Aristotle's Measurement Dilemma

1. Introduction

Aristotle is the first thinker in the Western tradition handed down to us who explicitly discusses the topic of measurement systematically within a context relevant for natural philosophy.[1] His account of measurement is, nevertheless, very much an underex-

[1] Of course the practise of measurement long preceded Aristotle, and had seen important developments in near Eastern cultures, in Egypt, Mesopotamia, etc. With respect to the conceptual side of measurement – for reasons of space I only look at the Hellenistic world here – there are questions of measurement connected with natural philosophy to be found in earlier thinkers, for example, in Anaximander, where we find the distances between the earth, moon, and sun measured in terms of the earth radius or circumference, cf., for example, D. O'Brien, "Anaximander's measurements", *Classical Quarterly* 17 (1967), pp. 423-432 and A. Gregory, *Anaximander, a Re-assessment*, London 2016. Also Heraclitus B 94 (which claims that the sun will not overstep his measure, otherwise the Erinyes, ministers of Justice, will find him out) and Diogenes of Apollonia B3 (telling us that the underlying substance has a measure of everything, of winter and summer, of night and day, of rains and winds, and of fair weather) can be seen in connection with natural philosophy. There probably was some discussion about measurement in the Pythagoreans, and Philolaos' basic pair of limiters and unlimiteds may be related to a measurement context (see Philolaos fragments 1, 2, and 9A in C. Huffman, *Philolaus of Croton* (Cambridge 1993), and G. Lloyd, *Revolutions of Wisdom* (Berkeley 1987), 276-278, who thinks that the Pythagorean idea that all things are numbers or are like numbers acted "as a stimulus to find those numbers, by measurement, in the phenomena"). But neither in Anaximander, nor in Heraclitus, Apollonius, or the Pythagoreans, as handed down to us, do we find a full account of what measurement requires and means.

L. Elders, *Aristotle's Theory of the One - A Commentary on Book X of the Metaphysics* [*One*] (Assen 1961). 71 thinks that we can explain "Aristotle's enthusiasm for 'measure'" only "by considering it as a remainder of Platonism". However, while we find substantial discussions of measurement in Plato, especially in the *Philebus* and the *Laws*, it is there discussed mainly as a metaphysical and ethical notion, and not as what we can see as a forerunner to contemporary measurement theory (in *Philebus* 55d ff. Plato uses measurement for a division of different sciences, and also the *Laws* sometimes mention measure in the context of the sciences, for example, in 746eff.; but again Plato is not developing any kind of a measurement theory there). In the *Timaeus* Plato uses the notion of a measure within the context of astronomy, but also here the notion of measurement is not further analysed, cf. my *Natural Philosophy in Ancient Greece - Logical, Methodological, and Mathematical Foundations for the Theory of Motion* [*Motion*], chapter 6. We are provided with a kind of classification of measurement in *Republic* VII (in 526cff. where Plato talks about *geômetria* plored topic, with hardly any research done on it. The research there has been focuses on time as a measure of motion,[2] but does not discuss Aristotle's general account of measurement and how this general account informs his idea of time as a *measure*.

This paper investigates Aristotle's account of measurement in two respects. First, it reconstructs Aristotle's account of measurement in his *Metaphysics* and shows how it connects to modern notions of measurement – it shows in how far Aristotle's account can be seen as providing some basis for our contemporary notion of a measure and where the two come apart. Aristotle's notion of measure is not simply the same as the one we are working with today, but it can be understood as a special case of our notion and thus, to some degree, as an important predecessor for a contemporary notion of measurement. Second, the paper demonstrates that while Aristotle's notion of measurement is a useful notion in general, it only works for simple measures. Consequently, it leads Aristotle into a dilemma once it comes to measuring complex phenomena, such as motion, where two or more different aspects have to be taken into account (in the case of locomotion, the temporal as well as the spatial aspect have to be considered). Modern theories are not beset by this dilemma, since the

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which includes the art of measurement or land-surveying). But all we find there is the distinction between a less precise branch that deals with practical matters, such as setting up a camp, and a precise one, that is directed towards what always is.

The ancient thinker who is perhaps most famously connected with the idea of a measure, Protagoras, claims that man is the measure of all things - a claim, which, according to the discussions we find of it in Plato's *Theaetetus* and indeed in Aristotle's *Metaphysics*, is an epistemological claim. We will have to leave out any discussions of ethical, epistemological, and metaphysical notions of a measure here. And within the context of natural philosophy, it is only Aristotle who discusses the theoretical problems the notion of measurement raises explicitly. Unfortunately, in this paper I will not be able to look at one area that is quite important for measurement theory in this context, astronomy, and also not at the relationship between measurement theory and proportion theory.

[2] Cf., for example, *Aristotle, Physics Books III and IV*. Translated with introduction and notes by Edward Hussey [*Physics*] (Oxford 1993); U. Coope, *Time for Aristotle*. Physics IV.10-14 [*Time*] (Oxford 2005); and T. Roark, *Aristotle on Time, A Study of the Physics* (Cambridge 2011).

restriction to simple measures that we find in Aristotle's account was ultimately modified in the history of measurement theory.

