any belief formed via this method is true conditional on the premiss beliefs being true. By contrast, on a fallibilist individuation of deduction, forming beliefs via invalid deductive inferences may qualify as tokens of deduction and so a belief formed via deduction may not be true even when the premiss beliefs are because the agent may have made a mistake and reasoned invalidly.

Crucially, on pain of vacuity, deduction must be individuated *fallibly* in *IBP**$_{BP}$. To see why, suppose that a relevant logic is the one true logic and $\Gamma \not\models_R \psi$. By *IBP**$_{BP}$, the normative constraint induced by this invalidity fact states that agents are forbidden from or have reason not to form the belief that ψ via deduction from Γ. Were deduction individuated *infallibly*, *IBP**$_{BP}$ would be vacuous as it would forbid them from (or provide a reason against) doing the impossible: namely, forming the belief that ψ via relevantly valid inference from Γ. Fortunately, fallibilist method individuation is a mainstream position in contemporary epistemology – endorsed by, *inter alia*, Goldman (1979, 1986), Nozick (1981), and Sosa (1991) – and cogent arguments have been given in its favour.[10]

First, infallibilist method individuation divorces methods' reliability from ordinary agents' cognitive capacities, which should not be the case given epistemology aims to understand how and when ordinary agents have knowledge (Sosa, 1991, Ch. 13). To see how this argument can be wielded against infallibilist individuations of deduction, suppose that Ruth is an impeccable

[10] Even if one thinks that deduction ought to be individuated infallibly, one might still accept an amended version of *IBP**$_{BP}$ on which m denotes a *monotonic* inferential process rather than deduction. Given other inferential processes such as induction and abduction are non-monotonic, this variant would still satisfy *Alternative Methods*.

logician whilst Jacques is deductively inept. On an infallibilist individuation of deduction, none of the instances in which Jacques reasons invalidly qualify as tokens of deduction, and therefore the reliability of forming beliefs via deduction for him is unattenuated by his deductive incompetence. Thus, given an infallibilist individuation, forming beliefs via deduction is an equally reliable method of belief formation for the two of them, despite Ruth's proficiency and Jacques' ineptitude. Since this is "absurd" (1991, p. 234), Sosa concludes methods ought to be individuated fallibly.[11]

A second argument against infallibilist method individuation centres on the phenomenon of *epistemic defeat*, where a belief loses an epistemic status such as justification or knowledge (Lasonen-Aarnio, 2010, p. 1). For instance, suppose that, on the basis of a reliable testimony, I come to know that flamingos are not born pink and only become pink because their diet contains a naturally occurring pink dye. If I later encounter a world-renowned zoologist who mistakenly testifies that this is actually a myth then, assuming that I have no other justification for my belief, it ceases to be knowledge because the zoologist's testimony *defeats* the initial justification. Epistemic defeat has played a central role in epistemology during the post-Gettier era, and is regarded as part of "epistemic orthodoxy" (Brown, 2018, p. 103). Despite this, infallibilist method individuation disallows epistemic defeat as a belief that is initially produced via a perfectly reliable method cannot cease to be justified, even in the presence of defeating evidence (Brown, 2018, pp. 115–177; Lasonen-Aarnio, 2010).

This argument can be straightforwardly converted into an argument against an infallibilist individuation of deduction. Suppose I justifiably and correctly believe a set of propositions which entail that flamingos are not born pink, and form this belief via deduction. As in the previous case, if a world-renowned zoologist subsequently tells me this is wrong, my belief is no longer justified or qualifies as knowledge. However, this verdict cannot be accommodated

[11] Sosa presents a further argument in favour of individuating deduction fallibly, namely, it is possible for a belief to be justified even when produced via an invalid deductive inference: "Take again a logician gifted with excellent deductive powers who goes through a relatively simple proof and somehow fails for once to detect an invalidating flaw. Might he not be justified still in believing the (false) conclusion on the basis of his inference from the (true) premises? And are we not thereby forced to admit the fallibility of deduction?" (1991, p. 232).

if deduction is individuated infallibly. Thus, not only does an infallibilist method individuation sever the tie between agents' deductive aptitude and the reliability of forming beliefs via deduction, it also flies in the face of epistemic orthodoxy.

Accordingly, in what follows, I take m to denote a fallibilist individuation of deduction. Moreover, any fallibilist individuation of deduction employed in epistemology will not count instances of forming beliefs via non-deductive methods as tokens of deduction. Thus, when m is an fallibilist individuation of deduction, IBP^*_{BP} is both substantive and satisfies *Alternative Methods*. I will therefore remain neutral on exactly how deduction is internally individuated, and invite the reader to plug in their preferred fallibilist account of method individuation.[12] Having clarified how the prohibited method of belief formation must be specified, we are finally ready to ascend the summit and ascertain which variants of IBP^*_{BP} can serve as evaluative or directive invalidity bridge principles.

3.4 *Evaluative Invalidity Bridge Principles*

With *Alternative Methods* satisfied, all that remains to be done is to apply the remaining relevant adequacy criteria to the space of evaluative invalidity bridge principles – which, recall from §2.3.2, are *Priority Question* and *Strictness*. In doing so we settle the values that must be taken by IBP^*_{BP}'s remaining parameters – namely, whether its antecedent is factual or attitudinal, its deontic operator, and the scope of the negation with respect to this operator – and thereby identify a true evaluative invalidity bridge principle.

Priority Question favours a factual antecedent in order to reflect the fact that we are subject to the normative constraints induced by invalidity facts irrespective of whether we think these facts obtain. *Strictness*, on the other hand, is doubly relevant to invalidity bridge principles. First, as before, it recommends that the deontic operator be the indefeasible 'ought to' operator (O). Second, *Strictness* favours the negation taking narrow scope with respect to the deontic operator

[12] I favour Sosa's (1991) individuation on which a belief is formed via deduction is one produced by the faculty of deduction. The existence of a distinct deductive faculty is corroborated by recent findings in neuropsychology – see Coetzee & Monti (2018).

so that, given $\Gamma \not\models \psi$, agents are forbidden from deducing ψ from Γ, rather than the normative constraint merely saying that it is not the case that agents ought to deduce ψ from Γ.

Putting all this together yields the following evaluative invalidity bridge principle:

IBP$_E$: If $\Gamma \not\models \psi$, then $O\neg(Fs\psi m_\Gamma)$.

That is, if Γ does not entail ψ, then *s* is forbidden from forming the belief that ψ via deduction from Γ. Recall Max, who formed his belief that an eagle dropped a tortoise on Aeschylus' head by affirming the consequent. *IBP$_E$* states that, from an objective and ideal standpoint, Max's reasoning is incorrect because this belief's content is not entailed by the sentences from which he deduced it.

3.5 *Directive Invalidity Bridge Principles*

With directive invalidity bridge principles, matters are somewhat different. *Priority Question* is irrelevant to directives, whilst *Excessive Demands* joins *Strictness* as the remaining adequacy criteria. Since *Strictness* applies just as it did to evaluative invalidity bridge principles, this leaves *Excessive Demands*.

Excessive Demands and the normative function of directive invalidity bridge principles both dictate that their triggering conditions be cognitively accessible, and therefore that their antecedents be attitudinal. This still leaves open the question of what attitude δ should be. Per the discussion of directive validity bridge principles in §2.2.4, δ must refer to an attitude that does not presuppose explicit belief or possession of the requisite concepts so as to avoid entailing that the logically uninitiated are not subject to logical norms. Again, I follow Steinberger in opting for 'In *s*'s best estimation…'. Thus, the following directive invalidity bridge principle satisfies all the relevant adequacy criteria:

IBP$_D$: If in *s*'s best estimation $\Gamma \not\models \psi$, then $O\neg(Fs\psi m_\Gamma)$.

This asserts that, if in *s*'s best estimation Γ does not entail ψ, then *s* is forbidden from forming the belief that ψ via deduction from Γ. Returning to Max, and assuming that in his best estimation affirming the consequent is invalid, IBP_D directs him not to form the belief that an eagle dropped a tortoise on Aeschylus' head via deduction from 'If an eagle dropped a tortoise on Aeschylus' head, then Aeschylus is dead' and 'Aeschylus is dead'.

CONCLUSION

This chapter began in §2.1 by defending the *Incompleteness Thesis* that the prevailing conception of logic's normativity is incomplete because, in addition to constraining the *combination of beliefs* that we may have, logic also constrains the *methods* by which we may form them. The rest of the chapter then set about articulating the bridge principles which together captured both dimensions of logic's normativity in the evaluative and directive senses. §2.2 identified two validity bridge principles capturing the normativity of logic for belief combinations, VBP_E and VBP_D. §2.3 then broached the hitherto unexplored terrain of invalidity bridge principles, and argued that IBP_E and IBP_D captured the normativity of logic for methods of belief formation. The next chapter uses the fact that logic is doubly normative for reasoning to establish the *Inconsistency Thesis*: almost all the existing logical pluralisms are untenable because they are inconsistent.

3

Double Trouble for Logical Pluralists

This chapter begins the defence of the upper bound thesis that there is *at most* one correct logic. It does so by showing that, given logic is doubly normative for reasoning, almost all of the logical pluralists' proposals for cashing out the claim that there is more than one correct logic entail contradictions. That is, this chapter argues for the following *Inconsistency Thesis*:

Inconsistency Thesis: Given that logic is doubly normative for reasoning, almost all the existing logical pluralisms are inconsistent.

Let us call the argument for this thesis the *normative contradiction argument* (*NCA*). The basic idea behind *NCA* is that, given logic is doubly normative, most pluralisms entail logically contradictory claims about how one ought to reason when one ought to believe some set of propositions, Γ, and φ follows from Γ on one of the pluralist's logics but not another.

Here's the plan. §3.1 reviews two objections to logical pluralism based on the normativity of logic – namely, the upwards collapse objection due to Priest (2006a, p. 203) and Read (2006, pp. 194–195), and Kellen's (2020) early version of *NCA* – and argues that neither succeed. §3.2 introduces a further principle needed to get *NCA* off the ground before formulating *NCA*. §3.3 ascertains which of the pluralisms introduced in Chapter 1 succumb to *NCA*. We finish in §3.4 by considering a number of replies on the pluralists' behalves, and argue that none succeed.

1 PLURALISM AND THE NORMATIVITY OF LOGIC: THE STORY SO FAR

Perhaps the most well-known objection to logical pluralism based on the normativity of logic is the upwards collapse objection propounded by Keefe (2014, pp. 1384–1385), Priest (2006a, p. 203), and Read (2006, pp. 194–195) which goes like this:

> [S]uppose there really are two equally good accounts of deductive validity, K_1 and K_2, that β follows from α according to K_1 but not K_2, and we know that α is true. Is β true?... It follows K_1-ly that β is true, but not K_2-ly. Should we, or should we not conclude that β is true? The answer seems clear: K_1 trumps K_2... K_1 answers a crucial question which K_2 does not (Read, 2006, pp. 194–195).[1]

Since the strongest of the pluralist's logics – if there is a uniquely strongest one – trumps all the others, the pluralist's position collapses upwards to the strongest logic. Notice, however, that none of this requires logic to be normative. The objection begins with the thesis that one of our epistemic goals is to have true beliefs. Since K_1 tells us *as a matter of fact* that β is true whereas K_2 does not, it follows that K_1 is superior to K_2 in advancing this aim. Hence, K_1 trumps K_2, with no bridge principles anywhere to be seen. Thus, although the collapse objection to pluralism may succeed, it does not demonstrate that pluralism is inconsistent with the normativity of logic.[2]

Kellen (2020) provides a different normativity objection to pluralism. Like *NCA*, Kellen's argument aims to establish this conclusion by showing that pluralism gives rise to inconsistent claims about how agents ought to reason (2020, p. 268). However, as we shall now see, his argument is invalid.

Kellen's argument begins with the following validity bridge principle:

[1]Steinberger (2019b) also endorses this criticism, whilst Caret (2017) provides a response.

[2]I am pessimistic about its prospects for success. At most, it demonstrates that the strongest of the pluralist's logics is the most useful for advancing the aim of having true beliefs. But the pluralist's position is not that there are multiple *equally useful* logics. It is that there are multiple *correct* logics, where a logic $\mathscr{L}_i$ is correct iff all and only valid natural language arguments have $\mathscr{L}_i$-valid formal counterparts, and correctness so construed has nothing to do with usefulness.

VBP$_K$: If $\Gamma \models \varphi$, then $Bs\Gamma \supset O(Bs\varphi)$ (2020, p. 267).[3]

That is, if $\Gamma \models \varphi$, then *s* ought to believe φ if they believe Γ. Kellen then has us consider a pluralist who maintains that classical and intuitionistic logic are both correct, and believes $\neg\neg\varphi$. Taking the argument from $\neg\neg\varphi$ to φ, he argues:

> Because the argument is classically valid, and classical logic is normative in our reasoning, she seemingly ought to believe [φ]. On the other hand, her other logic demurs from the validity of the above argument... intuitionistic logic, which again is normative in our reasoning, says she need not believe [φ] (2020, p. 267).

Since it both is and is not the case that *s* ought to believe φ, Kellen concludes that logical pluralism is false. As things stand, however, Kellen's argument is invalid. Granted, given *VBP*$_K$, that *s* ought to believe φ follows from the conjunction of $\neg\neg\varphi \models_C \varphi$ and *s* believes $\neg\neg\varphi$. However, in moving from *VBP*$_K$ and the conjunction of $\neg\neg\varphi \not\models_I \varphi$ and *s* believes $\neg\neg\varphi$ to it is not the case that *s* ought to believe φ, Kellen has denied the antecedent.

The following remedy suggests itself: the missing ingredient is an *invalidity* bridge principle. Since an invalidity bridge principle's antecedent is an invalidity fact, together with $\neg\neg\varphi \not\models_I \varphi$, such a principle *would* entail a normative constraint on the agent's reasoning. Provided this constraint is the contradictory of the constraint entailed by the validity bridge principle and $\neg\neg\varphi \models_C \varphi$, it will have been shown that logical pluralism is inconsistent. Thus, the prospects for a normativity objection to pluralism are dim if logic is only thought to constrain the combinations of beliefs that we may have. However, there is hope for showing that pluralism is inconsistent given that logic is doubly normative for reasoning, as argued in the previous chapter. And this brings us to *NCA*.

[3] That *VBP*$_K$ differs from the validity bridge principles of the previous chapter is unimportant for the ensuing criticism.

2 THE NORMATIVE CONTRADICTION ARGUMENT

NCA aims to demonstrate that a wide variety of pluralisms are inconsistent because, given that logic is doubly normative for reasoning, they entail logically contradictory claims about how agents ought to reason whenever two of their logics conflict over an argument's validity. This contradiction can then be used in a *reductio* against these pluralisms, thereby establishing the *Inconsistency Thesis*. Before we can articulate *NCA*, however, some further details are needed.

2.1 *Which Bridge Principles?*

As stated, *NCA* begins with a validity and an invalidity bridge principle that together capture both dimensions of logic's normativity. But are these the evaluative bridge principles stating which belief combinations and methods of belief formation are permissible from an objective and ideal standpoint? Or the directive bridge principles which guide agents' selection of belief combinations and methods of belief formation?

We will start from the evaluative validity and invalidity bridge principles articulated in the previous chapter. In §2.2.3 I argued that the correct evaluative validity bridge principle is *VBP*$_E$:

VBP$_E$: If $\Gamma \models \varphi$ then $O(Bs\Gamma \supset Bs\varphi)$.

That is, if $\Gamma \models \varphi$ then *s* ought to see to it that if they believe Γ then they believe φ. And in §2.3.4 I argued that the correct evaluative invalidity bridge principle is *IBP*$_E$:

IBP$_E$: If $\Gamma \not\models \psi$, then $O\neg(Fs\psi m_\Gamma)$.

That is, if $\Gamma \not\models \psi$ then *s* is forbidden from forming the belief that ψ via deduction from Γ.

It is worth highlighting at the outset that although *NCA* relies on logic being doubly normative for reasoning, it is somewhat independent of *VBP*$_E$ and *IBP*$_E$ being the correct evaluative validity and invalidity bridge principles. Bridge principles are typically intended to be *unrestricted* – for instance, the constraint in a validity bridge principle's consequent is supposed to hold for *any* Γ, φ such that $\Gamma \models \varphi$. However, for *NCA* to get off the ground, all that is required is for VBP_E and IBP_E to hold for a very narrow range of arguments. This is because *NCA* aims to show that many pluralisms entail contradictions when two of the pluralist's logics conflict over an argument's validity. And since these conflicts can arise for arguments whose conclusions are both of interest to the agent and easily deducible from their premisses – like basic disjunctive syllogisms – VBP_E and IBP_E only need to hold in these 'simple' cases for *NCA* to proceed.

This further increases VBP_E's and IBP_E's plausibility – after all, it is hardly disputable that, from an objective and ideal standpoint, an agent ought to believe the straightforward logical consequences of their belief set that are of interest to them (or else revise their belief set). More importantly, however, it also renders them plausible as directive norms. For instance, the principle problem with VBP_E *qua* directive was that, by requiring that we believe *all* the logical consequences of our beliefs (or else revise them), it is *too demanding*, abiding by it would *clutter* our minds with irrelevancies, and it ran roughshod over the author's obligations in the Preface Paradox. However, these worries are dispelled by restricting VBP_E to 'simple' non-paradoxical cases in which the consequences are limited to those which the agent has reason to consider and which follow straightforwardly from their belief set. So although *NCA* begins with VBP_E and IBP_E, the normative contradiction which *NCA* derives from them manifests itself both at the level of objective and ideal oughts but also at the level of action-guiding oughts.

2.2 *Transmission*

The strategy of showing that pluralism is inconsistent with the normativity of logic by teasing out a contradiction from VBP_E, IBP_E, and pluralism immediately runs into a problem: VBP_E's and IBP_E's consequents are not contradictories. Whereas the normative constraint in VBP_E's consequent says that *s* ought to see to it that if they believe Γ then they believe φ, IBP_E's consequent says that *s* is forbidden from forming the belief that φ via deduction from Γ. To jump this hurdle, we require a principle that somehow links the fact that one ought to have a belief to its being permissible to form it via deduction.

Principles delineating how oughts transmit from ends to the means by which they can be accomplished – so-called *instrumental transmission principles* – are familiar from the literature on instrumental rationality (see Kolodny (2018) for an overview). The following 'epistemic' transmission principle fits the bill:

> *Transmission*: If s ought to believe φ and m is a reliable method by which s can form the belief that φ, then s is permitted to form the belief that φ via m.

Besides its intuitive plausibility, *Transmission* satisfies the two desiderata that Kolodny argues an instrumental transmission principle must satisfy. First, the means which the principle permits one (or requires, or gives reason) to employ must *increase the probability* of one accomplishing the end in question (2018, pp. 734–735). Second, the means must not be *superfluous* in the sense that part of the permitted means alone is sufficient to accomplish the end (2018, pp. 747–748). For example, if my aim is to make my dog wag his tail and petting him is a sufficient means to doing so, petting him whilst telling him a joke is a superfluous means. *Transmission* satisfies the first desideratum because employing a reliable method to form a belief is a *sufficient* means to doing so. And it does not permit one to employ superfluous means because no part of employing a reliable method to form a new belief is sufficient for doing so.

