Velocity in the Long Run:
Money and Structural Transformation\textsuperscript{1}

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Abstract
Monetary velocity declines as economies grow. We demonstrate that this is due to the process of structural transformation - the shift of workers from agricultural to non-agricultural production associated with rising income. A calibrated, two-sector model of structural transformation with monetary and non-monetary trade accurately generates the long run monetary velocity of the US between 1869 and 2013 as well as the velocity of a panel of 102 countries between 1980 and 2010. Three lessons arise from our analysis: 1) Developments in agriculture, rather than non-agriculture, are key in driving monetary velocity; 2) Inflationary policies are disproportionately more costly in richer than in poorer countries; and 3) Nominal prices and inflation rates are not ‘always and everywhere a monetary phenomenon’: the composition of output also influences money demand and hence the secular trends of price levels.

JEL codes: O1; O4; E4; E5; N1
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1 Introduction

How does a country’s long-run money demand change with its economic development? An extensive literature\(^2\) finds that for broad enough measures of the money stock and over long periods of time, increases in income per capita tend to be associated with increases in the money-to-GDP ratio (or, equivalently, a falling monetary velocity).\(^3\) The possible sources of this stylized fact have been widely debated and include institutional changes, financial innovations, improvements in communication and information-gathering technologies as well as changes in the composition of output.\(^4\) Perhaps surprisingly, this research has been almost entirely empirical in nature\(^5\) which has made it challenging to quantify the role played by individual channels due to associated issues with endogeneity and causality (Bosworth and Collins, 2003).

In this paper we quantify the role of a single mechanism driving the income-velocity relationship: \textit{structural transformation}. This process, also known as industrialization, is the systematic change in the composition of an economy’s employment and output, from agriculture towards non-agriculture, associated with economic growth. Although structural transformation is certainly known to influence money demand (e.g. Jonung (1983), Friedman (1959), Chandavarkar (1977)), no theoretical models of the process exist, and the quantitative importance of this channel is unclear. Rather than taking a purely empirical or accounting approach, we construct and calibrate a multi-sector model of long-run monetary demand and compare our model’s predictions to those of a standard one-sector model of money demand. This allows us to isolate and quantify the role of structural transformation on the money-share whilst avoiding most of the issues of endogeneity and causality usually encountered in the empirical work.

Our model is motivated by two facts found in the data. First, agriculture - especially traditional agriculture - tends to be largely non-monetary due to the dominance of compensation in kind, home production and barter. Non-agriculture on the other hand, is more likely to require money to enable exchange due to the greater variety of goods within that sector (Chandavarkar, 1977). Second, as an economy grows, the relative size of the agricultural sector (in terms of employment and output) tends to shrink whilst that of the non-agricultural sector tends to rise. Together, these empirical

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\(^3\) Income velocity of money, \(V\), is defined as the ratio of gross domestic product, \(Y\) (measured in current prices, \(P\)) to the nominal money stock, \(M\): \(V \equiv \frac{P \times Y}{M}\). The inverse of velocity is the money-share: \(V^{-1} = \frac{M}{P \times Y}\). Velocity is thus a convenient measure of how money demand compares to income, with lower relative money demand translating into higher velocity and vice-versa. This also means that a rising money share is equivalent to a falling velocity.

\(^4\) Wicksell (1936), for example, argues that this trend stems from an increase in the prominence of organized markets, a shift from home to market production as well as the increase in both worker specialization and the complexity of goods. Friedman and Schwartz (1963) suggest that real money balances are luxury goods and hence have an income elasticity larger than unity. Bordo and Jonung (1987, 1990) emphasize institutional changes and financial innovations as systematically influencing velocity over long periods, whilst Townsend (1987) and Goodfriend (1991) highlight that improvements in communications and information-gathering technologies can contribute to a falling velocity. Finally, Friedman (1959), Chandavarkar (1977) and Jonung (1983), assert that the composition of an economy can be important in driving money demand.

\(^5\) One notable exception is Ireland (1994), who constructs a theoretical model of changes in the composition of monetary aggregates. Our paper instead focuses on the changes in the total share of money-stock-to-GDP and abstracts from compositional effects.
facts suggest that the changing composition of economies associated with growth will contribute to a rising demand for money, a rising money-to-GDP ratio and consequently a falling velocity.

To capture these regularities, we construct a model with two sectors: agriculture, which we assume produces goods traded purely without money, and non-agriculture, in which an endogenous share of goods is exchanged using money.\(^6\) The demand for money is introduced through a cash-in-advance constraint on non-agricultural consumption goods following Cole and Kocherlakota (1998). Structural transformation is generated by introducing non-homothetic preferences: consumers are assumed to have a subsistence level of agricultural consumption.\(^7\) As agricultural productivity increases, fewer workers are needed to satisfy subsistence consumption, prompting workers to move into the non-agricultural sector and thus increasing that sector’s share in total employment and output. Given that a part of non-agricultural goods are traded with money, the shift in the composition of the economy towards the non-agricultural sector will result in the model predicting an increase in monetary transactions, an increase in the money-to-GDP ratio and hence in lower velocity.

By contrast, in a standard one-sector model, money demand is only affected by the nominal interest rate. Such a model is thus unable to reproduce the trend in money-share and exhibits a stable velocity with respect to income. We compare our multi-sector model - which includes both the interest-rate money-demand mechanism as well as the additional compositional money-demand mechanism - with a standard one-sector model - which only contains the interest-rate money-demand mechanism. This comparison allows us 1) to disentangle the effects of structural change on money shares from other potential drivers, such as different monetary policies (i.e. different nominal interest rates), different productivity levels and growth rates, and differences in capital accumulation and 2) to quantify how much of the trend in the money share is explicitly explained by structural change. Finally, since we explicitly model the sources of structural transformation and money-demand, our model-based approach avoids endogeneity and causality pitfalls associated with purely empirical attempts to isolate the role of structural transformation in driving money demand.\(^8\)

We calibrate our multi-sector model to the 1869-2007 patterns of US growth, structural transformation, and monetary supply. This simple framework successfully replicates several features of the long run data including agricultural labor shares, GDP per worker, sectoral prices, nominal interest rates, and aggregate inflation. Most importantly, the model reproduces the evolution of the US long run money-to-GDP ratio over 140 years, capturing 75.2% of the increase in the data. A traditional, similarly calibrated one-sector model fails to replicate observed money-share, as it is unable to match

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\(^6\) In an extension we show our results remain unchanged when an endogenous fraction of agricultural output is exchanged with money.

\(^7\) See for example Herrendorf et al. (2013), Buera and Kaboski (2012), Gollin et al. (2002), Restuccia et al. (2008), Yang and Zhu (2013) and Stefanski (2014a,b).

\(^8\) Money share can be decomposed as \(\frac{M}{PY} = \frac{M}{PY_c} \times \frac{PY_c}{PY}\), where \(P_c\) and \(Y_c\) are the non-agricultural price and value-added levels respectively. In principle, the contribution of structural change on the money-share could be quantified by estimating the money-share elasticity with respect to \(\frac{PY_c}{PY}\). In equilibrium however, \(\frac{M}{PY_c}\), \(\frac{PY_c}{PY}\), \(P\), \(P_c\), and \(Y\) are all jointly determined. As such, the two components of the decomposition \(\frac{M}{PY}\) and \(\frac{PY_c}{PY}\) are not orthogonal to each other. Without specifying a mechanism that relates these individual terms, reduced-formed regressions of this type will necessarily give biased estimates. This is a standard problem in the growth accounting literature - see, for example, Bosworth and Collins (2003).
the observed price dynamics - and in particular the so-called ‘Great-Deflation’ of the late 19th century. Our model also accurately predicts the variability in money-shares across countries. Keeping preference parameters of the US, we recalibrate the model to a panel of 102 countries between 1980 and 2010. The model accurately captures cross-country differences in incomes, employment shares, interest rates, and inflation rates. The baseline model captures 83% of the increase in money-share between the top and bottom deciles of cross-country data and does exceptionally well in replicating the income-velocity relationship, whereas a one-sector model fails to capture any part of the relationship.

Finally, we examine how the costs of suboptimal monetary policies vary with income. Inflation is more costly in richer countries than in poorer countries - where the monetary part of the economy is smaller and therefore distortions from the inflation tax are less damaging. For example, a hyperinflation of approximately 400% a year in a poor country like Zimbabwe (where GDP per worker is 2% of that of the US) will have negligible welfare costs. By contrast, the same hyperinflation in a country like Argentina (30% of US GDP per worker) will have very large welfare costs. Argentinean incomes would have to rise by approximately 29% to deliver the same expected flow utility as without the hyperinflation. The low cost of inflation in poorer countries may help explain why they tend to have much higher inflation rates than richer countries.

There are three lessons from our work. First, monetary velocity is not constant over the development process but falls with income. Most existing explanations of this observation are rooted in the non-agricultural sector - and focus on factors such as financial innovation or the expansion of the banking sector. The surprising finding of this paper is that the evolution of monetary velocity is driven largely by developments in the agricultural sector. It is the variation in agricultural productivity that influences the size of the non-agricultural sector and in turn influences monetary demand and hence velocity. Second, the cost of inefficient monetary policy varies with income and is higher in richer countries than in poorer countries. An inflation tax offers a relatively cheap source of income to poor-country governments, and its distortive effects are relatively small in economies that are dominated by large, non-monetary, agricultural sectors. This may help explain why we observe persistently higher inflation in poorer countries than in richer countries - despite recommendations of strong anti-inflationary policies by international financial institutions such as the International Monetary Fund (IMF). Finally, the third lesson is that, since velocity depends systematically on the composition of output, a country’s price levels and inflation rates may not ‘always and everywhere be a monetary phenomenon’, as suggested for example by the work of Friedman and Schwartz (1963). The price level in an economy, \( P \), is defined as \( P \equiv (M/Y)V \) where \( M \) is the money stock, \( Y \) is output and \( V \) is velocity. Two countries, with identical money stocks and identical output levels - but with different output compositions - will have entirely different velocities, \( V \), and hence

\[9\] In particular, a one percent increase in GDP per worker is found to increase the money-to-GDP ratio by 0.140 percentage points in the data and 0.145 percentage points in the model.

\[10\] Weisbrot et al. (2009), for example, examine existing IMF loan agreements with 41 developing countries in 2009. They find that the IMF took a stance on monetary policy in 25 of these countries and in 22 of those 25 countries it recommended a contractionary or anti-inflationary monetary policy.
different price levels. The message to researchers from these findings is that a one-sector model cannot be successfully used to understand the long run dynamics of monetary velocity and hence the evolution of price levels or inflation rates. These findings should also be of interest to policymakers in developing countries who may have overlooked the importance of the agricultural sector in their monetary policy decisions.

In the next section we document the main facts regarding structural transformation, monetary velocity, and the extent of non-monetary production in agriculture. In section 3 we construct and solve our simple baseline model. In section 4 we calibrate the model to the experience of the United States and in section 5 we show the results for the US. Section 6 carries out the cross-country analysis by examining how the model performs in international, cross-country data and by running a number of counterfactuals to quantify the importance of the different mechanisms of the model. Section 7 performs a number of robustness checks and extensions. Section 8 examines the different costs of inflation in rich and poor countries. Finally, section 9 offers some concluding remarks on the importance of our findings to researchers and policy makers.

2 Facts

In this section we present three stylized facts. First, we show that the the money-to-GDP ratio (or the inverse of monetary velocity) increases with income per capita. Second, we show that non-monetary activities tend to be most prevalent in countries with large agricultural sectors. Third, we show that the relative size of the agricultural sector tends to shrink systematically as income rises. Taken together these facts suggest that as countries grow richer, they tend to use more money relative to the size of their economy, and that this process is potentially linked to the decline of a predominantly non-monetary sector - agriculture.\[11\]

Money Shares

We are interested in the pattern of the ratio of the stock of money to nominal GDP, over time and across countries. We choose M2 as our measure of money stock.\[12\] We plot the average money share (M2 divided by nominal GDP) as a function of the average GDP per worker (relative to the US average GDP) over the 1980-2010 period in order to focus on the long run relationship between these two variables in Figure 1(a). A strikingly positive relationship emerges: richer countries tend to have a higher money share (or a lower velocity) than poorer countries. This fact is statistically significant, as apparent from the 5% error bands on the regression line.

\[11\] See Online Appendix A for sources and construction of all data.

\[12\] We are not directly interested in the narrowest definition of the monetary stock such as M0 or M1: as argued by Ireland [1994], economies undergo a change in monetary technologies as they develop, and they therefore use less currency and more sophisticated forms of exchange such as electronic currency or time deposits. These technological changes and the subsequent changes in the composition of money are not the focus of this paper. Instead we are interested in how the quantity of money evolves with income. This suggests that we should focus on the wider definitions of money. The data for M3 however is relatively scarce - both across countries and for longer periods of time. M2 is reasonably broad and we have reliable data for a large number of countries. Dorich [2009] and McCallum and Nelson [2010] discuss which monetary aggregate better captures the liquidity services of money.
Quantitatively, a 1% increase in GDP per worker is associated with a 0.16 percentage point increase in the money share. A similar fact can be observed in long run time series data. Figure 1(b) shows the share of M2 money stock relative to nominal GDP in the United States between 1869 and 2014. There is a clear rising trend in the money stock to GDP ratio. Since GDP per worker in the US roughly grew at a constant rate over the period, there is also a positive relationship between money share and income per worker in the US time-series data. Bordo et al. (1993) have documented similar historical patterns in long run data for Canada, the United Kingdom, Norway and Sweden.

The non-monetary economy Next, we present evidence on the extent of non-monetary or subsistence activities, and their relationship with the agricultural sector. Blades (1975a,b) carries out an analysis for the OECD using survey data for 48 developing countries over the 1970-1975 period. The semi-elasticity of the money share with respect to GDP per worker in the US over the period is 0.19.
period, documenting the proportion of GDP that can be classified as a non-monetary or subsistence activity.\textsuperscript{14} His conclusions are quite strong: “Agriculture is obviously the main item in non-monetary production, accounting often for over 80 percent of the total.” Figure 2 plots the share of non-monetary value added in GDP from \textsuperscript{15}Blades (1975a) versus agricultural employment share in 1980. Agricultural economies tend to have a far greater proportion of their value added in non-monetary sectors than more industrialized countries.

\textbf{Structural Transformation} Finally, we present the well known fact that the relative size of the agricultural sector falls with rising income in a process referred to as structural transformation.\textsuperscript{16} Figure 3\textsuperscript{(a)} documents this pattern for a cross-section of 171 countries by plotting the average agricultural employment shares over the 1980-2010 period versus the average (1990, PPP) GDP per worker of each country. A similar pattern is visible within countries over time as their income per capita increases. Figure 3\textsuperscript{(b)}, for example, depicts falling agricultural employment shares in the US between 1850 and 2015 from \textsuperscript{14}FAOSTAT, Alvarez-Cuadrado and Poschke (2011) and Lebergott (1966). Similar patterns hold not only with respect to labor shares but also with respect to sectoral value-added shares.

These three facts suggest that a model with two sectors (agriculture and non-agriculture), with differing sectoral money demands and endogenous relative sectoral output composition would potentially be able to explain the trend in money shares.