We will start with a brief sketch of the crucial features a measure exhibits according to modern notions of measurement, as this will give us some conceptual tools for understanding Aristotle's account of measurement. Subsequently, we will discuss the essential parts of the most extensive and important passage on measurement in Aristotle's corpus, the most relevant bits of a passage from *Metaphysics* book Iota. With the reconstruction of Aristotle's understanding of measurement in hand, I will then show the dilemma Aristotle's notion leads into for measuring motion. For this part I will look at Aristotle's *Physics*, book IV and VI, where we find a contrast between Aristotle's implicit and his explicit understanding of the measure of motion: Aristotle *implicitly* uses a *complex measure* in several passages, taking into account the time taken as well as the distance covered, very prominently so when solving one of the problems Zeno's dichotomy paradox raises, which allows him to offer a good resolution to Zeno's paradox. The problem is that Aristotle is not entitled to this response given his official account of measure, since his explicit definition in the *Metaphysics* characterizes the measure of motion as a simple measure. In accordance with this account, it is only time which is *explicitly* claimed to be the measure of motion in the *Physics*;[3] as we will see, there is no other magnitude that is explicitly called a measure of motion and no combination of magnitudes that would make for a complex measure. While a complex measure is in tension with Aristotle's official account of measurement, a simple measure will turn out to be insufficient for measuring motion and for resolving Zeno's paradox.

[3] This is not to deny that Aristotle also talks about measuring time by motion (cf. *Phys*. Δ 12), but simply to point out that when we measure motion, only time is explicitly called a measure of motion.

2. Basic Features of Contemporary Notions of Measurement

While Aristotle discusses measure in quite some detail, it is in fact not easy to give an account of his notion of measurement. In order to get a better grasp of Aristotle's account, it will be helpful first to sketch some basic structures of a contemporary understanding of measurement, as we find it in the philosophical literature on measurement theory. This will help to make explicit certain features that Aristotle assumes implicitly and it will show how certain features of the measurement procedure that may sound odd in Aristotle are in fact predecessors of features we also assume when giving an account of measurement. In this way a sketch of some of the basic features of a modern notion of measurement will allow us to navigate the difficult texts of Aristotle on measurement more easily. And we have to introduce a modern conception of measurement anyway if we want to compare it to Aristotle's account.

I will sketch important features of a contemporary conception of measurement that is in accordance with the "Foundations of Measurement" by Krantz/Luce/Suppes/Tversky and Ellis's "Basic Concepts of Measurement".[4] I will, however, leave out a lot of the technicalities that are not relevant for a comparison with Aristotle.

It may seem that this is a hopelessly anachronistic approach and that introducing Krantz et al. is real overkill – not only are there important conceptual differences, but also enormous technical ones. And in fact both Ellis and Krantz et al. seem to have very different projects than Aristotle: Ellis' aim is to give "a consistent positivist account of the nature of measurement". And Krantz et al. want to build an axiom-

[4] D. Krantz, R. Luce, P. Suppes, and A. Tversky, *Foundations of Measurement* [*Foundations*] (Dover 2006) (a first edition was published in 1971); B. Ellis, *Basic Concepts of Measurement* [*Concepts*] (Cambridge 1968).

system that allows them to establish representation and uniqueness theorems that hold for all kinds of different measures.[5] I want to show, however, that while all three projects are based on very different motivations and prerequisites, nevertheless they all are concerned with what we may call foundations of measurement,[6] they all want to provide conceptual clarifications of our understanding of measurement and, to some degree, clean up potential conceptual confusions.[7]

Thus, introducing Ellis and Krantz et al. is not to deny that modern approaches have a very different starting point than Aristotle,[8] but an attempt to show that it can be fruitful to keep some of our modern notions consciously in mind for understanding Aristotle – not only in order to find interesting conceptual similarities, but also to get a better understanding of the differences. The sketch I give here will be in accordance with the account of Ellis and Krantz et al., but it will not simply be their account, as this is too abstract for our purposes. Instead I will extract three essential features of measurement that allow us to compare Aristotle with a modern account.

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[5] See Krantz et al., *Foundations*, 12: "We conclude, then, that an analysis into the foundations of measurement involves, for any particular empirical structure, the formulation of a set of axioms that is sufficient to establish two types of theorems: a representation theorem, which asserts the existence of a homomorphism φ into a particular numerical relational structure, and a uniqueness theorem, which sets forth the permissible transformations φ → φ' that also yields homomorphisms into the same numerical relational structure. A measurement procedure corresponds to the construction of a φ in the representation theorem." Their basic idea is to construct a homomorphism into the real numbers. Cf. also *Foundations*, 13.

[6] Krantz et al. point out on p. XVIII that they are concerned with foundations of measurement and "not with historic or current practice of measurement in any field". And they give a foundational theory of the practices and conceptions that can be understood as part of the ancient or modern practice of measurement.

