ON THE FACTORS OF INNOVATION AND ECONOMIC GROWTH

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Abstract

One could hardly overstate the importance of economic growth and innovation as the engine of continuous improvement in the quality of life and wellbeing. This dissertation seeks to improve our understanding of factors impacting on economic growth and the process of innovating.

Chapter 1 underscores that exploring the relationship between competition and innovation requires consistency in the way innovation is measured and proxied. It shows that a failure to do so can lead to conflicting conclusions on how competition affects innovation.

Chapter 2 explores the literature on the relationship between business cycles and innovation to argue that existing evidence is conflicting and implies that research and development (R&D) activities respond positively to downturns at the level of individual firms (i.e. are countercyclical) but negatively at that of sectors and economies (i.e. are procyclical). The chapter explains this regularity through the procyclicality of fluctuations in the number of R&D performers, which offsets the countercyclicality of individual R&D profiles and makes it look procyclical at the aggregate level.

Chapter 3 documents and analyses the positive link between the strength of intersectoral connections within an economy, and its growth. To explain this regularity, it develops a tractable theoretical framework whereby tighter connections bring about stronger propagation downstream of productivity growth episodes in individual firms and sectors. Analysing the framework yields stark testable predictions on the most growth-enhancing structure of the intersectoral linkage.

Preface

Since the seminal works due to Solow (1956, 1957) and Swan (1956), technological improvements have been shown to be the key engine of economic growth. Even though this phenomenon is ultimately an outcome of individual decisions to innovate and individual research and development effort, it had not been until the contributions by Romer (1990), Rivera-Batiz and Romer (1991), Grossman and Helpman (1991a) and Aghion and Howitt (1992) that the economic profession has acquired a theoretical perspective on tracing technological growth down to its micro-origins of economic agents' innovations. Collectively, these works laid the foundations of what came to be named endogenous growth theory.[1]

The establishment of micro-foundations for growth theory linked it with firm- and sector-level studies of R&D and productivity within the industrial organisation literature, pursued since at least the 1950s – 1960s.[2] Coupled with the increasing availability of sector- and firm-level data, this paradigm development has stimulated research into the role of various sectoral factors impacting on the process of innovation within firms,[3] and how this relates to aggregate outcomes

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[1]See (Aghion and Howitt, 1998, Chapter 1) for a summary of the field's early evolution; see Gancia and Zilibotti (2005), Aghion and Howitt (2005), Acemoglu (2009) Chapters 13, 14, Aghion, Akcigit, and Howitt (2014) for reviews of more recent developments in the area.

[2]See Griliches (1998).

[3]Alongside studying sectoral factors of innovation, another broad strand of the IO literature has focused on the individual firm level. Examples of relevant factors of innovation investigated in this literature include the size of a firm (literature on Gibrat's law, see detailed reviews in Symeonidis (1996) and Sutton (1997)), different dimensions of a firm's ownership structure, i.e. concentrated vs. disperse, state-owned vs. private, with foreign participation vs. without

such as economic growth.

Sectoral factors include, the intensity of intrasectoral competition, which has been analysed since early theoretical contributions by Schumpeter (1943) and Arrow (1962). The inconclusiveness of empirical evidence on the direction of the link between competition and innovation has pushed the research in the direction of exploring (both empirically and theoretically) nonlinear characterisations of the relationship between the two variables. Two early examples are Scherer (1965) and Scherer (1967), with a more recent contribution being Aghion, Bloom, Blundell, Griffith, and Howitt (2005). See Symeonidis (1996) and Gilbert (2006) for a extended discussion of the literature.

Business cycles have been studied as another factor of innovation, following the exploration of the topic by Schumpeter (1943). As Chapter 2 argues, this branch of research has been characterised by a lack of agreement on whether crises can spur innovative effort, which has led to the creation of approaches explaining the possibility of business cycles' both positive and negative impact on innovation within a single framework. For a review of recent contributions to this branch of research see Section 2.1.1 in Chapter 2 of the present work.

Among other sectoral factors of innovation under recent study one could mention relative abundance/scarcity of different production factors as studied within directed technical (technological) change literature initiated by the works of Acemoglu (1998, 2002).[4] The distribution of government spending across sectors must be also considered another factor of innovation, as argued by Cozzi and Impullitti (2010). Finally, it is worth pointing out at the configuration of production links between sectors (also known as production networks), which Carvalho

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such etc. (see discussions in Bishop and Wiseman (1999) and Choi, Hee, and Williams (2011)). Another feature explored is the location of a firm, as studied in the context of agglomeration and spatial knowledge spillovers (see Carlino and Kerr (2015)).

[4]A review of the literature and an introduction to its workhorse framework can be found in (Acemoglu, 2009, Chapter 15). See also Acemoglu (2007) for a discussion of the framework's general analytical properties.

and Voigtländer (2015) show to be a significant contributor to technology adoption by firms.

The first two chapters of this work belong directly to the first of the two broad strands of literature outlined above: Chapter 1 explores further the relationship between competition and innovation by studying the hypothesis of a hump-shaped (inverted-U) relationship between the two phenomena due to Aghion et al. (2005). The chapter suggests that observable properties of innovation as function of competition depend crucially on how the former is proxied/measured. It shows that the hypothesis is sensitive to whether innovation is measured by effort (i.e., R&D spending) or outcomes/accomplishment, thereby acting as a guidance tool for future attempts to validate the theory.

Chapter 2 concentrates on the link between business cycles and innovation. It reconciles mixed evidence on the cyclicality of R&D spending by arguing that the cyclical nature of innovative activities changes depending on the level of aggregation: while R&D spending is countercyclical on the level of individual firms, it becomes procyclical for sectors and economies. The chapter's key argument is the idea of intensive versus extensive margin – the countercyclicality of an individual firm's R&D spending (intensive margin) becomes its opposite through procyclical fluctuations in the number of R&D performers, which make total R&D spending in a sector/economy (extensive margin) procyclical as well.

Chapter 3, while maintaining an intimate connection with studying how certain aspects of sectoral and macroeconomic environment affect the dynamics of innovative activities and economic growth, departs from the overarching theme of the first two: in its focus is the interplay between the structure of an economy's intersectoral linkage and its growth rate. The chapter uses tools from network theory[5] to show that a greater strength of connections between sectors (i.e., their greater reliance on other industries' products used as intermediates) leads to higher growth rates through the effect of network multiplier; in addition, it pins

[5]See Carvalho (2014) and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) for an introduction to the fundamental tools and concepts of theory of macroeconomic networks.

down the optimal growth-enhancing structure of a sectoral interlinkage.

The last chapter of this dissertation contains a brief summary of the dissertation's main results and conclusions, as well as suggestions for potential directions for future research.

Chapter 1

The Relationship between R&D and Competition: Reconciling Theory and Evidence

The hypothesis of an inverted-U (hump-shaped) relationship between innovation and competition due to Aghion et al. (2005), has been tested for different data sets (Tingvall and Poldahl (2006), Askenazy, Cahn, and Irac (2013)) without garnering conclusive support.

In this chapter we argue that this lack of agreement occurs owing to the incompatibility of approaches employed in the papers (measuring innovation in terms of R&D outcomes in Aghion et al. (2005), and by R&D effort in the other two). We illustrate our point by developing a tractable general-equilibrium model, in which a multitude of industries are populated by Cournot-competing firms engaging in productivity-increasing R&D. While R&D outcomes in our model are a hump-shaped function of competition, R&D effort can be observed to be either increasing, decreasing, or hump-shaped, thus reproducing compatibly and reconciling the results derived in both Aghion et al. (2005) and in Tingvall and Poldahl (2006) and Askenazy et al. (2013). Furthermore, this result can act as a guidance for further attempts to identify the hump-shaped pattern in data.

1.1 Introduction

The traditional view on how innovation is affected by competition has been that the two have an inverse relationship. This conclusion naturally follows from the Schumpeterian-style idea that heavier competitive pressure hampers innovators from reaping all the fruits of their effort, thus discouraging R&D. This line of reasoning can be traced back to the work of Schumpeter (1943), who argued that ensuing monopoly power (and resulting profits) is a necessary reward for bearing the costs of innovation. In addition, this idea follows from standard industrial organisation models of product differentiation (e.g., Dixit and Stiglitz (1977) and Salop (1979)) and has been encapsulated in the first-generation models of endogenous growth (e.g., Romer (1990), Grossman and Helpman (1991a) and Aghion and Howitt (1992)).

This theoretical prediction, however, has grown to be challenged by the body of evidence available: for example, empirical findings of Nickell (1996) and Blundell, Griffith, and van Reenen (1999) show competition and innovation to have a positive relationship. In order to reconcile the theoretical prediction of a negative relationship with these results, Aghion, Bloom, Blundell, Griffith, and Howitt (2005) used data on a panel of firms listed on the London Stock Exchange to estimate a non-linear model of the relationship between the number of registered patents and the degree of competition in an industry, which has shown the two variables to be related in the inverted-U (hump-shaped) fashion.[1]

Several attempts to detect the presence of the hump-shaped pattern in other datasets have been at most only partly successful. Tingvall and Poldahl (2006), using a panel of Swedish firms, have shown the pattern to be fragile with respect to the choice of the competition measure[2] and the estimation procedure.[3]

[1]The authors also introduce a theoretical framework for their empirical results – see discussion below in this section.

[2]Specifically, the hump-shaped pattern has been confirmed for the Herfindahl index, but not for the Lerner (price to cost margin, PCM) index.

[3]The relationship between the Herfindahl index and R&D becomes insignificant if a fixed-

Askenazy, Cahn, and Irac (2013), using a panel of French firms, have confirmed the presence of the hump-shaped pattern only for a subsample of large firms in their dataset.[4]

A notable feature of both aforementioned studies is the use of R&D expenditure as the measure of innovation, or, put differently, *R&D effort* – as opposed to *R&D outcomes* (*R&D accomplishment*),[5] which is the case with Aghion et al. (2005), who proxy R&D with the number of patents.[6] Given the difference in the approaches to gauging R&D, one can argue that the methods used by Aghion et al. (2005) on one hand, and Tingvall and Poldahl (2006) and Askenazy et al. (2013) on the other, *can possibly* be compatible with each other (and, hence, the respective authors' conclusions can be considered together) *only* if R&D effort and R&D outcomes, *as functions of competition*, are related monotonically to each other.

This chapter argues that such a premise is not safe, by introducing a novel theoretical framework whereby under some plausible assumptions, R&D outcomes behave as a hump-shaped function of competition, while R&D effort *can be observed* to depend on competition either monotonically (as either an increasing or a decreasing function) or in a hump-shaped fashion. Introducing such a discrepancy in patterns serves two purposes. First of all, it naturally reconciles the contradictory conclusions by Aghion et al. (2005), Tingvall and Poldahl (2006) and Askenazy et al. (2013) by pointing out that those are drawn using variables behaving inconsistently with each other. Second – and more importantly – this

effects estimator is used.

[4]In order to explain the discrepancy in their results and those by Aghion et al. (2005), Askenazy et al. (2013) advance a model in which the hump-shaped pattern manifests itself more pronouncedly for large companies – see discussion below in this Section.

[5]The latter term is used by Griliches (1998).

[6]It is worth mentioning here that Aghion et al. (2005) admit they could not obtain a statistically significant confirmation of their results when using R&D investment as a measure of innovation either, which they explain by an insufficient number of observation periods for that variable in their sample – see (Aghion et al., 2005, pp. 707–708).

chapter has the merit of bringing additional methodological awareness (with regards to approaches to proxying innovation) to studying the relationship between competition and innovation in general, and the efforts to confirm the hypothesis of the hump-shaped relationship between the two, in particular.

To illustrate the chapter's key intuition, suppose that R&D effort γ is a function $g(m;\chi)$ of competition (as measured by some parameter $m \in [\underline{m};\bar{m}]$) and a set of other parameters χ. The standard logic, which implicitly underlies the shift from R&D outcomes to R&D effort in Tingvall and Poldahl (2006) and Askenazy et al. (2013), suggests that if q stands for R&D outcomes, it is technologically related to γ through an increasing function $R(\gamma;\theta)$ (where θ are other affecting parameters). Thus, the signs of m's effects on q ($\frac{\partial q}{\partial m} = \frac{\partial R}{\partial \gamma} \cdot \frac{\partial g}{\partial m}$) and on γ ($\frac{\partial \gamma}{\partial m} = \frac{\partial g}{\partial m}$) have to coincide, which allows one to treat γ and q equivalently. By contrast, R&D outcomes in our model are related to R&D effort via function $R'(\gamma;m;\theta)$, which increases in γ and in addition, *directly* depends on competition. In this case, the signs of competition's effects on the two aspects of innovation are determined by expressions: $\frac{\partial q}{\partial m} = \frac{\partial R'}{\partial \gamma} \cdot \frac{\partial g}{\partial m} + \frac{\partial R'}{\partial m} \gtreqless 0$ and $\frac{\partial \gamma}{\partial m} = \frac{\partial g}{\partial m} \gtreqless 0$, and no longer have to coincide. In particular, when $\frac{\partial \gamma}{\partial m}$ maintains the same sign for any $m \in [\underline{m};\bar{m}]$, whereas that of $\frac{\partial q}{\partial m}$ changes from positive to negative at some point $\hat{m}_q \in [\underline{m};\bar{m}]$, a situation of discrepancy in the observed behaviour of γ and q occurs.[7]

The logic above drives the general-equilibrium framework presented in the chapter. In the framework, R&D effort is exerted by Cournot-competing firms populating a multitude of homogeneous industries, and R&D outcome is the resulting increase in a firm's productivity. First of all, R&D effort is a hump-shaped function of competition, which reflects the interaction of two forces: on one hand, the upward locus of the hump is driven by the escape-costs effect: increasing competition (which we model as the number of firms in an industry) translates into more intense rivalry for production factors, which pushes up factor prices and, thereby, producers' marginal costs. In this situation, as a counteracting

[7] The author wishes to thank Prof. David Ulph for this interpretation.

response, firms are prompted to invest more in R&D. This force is opposed by the division effect, whereby more competition (as expressed through a larger number of firms) implies a smaller size of each one of them, thus driving down the individual amount of R&D effort.[8]

R&D outcomes in our framework experience the impact of competition both through R&D effort, and directly through the absence of the scale effect (as opposed to R&D effort) and congestion. The former encapsulates Gibrat's law, whereby the growth rate of a firm is independent of its size,[9] while the congestion ('fishing-out') effect reflects the duplication of firms' R&D effort, as it is exerted simultaneously by all firms in every industry, and "...when many firms are undertaking R&D, ...they are likely to try similar ideas; thus there will be some amount of external diminishing returns", – put differently, in the situation of simultaneous engagement in R&D firms are 'fishing out of the same pond' (Acemoglu, 2009, p. 473).[10,11]

In light of the above, switching from considering R&D effort to R&D outcomes removes one impending effect (division) and introduces another (congestion), thus creating a leftward shift (if the congestion effect is relatively weaker than the division effect) or a rightward one (otherwise) in the R&D outcomes as a function of competition. Analysing the framework pins down exactly conditions when these shifts are such that R&D outcomes have the hump region within the range competition indicator's range of possible values, while the opposite is

[8]The last argument relies on the stylised fact that the intensity of R&D effort (as usually measured by the ratio of R&D spending to total sales) is independent of a firm's size, i.e. R&D on average makes up a fixed proportion of an innovator's size (see Klette and Kortum (2004)).

[9]See Box 1 for a discussion.

[10]The logic of this assumption follows the one employed in the premises of the framework developed in (Acemoglu, 2009, Sec. 14.3) and further generalised in Acemoglu and Cao (2015).

[11]Note that the presence of the congestion effect does not rule out the possibility of knowledge spillovers: while those imply the presence of intertemporal dimension (i.e., benefiting from someone else's *previous* research), our setting describes firms' conducting R&D simultaneously, which results in duplication of their effort.

true for R&D effort, so that a situation of discrepancy in innovation measures' patterns occurs.[12]

1.1.1 Related Literature

This chapter is related to several strands of literature. First and foremost, similarly to Aghion et al. (2005), d'Aspremont, Dos Santos Ferreira, and Gérard-Varet (2010), Askenazy et al. (2013), Onori (2015), we study the phenomenon of a hump-shaped relationship between competition and innovation in the context of a general equilibrium model.

Aghion et al. (2005) put forward a model of step-by-step innovation, in which a multitude of an economy's industries (each populated by two firms) can exist in two states: neck-and-neck (both firms employ technologies of the same level) or levelled (one of firms has a technological lead over the other). In the former state, tougher competition prompts firms to step up their R&D efforts in order to escape competition; in the latter R&D decreases in competition: due to the Arrow replacement effect (introduced by Arrow (1962)), the technological leader does not innovate, whereas the laggard is discouraged from undertaking R&D by the prospect of facing (in the case of his success) fiercer competition with the former leader. The hump-shaped pattern emerges in the model on the economy-wide level: if the degree of competition is low, firms' incentives to escape competition are weak, so most industries are in the neck-and-neck state: therefore, tougher competition intensifies R&D in the majority of industries and impedes it in the minority represented by the levelled industries, thus having a positive overall impact on R&D on the economy scale (the reverse logic applies when the initial level of competition is already high).[13]

[12]See Figures 1.3a and 1.3b for an illustration.

[13]It is worth noting here that the authors' framework suggests a firm's R&D intensity to increase with its size, which is generally not supported by empirical evidence available (see e.g. Cohen and Klepper (1996), Klette and Griliches (2000)). This property is also inherited by the model of Askenazy et al. (2013), which builds upon the framework by Aghion et al. (2005) (see

In the framework due to d'Aspremont et al. (2010), a multitude of firms in every industry simultaneously exert R&D effort, which with a fixed probability yields a decrease in the marginal costs of a successful innovator. If innovations are not drastic (i.e. the gap between R&D winners' and losers' marginal costs is not too wide, so that the latter stay in the industry), fiercer competition on one hand raises the sales of successful innovators (at the expense of losers), but, on the other hand, drags down all firms' price mark-ups. The superposition of the two effects brings about a hump-shaped pattern in firms' expected profits, which translates into the relationship between firms' R&D effort and competition.

Onori (2015) develops a modification of the standard Schumpeterian creative destruction model due to Aghion and Howitt (1992), in which R&D laboratories create and sell the blueprints of new technologies to Cournot-competing producers of intermediate goods, who experience spillover effects of sales volumes (both their own and those of the other firms in the industry) on their marginal cost of production. As the author shows, if the effect of learning from other firms is stronger than that of intra-firm ('in-house') learning, the economy's aggregate innovation rate can be an inverted-U function of competition, as measured by the number of firms. If the initial number of competitors is low, its increase allows each one of them to learn from a larger number of sources, thus raising their profits (by driving down their costs) and in turn providing the R&D laboratories with greater incentives to innovate.[14]

The work by Askenazy et al. (2013) attempts to reconcile their empirical conclusions with those by Aghion et al. (2005) by building a model upon the latter authors' framework, in which the cost of innovation is assumed to decrease with a firm's size. Thus, the hump-shaped pattern becomes more pronounced for

below).

[14]As is commonly assumed in the literature, blueprints created by R&D laboratories are sold to the intermediate goods producers at the competitive price, which in this case equals the increase in the latter's profits. Therefore, such an increase directly translates into that of the equal magnitude (and, hence, stronger incentives to innovate) for the R&D laboratories.

larger firms, as observed in the authors' empirical results.

We believe that this chapter contributes to the reviewed body of literature by stressing the difference in the behaviour of R&D effort and R&D outcomes with regards to competition intensity, through demonstrating that the latter can be a hump-shaped function of competition even though the former is not – a distinction that is not made in any of the aforementioned papers, which prevents them from accommodating the inconsistency of the results by Aghion et al. (2005) on one hand, and Tingvall and Poldahl (2006) and Askenazy et al. (2013) on the other.

Furthermore, our theoretical contribution brings additional methodological awareness with regards to approaches to proxying innovation, to the empirical literature inspired by the work of Aghion et al. (2005) discussed above, and, as we hope, will help to guide future efforts to confirm the hypothesis of the hump-shaped relationship between competition and innovation.

Additionally, in our model we suggest a novel mechanism producing the hump-shaped pattern, which comprises the superposition of the escape-costs effect and the division effect (in the case of R&D effort) and that of escape-costs effect and congestion (for R&D outcomes).

Apart from the body of literature presented above, this chapter is related to a few other strands of research. First of all, it belongs to the literature investigating the link between market structure and innovation in the general-equilibrium macroeconomic context (examples of contributions in the area are Peretto (1996) and Impullitti and Licandro (2018)). In addition, by highlighting the differences between the behaviour of R&D effort and that of R&D outcomes on the theoretical level, our model is related to the works by Link (1980) and Pohlmeier (1992), where those differences are underscored in the context of empirical IO studies.

Lastly, our model can be thought of as a general-equilibrium generalisation of the stylised model by Cohen and Klepper (1996), which allows it, similarly to the authors' model, to match a number of stylised regularities of R&D, including:[15]

1. R&D expenditure increases in a firm's output size (Cohen and Klepper, 1996, Stylised fact 2, p. 928);

2. The elasticity of R&D expenditure with respect to the volume of a firm's output is unity (Cohen and Klepper, 1996, Stylised fact 3, p. 929);

3. R&D productivity (defined as the ratio of R&D accomplishment to R&D effort) decreases with a firm's size (Cohen and Klepper, 1996, Stylised fact 4, p. 930): in our model, the resulting increase in productivity is independent of the firm's output, whereas R&D effort is linear in the latter, so that in the long-run their ratio diminishes to zero.

1.1.2 Structure of the Chapter

The rest of this chapter is organised as follows: in Section 1.2 the model is presented and solved: Sections 1.2.1 and 1.2.2 introduce the model's equations; in Section 1.2.3 we describe the equilibrium conditions in intermediate-good markets, solve for firms' optimal decisions on production and R&D investment, and establish the conditions under which the hump-shaped patterns in R&D effort and R&D accomplishment emerge; Section 1.2.4 enquires into the behaviour of the aggregate household to pin down the economy's behaviour in the long-run. Section 1.3 is devoted to investigating the conditions under which the situations of discrepancy in the behaviour of R&D effort and R&D outcomes occur, and checking the empirical compatibility of the model's predictions. In particular, given that our results depend crucially on the value of the elasticity of substitution between inputs, we start with assessing the range of its values compatible with our model's setting (Section 1.3.1), after which we proceed to deriving the discrepancy conditions in Section 1.3.2 and discussing their empirical plausibility in Section 1.3.3. The last section concludes.

[15] More recently these regularities were revisited and confirmed by Klette and Kortum (2004).

1.2 The Model

The model presented in this section describes an economy where the final good is produced competitively using inputs supplied by industries populated by homogeneous Cournot-competitive firms. Each firm engages in production and R&D (which leads to a decrease in a firm's marginal costs). Both R&D effort (R&D spending in the model) and R&D outcomes (a drop in marginal costs) are shown to be hump-shaped functions of competition in an industry, whose peaks do not generally coincide. This can bring about a situation of discrepancy in the observed behaviour of R&D effort and R&D outcomes, exact conditions for which are formally investigated in Section 1.3.

1.2.1 Aggregate Production

Suppose that final output is produced competitively using the CES technology

$$Y(t) = \left(\int_0^N y(i;t)^{\frac{\xi-1}{\xi}} di \right)^{\frac{\xi}{\xi-1}} \tag{1.1}$$

where N is the number of homogeneous industries, $y(i;t) = y(t)$ is the i-th industry's output at t, and $\xi > 0$ is the elasticity of substitution between inputs. We assume the final good to be the numeraire, so that its price equals 1 at every t.

Each industry is populated by a constant mass $m(i) = m$ of homogeneous

Cournot-competing firms,[16] so that an industry's output equals

$$y(i;t) = \int_0^{m(i)} \tilde{y}(i;j;t)\,dj = m\tilde{y}(t) \tag{1.2}$$

Throughout the chapter, we use tildes to denote firm-specific variables.

In what follows, we use m as a measure of competition (similarly to Onori (2015)): although the number of firms in an industry is not among standard measures of competition, the model's m maps monotonically into those – in particular, the Herfindahl[17] and Lerner[18] indices.[19] Since we are interested in investigating the relationship between competition and different aspects of R&D, we consider the latter in strictly oligopolistic environment, so that $m \geqslant \underline{m} \equiv 2$ throughout the chapter, similarly to existing theoretical literature (Aghion et al. (2005), d'Aspremont et al. (2010), Askenazy et al. (2013), Onori (2015)).

[16]The main reason we use the Cournot-competition mechanism in our model is because it establishes a negative relationship between the number of firms per industry m and their mark-ups (see (1.15) and the following discussion), which in turn gives rise to the escape-costs effect introduced below. Alternatively, one could use the direct approach suggested in Galí (1994) and Galí (1995), and later used in Comin and Gertler (2006), wherein the elasticity of substitution $\xi(m)$ is posited to be an increasing function of the number of firms (in particular, Comin and Gertler (2006) use the following functional form: $\xi(m) = \frac{1+Dm^{\chi}}{Dm^{\chi}}$, so that the mark-up is a constant elasticity function of m: $\mu(m) = Dm^{\chi}$).

[17]The Herfindahl index is calculated as $I_H = \sum_{i=1}^{m} \delta_i^2$, where δ_i is the market share of the i-th firm. Since in our framework all firms are homogeneous, the Herfindahl index equals $\frac{1}{m}$.

[18]A firm's Lerner index equals $I_L = \frac{p-\psi}{p}$ (the notations are taken from the model). As follows from equation (1.15), the model's Lerner index for each firm equals $\frac{1}{\xi m}$.

[19]For a detailed discussion, see (Martin, 2002, pp. 335–338) and references cited therein.

1.2.2 Individual Firms

Each firm seeks to maximise its profits by choosing the volume of output and the amount of R&D effort $\tilde{\gamma}(i;j;t)$

$$\tilde{\pi}(i;j;t) = \Big(p(i;t) - \tilde{\psi}(i;j;t)\Big)\,\tilde{y}(i;j;t) - \tilde{\gamma}(i;j;t) \tag{1.3}$$

where $\tilde{\psi}(i;j;t)$ is the average/marginal cost of producing the i-th intermediate good by the j-th firm.[20]

A firm produces its output using a Cobb-Douglas technology of unitary homogeneity by employing capital ($\tilde{k}(i;j;t)$) and labour ($\tilde{l}(i;j;t)$), both provided by the economy's households in competitive markets

$$\begin{aligned} \tilde{y}(i;j;t) &= \tilde{q}(i;j;t)\,Q(t)\,F\Big(\tilde{k}(i;j;t)\,;\tilde{l}(i;j;t)\Big) = \\ &= \tilde{q}(i;j;t)\,Q(t)\,\tilde{k}(i;j;t)^{\,\nu}\,\tilde{l}(i;j;t)^{\,1-\nu} \end{aligned} \tag{1.4}$$

where $Q(t)$ is the economy-wide total factor productivity (TFP) level, and $\tilde{q}(i;j;t)$ is its increase for the j-th firm, generated by its individual R&D effort by means of the following technology

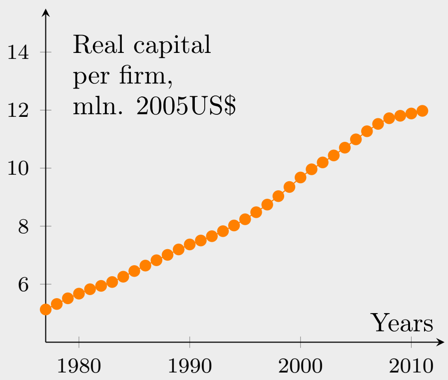
$$\begin{aligned} \tilde{q}(i;j_0;t) = \eta &\left(\frac{\tilde{\gamma}(i;j_0;t)}{Q(t)\,F\Big(\tilde{k}(i;j;t)\,;\tilde{l}(i;j;t)\Big)}\right)^{\alpha} \times \\ &\times \left(\int_0^{m(i)} \frac{\tilde{\gamma}(i;j;t)}{Q(t)\,F\Big(\tilde{k}(i;j;t)\,;\tilde{l}(i;j;t)\Big)} dj\right)^{-\beta}, \\ &\alpha > \beta > 0 \end{aligned} \tag{1.5}$$

where α and $-\beta$ are, respectively, the perceived elasticity of $\tilde{q}$ with respect to a firm's own R&D effort, and the elasticity of $\tilde{q}$ with respect to aggregate R&D effort in the industry. In what follows, we interpret $\tilde{q}(i;j;t)$ as the R&D outcome being brought about by R&D effort $\tilde{\gamma}(i;j;t)$. We assume that firms do not take into account their impact on aggregate R&D effort within an industry $\int_0^{m(i)} \tilde{\gamma}^*(i;t)\,di$.

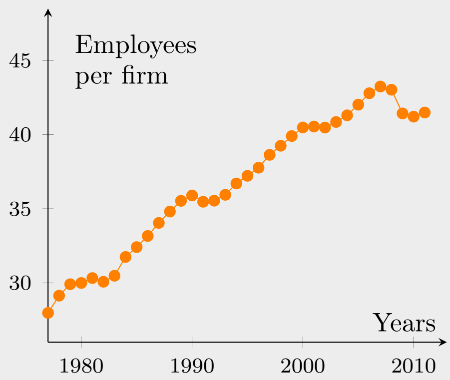
[20] We slightly abuse notation by equating average costs to marginal costs, but this claim is valid in this instance, since firms in our model use a linearly homogeneous technology.

Box 1.1: Factor Scale Independence

Despite the fact that the average quantities of production factors employed by firms, have been growing steadily in the last thirty years (see Figures 1.1a and 1.1b), that dynamics has not translated into a persistent growth of TFP growth rates (see fig. 1.2), which have remained stationary throughout the observation period (1977 – 2013) (see Figure 1.2).



(a) Average real capital per firm.



(b) Average workers per firm.

Figure 1.1: Average quantities of production factors employed, US data.
Data source: Feenstra et al. (2015).

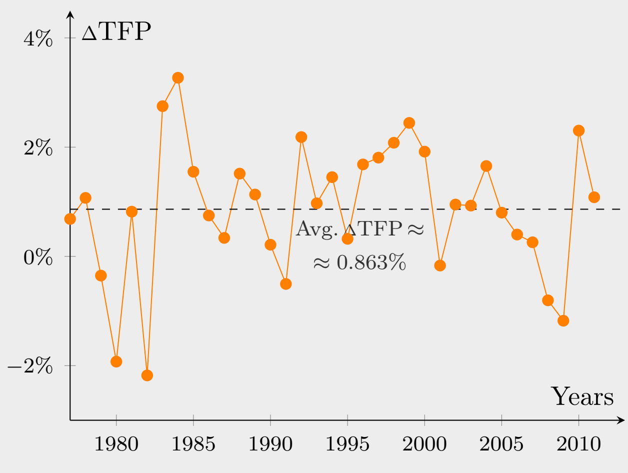


Figure 1.2: Percentage changes in TFP levels, US data.
Data sources: Feenstra et al. (2015), Jarmin and Miranda (2002)

We would also like to mention a few additional supporting considerations in favour of our assumption of TFP growth's factor scale independence. First of all, from the IO perspective, we can refer to Gibrat's law stating that a firm's growth rate is independent of its size.[21] As shown below, a firm's growth rate is that of the economy, which (in the long-run) is linear in the natural logarithm of $\tilde{q}$ – therefore, its behaviour is compatible with Gibrat's law if the latter is independent of a firm's size.[22]

From the growth perspective, the scale-independence assumption separates the economy's growth rate from the amounts of production factors it possesses, thus removing scale effects, whose presence in models of economic growth (especially those of endogenous growth) has been criticised.[23]

In order to bound $\tilde{q}$ from below we assume that zero R&D effort does not change the current level of productivity, so that the latter's incremental multiplier $\tilde{q}(i;j;t)$ equals one: $\tilde{\gamma}(i;j;t) = 0 \Rightarrow \tilde{q}(i;j;t) = 1$, i.e. a firm's productivity does not change. In what follows, we assume that η is suitably high, so that the situation when innovation is absent, never occurs.

Dividing every instance of $\tilde{\gamma}$ by $F\left(\tilde{k}(i;j;t)\,;\tilde{l}(i;j;t)\right)$ in (1.5) results in $\tilde{q}$'s being independent of the amount of production factors employed by a firm, which we motivate by empirical regularities observed in US data (see Box 1.1).

We follow existing literature (see, e.g., (Acemoglu, 2009, Section 14.3), Acemoglu and Cao (2015)) in assuming that ideas are fished out: in terms of our model, β is strictly positive, so that it reflects the inhibiting effect of aggregate

[21] See a detailed discussion in Sutton (1997). Although the consensus appears not to support the law for small/young firms, it seems to apply to large/old companies (see, e.g., Sutton (1997), Klette and Griliches (2000), Lotti, Santarelli, and Vivarelli (2009)). Given that our model does not allow for entry/exit of new producers, every existing firm eventually becomes 'old', which ultimately makes it amenable to Gibrat's law.

[22] Although $F\left(\tilde{k}(i;j;t)\,;\tilde{l}(i;j;t)\right)$ is not equal to a firm's amount of production $\tilde{y}(i;j;t)$, the assumption of factor scale independence results in the optimal value of $\tilde{q}(i;j;t)$ being independent of $\tilde{y}(i;j;t)$ as well (see (1.22)).

[23] This line of criticism is also known as Jones' critique after Jones (1995) – among other works on the problem, see Young (1998), Howitt (1999), Jones (1999).

research effort on individual R&D productivity within a firm. Thus, β can be interpreted as the congestion parameter, which captures the depletion of the stock of available ideas as those are being searched for simultaneously by a multitude of firms (as put in (Acemoglu, 2009, p. 472), 'fishing from the same pond').[24] Note that, in spite of the congestion component being present in (1.5), $\tilde{q}(t)$ is still an increasing function of individual research effort $\tilde{\gamma}(i;j;t)$ for every fixed level of competition m (see (1.9)), which is achieved by setting $\alpha > \beta$.

Similarly to Aghion and Howitt (1992), Howitt (1999), d'Aspremont et al. (2010), the economy-wide TFP level $Q(t)$ grows as a by-product of the individual research activity. In particular, we model the growth rate of $Q(t)$ as the logarithm of the average of individual $\tilde{q}(i;j;t)$-s across the economy

$$g_Q(t) \equiv \frac{\dot{Q}(t)}{Q(t)} = \ln\left(\frac{1}{N}\int_0^N \int_0^{m(i)} \frac{\tilde{q}(i;j;t)}{m(i)} dj\, di\right) \tag{1.6}$$

We opt for the logarithmic function in (1.6) primarily because the equation's corresponding discrete-time (i.e., observable) version takes the natural form $Q_{t+1} = \left(\frac{1}{N}\int_0^N \int_0^{m(i)} \frac{\tilde{q}(i;j;t)}{m(i)} dj\, di\right) Q(t)$,[25] which makes our results comparable to pieces of empirical evidence used later in the chapter (see Section 1.3.3). Equation (1.6) completes the introduction of the model's production side.

1.2.3 Industry Equilibrium

We shall start solving the model by transforming formulae (1.2), (1.4)–(1.6) given the symmetry of all firms and industries ($y(i;t) = y(t)\ \ \forall i$)

$$Y(t) = N^{\frac{\xi}{\xi-1}} y(t) \tag{1.7}$$

$$\tilde{y}(i;j;t) = \tilde{y}(t) = \tilde{q}(t)\, Q(t)\, F\left(\frac{K(t)}{Nm}; \frac{L(t)}{Nm}\right) = \tilde{q}(t)\, Q(t)\, \frac{K(t)^{\,\nu} L(t)^{\,1-\nu}}{Nm} \tag{1.8}$$

[24]A similar assumption is made in, e.g., Impullitti (2016).

[25]See footnote 38.

$$\tilde{q}(i;j;t) = \tilde{q}(t) = \eta m^{-\beta} \left(\frac{\tilde{\gamma}(t)}{Q(t)\, F\left(\frac{K(t)}{Nm}; \frac{L(t)}{Nm}\right)} \right)^{\alpha-\beta} \tag{1.9}$$

$$g_Q(t) = \ln \tilde{q}(t) \tag{1.10}$$

Note that function (1.9) represents a particular form of $R'(\gamma; m; \theta)$ from the stylised model discussed in the Introduction: $\tilde{q}(t) = \eta \tilde{\gamma}(t)^{\alpha-\beta} m^{\alpha-2\beta} \left(\frac{N}{Q(t)F(K(t);L(t))}\right)^{\alpha-\beta}$.

Before specifying the behaviour of intermediate firms, we will derive the demand function for an intermediate input from profit maximisation in the the final-good sector

$$p(i;t) = p(t) = \frac{\partial Y(t)}{\partial y(i;t)} = Y(t)^{\frac{1}{\xi}} y(t)^{-\frac{1}{\xi}} \tag{1.11}$$

Since $Y(t)$ is a linear homogeneous function of $y(i;t)$, the Euler theorem can be brought to bear to obtain the expression: $Y(t) = \int_0^N \frac{\partial Y(t)}{\partial y(i;t)} y(i;t)\, di = Np(t)\, y(t)$. Combining the last equation with the assumption of firms' homogeneity pins down the price of an intermediate good (in terms of the final good's price)

$$\begin{gathered} Y(t) = N^{\frac{\xi}{\xi-1}} y(t) = Np(t)\, y(t) \Leftrightarrow \\ \Leftrightarrow p(t) = p = N^{\frac{1}{\xi-1}} \end{gathered} \tag{1.12}$$

Given the constancy of each intermediate input's price, we can solve a firm's problem. We shall start with rewriting the cost function. Since the production function is Cobb-Douglas, a firm's cost function takes the form[26]

$$\tilde{\psi}(i;j;t) = \frac{1}{\tilde{q}(i;j;t)\, Q(t)} \left(\frac{R(t)}{\nu}\right)^{\nu} \left(\frac{w(t)}{1-\nu}\right)^{1-\nu} \equiv \frac{\psi(t)}{\tilde{q}(i;j;t)} \tag{1.13}$$

where $w(t)$ and $R(t)$ are the factor prices of labour and capital, respectively. Given (1.13), the profit function can be rewritten as follows

$$\tilde{\pi}(i;j;t) = p(i;t)\, \tilde{y}(i;j;t) - \frac{\psi(t)}{\tilde{q}(i;j;t)} \tilde{y}(i;j;t) - \tilde{\gamma}(i;j;t) \tag{1.14}$$

[26]See (Mas-Colell, Whinston, and Greene, 1995, p. 142).

Since all firms are homogeneous, maximising (1.14) with respect to $\tilde{y}$ yields the standard result for the price charged in each industry

$$\frac{\partial\tilde{\pi}(t)}{\partial\tilde{y}(t)} \equiv Y(t)^{\frac{1}{\xi}} y(t)^{-\frac{1}{\xi}} - \frac{1}{\xi} Y(t)^{\frac{1}{\xi}} y(t)^{-\frac{1}{\xi}} \frac{\tilde{y}(t)}{y(t)} - \frac{\psi(t)}{\tilde{q}(t)} = 0$$

$$p^*(t) - \frac{1}{\xi m} p^*(t) = \frac{\psi(t)}{\tilde{q}(t)}$$

$$p^*(t) = \frac{\xi m}{\xi m - 1} \cdot \frac{\psi(t)}{\tilde{q}(t)} \Leftrightarrow \psi(t) = \frac{\xi m - 1}{\xi m} N^{\frac{1}{\xi - 1}} \tilde{q}(t) \tag{1.15}$$

where the second expression in (1.15) follows from (1.12). Given the first expression in (1.15), $p^*(t)$ can be rewritten as $p^*(t) = (1+\mu)\frac{\psi(t)}{\tilde{q}(t)}$, where $\mu = \frac{1}{\xi m - 1}$ is the price mark-up (price-cost margin), whose inverse relationship with the mass of firms in an industry m is an implication of Cournot-competition between firms. Given the functional form of μ and our premise that $m \geqslant 2$, we restrict ξ to be strictly greater than $\frac{1}{2}$.

Since the factor markets are perfectly competitive, the problem of maximising $\tilde{\pi}$ with respect to $\tilde{y}$ is equivalent to maximising it with respect to amounts of capital and labour employed (without taking into consideration the impact of $\tilde{k}(t)$ and $\tilde{l}(t)$ on $\tilde{q}(t)$), which allows one to express the economy's equilibrium factor prices

$$\max_{\tilde{y}} \left\{ p(i;t)\,\tilde{y}(i;j;t) - \frac{\psi(t)}{\tilde{q}(i;j;t)}\tilde{y}(i;j;t) - \tilde{\gamma}(i;j;t) \right\} \Leftrightarrow \tag{1.16}$$

$$\begin{aligned} \Leftrightarrow \max_{\tilde{k};\tilde{l}} \Big\{ & p(i;t)\,\tilde{q}(i;j;t)\,Q(t)\,\tilde{k}(i;j;t)^{\nu}\tilde{l}(i;j;t)^{1-\nu} - \\ & - R(t)\,\tilde{k}(i;j;t) - w(t)\,\tilde{l}(i;j;t) - \tilde{\gamma}(i;j;t) \Big\} \end{aligned} \tag{1.17}$$

$$\frac{\partial\tilde{\pi}}{\partial\tilde{k}} = 0 \Leftrightarrow R(t) = \frac{\xi m - 1}{\xi m}\, p(t)\, \tilde{q}(t)\, Q(t)\, F'_{\tilde{k}}\Big(\tilde{k}(t)\,; \tilde{l}(t)\Big) = \\ = \nu \frac{\xi m - 1}{\xi m} N^{\frac{1}{\xi - 1}} \tilde{q}(t)\, Q(t) \left(\frac{L(t)}{K(t)}\right)^{1-\nu} = \frac{\xi m - 1}{\xi m} \mathit{MPK}(t) \tag{1.18}$$

$$\frac{\partial\tilde{\pi}}{\partial\tilde{l}} = 0 \Leftrightarrow w(t) = \frac{\xi m - 1}{\xi m}\, p(t)\, \tilde{q}(t)\, Q(t)\, F'_{\tilde{l}}\Big(\tilde{k}(t)\,; \tilde{l}(t)\Big) = \\ = (1-\nu) \frac{\xi m - 1}{\xi m} N^{\frac{1}{\xi - 1}} \tilde{q}(t)\, Q(t) \left(\frac{K(t)}{L(t)}\right)^{\nu} = \frac{\xi m - 1}{\xi m} \mathit{MPL}(t) \tag{1.19}$$

The term $\frac{\xi m-1}{\xi m} = \frac{1}{1+\mu}$ reflects the distortive impact of monopoly power on factor prices in the economy: the greater it is (or, put equivalently, the higher oligopolists' mark-ups are) the more pronounced the deviation of $R(t)$ and $w(t)$ becomes from the marginal products of capital and labour ($\mathit{MPK}(t)$ and $\mathit{MPL}(t)$, respectively), determining factor prices in a competitive economy.

Turning to characterising firms' decisions on R&D investment, differentiating (1.14) with respect to $\tilde{\gamma}$ leads to the expression

$$\frac{\tilde{q}'_{\tilde{\gamma}}(\tilde{\gamma}(i; j_0; t))}{\tilde{q}(\tilde{\gamma}(i; j_0; t))^{\,2}}\, \tilde{y}(i; j_0; t) = \frac{1}{\psi(t)} \Leftrightarrow \frac{\tilde{q}'_{\tilde{\gamma}}(\tilde{\gamma}(i; j_0; t))}{\tilde{q}(\tilde{\gamma}(i; j_0; t))}\, \tilde{y}(i; j_0; t) = \frac{\tilde{q}(\tilde{\gamma}(i; j_0; t))}{\psi(t)}$$

$$\alpha \frac{\tilde{y}(i; j_0; t)}{\tilde{\gamma}(i; j_0; t)} = \frac{\tilde{q}(\tilde{\gamma}(i; j_0; t))}{\psi(t)} \Leftrightarrow \tilde{\gamma}(i; j_0; t) = \alpha \tilde{y}(i; j_0; t)\, \frac{\psi(t)}{\tilde{q}(\tilde{\gamma}(i; j_0; t))} \tag{1.20}$$

$$\tilde{\gamma}^*(t) = \alpha N^{\frac{1}{\xi - 1}}\, \tilde{y}(t)\, \frac{\xi m - 1}{\xi m} \tag{1.21}$$

where expression (1.21) follows from (1.12). A noteworthy feature of equation (1.21) is that the elasticity of $\tilde{\gamma}^*(t)$ with respect to $\tilde{y}(t)$ is unity, which matches a widely recognised stylised fact on R&D.[27] This result can be interpreted along the lines of reasoning advanced by Cohen and Klepper (1996): as a firm's output increases, so does, for two reasons, the total effect of R&D. Firstly,

[27] See, e.g. Klette and Griliches (2000).

with a larger scale of production, the costs of R&D can be spread across a larger level of output; secondly, the drop in production costs resulting from R&D, applies to a larger number of items produced, thus increasing a firm's gains, thereby encouraging further R&D effort.

