Job Search Costs and Incentives*

Andriy Zapechelnyuk[†] Ro'i Zultan[‡]

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Abstract

We demonstrate that policies aimed at reducing frictional unemployment may lead to the opposite results. In a labor market with long-term wage contracts and moral hazard, any such policy reduces employees' opportunity costs of staying on a job. As employees are less worried about losing their job, a smaller share of employees is willing to exert effort, leading to a lower average productivity. Consequently, firms create fewer vacancies, resulting in lower employment and decreased welfare.

Keywords: job search, moral hazard, labor market, unemployment insurance
JEL codes: D82, J64, J65

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[†]Zapechelnyuk: School of Economics and Finance, University of St Andrews, St Andrews KY16 9AR, UK. *Tel:* +44 (0) 1334 46 2441. ORCID: 0000-0001-5033-3848. *E-mail:* az48@st-andrews.ac.uk

[‡]Zultan: Department of Economics, Ben-Gurion University of the Negev, P.O. Box 653, Beer-Sheva 84105, Israel. *Tel:* +972 (0) 8 647 2306. ORCID: 0000-0002-4256-8601. *E-mail:* zultan@bgu.ac.il

1 Introduction

Mechanisms that target reduction of out-of-pocket expenses (or increase of disposable income) of job seekers but have no effect on the job matching technology are popular in many countries and appear in various forms.[1] The costs associated with active job search are closely linked to job market participation, as they deter potential workers from active search, increase unemployment spells, and thus reduce welfare (Pries and Rogerson, 2009). Accordingly, mechanisms designed to mitigate these costs are typically seen as means to reduce frictions and to provide better incentives for participation in the job market (Grubb, 2001), or for avoiding unsuitable jobs, thus increasing the job matching efficiency (Chetty, 2008). The introduction of the UK Jobseeker's Allowance in 1996, for example, was to some extent aimed at increasing search intensity among the unemployed and, consequently, the flow into employment (Rayner et al., 2000). However, although the search intensity of job seekers increased as a result, this could be attributed to weeding out of low-intensity job seekers, who opt out of the job market (Manning, 2009). Therefore, the social implications of benefit programs for job seekers appear to be non-trivial.

We demonstrate that, counterintuitively, jobseeker benefits and policies that target reduction of job search costs may have welfare damaging effect and, moreover, may lead to a collapse of the labor market if taken to the extreme.[2] Our model is applicable to labor markets with long-term wage contracts where an employee's performance is observed after a certain period of time—as in public administration jobs, academic and medical jobs, and many types of professional employment. Our results suggest that policy makers should take into consideration the specific characteristics of the targeted labor market in determination of optimal jobseeker benefits. Otherwise, a careless use of jobseeker or unemployment benefits can be a welfare-harmful misspending of the taxpayer money.

Consider the situation where an employer wants to fill a qualified job position. It is not observable at the stage of job interviews whether a potential employee would be willing to exert enough effort to cope with assigned tasks. Furthermore, an employee's productivity becomes observable only after a certain period of time. Thus, in the initial period of employment, an employee can potentially work hard to demonstrate her abilities and to obtain permanent employment, or, alternatively, she can shirk and, when her real productivity

[1] Prominent examples are tax deductibility for job search expenses in the US (IRS Publication 529, 2011, p. 5; see also Garrison and Cummings, 2010), and unemployment benefit programs with provisions that an individual maintains the status of "job seeker" such as Jobseeker's Allowance (UK), Newstart Allowance (Australia), Employment Insurance (Canada), and the Unemployment Benefit (New Zealand), to name a few.

[2] This is without taking into account the cost of running such schemes which is an additional toll on the social welfare.

becomes known, separate and search for a new vacancy at another firm.

In the described situation, job search costs play a crucial role. The higher the costs of searching for a new job, the weaker the incentives an employee has to "cheat" in the initial period of employment and move on to another employer. Therefore, a decrease in the job search costs could be welfare damaging, since fewer employees would be willing to exert effort and to continue being employed by the same firm, and more individuals would be searching for a job. This, in turn, reduces firms' benefits of opening a vacancy. Eventually, when these benefits drop below the cost of maintaining a vacancy, firms refrain from offering vacancies, and the labor market collapses.

Our paper contributes to the literature concerned with the relationship between unemployment and incentives. A substantial part of the literature focuses on the adverse effects of unemployment benefits on job search effort (Holmlund, 1998; Pissarides, 2000; Fredriksson and Holmlund, 2006). Unemployment benefits provide an income insurance for risk-averse workers on the one hand, but reduce incentives for job search (thus leading to higher unemployment rates) on the other hand.[3] A solution for the incentive problem that the existing literature largely supports is conditioning unemployment benefits on active job search, in other words, restricting the benefits to individuals who seek employment, and only for a limited time after losing the previous job (e.g., Fredriksson and Holmlund, 2006). There remain adverse effects of unemployment insurance on the intensity of search, as unemployed workers reduce the effort invested in finding vacancies (Baily, 1978; Shavell and Weiss, 1979). In comparison, in our model, unemployment benefits may be welfare damaging even when only job seekers are targeted and search intensity is fixed. An increase in the job seekers' utility undermines incentives for workers to exert *on-the-job effort* and makes individuals more eager to quit one job and seek another.

Our paper is also closely related to the efficiency wage literature. Shapiro and Stiglitz (1984) suggested *involuntarily* unemployment as an incentive device. In their model, firms strategically reduce the number of vacancies, so that some workers remain involuntarily unemployed, so the threat of being laid off motivates employed workers to exert effort. The main insight of Shapiro and Stiglitz (1984) is that "Imperfect monitoring [of workers' on-the-job effort] necessitates unemployment." In contrast, in our model involuntary unemployment is not a necessity. In fact, all unemployment is *voluntary* in equilibrium, as any unemployed worker can always secure a position with certainty. Our model can be viewed as a simplified

[3]Under certain assumptions an increase in unemployment benefits can mitigate, rather than aggravate, the incentive problem, by promoting flow into (and creation of) high-productivity jobs (Acemoglu and Shimer, 2000) or through increasing the equilibrium performance-based component of the wage contracts (Demougin and Helm, 2011).

environment revealing the distinct role of job search costs.[4] Thus, job search costs play a complementary role to the utility loss of being unemployed. In this sense, we go further than Shapiro and Stiglitz (1984) in exploring policy implications with regard to job search costs, an issue that has not been raised in previous research. Furthermore, by considering participation choices of heterogeneous workers, we can study the effects of search costs as a discipline mechanism on a labor market that allows for simultaneous productive participation, non-productive participation, and non-participation in equilibrium.[5]

2 Model

We consider a discrete-time labor market populated by a unit mass of heterogeneous individuals and a large set of homogeneous firms that create job positions. Individuals and firms are risk neutral, infinitely lived and maximize their total lifetime utility with discount factor $\delta \in (0, 1)$.

Each individual is characterized by her cost of effort $c \in \mathbb{R}_+$, which is private information. At the beginning of every period, each individual can be in one of two states, employed or unemployed.[6] Each unemployed individual decides whether to participate in the labor market or to stay out. An individual who stays out (called *inactive*) receives unemployment income $u_0 \geq 0$ in that period. An individual who searches for a job incurs some matching cost and may receive some form of jobseeker's benefits. The net expenses of an individual for finding a job match will be denoted by s and referred to as the *search cost*.[7] This cost is sunk and cannot be recovered. The matching technology is given by matching function $m(x, y) = \min\{x, y\}$,[8] where x and y represent the masses of participating individuals and firms, respectively. Thus, the probability that a job seeker finds a match is given by $\min\{y/x, 1\}$.

At the beginning of every period, each firm has a single job position that can be either filled or vacant. A firm that has a vacancy and is actively searching to fill it incurs a fee s_F in each period, until the vacancy is filled or withdrawn. To make a distinction from the

[4] This role remains qualitatively unchanged when incorporating matching friction and involuntary unemployment, as we discuss in Section 2 below.

[5] Other related papers are Guerrieri (2008) and Matouschek et al. (2009). Guerrieri (2008) is concerned about inefficiencies in a dynamic search model where workers' disutility from labor is subject to random shocks; Matouschek et al. (2009) study the effect of the separation cost on the willingness of firms and employees to invest into continuation of the relationship after an exogenous shock.

[6] Here *unemployed* has its literal meaning of not being employed. This term does not make a distinction between jobless individuals who are looking for a job and who are not.

[7] The search cost is exogenous and it does not include any foregone wages while being unemployed.

[8] This assumption is made for clarity of exposition, it is not crucial for the results (as discussed at the end of this section).

individuals' job search cost, we shall refer to s_F as the *advertising cost*. Because maintaining a vacancy is costly, the total mass of vacant job positions will be endogenously determined (the firms will create jobs so long as the cost of maintaining a vacancy does not exceed the expected benefit).

After an individual has been matched with a firm, she becomes an employee. In every period of employment, the employee is paid a constant wage w that divides the expected surplus from the job match between the individual and the firm in a fixed proportion. The share of the employee is β.[9] At the end of the period, with exogenous probability $1-\alpha$ (job destruction rate) the employee quits the job and returns to the market.

Let us now introduce a moral hazard aspect to our story. In each period, an employee decides either to exert effort or to shirk. High effort results in high revenue π for the firm, $\pi > u_0$; low effort results in low revenue for the firm, normalized to zero. We assume that the wage is a binding contract, so the firm has to pay the same wage irrespective of productivity. However, the firm can motivate its employee to exert high effort by deciding whether to keep her on the job or to fire her on the basis of her past performance. The history of an employee's performance that preceded employment in the current firm is not observable and cannot be conditioned upon. An employee that lost her job becomes unemployed and immediately, before the beginning of the next period, makes a decision whether to participate in the labor market or to stay out, and so on.

