Online Appendix not for publication

A Data Appendix

A.1 US and Cross-Country Data

GDP (PPP) and Labor Force Data We use data from Penn World Tables version 7.1. (see Heston et al. (2012)) to construct annual time series of PPP-adjusted GDP in constant 1990 prices, PPP-adjusted GDP per worker in 1990 constant prices, and total employment between 1950-2010. For each country, we construct total employment L using the variables Population (POP), PPP Converted GDP Chain per worker at 2005 constant prices (RGDPWOK), and PPP Converted GDP Per Capita (Chain Series) at 2005 constant prices (RGDPCH) as $L = (RGDPCH \times POP)/RGDPWOK$. We then construct PPP-adjusted GDP in constant 2005 prices using the variables "PPP Converted GDP Per Capita (Laspeyres), derived from growth rates of c, g, i, at 2005 constant prices (RGDPL)" and "Population (POP)" as $GDPK = RGDPL \times POP$. We then re-base the 2005 data to 1990 prices.

We extend the labor force and GDP data calculated above back in time for the period 1869-1950 using growth rates from Maddison (2007). In particular, we calculate the pre-1950 growth rates for Maddison's GDP measure (in 1990 Geary-Khamis dollars) and use it to extend the GDP measure calculated above. We also calculate the population growth rates from Maddison's data and assume that the growth rate of the labor force pre-1950 is the same as the growth rate of the population (which is true if the labor-force-to-population ratio stays constant over time). We then use these population growth rates to extend the labor force data calculated above back in time. Finally, we extend the series for GDP and labor force forward in time between 2011 and 2014 using growth rates for labor force and constant price PPP GDP from the WDI (2016). The series for PPP-adjusted GDP per worker in constant prices is then computed as $y = GDP/L$.

Agricultural and Non-Agricultural Employment We construct contemporary (1980-2014) agricultural employment share using data from the FAOSTAT (2012) by taking the ratio of economically active population (labor force) in agriculture to total economically active population (labor force). For the US, we then combine this with the agricultural employment share from Alvarez-Cuadrado and Poschke (2011) for 1909-1980 and Lebergott (1966) for 1869-1908. Missing data is interpolated. We then obtain total employment in agriculture by multiplying the agricultural employment shares calculated above by the labor force data found in the previous paragraph. Non-agricultural employment is then the difference between total employment and agricultural employment. These are the $L_{a,t}$ and $L_{c,t}$ referred to in the main body of the text.

Sectoral Value Added, Constant price For the period 1970-2014 we construct constant price (1990) value added data for agriculture and total value added in Local Currency Units from UN (2016). For the US, we then extend this data backwards using sectoral growth rates. First, we obtain constant price agricultural and total value added data for the period 1947-1970 from the 10-sector database constructed by Timmer et al. (2014) and use this data to extend our value added measures back to 1947. Then, we extend these concatenated data back to 1869 using the Historical National Accounts database constructed by the Timmer et al. (2014). This database provides historical constant price indices of GNP for agriculture and the total economy. Missing values are interpolated. Notice that - like Duarte and Restuccia (2010) - we do *not* directly use the resulting series in our model. Instead, we use the above constant (1990) price agricultural and total value added data to calculate constant price shares of agricultural value added in total value added. Then, in order to remain consistent with out aggregate GDP data calculated above, we multiply these constant price shares by the aggregate PPP GDP data calculated above. This gives us a consistent estimate of 1990 value added of agriculture. Subtracting this estimate from the GDP PPP gives us an estimate of non-agricultural value added. These are the $Y_{a,t}$ and $Y_{c,t}$ referred to in the main body of the text.

Labor Productivity Growth Rates To calculate labor productivity growth rates we first calculate sectoral labor productivity for each sector $s = a, c$ as $y_{s,t} \equiv Y_{s,t}/L_{s,t}$ using values described in the above paragraphs. Next, we HP-filter these series and for each country and sector $s = a, c$ we calculate the average annualized sectoral labor productivity growth rates, $g_s - 1$, between 1980 and 2010. For the US we follow the same procedure but focus on the 1869-2007 period.

Price Indices For the cross country data and the period 1970-2014 we can construct agriculture and non-agriculture price indices by dividing sectoral value added in current prices by constant price (1990) value added data from UN (2016). Sectoral prices between 1929 and 2013 for the United States are obtained by dividing sectoral value added in current prices (obtained from the BEA and (Timmer et al., 2014)) by constant price sectoral value added found in the previous paragraphs. For the 1970-2014 period, these are identical to the indices that can be obtained using UN data. Obtaining pre-1929 prices is more complicated. In particular, as far as we know, no dependable series of data on sectoral prices exists. As such, we use nominal wheat prices obtained from Table Cc205-266 in Carter et al., eds (2006) to extend the agricultural price index back in time between 1869 and 1929. Then, using data on constant- and current-price aggregate GDP, constant price sectoral value added and the agricultural price index calculated above, we can infer a non-agricultural price index as well. In particular, multiplying the constant-price agricultural value added by the agricultural price index, gives us an estimate of current price agricultural value added, $p_{a,t}Y_{a,t}$. We can then subtract this from current price GDP, to obtain an estimate of current price non-agricultural value added, $p_{c,t}Y_{c,t}$. Then, taking the ratio of current price non-agricultural value added to constant price non-agricultural value added we obtain an estimate of the price index of the non-agricultural sector. We obtain price indices for the economy as a whole in the same manner -

Money Data on M2 for the period 1980-2014 comes from the IMF (2015) and is conveniently collected in the World Bank's WDI database. For the US, we follow Anderson (2006) in the construction of the long run money stock series. The source of the data for the years 1959-2014 are lines 34 and 35 in the International Financial Statistics (IMF, 2015).[45] For the year 1948-1958 we use data constructed by Rasche (1990) and available online on the Historical Statistics of the United States website (Carter et al., eds, 2006). We extend this for 1869-1947 using data from Friedman and Schwartz (1963) also reported in the Historical Statistics of the United States.[46]

Nominal Interest Rates Nominal interest rates for the period 1980-2014 are constructed as the sum of the real interest rates from the IMF (2015) and the GDP deflator. Both data series are conveniently collected in the World Bank's WDI. For the US, we take nominal interest rates for the period 1871 and 2014 as the nominal returns (including dividends) on the Standard and Poor Composite Stock Price Index from an updated version of Chapter 26 of Shiller (1989), available online at `http://www.econ.yale.edu/~shiller/data/chapt26.xlsx`.

Aggregate Capital We follow Kuralbayeva and Stefanski (2013) and Caselli (2005) in constructing capital and make use of the perpetual inventory method. Capital is accumulated according to:

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{A.1}$$

where I_t is investment and δ is the depreciation rate. We measure I_t from the PWT 7.1 as real aggregate investment in PPP terms.[47] As is standard, we compute the initial capital stock K_0 as $I_0/(g+\delta)$, where I_0 is the value of the above investment series in the first period that it is available, and g is the average geometric growth rate for the investment series in the first twenty years the data is available. Finally, we set the depreciation rate, δ, to 0.05 to match the depreciation rates in the US.[48] The results are not very sensitive to choices in either δ, g or initial capital stock. The above process gives us sequences of capital stocks, K_t, derived from PWT data.

[45] Notice, that for 1980-2014 this coincides with the IFS/WDI data used above.

[46] Notice that Friedman and Schwartz (1963) have slightly different definitions of monetary aggregates to those currently used. According to Anderson (2006), Friedman and Schwartz's 'M4 resembles, in many respects, the Federal Reserve's current M2' and '(h)ence, from an economic viewpoint, Friedman and Schwartz's M4 and the currently published Federal Reserve M2 aggregates are more similar than first appearances might suggest'. As such, for the 1898-1947 period - when it is available - we use Friedman and Schwartz's measure of M4. For the period 1869-1897 only the M3 measure is available. As such, we use the growth rate of M3, to extend the data back in time.

[47] So that $I_t \equiv RGDPL \cdot POP \cdot KI$, where $RGDPL$ is real income per capita obtained with the Laspeyres method, POP is the population and KI is the investment share in real income. In the above we have re-based $RGDPL$ into 1990 dollars.

[48] The value of δ is chosen so that the average ratio of depreciation to GDP using the constructed capital stock series matches the average ratio of depreciation to GDP in the data over the corresponding period. The OECD's Annual National Accounts report depreciation in the data as "Consumption of Fixed Capital."

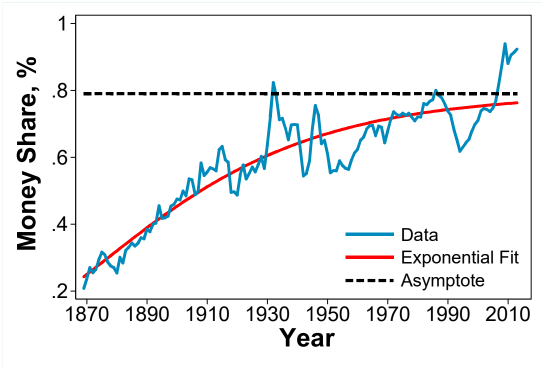


Figure A.1: Money Share in US and exponential best-fit line.

We extend our US capital data back in time using Piketty (2014) who provides data on the capital-output ratio in the US between 1770 and 2010 (measured in current-period prices). We consider only the reproducible part of his capital measure by subtracting the value of land from his measure of 'national capital'. The resulting capital series corresponds much more closely to our modern measure of capital (Jones, 2015). We use the implied growth rates in the capital-output ratio from Piketty (2014) to extend our capital-output ratio data backwards in time and maintain comparability. We find that the implied (reproducible) capital-output ratio in the US in 1869 (in current-period prices) was approximately 1.97. This is of course is a very crude estimate and should be approached cautiously - nonetheless this is the best we can do over this long time horizon.

Capital Shares Caselli and Feyrer (2007) estimate reproducible capital shares for 57 countries. They start by taking aggregate capital shares using data from Gollin (2002) and Bernanke and Gurkaynak (2001). They then make use of the World bank's "Where is the Wealth of Nations?: Measuring Capital for the 21st Century" database (WB, 2005) to adjust these shares by excluding non-reproducible capital. We take these capital shares as our ν's. For countries not included in their data set, we assign the average capital shares value of the 57 countries, which is 0.19.

TFP growth rates In the version of the model with capital, we use total factor productivity growth rates of the non-agricultural sector directly from the data. To calculate these, we assign all capital in the economy to the non-agricultural sector. Then, taking $Y_{c,t}$, $L_{c,t}$ and K_t from the above, and using the country-specific capital share, ν, the TFP in non-agriculture in a country is simply calculated as a Solow residual: $Y_{c,t}/(K_t^{\nu}L_{c,t}^{1-\nu})$. Calculating the annualized average growth rate of this term between 1980 and 2010, gives us the TFP growth rate used in the model.