Plugging (1.21) into (1.9) yields the final expression for $\tilde{q}^*$

$$\tilde{q}(t) = \eta m^{-\beta}\left(\frac{\tilde{\gamma}(t)}{F\left(\frac{K(t)}{Nm};\frac{L(t)}{Nm}\right)}\right)^{\alpha-\beta} = \eta m^{-\beta}\left(\frac{\alpha N^{\frac{1}{\xi-1}}\frac{\xi m-1}{\xi m}\tilde{q}(t)\,F\left(\frac{K(t)}{Nm};\frac{L(t)}{Nm}\right)}{F\left(\frac{K(t)}{Nm};\frac{L(t)}{Nm}\right)}\right)^{\alpha-\beta}$$

$$\tilde{q}^*(t) = \tilde{q}^* = E\left(\frac{\xi m-1}{\xi m^{\frac{\alpha}{\alpha-\beta}}}\right)^{\frac{\alpha-\beta}{1-(\alpha-\beta)}},\ E \equiv \left(\alpha^{\alpha-\beta}\eta\, N^{\frac{\alpha-\beta}{\xi-1}}\right)^{\frac{1}{1-(\alpha-\beta)}} \tag{1.22}$$

Together, equations (1.21) and (1.22) pin down the equilibrium level of profits accruing to a firm

$$\tilde{\pi}^*(t) = \left((1+\mu)\,\frac{\psi(t)}{\tilde{q}^*} - \frac{\psi(t)}{\tilde{q}^*}\right)\tilde{y}(t) - \alpha\frac{\psi(t)}{\tilde{q}^*}\tilde{y}(t)$$

$$\tilde{\pi}^*(t) = (\mu-\alpha)\,\frac{\psi(t)}{\tilde{q}^*}\tilde{y}(t) = (\mu-\alpha)\,\psi(t)\,Q(t)\,F\left(\frac{K(t)}{Nm};\frac{L(t)}{Nm}\right)$$

$$\tilde{\pi}^*(t) = \frac{\mu-\alpha}{Nm}\left(\frac{R(t)\,K(t)}{\nu}\right)^{\nu}\left(\frac{w(t)\,L(t)}{1-\nu}\right)^{1-\nu}$$

$$\tilde{\pi}^*(t) = \frac{\mu-\alpha}{Nm}\,\frac{\xi m-1}{\xi m}\,N^{\frac{1}{\xi-1}}\tilde{q}^* Q(t)\,K(t)^{\nu}L(t)^{1-\nu} = \frac{\mu-\alpha}{Nm}\,\frac{\xi m-1}{\xi m}\,Y(t) \tag{1.23}$$

where the last equation follows from substituting expressions (1.18) and (1.19) for, respectively, $R(t)$ and $w(t)$.

Note that the optimal solution for $\tilde{\pi}(t)$ can potentially be negative if term $\mu-\alpha$ is so. This property comes from the fact that R&D effort enters the profit function in the fixed-cost form. In what follows, we require that the term be non-negative, which sets the upper bound on the number of firms per industry

$$\alpha \leqslant \mu = \frac{1}{\xi m - 1} \Leftrightarrow m \leqslant \bar{m} = \frac{1+\alpha}{\alpha \xi} \tag{1.24}$$

As the final step in deriving a firm's optimal solution, combining (1.21) and (1.22) allows one to express $\tilde{\gamma}^*(t)$ as a function of the economy's production factors. By plugging (1.22) into (1.21) and using $\tilde{y}(t) = \frac{\tilde{q}(t)\,Q(t)}{Nm} F(K(t)\,;L(t))$, we have

$$\tilde{\gamma}^*(t) = \frac{\alpha E}{N^{\frac{\xi-2}{\xi-1}}} \left(\frac{\xi m - 1}{\xi m^{2-\alpha+2\beta}} \right)^{\frac{1}{1-(\alpha-\beta)}} Q(t)\, F(K(t)\,;L(t)) \tag{1.25}$$

One can show that both $\tilde{q}^*$ and $\tilde{\gamma}^*(t)$ (once the dynamics of $Q(t)$, $K(t)$ and $L(t)$ is controlled for)[28] may be related to the degree of competition in the hump-shaped fashion – the following propositions specify the conditions under which this obtains.

Proposition 1.1. *Let* $\hat{m}_q = \frac{\alpha}{\beta\xi}$ *and* $\alpha < 1+\beta$*. Then* $\tilde{q}^*$ *is increasing* $\forall m < \hat{m}_q$ *and is decreasing otherwise, so that* $\hat{m}_q$ *is* $\tilde{q}^*(t)$*'s global maximum. Therefore, when* $\alpha > 2\beta\xi \Leftrightarrow \hat{m}_q > 2$ *and* $\hat{m}_q < \bar{m}$*, the hump-shaped pattern in the relationship between competition and innovation outcomes becomes observable.*

Proof. Follows from calculating $\frac{d\tilde{q}}{dm}$ and applying the method of intervals. ∎

The hump-shapedness of $\tilde{q}^*$ is achieved through the superposition of two forces: on one hand, if we assume for a moment that $\beta = 0$, $\tilde{q}^*$ becomes an increasing function of the mass of firms in an industry: as follows from (1.12), each industry's price is fixed (relative to the price of the consumption good), which, together with a firm's pricing rule (1.15), has the general-equilibrium implication that for any fixed value of $\tilde{q}^*$, firms' effective marginal costs $\frac{\psi(t)}{\tilde{q}^*}$ increase in the

[28]Controlling for industry-specific and time-specific effects is a standard feature of empirical analyses in the field – see Aghion et al. (2005), Tingvall and Poldahl (2006), Askenazy et al. (2013).

degree of competition, which prompts them to invest more in R&D as an attempt to drive effective costs down through increasing their productivity, and greater R&D effort translates into better R&D outcomes (the escape-costs effect).[29] On the other hand, the effect's impact is counteracted by the 'fishing-out' effect discussed above. As suggested by Proposition 1.1, the escape-costs effect prevails for lower values of m (below $\hat{m}_q$), whereas the opposite is true for $m \geqslant \hat{m}_q$.

Note that, as suggested by the formula for $\hat{m}_q$, the relative strength of the escape-costs effect decreases in the elasticity of substitution ξ: if it is low, then the mark-up wedge μ between costs and prices is further from zero, and hence an increase in m has a greater impact on μ, prompting firms to invest more in R&D. In addition, rather naturally, the markedness of the hump-shaped pattern increases in α, since higher productivity of innovation induces more R&D effort.

Proposition 1.2. *Let $\hat{m}_\gamma = \frac{2-\alpha+2\beta}{1-\alpha+2\beta}\frac{1}{\xi}$. If $\alpha < 1+2\beta$, then $\tilde{\gamma}^*(t)$ is increasing $\forall m < \hat{m}_\gamma$ and is decreasing otherwise, so that $\hat{m}_\gamma$ is $\tilde{\gamma}^*(t)$'s global maximum. Therefore, when $2 < \hat{m}_\gamma < \bar{m}$, the hump-shaped pattern in the relationship between competition and innovation effort becomes observable. Otherwise when $\alpha > 1+2\beta$, $\tilde{\gamma}^*(t)$ is an increasing function of m.*

Proof. Follows from calculating $\frac{d\tilde{\gamma}^*(t)}{dm}$ and applying the method of intervals. ■

In the case of the hump-shaped pattern in R&D effort, the escape-costs effect is present in $\tilde{\gamma}^*(t)$ both directly, as reflected by term $\frac{\psi(t)}{\tilde{q}}$ in (1.20) (and equivalently, term $\frac{\xi m-1}{\xi m}$ in (1.21)) and indirectly, as encapsulated in productivity term $\tilde{q}^*$ entering $\tilde{y}(t) = \tilde{q}^* F\left(\frac{K(t)}{m}; \frac{L(t)}{m}\right)$. In addition, the presence of $\tilde{q}^*$ also serves as a channel for the indirect depletion effect, which is further reinforced by the division effect: a larger number of producers m entails that each one of them can attract a smaller share of the economy's capital and labour, which in turn

[29] Naturally, greater costs create stronger incentives to innovate in our model: if, for the sake of the argument, a firm's costs amount to £100, then doubling its productivity level reduces effective costs by £50. By contrast, if the costs' level is £2, then the impact a two-fold increase in $\tilde{q}$ saves a firm only £1 per unit of output.

reduces a firm's scale of production and, by that means, shrinks its opportunities to spread R&D costs across their output.

Comparing the mechanics of the hump-shaped patterns in $\tilde{\gamma}^*(t)$ and $\tilde{q}^*$ suggests that since in the case of the former the negative (division) effect replaces the depletion effect, as well as both are enhanced (through the indirect escape-costs effect and a combination of the indirect depletion effect, respectively), depending on which of them is reinforced more strongly, we may expect either of the situations $\hat{m}_q \geqslant \hat{m}_\gamma$ and $\hat{m}_\gamma \geqslant \hat{m}_q$ to occur. This gives rise to the possibility of the discrepancy in the behaviour of the two functions when the turning point of one of them is outside the range $[\underline{m}; \bar{m}]$. The situations of particular interest for us are those when the hump-shapedness in R&D outcomes is observable, while that in R&D effort is not. This can be the case if either $\hat{m}_\gamma \leqslant 2 < \hat{m}_q$ (R&D effort detectably decreases in m, R&D accomplishment is hump-shaped) or $\hat{m}_q < \bar{m} \leqslant \hat{m}_\gamma$ (R&D effort detectably increases in m, R&D accomplishment is hump-shaped). We set a detailed discussion of these conditions aside until Section 1.3, after we specify the steady-state dynamics of the model's economy, for which we turn to enquiring into the behaviour of the aggregate household.

1.2.4 Representative Household and Long-Run Equilibrium

Moving on to the consumption side of the economy, we model it as the aggregate household comprising $L(t) = L_0 e^{nt}$ individuals, with a CRRA-type instantaneous utility function, so that its lifetime utility equals

$$U = \int\limits_0^{+\infty} e^{-\rho t} \, \frac{c(t)^{\,1-\theta} - 1}{1-\theta} L(t) \, dt \tag{1.26}$$

where $c(t)$ is consumption per capita, $C(t) = c(t)\, L(t)$ is total consumption, and $\rho > n$ is the consumers' time-discount factor.

The household's members are assumed to own together all firms and production factors (capital and labour) in the economy, so that their income is composed of firms' profits $\tilde{\pi}(t)\, Nm$ and total factor payments ($w(t)\, L(t)$ – for labour and $(R(t)-\delta)\,K(t)$ – for capital, where δ is the rate of capital depreciation).

The household splits its assets between consumption and investment, taking firms' profits and factor prices as given, which gives rise to the standard intertemporal budget constraint

$$\dot{K}(t) = (R(t)-\delta)\,K(t) + w(t)\,L(t) + \tilde{\pi}(t)\,Nm - C(t) \tag{1.27}$$

Equation (1.27) can be transformed using (1.18), (1.19) and (1.23)

$$\dot{K}(t) = \frac{\xi m\,(1-\alpha)+\alpha}{\xi m}\,N^{\frac{1}{\xi-1}}\tilde{q}^{*}Q(t)\,K(t)^{\,\nu}L(t)^{\,1-\nu} - C(t) \tag{1.28}$$

Maximising (1.26) with respect to (1.28) constitutes a canonical Ramsey-Cass-Koopmans dynamic optimisation problem with a Cobb-Douglas production function and CRRA preferences. Therefore, one can argue that the model exhibits saddle-path convergence to the unique steady state, in which the economy's wage rate and per capita variables (namely output, capital and consumption) grow at the rate of $\frac{\ln\tilde{q}^{*}}{1-\nu}$.[30] In addition, at the steady state the effective capital to labour ratio $\frac{K(t)}{Q(t)^{\frac{1}{1-\nu}}L(t)}$ equals a fixed number k^{SS} determined by the Euler equation[31]

$$\begin{aligned} R^{SS} \equiv \nu\tilde{q}^{*}\frac{\xi m-1}{\xi m}\,N^{\frac{1}{\xi-1}}\left(k^{SS}\right)^{\nu-1} = \rho+\delta+\frac{\theta\ln\tilde{q}^{*}}{1-\nu} \Rightarrow \\ \Rightarrow k^{SS} = \left(\frac{\nu N^{\frac{1}{\xi-1}}\,\frac{\xi m-1}{\xi m}\,\tilde{q}^{*}}{\rho+\delta+\frac{\theta\ln\tilde{q}^{*}}{1-\nu}}\right)^{\frac{1}{1-\nu}} \end{aligned} \tag{1.29}$$

Pinning down the long-run growth rates of the economy's variables and its effective capital to labour ratio (1.29) completes specifying the solution of the

[30]Naturally, the last conclusion suggests that the economy's total output, capital and consumption grow in the long-run at the rate of $\frac{\tilde{q}^{*}}{1-\nu}+n$. Therefore, the growth rates of both aggregate and per-capita quantities in the economy inherit the hump-shapedness properties of $\tilde{q}^{*}$.

[31]For a detailed derivation and discussion of the Ramsey-Cass-Koopmans model's properties see, e.g., (Aghion and Howitt, 1998, Section 1.2), (Barro and Sala-i-Martin, 2004, Chapter 2), (Acemoglu, 2009, Chapter 8).

model. In the next section, we turn to investigating the situations of discrepancy in the shape of relations between $\tilde{q}^*$ and $\tilde{\gamma}^*(t)$ on one hand, and m on the other.

1.3 Quantitative Assessment of the Model

Despite the fact that both $\tilde{\gamma}^*(t)$ and $\tilde{q}^*$ are hump-shaped with respect to the degree of competition m, a situation of discrepancy in the behaviour of these functions can occur if the maximum point of one of them is outside the interval $[\underline{m};\bar{m}]$.

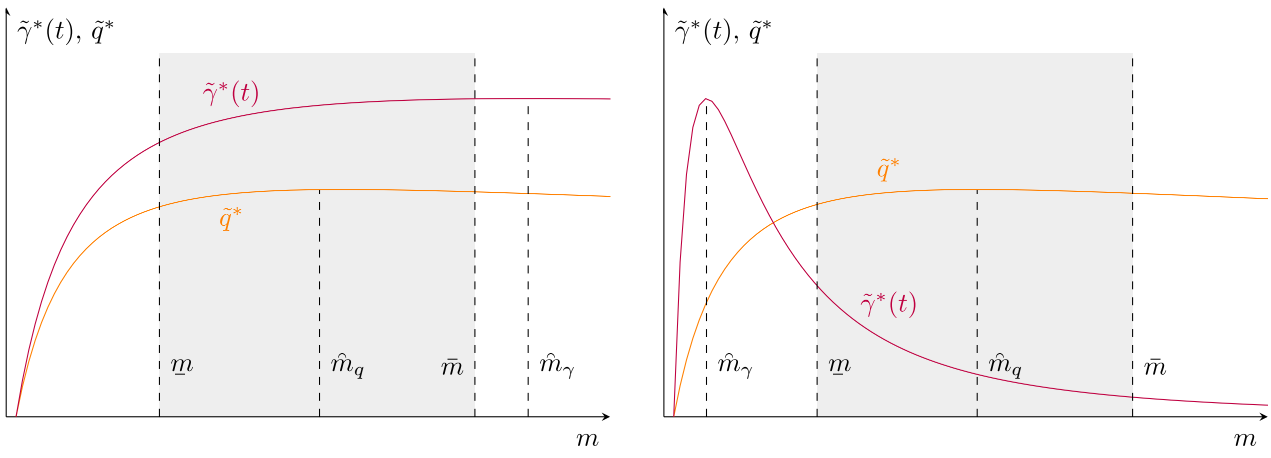
Given the motivation of this chapter, we are interested in looking into the conditions under which $\tilde{q}^*$ is hump-shaped, whereas $\tilde{\gamma}^*(t)$ is not (which, given that $\tilde{\gamma}^*(t)$ is a hump-shaped function, suggests that it has to be observably monotone in the range $[\underline{m};\bar{m}]$). This can be the case if $\hat{m}_q$ (the turning point of $\tilde{q}^*$) lies within the interval of m's permissible values $[\underline{m};\bar{m}]$, while $\hat{m}_\gamma$ (the turning point of $\tilde{\gamma}^*(t)$) is outside it. In light of these considerations, we formally define the discrepancy in the behaviour of $\tilde{\gamma}^*(t)$ and $\tilde{q}^*$ as follows.

Definition. Functions $\tilde{q}^*$ and $\tilde{\gamma}^*(t)$ exhibit a discrepancy in their behaviour if the turning point $\hat{m}_q$ of the former belongs to the interval $[\underline{m};\bar{m}]$, while the turning point $\hat{m}_\gamma$ of the latter resides outside it. Equivalently, a situation of discrepancy occurs when either of the two following conditions holds.

$$\underline{m} < \hat{m}_q < \bar{m} \leqslant \hat{m}_\gamma \tag{1.30}$$

$$\hat{m}_\gamma \leqslant \underline{m} < \hat{m}_q < \bar{m} \tag{1.31}$$

Equations (1.30) and (1.31) describe the cases when $\tilde{\gamma}^*(t)$ increases (respectively, decreases) for any $m \in [\underline{m};\bar{m}]$, while $\tilde{q}^*$ retains its hump-shapedness (see Figures 1.3a and 1.3b for the illustration). For the sake of brevity, we refer to these situations as HS-I (hump-shaped, increasing) and HS-D (hump-shaped, decreasing) discrepancies, respectively.



(a) Discrepancy case №1, eq. (1.30).

(b) Discrepancy case №2, eq. (1.31).

Figure 1.3: The cases of discrepancy in the behaviour of $\tilde{\gamma}^*(t)$ and $\tilde{q}^*$.

Given that $\hat{m}_q$, $\bar{m}$, $\hat{m}_\gamma$ depend on three parameters (namely the productivity of individual R&D effort α, congestive impact of aggregate R&D effort β, elasticity of substitution between sectors' products ξ), we are going to simplify enquiring into conditions (1.30), (1.31) by restricting the range of ξ's values to those implied by existing literature, and thus concentrating on mapping out the relationships between α and β, which underlie (1.30) and (1.31). Unfortunately, our task is hampered by the incompatibility of our model (where each industry is populated with a multitude of firms) with existing estimates in macro literature (see, e.g., Smets and Wouters (2003), Christiano, Eichenbaum, and Evans (2005), Broda and Weinstein (2006), Primiceri, Schaumburg, and Tambalotti (2006), Broda and Weinstein (2010)), where the estimates of ξ are obtained under the assumption of each industry's being monopolised. This consideration motivates the next section, where we derive, in a stylised fashion, the values of ξ compatible with our model's setting.

1.3.1 Preliminary Considerations – the Value of the Elasticity of Substitution

Our approach to assessing the value of ξ is akin to the line of argumentation used in Atkeson and Burstein (2008): we are going to equate the model's mark-up μ to mark-up values inferred from existing data and backup ξ from them, for which, given μ's functional form, we need to gauge the number of firms in an industry m first.

We would like to precede the assessment of m with a discussion of what real-life concepts may match our model's notion of industry. Clearly, it cannot be equated to the industry in the sense of a unit in an industrial classification table: our assumption of Cournot-competition on the intra-industry level implies that each firm takes into account how its decisions affect the whole industry, which clearly suggests that, while determining the value of m, we need to take into account the 'compactness' of the corresponding group of firms. In the spirit of the models due to Hotelling (1929) and Salop (1979), we focus on interpreting this 'compactness' in spatial terms, which is why we assess the value of m as the average number of firms[32] per 'industrial-table' industry *per* local economy unit. The last consideration has led us to using US business data, because (primarily for the purposes of labour market research) the US territory has been split into 709 the so-called commuting zones,[33] which are interpreted as local economy units (see, e.g., (Killian and Hady, 1988, pp. 3–5), Tolbert and Sizer (1996), (Walden, 2008, Ch. 5)).

In addition, in order to reflect the homogeneity of each industry's product, in our evaluation of m we use the number of six-digit NAICS industries, which constitute the most detailed level of the US industrial classification and, hence, are expected to be comprised of the most substitutable products.[34] Resorting to

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[32]We take the latest available data on the total number of firms (for year 2013) from the Statistics of US Businesses and Business Dynamics Statistics databases.

[33]Data source: U.S. Commuting Zones and Labour Market Areas: Documentation.

[34]As an example, mayonnaise and ketchup are likely to be less substitutable than two ketchup

Table 1.1: The ranges of ξ's values for a selection of countries (mark-up estimates from (Oliveira Martins et al., 1996, Tables 1, 2)).

Country	$m = 7$			$m = 10$		
	$\underline{\xi}$	ξ_{med}	$\bar{\xi}$	$\underline{\xi}$	ξ_{med}	$\bar{\xi}$
France	0.350	0.937	3.714	0.245	0.656	2.6
Sweden	0.475	1.036	2.184	0.333	0.725	1.529
USA	0.407	1.571	4.905	0.285	1.100	3.433

data from the Statistics of US Businesses database yields the total of 978 six-digit industries.

Depending on whether the figures on the total number of US firms are taken from the Statistics of US Businesses database or the Business Dynamics Statistics database,[35] the resulting number of firms per model's industry m equals either 7 or 10.

We recover the range of ξ's values by drawing upon the body of literature on mark-up estimation (see, e.g., Oliveira Martins et al. (1996), Klette (1999), De Loecker and Warzynski (2012)). In particular, we use mark-up estimates for samples of industries in France and Sweden from Oliveira Martins et al. (1996), which, when coupled with the definition of μ, generate the estimates for ξ ranging from 0.245 to 3.714 (for France) and from 0.333 to 2.184 (for Sweden, see Table 1.1).

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brands. The assumption of higher substitutability is in line with existing evidence – see Broda and Weinstein (2006), Broda and Weinstein (2010).

[35]The discrepancy in numbers partly occurs because firms entering the Business Dynamics Statistics database are those active during the pay period which covers the 12^{th} of March, whereas firms in the other database are active at some point in a year.

1.3.2 Theoretical Considerations

First of all, in the discussion to follow we assume that there are possibilities for firms to make profits, viz. $\underline{m} < \bar{m}$, or, equivalently

$$2 < \frac{1+\xi}{\alpha\xi} \Leftrightarrow \alpha < \frac{1}{2\xi - 1} \tag{1.32}$$

In the case of both HS-D and HS-I discrepancy we require the following condition to hold: $\underline{m} < \hat{m}_q < \bar{m}$. Given Proposition 1.1, this implies the following system of inequalities

$$\begin{cases} \alpha & > & 2\beta\xi \\ \frac{\alpha}{\beta\xi} & < & \frac{1+\alpha}{\alpha\xi} \end{cases} \Leftrightarrow \begin{cases} \alpha & > & 2\beta\xi \\ \alpha^2 & < & \beta\alpha + \beta \end{cases} \tag{1.33}$$

Note that system (1.33) implies that for its solution $2 < \frac{\alpha}{\beta\xi} <$ – i.e. condition (1.32) is satisfied automatically. Solving (1.33) for α yields the result

$$\alpha \in \left(2\beta\xi; \frac{\beta + \sqrt{\beta^2 + 4\beta}}{2}\right) \tag{1.34}$$

A graphic representation of condition (1.34) is shown in Figure 1.4. Naturally, the range of α's suitable values is nonempty whenever $2\beta\xi < \frac{\beta+\sqrt{\beta^2+4\beta}}{2}$, which is the case when the following condition holds

$$\beta < \frac{1}{2\xi\,(2\xi - 1)} \tag{1.35}$$

Expression (1.35) imposes the upper limit on the value of 'fishing-out' coefficient β, above which the level of α required to generate a detectable hump-shaped pattern, exceeds the mark-up. A noteworthy feature of the conditions obtained is that, since the derivative of the upper limit in (1.34) increases to infinity for $\beta \to 0$, a suitable pair of α (in the interval specified in (1.34)) and β (satisfying (1.35)) can be chosen for any given value of ξ – put differently, for any level of ξ the shaded area in Figure 1.4 is nonempty.

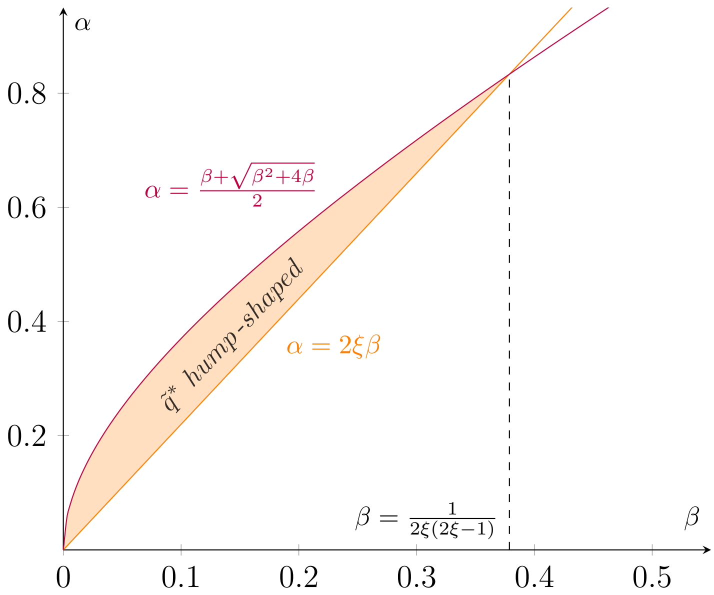

Figure 1.4: The hump-shapedness conditions for $\tilde{q}^*$ (for $\xi = 1.1$).

Turning to the conditions of observed monotonicity of R&D effort, it is decreasing in m if the following condition holds

$$\frac{2-\alpha+2\beta}{1-\alpha+2\beta}\frac{1}{\xi} \leqslant 2 \Leftrightarrow \alpha\,(2\xi-1) \leqslant 2\,(\xi-1) + 2\beta\,(2\xi-1) \tag{1.36}$$

Depending on whether ξ is greater than $1/2$ or not, the final condition for α takes either of two forms: $\alpha \leqslant \frac{2(\xi-1)}{2\xi-1} + 2\beta$ for $\xi > 1/2$, and $\alpha \geqslant \frac{2(\xi-1)}{2\xi-1} + 2\beta$ for $\xi < 1/2$. We can omit the second inequality however, as the corresponding value of $\hat{m}_\gamma$ is outside $\tilde{\gamma}^*(t)$'s domain. Thus, a decreasing pattern in $\tilde{\gamma}^*(t)$ can occur only if $\xi > 1/2$, and the final expression for the corresponding condition is

$$\alpha \leqslant \frac{2\,(\xi-1)}{2\xi-1} + 2\beta,\ \ \xi > 1/2 \tag{1.37}$$

Naturally, if α is too large (in the sense of (1.37)), the productiveness of R&D is sufficiently high to induce further R&D effort and reinforce the escape-costs effect to the degree when $\hat{m}_\gamma$ is pushed further right into interval $[\underline{m}; \bar{m}]$, which makes $\tilde{\gamma}^*(t)$ observably hump-shaped.

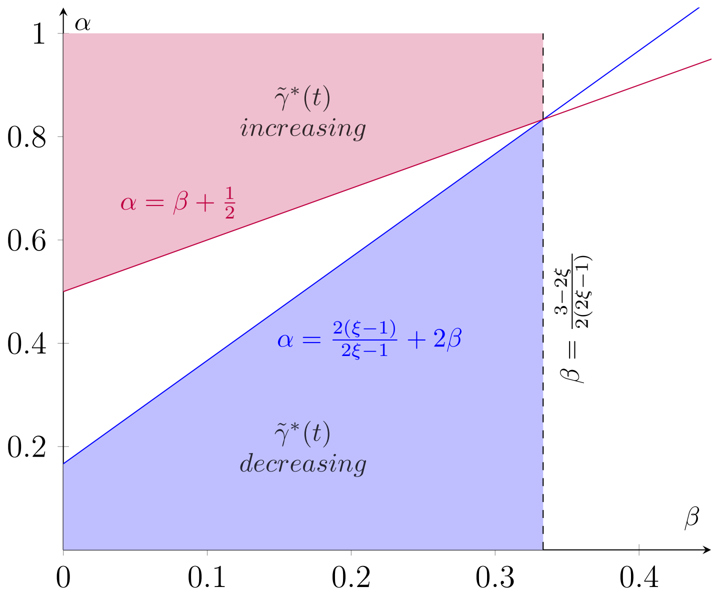

Figure 1.5: The unobserved hump-shapedness conditions for $\tilde{\gamma}^*(t)$ (for $\xi = 1.1$). The intersection point and values of β above it correspond to the situation in which condition (1.39) is violated.

An increasing pattern in R&D effort obtains if

$$\frac{2-\alpha+2\beta}{1-\alpha+2\beta}\frac{1}{\xi} \geqslant \frac{1+\alpha}{\alpha}\frac{1}{\xi} \Leftrightarrow \alpha - \beta \geqslant \tfrac{1}{2} \tag{1.38}$$

Note that unlike all previous conditions, equation (1.38) imposes a constraint on α not only in relative, but also in absolute terms: since $\beta > 0$, α cannot be smaller than $^1/_2$. The last consideration, in conjuncture with the stipulation that $\underline{m} < \bar{m}$, imposes the upper limit on ξ (in the form of a necessary condition) – even though ξ does not enter (1.38) directly. Given that $\frac{1+\alpha}{\alpha\xi}$ is a decreasing function of α, replacing it with $\beta + \frac{1}{2}$ yields the inequality $\frac{1+\beta+\frac{1}{2}}{\left(\beta+\frac{1}{2}\right)\xi} = \frac{3+2\beta}{(1+2\beta)\xi} \geqslant \frac{1+\alpha}{\alpha\xi}$, which suggests the following necessary condition for ξ

$$\frac{3+2\beta}{(1+2\beta)\,\xi} > 2 \Leftrightarrow \xi < \frac{3+2\beta}{2\,(1+2\beta)} < \frac{3}{2} \tag{1.39}$$

As the final step in this section, let us specify the conditions for the presence of HS-I and HS-D discrepancies. As to the former, it occurs in the event of (at least partial) overlap of the orange area in Figure 1.4 and the blue area in Fig-

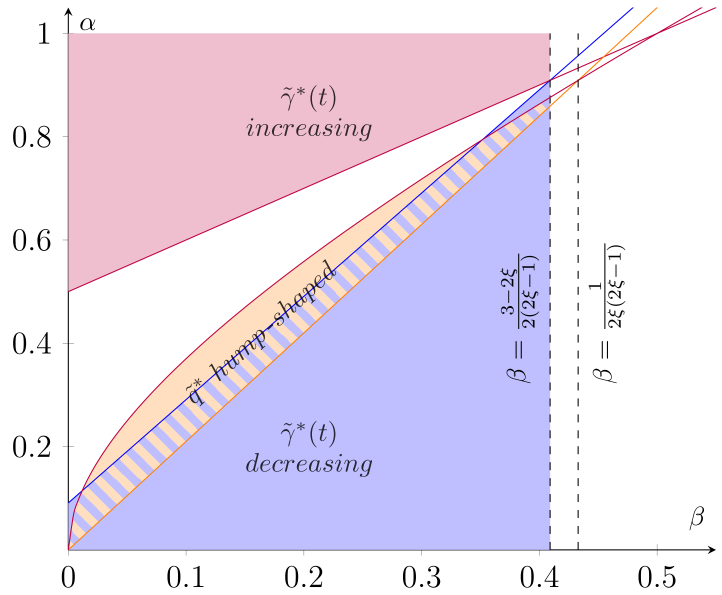

Figure 1.6: The HS-D discrepancy region (the hatched area) for $\xi = 1.05$.

ure 1.5, which is the case when $2\xi\beta \leqslant \frac{2(\xi-1)}{2\xi-1} + 2\beta$. Depending on whether $\xi \geqslant 1$ or otherwise, the last expression becomes either $\beta \leqslant \frac{1}{2\xi-1}$ or $\beta \geqslant \frac{1}{2\xi-1}$, respectively. The latter condition can be omitted though, as it implies that $\alpha > 2\xi\beta = \frac{2\xi}{2\xi-1} > \frac{1}{2\xi-1}$ – the latter result is incompatible with the requirements that $\tilde{q}^*$'s hump-shapedness is observable: $\alpha < \frac{1}{\xi\hat{m}_q-1} < \frac{1}{\xi\underline{m}-1} = \frac{1}{2\xi-1}$. Thus we can argue that for HS-D discrepancy to occur, the elasticity of substitution has to exceed one, and β has to be smaller than $\frac{1}{2\xi-1}$. Given our assumption that $\xi > 1/2$, the condition obtained for β is weaker than that required for the observability of $\tilde{q}^*$'s hump-shapedness (1.35): $\tilde{q}^* - \textit{hump-shaped} \Rightarrow \beta < \frac{1}{2\xi(2\xi-1)} < \frac{1}{2\xi-1}$, which suggests that whenever $\tilde{q}^*$ is hump-shaped and $\xi \geqslant 1$, HS-D discrepancy is observable

$$\left.\begin{array}{c} \tilde{q}^* - \textit{hump-shaped} \\ \xi \geqslant 1 \end{array}\right\} \Rightarrow \textit{HS-D discrepancy} \tag{1.40}$$

In light of the conclusions by Tingvall and Poldahl (2006), we expect condition $\xi \geqslant 1$ to hold for Sweden, and, as suggested by our estimates in Table 1.1, it is indeed satisfied for the median estimate of ξ when $m = 7$.

As regards HS-I discrepancy, we require, in graphic terms, that the orange area in Figure 1.4 overlap with the red area in Figure 1.5, which is the case when

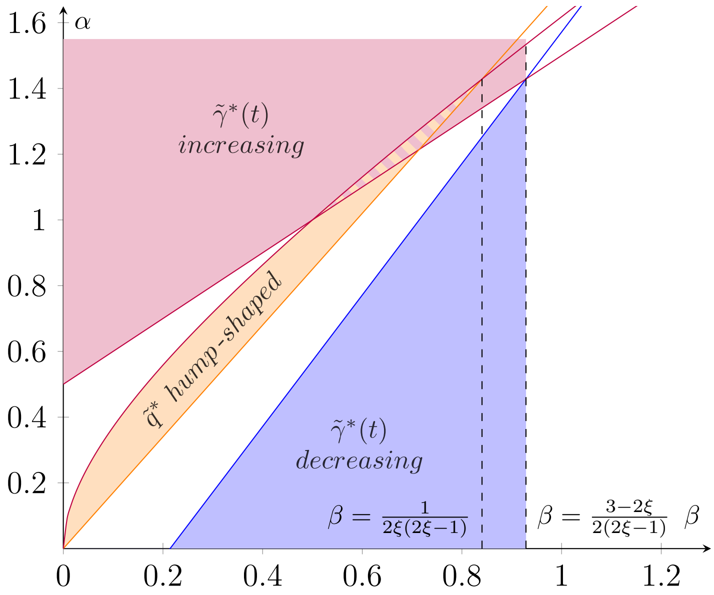

Figure 1.7: The HS-D discrepancy region (the hatched area) for $\xi = 0.85$.

$$\beta + \frac{1}{2} \leqslant \frac{\beta + \sqrt{\beta^2 + 4\beta}}{2} \Leftrightarrow \beta + 1 \leqslant \sqrt{\beta^2 + 4\beta}$$

$$\beta^2 + 2\beta + 1 \leqslant \beta^2 + 4\beta$$

$$\beta \geqslant \frac{1}{2} \tag{1.41}$$

Given that we consider the situation of HS-I discrepancy, condition (1.39) of $\tilde{q}^*$'s hump-shapedness has to hold, so that

$$\xi < 1 \tag{1.42}$$

Note that condition (1.42) is satisfied for the assessed median values of ξ for France (see Table 1.1), where, given the results by Askenazy et al. (2013), we expect HS-I discrepancy to occur.

Putting together conditions (1.41) and (1.42) alongside the stipulation that $\tilde{q}^*$ is hump-shaped, yields the final result

$$\left.\begin{array}{c} \tilde{q}^* - \textit{hump-shaped} \\ \beta \geqslant \frac{1}{2} \\ \xi < 1 \end{array}\right\} \Rightarrow \textit{HS-I discrepancy} \tag{1.43}$$

Combining conditions (1.40) and (1.43) suggests that, conditional on $\tilde{q}^*$ being observably hump-shaped, $\tilde{\gamma}^*(t)$ is so as well (and, thus, no discrepancy occurs), if $\xi < 1$ and $\beta \leqslant \frac{1}{2}$.

Having specified the analytical conditions for both HS-D and HS-I discrepancy, we would naturally like to check whether the restrictions imposed on α, β and ξ are consistent with available pieces of empirical evidence – we address this question in greater detail in the next section.

1.3.3 Empirical Plausibility of the Results

In order to check the validity of our theoretical conclusions, we need to relate α and β to a variable whose range of values can be inferred from existing empirical literature, and check whether the constraints we impose on the parameters, are compatible with that range. Our candidate to that end is the parameter known in the empirical literature as 'the return on R&D'[36] ζ which is estimated using the following equation[37]

$$\ln\left(\frac{\tilde{y}_{t+1}}{\tilde{y}_t}\right) = b_0 + b_1 \ln\left(\frac{\tilde{k}_{t+1}}{\tilde{k}_t}\right) + b_2 \ln\left(\frac{\tilde{l}_{t+1}}{\tilde{l}_t}\right) + \zeta \frac{\tilde{\gamma}_t}{\tilde{y}_t} + u_t \tag{1.44}$$

$$E_t\left\{\ln\left(\frac{\tilde{y}_{t+1}}{\tilde{y}_t}\right) - b_1 \ln\left(\frac{\tilde{k}_{t+1}}{\tilde{k}_t}\right) - b_2 \ln\left(\frac{\tilde{l}_{t+1}}{\tilde{l}_t}\right)\right\} = b_0 + \zeta \frac{\tilde{\gamma}_t}{\tilde{y}_t} \tag{1.45}$$

[36]We use the quotation marks here, as ζ can be interpreted as the return on R&D investment if R&D effort is assumed to stack up in the form of research capital. In our model though the latter is not present directly (although one can potentially interpret total factor productivity $Q(t)$ in this vein) – see (Mairesse and Sassenou, 1991, pp. 2–3) for a detailed discussion.

[37](Mairesse and Sassenou, 1991, p. 3).

where u_t is a temporal sequence of independent identically distributed random variables. Given the model's parameterisation and the absence of stochasticity in it, $b_1 = \nu$, $b_2 = 1 - \nu$, which reduces equation (1.45) to

$$\ln \frac{Q_{t+1}}{Q_t} = b_0 + \zeta \frac{\tilde{\gamma}_t}{\tilde{y}_t} \tag{1.46}$$

Given that discrete data for Q_t is generated by $Q(t)$ (i.e., $Q_t = Q(t)$ at any discrete point of time t), we have that $Q_{t+1} = \tilde{q}^* Q_t$.[38] Using the definition of $\tilde{y}(t)$, equation (1.22) can be transformed as follows: $\tilde{q}^{*\,1-(\alpha-\beta)} = \left(\frac{\tilde{\gamma}^*(t)}{\tilde{y}(t)}\right)^{\alpha-\beta} m^{-\beta}$. Plugging these results into (1.46) establishes the connection between α and β on one hand, and ζ on the other

$$\frac{\alpha - \beta}{1 - (\alpha - \beta)} \frac{\tilde{\gamma}^*(t)}{\tilde{y}(t)} - \frac{\beta}{1 - (\alpha - \beta)} m = b_0 + \zeta \frac{\tilde{\gamma}^*(t)}{\tilde{y}(t)} \tag{1.47}$$

As follows from (1.47), $\frac{\alpha-\beta}{1-(\alpha-\beta)} = \zeta \Leftrightarrow \alpha - \beta = \frac{\zeta}{1+\zeta}$, which allows us to take difference $\alpha - \beta$ to data.

In a comprehensive survey of ζ's estimates, Hall, Mairesse, and Mohnen (2010) list a sufficiently wide range of values for France from 16% to 128%;[39] as regards Sweden, we use the estimate of 50.7% from Griffith, Redding, and Van Reenen (2004).[40] These values can support empirically the situations of both HS-D (Aghion et al. (2005) vs. Tingvall and Poldahl (2006)) and HS-I (Aghion et al. (2005) vs. Askenazy et al. (2013)) discrepancy. Starting with the former, condition (1.40) stipulates that $\tilde{q}^*$ be hump-shaped, and that the elasticity of substitution be no smaller than one. As regards the former, given (1.47), it is

[38] To see that, consider two functions $A_t : A_{t+1} = \lambda A_t \Leftrightarrow A_t = \lambda^t A_0$ and $A'(t) : \dot{A}'(t) = \omega A'(t) \Leftrightarrow A'(t) = A'_0 e^{\omega t}$. Discrete data for A_t is generated by $A'(t)$ if $A_0 = A'_0$ and $e^{\omega} = \lambda$. With regards to Q_t this implies $Q_{t+1} = e^{\ln \tilde{q}^*} Q_t = \tilde{q}^* Q_t$.

[39] See (Hall et al., 2010, Tables 2–5) for the full list of estimates for different countries.

[40] We use the authors' estimate of return on innovation net of technology transfer contribution – see (Griffith et al., 2004, Table 3).

satisfied for the values of α such that

$$\begin{cases} \frac{\alpha}{\left(\alpha-\frac{\zeta}{1+\zeta}\right)\xi} > 2 \\ \alpha < \frac{\alpha-\frac{\zeta}{1+\zeta}+\sqrt{\left(\alpha-\frac{\zeta}{1+\zeta}\right)^2+4\left(\alpha-\frac{\zeta}{1+\zeta}\right)}}{2} \end{cases} \Leftrightarrow \zeta < \alpha < \frac{\zeta}{1+\zeta} \cdot \frac{2\xi}{2\xi-1} \tag{1.48}$$

The set of α's values is non-empty (or, equivalently, (1.48) is compatible with condition (1.32) for the non-negativity of firms' profits) when

$$\zeta < \frac{\zeta}{1+\zeta} \cdot \frac{2\xi}{2\xi-1} \Leftrightarrow \zeta < \frac{1}{2\xi-1} \tag{1.49}$$

The range of ξ's values prescribed by condition (1.49) covers the empirical estimate of ζ for Sweden 50.7%, when $0.507 < \frac{1}{2\xi-1} \Leftrightarrow \xi < \bar{\xi} \approx 1.486$. Given that $\xi > 1$ (as stated in (1.40)), the last consideration suggests the range of ξ's values of $(1; 1.486)$, which fits our estimate of ξ's median value for Sweden (for $m = 7$), thus suggesting that the situation of HS-D discrepancy is compatible with the pieces of empirical evidence presented.

As to the HS-I discrepancy, we still require $\tilde{q}^*$ to be hump-shaped, so that condition (1.49) holds. In addition, given (1.43), $\alpha - \beta = \frac{\zeta}{1+\zeta} > \frac{1}{2} \Leftrightarrow \zeta > 1$. Thus, the range of ζ's suitable values is $\left(1; \frac{1}{2\xi-1}\right)$, which, given (1.43), is non-empty: $\xi < 1 \Rightarrow \frac{1}{2\xi-1} > \frac{1}{2-1} = 1$. Thereby, so long as $\xi < 1$, the range obtained overlaps with the upper tail of the interval of empirical estimates [16%; 128%], which confirms the plausibility of the conditions for the HS-I discrepancy as well. This inference concludes the section.

1.4 Summary

In this chapter we have explored theoretically the possibility of discrepancy in the behaviour of R&D effort and R&D outcomes (R&D accomplishment) as functions of competition.

We have shown that, since in the context of their relationship with competition, R&D outcomes and R&D effort are affected by non-identical sets of factors (viz. the escape-costs effect and congestion ('fishing-out') effect for the former, and the escape-costs effect and the division effect for the latter), the two can exhibit different kinds of detectable behaviour: even though both functions are hump-shaped with respect to the degree of competition m, the turning point of R&D effort can reside outside the permissible range of m's values, which makes it observably increasing or observably decreasing function, thus, coupled with the hump-shapedness of R&D outcomes, producing observed discrepancy in the behaviour of the two aspects in innovation with respect to the degree of competition. Conditions imposed on our model's parameters in order to generate the discrepancy, seem to comply with existing ranges of their empirical counterparts' estimates.

One merit of our approach is that it reconciles the contradictory conclusions drawn in Aghion et al. (2005) on one hand (the presence of the hump-shaped pattern in the relationship between innovation (measured in terms of R&D outcomes) and competition) and in Tingvall and Poldahl (2006), Askenazy et al. (2013) on the other (rejection of the hump-shaped pattern hypothesis in the relationship between innovation, as proxied by R&D effort, and competition).

We hope that our illustration of the possibility that the two aspects of innovation cannot necessarily be equated to each other (in terms of their relations with competition), will inform further attempts to confirm empirically the hypothesis of the hump-shaped pattern.