\textsuperscript{14} See Online Appendix A for more details on this data.
\textsuperscript{15} We use the 1980 agricultural employment share, since earlier data is not available for all countries.
\textsuperscript{16} See for example, Maddison (1982), Echevarria (1997) or Duarte and Restuccia (2010).
3 The Baseline Model

We illustrate our theory with a simple cash-in-advance model of structural transformation guided by the three empirical facts presented in the previous section. Our model is a version of Cole and Kocherlakota (1998) but with two sectors: a completely non-monetary sector (agriculture) and a partially monetary sector (non-agriculture). To generate structural transformation, we impose non-homothetic preferences over agricultural goods which generates a shift of labor from agriculture to non-agriculture as agricultural productivity rises. A calibrated version of the model is used to quantify the role that structural transformation plays in driving across- and within-country differences in monetary shares.

**Households** A representative household maximizes welfare subject to a budget constraint, cash-in-advance constraint, non-negativity constraint on money holdings, and a non-Ponzi condition:

\[
\max_{a_t, c_{m,t}, c_{n,t}, b_{t+1}, m_{t+1}} \sum_{t=0}^{\infty} \beta^t \left( \alpha \log(a_t - \bar{a}) + (1 - \alpha - \gamma) \log(c_{m,t}) + \gamma \log(c_{n,t}) \right)
\]

\[
\text{s.t. } p_{a,t} a_t + p_{c,t} (c_{m,t} + c_{n,t}) + b_{t+1} + m_{t+1} \leq w_t + (1 + r_t) b_t + m_t + T_t
\]

\[
p_{c,t} c_{m,t} \leq m_t
\]

\[
m_t \geq 0 \text{ and } b_t \geq -\bar{B}
\]

The household owns a unit of labor which it sells on the market for a wage \(w_t\). In addition, the household comes into the period with money holdings \(m_t\), bond holdings \(b_t\) (which it sells at a price \(1 + r_t\)) and it receives a helicopter transfer of money from the government of \(T_t\). Given this income, the household purchases agricultural goods \(a_t\) (at the price \(p_{a,t}\)) as well as non-agricultural goods \(c_{m,t}\) and \(c_{n,t}\) (at the price \(p_{c,t}\)). It also purchases bonds \(b_{t+1}\) that promise to pay out \(1 + r_{t+1}\) dollars next period, and chooses its (non-negative) stock of money, \(m_{t+1}\), for the next period. We assume that there are two kinds of non-agricultural goods: those that can be bought without money \(c^p_{n,t}\) and those that must be bought with money \(c_{m,t}\). This establishes a role for money: the household needs to put aside a part of its income, \(m_{t+1}\), each period in order to be able to buy monetary goods in the following period. To capture this idea, we impose the cash-in-advance (CIA) constraint \([1]\) on \(c_{m,t}\) goods. Only cash held from the previous period can be used to purchase monetary goods in the current period and cash transfers from government can only be used in the subsequent period. Finally, we impose a lower bound, \(-\bar{B}\), on bond holdings to avoid Ponzi schemes.

Households have non-homothetic preferences for agricultural goods \(a_t\). In other words, there exists a subsistence level of agriculture, \(\bar{a}\), that must be consumed every period. Intuitively, households need to obtain a minimum quantity of food or calories in order to survive. An important assumption

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\[17\] Online Appendix B.5 relaxes the assumption that agricultural goods are only exchanged without money, introducing a money demand in agriculture; the results are very similar, therefore here we focus on the simplest version of the model.
of our baseline model is that agriculture is a non-cash good. The intuition here is that agriculture - especially traditional agriculture - can be characterized by compensation in kind or home production (Blades, 1975a,b). Barter is also far more likely in agriculture than in non-agriculture. Most people wish to eat a diversified basket of food making double coincidence of wants for agricultural producers relatively likely and money less useful. Money use in non-agriculture, however, will be important. The massive variety of goods makes the probability of double-coincidence far smaller. Money becomes necessary to overcome this mismatch problem. Of course, some goods in non-agriculture are nonetheless still traded without cash using credit arrangements or payments in kind. To capture this, we follow Chari et al. (1996), by assuming that a fixed proportion $\gamma$ of non-agricultural goods do not require cash.

**Firms** There are two representative firms: agricultural ($a$) and non-agricultural ($c$). Each firm $s = a, c$, hires labor, $L_{s,t}$, and produces output, $Y_{s,t}$, using a simple linear technology that combines labor with exogenous, sector-specific total factor productivity, $B_{s,t}$. The profit maximization problems are:

$$\max_{L_{s,t}} p_t Y_{s,t} - w_t L_{s,t} \quad \text{s.t.} \quad Y_{s,t} = B_{s,t} L_{s,t}. \tag{2}$$

Output of non-agricultural firms, $Y_{c,t}$, can be sold as both a monetary and a non-monetary good, whereas all agricultural output is assumed to be non-monetary.

**Money Supply** The government is assumed to have a so-called helicopter monetary policy:

$$M_{t+1} = T_{t+1} + M_t. \tag{3}$$

**Market Clearing** Finally, in each period, markets clear in a standard fashion.

$$a_t = Y_{a,t}, \quad c_{m,t} + c_{n,t} = Y_{c,t}, \quad L_{a,t} + L_{c,t} = 1 \quad m_t = M_t, \quad b_t = 0. \tag{4}$$

**Solution** The definition of competitive equilibrium is standard and shown in Online Appendix B.4.

To solve the model, we follow Cole and Kocherlakota (1998), and impose the following assumption:

**Assumption 3.1.** Assume that interest rates are always positive, i.e. $r_t > 0$.  

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18 In particular, agricultural workers are often compensated in kind, either through share-cropping or other informal credit arrangements whilst households in poorer countries tend to home-produce their agricultural consumption.

19 For an in-depth discussion of this transactional role of money see Ostroy and Starr (1990).

20 In Online Appendix B.4 we follow Cole and Kocherlakota (1998) by adding capital to the model, and show that if we treat investment as a non-monetary, credit good we can go a long way in endogenizing the cash-credit split of the non-agricultural sector.

21 Online Appendix B.4 extends the model to include capital accumulation in the non-agricultural sector. Since the results remain almost unchanged, we focus on the simpler version.
The consequence of this assumption is that the CIA constraint, \( \frac{M}{M_t} \), always binds since there is a positive opportunity cost of holding money. Given a sequence of government policies, in Online Appendix B.1 we then solve for the competitive equilibrium using first order conditions and market clearing. We show that sectoral employment is given by:

\[
L_{a,t} = \alpha \tau_t + \frac{(1 - \alpha - \gamma(1 - \tau_t)) \bar{a}}{1 - (\alpha + \gamma)(1 - \tau_t)} \quad \text{and} \quad L_{c,t} = 1 - L_{a,t},
\]

where \( \tau_t \equiv \frac{1}{\beta} \frac{M_{a,t}}{M_t} \). Non-agricultural output is divided into cash and non-cash goods according to:

\[
c_{n,t} = (1 - \phi_t)Y_{c,t} \quad \text{and} \quad c_{m,t} = \phi_t Y_{c,t},
\]

where \( \phi_t = \frac{1 - \alpha - \gamma}{(1 - \alpha - \gamma) + \gamma \tau_t} \). Finally, sectoral prices - in terms of local currency - are:

\[
p_{a,t} = \frac{M_t}{\phi_t B_{a,t} L_{c,t}} \quad \text{and} \quad p_{c,t} = \frac{M_t}{\phi_t B_{c,t} L_{c,t}}
\]

and the nominal interest rate and wage rate are:

\[
r_t = \tau_t - 1 \quad \text{and} \quad w_t = \frac{M_t}{\phi_t L_{c,t}}.
\]

Due to the existence of the non-homotheticity, \( \bar{a} > 0 \) in equation (5), higher agricultural productivity, \( B_{a,t} \), is associated with a lower fraction of the labor force employed in agriculture as fewer workers are needed to produce enough food for subsistence. The composition of the economy also affects nominal sectoral prices in (7). Rising agricultural productivity increases employment in the non-agricultural, monetary sector and hence overall demand for money. This, in turn, decreases the nominal, dollar price of each sector’s goods - whilst leaving the relative price of goods unchanged. Thus it is not only the supply of money, \( M_t \), that drives nominal prices as has been suggested by the literature [Friedman and Schwartz, 1963]. The demand for money - as captured by \( L_{c,t} \) - also plays a crucial role. Section 5 shows that this process was key in generating the so-called “Great Deflation” in the late 19th century United States.

Finally, monetary policy has an impact both on sectoral employment and the consumption of monetary goods. Government directly controls the evolution of the money stock, \( M_t \), and hence the variable \( \tau_t \). This so-called inflation tax determines how costly it is to hold cash from one period to the next.\(^{22}\) The higher the cost of cash, the fewer the workers employed in cash-dominated sectors, \( \partial L_{c,t} / \partial \tau_t < 0 \) and the lower the consumption of cash goods, \( \partial c_{m,t} / \partial \tau_t < 0 \). This highlights a new cost of monetary policy: higher inflation taxes can reverse or delay structural transformation. Interestingly, this cost itself depends on the level of agricultural productivity in a country, with greater distortionary effects found in countries with higher agricultural productivity levels.\(^{23}\) We quantify this differing effect in rich and poor countries in section 8 where we look at the welfare cost of inflation.

\(^{22}\) From equation (6), the higher the \( \tau_t \), the higher the nominal interest rate, and hence the greater the opportunity cost of holding cash.

\(^{23}\) Countries with high \( B_{a,t} \) have larger non-agricultural and hence monetary sectors. The inflation tax will therefore impact a larger fraction of those economies. Mathematically we write: \( \frac{\partial (\partial L_{a,t} / \partial \tau_t)}{\partial B_{a,t}} > 0 \).
**Velocity** The share of monetary stock relative to the nominal GDP (or the inverse of monetary velocity) can be written as:

\[ V_t^{-1} = \frac{M_t}{p_{a,t}Y_{a,t} + p_{c,t}Y_{c,t}} = \phi_t \frac{p_{c,t}Y_{c,t}}{p_{a,t}Y_{a,t} + p_{c,t}Y_{c,t}} = \phi_t L_{c,t}. \] (9)

The first equality follows by definition. The second equality follows from the assumption that the CIA constraint binds, and from equation (6), which splits non-agricultural consumption into its monetary and non-monetary components. The final equality follows from equations (5)-(8).

There are two channels driving the money share. First, the term \( \phi_t \) determines what proportion of non-agriculture is bought with cash. This variable itself is influenced by the preference parameter, \( \gamma \), which captures (in a reduced form) the non-monetary activity in the non-agricultural sector, as well as by \( \tau_t \). Since \( \partial \phi_t / \partial \tau_t < 0 \), a higher inflation tax results in households wanting to purchase fewer cash goods, which in turn lowers money demand and the money share. Second, the money share crucially and positively depends on \( L_{c,t} \) - the share of employment in the non-agricultural sector. As a greater proportion of workers shifts to a largely cash sector, the share of money in the economy rises - and the velocity falls.

From (9), two facts of interest emerge. First, since higher agricultural productivity results in greater non-agicultural employment, it also implies a higher money share. In other words, \( \frac{\partial V_t^{-1}}{\partial B_{a,t}} > 0 \). Second, a higher inflation tax means it is more costly to hold cash, which leads to a lower employment share in non-agriculture (as workers move away from a cash to a non-cash sector) and a lower \( \phi_t \) (as consumers want to consume less cash goods). Together this means that the money share decreases in response to a higher \( \tau_t \) so that \( \frac{\partial V_t^{-1}}{\partial \tau_t} < 0 \).

These two facts highlight that both monetary policy and agricultural productivity can influence a country’s monetary demand and hence monetary shares. We quantify the role that each of these channels plays in explaining within and across country variation in monetary shares, by calibrating the model in the next section.

### 4 US Calibration

We calibrate a benchmark economy to US data for the period from 1869 to 2007.\(^{24}\) Our calibration strategy involves choosing parameter values so that the equilibrium of the model matches the most important features of the structural transformation in the United States over this period.\(^{25}\) To stay consistent with the literature we follows as closely as possible the calibration of Duarte and Restuccia (2010). We assume that a period in the model is one year. We need to select parameter values for \( \alpha, \bar{a}, \beta, \gamma \), the time series of the money stock \( M_t \) for \( t \) from 1869 to 2007 and the time series of productivity for each sector \( B_{s,t} \) for \( t \) from 1869 to 2007 and \( s \in \{a, c\} \).

\(^{24}\) We stop at 2007 to avoid the financial crisis period, when emergency monetary policy was implemented.

\(^{25}\) See Online Appendix A for details of sources and construction of all data.
Table 1: Baseline Model, calibrated parameters. See Online Appendix \textit{A} for detailed sources.

We proceed as follows. First, we normalize the level of money to one in 1869; that is, \( M_{1869} = 1 \). Then we use data on the growth rate of M2 per worker in the United States to obtain the time paths of money per worker in the model. In particular, denoting as \( g_{m,t} \) the growth factor of M2 per worker at time \( t \), we obtain the time path of money in the model as \( M_{t+1} = g_{m,t} M_t \).\(^{26}\)

Second, we use data on average money per worker growth rates and nominal interest rates in the United States to choose the discount factor. In particular, from equation (8), it follows that \( \beta = \frac{g_{m,t}}{1 + \bar{r}} \). Denoting by \( \bar{g}_m \) the average annual growth factor of M2 per worker in the US between 1869 and 2007 of 1.0478 and by \( \bar{r} \) the average annual nominal returns of the Standard and Poor Composite Stock Price Index\(^{27}\) between 1871 and 2007 of 0.0858, we set \( \beta = \frac{\bar{g}_m}{1 + \bar{r}} \).

Third, we normalize productivity levels across sectors to one in 1869; that is, \( B_{s,1869} = 1 \) for \( s \in \{a, c\} \). Then we use data on the average growth rate of sectoral value added per worker in the United States to obtain the time paths of sectoral labor productivity. In particular, denoting by \( g_s \) the average (annualized) growth rate of labor productivity between 1869 and 2007 in sector \( s \), we obtain the time path of labor productivity in each sector as \( B_{s,t+1} = (1 + g_s) B_{s,t} \).\(^{28}\)

Fourth, with positive productivity growth in all sectors and a monetary policy that converges to the optimum, the share of employment in agriculture converges to \( \alpha \) whilst the ratio of money to nominal GDP converges to \( 1 - \gamma - \alpha \).\(^{29}\) Because the share of employment in agriculture has been falling systematically and was about 1.7% in 2007, we assume a long-run share of 0.5%. The share of M2 relative to value added, on the other hand, has been rising and was about 75% in the

\(^{26}\) In Online Appendix \textit{B.2} we show that calibrating the model to observed nominal interest rates instead of money growth rates leaves the results unchanged.

\(^{27}\) We have performed the same analysis by using nominal interest rates on government bonds and the results are unchanged. However, given that we are considering the very long run, and a broad concept of money, it is sensible to assume that the opportunity cost of money is given by the returns obtained from long run investments in stock markets. For example, Mulligan and Sala-I-Martin \cite{mulligan1992}, page 295, suggest that "[f]or demand deposits, the appropriate asset might be Treasury bills. For a broader concept of money, corporate bonds or equities might be appropriate".

