

Interest Rate Pegs in New Keynesian Models*

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Abstract

The conventional policy perspective is that lowering the interest rate increases output and inflation in the short run, while maintaining inflation at a higher level requires a higher interest rate in the long run. In contrast it has been argued that a Neo-Fisherian policy of setting an interest-rate peg at a fixed higher level will increase the inflation rate. We show that adaptive learning argues against the neo-Fisherian approach. Pegging the interest rate at a higher level will induce instability and most likely lead to falling inflation and output over time. Eventually, this would precipitate a change of policy.

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1 Introduction

The Fisher equation connecting interest rates to inflation is a well-established long-run relationship. It is often described as stating that in a steady state (and ignoring stochastic uncertainty, tax effects and other distortions), the net nominal interest rate equals the sum of the net real interest rate and the net inflation rate, where

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the net real interest rate is in turn determined by the preferences of households and by the steady-state growth rate of output per capita. Taking the steady-state real interest rate as given, the Fisher equation implies that inflation and interest rates move together one for one, i.e. a one percentage point increase in the inflation rate must be accompanied by a one percentage point increase in the nominal interest rate. The Fisher equation can be grounded theoretically in the household consumption Euler equation, and there is considerable empirical cross-country and long-run time-series support for it.

The conventional view, however, is that (i) the one-for-one positive relationship between inflation and (nominal) interest rates does not hold in the *short run* and (ii) higher interest rates do not *cause* higher inflation rates. For example, starting from a steady-state equilibrium with a positive (nominal) interest rate, if policy-makers use a standard Taylor rule to implement a higher inflation target, this may require lower interest rates in the short run while eventually leading to higher inflation and higher interest rates in the long run.

What has been called the neo-Fisherian view essentially disputes this causal chain and argues instead that an increase in the inflation rate can be obtained by the central bank raising interest rates. A particularly sharp interpretation, emphasized by Cochrane, suggests a policy of announcing and implementing a fixed peg of the nominal interest rate at a higher level. The Fisher equation then appears to imply that inflation will necessarily increase in order to leave the real interest rate unchanged. Cochrane (2015) argues that the Fisher relation identifies a stable steady state, and thus, “under an interest rate peg ... were the Fed to raise interest rates, sooner or later inflation must rise. This prediction has been dubbed the ‘Neo-Fisherian’ hypothesis.” A more flexible interpretation of the neo-Fisherian view, which need not necessarily involve a fixed peg, has been expressed by other economists, including Williamson (2016b), who writes “The key Neo-Fisherian principle is that central banks can increase inflation by increasing their interest-rate targets ...”. In this paper we assess the claims made by the sharper view; however, we return to a more general discussion later.¹

Our central claim is that the sharp neo-Fisherian viewpoint is incorrect because it relies on a reduced-form rational expectations (RE) logic that neglects the mechanisms by which equilibrium is achieved. While RE is the standard benchmark in macroeconomics, an RE solution is an equilibrium object, and in order for it to be

¹Cochrane (2014) distinguishes among “super-pure,” “pure” and “mild” neo-Fisherian views. All views involve an interest-rate peg, and are distinguished by the implied possible paths of inflation, reflecting the multiplicity of perfect-foresight solutions. For example, in response to an increase in the interest-rate peg, the super-pure view requires that inflation immediately jump to its new, higher long-run steady-state level; the pure view has inflation monotonically converging to this new long-run steady state; and the mild view allows for more conventional short-run responses before converging to the new steady state.

viewed as a plausible outcome a mechanism needs to be specified that shows how a rational expectations equilibrium (REE) is attainable. The adaptive learning approach developed, for example, by Bray and Savin (1986), Marcet and Sargent (1989) and Evans and Honkapohja (2001), provides a natural mechanism for studying the stability of an REE.²

The results of the adaptive learning approach are directly applicable to the issue at hand. Using these tools we show that the neo-Fisherian policy delivers an REE that is not stable under learning, while conventional policy, in which the interest rate rule obeys the Taylor principle, delivers an REE that is stable under learning. We emphasize that many of the key results are essentially already available in the adaptive learning literature. In particular, the instability of an interest rate peg can be seen in Howitt (1982) and Evans and Honkapohja (2003), and the importance of the Taylor principle for stability of an interest rate rule is examined in Bullard and Mitra (2002). In the current paper we revisit this issue in the context of current monetary policy discussions and neo-Fisherian proposals. We therefore begin by situating the discussion within the US monetary policy environment of 2008 - 2016.

Following the Financial Crisis of 2008-9 and the subsequent Great Recession, policy interest rates have been at or close to zero for extended periods. For example, in the US the federal funds rate was in the zero to 0.25% range from late 2008 through 2015. Although the federal funds rate was lifted by 0.25% in December 2015, in Europe as of summer 2016 there were several countries with near zero (or even negative) interest rates paid by central banks on bank reserves. Despite these low interest rates, the rate of inflation, while positive, has tended to remain below target, and there has been concern by policymakers that the economy may be evolving toward an unintended low-inflation equilibrium consistent with the neo-Fisherian view that low interest rates lead to low inflation. This view was discussed by Bullard (2010, 2015), and has been forcefully argued by Cochrane (2015) who states, in his abstract, “Perhaps both theory and data are trying to tell us that, when conditions including adequate fiscal-monetary coordination operate, pegs can be stable and inflation responds positively to nominal interest rate increases.”

In more recent commentary, Bullard (2016) noted that “market-based measures of inflation expectations have been declining in the US since the summer of 2014.”³ However, in this context, instead of advocating higher interest rates in line with Cochrane (2015), Bullard (2016) argued that the deteriorating inflation expectations make a case for a slower pace of normalization to higher rates.

²McCallum (2007, 2009) has argued that in analysis of monetary policies, stability under least-squares learning is a necessary condition for plausibility of an REE. See also Lucas (1985), who used adaptive learning in an overlapping generations model with money to argue that the monetary equilibrium should be selected over the autarky equilibrium.

³A similar phenomenon has been observed recurrently in Europe and Japan, in which actual and expected inflation move close to the target inflation rate, only to then begin to decline.

The conflicting policy recommendations of conventional and neo-Fisherian approaches reflect also the multiplicity of rational expectations equilibria (REE) noted by Benhabib, Schmidt-Grohe and Uribe (2001). They emphasized that because of the zero lower bound (ZLB) on the nominal interest rate, an active Taylor rule at the inflation target implies a second, unintended, low-inflation (or deflation) equilibrium corresponding to an interest-rate peg at zero or a near-zero level.⁴ This might be viewed as implying that, if inflation and interest rates are low, then pegging the interest rate at the higher level consistent with the inflation target will eliminate the low-inflation steady state and push the economy to the higher target inflation rate.

However, the adaptive learning viewpoint casts doubt on the stability and relevance of the unintended low-inflation steady state. This steady state corresponds either strictly to an interest-rate peg at the ZLB or, less strictly, to a low-inflation steady state at which the Taylor principle is not satisfied, i.e. in which the policy interest rate responds only weakly to the inflation rate. As noted above, instability under an interest-rate peg was first demonstrated in a monetary model by Howitt (1992). In New Keynesian (NK) models an analogous result appears in Evans and Honkapohja (2003); and, for a discrete-time version of the Benhabib, Schmidt-Grohe and Uribe (2001) model, the instability result for the unintended low-inflation steady state is provided in Evans, Guse and Honkapohja (2008) and Benhabib, Evans and Honkapohja (2014).⁵

In this paper we begin in Section 2 by using the simplified NK model of Kocherlakota (2016) to develop and discuss the theory of monetary policy in the presence of adaptive agents. In particular we establish the generic instability of an interest rate peg in this model and analyze the robustness of the instability result to more sophisticated modeling environments, including alternative information assumptions, multivariate and stochastic extensions, and the zero lower bound. We also use this model to address the robustness concerns raised by Williamson (2016a).

In Section 3 we turn to a standard bivariate NK model, using the long-horizon decision-making formulation of Eusepi and Preston (2010) and employed by Evans, Honkapohja and Mitra (2016). An advantage of this set-up is that it allows us to incorporate announced changes in interest-rate policy into household decisions. This section reviews the key formal results for stability under adaptive learning.

Section 4 then takes up the policy issue of how to implement a change in the target inflation rate. For an economy initially in a steady state, we compare an increase in the inflation target that is implemented by an active Taylor rule around the new target with implementation by the neo-Fisherian policy of setting a fixed interest rate peg at a level consistent with the new, higher inflation target. We

⁴By an active Taylor rule is meant one that locally satisfies the Taylor principle that net interest rates respond for than one for one to deviations from target of the inflation rate.

⁵Concerns about REE of the type studied by Cochrane (2015) have been raised also by Garcia-Schmidt and Woodford (2015) and Evans and McGough (2015).

show that the neo-Fisherian policy leads to instability. How soon this instability manifests itself depends on the extent to which expected inflation immediately adjusts upon announcement and implementation of the policy, but if the initial expectation adjustment falls even slightly short of full adjustment then under adaptive learning the expected path of the economy will be one of declining inflation and recession. The mechanism is straightforward: under an interest rate peg, lower inflation expectations lead to higher ex-ante real interest rates and hence to declining output and inflation. In contrast an increase in the inflation target implemented by a Taylor rule that satisfies the Taylor principle will lead to convergence to the new steady state at the higher inflation target regardless of how inflation expectations initially adjust.

Finally, in Section 5 we turn to the issue of policy normalization in an environment – like the one that was present in the US in late 2015 and the first half of 2016 – in which the interest rate remains near the ZLB, output growth is steady, though low, and labor markets are strengthening, but in which inflation and inflation expectations remain below the 2% policy target. In considering the policy question we allow for concerns by policymakers that the economy might experience a sequence of adverse demand shocks over the next six quarters. In this context we first compare normalization implemented by an immediate return to the Taylor rule with a delayed normalization to a Taylor rule after a specified period. The results depend on whether or not the adverse shocks are realized, but we find a significant asymmetry: an immediate normalization when the adverse shocks do materialize leads to a significant recession, whereas a delayed normalization when the shocks do not materialize leads to a modestly larger temporary overshooting of the inflation target.