We focus on stationary Bayesian Nash equilibria in steady state only.[10] In addition, we make the following assumptions on the distribution of individuals' types (costs of high effort). Denote by F the cumulative distribution function of individuals' types. Assume that F is differentiable, and denote by f its density. Further, assume that F first order stochastically dominates the uniform distribution on domain $[0, \alpha\delta\beta(\pi - u_0)]$. Roughly speaking, this assumption rules out the case that individuals with relatively high cost of effort (those who prefer to shirk when employed) are so small a minority that their participation can be accommodated in equilibrium, because the average productivity is high enough. The effect of this assumption on the results is discussed in Remark 1 (Section 4.3).

Discussion of assumptions. Our model is deliberately simple and stylized, yet it delivers a robust insight. There are, however, two key assumptions that drive the results. First, we assume that individuals have no reputation, as a firm cannot observe an employee's history prior to employment at that firm. In practice, reputation often plays an important role in

[9]The bargaining approach to wage determination is very common in the search literature (see, e.g., Diamond (1982), Mortensen (1982), and Pissarides (2000) for justification and discussions.)

[10]We do not restrict strategies of the market participants to stationary ones—we consider equilibria where deviations to nonstationary strategies are not beneficial.

labor markets, imposing an additional cost on job search for individuals who have shirked in the past. Yet, so long as there is a fair degree of non-transparency of employment histories (so that the cost imposed by bad reputation is not too high), our results remain qualitatively unchanged.

Second, we assume that an employee's compensation cannot be conditioned on output. This assumption holds in many industries where salaries and employment contracts are negotiated between employers and employee's unions. If deferred compensation were possible, as in Moen and Rosén (2011) and Tsuyuhara (2016), it would solve the moral hazard problem, so the issue that we address in this paper would not arise.In fact, what really matters for our results is (i) the initial-period wage, and (ii) the present value of all subsequent wages net of the costs of effort. If the initial-period wage is lower than wages in the subsequent periods, our results are mitigated but qualitatively remain intact, as long as the initial period wage is greater than the search cost.[11]

There are also two stylized assumptions that can be easily relaxed. The assumption that a firm perfectly observes its employee's effort simplifies a sizable part of the analysis, but it is not necessary for the results. In fact, imperfect observability of effort would only aggravate the moral hazard problem, making our argument even stronger. The assumption that the matching function is $m(x, y) = \min\{x, y\}$, so job seekers are instantly matched to available vacancies, is also made for expositional purposes. Allowing for more general matching functions mitigates the moral hazard problem, since it creates *congestion externalities*, i.e., job search is increasingly more expensive as the pool of unemployed individuals grows (Pissarides, 2000). However, it cannot eliminate the problem completely, as the congestion externality is a second-order effect. The implied increase in the search length will partially offset the effect of decreases in search costs, but will not reverse it. Thus, the role of the exogenous search cost will remain qualitatively unchanged.

3 Results

To start with, consider a conventional performance-pay model, where a firm can decide, conditional on the recent performance of its employee, whether to pay the wage and keep the employee on the job or to fire the employee *without pay*. This is a model without moral hazard, as low performance is observable and punishable. In this benchmark model, reducing job search costs is always beneficial as it diminishes the job search friction.

[11] Assuming that there is a sufficient mass of individuals with high cost of effort who prefer to shirk even if the differential between the initial and subsequent wages is large.

Proposition 1. *In the model with no moral hazard, a reduction of the job search cost is welfare improving.*

See Appendix A.1 for details.

We now consider the model *with* moral hazard. In this model the wage must be paid to an employee irrespective of her performance, but the firm can motivate its employee to exert high effort by deciding to keep her on the job or not after having observed the performance.

3.1 Interaction of a Firm and an Employee

Let us begin with describing the optimal employment strategy for firms.

Lemma 1. *A firm's optimal strategy is to lay off its employee if and only if the last-period performance is low, irrespective of the previous history of her performance in that firm.*

Proof. The described strategy maximizes the difference between the payoff of one who always exerts high effort and the payoff of one who shirks in a single period, thus maximizing the set of types of individuals who would choose to exert high effort in all periods. ■

Let us derive an employee's equilibrium effort decision. By assumption, the employee's effort decision is stationary. Let us now demonstrate that an employee exerts high effort if and only if her type (cost of effort) is below some threshold level which is increasing in s. That is, as search cost s increases, more individuals are willing to exert effort. The intuition behind the result is that a higher search cost makes the strategy of shirking and searching for a new job in every period less attractive as compared to the strategy of exerting high effort and staying on the job with some probability.

Lemma 2. *An employee of type $c \in \mathbb{R}_+$ exerts high effort if and only if*

$$c < \alpha\delta s.$$

Proof. Let us compare the lifetime utilities $U_H(c)$ and U_L of an active individual who exerts, respectively, high and low effort whenever employed, starting from a period in which she is unemployed. Note that in the period where she is employed, her lifetime utility is simply $U_H(c) + s$, as in a steady state the only difference between being employed and unemployed is the job search cost. The former is given by

$$\begin{aligned} U_H(c) &= -s + (w - c) + \delta\left[\alpha(U_H(c) + s) + (1 - \alpha)U_H(c)\right] \\ &= \frac{w - c - (1 - \alpha\delta)s}{1 - \delta}. \end{aligned} \tag{1}$$

The latter is given by

$$U_L = -s + w + \delta U_L = \frac{w - s}{1 - \delta}, \tag{2}$$

since by Lemma 1 the individual is being laid off after one period of employment and hence re-enters the labor market and pays search cost s in every period. Note that U_L is independent of c, as this person never bears the cost of high effort. Thus, an employee of type c will exert high effort if and only if $U_H(c) > U_L$,[12] or

$$\frac{w - c - (1 - \alpha\delta)s}{1 - \delta} > \frac{w - s}{1 - \delta},$$

which is equivalent to $c < \alpha\delta s$. ■

3.2 Labor Supply

Let us now analyze the equilibrium participation decision of individuals. Denote by $c^*(s)$ the threshold individual type who is indifferent between exerting high or low effort,

$$c^*(s) = \alpha\delta s.$$

Lemma 3. *Let w be a steady-state equilibrium wage and denote*

$$\bar{w}(c) = \begin{cases} u_0 + s - (c^*(s) - c), & \text{if } c < c^*(s), \\ u_0 + s, & \text{if } c \geq c^*(s). \end{cases}$$

An individual of type c will participate in the labor market if w exceeds threshold wage $\bar{w}(c)$, she will be indifferent if $w = \bar{w}(c)$, and she will stay out otherwise.

Function $\bar{w}(c)$ represents the inverse labor supply curve. In other words, for a given wage $w < u_0 + s$, individuals of types below $c = \bar{w}^{-1}(w)$ will participate. At $w = u_0 + s$, the labor supply is discontinuous and jumps up, and for $w > u_0 + s$ it includes *all* types, since the individuals who shirk (with $c \geq c^*(s)$) are now willing to participate too, and their utility does not depend on their cost of effort.

Proof. Let us compare the lifetime utility of an individual from always participating and always staying out. The utility of always staying out, U_0, is given by

$$U_0 = u_0 + \delta U_0 = \frac{u_0}{1 - \delta}. \tag{3}$$

[12] As in the proof of Lemma 6, the tie is a zero probability event and thus can be ignored.

For an individual with $c < c^*(s)$, the utility of always participating, $U_H(c)$, is given by (1). This individual participates if $U_H(c) > U_0$, or

$$w > u_0 + c + (1 - \alpha\delta)s = u_0 + s - (c^*(s) - c).$$

For an individual with $c \geq c^*(s)$, the utility of always participating, U_L, is given by (2). This individual participates if $U_L > U_0$, or $w > u_0 + s$. ■

Putting together Lemma 2 and Lemma 3, we obtain the following result.

Corollary 1. *Let w be a steady-state equilibrium wage. Then:*

(a) *if $w < u_0 + s$, then every individual of type $c < c^*(s) + w - u_0 - s$ participates with probability one and exerts high effort; every other individual stays out.*[13]

(b) *if $w > u_0 + s$, then all individuals participate with probability one, and only individuals of type $c < c^*(s)$ exert high effort;*

(c) *if $w = u_0 + s$, then every individual of type $c < c^*(s)$ participates with probability one and exerts high effort; every other individual is indifferent between participating or not, and in the event of participating she exerts low effort.*

3.3 Equilibrium

Let X and Y denote the masses of active individuals (that are searching or producing) and active firms (that have a filled job or a vacancy), respectively. We are interested in equilibria *with positive participation* where there are active participants of the labor market, $X, Y > 0$. Note that there always exists an equilibrium with zero participation, since no labor market activity ($X = Y = 0$) is a steady state.

Define

$$\underline{s} = \frac{\beta(\pi - \alpha\delta u_0)s_F}{(1-\beta)\pi + \alpha\delta s_F} \quad \text{and} \quad \bar{s} = \frac{\beta(\pi - u_0)}{1 - \alpha\delta}.$$

We can now describe the equilibrium.

Theorem 1.
(A) *There exists an equilibrium with positive participation if and only if $\underline{s} \leq s < \bar{s}$.*
(B) *The equilibrium with positive participation is unique. If $s \geq \beta(\pi - u_0)$, then it coincides with that in the benchmark model. If $s \leq \beta(\pi - u_0)$,*[14] *then it is characterized by:*

(i) *the equilibrium wage is $w = u_0 + s$;*

[13] We ignore a measure zero of individuals who are indifferent.
[14] For $s = \beta(\pi - u_0)$ both statements are true.

(ii) *every individual of type $c < \alpha\delta s$ participates with probability one and exerts high effort; every individual of type $c \geq \alpha\delta s$ participates with probability λ^* and exerts low effort,*[15] *where*

$$\lambda^* = \frac{(1-\alpha)F(\alpha\delta s)}{(1-\alpha\delta)(1-F(\alpha\delta s))} \cdot \frac{\beta(\pi - u_0) - s}{\beta u_0 + s}. \quad (4)$$

The proof is deferred to the Appendix.