[7] Ellis' main thesis is that "certain metaphysical assumptions have played havoc with our understanding of many of the basic concepts of measurement, and concealed the existence of certain more or less arbitrary conventions" and thus he wants to examine what he considers to be the basic concepts of measurement. Krantz et al. take it that the practices and conceptions of measurement can be freed from conceptual messes by putting them into a formal theory and axiomatising them. And we will see Aristotle in his account of measurement ready to point out potential conceptual confusions (cf., for example, 1053a27-30 and the discussion of this passage below).

[8] For example, a different mathematics, a different account of quantities, etc.

Let us start now by asking what we normally take the task of a measure to be. Measuring somehow allows us to connect the physical world with numbers. But measuring does not mean to assign some numbers to the perceptible world in a random fashion. Rather, a measure enables us to quantify physical things, processes, and states of affairs systematically and is thus one way to understand the perceptible world as something intelligible. In order to quantify something, we have to decide first which aspect of a thing we want to quantify – for example, do we want to measure its volume, its temperature, or its density. How much of this aspect a thing possesses, that is, the amount of this aspect, can then be measured by assigning it to numbers with the help of measurement units. So in order to measure something, three things have to be taken into account:

(1) The respect in which something is meant to be measured has to be determined, that is, we have to decide whether we are going to measure the weight or rather the length of a table. This is what I want to call the *dimension* measured in the following, as is common in measurement literature.[9] (In fact, the English word "dimension" derives from Latin "dimetiri", which means "to measure out").[10] Thus by dimension we should not just understand spatial dimensions, but all kinds of respect that we may want to measure.[11] In modern literature on measurement dimension can include simple as well as complex dimensions, that is, dimensions that are derived from a combination of simple dimensions.[12] By contrast, we will see below that ancient Greek

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[9]Following Fourier's introduction of the notion of physical dimensions; see, for example, also Krantz et al. p. 455.

[10] See the OED entry on "dimension".

[11] Cf. also Ellis, who points out on that we speak of "the dimension of length, mass, time-interval and so forth" (*Concepts*, 139).

[12] In the literature, we often find a distinction between fundamental (or primary) and derived (or secondary) measures, quantities or scales, whereby a "derived scale is one which in the procedure of measurement presupposes and uses the numerical results of at least one other scale", while a fundamental scale does not depend on other scales (P.

treatment of measurement focuses on simple dimensions,[13] the only complex dimensions that seem to come into view are two- and three dimensional spatial magnitudes, planes and solids.[14] Thus, the modern notion of dimension can include more than the ancient will, but the core notion stays the same, that we determine which aspect or property we want to measure.

The thing to be measured with respect to this dimension (that is the thing qua possessing a certain feature) will be called the measurand. So if we decide to measure the weight of a table, the dimension to be measure is weight, and the measurand is the weight of the table.

(2) The measurand must be quantified by *assigning* it to the number series in a systematic way.

(3) *Units* have to be defined to carry out this quantification of a certain dimension. A certain amount of this dimension is taken as a unit, for example, a metre or a foot is take as a unit for length, so that the measurand, for example, the length of my table, can be determined as a multiple of it.

Let me give a slightly fuller discussion of each of these three requirements (this more detailed account can be skipped by anybody who is satisfied with the little sketch above):

(1) Given that we often understand the term "dimension" in a merely spatial sense, capturing length, breadth, and depth, it may seem unclear what it in fact means within

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Suppes' article on "Theory of Measurement" in the *Routledge Encyclopedia of Philosophy* 2003 [*Measurement*], cf. also Krantz et al., *Foundations,* chapter 10). A derived or secondary scale need not be the combination of two or more fundamental scales, but could in principle also be derived from just one fundamental or primary one, while a complex scale is the product or ratio of at least two other ones.

[13] And this paper should shed a bit of light for some of the reasons for this focus on simple dimensions.

[14] Two- and three-dimensional spatial magnitudes are, however, not necessarily seen as combinations of simpler dimensions.

the context of measurement. And even in the literature on measurement, it is sometimes seen as an unclear notion; as Ellis claims: "It is difficult to say how dimensions are usually regarded. For no one seems to have any clear conception."[15] As suggested above, I will understand "dimension" here as the aspect under which something is regarded in the process of measuring, hence as the feature, property or quality[16] that is quantified. This includes spatial dimensions, but is not restricted to them. Thus, for example, a golf club may be regarded from the point of view of its mass, its length, or the force it exerts on a golf ball – these are all different dimensions that we can measure. Which respect is taken into account depends on what we want to examine, and it has to be chosen before we can start measuring (there is no point in bringing my weights for measuring if it then turns out that we are actually going to measure length).