Long Run US Money Share To obtain an estimate of long-run US money share, we fit an exponential function with an asymptote to the US money-share data. In particular we choose a non-linear first order difference equation with an asymptote defined by: $m_{t+1} = (ae^{-b(t-1869)}+1)m_t$ and $m_{1869} = c$ and find parameters a, b and c that minimize the mean squared error between the fitted money share, m_t, and the money share in the US between 1869 and 2013. We find that $a = 0.0287$, $b = 0.024$ and $c = 0.243$. The predicted function and the data are shown in Figure A.1. The implied asymptote of this difference equation is approximately 0.79.

Cross-country, non-monetary value added Blades (1975a,b) collect their data through a survey questionnaire sent to national statistical offices in 1973. They describe the process as follows: "The information on country practices in this report was mainly obtained from a mail enquiry made by the OECD Development Centre during the first part of 1973. Questionnaires requesting information on the coverage, valuation, and relative importance of non-monetary activities were sent to national statistical offices in just under 100 of the more important countries on the DAC list and 70 of them returned, completed questionnaires. (...) During the course of this study data were obtained from 48 countries on the percentage contribution of non-monetary activities to total GDP for a recent year. This information is, of course, readily available for countries which make separate estimates for non-monetary activities, but the majority of countries do not attempt to distinguish the non-monetary component of GDP. These countries were asked to make a 'best guess', or to indicate the probable range for the share of non-monetary output." The estimated shares for the 48 countries are the ones used in Figure 2.

Importantly, in their analysis Blades and his team focus on a particular type of non-monetary activity: goods produced for own-use or subsistence only. They characterize goods as follows: "The most important of these activities is the production for own consumption of crops and livestock, but we also consider a number of other activities which are related to primary production and which are undertaken mostly in rural areas. These include such things as fishing, hunting and forestry activities, building and construction, manufacture of simple household articles, ownership of dwellings, water collection and crop storage."

Thus, the Blades data does *not* consider payments in kind received by employees, or barter trade. As such - if anything - the data presented in the paper represent an under-estimate of the importance of non-monetary goods used across countries. Since we expect barter and payments in kind to be more prevalent in poorer countries with larger agricultural sectors, including these transactions would likely make Figure 2 even steeper, supporting further our results.

B Theoretical Appendix

B.1 Details of Baseline Model

Competitive Equilibrium For a given monetary policy, $\{T_t\}_{t=0}^{\infty}$, a competitive equilibrium in this economy is a sequence of prices, $\{p_{a,t}, p_{c,t}, w_t, r_t\}$, and quantities, $\{a_t, c_{m,t}, c_{n,t}, b_t, m_t, L_{a,t}, L_{c,t}\}_{t=0}^{\infty}$, such that (1) given prices and monetary policy, households and firms solve their optimization problem, (2) the government budget constraint is satisfied and (3) markets clear.

Solution The first order conditions for the household are given by:

$$a_t\colon \frac{\alpha\beta^t}{a_t - \bar{a}} = \lambda_t p_{a,t}; \qquad c_{m,t}\colon \frac{(1-\alpha-\gamma)\beta^t}{c_{m,t}} = p_{c,t}(\lambda_t + \mu_t); \qquad c_{n,t}\colon \frac{\gamma\beta^t}{c_{n,t}} = p_{c,t}\lambda_t \tag{B.1}$$

$$b_{t+1}\colon \lambda_t = (1 + r_{t+1})\lambda_{t+1}; \qquad m_{t+1}\colon \lambda_t = \lambda_{t+1} + \mu_{t+1} \tag{B.2}$$

$$\text{CIA: } \mu_t(p_{c,t}c_{m,t} - m_t) = 0 \quad \text{and} \quad \mu_t \geq 0 \tag{B.3}$$

In the above, λ_t and μ_t are multipliers on the budget and CIA constraints respectively. The firms' first-order conditions are:

$$L_{a,t}\colon p_{a,t}B_{a,t} = w_t \quad \text{and} \quad L_{c,t}\colon p_{c,t}B_{c,t} = w_t. \tag{B.4}$$

The market clearing conditions are given by the equations in (4). Finally, the transversality conditions for the above problem are:

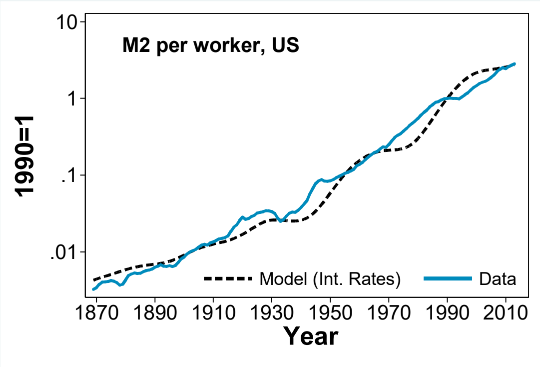
$$\lim \inf_{t\to\infty} \lambda_t(b_{t+1} + \bar{B}) = 0 \quad \text{and} \quad \lim \inf_{t\to\infty} \lambda_t m_{t+1} = 0. \tag{B.5}$$

Binding CIA To see that the CIA binds if we assume Assumption (3.1), divide the equations in (B.2) by each other to obtain the expression for interest rates, $r_{t+1} = \frac{\mu_{t+1}}{\lambda_{t+1}}$. Thus, the nominal interest rate is positive if and only if money yields liquidity services ($\mu_{t+1} > 0$). In particular, if the nominal interest rate is positive, the CIA constraint is binding.

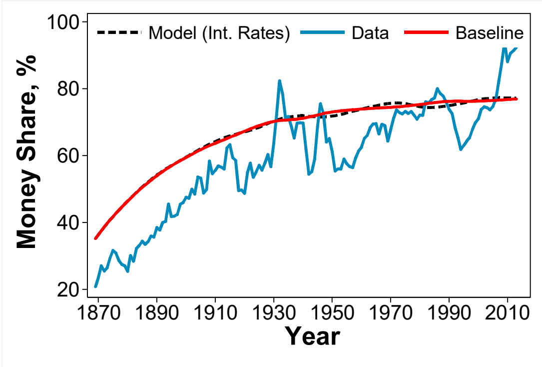
B.2 Matching interest rates

The per-capita money-stock in our baseline calibration is chosen to match money growth rates in data. Here we recalibrate the baseline model by matching observed nominal interest rates instead of money growth rates. Normalizing the initial level of money per worker to one, $M_{1869} = 1$, taking nominal interest rates from the data and using equation (8), the model implies a series for money per worker over time pictured in Figure B.1(a).[49] This derived money series is very similar to

[49] Since the calibration of the remaining parameters is independent from the monetary growth rate, the other parameters remain identical to the baseline.



(a) Money (M2) per worker in the data and implied by matching interest rates.



(b) Implied money share.

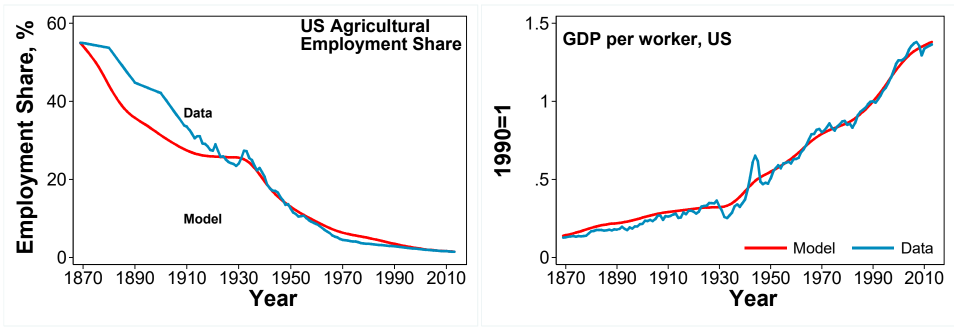
Figure B.1: Results for US when matching nominal interest rate rather than money, 1869-2012.

the observed monetary series. Figure B.1(b) then compares the evolution of the money share under this calibration, under the baseline calibration and in the data. Results are nearly indistinguishable between the two calibrations. The key reason is that the evolution of money shares is driven by changes in money demand stemming from structural transformation rather than money supply.

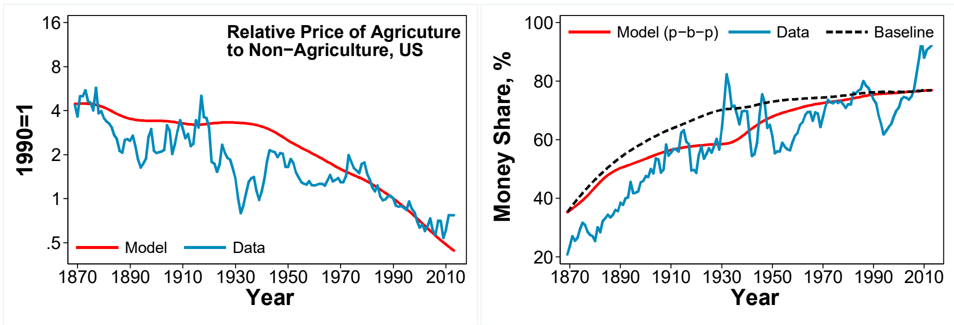
B.3 Observed Productivity Results

In the baseline model we chose to match *average* sectoral productivity growth. We did this since much of the early (pre-1929) data was interpolated. Here we show the results for a version of the baseline model calibrated to reproduce the period-by-period evolution of (interpolated) sectoral productivity. We show that qualitatively and quantitatively all previous results go through.

For the calibration, all parameters (besides sectoral productivity) remain identical to those of the baseline. To calibrate sectoral productivity, we normalize productivity levels across sectors to one in 1869; that is, $B_{s,1869} = 1$ for $s \in \{a, c\}$. Then we use data on the period-by-period growth rate of sectoral value added per worker in the United States to obtain the time paths of sectoral labor productivity. In particular, denoting by $g_{s,t}$ the growth factor of labor productivity between period t and period $t+1$ in sector s, we obtain the time path of labor productivity in each sector as $B_{s,t+1} = g_{s,t} B_{s,t}$. Figure B.2 shows the key results of this version of the model. First, in Figure B.2(a) we reproduce the employment share in agriculture. With this calibration strategy we do well - especially in the second part of the sample - where data has not been interpolated. This is as expected, since labor productivity is the major determinant of employment in agriculture in our model. Matching productivity more closely allows us to reproduces employment shares more closely. Analogously, this calibration strategy matches the GDP per worker slightly better than the baseline model (Figure B.2(b)). Figure B.2(c) shows the relative price of agriculture to non-agriculture goods. Once more, since relative prices are driven by relative productivity (as in equation (10)), we now do a



(a) Agricultural Employment Share, US (b) GDP per worker, US



(c) Relative prices agriculture to non-agriculture, US (d) Money (M2) Share in current price GDP, US

Figure B.2: Simulations (period-by-period model) and data for US, 1869-2012.

better job at capturing the evolution of relative prices. Nominal prices and factor payments evolve in a very similar fashion to the baseline, and are omitted to save space. Finally, we compare the money share from this calibration with the baseline version and the data in Figure B.2(d). Notice that the model with data calibrated period-by-period does better in the second half of our sample, when data is not interpolated. Importantly however, as expected, the money shares predicted by both models in 1869 and 2007 are the same. Thus, both qualitatively and quantitatively this extension makes very little difference to our results.