Chapter 2

R&D Cyclicality and Composition Effects: A Unifying Approach

Existing empirical studies do not concur on whether R&D spending is procyclical or countercyclical: the former hypothesis is supported by studies of aggregate R&D spending, whereas the latter is vindicated by firm-level evidence.

In this chapter, we reconcile the two facts by advancing a general equilibrium framework, in which, while a single firm's R&D spending profile is countercyclical, aggregate R&D spending is procyclical owing to procyclical fluctuations in the number of R&D performers.

Our findings suggest that economic crises might be beneficial for economic performance by fostering individual R&D effort. An advantage of our framework is that it brings together conflicting pieces of empirical evidence, while incorporating and building upon Schumpeter's hypothesis of countercyclical innovation.

2.1 Introduction

An influential part of Joseph Schumpeter's legacy is the idea that economic crises allow an economy to restructure itself on a more efficient basis (Schumpeter (1943)). This leads one to think of economic downturns as a way to induce research and development (R&D) activities, thereby suggesting that, in accordance with Schumpeter's view, R&D spending should exhibit countercyclical behaviour. This prediction has been explored extensively on both theoretical[1] and empirical levels. A noteworthy feature of the latter strand of research is the micro/macro dichotomy of the results obtained. Macro-data (economy-wide and industry data) based studies (see, e.g., Fatás (2000), Rafferty (2003), Wälde and Woitek (2004), Comin and Gertler (2006)) show R&D spending to be procyclical. By contrast, firm-data based results in Aghion, Askenazy, Berman, Cette, and Eymard (2012), Lopéz-García, Motero, and Moral-Benito (2012), Beneito, Rochina-Barrachina, and Sanchis-Llopis (2015) point in the opposite direction.

One theory proposed to accommodate both procyclicality and countercyclicality of R&D within a single framework is based on the liquidity constraint hypothesis advanced in Aghion, Angeletos, Banerjee, and Manova (2010). Even though firms would like to make their R&D spending profiles countercyclical, they are unable to do so because of insufficient access to loanable funds. In all aforementioned firm-data based papers, taking into account a measure of credit tightness indeed makes R&D spending procyclical.

Of importance, however, is the feature of the liquidity constraint approach that, while bringing together pro- and countercyclicality of R&D, the liquidity constraint hypothesis does not provide one with a way of explaining – in procyclical terms – countercyclical R&D spending by financially unconstrained firms, which constitute a significant share of the total number of firms in the firm-data based papers discussed.[2]

[1]An early example of the mathematical formalisation of Schumpeter's hypothesis is Caballero and Hammour (1994), who develop a creative-destruction model of an industry to show that a fall in demand and the ensuing shakedown can cleanse an industry of less efficient firms/plants, thus shifting resources in it to more productive units. Another example is the opportunity cost theory advanced by Aghion and Saint-Paul (1998), which is discussed below in this section.

[2]In particular, financially unconstrained firms make approx. 67%, 77% and 46% of the total number of firms in the datasets employed, respectively, by Aghion et al. (2012), Lopéz-García et al. (2012) and Beneito et al. (2015). See (Aghion et al., 2012, pp. 1008–1009), (Lopéz-García

In addition, the liquidity constraint hypothesis leaves unanswered the question why, by contrast with the firm level, economy-wide and industry-wide R&D spending exhibits procyclical properties even without considering credit constraints. In addition, and related to the previous point, it does not consider the micro/macro dimension of the discrepancy in R&D's cyclical behaviour.

Finally, the relevance of the liquidity constraint hypothesis for explaining the micro/macro discrepancy in the cyclical behaviour of R&D, however, is further undermined by limited industry-level evidence to support it: Ouyang (2011) finds the predictions of the hypothesis to be valid only in the case of demand-driven cyclical fluctuations. Yet, a few manifestations of the opportunity cost theory (e.g. Bental and Peled (1996), Matsuyama (1999, 2001), Francois and Lloyd-Ellis (2003), see discussion below) suggest that the opportunity cost effect can be induced by supply side dynamics as well. The results obtained by Ouyang (2011) further strengthen the idea that the procyclicality of R&D spending on the macro-level is due to factors other than credit constraints.

In this chapter, we argue that the difference between macro- and micro-based results cited above is because of a composition effect: even though an individual firm's R&D spending is countercyclical, its aggregate dynamics (on the industry or economy level) can be procyclical owing to changes in the number of firms investing in R&D (because of, for example, entry/exit dynamics). For instance, if an economy is in a trough, an increase in individual R&D can be offset by a drop in the number of firms engaging in R&D (the opposite is true for booms).

We base our conjecture on the combination of the following motivating observations: first of all, it is widely recognised in industrial organisation (IO) literature that the probability of a firm's engaging in R&D depends positively on its size.[3] Naturally, in the situation of a crisis one would expect firms' sizes (as measured by, e.g., sales or employment) to drop, thereby driving down both the

et al., 2012, Table 2, p. 32), (Beneito et al., 2015, Table 1, p. 352).

[3]See, e.g., (Cohen and Klepper, 1996, Stylised fact 1, p. 928).

volume of each cohort of firms of a given size, and the share of R&D performers within it – together the two observations suggest that during crises a smaller share of a lower number of firms engages in R&D in an industry, thus enabling one to expect that economic cycles produce a procyclical aggregation-based effect on the amount of R&D in an industry, as channelled through procyclical dynamics in the number of R&D-performing firms.

The key difference between the approach adopted in this chapter and the one encapsulated in the liquidity constraint hypothesis lies in that we do not try to present a framework consistently accommodating contradicting pieces of empirical evidence in opposition to Schumpeter's hypothesis. Rather, we develop a theory, which, on one hand, reconciles discrepancies in macro- and micro- empirical results, and, on the other hand, embeds and builds upon Schumpeter's hypothesis.

We illustrate our point by presenting a tractable general-equilibrium model, in which firms' R&D spending shows countercyclical behaviour, whereas that on the industry and economy-wide levels is procyclical. This results from introducing a two-level structure in an economy, whereby the final good is produced using industries' outputs, which themselves are aggregated from differentiated products made by competing monopolist firms. Each monopolist engages in two (limitedly substitutable) activities: production and R&D, which are, respectively, procyclical and countercyclical owing to the opportunity cost effect. If, however, substitutability between the two activities is not high enough (so that not too large a share of a firm's resources is shifted between production and R&D during an economic cycle), drops in individual R&D spending during upturns are dampened sufficiently to be offset by increases in aggregate industry R&D spending resulting from the entry of new firms (the opposite dynamics obtains during downturns).

The rest of the chapter is structured as follows. In Section 2.1.1 we review literature relevant to our research; in Sections 2.2.1–2.2.4 the baseline model is introduced (2.2.1–2.2.3) and solved (2.2.4); in Section 2.3 we examine empirical validity of the results obtained. In Sections 2.4, 2.5 the effect of technology accumulation is investigated and evaluated. In Section 2.6 we consider the procyclicality of aggregate R&D spending, when a firm's decisions to do R&D and to stay in business are separated from each other. The last section concludes.

2.1.1 Related Literature

Our paper is related to the rich literature on cyclicality of innovation. In the empirical dimension, one could list a number of works largely characterised by the macro/micro dichotomy discussed above (see Table 2.1).[4]

On the theoretical front, given the reconciliatory purpose of our paper and its general-equilibrium setting, it is related most closely to the branches of theoretical general-equilibrium literature on both countercyclicality and procyclicality of innovation. With regards to the former, we can mention the works by Bental and Peled (1996), Aghion and Saint-Paul (1998), Matsuyama (1999, 2001) and Francois and Lloyd-Ellis (2003).

Aghion and Saint-Paul (1998) have introduced explicitly the opportunity cost hypothesis by considering the dynamics of R&D in two settings: in one (the 'World 1' model) R&D has disruptive impact on production (i.e., engaging in R&D requires channelling some resources from production), which makes it countercyclical, since a firm's costs of diverting funds from production to R&D are procyclical. By contrast, the situation when R&D is non-disruptive (viz., R&D can be funded directly by buying the final good instead of reallocating resources, in the 'World 2' model) leads to the procyclical behaviour of innovation, since

[4] Barlevy (2007) uses firm-level data to show a firm's growth rate of R&D spending to be an increasing function of the industry's growth (i.e. suggesting R&D's being procyclical). One could argue though that an industry's growth, being an industry-wide aggregate indicator, can channel the impact coming from other firms in the industry through, e.g., intrasectoral competition; in addition, sectoral growth can pick the impact of targeted government spending in the sector (see Cozzi and Impullitti (2010) for an investigation into the effects of sectoral government spending on R&D).

Table 2.1: Empirical findings on the cyclicality of R&D.

Study	Level of data	Sample	R&D cyclicality	Salient findings
Fatás (2000)	Country	USA, 1961 –1996	Procyc.	Growth rates of total R&D and GDP are positively correlated
Rafferty (2003)	Country	USA, 1953 –1999	Procyc.	Real firm-financed R&D and GDP are positively cointegrated
Wälde and Woitek (2004)	Country	G7 countries, 1973 –2000	Procyc.	Cyclical components of R&D per capita and GDP per capita are positively correlated
Comin and Gertler (2006)	Country	USA, 1948 –2001	Procyc.	Short- and medium-run cyclical components of R&D and GDP are positively correlated
Barlevy (2007)	Industry	7 719 firms, USA, 1978 –2004[5]	Procyc.	Growth rates of firms' real R&D expenditures and the industry's real output/value added are positively correlated
Aghion et al. (2012)	Firms	≈13 000 firms, France, 1994 –2004	Ctrcyc.	R&D[6] is negatively correlated with changes in a firm's sales; the relationship becomes procyclical for financially constrained firms
Lopéz-García et al. (2012)	Firms	3 278, Spain, 1991 –2010	Ctrcyc.	
Beneito et al. (2015)	Firms	3 361 firm, Spain, 1990 –2006	Ctrcyc.	

there is no opportunity cost based trade-off between the two activities.

In Matsuyama (1999, 2001),[7] the model's economy alternates between the

[7]The key difference between the two papers is that in Matsuyama (1999) the Solow-type assumption is made of a fixed saving rate, whereas in Matsuyama (2001) it is endogenous, in the style of the Ramsey-Cass-Koopmans model.

states of capital accumulation ('the Solow mode', after the celebrated model of a relationship between economic growth and factor accumulation due to Solow (1956), when the economy's resources are devoted exclusively to competitive production of existing goods) and that of product expansion ('the Romer mode', after the model of endogenous product expansion due to Romer (1990), when, alongside producing existing varieties, part of the economy's output is directed to the creation of new goods, whose supply is monopolised for one period). During intervals of accumulation, the build-up of capital reduces the economy's interest rate and, thus, the cost of creating new varieties relative to the value of innovation, which ultimately induces the expansion phase when the interest rate reaches a certain threshold value.[8] This mechanism can be thought of as an indirect manifestation of the opportunity cost reasoning as well, since expansion in the model begins once the opportunity cost of diverting funds from production to the creation of new varieties becomes sufficiently low (i.e., non-positive, which occurs when one-period profits from innovation become non-negative).

Francois and Lloyd-Ellis (2003) advance a model of cyclically clustered innovation, in the spirit of the implementation cycles approach due to Shleifer (1986). In the model, 'disruptiveness' of R&D manifests itself in labour's being split between production and R&D. Within each endogenous cycle, the initial wage level (which also acts as the (scaled) cost of innovating) is too high for R&D to pay off, so no innovation occurs, all labour is directed to production, and the economy stagnates, which in turn results in wages starting to fall relatively to the value of innovation. Once the two quantities are equal, the opportunity cost effect comes into operation, and an increasingly higher share of labour is diverted to R&D, so that new discoveries are made, but not implemented. At the end of the cycle, implementation of accumulated discoveries occurs en masse, thus resulting in an instantaneous expansion, which pushes up the economy's wage rate, and thereby initiates a new cycle.

[8] In addition to the case of alternating between the two modes, the economy can perpetually stay in either of them, depending on the model's parameterisation.

The model due to Bental and Peled (1996) has features present in three other papers discussed above: as in Aghion and Saint-Paul (1998), direct 'disruptiveness' of R&D obtains: firms spend capital on either searching for new discoveries or on production using the most advanced technology from the last period; as in Matsuyama (1999, 2001), the dynamics of the opportunity cost of searching for and creating new technologies is driven by accumulation of capital: as a result of its build-up, firms' returns on production increasingly diminish, thus ultimately making R&D attractive enough for firms to embark on; as in Francois and Lloyd-Ellis (2003) the discovery and implementation of new technologies (the two in Bental and Peled (1996) occur simultaneously) are followed by a halt in exerting R&D effort for some time owing to the opportunity cost effect.

Similarly to all aforementioned papers, we consider the behaviour of R&D on the firm level in the framework of the opportunity cost theory.[9] Since, however, we are not interested in mechanisms behind economic fluctuations per se, our model does not generate endogenous cycles (as in Bental and Peled (1996), Matsuyama (1999, 2001) and Francois and Lloyd-Ellis (2003)), but rather uses the cyclicality of productivity in production as a modelling 'input', which induces cyclical reallocation of funds between production and R&D.[10]

As regards theoretical papers on the procyclicality of R&D spending, the following can be mentioned: Wälde (2005), Barlevy (2007), Francois and Lloyd-Ellis (2009), Aghion et al. (2010), Bambi, Gozzi, and Licandro (2014).

In Wälde (2005), investing in R&D results in the stochastic arrival of a new technology (embodied in a new more productive vintage of the capital good). It has a by-product of increasing the economy-wide level of labour pro-

[9]In particular, by making producers choose between allocating their facilities to production and to R&D, we make the latter 'disruptive', as in the 'World 1' model in Aghion and Saint-Paul (1998) and the model in Bental and Peled (1996) discussed above.

[10]Aghion and Saint-Paul (1998) employ a technically similar approach by allowing the dynamics of aggregate demand in their model to be driven by a two-state Markov process, which results in cyclical reallocation of funds between production and R&D.

ductivity, which in turn pushes the economy's capital-to-effective labour ratio below the equilibrium level. Thus, the economy's dynamics is represented by periods of convergence to steady-state capital to effective labour ratios, which are interrupted by intermittent arrivals of new technologies. Under the model's parameterisations investigated in the paper, R&D spending is a linear function of the economy's stock of capital,[11] which leads to its dynamics replicating that of capital and output.

Barlevy (2007) considers procyclicality and countercyclicality of R&D in a single framework by investigating a model of Schumpeterian growth with 'disruptive' R&D, which requires labour to be diverted from production. While it is socially optimal for R&D to be countercyclical (because of its 'disruptiveness', which induces the opportunity cost effect), the market outcome can result in procyclical R&D, which is driven by the presence of fixed costs incurred by firms. As those are measured in units of the final good, they inherit the countercyclical properties of the latter's price.[12] The resulting countercyclicality of fixed costs exerts procyclical pressure on firms' expected profits, which in turn brings about procyclicality of R&D in the model.

Francois and Lloyd-Ellis (2009) suggest a model of R&D procyclicality, which builds upon the authors' earlier theory introduced in Francois and Lloyd-Ellis (2003). The key addition to the framework in Francois and Lloyd-Ellis (2003) concerns splitting the process of innovating into three subsequent stages: R&D, commercialisation, and implementation. R&D is performed by using the final good, which makes it 'non-disruptive' and thereby procyclical (in line with the logic of the 'World 2' model in Aghion and Saint-Paul (1998)); by contrast,

[11]The linearity of R&D spending R with respect to the stock of capital K comes from the fact that the expected technology arrival rate λ is modelled as a function of the ratio of the two quantities $\lambda = f\left(\frac{R}{K}\right)$: under special parameter restrictions it becomes fixed, which establishes the result stated: $R = f^{-1}(\lambda)\, K$.

[12]This property follows from extensively documented countercyclical behaviour of price mark-ups – see, e.g., Christiano et al. (2005), Comin and Gertler (2006), Galí, Gertler, and López-Salido (2007), Justiniano, Primiceri, and Tambalotti (2010).

commercialisation (during which entrepreneurs draw upon the stock of ideas generated by R&D) requires diverting labour from production, which results in its inheriting the countercyclical properties of R&D in Francois and Lloyd-Ellis (2003), owing to the reasons outlined above in the discussion of the paper.

Aghion et al. (2010) consider pro- and countercyclicality of R&D within a unified framework (similarly to Barlevy (2007)), in which the cyclical behaviour of R&D is driven by the presence of credit constraints.[13] In the model, each entrepreneur allocates his funds between short-run capital investment and long-run R&D investment (i.e., R&D is 'disruptive'), which, when he is not bound by credit constraints, brings about standard countercyclical dynamics in R&D owing to the opportunity cost effect. If, however, an entrepreneur is financially constrained and expects to be hit by adverse liquidity shocks, R&D spending can become procyclical: since there is no unlimited access to loanable funds, entrepreneurs have to cover liquidity risks by drawing upon their own assets generated by short-run investment projects (as funds allocated to long-run R&D investment are immobilised). In the model, downturns are characterised by a higher probability of a liquidity shock, which prompts entrepreneurs to shift their wealth to short-run investment, which induces procyclicality of R&D.

Bambi et al. (2014) consider a standard Romer-type model of expanding input varieties with an exogenous implementation lag of new inputs, which, by rendering the model's variety growth equation delayed, can induce cyclical behaviour in the growth rate of the economy's mass of inputs.[14] Given that the model is based on the framework introduced by Romer (1990), total output is a power function of the mass of varieties, which allows it to inherit the cyclical patterns in the dynamics of the mass of inputs. Thus, the two quantities co-oscillate, thereby making the mass of varieties (which can be interpreted as accumulated R&D) procyclical.

This chapter contributes to the last body of literature reviewed by intro-

[13] A simplified version of the model is presented in Aghion et al. (2012).

[14] See (Banks, 1994, Ch. 6) for an introduction to lagged differential equations.

ducing a novel mechanism generating R&D procyclicality on the aggregate level through the composition effect, as embodied in firm entry/exit dynamics. The key difference between the papers discussed above and our work is that R&D procyclicality manifests itself not on the individual firm level, but on that of industries, alongside R&D countercyclicality on the firm level. In other words, by contrast with the papers discussed above, we do not seek to present a mechanism that makes firms' R&D spending procyclical – instead, our approach allows firms' R&D spending to be countercyclical (in line with the empirical micro-based results cited above), while producing procyclical R&D dynamics for industries by adding the composition effect.

2.2 The Baseline Model

The model below introduces a three-level economy where the final good is produced by competitive firms using a Cobb-Douglas technology to combine labour with the composite of outputs provided by homogeneous competitive industries. Each industry's output is made from products supplied by competing monopolist firms engaging in both production and R&D, of which the latter is countercyclical. The mass of monopolist firms in each industry varies procyclically, and will be shown to act as the driving force of the composition effect behind the procyclicality of aggregate R&D spending.

The model captures a number of stylised facts on innovation within the strands of growth and IO literature

1. Macro facts

    (a) Aggregate output and productivity are procyclical (RBC literature);

    (b) Price mark-ups are countercyclical (see, e.g., Christiano et al. (2005), Comin and Gertler (2006), Galí et al. (2007), Justiniano et al. (2010));

    (c) Net entry of firms is procyclical (see Campbell (1998), Clementi and Palazzo (2016)).

2. IO facts

   (a) A firm's R&D spending increases monotonically in the firm's size (see, for example, (Cohen and Klepper, 1996, Stylised fact 2));

   (b) The elasticity of R&D spending with respect to the firm's size is unity (see, e.g., (Cohen and Klepper, 1996, Stylised fact 3)).

2.2.1 Aggregate Production

Suppose that the final (consumable) good $Y(t)$ is produced by competitive firms using fixed amount of labour L and the composite capital good aggregated from intermediate inputs supplied by the constant mass N of symmetric industries. The production technology is linear homogeneous and takes the form

$$Y(t) = \tfrac{1}{1-\nu}\left(\int_0^N y(i;t)^{\,1-\nu} di\right) L^{\nu} = \frac{Ny(t)^{\,1-\nu} L^{\nu}}{1-\nu} \tag{2.1}$$

where $y(i;t)$ is the product of the i-th industry, L is the economy's labour force, and ν is the elasticity of $Y(t)$ with respect to L (and the share of wage income in the economy's output). We assume that all industries are homogeneous, so that $\forall i \; y(i;t) = y(t)$, which gives rise to the last expression in (2.1). The term $\frac{1}{1-\nu}$ is used for normalisation purposes. The price of the final good is chosen as the numeraire. Time is continuous.

Each industry's output $y(i;t)$ is produced competitively by means of a CES production technology, using intermediate inputs $\tilde{y}(i;j;t)$ supplied by homogeneous monopolist firms

$$y(i;t) = y(t) = \left(\int_0^{m(i;t)} \tilde{y}(i;j;t)^{\,\frac{\xi-1}{\xi}} dj\right)^{\frac{\xi}{\xi-1}} = m(t)^{\frac{\xi}{\xi-1}}\,\tilde{y}(t) \tag{2.2}$$

where $m(i;t) = m(t)$ is the dynamically changing mass of intermediate producers in the i-th industry (which is the same across all industries), and ξ is the elasticity of substitution between the products of each two producers. In the next two sections, $m(t)$ embodies the composition effect, and it is its procyclical fluctuations that act as the force offsetting the countercyclicality of individual R&D effort (to be introduced in Section 2.2.2). Throughout the chapter, we use tildes to denote firm-specific quantities. We assume that there are no barriers for the entry/exit of firms to industries.

In line with existing empirical evidence,[15] we assume that the elasticity of substitution between firms' products exceeds that between industries' goods

$$\xi > \frac{1}{\nu} \Leftrightarrow \nu\xi > 1 \tag{2.3}$$

The reason for our choice of assigning production technologies (i.e. aggregation with labour on the economy-wide level and a simple CES aggregator on the industry level) is that doing it otherwise by applying technology (2.1) on the industry level, results in ν playing the double role of determining both the elasticity of output with respect to labour and a firm's relative mark-up (ν and $\frac{\nu}{1-\nu}$, respectively), which would bring ambiguity in the quantitative assessment of the model carried out in Section 2.3.

2.2.2 Individual Firms

Suppose that intermediate inputs are made by firms using production facilities $\tilde{x}(i;j;t) = \tilde{x}(t)$, which can be maintained at the constant marginal costs of ψ. The facilities are created using a one-to-one linear production technology from the final good acquired from the economy's consumers. If the facilities are used exclusively for production, a firm's output equals $z(t)\,\tilde{x}(t)$, where $z(t) = \bar{z} + \hat{z}(t) > 0\ \forall t$ is the economy-wide productivity level, which has the fixed component ($\bar{z}$) and the cyclical component with a bounded image ($\hat{z}(t) \in [z_L; z_H]\ \ \forall t$).[16,17] Follow-

[15]See, e.g., Broda and Weinstein (2006, 2010).

[16]We do not specify whether $\hat{z}(t)$ is stochastic (e.g. a Markov stochastic process with two states as in Aghion and Saint-Paul (1998) and Barlevy (2007)) or a deterministic (e.g. trigonometric) function, as it does not affect the model's key conclusions.

[17]Without loss of generality, the cyclical component $\hat{z}(t)$ is stipulated to have zero mean $\lim\limits_{t\to+\infty} \frac{1}{t} E_0 \left(\int_0^t \hat{z}(\tau)\, d\tau \right) = 0 \Leftrightarrow \lim\limits_{t\to+\infty} \frac{1}{t} E_0 \left(\int_0^t z(\tau)\, d\tau \right) = \bar{z}$.

ing the spirit of real business cycle (RBC) literature (see, e.g., Kydland and Prescott (1982), King, Plosser, and Rebelo (1988)), we assume $z(t)$ to be the source of fluctuations in the model's economy. In what follows, we use $z(t)$ as the cycle indicator, so that some function $B(t)$ is procyclical if $(B(t))'_{z(t)} > 0$, and is countercyclical otherwise.[18] Although cyclicality is usually understood in terms of a variable's alignment with fluctuations of output, the shift to $z(t)$ in our model is justified by the procyclicality of output in terms of $z(t)$ (see equations (2.24), (2.25)).

If a firm allocates part of its facilities $\tilde{\gamma}(i;j;t) = \tilde{\gamma}(t) < \tilde{x}(t)$ to R&D, its production function takes the form[19]

$$\tilde{y}(t) = \left(\left(z(t) \left(\tilde{x}(t) - \tilde{\gamma}(t) \right) \right)^{\frac{\eta-1}{\eta}} + \left(\zeta \tilde{\gamma}(t) \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{2.4}$$

where η is the elasticity of substitution between production and R&D; ζ stands for the intrinsic productivity of R&D.[20] Motivated by empirical evidence (see (Griliches, 1998, Ch. 13)) and similarly to other theoretical works in the field (see, e.g., Comin and Gertler (2006), Barlevy (2007)), we keep ζ constant across the cycle.

[18]Our approach is similar to the one employed in Aghion et al. (2010) (see (Aghion et al., 2010, p. 252)).

[19]Equation (2.4) implies equal importance of production and R&D in creating output – while this might appear restrictive, attaching arbitrary weights ς and $1-\varsigma$ to the two activities does not affect any of the model's conclusions.

[20]By assuming R&D outcomes to be a linear function of R&D spending, we leave outside consideration the stochastic nature of R&D (which is usually modelled as a Poisson process with the arrival rate of $\eta\tilde{\gamma}(t)$ – see, e.g., Grossman and Helpman (1991a), Aghion and Howitt (1992)). Our reasoning for this is that the absence of individual stochasticity keeps all monopolist firms homogeneous, thus significantly improving the tractability of our model and keeping it focused on conveying its key message on the role of aggregation in generating procyclical R&D. In addition, the assumption of linear R&D technology can be reconciled with that of stochastic R&D outcomes if each firm is posited to have access to a sufficiently large number of R&D projects, so that the individual uncertainty of each one of them does not affect the dynamics of the firm's aggregate R&D portfolio (because of, for example, the law of large numbers).

Expression (2.4) allows one to think of the model's R&D as an activity that, only so long as carried out, generates synergy effects with production and has no effect on a firm's future productivity, i.e., it ignores the impact of technology accumulation. In order to focus on our key results though, we leave aside dealing with this consideration until Section 2.4.

In what follows, our focus is on the situation where $\eta > 1$, so that investment in production facilities and in R&D facilities are gross substitutes, and the predictions of the opportunity cost theory become operational: indeed, as will be shown below, during downturns (viz. when $z(t)$ is low), a firm can use substitutability between research and production to mitigate (at least partly) the impact of a slowdown by reassigning a larger share of its facilities to R&D (the opposite is true for intervals of $z(t)$'s high values).[21,22]

Each firm seeks to maximise its profits by choosing the level of output $\tilde{y}(t)$ and the share of facilities devoted to R&D

$$\tilde{\pi}(t) = \tilde{p}(t)\, \tilde{y}(t) - \psi \tilde{x}(t) - \Phi \tag{2.5}$$

$$\begin{gathered} \max_{\tilde{y}(t), \tilde{\gamma}(t)} \{\tilde{p}(t)\, \tilde{y}(t) - \psi \tilde{x}(t) - \Phi\} \\ 0 \leqslant \tilde{\gamma}(t) < \tilde{x}(t) \end{gathered} \tag{2.6}$$

where Φ is the fixed cost of staying in an industry, expressed in units of the final

[21]Mathematically, expression (2.4) can be thought of as a generalisation of the approach used in the model due to Aghion et al. (2010) (see equations (2)–(4) therein) and the stylised model in (Aghion et al., 2012, Sections 2.1, 2.2), wherein the elasticity of substitution between short-run investment and long-run R&D investment is infinity. In our model, however, for the sake of tractability the trade-off between producing and researching is not inter-, but intratemporal, since this chapter does not focus on the role of intertemporal factors affecting R&D decisions (i.e. credit constraints).

[22]Our mechanism of inducing cyclical reallocation of facilities between production and R&D through combining substitutability between the two activities with the fluctuations of the productivity associated with one of them, is akin to those used by Ngai and Pissarides (2007) and Duarte and Restuccia (2010) in studying the phenomenon of structural transformation.

good. In addition, when maximising (2.6), each firm is assumed to ignore its impact on the economy's and an industry's aggregate quantities $Y(t)$ and $y(t)$.

2.2.3 Households

To close the model, we assume that the representative household of size L supplies inelastically the economy's labour force, and owns collectively all firms in the economy. The household's preferences are characterised by a standard twice differentiable instantaneous utility function: $u(c(t))$, $u'(c(t)) > 0$, $u''(c(t)) < 0$. The household's lifetime utility takes the form

$$U = \int_0^{+\infty} e^{-\rho t} u(c(t))\, dt \tag{2.7}$$

where ρ is the intertemporal discount factor.

Finally, the household's total income comprises firms' profits and labour income, and, since the economy has no investment goods, is spent exclusively on consumption, which gives rise to the budget constraint

$$C(t) \equiv c(t)\, L = Nm(t)\, \tilde{\pi}(t) + w(t)\, L \tag{2.8}$$

where $C(t)$ and $c(t)$ are, respectively, total and per capita consumption, and $w(t)$ is the wage rate.

2.2.4 Solution

We shall start with stating competitive producers' inverse demand functions, which can be derived from the corresponding profit maximisation problems. In the case of the final good, the functions are

$$p(t) = \left(\frac{L}{y(t)}\right)^{\nu} \tag{2.9}$$

$$w(t) = \frac{\nu Y(t)}{L} \tag{2.10}$$

for, respectively, each intermediate good and labour, where $p(t)$ is the price of an industry's output. As regards intra-industry demand functions, those take the form

$$\tilde{p}(t) = \frac{p(t)\, y(t)^{\frac{1}{\xi}}}{\tilde{y}(t)^{\frac{1}{\xi}}} \tag{2.11}$$

Using (2.9) and (2.11) allows one to solve an intermediate producer's problem (2.6). First of all, the division of facilities between production and R&D can be pinned down by solving the following cost-minimisation problem

$$\begin{gathered}\min_{\tilde{x}(t),\tilde{\gamma}(t)} \{\psi\ \tilde{x}(t)\}\ \ s.t. \\ \left((z(t)\,(\tilde{x}(t)-\tilde{\gamma}(t)))^{\frac{\eta-1}{\eta}} + (\zeta\tilde{\gamma}(t))^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \leqslant \tilde{y}(t)\end{gathered} \tag{2.12}$$

The optimal allocation of a firm's facilities, as implied by (2.12), is

$$\tilde{y}(t) = Z(t)\,\tilde{x}^*(t) \Leftrightarrow \tilde{x}^*(t) = \frac{\tilde{y}(t)}{Z(t)},\ \ Z(t) \equiv \left(\zeta^{\eta-1} + z(t)^{\,\eta-1}\right)^{\frac{1}{\eta-1}} \tag{2.13}$$

$$\tilde{\gamma}^*(t) = \frac{\zeta^{\eta-1}}{\zeta^{\eta-1}+z(t)^{\,\eta-1}}\tilde{x}^*(t) = \frac{\zeta^{\eta-1}}{\left(\zeta^{\eta-1}+z(t)^{\,\eta-1}\right)^{\frac{\eta}{\eta-1}}}\tilde{y}^*(t) = \left(\frac{\zeta}{Z(t)}\right)^{\eta-1}\frac{\tilde{y}^*(t)}{Z(t)} \tag{2.14}$$

where, given the absence of technology accumulation, $Z(t)$ is the total productivity of a firm's facilities arising from their optimal allocation across production and R&D. $Z(t)$'s functional form suggests it to be procyclical, which is why it will be used thereafter as cycle indicator instead of $z(t)$ for tractability's sake.

The first feature to note in (2.14) is that the share of R&D facilities $\frac{\tilde{\gamma}^*(t)}{\tilde{x}^*(t)}$ is always smaller than 1 or, equivalently, a firm always engages in both production and R&D. As suggested by (2.14), when investing in R&D is a substitute for investing in production facilities (i.e. $\eta > 1$), $\tilde{\gamma}^*(t)$ exhibits countercyclical properties (viz. $(\tilde{\gamma}^*(t))'_{z(t)} < 0$), in accordance with the prescriptions of the opportunity cost theory. Note also that, in line with stylised fact 2.a (see p. 48), a firm's R&D spending $\psi\tilde{\gamma}^*(t)$ increases in its scale of production $\tilde{y}(t)$, and the two quantities are proportional to each other (i.e., the elasticity of the former with respect to the latter is unity, as stipulated by stylised fact 2.b, p. 48).

Expression (2.13) reflects the synergy effect of R&D as embedded in the 'preference for diversity' (or, alternatively, 'taste for variety') feature of the CES production technology (2.4): as long as substitution between investing in production and R&D is not complete (i.e. $\eta < +\infty$), for any level of R&D productivity ζ and any size of production facilities $\tilde{x}(t)$ optimal engaging in both production and R&D results in a higher level of output than one stemming exclusively from production: $(\zeta^{\eta-1} + z(t)^{\eta-1})^{\frac{1}{\eta-1}} \tilde{x}(t) > z(t)\, \tilde{x}(t)$. Put differently, optimal engaging in R&D brings about a boost of productivity (as compared to the situation when no R&D is performed) of the size of $(\zeta^{\eta-1} + z(t)^{\eta-1})^{\frac{1}{\eta-1}} - z(t) > 0$ for any levels of intrinsic productivity in R&D and production.

A noteworthy property of production technology (2.4) is that R&D-induced productivity level $Z(t)$ decreases in η, which reflects the fact that as production and R&D become more easily substitutable, the importance of each separate activity diminishes, thereby dragging down the synergy of their joined use.

Plugging (2.11) and (2.14) in profit maximisation problem (2.6) and deriving the FOC pins down firms' prices and volumes

$$\tilde{p}^*(t) = \frac{\xi\psi}{\xi - 1} \cdot \frac{1}{Z(t)} \tag{2.15}$$

It is worth noting that expression (2.15) implies a firm's mark-up $\mu(t) = \frac{\tilde{p}^*(t)}{\psi} - 1$ to be countercyclical, which is widely supported by existing macroeconomic literature (see stylised fact 1.b, p. 47).

A firm's sales volume can be derived using (2.2), (2.11) and (2.15)

$$\tilde{y}^*(t) = \frac{L^{\nu\xi} y(t)^{1-\nu\xi}}{\tilde{p}^*(t)^{\xi}} \tag{2.16}$$

$$\tilde{y}^*(t) = \frac{Lm(t)^{\frac{\xi}{\xi-1}\frac{1-\nu\xi}{\nu\xi}}}{\tilde{p}^*(t)^{\frac{1}{\nu}}} \tag{2.17}$$

Free entry to industries entails zero profits for every firm, which enables one to express the number of firms per industry as a function of $z(t)$. To that end, one can calculate a firm's output first

$$\tilde{p}^*(t)\,\tilde{y}^*(t) - \psi\tilde{x}^*(t) = \Phi$$

$$\tilde{y}^*(t) = \frac{\xi\Phi}{\tilde{p}^*(t)} = \frac{(\xi-1)\,Z(t)\,\Phi}{\psi} \tag{2.18}$$

Combining (2.17) and (2.18) yields the final expression for the equilibrium number of firms per industry $m^*(t)$

$$m^*(t) = \left(\frac{L}{\xi\Phi\,\tilde{p}^*(t)^{\frac{1-\nu}{\nu}}}\right)^{\frac{\xi-1}{\xi}\frac{\nu\xi}{\nu\xi-1}} = \left(\frac{L}{\xi\Phi}\left(\frac{\xi-1}{\xi\psi}Z(t)\right)^{\frac{1-\nu}{\nu}}\right)^{\frac{\nu(\xi-1)}{\nu\xi-1}} \tag{2.19}$$

Naturally, (2.19) shows $m^*(t)$ to depend positively on the size of the economy's labour force (which effectively determines the scale of the economy – thereby a larger one has more firms), and to decrease in both cost parameters ψ and Φ. In addition, as expression (2.19) suggests, firm entry is procyclical: $(m^*(t))'_{z(t)} = (m^*(t))'_{Z(t)} \cdot (Z(t))'_{z(t)} > 0$, in line with existing empirical evidence (see stylised fact 1.c on page 47).

Given (2.18), one can derive the closed-form expression for $\tilde{\gamma}(t)^*$

$$\tilde{\gamma}^*(t) = \left(\frac{\zeta}{Z(t)}\right)^{\eta-1}\frac{(\xi-1)\,\Phi}{\psi} \tag{2.20}$$

Finally, given that the cost of maintaining the facilities of the unitary size is ψ units of the final good, a firm's individual R&D spending amounts to

$$\psi\tilde{\gamma}^*(t) = \left(\frac{\zeta}{Z(t)}\right)^{\eta-1}(\xi-1)\,\Phi \tag{2.21}$$

Note that since $Z(t)$ increases in $z(t)$ and since $\eta > 1$, a firm's R&D spending is countercyclical – in line with the predictions of Schumpeter's hypothesis.

Expression (2.21), together with (2.14), suggests a way of understanding how a larger amount of individual R&D can be compatible with a smaller number of R&D performers during a downturn: as the intrinsic productivity in production

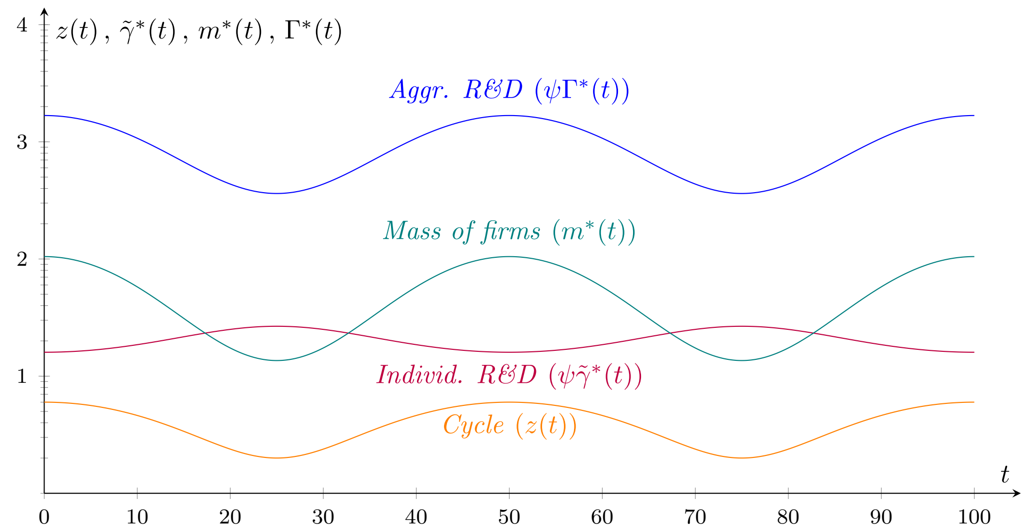

Figure 2.1: Trajectories of $z(t)$,[23] $\psi\tilde{\gamma}(t)$, $\psi\Gamma(t)$, $m^*(t)$ on the $\log_{10}$ scale. The values of parameters used are: $a = 4$, $b = 2$, $\omega = 50$, $\eta = 2$, $\xi = 5$, $\nu = \frac{1}{3}$, $\psi = 1$, $\Phi = 10$, $\zeta = 4$, $L = 10$.

falls, a firm reacts by shifting part of its facilities to R&D. These actions, while not being able to completely nullify the adverse impact of a slowdown, maximally cushion it, thus putting the firm in the least harmful situation possible during a downturn and thereby reducing the number of firms ceasing to operate.

The behaviour of aggregate R&D spending in an industry $\psi\Gamma^*(i;t) = \psi\Gamma^*(t) \equiv \psi\tilde{\gamma}^*(t)\, m^*(t)$ is described by the expression

$$\begin{aligned} \psi\Gamma^*(t) &= \psi\Gamma Z(t)^{\frac{(1-\nu)(\xi-1)}{\nu\xi-1}-(\eta-1)}, \\ \Gamma &\equiv \frac{\zeta^{\eta-1}(\xi-1)\,\Phi L^{\frac{\nu\xi-\nu}{\nu\xi-1}}}{\left(\xi\Phi\left(\frac{\xi-1}{\xi}\psi\right)^{\frac{1}{\nu}-1}\right)^{\frac{\nu\xi-\nu}{\nu\xi-1}}} \end{aligned} \tag{2.22}$$

The pro-/countercyclicality properties of $\Gamma^*(t)$ are determined by whether the power term $\frac{(1-\nu)(\xi-1)}{\nu\xi-1}-(\eta-1)$ is positive (procyclicality) or negative (countercyclicality). Given the motivation of this chapter, we are interested in specifying the conditions for the former case

[23] The economy's cycle driver $z(t)$ is paramterised as a trigonometric function $z(t) = a + b\cos\left(\frac{2\pi t}{\omega}\right)$.

$$\eta - 1 < \frac{(1-\nu)(\xi-1)}{\nu\xi-1}$$

$$\eta < \frac{\xi+\nu-2}{\nu\xi-1} \equiv \hat{\eta}_0 \tag{2.23}$$

Condition (2.23) constrains the range of η's values from above, so that $\psi\Gamma^*(t)$ is procyclical when $\eta \in (1; \hat{\eta}_0)$. Naturally, if the degree of substitutability between production and R&D is limited, shifts between the two activities during the cycle are less pronounced on the firm level and, hence, are reversed on the industry level by firm entry/exit dynamics (see Figure 2.1).[24]

To illustrate the last point, suppose to the contrary that $\eta \to +\infty$ (i.e., nearly complete substitution between production and R&D takes place), so that the production technology asymptotically becomes $\lim_{\eta\to+\infty} \tilde{y}(t) = \max\{z(t); \zeta\}\, \tilde{x}(t)$. Suppose that $z(t) > \zeta$ during upturns and vice versa during downturns. In this case, given (2.20), a firm's R&D spending is going to be $\lim_{\eta\to+\infty} \tilde{\gamma}^*(t) = 0$ and $\lim_{\eta\to+\infty} \tilde{\gamma}^*(t) = \frac{(\xi-1)\Phi}{\psi}$ during upturns and downturns, respectively. Obviously, for any trajectory of the mass of firms per industry[25] $\{m^*(t)\}_{t=0}^{+\infty}$, an industry's R&D spending is going to be zero during upturns, and positive during downturns, since shifts in $m(t)$ cannot overcome (asymptotically) complete substitution of production facilities for R&D ones on the firm level (see an example in Figure 2.2).

As the last step in solving the model, one can derive the closed-form expressions for industrial and economy-wide aggregates. The equilibrium output of an industry can be pinned down by combining (2.18) and (2.19) with the fact

[24] Note that condition (2.23) equally applies to R&D spending in the whole economy, as the latter equals R&D spending within an industry, scaled by N.

[25] One can show that $\lim_{\eta\to+\infty} m^*(t) = \max\{\zeta; z(t)\}^{\frac{1-\nu}{\nu}} \left(\frac{L}{\xi\Phi\left(\frac{\xi}{\xi-1}\psi\right)^{\frac{1-\nu}{\nu}}} \right)^{\frac{\xi-1}{\xi}\frac{\nu\xi}{\nu\xi-1}}$.

[26] The functional form of $z(t)$ and all other parameters' values are as in Figure 2.1.