Moreover, *Transmission*'s inclusion of the reliability clause renders it impervious to the criticism that the means it permits one to employ are objectionable in some sense. Consider the more general instrumental transmission principle that *Transmission* resembles, namely: 'If one ought to E, and M-ing is a sufficient means to E-ing, then it is permissible to M'. This can be objected to on the grounds that a mass population cull is a sufficient means to reducing CO_2 emissions, yet even if one ought to reduce CO_2 emissions one is forbidden from culling people. However, the inclusion of the reliability clause protects *Transmission* from the analogous objection. If Bobby ought to believe that the earth is not flat and consulting the tea leaves is a sufficient means for him to do so, *Transmission* does not entail that it is permissible to consult the tea leaves since this is not a reliable method of belief formation. With *Transmission* in place, we are finally ready to articulate *NCA*.

2.3 *The Normative Contradiction Argument Articulated*

NCA can be informally stated as follows. Take a simple argument from Γ to φ, like a disjunctive syllogism, whose premisses *s* ought to believe. Suppose that logics $\mathscr{L}_1$ and $\mathscr{L}_2$ are correct and the argument is $\mathscr{L}_1$-valid but $\mathscr{L}_2$-invalid. Since *s* ought to believe Γ and $\Gamma \models_{\mathscr{L}_1} \varphi$, by *VBP$_E$* it follows that *s* ought to believe φ. Given that $\mathscr{L}_1$ is correct, forming the belief that φ via $\mathscr{L}_1$-valid deduction from Γ is a reliable method for *s*. In conjunction with *Transmission* this entails that it is permissible for *s* to form the belief that φ via deduction from Γ. However, since $\mathscr{L}_2$ is also correct and $\Gamma \not\models_{\mathscr{L}_2} \varphi$, by *IBP$_E$* it follows that *s* is forbidden from forming the belief that φ via deduction Γ. Contradiction. Thus $\mathscr{L}_1$ and $\mathscr{L}_2$ cannot both be correct.

More formally:

(1)	If $\mathscr{L}_1$ is correct, then: If $\Gamma \models_{\mathscr{L}_1} \varphi$ then $O(Bs\Gamma \supset Bs\varphi)$.	(Normativity)
(2)	If $\mathscr{L}_2$ is correct, then: If $\Gamma \not\models_{\mathscr{L}_2} \varphi$ then $O\neg(Fs\varphi m_\Gamma)$.	(Normativity)
(3)	$\mathscr{L}_1$ and $\mathscr{L}_2$ are correct.	(Pluralism)
(4)	If $\Gamma \models_{\mathscr{L}_1} \varphi$ then $O(Bs\Gamma \supset Bs\varphi)$.	(1,3, MP)
(5)	If $\Gamma \not\models_{\mathscr{L}_2} \varphi$ then $O\neg(Fs\varphi m_\Gamma)$.	(2,3, MP)
(6)	If $O(Bs\varphi)$ and m_Γ is a reliable method by which *s* can form the belief that φ, then $\neg O\neg(Fs\varphi m_\Gamma)$.	(*Transmission*)
(7)	$\Gamma \models_{\mathscr{L}_1} \varphi$.	(Assumption)
(8)	$\Gamma \not\models_{\mathscr{L}_2} \varphi$.	(Assumption)
(9)	$O(Bs\Gamma)$.	(Assumption)
(10)	m_Γ is a reliable method by which *s* can form the belief that φ.	(Pluralism)
(11)	$O(Bs\Gamma \supset Bs\varphi)$.	(4,7, MP)
(12)	$O(Bs\varphi)$.	(9,11, MP)
(13)	$\neg O\neg(Fs\varphi m_\Gamma)$.	(6,10,12, MP)
(14)	$O\neg(Fs\varphi m_\Gamma)$.	(5,8, MP)
(15)	$\mathscr{L}_1$ and $\mathscr{L}_2$ are not correct.	(3,13,14, RAA)

Thus, given that logic is doubly normative for reasoning, logical pluralism is inconsistent because it entails logical contradictions *whenever* an adept reasoner ought to believe some set of propositions, Γ, and φ follows from Γ according to one of the correct logics but not the other. Hence the *Inconsistency Thesis*.

3 IS ANYONE HURT?

This section articulates a necessary and sufficient condition for a pluralism to be susceptible to *NCA* before arguing that a wide range of pluralisms satisfy this condition.

NCA shows that, given logic is doubly normative for reasoning, if there are two logics that differ over an argument's validity then accepting both logics as correct with respect to that argument entails logically contradictory claims about how agents ought to reason. Accordingly, the following condition for a pluralism to be susceptible to *NCA* suggests itself:

Susceptible: A pluralism on which logics $\mathscr{L}_1$ and $\mathscr{L}_2$ are correct is susceptible to *NCA* iff there is an argument from set of premisses Γ to conclusion φ, such that:

(i) $\Gamma \models_{\mathscr{L}_1} \varphi$.

(ii) $\Gamma \not\models_{\mathscr{L}_2} \varphi$.

(iii) $\mathscr{L}_1$ and $\mathscr{L}_2$ are correct with respect to the argument from Γ to φ.

Recall from §1.1 that pluralists maintain that logics are correct relative to an independent parameter, $\mathscr{P}$. Accordingly, there are two ways that multiple conflicting logics could end up being correct with respect to a single argument. First, the pluralism may be a *one-many* pluralism on which the argument determines a single value for $\mathscr{P}$, p_1, but there are multiple conflicting logics, $\mathscr{L}_1$ and $\mathscr{L}_2$, which are correct relative to p_1. Diagrammatically:

$$\begin{tikzcd} \mathscr{L}_1 & & \mathscr{L}_2 \\ & p_1 \arrow[ul] \arrow[ur] & \\ & \text{Argument} \arrow[u] & \end{tikzcd}$$

Figure 3.1: One-many pluralism.

Second, the pluralism may be a *many-one* pluralism on which the argument does not determine a unique value for $\mathscr{P}$, and instead $\mathscr{P}$ takes multiple values, say, p_1 and p_2, relative to which conflicting logics $\mathscr{L}_1$ and $\mathscr{L}_2$ are correct. Diagrammatically:

$$\begin{tikzcd}
\mathscr{L}_1 & & \mathscr{L}_2 \\
p_1 \arrow[u] & & p_2 \arrow[u] \\
& \text{Argument} \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

Figure 3.2: Many-one pluralism.

We can now use this typology to determine which of the pluralisms presently on offer are vulnerable to *NCA*.[4]

Case-Relative Pluralism. Beall & Restall (2006) argue that logics are correct relative to *cases*, so that classical logic is correct when cases are Tarskian models, intuitionistic logic when cases are constructions, and relevant logics when cases are situations. Crucially, Beall & Restall endorse the formality of logic (2006, pp. 18–23) – that is, the logics they endorse are correct across all domains of inquiry – and they "do not take *logical* consequence to be relative to languages, communities of inquiry, contexts, or anything else" (2006, p. 88).[5] Accordingly, for any given argument, cases can be models, constructions, or situations, and so any of their logics can be correctly applied to any given argument. Accordingly, theirs is a *many-one* pluralism.[6]

Distinction-Relative Pluralism. On Varzi's (2002) pluralism, logics are correct relative to where the distinction between logical and non-logical terms is drawn. Since any term may in principle be treated as a logical term, for any given argument the logical–non-logical distinction can be drawn in multiple places relative to which different logics are correct. Thus, Varzi's pluralism is *many-one*.

[4] Since none of the pluralisms presently on offer are *many-many*, I have set aside this possibility.
[5] That is, anything else bar cases.
[6] The same goes for Restall's (2014) proof-theoretic version of their pluralism.

Truthbearer-Relative Pluralism. Russell (2008) maintains that logics are correct relative to what kinds of truthbearers arguments' premisses and conclusions are taken to be. Crucially, Russell states that any argument can be understood as being comprised of interpreted sentences, Kaplanian characters, or propositions. For instance, in relation to the argument, 'Gillian Russell is in Banff. Therefore: I am in Banff', she writes:

> If one simply stipulates that arguments are made up of sentences. . . [then] that argument is valid, or invalid, relative to different interpretations, or even, less platitudinously, the question of its validity depends on the depth of the interpretation intended. Assign mere characters to the sentences, and it is possible for the premises to be true and the conclusion false, so the argument is not valid. Assign propositions to them (relative to the context in which this paper was presented) and that is no longer possible, and so the argument is valid. That looks like a stripe of logical pluralism (2008, p. 609).

It follows that Russell's pluralism is a *many-one* pluralism since, for a given argument, it can be legitimately construed as being comprised of different kinds of truthbearers, and different logics are correct depending on the truthbearers one has in mind.

Framework-Relative Pluralism. Kouri Kissel (2018) argues that logics are correct relative to *linguistic frameworks*, which are in turn chosen on pragmatic grounds according to which is best suited for achieving a given theoretical aim. Both the criteria for framework selection – such as "convenience, fruitfulness, and simplicity" (Carnap, 1963, p. 66) – and how they are weighted against one another leave plenty of room for multiple frameworks employing different logics to be equally good for accomplishing a theoretical aim. Steinberger (2016, p. 650) echoes this point: "[I]t is plausible that there is no one framework that best negotiates these demands. It may well be the case that many frameworks perform equally well". Since there are theoretical aims relative to which two linguistic frameworks with different logics are equally appropriate, multiple logics are correct relative to arguments encountered during the course of pursuing this aim. Thus, Kouri Kissel's pluralism is *many-one*. We now turn to one-many pluralisms.

Aim-Relative Pluralisms. Field (2009a) and Blake-Turner & Russell (2018) both argue that logics are correct relative to one's *epistemic goals*. Field's pluralism is a *one-many* pluralism because he allows that "it isn't obvious that there need be a uniquely best logic for a given goal" (Field, 2009a, p. 356). I am also inclined to say Blake-Turner & Russell's (2018) pluralism is *one-many*. Given that there are many relevant and intermediate logics, if one's aim is to draw true and relevant or true and demonstrable conclusions from a set of premisses, then there are multiple logics which further this aim equally well. However, since they do not explicitly address this point, this suggestion is somewhat speculative.

Structure-Relative Pluralism. Shapiro (2014) maintains that logics are correct relative to *mathematical structures*. A logic $\mathscr{L}_i$ is correct relative to a mathematical theory if, at the very least, the theory is $\mathscr{L}_i$-consistent. There are also grounds for thinking that Shapiro's pluralism is a *one-many* pluralism. To see why, recall that Shapiro argues that classical logic is incorrect relative to smooth infinitesimal analysis (*SIA*) because *SIA* is classically inconsistent. Thus, at the very least, a theory, T, being $\mathscr{L}_i$-consistent is *necessary* for $\mathscr{L}_i$ to be correct relative to T. Shapiro is equivocal on the matter of whether $\mathscr{L}_i$-consistency is also *sufficient* for $\mathscr{L}_i$ to be correct relative to T. At points he implies that it is sufficient, such as when he states that "the weaker the logic, the more theories that are consistent, and thus legitimate" (2014, p. 67). However, at other points he says that only "the strongest logic possible that renders the target principles consistent" is correct relative to those principles (i.e. theory).[7]

Suppose that T being $\mathscr{L}_i$-consistent is sufficient for $\mathscr{L}_i$ to be correct relative to T. Given that the Peano axioms are classically consistent and intuitionistic logic is a sublogic of classical logic, they are also intuitionistically consistent. Thus, both classical and intuitionistic logic are correct when reasoning about a single theory, making his pluralism *one-many*. Now suppose that only the strongest logic on which T is consistent is correct relative to T. In general, there is no

[7] It is unclear if this latter option is compatible with Shapiro's pluralism. One of his central claims is that intuitionistic logic is correct relative to *SIA*. But if only the strongest logic on which a theory is consistent is correct relative to that theory, then intuitionistic logic is not correct relative to *SIA* because there are intermediate logics stronger than intuitionistic logic on which *SIA* is consistent.

guarantee that there is a uniquely strongest logic on which T is consistent. Rather, there might be multiple logics on which T is consistent but none of which is stronger than the other. For instance, a theory might be consistent on multiple intermediate logics that can only be partially ordered according to their strength, and so there are multiple 'strongest logics' on which the theory is consistent. Thus, there will be multiple logics that are correct when reasoning about this theory, thereby ensuring that Shapiro's pluralism is *one-many* regardless of whether all logics or only the strongest logics on which a theory is consistent are correct relative to it.

Domain-Relative Pluralisms. Bueno & Shalkowski (2009) and Lynch (2009) both contend that logics are correct relative to *domains of inquiry*, although they carve up these domains differently. Bueno & Shalkowski's pluralism is a *one-many* pluralism because they admit that "often several such logics would be adequate to reason about the objects in a given domain" (Bueno & Shalkowski, 2009, p. 312).[8]

However, Lynch's pluralism is immune to *NCA*. Recall that Lynch's domain-relative logical pluralism is built atop his domain-relative alethic pluralism. Since he states that "there can be at at best only one property that manifests truth for every domain" (2009, p. 80), I think it fair to say that his pluralism cannot be one-many as this precludes there being more than one correct logic per domain.[9] Nor is Lynch's pluralism a many-one pluralism. One might think it is because there are arguments whose premisses belong to multiple domains and, for these arguments, all the logics which are correct relative to these domains can be correctly applied to the argument. But this rests upon a misunderstanding of Lynch's position. Suppose an argument's premisses belong to domains D_1 and D_2, and logics $\mathscr{L}_1$ and $\mathscr{L}_2$, respectively, are correct relative to these domains. Rather than $\mathscr{P}$ taking two values corresponding to D_1 and D_2 and therefore both $\mathscr{L}_1$ and $\mathscr{L}_2$ being correct with respect to the argument, $\mathscr{P}$ takes a single

[8]The vulnerability of Bueno & Shalkowski's and Shapiro's pluralisms to *NCA* despite both rejecting the formality of logic falsifies Kouri Kissel & Shapiro's (2020, p. 391) contention that pluralists can inoculate themselves against normativity objections by jettisoning formality. After all, even if not *all* the logics that are correct relative to a value of $\mathscr{P}$ can be correctly applied to *any* argument irrespective of its premisses' subject-matter, there might still be two logics that can be correctly applied to a single argument, which is all that *NCA* requires.

[9]The same holds for Pedersen's (2014) domain-relative pluralism.

value corresponding to $D_1 \wedge D_2$. In such situations, Lynch's modesty principle comes into play, according to which the correct logic relative to $D_1 \wedge D_2$ is whichever of $\mathscr{L}_1$ and $\mathscr{L}_2$ is weakest (or, if neither is weaker than the other, their intersection) (2009, p. 100). Thus, Lynch's pluralism is not many-one either, and therefore safe from *NCA*.

Meaning-Relative Pluralism. Haack (1978, pp. 230–231) maintains that logics are correct relative to the *meanings of their logical terms*. By giving logical terms different truth-conditions and/or proof rules, different logics give logical terms different meanings; provided these meanings respect the central features of the corresponding natural language logical terms, the resulting logic is correct. Although Haack's pluralism appears to be a many-one pluralism because the meanings of the natural language logical terms in an argument's premisses underdetermine the correct formalisation, this is not so. Crucially, $\mathscr{L}_1$ and $\mathscr{L}_2$ will only clash over an argument's validity when its premisses employ at least one natural language logical term whose formal counterparts in $\mathscr{L}_1$ and $\mathscr{L}_2$ are assigned different truth-conditions or proof rules. Yet, because the logics assign different meanings to logical terms, the argument which is $\mathscr{L}_1$-valid has different premisses to the argument which is $\mathscr{L}_2$-invalid. Thus, on Haack's pluralism there cannot be an argument from Γ to φ satisfying *Susceptibility,* thereby inoculating it against *NCA*.

Thus, if logic is doubly normative for reasoning, *NCA* will have shown that almost all of the existing pluralisms are inconsistent – that is, unless the pluralist can find some reply to *NCA*. It is to these that we now turn.

4 REPLIES AND REJOINDERS

Reply 1. The pluralist can avoid contradiction by rejecting *VBP*$_E$ or *IBP*$_E$ in favour of a bridge principle from which no contradiction can be derived. Indeed, one might think that Beall & Restall can adopt the latter strategy since they seem to endorse the following validity bridge principle: If $\Gamma \models \varphi$, then *s* is forbidden from believing Γ and $\neg\varphi$ (2006, p. 16). This cannot be used to obtain a contradiction because, in conjunction with $\Gamma \models \varphi$, it does not follow that an agent who ought to believe Γ must also believe φ as they are permitted to stay neutral on φ.

Rejoinder. Rejecting VBP_E is not a straightforward matter because it satisfies all the relevant adequacy criteria and, as an evaluative norm, it is inoculated against criticisms such as that it is overdemanding (see §2.2.2). Furthermore, as we noted in §3.2.1, even if VBP_E does not hold unrestrictedly, all *NCA* requires is the far weaker claim that VBP_E holds for the straightforward logical consequences of agents' belief sets that are of interest to them. Finally, Beall & Restall's bridge principle is problematic because, unlike VBP_E, it does not satisfy all the relevant adequacy criteria. Indeed, their principle violates *Obtuseness*: due to its negative polarity, it incorrectly states that, from an objective and ideal standpoint, there is nothing amiss with an agent who refuses to take a stance on a conjunction whilst believing both conjuncts.

Rejecting IBP_E runs into similar problems. An invalidity bridge principle *qua* evaluation must employ the indefeasible 'forbidden to' operator as opposed to a defeasible reasons-based operator because, from an objective and ideal standpoint, there is something wrong with an agent who forms a belief via an invalid inference irrespective of their reasons for doing so. Finally, it is unlikely that adopting an entirely different invalidity bridge principle would help. This principle would still state that agents are forbidden from forming beliefs via invalid inferences, albeit in a different way to IBP_E, and there is no reason to think that *NCA* could not be reconfigured to deliver the same conclusion.

Reply 2. *NCA* is invalid because it equivocates with respect to the disputed argument's premisses and conclusions. Since different logics give different truth-conditions to logical terms, the meanings of logical terms changes between logics. Thus, (13) and (14) are not contradictories since they permit/forbid agents from deducing φ from different sets of premisses.

Rejoinder. This reply relies on the Quinean 'change in logic, change in meaning' thesis (*CLCM*) and, as far as it goes, is correct. As acknowledged previously, Haack's (1978) meaning-relative pluralism accepts *CLCM* which renders it immune from *NCA*. However, as I will now argue, this response is limited in scope and therefore *NCA* remains of interest.