In contrast to both of these cases are the results of a normalization based on the neo-Fisherian policy of an increase in the interest rate to a fixed peg. This leads to instability whether or not the bad shocks materialize. If expectations evolve fully according to adaptive learning the result will be a rapidly developing recession due to high real interest rates. Even in the extreme case of full or near-full initial adjustment of inflation expectations, and assuming the bad shocks do not materialize, the results of Sections 3 and 4 imply that under a fixed peg the smallest disturbance will eventually lead to economic instability.

2 A Simplified NK Model

To expose the principal mechanisms at play, in this Section we adopt a simplified NK model proposed by Kocherlakota (2016) and used by Williamson (2016a) in discussing an earlier version of our paper. This model is also convenient for demonstrating the

robustness of our results. The simplified NK model may be written

$$y_t = -\sigma^{-1}(R_t - \hat{E}_t\pi_{t+1} - r) + \hat{E}_ty_{t+1} \quad (1)$$

$$\pi_t = by_t + v_t \quad (2)$$

$$R_t = R^* + \phi_\pi \hat{E}_t(\pi_{t+1} - \pi^*), \quad (3)$$

where now y is the output gap, π is the net inflation rate, π^* is the inflation target, R is the net nominal interest rate, r is the natural interest rate and R^* is the interest-rate target. Equation (1) is the IS-relation. Equation (2) is a non-accelerationist Phillips curve with slope $b > 0$; this curve can be viewed as an NK Phillips curve with the usual dependence on inflation expectations eliminated. In order to discuss the importance of stochastic shocks in a learning context, we have introduced a white noise inflation shock v_t . Equation (3) is the policy rule, with $\phi_\pi = 0$ corresponding to an interest-rate peg, and with $\phi_\pi > 1$ providing a rule that satisfies the Taylor principle. The interest-rate and inflation targets are assumed compatible with a long-run steady state: $\pi^* = R^* - r$. The non-stochastic steady state of this model (for $v_t \equiv 0$) is given by $y_t = y^* = b^{-1}\pi^*$, $R_t = R^*$, and $\pi_t = \pi^*$.

This model is simplified, relative to the model used in our Sections 3 and 4, in two distinct ways. First, the inflation equation (2) has in effect been reduced to a traditional Phillips curve. Second, when these equations are interpreted as behavioral equations, as they are under adaptive learning, decisions are based on one-step ahead forecasts. With an additional assumption, given below, the system can be further reduced to a univariate linear system in which analytical results on learning dynamics can be readily obtained.

2.1 Adaptive learning in the simplified model

The simplified model (1)-(3) has, as an REE, the stochastic steady state $y_t = y^*$ and $\pi_t = \pi^* + v_t$, so that $E_t\pi_{t+1} = \pi^*$. To determine whether this REE is stable under adaptive learning it is convenient to reduce the model to a single endogenous variable by assuming that agents understand and make-use of (2), so that their expectations of future output satisfy

$$\hat{E}_ty_{t+1} = b^{-1}\hat{E}_t\pi_{t+1}.$$

Combining equations we have

$$\pi_t = (1 - \psi)\pi^* + \psi\hat{E}_t\pi_{t+1} + v_t, \text{ where } \psi = 1 - \sigma^{-1}b(\phi_\pi - 1). \quad (4)$$

This equation is often called the “temporary equilibrium” equation – an approach introduced by Hicks (1939) – because it specifies how inflation is determined in equilibrium by expectations and the exogenous shocks.

Under adaptive learning we assume that agents do not fully trust the inflation target π^* because they have doubts about the central bank’s commitment and/or its

ability to hit the target. Assume, therefore, that private agents update their inflation forecasts using observed data according to the statistical rule

$$\hat{E}_t \pi_{t+1} = a_t, \text{ where } a_t = a_{t-1} + \kappa_t(\tilde{\pi}_t - a_{t-1}). \quad (5)$$

Here $\tilde{\pi}_t$ can be viewed as the agents' estimate of inflation at the beginning of time t , and we will consider two cases $\tilde{\pi}_t = \pi_{t-1}$ and $\tilde{\pi}_t = \pi_t$.

The adjustment coefficient $\kappa_t > 0$ is called the “gain” and two common assumptions are standard. Under decreasing gain $\kappa_t \rightarrow 0$ as $t \rightarrow \infty$ at a suitable rate. The most widely used specification is $\kappa_t = t^{-1}$, or more generally $\kappa_t = (N + t)^{-1}$ for some $N \geq 0$. Decreasing gain is usually used in situations where the model dynamics are stationary, or asymptotically stationary, and corresponds to equal weighting of current and past data since $\kappa_t = t^{-1}$ implies that each of the t data points receives weight t^{-1} . (Including $N > 0$ can be viewed as beginning learning with a *prior* equivalent to an initial sample size of N). An alternative assumption, also widely used, is constant gain, i.e. $\kappa_t = \kappa \in (0, 1]$. This is typically employed when model dynamics are viewed as subject to occasional or evolving structural change, and corresponds to discounting past data at geometric weight $1 - \kappa$.

Using stochastic approximation theory a well-known strong and sharp result can be obtained that covers both cases $\tilde{\pi}_t = \pi_{t-1}$ and $\tilde{\pi}_t = \pi_t$:

Proposition 1 *Consider the model (4) with adaptive learning dynamics (5). Assume either $\tilde{\pi}_t = \pi_{t-1}$ or $\tilde{\pi}_t = \pi_t$ and gain sequence $\kappa_t = (N + t)^{-1}$. If $\phi_\pi > 1$ then $a_t \rightarrow \pi^*$ as $t \rightarrow \infty$ with probability one. If $\phi_\pi < 1$ then a_t converges to π^* or any other value with probability zero.*

The stable case corresponds to the Taylor principle $\phi_\pi > 1$. An exogenous interest rate peg, $\phi_\pi = 0$, is always unstable under learning. This is the case even if the initial estimate is $a_0 = \pi^*$: random shocks v_t will move the estimate a_t away from π^* and the sequence of estimates a_t will then diverge.

The proof of the Proposition requires putting the system into stochastic recursive algorithm (SRA) form; see Marcet and Sargent (1989) or Evans and Honkapohja (2001). For the case $\tilde{\pi}_t = \pi_{t-1}$, this is particularly straightforward: the system (4)-(5) can be written

$$a_t = a_{t-1} + (N + t)^{-1}(\sigma^{-1}b(\phi_\pi - 1)\pi^* - \sigma^{-1}b(\phi_\pi - 1)a_{t-1} + v_t),$$

which is in standard SRA form. The stability condition from stochastic approximation results is $-\sigma^{-1}b(\phi_\pi - 1) < 0$, i.e. $\phi_\pi > 1$. For the case $\tilde{\pi}_t = \pi_t$, it can be verified that the recursion in a_t is the same except that N is replaced by $N' = N - 1 + \sigma^{-1}b(\phi_\pi - 1)$. Although the transitional paths will be somewhat different, the stability results are identical for the cases $\tilde{\pi}_t = \pi_{t-1}$ and $\tilde{\pi}_t = \pi_t$.

Because the model is stochastic, convergence of learning to the REE *requires* decreasing gain. Consider the case $\tilde{\pi}_t = \pi_{t-1}$. (We focus on $\tilde{\pi}_t = \pi_t$ in Section 2.2.) With constant gain κ inflation expectations $\hat{E}_t \pi_{t+1} = a_t$ follow the AR(1) process

$$a_t = (1 - \kappa \sigma^{-1} b(\phi_\pi - 1)) a_{t-1} + \kappa (\sigma^{-1} b(\phi_\pi - 1) \pi^* + v_t). \quad (6)$$

If $\phi_\pi < 1$ this process is explosive for all $0 < \kappa \leq 1$. The Taylor principle $\phi_\pi > 1$ is thus necessary for a non-explosive process, and for any given gain $0 < \kappa \leq 1$ there is an open set of policies $1 < \phi_\pi < \bar{\phi}_\pi(\kappa)$ such that expectations converge to a stochastic process centered on RE.⁶

Full convergence to RE is prevented by the constant gain, which induces persistent stochastic perturbations to expectations in response to the *iid* shocks v_t . Furthermore, the variance of the ergodic distribution is

$$\text{var}(a_t) = \frac{\text{var}(v_t)}{\sigma^{-1} b(\phi_\pi - 1)} \frac{\kappa}{(2 - \kappa \sigma^{-1} b(\phi_\pi - 1))},$$

which is increasing in κ over the range $0 < \kappa < 2\sigma b^{-1}(\phi_\pi - 1)^{-1}$. A large gain κ leads to large persistent deviations from RE. In a stationary environment agents will want to either use decreasing gain or a small constant gain to minimize forecast errors.

Constant gain is often used, despite the lack of full convergence to RE, because it makes forecast parameters more alert to structural change of unknown form. If structural change is present or potentially present, there is a trade-off between tracking change and filtering random shocks. The optimal choice of constant gain depends on the degree of structural change. In practice plausible empirical values for the gain appear to be relatively small, e.g. Branch and Evans (2006) find that appropriate values for quarterly US output and inflation data are in the range $\kappa = 0.01$ to $\kappa = 0.06$.

The results just given assume a single white-noise exogenous shock, but the results are more general. For example, if v_t is assumed to be an observable stationary AR(1) process, $v_t = \rho v_{t-1} + \varepsilon_t$, where $|\rho| < 1$, the corresponding RE takes the form $\pi_t = \pi^* + \bar{c} v_t$, where $\bar{c} = (1 - \psi \rho)^{-1}$. Under adaptive learning agents estimate the model $\pi_t = a + c v_t$ using recursive least squares to update estimate (a_t, c_t) over time. Again, the Taylor principle $\phi_\pi > 1$ is needed for stability of the REE under learning.

The results also hold for the standard bivariate NK model in which the Phillips curve takes the more usual form $\pi_t = \beta \hat{E}_t \pi_{t+1} + b y_t + v_t$ and the IS curve (1) includes an observable AR(1) exogenous shock. Bullard and Mitra (2002) show the important role of the Taylor principle under alternative interest rate rules for obtaining stability under learning of the REE. Closely related results are given in Evans and Honkapohja (2003). Section 3 below gives the analogous results when households and firms make decisions based on infinite-horizon decision rules and forecasts based on adaptive

⁶Policies $\phi > \bar{\phi}_\pi(\kappa)$ lead to cyclical overshooting. Note that $\bar{\phi}_\pi(\kappa) \rightarrow \infty$ as $\kappa \rightarrow 0$.

learning. Our presentation in Section 3 omits random shocks in order to simplify the argument, but the results can readily be extended to the stochastic case along the lines discussed in the current section.