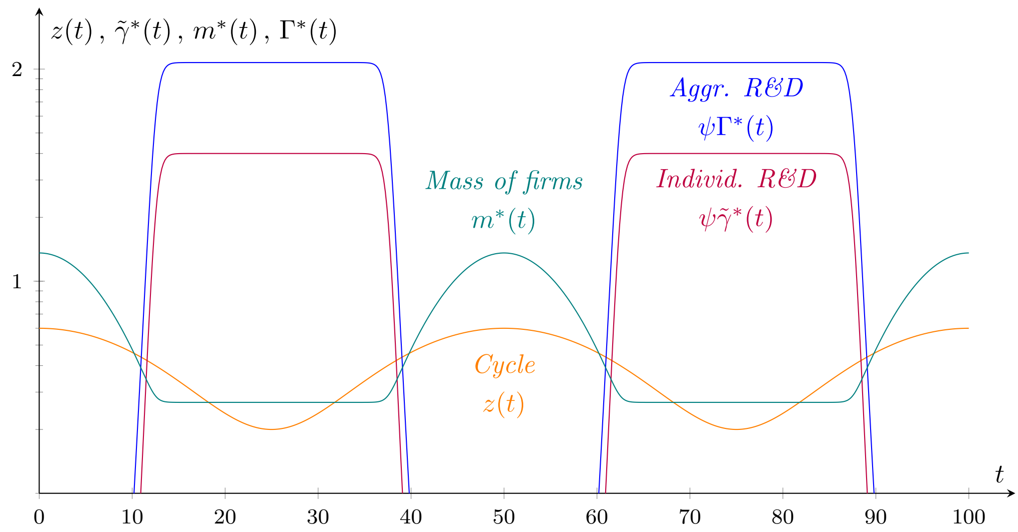

Figure 2.2: Time trajectories of $z(t)$, $\psi\tilde{\gamma}^*(t)$, $\psi\Gamma^*(t)$, $m^*(t)$ on the $\log_{10}$ scale for high substitutability between production facilities and R&D facilities ($\eta = 40$).[26]

that $y(t) = m(t)^{\frac{\xi}{\xi-1}}\ \tilde{y}(t)$

$$y(t) = \Phi^{-\frac{1}{\nu\xi-1}} \left(\frac{\xi-1}{\xi\psi} Z(t)\right)^{\frac{\xi-1}{\nu\xi-1}} \left(\frac{L}{\xi}\right)^{\frac{\nu\xi}{\nu\xi-1}} \tag{2.24}$$

Plugging (2.24) into (2.1) and (2.10) yields the closed-form results for $Y(t)$ and $w(t)$

$$Y(t) = \frac{\Phi^{-\frac{1-\nu}{\nu\xi-1}}}{1-\nu} \left(\xi^{-\frac{\xi+\nu-1}{\xi-1}} \left(\frac{\xi-1}{\xi\psi} Z(t)\right)^{1-\nu} L^{\nu}\right)^{\frac{\xi-1}{\nu\xi-1}} \tag{2.25}$$

$$w(t) = \frac{\nu}{1-\nu} \left(\frac{L}{\Phi}\right)^{1-\nu} \left(\xi^{-\frac{\xi+\nu-1}{\xi-1}} \left(\frac{\xi-1}{\xi\psi} Z(t)\right)^{1-\nu}\right)^{\frac{\xi-1}{\nu\xi-1}} \tag{2.26}$$

Equations (2.24)–(2.26) formally establish the positivity of the relationships between $z(t)$ on one hand, and $y(t)$, $Y(t)$, $w(t)$ on the other. Finally, given that (because of free entry) firms accrue zero profits, the representative household's income comprises only its labour component $C(t) = w(t)\, L = \nu Y(t)$. This result completes the solution of the model.

2.3 Evaluating the Model

Having solved the model, we conclude its discussion with assessing the empirical plausibility of its key result – aggregated procyclicality condition (2.23). In particular, our approach splits into two steps: firstly, we retrieve the range of η's values from existing empirical literature, after which we compare it against an estimate of $\hat{\eta}_0$.

The first step can be achieved using the estimates of the econometric model introduced in Aghion et al. (2012) and later employed by Beneito et al. (2015), whereby the natural logarithm of a firm's R&D ($\ln \tilde{\gamma}^*(t) = b_0 - (\eta - 1)\ln Z(t)$ in our model's notations)[27] is regressed, among others, on the increment of the natural logarithm of the firm's sales volume ($(\ln \tilde{y}^*(t))'_t = \frac{(\tilde{y}^*(t))'_t}{\tilde{y}^*(t)} = \frac{\dot{Z}(t)}{Z(t)}$ in our model's notations). If $-b_1$ is the coefficient at $(\ln \tilde{y}^*(t))'_t$ in the regression in hand, it determines the marginal effect of $(\ln \tilde{y}^*(t))'_t$ on $\ln \tilde{\gamma}^*(t)$. In our model, this effect can be replicated by differentiating $\ln \tilde{\gamma}^*(t)$ at fixed time t, with respect to an external variation in $(\ln \tilde{y}^*(t))'_t$, denoted as Δ

$$(\ln \tilde{\gamma}^*(t))'_\Delta = -(\eta - 1)\left(\int_0^t (\ln \tilde{y}^*(\tau))'_\tau \, d\tau\right)'_\Delta = -(\eta - 1) = -b_1 \qquad (2.27)$$

Equation (2.27) allows one to recover the value of η from b_1 as $\eta = 1 + b_1$. Given this result, one can turn to the empirical studies mentioned above to land η's value in the intervals listed in Table 2.2. Overall, the range of η's empirical values is bounded by approximately 2 from above.

Moving on to assessing $\hat{\eta}_0$, following Matsuyama (1999) and Wälde (2005), we interpret the aggregate capital good as a combination of both physical and human capital, which puts the estimate of ν at the approximate level of $1/3$ (see, e.g., Parente and Prescott (2000)). As concerns ξ, its estimates are usually placed

[27] In order to guarantee that $\ln \tilde{\gamma}^*(t)$ be well-behaved when R&D is zero, 1 is added to it in the studies cited in the text. We ignore this transformation in our calculations, as in our model $\tilde{\gamma}^*(t)$ is always positive.

Table 2.2: Deduced empirical ranges of η's values.

Study	Country	η's values
Aghion et al. (2012)	France	$[1.032; 1.11]$
Beneito et al. (2015)	Spain	2.055

in the interval from 3 to 7 (see, e.g., Montgomery and Rossi (1999), Dubé and Manchanda (2005), Broda and Weinstein (2006), Broda and Weinstein (2010)). Together, the two estimates suggest that individual countercyclicality of R&D is reversed on the industry level if η belongs to the interval whose lower bound is 1, and whose upper bound $\hat{\eta}$ diminishes from $+\infty$ to 4 as ξ goes from 3 to 7. Regardless of the exact value of ξ, our estimates of η are below $\hat{\eta}_0$, which validates empirical plausibility of condition (2.23).

2.4 Extension №1 – Technology Accumulation

2.4.1 Mechanics of Technology Accumulation

In this section, we extend the baseline model by allowing innovation to have lasting effects on productivity levels. In particular, we assume that a firm's production technology takes the form

$$\tilde{y}(t) = Q(t)\left(\left(z(t)\left(\tilde{x}(t) - \tilde{\gamma}(t)\right)\right)^{\frac{\eta-1}{\eta}} + \left(\zeta\tilde{\gamma}(t)\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{2.28}$$

where $Q(t)$ is the economy-wide technology level, whose growth is a spillover of individual R&D effort,[28] in the spirit of Romer (1986):

$$g_Q(t) \equiv \frac{\dot{Q}(t)}{Q(t)} = \lambda\left(\frac{\tilde{\gamma}(t)}{Q(t)^{\,1+\chi}}\right) \tag{2.29}$$

where $\lambda(\cdot)$ is an increasing differentiable function $\lambda'(\cdot) > 0$ of bounded mean

[28]One can think of $\tilde{\gamma}(t)$ in (2.29) as the average R&D effort across firms: $\bar{\gamma}(t) = \int_0^N \int_0^{m(i;t)} \frac{\tilde{\gamma}(i;j;t)}{Nm(i;t)} dj di$, which collapses to $\tilde{\gamma}(t)$ given firms' homogeneity.

oscillation (BMO).[29] Power term $\chi > 0$ connects the dynamics of $Q(t)$ with that of a firm's fixed costs

$$\Phi(t) = \phi Q(t)^{\chi} \tag{2.30}$$

The relationship between $Q(t)$ and $\Phi(t)$, as expressed in (2.30), is introduced so that we can gain an additional degree of freedom that will be used in the quantitative assessment of the extension in hand, carried out in Section 2.5. We impose the following restriction on the range of χ's values

$$\chi < \frac{1-\nu}{\nu} \tag{2.31}$$

Condition guarantees that the number of firms $m(t)$, each industry's output $y(t)$ and total output $Y(t)$ increase in time in the long-run.[30]

We divide $\tilde{\gamma}(t)$ by $Q(t)^{1+\chi}$ to reflect the idea that new ideas are harder to obtain as the economy develops and becomes more complex.[31] From the mathematical standpoint, this assumption ascertains that the economy attains a balanced growth path with stationary growth rates. All other equations are as in the baseline model.

As suggested by (2.28) and (2.29), engaging in R&D creates two effects: a temporary synergetic one (introduced and discussed in Section 2.2) and a permanent cost-decreasing one. The latter is channelled through the continuous instantaneous embedding of individual research effort in the aggregate stock of public knowledge – i.e., newly discovered technologies become instantly available for general use, which enhances public knowledge, based upon which further

[29]The fact that $\lambda(\cdot)$ is BMO guarantees the existence of $Q(t)$'s average growth rate (to be derived below, see (2.40)–(2.42)).

[30]Formally the results we obtain below (see (2.34), (2.36), (2.37)) suggest that χ's upper bound should be $\min\left\{\xi - 1; \frac{1-\nu}{\nu}\right\}$, but the latter expression collapses to $\frac{1-\nu}{\nu}$ given condition (2.3).

[31]A similar assumption is made in, e.g., Jones (1995), Bental and Peled (1996), Howitt (1999).

discoveries are made.

2.4.2 Solution

Going through the same steps as in solving the baseline model, yields the following results

$$\tilde{p}^*(t) = \frac{\xi\psi}{\xi - 1} \cdot \frac{1}{Q(t)\, Z(t)} \tag{2.32}$$

$$\tilde{y}^*(t) = \frac{\xi - 1}{\psi} \Phi(t)\, Q(t)\, Z(t) \tag{2.33}$$

$$m^*(t) = \left(\frac{L}{\xi\Phi(t)} \left(\frac{(\xi - 1)\, Q(t)\, Z(t)}{\xi\psi} \right)^{\frac{1-\nu}{\nu}} \right)^{\frac{\nu(\xi-1)}{\nu\xi-1}} \tag{2.34}$$

$$\tilde{\gamma}^*(t) = \left(\frac{\zeta}{Z(t)} \right)^{\eta-1} \frac{(\xi - 1)\, \Phi(t)\, Q(t)}{\psi} \tag{2.35}$$

The industry and economy-wide aggregates are

$$y(t) = \Phi(t)^{-\frac{1}{\nu\xi-1}} \left(\frac{\xi - 1}{\xi\psi} Q(t)\, Z(t) \right)^{\frac{\xi-1}{\nu\xi-1}} \left(\frac{L}{\xi} \right)^{\frac{\nu\xi}{\nu\xi-1}} \tag{2.36}$$

$$Y(t) = \frac{\Phi(t)^{-\frac{1-\nu}{\nu\xi-1}}}{1 - \nu} \left(\xi^{-\frac{\xi+\nu-1}{\xi-1}} \left(\frac{\xi - 1}{\xi\psi} Q(t)\, Z(t) \right)^{1-\nu} L^{\nu} \right)^{\frac{\xi-1}{\nu\xi-1}} \tag{2.37}$$

The only respect in which the extension's solution (2.32)–(2.37) differs from that of the baseline model in Section 2.2.4, is the temporal variability of firms' fixed costs $\Phi(t)$ and the presence of term $Q(t)$ for the economy's aggregate accumulated technology.

As follows from (2.32)–(2.37), by calculating the growth rate of $Q(t)$ one can pin down those of the economy's variables. Combining (2.29) with (2.35) suggests that $g_Q(t)$ takes the form

$$g_Q(t) = \lambda \left(\left(\frac{\zeta}{Z(t)} \right)^{\eta-1} \frac{(\xi - 1)\, \phi}{\psi} \right) \tag{2.38}$$

First of all, given that $g_Q(t)$ derives from individual R&D spending $\tilde{\gamma}^*(t)$, it inherits the latter's countercyclical properties. In addition, note that $g_Q(t)$ depends positively on the fixed cost multiplier ϕ, as higher fixed costs lead to a drop in the mass of firms $m^*(t)$ and, in turn, an increase in sales volumes (and, as a result, R&D spending – as follows from stylised fact 2.b) of those remaining in the market. Since $g_Q(t)$ depends on the level of individual R&D effort, it remains unaffected by the dynamics of $m(t)$, and thus reflects only the positive impact of a higher ϕ on individual R&D spending.

Combining (2.29) with (2.38) yields the expression for the technology level $Q(t)$

$$Q(t) = Q_0 e^{\int_0^t g_Q(\tau)d\tau} = Q_0 e^{\int_0^t \lambda\left(\left(\frac{\zeta}{Z(\tau)}\right)^{\eta-1}\frac{(\xi-1)\phi}{\psi}\right)d\tau} \tag{2.39}$$

where Q_0 is the initial technology level. Following Wälde (2005), we treat $Q(t)$ as the product of the trend $\bar{Q}(t)$ and cyclical component $\hat{Q}(t)$[32]

$$\bar{Q}(t) \equiv Q_0 e^{\bar{g}_Q t} \tag{2.40}$$

$$\hat{Q}(t) \equiv \frac{Q(t)}{\bar{Q}(t)} = e^{\int_0^t \left(\lambda\left(\left(\frac{\zeta}{Z(\tau)}\right)^{\eta-1}\frac{(\xi-1)\phi}{\psi}\right)-\bar{g}_Q\right)d\tau} \tag{2.41}$$

$$\bar{g}_Q \equiv \lim_{t\to+\infty}\frac{1}{t}\int_0^t \frac{\dot{Q}(\tau)}{Q(\tau)}d\tau = \lim_{t\to+\infty}\frac{1}{t}\int_0^t \lambda\left(\left(\frac{\zeta}{Z(\tau)}\right)^{\eta-1}\frac{(\xi-1)\,\phi}{\psi}\right)d\tau \tag{2.42}$$

where $\bar{g}_Q$ is the average (or, equivalently, the long-run) growth rate of $Q(t)$. By analogy with $\bar{g}_Q$, the average growth rates of the economy's other level variables can be defined as $\bar{g}_X = \lim\limits_{t\to+\infty}\frac{1}{t}\int_0^t \frac{\dot{X}(\tau)}{X(\tau)}d\tau$, and shown to be as follows

Observation 2.1.

$$\bar{g}_{\tilde{y}} = \bar{g}_{\tilde{\gamma}} = (1+\chi)\,\bar{g}_Q \tag{2.43}$$

[32]The existence of $\bar{g}_Q$ follows from $\lambda(\cdot)$'s being a BMO function.

$$\bar{g}_m = \frac{\xi - 1}{\xi} \frac{\nu\xi}{\nu\xi - 1} \left(\frac{1-\nu}{\nu} - \chi \right) \bar{g}_Q \tag{2.44}$$

$$\bar{g}_y = \frac{\xi}{\xi - 1} \bar{g}_m + \bar{g}_{\tilde{y}} = \frac{\xi - \chi - 1}{\nu\xi - 1} \bar{g}_Q \tag{2.45}$$

$$\bar{g}_Y = (1-\nu)\,\bar{g}_y = (1-\nu)\,\frac{\xi - \chi - 1}{\nu\xi - 1} \bar{g}_Q \tag{2.46}$$

Proof. See Appendix A.1.1. ■

As regards the cyclical components of the economy's variables, starting with investigating $\hat{Q}(t)$ suggests that it can be potentially unsynchronised with $Z(t)$. To see that (in a heuristic fashion), one can calculate the implicit derivative of $\hat{Q}(t)$ with respect to $Z(t)$

$$\begin{aligned} \frac{d\hat{Q}(t)}{dZ(t)} =& \frac{\left(\lambda \left(\left(\frac{\zeta}{Z(t)} \right)^{\eta-1} \frac{(\xi-1)\phi}{\psi} \right) - \bar{g}_Q \right) \hat{Q}(t)}{\dot{Z}(t)} = 0 \Leftrightarrow \\ \Leftrightarrow& \lambda \left(\left(\frac{\zeta}{Z(t)} \right)^{\eta-1} \frac{(\xi-1)\,\phi}{\psi} \right) = \bar{g}_Q \end{aligned} \tag{2.47}$$

As follows from (2.47), intervals of $\hat{Q}(t)$'s monotonicity in general do not coincide with those of $Z(t)$'s. For the sake of the extension's results' generality and analytical tractability, we rule out this situation by assuming that a downturn in the economy's dynamics starts with a discrete jump in the value of $Z(t)$ such that $Z(t) < \zeta \left(\frac{(\xi-1)\phi}{\psi \lambda^{-1}(\bar{g}_Q)} \right)^{\frac{1}{\eta-1}} \equiv \bar{Z}$, whereas an upturn is initiated by a jump in $Z(t)$ putting its value above $\bar{Z}$, so that the cyclical patterns in $Z(t)$ and $\hat{Q}(t)$ coincide.

As in Wälde (2005), other variables' cyclical components can be defined by replacing $Q(t)$ with $\hat{Q}(t)$ in formulae (2.32)–(2.37). In particular, in order to investigate the cyclical behaviour of $\Gamma^*(t) \equiv \tilde{\gamma}^*(t)\, m^*(t)$, one can express its cyclical component as follows

$$\hat{\Gamma}^{*}(t)=\frac{\zeta^{\eta-1}\left(\xi-1\right)\phi L^{\frac{\nu(\xi-1)}{\nu\xi-1}}}{\left(\xi\phi\left(\frac{\xi-1}{\xi}\psi\right)^{\frac{1}{\nu}-1}\right)^{\frac{\nu(\xi-1)}{\nu\xi-1}}}\hat{Q}(t)^{\frac{(\xi-1)-(1+\chi)(1-\nu)}{\nu\xi-1}}Z(t)^{\frac{(\xi-1)(1-\nu)}{\nu\xi-1}-(\eta-1)} \tag{2.48}$$

As equation (2.48) suggests, R&D spending on the industry level is procyclical if

$$\begin{aligned}\frac{\left(\hat{\Gamma}^{*}(t)\right)'_{z(t)}}{\hat{\Gamma}^{*}(t)}=\frac{(\xi-1)-(1+\chi)(1-\nu)}{\nu\xi-1}\frac{\left(\hat{Q}(t)\right)'_{z(t)}}{\hat{Q}(t)}+\\+\left(\frac{(\xi-1)(1-\nu)}{\nu\xi-1}-(\eta-1)\right)\frac{(Z(t))'_{z(t)}}{Z(t)}>0\end{aligned} \tag{2.49}$$

Although the exact specification of (2.49) depends on the functional form of $\lambda(\cdot)$, a necessary and a sufficient condition for (2.49) can be derived even without this piece of information. We shall start with the former. First of all, one may note that term $\frac{\xi-1-(1+\chi)(1-\nu)}{\nu\xi-1}\frac{\left(\hat{Q}(t)\right)'_{z(t)}}{\hat{Q}(t)}$ is negative since $\frac{\left(\hat{Q}(t)\right)'_{z(t)}}{\hat{Q}(t)}<0$ and $\frac{\xi-1-(1+\chi)(1-\nu)}{\nu\xi-1}>0$,[33] which implies that $\frac{\left(\hat{\Gamma}^{*}(t)\right)'_{z(t)}}{\hat{\Gamma}^{*}(t)}<\left(\frac{(\xi-1)(1-\nu)}{\nu\xi-1}-(\eta-1)\right)\frac{(Z(t))'_{z(t)}}{Z(t)}$, thereby suggesting the necessary condition

$$\left(\frac{(\xi-1)(1-\nu)}{\nu\xi-1}-(\eta-1)\right)\frac{(Z(t))'_{z(t)}}{Z(t)}>0\Rightarrow\eta<\frac{\xi+\nu-2}{\nu\xi-1}\equiv\hat{\eta}_{1}^{N}=\hat{\eta}_{0} \tag{2.50}$$

which coincides with condition (2.23) obtained for the economy without technology accumulation. This result comes from the fact that condition (2.50) is derived effectively by omitting term $\frac{\left(\hat{Q}(t)\right)'_{z(t)}}{\hat{Q}(t)}$, through which the impact of technology accumulation is projected, and without which the cyclical behaviour of $\hat{\Gamma}^{*}(t)$ is affected only by the dynamics of $Z(t)$, as in the baseline model.

In order to derive a sufficient condition for (2.49), we will use the countercyclicality of mark-ups (and, equivalently, the model's prices) to show first that $\frac{\left(\hat{Q}(t)\right)'_{z(t)}}{\hat{Q}(t)}>-\frac{(Z(t))'_{z(t)}}{Z(t)}$. To that end, note that, as the cyclical form of (2.32)

[33] The last assertion follows from the assumptions that $\chi<\frac{1-\nu}{\nu}$ and $\nu\xi-1>0\Leftrightarrow\xi>\frac{1}{\nu}$: $\xi-1>\frac{1}{\nu}-1=(1-\nu)\left(\frac{1}{\nu}-1+1\right)>(1-\nu)(1+\chi)\Rightarrow\xi-1-(1-\nu)(1+\chi)>0$.

($\hat{p}^*(t) = \frac{\xi\psi}{\xi-1} \cdot \frac{1}{\hat{Q}(t) Z(t)}$) suggests, since prices are countercyclical, and since their cyclical behaviour is determined by that of $\hat{Q}(t)\, Z(t)$, the latter has to be procyclical. As the product's components fluctuate in the opposite directions – i.e., $\hat{Q}(t)$ is countercyclical, $Z(t)$ is procyclical – for its overall procyclicality to obtain, $Z(t)$'s procyclicality has to dominate $\hat{Q}(t)$'s countercyclicality, viz. $\left(\hat{Q}(t)\, Z(t)\right)'_{z(t)} > 0 \Leftrightarrow \left(\hat{Q}(t)\right)'_{z(t)} Z(t) + \left(Z(t)\right)'_{z(t)} \hat{Q}(t) > 0$, which gives rise to the desired inequality stated above. Combining it with (2.49) yields the sufficient condition

$$\frac{\left(\hat{\Gamma}^*(t)\right)'_{z(t)}}{\hat{\Gamma}^*(t)} > \left(\frac{(\xi-1)(1-\nu)}{\nu\xi-1} - \frac{\xi-1-(1+\chi)(1-\nu)}{\nu\xi-1} - (\eta-1)\right) \frac{\left(Z(t)\right)'_{z(t)}}{Z(t)}$$

$$\left(\frac{(1+\chi)(1-\nu)-\nu(\xi-1)}{\nu\xi-1} - (\eta-1)\right) \frac{\left(Z(t)\right)'_{z(t)}}{Z(t)} > 0$$

$$\eta < \frac{\chi(1-\nu)}{\nu\xi-1} \equiv \hat{\eta}_1^S \tag{2.51}$$

Given that condition (2.51) is sufficient, it is more stringent than the necessary condition (2.50) – i.e., the upper limit it imposes on η, is lower than that implied by (2.50)), since $\xi - 1 > (1+\chi)(1-\nu)$ (see footnote 33). Such a result comes from the fact that the sufficient condition deals with the case of the highest permissible degree of $\hat{Q}(t)$'s countercyclicality feeding into and reinforcing that of $\tilde{\gamma}(t)$. Since therefore the countercyclicality of $\tilde{\gamma}(t)$ is more pronounced (as compared to the baseline case), it can be offset by the dynamics of $m(t)$, if a smaller share of firms' facilities is shifted between production and R&D, which is controlled by a lower η.

2.5 Evaluating the Extension

Following the logic and structure of Section 2.3, we focus on the range of η's values first. Despite the presence of additional temporal terms $\Phi(t)$ and $Q(t)$ in the expressions for a firm's output (2.33) and R&D spending (2.35), one could argue that η's deduced values obtained in Section 2.3 still carry through, as the impact of both of these terms is, in essence, economy-wide and, as a result, would be captured by time-specific fixed effects used by both Aghion et al. (2012) and Beneito et al. (2015) in their estimating procedures.[34] Thus, the relationship between a firm's output fluctuations and R&D, as characterised in the cited studies' results, is driven, in our model's terms, by the interaction between $-(\eta-1)\ln Z(t)$ and $\frac{\dot{Z}(t)}{Z(t)}$, as in Section 2.3.[35]

The remainder of this section focuses on the evaluation of condition (2.51), for which one first needs to assess the range of χ's values. To that end, we first combine data on the growth rates of U.S. total output ($\bar{g}_Y \approx 0.027$)[36] and those of the number of U.S. firms ($\bar{g}_m \approx 0.011$)[37] during the period from 1977 to 2013, to express that of a firm's production levels $\bar{g}_{\tilde{y}} = \frac{\bar{g}_Y}{1-\nu} - \frac{\xi \bar{g}_m}{\xi-1}$. Parameter χ can be evaluated by assuming a linear relationship between $\bar{g}_{\tilde{y}}$ and $\bar{g}_m$: $\bar{g}_{\tilde{y}} = a\bar{g}_m = \left(\frac{g_{\bar{Y}}/g_{\bar{m}}}{1-\nu} - \frac{\xi}{\xi-1}\right) g_{\bar{m}}$, and then by pinning down χ as a function of $g_{\bar{Y}}/g_{\bar{m}}$, ξ and ν[38]

$$\begin{aligned} \frac{\bar{g}_{\tilde{y}}}{\bar{g}_m} &= \frac{1+\chi}{\frac{\xi-1}{\xi}\frac{\nu\xi}{\nu\xi-1}\left(\frac{1-\nu}{\nu}-\chi\right)} = \frac{g_{\bar{Y}}/g_{\bar{m}}}{1-\nu} - \frac{\xi}{\xi-1} \Leftrightarrow \\ &\Leftrightarrow \chi = \frac{\left(\frac{g_{\bar{Y}}}{g_{\bar{m}}}-1\right)(1-\nu)(\xi-1)}{\frac{g_{\bar{Y}}}{g_{\bar{m}}}\nu(\xi-1)-(1-\nu)} < \frac{1-\nu}{\nu} \quad \forall \nu, \xi :\ \nu\xi > 1 \end{aligned} \tag{2.52}$$

Combining (2.51) and (2.52) casts $\hat{\eta}_1^S$ as a monotonically decreasing function of ξ, which drops from $+\infty$ to 0.685 as ξ goes from 3 to 7. In particular, the estimates of η derived from the results by Aghion et al. (2012) and Beneito et al. (2015) are guaranteed to be accommodated by condition (2.51) – regardless of $\lambda(\cdot)$'s

[34] (Aghion et al., 2012, Table 3), (Beneito et al., 2015, Table 2).

[35] We keep our argument in the text more heuristic, with a more formal proof banished to Appendix A.1.2.

[36] We retrieve the growth rates of $Y(t)$ from data on real output in the U.S. in Feenstra et al. (2015).

[37] Data source: Jarmin and Miranda (2002).

[38] Note that regardless of $\frac{\bar{g}_{\tilde{y}}}{\bar{g}_m}$'s exact value, expression (2.52) satisfies restriction (2.31), so long as condition (2.3) holds.

functional form – for $\xi < 5.57$ and $\xi < 4.5$, respectively.

2.6 Extension №2 – Separate Decisions on R&D and Staying in Business

Throughout the last two sections, the driving force behind the reversal of R&D's countercyclical behaviour on the aggregate level, has been modelled to be the entry/exit of firms in an industry. In this section we generalise our framework to allow for a situation when firms that do not perform R&D, can still remain active – i.e. when engaging in R&D is decoupled from staying in business.

The extension in hand can be motivated by the fact that R&D performers do not make the majority of firms active in an industry: as an illustration, turning to data on R&D personnel numbers in OECD countries, and assuming (quite unrealistically) that each R&D performer employs just one staff member to do all R&D, leaves one with a very rough upper estimate of $12-210$ R&D performers per thousand firms for a subsample of OECD countries (see Figure 2.3), which suggests that at least three quarters of firms in an OECD country's average industry do not engage in R&D.

2.6.1 Introducing R&D Non-Performing Firms

To incorporate the separation of the decisions on performing R&D and staying in business in the simplest form possible, we first introduce the explicit cost of R&D in the shape of an increase in an R&D performer's fixed costs to Φ^+ from the level of Φ^- incurred by firms focusing exclusively on production (R&D non-performers).[40]

[39] Data on R&D personnel and numbers of firms were retrieved, respectively, from OECD Research and Development Statistics Database and OECD SDBC Structural Business Statistics Database.

[40] A similar assumption of a positive differential in fixed costs between an industry's sole R&D performer and R&D non-performing fringe (with the latter enjoying zero fixed costs) is

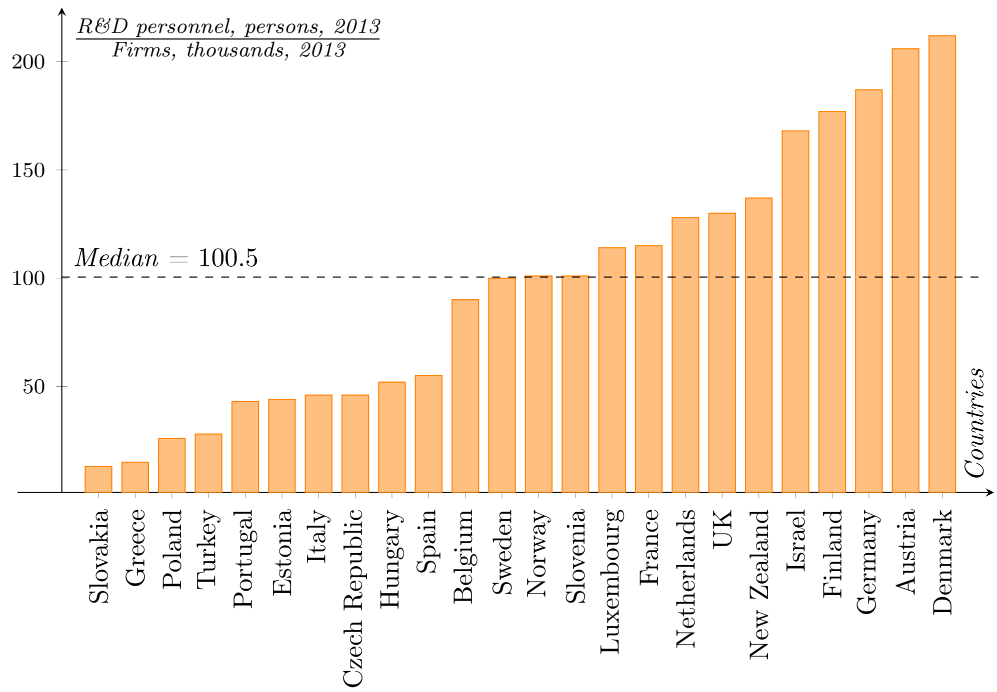

Figure 2.3: R&D personnel per 1000 firms for selected OECD countries, 2013.[39]

Secondly, we assume that two goods produced within the same group (i.e. R&D performers or R&D non-performers) are closer substitutes for each other than goods from different groups, which can be interpreted in the sense that the difference between R&D performers' and R&D non-performers' goods is more fundamental than between two goods by produced within the same group. Mathematically, we reflect this consideration by introducing the nested structure to the production technology of an industrial good (2.4)

$$\begin{aligned} y(t) &= \left(\int_0^{m_R(t)} \tilde{y}_R(j;t)^{\frac{\xi-1}{\xi}} dj + \left(\int_0^{m_P(t)} \tilde{y}_P(s;t)^{\frac{\mu-1}{\mu}} ds \right)^{\frac{\mu}{\mu-1}\cdot\frac{\xi-1}{\xi}} \right)^{\frac{\xi}{\xi-1}} = \\ &= \left(\left(m_R(t)^{\frac{\mu}{\mu-1}} \tilde{y}_R(t) \right)^{\frac{\xi-1}{\xi}} + \left(m_P(t)^{\frac{\mu}{\mu-1}} \tilde{y}_P(t) \right)^{\frac{\xi-1}{\xi}} \right)^{\frac{\xi}{\xi-1}} \equiv \\ &\equiv \left(y_R(t)^{\frac{\xi-1}{\xi}} + y_P(t)^{\frac{\xi-1}{\xi}} \right)^{\frac{\xi}{\xi-1}} \end{aligned} \tag{2.53}$$

made in Barlevy (2007).

where $\mu \equiv \kappa\xi$, $\kappa > 1$ is the elasticity of substitution between products within the groups of R&D performers and R&D non-performers; the R and P indices denote variables pertaining to, respectively, R&D performers and R&D non-performers.

Therefore, the choice each firm faces is between engaging in R&D and, as a result, producing its good with lower marginal costs at the price of higher fixed costs, or refraining from R&D – and thus saving on the fixed costs – but losing effectiveness in production (in terms of marginal costs). In either case, a firm faces stronger competition from other producers in the same group (R&D performers or R&D non-performers).

The rest of equations and parameter restrictions are as in the baseline model (see Sections 2.2.1–2.2.3).

2.6.2 Solution

Given that all firms still employ production technology (2.4), a firm allocates its facilities to production and R&D in the same way as prescribed by (2.14), which yields the expressions for a firm's output and the share of a firm's facilities allocated to R&D

$$\tilde{y}_s(t) = \mathcal{Z}_s(t)\ \tilde{x}_s(t)\,,\ \mathcal{Z}_s(t) = \begin{cases} Z(t)\,, & s = R \\ z(t)\,, & s = P \end{cases} \tag{2.54}$$

$$\tilde{\gamma}_s(t) = \begin{cases} \left(\frac{\zeta}{Z(t)}\right)^{\eta-1} \frac{\tilde{y}_R(t)}{Z(t)}, & s = R \\ 0, & s = P \end{cases} \tag{2.55}$$

In order to derive the expression for a firm's price and output, one can first note that, given the homogeneity of the economy's industries, the formulae for inverse factor demand functions (2.9) and (2.10) still carry through, so $p(t)$ is as expressed in (2.9). Given that, an analogue of the baseline model's expression for inverse demand for a firm's product (2.11) still holds

$$\tilde{p}_s(t) = \frac{y(t)^{\frac{1-\nu\xi}{\xi}}\, y_s(t)^{\frac{1}{\mu}-\frac{1}{\xi}}}{\tilde{y}_s(t)^{\frac{1}{\mu}}} \tag{2.56}$$

Solving a firm's profit maximisation problem given (2.54) and (2.56), pins down a firm's price

$$\tilde{p}_s^*(t) = \frac{\mu\psi}{\mu - 1} \cdot \frac{1}{\mathcal{Z}_s(t)} \tag{2.57}$$

Combining (2.56) with (2.57) enables one to re-express a firm's and an industry's outputs as follows (see Appendix A.1.3)

$$y(t) = L\left(\frac{\mu - 1}{\mu\psi}\right)^{\frac{1}{\nu}} \mathcal{I}(t)^{\frac{1}{\nu(\xi-1)}} \tag{2.58}$$

$$\tilde{y}_s^*(t) = \frac{L}{m_s(t)^{\frac{\mu-\xi}{\mu-1}}} \left(\frac{\mu - 1}{\mu\psi}\right)^{\frac{1}{\nu}} \mathcal{Z}_s(t)^{\xi} \mathcal{I}(t)^{\frac{1-\nu\xi}{\nu(\xi-1)}} \tag{2.59}$$

where $\mathcal{I}(t) \equiv Z(t)^{\xi-1} m_R(t)^{\frac{\xi-1}{\mu-1}} + z(t)^{\xi-1} m_P(t)^{\frac{\xi-1}{\mu-1}}$. Combining (2.57) with (2.59) allows one to obtain the expression for the level of profits a firm accrues

$$\begin{gathered}\tilde{\pi}_s^*(t) = \frac{\tilde{p}_s^*(t)\,\tilde{y}_s^*(t)}{\mu} - \Phi_s = \frac{L}{\mu m_s(t)^{\frac{\mu-\xi}{\mu-1}}} \left(\frac{\mu - 1}{\mu\psi}\right)^{\frac{1-\nu}{\nu}} \mathcal{Z}_s(t)^{\xi-1} \mathcal{I}(t)^{\frac{1-\nu\xi}{\nu(\xi-1)}} - \Phi_s, \\ \Phi_s = \begin{cases} \Phi^+, & s = R \\ \Phi^-, & s = P \end{cases}\end{gathered} \tag{2.60}$$

The assumption of free entry to an industry gives rise to the final expressions for a firm's output and R&D spending

$$\tilde{y}_s^*(t) = \frac{(\mu - 1)\,\Phi^+}{\psi} \mathcal{Z}_s(t) \tag{2.61}$$

$$\tilde{\gamma}_R^*(t) = \left(\frac{\zeta}{Z(t)}\right)^{\eta-1} \frac{(\mu - 1)\,\Phi^+}{\psi} \tag{2.62}$$

As before, R&D spending of an individual firm is countercyclical, as $Z(t)$ is procyclical, and $\eta > 1$. Combining (2.59) with (2.61) enables one to re-express the mass of firms of each type as a function of $\mathcal{I}(t)$

$$\begin{aligned} m_s^*(t) = &\left(\frac{L}{\mu\Phi_s}\left(\frac{\mu-1}{\mu\psi}\right)^{\frac{1-\nu}{\nu}} \mathcal{Z}_s(t)^{\,\xi-1}\,\mathcal{I}(t)^{\,\frac{1-\nu\xi}{\nu(\xi-1)}}\right)^{\frac{\mu-1}{\mu-\xi}} \Rightarrow \\ \Rightarrow m_R^*(t) = &\left(\frac{\Phi^-}{\Phi^+}\cdot\left(\frac{Z(t)}{z(t)}\right)^{\xi-1}\right)^{\frac{\mu-1}{\mu-\xi}} m_P^*(t) \end{aligned} \tag{2.63}$$

As suggested by the second line in (2.63), the share of R&D producers in the total mass of firms in an industry $\frac{m_R^*(t)}{m_P^*(t)+m_R^*(t)} = \left(1+\left(\frac{\Phi^+}{\Phi^-}\cdot\left(\frac{z(t)}{Z(t)}\right)^{\xi-1}\right)^{\frac{\mu-1}{\mu-\xi}}\right)^{-1}$ decreases in $z(t)$: such a result can be put down to the fact that R&D non-performers' profits oscillate more widely than those of their R&D performing counterparts – in particular, the former grow faster during upturns, and decline more during downturns. This consideration can be ascribed in turn to the diversifying nature of engaging in R&D, as embedded in the 'preference for diversity' property of production technology (2.59): unlike R&D non-performers, during an upturn, R&D-performing firms cannot reap in full the benefits of higher productivity in production $z(t)$, as part of their facilities is diverted to R&D, whose productivity stays constant. On the other hand, engaging in R&D during a downturn allows R&D performers to soften the impact of lower productivity in production $z(t)$ through diversification between production and R&D.

In order to fully characterise the solution of the model, one needs to express $\mathcal{I}(t)$, which can be done by combining its definition with (2.63)

$$\mathcal{I}(t) = \left(\frac{L}{\mu}\left(\frac{\mu-1}{\mu\psi}\right)^{\frac{1-\nu}{\nu}}\right)^{\frac{\nu(\xi-1)}{\nu\mu-1}} \left(\left(\frac{Z(t)^{\,\mu-1}}{\Phi^+}\right)^{\frac{\xi-1}{\mu-\xi}} + \left(\frac{z(t)^{\,\mu-1}}{\Phi^-}\right)^{\frac{\xi-1}{\mu-\xi}}\right)^{\frac{\nu(\mu-\xi)}{\nu\mu-1}} \tag{2.64}$$

Bringing together (2.63) and (2.64) enables one to pin down the closed-form expressions for $m_R^*(t)$ and $m_P^*(t)$

$$m_R^*(t) = \left(\frac{L}{\mu\Phi^+}\left(\frac{\mu-1}{\mu\psi}Z(t)\right)^{\frac{1-\nu}{\nu}}\right)^{\frac{\nu(\mu-1)}{\nu\mu-1}}\left(1+\left(\frac{\Phi^+}{\Phi^-}\left(\frac{z(t)}{Z(t)}\right)^{\mu-1}\right)^{\frac{\xi-1}{\mu-\xi}}\right)^{-\frac{\nu\xi-1}{\nu\mu-1}\cdot\frac{\mu-1}{\xi-1}} \tag{2.65}$$

$$m_P^*(t) = \left(\frac{L}{\mu\Phi^-}\left(\frac{\mu-1}{\mu\psi}z(t)\right)^{\frac{1-\nu}{\nu}}\right)^{\frac{\nu(\mu-1)}{\nu\mu-1}}\left(1+\left(\frac{\Phi^-}{\Phi^+}\left(\frac{Z(t)}{z(t)}\right)^{\mu-1}\right)^{\frac{\xi-1}{\mu-\xi}}\right)^{-\frac{\nu\xi-1}{\nu\mu-1}\cdot\frac{\mu-1}{\xi-1}} \tag{2.66}$$

Together conditions (2.55) and (2.65) determine aggregate R&D spending in an industry $\psi\Gamma^*(t)$. Although the algebraic complexity of the resulting expression precludes one from analysing the cyclical properties of $\psi\Gamma^*(t)$ directly, one can still gain an insight into its behaviour by deriving a necessary and a sufficient condition for $\psi\Gamma^*(t)$'s being procyclical, which can be shown to take the following forms

Observation 2.2 (Necessary Condition for $\psi\Gamma^*(t)$'s procyclicality). *If function $\psi\Gamma^*(t)$ is procyclical, the following condition must hold*

$$\eta \leqslant \frac{\mu+\nu-2}{\nu\mu-1} \equiv \hat{\eta}_2^N < \hat{\eta}_0 \tag{2.67}$$

Proof. See Appendix A.1.4. ∎

Observation 2.3 (Sufficient Condition for $\psi\Gamma^*(t)$'s procyclicality). *Suppose that condition (2.67) holds. Then function $\psi\Gamma^*(t)$ is procyclical if the following condition holds*

$$\eta \leqslant 1 + \frac{(1-\nu)(\mu-1)}{\nu\mu-1}\cdot\frac{1}{1+\frac{\nu\xi-1}{\nu\mu-1}\cdot\frac{\mu-1}{\xi-1}\left(\frac{\Phi^+}{\Phi^-}\right)^{\frac{\xi-1}{\mu-\xi}}} \equiv \hat{\eta}_2^S < \hat{\eta}_2^N < \hat{\eta}_0 \tag{2.68}$$

Proof. See Appendix A.1.4. ∎

One can verify that condition (2.67) is stronger than its counterpart from the baseline model (2.23): a higher substitutability between R&D producers' goods μ suggests fiercer competition between them, which in turn reduces their numbers, thus dampening the composition effect on aggregate R&D spending. Sufficient condition (2.68) is, naturally, more restrictive than (2.67), as it also takes into account countercyclical competitive pressure exerted on R&D performers by R&D non-performers, whose entry to the industry is procyclical.

One can finish solving the model by plugging the formula for $\mathcal{I}(t)$ (2.64) into the expression for an industry's output $y(t)$ (2.58)

$$y(t) = \frac{(\mu-1)\,L}{\psi}\left(\frac{L}{\mu}\left(\frac{\mu-1}{\mu\psi}\right)^{\frac{1-\nu}{\nu}}\left(\left(\frac{Z(t)^{\,\mu-1}}{\Phi^{+}}\right)^{\frac{\xi-1}{\mu-\xi}}+\left(\frac{z(t)^{\,\mu-1}}{\Phi^{-}}\right)^{\frac{\xi-1}{\mu-\xi}}\right)\right)^{\frac{\nu(\mu-\xi)}{\nu\mu-1}} \tag{2.69}$$

Given (2.1), (2.10) and (2.69), the equilibrium expressions for total output $Y(t)$, the wage rate $w(t) = \frac{\nu Y(t)}{L}$ and consumption $C(t) = \nu Y(t)$ can be derived, thus completing the extension's solution.

2.7 Summary

In this chapter, we have explored the role of the composition effect, as manifesting itself in fluctuations of the numbers of R&D performers, in reconciling contradictory results in empirical macro- and micro-studies on the cyclicality of R&D spending.

In all three versions of the chapter's framework, our results suggest that when the amplitude of shifts between production and R&D a firm's resources undergo across an economic cycle, is sufficiently low, the predictions of Schumpeter's hypothesis, while operational on the firm level, are reversed on the industry and the economy-wide level through changes in the numbers of R&D performers, which, by being procyclical, thereby offset countercyclical fluctuations of R&D spending on the individual firm level, and transform them into procyclical macrooscillations.

Furthermore, our quantitative assessments suggest that the shift in cyclicality between the micro-level and macro-level is empirically plausible, which leads to two major implications. From the methodological perspective, the chapter suggests that exploring the relationship between business cycles and R&D must take place on the level of individual firms rather than economies/sectors, as in the latter case the results are likely to be confounded by fluctuations of the extensive margin. The other upshot is that downturns have their redeeming value of fostering innovation. This conclusion brings about a major policy implication of countercyclical R&D stimulation.

Chapter 3

Intersectoral Linkage as a Factor of Economic Growth

This chapter documents the presence of a positive link between the intensity of using intermediates by a country's industries and its economic growth. We explain the finding by advancing a framework of endogenous growth in an economy with interconnected industries, whereby sectoral productivity growth is amplified by the interconnection, and the degree of amplification grows in the strength of sectoral connections. We use the framework to derive the optimal growth-enhancing structure of an economy, which is when all industries source their intermediates from a single sector (the star network) characterised by the largest concentration potential – a novel indicator which captures how fast is a sector's productivity growth and how much it itself relies on intermediates.

3.1 Introduction

There has been a growing recognition of the impact the system of links between sectors in an economy (usually termed 'intersectoral linkage' or 'production network') exerts on its macroeconomic outcomes by serving as an amplifying channel for various micro-level phenomena. A particular emphasis has been put on investigating the macro-propagation of micro-shocks[1] and amplification of disparities in living standards across countries.[2] To the best of our knowledge, however, existing literature has not studied systematically the relationship between intersectoral linkage and one of key macro-indicators with micro-origins – namely, economic growth.