B.4 Introducing Capital

In this section we show that our results remain unchanged when we allow for capital accumulation in the non-agricultural sector. Adding capital allows us to capture a potentially important part of the development process and to endogenize the non-monetary demand of the non-agricultural sector which in the baseline is captured entirely by preference for credit goods. We find that adding capital does not significantly change our results.

Household's problem The representative households's problem is now given by:

$$\max_{\substack{a_t, c_{m,t}, c_{n,t}, \\ b_{t+1}, m_{t+1}, K_{t+1}}} \sum_{t=0}^{\infty} \beta^t \left(\alpha \log(a_t - \bar{a}) + (1-\alpha-\gamma)\log(c_{m,t}) + \gamma \log(c_{n,t})\right)$$

$$\text{s.t. } p_{a,t}a_t + p_{c,t}(c_{m,t} + c_{n,t} + x_t) + b_{t+1} + m_{t+1} \leq w_t + r_{k,t}K_t + (1+r_t)b_t + m_t + T_t$$

$$K_{t+1} = x_t + (1-\delta)K_t \tag{B.6}$$

$$p_{c,t}c_{m,t} \leq m_t \tag{B.7}$$

$$K_t, m_t \geq 0 \text{ and } b_t \geq -\bar{B} \text{ and } K_0 \text{ given}$$

The problem is very similar to the baseline model. The household owns a unit of labor and a stock of capital K_t which it rents out on the market for a wage w_t and a rental rate $r_{k,t}$ respectively. In addition, the household enters period t with money holdings m_t, bond holdings b_t, and a helicopter transfer of money from the government, T_t. With these resources, it can buy agricultural goods a_t (at the price $p_{a,t}$), cash- and non-cash non-agricultural goods (respectively, $c_{m,t}$ and $c_{n,t}$, at the price $p_{c,t}$), investment goods x_t (at the price $p_{c,t}$), bonds b_{t+1} for a gross return of $1 + r_{t+1}$ dollars next period, and it can decide to hold a (non-negative) stock of money m_{t+1} for the next period. Notice that we follow Cole and Kocherlakota (1998) by treating investment as a non-monetary, credit good which helps endogenize the cash-credit split of the non-agricultural sector.

Firms There are representative agricultural and non-agricultural firms. As in the baseline model, the agricultural firm hires labor, $L_{a,t}$, and produces output, $Y_{a,t}$, using a simple linear technology that combines labor with exogenous, sector-specific total factor productivity, $B_{a,t}$. Its profit maximization problem is given by equation (2). Non-agricultural firms however, are now assumed to hire labor, $L_{c,t}$, and to rent capital, K_t^f, from households. With these inputs they produce output, $Y_{c,t}$, using a Cobb-Douglas technology that combines labor and capital with exogenous, sector-specific total factor productivity, $B_{c,t}$. Their profit maximization problem is therefore given by:

$$\max_{K_t^f, L_{c,t}} \; p_{c,t}Y_{c,t} - w_t L_{c,t} - r_{k,t}K_t^f \qquad \text{s.t. } Y_{c,t} = B_{c,t}(K_t^f)^{\nu}(L_{c,t})^{1-\nu} \tag{B.8}$$

Output of non-agricultural firms, $Y_{c,t}$, can be sold as both a monetary and non-monetary consumption good as well as a non-monetary investment good. As in the baseline model, agricultural output is assumed to be a non-monetary, consumption good.

Money Supply The government is assumed to have a helicopter monetary policy as before:

$$M_{t+1} = T_{t+1} + M_t. \tag{B.9}$$

Market Clearing Finally, market clearing is standard.

$$\begin{gathered} a_t = Y_{a,t},\; c_{m,t} + c_{n,t} + x_t = Y_{c,t} \\ m_t = M_t,\; b_t = 0 \\ L_{a,t} + L_{c,t} = 1,\; K_t = K_t^f. \end{gathered} \tag{B.10}$$

Solution We follow the same solution strategy as in the baseline model. In particular, we assume that nominal interest rates are always positive and we use the first order conditions of the household and firms problems, in combination with the market clearing conditions and government's money supply to obtain three equations that (together with transversality conditions for bonds, capital and money) pin down the equilibrium solutions of the problem.

The first equation defines a split of non-agricultural consumption between monetary and non-monetary goods. Defining $C_t \equiv c_{m,t} + c_{n,t} = Y_{c,t} - (K_{t+1} - (1-\delta)K_t)$ we can write:

$$c_{n,t} = (1-\phi_t)C_t \qquad \text{and} \qquad c_{m,t} = \phi_t C_t, \tag{B.11}$$

where $\phi_t = \frac{1-\alpha-\gamma}{(1-\alpha-\gamma)+\gamma\tau_t}$. Second, given initial capital endowment, K_0, the path of capital is pinned down by a transversality condition and the following Euler equation:

$$\frac{\tau_{t+1}}{\tau_t}\frac{\phi_{t+1}}{\phi_t}\frac{C_{t+1}}{C_t} = \beta\left(\nu B^c_{t+1}(K_{t+1})^{\nu-1}(L^c_{t+1})^{1-\nu} + 1 - \delta\right). \tag{B.12}$$

Finally, employment in the non-agricultural sector, $L_{c,t}$, is determined by the following:

$$\frac{\alpha\tau_t}{(1-L_{c,t}) - \frac{\bar{a}}{B_{a,t}}} = \frac{(1-\nu)(1-\alpha-\gamma)B_{c,t}K_t^{\nu}(L_{c,t})^{-\nu}}{\phi_t C_t}. \tag{B.13}$$

We solve the model following the strategy of Echevarria (1997). We assume that after some point in time, T, the variables M_t, $B_{a,t}$ and $B_{c,t}$ grow at constant rates ($g_m - 1$, $g_a - 1 > 0$ and $g_c - 1$ respectively) and hence $\tau_t \to \bar{\tau}$ and $\phi_t \to \bar{\phi}$ converge to constants. Given these assumptions, the role of the non-homotheticity disappears in the limit as $\lim_{t\to\infty}\frac{\bar{a}}{B_{a,t}} = 0$ in equation (B.13). The model thus converges asymptotically to a balanced growth path where capital, investment and non-agricultural consumption grow at the rate $g_c^{\frac{1}{1-\nu}}$, agricultural consumption grows at a rate g_a and employment in agriculture and non-agriculture are constant. Consequently, we can re-write the model in terms of variables that are stationary in the long run. In particular, defining $\tilde{k}_t \equiv K_t/B_{c,t}^{\frac{1}{1-\nu}}$ and $\tilde{c}_t \equiv C_t/B_{c,t}^{\frac{1}{1-\nu}}$ equations (B.12) and (B.13) become:

$$\left(\frac{B_{c,t+1}}{B_{c,t}}\right)^{\frac{1}{1-\nu}}\frac{c_{t+1}}{c_t}\frac{\tau_{t+1}}{\tau_t}\frac{\phi_{t+1}}{\phi_t} = \beta\left(\nu\tilde{k}_{t+1}^{\nu-1}(L^c_{t+1})^{1-\nu} + 1 - \delta\right) \text{ and} \tag{B.14}$$

$$\frac{\alpha\tau_t\phi_t}{(1-L_{c,t}) - \frac{\bar{a}}{B_{a,t}}} = \frac{(1-\nu)(1-\alpha-\gamma)\tilde{k}_t^{\nu}(L_{c,t})^{-\nu}}{(1-\delta)\tilde{k}_t - \left(\frac{B_{c,t+1}}{B_{c,t}}\right)^{\frac{1}{1-\nu}}\tilde{k}_{t+1} + \tilde{k}_t^{\nu}(L_{c,t})^{1-\nu}} \tag{B.15}$$

Since $\lim_{t\to\infty}\frac{\bar{a}}{B_{a,t}} = 0$, using the above it is easy to show that $\tilde{k}_t \to \tilde{k}^*$ and $L^c_t \to L^*_c$, where:

$$L^*_c = \frac{(1-\nu)\left(g_c^{\frac{1}{1-\nu}} - \beta(1-\delta)\right)}{\left(g_c^{\frac{1}{1-\nu}} - \beta(1-\delta)\right) + \frac{\alpha}{1-\alpha-\gamma}\bar{\tau}\bar{\phi}\left(g_c^{\frac{1}{1-\nu}}(1-\beta\nu) - \beta(1-\delta)(1-\nu)\right)}, \text{ and} \tag{B.16}$$

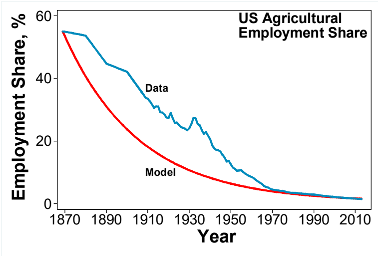
$$\tilde{k}^* = \beta^{-\frac{1}{1-\nu}}(g_c^{\frac{1}{1-\nu}} - \beta(1-\delta))^{\frac{1}{1-\nu}}\nu^{\frac{1}{1-\nu}}L^*_c. \tag{B.17}$$

Parameter	Values	Target	Target Value
$B_{s,1869}, M_{1869}$	1	Normalization, $s \in \{a, c\}$.	-
$g_a - 1$	0.028	Labor productivity growth in agriculture, 1869-2007.	2.79%
$g_c - 1$	0.009	Labor productivity growth in non-agriculture, 1869-2007.	1.23%
$\{B_{s,t}\}_{t=1869}^{2014}$	$B_{s,t} = g_s^{t-1869}$	Constant productivity growth in sector $s \in \{a, c\}$.	-
$\{M_t\}_{t=1869}^{2014}$	$\{\cdot\}$	Growth in money stock per worker.	-
K_0	1.298	Reproducible-capital to output ratio in 1869, Piketty (2014).	1.97
α	0.004	Long-run employment share in agriculture.	0.500%
$\bar{a}$	0.547	Employment share in agriculture in 1869.	55.%
β	0.963	Average annual nominal interest & money growth rates, 1869-2007.	8.85% & 4.78%
δ	0.050	Consumption of fixed capital relative to GDP, 1964-2007.	0.050
ν	0.229	Reproducible-capital income share, Caselli and Feyrer (2007).	22.9%
$1-\gamma$	0.923	Long-run share of money in nominal value-added.	79.%

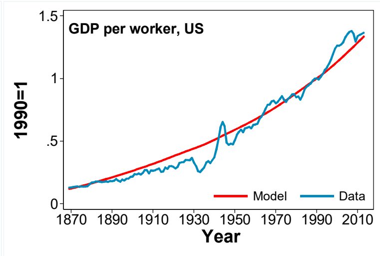
Table B.1: Model with capital, calibrated parameters. See Online Appendix A for detailed sources.