2.2 Contemporaneous timing and TE-stability

We now discuss the contemporaneous-timing case, i.e. $\tilde{\pi}_t = \pi_t$ emphasized by Williamson (2016a). With constant gain $0 < \kappa \leq 1$ we have $a_t = a_{t-1} + \kappa((1 - \psi)\pi^* + \psi a_t - a_{t-1})$. Recalling $\psi = 1 - \sigma^{-1}b(\phi_\pi - 1)$, stability obtains when $1 - \kappa < |1 - \kappa + \kappa\sigma^{-1}b(\phi_\pi - 1)|$. Once again we have stability if $\phi_\pi > 1$ so that the Taylor principle is satisfied. However, as Williamson points out, if $\phi_\pi < 1$ then stable cases can arise for large κ . In particular, if $\phi_\pi = 0$ stability obtains when $2\sigma/(2\sigma + b) < \kappa < 1$. However, temporary equilibrium (TE) analysis shows that this condition confers serious theoretical deficiencies.

Temporary equilibrium, which, using $\pi_{t+1}^e = \hat{E}_t\pi_{t+1}$ in (4)-(5), is characterized by

$$\text{RF: } \pi_t = \pi^* + \psi(\pi_{t+1}^e - \pi^*) + v_t \text{ and AE: } \pi_{t+1}^e = (1 - \kappa)\pi_t^e + \kappa\pi_t, \quad (7)$$

yields an evident simultaneity: given the current exogenous shock v_t (if present) and last period's inflation expectation π_t^e , current inflation π_t depends on inflation expectations π_{t+1}^e which in turn depends on inflation π_t . It is natural to view coordination on the solution to this system as arising from an implicit TE dynamic, with π_{t+1}^e adjusting to solve the system in a manner similar to the way prices adjust to clear competitive markets. For the solution to be viewed as an attainable outcome, it must be TE-stable, that is, robust to perturbations in π_{t+1}^e .

To analyze TE-stability, take π_t^e and v_t as given and let $(\pi_{t+1}^e, \pi_t) = (\tilde{\pi}, \hat{\pi})$ solve the system (7). Equation AE may be interpreted as the inflation expectation π_{t+1}^e needed to support the inflation rate π_t . In Figure 1, the lines corresponding to equations (7) are plotted in (π_{t+1}^e, π_t) -space and the intersection corresponds to the temporary equilibrium. Now suppose agents' expectations $\pi_{t+1}^e = \pi^e$ are too high, i.e. $\pi^e > \tilde{\pi}$, and let π be the inflation rate generated by these beliefs via RF. Finally, let $\tilde{\pi}^e$ be the inflation expectations needed to support the inflation rate π . In the left panel of Figure 1 the slope of AE is flatter than RF, whence $\tilde{\pi}^e > \pi^e$. It follows that the TE-dynamic would result in inflation expectations being “pushed” away from $\tilde{\pi}$: the solution is not robust to (even very small) perturbations in expectations. The condition for robustness to perturbations in expectations is $\kappa < \sigma(\sigma + b(1 - \phi_\pi))^{-1}$, which corresponds to the the right panel of Figure 1.

The theoretical deficiency of the asymptotic stability condition $2\sigma/(2\sigma + b) < \kappa < 1$ is now evident. When this condition holds, the system (7) is not TE-stable, and thus the solution should not be viewed as an attainable outcome.⁷

⁷TE-instability also leads to implausible comparative statics: in the left panel, a positive inflation

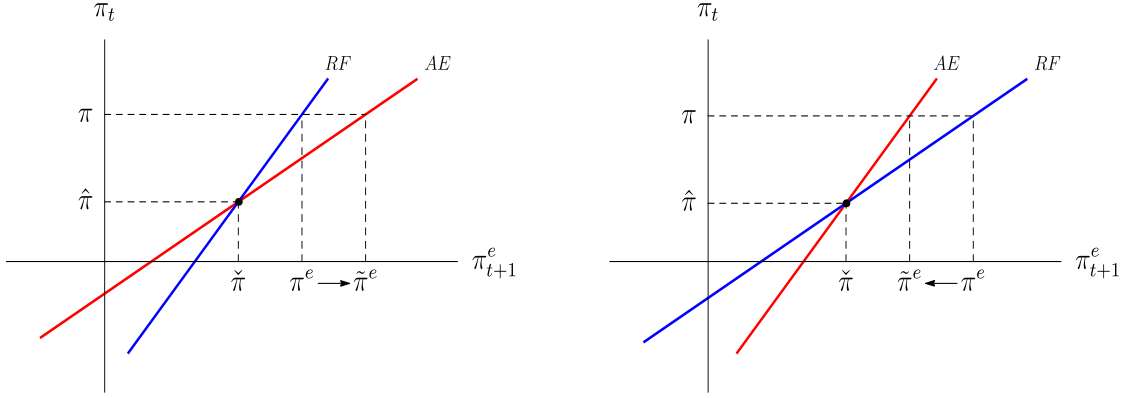


Figure 1: Temporary equilibrium with simultaneity of expectations.

2.3 The zero lower bound

A final issue to be addressed is how our arguments are affected by the interest rate zero lower bound (ZLB). We have so far managed to side-step this issue, in order to not distract from our central points, but we now discuss how our analysis can take the ZLB into account. The interest rate rule (3) is now modified to the following rule:

$$R_t = \max \left\{ 0, \pi^* + r + \phi_\pi \hat{E}_t(\pi_{t+1} - \pi^*) \right\},$$

which implies that the reduced form equation (4) is replaced by

$$\pi_t = \max \left\{ \begin{array}{l} b\sigma^{-1}r + (1 + b\sigma^{-1}) \hat{E}_t\pi_{t+1} \\ b\sigma^{-1}\pi^*(\phi_\pi - 1) + (1 - b\sigma^{-1}(\phi_\pi - 1)) \hat{E}_t\pi_{t+1} \end{array} \right\}, \quad (8)$$

where for simplicity we now omit the exogenous shock v_t . When $\phi_\pi > 1$, so that the Taylor principle is satisfied, it can be seen that there are now two REE steady states. We again have a steady state at $\pi_t = \pi^*$ with $R_t = R^* = \pi^* + r$. However there is also a steady state $\pi_t = \pi_L \equiv -r$ with $R_t = 0$.

Using arguments analogous to those of Section 2.1 it can be shown that while the $\pi_t = \pi^*$ REE continues to be stable under learning, the REE $\pi_t = \pi_L < \pi^*$ is not stable under learning: near $\hat{E}_t\pi_{t+1} = \pi_L$, and for all $\hat{E}_t\pi_{t+1} < \pi_L$ we have $\phi_\pi = 0$ and Proposition 1 applies. For initial expectations $\pi_{t+1}^e > \pi_L$, expectations will be revised upward over time and there will be convergence to $\pi_t = \pi^*$. However, if initial inflation expectations are sufficiently pessimistic, so that $\pi_{t+1}^e < \pi_L$, then expectations will be revised downward over time, so that $\pi_{t+1}^e \rightarrow -\infty$. Under our

shock v_t shifts RF upward, leading in equilibrium to lower inflation and expected inflation.

assumptions it also follows that π_t and y_t fall without bound. This is the deflationary trap emphasized as a possible outcome in Evans, Guse and Honkapohja (2008).⁸

2.4 Discussion

To summarize, Section 2.1 shows that under adaptive learning with decreasing gain, the Taylor principle $\phi_\pi > 1$ is critical for stability and asymptotic convergence to the RE, while a fixed interest rate peg is never stable. These results hold regardless of which information timing assumption is made for updating expectations using observed inflation. These results carry over to the use of a constant gain, which discounts older data. Convergence in this case is to a stochastic process near to and centered on the RE, provided the gain is not large. Agents will have an incentive to use a small constant gain or a decreasing gain, because large constant gains will lead to a high forecast error variance that can be avoided by using a smaller gain.

In Section 2.2, an apparent exception to the importance of the Taylor principle for stability, and to the universal instability of a fixed interest-rate peg, is considered. However, this case is shown to be theoretically defective. Precisely in this case the temporary equilibrium itself is not stable: there is no natural mechanism by which the temporary equilibrium could be achieved.

Finally, in Section 2.3, we allow for the ZLB and show that the π^* REE is *locally* stable under learning: there will be convergence to the RE steady state provided initial expectations $\hat{E}_t \pi_{t+1}$ are not too pessimistic. In the model in Section 3, to which we now turn, we will also incorporate a ZLB.

3 The Benchmark NK Model

We now turn to a standard NK model based on a Rotemberg price friction. There is a continuum of identical, infinitely-lived households. The representative household chooses its consumption bundle and labor supply based on current and expected values of income, inflation and interest rates. There is a continuum of monopolistically competitive firms that hire labor and set prices for their differentiated goods based on current and expected values of output, inflation and interest rates. A quadratic adjustment cost is imposed to impart the nominal pricing friction. When closed with a Taylor rule, the symmetric REE of this entirely standard model satisfies the usual 3-equation system: see, e.g., Evans, Guse and Honkapohja (2008) for details.

We now modify the model to allow for boundedly-rational households and firms.

⁸Clearly it is not realistic for output and inflation to fall without bound. In a modified NK model, Evans, Honkapohja and Mitra (2016) show that if policy is not altered there can be convergence to a third, “stagnation,” steady state with deflation and a low level of steady state output.

Our modification is based on Evans, Honkapohja and Mitra (2016) (EHM) who employ the framework developed in Preston (2005) and Eusepi and Preston (2010). We refrain from elaborating on the details: see EHM for a full development. For consistency with Eusepi and Preston (2010) and EHM we follow their notation in which the variables π and R now denote the inflation and interest-rate factors (i.e. gross rates) and c and y denote the levels of consumption and output. We then use tildes to denote deviations from steady-state form.