As a motivating example for the existence of this relationship, consider Figure 3.1 which plots the average intensity of intermediate consumption (industries' using each other's products) in 1995 (i.e., the average strength of intersectoral connections in an economy), against economies' growth rates in 1995–2011. A precursory examination of the graph suggests the presence of a clear positive link between how strongly sectors rely on each other's products, and economic growth. The existence of this link poses three natural questions:

1. How robust is its presence empirically?
2. What are the underpinnings of this positive relationship?
3. If there is a relationship between intersectoral connections and growth, which structure of the former is the most growth-enhancing?

To answer the first question, we assemble a panel dataset by combining data from Penn World Tables by Feenstra et al. (2015) with information on flows of products between sectors (for different countries) from OECD Input-Output Tables compiled and harmonised by the OECD.[3] We document a statistically significant positive link between the average intensity of intermediate consumption

[1]To mention a few examples, Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017) investigate the role of production networks in, respectively, transforming micro-shocks into macro-volatility and the formation of heavy tails in macro-indicators' distributions.

[2]See Jones (2011).

[3]To the best of our knowledge, this is the first paper to employ OECD's Input-Output Tables in the context of studying macroeconomic production networks.

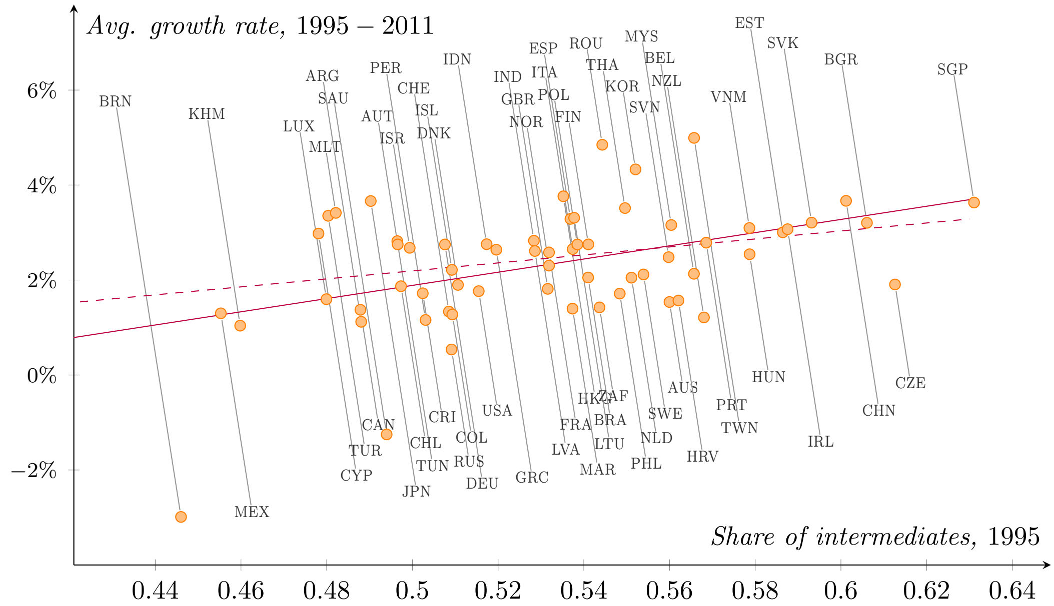

Figure 3.1: The relationship between the average share of intermediates in 1995 and the average growth rate in 1995–2011 for a cross-section of economies.[4]

Data source: see Section 3.2.1.

and economic growth in the presence of a wide range of standard explanatory variables from growth regressions.

To tackle the last two questions, we advance a novel theoretical framework of an endogenously growing interconnected economy, which builds upon the canonical model of an interconnected economy with perfect competition, first developed by Long and Plosser (1983) and lately popularised by Jones (2011) and Acemoglu et al. (2012). The paper's framework explains the link depicted in Figure 3.1 through the presence of a multiplier effect inherent in production networks, first posited theoretically by Hulten (1978): an increase in the productivity of a firm enhances that of its downstream counteragents, and if the original firm itself uses their products as inputs, it benefits also indirectly from its own enhanced productivity which further affects firms downstream etc., which

[4] The solid line is the regression line derived for the whole sample, the dashed line is that for the sample without two outliers: Brunei (BRN) and Saudi Arabia (SAU). Both lines' slopes are statistically significant at the 5% level.

increases further the economy's observable aggregate growth rate. One of this paper's contributions is to show both empirically and theoretically that the strength of Hulten's multiplier effect increases as industries become more reliant on using each other's products as intermediates.

The paper's framework makes very sharp analytical predictions on which structure of the intersectoral linkage enhances its growth the most. In particular, it shows that an economy grows at its fastest when all industries use the product of one sector[5] characterised by high productivity growth rates and sufficient reliance on intermediates. Quantitatively, these two criteria are captured by the notion of a sector's concentration potential introduced in this paper.

This work is related to a few strands of literature. Its closest link is with papers studying different aspects of the growth-interconnection nexus. A seminal work in the field is by Hulten (1978), who uses a reduced-from model to introduce the idea of the multiplier driven by intermediate consumption. Modelling explicitly the structure of connections between sectors, allows us to advance further Hulten's results by showing that stronger interconnections between sectors make the multiplier effect more pronounced, as well as pinning down the optimal growth-enhancing structure of an economy's interlinkage. Other papers in the area include Ngai and Samaniego (2009), who show that ignoring the effect of production networks can bias sectoral productivity growth accounting, since in this case a productivity shift in a supplying industry can be wrongly ascribed to the consuming sector. Carvalho and Voigtländer (2015) demonstrate that technology adoption (in the form of implementing new inputs in production) is facilitated by the presence of a linkage between firms, since firms are likelier to use inputs employed by their suppliers.

Furthermore, our work is connected with the branch of research initiated by the contributions of Hidalgo and Hausmann (2009) and Hausmann and Hidalgo (2011), who use network analysis techniques to explore the relationship between growth and development on one hand, and features of the network which connects

[5] In network studies, such a configuration of an interlinkage is called the star network.

economies to varieties of products they export.

Additionally, this paper is connected with the strand of research on the interplay between the use of intermediates and economic growth and development, where one could mention the following works: Jones (2011) (cited above) shows how misallocation of resources between sectors can propagate in the presence of an intersectoral linkage through Hulten's multiplier, to generate cross-country income disparities on the scale observed in data.[6] Moro (2012a) argues that the dynamics of intrinsic productivity in the use of intermediates is an important factor affecting the evolution of observed total factor productivity. Moro (2012b) studies the connection between a drop in the volatility of the U.S. economy's aggregate productivity in the course of its structural transformation, and connects it with heavier reliance on intermediates in manufacturing, as compared to services. Grobovšek (2013) shows that a significant share of cross-country differences in levels of labour productivity is accounted for by disparities between their efficiencies in the production of intermediates. Cavalcanti and Giannitsarou (2017) investigate the accumulation of human capital in an economy comprising a set of interconnected households to show that homogeneous distribution of human capital can be achieved in the long-run in a well-connected network.

More generally, the paper belongs to the growing body of literature on the macro-impact of the interlinkage; apart from the aforementioned works by Acemoglu et al. (2012) and Acemoglu et al. (2017), recent contributions to the area include Acemoglu et al. (2015) who show that the network of interbank loans can act as a stabiliser for a bank system if shocks hitting it are relatively low in the magnitude – otherwise mutual connections become an amplifier spreading financial contagion across the system. See Carvalho (2014) for an introduction to

[6] The theory advanced by Jones (2011) is closely connected with the O-ring theory development due to Kremer (1993). According to the latter, just like a fault in a component can render the whole product dysfunctional, economic development can be hampered by a small deficiency in even one of its factors. In the case of Jones (2011), such a deficiency takes the form of misallocation within a sector, which propagates to have a significant effect on the overall economic performance.

the literature's core theoretical concepts, and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2016) for a detailed review of the literature.

The remainder of the paper is structured as follows. Section 3.2 explores further evidence on the link between sectoral interconnection and economic growth; Section 3.3 introduces the paper's framework of endogenous growth in an interconnected economy. Section 3.4 spells out the framework's key theoretical predictions, while Section 3.5 reports current results on testing them for a calibrated version of the framework. The last section concludes.

3.2 Evidence on the Link between Interconnection and Growth

This section starts with introducing the dataset used in the section's empirical exercises (Section 3.2.1), and then proceeds with discussing the methodology and presenting the findings (Section 3.2.2).

3.2.1 Dataset Description

The dataset comprises a balanced panel of 63 countries (including all OECD members and all G20 members)[7], covering the period from 1995 to 2011. In total, the dataset encompasses no less than 66% and 70% of, respectively, the world's total output and employment. The data come from two sources: we use the information on intersectoral connection from the OECD Input-Output Tables – a set of harmonised 33 industries by 33 industries input-output tables[8] compiled by the OECD – to gauge sectoral reliance on intermediates RI by the ratio of total sales of intermediates to a sector, to its total sales. All other socio-economic indicators used in the paper are from Penn World Tables 9.0 by Feenstra et al. (2015).

[7]See Table A.1 for the full list of countries.

[8]See Table A.2 for the list of industries included.

3.2.2 Methodology and Results

We test the presence of a relationship between the strength of intersectoral links and economic growth by running the regression as follows

$$Gr_{i;t}^{\Delta t} = const + b_1 RI_{i;t-1}^{a} + \mathbf{b}\mathbf{x}_{i;t-1} \tag{3.1}$$

where $Gr_{i;t}^{\Delta t}$ is economy i's growth rate averaged over Δt years starting with year t, $RI_{i;t-1}^{a}$ is the level of reliance on intermediates (measured as the ratio of intermediate inputs to a sector, to the sector's gross output) averaged across its industries for year $t-1$: $RI_{i;t-1}^{a} \equiv \frac{1}{N}\sum_{j=1}^{N}\frac{Int.\,Sales_{i;t-1}^{j}}{Total\,Sales_{i;t-1}^{j}}$ (N is the number of industries, j is a sector's index), which is the same as the measure used in plotting Figure 3.1. Finally, $\mathbf{x}_{i;t-1}$ is the list of controlling regressors' values in $t-1$.

We compile $\mathbf{x}$ in such a way as to rule out the impact of other common growth factors:[9] in particular, we include the logarithms of initial levels of GDP per capita, the logarithms of human capital, population growth rates and saving rates.

In addition, the findings by Acemoglu et al. (2012) reveal a connection between an economy's sectoral interlinkage and volatility, of which the latter is shown to impede growth (see Ramey and Ramey (1995)). To control for the impact of this potential channel, we incorporate aggregate volatility in our estimates by adding the moving standard deviation of an economy's growth rate to the list of controls $\mathbf{x}$.

Furthermore, as we do not distinguish between domestic and foreign intermediates when calculating RI, one could argue that regression (3.1) can pick the positive relationship between growth and openness to international trade.[10]

[9] See Durlauf, Johnson, and Temple (2005) for a detailed review of common growth regressors, and Levine and Renelt (1992) for a discussion of their robustness.

[10] The idea dates back to Adam Smith. More recent contributions focus on trade as a conduit for dissemination of new ideas and include Rivera-Batiz and Romer (1991), Grossman and Helpman (1991b), Eaton and Kortum (2002). See Grossman and Helpman (2015) for a review.

To rule this out, we use different measures of trade openness (the results below are presented for the share of real total trade in real output). Finally, we include dummies for world geographical regions as defined by the World Bank.[11]

In order to dampen the impact of short-run fluctuations, we set the averaging period Δt in (3.1) equal to $\Delta t \in \{5; 6; 7; 8\}$. We choose 8 as Δt's maximal length so that there is more than one time observation per country in our dataset. In addition to working with the panel, we repeat the exercise in the cross-section setting, where all regressors' values are taken for the dataset's initial year 1995, and the growth rates are averaged for the $1995 - 2011$ period (i.e., $\Delta t = 16$). The

[11]See https://data.worldbank.org/country.

results are presented in Table 3.1.

Table 3.1: The relationship between economic growth in 1995–2011 and the average reliance on intermediates RI^a.

Dep. var.	*Growth rate of GDP per worker averaged across Δt years*				
	$\Delta t = 5\,yrs$	$\Delta t = 6\,yrs$	$\Delta t = 7\,yrs$	$\Delta t = 8\,yrs$	$\Delta t = 16\,yrs$
RI^a	0.032*** (0.011)	0.047*** (0.006)	0.049*** (0.000)	0.047*** (0.010)	0.037** (0.017)
$\ln(hc)$	0.016** (0.007)	0.030*** (0.001)	0.022*** (0.004)	0.018*** (0.006)	0.020*** (0.007)
$\ln(Y/L)$	−0.015*** (0.001)	−0.017*** (0.000)	−0.017*** (0.000)	−0.015*** (0.001)	−0.018*** (0.002)
g_L	−0.536*** (0.037)	−0.388*** (0.113)	−0.225*** (0.034)	−0.299*** (0.023)	0.400** (0.185)
s	0.004 (0.010)	−0.012*** (0.003)	−0.015*** (0.000)	−0.012*** (0.001)	−0.011 (0.009)
const	0.144*** (0.012)	0.147*** (0.009)	0.149*** (0.006)	0.136*** (0.014)	0.150*** (0.016)
Regional dummies	Yes	Yes	Yes	Yes	Yes
Countries	63	63	63	63	63
Periods	3	2	2	2	1
R^2	0.683	0.786	0.893	0.937	0.987
R^2_{adj}	0.658	0.762	0.880	0.930	0.984

NOTES $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

1. $\ln(hc)$ and $\ln(Y/L)$ are the logarithms of initial levels of human capital and GDP per capita, g_L is the population growth rate, s is the savings rate;
2. In order to control for cross-country heteroskedasticity, cross-sectional weighting was used alongside calculating standard errors robust to cross-section heteroskedasticity;
3. The case of $\Delta t = 16$ corresponds to a simple cross-country regression estimated using the weighted LS.

Table 3.2: The relationship between economic growth in 1995–2011 and the median reliance on intermediates RI^m.

Dep. var.	*Growth rate of GDP per worker averaged across Δt years*				
	$\Delta t = 5\,yrs$	$\Delta t = 6\,yrs$	$\Delta t = 7\,yrs$	$\Delta t = 8\,yrs$	$\Delta t = 16\,yrs$
RI^m	0.024*** (0.009)	0.029*** (0.008)	0.048*** (0.001)	0.041*** (0.009)	0.041*** (0.015)
$\ln(hc)$	0.016** (0.007)	0.029*** (0.001)	0.021*** (0.004)	0.016** (0.006)	0.021*** (0.004)
$\ln(Y/L)$	−0.015*** (0.001)	−0.017*** (0.000)	−0.016*** (0.000)	−0.015*** (0.001)	−0.018*** (0.001)
g_L	−0.533*** (0.045)	−0.367** (0.152)	−0.226*** (0.037)	−0.295*** (0.007)	0.425*** (0.153)
s	0.004 (0.010)	−0.009*** (0.002)	−0.018*** (0.000)	−0.016*** (0.002)	−0.013*** (0.005)
$const$	0.146*** (0.011)	0.156*** (0.012)	0.138*** (0.002)	0.135*** (0.015)	0.140*** (0.018)
Regional dummies	Yes	Yes	Yes	Yes	Yes
Countries	63	63	63	63	63
Periods	3	2	2	2	1
R^2	0.683	0.786	0.893	0.937	0.987
R^2_{adj}	0.658	0.762	0.880	0.930	0.984

NOTES

See Table 3.1.

The table shows the presence of a strong positive link between the average reliance on intermediates and the growth rate. As a robustness check, we repeat the exercise above for the median reliance on intermediates $RI^m \equiv median\left(\frac{Int.\,Sales^1_{i;t-1}}{Total\,Sales^1_{i;t-1}}; \ldots ; \frac{Int.\,Sales^N_{i;t-1}}{Total\,Sales^N_{i;t-1}}\right)$, with the results (see Table 3.2) consistent with the findings above for RI^a.

The concurrence of the empirical results on the presence of a relationship between the strength of sectoral interlinkage and economic growth motivates a theoretical exploration of the matter, to which we move in the next section.

3.3 Theoretical Framework

We begin the discussion of the paper's theoretical framework by introducing in Section 3.3.1 a simplified model capturing the paper's intuition that stronger intersectoral connection implies stronger amplification of productivity growth on the economy-wide level, as increased productivity of a firm can benefit not only firms further downstream, but the original firm as well if it draws some its inputs from those, thus creating a virtuous cycle increasing an economy's aggregate growth. In Section 3.3.2 we proceed to consider a richer theoretical setting with endogenously growing interconnected economy, whereby technological growth is an outcome of optimal innovation choices by competing monopolist firms.

3.3.1 Introductory Theoretical Illustration

As a means of illustrating the paper's intuition, consider the following stylised model. Suppose that an economy operates in continuous time, and is represented by N perfectly competitive firms, each using a Cobb-Douglas production technology with labour and other firms' products (intermediate goods) as inputs

$$y_i(t) = \frac{q_i(t)}{\zeta_i}\, l_i(t)^{1-\nu} \left(\prod_{j=1}^{N} y_{ij}(t)^{\alpha_{ij}} \right)^{\nu} \tag{3.2}$$

where y_i is the i-th firm's output, $q_i(t)$ is its productivity, $l_i(t)$ and $y_{ij}(t)$ are, respectively, the amounts of labour and other firms' products it employs in period t; $\nu \in [0;1)$ is the total share of intermediate products in the i-th firm's sales, and $\alpha_{ij} \in [0;1]$ is j-th firm's share within that share; coefficient $\zeta_i \equiv (1-\nu)^{1-\nu} \prod_{j=1}^{N} (\nu\alpha_{ij})^{\nu\alpha_{ij}}$ is introduced for normalisation purposes. In this section, we simplify the exposition by assuming that all intermediate inputs are used with the same intensity, so that $\forall i, j = 1 \div N$, $\alpha_{ij} = \alpha = \frac{1}{N}$. Together all α_{ij}s can be represented jointly by a technological matrix $\mathbf{A} \equiv \alpha \mathbb{1}_N = \frac{1}{N}\mathbb{1}_N$, where $\mathbb{1}_N$ is an $N \times N$ matrix of ones.

In light of (3.2), and since the firms are perfect competitors, one can straightforwardly show that the vector of equilibrium prices' natural logarithms $\mathbf{p}^*(t)$ is described by the matrix expression as follows[12]

$$\mathbf{p}^*(t) = \ln w(t)\,\mathbf{e}_N - (\mathbf{E}_N - \nu\mathbf{A})^{-1}\,\mathbf{q}(t) = \ln w(t)\,\mathbf{e}_N - \left(\mathbf{E}_N + \frac{\nu}{1-\nu}\,\mathbf{A}\right)\mathbf{q}(t) \tag{3.3}$$

where $\mathbf{e}_N$ and $\mathbf{E}_N$ are, respectively, the $N \times 1$ vector of ones and the $N \times N$ identity matrix; $\mathbf{q}(t)$ is the vector of the natural logarithms of firms' productivities.

Assume that each firm's product is consumed by the representative household with symmetric Cobb-Douglas preferences

$$C(t) = N \prod_{i=1}^{N} c_i(t)^{\frac{1}{N}} \tag{3.4}$$

where $c_i(t)$ is the consumption of the i-th firm's product. The expression is multiplied by N for normalisation purposes. The household comprises $L(t)$ individuals, each supplying inelastically one unit of labour.

The household's optimising behaviour entails that aggregate spending on consumption (which is equivalent to the economy's GDP, given the absence of capital investment) amounts to $P(t)\,C(t)$, where $P(t) \equiv \left(\prod_{i=1}^{N} p_i(t)\right)^{\frac{1}{N}}$ is the economy's price index. In what follows, we use the household's optimal consumption bundle as the numeraire, which entails that $P(t) = 1\ \forall t$.

Combining the solution for the system of equilibrium prices (3.3) with the definition of the aggregate price index $P(t)$ yields the following expression for the

[12]See the derivation in Acemoglu et al. (2012).

natural logarithm of the equilibrium wage rate

$$\begin{aligned} \ln P^{*}(t) = 0 = \ln w(t) - \frac{1}{N}\mathbf{e}_N^T \left(\mathbf{E}_N + \frac{\nu}{1-\nu}\mathbf{A}\right)\mathbf{q}(t) \Leftrightarrow \\ \Leftrightarrow \ln w(t) = \frac{1}{1-\nu}\sum_{i=1}^{N}\frac{\ln q_i(t)}{N} \end{aligned} \tag{3.5}$$

where $(\cdot)^T$ is the transposition operator.

As labour is the only primary production factor in the economy, total labour compensation constitutes the economy's value added, so that $w(t)\,L(t)$ is the total output, and $w(t) = y(t)$ is GDP per capita. When combined with (3.5), this suggests the following expression for the economy's growth rate at t

$$(\ln \dot{w}(t)) = \frac{\dot{w}(t)}{w(t)} \equiv g(t) = \frac{1}{1-\nu}\sum_{i=1}^{N}\frac{g_i(t)}{N} \tag{3.6}$$

where $g_i(t) \equiv \frac{\dot{q}_i(t)}{q_i(t)}$ is the growth rate of firm i's productivity. The term $\frac{1}{1-\nu}$ in the rightmost expression in (3.6) reflects the impact of the interconnection between firms on the growth rate. The key feature of the term is that it is strictly greater than one, and increases in ν. This implication comes directly from the amplification effects inherent in production networks and encapsulated in the idea of Hulten's multiplier introduced by Hulten (1978): an increase in firm i's productivity translates into that of its output, which leads to the expansion for all firms downstream from i.[13] This in turn induces second- and higher-order effects for all firms further downstream.[14] Thereby, the resulting overall growth in the economy exceeds the productivity growth rate of any single industry. This result is further generalised in Section 3.3.2.

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[13]Potentially, a firm's higher productivity can have an impact on its upstream counteragents as well, but this effect is reflected only when the elasticity of substitution between a firm's inputs is different from unity, i.e. when production technology (3.2) is CES instead of Cobb-Douglas – see Baqaee (2016) for a discussion.

[14]The same mechanism is explored by Acemoglu et al. (2012) and Acemoglu et al. (2017) in the context of micro-shock propagation, and by Jones (2011) for the amplification of misallocation-driven distortions.

Suppose that at t all firms' productivities are characterised by vector $\mathbf{q}(t)$. During a period starting at t, an episode of technological growth occurs, within which the i_0-th firm sees an increase in its productivity at the rate of $g_{i_0}(t) = \eta$. Given (3.6), the growth rate of the economy's output per capita $g(t)$ during the episode is

$$g(t) = g = \frac{1}{1-\nu}\frac{\eta}{N} \tag{3.7}$$

Note that in the case when intersectoral linkage is absent in the economy – that is, when $\nu = 0$ (so that industries use only primary production factors represented by labour) – expression (3.6) transforms as follows

$$\hat{g}(t) = \hat{g} = \frac{\eta}{N} \Leftrightarrow g = \frac{\hat{g}}{1-\nu} \tag{3.8}$$

Comparing expressions (3.7) and (3.8) confirms the intuition of (3.6): the presence of interlinkage serves as an amplifier of economic growth, as the growth rate of an economy with interconnections g always exceeds that of its counterpart without a linkage $\hat{g}$. In the next section, we explore the amplifying properties of production networks in the context of a richer framework by introducing endogenous productivity choices and generalising the structure of intersectoral production networks.

3.3.2 Extended Theoretical Framework

Aggregate Household and Industries

Consider an economy with N industries. All industries' goods are consumed by a representative household with a weighted version of Cobb-Douglas preferences introduced in (3.4)

$$C(t) = \prod_{i=1}^{N}\left(\frac{c_i(t)}{\beta_i}\right)^{\beta_i}, \ \sum_{i=1}^{N}\beta_i = 1 \tag{3.9}$$

If $p_i(t)$s denote the prices of industries' goods, one can show that β_is determine the shares of industries' products in the household's total spending (which still equals

GDP in the absence of capital investment): $p_i(t)\, c_i(t) = p_i(t)\, y_i(t) = \beta_i P(t)\, C(t)$. The aggregate price index $P(t)$ is equal to $P(t) = \prod_{i=1}^{N} p_i(t)^{\beta_i} \Leftrightarrow \ln P_t = \boldsymbol{\beta}^T \mathbf{p}_t$, where β is the $m \times 1$ vector of β_is. We assume that each product is consumed, so that $\beta_i > 0\ \forall i$. As before, $P(t) = 1\ \forall t$.

The household is assumed to comprise $L(t)$ individuals, each one of whom supplies inelastically one unit of labour at any t. The population size changes over time at a fixed rate g_L

$$\dot{L}(t) = g_L L(t) \Leftrightarrow L(t) = L_0 e^{g_L t} \tag{3.10}$$

where L_0 is the economy's initial population.

Collectively, all individuals supply labour and own every firm in the economy; all resulting income is spent on consumption

$$C(t) = Y(t) = w(t)\, L(t) + \Pi(t) \tag{3.11}$$

where $w(t)$ is the wage rate, and $\Pi(t)$ is the income from receiving the profits of all firms in the economy.

Each industry operates by aggregating goods supplied by monopolistically competitive firms, using the following symmetric CES technology

$$y_i(t) = m_i(t)^{\,1+\kappa_i} \left(\frac{1}{m_i(t)} \int_0^{m_i(t)} \tilde{y}_i(k;t)^{\,\frac{\xi_i - 1}{\xi_i}}\, dk \right)^{\frac{\xi_i}{\xi_i - 1}} \tag{3.12}$$

where i is the industry's index, $m_i(t)$ is the mass of monopolist firms in the industry, $\tilde{y}_i(k;t)$ is the output of the k-th firm, and $\xi_i > 1$ is the elasticity of substitution between the products of any two firms in the industry.[15,16] In what

[15]Note that by positing that the elasticity of substitution between firms' products is strictly greater than one, we indirectly assume that firms' goods are more substitutable than industries', as the elasticity of substitution between the latter is unity (which follows from the household's preferences being Cobb-Douglas). Such a relationship between the elasticities is borne out by existing evidence – see, e.g., Broda and Weinstein (2006, 2010).

[16]While ξ is fixed in this section for greater ostensivity, we take into account its potential

follows, we use tildes to denote firm-level variables.

We follow Ethier (1982), Bènassy (1998), Acemoglu, Antrás, and Helpman (2007) in adding the term $m_i(t)^{1+\kappa_i-\frac{\xi_i}{\xi_i-1}}$, $\kappa_i > -1$ to the standard CES aggregator function in (3.12) to have separate modelling control over the elasticity of substitution between firms' products (captured by ξ_i) on one hand, and the so-called returns from specialisation (expressed by κ_i – see equations (3.32) and (3.33)) on the other.[17] This choice is justified by the fact that, as will be argued below, the nature of returns to specialisation (i.e. increasing, constant or decreasing) determines how changes in the mass of firms in industries impact on the economy's growth rate.

Given (3.12) together with the standard assumption of price-taking behaviour by producers of an industry's good, one can invoke the standard result that the demand for the product of each firm is

$$\tilde{y}_i(k;t) = \left(\frac{p_i(t)}{\tilde{p}_i(k;t)}\right)^{\xi_i} y_i(t) \tag{3.13}$$

where $\tilde{p}_i(k;t)$ is the price of k-th variety, and $p_i(t)$ is the equilibrium price of an industrial good

$$p_i(t) = m_i(t)^{-\kappa_i} \left(\frac{1}{m_i(t)} \int_0^{m_i(t)} \tilde{p}_i(k;t)^{1-\xi_i} dk\right)^{\frac{1}{1-\xi_i}} \tag{3.14}$$

Finally, we make the following assumptions on the firm dynamics within an industry. Firstly, while each single firm is the only supplier of its good, there is free entry and exit for monopolist suppliers in an industry *á la* Dixit and Stiglitz

temporal variability in the quantification exercises in Section 3.5.1.

[17]The notion of returns to specialisation has its counterpart in consumer behaviour theory, known as preference for diversity ('taste for variety'). They capture how more efficient (in terms of produced output or derived utility) is the use of many inputs relative to that of a sole input. Formally, the degree of returns to specialisation can be calculated as the elasticity of function $s(m)$, which is defined as the ratio of output produced with a multitude m of inputs supplied in quantity of s/m each, to that resulting from a single input supplied at amount s, see Bènassy (1996, 1998) for further details.

(1977). Each firm is 'small', so that it does not take into account the impact of its decisions on industry aggregates $p_i(t)$ and $y_i(t)$. We assume that the productivity level of each firm entering an industry at t (denoted as $\tilde{q}_i^E(t)$) is determined by the mean of the industry's current productivity distribution

$$\tilde{q}_i^E(t) = \int_0^{m_i(t)} \frac{\tilde{q}_i(k;t)}{m_i(t)} dk \tag{3.15}$$

In addition, we normalise the productivity of all firms active in all industries at 0, to unity.

Thirdly, each industry is characterised by an exogenous rate of obsolescence $\delta_i > 0$, so that at each moment an active firm can permanently cease to operate with the probability δ_i. Put equivalently, of $m_i(t)$ firms active in the industry at t, only $(1 - \delta_i)\, m_i(t)$ continue to operate further. More formally, the dynamics of the mass of firms in industry i is described by the differential equation as follows

$$\dot{m}_i(t) = m_i^E(t) - \delta_i m_i(t) \tag{3.16}$$

where $m_i^E(t)$ is the mass of entrants to an industry. Parameter δ_i plays the role of the discount factor in a firm's problem introduced in the next two sections.

Firms

Production Technology and Innovation

Each firm produces its output with a Cobb-Douglas technology, using labour and industries' goods

$$\tilde{y}_i(k;t) = \frac{\tilde{q}_i(k;t)}{\zeta_i} \tilde{l}_i(k;t)^{1-\nu_i} \left(\prod_{j=1}^{N} \tilde{y}_{ij}(k;t)^{\alpha_{ij}} \right)^{\nu_i} \tag{3.17}$$

where $\tilde{q}_i(k;t)$ is the productivity level of k-th firm, and $\tilde{l}_i(k;t)$ and $\tilde{y}_i(k;t)$ are the amounts of labour and each of intermediate inputs it employs; for each i, all α_{ij}s are assumed to add up to one, so that each firm operates a technology with constant returns to scale

$$\sum_{j=1}^{m} \alpha_{ij} = 1, \ \forall i = 1 \div n \tag{3.18}$$

Given (3.18), together α_{ij}s comprise a stochastic matrix $\mathbf{A}$.[18] Coefficient ν_i still stands for the share of intermediate inputs in the k-th firm's sales, but is now industry specific. Together all ν_is comprise vector $\boldsymbol{\nu}$. Note that the weights assigned to each intermediate input are not firm-, but industry-specific.

Each firm can engage in research and development activities (R&D) to increase its productivity level. Suppose that through spending $z_i(k;t)$, units of the numeraire good, a firm sees the increase in its productivity level as follows

$$\dot{\tilde{q}}_i(k;t) = \frac{\gamma_i}{\chi_i} z_i(k;t)^{\,\omega_i} \tilde{q}_i(k;t)^{\,1-\chi_i} \tag{3.19}$$

where $\gamma_i > 0$, the term $\tilde{q}_i(k;t)^{\,1-\chi_i}$ reflects the increasing difficulty of productivity enhancement, and $0 < \omega_i \leqslant 1$ captures the degree of diminishing returns from a firm's innovative activities. We assume χ_i to be sufficiently large: $\chi_i \geqslant \xi_i - 1$, so that the difficulty of further innovation grows fast enough to keep a firm facing the trade-off between extracting profits and innovating;[19] $\frac{1}{\chi_i}$ is a normalising coefficient.

In light of (3.17) and the discussion above, a firm's profits take the following form

$$\tilde{\pi}_i(k;t) \equiv \tilde{p}_i(k;t)\,\tilde{y}_i(k;t) - w(t)\,\tilde{l}_i(k;t) - \sum_{j=1}^{N} p_j(t)\,\tilde{y}_{ij}(k;t) - z_i(k;t) \tag{3.20}$$

where the first term represents a firm's sales, whereas the remaining three are its spending on, respectively, primary inputs (labour), intermediates and R&D.

[18]The stochastic matrix is a nonnegative square matrix characterised by the property that $\mathbf{Ae} = \mathbf{e}$, where $\mathbf{e}$ is the vector of ones of the compatible dimensionality.

[19]From the mathematical perspective, stipulating that $\chi_i \geqslant \xi_i - 1$ allows a firm's problem (introduced in Section 3.3.2) to have a solution.

Optimal Behaviour

Each firm seeks to maximise the expected flow of its profits $\tilde{\Pi}_i(k;t)$[20] by setting the optimal price $\tilde{p}_i^*(k;t)$ and deciding on optimal amounts of inputs (i.e., labour and intermediate products) given their prices, and on the optimal R&D strategy

$$\tilde{\Pi}_i(k;t_0) = \int_{t_0}^{+\infty} e^{-\delta_i(t-t_0)}\tilde{\pi}_i(k;t_0)\,dt \longrightarrow \max_{\substack{\tilde{p}_i(k;t_0),\,\tilde{l}_i(k;t_0),\\ \{\tilde{y}_{ij}(k;t_0)\}_{j=1}^n,\,z_i(k;t_0)}} \tag{3.21}$$

subject to (3.13) and (3.19), where t_0 is the time of entry.

Before turning to the solution of a firm's problem, it is worth noting that, since there is free entry to each industry, the expected flow of a firm's profits has to be zero at every t, which can be straightforwardly shown to imply that $\tilde{\pi}_i(k;t) = 0$ at any t, or equivalently

$$z_i(k;t)^{\,\omega_i} = \tilde{p}_i(k;t)\,\tilde{y}_i(k;t) - w(t)\,\tilde{l}_i(k;t) - \sum_{j=1}^{N} p_j(t)\,\tilde{y}_{ij}(k;t)\,,\ \forall t \tag{3.22}$$

Starting with the static aspect of problem (3.21), given the demand equation (3.13) and production technology (3.17), one can derive the following result.

Proposition 3.1 (Optimal pricing and cost structure). *The share of each production factor in a firm's sales is described by the formulae*[21]

$$\frac{p_j(t)\,\tilde{y}_{ij}^*(k;t)}{\tilde{p}_i^*(k;t)\,\tilde{y}_i^*(k;t)} = \tfrac{\xi_i-1}{\xi_i}\nu_i\alpha_{ij} \equiv \frac{\nu_i\alpha_{ij}}{\mu_i} \tag{3.23}$$

$$\frac{w(t)\,\tilde{l}_i^*(k;t)}{\tilde{p}_i^*(k;t)\,\tilde{y}_i^*(k;t)} = \tfrac{\xi_i-1}{\xi_i}\,(1-\nu_i) \equiv \frac{1-\nu_i}{\mu_i} \tag{3.24}$$

[20] Alternatively, one can think of $\tilde{\Pi}_i(k;t)$ as the discounted flow of profits with δ_i's being the discount factor.

[21] Expressions (3.23), (3.24) enable one to interpret the ad hoc *RI* indicator in Section 3.2 as the elasticity of a sector's output with respect to the composite intermediate input.

where $\mu_i \equiv \frac{\xi_i}{\xi_i - 1}$ is the constant relative mark-up optimally charged by each firm over its marginal cost $\tilde{\psi}_i(k;t)$; the latter depends on a firm's productivity $\tilde{q}_i(k;t)$ and the industry-specific component $\psi_i(t)$

$$\begin{aligned} \tilde{p}_i^*(k;t) &= \frac{\xi_i}{\xi_i - 1}\,\tilde{\psi}_i(k;t) = \frac{\xi_i}{\xi_i - 1}\,\frac{\psi_i(t)}{\tilde{q}_i(k;t)} \\ \psi_i(t) &\equiv w(t)^{\,1-\nu_i}\left(\prod_{j=1}^{N} p_j(t)^{\,\alpha_{ij}}\right)^{\nu_i} \end{aligned} \tag{3.25}$$

Furthermore, under the optimal pricing strategy (3.25), the free-entry condition (3.22) transforms as follows

$$z_i(k;t) = \frac{\tilde{p}_i^*(k;t)\,\tilde{y}_i^*(k;t)}{\xi_i} \tag{3.26}$$

Proof. See Appendix A.2.1. ■

Turning to the characterisation of a firm's R&D decisions, one can show that in the optimum a firm's productivity grows over time at a constant rate specified in the proposition below.

Proposition 3.2 (Optimal productivity dynamics). *The optimal solution of problem (3.21) given (3.19), (3.20), (3.25) and the free-entry condition (3.22), implies that a firm's productivity $\tilde{q}_i^*(k;t)$ and R&D investment profile $z_i^*(k;t)$ are firm-independent industry-specific exponential functions of t, which assume the form*[22]

$$\tilde{q}_i^*(k;t) = \tilde{q}_i^*(t) = \tilde{q}_i^E(t_0)\,e^{\frac{\delta_i}{\phi_i}(t-t_0)} \tag{3.27}$$

$$z_i^*(k;t) = z_i^*(t) = \gamma_i^{-\frac{1}{\omega_i}}\,\tilde{q}_i^E(t_0)^{\frac{\chi_i}{\omega_i}}\,e^{\frac{\delta_i\chi_i}{\phi_i\omega_i}(t-t_0)} \tag{3.28}$$

where ϕ_i is defined as $\phi_i \equiv (\xi_i - 1)\,\omega_i + \frac{\chi_i}{\omega_i}\,(1-\omega_i)$. Thereby, the productivity of all firms in industry i grows at a constant rate equal to $\frac{\delta_i}{\phi_i}$.

[22] We slightly abuse notation by not stating $\tilde{q}_i^*(t)$ and $\tilde{z}_i^*(t)$ to be functions of the time of entry t_0: this is done in anticipation of the result that in fact both $\tilde{q}_i^*(k;t)$ and $\tilde{z}_i^*(t)$ are independent of t_0, so that all firms in an industry are homogeneous – see Corollary 3.2.1.

Proof. See Appendix A.2.2. ∎

Note that a firm's productivity growth rate $\frac{\delta_i}{\phi_i}$ depends positively on the obsolescence rate δ_i. This result is brought about by the fact that expression (3.27) embeds the assumption of free entry: as δ_i increases, a firm's expected flow of profits drops, thus reducing entry in industry. This results in fewer firms each with higher sales, which prompts them to invest in R&D at a higher rate, so that each unit of its increased output is produced at a lower cost.[23]

Conversely, an increase in χ_i or ω_i reduces the productivity growth rate $\frac{\delta_i}{\phi_i}$. A higher χ_i implies that current productivity growth is impeded more by the existing stock of knowledge (i.e. it is harder for firms to come by new ideas). Similarly, a higher ω_i entails that returns from R&D diminish faster, thus prompting firms to invest in R&D at a smaller rate. In both cases, lower effectiveness of innovation discourages firms from engaging in R&D, which reduces the resulting productivity growth rate.

An important implication of the fact that firms' growth rates are industry-specific, is the homogeneity of firms within an industry, as argued in the corollary below.

Corollary 3.2.1. *All firms in every industry at any t are homogeneous, and a firm's productivity and R&D spending are described by the formulae*

$$\tilde{q}_i^*(t) = \tilde{q}_i(0)\, e^{\frac{\delta_i}{\phi_i}t} = e^{\frac{\delta_i}{\phi_i}t} \tag{3.29}$$

$$\tilde{z}_i^*(t) = \gamma_i^{-\frac{1}{\omega_i}}\, e^{\frac{\delta_i \chi_i}{\phi_i \omega_i}t} \tag{3.30}$$

Proof. See Appendix A.2.3. ∎

The homogeneity of firms within each industry simplifies significantly the characterisation of industrial and economy-wide aggregates, to which we turn

[23]As an alternative argument, one could follow Cohen and Klepper (1996) by stating that larger sales allow a firm to spread further its R&D costs, thus enabling it to innovate more.

next.

Equilibrium Aggregate Variables

The implications of the intra-industry homogeneity of firms are two-fold: first of all, in conjunction with the assumption of free entry, this suggests that each firm breaks even exactly, thus leaving labour compensation $w(t)\,L(t)$ as the only source of the representative household's income. Given that, total consumption in the economy $C(t)$ equals total labour income, thereby changing equation (3.11) accordingly

$$C(t) = Y(t) = w(t)\,L(t) \tag{3.31}$$

As in the illustrating example in Section 3.3.1, since the economy's total value added comprises only labour compensation, the growth rate of output per capita equals that of the wage rate.

Secondly, given the homogeneity of firms in each industry, industrial aggregates $y_i(t)$ and $p_i(t)$ take the form

$$y_i(t) = m_i(t)^{1+\kappa_i}\,\tilde{y}_i(t) \tag{3.32}$$

$$p_i(t) = m_i(t)^{-\kappa_i}\,\tilde{p}_i(t) \tag{3.33}$$

As follows from (3.32), parameter κ_i is by definition the degree of returns to specialisation in industry i.[24] Expressions (3.32) and (3.33) connect returns to specialisation κ_i with the response of industrial price indices to entry: if the returns are decreasing (i.e. $\kappa_i \in (-1;0)$), the entry of more firms increases the marginal cost of producing the industrial good, which, given that its producers are competitive, directly translates into a higher price index $p_i(t)$. Symmetric arguments apply to two other cases of constant ($\kappa_i = 0$) and increasing ($\kappa_i \in (0;1)$) returns to specialisation.

[24]See footnote 17 and the references cited there.

Combining (3.33) and (3.32) allows one to express $m_i(t)$ as a function of aggregate output $Y(t) = C(t)$

$$p_i(t)\, y_i(t) = m_i(t)\, \tilde{p}_i^*(t)\, \tilde{y}_i^*(t) \Leftrightarrow m_i(t) = \frac{p_i(t)\, y_i(t)}{\tilde{p}_i^*(t)\, \tilde{y}_i^*(t)} = \frac{\beta_i Y(t)}{\xi_i z_i^*(t)^{\,\omega_i}} \tag{3.34}$$

where the second equality follows from applying the free-entry condition (3.26). As follows from (3.34), the mass of firms in an industry decreases in a firm's R&D spending, which stems from the fact that R&D, being effectively a fixed cost (from the production perspective), determines the size of a firm under free entry, which naturally suggests that more R&D translates into larger firms, of which fewer can be accommodated within an industry.

As in the introductory example in Section (3.3.1), one can derive the economy's growth rate by pinning down its system of equilibrium industrial price indices. To that end, combining equations (3.25) and (3.33) yields

$$p_i(t) = \frac{\xi_i}{\xi_i - 1}\, m_i(t)^{-\kappa_i} \frac{w(t)^{\,1-\nu_i} \left(\prod_{j=1}^{N} p_j(t)^{\,\alpha_{ij}}\right)^{\nu_i}}{\tilde{q}_i^*(t)} \tag{3.35}$$

One can use the solution of the system of equations implied by (3.35) to show that the growth rate of the economy takes the form specified in the proposition below.

Proposition 3.3 (The economy's equilibrium growth rate). *Given the system of equations generated by (3.35) $\forall i$, the economy's output per capita grows at the constant rate g*

$$g = \boldsymbol{\beta}^T \boldsymbol{\Lambda}_\nu \left(\mathbf{g}_q + \mathrm{dg}(\boldsymbol{\kappa})\, \mathbf{g}_m\right) \tag{3.36}$$

where $\boldsymbol{\Lambda}_\nu \equiv \left(\mathbf{E}_N - \mathrm{dg}(\boldsymbol{\nu})\, \mathbf{A}\right)^{-1}$ is the Leontiev inverse, $\mathbf{g}_q$ and $\mathbf{g}_m$ are, respectively, the vectors of firm productivity growth rates $\frac{\delta_i}{\phi_i}$ and firm mass growth rates (specified below in (3.38)) across all industries; $\boldsymbol{\kappa}$ is the vector of κ_is (degrees of returns to specialisation); dg$(\cdot)$ *is the diagonalisation operator transforming an $N \times 1$ vector $\mathbf{x}$ into the $N \times N$ matrix $\mathbf{X}$ with $\mathbf{x}$'s elements on its main diagonal and zeros elsewhere. Both $\mathbf{g}_q$ and $\mathbf{g}_m$ are time-independent.*

Proof. See Appendix A.2.4. ■

Note that the impact of a change in the mass of firms in an industry can be either positive or negative depending on the returns to specialisation κ_is. Naturally, if those are positive, an increase in the mass of firms operating in an industry lead to a drop in the industry's price index, thereby producing an expansionary impact on firms further downstream through the decrease in their costs.