First off, pluralists like Beall & Restall (2006, p. 79) and Field (2009a, p. 345) explicitly disavow *CLCM* and therefore cannot use this reply. Second, some pluralists, including Shapiro (2014, p. 154) and Kouri Kissel (2018, p. 578), maintain that, while *CLCM* holds in general, in some contexts logical terms have the same meanings in different logics. For *NCA* to apply to these pluralisms all that is required is the addition of a premiss stating that the subject is in such a context. Third, this move will not help pluralists such as Varzi (2002) and Russell (2008) because both allow conflicts to arise between logics which do not assign different truth-conditions to their logical terms. On the one hand, Varzi's pluralism allows conflicts over an argument's validity to arise because the logics count different terms as logical rather than because they give the same logical terms different meanings.[10] On the other hand, Russell's pluralism allows conflicts over an argument's validity to arise even though its premisses and conclusion feature no logical terms. Recall from §1.4 that, according to Russell's pluralism, the logical consequence relation holding between standard propositions is not reflexive but that the relation holding between hyperintensional propositions is. The corresponding logics will therefore conflict over the validity of an argument whose premiss and conclusion are identical but feature no logical terms. Finally, adopting *CLCM* alongside the claim that natural language logical terms are polysemous is *sufficient* to establish a variety of logical pluralism – namely, Haack's meaning-relative pluralism. Thus, the cost of adopting *CLCM* to proponents of other pluralisms is high: they remain pluralists, but their own pluralism is rendered obsolete in the process.

Reply 3. *NCA* is invalid because it equivocates with respect to the permitted and forbidden methods of belief formation, which are indexed to the logics in question. Thus, *s* is permitted to form the belief that φ via $\mathscr{L}_1$-deduction from Γ, $m_{\Gamma,\mathscr{L}_1}$, but forbidden from forming the belief that φ via $\mathscr{L}_2$-deduction from Γ, $m_{\Gamma,\mathscr{L}_2}$. Accordingly, (13) and (14) read:

(13') $\neg O \neg (Fs\varphi m_{\Gamma,\mathscr{L}_1})$

[10]Note that designating a term 'logical' does not change its meaning, but simply means that this meaning is held constant when evaluating arguments' validity.

(14') $O\neg(Fs\varphi m_{\Gamma,\mathscr{L}_2})$

This renders the *reductio* invalid because (13') and (14') are not contradictories.

Rejoinder. This reply relies on a bridge principle of the following form being true:

$IBP_{\mathscr{L}_i}$: If $\Gamma \not\models \varphi$ then $O\neg(Fs\varphi m_{\mathscr{L}_i})$

Whether $IBP_{\mathscr{L}_i}$ is true turns on how $\mathscr{L}_i$-deduction is individuated. Since deduction must be individuated fallibly, $\mathscr{L}_i$-deduction cannot simply be to form a belief via a $\mathscr{L}_i$-valid inference. Perhaps the most natural way to fallibly individuate logic-indexed methods of belief formation is according to the intentions of the reasoner. Roughly, *s* forms the belief that φ via $\mathscr{L}_i$-deduction from Γ just in case *s* intends to infer φ from Γ via the rules of $\mathscr{L}_i$-valid deduction. However, this has the deleterious consequence that, from an objective and ideal standpoint, there is nothing logically wrong with Max's reasoning when he affirms the consequent. Suppose that he, like most laypersons, has neither encountered any of the pluralist's logics nor been 'socialised' into accepting a particular logic. Then, for all $\mathscr{L}_i$, Max cannot form beliefs via $\mathscr{L}_i$-deduction because he cannot have the requisite intention. Consequently, Max forming his belief by affirming the consequent is not a token of any kind of deduction and therefore his reasoning cannot be logically erroneous because it cannot violate any instance of $\mathrm{IBP}_{\mathscr{L}_i}$. That is, indexing deduction as a method of belief formation to logics yields the falsehood that, from an objective and ideal standpoint, there is nothing wrong with Max's reasoning.

Reply 4. The argument is invalid because it equivocates with respect to the deontic operators, which differ as they are indexed to the logics in question. So (13) and (14) are:

(13") $\neg O_{\mathscr{L}_1}\neg(Fs\varphi m_{\Gamma})$

(14") $O_{\mathscr{L}_2}\neg(Fs\varphi m_{\Gamma})$

Since (13") and (14") are not contradictories, the *reductio* is invalid.

Rejoinder. Since other paradoxes arising from conflicting norms cannot be resolved in this way, neither can this one. Take the Preface Paradox, where an author rationally believes each sentence in her book. Deductive norms require her to believe their conjunction, but inductive norms require her to believe their conjunction's negation as she has made mistakes before. Given that this conflict cannot be satisfactorily resolved by simply distinguishing between deductive and inductive oughts, *a fortiori* one cannot legitimately resolve conflicts between different deductive norms in this way either.[11]

Reply 5. Maybe logical pluralism entails contradictory normative claims, but normative contradictions cannot be used in *reductios*. After all, we are frequently subject to conflicting moral and epistemic requirements, as illustrated by moral dilemmas and the Preface Paradox.

Rejoinder. Not only does this response contravene the idea that logic is formal, it cannot appeal to moral and epistemic dilemmas as 'companions-in-guilt' because neither involve logically contradictory obligations. For instance, in such dilemmas, one usually either has an obligation to ϕ and an obligation to ψ but cannot do both; or else one is both required to and forbidden from ϕ-ing, as in the Preface Paradox. However, as Ruth Marcus (1980) highlighted, neither case involves a logical contradiction because the negation of $O\phi$ is $\neg O\phi$, not $O\psi$ or $O\neg\phi$. These conflicting obligations only entail contradictions in conjunction with further deontic principles, and friends of dilemmas deny these principles *precisely* to avoid logical contradiction.

Reply 6. Perhaps your argument does show that many logical pluralisms are inconsistent given that logic is doubly normative. But Russell (2020) has persuasively argued that logic is not normative and therefore this conclusion is of little importance.

Rejoinder. To see why *NCA* is immune from Russell's argument, let us distinguish between two questions that one can ask about the normativity of logic. First, is logic normative for

[11] Strictly speaking, unlike *NCA*, the Preface Paradox does not involve logically contradictory norms but this is inconsequential to the point being made here.

reasoning? To answer affirmatively is merely to say that there are true bridge principles of either kind. Second, is logic *inherently* normative for reasoning? To answer affirmatively is not only to say that there are true bridge principles, but that these principles are true in virtue of the logical consequence relation *itself* being normative. Crucially, logic can be normative in the first weaker sense but not the second stronger sense because bridge principles may be true in virtue of the normativity of truth rather than because the consequence relation itself is normative. Indeed, this is precisely what Russell argues: '[It is only] in conjunction with common normative commitments concerning truth and falsity [that] logics. . . have normative consequences' (2020, p. 380). But since *NCA* merely requires that VBP_E and IBP_E be true – that is, it only requires that logic be normative in the weaker but not the stronger sense – it is impervious to Russell's argument that logic is not strongly normative. Thus, Russell's conclusion that "logical pluralists needn't worry about objections from the normativity of logic – logic isn't normative" (2020, p. 387) is mistaken. Many pluralists should be worried by *NCA*.

Reply 7. *NCA* uses *reductio ad absurdum* to move from the contradictory normative claims to the conclusion that $\mathscr{L}_1$ and $\mathscr{L}_2$ are not both correct. But the pluralist can easily sidestep this conclusion by simply claiming that the correct metalogic is a paraconsistent logic that invalidates *reductio ad absurdum*.

Rejoinder. Notice that this reply forces the pluralist to reject the formality of logic as it commits them to a domain-relative pluralism on which *only* paraconsistent logics are correct in the metalogical domain. As such, this avenue of response is simply incompatible with many pluralisms, which allow that explosive logics are correct in metalogical settings. This might happen because they affirm the formality of logic whilst accepting that some explosive logics are correct (e.g. Beall & Restall, 2006, pp. 18–23; Blake-Turner & Russell, 2018, p. 14; Field, 2009a, p. 256). Or it might happen irrespective of whether formality is endorsed because, in metalogical settings, the parameter, $\mathscr{P}$, to which they relativise correctness may take a value for which an explosive logic is correct. Taking Varzi's (2002) pluralism as an example, in metalogical contexts we may draw the distinction between logical and non-logical terms so that classical logic is correct in the metalogical domain and thus the argument remains valid.[12]

Moreover, even if the correct metalogic is paraconsistent and so *reductio* is invalid, it remains truth-preserving provided that none of the premisses are dialetheias – in Priest's (2006) terminology, *reductio* is *quasi-valid*. Since there is no reason to think that any of *NCA*'s premisses are dialetheias and consistency is the default position even amongst paraconsistent logicians, the argument remains truth-preserving (albeit not solely in virtue of its logical form). Consequently, even if the only correct metalogics are paraconsistent, provided *NCA*'s premisses are true as I have argued, so too is its conclusion.

CONCLUSION

This chapter began the defence of the upper bound thesis that there is *at most* one correct logic. I argued for the *Inconsistency Thesis* that, given logic is doubly normative for reasoning, almost all of the pluralists' proposals for cashing out the claim that there is more than one correct logic are inconsistent, thus spelling double trouble for these pluralists. To show this, I constructed the *normative contradiction argument* (*NCA*). *NCA* demonstrated that most pluralisms entail logically contradictory claims about how subjects ought to reason whenever they ought to believe some set of propositions, Γ, and φ follows from Γ according to one of the correct logics but not another. Finally, a number of replies were considered on the pluralists' behalves and found wanting. Nonetheless, in *NCA*'s wake two species of pluralism are still alive and kicking: namely, Lynch's domain-relative pluralism and Haack's meaning-relative pluralism. Accordingly, if we are to fully justify the upper bound thesis, something must be said about why these pluralisms are either untenable or not pluralisms at all, and this is the subject of the next chapter.

[12] Note that this is quite compatible with some paraconsistent logics also being correct in the metalogical domain – after all, the argument only needs to be valid on *one* of the correct metalogics for its conclusion to follow.

4

Loose Ends

The normative contradiction argument of the previous chapter went a long way to defending the upper bound thesis that there is at most one correct logic. However, two pluralisms remained viable in its wake, namely, Haack's meaning-relative pluralism and some domain-relative pluralisms. This chapter completes the defence of the upper bound thesis by suggesting that the arguments profferred by neither kind of pluralist demonstrate that there is more than one correct logic. By necessity this chapter will be extremely brief, and the arguments are intended to be sketches rather than decisive. §4.1 considers Haack's meaning relative pluralism, whilst §4.2 tackles the domain-relative pluralisms.

1 MEANING-RELATIVE PLURALISM

On Haack's (1978) pluralism, logics are correct relative to the meanings of logical terms. As described in §1.9, Haack's pluralism originates in the claim that natural language logical terms like 'or' and 'if' are ambiguous – that is, in Haack's parlance, they have multiple 'aspects'. For instance, on one aspect of 'if', 'If *A* then *B*' is true just in case 'not-*A* or *B*' is also. But a second aspect of 'if' requires that a true conditional's antecedent to be relevant to its consequent, and so 'If 5G masts cause coronavirus then Australia lost the Great Emu War' is false despite '5G masts do not cause coronavirus or Australia lost the Great Emu War' being true. These different aspects can then be represented by different logical terms as found in formal logics:

> [F]or instance, material implication, strict implication, relevant implication, and other formal conditionals might all have some claim to represent some aspect of 'if' (1978, p. 230).

A logic is correct, then, just in case its logical terms correctly represent an aspect of their natural language counterparts, and different logics are correct depending on which of the natural language logical terms' aspects its logical terms represent. Persisting with the above example, the aspect of 'if' on which 'If A then B' is equivalent to 'not-A or B' is correctly represented by the material conditional, $\supset$, relative to which classical logic is correct. However, the aspect of 'if' which incorporates relevance is correctly represented by a relevant conditional, $\rightarrow$, relative to which a relevant logic is correct.

However, that there is more than one correct logic does not follow from natural language logical terms being ambiguous and having multiple aspects. Supposing that 'if' really is ambiguous between two distinct natural language logical terms, the appropriate response is not endorse two logics each of which includes a conditional representing only one aspect. Rather, the appropriate response is to include both '$\supset$' and '$\rightarrow$' as separate logical terms within a single logic – just as we do for other distinct natural language logical terms like 'and' and 'or'. In relation to a different pluralism, Graham Priest puts the point thus:[1]

> [I]t may be argued that vernacular negation sometimes means classical negation and sometimes means intuitionistic negation. But if this were right, we would have two legitimate meanings of negation, and the correct way to treat this formally would be to have two corresponding negation signs in the formal language... In exactly this way, it is often argued that the English conditional is ambiguous, between the subjunctive and indicative. This does not cause us to change logics, we simply have a formal language with two conditional symbols, say $\supset$ and $>$, and use both (2006a, pp. 198–199).

Including multiple logical terms within a single logic is the correct response because a logic

[1]The pluralism in question that espoused by Newton da Costa (1997), which I have omitted because da Costa's paper is in French et je ne parle pas français.

which does not include a logical term for each of the disambiguations of each natural language logical term cannot be correct. For example, take a valid natural language argument in which 'if' should be disambiguated as a material conditional, and is rendered invalid when 'if' is understood as a relevant conditional. If 'if' is ambiguous as Haack claims, then a relevant logic cannot be correct because such an argument lacks a formally valid counterpart in said relevant logic. Thus, even if natural language logical terms are ambiguous as Haack maintains, this does not provide a reason to think that there is more than one correct logic.

2 DOMAIN-RELATIVE PLURALISMS

As we saw in §1.8, the basic idea behind Lynch and Pedersen's domain-relative pluralism is that, owing to the differing natures of entities belonging to different domains, whether an argument is truth-preserving is domain sensitive.[2] For instance, mind-independent and mind-dependent entities are very different and, in virtue of these differences, double-negation elimination is truth-preserving when reasoning about the former but not the latter. From here, Lynch and Pedersen go on to conclude that classical logic is correct in the mind-independent domain whilst intuitionistic logic is correct in the mind-dependent domain.

Monists can agree with the domain-relative pluralist's observation that different arguments are truth-preserving in different domains whilst resisting their conclusion that different logics are correct in these domains. This is because, unlike domain-relative pluralists, monists affirm the *formality* of logical consequence. Logical consequence is said to be formal in the sense that valid arguments not only preserve truth but do so *solely in virtue of their logical form*, and so if an argument is valid then every argument of the same form is too regardless of their premisses' subject-matter.[3] Since domain-relative pluralists claim that some arguments are valid in some domains but not others, they explicitly disavow the formality of logical consequence. By contrast, monists are champions of logical orthodoxy and not only affirm the formality of logical

[2]For present purposes, we can set aside the fact that the relationship between the nature of a domain's entities and the correct logic for that domain is mediated by the property that realises truth in that domain.

[3]How exactly formality is to be spelt out is somewhat controversial, and for more detailed discussions see MacFarlane (2000), Novaes (2011), Sagi (2015), Sher (1991, 1996, 2001, 2010), and Tarski (1986).

consequence, but take it to be *constitutive* of logical consequence – a tradition beginning at least with Kant (1787) and passing through Frege (1879), Russell (1914), and Tarski (1956a, 1986) to contemporary adherents such as McGee (1996), Sher (1991, 1996, 2001, 2010), and van Benthem (1989).

Supposing that logical consequence is formal, pluralism does not follow from the domain-relative pluralist's observation that different arguments are truth-preserving in different domains. Rather, all that follows is that the only arguments which are valid are those which the domain-relative pluralist accepts as valid in all domains, and those which fail to preserve truth in some domains are invalid.[4] Accordingly, the one true logic is the logic which validates all and only those arguments that are validated by all the domain-relative pluralist's logics – that is, the one true logic is the intersection of all the domain-specific logics. For instance, on Lynch's and Pedersen's domain-relative pluralisms, classical and intuitionistic logic are correct in the mind-independent and mind-dependent domains, and so insisting upon the formality of logic simply results in their intersection – namely, intuitionistic logic – being the one true logic. Thus, at the very least, the domain-relative pluralist's arguments will be unpersuasive to those who, like monists, affirm the formality of logic; and if formality is constitutive of logic as so many have thought, then domain-relative pluralisms immediately collapse into monism.[5]

Lynch (2009, p. 103) anticipates a version of this reply, and responds as follows:

> [I]n domains which, according to this suggestion, nonetheless *appear* classical – and therefore abide by LEM – LEM must be true for some *non-logical* reason. And one might wonder what that reason might be.

However, Lynch does not say why this cannot be explained in exactly same way as he explains why classical logic holds in the mind-independent domain, namely, in terms of the nature of

[4] Of course, even if an argument is invalid this does not mean we should not use it when reasoning about a domain in which it preserves truth. Such arguments can, in Priest's (2006a, p. 198) idiom, be "recovered enthymematically".

[5] Some commentators have suggested that the dispute at this point is entirely verbal as it simply concerns which arguments are to be labelled valid (e.g. Shapiro, 2014, pp. 93–94). However, if one thinks that formality is constitutive of logic, as the monist does, then this dispute is no more verbal than one concerning whether whales are mammals or fish.

mind-independent entities. A potentially more serious worry is that, by insisting that the one true logic is the intersection of all the domain-relative pluralist's logics, the monist inadvertently commits themselves to a version of logical nihilism on which there are no (or very few) valid arguments. Steinberger puts the point as follows:

> According to [the monist], the only *bona fide* laws of logic are those that hold good in all domains. But here's the rub: scarcely any logical principle has gone unchallenged in one context or another. Hence, if for sufficiently many domains our best overall theory requires weakening our logic, the monist runs the risk of finding herself with an unworkably weak or even empty consequence relation. Call this the *Objection From the Threat of Logical Nihilism* (2019b, p. 16).[6]

However, there are good reasons for thinking that this threat is fairly minimal. First, none of the extant domain-relative pluralisms countenance enough domains with sufficiently different logics for this threat to materialise – as we have seen, the intersection of Lynch's and Pedersen's domain-specific logics is not empty, but is intuitionistic logic.

Second, this threat is somewhat exaggerated because, when advancing the threat of nihilism, pluralists sometimes postulate additional domains governed by more exotic logics, but do little to substantiate these suggestions. For instance, Bueno & Shalkowski (2009, p. 300) and Kouri Kissel & Shapiro (2020, p. 392) both mention in passing that nihilism threatens monists because – in addition to there being domains governed by classical, intuitionistic, and relevant logics – a quantum logic governs the quantum domain. This inclusion would drastically empty the intersection of the domain-specific logics because quantum logics invalidate laws that hold in all these other logics, notably, the distributivity laws.[7] However, neither provides this suggestion with any support. Indeed, much of the contemporary discussion of quantum logics denies that the distributivity laws actually fail when reasoning about quantum phenomena, and

[6] Versions of this objection are given by Bueno & Shalkowski (2009, pp. 299–300) and Kouri Kissel & Shapiro (2020, p. 392).

[7] Namely, $\varphi \wedge (\psi \vee \delta) \models (\varphi \wedge \psi) \vee (\varphi \wedge \delta)$ and $\varphi \vee (\psi \wedge \delta) \models (\varphi \vee \psi) \wedge (\varphi \vee \delta)$.

instead contends that they only appear to fail because we are unable to simultaneously observe particles' positions and momentum. As Wilce (2017, §2.2) puts it:

> [Quantum mechanics'] non-classical 'logic' simply reflects the fact that not all observable phenomena can be observed simultaneously... but [this] in no way requires any deep shift in our understanding of logic (see also Dalla Chiara & Giuntini, 2002, p. 225; Gibbins, 1987, Ch. 10).