We start with behavioral rules for consumption and price-setting. The consumption function for household i is

$$\tilde{c}_t^i = (1 - \beta) \hat{E}_t \sum_{s \geq 0} \beta^s \tilde{y}_{t+s}^i - \frac{\beta^2 \bar{y}}{\pi^*} \hat{E}_t \sum_{s \geq 0} \beta^s \tilde{R}_{t+s} + \frac{\bar{y}}{\pi^*} \hat{E}_t \sum_{s \geq 1} \beta^s \tilde{\pi}_{t+s},$$

where \tilde{y}_t^i is the income of household i , \tilde{R}_t is the nominal interest rate factor and $\tilde{\pi}_t$ is the inflation factor. We use \hat{E}_t to denote the subjective expectations of agents, where for simplicity we assume expectations are the same for all agents. We are studying behavior under learning so these expectations need not coincide with rational expectations. We assume that $\hat{E}_t x_t = x_t$ for generic variable x_t . We continue to use $0 < \beta < 1$ to denote the discount factor. \bar{y} denotes the steady-state level of output, and π^* is the inflation target.

The firm j chooses its inflation rate, $\tilde{\pi}_t^j$, to satisfy

$$\tilde{\pi}_t^j = (1 - \gamma_1) \hat{E}_t \sum_{s \geq 0} (\beta \gamma_1)^s \tilde{\pi}_{t+s} + \frac{a_2 \pi^*}{\bar{y}} \hat{E}_t \sum_{s \geq 0} (\beta \gamma_1)^s \tilde{y}_{t+s},$$

where \tilde{y}_t denotes aggregate output. Here $0 < \gamma_1 < 1$ and $a_2 > 0$ depend on deep parameters and are given in EHM. For simplicity we omit exogenous shocks, which would be straightforward to include.

Assuming identical agents, $\tilde{y}_t^i = \tilde{y}_t$, $\tilde{c}_t^i = \tilde{c}_t$, and also that firms set the same price so that $\tilde{\pi}_t^j = \tilde{\pi}_t$, we can impose market clearing, i.e. $\tilde{y}_t = \tilde{c}_t$, and solve for the temporary equilibrium values of \tilde{y}_t and $\tilde{\pi}_t$ in terms of the current interest rate and expectations:

$$\tilde{y}_t = -\frac{\beta \bar{y}}{\pi^*} \tilde{R}_t - \frac{\beta \bar{y}}{\pi^*} \hat{E}_t \sum_{s \geq 1} \beta^s \tilde{R}_{t+s} + \frac{1 - \beta}{\beta} \hat{E}_t \sum_{s \geq 1} \beta^s \tilde{y}_{t+s} + \frac{\bar{y}}{\beta \pi^*} \hat{E}_t \sum_{s \geq 1} \beta^s \tilde{\pi}_{t+s}, \quad (9)$$

$$\begin{aligned} \tilde{\pi}_t &= -\frac{\beta a_2}{\gamma_1} \tilde{R}_t - \frac{\beta a_2}{\gamma_1} \hat{E}_t \sum_{s \geq 1} \beta^s \tilde{R}_{t+s} + \frac{1 - \gamma_1}{\gamma_1} \hat{E}_t \sum_{s \geq 1} (\beta \gamma_1)^s \tilde{\pi}_{t+s} \\ &+ \frac{a_2 \pi^*}{\gamma_1 \bar{y}} \left(\frac{1 - \beta}{\beta} \hat{E}_t \sum_{s \geq 1} \beta^s \tilde{y}_{t+s} + \frac{\bar{y}}{\beta \pi^*} \hat{E}_t \sum_{s \geq 1} \beta^s \tilde{\pi}_{t+s} + \hat{E}_t \sum_{s \geq 1} (\beta \gamma_1)^s \tilde{y}_{t+s} \right). \end{aligned} \quad (10)$$

We assume the policy rule takes the form $\tilde{R}_t = \frac{R^* \phi_\pi}{\pi^*} \hat{E}_t \tilde{\pi}_{t+1}$, where $\phi_\pi \geq 0$ and R^* is the target nominal interest-rate factor consistent with the inflation target and the

steady state real interest-rate factor: $R^* = \pi^*/\beta$. If the ZLB were not an issue this would yield the interest-rate rule

$$\tilde{R}_t = \frac{\phi_\pi}{\beta} \hat{E}_t \tilde{\pi}_{t+1}.$$

As before, $\phi_\pi > 1$ corresponds to the Taylor principle⁹ and setting $\phi_\pi = 0$ corresponds to an interest-rate peg.¹⁰

In this nonstochastic setting there is a perfect-foresight REE given by $\tilde{y}_t = \tilde{\pi}_t = \tilde{R}_t = 0$, and if $\phi_\pi > 1$ and not too large it can be shown that this is the unique non-explosive perfect foresight solution: see Bullard and Mitra (2002).¹¹ If $0 \leq \phi_\pi < 1$ the steady state is indeterminate and the set of REE includes a continuum of perfect-foresight solutions that converge to it.

To address recent and current policy issues, it is important to incorporate the ZLB into the policy rule. In doing so we also allow for credit frictions, as in EHM, which create a spread $\varphi > 0$ between the market interest rate relevant for aggregate private decision-making, and the policy rate controlled by the Central Bank. The existence of such a spread is formally modeled in Curdia and Woodford (2015), and we use their steady-state calibration of one percentage point at annual rates, i.e. $\varphi = 0.0025$ at quarterly rates. It is convenient to use R_t now to denote the market interest rate. Thus a lower bound of zero for the policy rate corresponds to a lower bound of φ for the market interest rate R_t . Incorporating the ZLB thus leads to

$$\tilde{R}_t = \max \left\{ \frac{\phi_\pi}{\beta} \hat{E}_t \tilde{\pi}_{t+1}, 1 + \varphi - R^* \right\}. \quad (11)$$

We now introduce adaptive learning into the NK model following Evans and Honkapohja (2001), Eusepi and Preston (2010) and EHM. Given our simple purely forward-looking set-up without the presence of stochastic shocks it is sufficient to assume that agents use a Perceived Law of Motion (PLM) for $(\tilde{y}_t, \tilde{\pi}_t)$ of the form

$$\tilde{y}_t = \delta^y + noise_t \text{ and } \tilde{\pi}_t = \delta^\pi + noise_t,$$

where $noise_t$ indicate unforecastable perceived white-noise disturbances, and thus $\hat{E}_t \tilde{y}_{t+s} = \delta^y$ and $\hat{E}_t \tilde{\pi}_{t+s} = \delta^\pi$. Under adaptive learning agents at time t make forecasts using estimates δ_t^y, δ_t^π of δ^y, δ^π , so that

$$\hat{E}_t \tilde{y}_{t+s} = \delta_t^y \text{ and } \hat{E}_t \tilde{\pi}_{t+s} = \delta_t^\pi. \quad (12)$$

⁹For convenience we assume \tilde{R}_t depends on expected inflation rather than contemporaneous inflation and we do not include a dependence on contemporaneous or expected output.

¹⁰Throughout this paper we have assumed for simplicity that policymakers know the steady state real interest rate and that the interest-rate target is consistent with the inflation target: $R^* = \pi^*/\beta$. However, none of our results depend on this.

¹¹If stochastic productivity, preferences, mark-ups or government spending shocks are included then the corresponding REE also depends on these shocks.

We discuss below how their beliefs δ_t^y, δ_t^π evolve over time under learning.

Given time t beliefs δ_t^y, δ_t^π , output and inflation are determined by combining the temporary equilibrium equations (9)-(10) and the policy rule (11). This yields

$$\begin{pmatrix} \tilde{y}_t \\ \tilde{\pi}_t \end{pmatrix} = \begin{pmatrix} 1 & -\frac{\bar{y}}{\pi^*} \frac{\phi_\pi - 1}{1 - \beta} \\ \frac{a_2 \pi^*}{\bar{y} \gamma_1 (1 - \beta \gamma_1)} & \frac{\beta(1 - \gamma_1)}{1 - \beta \gamma_1} - \frac{a_2(\phi_\pi - 1)}{\gamma_1(1 - \beta)} \end{pmatrix} \begin{pmatrix} \delta_t^y \\ \delta_t^\pi \end{pmatrix} \equiv T(\delta_t),$$

where $\delta = (\delta^y, \delta^\pi)'$. We note that generically the unique fixed point of $T(\delta)$ is the REE $\delta^y = \delta^\pi = 0$.

Under adaptive learning the parameters δ_t^y, δ_t^π are updated over time using observed data according to

$$\delta_t^y = \delta_{t-1}^y + \kappa(\tilde{y}_{t-1} - \delta_{t-1}^y) \text{ and } \delta_t^\pi = \delta_{t-1}^\pi + \kappa(\tilde{\pi}_{t-1} - \delta_{t-1}^\pi). \quad (13)$$

It follows that

$$\delta_t = \delta_{t-1} + \kappa(T(\delta_{t-1}) - \delta_{t-1}).$$

Note that given initial beliefs δ_0 the above difference equation generates the time-path of both beliefs $(\delta_t^y, \delta_t^\pi)$ and realized aggregates $(\tilde{y}_t, \tilde{\pi}_t)$.

The asymptotic behavior of the economy is characterized by the following Proposition, which is straightforward to show:

Proposition 2 *Consider the NK model (9),(10),(11) with expectations (12) updated according to (13). The perfect foresight steady state $(\tilde{y}, \tilde{\pi}) = (\delta^y, \delta^\pi) = (0, 0)$ is locally stable under adaptive learning for $\kappa > 0$ sufficiently small if and only if $\phi_\pi > 1$.*

This result is well-known and fully in accordance with, e.g., Bullard and Mitra (2002) and EHM.¹²

Although for simplicity we have stated this proposition for the nonstochastic model, if exogenous random shocks are added the stability results are essentially unchanged and can be extended along the lines discussed in the simplified NK model of Section 2. Furthermore, as with the Section 2 model, it follows immediately from Proposition 2 that an interest-rate peg, i.e. $\phi_\pi = 0$, is unstable under adaptive learning.

We now use this framework first to study alternative ways to implement a policy to raise the inflation rate, and secondly to examine the implications of our results for how best to implement interest-rate normalization.