Switching to the growth rates in (3.34), and transforming the result to the vector form, yields $\mathbf{g}_m = (g + g_L)\,\mathbf{e} - \mathrm{dg}(\boldsymbol{\chi})\,\mathrm{dg}(\boldsymbol{\omega})\,\mathbf{g}_q$. When combined with (3.36), the last expression yields a closed-form solution for the economy's growth rate g

$$g = \boldsymbol{\beta}^T \boldsymbol{\Lambda}_\nu \left(\mathbf{g}_q + \mathrm{dg}(\boldsymbol{\kappa})\left((g + g_L)\,\mathbf{e} - \mathrm{dg}(\boldsymbol{\chi})\,\mathrm{dg}(\boldsymbol{\omega})\,\mathbf{g}_q\right)\right)$$

$$g = \frac{\sigma}{1-\sigma} g_L + \frac{1}{1-\sigma} \boldsymbol{\beta}^T \boldsymbol{\Lambda}_\nu \left(\mathbf{E}_N - \mathrm{dg}(\boldsymbol{\kappa})\,\mathrm{dg}(\boldsymbol{\chi})\,\mathrm{dg}(\boldsymbol{\omega})\right)\mathbf{g}_q \tag{3.37}$$

where $\sigma \equiv \boldsymbol{\beta}^T \boldsymbol{\Lambda}_\nu \boldsymbol{\kappa}$. As the final step, the growth rates of firm masses in industries $\mathbf{g}_m$ can be recovered by combining the vectorised growth rate version of (3.34) mentioned above, with (3.37)

$$\mathbf{g}_m = (g + g_L)\,\mathbf{e} - \mathrm{dg}(\boldsymbol{\chi})\,\mathrm{dg}(\boldsymbol{\omega})\,\mathbf{g}_q \tag{3.38}$$

Expression (3.38) completes the solution of the model.

A particularly simple and analytically tractable characterisation of equation (3.36) emerges in the case of an economy with constant returns to specialisation in each industry (i.e. $\kappa_i = 0\ \forall i$), so that the growth rate in (3.36) becomes a function of productivity growth rates $\frac{\delta_i}{\phi_i}$ in industries, and equation (3.36) reduces to

$$g = \boldsymbol{\beta}^T \boldsymbol{\Lambda}_\nu \mathbf{g}_q \equiv \boldsymbol{\beta}^T \left(\mathbf{E}_N - \mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A}\right)^{-1} \mathbf{g}_q \tag{3.39}$$

If one was to interpret $\mathbf{g}_q$ as a vector of productivity shocks' logarithms instead of productivity growth rates, equation (3.39) is the matrix equivalent of expressions (2), (3) in Acemoglu et al. (2017).

Despite being a particular case of expression (3.37), equation (3.39) captures two key aspects of the interaction between the intersectoral linkage and growth: first of all, the exact structure of connections between sectors in the economy (as expressed through $\mathbf{A}$) affects the economy's growth rate; secondly, so does the extent of industries' products being used as intermediates (as controlled through the vector of intermediates' shares $\boldsymbol{\nu}$). The sections to follow focus on studying more closely an economy described by (3.39).

3.4 Theoretical Implications of the Model

The examination of equation (3.39) suggests two sharp predictions on the relationships between the growth rate of an economy on one hand, and the shares of intermediates $\boldsymbol{\nu}$ and the structure of technology matrix $\mathbf{A}$ on the other. Starting with the former, one can formally show that an interconnected economy grows faster than its counterpart without an intersectoral linkage. Furthermore, an increase in industries' shares of intermediates translates into faster growth. To see that, first note that in the absence of interlinkage (i.e. when $\boldsymbol{\nu} = \mathbb{0}_N$), the economy's growth rate $\hat{g}$ becomes

$$\hat{g} = \boldsymbol{\beta}^T \left(\mathbf{E}_N - \mathrm{dg}(\mathbb{0}_N)\,\mathbf{A}\right)^{-1} \mathbf{g}_q = \boldsymbol{\beta}^T \mathbf{g}_q \tag{3.40}$$

One can use expressions (3.39) and (3.40) to formally show that an interconnected economy is guaranteed to grow faster than an economy without intersectoral linkage (which we will term a basic economy), and furthermore, an economy grows faster as its production becomes more reliant on intermediate products (i.e. g increases as $\boldsymbol{\nu}$ becomes larger, everything else held constant).

Proposition 3.4. *Let $\hat{g} = \boldsymbol{\beta}^T \mathbf{g}_q$ be the growth rate of a basic economy. Then for any choice of production matrix $\mathbf{A}$, $g > \hat{g}$. Furthermore, the gradient of the growth* *rate with respect to shares of intermediates* $\frac{\partial g}{\partial \boldsymbol{\nu}}$ *is strictly nonnegative* $\frac{\partial g}{\partial \boldsymbol{\nu}} \geqslant 0$ *so long as at least one industry experiences growth.*

Proof. See Appendix A.2.5. ■

This result is a direct generalisation of the conclusion in Section 3.3.1: through transmitting downstream the impact of productivity growth in an industry, a production network amplifies the effect of intra-industry growth episodes, thereby making an interconnected economy grow faster. Moreover, if the interlinkage plays a more pronounced role in production (i.e. when ν_is are larger), the amplification effects stemming from it, intuitively become more marked.

Turning to the link between technology matrix $\mathbf{A}$ and growth rate g, it is instructive to consider first a particular case of an economy described in (3.39), for which all industries are characterised by the same share of intermediates in production $\nu_i = \nu$, $\forall i$. In this setting, one can formally establish that an interconnected economy grows at its fastest when all its sectors are connected only with the fastest-growing industry, and the latter uses only its own product as the only intermediate. Such a configuration of a linkage is known as the star network, whose corresponding production matrix (denoted as $\mathbb{1}_N(i_0)$ thereafter) has a column of ones in the i_0th position, and zeros elsewhere. The result is established in the following proposition.

Proposition 3.5. *Suppose that* $\boldsymbol{\nu} = \nu\mathbf{e} \Leftrightarrow \mathrm{dg}(\boldsymbol{\nu}) = \nu\mathbf{E}_N$*, and let* i_0 *denote the index of the industry with the maximal growth rate* $g_{max} = g_{i_0} \equiv \max\limits_{i=1\div N}\left\{\frac{\delta_i}{\phi_i}\right\}$*. Then the growth rate of an economy described by (3.39), attains its maximal value when the economy's underlying structure is described by production matrix* $\mathbb{1}_N(i_0)$*. Furthermore, the maximum is strict if only one sector grows at the rate of* g_{max}*.*

Proof. See Appendix A.2.6. ■

One should note that Proposition 3.5 mirrors the conclusions drawn by Acemoglu et al. (2012) and Acemoglu et al. (2017) that the impact of microshocks is propagated more strongly in economies with a more concentrated interlinkage. If one were to liken productivity shifts to perfectly persistent nonrandom shocks (however oxymoronic this may sound), then our framework suggests that the impact of such a 'shock' on the economy is maximal if the technology structure of the economy is aligned in such a way as to propagate it maximally from the source. This is naturally the case when an economy's interlinkage is focused entirely on the industry within which productivity growth is the fastest. This result can be generalised for the case when industries' shares of intermediates are heterogeneous. One can first introduce the following notation

Definition. The concentration potential of industry i is $d_i \equiv \frac{\delta_i/\phi_i}{1-\nu_i}$.

Proposition 3.6. *Let* i_1 *denote the index of an industry with the highest concentration potential* $d_{i_1} \equiv \frac{\delta_{i_1}/\phi_{i_1}}{1-\nu_{i_1}} = d_{max} \equiv \max\limits_{i=1\times N}\left\{\frac{\delta_i/\phi_i}{1-\nu_i}\right\}$. *Then the growth rate of an economy described by (3.39), attains its maximal value when the economy's underlying structure is described by production matrix* $\mathbb{1}_N(i_1)$. *Furthermore, the maximum is strict if only one sector is characterised by the maximal value* d_{max}.

Proof. See Appendix A.2.6. ∎

The gist of Proposition 3.6 is that maximal growth is achieved when an economy's interlinkage is concentrated on the industry which, when being the focal point of the interlinkage, produces the maximal impact on the economy's growth rate. As follows from the definition of the concentration potential and Proposition 3.6, such a status is determined by two considerations: how fast an industry grows, and how much its production relies on intermediates. To see the importance of the latter factor, consider an industry characterised by fast productivity growth and low reliance on intermediates.[25] In this case, if the industry becomes the focus of the economy's productive network (and, in particular, its own supplier), the indirect impact of productivity growth within it on the economy is channelled only through the industry itself, since it draws only

[25]Formally the requirement of weak reliance on intermediates is equivalent to saying that there exists another industry with a higher concentration potential.

on its own product as the intermediate. Since the industry does not rely much on intermediates, its indirect impact on itself is liable to stay weak, thus limiting the strength of the overall effect of its productivity growth on the economy.

Therefore, while the productivity growth rate δ_i/ϕ_i captures the strength of an industry's initial impact on economic growth, that of its indirect impact is also affected by how intensely the industry uses intermediates, which in particular determines how much it itself is affected by productivity shifts within it.

Given the model's theoretical predictions, the next step is naturally to validate them empirically. As a preceding step, however, one needs to map the model's elements to data – we turn to this question in the next section.

3.5 Testing the Predictions

3.5.1 Calibration of the Model

This section discusses the paper's approach to quantifying the theoretical framework developed in Section 3.3.2, using the dataset introduced in Section 3.2.1. We start with a few brief introductory remarks concerning how values for the model's parameters $\boldsymbol{\beta}$ and $\mathbf{A}$ can calculated from input-output tables (Section 3.5.1), after which Section 3.5.1 describes extracting values for sectoral mark-ups μ_is and shares of intermediates $\boldsymbol{\nu}$. Section 3.5.1 focuses on various approaches to calculating technology growth rates.

First Step: Consumption Shares $\boldsymbol{\beta}_i$ and Technology Matrices $\mathbf{A}$

The calibration of the model requires quantifying five of its objects: vector of consumption shares $\boldsymbol{\beta}$, technology matrix $\mathbf{A}$, vector of sectoral mark-ups $\boldsymbol{\mu}$, vector of intermediate shares $\boldsymbol{\nu}$ and vector of productivity growth rates in industries $\mathbf{g}_q$. This section focuses on the first three, as those can be derived directly from the same statistical object – a country's input-output table, which contains data on flows of goods and services between industries, as well as the amount

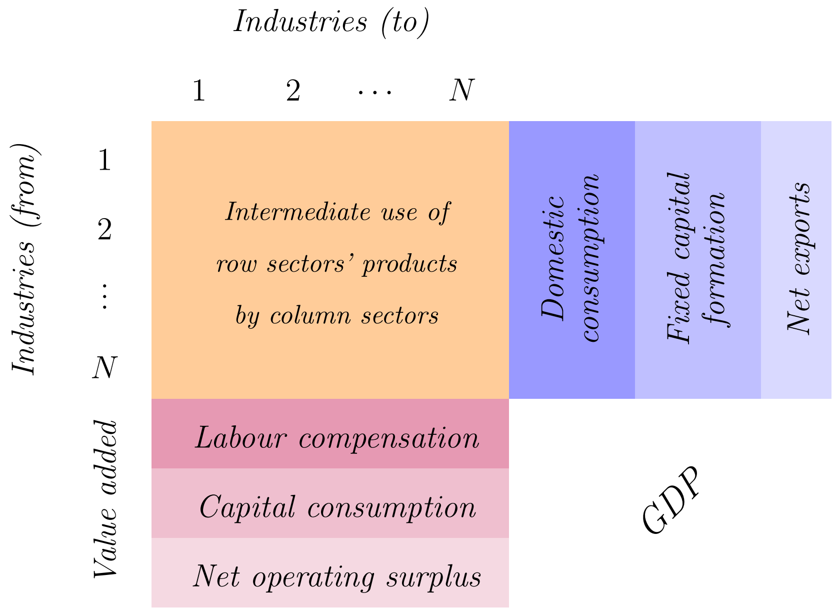

Figure 3.2: A schematic example of an input-output table.

of industries' production used in final consumption, and the volume of value added within each industry.[26] A schematic example of an input-output table is presented in Figure 3.2.

From Figure 3.2, consumption shares β_is can be derived using their definition by dividing consumption of an industry's product by total consumption. In particular, we derive the consumption of a sector's product as the sum of consumption by households, non-profit organisations and the government (domestic consumption) plus imports minus changes in inventories.

The key to extracting an economy's technology matrix $\mathbf{A}$ is equation (3.23), which connects the observable intensity in sectors' use of other sectors' products with the an industry's share of intermediates ν_i, mark-up μ_i and components of the technology matrix $\mathbf{A}$ as follows: $\frac{Sales_{ij}}{Sales_i} = \frac{\nu_i \alpha_{ij}}{\mu_i}$. Summing the expression across all js yields $\sum_{j=1}^{N} \frac{Sales_{ij}}{Sales_i} = \frac{\nu_i}{\mu_i}$, thereby pinning down α_{ij} as the ratio of the

[26]The description in the text concerns the so-called industry-by-industry input-output table, whose alternative is the product-by-product input-output table capturing flows between production of different goods. We focus on the former characterisation following the structure of datasets available for this paper.

two expressions' left-hand sides

$$\alpha_{ij} = \frac{\frac{\nu_i \alpha_{ij}}{\mu_i}}{\sum_{j=1}^{N} \frac{\nu_i \alpha_{ij}}{\mu_i}} = \frac{\frac{Sales_{ij}}{Sales_i}}{\sum_{j=1}^{N} \frac{Sales_{ij}}{Sales_i}} \tag{3.41}$$

Given (3.41), **A** can be extracted from the matrix of intermediate uses (marked orange in Figure 3.2) by normalising each of its columns by the sum of its elements, and then transposing the result.

A closer examination of equation (3.23) shows why, while all α_{ij}s can be extracted directly from an input-output table, ν_is cannot: as firms in the paper's framework are not competitive, production factors they use are remunerated at levels below their marginal products – this introduces the wedge between elasticities of output $\nu_i \alpha_{ij}$ and $1 - \nu_i$, and corresponding factor shares in sales $\frac{\nu_i \alpha_{ij}}{\mu_i}$ and $\frac{1-\nu_i}{\mu_i}$, making it impossible to infer ν_is directly from the last two conditions without the knowledge of sectoral mark-ups μ_i.

Second Step: Sectoral Mark-ups μ_i and Shares of Intermediates ν_i

The paper's main approach to estimating mark-ups is by calculating the share of an industry's net operating surplus (NOPS, which can be broadly thought of as a macro analogue of operating profits) in its total output net of taxes. Unfortunately, our data sources contain information on NOPS for approximately 64% of observations, which has led us to using cruder estimates for remaining μ_is. In particular, data on gross operating surplus (GOPS) have larger coverage than those on NOPS, but this indicator comprises both NOPS and industries' capital consumption (i.e. capital depreciation in sectors), thus producing positively biased estimates of mark-ups. To compensate for the bias, we take the average of GOPS-based estimates of μ_is and a lower estimate of an industry's operating surplus, for which we use the share of an industry's domestic investment in fixed capital (the so-called gross fixed capital formation, GFCF), in its total output.[27]

[27] Our reasoning for this choice of lower estimate is motivated by the fact that GFCF, being a form of investment, has to be ultimately financed from the revenues generated by an industry's operations.

Finally, whenever information on GOPS was missing, we estimated μ_is either using the measure based on domestic GFCF or (in the absence of information on the latter), the ratio of total GFCF to total sales of an industry's products including imports.

While differing from more analytical methods developed in the specialised literature (see, e.g., Hall (1988), Klette (1999), De Loecker and Warzynski (2012)), the paper's approach produces results remarkably consistent with those in the papers cited above, with the average and median mark-ups' equalling, respectively, 1.131 and 1.100 (see Figure A.2 for the distribution of mark-up estimates).

With μ_is' estimates, condition (3.23) becomes overidentified, as for every industry i its share of intermediates ν_i can be calculated as $\nu_i = \frac{Sales_{ij}}{Sales_i}\frac{\mu_i}{\alpha_{ij}}$ for any choice of the intermediate j. This ambiguity can be resolved by regressing the vector of $\mu_i \frac{Sales_{ij}}{Sales_i}$ on the vector of technological coefficients α_{ij} for all j, thereby pinning ν_i down as follows

$$\nu_i = \mu_i \frac{\sum_{j=1}^N \alpha_{ij}\frac{Sales_{ij}}{Sales_i}}{\sum_{j=1}^N \alpha_{ij}^2} = \mu_i \sum_{j=1}^N \frac{Sales_{ij}}{Sales_i} \tag{3.42}$$

where the rightmost expression in (3.42) follows from (3.41).[28]

As the four parameters $\boldsymbol{\beta}$, $\mathbf{A}$, $\boldsymbol{\mu}$, and $\boldsymbol{\nu}$ can be extracted from input-output tables, one can quantify them using the dataset introduced in Section 3.2.1 – see Appendix A.4 for the description of the results.

As the final step in quantifying the model's parameters, one can use the acquired knowledge of $\boldsymbol{\beta}$, $\boldsymbol{\nu}$ and $\mathbf{A}$ to derive industries' productivity growth rates, which is the focus of the next section.

[28]Alternatively, expression (3.42) can be derived by estimating ν_i for every intermediate j as $\frac{Sales_{ij}}{Sales_i}\frac{\mu_i}{\alpha_{ij}}$, and then aggregating the results using the weighted average with weights equal to the technological coefficients of the corresponding intermediates $\nu_i = \sum_{j=1}^N \alpha_{ij}\left(\frac{Sales_{ij}}{Sales_i}\cdot\frac{\mu_i}{\alpha_{ij}}\right) = \mu_i \sum_{j=1}^N \frac{Sales_{ij}}{Sales_i}$.

Third Step: Productivity Growth Rates $\frac{\delta_i}{\phi_i}$

While data for $\boldsymbol{\beta}$ and $\mathbf{A}$ can be extracted directly from a country's standard input-output table, recovering industries' shares of intermediates and growth rates can be achieved only indirectly through using either a firm's production function (3.17) (the direct approach)[29] or its price function (3.25) (the dual approach).

The first method relies on the industry-level version of (3.17), which can be derived by multiplying both sides of the expression by the mass of firms $m_i(t)$

$$m_i(t)\,\tilde{y}_i(t) = y_i(t) = \frac{\tilde{q}_i(t)}{\zeta_i}\left(m_i(t)\,\tilde{l}_i(t)\right)^{1-\nu_i}\left(\prod_{j=1}^{N}\left(m_i(t)\,\tilde{y}_{ij}(t)\right)^{\alpha_{ij}}\right)^{\nu_i} \tag{3.43}$$

where $m_i(t)\,\tilde{l}_i(t) \equiv l_i(t)$ is an industry's consumption of primary production factors, and $m_i(t)\,\tilde{y}_{ij}(t) \equiv y_{ij}(t)$ is the amount of industry j's real output used in industry i. Given (3.43), the growth rates of $\tilde{q}_i^*(t)$s can be quantified empirically as follows

$$\ln\left(\frac{\tilde{q}^*_{i;t+1}}{\tilde{q}^*_{i;t}}\right) = \ln\left(\frac{y_{i;t+1}}{y_{i;t}}\right) - (1-\nu_i)\ln\left(\frac{l_{i;t+1}}{l_{i;t}}\right) - \nu_i\sum_{j=1}^{N}\alpha_{ij}\ln\left(\frac{y_{ij;t+1}}{y_{ij;t}}\right) \tag{3.44}$$

All N productivity growth rates $\ln\left(\frac{\tilde{q}^*_{i;t+1}}{\tilde{q}^*_{i;t}}\right)$ can be exactly identified from condition (3.44) applied to every i.

The main complication in employing the method above is that an accurate assessment of primary factors' volumes would involve imputing amounts of capital services used in industries. The latter is approximated by estimates of the stock of capital in an industry, which itself requires making assumptions on capital depreciation rates, as well as asset composition and initial levels of capital.

An alternative to the approach above is to use a firm's pricing condition (3.25) which, when multiplied by price index $P(t)$, brings about the expres-

[29]The approach is used in such productivity databases as PDB and PDBi by OECD, and EU KLEMS.

sion

$$P(t)\,p_i(t) = \mu_i \frac{\left(P(t)\,w(t)\right)^{1-\nu_i}\left(\prod_{j=1}^{N}\left(P(t)\,p_j(t)\right)^{\alpha_{ij}}\right)^{\nu_i}}{\tilde{q}_i^*(t)} \tag{3.45}$$

where the absence of tildes at $p_i(t)$ and $p_j(t)$s underscores that formula (3.45) holds for industries' price indices as much as for individual firms, owing to the assumption that $\boldsymbol{\kappa} = \mathbb{0}$. With this equivalence in mind, one can recover $P(t)\,p_i^*(t)$s from data as sectoral price indices, and $P(t)\,w(t)$ as nominal GDP per capita. Thereby, the empirical equivalent of (3.45) is

$$\varpi_{i;t} = \mu_i \frac{\left(\frac{nGDP_t}{L_t}\right)^{1-\nu_i}\left(\prod_{j=1}^{N}\varpi_{j;t}{}^{\alpha_{ij}}\right)^{\nu_i}}{\tilde{q}_{i;t}^*} \tag{3.46}$$

where term $\prod_{j=1}^{N}\varpi_{j;t}{}^{\alpha_{ij}}$ can be interpreted as the price index of intermediates used in industry i. Switching to the growth-rate form of (3.46) enables one to isolate the growth rate of $\tilde{q}_{i;t}$, thus yielding the final result

$$\ln\left(\frac{\varpi_{i;t+1}}{\varpi_{i;t}}\right) = (1-\nu_i)\ln\left(\frac{nGDP_{t+1}/L_{t+1}}{nGDP_t/L_t}\right) + \nu_i \ln\left(\frac{\sum_{j=1}^{N}\alpha_{ij}\varpi_{j;t+1}}{\sum_{j=1}^{N}\alpha_{ij}\varpi_{j;t}}\right) - \ln\left(\frac{\tilde{q}_{i;t+1}^*}{\tilde{q}_{i;t}^*}\right) \tag{3.47}$$

$$\ln\left(\frac{\tilde{q}_{i;t+1}^*}{\tilde{q}_{i;t}^*}\right) = \nu_i \ln\left(\frac{\sum_{j=1}^{N}\alpha_{ij}\hat{\varpi}_{j;t+1}}{\sum_{j=1}^{N}\alpha_{ij}\hat{\varpi}_{j;t}}\right) - \ln\left(\frac{\hat{\varpi}_{i;t+1}}{\hat{\varpi}_{i;t}}\right) \tag{3.48}$$

where $\hat{\varpi}_{i;t} \equiv \frac{\varpi_{i;t}}{nGDP_t/L_t}$ is the ratio of an industry's price index to the country's output per capita. Together equations of type (3.48) form a system from which $\ln\left(\frac{\tilde{q}_{i;t+1}^*}{\tilde{q}_{i;t}^*}\right)$ can be exactly identified.

Note that irrespective of the method used, each of the exactly identified systems of equations (either based on (3.44) or on (3.48)) can be transformed into an over-identified one by adding the expression for the growth rate (3.39), which opens the way to calculating $\ln\left(\frac{\tilde{q}_{i;t+1}^*}{\tilde{q}_{i;t}^*}\right)$ using the GMM.

Comparing the two approaches above suggests that while both require the knowledge of industrial price indices – either direct for $\varpi_{i;t}$s in the dual approach, or for the imputation of real quantities of intermediates $y_{ij;t}$, the latter method also involves making further assumptions concerning capital assets used in industries. For this reason the indirect approach seems a more desirable option.

3.5.2 Empirical Analysis

As mentioned in the last section, values for four out five primitives in the framework (namely, $\boldsymbol{\beta}$, $\mathbf{A}$, $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$) can be extracted from countries' input-output tables (i.e. from the dataset introduced in Section 3.2.1), while calculating sectoral growth rates requires the extra input of sectoral prices. Even though the last aspect is still work in progress, having the values for shares of intermediates ν_is enables one to revisit exploring the relationship between reliance on intermediates and economic growth, as the paper's model makes a more specific prediction of the presence of a positive link between $\boldsymbol{\nu}$ and g, as follows from Proposition 3.4.

We test this implication by repeating the exercise from Section 3.2.2 with average value of ν_i used instead of the ad hoc RI^a indicator

$$Gr_{i;t}^{\Delta t} = const + b_1 \bar{\nu}_{i;t-1} + \mathbf{b}\mathbf{x}_{i;t-1} \tag{3.49}$$

Applying the methodology of Section 3.2.2 to equation (3.49) yields results strongly consistent with the findings presented in Section 3.2.2,[30] with the signs as predicted by the model (see Table 3.3). As in Section 3.2.2, the positive link between the use of intermediates and economic growth passes the robustness check of substituting the median share of intermediates ν_{med} for $\bar{\nu}$, as demonstrated in

[30]While the insignificance of $\bar{\nu}$ and ν_{med} for the five-year period specification admittedly does not support our theoretical predictions, it is clearly outweighed by the strong significance of both $\bar{\nu}$ and ν_{med} across all other averaging intervals, and the coefficients' consistence with the results presented in Tables 3.1, 3.2.

Table 3.4.

Table 3.3: The relationship between economic growth in 1995–2011 and the mean share of intermediates $\bar{\nu}$.

Dep. var.	*Growth rate of GDP per worker averaged across Δt years*				
	$\Delta t = 5\,yrs$	$\Delta t = 6\,yrs$	$\Delta t = 7\,yrs$	$\Delta t = 8\,yrs$	$\Delta t = 16\,yrs$
$\bar{\nu}$	0.009 (0.015)	0.041*** (0.007)	0.057*** (0.001)	0.063*** (0.004)	0.029*** (0.007)
$\ln(hc)$	0.017** (0.008)	0.037*** (0.002)	0.028*** (0.003)	0.022*** (0.008)	0.025*** (0.008)
$\ln(Y/L)$	−0.016*** (0.001)	−0.018*** (0.000)	−0.018*** (0.001)	−0.016*** (0.000)	−0.019*** (0.002)
g_L	−0.540*** (0.054)	−0.352*** (0.087)	−0.276*** (0.006)	−0.310*** (0.050)	0.325* (0.170)
s	0.009 (0.011)	−0.002 (0.003)	−0.003 (0.008)	−0.019*** (0.000)	−0.005 (0.008)
const	0.161*** (0.003)	0.150*** (0.004)	0.142*** (0.006)	0.124*** (0.003)	0.158*** (0.013)
Regional dummies	Yes	Yes	Yes	Yes	Yes
Countries	63	63	63	63	63
Periods	3	2	2	2	1
R^2	0.658	0.909	0.791	0.941	0.989
R^2_{adj}	0.631	0.898	0.767	0.935	0.986

NOTES $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

1. $\ln(hc)$ and $\ln(Y/L)$ are the logarithms of initial levels of human capital and GDP per capita, g_L is the population growth rate, s is the savings rate;
2. In order to control for cross-country heteroskedasticity, cross-sectional weighting was used alongside calculating standard errors robust to cross-section heteroskedasticity;
3. The case of $\Delta t = 16$ corresponds to a simple cross-country regression estimated using the weighted LS.

Table 3.4: The relationship between economic growth in 1995–2011 and the median share of intermediates ν_{med}.

Dep. var.	*Growth rate of GDP per worker averaged across Δt years*				
	$\Delta t = 5\,yrs$	$\Delta t = 6\,yrs$	$\Delta t = 7\,yrs$	$\Delta t = 8\,yrs$	$\Delta t = 16\,yrs$
ν_{med}	-0.001 (0.011)	0.026^{***} (0.002)	0.037^{***} (0.000)	0.034^{***} (0.000)	0.026^{***} (0.007)
$\ln(hc)$	0.017^{**} (0.007)	0.033^{***} (0.000)	0.025^{***} (0.003)	0.020^{**} (0.008)	0.027^{***} (0.004)
$\ln(Y/L)$	-0.016^{***} (0.001)	-0.017^{***} (0.000)	-0.017^{***} (0.001)	-0.015^{***} (0.000)	-0.019^{***} (0.001)
g_L	-0.538^{***} (0.063)	-0.344^{***} (0.091)	-0.307^{***} (0.027)	-0.281^{***} (0.036)	0.391^{***} (0.143)
s	0.009 (0.013)	-0.003 (0.003)	-0.004 (0.010)	-0.013^{***} (0.002)	-0.008^{*} (0.005)
const	0.168^{***} (0.001)	0.155^{***} (0.004)	0.150^{***} (0.005)	0.138^{***} (0.001)	0.159^{***} (0.013)
Regional dummies	Yes	Yes	Yes	Yes	Yes
Countries	63	63	63	63	63
Periods	3	2	2	2	1
R^2	0.650	0.838	0.777	0.832	0.989
R^2_{adj}0.623	0.819	0.751	0.813	0.987	

NOTES $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

See Table 3.3.

3.6 Summary

This paper has explored the role of intersectoral connection as an amplifying factor of economic growth. Our results suggest that stronger connections between industries result in stronger growth through the presence of Hulten's multiplier – an increase in the productivity of a firm increases that of its downstream counteragents, which can then affect the original firm indirectly if it sources some of its inputs from them.

This work has two direct avenues for further development. First of all, the author's ongoing project is the empirical testing of the framework's predictions on the optimal structure of the intersectoral interlinkage. This validation is based on the knowledge of sectoral productivity growth rates (which can be derived from sectoral prices along the lines discussed in the paper), which can be used to identify industries with the highest concentration potential, and to examine whether economies concentrated on those, tend to grow faster.

The second direction concerns studying the properties of the general model introduced in the framework, and in particular, examining how robust the star network is – as the optimal growth-enhancing configuration of the sectoral interlinkage – in these conditions.

Finally, throughout this work sectoral technologies (as jointly represented by technology matrix $\mathbf{A}$) have been assumed to be exogenous – while this suffices for the scope of the present research, in future it would be instructive to consider a framework with endogenised technology choice following the frameworks advanced by Acemoglu (2002, 2007), Jones (2005) and Caselli and Coleman (2006).

Conclusion

As one famous but variably attributed[31] saying goes, to believe that infinite growth is possible with finite resources, one has to be an economist or a madman. Perhaps, productivity growth and, in particular, technological growth constitutes the main reason why economists are included as a separate group in this insightful quote. Indeed, humanity's ability to produce more and better from a given amount of inputs, has been arguably one of the most important long-run drivers of socio-economic development starting from the Neolithic revolution about 12 000 years ago.

This dissertation has sought to improve the state of our understanding of three innovation- and technology-related phenomena. The first two are innovation and technology growth factors: competition between innovating firms and business cycles, and one is a transmission channel from technological growth to economic growth – the presence of production interdependencies between firms within an economy.

In particular, Chapter 1 suggests a way of understanding how a nonlinear relationship between competition and innovation can remain hidden if there is no consensus on which indicator is used to measure the innovative activity. Chapter 2 argues that innovation effort can be both procyclical and countercyclical – that is, business cycles can appear to contribute both positively and negatively to it, depending on whether it is considered on the macro-level of industries and economies, or the micro-level of individual firms. Finally, Chapter 3 offers a way

[31] To the best of the author's knowledge, this apophthegm has been ascribed to either American economist Kenneth E. Boulding or British broadcaster Sir David F. Attenborough.

of thinking about production links between industries as – in its own right – a magnifying factor for productivity growth occurring within firms, which amplifies and channels it to the level of the whole economy, as well as pinning down the link structure which delivers the maximal strengthening of sectoral growth.

A natural way to extend the first two chapters is to seek more direct empirical support to theories advanced therein. This would require working with firm-level datasets which include information on R&D (i.e., Compustat for the U.S. or AMADEUS for Europe). As concerns the last chapter, three key avenues for further exploration are, first, the investigation of the framework's generalised version with scaled effects in an economy's sectors, including its calibration; second, empirical validation of the framework's predictions on the optimal structure of an intersectoral linkage, and finally, endogenising the structure of connections between sectors.

The author hopes that the findings arrived at and presented in this dissertation will stand the profession in good stead in its continuous quest for bettering our knowledge on the phenomenon of productivity growth and technological innovation.

Bibliography

Daron Acemoglu. Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality. *The Quarterly Journal of Economics*, 113 (4):1055–1089, November 1998.

Daron Acemoglu. Directed Technical Change. *The Review of Economic Studies*, 69(4):781–809, October 2002.

Daron Acemoglu. Equilibrium Bias of Technology. *Econometrica*, 75(5):1371–1410, October 2007.

Daron Acemoglu. *Introduction to Modern Economic Growth*. Princeton University Press, Princeton, NJ, 2009.

Daron Acemoglu and Dan Cao. Innovation by Entrants and Incumbents. *Journal of Economic Theory*, 157:255–294, May 2015.

Daron Acemoglu, Pol Antrás, and Elhanan Helpman. Contracts and Technology Adoption. *The American Economic Review*, 97(3):916–943, June 2007.

Daron Acemoglu, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. The Network Origins of Aggregate Fluctuations. *Econometrica*, 80(5): 1977–2016, September 2012.

Daron Acemoglu, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. Systemic Risk and Stability in Financial Networks. *The American Economic Review*, 105(2): 564–608, February 2015.

Daron Acemoglu, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. Networks, Shocks and Systemic Risk. In Yann Bramoullé, Andrea Galeotti, and Brian W. Rogers, editors, *The Oxford Handbook of the Economics of Networks*, chapter 21, pages 569–607. Oxford University Press, Oxford, UK, 2016.

Daron Acemoglu, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. Microeconomic Origins of Macroeconomic Tail Risks. *The American Economic Review*, 107 (1):54–108, January 2017.

Philippe Aghion and Peter Howitt. A Model of Growth Through Creative Destruction. *Econometrica*, 60(2):323–351, March 1992.

Philippe Aghion and Peter Howitt. *Endogenous Growth Theory*. MIT Press, Cambridge, Mass., 1998.

Philippe Aghion and Peter Howitt. Growth with Quality-Improving Innovations: An Integrated Framework. In Philippe Aghion and Steven N. Durlauf, editors, *Handbook of Economic Growth, Vol. 1*, chapter 8, pages 67–110. North Holland, Oxford, UK, 2005.

Philippe Aghion and Gilles Saint-Paul. Virtues of Bad Times. *Macroeconomic Dynamics*, 2(3):322–344, September 1998.

Philippe Aghion, Nick Bloom, Richard Blundell, Rachel Griffith, and Peter Howitt. Competition and Innovation: An Inverted-U Relationship. *The Quarterly Journal of Economics*, 120(2):701–728, May 2005.

Philippe Aghion, George-Marios Angeletos, Abhijit Banerjee, and Kalina Manova. Volatility and Growth: Credit Constraints and the Composition of Investment. *Journal of Monetary Economics*, 57(3):246–265, April 2010.

Philippe Aghion, Philippe Askenazy, Nicolas Berman, Gilbert Cette, and Laurent Eymard. Credit and the Cyclicality of R&D Investment: Evidence from France. *Journal of the European Economic Association*, 10(5):1001–1024, October 2012.

Philippe Aghion, Ufuk Akcigit, and Peter Howitt. Growth Econometrics. In Philippe Aghion and Steven N. Durlauf, editors, *Handbook of Economic Growth, Vol. 2B*, chapter 1, pages 515–563. North Holland, Oxford, UK, 2014.

Kenneth J. Arrow. The Economic Implications of Learning by Doing. *The Review of Economic Studies*, 29(3):155–173, June 1962.

Philippe Askenazy, Christophe Cahn, and Delphine Irac. Competition, R&D, and the Cost of Innovation: Evidence form France. *Oxford Economic Papers*, 65(2):293–311, April 2013.

Andrew Atkeson and Ariel Burstein. Pricing-to-Market, Trade Costs, and International Relative Prices. *The American Economic Review*, 98(5):1998–2031, December 2008.

Mauro Bambi, Fausto Gozzi, and Omar Licandro. Endogenous Growth and Wave-Like Business Fluctuations. *Journal of Economic Theory*, 154(5):68–111, September 2014.

Robert B. Banks. *Growth and Diffusion Phenomena: Mathematical Frameworks and Applications*. Texts in Applied Mathematics. Springer-Verlag, New York, NY, 1994.

David R. Baqaee. Cascading Failures in Production Networks. Working paper, Harvard University, September 2016.

Gadi Barlevy. On the Cyclicality of Research and Development. *The American Economic Review*, 97(4):1131–1164, September 2007.

Robert J. Barro and Xavier Sala-i-Martin. *Economic Growth*. MIT Press, Cambridge, Mass., 2 edition, 2004.

Jean-Pascal Bènassy. Taste for Variety and Optimum Production Patterns in Monopolistic Competition. *Economics Letters*, 52(1):41–47, July 1996.

Jean-Pascal Bènassy. Is There Always Too Little Research in Endogenous Growth with Expanding Product Variety? *European Economic Review*, 42(1):61–69, January 1998.

Pilar Beneito, María Engracia Rochina-Barrachina, and Amparo Sanchis-Llopis. Ownership and the Cyclicality of Firms' R&D Investment. *International Entrepreneurship and Management Journal*, 11(2):343–359, June 2015.

Benjamin Bental and Dan Peled. The Accumulation of Wealth and the Cyclical Generation of New Technologies: A Search Theoretic Approach. *International Economic Review*, 53(2):687–718, August 1996.

Paul Bishop and Nick Wiseman. External Ownership and Innovation in the United Kingdom. *Applied Economics*, 31(4):443–450, April 1999.

Richard Blundell, Rachel Griffith, and John van Reenen. Market Share, Market Value and Innovation in a Panel of British Manufacturing Firms. *Journal of Economic Perspectives*, 66(3):529–554, 1999.

Christian Broda and David E. Weinstein. Globalization and the Gains from Variety. *The Quarterly Journal of Economics*, 121(2):541–585, May 2006.

Christian Broda and David E. Weinstein. Product Creation and Destruction: Evidence and Price Implications. *The American Economic Review*, 100(3): 691–723, June 2010.

Ricardo J. Caballero and Mohamad L. Hammour. The Cleansing Effect of Recessions. *The American Economic Review*, 84(5):1350–1368, December 1994.

Jeffrey R. Campbell. Entry, Exit, Embodied Technology, and Business Cycles. *Review of Economic Dynamics*, 1(2):371–408, April 1998.

Gerald Carlino and William R. Kerr. Agglomeration and Innovation. In Gilles Duranton, J. Verner Henderson, and William C. Strange, editors, *Handbook of Regional and Urban Economics, Vol. 5*. North Holland, Oxford, UK, 2015.

Vasco M. Carvalho. From Micro to Macro via Production Networks. *The Journal of Economic Perspectives*, 28(4):23–47, Autumn 2014.

Vasco M. Carvalho and Nico Voigtländer. Input Diffusion and the Evolution of Production Networks. Working Paper 20025, NBER, March 2015.

Francesco Caselli and John Coleman. The World Technology Frontier. *The American Economic Review*, 96(3):499–522, June 2006.

Tiago V.V. Cavalcanti and Chryssi Giannitsarou. Growth and Human Capital: A Network Approach. *The Economic Journal*, 127(603):1279–1317, August 2017.

Suk Bong Choi, Lee Soo Hee, and Christopher Williams. Ownership and Firm Innovation in a Transition Economy: Evidence from China. *Research Policy*, 40(3):441–452, April 2011.

Lawrence J. Christiano, Martin Eichenbaum, and Charles L. Evans. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 113(1):1–45, February 2005.

Gian Luca Clementi and Berardino Palazzo. Entry, Exit, Firm Dynamics, and Aggregate Fluctuations. *American Economic Journal: Macroeconomics*, 8(3): 1–41, July 2016.

Wesley M. Cohen and Steven Klepper. A Reprise of Size and R&D. *The Economic Journal*, 106(437):925–951, July 1996.

Diego Comin and Mark Gertler. Medium-Run Economic Cycles. *The American Economic Review*, 96(3):523–551, June 2006.

Guido Cozzi and Giammario Impullitti. Government Spending Composition, Techical Change and Wage Inequality. *Journal of the European Economic Association*, 8(6):1325–1358, December 2010.

Claude d'Aspremont, Rodolphe Dos Santos Ferreira, and Louis-André Gérard-Varet. Strategic R&D Investment, Competitive Toughness and Growth. *International Journal of Economic Theory*, 6(3):273–295, September 2010.

Jan De Loecker and Frederic Warzynski. Markups and Firm-Level Export Status. *The American Economic Review*, 102(6):2437–2471, October 2012.

Avinash K. Dixit and Joseph E. Stiglitz. Monopolistic Competition and Optimum Product Diversity. *The American Economic Review*, 67(3):297–308, June 1977.

Margarida Duarte and Diego Restuccia. The Role of the Structural Transformation in Aggregate Productivity. *Marketing Science*, 125(1):129–173, February 2010.

Jean-Pierre Dubé and Puneet Manchanda. Differences in Dynamic Brand Competition across Markets: An Empirical Analysis. *Marketing Science*, 24(1): 81–95, Winter 2005.

Steven N. Durlauf, Paul A. Johnson, and Jonathan R.W. Temple. Growth Econometrics. In Philippe Aghion and Steven N. Durlauf, editors, *Handbook of Economic Growth, Vol. 1*, chapter 8, pages 555–677. North Holland, Oxford, UK, 2005.

Jonathan Eaton and Samuel Kortum. Technology, Geography, and Trade. *Econometrica*, 70(5):1741–1779, September 2002.

Wilfred J. Ethier. National and International Returns to Scale in the Modern Theory of International Trade. *The American Economic Review*, 72(3):389–405, June 1982.

Antonio Fatás. Do Business Cycles Cast Long Shadows? Short-Run Persistence and Economic Growth. *Journal of Economic Growth*, 5(2):147–162, June 2000.

Robert C. Feenstra, Robert Inklaar, and Marcel P. Timmer. The Next Generation of the Penn World Table. *The American Economic Review*, 105(10):3150–3182, October 2015.

Patrick Francois and Huw Lloyd-Ellis. Animal Spirits through Creative Destruction. *The American Economic Review*, 93(3):530–550, June 2003.

Patrick Francois and Huw Lloyd-Ellis. Schumpeterian Cycles with Pro-Cyclical R&D. *Review of Economic Dynamics*, 12(3):550–530, October 2009.

Xavier Gabaix and Rustam Ibragimov. Rank – 1/2: A Simple Way to Improve the OLS Estimation of Tail Exponents. *Journal of Business and Economic Statistics*, 29(1):24–39, January 2011.

David Gale. *The Theory of Linear Economic Models*. McGraw-Hill, New York, NY, 1960.

Jordi Galí. Monopolistic Competition, Endogenous Markups, and Growth. *European Economic Review*, 38(3–4):748–756, April 1994.

Jordi Galí. Product Diversity, Endogenous Markups and Development Traps. *Journal of Monetary Economics*, 36(1):39–63, February 1995.

Jordi Galí, Mark Gertler, and J. David López-Salido. Markups, Gaps, and the Welfare Costs of Business Fluctuations. *The Review of Economics and Statistics*, 89(1):44–59, February 2007.

Gino Gancia and Fabrizio Zilibotti. Horizontal Innovation in the Theory of Growth and Development. In Philippe Aghion and Steven N. Durlauf, editors, *Handbook of Economic Growth, Vol. 1*, chapter 3, pages 111–170. North Holland, Oxford, UK, 2005.

Richard Gilbert. Looking for Mr. Schumpeter: Where Are We in the Competition–Innovation Debate? *Innovation Policy and the Economy*, 6:159–215, 2006.

Rachel Griffith, Stephen Redding, and John Van Reenen. Mapping the Two Faces of R&D: Productivity Growth in a Panel of OECD Industries. *The Review of Economics and Statistics*, 86(4):883–895, November 2004.

Zvi Griliches. *R&D and Productivity: The Econometric Evidence*. University of Chicago Press, Chicago, IL, 1998.

Jan Grobovšek. Development Accounting With Intermediate Goods. SIRE Discussion Paper SIRE-DP-2013-42, Scottish Institute for Research in Economics, 2013.

Gene M. Grossman and Elhanan Helpman. Quality Ladders in the Theory of Growth. *The Review of Economic Studies*, 58(1):43–61, January 1991a.

Gene M. Grossman and Elhanan Helpman. Trade, Knowledge Spillovers and Growth. *European Economic Review*, 35(2–3):517–526, April 1991b.

Gene M. Grossman and Elhanan Helpman. Globalization and Growth. *The American Economic Review*, 105(5):100–104, May 2015.

Bronwyn H. Hall, Jacques Mairesse, and Pierre Mohnen. Measuring the Returns to R&D. In Bronwyn H. Hall and Nathan Rosenberg, editors, *Handbook of the Economics of Innovation*, volume 2, chapter 24, pages 1033–1082. Elsevier, Oxford, UK, 2010.

Robert E. Hall. The Relation between Price and Marginal Cost in U.S. Industry. *Journal of Political Economy*, 96(5):921–947, October 1988.

Ricardo Hausmann and César A. Hidalgo. The Network Structure of Economic Output. *Journal of Economic Growth*, 16(4):309–342, December 2011.

César A. Hidalgo and Ricardo Hausmann. The Building Blocks of Economic Complexity. *Proceedings of the National Academy of Sciences*, 106(26):10570–10575, June 2009.

Harold Hotelling. Stability in Competition. *The Economic Journal*, 39(153): 41–57, March 1929.