Determining the respect in which something is going to be measured includes determining whether the dimension is simple (for example, in the case of measuring length) or complex (as for instance with speed, where we deal with two dimensions, time and space). If we are dealing with a complex dimension, we also have to determine the relation of the different dimensions (for example, for our modern notion of speed we *divide* the space covered by the time taken, $v = s/t$,[17] it is displacement *per*

[15] Ellis, *Concepts*, 139. He understands by 'dimension' generalized unit names: "just as a unit name refers to a particular scale, so a dimension name refers to a particular class of scales" (142).
[16] Note that the notion of quality we will come across later on in Aristotle's account of measurement units is a different notion.
[17] Usually, we talk about $v = (ds/dt)$, that is, about instantaneous speed, which is not a concept the ancient Greeks worked with. However, while we can talk about movement at a point nowadays, the way we get to this instantaneous speed is still to take a period and distance in which a movement takes place and make the period and distance shorter and shorter so as to let them converge to the limit of the initial period and distance. Our notion of an instantaneous speed, hence, depends also on extended periods and distances (even if the limit is distinct from any member of the series); cf. also Lear, "A note on Zeno's arrow", *Phronesis* 26 (1981).

time, while in order to calculate electric charge we *multiply* amperes with seconds, c = A *times* s). Thus we determine the dimension to be measured in such a way that it can guide the process of measurement. If we are, for example, interested in the force your golf club exerts on a golf ball, our measure will be complex, reflecting that force is the product of mass and acceleration (F = m • a).[18] After a certain aspect of the golf club is measured we are able to compare the result of this measurement to the result of a measurement with regard to the very same aspect of other bodies or of the same body at a different time.

It may sometimes seem trivial to determine the dimension to be measured and there are modern accounts of measurement which do not seem to refer to anything that corresponds to what I call 'dimension'. This we find for instance in the *Encyclopaedia Britannica*[19] where "measurement" is defined as the "process of associating numbers with physical quantities and phenomena". In this account nothing like dimension is mentioned. What we call dimension is, however, introduced with the measurand: "Measurement begins with a *definition* of the measurand, the quantity that is to be measured" (*Encyclopaedia Britannica*, article "Measurement", my italics). Here we obtain the dimension needed for measurement with the definition of the measurand.[20] Determining the dimension is nevertheless unproblematic in many cases

[18] Not all dimensions will be measured directly, rather some will be measured through calculations and theories based on other measures; for example, our measurement of force will usually rely on our measurement of mass and acceleration, and in some cases we may measure momentum and then calculate the force.

[19] In the article "Measurement", 2004 edition (updated 2015).

[20] Similarly, the first characterization of measurement given in Krantz et al., *Foundations* does not seem to refer to something corresponding to what I called 'dimension', that is, the certain respect, which is chosen for comparison. However, the authors presuppose what I call 'dimensionality' in that they understand "the measuring of some attributes of a class of objects or events" as the process of associating "numbers (or other familiar mathematical entities such as vectors) with the objects in such a way that the properties of the attributes are faithfully represented as numerical properties" (p.1). The measurands here are "some attributes of a class of objects or events", and

in a modern context. Therefore, many modern conceptions of measurement can just presuppose dimensionality without discussing it. The fact that dimensionality may be difficult to deal with, however, will become obvious once we investigate the measure of motion in Aristotle.

(2) In order to measure something, we have to assign its structure in a certain respect (the dimension we are measuring) to a mathematical structure, i.e. to the real numbers, the natural number series, or to a certain set of numbers.[21] Hence Krantz et al. understand measurement as the "construction of homomorphisms […] *from empirical relational structures* of interest *into numerical relational structures* that are useful".[22]

This is a rather abstract way of conceptualising the human activity of measuring that I perform, for example, when I measure the length of the walls of a room with a ruler, which allows me to assign numbers of feet or metres to the length of the walls that I can then compare (I can determine which wall is longer), multiply (I can determine the square metre or square feet of the room), etc.

The assignment used for measurement purposes is a homomorphism, that is, a structure-preserving map;[23] but it is not normally an isomorphism, that is, a homo-

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the dimension is the kind of attribute measured. Moreover, the authors discuss dimension explicitly in chapter 10.

[21] Krantz et al. take the co-domain to be real numbers, but Suppes, *Measurement* talks about "some set of numbers", and in simple cases the natural numbers will suffice. The ancient Greeks had a different conception of number, and real numbers do not occur in it, but for the assignment it is enough to have some conception of numbers.

[22] Cf. Krantz et al., *Foundations*, 9, my italics. By a relational structure they understand "a set together with one or more relations on that set", for example, a numerical relational structure is the set of real numbers, together with a "greater than" relation and an operation of addition, see p. 8.

[23] To express 'preserving structure' in slightly more formal terms: we have a domain of physical objects, a concatenation function ($\circ$) and a relation ('bigger than') on the one hand, and for the codomain the real numbers with addition (+) and a relation (>) on the other hand. Then a homomorphism is a function from the structure of what is to be measured into the real numbers, such that $F(g1 \circ g2) \rightarrow F(g1) + F(g2)$, $g1 \gtrsim g2 \rightarrow F(g1) \geq F(g2)$, etc. For Krantz et al. F is essentially not a surjection. I owe this clarification to an anonymous reader for OSAP.