Of course, some do think that quantum phenomena require us to revise our logics (e.g. French & Krause, 2006, Ch. 8). However, unless domain-relative pluralists can provide decisive reasons in favour of this approach to quantum logic – and more generally, until more is done to demonstrate that the world is comprised of many domains governed by logics so different that their intersection is empty – the threat of nihilism to monists remains academic.

Finally, it is doubtful that there *could* be a plausible domain-relative pluralism for which the threat of nihilism arises. This is because insisting upon the formality of logic only threatens monists with nihilism when the intersection of the domain-specific logics is empty. But this very feature renders such a domain-relative pluralism incapable of accounting for our ability to validly reason across different domains. Let us call such a domain-relative pluralism *promiscuous*, since it must countenance many different domains governed by radically different logics for their intersection to be empty.

Recall from §1.8 that the problem of which arguments are valid and which logics are correct when reasoning across multiple domains is the *problem of mixed inferences*. Domain-relative pluralists take this worry very seriously – indeed, Lynch (2006, p. 85) labels it "the fundamental worry" facing domain-relative pluralists – because they take it as given that we are able to validly reason across domains and that our ability to do so is paramount to our pursuit of knowledge. Lynch's (2009, p. 102) solution was to adopt the *modesty principle*: an argument whose premisses belong to multiple domains is valid iff it is valid in all the domains to which its premisses belong, and so the correct logic for reasoning across multiple domains is the intersection of the domain-specific logics in play.

However, the promiscuous domain-relative pluralist cannot commandeer Lynch's solution to account for our ability to reason validly across domains. By countenancing domains whose logics' intersections are close to empty or empty, the promiscuous domain-relative pluralist increases the likelihood that an argument whose premisses belong to multiple domains is invalid. And by countenancing more domains than Lynch or Pedersen, the promiscuous domain-relative pluralist increases the likelihood that a given argument's premisses belong to multiple domains. Together these two factors conspire against the promiscuous domain-relative pluralist being able to account for the validity of many arguments which, as other domain-relative pluralists have acknowledged, are so central to acquiring knowledge.

Moreover, potential alternative solutions to the problem of mixed inferences are of little help to the promiscuous domain-relative pluralist. One solution is that *all* the logics corresponding to the domains to which an argument's premisses belong are correct. But this means that multiple logics can be correctly applied to a single argument and therefore, by the normative contradiction argument, such a pluralism is inconsistent. Another solution is that, when reasoning across domains, only the *strongest* of the corresponding domain-specific logics is correct. However, as argued by Steinberger (2019b, p. 14), this causes the domain-relative pluralism to collapse upwards into monism. To see how, suppose that intuitionistic logic is correct in the mathematical domain and classical logic is correct in all other domains. Letting φ be some mathematical proposition, given that intuitionistic logic is correct in the mathematical domain, $\neg\neg\varphi \not\models \varphi$. But if the correct logic when reasoning across multiple domains is the strongest logic, we can show that $\neg\neg\varphi \models \varphi$ and thereby show that classical logic holds in the mathematical domain. Suppose $\neg\neg\varphi$. By disjunction introduction, it follows that $\neg\neg\varphi \vee \psi$, where ψ is a random non-mathematical falsehood. Classical logic is correct relative to $\neg\neg\varphi \vee \psi$ because it is the strongest of the two logics corresponding to the domains to which the disjuncts belong. Thus, from $\neg\neg\varphi \vee \psi$ we can infer $\varphi \vee \psi$ since they are classically equivalent. Given that, *ex hypothesi*, ψ is false, it follows by disjunctive syllogism that φ. Consequently, double-negation elimination holds in the mathematical domain, and the domain-relative pluralism has collapsed into a monism in which classical logic is correct in all domains.

In summary, even if monists accept domain-relative pluralists' claims about which arguments are truth-preserving in which domains, their endorsement of the widely-held view that formality is constitutive of logical consequence entails that the one true logic is the intersection of the domain-specific logics. Moreover, I have argued that making this move does not threaten the monist with nihilism. This was because the extant domain-relative pluralisms do not substantiate the claim that many domains are governed by radically different logics and, even if there were such a pluralism, it would be implausible because it is unable to resolve the problem of mixed inferences.

CONCLUSION

The aim of the first part of this thesis has been to defend the upper bound thesis that the number of correct logics is at most one. This chapter has tackled the two remaining logical pluralisms that resisted the normative contradiction argument's advances – Haack's meaning-relative pluralism and the domain-relative pluralism of Lynch and Pedersen. Regarding Haack's meaning-relative pluralism, I suggested that monists can accommodate her claim that natural language logical terms are ambiguous by including logical terms for each of the natural language logical terms' disambiguations. Similarly, I argued that monists can accommodate much of the domain-relative pluralist's insights by insisting upon the formality of logic and maintaining that the only valid arguments are the ones which are valid across all domains, and can do so without collapsing into nihilism. With the defence of the upper bound thesis now complete, the monist is halfway towards the holy grail of establishing that there is but one true logic. All that remains is to defend the lower bound thesis that there is at least one correct logic against logical nihilists who claim that there *no* correct logics. It is to them that we now turn.

Part II

Logical Nihilism

5

Obliterative Nihilism

Our strategy for defending logical monism is to defend two intermediate theses which together entail that there is one true logic. Namely, the *upper bound thesis* that there is *at most one* correct logic, and the *lower bound thesis* that there is *at least one* correct logic. Part I defended the upper bound thesis, and Part II now aims to defend the lower bound thesis.

Part II is based upon the recognition that the following two claims must hold for the lower bound thesis to be true. First, the *non-emptiness claim* that the logical consequence relation proper is non-empty. That is, given our definition of correctness, for there to be at least one correct logic with a non-empty consequence relation, the logical consequence relation proper cannot be empty. Second, the *existence claim* that, supposing there are valid natural language arguments, there actually is at least one logic in which all and only these arguments have formally valid counterparts.

This chapter and the next defend the non-emptiness and existence claims, each of which has recently come under fire from logical nihilists who deny them precisely because they think that there are no correct logics. The non-emptiness claim that the logical consequence relation is non-empty – or, equivalently, that there are laws of logic – is disputed by Gillian Russell (2017, 2018a, 2018b). Russell maintains that there are no laws of logic – a position which we shall call *obliterative nihilism* – because there are counterexamples to even the most basic of putative logical laws, such as conjunction introduction and identity. The existence claim is contested by Aaron Cotnoir (2018), and we shall call the resulting position *pessimistic nihilism*. Cotnoir claims that there cannot be a correct logic despite there being valid natural language arguments because the properties that a logic's consequence relation must instantiate to be correct, along with the inherent limitations of formal languages, preclude every valid natural language argument from having a $\mathscr{L}_i$-valid formal counterpart.

This chapter aims to vindicate the non-emptiness claim by showing that, *pace* Russell, the consequence relation is non-empty and there are logical laws. §5.1 explains why the non-emptiness claim must be true for there to be at least one correct logic before introducing obliterative nihilism. §5.2 presents Russell's counterexamples to conjunction introduction and identity, whilst §5.3 defuses them. I then go on the offensive, and the remainder of the chapter employs an anti-exceptionalist stance towards logic to construct an abductive argument for the existence of certain logical laws. §5.4 introduces anti-exceptionalism and abduction, whilst §5.5 presents the evidence which §5.6 argues is better explained by the hypothesis that there are logical laws than by the hypothesis that there are none.

1 PRELIMINARIES

As usual, for a logic to be correct it must satisfy:

> *Correct*: $\mathscr{L}_i$ is correct iff, for any *P*, *C*: *C* is a logical consequence of *P* iff $\mathscr{T}(C)$ is a $\mathscr{L}_i$-consequence of $\mathscr{T}(P)$.

That is, for $\mathscr{L}_i$ to be correct, every valid natural language argument must have a $\mathscr{L}_i$-valid formal counterpart and, more importantly for present purposes, *every* $\mathscr{L}_i$-valid argument in the formal language must be the formal counterpart of a valid natural language argument. Consequently, since we are only counting structures with non-empty consequence relations as logics, for at least one logic to be correct there must be valid natural language arguments. That is, the lower bound thesis can only be true if the non-emptiness claim is true.

Lately, however, the non-emptiness claim – and consequently the idea that there are *any* correct logics – has come under attack from Russell's (2017, 2018a, 2018b) obliterative nihilism. According to the obliterative nihilist:

> [T]he extension of the relation of logical consequence is *empty*; there is no pairing of premises and conclusion such that the second is a logical consequence of the first (2018a, pp. 310–311).

Before we elucidate the obliterative nihilist's arguments for this radical conclusion, a couple of preliminary remarks are needed. A point that will raise its head throughout this chapter is that, despite their repudiation of valid arguments, obliterative nihilists accept that many individual arguments are necessarily truth-preserving – that is, it is impossible for their premisses to be true but their conclusions false. For instance, regarding the argument, 'If snow is white then grass is green. Snow is white. Therefore: Grass is green', Russell writes:

> [T]his instance of modus ponens might be perfectly acceptable to the nihilist – in the sense that the truth of the premises *guarantees* [my emphasis] that of the conclusion – even if she believes that modus ponens is not a law of logic, thanks to some recherché counterexamples. . . To claim that modus ponens is *logically* valid is to make a claim of great generality, whereas to claim that the above argument preserves truth is merely to claim that some *instance* of modus ponens is good (2017, pp. 126–127).

Indeed, the obliterative nihilist's admission of necessarily truth-preserving arguments is central to the tenability of their position as it enables them to explain away why arguments like the following appear valid:

> *Dogs*: Guinness is a collie. Oscar is a Labrador. Therefore: Guinness is a collie and Oscar is a Labrador

Namely, *Dogs* appears valid because it is necessarily truth-preserving and *most* arguments of the same form are too. More generally, so-called laws like conjunction introduction and *modus ponens* appear valid because *almost all* of their instances are necessarily truth-preserving.

Second, a terminological matter needs clarifying. When arguing for obliterative nihilism, Russell primarily frames her position as "the view that there are no *laws of logic*" (2018a, p. 308).

Logical laws are expressed via so-called entailment sentences of the form 'P entails C', where P is a set of natural language sentences and C is a single natural language sentence.[1] For instance, the law of conjunction introduction states: for any natural language sentences A, B: A, B entails A and B.[2] Thus, by stating that there are no laws of logic, the obliterative nihilist is saying that all such entailment sentences are false: "For any arbitrary set of premises and conclusion, the nihilist holds that the premises do not entail that conclusion" (Russell, 2018b, p. 340). This, of course, is equivalent to saying that the logical consequence relation is empty. Accordingly, the non-emptiness claim may be rephrased as stating that there are laws of logic. The next section outlines the obliterative nihilist's argument against this claim.

2 THE COUNTEREXAMPLES

To demonstrate that there are no laws of logic by presenting a counterexample to every putative logical law is a Herculean undertaking. To cut this task down to size Russell focusses on a pair of extremely basic alleged logical laws, conjunction introduction and identity:

Conjunction Introduction: A, B entails A and B.

Identity: A entails A.

The obliterative nihilist's central argument against the non-emptiness claim is that there are counterexamples to both, and "if this is the case for laws as basic as identity and conjunction introduction, then nothing is safe: logical nihilism could well be true" (Russell, 2018a, p. 315).

These counterexamples revolve around two relatively unknown sentences called 'SOLO' and 'PREM'. What is unique about SOLO and PREM is that their truth-conditions are sensitive to the linguistic context in which they occur:

T-C_{SOLO}: SOLO is true iff it occurs as an atomic sentence; false otherwise.

[1] Russell (2018a, p. 311) says that "a law of logic takes the form: $\Gamma \models \varphi$", where Γ and φ are (sets of) sentences in the formal language. However, I take sentences of this form to be formal representations of logical laws in the same way that $\models_{\mathscr{L}_i}$ is a formal representation of the logical consequence relation proper.

[2] Henceforth I leave the quantification over natural language sentences implicit.

T-C_{PREM}: PREM is true iff it occurs as a premiss in an argument; false otherwise.

They can then be used to give counterexamples to both *Conjunction Introduction* and *Identity*:

Conjunction-Counter: SOLO. Guinness is a collie. Therefore: SOLO and Guinness is a collie.

Identity-Counter: PREM. Therefore: PREM.

Given SOLO's truth-conditions and that Guinness is indeed a collie, *Conjunction-Counter* has true premisses and a false conclusion because SOLO is false whenever it is part of a compound sentence, like a conjunction. Thus, although most instances of conjunction introduction are necessarily truth-preserving, *Conjunction Introduction* cannot be a law because it has non-truth-preserving instances. Similarly, *Identity* fails because PREM's truth-conditions ensure that *Identity-Counter* has a true premiss but a false conclusion. Given that *Conjunction Introduction* and *Identity* are logical laws if any are, *a fortiori* there are no laws and the consequence relation is empty.[3]

One line of response to these couterexamples is to argue, as Bogdan Dicher (2020, p. 7) does, that SOLO and PREM do not *entirely* empty the consequence relation as they must if they are to establish obliterative nihilism. For instance, the argument, 'Guinness is a collie. Therefore: It is not the case that PREM' comes out as valid because, given PREM's truth-conditions, its negation must be true whenever it is an argument's conclusion. Thus, Dicher concludes, "emptying the consequence relation is slightly trickier than it seems at first blush" (2020, p. 8). This, I submit, is too concessive a response to the obliterative nihilist. *Conjunction Introduction* and *Identity* are extremely basic laws that hold in virtually every widely accepted logic; we should not abandon them – and all the logics which validate them – without a fight.

[3] I have set aside Russell's (2018a, pp. 313–315) argument that allowing the consequence relation to quantify over cases in which sentences can be both true and false or neither invalidates laws such as excluded middle and non-contradiction. Since this only weakens our logic as far as the logic of first-degree entailment (FDE) and examples of the style discussed here could also be used to invalidate the classical laws rejected by FDE, this component of Russell's argument is superfluous.

The next section argues that Russell's overarching strategy for generating counterexamples to basic logical laws cannot empty the consequence relation *at all*.

3 COUNTEREXAMPLES DEFUSED

Recall that the logical consequence relation holds between *sets of natural language sentences* and a logic's formal representation of this relation holds between sets of sentences in the logic's formal language. So too for the laws of logic: an instance of a law of logic is not a formal argument but a *natural language argument* exhibiting the relevant form. The importance of this distinction now crystallises: to show that *Conjunction Introduction* and *Identity* are not logical laws, the obliterative nihilist must show that there are instances of both laws – that is, natural language arguments of the appropriate forms – that fail to preserve truth. But Russell has done no such thing, for SOLO and PREM are not natural language sentences and therefore *Conjunction-Counter* and *Identity-Counter* cannot be instances of *Conjunction Introduction* and *Identity*.[4]

Rather, to invalidate *Conjunction Introduction* and *Identity*, the obliterative nihilist requires natural language sentences whose truth-conditions emulate SOLO's and PREM's. Two obvious candidates are:

SOLO*: This sentence is atomic.

PREM*: This sentence is a premiss.

The original arguments against *Conjunction Introduction* and *Identity* can then be reconstructed by substituting SOLO* and PREM* for SOLO and PREM:

*Conjunction-Counter**: This sentence is atomic. Guinness is a collie. Therefore: This sentence is atomic and Guinness is a collie.

*Identity-Counter**: This sentence is a premiss. Therefore: This sentence is a premiss.

[4] Dicher (2020, p. 8) seems to make a similar point in passing, but does not go on to consider natural language sentences that might mimic SOLO and PREM.

The sentence 'This sentence is atomic' is atomic and therefore true, but 'This sentence is atomic and Guinness is a collie' is non-atomic and therefore false. Thus, *Conjunction-Counter** is a counterexample to *Conjunction Introduction* since it is an instance of this law that has true premisses but a false conclusion. Similarly, *Identity-Counter** is a counterexample to *Identity* since it is an instance of *Identity* and, given that the premiss token of 'This sentence is a premiss' is a premiss but the conclusion token is not, it has a true premiss but a false conclusion. So it seems that Russell's original argument is unaffected by pointing out that SOLO and PREM are not natural language sentences because natural language sentences displaying the same sensitivity to linguistic context can be easily identified.

But this is too quick. To see where *Conjunction-Counter** and *Identity-Counter** go astray, consider the following arguments:

Beer: My beer has a head. Guinness is a collie. Therefore: My beer has a head and Guinness is a collie.

Umbrella: This is an umbrella. Therefore: This is an umbrella.

Suppose that the premiss occurrence of 'head' in *Beer* refers to the frothy layer of foam but the conclusion occurrence refers to the part of an organism that usually houses its eyes and ears. And suppose that the premiss occurrence of 'This' in *Umbrella* refers to an umbrella but the conclusion occurrence of 'This' refers to an emu. So understood, both *Beer* and *Umbrella* have true premisses and false conclusions. But do they invalidate *Conjunction Introduction* and *Identity*?

The answer, of course, is that they do not. Take *Beer*. Although the premiss and conclusion occurrences of 'My beer has a head' are *tokens of the same orthographic type*, given that 'head' is being used to refer to two different things, they are distinct sentences with distinct meanings. That is, they are *tokens of distinct interpereted sentence types* since they contain expressions which refer to different things.[5] Accordingly, the first premiss and conclusion are more perspicuously

[5]Note that the same point can be made if the relata of the logical consequence relation proper are propositions instead of interpreted sentences.

stated as, 'My beer has a head_1' and 'My beer has a head_2 and Guinness is a collie'. But now *Beer* is not an instance of *Conjunction Introduction* as it has the form, 'A. B. Therefore: C and B'. So too for *Umbrella*. Since 'This' refers to different things in the premiss and conclusion, they are distinct sentences despite being tokens of the same orthographic type. Thus, *Umbrella* is not an instance of *Identity* – rather, it has the form, 'A. Therefore: B'. The remainder of this section argues that if *Beer* and *Umbrella* do not invalidate *Conjunction Introduction* and *Identity* then, for exactly the same reasons, nor do any natural language renderings of SOLO and PREM.

Returning to *Conjunction-Counter** and *Identity-Counter**, careful attention needs to paid to what the premiss and conclusion occurrences of 'This' refer to. To see the problem, consider *Conjunction-Counter**. For this argument to have true premisses and a false conclusion, the premiss occurrence of 'This' must refer to the sentence, 'This sentence is an atomic sentence' but the conclusion occurrence of 'This' must refer to the sentence, 'This sentence is an atomic sentence and Guinness is a collie'. Were both the premiss and conclusion occurrences of 'This' to refer to the sentence, 'This sentence is an atomic sentence' then both the premisses and conclusion would be true. Conversely, were both the premiss and conclusion occurrences of 'This' to refer to the sentence, 'This sentence is an atomic sentence and Guinness is a collie' then the conclusion would be false but so is one of the premisses. Either way, we do not have a counterexample to *Conjunction Introduction*. As such, *Conjunction-Counter** succeeds *only* if the two occurrences of 'This' refer differently.