Peter Howitt. Steady Endogenous Growth with Population and R&D Inputs Growing. *Journal of Political Economy*, 107(4):715–730, August 1999.

Charles R. Hulten. Growth Accounting with Intermediate Inputs. *The Review of Economic Studies*, 45(3):511–518, October 1978.

Giammario Impullitti. Global Innovation Races, Offshoring and Wage Inequality. *Review of International Economics*, 24(1):171–202, February 2016.

Giammario Impullitti and Omar Licandro. Trade, Firm Selection and Innovation: The Competition Channel. *The Economic Journal*, 128(608):189–229, February 2018.

Ron S. Jarmin and Javier Miranda. The Longitudal Business Database. Centre for Economic Studies Discussion Paper CES-WP-02-17, Centre for Economic Studies, US Census Bureau, 2002.

Charles I. Jones. R&D-Based Models of Economic Growth. *Journal of Political Economy*, 103(4):759–784, August 1995.

Charles I. Jones. Growth: With or without Scale Effects? *The American Economic Review*, 89(2):139–144, May 1999.

Charles I. Jones. The Shape of Production Functions and the Direction of Technical Change. *The Quarterly Journal of Economics*, 120(2):517–549, May 2005.

Charles I. Jones. Misallocation, Economic Growth and Input-Output Economics. *American Economic Journal: Macroeconomics*, 3(2):1–28, April 2011.

Alejandro Justiniano, Giorgio E. Primiceri, and Andrea Tambalotti. Investment Shocks and Business Cycles. *Journal of Monetary Economics*, 57(2):132–145, March 2010.

Molly Sizer Killian and Thomas F. Hady. The Economic Performance of Rural Labor Markets. In David L. Brown, Norman Reid, Herman Bluestone, David A. McGranahan, and Sara M. Mazie, editors, *Rural Economic Development in the 1980's: Prospects for the Future*, chapter 8, pages 181–200. U.S. Department of Agriculture, Economic Research Service, Washington, DC, 1988.

Robert G. King, Charles I. Plosser, and Sergio T. Rebelo. Production, Growth and Business Cycles: I. The Basic Neoclassical Model. *Journal of Monetary Economics*, 21(2–3):195–232, March–May 1988.

Tor Jakob Klette. Market Power, Scale Economies and Productivity: Estimates from a Panel of Establishment Data. *The Journal of Industrial Economics*, 47 (4):451–476, December 1999.

Tor Jakob Klette and Zvi Griliches. Empirical Patterns of Firm Growth and R&D Investment: A Quality Ladder Model Interpretation. *The Economic Journal*, 110(463):363–387, April 2000.

Tor Jakob Klette and Samuel Kortum. Innovating Firms and Aggregate Innovation. *Journal of Political Economy*, 112(5):986–1018, October 2004.

Michael Kremer. The O-Ring Theory of Economic Development. *The Quarterly Journal of Economics*, 108(3):551–575, August 1993.

Finn E. Kydland and Edward C. Prescott. Time t Build and Aggregate Fluctuations. *Econometrica*, 50(6):1345–1370, November 1982.

Ross Levine and David Renelt. A Sensitivity Analysis of Cross-Country Growth Regressions. *The American Economic Review*, 82(4):942–963, September 1992.

Albert N. Link. Firm Size and Efficient Entrepreneurial Activity: A Reformulation of the Schumpeter Hypothesis. *Journal of Political Economy*, 88(4): 771–782, August 1980.

John B. Long and Charles I. Plosser. Real Business Cycles. *Journal of Political Economy*, 91(1):39–69, February 1983.

Paloma Lopéz-García, José Manuel Motero, and Enrique Moral-Benito. Business Cycles and Investment in Intangibles: Evidence from Spanish Firms. Working Paper 1219, Banco de España, May 2012.

Francesca Lotti, Enrico Santarelli, and Marco Vivarelli. Defending Gibrat's Law as a Long-Run Regularity. *Small Business Economics*, 32(1):31–44, January 2009.

Jaques Mairesse and Mohamed Sassenou. R&D and Productivity: A Survey of Econometric Studies at the Firm Level. Working Paper 3666, NBER, March 1991.

Stephen Martin. *Advanced Industrial Economics*. Blackwell Publishers, Oxford, UK, 2 edition, 2002.

Andreu Mas-Colell, Michael D. Whinston, and Jerry R. Greene. *Microeconomic Theory*. Oxford University Press, New York, NY, 1995.

Kiminori Matsuyama. Growing through Cycles. *Econometrica*, 67(2):335–347, March 1999.

Kiminori Matsuyama. Growing through Cycles in an Infinitely Lived Agent Economy. *Journal of Economic Theory*, 100(2):220–234, October 2001.

Alan L. Montgomery and Peter E. Rossi. Estimating Price Elasticities with Theory-Based Priors. *Journal of Marketing Research*, 36(4):413–423, November 1999.

Alessio Moro. Biased Technical Change, Intermediate Goods and Total Factor Productivity. *Macroeconomic Dynamics*, 16(2):184–203, April 2012a.

Alessio Moro. The Structural Transformation between Manufacturing and Services and the Decline in the US GDP Volatility. *Review of Economic Dynamics*, 15(3):402–415, July 2012b.

Liwa Rachel Ngai and Christopher A. Pissarides. Structural Change in a Multi-sector Model of Growth. *The American Economic Review*, 97(1):429–443, March 2007.

Liwa Rachel Ngai and Roberto M. Samaniego. Mapping Prices into Productivity in Multisector Growth Models. *Journal of Economic Growth*, 14(3):183–204, September 2009.

Stephen J. Nickell. Competition and Corporate Performance. *Journal of Political Economy*, 104(4):724–746, August 1996.

OECD. Research and Development Statistics, 2016a. URL `http://stats.oecd.org/`. (Accessed on 25.11.2016).

OECD. SDBC Structural Business Statistics (ISIC Rev. 4), 2016b. URL `http://stats.oecd.org/`. (Accessed on 25.11.2016).

OECD. Input-Output Tables, 2017. URL `http://oe.cd/i-o`. (Accessed on 10.08.2017).

Joachim Oliveira Martins, Stefano Scarpetta, and Dirk Pilat. Mark-Up Ratios in Manufacturing Industries: Estimates for 14 OECD Countries. OECD Economics Department Working Paper 162, OECD Economics Department, 1996.

Daria Onori. Competition and Growth: Reinterpreting their Relationship. *The Manchester School*, 83(4):398–422, July 2015.

Min Ouyang. On the Cyclicality of R&D. *The Review of Economics and Statistics*, 93(2):542–553, May 2011.

Stephen L. Parente and Edward C. Prescott. *Barriers to Riches*. MIT Press, Cambridge, Mass., 2000.

Pietro F. Peretto. Sunk Costs, Market Structure and Growth. *International Economic Review*, 37(4):895–923, November 1996.

Winfried Pohlmeier. On the Simultaneity of Innovations and Market Structure. *Empirical Economics*, 17(2):253–272, June 1992.

Giorgio E. Primiceri, Ernst Schaumburg, and Andrea Tambalotti. Intertemporal Disturbances. Working Paper 12243, NBER, May 2006.

Matthew C. Rafferty. Do Business Cycles Influence Long-Run Growth? The Effect of Aggregate Demand on Firm-Financed R&D Expenditures. *Eastern Economic Journal*, 29(4):607–618, Autumn 2003.

Garey Ramey and Valerie A. Ramey. Cross-Country Evidence on the Link Between Volatility and Growth. *The American Economic Review*, 85(5):1138–1151, December 1995.

Luis A. Rivera-Batiz and Paul M. Romer. Economic Integration and Endogenous Growth. *The Quarterly Journal of Economics*, 106(2):531–555, May 1991.

Paul M. Romer. Increasing Returns and Long-Run Growth. *Journal of Political Economy*, 94(5):1002–1037, October 1986.

Paul M. Romer. Endogenous Technological Change. *Journal of Political Economy*, 98(5):S71–S102, October 1990.

Steven C. Salop. Monopolistic Competition with Outside Goods. *The Bell Journal of Economics*, 10(1):141–156, Spring 1979.

Frederic M. Scherer. Firm Size, Market Structure, Opportunity and the Output of Patented Inventions. *The American Economic Review*, 55(5):1097–1125, December 1965.

Frederic M. Scherer. Market Structure and the Employment of Scientists and Engineers. *The American Economic Review*, 57(3):524–531, June 1967.

Joseph A. Schumpeter. *Capitalism, Socialism and Democracy*. Allen and Unwin, London, UK, 1 edition, 1943.

Andrei Shleifer. Implementation Cycles. *Journal of Political Economy*, 94(6):1163–1190, December 1986.

Frank Smets and Raf Wouters. An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, 1(5):1123–1175, September 2003.

Robert M. Solow. A Contribution to the Theory of Economic Growth. *The Quarterly Journal of Economics*, 70(1):65–94, February 1956.

Robert M. Solow. Technical Change and the Aggregate Production Function. *The Review of Economics and Statistics*, 39(3):312–320, August 1957.

John Sutton. Gibrat's Legacy. *Journal of Economic Literature*, 35(1):40–59, March 1997.

Trevor W. Swan. Economic Growth and Capital Accumulation. *Economic Record*, 32(2):334–361, November 1956.

George Symeonidis. Innovation, Firm Size and Market Structure: Schumpeterian Hypotheses and Some New Themes. Working Paper 161, OECD Economics Department, September 1996.

Patrik Gustavsson Tingvall and Andreas Poldahl. Is There Really an Inverted U-Shaped Relation between Competition and R&D? *Economics of Innovation and New Technology*, 15(2):101–118, 2006.

Charles M. Tolbert and Molly Sizer. U.S. Commuting Zones and Labor Market Areas: A 1990 Update. ERS Staff Paper AGES-9614, U.S. Department of Agriculture, Economic Research Service, Rural Economy Division, September 1996.

U.S. Department of Agriculture, Economic Research Service. Commuting Zones and Labor Market Areas: Documentation, July 2012. URL `http://www.ers.usda.gov/data-products/commuting-zones-and-labor-market-areas/documentation.aspx`. (Accessed on 24.01.2016).

Klaus Wälde. Endogenous Growth Cycles. *International Economic Review*, 46 (3):867–894, August 2005.

Klaus Wälde and Ulrich Woitek. R&D Expenditure in G7 Countries and the Implications for Endogenous Fluctuations and Growth. *Economic Letters*, 82 (1):91–97, January 2004.

Michael L. Walden. *North Carolina in the Connected Age: Challenges and Opportunities in a Globalizing Economy*. The University of North Carolina Press, Chapel Hill, NC, 2008.

Alwyn Young. Growth without Scale Effects. *Journal of Political Economy*, 106 (1):41–63, February 1998.

Appendix A

A.1 Proofs for Chapter 2

A.1.1 Proof of Observation 2.1

Before establishing the asserted result, we shall prove the following Lemma

Lemma A.1.1.1. *From* $\lim_{t\to+\infty} \frac{1}{t} E_0 \left(\int_0^t z(\tau)\, d\tau \right) = \bar{z}$ *follows that* $\lim_{t\to+\infty} \frac{E_0 z(t)}{t} = 0.$

Proof. The lemma can be proven by differentiating $\lim_{t\to+\infty} \frac{1}{t} E_0 \left(\int_0^t z(\tau)\, d\tau \right) = \bar{z}$ with respect to t and applying the Leibniz integral rule

$$\frac{d}{dt} \lim_{t\to+\infty} \frac{1}{t} E_0 \left(\int\limits_0^t z(\tau)\, d\tau \right) = \frac{d\bar{z}}{dt}$$

$$\lim_{t\to+\infty} \frac{E_0 z(t)}{t} - \lim_{t\to+\infty} \left(\frac{1}{t} \right) \cdot \lim_{t\to+\infty} \left(\frac{1}{t} E_0 \left(\int\limits_0^t z(\tau)\, d\tau \right) \right) = 0$$

$$\lim_{t\to+\infty} \frac{E_0 z(t)}{t} = \lim_{t\to+\infty} \frac{\bar{z}}{t} = 0$$

■

Given formulae (2.33)–(2.37), the exact growth rates of the economy's variables take the general form

$$\frac{\dot{X}(t)}{X(t)} = a \frac{\dot{Z}(t)}{Z(t)} + b \frac{\dot{Q}(t)}{Q(t)} \tag{A.1}$$

Applying the definition of the long-run growth rate to (A.1) yields the expression

$$\bar{g}_X = a \lim_{t \to +\infty} \frac{1}{t} E_0 \left(\int_0^t \left(\frac{z(\tau)}{Z(\tau)} \right)^{\eta - 1} \frac{\dot{z}(\tau)}{z(\tau)} d\tau \right) + b\bar{g}_Q \tag{A.2}$$

Given that $\eta > 1$ and $z(t) \in [z_L; z_H]$, expression (A.2) gives rise to the following double inequality

$$\frac{a}{\left(\frac{\zeta}{z_L}\right)^{\eta-1} + 1} \lim_{t \to +\infty} \frac{1}{t} E_0 \left(\int_0^t \frac{\dot{z}(\tau)}{z(\tau)} d\tau \right) \leqslant \bar{g}_X - b\bar{g}_Q \leqslant$$

$$\leqslant \frac{a}{\left(\frac{\zeta}{z_H}\right)^{\eta-1} + 1} \lim_{t \to +\infty} \frac{1}{t} E_0 \left(\int_0^t \frac{\dot{z}(\tau)}{z(\tau)} d\tau \right)$$

$$\frac{a/z_H}{\left(\frac{\zeta}{z_L}\right)^{\eta-1} + 1} \lim_{t \to +\infty} \frac{1}{t} E_0 \left(\int_0^t \dot{z}(\tau)\, d\tau \right) \leqslant \bar{g}_X - b\bar{g}_Q \leqslant$$

$$\leqslant \frac{a/z_L}{\left(\frac{\zeta}{z_H}\right)^{\eta-1} + 1} \lim_{t \to +\infty} \frac{1}{t} E_0 \left(\int_0^t \dot{z}(\tau)\, d\tau \right)$$

$$\frac{a/z_H}{\left(\frac{\zeta}{z_L}\right)^{\eta-1} + 1} \lim_{t \to +\infty} \frac{E_0 z(t)}{t} \leqslant \bar{g}_X - b\bar{g}_Q \leqslant \frac{a/z_L}{\left(\frac{\zeta}{z_H}\right)^{\eta-1} + 1} \lim_{t \to +\infty} \frac{E_0 z(t)}{t} \tag{A.3}$$

Combining (A.3) with Lemma A.1.1.1 suggests that $0 \leqslant \bar{g}_X - b\bar{g}_Q \leqslant 0 \Leftrightarrow \bar{g}_X = b\bar{g}_Q$. Applying the last result to formulae (2.33)–(2.37) brings about the expressions listed in Observation 2.1. ■

A.1.2 The range of η's empirical values in Extension №1

The natural logarithm of a firm's R&D spending (2.35) and the time derivative of that of its output (2.33) and are equal to, respectively, $\ln \tilde{\gamma}^*(t) = b_0^1 - (\eta - 1) \ln Z(t) + \ln \Phi(t) + \ln Q(t)$ and $\frac{(\tilde{y}^*(t))'_t}{\tilde{y}^*(t)} = \frac{\dot{Z}(t)}{Z(t)} + \frac{\dot{\Phi}(t)}{\Phi(t)} + \frac{\dot{Q}(t)}{Q(t)}$. The former can be transformed as follows

$$\begin{aligned}\ln\tilde{\gamma}(t) =& b_0^1-(\eta-1)\left(\ln Z(t)+\ln\Phi(t)+\ln Q(t)\right)+\eta\left(\ln\Phi(t)+\ln Q(t)\right)=\\ =& b_0^1-(\eta-1)\int\limits_0^t\frac{(\tilde{y}(\tau))'_\tau}{\tilde{y}(\tau)}dt+\eta\left(1+\chi\right)\ln Q(t)+\eta\ln\phi\end{aligned} \tag{A.4}$$

Note that in (A.4), the economy's technology level $Q(t)$, being a force affecting the whole economy, is bound to have its impact captured by time-specific fixed effects used in both Aghion et al. (2012) and Beneito et al. (2015). Thereby the impact of $Q(t)$ cannot feed into the estimates of $\frac{(\tilde{y}(t))'_t}{\tilde{y}(t)}$'s effect on $\ln\tilde{\gamma}(t)$, which leaves one with the derived empirical values of η from Section 2.3. ■

A.1.3 A firm's and an industry's output in Extension №2

We shall start with deriving expression (2.58) for an industry's output. To that end, one can use (2.56) to express an individual firm's output $\tilde{y}_s(t)$: $\tilde{y}_s(t) = L^{\nu\mu}y(t)^{\frac{\mu}{\xi}(1-\nu\xi)}y_s(t)^{1-\frac{\mu}{\xi}}\tilde{p}_s(t)^{-\mu}$, multiply both sides of the formula by $m_s(t)^{\frac{\mu}{\mu-1}}$ and derive $y_s(t)$ from the result

$$m_s(t)^{\frac{\mu}{\mu-1}}\tilde{y}_s(t)=y_s(t)=L^{\nu\mu}y(t)^{\frac{\mu}{\xi}(1-\nu\xi)}y_s(t)^{1-\frac{\mu}{\xi}}m_s(t)^{\frac{\mu}{\mu-1}}\tilde{p}_s(t)^{-\mu}$$

$$y_s(t)=L^{\nu\xi}y(t)^{1-\nu\xi}m_s(t)^{\frac{\xi}{\mu-1}}\tilde{p}_s(t)^{-\xi} \tag{A.5}$$

Raising both sides of equation (A.5) to power $\frac{\xi-1}{\xi}$, and summing it up for $s = R$ and $s = P$ yields the expression

$$y(t)^{\frac{\xi-1}{\xi}}=\left(L^{\nu\xi}y(t)^{1-\nu\xi}\right)^{\frac{\xi-1}{\xi}}\left(\frac{m_R(t)^{\frac{\xi-1}{\mu-1}}}{\tilde{p}_R(t)^{\xi-1}}+\frac{m_P(t)^{\frac{\xi-1}{\mu-1}}}{\tilde{p}_P(t)^{\xi-1}}\right) \tag{A.6}$$

Plugging (2.57) into (A.6), and rearranging for $y(t)$ brings about formula (2.58). Expression (2.59) for a firm's output $\tilde{y}^*_s(t)$ can be obtained by combining (2.56), (2.57) and (2.58). ■

A.1.4 Proof of Observations 2.2 and 2.3

We shall start with proving Observation 2.2. Taking the derivative of $\ln \psi\Gamma^*(t)$ and stipulating that it be positive, yields the inequality

$$\begin{aligned}(\ln \Gamma^*(t))'_{z(t)} = \left(\frac{(1-\nu)(\mu-1)}{\nu\mu-1} - (\eta-1)\right)\frac{(Z(t))'_{z(t)}}{Z(t)} - \\ - \frac{\nu\xi-1}{\nu\mu-1}\cdot\frac{(\mu-1)^2}{\mu-\xi}\cdot\mathcal{V}_1(t)\,\mathcal{V}_2(t)\,\frac{(Z(t))'_{z(t)}}{Z(t)} > 0\end{aligned} \tag{A.7}$$

where $v(t) \equiv \frac{Z(t)}{z(t)} > 1$, and $\mathcal{V}_1(t)$ and $\mathcal{V}_2(t)$ are defined as follows

$$\mathcal{V}_1(t) \equiv \frac{v(t)^{\frac{(\mu-1)(\xi-1)}{\mu-\xi}} - 1}{\left(\frac{\Phi^-}{\Phi^+}\right)^{\frac{\xi-1}{\mu-\xi}} v(t)^{\frac{(\mu-1)(\xi-1)}{\mu-\xi}} + 1} \tag{A.8}$$

$$\mathcal{V}_2(t) \equiv \frac{v(t)^{\eta-1} - 1}{v(t)^{\frac{(\mu-1)(\xi-1)}{\mu-\xi}} - 1} \tag{A.9}$$

Given that $\mathcal{V}_1(t) > 0$ and $\mathcal{V}_2(t) > 0$, necessary condition (2.67) can be derived from (A.7) by omitting the second term on its right-hand side

$$(\ln \Gamma^*(t))'_{z(t)} < \left(\frac{(1-\nu)(\mu-1)}{\nu\mu-1} - (\eta-1)\right)\frac{(Z(t))'_{z(t)}}{Z(t)} \Leftrightarrow \eta < \frac{\mu+\nu-2}{\nu\mu-1}$$

$$(\ln \Gamma^*(t))'_{z(t)} > 0 \Rightarrow \eta < \frac{\mu+\nu-2}{\nu\mu-1} \tag{A.10}$$

Turning to sufficient condition (2.68), since we are interested in specifying a sufficient condition for the positivity of (A.7), our approach is going to be to replace $\mathcal{V}_1(t)$ and $\mathcal{V}_2(t)$ with their estimates from above, and then to derive the condition for the resulting expression being positive. Starting with the former, one can consider the chain of inequalities as follows

$$\mathcal{V}_1(t) < \frac{v(t)^{\frac{(\mu-1)(\xi-1)}{\mu-\xi}}}{\left(\frac{\Phi^-}{\Phi^+}\right)^{\frac{\xi-1}{\mu-\xi}} v(t)^{\frac{(\mu-1)(\xi-1)}{\mu-\xi}}} = \left(\frac{\Phi^+}{\Phi^-}\right)^{\frac{\xi-1}{\mu-\xi}} \tag{A.11}$$

As concerns term $\mathcal{V}_2(t)$, it can be replaced with its supremum, which is derived using the following Lemma

Lemma A.1.4.1. *Let* $h(x) \equiv \frac{x^a-1}{x^b-1}$, $x \geqslant 1$.

1. $\lim_{x\to 1} h(x) = \frac{a}{b}$.

2. *If* $a < b$, $h(x)$ *is decreasing for any* $x \geqslant 1$; *if* $a > b$, $h(x)$ *is increasing for any* $x \geqslant 1$.

Proof.

1. Follows directly from applying L'Hôpital's rule.

2. Differentiating $h(x)$ yields the following formula

$$h'(t) = \frac{(a-b)\,x^a - (a-1)\,x^{a-b} + (b-1)}{x^{1-b}\left(x^b-1\right)^2} \tag{A.12}$$

Since one needs to prove the monotonicity of $h(x)$, and since (as suggested by (A.12)) the sign of $h'(t)$ is determined by its numerator, one needs to show that the sign of the latter does not change. To that end, one can use the fact that the numerator has a root at $x = 1$ in conjunction with showing that it is monotonic for any $x \geqslant 1$

$$\left((a-b)\,x^a - ax^{a-b} + b\right)' \gtreqless 0$$

$$(a-b)\,ax^{a-1} - a\,(a-b)\,x^{a-b-1} \gtreqless 0$$

$$\left[\begin{array}{l} \left\{\begin{array}{l} x^a \lesseqgtr 1 \Leftrightarrow x \lesseqgtr 1 \\ a \leqslant b \end{array}\right. \\ \left\{\begin{array}{l} x^a \gtreqless 1 \Leftrightarrow x \gtreqless 1 \\ a > b \end{array}\right. \end{array}\right. \tag{A.13}$$

Since $x \geqslant 1$, the first system of conditions in (A.13) suggests that when $a < b$, the numerator of $h'(t)$ monotonically decreases from zero, and thereby is negative, which (given the considerations above) entails that $h(x)$ is a decreasing function. A symmetric argument can be applied to the second system in (A.13) to prove the statement in hand for the case of $a > b$. ∎

Given the stipulation that the necessary condition for $\Gamma^*(t)$'s procyclicality (2.67) holds, one can straightforwardly show that $\eta - 1 < \hat{\eta}_2^N - 1 < \frac{(\mu-1)(\xi-1)}{\mu-\xi}$, which, in light of Lemma A.1.4.1 suggests that $\mathcal{V}_2(t)$ is a decreasing function bounded from above by $\frac{(\mu-\xi)(\eta-1)}{(\mu-1)(\xi-1)}$. Substituting the result, alongside equation (A.11), for $\mathcal{V}_2(t)$ and $\mathcal{V}_1(t)$, respectively, leads to the expression

$$\begin{aligned}(\ln \Gamma^*(t))'_{z(t)} > &\left(\frac{(1-\nu)(\mu-1)}{\nu\mu-1} - \right.\\ &\left. - \left(1 + \frac{\nu\xi-1}{\nu\mu-1} \cdot \frac{\mu-1}{\xi-1}\left(\frac{\Phi^+}{\Phi^-}\right)^{\frac{\xi-1}{\mu-\xi}}\right)(\eta-1)\right)\frac{(Z(t))'_{z(t)}}{Z(t)}\end{aligned} \tag{A.14}$$

The final version of condition (2.68), as stated in Observation 2.3, can be established by setting the right-hand side of (A.14) non-negative, and solving the resulting inequality for η. ∎

A.2 Proofs for Chapter 3

A.2.1 Proof of Proposition 3.1

The static optimisation of problem (3.21) given (3.19), (3.20) and (3.25), can be achieved through calculating a firm's price $\tilde{p}_i^*(k;t)$ and demands for labour $\tilde{l}_i^*(k;t)$ and intermediate goods $\left\{\tilde{y}_{ij}^*(k;t)\right\}_{j=1}^N$, which deliver the maximum value for its profit $\tilde{\pi}_i(k;t)$ at any t for a given level of productivity $\tilde{q}_i(k;t)$. The solution of the static optimisation can be derived in two steps. First, one can characterise a firm's optimal production policy (i.e. the amounts of production factors minimising the costs of producing a given level of output $\tilde{y}_i(k;t)$), which formally requires solving the following cost minimisation problem

$$\Psi_i(k;t) \equiv w(t)\,\tilde{l}_i(k;t) + \sum_{j=1}^N p_j(t)\,\tilde{y}_{ij}(k;t) \underset{\tilde{l}_i(k;t),\{\tilde{y}_{ij}(k;t)\}_{j=1}^n}{\longrightarrow \min} \tag{A.15}$$

given production technology (3.17). Since a firm's production function is Cobb-Douglas, and given the normalisation embedded in ζ_i in (3.17), problem (A.15) has a standard solution, whereby the demand for each production factor is as follows

$$\tilde{l}_i^*(k;t) = \frac{(1-\nu_i)\,\tilde{y}_i(k;t)}{\tilde{q}_i(t)\,w(t)} w(t)^{\,1-\nu_i}\left(\prod_{j=1}^N p_j(t)^{\,\alpha_{ij}}\right)^{\nu_i} \tag{A.16}$$

$$\tilde{y}_{ij}^*(k;t) = \frac{\nu_i\alpha_{ij}\tilde{y}_i(k;t)}{\tilde{q}_i(t)\,p_j(t)} w(t)^{\,1-\nu_i}\left(\prod_{j=1}^N p_j(t)^{\,\alpha_{ij}}\right)^{\nu_i} \tag{A.17}$$

Given optimal factor demands (A.16), (A.17) and the linear homogeneity of a firm's technology, its optimal production costs $\tilde{\Psi}_i(k;t)$ are linear in the output $\tilde{\Psi}_i(k;t) = \tilde{\psi}_i(k;t)\,\tilde{y}_i(k;t)$. Furthermore, expressions (A.16), (A.17) suggest that, as specified in (3.25), a firm's marginal cost splits into an industry-specific component $\psi_i(t)$ and a firm-specific component comprising the productivity level $\tilde{q}_i(k;t)$.

As the second step of the static optimisation, one needs to pin down a firm's pricing policy, which can be done by solving the profit maximisation problem with respect to the price level $\tilde{p}_i(k;t)$, given demand for a firm's product (3.13) and its optimal production policy embodied in the optimal cost structure $\tilde{\psi}_i(k;t)\,\tilde{y}_i(k;t)$

$$\left(\frac{p_i(t)}{\tilde{p}_i(k;t)}\right)^{\xi_i} y_i(t)\left(\tilde{p}_i(k;t)-\tilde{\psi}_i(k;t)\right) \longrightarrow \max_{\tilde{p}_i(k;t)} \tag{A.18}$$

Calculating the first-order condition for (A.18) yields the expression for optimal price $\tilde{p}_i^*(k;t)$ in (3.25). Since $\xi_i > 1$ the objective in (A.18) is strictly concave, which thereby makes $\tilde{p}_i^*(k;t)$ the global maximum of problem (A.18).

As the final step of static optimisation, one can derive expression (3.26) from (3.22) by substituting there (3.25) for $\tilde{p}_i(k;t)$, and replacing formula (3.22)'s last two terms with $\tilde{\psi}_i(k;t)\,\tilde{y}_i(k;t)$. ■

A.2.2 Proof of Proposition 3.2

Given the expressions (3.13) and (3.25) (for demand for a firm's product and its optimal price), the objective of the dynamic problem in hand can be rewritten in the following way

$$\begin{aligned}\int_{t_0}^{+\infty} e^{-\delta_i(t-t_0)}\left(\left(\frac{p_i(t)}{\tilde{p}_i^*(k;t)}\right)^{\xi_i} y_i(t)\left(\tilde{p}_i^*(k;t)-\tilde{\psi}_i(k;t)\right)-z_i(k;t)\right)dt = \\ = \int_{t_0}^{+\infty} e^{-\delta_i(t-t_0)}\left(\mathcal{A}_i(t)\,\tilde{q}_i(k;t)^{\,\xi_i-1}-z_i(k;t)\right) \longrightarrow \max_{z_i(k;t_0)}\end{aligned} \tag{A.19}$$

where $\mathcal{A}_i(t) \equiv \frac{p_i(t)^{\xi_i} y_i(t)}{\xi_i}\left(\frac{\xi_i-1}{\xi_i}\right)^{\xi_i-1}$. A change of variable $x_i(k;t) \equiv \tilde{q}_i(k;t)^{\,\chi_i}$ transforms the dynamic problem in question as follows

$$\int_{t_0}^{+\infty} e^{-\delta_i(t-t_0)}\left(\mathcal{A}_i(t)\,x_i(k;t)^{\,\varsigma_i}-z_i(k;t)\right) \longrightarrow \max_{z_i(k;t_0)} \tag{A.20}$$

$$\dot{x}_i(k;t) = \gamma_i z_i(k;t)^{\,\omega_i} \tag{A.21}$$

where $\varsigma_i \equiv \frac{\xi_i - 1}{\chi_i}$. Problem (A.20), (A.21) generates the Hamiltonian function

$$\mathcal{H}(z;x) \equiv e^{-\delta_i(t-t_0)} \left(\mathcal{A}_i(t)\, x_i(k;t)^{\,\varsigma_i} - z_i(k;t)\right) + \gamma_i \mu_i(k;t)\, z_i(k;t)^{\,\omega_i} \tag{A.22}$$

where $\mu_i(k;t)$ is the Lagrangian multiplier associated with problem (A.20), which will be shown to be strictly positive (see equation (A.25)). In light of that, Hamiltonian $\mathcal{H}(z;x)$ satisfies Mangasarian's sufficiency condition, and therefore the pair of functions $(z_i^*(k;t)\,; x_i^*(k;t))$ satisfying the first-order conditions for dynamic problem (A.20) and (A.21), delivers it a global maximum.[1,2]

Calculating the first-order conditions for the maximisation of (A.22) yields

$$\frac{\partial \mathcal{H}}{\partial x_i(k;t)} \equiv e^{-\delta_i(t-t_0)} \varsigma_i \mathcal{A}_i(t)\, x_i(k;t)^{\,\varsigma_i - 1} = -\dot{\mu}_i(k;t) \tag{A.23}$$

$$\frac{\partial \mathcal{H}}{\partial z_i(k;t)} \equiv -e^{-\delta_i(t-t_0)} + \gamma_i \omega_i \mu_i(k;t)\, z_i(k;t)^{\,-(1-\omega_i)} = 0 \tag{A.24}$$

Expressing $\mu_i(k;t)$ from (A.24) yields the following formulae for $\mu_i(k;t)$ and $-\frac{\dot{\mu}_i(k;t)}{\mu_i(k;t)}$

$$\mu_i(k;t) = e^{-\delta_i(t-t_0)} \frac{z_i(k;t)^{\,1-\omega_i}}{\gamma_i \omega_i} \Rightarrow -\frac{\dot{\mu}_i(k;t)}{\mu_i(k;t)} = \delta_i - (1-\omega_i)\frac{\dot{z}_i(k;t)}{z_i(k;t)} \tag{A.25}$$

Dividing (A.23) by $\mu_i(k;t)$ and plugging in (A.25) allows one to express the growth rates of $x_i(k;t)$ and $z_i(k;t)$ through each other

$$\begin{aligned} \gamma_i \varsigma_i \omega_i \cdot \frac{\mathcal{A}_i(t) \cdot x_i(k;t)^{\,\varsigma_i}}{z_i(k;t)} \cdot \frac{z_i(k;t)^{\,\omega_i}}{x_i(k;t)} = \varsigma_i \omega_i \cdot \frac{\dot{x}_i(k;t)}{x_i(k;t)} = \\ = \delta_i - (1-\omega_i)\frac{\dot{z}_i(k;t)}{z_i(k;t)} \end{aligned} \tag{A.26}$$

where the equality in (A.26)'s first line follows from applying the free-entry condition (3.26) in conjunction with the dynamic equation (A.21) for the accumulation of $x_i(k;t)$. Solving (A.26) for $z_i(k;t)$ yields

[1] Furthermore, the maximum is guaranteed to be unique for any $\omega_i < 1$.

[2] See (Acemoglu, 2009, pp. 236–239) for a detailed exposition of sufficiency conditions in dynamic continuous optimisation.

$$\varsigma_i \omega_i \ln x_i(k;t) = \ln C + \delta_i (t - t_0) - (1 - \omega_i) \ln z_i(k;t)$$

$$z_i(k;t) = C e^{\frac{\delta_i}{1-\omega_i}(t-t_0)} x_i(k;t)^{-\frac{\varsigma_i \omega_i}{1-\omega_i}} \tag{A.27}$$

where C is a constant of integration. Substituting (A.27) for $z_i(k;t)$ in the problem's dynamic constraint (A.21), and solving the resulting differential equation of $x_i(k;t)$ brings about the final expression for the variable

$$x_i^*(k;t) = x_i^*(k;t_0)\, e^{\frac{\delta_i \chi_i}{\phi_i}(t-t_0)} \tag{A.28}$$

where ϕ_i is as defined in the statement of Proposition 3.2, and $x_i^*(k;t_0)$ is $x_i^*(k;t)$'s initial value, which, given the definition of $x_i(k;t)$, equals $\tilde{q}_i^E(t_0)^{\chi_i}$, where $\tilde{q}_i^E(t_0)$ is the industry-specific initial level of a firm's productivity upon entry, defined in (3.15). Expression (3.27) for $\tilde{q}_i^*(t)$ can be immediately derived from (A.28), while formula (3.28) for $z_i^*(t)$ in the main text comes directly from plugging equation (A.28) into (A.27) or into (A.21). ■

A.2.3 Proof of Corollary 3.2.1

As follows from equations (3.13), (3.25) the only potential source of heterogeneity between firms within an industry is their productivity levels $\tilde{q}_i(k;t)$. Since, however, all firms grow at the same rate according to (3.27), firms' productivities can differ only in terms of firms' initial entry productivity levels $\tilde{q}_i^E(t_0)$. In order to prove the opposite, in what follows, we will explicitly derive the dynamics of $\tilde{q}_i(t_0)$ to show that $\tilde{q}_i(t_0) = e^{\frac{\delta_i}{\phi_i} t_0}$, so that (3.27) collapses to (3.29). To that end, note that, given (3.15) and (3.16), for some small Δt, $m_i(t_0 + \Delta t)$ and $\tilde{q}_i(t_0 + \Delta t)$ are

$$m_i(t_0 + \Delta t) = (1 - \delta_i \Delta t)\, m_i(t_0) + m_i^E(t_0)\, \Delta t \tag{A.29}$$

$$
\begin{aligned}
\tilde{q}_i^E(t_0+\Delta t) &= \int\limits_0^{m_i(t_0+\Delta t)} \frac{\tilde{q}_i(k;t_0+\Delta t)}{m_i(t_0+\Delta t)} dk = \\
&= \frac{\left(1+\frac{\delta_i}{\phi_i}\Delta t\right)\int_0^{(1-\delta_i\Delta t)m_i(t_0)} \tilde{q}_i(k;t_0)\,dk + m_i^E(t_0)\,\Delta t\left(1+\frac{\delta_i}{\phi_i}\Delta t\right)\tilde{q}_i^E(t_0)}{m_i(t_0+\Delta t)}
\end{aligned} \tag{A.30}
$$

where the first and second terms in the numerator in (A.30)'s second line capture the contribution of, respectively, surviving firms and newcomers to the dynamics of $\tilde{q}_i^E$. Since every firm faces the same chance of its product's becoming obsolescent, the integral term in (A.30) equals $(1-\delta_i\Delta t)\int_0^{m_i(t_0)} \tilde{q}_i(k;t_0)\,dk = (1-\delta_i\Delta t)\,m_i(t_0)\,\tilde{q}_i^E(t_0)$. Plugging the result back into (A.30) and making use of (A.29) yields

$$
\tilde{q}_i^E(t_0+\Delta t) = \left(1+\frac{\delta_i}{\phi_i}\Delta t\right)\tilde{q}_i^E(t_0) \tag{A.31}
$$

Subtracting $\tilde{q}_i^E(t_0)$ from both sides of (A.31) and dividing by Δt brings about the equation $\frac{\tilde{q}_i^E(t_0+\Delta t)-\tilde{q}_i^E(t_0)}{\Delta t} = \frac{\delta_i}{\phi_i}\tilde{q}_i^E(t_0)$, which in the limit for $\Delta t \to 0$ yields a simple differential equation of the desired exponential function

$$
\dot{\tilde{q}}_i^E(t_0) = \frac{\delta_i}{\phi_i}\tilde{q}_i^E(t_0) \Leftrightarrow \tilde{q}_i^E(t_0) = \tilde{q}_i^E(0)\,e^{\frac{\delta_i}{\phi_i}t_0} \tag{A.32}
$$

Since all firms start at 0 with their productivities equal to unity, $\tilde{q}_i^E(0)=1$, this establishes the result that $\tilde{q}_i^E(t_0) = e^{\frac{\delta_i}{\phi_i}t_0}$, thus completing the proof. ■

A.2.4 Proof of Proposition 3.3

In the course of the proof, one can make use of the two following lemmata

Lemma A.2.4.1. *Let* $\mathbf{A}$ *be a stochastic matrix, and* $\boldsymbol{\nu}$ *be a vector of values strictly smaller than one. Then matrix* $\mathbf{E}_N - \mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A}$ *is invertible.*

Proof. The proof relies on the result due to Gale (1960) (see (Gale, 1960, Theorem 9.1)), stating that matrix $\mathbf{E}_N - \mathbf{X}$ is invertible if there exists a vector $\mathbf{b}$

such that $\mathbf{Xb} < \mathbf{b}$. One can show that the vector of ones $\mathbf{e}$ satisfies this criterion

$$\mathrm{dg}(\boldsymbol{\nu})\,\mathbf{Ae} = \mathrm{dg}(\boldsymbol{\nu})\,\mathbf{e} = \boldsymbol{\nu} < \mathbf{e} \tag{A.33}$$

The expression above completes the proof. ■

Lemma A.2.4.2. *Matrix* $\boldsymbol{\Lambda}_\nu\left(\mathbf{E}_N - \mathrm{dg}(\boldsymbol{\nu})\right)$ *has an eigenvalue equal to unity, whose associated eigenvector is* $a \cdot \mathbf{e}\ \forall a \in \mathbb{C}$, *or equivalently* $\boldsymbol{\Lambda}_\nu\left(\mathbf{E}_N - \mathrm{dg}(\boldsymbol{\nu})\right)\mathbf{e} = \mathbf{e}$.

Proof. By definition, $\mathbf{Ae} = \mathbf{e}$. Consider next the following chain of transformations

$$\mathrm{dg}(\boldsymbol{\nu})\,\mathbf{Ae} = \mathrm{dg}(\boldsymbol{\nu})\,\mathbf{e}$$

$$\left(\mathbf{E}_N - \mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A}\right)\mathbf{e} = \left(\mathbf{E}_N - \mathrm{dg}(\boldsymbol{\nu})\right)\mathbf{e}$$

As matrix $\left(\mathbf{E}_N - \mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A}\right)$ is invertible (see Lemma A.2.4.1), one can establish the desired result by premultiplying both sides of the last expression by $\boldsymbol{\Lambda}_\nu \equiv \left(\mathbf{E}_N - \mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A}\right)^{-1}$. ■

As the first step of the proof, one can switch to the growth-rate version of equation (3.35) by taking natural logarithms of both its sides, and differentiating the result with respect to time

$$\frac{\dot{p}_i(t)}{p_i(t)} = -\kappa_i \frac{\dot{m}_i(t)}{m_i(t)} + (1-\nu_i)\,\frac{\dot{w}(t)}{w(t)} + \nu_i \sum_{j=1}^{N} \alpha_{ij} \frac{\dot{p}_j(t)}{p_j(t)} - \frac{\dot{\tilde{q}}_i^*(t)}{\tilde{q}_i^*(t)} \tag{A.34}$$

Denoting $\frac{\dot{x}(t)}{x(t)}$ as $g_x(t)$ and combining all versions of expression (A.34) for each i yields a chain of matrix equations

$$\mathbf{g}_p(t) = \left(\mathbf{E} - \mathrm{dg}(\boldsymbol{\nu})\right)\mathbf{e}g_w(t) + \mathrm{dg}(\boldsymbol{\nu})\,\mathbf{Ag}_p(t) - \left(\mathbf{g}_q + \mathrm{dg}(\boldsymbol{\kappa})\,\mathbf{g}_m(t)\right)$$

$$(\mathbf{E} - \mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A})\,\mathbf{g}_p(t) = (\mathbf{E} - \mathrm{dg}(\boldsymbol{\nu}))\,\mathbf{e}g_w(t) - (\mathbf{g}_q + \mathrm{dg}(\boldsymbol{\kappa})\,\mathbf{g}_m(t))$$

where $\mathbf{g}_x(t)$ is the vector of all $g_{x_i}(t)$s for each i. Given Lemma A.2.4.1, matrix $(\mathbf{E} - \mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A})$ in the expression above is invertible, which brings about the following formula

$$\begin{aligned}\mathbf{g}_p(t) &= (\mathbf{E} - \mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A})^{-1}\,(\mathbf{E} - \mathrm{dg}(\boldsymbol{\nu}))\,(\mathbf{e}g_w(t) - \mathrm{dg}(\boldsymbol{\kappa})\,\mathbf{g}_m(t) - \mathbf{g}_q) = \\ &= g_w(t)\,\mathbf{e} - \boldsymbol{\Lambda}_\nu\,(\mathbf{g}_q + \mathrm{dg}(\boldsymbol{\kappa})\,\mathbf{g}_m(t))\end{aligned} \tag{A.35}$$

where the second line in (A.35) follows from Lemma A.2.4.2 and the definition of the Leontiev inverse $\boldsymbol{\Lambda}_\nu$. Note that given Proposition 3.2, $\mathbf{g}_q$ is time-independent. Pre-multiplying equation (A.35) by $\boldsymbol{\beta}^T$ yields the expression

$$\boldsymbol{\beta}^T\mathbf{g}_p(t) = 0 = g_w(t) - \boldsymbol{\beta}^T\boldsymbol{\Lambda}_\nu\,(\mathbf{g}_q + \mathrm{dg}(\boldsymbol{\kappa})\,\mathbf{g}_m(t))$$

where the second expression in the formula above follows from the definition of an economy's aggregate price index $P(t) = \prod_{i=1}^N p_i(t)^{\,\beta_i} \Leftrightarrow \ln P(t) = \sum_{i=1}^N \beta_i \ln p_i(t)$ and the fact that $P(t)$ is the economy's numeraire. Since $w(t) = y(t) \Rightarrow g_w(t) = g_y(t)$, the expression above can be recast as

$$g_y(t) = \boldsymbol{\beta}^T\boldsymbol{\Lambda}_\nu\,(\mathbf{g}_q + \mathrm{dg}(\boldsymbol{\kappa})\,\mathbf{g}_m(t)) \tag{A.36}$$

Finally, the time-independence of $g_y(t)$ and $\mathbf{g}_m(t)$ can be shown by combining (A.36) with the growth-rate version of the formula for equilibrium masses of firms in industries (3.34)

$$\begin{cases} g_y(t) = \boldsymbol{\beta}^T\boldsymbol{\Lambda}_\nu\,(\mathbf{g}_q + \mathrm{dg}(\boldsymbol{\kappa})\,\mathbf{g}_m(t)) \\ g_m(t) = (g_y(t) + g_L)\,\mathbf{e} - \mathrm{dg}(\boldsymbol{\chi})\,\mathrm{dg}(\boldsymbol{\omega})\,\mathbf{g}_q \end{cases} \tag{A.37}$$

Substituting the second line in (A.37) for $g_m(t)$ in the first, yields an expression featuring $g_y(t)$ and a series of time-independent variables (whence the final formula (3.37) for g_y is expressed), which directly suggests that $g_y(t)$ is time-independent as well. When applied to the second line in (A.37), the last result leads to the same conclusion for $g_m(t)$. ■

A.2.5 Proof of Proposition 3.4

Consider the difference $g - \hat{g} = \boldsymbol{\beta}^T \left((\mathbf{E}_N - \mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A})^{-1} - \mathbf{E}_N \right) \mathbf{g}$. In what follows, we will show that it is strictly positive by proving that the difference of the Leontiev inverse and the identity matrix $(\mathbf{E}_N - \mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A})^{-1} - \mathbf{E}_N$ is so. The last result follows from using the geometric series representation of the Leontiev inverse[3]

$$(\mathbf{E}_N - \mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A})^{-1} = \sum_{s=0}^{+\infty} (\mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A})^s = \mathbf{E}_N + \sum_{s=1}^{+\infty} (\mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A})^s \qquad \text{(A.38)}$$

Subtracting the identity matrix from (A.38) leaves $\sum_{s=1}^{+\infty} (\mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A})^s$, which, being the sum of a nonnegative matrix's powers, is nonnegative, thereby proving the first statement in the proposition in hand. Note that expression (A.38) suggests to interpret $\mathrm{dg}(\boldsymbol{\nu})$ as the attenuation factor for the strength of interactions between industries, which guarantees the convergence of the infinite matrix power series in (A.38). Naturally, if industries rely largely on primary inputs (so that the components of $\boldsymbol{\nu}$ are close to zero), technological advances in the production of intermediates will have only a limited expansionary impact on downstream firms, which for the same reason will lead to a weak response further downstream, thereby producing a limited total impact on the economy, as expressed in faster convergence of the power series term in (A.38).