morphism which is bijective (it is both one-to-one and onto).[24] In modern science, this systematic homomorphic mapping transfers the empirical realm to an abelian group[25] of real numbers. As Suppes writes: "What we can show is that the structure of a set of phenomena under certain empirical operations and relations is the same as the structure of some set of numbers under corresponding arithmetical operations and relations".[26] We may thus use our knowledge of the arithmetical structure to infer information about the homomorphic empirical structure; for example, if we measure two tables and find that one is 80cm in length and the second is 90cm in length, then we may infer from the arithmetical realm that if we put them together (and thus "concatenate" them), together they should be 170 cm in length.[27]

Finally, the rule of assignment has to guarantee that under the same conditions the same assignments will be made, and under different conditions different assignments are possible.[28]

(3) What we measure in the physical world will usually be a continuum in the sense that it is not yet divided into given parts, for example, if you want to measure the

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[24] For this see Krantz et al. who explicitly talk of a homomorphism in the definition given above, and who note on p. 8: "We speak of a homomorphism (rather than of an isomorphism) because Φ [i.e. the used numerical function] is not normally one-to-one."
[25] That is, to a group where commutativity holds, so that the result of an operation does not depend on the order in which the elements of the group are written.
[26] Cf. Suppes, *Measurement*.
[27] We do this via the injection F: F(80 cm length ∘ 90 cm length) = F(80 cm length) + F(90 cm length) = 80.0 + 90.0 = 170.0 and F (170cm) = 170.0 (or the inverse: F-1(170.0) = 170 cm). According to Suppes, this does not mean that all systems of measurement will work in a simple compositional way, but for our purposes we will leave non-additive scales to the side.
[28] Ellis, *Concepts*, 41 characterizes these conditions as follows: "Measurement is the assignment of numerals to things according to a determinative, non-degenerate rule", where "determinative" means that "provided sufficient care is exercised the same numerals (or range of numerals) would always be assigned to the same things under the same conditions", and "non-degenerate" means that the rule "allows for the possibility of assigning different numerals (or ranges of numerals) to different things, or to the same thing under different conditions". Krantz/Luce/Suppes/Tversky call this a "uniqueness theorem".

length of your table, you are normally dealing with a continuous length without any actual parts.[29] Consequently, the structure of such continua cannot be assigned to a numerical structure straight away, since that would require given discrete chunks that we can assign to numbers. Instead, we need basic units to mark off discrete parts. These basic units have to be constant, otherwise we cannot make sensible comparisons.

For some dimensions the units used can be defined once and then kept for further measurement procedures. For example, in order to measure weight, we can define a stone as our unit and then use the very same stone for measuring weight over and over again, for different measurement procedures at different times; or, to use a different example, we can always use the same measuring cup (where the volume that fits into the cup is the unit of measurement) for different baking occasions. And this is similar in the case of measuring length, I can use the very same centimetre on a ruler to measure out different lengths. What is specific to time-measurement, which is crucial for measuring motion, is that here the measurement units always have to be "produced" anew – at each moment when we want to measure something we need a motion or change going on from which we can derive a unit (we cannot use a past second to measure a time right now). This process of creating units has to be regular (as, for instance, the regular vibration of a quartz in our watches) to secure the units being constant.[30]

[29] Whatever your metaphysics of continua is on the micro-level.

[30] While the units used to measure and the process used to gain measurement units have to be regular, regularity (in the sense of having always the same quantitative size) of the thing inquired is not a necessary condition for countability or measurability, since also irregular pieces can be counted and measured. For example, if I drop my tea cup on the floor and it breaks, I can count the number of pieces that are now left of my cup, even if they are irregular pieces; and I can measure the duration of a motion even if it changes direction or speed. And counting or measuring something does not "produce" a regular order, either.

The units used for measurement have to be chosen in accordance with the measure: what is to be measured and the units chosen to measure have to be of the same dimension (we cannot measure the weight of a table with centimetres). And they also need the same degree of complexity. For example, as the measure of the force exerted by a golf club is complex – it is the product of mass and acceleration ($F = m \bullet a$) – the units have to be complex as well, usually $kg \cdot m/sec^2$. In Aristotle we will see this requirement, with an important restriction, spelt out as a "homogeneity" requirement.

The units used for measuring are in principle arbitrary in the sense that we can use units of different size – I can use centimetres as well as inches for measuring the length of my table, the one is no more natural than the other. Centimetres will divide the length of my table in different (potential) parts than inches, but both will allow me to make reliable comparisons equally.

By understanding the arbitrary choice of a dimensional unit as an important feature of measurement, we are distinguishing measurement from mere counting where what is quantified is given as discrete elements. Counting can be understood as determining the cardinality of a plurality of given discrete things by coordinating two procedures: the operation that allows us to consider each element of the plurality singly, no matter in which order, is coordinated with the operation that takes us through the series of the natural numbers. This has to be done in such a way that whenever I take up a new element I move ahead one step in the number series; for example, when counting my chickens, I take up the first chicken and start with the first element in the number series, 1, then I take up the next chicken and move ahead one step in the number series, to 2, etc. In the procedure of counting we do not need to break down a continuum first into parts that can then be assigned to numbers, as we do when we measure something; rather, with counting there are already given discrete parts. While with measuring we use arbitrary units and we first have to "divide" the continuous quantity into parts, with counting the individual elements we count are given as is the unit with which we count (as, for example, the unit "chicken"), that cannot be made smaller or bigger arbitrarily, since then we are counting something else.[31] And with mere counting, like counting up to a hundred, no dimension whatsoever plays a role,[32] since we normally do not understand numbers to have any dimension of their own[33] – thus they are suitable to be used for operations on all sorts of dimensions.[34]