The same point holds *mutatis mutandis* for *Identity-Counter**, although this is partially obscured by the premiss and conclusion being tokens of the same orthographic type. For *Identity-Counter** to have a true premiss but a false conclusion, the premiss and conclusion occurrences of 'This' must refer to distinct tokens. The premiss occurrence must refer to the premiss token of 'This sentence is a premiss' but the conclusion occurrence must refer to the conclusion token. Were both occurrences of 'This' to refer to the premiss token then both the premiss and conclusion would be true. Conversely, if both occurrences of 'This' were to refer to the conclusion token then both the premiss and conclusion would be false. Again, either way we do not have an instance of *Identity* that fails to preserve truth. Thus, *Identity-Counter**'s success also relies upon the two occurrences of 'This' referring differently.

However, as was the case with *Beer* and *Umbrella*, allowing an expression to refer differently between the premisses and conclusion bars *Conjunction-Counter** and *Identity-Counter** from being counterexamples to *Conjunction Introduction* and *Identity*. Since 'This' refers differently, it follows that the premiss and conclusion tokens of SOLO* and PREM* in *Conjunction-Counter** and *Identity-Counter** respectively are not tokens of the same sentence type. That is, *Conjunction-Counter** and *Identity-Counter** are more perspicuously stated as:

*Conjunction-Counter**: This$_1$ sentence is atomic. Guinness is a collie. Therefore: This$_2$ sentence is atomic and Guinness is a collie.

*Identity-Counter**: This$_1$ sentence is a premiss. Therefore: This$_2$ sentence is a premiss.

But now neither argument is an instance of *Conjunction Introduction* or *Identity*, and therefore neither can be counterexamples to either law. Given that 'This$_1$ sentence is atomic' and 'This$_2$ sentence is atomic' are distinct sentences, *Conjunction-Counter** has the form '*A*. *B*. Therefore: *C* and *B*' and so is not an instance of *Conjunction Introduction*. And given that 'This$_1$ sentence is a premiss' and 'This$_2$ sentence is a premiss' are distinct, *Identity-Counter** has the form '*A*. Therefore: *B*' and so is not an instance of *Identity*. Thus, the most obvious natural language counterparts of SOLO and PREM are not counterexamples to *Conjunction Introduction* and *Identity*.

Of course, this *only* shows that *one* attempt at providing natural language equivalents to SOLO and PREM fails. However, it is difficult to see how Russell's recipe for constructing counterexamples could succeed because the foregoing problem generalises. Suppose that the obliterative nihilist wishes to construct a counterexample to a law, *L*. To succeed, the counterexample must satisfy two constraints: it must have true premisses but a false conclusion, and it must be an instance of *L*. Russell's recipe begins by identifying a property, *P*, possessed by the premisses of an instance of *L* but not by the conclusion. In the counterexample to *Conjunction*

Introduction this property was being an atomic sentence and for *Identity* it was being a premiss. To obtain a sentence, A, that is true when it occurs as a premiss in an instance of L but false when it occurs as a conclusion, A must be such that each of its tokens ascribes P to itself. In turn, to ascribe P to itself A needs to self-refer, and must therefore include an indexical expression like 'This' or a definite description such as 'The sentence…'.[6] A counterexample can then be obtained by constructing an instance of L in which A appears in the premisses and the conclusion. However, there is good reason to think that the counterexamples this recipe produces cannot satisfy both constraints.

Suppose that self-reference is achieved via an indexical expression like 'This sentence'. To satisfy the first constraint, A's premiss token must be true but its conclusion token false. Given that the premisses of an instance of L are P but the conclusion is not, for A's premiss token to be true but its conclusion token false, the indexical in the premiss token must refer to the premiss token but the indexical in the conclusion token must refer to the conclusion token. But if the indexical is referring to different things then A's premiss and conclusion tokens are different sentences. And as we saw in relation to *Conjunction Introduction* and *Identity*, A's premiss and conclusion tokens being different sentences precludes the argument from being an instance of L, thereby violating the second constraint.

Attempts at using definite descriptions to attain self-reference fare no better, albeit for different reasons. To self-refer via a definite description, A must take the general form, 'The sentence with property X is P' where X is the property featuring in the definite description that is satisfied by A and, as before, P is the property possessed by premisses of an instance of L but not by the conclusion. On any account of definite descriptions, for a sentence of the form 'The sentence with property X is P' to be true, the definite description 'The sentence with property X' must be satisfied by *at least one* sentence that is P. Thus, to satisfy the first constraint – that

[6]Could one attain self-reference in natural language without employing an indexical or definite description? It is controversial whether there are examples of non-indexical self-reference in natural language. The most famous example is Quine's Paradox, "'Yields a falsehood when appended by its quotation' yields a falsehood when appended by its quotation' (Quine, 1976, p. 9). However, since 'its quotation' is short for 'the quotation of it' and 'it' is widely taken to be an indexical (e.g. Kaplan, 1989, p. 489), this is not so. I therefore set this possibility aside.

is, for *A*'s premiss token to be true but its conclusion token false – the premiss occurrence of the definite description *must* be satisfied by *A*'s premiss token, but the conclusion occurrence of the definite description *cannot* be satisfied by *A*'s premiss token.[7]

However, the satisfaction of this condition – and therefore the first constraint – is precluded by the fact that both the premiss and conclusion occurrences of the definite description are tokens of the same definite description type. Given they are of the same type, barring any semantically relevant changes in context, any sentence satisfying the premiss occurrence satisfies the conclusion occurrence and *vice versa*. Consequently, if *A*'s premiss token is true as the first constraint requires, then *A*'s premiss token satisfies the not only the premiss occurrence of the definite description but also the conclusion occurrence, thereby rendering *A*'s conclusion token true in violation of the first constraint. One could in principle satisfy the first constraint by changing the context so that the premiss occurrence of the definite description refers to *A*'s premiss token but the conclusion occurrence refers to the conclusion token. Since the former is *P* but the latter is not, the resulting argument would have a true premiss but a false conclusion. However, just as two tokens of 'This is an umbrella' are distinct sentences when the context is shifted so that 'This' refers to an umbrella in one but an emu in the other, so too for definite descriptions. Thus, shifting the context so the definite description's premiss and conclusion occurrences refer differently yields distinct sentences, thereby precluding the resulting argument from being an instance of *L* and thus violating the second constraint.

Accordingly, it is doubtful that there *can* be counterexamples to *Conjunction Introduction* and *Identity* of the kind that Russell originally had in mind. As such, we may conclude that the obliterative nihilist's attack on the non-emptiness claim is unsuccessful.

[7] One might wonder whether one could obtain an appropriate self-referential natural language sentence using Gödel coding, whereby distinct sentences are mapped to distinct integers, their Gödel numbers. Suppose that the Gödel number of the sentence 'The sentence with Gödel number *g* is a premiss' is *g*. We then have the following counterexample to *Identity*: 'The sentence with Gödel number *g* is a premiss. Therefore: The sentence with Gödel number *g* is a premiss'. However, for the premiss to be true, the definite description 'The sentence with Gödel number *g*' must refer to the sentence type as there are many tokens satisfying the description. But since this type does occur as a premiss, the conclusion is true, thereby violating the first constraint.

4 ANTI-EXCEPTIONALISM & ABDUCTION

It is one thing to demonstrate that the obliterative nihilist has not undermined the non-emptiness claim, but quite another to show that it is true – that is, without tacitly relying on the claim that the burden of proof lies with the obliterative nihilist. This remainder of this chapter employs an anti-exceptionalist stance towards logic to construct an abductive argument for the existence of certain basic logical laws. Although my primary purpose is to provide a positive argument for the non-emptiness claim, in doing so I also hope to show that anti-exceptionalism is perfectly compatible with the piecemeal justification of individual logical laws – something that many contemporary anti-exceptionalists have denied. This section briefly outlines anti-exceptionalism and the structure of abductive arguments, and §5.5 presents a body of evidence which §5.6 argues is better explained by the hypothesis that there are logical laws than by the hypothesis that there are none.

Our starting point is the anti-exceptionalist stance towards logic, according to which:

> Logic isn't special. Its theories are continuous with science; its method continuous with scientific method. Logic isn't a priori, nor are its truths analytic truths. Logical theories are revisable, and if they are revised, they are revised on the same grounds as scientific theories (Hjortland, 2017, p. 632).[8]

Given that scientific theories are revised and justified abductively, this is to say that logical theories are too (Williamson, 2017, p. 334). Hitherto, anti-exceptionalists have used abduction to justify *entire* logical theories – for instance, Williamson (2017) and Priest (2006a) use abduction to justify classical and a paraconsistent logic, respectively. Indeed, a number of anti-exceptionalists take their position's defining feature to be that it is logical theories that are justified abductively as opposed to individual laws:

> [C]onfirmational holism is part of the account. A logical theory is confirmed *en bloc*, not

[8] Anti-exceptionalists include Hjortland (2017, 2019), Priest (2006a, 2014, 2016), Quine (1951, 1986), and Williamson (2013, 2017). Crucially, Russell (2014, 2015, 2019) also endorses anti-exceptionalism.

by individual confirmation of logical laws (Hjortland, 2019, p. 256).[9]

However, this chapter employs abduction in a novel way to atomistically justify *particular* logical laws such as *Conjunction Introduction* and *Identity*, thereby both demonstrating that the logical consequence relation is non-empty and challenging this component of anti-exceptionalism. First, however, we turn to the nature of abduction.

Abduction – or 'inference to the best explanation' – is a form of ampliative inference ubiquitous in both ordinary and scientific reasoning, in which one reasons from a body of evidence to the hypothesis which best explains that evidence. For instance, suppose that I find 'Clean me!' written in the dirt on the family car. Three hypotheses which explain this evidence are that Mum did it, that the dirt miraculously landed on the car in that pattern, and that my mischievous younger brother Max did it. I conclude that the culprit is Max. This inference cannot be deductive because my evidence does not entail that he did it, but nor can it be inductive because this has never happened before. Rather, I reach this conclusion because it offers a better explanation of the evidence than the alternatives – after all, the handwriting looks like his and it is *exactly* the kind of joke Max would pull.

More generally, letting *e* be some evidence and $h_1, h_2, \ldots, h_n$ be competing hypotheses purporting to explain *e*, abductive arguments take the form:

$$\begin{array}{l} e \\ h_1 \text{ is a better explanation of } e \text{ than any of } h_2, \ldots, h_n \\ \hline \therefore h_1 \end{array}$$

What is it for one hypothesis to be a better explanation of some evidence than another? Explanatory goodness is cashed out in terms of the so-called *explanatory virtues* and, roughly speaking, one explanation is better than another if it instantiates these virtues to a greater degree than its competitors. Whilst different proponents of abduction give different albeit largely overlapping lists of the explanatory virtues, we will concentrate on the virtues of *explanatory power*,

[9] In a similar vein, Russell states: "The heart of the abductivist approach consists in two claims. The first is holism about the justification of logic: it is entire logics – rather than isolated claims of consequence – that are justified (or not)" (2019, p. 550) and, "One view that is incompatible with abductivism is a view on which individual claims about entailment are justified atomistically, rather than in the context of a whole theory" (2019, p. 552).

simplicity, and *unification*.[10]

A hypothesis' explanatory power over some evidence, the explanandum, is its "ability to decrease the degree to which we find the explanandum surprising" (Schupbach & Sprenger, 2011, p. 108). That is, the less surprising the evidence is given the assumption that the hypothesis is true, the greater the power that hypothesis has over the evidence. For instance, General Relativity has more explanatory power over evidence concerning the gravitational deflection of light by the sun than Newton's theory of gravity. Whereas General Relativity predicts almost exactly the observed deflection, Newton's theory predicts only half the observed amount and therefore the evidence is less surprising given the former than the latter.[11]

When defining simplicity it is important to distinguish between *syntactic simplicity* and *ontological simplicity*. Syntactic simplicity refers to the number of adjustable parameters a hypothesis uses to explain the evidence, whereas ontological simplicity refers to the number and kind of entities that a hypothesis postulates (Sober, 2002, pp. 16–17). Since the obliterative nihilist and friends of logical laws are not committed to any entities that the other is not, it is syntactic simplicity that is relevant here.[12] To see syntactic simplicity in action, suppose that we have two hypotheses that aim to explain the incidence of lung cancer in a population. The first explains this in terms of individuals' cigarette consumption, whilst the second explains it in terms of their cigarette consumption and the number of games of tiddlywinks they have won. Assuming that both offer equally powerful explanations of the evidence, syntactic simplicity demands that we prefer the former over the latter as it explains the evidence using one fewer parameter.

The degree of unification exhibited by a hypothesis is the extent to which that hypothe-

[10]Some have endorsed additional virtues – for instance, Psillos (2002, p. 615) includes a hypothesis' fit with background data whilst Lipton (2004, p. 122) includes the extent to which a hypothesis provides a precise mechanism explaining the evidence. I focus on power, simplicity, and unification because they are common to all the various lists of virtues (see Cabrera (2017, p. 1249) for an overview), have received most attention within the literature on abduction, and have been defined most precisely. Moreover, there are compelling arguments that power, simplicity, and unification have confirmational import whereas other virtues are purely pragmatic (see Cabrera (2017, pp. 1254–1255)).

[11]For arguments that explanatory power has confirmational import, see Crupi & Tentori (2012), Schupbach & Sprenger (2011), and Schupbach (2017).

[12]For arguments that syntactic simplicity has confirmational import, see Forster & Sober (1994), Sober (2015, Ch.2), and Cabrera (2017, p. 1252).

sis "provides a unified account of what might otherwise seem to be independent phenomena" (Myrvold, 2003, p. 399). That is, a unifying hypothesis is one which is able to provide the same explanation of seemingly independent pieces of evidence, and the more unifying a hypothesis is, the better. For example, Newton's theory of planetary motion was more unifying than Kepler's because Newton's theory gives the same explanation of both the evidence that we have about the motion of objects on Earth and the motions of celestial bodies, whereas Kepler's theory cannot.[13]

With these virtues to hand it becomes apparent that there are three ways that one hypothesis, h_1, can outpoint another, h_2, on the abductive score board. First, h_1 might *strictly dominate* h_2 by possessing all the explanatory virtues to a higher degree than h_2. Alternatively, h_1 might *weakly dominate* h_2 by possessing the all the explanatory virtues to at least the same degree as h_2 and at least one to a higher degree. Finally, there might be some virtues on which h_1 scores more highly than h_2 and *vice versa* but, given the relative importance of the virtues and the margins by which h_1 and h_2 outpoint one another, h_1 is the better hypothesis *all things considered*. In what follows, I establish the non-emptiness claim by demonstrating that the hypothesis that there are logical laws *weakly dominates* any of the hypotheses available to the obliterative nihilist.

5 THE EVIDENCE

What is the evidence that the hypothesis that there are logical laws is supposed to better explain than the hypothesis that there are no such laws? To narrow down this question and make matters more concrete, we will focus on constructing an abductive argument for *Conjunction Introduction*, although the ensuing argument can easily be adapted to establish other basic laws like *Identity*.

As we saw in §5.1, it is integral to the plausibility of the obliterative nihilist's position that they admit that arguments like *Dogs* are necessarily truth-preserving. This admission forms the basis of the evidence used in the ensuing abductive argument. More specifically, we will focus on two pieces of evidence. First, not only are there infinitely many instances of *Conjunction*

[13]For arguments that unification has confirmational import, see Myrvold (2003, 2017) and Schupbach (2005).

Introduction that are necessarily truth-preserving, but many of these instances are necessarily truth-preserving despite sharing little in common except their form. Consider the arguments:

> Grass is green. Snow is white. Therefore: Snow is white and grass is green.
>
> Two is even or two is odd. There is no set of all non-self-membered sets. Therefore: Two is even or two is odd and there is no set of all non-self-membered sets.

Both arguments are necessarily truth-preserving despite their conjuncts having completely different logical forms, truth-conditions, and subject-matters. Indeed, the *only* properties that these arguments share, besides being instances of *Conjunction Introduction*, are those that they *must* instantiate to be arguments in the first place, such as being composed of declarative natural language sentences.[14] Let an argument's *non-form properties* be all the properties that it may instantiate besides its form and the properties which it must instantiate to be an argument. Accordingly, the non-form properties of an instance of *Conjunction Introduction* will include its subject-matter, its conjuncts' logical forms, its conjuncts' truth-conditions and the like. Our first piece of evidence, then, is that *there is an infinite number of necessarily truth-preserving instances of Conjunction Introduction, many of which share no non-form properties*.

Our second piece of evidence concerns the lack of counterexamples to *Conjunction Introduction*. Although §5.3 demonstrated that Russell's counterexamples cannot succeed, we have not shown that there cannot be *any* counterexamples to *Conjunction Introduction* because we have not shown that *every* strategy for generating counterexamples cannot succeed. Indeed, if we had, there would be no need for an abductive argument supporting *Conjunction Introduction*. Consequently, we may not include in our evidence that there are no counterexamples whatsoever to *Conjunction Introduction*. However, given that Russell's counterexamples falter and in the absence of any other counterexamples, we may include in our evidence that *there are no known counterexamples to Conjunction Introduction*.

[14] We will see later on why the properties that an argument must instantiate cannot be used by the nihilist to explain this evidence.

Crucially, the obliterative nihilist *already accepts* the first piece of evidence and *must accept* the second piece in light of the arguments of §5.3. The obliterative nihilist is committed to accepting the first piece of evidence by their strategy for explaining away the fact that arguments like *Dogs* appear to be valid, as outlined in §5.1. Namely, *Dogs* seems valid because it and almost all arguments of the same form are necessarily truth-preserving. Consequently, if the ensuing abductive argument is successful, even obliterative nihilists will have to endorse the non-emptiness claim – unless, of course, they can point to a flaw in its reasoning.[15]

6 THE ABDUCTION

This section argues that the hypothesis that *Conjunction Introduction* is a logical law better explains the foregoing evidence than any hypothesis available to the obliterative nihilist, thereby showing that *Conjunction Introduction* is a logical law and that the non-emptiness claim is true.

Let h_{CI} be the hypothesis that *Conjunction Introduction* is a logical law:

h_{CI}: A, B entails A and B.

Crucially, h_{CI} gives a straightforward explanation of our evidence that there are infinitely many necessarily truth-preserving instances of *Conjunction Introduction*, many of which share no non-form properties, and there are no known counterexamples. Supposing that *Conjunction Introduction* is a law, it follows that *all* arguments of the form 'A. B. Therefore: A and B' are necessarily truth-preserving. In turn, this is explained in terms of the fact that the meaning of 'and' is *sufficient* for an argument of this form to be necessarily truth-preserving, regardless of its premisses' subject-matter and any other non-form properties that it may instantiate. Moreover, this explanation is powerful, simple, and unifying. It is powerful because the evidence is entirely unsurprising given that instantiating this form and the meaning of 'and' is sufficient

[15] This is to say that our evidence satisfies the principle of evidence neutrality articulated by Williamson (2007, p. 210). Although Williamson himself rejects evidence neutrality, he readily admits that arguments employing neutral evidence will be more dialectically effective: "Relying on a premise one's opponents have already refused to accept tends to be dialectically useless. They will probably deny that it constitutes evidence; one's argument will make no headway" (2007, p. 210).

for, and thus guarantees, necessary truth-preservation. It is simple because h_{CI} explains this evidence using a single parameter, namely, an argument's form. And it is unifying because, for any two instances of *Conjunction Introduction* that share no non-form properties, h_{CI} provides the same explanation of the seemingly independent facts that both instances are necessarily truth-preserving.