The second result can be established by calculating the matrix derivative of g with respect to vector of shares of intermediates in industries' production

[3]The Leontiev inverse $(\mathbf{E} - \mathbf{X})^{-1}$ admits the geometric series representation if the absolute value of $\mathbf{X}$'s largest eigenvalue $|ev(\mathbf{X})|$ is strictly smaller than unity, which is the case with the Leontiev inverse in hand: $|ev(\mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A})| = ev(\mathrm{dg}(\boldsymbol{\nu}))\,ev(\mathbf{A}) = \nu_{\max} \cdot 1 < 1$. The fact that $ev(\mathbf{A}) = 1$ is an implication of the Perron-Frobenius theorem for stochastic matrices.

technologies $\boldsymbol{\nu}$

$$\frac{\partial g}{\partial \boldsymbol{\nu}} = \mathbf{e}^T \left(\mathbf{E}_N \circ \left(\mathbf{A} \boldsymbol{\Lambda}_\nu \mathbf{g} \boldsymbol{\beta}^T \boldsymbol{\Lambda}_\nu \right) \right) \tag{A.39}$$

where $\circ$ denotes the Hadamard product. As $\frac{\partial g}{\partial \boldsymbol{\nu}}$ constitutes a diagonal matrix with the diagonal elements of the product of nonnegative matrices, it must be nonnegative as well, thus establishing the desired result. ■

A.2.6 Proofs of Propositions 3.5 and 3.6

Starting with Proposition 3.5, its proof proceeds in two steps: first of all, one can use the geometric series representation of the Leontiev inverse (A.38) to transform equation (3.39) as follows

$$g = \sum_{s=0}^{+\infty} \boldsymbol{\beta}^T \left(\nu \mathbf{E}_N \mathbf{A} \right)^s \mathbf{g}_q = \nu \boldsymbol{\beta}^T \mathbf{g}_q + \boldsymbol{\beta}^T \sum_{s=1}^{+\infty} \left(\nu \mathbf{A} \right)^s \mathbf{g}_q \tag{A.40}$$

As suggested by (A.40), the economy's structure affects its growth rate through the infinite power series term $\boldsymbol{\beta}^T \sum_{s=1}^{+\infty} \left(\nu \mathbf{A} \right)^s \mathbf{g}_q$. As the second step, one can show that for any matrix $\mathbf{A}$ different from $\mathbb{1}_N(i_0)$, each of the series' terms is smaller than its counterpart generated by the latter matrix. To that end, one can show that for any $s \geqslant 1$ vector $\left(\nu \mathbb{1}_N(i_0) \right)^s \mathbf{g}_q$ is greater or equal to $\left(\nu \mathbf{A} \right)^s \mathbf{g}_q$. Starting with the case of $s = 1$, one can arrive at the expression

$$\mathbf{A}\mathbf{g}_q = \begin{bmatrix} \sum_{j=1}^{N} \alpha_{1j} g_j \\ \sum_{j=1}^{N} \alpha_{2j} g_j \\ \dots \\ \sum_{j=1}^{N} \alpha_{Nj} g_j \end{bmatrix} \leqslant \begin{bmatrix} \sum_{j=1}^{N} \alpha_{1j} g_{max} \\ \sum_{j=1}^{N} \alpha_{2j} g_{max} \\ \dots \\ \sum_{j=1}^{N} \alpha_{Nj} g_{max} \end{bmatrix} = \begin{bmatrix} g_{max} \\ g_{max} \\ \dots \\ g_{max} \end{bmatrix} = g_{max}\mathbf{e} = \mathbb{1}_N(i_0)\, \mathbf{g}_q$$

$$\nu \mathbb{1}_N(i_0)\, \mathbf{g}_q \geqslant \nu \mathbf{A} \mathbf{g}_q \tag{A.41}$$

Note that $\nu \mathbf{A} \mathbf{g}_q$ is strictly smaller or equal to $\nu \mathbb{1}_N(i_0)\, \mathbf{g}_q$ when only one sector in the economy grows at the rate of g_{max}. Moving on to values of s above unity, consider first the following chain of equalities

$$\mathbf{A}^{s-1}\mathbb{1}_N(i_0)\,\mathbf{g}_q = g_{max}\mathbf{A}^{s-1}\mathbf{e} = g_{max}$$

where the rightmost equality follows from the fact that for any production matrix $\mathbf{A}$, $\mathbf{Ae} = \mathbf{e}$. Note that the transformations above apply to any production matrix including $\mathbb{1}_N(i_0)$, thus yielding

$$\mathbb{1}_N(i_0)^s\mathbf{g}_q = \mathbf{A}^{s-1}\mathbb{1}_N(i_0)\,\mathbf{g}_q \Leftrightarrow (\nu\mathbb{1}_N(i_0))^s\,\mathbf{g}_q = \nu^s\mathbf{A}^{s-1}\mathbb{1}_N(i_0)\,\mathbf{g}_q \tag{A.42}$$

Given that $\mathbf{A}^{s-1}$ is a power of a transition matrix, it is a transition matrix as well, which makes it strictly nonnegative. This directly implies that multiplying by $\mathbf{A}^{s-1}$ preserves the inequality sign, so that if $\mathbf{x}_1 \leqslant \mathbf{x}_2$ then it must be that $\mathbf{A}^{s-1}\mathbf{x}_1 \leqslant \mathbf{A}^{s-1}\mathbf{x}_2$. In particular, the discussion of the case for $s = 1$ allows one to replace $\mathbf{x}_1$ and $\mathbf{x}_2$ with $\mathbf{Ag}_q$ and $\mathbb{1}_N(i_0)\,\mathbf{g}_q$, respectively

$$\mathbf{A}^s\mathbf{g}_q = \mathbf{A}^{s-1}\mathbf{A}\mathbf{g}_q \leqslant \mathbf{A}^{s-1}\mathbb{1}_N(i_0)\,\mathbf{g}_q = \mathbb{1}_N(i_0)^s\mathbf{g}_q$$

$$\nu\mathbb{1}_N(i_0)^s\mathbf{g}_q \geqslant \nu\mathbf{A}^s\mathbf{g}_q \tag{A.43}$$

Together equations (A.41), (A.43) prove that $(\nu\mathbb{1}_N(i_0))^s\,\mathbf{g}_q \geqslant (\nu\mathbf{A})^s\,\mathbf{g}_q$ for any $s \geqslant 1$. In conjunction with the fact that $\boldsymbol{\beta}^T$ is strictly positive, this naturally suggests that $\boldsymbol{\beta}^T\,(\nu\mathbb{1}_N(i_0))^s\,\mathbf{g}_q \geqslant \boldsymbol{\beta}^T\,(\nu\mathbf{A})^s\,\mathbf{g}_q$ for any $s \geqslant 1$, thus entailing that $\sum_{s=1}^N \boldsymbol{\beta}^T\,(\nu\mathbb{1}_N(i_0))^s\,\mathbf{g}_q \geqslant \sum_{s=1}^N \boldsymbol{\beta}^T\,(\nu\mathbf{A})^s\,\mathbf{g}_q$, which completes the proof. ∎

A slightly more intricate proof is readily available for Proposition 3.6 describing the case of heterogeneous shares of intermediates ν_i. It utilises the following lemma.

Lemma A.2.6.1 (Decomposition of the Leontiev Inverse). *Suppose that a technology matrix* $\mathbf{A}$ *is a generalised pre-weighted linear combination of technology matrices* $\mathbf{A}_k$, $k = 1 \div K$, *so that* $\mathbf{A} \equiv \sum_{k=1}^K \boldsymbol{\Gamma}_k\mathbf{A}_k$, *where* $\boldsymbol{\Gamma}_k$*s are nonnegative* *diagonal matrices adding up to the identity matrix* $\mathbf{E}$*:* $\sum_{k=1}^{K}\mathbf{\Gamma}_k = \mathbf{E}$*. Then* $\mathbf{A}$*'s Leontiev inverse can be written as*

$$\begin{aligned}\mathbf{\Lambda}_\nu &\equiv \sum_{k=1}^{K}\tilde{\mathbf{\Gamma}}_k\mathbf{\Lambda}_{\nu;k} \\ \tilde{\mathbf{\Gamma}}_k &\equiv \mathbf{\Lambda}_\nu\mathbf{\Gamma}_k\mathbf{\Lambda}_{\nu;k}^{-1}\end{aligned} \tag{A.44}$$

where $\tilde{\mathbf{\Gamma}}_k$*s are characterised by the following properties:*

1. *Together, all* $\tilde{\mathbf{\Gamma}}_k$*s add up to the identity matrix:* $\sum_{k=1}^{K}\tilde{\mathbf{\Gamma}}_k = \mathbf{E}$*;*
2. *For each row index* i*, the sum of* $\tilde{\mathbf{\Gamma}}_k$*s' i-th row sums* $\tilde{\gamma}_k^i$*s across all* $\tilde{\mathbf{\Gamma}}_k$*s is equal to* 1*:* $\sum_{k=1}^{K}\tilde{\gamma}_k^i = 1$, $\forall i$*;*
3. *For each row index* i*, the* i*-th row sum of every* $\tilde{\mathbf{\Gamma}}_k$ *is nonnegative:* $\tilde{\gamma}_k^i \geqslant 0$, $\forall i$.

Proof. Equation (A.44) can be validated by direct demonstration

$$\sum_{k=1}^{K}\tilde{\mathbf{\Gamma}}_k\mathbf{\Lambda}_{\nu;k} = \mathbf{\Lambda}_\nu\sum_{k=1}^{K}\mathbf{\Gamma}_k\mathbf{\Lambda}_{\nu;k}^{-1}\mathbf{\Lambda}_{\nu;k} = \mathbf{\Lambda}_\nu\sum_{k=1}^{K}\mathbf{\Gamma}_k = \mathbf{\Lambda}_\nu\mathbf{E} = \mathbf{\Lambda}_\nu$$

Analogously, $\tilde{\mathbf{\Gamma}}_k$s' first property can be proven by direct calculation using the definition of the Leontiev inverse together with that of $\mathbf{A}$

$$\sum_{k=1}^{K}\tilde{\mathbf{\Gamma}}_k = \mathbf{\Lambda}_\nu\sum_{k=1}^{K}\mathbf{\Gamma}_k\left(\mathbf{E} - \mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A}_k\right) = \mathbf{\Lambda}_\nu\left(\sum_{k=1}^{K}\mathbf{\Gamma}_k - \mathrm{dg}(\boldsymbol{\nu})\sum_{k=1}^{K}\mathbf{\Gamma}_k\mathbf{A}_k\right)$$

$$\sum_{k=1}^{K}\tilde{\mathbf{\Gamma}}_k = \mathbf{\Lambda}_\nu\left(\mathbf{E} - \mathrm{dg}(\boldsymbol{\nu})\,\mathbf{A}\right) = \mathbf{\Lambda}_\nu\mathbf{\Lambda}_\nu^{-1} = \mathbf{E}$$

The second property follows immediately from the first property combined with the equivalence of row summation to post-multiplication by a vector of ones

$$\begin{bmatrix}\sum_{k=1}^{K}\tilde{\gamma}_k^1 \\ \sum_{k=1}^{K}\tilde{\gamma}_k^2 \\ \dots \\ \sum_{k=1}^{K}\tilde{\gamma}_k^N\end{bmatrix} = \sum_{k=1}^{K}\begin{bmatrix}\tilde{\gamma}_k^1 \\ \tilde{\gamma}_k^2 \\ \dots \\ \tilde{\gamma}_k^N\end{bmatrix} = \sum_{k=1}^{K}\tilde{\mathbf{\Gamma}}_k\mathbf{e} = \mathbf{E}\mathbf{e} = \begin{bmatrix}1 \\ 1 \\ \dots \\ 1\end{bmatrix}$$

Finally, the last property uses the fact that matrices $\mathbf{A}$ and $\mathbf{A}_k$s are stochastic

$$\begin{bmatrix} \tilde{\gamma}_k^1 \\ \tilde{\gamma}_k^2 \\ \cdots \\ \tilde{\gamma}_k^N \end{bmatrix} = \mathbf{\Lambda}_\nu \mathbf{\Gamma} \left(\mathbf{E}\mathbf{e} - \mathrm{dg}(\boldsymbol{\nu})\, \mathbf{A}\mathbf{e} \right) = \mathbf{\Lambda}_\nu \mathbf{\Gamma} \left(\mathbf{e} - \boldsymbol{\nu} \right)$$

From the expression above, the vector of $\tilde{\gamma}_k^i$s is the product of a nonnegative matrix ($\mathbf{\Gamma}$) and two strictly nonnegative matrices $\mathbf{\Lambda}_\nu$ and $\mathbf{e} - \boldsymbol{\nu}$,[4] which makes it a nonnegative matrix as well, thereby suggesting that so are all its elements. ■

The proof of Proposition 3.6 relies on noting that any production matrix $\mathbf{A}$ can be represented as a pre-weighted combination of $K = N$ $\mathbb{1}_N(i)$-type matrices, where each weighting matrix $\mathbf{\Gamma}_k$ is the corresponding diagonalised column of $\mathbf{A}$: $\mathbf{\Gamma}_k \equiv \mathrm{dg}(\mathbf{A}_{\cdot k})$. This consideration allows one to bring to bear the results of Lemma A.2.6.1 by decomposing $\mathbf{A}$'s Leontiev inverse as

$$\mathbf{\Lambda}_\nu = \sum_{k=1}^{N} \tilde{\mathbf{\Gamma}}_k \left(\mathbf{E} - \mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k) \right)^{-1} = \sum_{k=1}^{N} \tilde{\mathbf{\Gamma}}_k \left(\mathbf{E} + \frac{1}{1-\nu_k} \mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k) \right) \quad \text{(A.45)}$$

where $\tilde{\mathbf{\Gamma}}_k$ is defined as in (A.44), and the rightmost expression follows by calculating $\left(\mathbf{E} - \mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k) \right)^{-1}$ either directly or using the geometric-series representation of Leontiev inverse (A.38). Applying the first property derived in Lemma A.2.6.1, equation (A.45) transforms as follows

$$\mathbf{\Lambda}_\nu = \sum_{k=1}^{N} \tilde{\mathbf{\Gamma}}_k + \sum_{k=1}^{N} \frac{1}{1-\nu_k} \tilde{\mathbf{\Gamma}}_k \,\mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k) = \mathbf{E} + \sum_{k=1}^{N} \frac{1}{1-\nu_k} \tilde{\mathbf{\Gamma}}_k \,\mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k) \quad \text{(A.46)}$$

The further course of the proof makes use of the fact that in expression (A.46) each matrix product $\tilde{\mathbf{\Gamma}}_k \,\mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k)$ is nonnegative (and is strictly nonnegative

[4]In the case of $\mathbf{\Lambda}_\nu$ this conclusion follows from the geometric-series representation of the Leontiev inverse (A.38), whereas for $\mathbf{e} - \boldsymbol{\nu}$ it is implied from the fact that $\nu_i < 1$ for all i.

if $\mathbf{\Gamma}_k$ is distinct from zero), as shown below

$$\begin{aligned}\tilde{\mathbf{\Gamma}}_k \,\mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k) &\equiv \mathbf{\Lambda}_\nu \mathbf{\Gamma}_k \left(\mathbf{E} - \mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k)\right) \mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k) = \\ &= \mathbf{\Lambda}_\nu \mathbf{\Gamma}_k \cdot (1-\nu_k)\, \mathbb{1}_N(k)\end{aligned} \tag{A.47}$$

where the second line in (A.47) follows from the direct post-multiplication of $\mathbf{E} - \mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k)$ by $\mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k)$. Naturally, $\tilde{\mathbf{\Gamma}}_k \,\mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k)$ is nonnegative, as so is each of its constituent terms in its representation in the second line of (A.47).

Plugging expression (A.46) into the equation for the growth rate (3.39) yields

$$g = \boldsymbol{\beta}^T \mathbf{g}_q + \boldsymbol{\beta}^T \sum_{k=1}^{N} \frac{1}{1-\nu_k} \tilde{\mathbf{\Gamma}}_k \,\mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k)\, \mathbf{g}_q$$

$$g = \boldsymbol{\beta}^T \mathbf{g}_q + \sum_{k=1}^{N} \frac{\delta_k/\phi_k}{1-\nu_k} \left(\frac{1}{\delta_k/\phi_k} \boldsymbol{\beta}^T \tilde{\mathbf{\Gamma}}_k \,\mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k)\, \mathbf{g}_q \right)$$

In light of the nonnegativity of $\tilde{\mathbf{\Gamma}}_k \,\mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k)$ as discussed above, each scalar multiplier $\frac{1}{\delta_k/\phi_k} \boldsymbol{\beta}^T \tilde{\mathbf{\Gamma}}_k \,\mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k)\, \mathbf{g}_q$ is also nonnegative, which enables one to evaluate g from above by substituting $\max_{k=1\div N} \left\{ \frac{\delta_k/\phi_k}{1-\nu_k} \right\}$ for $\frac{\delta_k/\phi_k}{1-\nu_k}$ in the above expression

$$g \leqslant \boldsymbol{\beta}^T \mathbf{g}_q + \sum_{k=1}^{N} \max_{k=1\div N} \left\{ \frac{\delta_k/\phi_k}{1-\nu_k} \right\} \left(\frac{1}{\delta_k/\phi_k} \boldsymbol{\beta}^T \tilde{\mathbf{\Gamma}}_k \,\mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k)\, \mathbf{g}_q \right)$$

In conjunction with the first property proven in Lemma A.2.6.1, the last expression leads to the result as follows

$$g \leqslant \boldsymbol{\beta}^T \mathbf{g}_q + \max_{k=1\div N} \left\{ \frac{\delta_k/\phi_k}{1-\nu_k} \right\} \boldsymbol{\beta}^T \left(\sum_{k=1}^{N} \frac{1}{\delta_k/\phi_k} \tilde{\mathbf{\Gamma}}_k \,\mathrm{dg}(\boldsymbol{\nu})\, \mathbb{1}_N(k)\, \mathbf{g}_q \right)$$

$$g \leqslant \boldsymbol{\beta}^T \mathbf{g}_q + \max_{k=1\div N} \left\{ \frac{\delta_k/\phi_k}{1-\nu_k} \right\} \boldsymbol{\beta}^T \left(\sum_{k=1}^{N} \frac{1}{\delta_k/\phi_k} \tilde{\mathbf{\Gamma}}_k \boldsymbol{\nu} \left(\delta_k/\phi_k\right) \right)$$

$$g \leqslant \boldsymbol{\beta}^T \mathbf{g}_q + \max_{k=1\div N} \left\{ \frac{\delta_k/\phi_k}{1-\nu_k} \right\} \boldsymbol{\beta}^T \left(\sum_{k=1}^{N} \tilde{\boldsymbol{\Gamma}}_k \right) \boldsymbol{\nu} = \boldsymbol{\beta}^T \mathbf{g}_q + \max_{k=1\div N} \left\{ \frac{\delta_k/\phi_k}{1-\nu_k} \right\} \boldsymbol{\beta}^T \mathbf{E} \boldsymbol{\nu}$$

$$g \leqslant \boldsymbol{\beta}^T \mathbf{g}_q + \max_{k=1\div N} \left\{ \frac{\delta_k/\phi_k}{1-\nu_k} \right\} \boldsymbol{\beta}^T \boldsymbol{\nu} \equiv \boldsymbol{\beta}^T \mathbf{g}_q + d_{max} \cdot \boldsymbol{\beta}^T \boldsymbol{\nu} \tag{A.48}$$

where the rightmost expression in (A.48) follows from the definition of the concentration potential $d_i \equiv \frac{\delta_i/\phi_i}{1-\nu_i}$.[5]

As the final step of the proof, one can derive the growth rate of an economy with the interlinkage focused on the sector i_1 with the maximal concentration potential, which equals

$$\mathring{g} = \boldsymbol{\beta}^T \left(\mathbf{E} - \mathrm{dg}(\boldsymbol{\nu}) \, \mathbb{1}_N(i_1) \right)^{-1} \mathbf{g}_q = \boldsymbol{\beta}^T \left(\mathbf{E} + \frac{1}{1-\nu_{i_1}} \mathrm{dg}(\boldsymbol{\nu}) \, \mathbb{1}_N(i_1) \right) \mathbf{g}_q$$

$$\mathring{g} = \boldsymbol{\beta}^T \mathbf{g}_q + d_{max} \cdot \boldsymbol{\beta}^T \boldsymbol{\nu} \tag{A.49}$$

A direct comparison of (A.48) and (A.49) suggests that any structure of an economy's interlinkage delivers the growth rate not exceeding $\mathring{g}$ (and the inequality is strict when at least one sector has the concentration potential smaller than d_{max}), thus establishing the desired result. ∎

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[5]Note that the derivation of equation (A.48) uses implicitly the fact that $\frac{\delta_k/\phi_k}{1-\nu_k} > 0 \; \forall k$, as follows from the properties of δ_k and ϕ_k. The argument however can be generalised immediately for the situation when at least one $\frac{\delta_k/\phi_k}{1-\nu_k} > 0$ (i.e., the maximal concentration potential d_{max} is positive).

A.3 Countries and Industries in Chapter 3's Dataset

Table A.1: List of countries in Chapter 3's dataset.

Code	Name	Code	Name
ARG	Argentina	JPN	Japan
AUS	Australia	KHM	Cambodia
AUT	Austria	KOR	Republic of Korea
BEL	Belgium	LTU	Lithuania
BGR	Bulgaria	LUX	Luxembourg
BRA	Brazil	LVA	Latvia
BRN	Brunei Darussalam	MAR	Morocco
CAN	Canada	MEX	Mexico
CHE	Switzerland	MLT	Malta
CHL	Chile	MYS	Malaysia
CHN	China	NLD	Netherlands
COL	Colombia	NOR	Norway
CRI	Costa Rica	NZL	New Zealand
CYP	Cyprus	PER	Peru
CZE	Czech Republic	PHL	Philippines
DEU	Germany	POL	Poland
DNK	Denmark	PRT	Portugal
ESP	Spain	ROU	Romania
EST	Estonia	RUS	Russian Federation
FIN	Finland	SAU	Saudi Arabia
FRA	France	SGP	Singapore
GBR	United Kingdom	SVK	Slovakia
GRC	Greece	SVN	Slovenia
HKG	China, Hong Kong SAR	SWE	Sweden
HRV	Croatia	THA	Thailand
HUN	Hungary	TUN	Tunisia
IDN	Indonesia	TUR	Turkey
IND	India	TWN	Taiwan
IRL	Ireland	USA	United States
ISL	Iceland	VNM	Viet Nam
ISR	Israel	ZAF	South Africa
ITA	Italy		

Table A.2: List of sectors in Chapter 3's dataset.

Code	**Name**
C01T05	Agriculture, hunting, forestry and fishing
C10T14	Mining and quarrying
C15T16	Food products, beverages and tobacco
C17T19	Textiles, textile products, leather and footwear
C20	Wood and products of wood and cork
C21T22	Pulp, paper, paper products, printing and publishing
C23	Coke, refined petroleum products and nuclear fuel
C24	Chemicals and chemical products
C25	Rubber and plastics products
C26	Other non-metallic mineral products
C27	Basic metals
C28	Fabricated metal products
C29	Machinery and equipment, n.e.c.
C30T33X	Computer, electronic and optical equipment
C31	Electrical machinery and apparatus, n.e.c.
C34	Motor vehicles, trailers and semi-trailers
C35	Other transport equipment
C36T37	Manufacturing n.e.c., recycling
C40T41	Electricity, gas and water supply
C45	Construction
C50T52	Wholesale and retail trade, repairs
C55	Hotels and restaurants
C60T63	Transport and storage
C64	Post and telecommunications
C65T67	Financial intermediation
C70	Real estate activities
C71	Renting of machinery and equipment
C72	Computer and related activities
C73T74	R&D and other business activities
C75	Public administration and defence, compulsory social security
C80	Education
C85	Health and social work
C90T93	Other community, social and personal services

A.4 Quantification of $\boldsymbol{\beta}$, $\boldsymbol{\mu}$, $\boldsymbol{\nu}$

This Appendix presents a brief summary of estimating parameters from Chapter 3's framework $\boldsymbol{\beta}$, $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$. The histograms of the parameters' pooled distributions (across all countries, years and industries) are presented below in Figures A.1, A.2, A.3.

Examining the histogram of sectors' relative sizes β_is suggests that those follow a power law-type distribution $\Pr(Size > s) \propto s^{-\zeta}$. Our estimates of the tail index ζ suggest the value of $\zeta \approx -2.242$,[6] for which the hypothesis of heavy tails $\zeta = 2$ cannot be rejected on any standard significance level – in line with the consensus in existing literature.[7]

As mentioned above, the pooled mean and median of sectoral mark-ups' distribution (1.131 and 1.100, respectively) are in line with existing estimates from the specialised literature. The mean and median shares of intermediates are 0.617 and 0.649 – above the conventional benchmark of 0.5, which we can attribute to two factors: firstly, our estimates take into account that the true values of ν_is are distorted away from the corresponding shares in sales by imperfect competition (see equation (3.23));[8] secondly, the aforementioned benchmark value is derived for the US economy, characterised by relatively low values of ν_i (its average share of intermediates occupies the 15th place from the bottom out of 63, see also the discussion in (Jones, 2011, Section V)). The final remark on the two distributions in hand concerns the noticeable spikes at 1 for both ν_is and μ_is. The former is created by the natural restriction that $\nu_i < 1$, while the latter occurs owing to the fact that some mark-ups are estimated from below using

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[6]The estimate was derived using the standard ln-ln regression with the correction due to Gabaix and Ibragimov (2011): $\ln(\Pr(Size > s)) = const - \zeta \ln\left(i - \frac{1}{2}\right)$, where i is the index of the histogram's bar.

[7]See Gabaix and Ibragimov (2011).

[8]For a comparison's sake, Figure A.3b presents the histogram of pooled distribution of ν_is implied by the competitive markets (i.e., RIs from the empirical exercise in Section 3.2), with the mean of 0.550 and the median of 0.580.

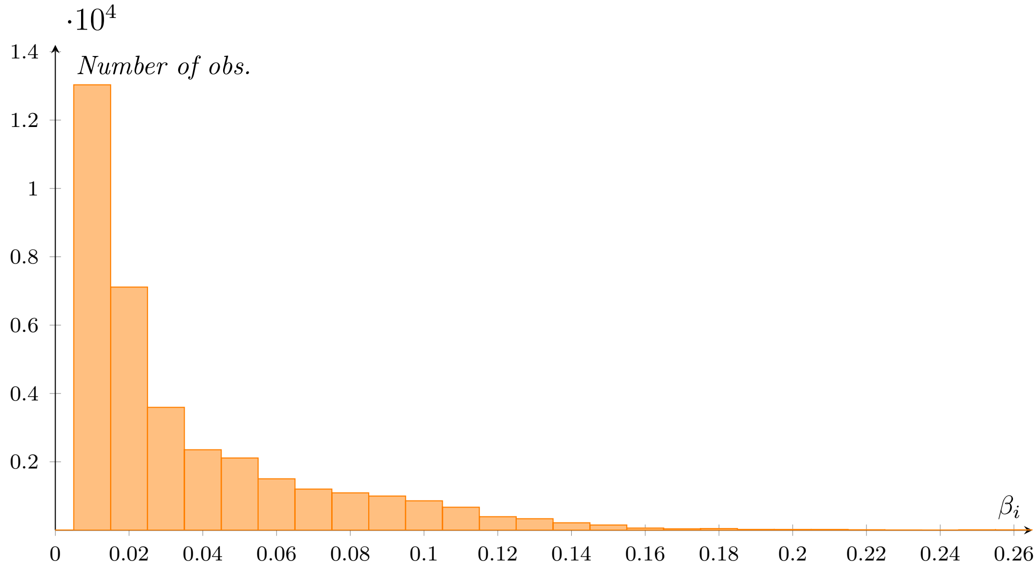


Figure A.1: The distribution of estimated β_is (pooled).

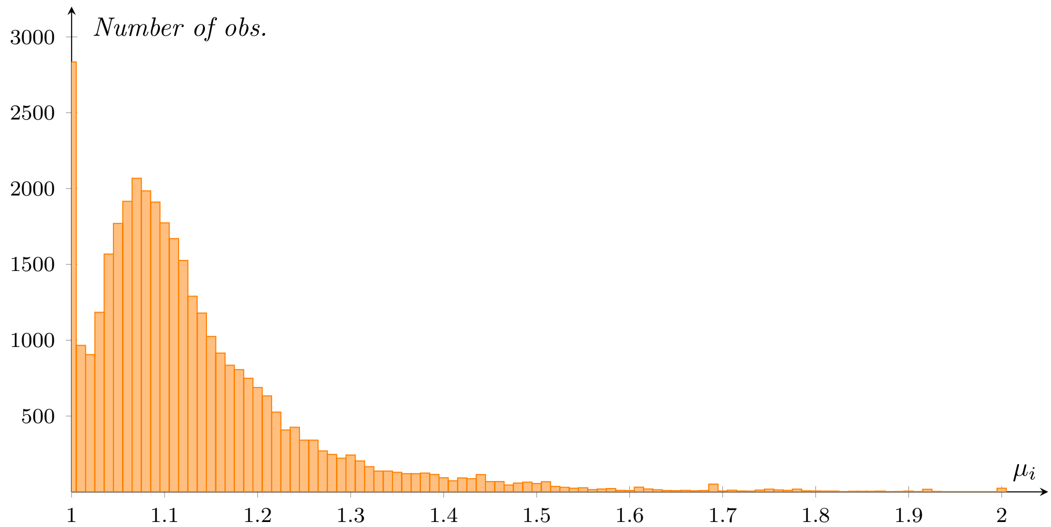
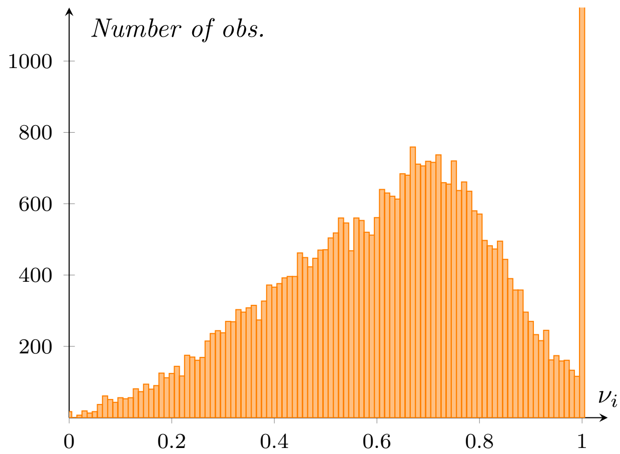


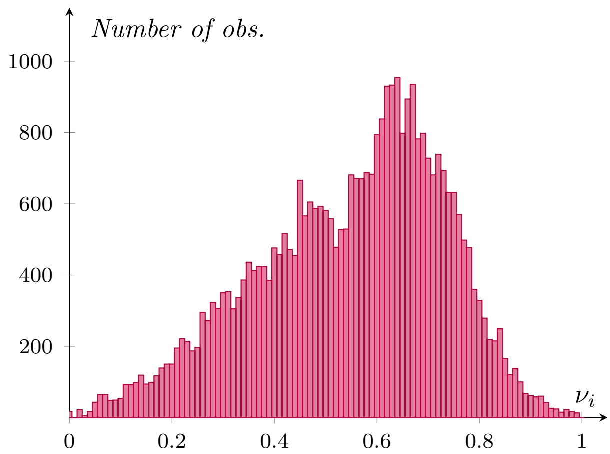
Figure A.2: The distribution of estimated mark-ups μ_is (pooled).

the data on GFCF. One should note that both represent the minority of all observations (approximately 9.3% and 3% for μ_is and ν_is respectively), and when averaged across years, the two spikes disappear – i.e., hitting the bounds by μ_is' and ν_is' estimates is not systematic.

Moving to the sectoral level, Figures A.6, A.7, A.8 present the scatter plots of country averages of, respectively, β_is, μ_is and ν_is for industries in dataset. On average, the smallest share of consumption expenditures is allocated to basic



(a) Imperfect competition ($\nu_i \alpha_{ij} = \mu_i \frac{Sales_{ij}}{Sales_i}$).



(b) Perfect competition ($\nu_i \alpha_{ij} = \frac{Sales_{ij}}{Sales_i}$).

Figure A.3: The distribution of estimated ν_is (pooled).

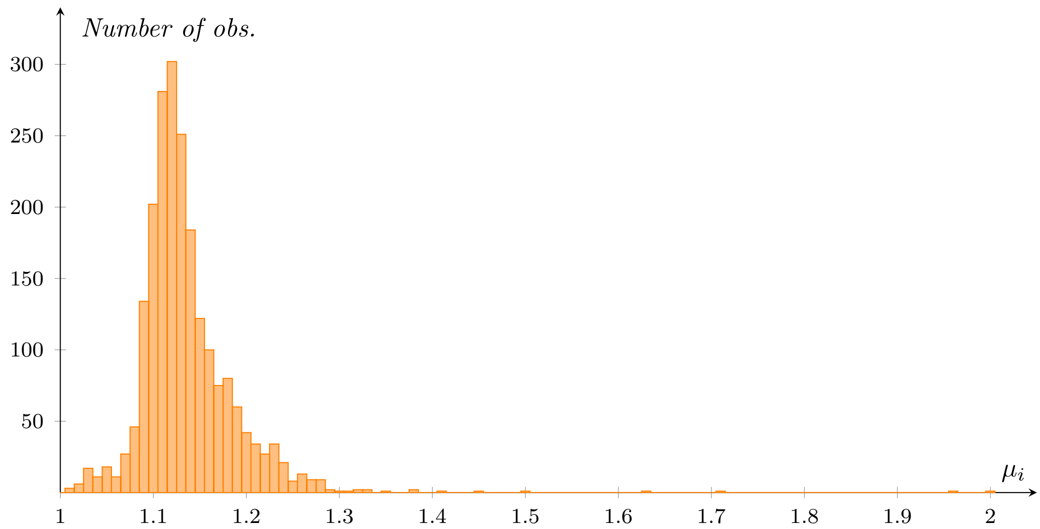


Figure A.4: The distribution of estimated mark-ups μ_is (averages across years).

metals (C27), while the largest – to sector C50C52: wholesale and retail trade and repairs, followed by housing (C45, construction sector), which is also characterised by the lowest average level of mark-ups. The highest mark-ups are a feature of sector C26: 'Other non-metallic mineral products'. Finally, with regards to ν_i, education and production of coke, refined petroleum products and nuclear fuel are the sectors least and most reliant on the use of intermediates, respectively.

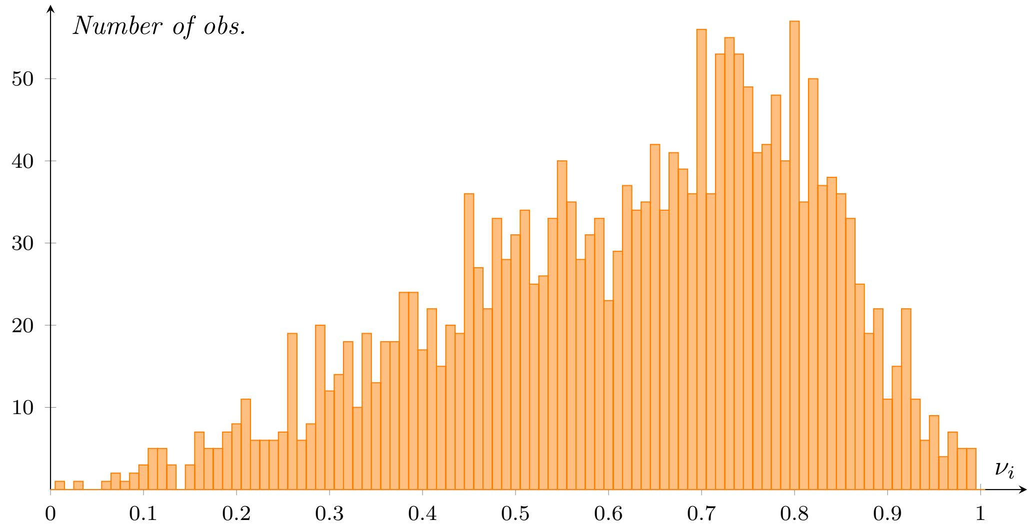


Figure A.5: The distribution of estimated ν_is (averages across years).

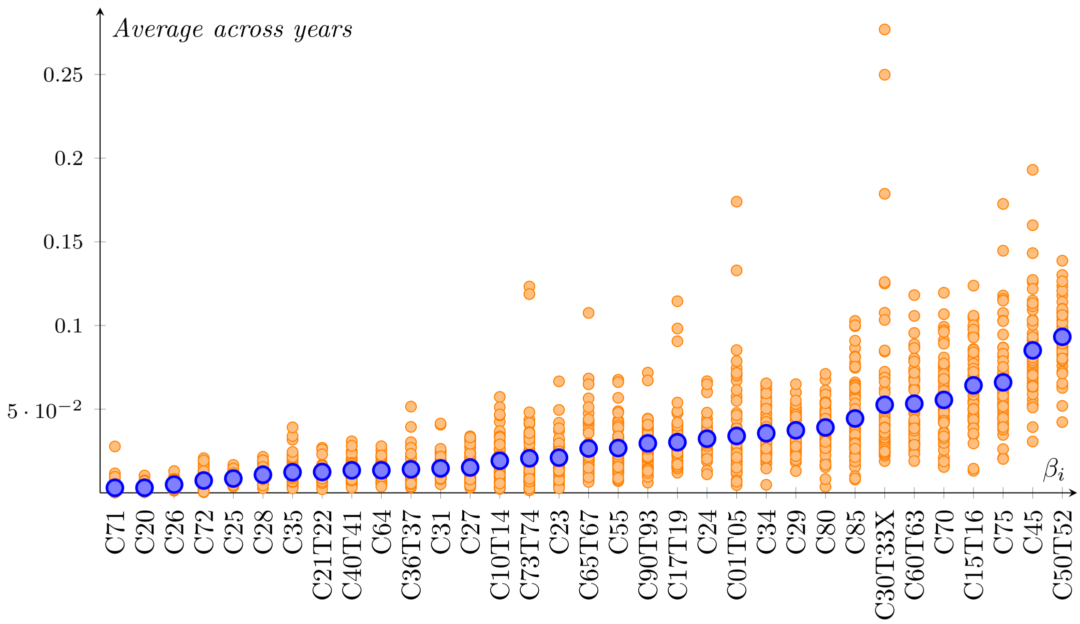


Figure A.6: Country averages of estimated β_is by sector (orange) and pooled averages by industry (blue).

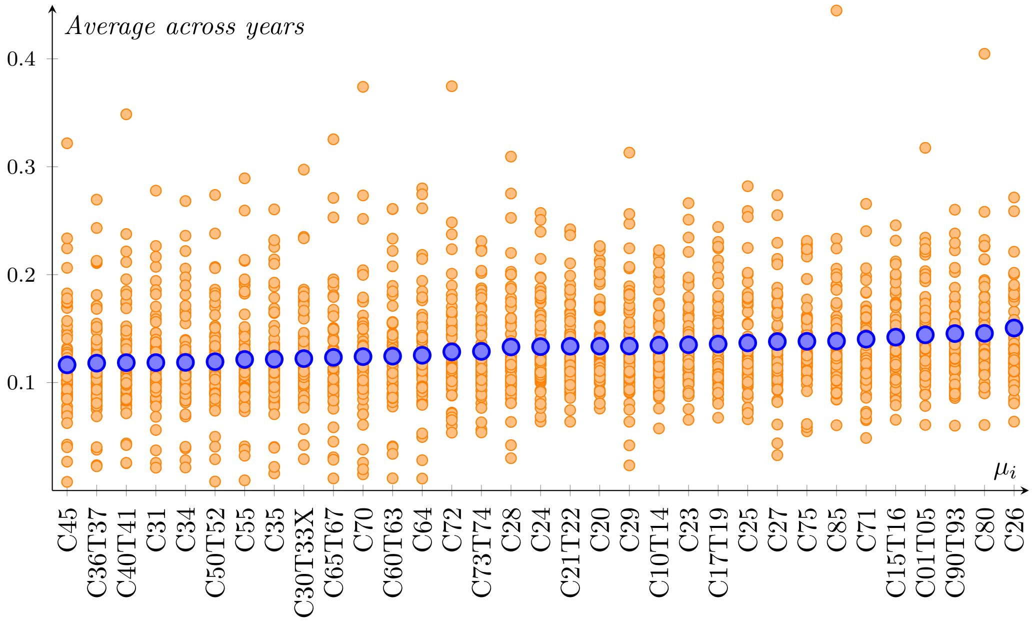


Figure A.7: Country averages of estimated μ_is by sector (orange) and pooled averages by industry (blue).

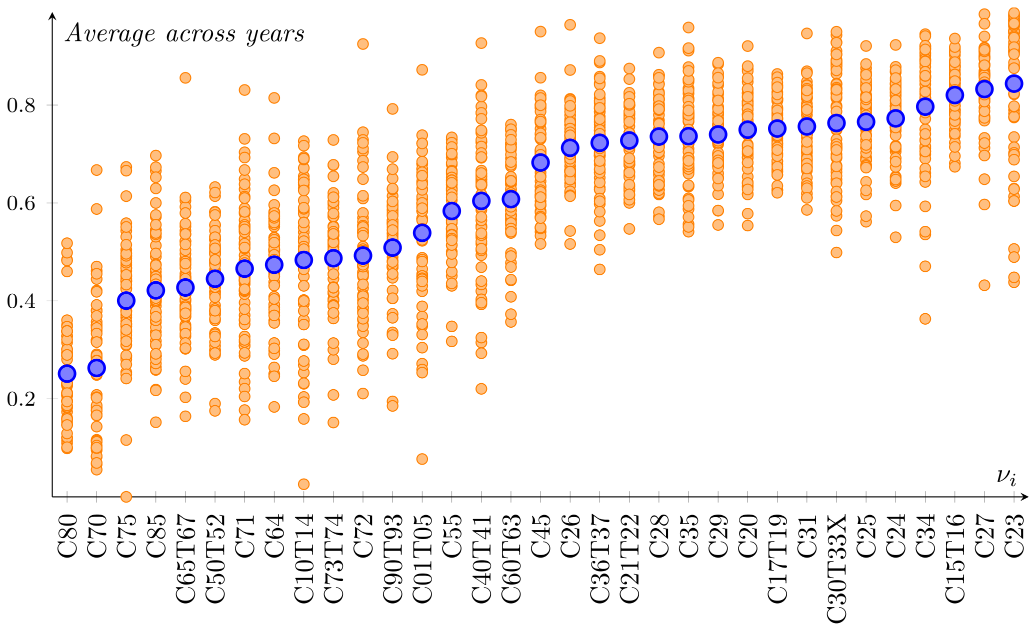


Figure A.8: Country averages of estimated ν_is by sector (orange) and pooled averages by industry (blue).

On the Factors of Innovation and Economic Growth

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