There are, however, authors like Suppes, *Measurement*, who treat counting as a measuring procedure, taking it as an absolute scale. This seems to be supported by the fact that we talk about two groups being equal in number just as we talk about

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[31] In case we are to count, that is, we quantify given discrete things, as, e.g., in Aristotle's example of quantifying horses in *Physics* 220b20-22?, the dimension is expressed by the very units – both are "horse". When we measure and thus quantify continua, on the other hand, dimension and units are not the very same thing: for example, our dimension measured may be time, in which case our units will be a certain amount of time, like seconds or minutes.

[32] In Aristotle's example of counting horses, however, dimension does play a role, since if we do not determine the dimension "horse" first, we may as well be counting chickens, see also *Physics* 223b13-14. If you want to count how many human beings are in a room, and we have not yet clarified the dimension "human beings" as what we count, the number we end up with may instead correspond to the number of animals in this room, see also below. In contrast to measuring, however, there is no arbitrary unit involved (I normally cannot use a slightly smaller unit than human being to quantify the number of human beings in the room in the way that I may use millimetres in some cases, rather than centimetres, or inches to measure length).

[33] Understanding number as Russell does in his *Principles of Mathematics* (London 1972), 116 as the class of similar classes: "Mathematically, a number is nothing but a class of similar classes".

[34] The difference we are interested in – between counting, where the parts we count are already given as elements of a group, and measuring, where such parts first have to be gained in some respect – we also find with Ellis, *Concepts*, 15 in the context of determining the rules of applying pure arithmetic: "[…] numeral terms are always interpreted as the *number* of things in a group or as the *measure* of something in some respect" (his italics). Consequently, Ellis is quite reluctant to call counting a measuring procedure, pp. 152-159, thinking it lacks the arbitrariness as there is no choice of unit with counting.

them as being equal in other qualities.[35] And indeed this makes good sense to the extent that one can compare two groups with respect to the number of their elements. (For reasons to do with his ontology, Aristotle will also belong in this group of authors who, at least sometimes, understand counting as a form of measuring.) However, for our purposes it will be useful to distinguish in principle between counting and measuring.

3. The general concept of measure in Aristotle's *Metaphysics*

Having sketched crucial features of a contemporary understanding of measurement, let us now look at Aristotle's discussion of measurement. The *Metaphysics* is the text where Aristotle gives an explicit account of his concept of measurement, especially in book Δ, chapter 6 (1016b17-25), book I, chapter 1 (1052b18-1053a8) and book N, chapter 1 (1087b33-1088a14). I will restrict myself for the most part to the essentials of book I, chapter 1 since this is the most elaborate passage concerning measurement. The other two passages will sometimes be referred to; they are basically variations of the passage in Iota.

The context in which Aristotle gives an account of measure in book Iota is his investigation of what it means to be one – a question that is explicitly distinguished from the question which things are one. To be one can mean different things,[36] but, according to Aristotle

> τὸ ἑνὶ εἶναι […] μάλιστα δὲ τὸ μέτρῳ εἶναι πρώτῳ ἑκάστου γένους
>
> καὶ κυριώτατα τοῦ ποσοῦ· ἐντεῦθεν γὰρ ἐπὶ τὰ ἄλλα ἐλήλυθεν.

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[35] Cf. also Ellis, *Concepts*, 152.

[36] Cf. also David Charles's distinction between what "one" signifies and what one is in his *Aristotle on Meaning and Essence*, Oxford, 2000, pp. 24-54.

> to be one is […] especially to be the first measure of a kind, and most strictly of quantity; for it is from this that it has been extended to the other cases (1052b16-20).

So in a primary sense being one means being a measure for quantities. A measure can be seen as a one in two respects, as we already saw in the previous section: it can be a one qua particular measurement unit (for example, one centimetre as the unit to measure length)[37] and a one qua determining the one respect that we are measuring (for example, the one respect we are measuring with regard to my desk is length, not weight).[38]

The most important passage for reconstructing Aristotle's explicit account of measurement is 1052b16-1053b8. This long passage can be seen to be structured as follows: (1) Aristotle starts out by claiming that being one means primarily to be the first measure of something (1052b16-20). (2) We then get a brief account of the function of a measure – in the strictest sense a measure is what allows us to know a quantity (1052b20-31). Subsequently, we learn the requirements for a measure:[39] (3) it is indivisible either in quality or quantity (1052b31-35); (4) it is most exact if nothing can be added or subtracted without it being noticed (1052b35-1053a14); (6) and it is homogenous with what is to be measured (1053a24-27). In between we are also told

[37] At least if we take a unit here to be a translation of μέτρον (things look differently if we take it to be a translation of μονάς – I will say more on the ancient notion of unit below).