What hypotheses can the obliterative nihilist use to explain this evidence? Besides giving no explanation whatsoever – and thereby rendering the fact that there are infinitely many necessarily truth-preserving instances of *Conjunction Introduction* a miracle of cataclysmic proportions – there are two varieties of explanatory hypotheses available to them. The first variety we shall call the '*Form + X*' variety. Although the obliterative nihilist cannot say that the instances of *Conjunction Introduction* are necessarily truth-preserving *solely* in virtue of their logical form, they can say that they are necessarily truth-preserving in virtue of their form *plus their instantiating some non-form properties*.[16] The only other option on the table is hypotheses of the '*X*' variety, which explain the necessary truth-preservingness of the instances of *Conjunction Introduction solely in terms of their instantiating some non-form properties*.[17] Different versions of both varieties can then be obtained by varying which non-form properties are used to explain the evidence. I will leave open which properties these are because I hope to show that, regardless of which properties the obliterative nihilist chooses, their hypothesis is explanatorily inferior to h_{CI}.

We begin with the *Form + X* variety. This variety comes in two species depending on whether, in addition *Conjunction Introduction*'s instances' form, they appeal to one non-form property, x, or multiple non-form properties, $x_1, x_2, \ldots, x_n$, to explain the evidence. Call a hypothesis of the first kind h_{F+x}, and one of the second kind h_{F+X}. According to h_{F+x}, in-

[16] Might they appeal to properties other than non-form properties? The only other properties that the obliterative nihilist can appeal to are those that arguments must instantiate to be arguments, such as being composed of declarative natural language sentences, and these are of no help. For one thing, since *all* arguments must instantiate these properties, claiming that being an instance of *Conjunction Introduction* and instantiating them is *sufficient* to be necessarily truth-preserving entails admitting that there *cannot* be counterexamples to *Conjunction Introduction*, which is the exact opposite of what the obliterative nihilist wants to say.

[17] Again, appealing to properties such as being composed of declarative natural language sentences is of no use, albeit for a different reason. Claiming that instantiating the properties that an argument must instantiate to be an argument is sufficient to be necessarily truth-preserving ensures that *all* arguments are necessarily truth-preserving.

stances of *Conjunction Introduction* are necessarily truth-preserving in virtue of their logical form and instantiating x, whilst h_{F+X} asserts that instances of *Conjunction Introduction* are necessarily truth-preserving in virtue of their logical form and instantiating one or more of $x_1, x_2, \ldots, x_n$. Naturally, adopting h_{F+x} or h_{F+X} requires the obliterative nihilist to be able to tell a story about how an argument's logical form and its instantiating x, or one or more of $x_1, x_2, \ldots, x_n$, are jointly (but not individually) sufficient for an argument to be necessarily truth-preserving. For the sake of argument let us assume that such a story can be told.

The problem with h_{F+x} is that there are instances of *Conjunction Introduction* sharing no non-form properties. Thus, for any non-form property x, there will be some instances that lack x but are necessarily truth-preserving, and these will fall outside the scope of the nihilist's explanation. Consequently, h_{CI} has greater explanatory power over the evidence than h_{F+x}, because it is less surprising that non-x instances of *Conjunction Introduction* are necessarily truth-preserving given h_{CI} than h_{F+x}. Moreover, h_{CI} is simpler because it explains the evidence solely in terms of the instances' logical form whereas h_{F+x} employs an additional parameter, whether or not an instance is x. Finally, h_{CI} is more unifying than h_{F+x} because, for any two instances of *Conjunction Introduction* that share no non-form properties and one of which is x, h_{CI} provides a single explanation of the fact that both instances are necessarily truth-preserving whereas h_{F+x} is unable to do so. As such, h_{CI} strictly dominates h_{F+x}.

The obliterative nihilist could try to circumvent h_{F+x}'s lack of explanatory power by switching to h_{F+X}, according to which instances of *Conjunction Introduction* are necessarily truth-preserving in virtue of their logical form and instantiating one or more of $x_1, x_2, \ldots, x_n$. Provided that $x_1, x_2, \ldots, x_n$ cover the set of all instances of *Conjunction Introduction*, so that every instance instantiates at least one of $x_1, x_2, \ldots, x_n$, then h_{F+X} can in principle explain why each and every one is necessarily truth-preserving. Consequently, h_{F+X} will be no less powerful than h_{CI}. However, h_{F+X} is no panacea: it is both less simple and less unifying than h_{CI}. It is less simple because h_{CI} employs a single parameter – namely, logical form – to explain the evidence whereas h_{F+X} employs $n+1$ parameters. Furthermore, h_{F+X} is a less unifying explanation than

h_{CI} because, for any two instances of *Conjunction Introduction* that share no non-form properties, h_{CI} explains the seemingly independent facts that both are necessarily truth-preserving in terms of their being instances of *Conjunction Introduction*. By contrast, h_{F+X} is unable to do so: given the two instances share no non-form properties, the necessary truth-preservingness of each instance must be explained in terms of their form plus the instantiation of different sets of non-form properties. Thus, h_{CI} weakly dominates h_{F+X}. Consequently, no matter which version of the *Form + X* variety the obliterative nihilist plucks for – that is, regardless of how many and which non-form properties the nihilist's hypothesis appeals to in addition to form – their explanation is at least weakly dominated by h_{CI}.

Turning to hypotheses of the *X* variety, we can make the same basic distinction between those which appeal to one non-form property, x, or multiple non-form properties, $x_1, x_2, \ldots, x_n$, to explain the evidence. Call a hypothesis of the first kind h_x, and one of the second kind h_X. h_x says that instances of *Conjunction Introduction* are necessarily truth-preserving solely in virtue of instantiating x, whilst h_X asserts that instances of *Conjunction Introduction* are necessarily truth-preserving in virtue of instantiating one or more of $x_1, x_2, \ldots, x_n$. Again, let us assume that the obliterative nihilist is able to show why instantiating x, or at least one of $x_1, x_2, \ldots, x_n$, is sufficient for an instance of *Conjunction Introduction* to be necessarily truth-preserving.

Since both h_x and h_X flaws are almost identical to those of h_{F+x} and h_{F+X}, respectively, I shall be brief. h_{CI} weakly dominates h_x because, although they are equally simple as they both employ a single parameter to explain the evidence, h_x is both less powerful and unifying than h_{CI} due to the existence of non-x instances of *Conjunction Introduction*. h_{CI} weakly dominates h_X because, although they are equally powerful, h_X is less simple and unifying than h_{CI} as it employs more parameters to explain the evidence and cannot give the same explanation of the fact that instances of *Conjunction Introduction* sharing no non-form properties are necessarily truth-preserving. Again, since this follows without making any assumptions about which non-form properties the obliterative nihilist employed in their explanation, h_{CI} weakly dominates every species of the *X* variety.

Thus, the hypothesis that *Conjunction Introduction* is a logical law weakly dominates every variant of the two varieties of explanatory hypotheses available to the obliterative nihilist. It therefore follows via abduction that *Conjunction Introduction* is a law of logic, and the non-emptiness claim is thereby vindicated. Moreover, as highlighted in §5.5, since the nihilist accepts the evidence from which this argument proceeded, they have little option but to accept this conclusion. Finally, although we have focussed on *Conjunction Introduction*, there is no reason to suppose that the same style of abductive argument could not be constructed for other basic logical laws like *Identity*, but also conjunction elimination, disjunction introduction and elimination, *modus ponens*, and the like. However, it is unlikely that the same approach could be extended to further laws such as double-negation elimination and explosion whilst remaining dialectically effective because it is more contentious whether there are counterexamples to them. This might lead one to worry that although the non-emptiness claim is true, the logical consequence relation proper is unpalatably weak. However, such worries are misguided because these more controversial laws may be true even though a dialectically effective abductive argument cannot be given in their favour. Moreover, given that our aim is to establish that there is one true logic without taking a stance on which logic it is, all that matters for present purposes is that the non-emptiness claim is true, which we have shown.

CONCLUSION

This chapter has defended the non-emptiness claim from Russell's obliterative nihilism. Russell motivates the claim that the consequence relation is empty by presenting counterexamples to *Conjunction Introduction* and *Identity*. In reply, I argued that the counterexamples are unsuccessful because they are either not instances of the laws that they were intended to invalidate or, if they are, they lack the true premisses and false conclusions required to invalidate them. I then provided a positive abductive argument in the non-emptiness claim's favour. Unlike existing applications of abduction by anti-exceptionalists, this abductive argument could be used to justify logical laws piecemeal rather than entire logics. In particular, I argued that the hypothesis that *Conjunction Introduction* is a logical law better explains evidence about which arguments are necessarily truth-preserving than any of the hypotheses available to the obliterative nihilist. Since analogous abductive arguments can be used to establish other logical laws, it follows that the logical consequence relation is far from empty. Nonetheless, it does not follow from the non-emptiness claim that there is at least one correct logic because there might not actually be a logic that validates all and only valid natural language arguments – that is, the existence claim might be false. This is the topic of the next chapter.

6

Pessimistic Nihilism

To defend the lower bound thesis that there is at least one correct logic, two intermediate claims needed justifying. First, the *non-emptiness claim* that the logical consequence relation proper is non-empty and there are valid arguments (or, equivalently, that there are logical laws). Second, the *existence claim* that there actually is at least one logic which correctly codifies the non-empty consequence relation. Although the non-emptiness claim was established in the previous chapter, this does not guarantee the existence claim because it might not be possible for there to be a logic in which *every* valid natural language has a formally valid counterpart.

This is precisely the position endorsed by pessimistic nihilists such as Cotnoir (2018). According to pessimistic nihilists, the existence claim is false despite there being valid natural language arguments because, for any logic $\mathscr{L}_i$, there are valid natural language arguments that cannot have $\mathscr{L}_i$-valid formal counterparts. Cotnoir deploys two groups of arguments in support of this claim. First, the *arguments from diversity*, according to which a correct logic's consequence relation must instantiate certain properties such as necessity and formality, but doing so imposes strictures upon its consequence relation that preclude it from validating every valid natural language argument. Second, the *arguments from expressive limitations*, which show that there are natural language sentences that cannot be translated into formal languages, and therefore any valid natural language argument in which they feature as premisses cannot have a formal counterpart, let alone one which is $\mathscr{L}_i$-valid. The purpose of this chapter is to defend the existence claim against these arguments, and thereby complete the defence of the lower bound thesis that there is at least one correct logic.

§6.1 outlines the common structure of the arguments from diversity and my responses to them, whilst §§6.2–6.3 present and neutralise both the necessity and formality variants. The same pattern is then repeated for the arguments from expressive limitations. §6.4 provides an overview of their structure and that of my responses, before §§6.5–6.7 expound and disarm all three variants.

1 AN OVERVIEW OF THE ARGUMENTS FROM DIVERSITY

The arguments from diversity can all be understood as dilemmas with the following structure. Both begin with an adequacy condition, A, which states that, for any $\mathscr{L}_i$, if $\mathscr{L}_i$ is correct then $\models_{\mathscr{L}_i}$ must instantiate a property such as necessity or formality. They then proceed to show that if $\mathscr{L}_i$ satisfies A and $\models_{\mathscr{L}_i}$ instantiates the property, then there are valid arguments that lack $\mathscr{L}_i$-valid formal counterparts. Thus, $\mathscr{L}_i$ is incorrect if it satisfies A and incorrect if it does not, and therefore the existence claim is false because there cannot be a correct logic.

Let us say that a logic, $\mathscr{L}_i$, *undergenerates* iff not all valid natural language arguments have $\mathscr{L}_i$-valid formal counterparts. Accordingly, the arguments from diversity's structure is as follows:

(1) For all $\mathscr{L}_i$: Either $\mathscr{L}_i$ satisfies A or $\mathscr{L}_i$ does not satisfy A.

(2) For all $\mathscr{L}_i$: If $\mathscr{L}_i$ does not satisfy A then $\mathscr{L}_i$ is incorrect.

(3) For all $\mathscr{L}_i$: If $\mathscr{L}_i$ satisfies A then $\mathscr{L}_i$ undergenerates.

(4) For all $\mathscr{L}_i$: $\mathscr{L}_i$ is incorrect.

Premiss (1) merely states that for any given logic, it either satisfies A or it does not – that is, its consequence relation either instantiates the relevant property or it does not. Premiss (2), which we shall call the *adequacy constraint*, merely restates that a necessary condition for a logic to be correct is that it satisfies A. Crucially, both these premisses will be accepted by the monist. Even those who reject the law of excluded middle will find (1) palatable since it does not possess any of the features that result in this law failing, such as denotation failure, vagueness, or effective undecidability. Almost all monists will accept (2) because the adequacy conditions which Cotnoir's arguments employ state that for $\mathscr{L}_i$ to be correct, $\models_{\mathscr{L}_i}$ must instantiate a property which is widely taken to be *constitutive* of logical consequence, such as necessity or formality. As Gila Sher puts it:

> [The] pretheoretic notion of logical consequence involves two intuitive ideas: the idea that *logical consequence is necessary* and the idea that *logical consequence is formal*. These ideas play the role of adequacy conditions: an adequate definition of logical consequence yields only consequences that are necessary and formal (1996, p. 654).

Thus, the crux of the arguments from diversity is (3), which we shall call their *incompatibility premiss*. According to the incompatibility premiss, $\mathscr{L}_i$ satisfying A ensures that $\mathscr{L}_i$ undergenerates because instantiating the property delineated in A imposes strictures upon $\models_{\mathscr{L}_i}$ that preclude $\mathscr{L}_i$ from validating every valid natural language argument. Both variants of the arguments from diversity support their incompatibility premisses in similar ways. They begin by introducing a pluralist claim and then demonstrate that natural language arguments of a certain form cannot have $\mathscr{L}_i$-valid formal counterparts if $\mathscr{L}_i$ is to satisfy A in light of this pluralist claim. However, since natural language arguments of this form are valid, it thereby follows that if $\mathscr{L}_i$ satisfies A then it undergenerates, as stated by the incompatibility premiss.

Since my responses to both arguments take the same form it is worth outlining their structure here as well. Although the most obvious line of response is to challenge the pluralist claim used to establish the incompatibility premiss, this is not the tack I shall take. Rather, I contend that Cotnoir's strategy for establishing the incompatibility premiss suffers from a structural flaw which ensures that it cannot succeed *by its own lights*. More specifically, I demonstrate that the argument used to show that certain valid natural language arguments cannot have $\mathscr{L}_i$-valid formal counterparts when $\mathscr{L}_i$ satisfies A simultaneously undercuts the claim that the original natural language arguments are valid. Thus, it has not been shown that if $\mathscr{L}_i$ satisfies A then $\mathscr{L}_i$ undergenerates because it transpires that, by the nihilist's own argument, the natural language argument that lacks a $\mathscr{L}_i$-valid formal counterpart is itself invalid. Since the pessimistic nihilist's arguments for the incompatibility premisses falter, the arguments from diversity cannot substantiate the conclusion that there cannot be a correct logic, and so pose little threat to the existence claim.

2 THE NECESSITY ARGUMENT FROM DIVERSITY

Cotnoir's first argument from diversity centres on the fact that one of the logical consequence relation's constitutive features is that it is *necessary*. If a conclusion is a logical consequence of a set of premisses, the truth of the premisses *necessitate* the truth of the conclusion – that is, it is impossible for the premisses to be true but the conclusion false. This yields the necessity adequacy constraint: if logic $\mathscr{L}_i$ is correct, then $\models_{\mathscr{L}_i}$ must be necessary in the sense that, for any Γ, φ, if $\langle \Gamma, \varphi \rangle \in \models_{\mathscr{L}_i}$ then every case in which Γ is true so is φ.

The necessity argument from diversity can then be stated as the following instance of the foregoing template:

(1_N) $\models_{\mathscr{L}_i}$ is necessary or $\models_{\mathscr{L}_i}$ is not necessary.

(2_N) If $\models_{\mathscr{L}_i}$ is not necessary, then $\mathscr{L}_i$ is incorrect.

(3_N) If $\models_{\mathscr{L}_i}$ is necessary, then $\mathscr{L}_i$ undergenerates.

(4_N) Therefore, $\mathscr{L}_i$ is incorrect.

As mentioned above, the bone of contention is the incompatibility premiss, (3_N). The rationale for (3_N) is that, for it to be impossible for Γ to be true but φ false whenever $\Gamma \models_{\mathscr{L}_i} \varphi$, $\models_{\mathscr{L}_i}$ must quantify over all possible cases. If it did not, and the excluded case were one in which Γ is true but φ false, then $\mathscr{L}_i$-validity would not be necessarily truth-preserving. As Bueno & Shalkowski put it:

> The point of quantifying over a domain of cases is to capture the idea that there is a logical, that is, necessary, connection between the premisses and conclusions of valid arguments. . . In order to guarantee that because something holds in every case it holds

> *necessarily*, we need to ensure not only that it indeed holds in all cases, but also that it holds in all *possible* cases (2009, p. 306).

Next, the pluralist claim is introduced. Cotnoir piggybacks on Beall & Restall's (2006) argument that complete and consistent Tarskian models, consistent but (potentially) incomplete constructions, and (potentially) incomplete and inconsistent situations are all possible cases. Consequently, if $\models_{\mathscr{L}_i}$ is necessary then it must quantify over models, constructions, and situations. However, if $\models_{\mathscr{L}_i}$ quantifies over them all – that is, $\mathscr{L}_i$ is the intersection of classical, intuitionistic, and a relevant logic – then $\mathscr{L}_i$ will be extremely weak. Indeed, Cotnoir contends that it will be *too* weak to be correct:

> [I]t seems clear that the logic resulting from quantifying over all such cases is still going to be far too weak to adequately account for natural language inference (2018, p. 307).

For instance, if $\models_{\mathscr{L}_i}$ quantifies over inconsistent situations, $\varphi \vee \psi, \neg\varphi \not\models_{\mathscr{L}_i} \psi$ since there are situations in which φ is both true and false, and ψ is false only. Consequently, $\mathscr{L}_i$ is too weak to be correct because the formal counterpart of any natural language disjunctive syllogism is $\mathscr{L}_i$-invalid, yet there are valid natural language disjunctive syllogisms, such as:

> *Fox*: Either the fox went left or the fox went right. The fox did not go left. Therefore: The fox went right.

Accordingly, if $\models_{\mathscr{L}_i}$ is necessary and quantifies over all possible cases – including models, constructions, and situations – then there are valid arguments, like *Fox*, lacking $\mathscr{L}_i$-valid formal counterparts. That is, if $\mathscr{L}_i$ satisfies the necessity adequacy constraint then it undergenerates and is incorrect, thereby establishing (3_N), the incompatibility premiss. Since $\mathscr{L}_i$ is incorrect if it satisfies the necessity adequacy constraint and incorrect if it does not, it follows that $\mathscr{L}_i$ is incorrect and there cannot be a correct logic.