\(^{28}\) We use average growth rates since earlier data exists only at decadal frequencies. In Online Appendix \textit{B.3} we allow for time varying growth rates by using interpolated data. This makes very little difference to our results.

\(^{29}\) With positive productivity growth, agricultural employment converges to \( \frac{1}{1 + (\alpha + \gamma - \alpha) (g_{m,t}/\bar{g}_m)} \) and the money share converges to \( \frac{1}{1 + (\alpha + \gamma - \alpha) (g_{m,t}/\bar{g}_m)} \). In section \textit{S} we show that monetary policy converging to the Friedman Rule, i.e. \( g_{m,t} \rightarrow \beta \), is optimal. Under this policy agricultural and money shares converge to the values given in the text.
2000’s. Using a non-linear regression, in Online Appendix we fit an exponential function to the money-share data and find an implied asymptote of approximately 79.%. We take this as our long run money share. Both targets are somewhat arbitrary, however our main results are not sensitive to these choices.

Finally, $\bar{a}$ is chosen to match the share of employment in agriculture in the United States in 1869 using equation (5). Table 1 summarizes the calibrated parameters and targets.

5 US Results

Despite its simplicity our model does well in capturing a number of real and monetary features of the US economy. We begin with the non-monetary or ‘real’ implications of the model. First, Figure 4(a) shows that the model predicts a decline of agricultural employment share from 55% in 1869 to 1.6% in 2013. Even though we only target the employment share in 1869, the agricultural employment share in 2013 is very close to the value in the data of 1.5%. This decline is driven by rising agricultural productivity in equation (5). Second, since the model matches the evolution of sectoral employment well and since productivity growth comes directly from the data, the model also matches sectoral output (the product of labor productivity and sectoral employment). Agricultural
Figure 5: Money share of GDP and summary statistics. In the Table, columns show statistics for: money shares from the data (Data), the baseline model ($m^{US}$), and the US counterfactual ($m^{US}_{cf}$). Rows (1)-(6) show: the number of observations, the mean, the top-decile, the bottom-decile, the standard deviation as well as the semi-elasticity of money-share to income.

The model predicts an average annualized growth rate of 1.68% a year versus 1.66% in the data. Third, the model also has strong implications for the evolution of relative prices. From equation (7), the relative price of sector $a$ to sector $c$ goods is given by the ratio of sectoral labor productivities:

$$\frac{p_{a,t}}{p_{c,t}} = \frac{B_{c,t}}{B_{a,t}}.$$  

Figure 4(c) shows that the relative price of agriculture in the data declined at approximately 1.52% a year between 1869 and 2013, whilst the model predicts a decline of 1.20% per year over the same period. The fit is thus remarkably close - even though the model is not calibrated to match the evolution of these prices. Finally, since the model matches relative prices and sectoral output well, it follows that it must also match the evolution of sectoral value added shares. Figure 4(d) shows that the share of non-agricultural value added in total value added in the data increased from approximately 59.7% in 1869 to 98.6% in 2013. The corresponding increase in the model was slightly larger from 45.0% in 1869 to 98.4% in 2013.

Next, we compare our predicted monetary or ‘nominal’ variables with the corresponding values in the data. Figure 5 shows the model’s striking ability to capture the evolution of US monetary share - our main variable of interest. This simple model generates $75.2\%$ ($\approx \frac{79.2 - 35.23}{79.2 - 20.81} \approx 75.2\%$) of the observed increase in the money share of the United States economy between 1869 and the implied long-run

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30 The numeraire in our model is current period dollars. Nominal GDP is thus defined as, $NGDP_t = p_{a,t}Y_{a,t} + p_{c,t}Y_{c,t}$. Constant price (or ‘real’ GDP) is evaluated in constant 1990 prices: $RGDP_t = p_{a,1990}Y_{a,t} + p_{c,1990}Y_{c,t}$. Importantly, the way we calculate GDP in the model corresponds exactly to the way GDP is calculated in the data.

31 This is because $p_{a,t}Y_{a,t} + p_{c,t}Y_{c,t} = \frac{Y_{a,t}}{p_{c,t}} + \frac{Y_{c,t}}{p_{c,t}}$. 

money share of approximately 79.3% and 61.5% ($\approx \frac{76.25-48.89}{76.25-32.25}$) of the observed increase between the top and bottom money-share deciles. The correlation between money shares in the model and the data is 0.91, the $R^2$ in a regression of the data on the model is 0.84 and the model captures 63% ($\approx \frac{10.88}{16.08}$) of the standard deviation in the data. Thus, whilst we do not capture short term fluctuations, the fit of the long-run trend is exceptional.

Importantly, notice that other than choosing $\gamma$ in the calibration to target the implied long-run money share of 79.3%, we do not target the evolution of money shares at all. Performing a variance decomposition, it is easy to show that the increase in money share over time emerges entirely from the mechanism of the model, i.e. from a productivity driven reallocation of labor out of agriculture and into non-agriculture.\footnote{To see this formally, recall that in the model, $\bar{V}^{-1}_t = \bar{\phi}_t L_{c,t}$. Taking logs of both sides of this equation we obtain, $\bar{V}^{-1}_t = \bar{\phi}_t + L_{c,t}$, where the bar represents the log of a variable. Next, denoting the sample variance by $var(\cdot)$ and the sample standard deviation by $std(\cdot)$, we can calculate two variance decompositions of the money shares implied by the model: $\frac{std(L_{c,t})}{var(\bar{\phi}_t)+var(L_{c,t})} = 1.01$ and $\frac{var(L_{c,t})}{var(\bar{\phi}_t)+var(L_{c,t})} = 1.00$. From this, we see that virtually all the variance in money share in the model is driven by variation in employment share. Importantly, this matches with what we observe in the US data where the corresponding values of the two decompositions between 1869 and 2013 are 0.780 and 0.776.} Finally, the fit of the model does not in any way come from the fact that the money stock is taken directly from the data. In Online Appendix B.2 we show that calibrating our baseline model to observed nominal interest rates rather than to the money stock directly, leaves our results quantitatively and qualitatively unchanged. Furthermore, to cement this intuition, in the following section we show that a one-sector model that also takes money stock directly from the data fails entirely in reproducing the observed dynamic of money shares. This is indicative that it is the model’s two-sector structure that enables it to capture closely the evolution of money share over time.

We also examine the predictions of the model for nominal producer prices: the quantity of dollars needed to purchase one unit of sector $s$ goods. From equations (7) and (9) we can derive an expression that pins down nominal prices for sectors $s \in \{a, c\}$:

$$p_{s,t} = \frac{M_t V_t}{B_{s,t}} = \frac{M_t}{B_{s,t}\bar{\phi}_t L_{c,t}}.$$  

This equation shows that the nominal price of sector $s$ goods depends on the quantity of money in the economy, sector $s$ productivity and the endogenously determined velocity of money. For a given velocity, nominal prices are driven by changes in the relative quantity of money and sectoral productivity: when money becomes relatively abundant the price of sector $s$ goods increases and vice-versa. In our model however, monetary velocity is not constant but falls endogenously as the economy shifts from non-monetary agriculture to monetary non-agriculture. This changing demand for money will play a key role in driving nominal prices - especially in the earlier parts of the sample when changes in velocity are largest. Comparing nominal prices in the model and the data provides us with a test of the model’s mechanism. Figures 6(a) and 6(b) show sectoral price indices (in dollar terms) in the data and the model. Even though nominal prices are not targeted directly, the model does exceptionally well in capturing the non-linear trends of sectoral prices - both the flat (or
falling) nominal sectoral prices in the initial part of the series and the subsequently growing prices in the second part of the series. The tight fit between model and data comes partially from the standard quantity theory of money: nominal prices are influenced by the quantity of money relative to sectoral productivity. However - as we shall see in the subsequent section - an important reason for the success (especially in the first part of the series) comes from the model’s ability to capture monetary velocity. Figure (c) and (d) then shows the evolution of the aggregate consumer prices index and the corresponding inflation rates. Again our model predictions fit very well with the data. Interestingly, we also capture the deflationary period in the late 19th century. This turns out to be a key feature of our model discussed in more detail below.

Finally, the model also has implications for nominal factor prices, via equation (8). Figures (a) and (b) compare factor prices generated by the model with data. Recall that $\beta$ was chosen to match the average nominal interest rates between 1869 and 2007 - and so it is unsurprising that the model does well in matching nominal interest rates. The model however also does well in matching the nominal wage rate, predicting an average annual increase of 4.2% between 1869 and 2013 (3.4% in the data). From equation (8), we see that nominal wages are given by $w_t = M_t V_t$. Thus, the good fit of nominal wages in the model and the data does not stem from the calibration, but rather emerges from the mechanism of the model.
US Counterfactual  To highlight the mechanism driving the results, let us consider how an (effectively) one-sector version of the model would perform in replicating monetary shares. We can do this by assuming that productivity in the agricultural sector in 1869 is set equal to the agricultural productivity of the US in 2010. We can then re-calibrate the non-agricultural productivity to match the same (period-by-period) GDP-per-worker as in the baseline model - whilst keeping all other parameters identical to the calibration in Table 1. Given the new, high agricultural productivity, equation (3) suggests that the employment share in agriculture will be close to $\alpha$ - and hence very small. In other words, the agricultural sector will be of a negligible size. Thus, this counterfactual effectively replicates a one-sector version of the baseline model. Figure 8(a) shows that monetary shares in the one-sector model are almost constant over the entire period. Variations in monetary growth rates generate some limited fluctuations, but these are not large or systematic enough to generate the observed changes in money shares (see Table 5(b)). Why does the one-sector model fail? By definition, money share is $\frac{M_t}{P_t \times Y_t}$ - where $M_t$ is the money stock, $P_t$ is the aggregate price level and $Y_t$ is GDP. In both models we take $M_t$ directly from the data, whilst $Y_t$ is chosen to be
identical in both models. Thus the baseline and the one-sector models differ only in their ability to reproduce the evolution of aggregate prices $P_t$ over time. Figure 8(b) shows the evolution of the price index in the baseline, the data and the one-sector counterfactual. The one-sector model fails to capture the evolution of aggregate prices in the early periods. In particular, from the equations in (7), an increase in the employment share of the non-agricultural sector in the baseline model decreases sectoral prices $p_{a,t}$ and $p_{c,t}$. This downward pressure on sectoral prices is stronger when $L_{c,t}$ is small, i.e. at early stages of the structural transformation process. This mechanism therefore helps the baseline model to closely match the observed deflation in the late 19th century, a task that the one-sector model cannot accomplish.

Historically, there has been an energetic debate among economists about the sources of this so-called ‘Great-Deflation’. One perspective, held by Tooke and Newmarch (1857), Wells (1891), Landes (1969), Rostow (1948, 1978) and Lewis (1978), highlights the importance of ‘real’ factors behind falling prices - such as the process of industrialization or globalization. The competing view - taken by Friedman and Schwartz (1963), Reti (1998), and Bordo and Schwartz (1980, 1981) - argues that it was inadequate growth of money supply relative to real output that drove observed prices at the end of the 19th century. Our model gives support to the proponents of the first explanation - by emphasizing the important role that structural transformation played in influencing monetary velocity and prices. Thus the lesson to be drawn from our results is that the multi-sector framework is essential in capturing the historical long-run evolution of monetary shares and prices in the US.

6 Cross-Country Analysis

In this section we test the model’s ability to explain variation in cross-country velocity, and we quantify the importance of structural transformation as a driver of the observed results.

Calibration We focus on an international panel of 102 countries for 1980-2010. We assume that each country is a closed economy with the same structural parameters of the US, with the exception of sectoral productivity and monetary policy. Money stock per worker in each country and each year is taken directly from the data. We assume that labor productivity in sector $s$ and in country $i$ grows at a constant rate, $g_i^s - 1$, and is given by $B_{i,s,t} = B_{i,s} \times (g_i^s)^{t-1980}$. We then choose $g_i^s$ to match observed average sectoral labor productivity growth rates in each country between 1980-2010, and pin down sectoral productivity levels, $B_{i,s}$, by following the approach of Duarte and Restuccia (2010). $B_{i,a}$ is chosen to match the agricultural employment share in country $i$ in 1980, whilst $B_{i,c}$ is chosen to match the ratio of GDP per worker in country $i$ to that of the USA in 1980.

[^33]: We consider all countries for which all necessary data is available. Unless otherwise stated all data sources and construction methodology is presented in Online Appendix A.

[^34]: Productivity levels are inferred using the model rather than directly from the data because internationally comparable sectoral price levels are not as readily available and perhaps not as accurate as aggregate price levels - especially in the service-dominated non-agricultural sector.
Results  We will focus on average values over the 1980-2010 period for each country, in order to emphasize the long-run cross-sectional fit of the model. Figure 9 compares the output per worker, employment shares, inflation rates and nominal interest rates in the model to those in the data.35

Despite some variance, countries lie close to the 45-degree line and the model does well on all fronts. Next, we examine the model’s performance in explaining money shares in the model and the data (presented in Figure 10(a)). Here too the model does well - although there is now more variation than in the US and the model is unable to generate money shares greater than 100% that occur in several European countries and Japan. Columns 1 and 2 of Table 2 illustrate the results by computing a few statistics. The model does well in predicting an average money share of 51.15% in the model (50.92% in the data) and a money share of 19.87% in the bottom decile (19.77% in the data). The model slightly under-predict money-share in the top decile at 74.42% (versus 75.5% in the data), however we nonetheless capture 83% (≈ 74.42−19.87/75.5−19.77) of the increase in money-shares observed in the data. There is more variation in money shares in the data, but the model...
The following two counterfactuals (1) and (2) respectively show: the number of observations, the mean, the top-decile, the bottom-decile and the standard deviation of money shares in the data as well as those predicted by the baseline model and both counterfactuals. The final row shows the semi-elasticity of money-share to income.

Table 2: Summary statistics for cross-section. Columns show statistics for: money shares from the data (Data), the baseline model (mb), and counterfactuals 1 (mcf1) and 2 (mcf2). Rows (1)-(5) respectively show: the number of observations, the mean, the top-decile, the bottom-decile and the standard deviation of money shares in the data as well as those predicted by the baseline model and both counterfactuals. The final row shows the semi-elasticity of money-share to income.

still captures 68% (≈ 19.95 / 29.30) of the standard deviation found in the data. Next, Figure 10(b) shows that the model performs exceptionally well in predicting the (semi-)elasticities of money-shares with respect to GDP per worker, which are approximately 0.14 in data and 0.145 in the model.36 The simple model thus successfully captures the cross-sectional variation in monetary shares and the long-run, cross-sectional trend between monetary shares and income.

**Variation in velocity** Countries are assumed to differ in their monetary policies $M_t^i$ (and hence $\tau_t^i$) and in their sectoral productivities, $B_{x,t}^i = B_t^i \times (g_x^i)^{t-1980}$. The following two counterfactuals quantify the extent to which variations in each of these factors contributes to explaining cross-country differences in velocities as well as in the velocity-income relationship.

36 In other words a doubling of GDP per worker in a country is associated with a 14 and 14.5 percentage point increase in the data and model money share respectively. The reason that this is different from the 0.16 elasticity found in Figure 8 is that we now have a smaller sample of countries.
(a) Counterfactual 1 (US agr. productivity and monetary policy): Money shares in data and model.