It is worthwhile to note a few features of the equilibrium described in Part B. First, if the job search cost is sufficiently high, $s \geq \beta(\pi - u_0)$, then all individuals who exert low effort when being employed prefer to stay out of the labor market. That is, all individuals who participate are willing to exert high effort, and the equilibrium coincides with that in the benchmark model.

Second, if $s < \beta(\pi - u_0)$, then both groups of individuals, those who exert high effort and those who shirk, are present on the market, $\lambda^* > 0$. Full separation (only high effort workers participate, case (a) in Corollary 1) is impossible, because in that case the expected productivity is high, which drives the wage up according to the wage determination rule, thus providing incentives for "shirkers" to participate as well.

In addition, in the case of $s < \beta(\pi - u_0)$ the equilibrium wage w is equal to $u_0 + s$, so it is increasing in search cost s. This is because the wage must be kept low enough to prevent individuals who shirk on the job from entering the market in their entire mass. If s goes down, then the strategy of shirking and switching jobs in every period becomes more attractive, so to counterbalance this effect the wage must go down as well.

Remark 1. The condition that F first order stochastically dominates the uniform distribution on domain $[0, \alpha\delta\beta(\pi - u_0)]$ is sufficient to guarantee that *pooling* (all types of individuals participate, case (b) in Corollary 1) cannot occur in equilibrium for any search cost level above $\underline{s}$. If this condition does not hold, then it may be the case that the relative mass of the individuals who shirk is small enough, so even if the entire mass of shirking individuals participates, the productivity is still high and the wage obtained by the wage determination rule is still attractive for these individuals.

Under pooling equilibrium, the search cost effect due to moral hazard is less substantial, because the mass of shirking individuals is small. Pooling equilibrium gives the lower bound on welfare, while separating equilibrium of the benchmark model (where the moral hazard problem is absent) gives the upper bound. The difference between these bounds approaches zero as the mass of shirking individuals decreases.

[15] This can be replaced with λ^* of individuals with types $c \geq \alpha\delta s$ participating and the rest staying out, or any of a continuum of payoff-equivalent equilibria that maintain a mass λ^* of this group participating in expectation. So long as the expected mass λ^* is preserved, which individuals participate and which stay out is arbitrary (recall that the type of these individuals is irrelevant since they shirk whenever employed).

Absent the aforementioned condition on F, complete characterization of equilibria is technically difficult. The reason is that as s increases, there are two opposing effects on the benefit from participation: (i) the negative effect of more expensive search; (ii) the positive effect of a higher payoff difference between exerting effort and shirking, leading to a smaller proportion of shirkers among job seekers, and hence a higher wage. Depending on the shape of F, these effects may interact nonmonotonically: As s varies, the *pooling wage* (above which all types of individuals are willing to participate) may cross the equilibrium wage multiple times. Hence, pooling equilibria may change to semi-separating equilibria and back at arbitrary points as s increases.

3.4 Comparative Statics

Now we demonstrate that, in sharp contrast to the benchmark model, a reduction of individuals' search cost in the model with moral hazard is welfare damaging for $s < \beta(\pi - u_0)$ and eventually leads to collapse of the labor market.

The intuition behind this result is as follows. As we noted before, a reduction of s to s' leads to higher incentives for 'shirkers' to participate. Thus, in equilibrium, wage goes down to counterbalance that effect. But all individuals who exert high effort pay the search cost rarely, while receiving the wage in every period. Thus their utility goes down, so some (who would have worked with high effort otherwise) leave the market.

As a result, under lower search cost there are fewer workers who exert high effort and contribute to welfare. The labor market shrinks, total production goes down, and this effect dominates the benefit of having lower search costs. Eventually, as s continues to go down, the expected benefit for firms from opening a vacancy becomes lower than the cost of maintaining the vacancy, thus labor demand drops to zero and the market collapses.

Proposition 2. *In the model with moral hazard, individual's search cost $s^* = \beta(\pi - u_0)$ maximizes the welfare. Any reduction of the search cost below s^* is welfare damaging, moreover, a reduction of the search cost below $\underline{s}$ leads to a collapse of the labor market.*

This proposition is straightforward by Lemma 4 below and Theorem 1, where we use the result in Part B that for $s \geq \beta(\pi - u_0)$ the equilibrium in the model with moral hazard coincides with that in the benchmark model.

Lemma 4. *For all $s \in [\underline{s}, \beta(\pi - u_0)]$ the welfare in the model with moral hazard is equal to*

$$W(s) = \int_0^{c^*(s)} \frac{\alpha s - c}{1-\delta} dF(c) + \frac{\alpha(\pi - u_0 - s)}{1 - \alpha\delta} F(c^*(s)) \tag{5}$$

and it is strictly increasing in s.

For all $s \in (0, \bar{s})$ the welfare in the benchmark model is equal to

$$\overline{W}(s) = \int_0^{\bar{c}(s)} \frac{[\beta(\pi - u_0) - s] + \alpha s - c}{1 - \delta} dF(c) + \frac{\alpha(1-\beta)(\pi - u_0)}{1 - \alpha\delta} F(\bar{c}(s)) \tag{6}$$

and it is strictly decreasing in s.

The proof is deferred to the Appendix.

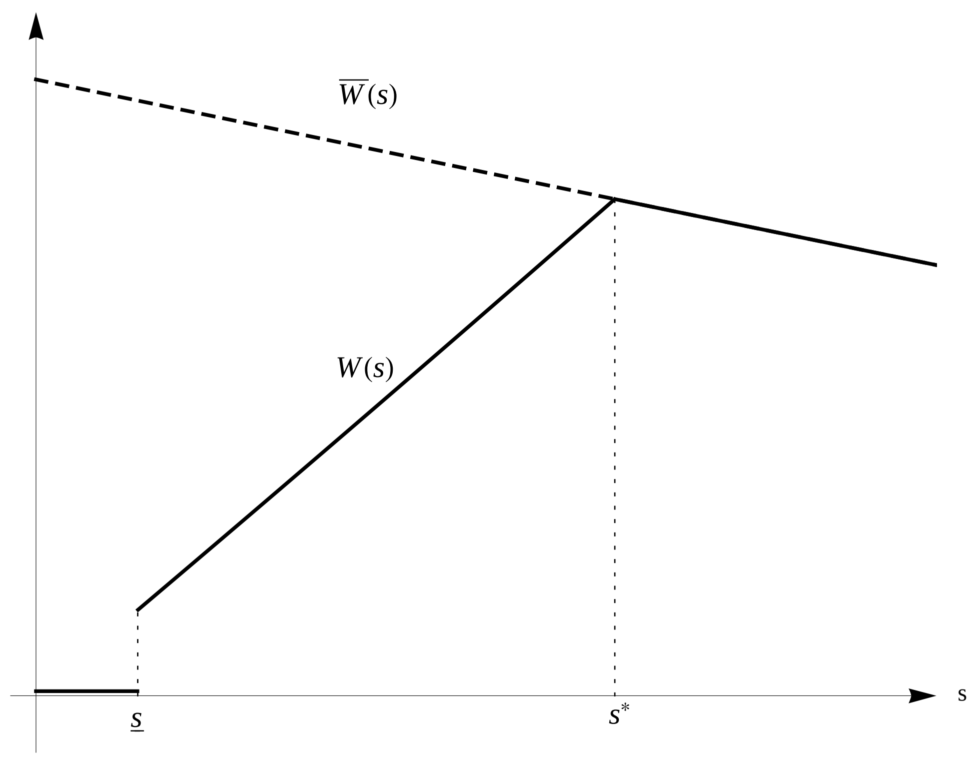

Fig. 1: Comparison of the total welfare in the two models

The difference in the welfare between the two models can be clearly seen by comparing (5) and (6) and is illustrated by Figure 1, where $W(s)$ is plotted as solid line and $\overline{W}(s)$ as dashed line.[16] The dominant factor that determines the welfare is the mass of individuals who participate in the labor market and exert high effort. In the benchmark model, the critical type who is indifferent between participating or not, $\bar{c}(s)$, is determined by comparing the expected wage, net of the unemployment income, and the expected costs of job search:

$$\bar{c}(s) = \beta(\pi - u_0) - (1 - \alpha\delta)s.$$

[16]The plot is done for the values of parameters $\alpha = \delta = 0.9$, $\beta = 1/2$, $\pi = 10$, $u_0 = s_F = 1$ and F uniform on $[0, \alpha\delta\beta(\pi - u_0)]$.

It follows that $c^*(s) < \bar{c}(s)$ for all $s < s^* \equiv \beta(\pi - v_0)$. The gap between $c^*(s)$ and $\bar{c}(s)$ (and consequently, between the quantities of individuals who contribute to production) is at maximum when $s = 0$ and decreases as s grows, up to the point $s = s^*$, where $c^*(s) = \bar{c}(s)$ and the equilibria in the two models become the same. The gap between the two welfare values essentially follows the same pattern, with the difference that for $s < \underline{s}$ another factor kicks in: the benefits of production are so low that firms are not willing to open vacancies. So for $s < \underline{s}$ we have $W(s) = 0$, as the labor market does not exist. At the other end of the scale, for $s > s^*$, the benefits from "shirking and switching jobs" are so low that every employee is willing to exert high effort. So the two models produce identical equilibria and identical welfare, $W(s) = \overline{W}(s)$. As the search cost goes up above s^*, it provides no additional incentives to exert effort, thus being a pure waste and hence reducing the welfare.