We then use standard numerical methods to solve for the sequences $\tilde{k}_t$ and $L_{c,t}$ that converge to L_c^* and $\tilde{k}^*$ using equations (B.14) and (B.15). Given these, we can obtain solutions for K_t and the other non-detrended variables.

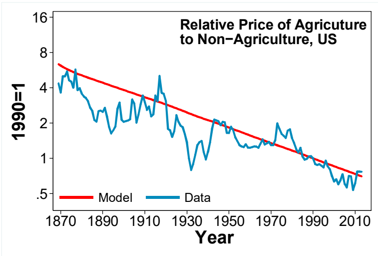
Calibration and Results We follow the same calibration strategy used to calibrate the baseline model. Table B.1 summarizes the parameters' values. The only additional parameters are δ, ν and initial capital stock. Total factor productivity growth in non-agriculture, $g_c - 1$, is also calibrated slightly differently. We choose δ to match the average ratio of consumption of fixed capital relative to GDP in the US between 1964 and 2007 from the BEA. We choose a country specific ν - or capital share - directly from Caselli and Feyrer (2007) who provide estimates of reproducible capital shares for 57 countries (see Online Appendix A for details). A crucial aspect of this calibration is the choice of initial capital in 1869. We use Piketty (2014)'s data to determine this value, by choosing the initial capital stock in the model to replicate his (reproducible-)capital-output ratio in the US in 1869 (details on our calculation are in Online Appendix A). Although this is most likely an imprecise measure of capital, it is the best we can do given the lack of historical data. Finally, since we do not have the entire capital stock series for the period, rather than calibrating total factor productivity growth in non-agriculture directly from the data, we instead choose the growth rate $g_c - 1$ so that our model replicates observed *labor* productivity growth between 1869 and 2007 in the data. Keeping in mind our caveats for capital stock data, Figure B.3 illustrates that our model with capital does a good job in fitting the long-run data for US, and that outcomes are not very different to the baseline model. In Figure B.4(a) we can see that our model predicts money shares trends quite accurately. Our initial money share in 1869 is slightly higher than before, however this is somewhat sensitive to the initial capital stock level - which is only roughly estimated. Importantly, we capture a large part of the increase in money shares- in particular we explain 62%($\approx \frac{79.-42.89}{79.-20.81}$) of the observed increase in the money share of the United States economy between 1869 and the implied long-run money share of approximately 79.%. This is quite remarkable especially in light of the caveats regarding



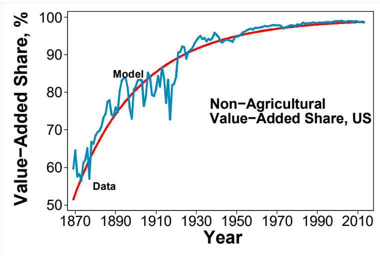
(a) Agricultural Employment Share



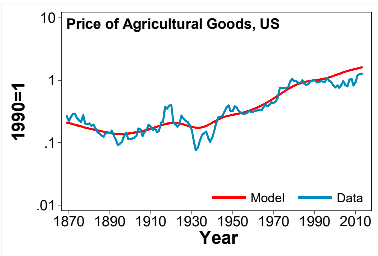
(b) GDP per worker



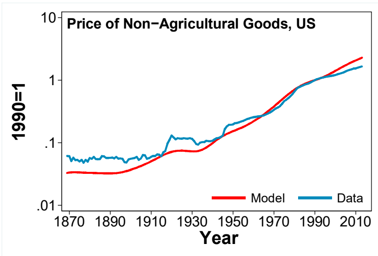
(c) Relative price of agriculture



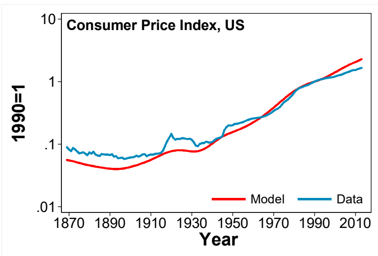
(d) Non-Agricultural value added share



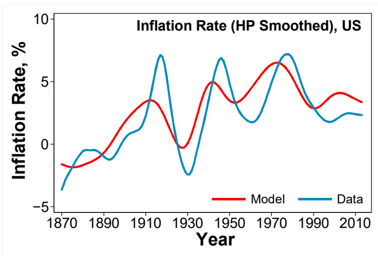
(e) Index of agricultural prices



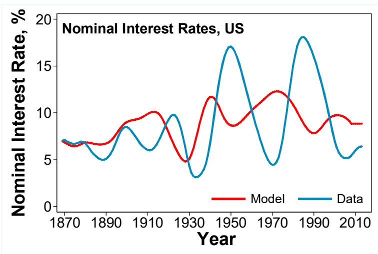
(f) Index of non-agricultural prices



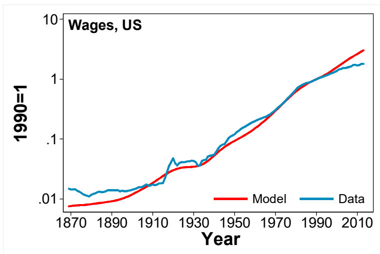
(g) Consumer Price Index



(h) Inflation Rate, (Smoothed)

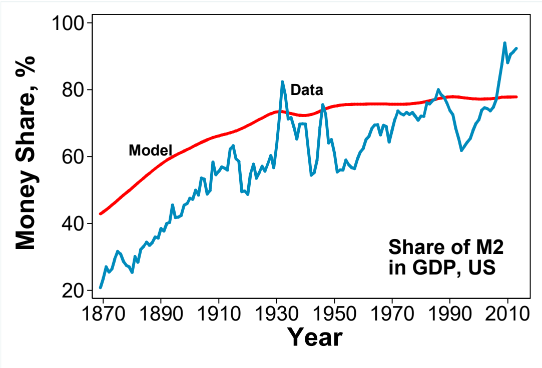


(i) Nominal interest rates



(j) Index of wages

Figure B.3: Simulations and data for the US in the Capital Model, 1869-2012.



(a) Money (M2) Share in current price GDP.

Money to GDP Ratio:	Data	m_b^{US}	m_k^{US}
Observations	144	144	144
Mean	58.97	66.88	69.18
$t = 1869$	20.81	35.23	42.89
p10	32.25	48.89	52.82
p90	76.07	76.25	77.49
Std. Dev.	16.43	10.89	9.61
M-Y Semi-Elast.	0.191	0.159	0.136

(b) Summary statistics for US.

Figure B.4: Money share of GDP and summary statistics for US. In the Table, columns show statistics for: money shares from the data (Data), the baseline model (m_b^{US}), and the capital model (m_k^{US}). Rows (1)-(6) show: the number of observations, the mean, the top decile, the bottom decile, the standard deviation as well as the semi-elasticity of money-share to income.

the measure of capital used.

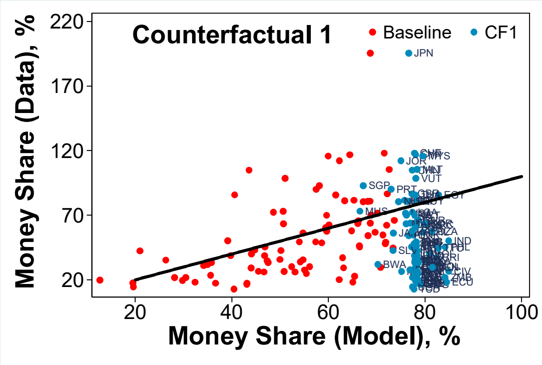
Cross-Country Calibration Next, we consider how well the model performs in the cross-country setting. As in the baseline we focus on an international panel of 89 countries for 1980-2010.[50] We assume that each country is a closed economy with the same structural parameters of the US, with the exception of sectoral productivity, monetary policy, capital share and initial capital stock. Money stock per worker in each country and each year is taken directly from the data. The country i specific capital share, ν_i, comes directly from Caselli and Feyrer (2007).We assume that total factor productivity in sector s and in country i grows at a constant rate, $g_s^i - 1$, and is given by:

$$B_{s,t}^i = B_s^i \times (g_s^i)^{t-1980}. \tag{B.18}$$

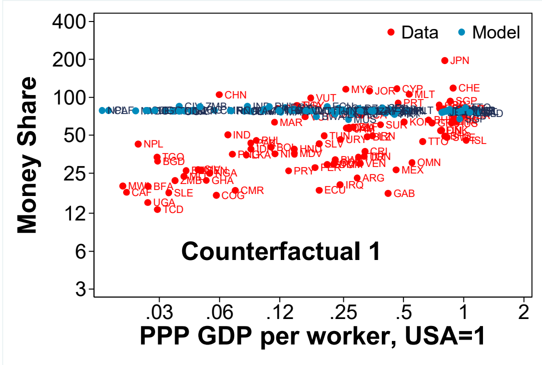
We then choose g_s^i to match observed average sectoral total factor productivity growth rates in each country between 1980-2010 directly from the data, and pin down sectoral productivity levels, B_s^i, as in the baseline: B_a^i is chosen to match the agricultural employment share in country i in 1980, whilst B_c^i is chosen to match the ratio of GDP per worker in country i to that of the USA in 1980.[51] For more details on how we construct the international series for sectoral total factor productivities, capital, capital shares and monetary aggregates, see Online Appendix A. Next, given this calibration and in order to disentangle the mechanisms at work, we perform a set of four counterfactuals.

[50] We consider all countries for which all necessary data is available. Unless otherwise stated all data sources and construction methodology is presented in Online Appendix A.

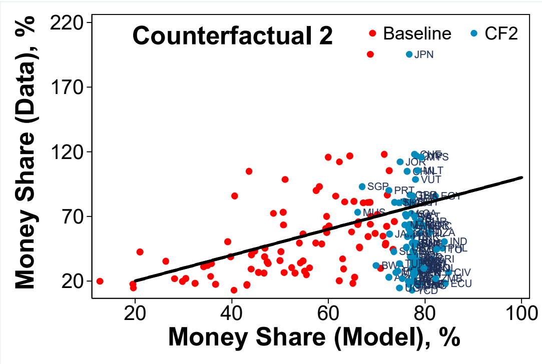
[51] Since this is a dynamic model, we must also specify future values of productivity and money stock. We assume that all exogenously growing variables continue to grow at their country specific averages for a hundred years after 2007. After this point it is assumed that the growth rates of these variables converge to the long run US average growth rates. This assumption is made for convenience and quantitatively plays no role in our results.