[38] A measure establishes a kind of unity by defining a uniform aspect to be measured and by determining numerically the amount of this aspect which the thing to be measured possesses.

[39] Aristotle's account here seems to be descriptive rather than normative, that is, he seems to given an account of how measures do work rather than of the way in which they should work. Thus, I do not take Aristotle to be revisionist or reformist here about geometry, etc. (though his conceptual clarification could turn out to be revisionist about people's actual practise).

that (5) sometimes there can be several basic "ones", that is, several measurement units for the same kind (1053a14-24).[40] Aristotle then (7) briefly tells us how we should understand his conception of measurement if we look at numbers (and counting (1053a27-30), before (8) he discusses in how far science and perception can be understood as measures (1053a31-1053b3). In this context he briefly takes up Protagoras' *homo mensura* statement, only to dismiss it as not saying anything new or special. Finally, he gives a brief summary (9) of some of the main points (1053b4-8).

We can see from this passage in Iota that for Aristotle a measure is characterized by four basic features – the task or function of a measure and three requirements needed for a measure to fulfil this function. To use a somewhat different order that will be useful for purposes of presentation we can say that for Aristotle

(1) Measure is that by which the quantity is known – this is its general task;

(2) A Measure always has to be homogenous with the measurand;

(3) It is simple either in quantity or quality;

(4) It is most exact if nothing can be added or subtracted without it being noticed.

[40] οὐκ ἀεὶ δὲ τῷ ἀριθμῷ ἓν τὸ μέτρον ἀλλ' ἐνίοτε πλείω, οἷον αἱ διέσεις δύο, αἱ μὴ κατὰ τὴν ἀκοὴν ἀλλ' ἐν τοῖς λόγοις, καὶ αἱ φωναὶ πλείους αἷς μετροῦμεν, καὶ ἡ διάμετρος δυσὶ μετρεῖται καὶ ἡ πλευρά, καὶ τὰ μεγέθη πάντα ("But the measure is not always one in number – sometimes there are several; for example, the semi-tones (not to the ear, but as determined by the ratios) are two, and the articulate sounds by which we measure are more than one, and the diagonal of the square and its side are measured by two quantities, and all magnitudes reveal similar varieties of unit", 1053a 14-18). These multiple units are incommensurable – strictly so in the case of the diagonal and side, with respect to harmony in the case of the semitones (*diesis*). A. Barker, *Greek Musical Writings*, volume II [*Musical Writings*] (Cambridge 1989), 72-73, n. 16 and 17 understands the first occurrence of *diesis* as a measure in the context of this passage (that is in 1053a12) as referring to the empirical harmonics, and the second one, the one just quoted, to ratio theory; for a different suggestion for how to understand the two kinds of *diesis* here can be found below.

We will see that these points only partly overlap with the three characteristics introduced from contemporary debates. Let us investigate these four points we found in Aristotle in turn:

(1) According to *Metaphysics Iota*, chapter 1, the essential task of measurement is to allow us to know the *quantity* of something:

> μέτρον γάρ ἐστιν ᾧ τὸ ποσὸν γιγνώσκεται
>
> For measure is that by which the quantity is known (1052b20).

Of a quantity we always want to know *how much* or *many* it is – that is what the Greek word that we normally translate as quantity, *poson*, actually means. And a measure obviously enables us to answer this question. This is a feature which fits with our contemporary accounts of measurement, even if we did not explicitly introduce it as a separate characteristic in our discussion above. To fulfil this function a measure needs certain characteristics which are expressed by the following conditions of Aristotle's concept of measurement.

(2) With Aristotle a measure – and by this Aristotle here seems to understand a measurement unit – always has to be homogenous (συγγενὲς) with the thing measured:

> ἀεὶ δὲ συγγενὲς τὸ μέτρον· μεγεθῶν μὲν γὰρ μέγεθος, καὶ καθ' ἕκαστον μήκους μῆκος, πλάτους πλάτος, φωνῆς φωνή, βάρους βάρος, μονάδων μονάς
>
> But the measure is always homogenous (with the measurand),[41] it is of magnitudes a magnitude, and in particular of length a length, of breadth a breadth, of sounds a sound, of weights a weight, of units a unit (1053a24-27).

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[41] The Greek text literally only says that the measure has to be συγγενὲς, but not what it is συγγενὲς with. However, something like "what is to be measured" or "the measurand" is the only reasonable addition, since the measure has to be of the same *genos* as the thing to be measured (otherwise no measurement is possible – we could not, for example, measure temperature with a kilogram). Accordingly, translations normally add something like this; for example, Bonitz translates "Immer ist das Maβ

According to this passage, the dimensionality has to be the same for the measure and the measurand – if we want to measure a length, for example, we need another (smaller) length to measure it, while for the measurement of weight we cannot use a length but rather need another weight. This homogeneity constraint corresponds to the requirement that the measurement unit has to be of the same dimension as the measurand we saw with our general notion of measure above.