(b) Counterfactual 1 (US agr. productivity and monetary policy): Money shares versus GDP per capita.

(c) Counterfactual 2 (US agr. productivity): Money shares in data and model.

(d) Counterfactual 2 (US agr. productivity): Money shares versus GDP per capita.

(e) Baseline Model: Money shares in data and model.

(f) Baseline Model: Money shares versus GDP per capita.

Figure 11: Money shares in the baseline model and the counterfactuals (average for 1980-2010). Drawn versus corresponding data and (log of) GDP per worker.
Counterfactual 1  In the first counterfactual, we assume that each country follows US monetary policy, i.e. $\tau^i_t = \tau^{US}_t$. We also assume that all countries have the same agricultural productivity as the US in 1980, so that $B^i_a = B^{US}_a$. We re-calibrate non-agricultural productivity in 1980, $B^i_c$, to match each country’s observed GDP per worker in 1980. Countries thus differ only in sectoral productivity growth rates and non-agricultural productivity levels. Since every country now has a high agricultural productivity (as the US), the agricultural sector will be vanishingly small in every country. As such, this experiment is akin to examining how well a one-sector model under US monetary policy in each country would match the data. Figure 11(a) compares money-shares in the counterfactual and the data, Figure 11(b) plots money-shares versus the (log) GDP per worker in the counterfactual and the data, whilst the column labelled $m_{cf1}$ of Table 2 computes some summary statistics. Predicted monetary shares are almost constant. Thus, an (effectively) one-sector version of the model with no variation in monetary policy cannot replicate the data. The reasons for this are clear from equation (9): $(V^i_t)^{-1} = \phi^i_L^i_{c,t}$. First, non-agricultural productivity does not influence velocity. Therefore, even if in this counterfactual there is cross-country heterogeneity in $B^i_c$, the effect on velocity is zero. Second, since monetary policies are the same across countries, $\phi_t = \frac{1-\alpha-\gamma}{(1-\alpha-\gamma)+\gamma\tau^{US}_t}$ will be identical across countries. Finally, since each country’s agricultural productivity is assumed to be high and the preference weight on agriculture, $\alpha$, is small under our calibration, we have that $L^i_{a,t} \approx \frac{\alpha^{US}_t}{1-(\alpha+\gamma)(1-\tau^{US}_t)} \approx 0$. Consequently, $L^i_{c,t} \approx 1$, and hence approximately constant. Thus, differences in productivity growth rates and non-agricultural productivity levels generate very little cross-country variation in monetary shares.

Counterfactual 2  In the second counterfactual we keep agricultural and non-agricultural productivity exactly as in Counterfactual 1, but we allow each country to have its own monetary policy taken directly from the data. Thus, this experiment examines whether differences in monetary policies in an (effectively) one sector model are capable of explaining cross-country differences in money shares and the velocity-income relationship. The results are shown in Figures 11(c) and 11(d) and column $m_{cf2}$ of Table 2. Whilst there is some additional variation coming from differences in monetary policies across countries - especially in some high inflation countries like Argentina or Peru - monetary shares do not change significantly with respect to the previous counterfactual. Thus, whilst differences in monetary growth rates do technically influence money shares across countries, the message from this experiment is that, quantitatively, velocity and the velocity-income relationship is overwhelmingly not driven by differences in monetary policies across countries.

Baseline  Finally, maintaining all assumptions from Counterfactual 2, we also allow countries to differ in their agricultural productivity levels - and hence in the size of their agricultural sectors. We thus revert to the baseline economy where countries differ in their monetary policies as well as in their productivity growth rates and levels. The results are striking and shown in Figures 11(e) and 11(f) and column $m_b$ of Table 2. The model does well in capturing the cross-country
variation in money-shares and the relationship between money-shares and income. We can therefore conclude that most of the variation in money shares in the model comes from the multi-sector framework, and in particular from the cross-country differences in agricultural productivity levels, which generate agricultural sectors of different sizes. Since the model captures practically the entire trend in cross-country velocity-income, this suggests - perhaps somewhat surprisingly - that structural transformation, and the decline of agricultural in particular, is a key driver of the long run monetary velocity.

7 Robustness

Our baseline results are robust to different setups and different calibration strategies. Here we summarize the main results, while leaving details to the Online Appendix. First, we re-calibrate the model by targeting observed nominal interest rates instead of money-growth rates which one may worry would not put enough distance between the calibration and the money-to-GDP ratio (Online Appendix B.2). We find the results nearly indistinguishable. Second, our baseline calibration matches average rather than period-by-period sectoral labor productivity growth rates due to a lack of annual data. An alternative approach would be to interpolate the missing data and match period-by-period productivity growth (Online Appendix B.3). The fit of the model is unchanged. Third, we endogenize the non-monetary demand by including capital accumulation in the non-agricultural sector so that capital plays the role of a credit-good (Online Appendix B.5). As capital accumulates over the growth process, non-agricultural non-monetary demand increases. The results remain largely unchanged, with a small role played by capital. Fourth, we provide additional evidence that the evolution of long-run monetary velocity originates largely in the non-agricultural sector by using a number of accounting-style procedures (Online Appendix B.6). Finally, in an extension we allow for monetary demand agriculture. We introduce two agricultural sub-sectors: a ‘traditional’, non-monetary agricultural sector with low productivity growth and a ‘modern’, monetary agricultural sector with higher productivity growth. The difference in productivity growth rates generates a transition from the non-monetary to the monetary agricultural sector as productivity increases and an endogenously changing demand for money in agriculture (Online Appendix B.5). Our main results remain unchanged, and this exercise confirms that agricultural monetary demand is not crucial to our results.

37 In particular, the (effectively) one-sector model with country-specific monetary growth can only explain 13% of the standard deviation of the baseline model. This means that 87% of the standard deviation of the baseline comes from the two-sector framework alone.

38 Of course, there are some striking outliers. Countries like Cyprus, Switzerland or Japan have monetary shares far above their predicted values. A limitation of our model is that it cannot generate money shares larger than 1. However, the focus here is on broad cross-country trends, rather than the specifics of individual countries - and in this respect the model does very well. A model with intermediate goods would presumably do even better. We leave this for future research.

39 We would like to thank an anonymous referee for the suggestion.
8 Welfare Cost of Inflation

In this section we examine the welfare costs of inflation in rich and poor countries. We first define optimality in our framework, and show that the Friedman rule holds. We then calculate a compensating variation measure of welfare that determines how much higher a household’s income would have to be in order to compensate for permanently higher inflation.

Optimality  The distortion in this environment arises - as in the standard CIA model - from the lag between households’ being paid their wage income and their ability to buy non-agricultural cash goods with that income. In particular, households can only use last period’s money holdings to purchase current period non-agricultural cash goods. This forces households to hold a low-yield asset (money) instead of a higher yield asset (bonds) in order to have money holdings to purchase cash goods in the future. Thus, as long as nominal interest rates are positive (i.e., the CIA constraint binds), the economy will not reach the first best. If, however, nominal interest rates were set to zero, then households would be indifferent between being paid today or being paid in the future (and indeed between holding money and a bond), and the distortion associated with the trading arrangement would be eliminated. Hence, since the CIA binds if and only if \( r_t > 0 \), and, as we showed above, \( r_t = \tau_t - 1 \), we need \( \tau_t \to 1 \) to eliminate the distortion. In other words, we need to implement the Friedman rule, i.e. we must have \( \frac{M_{t+1}}{M_t} \to \beta \).

40 More specifically, money does not expand the production possibility frontier. As such, the Pareto optimal allocations can be found by solving the corresponding social planner’s problem without money. It is then easy to show that the decentralized problem and the social planner’s problem are identical when nominal interest rates are zero.

41 A word of caution is needed here. Whilst it is true that as \( \tau_t \to 1 \), the equilibrium allocations approach the Pareto optimal allocations, directly setting \( \tau_t = 1 \) in equations (5)-(8) violates Assumption 5.1. It is relatively easy to show that it is nonetheless true that the allocations and prices implied by the above equations when \( \tau_t = 1 \) are also a competitive equilibrium. However, as is argued by Cole and Kocherlakota (1998), an equilibrium such as this (i.e. one where \( r_t = 0 \)) can be achieved with a large set of monetary policies - including but not restricted to the policy \( \frac{M_{t+1}}{M_t} = \beta \). Thus, whilst the limit is indeed an equilibrium, Pareto optimal, and can be implemented with a policy \( \tau_t = 1 \), it is not necessarily unique as other monetary policies could also sustain zero nominal interest rates.
Welfare cost of inflation  

The lifetime indirect utility of a household is given by:

$$V({w_t, p_{a,t}, p_{c,t}, \tau_t})_{t=T}^\infty = \sum_{t=T}^\infty \beta^t u(a(w_t, p_{a,t}, \tau_t), c_n(w_t, p_{a,t}, p_{c,t}, \tau_t), c_m(w_t, p_{a,t}, p_{c,t}, \tau_t)).$$

(11)

In equation (11), $a(\cdot)$, $c_n(\cdot)$ and $c_m(\cdot)$ are standard demand functions for agricultural goods as well as monetary and non-monetary non-agricultural goods.$^{42}$ Suppose that $V(\cdot; \lambda)$ denotes a household’s lifetime indirect welfare when income, $w_t$, is multiplied by a factor $\lambda$ in each time period:

$$V({w_t, p_{a,t}, p_{c,t}, \tau_t})_{t=T}^\infty = \sum_{t=T}^\infty \beta^t u(a(\lambda w_t, p_{a,t}, \tau_t), c_n(\lambda w_t, p_{a,t}, p_{c,t}, \tau_t), c_m(\lambda w_t, p_{a,t}, p_{c,t}, \tau_t)).$$

(12)

We wish to know by what factor, $\lambda$, income must be multiplied to make a household indifferent between living in a world with some sub-optimal monetary policy, $\{\tau_t\}_{t=0}^\infty$, and a world with optimal monetary policy. The answer to this question satisfies the following equation:

$$V({w_t, p_{a,t}, p_{c,t}, 1})_{t=T}^\infty = V({w_t, p_{a,t}, p_{c,t}, \hat{\tau}_t})_{t=T}^\infty; \lambda,$$

(13)

where prices are equilibrium prices for the optimal economy (i.e. one with $\tau_t = 1$) from equations $7$ and $8$. $^{43}$

We calculate such $\lambda$ for a hypothetical economy that is identical in all respects to the US economy at different stages of its development, with the exception of its inflation tax.$^{44}$ In particular, we set different values of this tax, ranging from $\tau_t = 1.09$, (the average rate in the US) up to $\tau = 5$. The results are shown in Figure 12. A higher inflation tax results in higher welfare costs - independent of a country’s level of income. However the cost of a sub-optimal policy will be lower in poorer countries. The reason for this is that most of the output in poorer countries is concentrated in (non-monetary) agriculture. An inflation tax on the small monetary sector will thus have relatively little effect. In richer countries, where more of the output is produced in (monetary) non-agriculture, inflation taxes can have larger welfare effects. For example, in a country with GDP per worker equal to the US GDP in 2010, a monetary policy of $\tau_t = 5$ will require an increase in income of approximately 31% to make a household indifferent to a world with optimal monetary policy. However, the same monetary policy in a country that has approximately 2% of the US’s GDP in 2010 will only require an increase in income of 3%. Thus, the same inflationary policies have very different costs in rich and poor countries. This suggests why we see higher inflation in poorer countries: these countries have a lower welfare cost of implementing inflationary policies.

An alternative approach for highlighting this result is to find the maximum inflation tax, $\bar{\tau}$, in each country that makes the country’s welfare cost of inflationary policy, $\lambda$, identical to the welfare

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$^{42}$ In particular, from the household’s first order conditions and its budget constraint we can show that: $a(w_t, p_{a,t}, \tau_t) = \frac{\bar{\alpha} - \bar{\alpha}(w_t - p_{a,t} \bar{\alpha})}{\bar{\alpha} + \frac{p_{a,t}(1 - \alpha - \gamma)(1 + \gamma)\gamma}{p_{c,t}(1 - \alpha - \gamma)(1 + \gamma)\gamma}}$, $c_n(w_t, p_{a,t}, \tau_t) = \frac{c_n(w_t, p_{a,t} \bar{\alpha})}{p_{c,t}(1 - \alpha - \gamma)(1 + \gamma)\gamma}$ and $c_m(w_t, p_{a,t}, \tau_t) = \frac{c_m(w_t, p_{a,t} \bar{\alpha})}{p_{c,t}(1 - \alpha - \gamma)(1 + \gamma)\gamma}$.

$^{43}$ It does not matter whether we choose prices from the optimal or the sub-optimal economy, as the demand functions $a(\cdot)$, $c_n(\cdot)$ and $c_m(\cdot)$ depend only on relative prices and these are entirely independent of $\tau_t$.

$^{44}$ Thus, for this exercise only, we assume each country follows the same growth path as the US. Poorer countries can thus be simply thought of as the US at some point in the past.
cost of the US’s inflationary policy, $\lambda^{US}$. This inflation tax satisfies the following equation:

$$V(\{w_t, p_{a,t}, p_{c,t}, 1\}_{t=T}^{\infty}) = V(\{w_t, p_{a,t}, p_{c,t}, \bar{\tau}\}_{t=T}^{\infty}; \lambda^{US}).$$  

(14)

The red line in Figure 13(a) shows this average welfare-equivalent $\bar{\tau}$ in each income quintile of the sample. In the lowest income quintile $\bar{\tau}_{q4} = 1.14$, whilst in the highest $\bar{\tau}_{q1} = 1.09$. The red line in Figure 13(b) then shows how each of these inflation taxes translate into observed average, annual inflation rates. Welfare-equivalent inflation rates in the lowest and highest income quintile are 8.0% and 3.4% a year, respectively. Thus the poorest countries can have inflation rates that are 2.3 times as high compared to rich countries, and yet have the same welfare costs as rich, low-inflation countries. The blue lines in Figure 13 show the corresponding inflationary taxes and the average, annual inflation rates in the data. Notice from Figure 13(b) that the observed inflation rate in the richest countries is 4.7 times higher than the inflation rate in the poorest countries. Thus, whilst inflation rates vary enormously between rich and poor countries, the welfare costs of these policies do not vary nearly as much.

9 Conclusions

We put forward a new theory of money demand in the long run, where velocity depends on a country’s GDP level. The main drivers are structural transformation - i.e. the reallocation of labor from agriculture to non-agriculture associated with development - and an endogenous share of monetary transactions in different sectors. Despite its stark simplicity, our theory is very successful at matching both within- and across-country changes in monetary velocity and the velocity-income relationship. We can replicate - almost perfectly - the long run monetary velocity in the United States between 1869 and 2013 and in a large cross-sectional data set of 102 countries.