4 Conclusion

In this paper we consider the role of job search costs as an incentive mechanism in the presence of moral hazard, and we study the effect of policies targeting reduction of job search friction on social welfare. Without moral hazard, or when job search costs are unnaturally high, a policy aimed at reducing unemployment and increasing productivity through a reduction in job search costs is likely to be successful. However, our results illustrate that in the presence of moral hazard and when the existing search costs are not very high, such policies may backfire by removing workers' incentives to exert effort on the job, leading to lower productivity, lower wages, and higher unemployment rates.

While unemployment benefits increase unemployment, they were found to increase average productivity levels (Blanchard, 2004). This has been explained by the fact that unemployment benefits enable workers to decline current job offers and continue searching for more productive jobs (e.g., Diamond, 1981; Acemoglu and Shimer, 2000; Marimon and Zilibotti, 1999) or to negotiate contracts that in turn lead to higher productivity (Demougin and Helm, 2011). By comparison, in our model unemployment benefits have two contrasting effects. Unconditional unemployment benefits undermine the incentives to participate in the job market for the individuals who exert low effort, so that the average productivity of employed workers increases, but the total output is unaffected (cf. Manning, 2009). Unemployment benefits with eligibility constraints that only affect active job seekers effectively reduce the job search costs, which according to our analysis, remove the incentives to exert effort for some proportion of the workers, thus resulting in lower total output.

Unemployment benefits with eligibility constraints are known to affect search intensity (Baily, 1978), leading to increased frictional unemployment. This adverse effect can be mitigated through an appropriate unemployment insurance schedule and taxation (Shavell and Weiss, 1979; Hopenhayn and Nicolini, 1997). However, such mechanisms are not conducive in dealing with the on-the-job moral hazard studied in our model. Thus, extending the model to allow workers to choose search intensity would not alter our main result.

Some empirical regularities are in line with our qualitative conclusions.[17] First, the model predicts that the workforce is composed of two broad types: workers who exert effort and remain employed with the same firm long-term and those who "shirk and switch". Consistent with this view, while most new jobs break down in just over a year, the workforce is dominated by workers who survive this initial period and remain employed for around ten years (Gregg and Wadsworth, 2002). Second, our model adds to the possible explanations behind the decrease in job stability over the last couple of decades (see, e.g., Farber, 2010; Macaulay, 2003; Gregg and Wadsworth, 2002), concurrent with technological changes that reduce search costs, e.g., the proliferation of websites such as LinkedIn designed to facilitate matching. Consistent with transitions to the "shirk and switch" strategy, the decrease in the *median* job tenure is more dramatic than the decrease in the *average* job tenure. For example, Macaulay (2003) reports a drop of 3.7% in average job tenure compared to a drop of 21.3% in the median between 1996 and 2001 in the UK.

Nineteen-Ninety-six also marked the Jobseeker's Allowance reform in the UK, which effectively reduced relative job search costs. Studying the long-term effects of the JSA reform, Petrongolo (2009) found that, although the reform was successful in moving workers into employment, it had an overall negative effect on job tenure after reemployment, again consistent with the "shirk and switch" strategy. Furthermore, the reform had a negative effect on post-unemployment annual earnings, possibly reflecting the negative effect on productivity in the labor market predicted by our model. Taken together, these stylized empirical facts, although far from providing conclusive evidence for the validity of the model, support the argument that reducing job search costs has a significant effect on workers' motivation on the job and on labor market outcomes.

We see a few interesting future extensions to the paper. We incorporate any costs associated with a lengthy search to the aggregate search costs and do not model explicitly frictional unemployment in the market as such. Disentangling the different effects could lead to a more complete assessment of different policies that influence job search costs. Another interesting avenue of research would be to include heterogeneous jobs and determine the effect of job search costs on the composition of jobs offered by firms and accepted by workers. Heterogeneity of jobs would also permit to model externalities between market

[17] That is not to say that the process we outline in this paper is the only one contributing to these regularities.

participants arising from search frictions and to study the effects of search costs in the presence of these externalities. Yet another interesting question arises, as in practice job search sometimes involves obtaining certain qualifications that can be costly and time consuming but improve productivity or reduce the cost of effort. Jobseeker benefits in the form of tuition subsidies thus have a productivity-improving element. So, the overall welfare impact becomes ambiguous and is subject to further studies. Finally, a more accurate welfare analysis requires to assume risk aversion of individuals and a more general form of the matching function (e.g., Cobb-Douglas).

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Appendix A. Proofs

A.1 Proof of Proposition 1

Proposition 1. *In the model with no moral hazard, a reduction of the job search cost is welfare improving.*

The proof is divided into four subsections.

A.1.1. Labor demand. Denote by x and y the masses of searching individuals and firms, respectively, and by X and Y the masses of active individuals (that are searching or producing) and active firms (that have a filled job or a vacancy), respectively. Let J and V be a firm's value of a filled job and a vacancy, respectively. Let μ_F be the probability to fill a vacancy in a given period, $\mu_F = \min\{x/y, 1\}$. Then

$$V = -s_F + \mu_F J + (1 - \mu_F)\delta V.$$

Assume that firms create vacancies so long as $V > 0$ and withdraw them if $V < 0$. If $J < s_F$, then $V < 0$ for all μ_F, hence there will be no labor demand, $Y = 0$. However, if $J \geq s_F$, then in steady state $V = 0$ must hold, hence μ_F must satisfy

$$-s_F + \mu_F J = 0. \tag{A.1}$$

Since $s_F/J = \mu_F \equiv \min\{x/y, 1\}$, then either $s_F/J < 1$, and hence $x/y < 1$, or $s_F/J = 1$, and hence $x/y \geq 1$. Moreover, in the latter case it must be $x/y = 1$, because we have assumed that firms create vacancies so long as the cost of maintaining one, s_F, does not exceed the expected benefit, J.[18] So, if $y < x$, then new vacancies will be created until $y = x$. Consequently, the mass of job searching individuals does not exceed the mass of vacancies, so every individual finds a job immediately with certainty.[19]

Let us now derive the mass of active firms, Y. Denote by γ the fraction of individuals that are employed at the beginning of each period. Then

$$\mu_F = \frac{x}{y} = \frac{X - \gamma X}{Y - \gamma X}. \tag{A.2}$$

[18] The assumption that firms create vacancies whenever indifferent is standard and can be justified, for example, that creating a vacancy is welfare improving. Note that this assumption has a bite only in the event of very low expected benefit from hiring an employee, $J = s_F$. The results presented below are unaffected so long as this low threshold is not reached.

[19] See the discussion at the bottom of Section 2.

Combining (A.1) and (A.2) yields

$$Y = X\left(\gamma + (1-\gamma)\frac{J}{s_F}\right). \tag{A.3}$$

A.1.2. Labor Supply. In the benchmark model there is no moral hazard, so all employees exert high effort.[20] Hence firms will never dismiss workers. The fraction of individuals who lose their jobs and return to the job market is given by job destruction rate $1-\alpha$. The total revenue created by a filled job position net of the unemployment income (for short, *surplus*) is therefore equal to

$$S = \pi - u_0 + \alpha\delta(\pi - u_0) + \ldots = \frac{\pi - u_0}{1-\alpha\delta}.$$

Similarly, a firm's gross value of filling a vacancy is equal to

$$J = \frac{\pi - w}{1-\alpha\delta}.$$

We assumed that a firm's share of the surplus is $1-\beta$, so $J = (1-\beta)S$, and hence the wage is given by[21]

$$w = u_0 + \beta(\pi - u_0). \tag{A.4}$$

Let us now determine the equilibrium behavior of an individual with cost of effort c.

Lemma 5. *An individual of type $c \in \mathbb{R}_+$ participates in the job market if and only if*

$$c < \beta(\pi - u_0) - (1-\alpha\delta)s.$$

Proof. Since we are considering equilibria in steady state only, it is sufficient to compare the lifetime utility of the individual who is *always active* and the one who is *always inactive.* The utility of an inactive individual of type c is given by (3) in the main text, restated here:

$$U_0 = u_0 + \delta U_0 = \frac{u_0}{1-\delta}. \tag{A.5}$$

Now consider an active individual of type c. Denote by $U_H(c)$ her lifetime utility starting from a period where she is unemployed (subscript H stands for "high effort" for consistency

[20]Individuals with type $c > w$ will not exert high effort. But these individuals experience negative utility from labor, thus staying out of the labor market in equilibrium.

[21]We assume this simple form of wage determination to keep the analysis straightforward and transparent. The interpretation is that w is a result of negotiations between firms and the employees union; from the perspective of an individual, w is exogenous. The results are not qualitatively affected if we model bargaining as a compromise of the two parties whose disagreement options are endogenously determined in the model.

with notations in further sections). Note that in the period where she is employed, her lifetime utility is simply $U_H(c) + s$, as in a steady state the only difference between being employed and unemployed is the job search cost (as we have established above that individuals find a job immediately with probability one). This yields (1) in the main text, restated here:

$$\begin{aligned} U_H(c) &= -s + (w - c) + \delta\left[\alpha(U_H(c) + s) + (1-\alpha)U_H(c)\right] \\ &= \frac{w - c - (1-\alpha\delta)s}{1-\delta}. \end{aligned} \tag{A.6}$$

Consequently, an individual of type c will participate in the labor market if and only if[22] $U_H(c) > U_0$, or

$$c < w - u_0 - (1-\alpha\delta)s,$$

which, together with (A.4), yields the result. ■

Intuitively, an individual will participate in the labor market if her cost of effort is small relative to the expected wage, net of the unemployment income and expected costs of job search. Denote by $\bar{c}(s)$ the critical type who is indifferent between participating or not,

$$\bar{c}(s) = \beta(\pi - u_0) - (1-\alpha\delta)s.$$

A.1.3. Steady State. We are now in position to find the masses of active individuals and active firms, X and Y, in steady state.