Having outlined the necessity argument from diversity, we may now apply the blueprint articulated in §6.1 to show why it falters. Namely, the nihilist fails to show that $\mathscr{L}_i$ undergenerates when $\models_{\mathscr{L}_i}$ is necessary because the very argument used to show that *Fox* lacks a $\mathscr{L}_i$-valid formal counterpart simultaneously undermines *Fox*'s validity. According to Cotnoir, *Fox* lacks a $\mathscr{L}_i$-valid formal counterpart because inconsistent situations are genuine cases. The key point argued for below is that, regardless of how cases are understood, if inconsistent situations are genuine cases then it is logically possible for a sentence and its negation to be true. Given that it is *logically impossible* for a valid natural language argument's premisses to be true but its conclusion false, it thereby follows that if inconsistent situations are genuine cases then *Fox* is invalid as it is logically possible for its premisses to be true but its conclusion false.[1]

Following Etchemendy (1990, 2010), cases can be understood *representationally* or *interpretationally*. On the representational account, cases are "mathematical models of *logically possible* [my emphasis] ways the world, or relevant portions of the world, might be or might have been" (Etchemendy, 2010, p. 287). That is, understood representationally, cases vary the non-linguistic facts but the meanings of logical and non-logical terms alike are held constant. By contrast, on the interpretational account, cases are interpretations of a language's non-logical vocabulary which vary the meanings of non-logical terms, whilst the meanings of logical terms and all the non-linguistic facts remain as they are in the actual world. Moreover, these interpretations are *uniform* insofar as every token of a non-logical term is assigned the same meaning.

On the representational account, the only cases which a logic's consequence relation should quantify over are those that represent genuine logical possibilities – after all, quantifying over cases representing logical impossibilities will result in valid arguments lacking formally valid counterparts. Accordingly, if inconsistent situations – such as the one used to show that *Fox*'s formal counterpart is $\mathscr{L}_i$-invalid – *genuinely* are cases as Cotnoir claims, then they must represent logically possible ways that relevant portions of the world might be.[2] However, if it is logically possible for a sentence to be both true and false, then *Fox* is invalid as it is logically possible for its premisses to be true and its conclusion false.

[1] This response does not hinge on disjunctive syllogisms being the valid arguments which lack $\mathscr{L}_i$-valid formal counterparts.

[2] Even if paraconsistent logicians must admit that it is logically possible that there are true contradictions they need not say that it is metaphysically possible (let alone actually so), just as it is logically but not metaphysically possible for water not to be H_2O.

On the interpretational account, admitting that there are inconsistent cases not only commits one to the logical possibility of true contradictions, but to there being true contradictions in the actual world (Asmus, 2012, pp. 12–14). To see this, suppose that there is a case in which $Fa \wedge \neg Fa$ is true – that is, there is an assignment of semantic values on which both Fa and $\neg Fa$ are true, where the semantic values for predicates and individual constants are properties and objects. Since all tokens of the same type are assigned the same semantic value, for $Fa \wedge \neg Fa$ to be true in this case, there must be a single object that both does and does not instantiate some property. And since the non-linguistic facts remain unchanged between the actual world and this case, this is to say that there is an object in the actual world that both does and does not instantiate a single property. But if contradictions are actual then they are logically possible, and again *Fox* is invalid.

Thus, Cotnoir's argument for the necessity argument's incompatibility premiss undercuts itself because the natural language argument that it has shown to lack a $\mathscr{L}_i$-valid formal counterpart is, by the argument's own lights, invalid. Since it has not been shown that $\mathscr{L}_i$ satisfying the necessity adequacy constraint ensures that there are valid natural language arguments that lack $\mathscr{L}_i$-valid formal counterparts, it has not been shown that there cannot be correct logics and the existence claim remains unscathed.

3 THE FORMALITY ARGUMENT FROM DIVERSITY

The second argument from diversity turns on another constitutive feature of logical consequence, that of *formality*. If an argument's conclusion is a logical consequence of its premisses, then the truth of the premisses not only necessitate the truth of the conclusion, but they do so *solely in virtue of their logical form*. Thus, if an argument is valid, then all arguments of the same form are too. This yields the formality adequacy constraint: if logic $\mathscr{L}_i$ is correct then $\models_{\mathscr{L}_i}$ must be formal in the sense that, for any Γ, φ, if $\Gamma \models_{\mathscr{L}_i} \varphi$, then every argument of the same form is also a member of $\models_{\mathscr{L}_i}$.

The formality argument from diversity can then be stated as follows:

(1_F) $\models_{\mathscr{L}_i}$ is formal or $\models_{\mathscr{L}_i}$ is not formal.

(2_F) If $\models_{\mathscr{L}_i}$ is not formal, then $\mathscr{L}_i$ is incorrect.

(3_F) If $\models_{\mathscr{L}_i}$ is formal, then $\mathscr{L}_i$ is incorrect.

(4_F) Therefore, $\mathscr{L}_i$ is incorrect.

Again, we concentrate on the incompatibility premiss, (3_F). The rationale for (3_F) is that, for $\models_{\mathscr{L}_i}$ to be formal, it must be the case that if $\Gamma \models_{\mathscr{L}_i} \varphi$ then *every* argument of the same form is a member of $\models_{\mathscr{L}_i}$. At this point, a claim made by domain-relative pluralists is introduced: different arguments are valid in different domains of inquiry. For instance, on Lynch's domain-relative pluralism, double-negation elimination is valid in domains where truth is not epistemically constrained, such as the realm of medium sized objects, but invalid in domains where truth is epistemically constrained, such as the realm of mathematics. If this is right then the first of the two arguments below is valid but the second is not:

Bottle: It is not the case that my water bottle is not blue. Therefore: My water bottle is blue.

Root: It is not the case that $\sqrt{2}$ is rational. Therefore: $\sqrt{2}$ is irrational.

Accordingly, intuitionistic logic is correct in domains where truth is epistemically constrained and classical logic is correct in domains where it is not (Lynch, 2009, p. 95). Consequently, if $\mathscr{L}_i$ satisfies the formality adequacy constraint then the only principles which are $\mathscr{L}_i$-valid are those which hold across all domains – that is, to satisfy the formality adequacy constraint, $\mathscr{L}_i$ must be the intersection of all the different logics which hold in all the various domains.

However, since very few logical principles are valid across all domains, if $\models_{\mathscr{L}_i}$ is formal then $\mathscr{L}_i$ will be too weak to be correct. For instance, if $\models_{\mathscr{L}_i}$ is formal and there are some domains in which double-negation elimination fails, then $\neg\neg\varphi \not\models_{\mathscr{L}_i} \varphi$. Consequently, the formal counterpart of any natural language argument employing double-negation elimination is $\mathscr{L}_i$-invalid. However, given that *Bottle* is valid, if $\models_{\mathscr{L}_i}$ is formal then there are valid arguments lacking $\mathscr{L}_i$-valid formal counterparts. That is, if $\mathscr{L}_i$ satisfies the formality adequacy constraint then it undergenerates and is incorrect, thereby establishing (3_F), the incompatibility premiss. Since $\mathscr{L}_i$ is incorrect if it satisfies the formality adequacy constraint and incorrect if it does not, it follows that $\mathscr{L}_i$ is incorrect and pessimistic nihilism prevails.[3]

As with the previous argument from diversity, I contend that the argument used to reach this conclusion undermines the claim that *Bottle* is valid. To see this, recall that the reason why $\models_{\mathscr{L}_i}$ must be formal if $\mathscr{L}_i$ is to be correct is that the logical consequence relation proper is formal. That is, a natural language argument's conclusion, *C*, is a logical consequence of its premisses, *P*, iff it is impossible for *P* to be true but *C* false *in virtue of its logical form*. Given that *Bottle* and *Root* have the same form, it follows from the formality of the logical consequence relation proper that both are valid or neither are. Since, *ex hypothesi*, *Root* is invalid, it follows that *Bottle* is too. However, if *Bottle* is invalid then it has not been shown that if $\mathscr{L}_i$ satisfies the formality adequacy constraint then there is a valid natural language argument which lacks a $\mathscr{L}_i$-valid formal counterpart.

Thus, Cotnoir's argument for the incompatibility premiss undercuts itself because the argument used to show that certain valid natural language arguments cannot have $\mathscr{L}_i$-valid formal counterparts simultaneously undermines the validity of said natural language arguments. As such, the formality argument from diversity falters and the existence claim remains intact.

4 AN OVERVIEW OF THE ARGUMENTS FROM EXPRESSIVE LIMITATIONS

The arguments from expressive limitations all aim to show that there cannot be a correct logic because, for any logic, the inherent limitations of formal languages preclude every valid natural language argument from having a formally valid counterpart in that logic. This is because there are natural language sentences that cannot be translated into any formal language, and so any valid natural language argument which includes these sentences are premisses cannot

[3] Note that Cotnoir's argument is distinct from the threat of nihilism discussed in §4.2. The latter sought to show that the logical consequence relation proper is empty because logical consequence is formal and no arguments are valid in all domains. Thus, if it were cogent – contrary to what I argued – this argument would support Russell's obliterative nihilism, not Cotnoir's pessimistic nihilism. By contrast, Cotnoir's argument is that there are valid natural language arguments which do not preserve truth in all domains, and so a logic which validates only those arguments that hold in all domains will be too weak to be correct.

have a formal counterpart, let alone one which is $\mathscr{L}_i$-valid. Thus, there cannot be a logic which validates all and only valid natural language arguments, and the existence claim is false.

Let us say that a formal language is *adequate* to natural language iff every natural language sentence can be translated into a formal language sentence with the same content – that is, every natural language sentence has a formal counterpart. The arguments from expressive limitations all take the following form:

(1) If no formal language is adequate to natural language then, for all $\mathscr{L}_i$, $\mathscr{L}_i$ is incorrect.

(2) No formal language is adequate to natural language.

(3) For all $\mathscr{L}_i$, $\mathscr{L}_i$ is incorrect.

Since (1) follows straightforwardly from the meaning of 'adequate' and our definition of what it is for a logic to be correct, the arguments from expressive limitations stand and fall with (2). In support of (2), the arguments from expressive limitations introduce certain features that can be exhibited by natural language sentences, such as including the predicate '. . . is true'. A paradox is then used to demonstrate that no formal language sentence can have the feature in question on pain of contradiction. It thereby follows that there are natural language sentences which cannot be translated into any formal language, and therefore that no formal language is adequate to natural language.

In each case, one avenue of response is to deny premiss (2) and argue that formal languages can exhibit the feature in question. However, this is not an avenue we shall go down. Rather, my responses to the arguments from expressive limitations will, to some extent, mirror the responses given to the arguments from diversity in that my strategy will be to use the nihilist's reasoning against them. More specifically, I contend that the same paradoxes that the nihilist uses to show that formal language sentences cannot have some feature on pain of contradiction can also be used to show that natural language sentences cannot have this same feature on pain of contradiction. Accordingly, if the fact that a language can only exhibit some feature on pain of contradiction *is* sufficient to show that it cannot exhibit that feature – as Cotnoir must maintain – then neither formal nor natural languages can exhibit it. Conversely, if the fact that a language can only exhibit some feature on pain of contradiction *is not* sufficient to show that it cannot exhibit that feature then, in the absence of further argument, both formal and natural languages can exhibit it. Either way, it has not been shown that there are natural language sentences that cannot be translated into the formal language, and therefore we have no grounds for thinking (2) is true.

It is crucial to acknowledge at the outset that the main benefit of this strategy is that it does not commit one to taking a stance on whether either natural or formal languages can exhibit the feature in question. The point is merely that, irrespective of whether they can, Cotnoir's arguments do not establish that formal languages cannot exhibit the feature but natural languages can. In what follows, we consider three features which Cotnoir argues are had by natural languages but lacked by formal languages: that they have their own truth predicates, validity predicates, and unrestricted quantifiers.[4]

5 THE EXPRESSIVE LIMITATIONS ARGUMENT FROM SEMANTIC CLOSURE I

The first argument from expressive limitations concerns *semantic closure*. A language, L, is semantically closed just in case it can express its own semantic concepts, such as truth and satisfaction, and therefore all truths about L can be expressed in L – that is, a semantic theory of L can be stated in L. For instance, English is widely taken to be semantically closed because it contains its own truth, satisfaction, and validity predicates, and we can therefore express the truth-conditions of English sentences in English.

Cotnoir (2018, p. 310) then argues as follows:

(1_{SC}) Natural languages are semantically closed.

(2_{SC}) No formal language is semantically closed.

[4] Cotnoir (2018, pp. 317–318) also argues that natural languages can exhibit vagueness whereas formal languages cannot, and so natural language sentences employing vague predicates cannot be translated into formal languages. However, since Cotnoir does not say why existing approaches to vagueness – such as epistemicism or supervaluationist and fuzzy logics – are inadequate, I have left this argument aside.

(3_{SC}) Therefore: No formal language is adequate to natural language.

In support of (1_{SC}) Cotnoir highlights that it is obvious that English can express its own semantic concepts – that is, we can talk about English in English. For instance, "'Snow is white' is true' is a perfectly meaningful English sentence. The justification for (2_{SC}) is that a formal language cannot be semantically closed on pain of inconsistency. More specifically, Cotnoir argues that a formal language cannot contain its own truth or validity predicates because doing so leads to contradiction. It follows that no formal language is adequate to natural language – that is, there are natural language sentences employing the natural language truth and validity predicates that cannot be translated into any formal language. This section focusses on the truth predicate and the next concentrates on the validity predicate.

The argument that a formal language cannot contain its own truth predicate is, of course, Tarski's (1956b) celebrated Undefinability Theorem.[5] This theorem begins with the Gödel-Tarski Diagonalisation Lemma, according to which, for any property expressible in a language, there is a sentence in that language which says of itself that it has that property. More formally:

Gödel-Tarski Diagonalisation Lemma. For any formula $C(v)$ in the language of arithmetic with 'v' as its only free variable, there is a sentence φ in that language such that $\varnothing \vdash \varphi \leftrightarrow C(\ulcorner\varphi\urcorner)$.

This Diagonalisation Lemma can then be used to prove Tarski's Undefinability Theorem, which entails that no formal language can define its own truth predicate:

Undefinability Theorem. For any formula in the language of arithmetic, $C(v)$ with 'v' as its only free variable, there is a sentence φ in that language such that $\varnothing \vdash \neg[\varphi \leftrightarrow C(\ulcorner\varphi\urcorner)]$.

Proof. Applying the Diagonalisation Lemma to the negation of $C(v)$ yields $\varnothing \vdash \varphi \leftrightarrow \neg C(\ulcorner\varphi\urcorner)$. Since $\varphi \leftrightarrow \neg C(\ulcorner\varphi\urcorner) \vdash \neg[\varphi \leftrightarrow C(\ulcorner\varphi\urcorner)]$, we have $\varnothing \vdash \neg[\varphi \leftrightarrow C(\ulcorner\varphi\urcorner)]$.

[5] This presentation of the Undefinability Theorem closely follows that of Field (2008, pp. 24–28). I follow Field in identifying sentences with their Gödel numbers for ease of exposition.

This proves that a formal language's truth predicate, Tr, cannot be defined in that language because, in order for a predicate to be the truth predicate, it must satisfy *T-Schema*:

T-Schema: For all φ: $\varphi \leftrightarrow Tr(\ulcorner\varphi\urcorner)$

However, letting C be Tr, the *Undefinability Theorem* says that there is a sentence, φ, for which *T-Schema* fails. In this context, the proof can be understood as stating that, by the *Gödel-Tarski Diagonalisation Lemma*, there is a sentence, φ, that says of itself that it is not true – that is, $\varnothing \vdash \varphi \leftrightarrow \neg Tr(\ulcorner\varphi\urcorner)$. Since this classically entails $\neg[\varphi \leftrightarrow Tr(\ulcorner\varphi\urcorner)]$, it follows that a formal language cannot contain its own truth predicate satisfying *T-Schema* on pain of inconsistency.[6]

There are, of course, a number of formal languages which are – or claim to be – semantically closed.[7] However, we need not become embroiled in disputes over their merits for it is straightforward to see that the same paradox can be used to show that natural languages cannot contain their own truth predicate satisfying *T-Schema*.[8] Although Tarski's Undefinability Theorem only applies to formal languages, one can use a Liar sentence to show that natural languages cannot be semantically closed on pain of inconsistency. Indeed, §1 of Tarski's *The Concept of Truth in Formalized Languages* is devoted to showing precisely this:

> A characteristic feature of colloquial language is its universality… it could be claimed that 'if we can speaking meaningfully about anything at all, we can also speak about it in colloquial language'. If we are to maintain this universality of everyday language in connexion with semantical investigations, we must admit into the language, in addition to its sentences and other expressions, also the names of these sentences and expressions, and sentences containing these names, as well as such semantic expressions as 'true sentence', 'name', 'denote', etc. But it is presumably just this universality of everyday language

[6] Cotnoir (2018, pp. 312–313) also appeals to (Beall, 2015) to argue that a formal language cannot be closed on not only on pain of inconsistency, but on pain of *triviality*. However, I will set this argument aside since Beall is explicit that his proof holds for natural languages – indeed, the claim he proves is that "any true English theory of English is either incomplete with respect to all truths about English or… trivial" (2015, p. 579).

[7] For instance, see Kripke (1975) and Field (2008).

[8] Indeed, Cotnoir's claim that the Liar undermines the semantic closure of formal but not natural languages is at odds with much of the existing literature. For instance, one of Priest's (2006b, Chs.1, 9) central arguments for dialetheism is that it allows natural languages to be semantically closed without descent into triviality.

> which is the primary source of all semantical antimonies, like the antimonies of the liar or of heterological words. *These antimonies seem to provide a proof that every language which is universal in the above sense, and for which the normal laws of logic hold, must be inconsistent* [my emphasis] (Tarski, 1956b, pp. 164–165).

Assuming the inference from '*A* iff it is not the case that *A*' to '*A* and it is not the case that *A*' is truth-preserving, Tarski (1956b, p. 165) contends that any semantically closed natural language cannot be consistent because a closed language must satisfy the following three conditions:

(1) For any sentence which occurs in the language a definite name of this sentence also belongs to the language.

(2) For any sentence *A*, '*A*' is true iff *A*.

(3) One can formulate a true and empirically established sentence stating that *A* is identical with '*A* is false'.

But it can then be proven that a semantically closed language satisfying these conditions is inconsistent. By (1), let '*A*' be the name of the sentence, '*A* is false'. By (2), '*A* is false' is true iff *A* is false. By (3), *A* is true iff *A* is false, which entails *A* and it is not the case that *A*. Accordingly, Tarski concludes:

> [T]he very possibility of a consistent use of the expression 'true sentence' which is in harmony with the laws of logic and the spirit of everyday language seems to be very questionable (1956b, p. 165)

Thus, like formal languages, natural languages can also only contain their own truth predicate satisfying *T-Schema* on pain of inconsistency. Accordingly, the Liar cannot be used to show that formal languages cannot contain their own truth predicate satisfying *T-Schema* if one wishes to simultaneously uphold that natural languages can. As such, it has not been shown that valid natural language arguments featuring a truth predicate satisfying *T-Schema* cannot have formally valid counterparts in any logic.