There are three lessons to be learned from our work. First, our findings suggest that the evolution
of monetary velocity is driven almost exclusively by developments in the agricultural rather than the non-agricultural sector. Second, we show that the costs of bad monetary policy are disproportionately higher in richer than in poorer countries. Third, we demonstrate that, since velocity depends systematically on the composition of output, a country’s price levels and inflation rates may not ‘always and everywhere be a monetary phenomenon’ [Friedman and Schwartz, 1963]. These lessons should be of utmost interest to central bankers, especially in developing countries, who may down-play the role of the agricultural sector in their policy decisions. Moreover, our findings may help explain why we observe persistently higher inflation rates in poorer countries than in richer countries - despite strong anti-inflationary policies recommended by international institutions. Finally, our results are also important to researchers, as they highlight that one-sector models cannot be successfully used to understand the long run dynamics of monetary velocity and nominal price levels. Our work on the evolution of long-run monetary velocity thus offers key insights into understanding the secular trends in sectoral and aggregate price levels, with strong implications for policy and future research.
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Online Appendix not for publication

A Data Appendix

A.1 US and Cross-Country Data

GDP (PPP) and Labor Force Data We use data from Penn World Tables version 7.1. (see Heston et al. (2012)) to construct annual time series of PPP-adjusted GDP in constant 1990 prices, PPP-adjusted GDP per worker in 1990 constant prices, and total employment between 1950-2010. For each country, we construct total employment \( L \) using the variables Population (POP), PPP Converted GDP Chain per worker at 2005 constant prices (RGDPWOK), and PPP Converted GDP Per Capita (Chain Series) at 2005 constant prices (RGDPCH) as \( L = \frac{RGDPCH \times POP}{RGDPWOK} \). We then construct PPP-adjusted GDP in constant 2005 prices using the variables “PPP Converted GDP Per Capita (Laspeyres), derived from growth rates of c, g, i, at 2005 constant prices (RGDPL)” and “Population (POP)” as \( GDPK = RGDPL \times POP \). We then re-base the 2005 data to 1990 prices.

We extend the labor force and GDP data calculated above back in time for the period 1869-1950 using growth rates from Maddison (2007). In particular, we calculate the pre-1950 growth rates for Maddison’s GDP measure (in 1990 Geary-Khamis dollars) and use it to extend the GDP measure calculated above. We also calculate the population growth rates from Maddison’s data and assume that the growth rate of the labor force pre-1950 is the same as the growth rate of the population (which is true if the labor-force-to-population ratio stays constant over time). We then use these population growth rates to extend the labor force data calculated above back in time. Finally, we extend the series for GDP and labor force forward in time between 2011 and 2014 using growth rates for labor force and constant price PPP GDP from the WDI (2016). The series for PPP-adjusted GDP per worker in constant prices is then computed as \( y = GDP/L \).

Agricultural and Non-Agricultural Employment We construct contemporary (1980-2014) agricultural employment share using data from the FAOSTAT (2012) by taking the ratio of economically active population (labor force) in agriculture to total economically active population (labor force). For the US, we then combine this with the agricultural employment share from Alvarez-Cuadrado and Poschke (2011) for 1909-1980 and Lebergott (1966) for 1869-1908. Missing data is interpolated. We then obtain total employment in agriculture by multiplying the agricultural employment shares calculated above by the labor force data found in the previous paragraph. Non-agricultural employment is then the difference between total employment and agricultural employment. These are the \( L_{a,t} \) and \( L_{c,t} \) referred to in the main body of the text.
Sectoral Value Added, Constant price  For the period 1970-2014 we construct constant price (1990) value added data for agriculture and total value added in Local Currency Units from UN [2016]. For the US, we then extend this data backwards using sectoral growth rates. First, we obtain constant price agricultural and total value added data for the period 1947-1970 from the 10-sector database constructed by Timmer et al. [2014] and use this data to extend our value added measures back to 1947. Then, we extend these concatenated data back to 1869 using the Historical National Accounts database constructed by the Timmer et al. [2014]. This database provides historical constant price indices of GNP for agriculture and the total economy. Missing values are interpolated. Notice that - like Duarte and Restuccia [2010] - we do not directly use the resulting series in our model. Instead, we use the above constant (1990) price agricultural and total value added data to calculate constant price shares of agricultural value added in total value added. Then, in order to remain consistent with out aggregate GDP data calculated above, we multiply these constant price shares by the aggregate PPP GDP data calculated above. This gives us a consistent estimate of 1990 value added of agriculture. Subtracting this estimate from the GDP PPP gives us an estimate of non-agricultural value added. These are the $Y_{a,t}$ and $Y_{c,t}$ referred to in the main body of the text.

Labor Productivity Growth Rates  To calculate labor productivity growth rates we first calculate sectoral labor productivity for each sector $s = a, c$ as $y_{s,t} \equiv Y_{s,t}/L_{s,t}$ using values described in the above paragraphs. Next, we HP-filter these series and for each country and sector $s = a, c$ we calculate the average annualized sectoral labor productivity growth rates, $g_{s-1}$, between 1980 and 2010. For the US we follow the same procedure but focus on the 1869-2007 period.

Price Indices  For the cross country data and the period 1970-2014 we can construct agriculture and non-agriculture price indices by dividing sectoral value added in current prices by constant price (1990) value added data from UN [2016]. Sectoral prices between 1929 and 2013 for the United States are obtained by dividing sectoral value added in current prices (obtained from the BEA and Timmer et al. [2014]) by constant price sectoral value added found in the previous paragraphs. For the 1970-2014 period, these are identical to the indices that can be obtained using UN data. Obtaining pre-1929 prices is more complicated. In particular, as far as we know, no dependable series of data on sectoral prices exists. As such, we use nominal wheat prices obtained from Table Cc205-266 in Carter et al., eds [2006] to extend the agricultural price index back in time between 1869 and 1929. Then, using data on constant- and current-price aggregate GDP, constant price sectoral value added and the agricultural price index calculated above, we can infer a non-agricultural price index as well. In particular, multiplying the constant-price agricultural value added by the agricultural price index, gives us an estimate of current price agricultural value added, $p_{a,t} Y_{a,t}$. We can then subtract this from current price GDP, to obtain an estimate of current price non-agricultural value added, $p_{c,t} Y_{c,t}$. Then, taking the ratio of current price non-agricultural value added to constant
price non-agricultural value added we obtain an estimate of the price index of the non-agricultural sector. We obtain price indices for the economy as a whole in the same manner.

Money Data on M2 for the period 1980-2014 comes from the [IMF 2015] and is conveniently collected in the World Bank’s [WDI] database. For the US, we follow [Anderson 2006] in the construction of the long run money stock series. The source of the data for the years 1959-2014 are lines 34 and 35 in the International Financial Statistics [IMF 2015].45 For the year 1948-1958 we use data constructed by [Rasche 1990] and available online on the Historical Statistics of the United States website [Carter et al., eds 2006]. We extend this for 1869-1947 using data from [Friedman and Schwartz 1963] also reported in the Historical Statistics of the United States.46

Nominal Interest Rates Nominal interest rates for the period 1980-2014 are constructed as the sum of the real interest rates from the [IMF 2015] and the GDP deflator. Both data series are conveniently collected in the World Bank’s [WDI]. For the US, we take nominal interest rates for the period 1871 and 2014 as the nominal returns (including dividends) on the Standard and Poor Composite Stock Price Index from an updated version of Chapter 26 of [Shiller 1989], available online at [http://www.econ.yale.edu/~shiller/data/chapt26.xlsx].

Aggregate Capital We follow [Kuralbayeva and Stefanski 2013] and [Caselli 2005] in constructing capital and make use of the perpetual inventory method. Capital is accumulated according to:

\[ K_{t+1} = (1 - \delta)K_t + I_t, \]  

(A.1) where \( I_t \) is investment and \( \delta \) is the depreciation rate. We measure \( I_t \) from the PWT 7.1 as real aggregate investment in PPP terms.47 As is standard, we compute the initial capital stock \( K_0 \) as \( I_0/(g + \delta) \), where \( I_0 \) is the value of the above investment series in the first period that it is available, and \( g \) is the average geometric growth rate for the investment series in the first twenty years the data is available. Finally, we set the depreciation rate, \( \delta \), to 0.05 to match the depreciation rates in the US.48 The results are not very sensitive to choices in either \( \delta \), \( g \) or initial capital stock. The above process gives us sequences of capital stocks, \( K_t \), derived from PWT data.

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45 Notice that for 1980-2014 this coincides with the IFS/WDI data used above.
46 Notice that [Friedman and Schwartz 1963] have slightly different definitions of monetary aggregates to those currently used. According to [Anderson 2006], Friedman and Schwartz’s ‘M4 resembles, in many respects, the Federal Reserve’s current M2’ and ‘(h)ence, from an economic viewpoint, Friedman and Schwartz’s M4 and the currently published Federal Reserve M2 aggregates are more similar than first appearances might suggest’. As such, for the 1898-1947 period - when it is available - we use [Friedman and Schwartz]’s measure of M4. For the period 1869-1897 only the M3 measure is available. As such, we use the growth rate of M3, to extend the data back in time.47 So that \( I_t \equiv RGDPL \cdot POP \cdot K1 \), where RGDPL is real income per capita obtained with the Laspeyres method, POP is the population and K1 is the investment share in real income. In the above we have re-based RGDPL into 1990 dollars.
48 The value of \( \delta \) is chosen so that the average ratio of depreciation to GDP using the constructed capital stock series matches the average ratio of depreciation to GDP in the data over the corresponding period. The OECD’s Annual National Accounts report depreciation in the data as “Consumption of Fixed Capital.”
We extend our US capital data back in time using Piketty (2014) who provides data on the capital-output ratio in the US between 1770 and 2010 (measured in current-period prices). We consider only the reproducible part of his capital measure by subtracting the value of land from his measure of ‘national capital’. The resulting capital series corresponds much more closely to our modern measure of capital (Jones, 2015). We use the implied growth rates in the capital-output ratio from Piketty (2014) to extend our capital-output ratio data backwards in time and maintain comparability. We find that the implied (reproducible) capital-output ratio in the US in 1869 (in current-period prices) was approximately 1.97. This is of course is a very crude estimate and should be approached cautiously - nonetheless this is the best we can do over this long time horizon.

Capital Shares Caselli and Feyrer (2007) estimate reproducible capital shares for 57 countries. They start by taking aggregate capital shares using data from Gollin (2002) and Bernanke and Gurkaynak (2001). They then make use of the World bank’s “Where is the Wealth of Nations?: Measuring Capital for the 21st Century” database (WB 2005) to adjust these shares by excluding non-reproducible capital. We take these capital shares as our $\nu$’s. For countries not included in their data set, we assign the average capital shares value of the 57 countries, which is 0.19.

TFP growth rates In the version of the model with capital, we use total factor productivity growth rates of the non-agricultural sector directly from the data. To calculate these, we assign all capital in the economy to the non-agricultural sector. Then, taking $Y_{c,t}$, $L_{c,t}$ and $K_t$ from the above, and using the country-specific capital share, $\nu$, the TFP in non-agriculture in a country is simply calculated as a Solow residual: $Y_{c,t}/(K_t^{\nu}L_{c,t}^{1-\nu})$. Calculating the annualized average growth rate of this term between 1980 and 2010, gives us the TFP growth rate used in the model.

Long Run US Money Share To obtain an estimate of long-run US money share, we fit an exponential function with an asymptote to the US money-share data. In particular we choose a non-linear first order difference equation with an asymptote defined by: $m_{t+1} = (ae^{-b(t-1869)} + 1)m_t$ and $m_{1869} = c$ and find parameters $a, b$ and $c$ that minimize the mean squared error between the
fitted money share, $m_t$, and the money share in the US between 1869 and 2013. We find that $a = 0.0287$, $b = 0.024$ and $c = 0.243$. The predicted function and the data are shown in Figure A.1. The implied asymptote of this difference equation is approximately 0.79.

**Cross-country, non-monetary value added** [Blades (1975a,b)] collect their data through a survey questionnaire sent to national statistical offices in 1973. They describe the process as follows: “The information on country practices in this report was mainly obtained from a mail enquiry made by the OECD Development Centre during the first part of 1973. Questionnaires requesting information on the coverage, valuation, and relative importance of non-monetary activities were sent to national statistical offices in just under 100 of the more important countries on the DAC list and 70 of them returned, completed questionnaires. (...) During the course of this study data were obtained from 48 countries on the percentage contribution of non-monetary activities to total GDP for a recent year. This information is, of course, readily available for countries which make separate estimates for non-monetary activities, but the majority of countries do not attempt to distinguish the non-monetary component of GDP. These countries were asked to make a ‘best guess’, or to indicate the probable range for the share of non-monetary output.” The estimated shares for the 48 countries are the ones used in Figure 2.

Importantly, in their analysis Blades and his team focus on a particular type of non-monetary activity: goods produced for own-use or subsistence only. They characterize goods as follows: “The most important of these activities is the production for own consumption of crops and livestock, but we also consider a number of other activities which are related to primary production and which are undertaken mostly in rural areas. These include such things as fishing, hunting and forestry activities, building and construction, manufacture of simple household articles, ownership of dwellings, water collection and crop storage.”

Thus, the Blades data does not consider payments in kind received by employees, or barter trade. As such, if anything, the data presented in the paper represent an under-estimate of the importance of non-monetary goods used across countries. Since we expect barter and payments in kind to be more prevalent in poorer countries with larger agricultural sectors, including these transactions would likely make Figure 2 even steeper, supporting further our results.
B Theoretical Appendix

B.1 Details of Baseline Model

Competitive Equilibrium  For a given monetary policy, \( \{T_t\}_{t=0}^{\infty} \), a competitive equilibrium in this economy is a sequence of prices, \( \{p_{a,t}, p_{c,t}, w_t, r_t\} \), and quantities, \( \{a_t, c_{m,t}, c_{n,t}, b_t, m_t, L_{a,t}, L_{c,t}\}_{t=0}^{\infty} \), such that (1) given prices and monetary policy, households and firms solve their optimization problem, (2) the government budget constraint is satisfied and (3) markets clear.

Solution  The first order conditions for the household are given by:

\[
\begin{align*}
\alpha \beta t \frac{a_t}{a_t - \bar{a}} & = \lambda_t p_{a,t}; \\
(1 - \alpha - \gamma) \beta t \frac{c_{m,t}}{c_{m,t}} & = p_{c,t} (\lambda_t + \mu_t); \\
\gamma \beta t \frac{c_{n,t}}{c_{n,t}} & = p_{c,t} \lambda_t
\end{align*}
\] (B.1)

\[
\begin{align*}
b_{t+1}: \lambda_t = (1 + r_{t+1}) \lambda_{t+1}; \\
m_{t+1}: \lambda_t = \lambda_{t+1} + \mu_{t+1}
\end{align*}
\] (B.2)

CIA: \( \mu_t (p_{c,t} c_{m,t} - m_t) = 0 \) and \( \mu_t \geq 0 \) (B.3)

In the above, \( \lambda_t \) and \( \mu_t \) are multipliers on the budget and CIA constraints respectively. The firms’ first-order conditions are:

\[
\begin{align*}
L_{a,t}: p_{a,t} B_{a,t} & = w_t \\
L_{c,t}: p_{c,t} B_{c,t} & = w_t.
\end{align*}
\] (B.4)

The market clearing conditions are given by the equations in (4). Finally, the transversality conditions for the above problem are:

\[
\lim_{t \rightarrow \infty} \lambda_t (b_{t+1} + \bar{B}) = 0 \quad \text{and} \quad \lim_{t \rightarrow \infty} \lambda_t m_{t+1} = 0.
\] (B.5)

Binding CIA  To see that the CIA binds if we assume Assumption (3.1), divide the equations in (B.2) by each other to obtain the expression for interest rates, \( r_{t+1} = \frac{\mu_{t+1}}{\lambda_{t+1}} \). Thus, the nominal interest rate is positive if and only if money yields liquidity services \( (\mu_{t+1} > 0) \). In particular, if the nominal interest rate is positive, the CIA constraint is binding.