First, by Lemma 6, only individuals of type $c < \bar{c}(s)$ will be active, hence

$$X = F(\bar{c}(s)).$$

Next, a firm's value of a filled job is given by

$$J = (1-\beta)S = \frac{(1-\beta)(\pi - u_0)}{1-\alpha\delta}.$$

Finally, after every period fraction α of active individuals remain employed, thus $\gamma = \alpha$. Consequently, by (A.3),

$$Y = F(\bar{c}(s))\left(\alpha + (1-\alpha)\frac{(1-\beta)(\pi - u_0)}{s_F(1-\alpha\delta)}\right).$$

Note that if the individuals' search cost is high enough, $s \geq \beta(\pi - u_0)/(1-\alpha\delta)$, then

[22] The tie is a zero probability event and thus can be ignored.

no individuals participate and the labor market collapses (since in that case $\bar{c}(s) \leq 0$ and $F(\bar{c}(s)) = 0$, so we have $X = 0$). Similarly, if the firms' advertising cost is high enough, $s_F > J = (1-\beta)(\pi - u_0)/(1-\alpha\delta)$, no firms are willing to open vacancies, so we have $Y = 0$ and the labor market collapses. For the rest of the paper we assume that $s_F \leq (1-\beta)(\pi - u_0)/(1-\alpha\delta)$.

A.1.4. Comparative Statics. Let us analyze the relationship between an individual's job search cost and the welfare. As in this model search cost s represents a pure waste, it is very intuitive that a reduction of s leads to welfare improvement.

Indeed, consider a reduction of the search cost from s to s'. Then the mass of the individuals who search for jobs weakly increases, as $s > s'$ entails $\bar{c}(s) < \bar{c}(s')$, and consequently $F(\bar{c}(s)) \leq F(\bar{c}(s'))$. Any individual who is participating under s' is strictly better off relative to s, as her utility has gone up due to the lower search cost. Any individual who is inactive under s' is indifferent between s and s', as her utility remains unchanged, $u_0/(1-\delta)$. Hence, the consumer surplus strictly increases as s goes down.

Next, firms with filled job positions make profit $(1-\beta)(\pi - u_0)$ in a single period. Under s' the mass of firms engaged in production is larger, and so is the total producers' surplus. ∎

A.2 Proof of Theorem 1

Theorem 1.
(A) *There exists an equilibrium with positive participation if and only if* $\underline{s} \leq s < \bar{s}$.
(B) *The equilibrium with positive participation is unique. If* $s \geq \beta(\pi - u_0)$*, then it coincides with that in the benchmark model. If* $s \leq \beta(\pi - u_0)$*,*[23] *then it is characterized by:*

(i) *the equilibrium wage is* $w = u_0 + s$*;*

(ii) *every individual of type* $c < \alpha\delta s$ *participates with probability one and exerts high effort; every individual of type* $c \geq \alpha\delta s$ *participates with probability* λ^* *and exerts low effort,*[24] *where*

$$\lambda^* = \frac{(1-\alpha)F(\alpha\delta s)}{(1-\alpha\delta)(1-F(\alpha\delta s))} \cdot \frac{\beta(\pi - u_0) - s}{\beta u_0 + s}. \tag{4}$$

A.2.1. Part (B). We will prove that if there exists an equilibrium with positive participation, then it is unique and satisfies the conditions stated in Part B.

[23] For $s = \beta(\pi - u_0)$ both statements are true.

[24] This can be replaced with λ^* of individuals with types $c \geq \alpha\delta s$ participating and the rest staying out, or any of a continuum of payoff-equivalent equilibria that maintain a mass λ^* of this group participating in expectation. So long as the expected mass λ^* is preserved, which individuals participate and which stay out is arbitrary (recall that the type of these individuals is irrelevant since they shirk whenever employed).

Recall that we focus on stationary equilibria. In particular, the best reply of a firm to a stationary strategy of the individuals (Lemma 1) and the best reply of individuals to a stationary strategy of firms (Theorem 1) are stationary. In other words, no market participants can benefit by deviating to nonstationary strategies.

Denote by p the probability that a newly hired worker has type $c < c^*(s)$ and thus will exert high effort when employed. Then, surplus S produced by a filled job vacancy is given by

$$S = p\frac{\pi - u_0}{1-\alpha\delta} + (1-p)(-u_0), \tag{A.7}$$

where $(\pi - u_0)/(1-\alpha\delta)$ is the surplus from an employee who always exerts high effort (as S in Section 3) and $(-u_0)$ is the surplus from an employee who shirks in the first period and loses the job at the end of that period.

Similarly, the value J of a filled job vacancy for a firm is given by

$$J = p\frac{\pi - w}{1-\alpha\delta} + (1-p)(-w) = S - p(\lambda)\frac{w - u_0}{1-\alpha\delta} - (1-p)(w-u_0),$$

where w is the equilibrium wage. Recall that wage determination rule requires

$$J = (1-\beta)S. \tag{A.8}$$

As J is strictly decreasing in w and $J = S$ for $w = u_0$, for any given p there exists a unique w that solves (A.8).

Suppose that $s > \beta(\pi - u_0)$. By the wage determination rule, the wage cannot exceed its feasible maximum $u_0 + \beta(\pi - u_0)$, hence $w < u_0 + s$. This corresponds to case (a) in Corollary 1, where all individuals who participate exert high effort, $p = 1$. Then we have $S = (\pi - u_0)/(1-\alpha\delta)$ and $J = (\pi - w)/(1-\alpha\delta)$. Solving (A.8) for w yields $w = u_0 + \beta(\pi - u_0)$. Note that this is the same equilibrium as in the benchmark model.

Next, suppose that $s < \beta(\pi - u_0)$. Corollary 1 implies the following labor market composition. The total mass of active individuals with type $c < c^*(s)$ is $F(c^*(s))$, of which only fraction $1-\alpha$ lose their jobs and return to the labor market. Hence the mass of individuals with type $c < c^*(s)$ on the labor market constitutes $(1-\alpha)F(c^*(s))$. Next, the total mass of individuals with type $c \geq c^*(s)$ is $1 - F(c^*(s))$, of which some fraction $\lambda \in [0,1]$ are active. Hence the mass of individuals with type $c \geq c^*(s)$ on the labor market constitutes $\lambda(1 - F(c^*(s)))$. Consequently, the probability that a newly hired worker has type $c < c^*(s)$ and thus will exert high effort is given by

$$p = p(\lambda) = \frac{(1-\alpha)F(c^*(s))}{(1-\alpha)F(c^*(s)) + \lambda(1-F(c^*(s)))} = \left(1 + \lambda\frac{1-F(c^*(s))}{(1-\alpha)F(c^*(s))}\right)^{-1} \tag{A.9}$$

Consider case (c) in Corollary 1 that stipulates $w = u_0 + s$, so condition (i) holds. Then there is a unique value of λ that solves (A.8) and it is equal to λ^* that can be easily verified. Thus we have proved that the equilibrium must satisfy condition (ii). Yet we need to verify that $\lambda^* \in [0,1]$. We have assumed that $s \leq \beta(\pi - u_0)$, thus $\lambda^* \geq 0$. Also we have assumed that F first order stochastically dominates the uniform distribution on $[0, \alpha\delta\beta(\pi - u_0)]$, i.e.,

$$F(c^*(s)) \leq \frac{c^*(s)}{\alpha\delta\beta(\pi - u_0)} = \frac{\alpha\delta s}{\alpha\delta\beta(\pi - u_0)} = \frac{s}{\beta(\pi - u_0)} \tag{A.10}$$

for all $s \in [0, \beta(\pi - u_0]$. Hence we have $F(c^*(s))\beta(\pi - u_0) \leq s$ for all $s \in [0, \beta(\pi - u_0)]$. Consequently,

$$\lambda^* = \frac{(1-\alpha)F(\alpha\delta s)}{(1-\alpha\delta)(1-F(\alpha\delta s))} \cdot \frac{\beta(\pi - u_0) - s}{\beta u_0 + s} \leq \frac{(1-\alpha)s}{\beta(1-\alpha\delta) + s} \leq 1.$$

Next, let us show that cases (a) and (b) in Corollary 1 cannot occur in equilibrium. It was shown above that case (a) entails $s > \beta(\pi - u_0)$, a contradiction. It remains to consider case (b) that stipulates $w > u_0 + s$ and $\lambda = 1$. Then we have

$$p = p(1) = \frac{(1-\alpha)F(c^*(s))}{1 - \alpha F(c^*(s))}.$$

Equation (A.8) can be rewritten as

$$p\frac{w - u_0}{1 - \alpha\delta} + (1-p)(w - u_0) = \beta\left(p\frac{\pi - u_0}{1-\alpha\delta} + (1-p)(-u_0)\right).$$

Solving for $(w - u_0)$ we obtain

$$\begin{aligned} w - u_0 &= \frac{p\beta(\pi - u_0)}{1 - \alpha\delta(1-p)} - \frac{(1-\alpha\delta)(1-p)u_0}{1-\alpha\delta(1-p)} \\ &\leq p\frac{\beta(\pi - u_0)}{1-\alpha\delta(1-p)} = \frac{(1-\alpha)F(c^*(s))}{1-\alpha F(c^*(s))} \cdot \frac{\beta(\pi - u_0)}{1-\alpha\delta(1-p)} \\ &\leq \frac{(1-\alpha)s}{(1-\alpha F(c^*(s)))(1-\alpha\delta(1-p))} \\ &\leq s, \end{aligned}$$

where the third line is by (A.10) and the last line is due to:

$$1 - \alpha\delta(1-p) = 1 - \alpha\delta\frac{1 - F(c^*(s))}{1-\alpha F(c^*(s))} = \frac{1 - \alpha(\delta + (1-\delta)F(c^*(s)))}{1 - \alpha F(c^*(s))} \geq \frac{1-\alpha}{1-\alpha F(c^*(s))}.$$

But we have assumed $w > u_0 + s$, a contradiction.