6 THE EXPRESSIVE LIMITATIONS ARGUMENT FROM SEMANTIC CLOSURE II

The second expressive limitations argument from sematic closure focusses on the validity predicate. The argument that a formal language cannot contain its own validity predicate centres on a novel version of Curry's Paradox articulated by Beall & Murzi (2013), which they dub 'v-Curry'.

To state v-Curry, we must first define the validity predicate in and for the formal language in question, $val(x, y)$. For $val(x, y)$ to be the validity predicate, it must unrestrictedly satisfy the following schema:

V-Schema: For all Γ, φ: $\Gamma \vdash \varphi$ iff $\varnothing \vdash val(\ulcorner\Gamma\urcorner, \ulcorner\varphi\urcorner)$

As Beall & Murzi (2013, p. 152) put it, "if [$\langle\Gamma, \varphi\rangle$] is in the validity relation, then saying as much – using the validity predicate – is true in a validity-strength fashion", and *vice versa*. Moreover, Beall & Murzi claim that the validity predicate is detachable in the sense that, from Γ and the validity of the argument $\langle\Gamma, \varphi\rangle$, one can validly infer φ:

V-Detach: $\Gamma, val(\ulcorner\Gamma\urcorner, \ulcorner\varphi\urcorner) \vdash \varphi$

After all, if Γ is true and the argument from Γ to φ is necessarily truth-preserving, then φ must also be true. Finally, let π be a sentence saying that the argument from itself to absurdity, $\langle\pi, \bot\rangle$, is valid:

$$\varnothing \vdash \pi \leftrightarrow val(\ulcorner\pi\urcorner, \ulcorner\bot\urcorner)$$

v-Curry may then be stated as follows:

(1) $\varnothing \vdash \pi \leftrightarrow val(\ulcorner\pi\urcorner, \ulcorner\bot\urcorner)$ (π def.)

(2) $\pi \vdash \pi$ (Assumption)

(3) $\pi \vdash val(\ulcorner\pi\urcorner, \ulcorner\bot\urcorner)$ (1,2, MP)

(4) $\pi \vdash \bot$ (2,3, *V-Detach*)

(5) $\varnothing \vdash val(\ulcorner\pi\urcorner, \ulcorner\bot\urcorner)$ (4, *V-Schema*)

(6) $\varnothing \vdash \pi$ (1,5, MP)

(7) $\varnothing \vdash \bot$ (5,6, *V-Detach*)

Thus, one cannot have a validity predicate satisfying *V-Schema* and *V-Detach* on pain of absurdity.[9] However, in much the same way that *T-Schema* is constitutive of the truth predicate, Beall & Murzi highlight that "both [*V-Schema*] and [*V-Detach*] are required in order for the validity predicate to express validity" (2013, p. 157). Thus, v-Curry shows that no formal language can contain its own validity predicate on pain of absurdity.

This argument suffers from the same flaw as the truth predicate argument. Namely, we can reconstruct v-Curry in a natural language to show it cannot contain its own validity predicate satisfying the natural language analogues of *V-Schema* and *V-Detach*:

V-Schema$_{NL}$: For all A, B: A entails B iff the argument from 'A' to 'B' is valid.

V-Detach$_{NL}$: A and the validity of the argument from A to B entail B.

Let our natural language v-Curry sentence, A, be, 'The argument from 'A' to '$0 = 1$' is valid'. We can then reconstruct v-Curry as follows. By the definition of A we have:

(1) A iff the argument from 'A' to '$0 = 1$' is valid.

Now, given (1) and supposing that A is true, we know:

(2) The supposition that A entails A.

(3) The supposition that A entails that the argument from 'A' to '$0 = 1$' is valid.

[9] That is, unless one is willing to forgo the claim that $\vdash$ structurally contracts – that $\vdash$ is such that: If $\alpha, \alpha \vdash \beta$ then $\alpha \vdash \beta$. However, as Cotnoir highlights, the logic resulting from abandoning structural contraction will be too weak to be correct.

Thus, under the supposition that A, we know that A and that the argument from 'A' to '$0 = 1$' is valid, from which it follows via *V-Detach*$_{NL}$ that:

(4) The supposition that A entails $0 = 1$.

Since A entails $0 = 1$, by *V-Schema*$_{NL}$ we can infer:

(5) The argument from 'A' to '$0 = 1$' is valid.

Given that, per (1), A is true iff the argument from 'A' to '$0 = 1$' is valid, it follows from (5) via *modus ponens* that:

(6) A.

Since (5) says that the argument from 'A' to '$0 = 1$' is valid and (6) says that A is true, it follows via *V-Detach*$_{NL}$ that:

(7) $0 = 1$.

Thus, the argument used to show that a formal language cannot contain its own validity predicate satisfying *V-Schema* and *V-Detach* equally shows that a natural language cannot contain its own validity predicate satisfying *V-Schema*$_{NL}$ and *V-Detach*$_{NL}$. Accordingly, v-Curry cannot be used to show that formal languages cannot contain their own validity predicate if one wishes to simultaneously uphold that natural languages can. Thus, it has not been shown that valid natural language arguments featuring a validity predicate satisfying *V-Schema*$_{NL}$ and *V-Detach*$_{NL}$ cannot have formally valid counterparts in any logic.

7 THE EXPRESSIVE LIMITATIONS ARGUMENT FROM UNRESTRICTED QUANTIFICATION

The final argument from expressive limitations concerns *unrestricted quantification*. Natural language sentences featuring quantificational expressions such as 'Every', 'Some', and 'Nothing' are evaluated relative to a domain of entities that they are said to quantify over. To borrow an example from Williamson (2003, p. 415), suppose that I am about to take a flight and say 'Everything is in my suitcase'. I do not mean that my suitcase contains absolutely everything that there is. Rather, the domain of entities over which 'Everything' quantifies is restricted by the context to those objects which are relevant – my toothbrush, phone charger, passport, and the like. An unrestricted quantifier is one which quantifies over *absolutely* everything that there is – that is, there is no entity that is not in its domain.

The unrestricted quantification argument from expressive limitations proceeds as follows:

(1_{UQ}) Natural languages have absolutely unrestricted quantifiers.

(2_{UQ}) No formal language has absolutely unrestricted quantifiers.

(3_{UQ}) Therefore, no formal language is adequate to natural language.

In support of (1_{UQ}), Cotnoir argues that there are many natural language sentences which are most plausibly understood as quantifying over absolutely everything. For instance, if I say, 'Everything is concrete', I am saying that *absolutely all* the objects which exist are concrete. Indeed, if this sentence was not quantifying over absolutely everything, we might end up in a situation where the sentence is true even though there are non-concrete entities which lie outside its domain of quantification. Moreover, denying (1_{UQ}) is incoherent – after all, the sentence, 'It is impossible to quantify over absolutely everything' entails, 'There is something over which one cannot quantify', which is self-contradictory since this sentence quantifies over that which it says cannot be quantified over (Williamson, 2003, pp. 427–435).

The argument for (2_{UQ}) centres upon the semantics for quantifiers in formal languages. Let a model, $\mathscr{M}$, be an ordered pair of a domain, $\mathscr{D}$, which contains the entities that exist in the model, and an interpretation function, $\mathscr{I}$, which maps constants and predicates to members of and relations over $\mathscr{D}$, respectively. A universally quantified formula such as $\forall x Fx$ is true in $\mathscr{M}$ just in case every member of the domain being quantified over, $\mathscr{D}$, is F – that is, iff for every $u \in \mathscr{D}$, $u \in \mathscr{I}(F)$. Crucially, domains are typically taken to be *sets*, and therefore for a quantifier in a formal language to be unrestricted – that is, for it to quantify over absolutely everything – there would have to be a universal set containing absolutely everything.

However, due to Russell's Paradox, there can be no universal set on pain of contradiction. One of the axiom schema of standard set theory, ZFC, is the *axiom schema of restricted comprehension* (henceforth, *Restricted Comprehension*). *Restricted Comprehension* states that, for any set A, there exists a subset of A, B, whose members are the members of A satisfying φ. More formally:

Restricted Comprehension: $\forall A \exists B \forall x[x \in B \leftrightarrow (x \in A \wedge \varphi(x))]$

Suppose that there is a universal set, $\mathscr{U}$. By *Restricted Comprehension*, there is a subset of $\mathscr{U}$, $\mathscr{R}$, comprised of all the non-self-membered sets in $\mathscr{U}$. Since $\mathscr{R} \in \mathscr{U}$, either $\mathscr{R} \in \mathscr{R}$ or $\mathscr{R} \notin \mathscr{R}$. By familiar reasoning, contradiction follows in either case. By contrast, if there is no universal set – and, in particular, if $\mathscr{R}$ is not a member of the set of which it is a subset – then no contradiction follows from $\mathscr{R} \notin \mathscr{R}$.[10] Thus, there cannot be a universal set, and since unrestricted quantification requires the existence of a universal set, no formal language can contain unrestricted quantifiers. Accordingly, any valid natural language argument featuring an unrestricted quantifier *cannot* have a $\mathscr{L}_i$-valid formal counterpart, and so no formal language is adequate to natural language.[11]

Notice that Cotnoir's argument rests entirely upon two claims: that quantification in formal languages *does* involve quantification over sets, but that quantification in natural languages *does not*. The former claim is a variant of Cartwright's (1994, p. 7) *All-in-One Principle* but restricted to formal languages, and can be stated thus:

All-in-One$_{FL}$: Quantification in formal languages involves quantification over sets or set-like domains.

[10] For a more formal exposition, see Goldrei (1996, p. 90).

[11] It is again worth noting that Cotnoir's contention that Russell's paradox precludes unrestricted quantification in formal but not natural languages diverges from the existing literature – as evidenced by the fact that generality relativists have gone to great lengths to explain the meaning of seemingly unrestrictedly quantified natural language sentences (e.g. Glanzberg, 2004, 2006).

Some have tried to call *All-in-One*$_{FL}$ into question by appealing to *plural quantification*, on which the domain quantified over is not a set but *some* objects without the further assumption that these objects form a set. However, it is a matter of great controversy whether plural quantification truly is as ontologically innocent as its proponents claim and circumvents commitment to set-like domains.[12] Happily, there is no need to become embroiled in these controversies, for I shall argue that Cotnoir's second claim – that natural language quantification does not involve quantification over sets – is false. The upshot of this is that, by Russell's paradox, unrestricted quantification can only be had in natural languages on pain of contradiction.

Recall that the motivation for *All-in-One*$_{FL}$ was that the semantics for quantifiers in formal languages are spelt out in terms of a domain, which is the set of entities being quantified over. Crucially, the thesis that natural language quantification involves quantification over sets, *All-in-One*$_{NL}$ can be motivated in the same way. According to *All-in-One*$_{NL}$:

All-in-One$_{NL}$: Quantification in natural languages involves quantification over sets or set-like domains.

Over the past few decades, *generalised quantifier theory* has emerged as the dominant theory of the meanings of natural language quantifiers (see Glanzberg, 2008; Keenan & Westerståhl, 2011; Peters & Westerståhl, 2006; Westerståhl, 1989). One of generalised quantifier theory's central theses concerning the meanings (or, semantic values) of natural language quantifiers is that "The semantic values of many quantifier expressions (determiners) in natural languages are relations between sets" (Glanzberg, 2008, p. 799).

For instance, according to generalised quantifier theory, there is some set, M, over which the natural language quantifiers 'Every' and 'Most' quantify. Letting 'X' and 'Y' respectively refer to the sets of all xs and ys in M, and '$|X|$' be the cardinality of X, the meanings of 'Every' and 'Most' in the sentences 'Every x is y' and 'Most xs are y' are given by:

[12] For further details on plural quantification, see Florio (2014), Rayo (2007), and Rayo & Uzquiano (2006). For arguments in favour of plural quantification's ontological innocence, see Boolos (1984, 1993), Cartwright (1994, p. 8), Lewis (1991, pp. 65–69), McKay (2006, Chs.1–2), Oliver & Smiley (2001), Uzquiano (2003), and Yi (2005, pp. 469–472); for arguments against, see Florio & Linnebo (2016), Hazen (1993), Linnebo (2003), Parsons (1990), and Resnik (1988).

Every: Every(x, y) iff $X \subseteq Y$

Most: Most(x, y) iff $|X \cap Y| > |X \setminus Y|$

Thus, the sentence 'Most species of mammals do not lay eggs' is true just in case the cardinality of the set of non-egg-laying species of mammals exceeds that of the set of egg-laying species of mammals – that is, there are more species of non-egg-laying than egg-laying mammals.

Now consider a natural language sentence featuring an unrestricted quantifier, such as, 'Everything is concrete'. By *Every*, this sentence is true just in case the set of entities quantified over by 'Everything', M, is a subset of the set of things that are concrete, C – that is, in this case, $M = C$. However, for 'Everything' to be unrestricted – that is, for 'Everything' to quantify over all there is – it must be the case that M is the set of all things. Thus, according to our best theory of the meanings of natural language quantifiers, unrestricted quantification in natural language is no less committed to the existence of a universal set than unrestricted quantification in formal languages. The upshot of this is that, by Russell's paradox, unrestricted quantification can only be had in natural language on pain of contradiction.

Thus, like formal languages, natural languages can also only contain unrestricted quantifiers on pain of inconsistency. Accordingly, Russell's paradox cannot be used to show that formal languages cannot contain unrestricted quantifiers if one wishes to simultaneously uphold that natural languages can. As such, it has not been shown that valid natural language arguments featuring truth and validity predicates or absolutely unrestricted quantifiers cannot have formally valid counterparts in any logic. That is, the arguments from expressive limitations have not shown that there cannot be correct logics, and the existence claim remains unscathed.

CONCLUSION

Our task in this chapter and the previous has been to defend the lower bound thesis that there is *at least* one correct logic. This task was broken down into two parts. First, justifying the *non-emptiness claim* that the logical consequence relation proper is non-empty. Second, justifying the *existence claim* that there actually is a logic that correctly codifies this non-empty relation. This chapter has defended the existence claim from Cotnoir's pessimistic nihilism, according to which there cannot be a correct logic even though the logical consequence relation proper is non-empty. Cotnoir supported this claim using two families of arguments, the arguments from diversity and the arguments from expressive limitations. Both arguments aimed to demonstrate that, for any logic $\mathscr{L}_i$, there are valid natural language arguments that cannot have $\mathscr{L}_i$-valid formal counterparts. For the arguments from diversity this was because any logic whose consequence relation is necessary or formal must be extremely weak in light of certain pluralist claims. For the arguments from expressive limitations this was because there are natural language expressions which cannot be translated into any formal language on pain of contradiction. I argued that neither threaten the existence claim because the reasons given for thinking that certain valid natural language arguments cannot have formally valid counterparts in any logic simultaneously undercut the validity of said natural language arguments. With the existence claim now safe from the pessimistic nihilist's clutches, our defence of the lower bound thesis that there is at least one correct logic is complete.

Conclusion

The present work has sought to defend logical monism, the position that there is exactly one correct logic. To do so, it has defended two intermediate theses which together entail monism, namely:

Upper Bound: There is at most one correct logic.

Lower Bound: There is at least one correct logic.

Whilst Part I defended *Upper Bound* from logical pluralists who contend that there is more than one correct logic, Part II defended *Lower Bound* from logical nihilists who maintain that there are no correct logics.

The groundwork for the defence of *Upper Bound* was laid in Chapter 2, where it was argued that the prevailing conception of the normativity of logic is *incomplete* because logic is *doubly* normative for reasoning. That is, logic not only constrains the combinations of beliefs that agents may have, as has been widely recognised, but it also constrains the methods by which they may form them. In particular, I argued that, from an objective and ideal standpoint, agents must not only either believe the logical consequences of their belief sets or else revise them, but are also forbidden from forming beliefs via invalid deductive inferences.

Chapter 3 then began the defence of the upper bound thesis in ernest. It argued that, given logic is doubly normative for reasoning, almost all of the logical pluralists' proposals for cashing out the claim that there is more than one correct logic are *inconsistent*. In support of this I articulated the *normative contradiction argument*, according to which most pluralisms entail logically contradictory claims about how subjects ought to reason whenever they ought to believe some set of propositions, Γ, and φ follows from Γ according to one of the correct logics but not another. The normative contradiction argument did leave some pluralisms unscathed – namely, Haack's meaning-relative pluralism and the domain-relative pluralisms of Lynch and Pedersen. Chapter 4 suggested that the arguments profferred by neither kind of pluralist demonstrate that there is more than one correct logic, thereby completing our defence of *Upper Bound*.

Part II sought to complete the defence of monism by defending *Lower Bound*. For there to be at least one correct logic, two further claims had to be true. First, the *non-emptiness claim* that the logical consequence relation proper is non-empty. Second, the *existence claim* that there actually is a logic which correctly codifies said non-empty relation.

Chapter 5 sought to establish the non-emptiness claim in spite of opposition from obliterative nihilists such as Russell. Obliterative nihilists claim that the logical consequence relation proper is empty because there are counterexamples to even the most basic of putative logical laws, such as conjunction introduction and identity. My defence of the non-emptiness claim began by disarming the obliterative nihilists' counterexamples, which I argued were either not instances of the laws that they were intended to invalidate or, if they were, that they lacked the true premisses and false conclusions needed to invalidate them. I then went on the offensive and constructed a positive abductive argument for the existence of certain logical laws. According to this argument, the hypothesis that there are logical laws better explains evidence about which arguments are necessarily truth-preserving than any of the hypotheses available to the obliterative nihilist, thereby establishing the non-emptiness claim.

Chapter 6 defended the existence claim from Cotnoir's pessimistic nihilism. According to pessimistic nihilists, there cannot be a logic which correctly codifies the non-empty consequence relation because, for any logic $\mathscr{L}_i$, there are valid natural language arguments that will lack $\mathscr{L}_i$-valid formal counterparts. Cotnoir gave two families of arguments in support of this contention. First, the *arguments from diversity*, according to which a correct logic's consequence relation must instantiate certain properties such as necessity and formality, but doing so imposes strictures upon its consequence relation that preclude it from validating every valid natural language argument. Second, the *arguments from expressive limitations*, which aimed to show that there are natural language sentences that cannot be translated into formal languages, and therefore any valid natural language argument in which they feature as premisses cannot have a formal counterpart, let alone one which is formally valid. In reply, I argued that neither family of arguments succeeded in establishing their intended conclusion because the reasons given for thinking that certain valid natural language arguments cannot have formally valid counterparts in any logic simultaneously undercut the validity of said natural language arguments.

Where does this leave monists? Although I do think that there are such things as conclusive philosophical arguments, those given here are not in their number. Even if the arguments that I have given in support of monism were sound – something which pluralists and nihilists will no doubt deny – they far from settle the question at hand. In particular, they leave plenty of room for new pluralisms which avoid the normative contradiction argument, and certainly do not preclude there from being other arguments in support of either kind of nihilism. What I do hope to have done, however, is bought monists some temporary reprieve and shown that monism remains defensible at the present time.

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