B.2 Matching interest rates

The per-capita money-stock in our baseline calibration is chosen to match money growth rates in data. Here we recalibrate the baseline model by matching observed nominal interest rates instead of money growth rates. Normalizing the initial level of money per worker to one, \( M_{1869} = 1 \), taking nominal interest rates from the data and using equation (8), the model implies a series for money per worker over time pictured in Figure B.1(a)49. This derived money series is very similar to

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49 Since the calibration of the remaining parameters is independent from the monetary growth rate, the other parameters remain identical to the baseline.
(a) Money (M2) per worker in the data and implied by matching interest rates.

Figure B.1: Results for US when matching nominal interest rate rather than money, 1869-2012.

(b) Implied money share.

the observed monetary series. Figure B.1(b) then compares the evolution of the money share under this calibration, under the baseline calibration and in the data. Results are nearly indistinguishable between the two calibrations. The key reason is that the evolution of money shares is driven by changes in money demand stemming from structural transformation rather than money supply.

B.3 Observed Productivity Results

In the baseline model we chose to match average sectoral productivity growth. We did this since much of the early (pre-1929) data was interpolated. Here we show the results for a version of the baseline model calibrated to reproduce the period-by-period evolution of (interpolated) sectoral productivity. We show that qualitatively and quantitatively all previous results go through.

For the calibration, all parameters (besides sectoral productivity) remain identical to those of the baseline. To calibrate sectoral productivity, we normalize productivity levels across sectors to one in 1869; that is, $B_{s,1869} = 1$ for $s \in \{a, c\}$. Then we use data on the period-by-period growth rate of sectoral value added per worker in the United States to obtain the time paths of sectoral labor productivity. In particular, denoting by $g_{s,t}$ the growth factor of labor productivity between period $t$ and period $t + 1$ in sector $s$, we obtain the time path of labor productivity in each sector as $B_{s,t+1} = g_{s,t}B_{s,t}$. Figure B.2 shows the key results of this version of the model. First, in Figure B.2(a) we reproduce the employment share in agriculture. With this calibration strategy we do well - especially in the second part of the sample - where data has not been interpolated. This is as expected, since labor productivity is the major determinant of employment in agriculture in our model. Matching productivity more closely allows us to reproduce labor shares more closely.

Analogously, this calibration strategy matches the GDP per worker slightly better than the baseline model (Figure B.2(b)). Figure B.2(c) shows the relative price of agriculture to non-agriculture goods. Once more, since relative prices are driven by relative productivity (as in equation 10), we now do a
better job at capturing the evolution of relative prices. Nominal prices and factor payments evolve in a very similar fashion to the baseline, and are omitted to save space. Finally, we compare the money share from this calibration with the baseline version and the data in Figure B.2(d). Notice that the model with data calibrated period-by-period does better in the second half of our sample, when data is not interpolated. Importantly however, as expected, the money shares predicted by both models in 1869 and 2007 are the same. Thus, both qualitatively and quantitatively this extension makes very little difference to our results.

### B.4 Introducing Capital

In this section we show that our results remain unchanged when we allow for capital accumulation in the non-agricultural sector. Adding capital allows us to capture a potentially important part of the development process and to endogenize the non-monetary demand of the non-agricultural sector which in the baseline is captured entirely by preference for credit goods. We find that adding capital does not significantly change our results.
Household’s problem  The representative household’s problem is now given by:

\[
\max_{a_t, c_{m,t}, c_{n,t}, b_{t+1}, m_{t+1}, K_{t+1}} \sum_{t=0}^{\infty} \beta^t \left( \alpha \log(a_t - \bar{a}) + (1 - \alpha - \gamma) \log(c_{m,t}) + \gamma \log(c_{n,t}) \right)
\]

s.t. \( p_{a,t} a_t + p_{c,t}(c_{m,t} + c_{n,t} + x_t) + b_{t+1} + m_{t+1} \leq w_t + r_{k,t}K_t + (1 + r_t)b_t + m_t + T_t \)

\[ K_{t+1} = x_t + (1 - \delta)K_t \] (B.6)

\[ p_{c,t} c_{m,t} \leq m_t \] (B.7)

\[ K_t, m_t \geq 0 \text{ and } b_t \geq -\bar{B} \text{ and } K_0 \text{ given} \]

The problem is very similar to the baseline model. The household owns a unit of labor and a stock of capital \( K_t \) which it rents out on the market for a wage \( w_t \) and a rental rate \( r_{k,t} \) respectively. In addition, the household enters period \( t \) with money holdings \( m_t \), bond holdings \( b_t \), and a helicopter transfer of money from the government, \( T_t \). With these resources, it can buy agricultural goods \( a_t \) (at the price \( p_{a,t} \)), cash- and non-cash non-agricultural goods (respectively, \( c_{m,t} \) and \( c_{n,t} \), at the price \( p_{c,t} \)), investment goods \( x_t \) (at the price \( p_{c,t} \)), bonds \( b_{t+1} \) for a gross return of \( 1 + r_{t+1} \) dollars next period, and it can decide to hold a (non-negative) stock of money \( m_{t+1} \) for the next period. Notice that we follow Cole and Kocherlakota (1998) by treating investment as a non-monetary, credit good which helps endogenize the cash-credit split of the non-agricultural sector.

Firms  There are representative agricultural and non-agricultural firms. As in the baseline model, the agricultural firm hires labor, \( L_{a,t} \), and produces output, \( Y_{a,t} \), using a simple linear technology that combines labor with exogenous, sector-specific total factor productivity, \( B_{a,t} \). Its profit maximization problem is given by equation (2). Non-agricultural firms however, are now assumed to hire labor, \( L_{c,t} \), and to rent capital, \( K_{t}^f \), from households. With these inputs they produce output, \( Y_{c,t} \), using a Cobb-Douglas technology that combines labor and capital with exogenous, sector-specific total factor productivity, \( B_{c,t} \). Their profit maximization problem is therefore given by:

\[
\max_{K_{t}^f, L_{c,t}} p_{c,t} Y_{c,t} - w_t L_{c,t} - r_{k,t}K_{t}^f \quad \text{s.t. } Y_{c,t} = B_{c,t}(K_{t}^f)^\nu(L_{c,t})^{1-\nu} \]

(B.8)

Output of non-agricultural firms, \( Y_{c,t} \), can be sold as both a monetary and non-monetary consumption good as well as a non-monetary investment good. As in the baseline model, agricultural output is assumed to be a non-monetary, consumption good.

Money Supply  The government is assumed to have a helicopter monetary policy as before:

\[ M_{t+1} = T_{t+1} + M_t \] (B.9)

Market Clearing  Finally, market clearing is standard.

\[
a_t = Y_{a,t}, c_{m,t} + c_{n,t} + x_t = Y_{c,t} \\
m_t = M_t, b_t = 0 \\
L_{a,t} + L_{c,t} = 1, K_t = K_{t}^f \]

(B.10)
Solution We follow the same solution strategy as in the baseline model. In particular, we assume that nominal interest rates are always positive and we use the first order conditions of the household and firms problems, in combination with the market clearing conditions and government’s money supply to obtain three equations that (together with transversality conditions for bonds, capital and money) pin down the equilibrium solutions of the problem.

The first equation defines a split of non-agricultural consumption between monetary and non-monetary goods. Defining \( C_t \equiv c_{m,t} + c_{n,t} = Y_{c,t} - (K_{t+1} - (1 - \delta)K_t) \) we can write:

\[
c_{n,t} = (1 - \phi_t)C_t \quad \text{and} \quad c_{m,t} = \phi_tC_t,
\]

where \( \phi_t = \frac{1-\alpha-\gamma}{(1-\alpha-\gamma) + \gamma \tau} \). Second, given initial capital endowment, \( K_0 \), the path of capital is pinned down by a transversality condition and the following Euler equation:

\[
\frac{\tau_{t+1} \phi_{t+1} \phi_{t+1} C_{t+1}}{\phi_t C_t} = \beta \left( \nu B_{t+1}^c (K_{t+1})^{\nu-1} (L_{t+1}^c)^{1-\nu} + 1 - \delta \right).
\]

(B.12)

Finally, employment in the non-agricultural sector, \( L_{c,t} \), is determined by the following:

\[
\frac{\alpha \tau_t}{(1 - L_{c,t}) - \frac{1}{B_{a,t}}} = \frac{(1 - \nu)(1 - \alpha - \gamma)B_{c,t}K_t^\nu (L_{c,t})^{-\nu}}{\phi_t C_t}.
\]

(B.13)

We solve the model following the strategy of [Echevarria (1997)]. We assume that after some point in time, \( T \), the variables \( M_t, B_{a,t} \) and \( B_{c,t} \) grow at constant rates \( (g_m - 1, g_a - 1 > 0 \text{ and } g_c - 1 \text{ respectively}) \) and hence \( \tau_t \rightarrow \bar{\tau} \) and \( \phi_t \rightarrow \bar{\phi} \) converge to constants. Given these assumptions, the role of the non-homotheticity disappears in the limit as \( \lim_{t \to \infty} \frac{\bar{a}}{B_{a,t}} = 0 \) in equation (B.13).

The model thus converges asymptotically to a balanced growth path where capital, investment and non-agricultural consumption grow at the rate \( \bar{g}_c \), agricultural consumption grows at a rate \( g_a \) and employment in agriculture and non-agriculture are constant. Consequently, we can re-write the model in terms of variables that are stationary in the long run. In particular, defining \( \tilde{k}_t \equiv K_t / B_{c,t}^{1/\nu} \) and \( \tilde{c}_t \equiv C_t / B_{c,t}^{1/\nu} \) equations (B.12) and (B.13) become:

\[
\left( \frac{B_{c,t+1}}{B_{c,t}} \right)^{1/\nu} \frac{\tau_{t+1} C_{t+1}}{c_t} = \beta \left( \nu \tilde{k}_{t+1}^{\nu-1}(L_{t+1}^c)^{1-\nu} + 1 - \delta \right) \quad \text{and}
\]

(B.14)

\[
\frac{\alpha \tau_t \phi_t}{(1 - L_{c,t}) - \frac{1}{B_{a,t}}} = \frac{(1 - \nu)(1 - \alpha - \gamma)\tilde{k}_t (L_{c,t})^{-\nu}}{(1 - \delta)\tilde{k}_t - \left( \frac{B_{c,t+1}}{B_{c,t}} \right)^{1/\nu} \tilde{k}_{t+1} + \tilde{k}_t (L_{c,t})^{1-\nu}}
\]

(B.15)

Since \( \lim_{t \to \infty} \frac{\bar{a}}{B_{a,t}} = 0 \), using the above it is easy to show that \( \tilde{k}_t \rightarrow \tilde{k}_* \) and \( L_{c,t} \rightarrow L_{c}^* \), where:

\[
L_{c}^* = \left( g_c \right)^{1/\nu} - \beta (1 - \delta)
\]

(B.16)

and:

\[
\tilde{k}_* = \beta^{-1/\nu} \left( g_c \right)^{1/\nu} - \beta (1 - \delta) (1 - \nu)
\]

(B.17)
Parameter Values Target Value

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
<th>Target Value</th>
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</thead>
<tbody>
<tr>
<td>$B_{s,1869}, M_{1869}$</td>
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<td>Normalization, $s \in {a, c}$.</td>
</tr>
<tr>
<td>$g_a - 1$</td>
<td>0.028</td>
<td>Labor productivity growth in agriculture, 1869-2007. 2.79%</td>
</tr>
<tr>
<td>$g_c - 1$</td>
<td>0.009</td>
<td>Labor productivity growth in non-agriculture, 1869-2007. 1.23%</td>
</tr>
<tr>
<td>${B_{s,t}}_{t=1869}^{2014}$</td>
<td></td>
<td>Constant productivity growth in sector $s \in {a, c}$. -</td>
</tr>
<tr>
<td>${M_t}_{t=1869}^{2014}$</td>
<td></td>
<td>Growth in money stock per worker. -</td>
</tr>
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<td>$K_0$</td>
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<td>Reproducible-capital to output ratio in 1869. Piketty (2014). 1.97</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.004</td>
<td>Long-run employment share in agriculture. 0.500%</td>
</tr>
<tr>
<td>$\bar{a}$</td>
<td>0.547</td>
<td>Employment share in agriculture in 1869. 55.5%</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.963</td>
<td>Average annual nominal interest &amp; money growth rates, 1869-2007. 8.85% &amp; 4.78%</td>
</tr>
<tr>
<td>$\delta$</td>
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<td>Consumption of fixed capital relative to GDP, 1964-2007. 0.050</td>
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<tr>
<td>$\nu$</td>
<td>0.229</td>
<td>Reproducible-capital income share. Caselli and Feyrer (2007). 22.9%</td>
</tr>
<tr>
<td>$1 - \gamma$</td>
<td>0.923</td>
<td>Long-run share of money in nominal value-added. 79.7%</td>
</tr>
</tbody>
</table>

| Table B.1: Model with capital, calibrated parameters. See Online Appendix [A] for detailed sources. |

We then use standard numerical methods to solve for the sequences $\hat{k}_t$ and $L_{c,t}$ that converge to $L^*_c$ and $\hat{k}^*$ using equations (B.14) and (B.15). Given these, we can obtain solutions for $K_t$ and the other non-detrended variables.

**Calibration and Results** We follow the same calibration strategy used to calibrate the baseline model. Table B.1 summarizes the parameters’ values. The only additional parameters are $\delta$, $\nu$ and initial capital stock. Total factor productivity growth in non-agriculture, $g_c - 1$, is also calibrated slightly differently. We choose $\delta$ to match the average ratio of consumption of fixed capital relative to GDP in the US between 1964 and 2007 from the BEA. We choose a country specific $\nu$ - or capital share - directly from Caselli and Feyrer (2007) who provide estimates of reproducible capital shares for 57 countries (see Online Appendix [A] for details). A crucial aspect of this calibration is the choice of initial capital in 1869. We use Piketty (2014)’s data to determine this value, by choosing the initial capital stock in the model to replicate his (reproducible-)capital-output ratio in the US in 1869 (details on our calculation are in Online Appendix [A]). Although this is most likely an imprecise measure of capital, it is the best we can do given the lack of historical data. Finally, since we do not have the entire capital stock series for the period, rather than calibrating total factor productivity growth in non-agriculture directly from the data, we instead choose the growth rate $g_c - 1$ so that our model replicates observed labor productivity growth between 1869 and 2007 in the data. Keeping in mind our caveats for capital stock data, Figure B.3 illustrates that our model with capital does a good job in fitting the long-run data for US, and that outcomes are not very different to the baseline model. In Figure B.4(a) we can see that our model predicts money shares trends quite accurately. Our initial money share in 1869 is slightly higher than before, however this is somewhat sensitive to the initial capital stock level - which is only roughly estimated. Importantly, we capture a large part of the increase in money shares- in particular we explain 62% (≈ 79.42 - 42.89) of the observed increase in the money share of the United States economy between 1869 and the implied long-run money share of approximately 79%. This is quite remarkable especially in light of the caveats regarding
Figure B.3: Simulations and data for the US in the Capital Model, 1869-2012.
Figure B.4: Money share of GDP and current price GDP from the data (Data), baseline model \( (m^{US}_b) \), and capital model \( (m^{US}_k) \). Rows (1)-(6) show: the number of observations, the mean, the top decile, the bottom decile, the standard deviation as well as the semi-elasticity of money-share to income.