A.2.2. Part (A). First, we show that $X = 0$ for $s \geq \bar{s}$. For $s > \beta(\pi - u_0)$, the equilibrium (if it exists) is the same as in the benchmark model, and by Lemma 6, an individual of type c participates in the labor market if and only if $c < \bar{c}(s)$. But for $s \geq \bar{s}$, $\bar{c}(s) = \beta(\pi - v_0) - (1 - \alpha\delta)s \leq 0$, hence there is no participation, $X = 0$.

Second, we show that $Y = 0$ for $s < \underline{s}$. Recall that in equilibrium $Y > 0$ only if $J \geq s_F$, where

$$J = (1 - \beta)S = (1 - \beta)\left(p\frac{\pi - u_0}{1 - \alpha\delta} - (1 - p)u_0\right).$$

Substituting the value of λ^* into (A.9), after some manipulations, yields

$$p = p(\lambda^*) = \frac{(1 - \alpha\delta)(\beta u_0 + s)}{\beta(\pi - \alpha\delta u_0) - \alpha\delta s}.$$

Next, substituting $p(\lambda^*)$ into the expression for J, after some manipulations, yields

$$J = \frac{(1 - \beta)\pi s}{\beta(\pi - \alpha\delta u_0) - \alpha\delta s}. \tag{A.11}$$

At last, solving inequality $J \geq s_F$ for s yields

$$s \geq \frac{\beta(\pi - \alpha\delta u_0)s_F}{(1 - \beta)\pi + \alpha\delta s_F} \equiv \underline{s}.$$ ■

A.3 Proof of Lemma 4

Lemma 4. *For all $s \in [\underline{s}, \beta(\pi - u_0)]$ the welfare in the model with moral hazard is equal to*

$$W(s) = \int_0^{c^*(s)} \frac{\alpha s - c}{1 - \delta} dF(c) + \frac{\alpha(\pi - u_0 - s)}{1 - \alpha\delta} F(c^*(s)) \tag{5}$$

and it is strictly increasing in s.

For all $s \in (0, \bar{s})$ the welfare in the benchmark model is equal to

$$\overline{W}(s) = \int_0^{\bar{c}(s)} \frac{[\beta(\pi - u_0) - s] + \alpha s - c}{1 - \delta} dF(c) + \frac{\alpha(1 - \beta)(\pi - u_0)}{1 - \alpha\delta} F(\bar{c}(s)) \tag{6}$$

and it is strictly decreasing in s.

Let us calculate the welfare in the benchmark model first. In every period an individual of type $c < \bar{c}(s) \equiv \beta(\pi - v_0) - (1 - \alpha\delta)s$ obtains $w - c$. Also, with probability $1 - \alpha$ she is unemployed at the beginning of the period, so she pays search cost s. Thus the lifetime utility of an individual of type $c < \bar{c}(s)$ is equal to

$$\frac{w - c - (1-\alpha)s}{1-\delta} = \frac{u_0 + \beta(\pi - u_0) - c - (1-\alpha)s}{1-\delta},$$

where we used that in equilibrium the wage satisfies $w = u_0 + \beta(\pi - u_0)$. The lifetime utility of an individual of type $c \geq \bar{c}(s)$ is the unemployment income, $u_0/(1-\delta)$. Hence, the consumer's surplus (net of the unemployment income) is equal to

$$\overline{CS}(s) = \int_0^{\bar{c}(s)} \frac{\beta(\pi - u_0) - (1-\alpha)s - c}{1-\delta} dF(c).$$

Next, at the beginning of the period, the mass of firms who have a filled position is $\alpha F(\bar{c}(s))$. The lifetime utility of any such firm is $(\pi - w)/(1-\alpha\delta)$. Any firm that has an unfilled vacancy or inactive has zero lifetime utility. Hence the producers' surplus is equal to

$$\begin{aligned}\overline{PS}(s) &= \alpha F(\bar{c}(s))\frac{\pi - w}{1-\alpha\delta} = \alpha F(\bar{c}(s))\frac{\pi - (u_0 + \beta(\pi - u_0))}{1-\alpha\delta} \\ &= \alpha F(\bar{c}(s))\frac{(1-\beta)(\pi - u_0)}{1-\alpha\delta}.\end{aligned}$$

Summing up $\overline{CS}(s)$ and $\overline{PS}(s)$ gives the expression for welfare $\overline{W}(s)$. As $\bar{c}(s)$ is strictly decreasing in s, it is easy to verify that $\overline{CS}(s)$ is strictly decreasing and $\overline{PS}(s)$ is decreasing. Hence $\overline{W}(s)$ is strictly decreasing.

Let us now calculate the welfare in the model with moral hazard. In every period an individual of type $c < c^*(s) \equiv \alpha\delta s$ obtains $w - c$ and also pays search cost s with probability $1-\alpha$ that she has become unemployed at the end of the previous period. Thus the lifetime utility of an individual of type $c < c^*(s) \equiv \alpha\delta s$ is equal to

$$\frac{w - c - (1-\alpha)s}{1-\delta} = \frac{(u_0 + s) - c - (1-\alpha)s}{1-\delta} = \frac{u_0 + \alpha s - c}{1-\delta},$$

where we used that in equilibrium the wage satisfies $w = u_0 + s$. The lifetime utility of an individual of type $c \geq c^*(s)$ is the unemployment income, $u_0/(1-\delta)$. Hence, the consumer's surplus (net of the unemployment income) is equal to

$$CS(s) = \int_0^{c^*(s)} \frac{\alpha s - c}{1-\delta} dF(c).$$

Next, similarly to in the benchmark model, at the beginning of the period, the mass of firms who have a filled position is $\alpha F(c^*(s))$. The lifetime utility of any such firm is $(\pi - w)/(1 - \alpha\delta)$. Any firm that has an unfilled vacancy or inactive has zero lifetime utility. Hence the producers' surplus is equal to

$$PS(s) = \alpha F(c^*(s))\frac{\pi - w}{1 - \alpha\delta} = \alpha F(c^*(s))\frac{\pi - (u_0 + s)}{1 - \alpha\delta}.$$

Summing up $CS(s)$ and $PS(s)$ gives the expression for welfare $W(s)$. Taking the derivative of $W(s)$ (where we have used $dc^*(s)/ds = \alpha\delta$) yields

$$\begin{aligned}\frac{d}{ds}W(s) = &\frac{\alpha}{1-\delta}F(c^*(s)) + \alpha\delta f(c^*(s)) + \frac{\alpha^2\delta}{1-\alpha\delta}f(c^*(s))(\pi - u_0 - s) - \frac{\alpha}{1-\alpha\delta}F(c^*(s)) \\ =&\frac{\alpha^2\delta}{1-\alpha\delta}f(c^*(s))(\pi - u_0 - s) + \alpha\delta f(c^*(s)) + \alpha F(c^*(s))\left(\frac{1}{1-\delta} - \frac{1}{1-\alpha\delta}\right) > 0,\end{aligned}$$

since by assumption $s \leq \beta(\pi - u_0) < \pi - u_0$ and $\frac{1}{1-\delta} - \frac{1}{1-\alpha\delta} > 0$. ■

Appendix A. Proofs

A.1 Proof of Proposition 1

Proposition 1. *In the model with no moral hazard, a reduction of the job search cost is welfare improving.*

The proof is divided into four subsections.

A.1.1. Labor demand. Denote by x and y the masses of searching individuals and firms, respectively, and by X and Y the masses of active individuals (that are searching or producing) and active firms (that have a filled job or a vacancy), respectively. Let J and V be a firm's value of a filled job and a vacancy, respectively. Let μ_F be the probability to fill a vacancy in a given period, $\mu_F = \min\{x/y, 1\}$. Then

$$V = -s_F + \mu_F J + (1 - \mu_F)\delta V.$$

Assume that firms create vacancies so long as $V > 0$ and withdraw them if $V < 0$. If $J < s_F$, then $V < 0$ for all μ_F, hence there will be no labor demand, $Y = 0$. However, if $J \geq s_F$, then in steady state $V = 0$ must hold, hence μ_F must satisfy

$$-s_F + \mu_F J = 0. \tag{A.1}$$

Since $s_F/J = \mu_F \equiv \min\{x/y, 1\}$, then either $s_F/J < 1$, and hence $x/y < 1$, or $s_F/J = 1$, and hence $x/y \geq 1$. Moreover, in the latter case it must be $x/y = 1$, because we have assumed that firms create vacancies so long as the cost of maintaining one, s_F, does not exceed the expected benefit, J.[25] So, if $y < x$, then new vacancies will be created until $y = x$. Consequently, the mass of job searching individuals does not exceed the mass of vacancies, so every individual finds a job immediately with certainty.[26]

Let us now derive the mass of active firms, Y. Denote by γ the fraction of individuals that are employed at the beginning of each period. Then

$$\mu_F = \frac{x}{y} = \frac{X - \gamma X}{Y - \gamma X}. \tag{A.2}$$

[25] The assumption that firms create vacancies whenever indifferent is standard and can be justified, for example, that creating a vacancy is welfare improving. Note that this assumption has a bite only in the event of very low expected benefit from hiring an employee, $J = s_F$. The results presented below are unaffected so long as this low threshold is not reached.

[26] See the discussion at the bottom of Section 2.

Combining (A.1) and (A.2) yields

$$Y = X\left(\gamma + (1-\gamma)\frac{J}{s_F}\right). \tag{A.3}$$

A.1.2. Labor Supply. In the benchmark model there is no moral hazard, so all employees exert high effort.[27] Hence firms will never dismiss workers. The fraction of individuals who lose their jobs and return to the job market is given by job destruction rate $1-\alpha$. The total revenue created by a filled job position net of the unemployment income (for short, *surplus*) is therefore equal to

$$S = \pi - u_0 + \alpha\delta(\pi - u_0) + \ldots = \frac{\pi - u_0}{1-\alpha\delta}.$$

Similarly, a firm's gross value of filling a vacancy is equal to

$$J = \frac{\pi - w}{1-\alpha\delta}.$$

We assumed that a firm's share of the surplus is $1-\beta$, so $J = (1-\beta)S$, and hence the wage is given by[28]

$$w = u_0 + \beta(\pi - u_0). \tag{A.4}$$

Let us now determine the equilibrium behavior of an individual with cost of effort c.