¹²Arifovic et al. (2012) find that under certain social learning mechanisms implemented by genetic algorithms the REE can be stable even when the Taylor principle is not satisfied. However, our preliminary investigations find that those results are not robust.

4 Policy experiments

Suppose the Central Bank (CB) wants to increase inflation and relies on the neo-Fisherian logic that an increase in the interest rate will necessarily lead, at least eventually, to a higher inflation rate. To study this under learning, suppose that initially the economy is in a steady state with $\pi^* = 1.0025$, i.e. an inflation rate of 1% per year, a zero output gap and nominal interest factor $R = \pi^* \beta^{-1}$. (Note that we now express variables in levels instead of deviation from mean form.) We set $\beta = 0.99$ and data is quarterly.¹³

Suppose that the CB at $t = 10$ announces an increase in the inflation target to $\pi^* = 1.0075$ quarterly, i.e. 3% per year, and in accordance with a sharp neo-Fisherian policy, raises the corresponding interest rate from $R^* = 1.0126$ to a fixed peg at $R^* = 1.0177$. There is an REE in which inflation immediately jumps to its new target and the output gap remains zero. Under learning we suppose agents place a high weight on this outcome, but retain a small doubt that inflation will increase by the full amount. To capture this, suppose that agents at $t = 10$ adjust their expectations to $\pi^e = \delta_0^\pi = 1.007$, i.e. slightly below the new inflation target of $\pi^* = 1.0075$. They then revise their expectations over time in response to data using the adaptive learning rule specified above. In this section we use a gain parameter of $\kappa = 0.03$. Figure 2 shows the resulting time paths for inflation, output and the interest rate.¹⁴ In this and the following figures we use \hat{y} to denote output in proportional deviation from mean form. It is also convenient to use π^e and \hat{y}^e to denote, respectively, expected inflation and expected output.

In period $t = 10$, R is pegged at the higher value consistent with the new inflation target. Because the agent's inflation expectation δ_{10}^π does not fully adjust to the new RE steady state inflation rate, in period $t = 10$ the real interest rate rises, output falls and inflation increases but falls short of inflation expectations. This leads to a process in subsequent periods in which inflation expectations adjusts downwards, leading to a further rise of the real interest rate and a decline in both inflation and output. Expectations of inflation and output then start to decline rapidly, pushing the economy into a serious recession accompanied by deflation. The central mechanism for instability is that, with a nominal interest rate peg, lower expected inflation leads to a higher ex-ante real interest rate that lowers consumption demand and equilibrium output.

¹³We use the same model parameters as in EHM, except that we set the price adjustment parameter $\psi = 250$ and we set government spending to zero for simplicity. For $\pi^* = 1.0075$ this leads to values $\bar{y} = 0.9545$, $\gamma_1 = 0.8570$, $a_2 = 0.06451$.

¹⁴In this and the following figures the top panel the horizontal solid lines give the steady-state interest rates corresponding to the inflation targets and the dotted horizontal line gives the market interest rate corresponding to the ZLB. In the middle panel the top two horizontal solid lines give the inflation targets and the lower solid horizontal lines corresponds to zero net inflation.

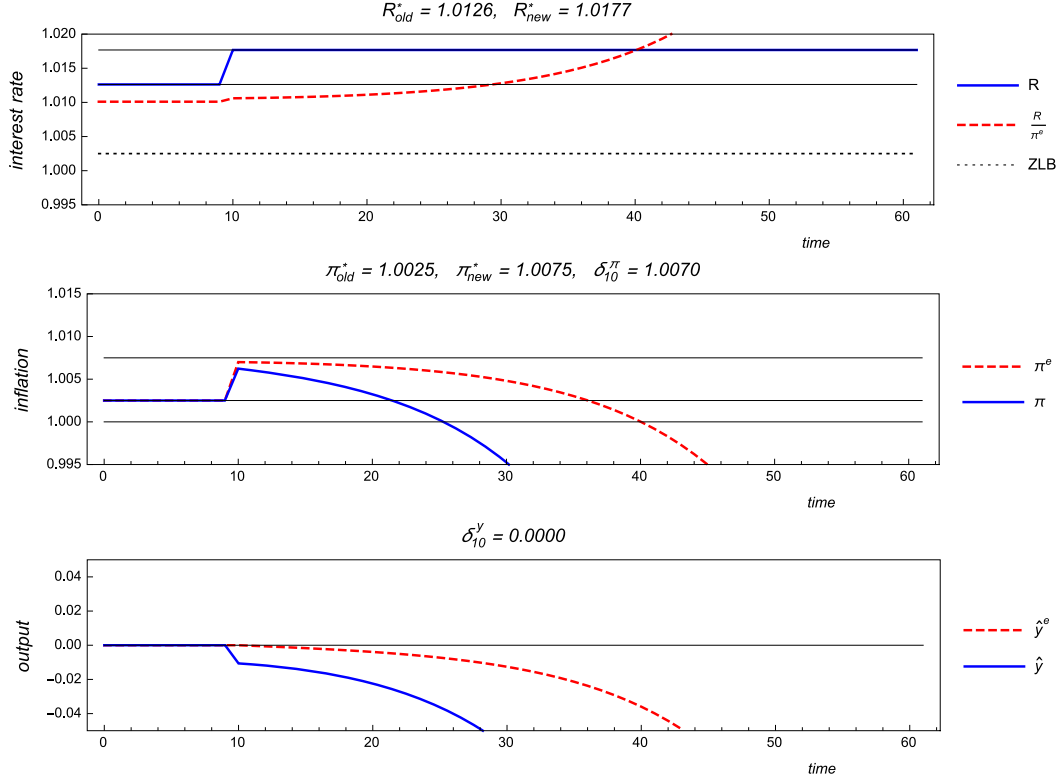


Figure 2: Increase in interest rate peg with almost full adjustment of inflation expectations.

It is important to note that we have assumed in Figure 2 that agents place a high weight on the ability of the CB to achieve its 3% annual inflation target: at the time of the policy change, i.e. $t = 10$, agents increase their expectations for annual inflation from 1% to 2.8%. Bearing in mind that CB policy only indirectly affects the inflation rate, we regard our assumption on initial expectations as strongly favoring the neo-Fisherian position.

Our demonstration of instability requires only that going forward in time agents revise their expectations in light of the actual evolution of inflation. Indeed, even if agents initially fully adjusted their initial expectations to the announced target, the slightest stochasticity will trigger a destabilizing path. Figure 3 demonstrates this possibility. At time $t = 10$ we now assume there is full adjustment of inflation expectations. Then in $t = 11$ there is small negative one-period shock to consumption. Because of the fragility of the neo-Fisherian outcome, this small shock, and indeed any disturbance at all, eventually destabilizes the economy.

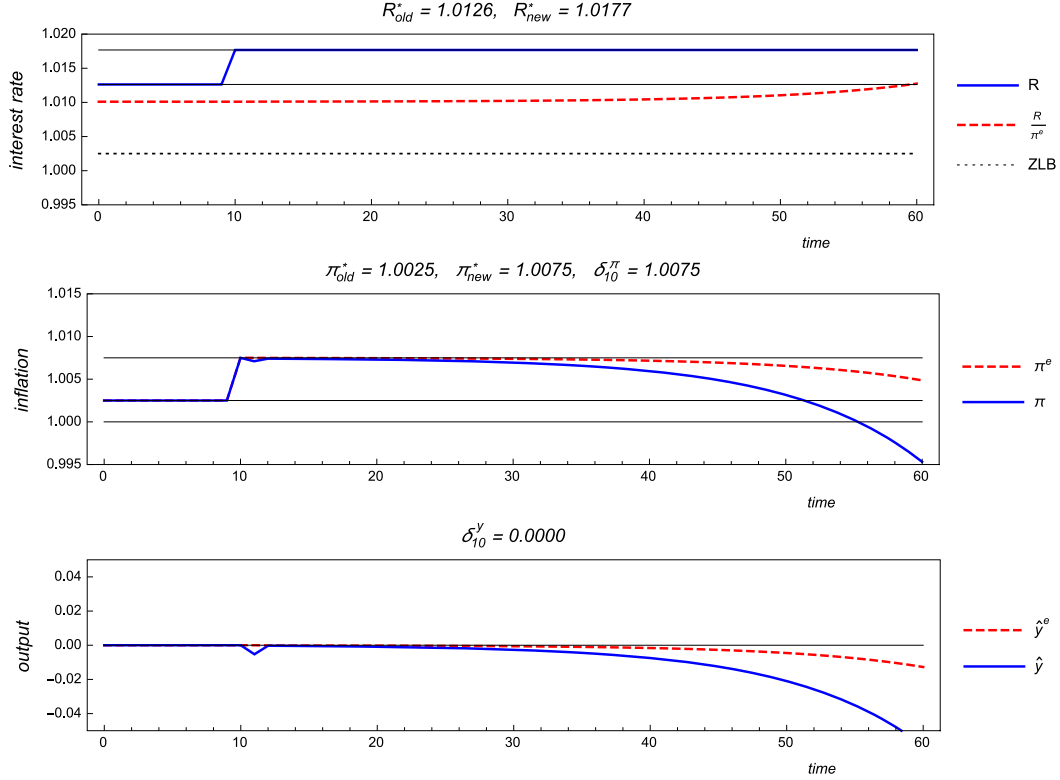


Figure 3: Increase in interest rate peg with full adjustment of inflation expectations.

The preceding does not, of course, imply that the inflation target is unattainable. Suppose that when the CB increases its inflation target it instead employs an interest-rate rule that satisfies the Taylor principle, e.g. $\phi_\pi = 1.3$. This policy experiment is shown in Figure 4, where we now set $\delta_{10}^\pi = 1.005$, i.e. inflation expectations adjust half-way to the new target (similar qualitative results obtain if expectations initially adjust almost all the way, as before, or not at all). Policy dictates an initial increase in interest rates smaller than recommended by the peg. In line with the Taylor principle, the resulting ex ante real interest rate falls, leading to initial increases in inflation and output. Over time the economy converges to the new REE as expectations adjust under the adaptive learning rule. Because of the partial adjustment of inflation expectations in Figure 4, the Taylor rule induces an increase in the nominal interest rate at $t = 10$ when the inflation target is increased. Monetary policy is still expansionary because the ex-ante real interest rate decreases. Whether the nominal interest rate increases or decreases, at the time the inflation target changes, depends on both the adjustment in expectations δ_{10}^π due to the announced change in policy and on the Taylor-rule coefficient ϕ_π . Figure 5 shows the results if inflation expectations do not adjust until higher realized inflation is actually observed. Note that in this case the nominal interest rate decreases at the time of the policy change.