Cross-Country Calibration Next, we consider how well the model performs in the cross-country setting. As in the baseline we focus on an international panel of 89 countries for 1980-2010.\(^{50}\) We assume that each country is a closed economy with the same structural parameters of the US, with the exception of sectoral productivity, monetary policy, capital share and initial capital stock. Money stock per worker in each country and each year is taken directly from the data. The country \( i \) specific capital share, \( \nu_i \), comes directly from Caselli and Feyrer (2007). We assume that total factor productivity in sector \( s \) and in country \( i \) grows at a constant rate, \( g^i_s - 1 \), and is given by:

\[
B^i_{s,t} = B^i_s \times (g^i_s)^{t-1980}.
\]

We then choose \( g^i_s \) to match observed average sectoral total factor productivity growth rates in each country between 1980-2010 directly from the data, and pin down sectoral productivity levels, \( B^i_s \), as in the baseline: \( B^i_a \) is chosen to match the agricultural employment share in country \( i \) in 1980, whilst \( B^i_c \) is chosen to match the ratio of GDP per worker in country \( i \) to that of the USA in 1980.\(^{51}\) For more details on how we construct the international series for sectoral total factor productivities, capital, capital shares and monetary aggregates, see Online Appendix A. Next, given this calibration and in order to disentangle the mechanisms at work, we perform a set of four counterfactuals.

\(^{50}\) We consider all countries for which all necessary data is available. Unless otherwise stated all data sources and construction methodology is presented in Online Appendix A.

\(^{51}\) Since this is a dynamic model, we must also specify future values of productivity and money stock. We assume that all exogenously growing variables continue to grow at their country specific averages for a hundred years after 2007. After this point it is assumed that the growth rates of these variables converge to the long run US average growth rates. This assumption is made for convenience and quantitatively plays no role in our results.
Figure B.5: Money shares in the baseline model and the counterfactuals (average for 1980-2010). Drawn versus corresponding data and (log of) GDP per worker.
of money demand. The USDA collects data on non-monetary value added for crops and animals in those countries anyway. In poor countries where agriculture is an important part of the economy, we would expect traditional, barter/home production-style and importantly non-cash agriculture to dominate and hence, again, agriculture would not play a large role in driving money demand.

Nonetheless, in this section we use US data to examine agriculture’s contribution to the evolution of money demand. The USDA collects data on non-monetary value added for crops and animals consumed in farms for 1910-2016, and for non-cash payment to labour for the period 1919-2016.
By summing up these two measures, we can obtain a rough estimate of the share of non-monetary transactions in the agricultural sector in the US.\footnote{We thank an anonymous referee for this suggestion.} According to these data, around 25% of agricultural value added was exchanged without cash in 1910. This figure declined at a roughly constant rate of 4.37% a year and was approximately 0.5% in 2016. Based on this evidence, we extend the model to incorporate cash and non-cash transactions in the agricultural sector.

We continue to assume there are two final-goods sectors (agriculture and non-agriculture). The setup of the non-agricultural sector remains identical to the baseline model.\footnote{For simplicity, we continue to abstract from capital accumulation in this sector.} However, we now assume that there are two intermediate sub-sectors in agriculture: a traditional and a modern sector. These two sub-sectors differ along two dimensions. First, the traditional sector is characterized by lower productivity growth. In fact, due to lack of better data, we will assume throughout that productivity in this sub-sector remains constant over time. This is in line with the argument put forward by Lucas (2004) that “traditional agricultural societies are very like one another”.\footnote{Notice that this is not a crucial assumption. All that is required is that productivity growth in the traditional sector is smaller than in the modern sector.} Second, we assume that households use cash to pay for the “modern” portion of their agricultural purchases whilst traditional agricultural goods are bought without cash.

The outputs of these two sub-sectors are close, but imperfect substitutes. The higher productivity growth in modern agriculture combined with the substitutability generates a transition from traditional to modern agriculture, on top of the standard shift from agriculture to non-agriculture. The changing composition of agriculture introduces an agriculture-specific money demand which lets us match the evolution of non-monetary transactions in the US. Crucially, we find that money shares and velocities are very similar to the benchmark model.\footnote{Notice that the above setup is stylized and adopted primarily for simplicity. We will get qualitatively and quantitatively very similar results - at the cost of greater complexity (i.e. corner solutions) - by assuming a transition from traditional to modern agricultural sector like in Gollin et al. (2007). There, both agricultural sub-sectors are perfect substitutes, but traditional agriculture is assumed not to use capital whilst modern agriculture does use capital. This results in poorer countries that have smaller capital endowments using traditional methods in agriculture, and later transitioning to modern farming techniques as capital holdings and modern agriculture productivity increase.} In what follows, we first present the setup and the analytical results of the model outlined above. We then explain how we calibrate the model to US data, and show that our results are close to the baseline model.

**Household’s problem** The representative household’s problem is now given by:

\[
\begin{align*}
\max_{a_t, c_{m,t}, c_{n,t}, b_{t+1}, m_{t+1}} \quad & \sum_{t=0}^{\infty} \beta^t \left( \alpha \log(a_t - \bar{a}) + (1 - \alpha - \gamma) \log(c_{m,t}) + \gamma \log(c_{n,t}) \right) \\
\text{s.t.} \quad & p_{a,t} a_t + p_{c,t} (c_{m,t} + c_{n,t}) + b_{t+1} + m_{t+1} \leq w_t + (1 + r_t) b_t + m_t + T_t \\
& p_{c,t} c_{m,t} + \theta_t p_{a,t} a_t \leq m_t \\
& b_t \geq -\bar{B}
\end{align*}
\]

(B.19)
The only difference relative to the baseline model is that now a proportion of agricultural goods purchased by the household, \(0 < \theta_t < 1\), will also be paid for with cash. We assume that this \(\theta_t\) is exogenous to the household (but not to the firm) to capture the idea that sellers - rather than buyers - decide whether to take cash for their goods or not.\(^5\)

**Firms** The final agricultural good is produced using two types of intermediate agricultural goods: modern (monetary) agricultural goods, denoted by \(m\) and traditional (non-monetary) agricultural goods denoted by \(n\). Production of the non-agricultural final good (denoted by \(c\)) remains exactly as in the baseline. Agricultural intermediate goods and non-agricultural final goods are produced using the following production functions: \(Y_{s,t} = B_{s,t}L_{s,t}\), for \(s \in \{m, n, c\}\). Here, \(B_{s,t}\) is labor productivity and \(L_{s,t}\) is the labor input. The profit maximization problem for each firm \(s \in \{m, n, c\}\) is given by:

\[
\max_{L_{s,t}} p_{s,t}Y_{s,t} - w_t L_{s,t} \quad \text{s.t.} \quad Y_{s,t} = B_{s,t}L_{s,t}.
\]

The final agricultural good is an aggregate of the intermediate \(m\) and \(n\) goods with a production function given by:

\[
Y_{a,t} = \left(\eta(a_{n,t})\frac{\sigma - 1}{\sigma} + (1 - \eta)(a_{m,t})\frac{\sigma - 1}{\sigma}\right)^{\frac{1}{\sigma - 1}}.
\]

In this expression \(\sigma\) is the elasticity of substitution between traditional and modern agriculture, whilst \(0 < \eta < 1\) determines the relative importance of traditional agriculture. Traditional and modern agriculture are assumed to be close substitutes so that \(\sigma > 1\). The profit maximization of the final good firm is standard:

\[
\max_{a_{n,t}, a_{m,t}} p_{a,t}Y_{a,t} - p_{n,t}a_{n,t} - p_{m,t}a_{m,t}.
\]

An endogenous proportion of the value of final agricultural goods, \(\theta_t \equiv \frac{p_{m,t}a_{m,t}}{p_{a,t}Y_{a,t}}\), will be ‘modern’ goods, whilst the remainder will be ‘traditional’.

**Money Supply** The government is assumed to have a helicopter monetary policy as before:

\[
M_{t+1} = T_{t+1} + M_t.
\]

**Market Clearing** Finally, markets clear in the usual way:

\[
an_{t}, a_m = Y_{n,t}, a_m = Y_{m,t}, a_t = Y_{a,t}, c_m + c_{n,t} = Y_{c,t}
\]

\[
Y_{a,t} = \left(\eta(a_{n,t})\frac{\sigma - 1}{\sigma} + (1 - \eta)(a_{m,t})\frac{\sigma - 1}{\sigma}\right)^{\frac{1}{\sigma - 1}}
\]

\[
m_t = M_t, b_t = 0
\]

\[
L_m + L_n + L_c = 1.
\]

\(^5\) Letting households rather than firms decide has almost no quantitative effect on the results but complicates the solution as changes in monetary growth will then effect \(\theta_t\) over time so that this variable becomes dependent on future and current outcomes rather than just current outcomes, much like the relative size of the monetary sector in non-agriculture, \(\phi_t\), described below.
Solution  Assuming positive nominal interest rates, we can solve for the optimal allocations. From
the first order conditions of the agricultural intermediate- and final-good firms we can derive an
expression for \( \theta_t \) given by:

\[
\theta_t = \frac{x_t}{1 + x_t}, \quad \text{where } x_t \equiv \left( \frac{1 - \eta}{\eta} \right)^\sigma \left( \frac{B_{m,t}}{B_{n,t}} \right)^{\sigma - 1}.
\]

Given our assumptions that \( \sigma > 1 \) and that productivity growth in modern agriculture is higher
than in traditional agriculture, the share of modern agriculture, \( \theta_t \), will rise over time, whilst the
share of traditional agriculture, \( (1 - \theta_t) = \frac{1}{1 + x_t} \), will fall. Denoting total agricultural employment
by \( L_{a,t} = L_{m,t} + L_{n,t} \), from the same first order conditions we can also show that:

\[
L_{m,t} = \theta_t L_{a,t} \quad \text{and} \quad L_{n,t} = (1 - \theta_t) L_{a,t}.
\]

Non-agricultural output is divided between cash and non-cash goods:

\[
c_{n,t} = (1 - \phi_t) Y_{c,t} \quad \text{and} \quad c_{m,t} = \phi_t Y_{c,t},
\]

We can then use households’ first order conditions and the previous equations to get the labor share
in agriculture

\[
L_{a,t} = \alpha \frac{\phi_t}{1 - \alpha - \gamma + \alpha \phi_t} + \eta \frac{\sigma}{\alpha} \frac{a}{B_{a,t}} \frac{1 - \alpha - \gamma + \alpha \phi_t}{(1 - \alpha - \gamma + \alpha \phi_t)} (1 - \theta_t)^{\frac{1}{\sigma}}.
\]

Using first order conditions for consumption in the non-agricultural sector, we get a dynamic first-
order difference equation that determines \( \phi_t \):

\[
\frac{\phi_{t+1} + L_{a,t+1} (\theta_{t+1} - \phi_{t+1})}{\phi_t + L_{a,t} (\theta_t - \phi_t)} = \frac{\gamma (1 - L_{a,t}) \phi_{t+1} \tau_t}{(1 - \alpha - \gamma) (1 - L_{a,t+1}) (1 - \phi_t)}
\]

In steady state, \( \phi_t = \phi^{SS} \), i.e.

\[
\phi^{SS} = \frac{1 - \alpha - \gamma}{1 - \alpha - \gamma + \gamma \tau^{SS}}
\]

where \( \tau^{SS} \) is determined by the steady state money growth rate. Given sequences for \( B_{i,t}, i = a, m, c \)
and \( M_t \), we can easily solve for the optimal sequence of \( \phi_t \) with a shooting algorithm, assuming that,
for a large enough \( T \), the economy is in steady state and hence \( \phi_T = \phi_{T+1} = \phi^{SS} \). Next, from firms
first order conditions, prices for sector \( s \in \{m, n, c\} \) goods are:

\[
p_{s,t} = \frac{w_t}{B_{s,t}}.
\]

Similarly, the price for final agricultural goods is given by:

\[
p_{a,t} = \frac{w_t}{B_{a,t}} \eta \frac{\sigma}{\alpha} \frac{1 - \theta_t}{(1 - \theta_t)^{\frac{1}{\sigma}}},
\]

and the nominal interest rate and wage rate are:

\[
r_t = \tau_t \left( \frac{1 + \theta_t L_{a,t}}{1 + \theta_{t+1} L_{c,t+1}} \right)^{\frac{1}{\sigma}} - 1 \quad \text{and} \quad w_t = \frac{M_t}{\theta_t L_{a,t} + \phi_t L_{c,t}}.
\]
Parameter | Values | Target | Target Value
--- | --- | --- | ---
$B_{s,1869}M_{1869}$ | 1 | Normalization, $s \in \{m,n,c\}$. | -
g_{m,a} - 1 | 0.038 | Labor productivity growth in agriculture, 1869-2007. | 2.79% 
g_{c} - 1 | 0.012 | Labor productivity growth in non-agriculture, 1869-2007. | 1.23% 
\{B_{s,t}\}_{t=1869}^{2014} \{g_{s}^{t-1869}\} | 0 | Constant productivity growth in sector $s \in \{m,n,c\}$. | -
\{M_{t}\}_{t=1869}^{2014} | 0 | Growth in money stock per worker. | -
$\alpha$ | 0.005 | Long-run employment share in agriculture. | 0.5% 
$\bar{a}$ | 0.306 | Employment share in agriculture in 1869. | 55.0% 
$\beta$ | 0.963 | Average annual nominal interest & money growth rates, 1869-2007. | 8.85% & 4.78% 
$1 - \gamma$ | 0.790 | Long-run share of money in nominal value-added. | 79.9% 
$\sigma$ | 2.331 | Change in proportion of traditional agriculture, 1910-2007. | -4.37% 
$\eta$ | 0.660 | Proportion of traditional agriculture, 2007. | 0.52%

Table B.2: Model with modern agriculture, calibrated parameters. See Online Appendix A for detailed sources.

Finally, the share of monetary stock relative to the nominal GDP (or the inverse of monetary velocity) can be written as:

$$V_t^{-1} = \frac{M_t}{p_{a,t}Y_{a,t} + p_{c,t}Y_{c,t}} = \theta_t \frac{p_{a,t}Y_{a,t}}{p_{a,t}Y_{a,t} + p_{c,t}Y_{c,t}} + \phi_t \frac{p_{c,t}Y_{c,t}}{p_{a,t}Y_{a,t} + p_{c,t}Y_{c,t}} = \theta_t L_{a,t} + \phi_t L_{c,t}. \quad \text{(B.32)}$$

The first equality follows by definition. The second equality follows from the assumption that the CIA constraint binds. The final equality follows from the above price equations and the perfectly competitive nature of the problem. In contrast to the baseline, the money share in this model now depends on money demand originating both in the agricultural sector ($\theta_t L_{a,t}$) and the non-agricultural sector ($\phi_t L_{c,t}$). The importance of money demand in each sector is determined by the monetary component and the size of the respective sector (in terms of sectoral value added shares or - equivalently - sectoral employment shares). The above clearly demonstrates why agricultural money demand is unlikely to be important. In poorer countries, where productivity tends to be low, we will observe employment in agriculture $L_{a,t}$ close to one (equation (B.26)). However, given low productivity in the modern agricultural sector, traditional agriculture will dominate this sector and hence $\theta_t$ will be close to zero (equation (B.24)). Thus, the product of the two terms will also be close to zero. In richer countries, productivity will tend to be higher, so we will observe $\theta_t$ closer to one, but $L_{a,t}$ closer to zero - thus again, the product of the two terms will be close to zero. As such, it is the second part of the expression, $\phi_t L_{c,t}$, that plays a key role in driving money shares over time both in the baseline and this model.