Lemma 6. *An individual of type $c \in \mathbb{R}_+$ participates in the job market if and only if*

$$c < \beta(\pi - u_0) - (1-\alpha\delta)s.$$

Proof. Since we are considering equilibria in steady state only, it is sufficient to compare the lifetime utility of the individual who is *always active* and the one who is *always inactive*. The utility of an inactive individual of type c is given by (3) in the main text, restated here:

$$U_0 = u_0 + \delta U_0 = \frac{u_0}{1-\delta}. \tag{A.5}$$

Now consider an active individual of type c. Denote by $U_H(c)$ her lifetime utility starting from a period where she is unemployed (subscript H stands for "high effort" for consistency

[27] Individuals with type $c > w$ will not exert high effort. But these individuals experience negative utility from labor, thus staying out of the labor market in equilibrium.

[28] We assume this simple form of wage determination to keep the analysis straightforward and transparent. The interpretation is that w is a result of negotiations between firms and the employees union; from the perspective of an individual, w is exogenous. The results are not qualitatively affected if we model bargaining as a compromise of the two parties whose disagreement options are endogenously determined in the model.

with notations in further sections). Note that in the period where she is employed, her lifetime utility is simply $U_H(c) + s$, as in a steady state the only difference between being employed and unemployed is the job search cost (as we have established above that individuals find a job immediately with probability one). This yields (1) in the main text, restated here:

$$U_H(c) = -s + (w - c) + \delta\left[\alpha(U_H(c) + s) + (1-\alpha)U_H(c)\right] \\ = \frac{w - c - (1-\alpha\delta)s}{1-\delta}. \tag{A.6}$$

Consequently, an individual of type c will participate in the labor market if and only if[29] $U_H(c) > U_0$, or

$$c < w - u_0 - (1-\alpha\delta)s,$$

which, together with (A.4), yields the result. ■

Intuitively, an individual will participate in the labor market if her cost of effort is small relative to the expected wage, net of the unemployment income and expected costs of job search. Denote by $\bar{c}(s)$ the critical type who is indifferent between participating or not,

$$\bar{c}(s) = \beta(\pi - u_0) - (1-\alpha\delta)s.$$

A.1.3. Steady State. We are now in position to find the masses of active individuals and active firms, X and Y, in steady state.

First, by Lemma 6, only individuals of type $c < \bar{c}(s)$ will be active, hence

$$X = F(\bar{c}(s)).$$

Next, a firm's value of a filled job is given by

$$J = (1-\beta)S = \frac{(1-\beta)(\pi - u_0)}{1-\alpha\delta}.$$

Finally, after every period fraction α of active individuals remain employed, thus $\gamma = \alpha$. Consequently, by (A.3),

$$Y = F(\bar{c}(s))\left(\alpha + (1-\alpha)\frac{(1-\beta)(\pi - u_0)}{s_F(1-\alpha\delta)}\right).$$

Note that if the individuals' search cost is high enough, $s \geq \beta(\pi - u_0)/(1-\alpha\delta)$, then

[29] The tie is a zero probability event and thus can be ignored.

no individuals participate and the labor market collapses (since in that case $\bar{c}(s) \leq 0$ and $F(\bar{c}(s)) = 0$, so we have $X = 0$). Similarly, if the firms' advertising cost is high enough, $s_F > J = (1-\beta)(\pi - u_0)/(1-\alpha\delta)$, no firms are willing to open vacancies, so we have $Y = 0$ and the labor market collapses. For the rest of the paper we assume that $s_F \leq (1-\beta)(\pi - u_0)/(1-\alpha\delta)$.

A.1.4. Comparative Statics. Let us analyze the relationship between an individual's job search cost and the welfare. As in this model search cost s represents a pure waste, it is very intuitive that a reduction of s leads to welfare improvement.

Indeed, consider a reduction of the search cost from s to s'. Then the mass of the individuals who search for jobs weakly increases, as $s > s'$ entails $\bar{c}(s) < \bar{c}(s')$, and consequently $F(\bar{c}(s)) \leq F(\bar{c}(s'))$. Any individual who is participating under s' is strictly better off relative to s, as her utility has gone up due to the lower search cost. Any individual who is inactive under s' is indifferent between s and s', as her utility remains unchanged, $u_0/(1-\delta)$. Hence, the consumer surplus strictly increases as s goes down.

Next, firms with filled job positions make profit $(1-\beta)(\pi - u_0)$ in a single period. Under s' the mass of firms engaged in production is larger, and so is the total producers' surplus. ■

A.2 Proof of Theorem 1

Theorem 1.

(A) *There exists an equilibrium with positive participation if and only if* $\underline{s} \leq s < \bar{s}$.

(B) *The equilibrium with positive participation is unique. If* $s \geq \beta(\pi - u_0)$, *then it coincides with that in the benchmark model. If* $s \leq \beta(\pi - u_0)$,[30] *then it is characterized by:*

(i) *the equilibrium wage is* $w = u_0 + s$;

(ii) *every individual of type* $c < \alpha\delta s$ *participates with probability one and exerts high effort; every individual of type* $c \geq \alpha\delta s$ *participates with probability* λ^* *and exerts low effort,*[31] *where*

$$\lambda^* = \frac{(1-\alpha)F(\alpha\delta s)}{(1-\alpha\delta)(1-F(\alpha\delta s))} \cdot \frac{\beta(\pi - u_0) - s}{\beta u_0 + s}. \tag{4}$$

A.2.1. Part (B). We will prove that if there exists an equilibrium with positive participation, then it is unique and satisfies the conditions stated in Part B.

[30] For $s = \beta(\pi - u_0)$ both statements are true.

[31] This can be replaced with λ^* of individuals with types $c \geq \alpha\delta s$ participating and the rest staying out, or any of a continuum of payoff-equivalent equilibria that maintain a mass λ^* of this group participating in expectation. So long as the expected mass λ^* is preserved, which individuals participate and which stay out is arbitrary (recall that the type of these individuals is irrelevant since they shirk whenever employed).

Recall that we focus on stationary equilibria. In particular, the best reply of a firm to a stationary strategy of the individuals (Lemma 1) and the best reply of individuals to a stationary strategy of firms (Theorem 1) are stationary. In other words, no market participants can benefit by deviating to nonstationary strategies.

Denote by p the probability that a newly hired worker has type $c < c^*(s)$ and thus will exert high effort when employed. Then, surplus S produced by a filled job vacancy is given by

$$S = p\frac{\pi - u_0}{1 - \alpha\delta} + (1 - p)(-u_0), \tag{A.7}$$

where $(\pi - u_0)/(1 - \alpha\delta)$ is the surplus from an employee who always exerts high effort (as S in Section 3) and $(-u_0)$ is the surplus from an employee who shirks in the first period and loses the job at the end of that period.

Similarly, the value J of a filled job vacancy for a firm is given by

$$J = p\frac{\pi - w}{1 - \alpha\delta} + (1 - p)(-w) = S - p(\lambda)\frac{w - u_0}{1 - \alpha\delta} - (1 - p)(w - u_0),$$

where w is the equilibrium wage. Recall that wage determination rule requires

$$J = (1 - \beta)S. \tag{A.8}$$

As J is strictly decreasing in w and $J = S$ for $w = u_0$, for any given p there exists a unique w that solves (A.8).

Suppose that $s > \beta(\pi - u_0)$. By the wage determination rule, the wage cannot exceed its feasible maximum $u_0 + \beta(\pi - u_0)$, hence $w < u_0 + s$. This corresponds to case (a) in Corollary 1, where all individuals who participate exert high effort, $p = 1$. Then we have $S = (\pi - u_0)/(1 - \alpha\delta)$ and $J = (\pi - w)/(1 - \alpha\delta)$. Solving (A.8) for w yields $w = u_0 + \beta(\pi - u_0)$. Note that this is the same equilibrium as in the benchmark model.

Next, suppose that $s < \beta(\pi - u_0)$. Corollary 1 implies the following labor market composition. The total mass of active individuals with type $c < c^*(s)$ is $F(c^*(s))$, of which only fraction $1 - \alpha$ lose their jobs and return to the labor market. Hence the mass of individuals with type $c < c^*(s)$ on the labor market constitutes $(1 - \alpha)F(c^*(s))$. Next, the total mass of individuals with type $c \geq c^*(s)$ is $1 - F(c^*(s))$, of which some fraction $\lambda \in [0, 1]$ are active. Hence the mass of individuals with type $c \geq c^*(s)$ on the labor market constitutes $\lambda(1 - F(c^*(s)))$. Consequently, the probability that a newly hired worker has type $c < c^*(s)$ and thus will exert high effort is given by

$$p = p(\lambda) = \frac{(1 - \alpha)F(c^*(s))}{(1 - \alpha)F(c^*(s)) + \lambda(1 - F(c^*(s)))} = \left(1 + \lambda\frac{1 - F(c^*(s))}{(1 - \alpha)F(c^*(s))}\right)^{-1} \tag{A.9}$$