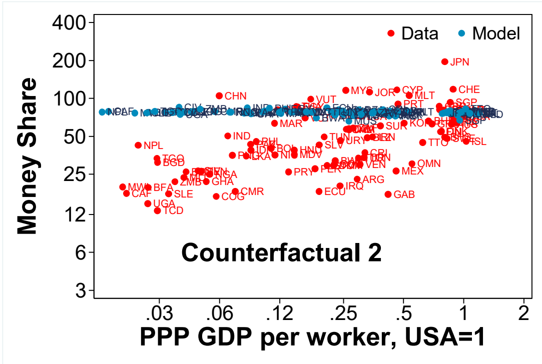
(a) Counterfactual 1 (US agr. productivity and monetary policy, steady state capital level): Money shares in data and model.



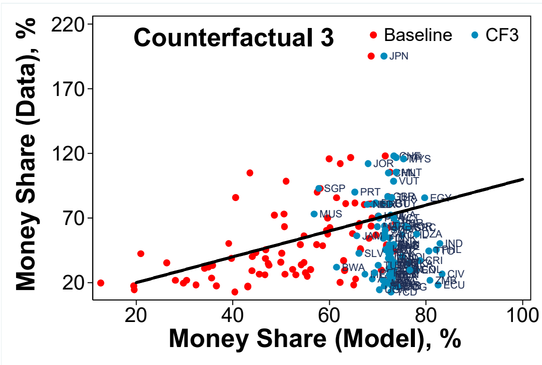
(b) Counterfactual 1 (US agr. productivity and monetary policy, steady state capital level): Money shares versus GDP per capita.



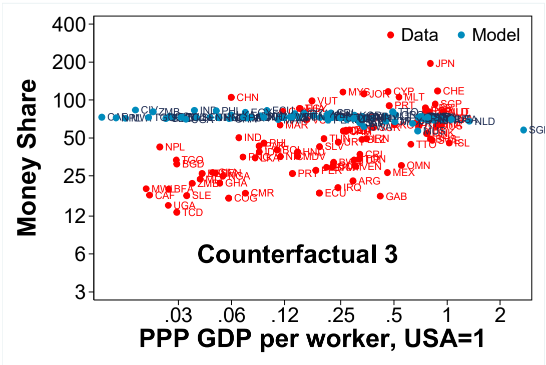
(c) Counterfactual 2 (US agr. productivity, steady state capital level): Money shares in data and model.



(d) Counterfactual 2 (US agr. productivity, steady state capital level): Money shares versus GDP per capita.

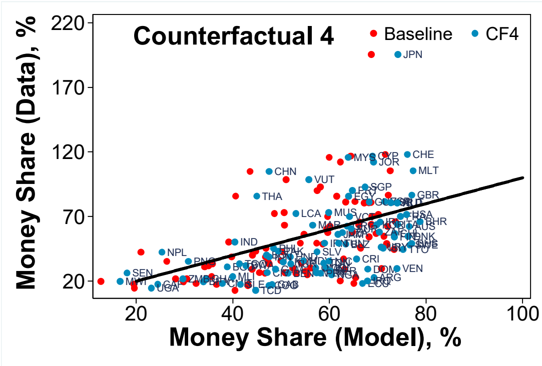


(e) Counterfactual 3 (US agr. productivity, country-specific capital level): Money shares in data and model.

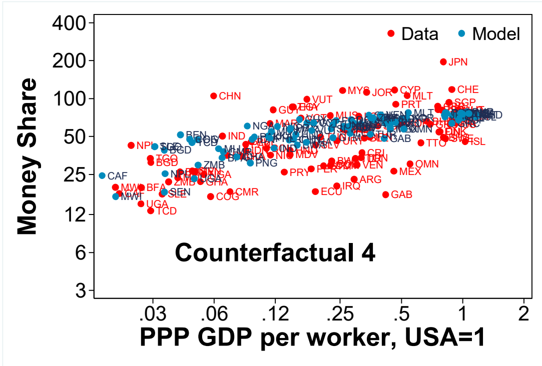


(f) Counterfactual 3 (US agr. productivity, country-specific capital level): Money shares versus GDP per capita.

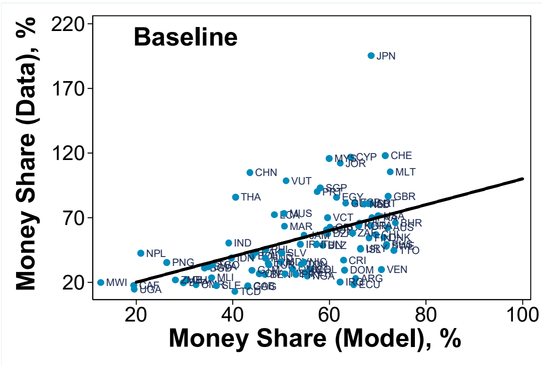
Figure B.5: Money shares in the the baseline model and the counterfactuals (average for 1980-2010). Drawn versus corresponding data and (log of) GDP per worker.



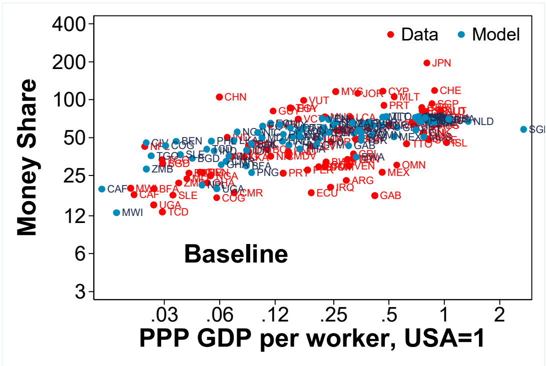
(a) Counterfactual 4 (Country-specific agr. productivity, steady state capital level): Money shares in data and model.



(b) Counterfactual 4 (Country-specific agr. productivity, steady state capital level): Money shares versus GDP per capita.



(c) Baseline Model: Money shares in data and model.



(d) Baseline Model: Money shares versus GDP per capita.

Figure B.6: Money shares in the baseline model and the counterfactuals (average for 1980-2010). Drawn versus corresponding data and (log of) GDP per worker.

B.5 Traditional and modern agriculture

The baseline model in section 3 assumes that all transactions in the agricultural sector are non-monetary. We make the assumption both for simplicity and since data on non-monetary agricultural consumption is generally unavailable - especially in the cross-sectional setting. Nonetheless, we do not expect this assumption to make too much of a difference to our results. The agricultural sector tends to be relatively small in rich countries, and hence will not contribute much to money demand in those countries anyway. In poor countries where agriculture is an important part of the economy, we would expect traditional, barter/home production-style and importantly non-cash agriculture to dominate and hence, again, agriculture would not play a large role in driving money demand.

Nonetheless, in this section we use US data to examine agriculture's contribution to the evolution of money demand. The USDA collects data on non-monetary value added for crops and animals consumed in farms for 1910-2016, and for non-cash payment to labour for the period 1919-2016.

By summing up these two measures, we can obtain a rough estimate of the share of non-monetary transactions in the agricultural sector in the US.[52] According to these data, around 25% of agricultural value added was exchanged without cash in 1910. This figure declined at a roughly constant rate of 4.37% a year and was approximately 0.5% in 2016. Based on this evidence, we extend the model to incorporate cash and non-cash transactions in the agricultural sector.

We continue to assume there are two final-goods sectors (agriculture and non-agriculture). The setup of the non-agricultural sector remains identical to the baseline model.[53] However, we now assume that there are *two* intermediate sub-sectors in agriculture: a traditional and a modern sector. These two sub-sectors differ along two dimensions. First, the traditional sector is characterized by lower productivity growth. In fact, due to lack of better data, we will assume throughout that productivity in this sub-sector remains constant over time. This is in line with the argument put forward by Lucas (2004) that "traditional agricultural societies are very like one another".[54] Second, we assume that households use cash to pay for the "modern" portion of their agricultural purchases whilst traditional agricultural goods are bought without cash.

The outputs of these two sub-sectors are close, but imperfect substitutes. The higher productivity growth in modern agriculture combined with the substitutability generates a transition from traditional to modern agriculture, on top of the standard shift from agriculture to non-agriculture. The changing composition of agriculture introduces an agriculture-specific money demand which lets us match the evolution of non-monetary transactions in the US. Crucially, we find that money shares and velocities are very similar to the benchmark model.[55] In what follows, we first present the setup and the analytical results of the model outlined above. We then explain how we calibrate the model to US data, and show that our results are close to the baseline model.

Household's problem The representative households's problem is now given by:

$$\max_{\substack{a_t,\, c_{m,t},\, c_{n,t},\\ b_{t+1},\, m_{t+1}}} \sum_{t=0}^{\infty} \beta^t \left(\alpha \log(a_t - \bar{a}) + (1-\alpha-\gamma)\log(c_{m,t}) + \gamma \log(c_{n,t})\right)$$

$$\begin{aligned} \text{s.t. } & p_{a,t}a_t + p_{c,t}(c_{m,t}+c_{n,t}) + b_{t+1} + m_{t+1} \leq w_t + (1+r_t)b_t + m_t + T_t \\ & p_{c,t}c_{m,t} + \theta_t p_{a,t} a_t \leq m_t \\ & b_t \geq -\bar{B} \end{aligned} \tag{B.19}$$

[52] We thank an anonymous referee for this suggestion.
[53] For simplicity, we continue to abstract from capital accumulation in this sector.
[54] Notice that this is not a crucial assumption. All that is required is that productivity growth in the traditional sector is smaller than in the modern sector.
[55] Notice that the above setup is stylized and adopted primarily for simplicity. We will get qualitatively and quantitatively very similar results - at the cost of greater complexity (i.e. corner solutions) - by assuming a transition from traditional to modern agricultural sector like in Gollin et al. (2007). There, both agricultural sub-sectors are perfect substitutes, but traditional agriculture is assumed not to use capital whilst modern agriculture does use capital. This results in poorer countries that have smaller capital endowments using traditional methods in agriculture, and later transitioning to modern farming techniques as capital holdings and modern agriculture productivity increase.