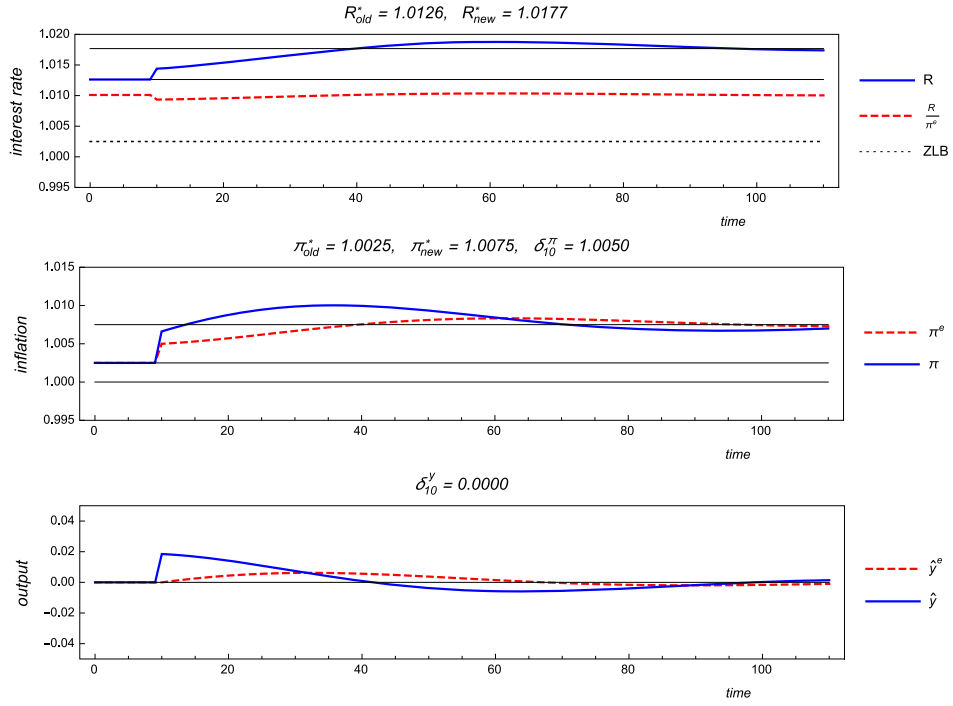


Figure 4: Increase in inflation target implemented by Taylor rule assuming partial adjustment of inflation expectations at time of policy change, i.e. $t = 10$.

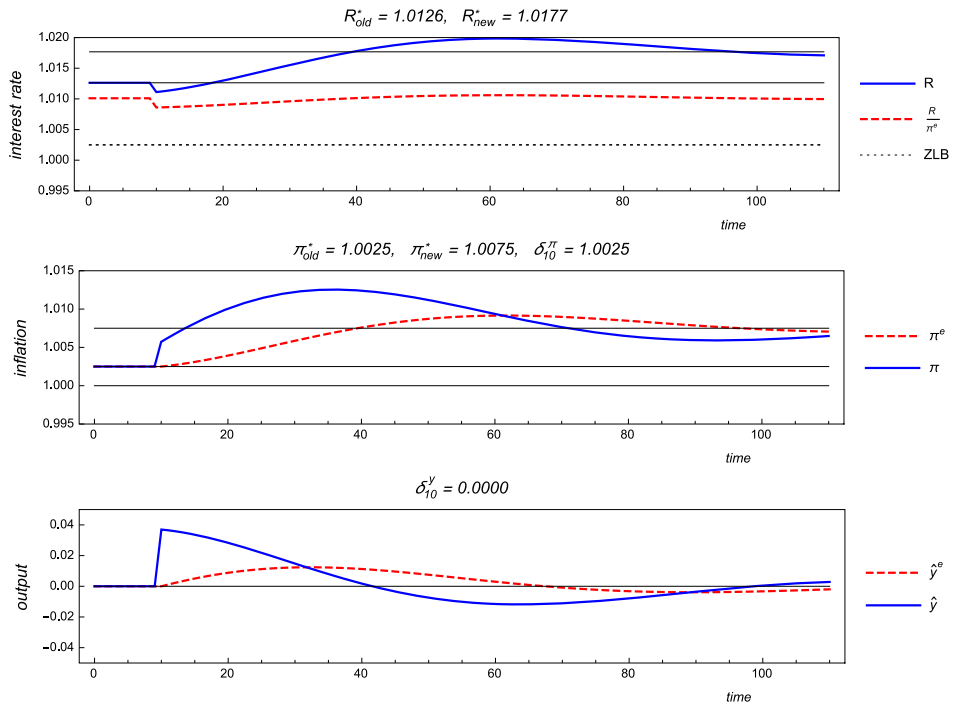


Figure 5: Increase in inflation target implemented by Taylor rule, assuming no adjustment of inflation expectations at time of policy change, i.e. $t = 10$.

Finally, returning to the case of the interest rate peg that was illustrated in Figure 2, suppose that the CB, after observing declining inflation expectations and reductions in both inflation and output, decides after a number of periods to shift from a peg to an interest-rate rule that satisfies the Taylor principle.

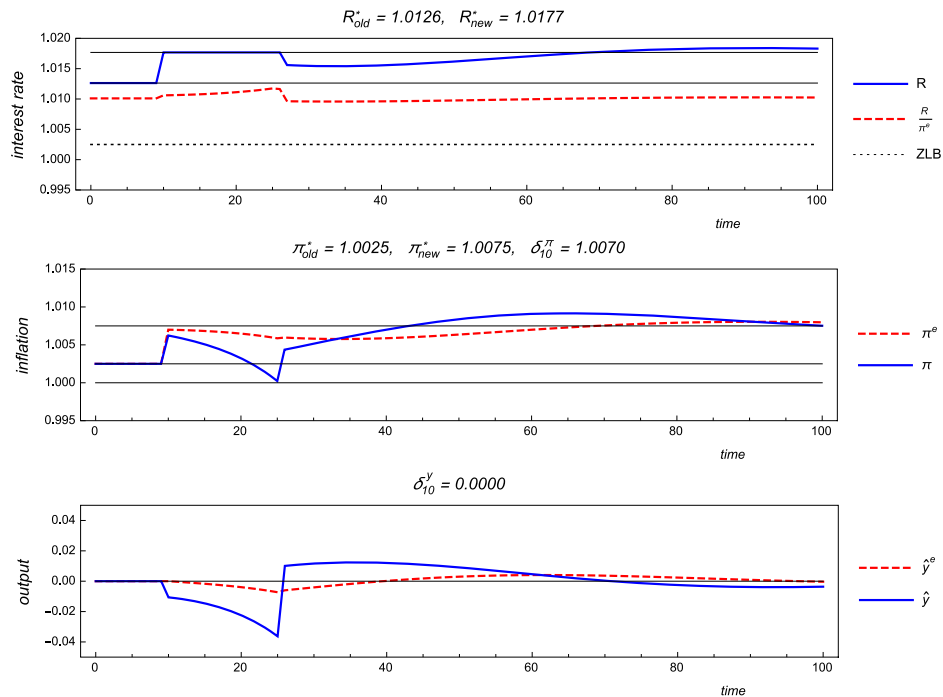


Figure 6: Increase in interest rate peg, with large initial adjustment of expectations, and with later implementation of Taylor rule at $t = 25$ with $\phi = 1.3$.

Figure 6 illustrates the results for initial $\delta_{10}^\pi = 1.007$, the value used in Figure 2. At time $t = 25$, with output around 4% below the steady-state level and with deflation now threatening the economy, the CB implements an interest-rate rule with $\phi_\pi = 1.3$. The adoption of this new policy leads to an immediate reduction in interest rates and a corresponding reduction in real interest rates to below steady state values. This stimulates demand, resulting in a contemporaneous increase in output and inflation, with eventual convergence to the new steady state.

We return now to the more flexible interpretation of the Neo-Fisherian view, by considering the following question: to raise inflation, should the central bank raise or lower nominal interest rates? In contrast to the standard prescription that rates should be lowered, a flexible neo-Fisherian would argue that to raise inflation the monetary authority should raise the nominal interest rate. The results from this paper, and the lessons from the learning literature more broadly, support a more nuanced, expectations-based prescription: an inflation target should be set and the nominal interest should then be adjusted based on a rule that satisfies the Taylor principle and that conditions on measures of inflation expectations – whether an announcement of a higher inflation target requires that interest rates be initially

raised or lowered will depend on the specific reaction of inflation expectations and the particulars of the rule's specification, and is a matter of no explicit importance.

5 Monetary policy normalization

Beginning in December 2015 the US Federal Reserve embarked on a normalization program in which the policy interest rate was increased above the 0 to 0.25% range for the first time since December 2008. This was in response to the recovery of the economy from the Great Recession, in particular to steady if unexciting growth of GDP and a substantial strengthening of labor markets in the 2014-2015 period. In December 2015 the Federal Reserve also indicated that it planned to follow a normalization policy in which interest rates were to be further increased over 2016. In early 2016 some concerns were raised regarding the pace of normalization. In particular, St Louis FRB President James Bullard, in his February 2016 talk to the CGA, noted that inflation expectations had been declining since July 2015 and in February 2016 were around 1.6% p.a., significantly below the 2% p.a. target. Bullard argued that this, together with other developments, suggested the need to slow the pace of normalization (as well as to make it more data dependent). This economic situation has therefore given increased urgency to the policy debate between the neo-Fisherian view that interest rates should be increased and the standard view that interest rates should be kept low when inflation expectations are below target.