Consider case (c) in Corollary 1 that stipulates $w = u_0 + s$, so condition (i) holds. Then there is a unique value of λ that solves (A.8) and it is equal to λ^* that can be easily verified. Thus we have proved that the equilibrium must satisfy condition (ii). Yet we need to verify that $\lambda^* \in [0, 1]$. We have assumed that $s \leq \beta(\pi - u_0)$, thus $\lambda^* \geq 0$. Also we have assumed that F first order stochastically dominates the uniform distribution on $[0, \alpha\delta\beta(\pi - u_0)]$, i.e.,

$$F(c^*(s)) \leq \frac{c^*(s)}{\alpha\delta\beta(\pi - u_0)} = \frac{\alpha\delta s}{\alpha\delta\beta(\pi - u_0)} = \frac{s}{\beta(\pi - u_0)} \tag{A.10}$$

for all $s \in [0, \beta(\pi - u_0]$. Hence we have $F(c^*(s))\beta(\pi - u_0) \leq s$ for all $s \in [0, \beta(\pi - u_0)]$. Consequently,

$$\lambda^* = \frac{(1-\alpha)F(\alpha\delta s)}{(1-\alpha\delta)(1-F(\alpha\delta s))} \cdot \frac{\beta(\pi - u_0) - s}{\beta u_0 + s} \leq \frac{(1-\alpha)s}{\beta(1-\alpha\delta) + s} \leq 1.$$

Next, let us show that cases (a) and (b) in Corollary 1 cannot occur in equilibrium. It was shown above that case (a) entails $s > \beta(\pi - u_0)$, a contradiction. It remains to consider case (b) that stipulates $w > u_0 + s$ and $\lambda = 1$. Then we have

$$p = p(1) = \frac{(1-\alpha)F(c^*(s))}{1 - \alpha F(c^*(s))}.$$

Equation (A.8) can be rewritten as

$$p\frac{w - u_0}{1 - \alpha\delta} + (1-p)(w - u_0) = \beta\left(p\frac{\pi - u_0}{1-\alpha\delta} + (1-p)(-u_0)\right).$$

Solving for $(w - u_0)$ we obtain

$$\begin{aligned} w - u_0 &= \frac{p\beta(\pi - u_0)}{1 - \alpha\delta(1-p)} - \frac{(1-\alpha\delta)(1-p)u_0}{1-\alpha\delta(1-p)} \\ &\leq p\frac{\beta(\pi - u_0)}{1-\alpha\delta(1-p)} = \frac{(1-\alpha)F(c^*(s))}{1-\alpha F(c^*(s))} \cdot \frac{\beta(\pi - u_0)}{1 - \alpha\delta(1-p)} \\ &\leq \frac{(1-\alpha)s}{(1-\alpha F(c^*(s)))(1-\alpha\delta(1-p))} \\ &\leq s, \end{aligned}$$

where the third line is by (A.10) and the last line is due to:

$$1 - \alpha\delta(1-p) = 1 - \alpha\delta\frac{1 - F(c^*(s))}{1 - \alpha F(c^*(s))} = \frac{1 - \alpha(\delta + (1-\delta)F(c^*(s)))}{1 - \alpha F(c^*(s))} \geq \frac{1-\alpha}{1 - \alpha F(c^*(s))}.$$

But we have assumed $w > u_0 + s$, a contradiction.

A.2.2. Part (A). First, we show that $X = 0$ for $s \geq \bar{s}$. For $s > \beta(\pi - u_0)$, the equilibrium (if it exists) is the same as in the benchmark model, and by Lemma 6, an individual of type c participates in the labor market if and only if $c < \bar{c}(s)$. But for $s \geq \bar{s}$, $\bar{c}(s) = \beta(\pi - v_0) - (1 - \alpha\delta)s \leq 0$, hence there is no participation, $X = 0$.

Second, we show that $Y = 0$ for $s < \underline{s}$. Recall that in equilibrium $Y > 0$ only if $J \geq s_F$, where

$$J = (1-\beta)S = (1-\beta)\left(p\frac{\pi - u_0}{1-\alpha\delta} - (1-p)u_0\right).$$

Substituting the value of λ^* into (A.9), after some manipulations, yields

$$p = p(\lambda^*) = \frac{(1-\alpha\delta)(\beta u_0 + s)}{\beta(\pi - \alpha\delta u_0) - \alpha\delta s}.$$

Next, substituting $p(\lambda^*)$ into the expression for J, after some manipulations, yields

$$J = \frac{(1-\beta)\pi s}{\beta(\pi - \alpha\delta u_0) - \alpha\delta s}. \tag{A.11}$$

At last, solving inequality $J \geq s_F$ for s yields

$$s \geq \frac{\beta(\pi - \alpha\delta u_0)s_F}{(1-\beta)\pi + \alpha\delta s_F} \equiv \underline{s}.$$ ■

A.3 Proof of Lemma 4

Lemma 4. *For all $s \in [\underline{s}, \beta(\pi - u_0)]$ the welfare in the model with moral hazard is equal to*

$$W(s) = \int_0^{c^*(s)} \frac{\alpha s - c}{1-\delta} dF(c) + \frac{\alpha(\pi - u_0 - s)}{1-\alpha\delta} F(c^*(s)) \tag{5}$$

and it is strictly increasing in s.

For all $s \in (0, \bar{s})$ the welfare in the benchmark model is equal to

$$\overline{W}(s) = \int_0^{\bar{c}(s)} \frac{[\beta(\pi - u_0) - s] + \alpha s - c}{1-\delta} dF(c) + \frac{\alpha(1-\beta)(\pi - u_0)}{1-\alpha\delta} F(\bar{c}(s)) \tag{6}$$

and it is strictly decreasing in s.

Let us calculate the welfare in the benchmark model first. In every period an individual of type $c < \bar{c}(s) \equiv \beta(\pi - v_0) - (1-\alpha\delta)s$ obtains $w - c$. Also, with probability $1 - \alpha$ she is unemployed at the beginning of the period, so she pays search cost s. Thus the lifetime utility of an individual of type $c < \bar{c}(s)$ is equal to

$$\frac{w - c - (1-\alpha)s}{1-\delta} = \frac{u_0 + \beta(\pi - u_0) - c - (1-\alpha)s}{1-\delta},$$

where we used that in equilibrium the wage satisfies $w = u_0 + \beta(\pi - u_0)$. The lifetime utility of an individual of type $c \geq \bar{c}(s)$ is the unemployment income, $u_0/(1-\delta)$. Hence, the consumer's surplus (net of the unemployment income) is equal to

$$\overline{CS}(s) = \int_0^{\bar{c}(s)} \frac{\beta(\pi - u_0) - (1-\alpha)s - c}{1-\delta} dF(c).$$

Next, at the beginning of the period, the mass of firms who have a filled position is $\alpha F(\bar{c}(s))$. The lifetime utility of any such firm is $(\pi - w)/(1-\alpha\delta)$. Any firm that has an unfilled vacancy or inactive has zero lifetime utility. Hence the producers' surplus is equal to

$$\begin{aligned}\overline{PS}(s) &= \alpha F(\bar{c}(s)) \frac{\pi - w}{1-\alpha\delta} = \alpha F(\bar{c}(s)) \frac{\pi - (u_0 + \beta(\pi - u_0))}{1-\alpha\delta} \\ &= \alpha F(\bar{c}(s)) \frac{(1-\beta)(\pi - u_0)}{1-\alpha\delta}.\end{aligned}$$

Summing up $\overline{CS}(s)$ and $\overline{PS}(s)$ gives the expression for welfare $\overline{W}(s)$. As $\bar{c}(s)$ is strictly decreasing in s, it is easy to verify that $\overline{CS}(s)$ is strictly decreasing and $\overline{PS}(s)$ is decreasing. Hence $\overline{W}(s)$ is strictly decreasing.

Let us now calculate the welfare in the model with moral hazard. In every period an individual of type $c < c^*(s) \equiv \alpha\delta s$ obtains $w - c$ and also pays search cost s with probability $1 - \alpha$ that she has become unemployed at the end of the previous period. Thus the lifetime utility of an individual of type $c < c^*(s) \equiv \alpha\delta s$ is equal to

$$\frac{w - c - (1-\alpha)s}{1-\delta} = \frac{(u_0 + s) - c - (1-\alpha)s}{1-\delta} = \frac{u_0 + \alpha s - c}{1-\delta},$$

where we used that in equilibrium the wage satisfies $w = u_0 + s$. The lifetime utility of an individual of type $c \geq c^*(s)$ is the unemployment income, $u_0/(1-\delta)$. Hence, the consumer's surplus (net of the unemployment income) is equal to

$$CS(s) = \int_0^{c^*(s)} \frac{\alpha s - c}{1-\delta} dF(c).$$

Next, similarly to in the benchmark model, at the beginning of the period, the mass of firms who have a filled position is $\alpha F(c^*(s))$. The lifetime utility of any such firm is $(\pi - w)/(1 - \alpha\delta)$. Any firm that has an unfilled vacancy or inactive has zero lifetime utility. Hence the producers' surplus is equal to

$$PS(s) = \alpha F(c^*(s)) \frac{\pi - w}{1 - \alpha\delta} = \alpha F(c^*(s)) \frac{\pi - (u_0 + s)}{1 - \alpha\delta}.$$

Summing up $CS(s)$ and $PS(s)$ gives the expression for welfare $W(s)$. Taking the derivative of $W(s)$ (where we have used $dc^*(s)/ds = \alpha\delta$) yields

$$\begin{aligned}\frac{d}{ds}W(s) =& \frac{\alpha}{1-\delta}F(c^*(s)) + \alpha\delta f(c^*(s)) + \frac{\alpha^2\delta}{1-\alpha\delta}f(c^*(s))(\pi - u_0 - s) - \frac{\alpha}{1-\alpha\delta}F(c^*(s)) \\ =& \frac{\alpha^2\delta}{1-\alpha\delta}f(c^*(s))(\pi - u_0 - s) + \alpha\delta f(c^*(s)) + \alpha F(c^*(s))\left(\frac{1}{1-\delta} - \frac{1}{1-\alpha\delta}\right) > 0,\end{aligned}$$

since by assumption $s \leq \beta(\pi - u_0) < \pi - u_0$ and $\frac{1}{1-\delta} - \frac{1}{1-\alpha\delta} > 0$. ■