The only difference relative to the baseline model is that now a proportion of agricultural goods purchased by the household, $0 < \theta_t < 1$, will also be paid for with cash. We assume that this θ_t is exogenous to the household (but not to the firm) to capture the idea that sellers - rather than buyers - decide whether to take cash for their goods or not.[56]

Firms The final agricultural good is produced using two types of intermediate agricultural goods: modern (monetary) agricultural goods, denoted by m and traditional (non-monetary) agricultural goods denoted by n. Production of the non-agricultural final good (denoted by c) remains exactly as in the baseline. Agricultural intermediate goods and non-agricultural final goods are produced using the following production functions: $Y_{s,t} = B_{s,t}L_{s,t}$, for $s \in \{m, n, c\}$. Here, $B_{s,t}$ is labor productivity and $L_{s,t}$ is the labor input. The profit maximization problem for each firm $s \in \{m, n, c\}$ is given by:

$$\max_{L_{s,t}} p_{s,t}Y_{s,t} - w_t L_{s,t} \qquad \text{s.t. } Y_{s,t} = B_{s,t}L_{s,t}. \tag{B.20}$$

The final agricultural good is an aggregate of the intermediate m and n goods with a production function given by: $Y_{a,t} = \left(\eta(a_{n,t})^{\frac{\sigma-1}{\sigma}} + (1-\eta)(a_{m,t})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$. In this expression σ is the elasticity of substitution between traditional and modern agriculture, whilst $0 < \eta < 1$ determines the relative importance of traditional agriculture. Traditional and modern agriculture are assumed to be close substitutes so that $\sigma > 1$. The profit maximization of the final good firm is standard:

$$\max_{a_{n,t}, a_{m,t}} p_{a,t}Y_{a,t} - p_{n,t}a_{n,t} - p_{m,t}a_{m,t}. \tag{B.21}$$

An *endogenous* proportion of the value of final agricultural goods, $\theta_t \equiv \frac{p_{m,t}a_{m,t}}{p_{a,t}Y_{a,t}}$ will be 'modern' goods, whilst the remainder will be 'traditional'.

Money Supply The government is assumed to have a helicopter monetary policy as before:

$$M_{t+1} = T_{t+1} + M_t. \tag{B.22}$$

Market Clearing Finally, markets clear in the usual way:

$$\begin{gathered} a_{n,t} = Y_{n,t},\ a_{m,t} = Y_{m,t},\ a_t = Y_{a,t},\ c_{m,t} + c_{n,t} = Y_{c,t} \\ Y_{a,t} = \left(\eta(a_{n,t})^{\frac{\sigma-1}{\sigma}} + (1-\eta)(a_{m,t})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\ m_t = M_t,\ b_t = 0 \\ L_{m,t} + L_{n,t} + L_{c,t} = 1. \end{gathered} \tag{B.23}$$

[56] Letting households rather than firms decide has almost no quantitative effect on the results but complicates the solution as changes in monetary growth will then effect θ_t over time so that this variable becomes dependent on future and current outcomes rather than just current outcomes, much like the relative size of the monetary sector in non-agriculture, ϕ_t, described below.

Solution Assuming positive nominal interest rates, we can solve for the optimal allocations. From the first order conditions of the agricultural intermediate- and final-good firms we can derive an expression for θ_t given by:

$$\theta_t = \frac{x_t}{1+x_t}, \text{ where } x_t \equiv \left(\frac{1-\eta}{\eta}\right)^{\sigma} \left(\frac{B_{m,t}}{B_{n,t}}\right)^{\sigma-1}. \tag{B.24}$$

Given our assumptions that $\sigma > 1$ and that productivity growth in modern agriculture is higher than in traditional agriculture, the share of modern agriculture, θ_t, will rise over time, whilst the share of traditional agriculture, $(1-\theta_t) = \frac{1}{1+x_t}$, will fall. Denoting total agricultural employment by $L_{a,t} \equiv L_{m,t} + L_{n,t}$, from the same first order conditions we can also show that:

$$L_{m,t} = \theta_t L_{a,t} \text{ and } L_{n,t} = (1-\theta_t)L_{a,t}.$$

Non-agricultural output is divided between cash and non-cash goods:

$$c_{n,t} = (1-\phi_t)Y_{c,t} \qquad \text{and} \qquad c_{m,t} = \phi_t Y_{c,t}, \tag{B.25}$$

We can then use households' first order conditions and the previous equations to get the labor share in agriculture

$$L_{a,t} = \alpha\frac{\phi_t}{1-\alpha-\gamma+\alpha\phi_t} + \eta^{\frac{\sigma}{1-\sigma}}\frac{\bar{a}}{B_{a,t}}\frac{1-\alpha-\gamma}{1-\alpha-\gamma+\alpha\phi_t}(1-\theta_t)^{\frac{1}{\sigma-1}} \tag{B.26}$$

Using first order conditions for consumption in the non-agricultural sector, we get a dynamic first-order difference equation that determines ϕ_t:

$$\frac{\phi_{t+1} + L_{a,t+1}(\theta_{t+1} - \phi_{t+1})}{\phi_t + L_{a,t}(\theta_t - \phi_t)} = \frac{\gamma(1-L_{a,t})\phi_{t+1}\tau_t}{(1-\alpha-\gamma)(1-L_{a,t+1})(1-\phi_t)} \tag{B.27}$$

In steady state, $\phi_t = \phi^{SS}$, i.e.

$$\phi^{SS} = \frac{1-\alpha-\gamma}{1-\alpha-\gamma+\gamma\tau^{SS}} \tag{B.28}$$

where τ^{SS} is determined by the steady state money growth rate. Given sequences for $B_{i,t}$, $i = a, m, c$ and M_t, we can easily solve for the optimal sequence of ϕ_t with a shooting algorithm, assuming that, for a large enough T, the economy is in steady state and hence $\phi_T = \phi_{T+1} = \phi^{SS}$. Next, from firms first order conditions, prices for sector $s \in \{m, n, c\}$ goods are:

$$p_{s,t} = \frac{w_t}{B_{s,t}}. \tag{B.29}$$

Similarly, the price for final agricultural goods is given by:

$$p_{a,t} = \frac{w_t}{B_{n,t}}\eta^{\frac{\sigma}{1-\sigma}}(1-\theta_t)^{\frac{1}{\sigma-1}}, \tag{B.30}$$

and the nominal interest rate and wage rate are:

$$r_t = \tau_t\left(\frac{1+\frac{\theta_t L_{a,t}}{\phi_t L_{c,t}}}{1+\frac{\theta_{t+1}L_{a,t+1}}{\phi_{t+1}L_{c,t+1}}}\right) - 1 \qquad \text{and} \qquad w_t = \frac{M_t}{\theta_t L_{a,t} + \phi_t L_{c,t}}. \tag{B.31}$$

Parameter	Values	Target	Target Value
$B_{s,1869}, M_{1869}$	1	Normalization, $s \in \{m, n, c\}$.	-
$g_{m,a} - 1$	0.038	Labor productivity growth in agriculture, 1869-2007.	2.79%
$g_{n,a} - 1$	0	Productivity growth in traditional agriculture.	0%
$g_c - 1$	0.012	Labor productivity growth in non-agriculture, 1869-2007.	1.23%
$\{B_{s,t}\}_{t=1869}^{2014}$	$\{g_s^{t-1869}\}$	Constant productivity growth in sector $s \in \{m, n, c\}$.	-
$\{M_t\}_{t=1869}^{2014}$	$\{\cdot\}$	Growth in money stock per worker.	-
α	0.005	Long-run employment share in agriculture.	0.5%
$\bar{a}$	0.306	Employment share in agriculture in 1869.	55.%
β	0.963	Average annual nominal interest & money growth rates, 1869-2007.	8.85% & 4.78%
$1 - \gamma$	0.790	Long-run share of money in nominal value-added.	79.%
σ	2.331	Change in proportion of traditional agriculture, 1910-2007.	-4.37%
η	0.660	Proportion of traditional agriculture, 2007.	0.52%

Table B.2: Model with modern agriculture, calibrated parameters. See Online Appendix A for detailed sources.

Finally, the share of monetary stock relative to the nominal GDP (or the *inverse* of monetary velocity) can be written as:

$$\begin{aligned} V_t^{-1} = \frac{M_t}{p_{a,t}Y_{a,t} + p_{c,t}Y_{c,t}} &= \theta_t \frac{p_{a,t}Y_{a,t}}{p_{a,t}Y_{a,t} + p_{c,t}Y_{c,t}} + \phi_t \frac{p_{c,t}Y_{c,t}}{p_{a,t}Y_{a,t} + p_{c,t}Y_{c,t}} \\ &= \theta_t L_{a,t} + \phi_t L_{c,t}. \end{aligned} \tag{B.32}$$

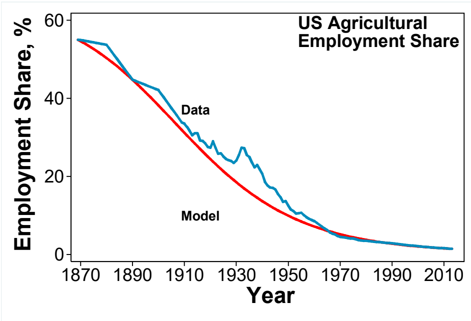
The first equality follows by definition. The second equality follows from the assumption that the CIA constraint binds. The final equality follows from the above price equations and the perfectly competitive nature of the problem. In contrast to the baseline, the money share in this model now depends on money demand originating both in the agricultural sector ($\theta_t L_{a,t}$) and the non-agricultural sector ($\phi_t L_{c,t}$). The importance of money demand in each sector is determined by the monetary component and the size of the respective sector (in terms of sectoral value added shares or - equivalently - sectoral employment shares). The above clearly demonstrates why agricultural money demand is unlikely to be important. In poorer countries, where productivity tends to be low, we will observe employment in agriculture $L_{a,t}$ close to one (equation (B.26)). However, given low productivity in the modern agricultural sector, traditional agriculture will dominate this sector and hence θ_t will be close to zero (equation (B.24)). Thus, the product of the two terms will also be close to zero. In richer countries, productivity will tend to be higher, so we will observe θ_t closer to one, but $L_{a,t}$ closer to zero - thus again, the product of the two terms will be close to zero. As such, it is the second part of the expression, $\phi_t L_{c,t}$, that plays a key role in driving money shares over time both in both the baseline and this model.

Calibration We calibrate the model to US data for the period from 1869 to 2007, following the same calibration strategy as in the baseline model. Table B.2 summarizes all the parameter values. To remain brief, below we focus only on the calibration of parameters that do not appear or differ from the baseline calibration: σ, η and the productivity parameters in modern and traditional agriculture: $B_{s,t}$ for $s \in \{m, n\}$.

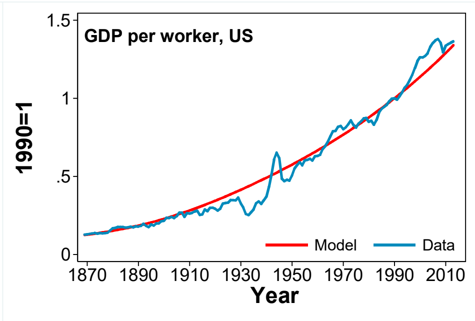
First, we normalize productivity in modern and traditional agriculture to one - so that $B_{s,1869} = 1$ for $s \in \{m, n\}$. As argued above we also assume that productivity growth in traditional agriculture is zero, so that $B_{n,t} = 1$. Then we use data on the average growth rate of agricultural value added per worker in the United States to obtain the time paths of labor productivity in the modern agricultural sector. In particular, we choose $g_m - 1$ - the average labor productivity growth in the modern agricultural sector in the model between 1869 and 2007 - so that the model reproduces the observed growth of labor productivity in *total* agriculture (both modern and traditional) in the data over the same period. The time path of labor productivity in the modern agricultural sector is then given by $B_{m,t+1} = g_m B_{m,t}$.