Our model can be used to explore this argument. In the formal presentation of Section 3, to ease the exposition, we suppressed the random fundamental shocks that are usually included in the adaptive learning approach. In addition, the learning literature sometimes incorporates expectation shocks into learning algorithms, e.g. Milani (2011). Inclusion of expectation shocks is a way of recognizing that expectations can reflect changes in sentiment about future economic conditions as well as respond to observed data. For example, there was considerable concern in the US during 2015 about the possibility of sovereign defaults in Europe, as well as the strength of the European economy generally. Concerns about decreasing growth in China were also a factor. These concerns could plausibly spill over into a deterioration in inflation and output expectations in the US. When considering practical monetary policy, account must also be taken of future fundamental shocks that policymakers may foresee or that may concern them. In 2016, recent examples include the implications for the economy of the European migration crisis, Brexit, further reductions in Chinese growth rates, tensions with Russia, instability in the price of oil, and continued fragility of the European economy.

With these observations in mind, we consider a stylized setting for recent policy discussions. Following EHM we set $\beta = 0.9975$ and the target inflation rate at 2% per year in line with current US monetary policy. Consistent with the situation in

late 2015 and early 2016 we assume inflation expectations are below target. For convenience we set π^e in $t = 0$ at 1% per year, i.e. $\delta_0^\pi = 1.0025$. Suppose also that the current market interest rate is 1% per year, i.e. $R = 1.0025$, given by a zero net policy rate plus the credit friction. These assumptions are meant to broadly reflect the state of monetary policy as of late 2015.¹⁵ We also allow for the possibility of anticipated adverse demand shocks. In the model of Section 3 these take the form of additive shocks u_t to the temporary equilibrium equation for \tilde{y}_t . These allow us to capture policymaker concerns, which may or may not be realized, about future adverse shocks to economic conditions.

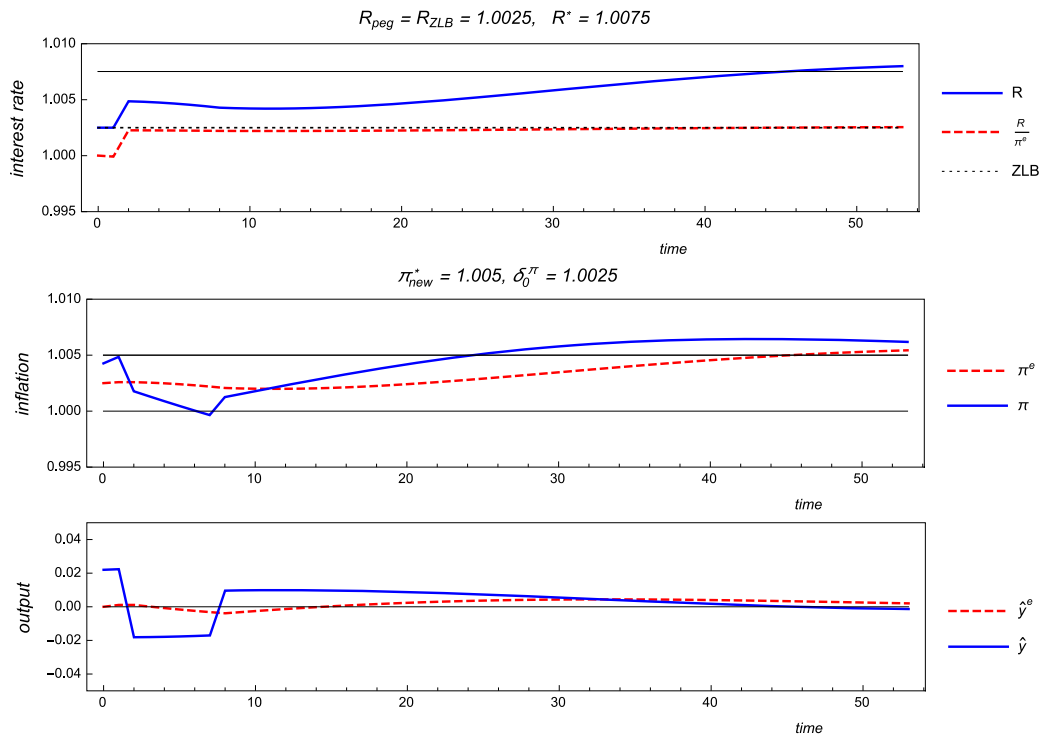


Figure 7: Immediate normalization to Taylor rule in stylized policy setting, with bad shocks.

In Figures 7-11 we consider simulations of policy normalization consistent with the above initial conditions. In periods $t = 0, 1$ the CB is assumed to hold the peg at the ZLB; however, because the CB is known to be contemplating a policy normalization in which interest rates are increased, agents form expectations of future interest rates by averaging the ZLB peg with the rate given by the standard Taylor rule. We assume that private agents in both $t = 0$ and $t = 1$ think there is a 75% probability that the ZLB peg will be replaced by a Taylor rule in future periods.

¹⁵For $\pi^* = 1.005$ this leads to values $\bar{y} = 0.9545$, $\gamma_1 = 0.8541$, $a_2 = 0.0643$. Also for the experiments in this Section we set $\phi = 1.1$ and the gain $\kappa = 0.05$.

This reflects statements by the CB that interest rates in the future are expected to be normalized upwards, and that this will likely occur sooner rather than later. In period $t = 2$, however, suppose that the CB becomes concerned that there will be adverse demand shocks over the next six quarters, and because of this considers whether to delay normalization. To examine this issue first suppose that the adverse shocks do materialize, and we compare an announced immediate normalization in Figure 7 with an announced delayed normalization in Figure 8. For the adverse demand shocks we specifically assume that $u_t = -0.025$ for periods $t = 2, \dots, 7$.

In Figure 7 at $t = 0, 1$ output is about 2% above steady state due to the low real interest rate associated with the ZLB and expected inflation of 1%. In period $t = 2$, the CB announces an immediate normalization, specifically adopting a Taylor rule with $\phi_\pi = 1.1$ and inflation target of 2% per year. This policy change results in an immediate increase in the market nominal interest rate to around $R = 1.005$, i.e. 2% per year. Consequently this results in an increase in the real interest rate, which lowers output. Output is further reduced by the adverse demand shocks that begin in period $t = 2$. The result is a recession $t = 2, \dots, 7$. This in turn leads to a sharp reduction in inflation in period $t = 2$ and a subsequent gradual fall over the coming periods, reaching a deflationary level in period $t = 7$. In period 8 the adverse shocks relent and the stability of the Taylor rule draws the economy back to the steady state.

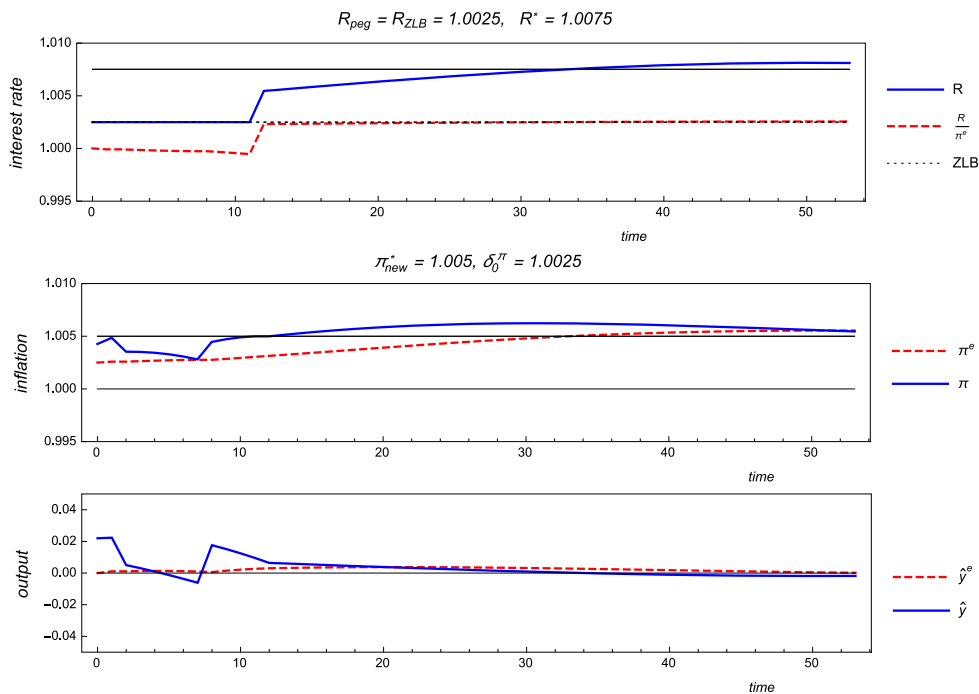


Figure 8: Delayed normalization to Taylor rule in stylized policy setting, with anticipated bad shocks.

In Figure 8 the first two initial periods $t = 0, 1$ are identical and we have the same sequence of adverse shocks. However we now assume that the CB, foreseeing the sequence of bad shocks, announce a delayed normalization of ten quarters, i.e. in periods $t = 2, \dots, 11$ the nominal interest rate will be held at the ZLB, and beginning in $t = 12$ policy will be normalized as in Figure 7. This delayed policy maintains a low interest rate, providing a stimulus to the economy to mitigate the impact of the adverse demand shocks. As a result the recession is almost entirely avoided and inflation never falls below 1% per year.

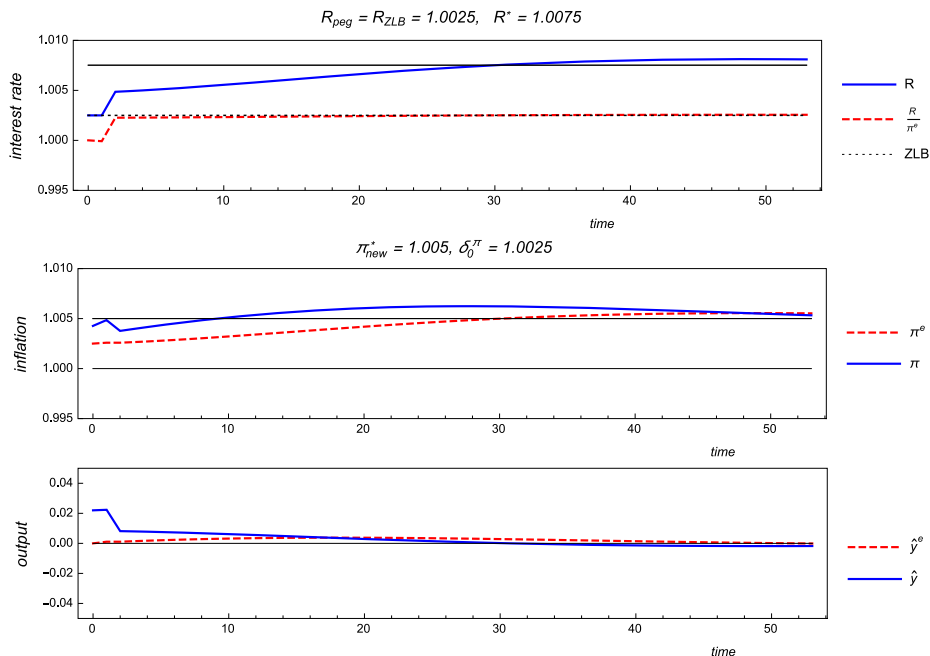


Figure 9: Immediate normalization to Taylor rule, in stylized policy setting, without bad shocks.