**Calibration** We calibrate the model to US data for the period from 1869 to 2007, following the same calibration strategy as in the baseline model. Table B.2 summarizes all the parameter values.

To remain brief, below we focus only on the calibration of parameters that do not appear or differ from the baseline calibration: $\sigma$, $\eta$ and the productivity parameters in modern and traditional
agriculture: \( B_{s,t} \) for \( s \in \{m,n\} \).

First, we normalize productivity in modern and traditional agriculture to one - so that \( B_{s,1869} = 1 \) for \( s \in \{m,n\} \). As argued above we also assume that productivity growth in traditional agriculture is zero, so that \( B_{n,t} = 1 \). Then we use data on the average growth rate of agricultural value added per worker in the United States to obtain the time paths of labor productivity in the modern agricultural sector. In particular, we choose \( g_m - 1 \) - the average labor productivity growth in the modern agricultural sector in the model between 1869 and 2007 - so that the model reproduces the observed growth of labor productivity in total agriculture (both modern and traditional) in the data over the same period. The time path of labor productivity in the modern agricultural sector is then given by \( B_{m,t+1} = g_m B_{m,t} \).

Second, we choose \( \sigma \) and \( \eta \) to match the evolution of the share of traditional agricultural value added in total agricultural value added (i.e. \( 1 - \theta_t \)) over time. We use USDA data to construct the series of traditional agriculture value added. From equation (B.24), for a given sequence of productivities in modern and traditional agriculture, \( \eta \) determines the level of \( 1 - \theta_t \), whilst \( \sigma \) determines the extent to which a change in relative productivity translates into a change in \( 1 - \theta_t \) over time. Consequently, we choose \( \eta \) so that the model reproduces the share of traditional agriculture in 2007 and we choose \( \sigma \) so that the model matches the average annual rate of decline in the share of traditional agriculture between 1910 (the first year available) and 2007. Notice that given greater productivity growth in modern agriculture than in traditional agriculture, the calibration implies an elasticity of substitution between modern and traditional agriculture that is greater than one. The calibration of all remaining parameters is identical to the baseline.

**Results** Results from this model are in line with the baseline setup. Furthermore, most explanations behind the results remain identical to those of the baseline. Figures B.7(a) and B.7(b) show a very good fit for the series for agricultural labor shares and GDP per worker. The fit in this case is even better than in the baseline setup - as we are now effectively allowing for an endogenously evolving productivity in agriculture, resulting from a changing composition of the agricultural sector from a (zero productivity growth) traditional sector to a (positive productivity growth) modern sector. Figure B.7(c) shows this changing composition of employment within agriculture. Notice that over time, traditional agriculture shrinks. This occurs for two reasons: not only because workers are moving away from traditional agriculture into modern agriculture due to the higher productivity growth in the modern sector, but also because of the flow of workers out of agriculture altogether. Notice also that modern agricultural employment share now forms a hump shape over time. This happens since traditional workers initially flow into modern agriculture, resulting in a rising employment share in that sector. However, as productivity in modern agriculture improves, fewer agricultural workers are needed to feed the population and so workers shift out of agriculture (including modern agriculture) altogether and into non-agriculture. Whilst we do not have data on the size of employment in traditional and modern agricultural sector, we do have data on the share
Figure B.7: Simulations and data for US, 1869-2012.
Money Share of M2 in GDP, US

Money to GDP Ratio: Data $m^G_{US}$ $m^US$

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<th>Observations</th>
<th>144</th>
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<tbody>
<tr>
<td>Mean</td>
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<td>66.88</td>
<td>72.36</td>
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<td>$t = 1869$</td>
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</tr>
<tr>
<td>Std. Dev.</td>
<td>16.43</td>
<td>10.89</td>
<td>9.76</td>
</tr>
<tr>
<td>M-Y Semi-Elast.</td>
<td>0.191</td>
<td>0.159</td>
<td>0.117</td>
</tr>
</tbody>
</table>

Figure B.8: Money share of GDP and summary statistics for US Alternative Measures. In the Table, columns show statistics for: money shares from the data (Data), the baseline model ($m^G_{US}$), and the model with modern and traditional agriculture ($m^{US}_{m}$). Rows (1)-(6) show: the number of observations, the mean, the top-decile, the bottom-decile, the standard deviation as well as the semi-elasticity of money-share to income.

of traditional agricultural value added in total agricultural value added. This is shown in Figure B.7(d) together with the predicted values for the model. This fit emerges largely from the calibration - recall that we have chosen parameters ($\eta$ and $\sigma$) to match the traditional agricultural share in 2007 as well as the average rate of decline of the traditional agricultural share. However, since our focus is on monetary velocity, this is not a big concern. The remaining figures show the evolution of selected price variables. The relative price of agriculture to non-agriculture shown in Figure B.7(e) is now nonlinear, first going up slightly at the end of the 19th century and then steadily declining. This feature arises from the fact that relative prices of agriculture to non-agriculture now not only depend on relative productivity in both sectors but also the composition of the agricultural sector - which is changing over time. This feature reduces the ability of the model to reproduce the Great Deflation at the end of the 19th century (see Figure B.7(f)): although a deflation is still present, the magnitude predicted by the model is smaller.

Finally, Figure B.8(a) shows the result for the money shares. Notice that the model in 1869 predicts a slightly higher money share than in the baseline. Importantly though, we still explain 59% of the increase of money share over the period relative to the long-run trend. Thus, even accounting for money demand in the agricultural sector, our results are in line with the those obtained from the baseline model.

B.6 Alternative tests of the model

Next, we present an alternative test of the baseline model. In the standard one-sector setup, money demand ($M$) is assumed to be proportional to nominal GDP ($\bar{Y}$) so that $M = \phi(r)\bar{Y}$. As an alternative, we propose that money demand originates (largely) in the non-agricultural sector. Our
theory can thus be summarized with the following equations describing the money share:

\[
\frac{M}{\bar{Y}} = \phi(r) \frac{\bar{Y}_c}{\bar{Y}_a + \bar{Y}_c}, \tag{B.33}
\]

\[= \phi(r) \frac{L_c}{L_a + L_c}. \tag{B.34}
\]

Equations (B.33)-(B.34) are identical to equation (9), where for notational convenience we have dropped time subscripts, we have defined \(\bar{Y}_s \equiv p_s Y_s\) for \(s = a, c\), and we have emphasized that \(\phi\) in equilibrium depends on the nominal interest rate. It is clear from both expressions that the money-to-GDP ratio is increasing in the level of development, since both the value-added share and the employment share of the non-agricultural sector increase with structural transformation. A straightforward check of our theory, therefore, would be to use equations (B.33) and (B.34) to plot the observed money-GDP ratio from the data (left hand side of the equation) against the predicted money-GDP ratios (right hand sides of the above equations). The only parameter that would then need to be calibrated is the interest elasticity of money demand.57 In this section we perform this check for US time series and the cross-country sample, and show that doing do not significantly change our baseline results.

Comparison We start by calibrating the interest elasticity of money demand. In our baseline model, \(\phi(r) = \frac{1}{1 + a(1 + r)}\), where \(a \equiv \frac{\gamma}{1 - \alpha - \gamma}\), which implies that interest elasticity of money demand is \(|\frac{\partial \log(\phi(r))}{\partial \log(r)}| = \frac{ar}{1 + a(1 + r)}\). To pin down this elasticity we only need to know the parameter \(a\). As we argued in the main body of the paper, under reasonable assumptions, the ratio of money to nominal GDP predicted by the model converges to \(1 - \alpha - \gamma\) in the long run and the agricultural employment share converges to \(\alpha\). If we assume - as in the baseline - that the money-to-GDP ratio converges to a long run value of 0.79 predicted by the US data and employment share in agriculture converges to a reasonable 0.05%, then it follows that, \(a = 0.259\), which allows us to pin down \(\phi(r)\).

Finally, taking nominal interest rates \(r\), current-price non-agriculture value added shares \((\frac{\bar{Y}_c}{\bar{Y}_a + \bar{Y}_c})\) and non-agriculture employment shares \((\frac{L_c}{L_a + L_c})\) directly from the data we can construct ‘predicted’ money shares, \(m_y \equiv \phi(r) \frac{\bar{Y}_c}{\bar{Y}_a + \bar{Y}_c}\) and \(m_l \equiv \phi(r) \frac{L_c}{L_a + L_c}\) and compare these with those predicted by our baseline model, \(m_b\), and the data.58

Figure B.9(a) examines these predictions for the US between 1869 and 2012: \(m_y\) and \(m_l\) are plotted in red, our baseline model money share, \(m_b\), is the black dashed line and the data is shown in blue. Table B.9(b) provides summary statistics for each estimate. Given our calibration, in the long run all three estimates converge to the same value of money share or 79%. We can thus measure our success by comparing the change in predicted and observed money share between 1869 and the long run value of 79%. The \(m_b\) and \(m_l\) explain the largest part of the increase - each capturing

57 We would like to thank an anonymous referee for this suggestion.
58 In our model \(\tau_t = \tau_t - 1\). Hence, instead of using nominal interest rates directly, we could use the fact that \(\tau_t = \frac{M_{t+1}}{M_t}\) to infer the model-predicted \(\tau_t\). This however, would require us to calibrate another parameter - \(\beta\) - so we will not go down this route. Moreover, choosing this option would give practically indistinguishable results.
Figure B.9: Money share of GDP and summary statistics for US Alternative Measures. In the Table, columns show statistics for: money shares from the data (Data), the baseline model ($m^{US}_b$), the model fitting employment shares directly ($m^{US}_l$) and the model fitting value-added shares directly ($m^{US}_y$). Rows (1)-(6) show: the number of observations, the mean, the top-decile, the bottom-decile, the standard deviation as well as the semi-elasticity of money-share to income.

75.2% of the increase. Next, $m_y$ generates a slightly smaller increase as it misses the money shares in the early part of the data - explaining approximately 56% of the increase - however it generates a better fit over the remainder of the period. Finally, $m_b$ captures 66% of the standard deviation of the data, $m_y$ captures 54% of the standard deviation whilst $m_l$ does best and captures 86% of the variation. Overall, the three predicted measures give qualitatively identical and quantitatively very similar results for the US - although $m_l$ and $m_b$ do best in capturing the evolution of money shares found in the data.

Next, we turn to the cross country data. As before we will focus on average values over the 1980-2010 period for each country, in order to emphasize the long-run cross-sectional fit of the model. Figure B.10 shows observed money shares versus predicted money shares (left column) and model predicted money shares versus GDP per worker (right panel) in the cross-section. Notice that all three measures capture a substantial portion of the variation of money shares in the data - as well as a positive money-share to GDP relationship. However, once more, it is the baseline model and $m_l$ that do the best - capturing most of the variation and best replicating the money-share income elasticity. This is highlighted in Table B.3 which provides summary statistics and allows us to quantify the success of individual models. The $m_y$ model captures about a third of the standard deviation in the data, 37-40% of the observed increase between the highest and the lowest money-shares deciles and it explains 49-54% of the money-share to income semi-elasticity. The $m_l$ and $m_b$ models fare about the same. They both capture about two-thirds of the standard deviation in the data, 81-88% of the observed increase between the highest and the lowest money-shares deciles and explain the entire money-share to income semi-elasticity. Thus, quantitatively $m_y$ does slightly worse in explaining...
Figure B.10: Money shares in different calibrations (average for 1980-2010). Drawn versus corresponding data and (log of) GDP per worker. ALL AVAILABLE DATA
Table B.3: Summary Statistics. Left part of table shows all data, whilst the right part of the table shows only data that is available for all measures.

<table>
<thead>
<tr>
<th>Observations</th>
<th>Data</th>
<th>$m_b$</th>
<th>$m_l$</th>
<th>$m_y$</th>
<th>Data</th>
<th>$m_b$</th>
<th>$m_l$</th>
<th>$m_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>50.09</td>
<td>51.15</td>
<td>50.16</td>
<td>64.52</td>
<td>52.40</td>
<td>52.50</td>
<td>53.03</td>
<td>66.02</td>
</tr>
<tr>
<td>p10</td>
<td>18.07</td>
<td>19.87</td>
<td>15.14</td>
<td>49.07</td>
<td>21.00</td>
<td>19.87</td>
<td>22.28</td>
<td>51.68</td>
</tr>
<tr>
<td>p90</td>
<td>85.81</td>
<td>74.42</td>
<td>74.88</td>
<td>75.85</td>
<td>86.80</td>
<td>74.51</td>
<td>75.25</td>
<td>76.32</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>42.94</td>
<td>19.95</td>
<td>21.85</td>
<td>10.58</td>
<td>30.06</td>
<td>19.88</td>
<td>19.48</td>
<td>9.88</td>
</tr>
<tr>
<td>M-Y Semi-Elast.</td>
<td>0.138</td>
<td>0.145</td>
<td>0.156</td>
<td>0.074</td>
<td>0.142</td>
<td>0.141</td>
<td>0.145</td>
<td>0.070</td>
</tr>
</tbody>
</table>

the cross-country data than either $m_b$ or $m_l$ - but qualitatively all three measures capture similar patterns. These findings support our choice to calibrate the baseline model to employment shares rather than value-added shares. Value added shares - especially in poorer countries and in historical US data - are of poor quality (they can be self-reported, based on surveys, or imputed) or they simply do not exist. Instead, by focusing on labor shares rather than value added shares, we avoid some of these problems as it is easier to count ‘bodies’ employed in a sector than the output and prices of hundreds of individual products.

Despite the simplicity of the above exercise, we believe that the baseline calibration in the main body of the paper is preferable as it helps us better understand the mechanism of the model and provides additional external validity. First, our baseline model specifies and quantifies the exact mechanism that drives employment, value added shares and nominal interest rates which in turn drive money shares. Instead, the exercise in this section takes shares and interest rates as exogenous. Knowing the mechanism is important as it allows us to determine - perhaps surprisingly - that agricultural productivity growth is largely responsible for the observed money-share patterns.

Second, having a fully calibrated and specified model allows us to perform counterfactuals and welfare analyses. This provide us with insights that the simple exercise in this section could not deliver. In section 8 for example, we find that inflationary policies tend to be far more damaging in rich countries than in poor countries, which helps to explain why we tend to observe higher inflation rates in poorer countries than in richer countries.

Finally, and most importantly, the calibrated baseline model has strong implications on the evolution of nominal prices, which can then be compared with the data. As we argue in section 5 this turns out to be especially interesting in light of the so-called ‘Great Deflation’ period in the United States at the end of the 19th century that our model, in contrast to more standard models, is able to replicate.
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