Second, we choose σ and η to match the evolution of the share of traditional agricultural value added in total agricultural value added (i.e. $1 - \theta_t$) over time. We use USDA data to construct the series of traditional agriculture value added. From equation (B.24), for a given sequence of productivities in modern and traditional agriculture, η determines the level of $1 - \theta_t$, whilst σ determines the extent to which a change in relative productivity translates into a change in $1-\theta_t$ over time. Consequently, we choose η so that the model reproduces the share of traditional agriculture in 2007 and we choose σ so that the model matches the average annual rate of decline in the share of traditional agriculture between 1910 (the first year available) and 2007. Notice that given greater productivity growth in modern agriculture than in traditional agriculture, the calibration implies an elasticity of substitution between modern and traditional agriculture that is greater than one. The calibration of all remaining parameters is identical to the baseline.

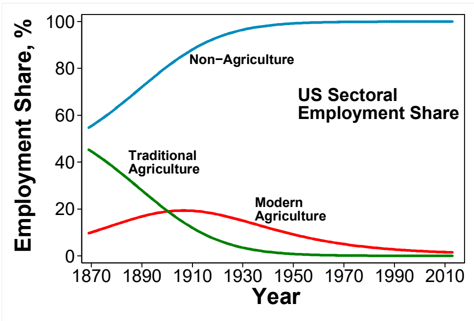
Results Results from this model are in line with the baseline setup. Furthermore, most explanations behind the results remain identical to those of the baseline. Figures B.7(a) and B.7(b) show a very good fit for the series for agricultural labor shares and GDP per worker. The fit in this case is even better than in the baseline setup - as we are now effectively allowing for an endogenously evolving productivity in agriculture, resulting from a changing composition of the agricultural sector from a (zero productivity growth) traditional sector to a (positive productivity growth) modern sector. Figure B.7(c), shows this changing composition of employment within agriculture. Notice that over time, traditional agriculture shrinks. This occurs for two reasons: not only because workers are moving away from traditional agriculture into modern agriculture due to the higher productivity growth in the modern sector, but also because of the flow of workers out of agriculture all together. Notice also that modern agricultural employment share now forms a hump shape over time. This happens since traditional workers initially flow into modern agriculture, resulting in a rising employment share in that sector. However, as productivity in modern agriculture improves, fewer agricultural workers are needed to feed the population and so workers shift out of agriculture (including modern agriculture) altogether and into non-agriculture. Whilst we do not have data on the size of employment in traditional and modern agricultural sector, we do have data on the share



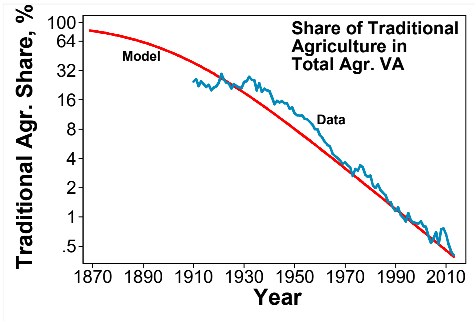
(a) Agricultural Employment Share, US



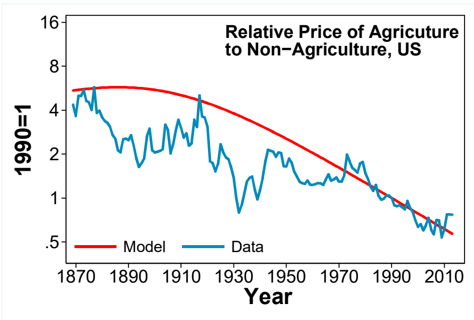
(b) GDP per worker, US



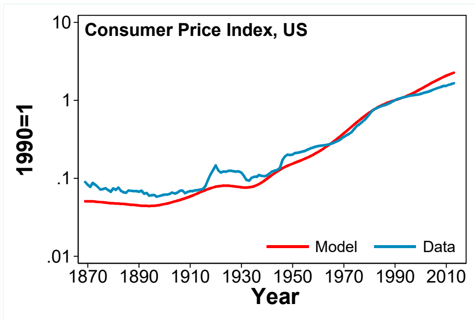
(c) Sectoral Employment, Model.



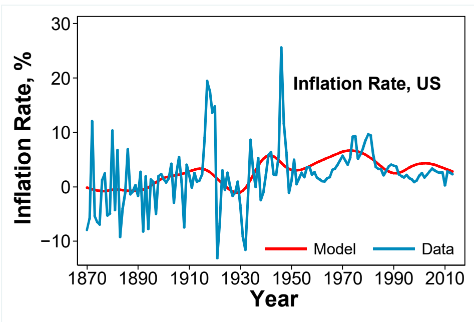
(d) Share of traditional agriculture in total agricultural value added.



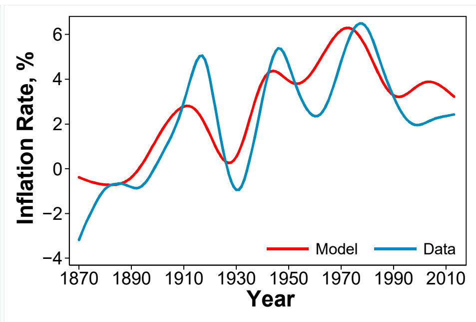
(e) Relative prices agriculture to non-agriculture, US



(f) Consumer Price Index, US

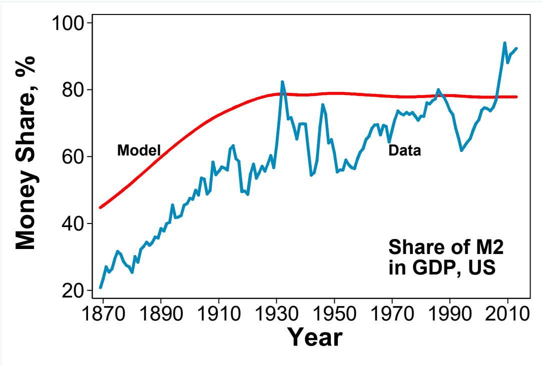


(g) Inflation Rate, US (Unsmoothed)



(h) Inflation Rate, US (Smoothed)

Figure B.7: Simulations and data for US, 1869-2012.



(a) Money (M2) Share in current price GDP.

Money to GDP Ratio:	Data	m_b^{US}	m_m^{US}
Observations	144	144	144
Mean	58.97	66.88	72.36
$t = 1869$	20.81	35.23	44.74
p10	32.25	48.89	54.45
p90	76.07	76.25	78.64
Std. Dev.	16.43	10.89	9.76
M-Y Semi-Elast.	0.191	0.159	0.117

(b) Summary statistics for alternative test, US.

Figure B.8: Money share of GDP and summary statistics for US Alternative Measures. In the Table, columns show statistics for: money shares from the data (Data), the baseline model (m_b^{US}), and the model with modern and traditional agriculture (m_m^{US}). Rows (1)-(6) show: the number of observations, the mean, the top-decile, the bottom-decile, the standard deviation as well as the semi-elasticity of money-share to income.

of traditional agricultural value added in total agricultural value added. This is shown in Figure B.7(d), together with the predicted values for the model. This fit emerges largely from the calibration - recall that we have chosen parameters (η and σ) to match the traditional agricultural share in 2007 as well as the average rate of decline of the traditional agricultural share. However, since our focus is on monetary velocity, this is not a big concern. The remaining figures show the evolution of selected price variables. The relative price of agriculture to non-agriculture shown in Figure B.7(e) is now nonlinear, first going up slightly at the end of the 19th century and then steadily declining. This feature arises from the fact that relative prices of agriculture to non-agriculture now not only depend on relative productivity in both sectors but also the composition of the agricultural sector - which is changing over time. This feature reduces the ability of the model to reproduce the Great Deflation at the end of the 19th century (see Figure B.7(f)): although a deflation is still present, the magnitude predicted by the model is smaller.

Finally, Figure B.8(a) shows the result for the money shares. Notice that the model in 1869 predicts a slightly higher money share than in the baseline. Importantly though, we still explain 59% of the increase of money share over the period relative to the long-run trend. Thus, even accounting for money demand in the agricultural sector, our results are in line with the those obtained from the baseline model.

B.6 Alternative tests of the model

Next, we present an alternative test of the baseline model. In the standard one-sector setup, money demand (M) is assumed to be proportional to nominal GDP ($\bar{Y}$) so that $M = \phi(r)\bar{Y}$. As an alternative, we propose that money demand originates (largely) in the non-agricultural sector. Our theory can thus be summarized with the following equations describing the money share:

$$\frac{M}{\bar{Y}} = \phi(r)\frac{\bar{Y}_c}{\bar{Y}_a + \bar{Y}_c} \tag{B.33}$$

$$= \phi(r)\frac{L_c}{L_a + L_c}. \tag{B.34}$$

Equations (B.33)-(B.34) are identical to equation (9), where for notational convenience we have dropped time subscripts, we have defined $\bar{Y}_s \equiv p_s Y_s$ for $s = a, c$, and we have emphasized that ϕ in equilibrium depends on the nominal interest rate. It is clear from both expressions that the money-to-GDP ratio is increasing in the level of development, since both the value-added share and the employment share of the non-agricultural sector increase with structural transformation. A straightforward check of our theory, therefore, would be to use equations (B.33) and (B.34) to plot the observed money-GDP ratio from the data (left hand side of the equation) against the predicted money-GDP ratios (right hand sides of the above equations). The only parameter that would then need to be calibrated is the interest elasticity of money demand.[57] In this section we perform this check for US time series and the cross-country sample, and show that doing do not significantly change our baseline results.