Of course, we can also consider the outcomes of the alternative policies if the bad shocks anticipated by policymakers do not in fact take place. Figure 9 gives the result when there is an immediate normalization to a Taylor rule, while Figure 10 shows the results for a delayed normalization. In both Figures, of course, the first two periods are the same and match Figures 7 and 8 for $t = 0, 1$. For Figure 9 the results from $t = 2$ are as would be expected. Normalization leads to an immediate partial adjustment of R toward its long-run target. This raises the real interest rate and lowers output, but because of the partial adjustment, output stays above its steady state. The stability of the Taylor rule ensures a smooth transition to the steady state.

If instead the normalization is delayed, R stays at the ZLB level through period $t = 11$: see Figure 10. The correspondingly low real interest rate stimulates the economy, leading to even higher output and to inflation above inflation expectations.

As a result, during the delayed implementation period real interest rates continue to decline. Note, however, that after the initial increase, output declines over periods $t = 2, \dots, 11$. The coincident decline in real interest rates and output reflects the anticipation of normalization in period $t = 12$. After normalization there is again a smooth transition to the steady state. However, the delayed normalization does result in inflation temporarily rising to almost 3.6% per year, in period $t = 10$.

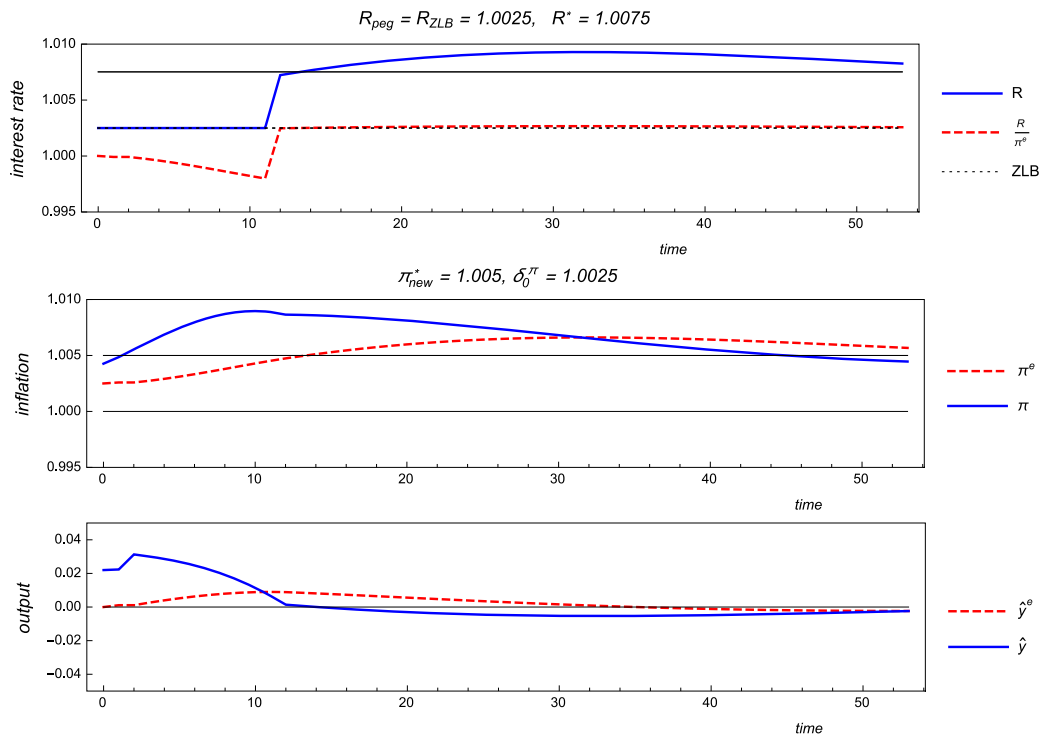


Figure 10: Delayed normalization to Taylor rule, in stylized policy setting, with anticipated bad shocks not realized.

Taking together the results of Figures 7 – 10 illustrate the argument that can be made for delayed normalization when policymakers observe circumstances that lead them to be concerned about future adverse shocks. There is an asymmetry in the response to policy when inflation expectations are below target and interest rates are already at or near the ZLB floor. To mitigate the impact of a recession resulting from the anticipated adverse shocks, the normalization to a Taylor rule would need to be delayed. If the normalization is not delayed and the bad shocks materialize then the outcome is a substantial recession. If instead the normalization is delayed and the bad shocks do not occur the overly strong expansion results in inflation temporarily above target. Policymakers arguing for delayed normalization would argue that the costs to the economy of recession from bad shocks under immediate normalization outweigh the costs of inflation temporarily above target if the bad shocks do not occur

and normalization is delayed.¹⁶ $R_{peg} = R_{ZLB} = 1.0025$, $R^* = 1.0075$

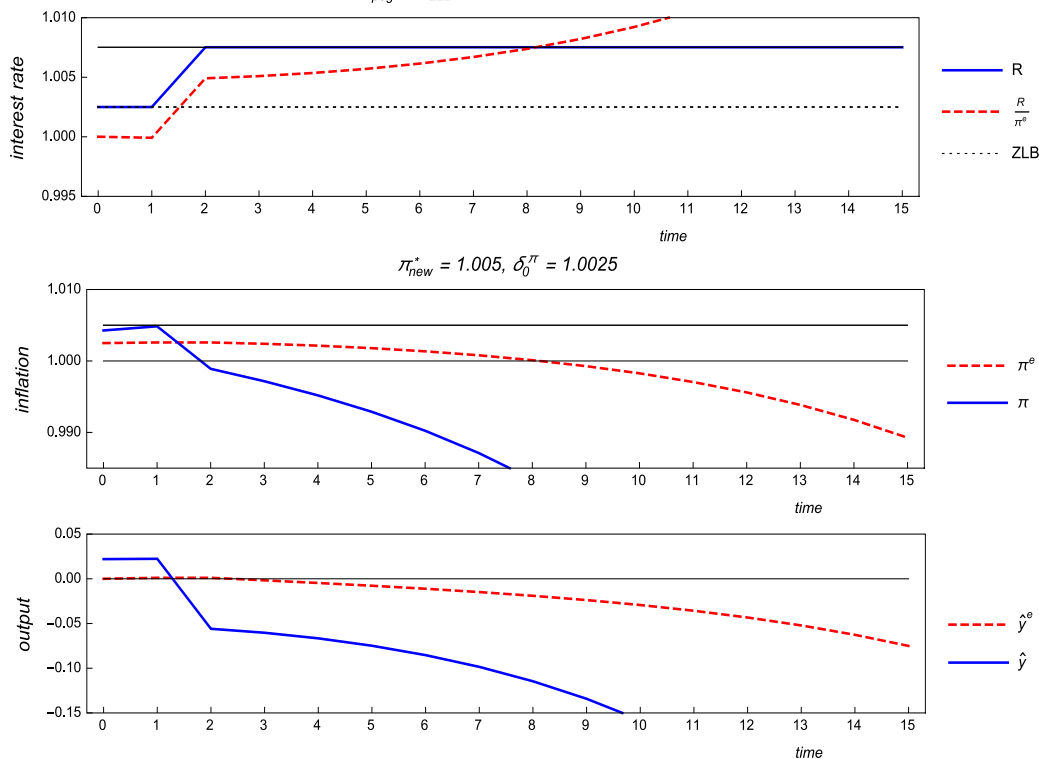


Figure 11: Immediate normalization to interest rate peg, in stylized policy setting, without bad shocks.

Finally, we return to the sharp neo-Fisherian alternative: beginning in $t = 2$, the nominal interest rate is pegged at R^* . Figure 11 provides the outcome when the adverse shocks do not materialize. The first two periods are again the same as in Figures 7 – 10. As would be anticipated from Figure 2 and 3 the results are not good: the policy results in recession, deflation and destabilization. In Figure 11 we have assumed that expectations evolve according to adaptive learning, i.e. there is no immediate expectational effect to a normalization to the peg. Hence the destabilization occurs very rapidly. However, it is clear from our earlier results that, even with the optimistic assumption that inflation expectations immediately adjust upward to the policy change, the policy will still lead to destabilization. It should also be noted that the presence of bad shocks would lead to an even more rapid descent into recessions and deflation. Note that the scale of the axes have been modified to accommodate the severity of the outcome.

¹⁶The case for delayed normalization is strengthened further if there is the possibility of adverse shocks large enough to push the economy into deflation and stagnation if interest rates are normalized early.

6 Conclusion

Following the Great Recession, many countries have experienced repeated periods with realized and expected inflation below target levels set by policymakers. Should policy respond to this by keeping interest rates near zero for a longer period or, in line with neo-Fisherian reasoning, by increasing the interest rate to the steady-state level corresponding to the target inflation rate? We have shown that sharp neo-Fisherian policies, in which interest rates are set according to a fixed peg, impart unavoidable instability. In contrast, normalization implemented by an interest-rate rule following the Taylor principle is locally stable and is likely to be successful if inflation expectations are not too far below target. If inflation expectations are significantly below target, and if there is the possibility of adverse shocks in the near future, then there is an argument for delayed normalization, keeping interest rates low until a later date, when inflation expectations are near target and the threat of adverse shocks has diminished, and then implementing normalization using a Taylor rule.

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