Comparison We start by calibrating the interest elasticity of money demand. In our baseline model, $\phi(r) = \frac{1}{1+a(1+r)}$, where $a \equiv \frac{\gamma}{1-\alpha-\gamma}$, which implies that interest elasticity of money demand is $|\frac{\partial \log(\phi(r))}{\partial \log(r)}| = \frac{ar}{1+a(1+r)}$. To pin down this elasticity we only need to know the parameter a. As we argued in the main body of the paper, under reasonable assumptions, the ratio of money to nominal GDP predicted by the model converges to $1 - \alpha - \gamma$ in the long run and the agricultural employment share converges to α. If we assume - as in the baseline - that the money-to-GDP ratio converges to a long run value of 0.79 predicted by the US data and employment share in agriculture converges to a reasonable 0.05%, then it follows that, $a = 0.259$, which allows us to pin down $\phi(r)$. Finally, taking nominal interest rates (r), current-price non-agriculture value added shares ($\frac{\bar{Y}_c}{\bar{Y}_a+\bar{Y}_c}$) and non-agriculture employment shares ($\frac{L_c}{L_a+L_c}$) directly from the data we can construct 'predicted' money shares, $m_y \equiv \phi(r)\frac{\bar{Y}_c}{\bar{Y}_a+\bar{Y}_c}$ and $m_l \equiv \phi(r)\frac{L_c}{L_a+L_c}$ and compare these with those predicted by our baseline model, m_b, and the data.[58]

Figure B.9(a) examines these predictions for the US between 1869 and 2012: m_y and m_l are plotted in red, our baseline model money share, m_b, is the black dashed line and the data is shown in blue. Table B.9(b) provides summary statistics for each estimate. Given our calibration, in the long run all three estimates converge to the same value of money share or 79%. We can thus measure our success by comparing the change in predicted and observed money share between 1869 and the long run value of 79%. The m_b and m_l explain the largest part of the increase - each capturing

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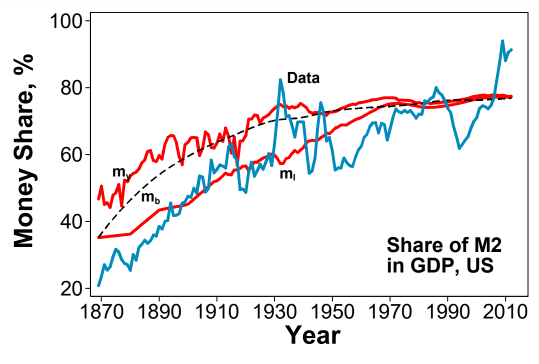
[57] We would like to thank an anonymous referee for this suggestion.

[58] In our model $r_t = \tau_t - 1$. Hence, instead of using nominal interest rates directly, we could use the fact that $\tau_t = \frac{M_{t+1}}{\beta M_t}$ to infer the model-predicted r_t. This however, would require us to calibrate another parameter - β - so we will not go down this route. Moreover, choosing this option would give practically indistinguishable results.



(a) Money (M2) Share in current price GDP. The black, dashed line is the baseline model. The two red lines are the alternative measures using value added and labor shares.

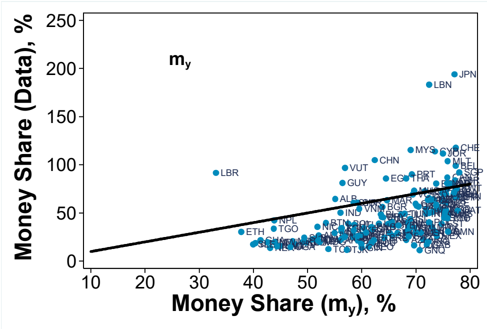
	Data	m_b^{US}	m_l^{US}	m_y^{US}
Observations	144	144	144	144
Mean	58.97	66.88	60.76	69.23
$t = 1869$	20.81	35.23	35.23	46.70
p10	32.25	48.89	38.43	55.67
p90	76.07	76.25	76.20	77.06
Std. Dev.	16.43	10.89	14.05	8.93
M-Y Semi-Elast.	0.191	0.159	0.187	0.106

(b) Summary statistics for alternative test, US.

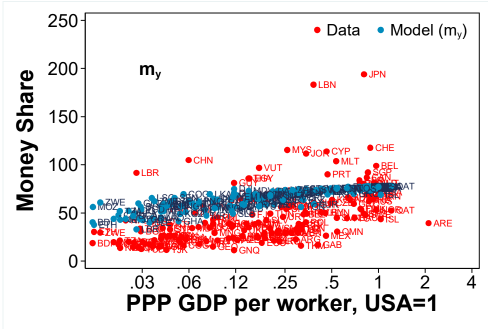
Figure B.9: Money share of GDP and summary statistics for US Alternative Measures. In the Table, columns show statistics for: money shares from the data (Data), the baseline model (m_b^{US}), the model fitting employment shares directly (m_l^{US}) and the model fitting value-added shares directly (m_y^{US}). Rows (1)-(6) show: the number of observations, the mean, the top-decile, the bottom-decile, the standard deviation as well as the semi-elasticity of money-share to income.

75.2% of the increase. Next, m_y generates a slightly smaller increase as it misses the money shares in the early part of the data - explaining approximately 56% of the increase - however it generates a better fit over the remainder of the period. Finally, m_b captures 66% of the standard deviation of the data, m_y captures 54% of the standard deviation whilst m_l does best and captures 86% of the variation. Overall, the three predicted measures give qualitatively identical and quantitatively very similar results for the US - although m_l and m_b do best in capturing the evolution of money shares found in the data.

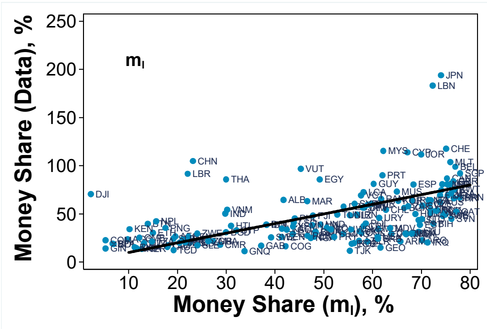
Next, we turn to the cross country data. As before we will focus on average values over the 1980-2010 period for each country, in order to emphasize the long-run cross-sectional fit of the model. Figure B.10 shows observed money shares versus predicted money shares (left column) and model predicted money shares versus GDP per worker (right panel) in the cross-section. Notice that all three measures capture a substantial portion of the variation of money shares in the data - as well as a positive money-share to GDP relationship. However, once more, it is the baseline model and m_l that do the best - capturing most of the variation and best replicating the money-share income elasticity. This is highlighted in Table B.3 which provides summary statistics and allows us to quantify the success of individual models. The m_y model captures about a third of the standard deviation in the data, 37-40% of the observed increase between the highest and the lowest money-shares deciles and it explains 49-54% of the money-share to income semi-elasticity. The m_l and m_b models fare about the same. They both capture about two-thirds of the standard deviation in the data, 81-88% of the observed increase between the highest and the lowest money-shares deciles and explain the entire money-share to income semi-elasticity. Thus, quantitatively m_y does slightly worse in explaining



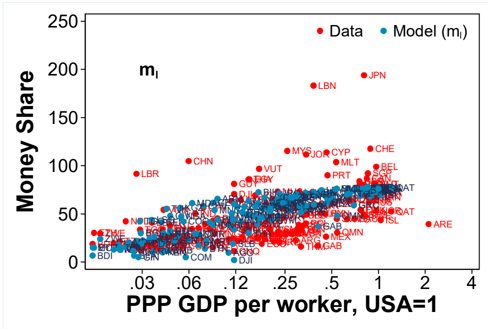
(a) Money Share, alternative measure versus Data. (VA)



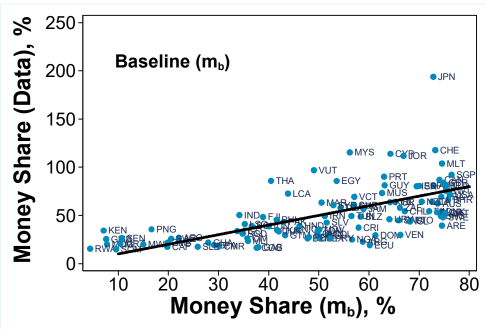
(b) Money Share, alternative measure and data versus GDP. (VA)



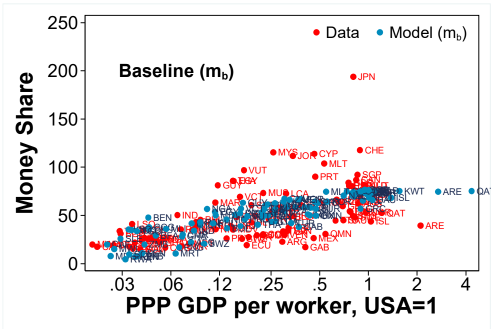
(c) Money Share, alternative measure versus Data. (EMP)



(d) Money Share, alternative measure and data versus GDP. (EMP)



(e) Money Share, alternative measure versus Data. (Baseline)



(f) Money Share, alternative measure and data versus GDP. (Baseline)

Figure B.10: Money shares in different calibrations (average for 1980-2010). Drawn versus corresponding data and (log of) GDP per worker. ALL AVAILABLE DATA

	Data	m_b	m_l	m_y	Data	m_b	m_l	m_y
Observations	166	102	156	155	95	95	95	95
Mean	50.09	51.15	50.16	64.52	52.40	52.50	53.03	66.02
p10	18.07	19.87	15.14	49.07	21.00	19.87	22.28	51.68
p90	85.81	74.42	74.88	75.85	86.80	74.51	75.25	76.32
Std. Dev.	42.94	19.95	21.85	10.58	30.06	19.88	19.48	9.88
M-Y Semi-Elast.	0.138	0.145	0.156	0.074	0.142	0.141	0.145	0.070

Table B.3: Summary Statistics. Left part of table shows all data, whilst the right part of the table shows only data that is available for all measures.

the cross-country data than either m_b or m_l - but qualitatively all three measures capture similar patterns. These findings support our choice to calibrate the baseline model to employment shares rather than value-added shares. Value added shares - especially in poorer countries and in historical US data - are of poor quality (they can be self-reported, based on surveys, or imputed) or they simply do not exist. Instead, by focusing on labor shares rather than value added shares, we avoid some of these problems as it is easier to count 'bodies' employed in a sector than the output and prices of hundreds of individual products.

Despite the simplicity of the above exercise, we believe that the baseline calibration in the main body of the paper is preferable as it helps us better understand the mechanism of the model and provides additional external validity. First, our baseline model specifies and quantifies the exact mechanism that drives employment, value added shares and nominal interest rates which in turn drive money shares. Instead, the exercise in this section takes shares and interest rates as exogenous. Knowing the mechanism is important as it allows us to determine - perhaps surprisingly - that agricultural productivity growth is largely responsible for the observed money-share patterns.

Second, having a fully calibrated and specified model allows us to perform counterfactuals and welfare analyses. This provide us with insights that the simple exercise in this section could not deliver. In section 8 for example, we find that inflationary policies tend to be far more damaging in rich countries than in poor countries, which helps to explain why we tend to observe higher inflation rates in poorer countries than in richer countries.

Finally, and most importantly, the calibrated baseline model has strong implications on the evolution of nominal prices, which can then be compared with the data. As we argue in section 5, this turns out to be especially interesting in light of the so-called 'Great Deflation' period in the United States at the end of the 19th century that our model, in contrast to more standard models, is able to replicate.

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