However, if we look a bit closer at this and surrounding passages, a restriction can be found in this homogeneity requirement that distinguishes Aristotle's account in a crucial way from a modern one: In Aristotle, the dimension to be measured turns out to be always conceived of as simple, that is, one-dimensional. A first indication for this is that in the passage just cited length is measured with one measure, and breadth with another one. By contrast, today we would trace back the measure of breadth to the measure of length plus what we may call its orientation in space or as given in position (θέσει), if we think of it as referring to a geometrical figure in the ancient context.[42] Aristotle uses what we would think of as a complex measure (length *plus* its orientation or position) like a simple one (breaths as its own, simple

dem Gemessenen gleichartig", Ross "the measure is always homogenous with *the thing measured*", Tricot "La mesure est toujours du même genre que *l'objet mesuré*", and Viano "Sempre la misura appartiene allo stesso genere delle *cose che debbono essere misurate*".

[42] When I am talking about what we may call "orientation in space" here, I am not suggesting that Aristotle could in the very same way talk about space. And for this idea it is not necessary that we can stand on the earth and say "this direction is length and that is breath or width", but only that once we determine one direction as length, the other side (in two-dimension) is breath or width (though for Aristotle there is an absolute orientation of the world, as he argues in *De Caelo* Book II, Chapter 2, because the world is a living being). Aristotle may in fact mean two different things by πλάτος here: either an area (as he does, for example, in *Metaphysics* Δ 13, 1020a10–12), or a length at right angles to a given length, what we may call "breath". But in both cases we would think of πλάτος as what we measure with the help of length plus something else (for the claim that for Aristotle we do not measure an area by a length see also below).

dimension, that is not derived from another dimension).[43] For Aristotle there is in fact no measurement dimension that is further divisible conceptually, i.e., that can be traced back to other, simpler dimensions. Nowhere in his *Metaphysics* do we find the possibility of a measure as the relation of two different qualities.

It is not just an inference from silence if I claim that for Aristotle the dimension to be measured is one-dimensional. Rather, this strict one-dimensionality is required, among other things, by his account of numbers, which seems to be driven by anti-Platonist concerns:[44] very roughly speaking, for Aristotle numbers are the multiple of a basic unit – for example, we do not have a two as such, but a *two* that is the multiple *of* the unit "cup", two cups,[45] or a *two* that is the multiple *of* the unit "second", two seconds, etc.[46] Thus numbers are necessarily tied to the dimension of this basic unit.[47] But if a number is always bound to a particular dimension, it is difficult to combine this number with a number of a different dimension or to assign one and

[43] This is not just a restriction we find in Aristotle, but in Greek treatments of measure more generally and goes back to early near-eastern mathematics, being based on the distinction of one side of a rectangle as length and the other as width. What is special about Aristotle, however, is that in principle he would have the conceptual tools to overcome the restriction, as we will see below.
[44] Cf. Aristotle's criticism of Plato's and Platonic conception of numbers in book M and N of his *Metaphysics*, his *Physics* 204a17-20, Hussey, *Physics*, 78 and 89, and J. Annas, "Aristotle, Number and Time", *Philosophical Quarterly* 25 (1975) who on 99-100 understands what I will call "the dimensionality of numbers" as an anti-Platonist reaction.
[45] This fits with the principle of homogeneity prevalent in the mathematics of Aristotle's time; thus it is not a problem only Aristotle's account of numbers would face. But it is not to be found with Plato and the early Academy against whom Aristotle is arguing in *Metaphysics* M and N. For Aristotle's distinction between numbers with which we count and numbers that we count in the *Physics* cf. my *Motion*, chapter 8.
[46] This is part of Aristotle's attempt to ensure that objects of mathematics do not exist separately from physical particulars; cf. also Mendell, "Aristotle and Mathematics", *Stanford Encyclopaedia of Philosophy* [*Aristotle*] (2004). For Aristotle, as I understand him, the mathematician works with numbers of indivisible units, a *two of monas*, i.e., two *monades*. So also the mathematician works with numbers *of* something, only this time it is not cups but *monades*.
[47] Compare Aristotle's account of the difference between counting ten horses and ten dogs at the end of book IV of his *Physics*, 224a3ff.

the same number to different dimensions (as we do it, for example, when we talk about 30km/h). In order to be able to take different dimensions into account in a single measurement procedure, we need something independent of both dimensions, so that both dimensions can be assigned to it; this is what we do nowadays when we work with a dimensionless number series.[48]

We may assume that Aristotle has to allow for complex dimensions when dealing with surfaces or bodies. However, the way we measure an area for Aristotle seems to be with the help of a smaller surface, and a cube is measured by a smaller cube.[49] Length, surface, and body are thus each a simple dimension for Aristotle.[50]

The strict one-dimensionality can also be seen in Aristotle’s account of the measure of motion just a few lines after the last quotation:

> καὶ δὴ καὶ κίνησιν [εἰδῶσι] τῇ ἁπλῇ κινήσει καὶ τῇ ταχίστῃ (ὀλίγιστον γὰρ αὕτη ἔχει χρόνον)· διὸ ἐν τῇ ἀστρολογίᾳ τὸ τοιοῦτον ἓν ἀρχὴ καὶ μέτρον (τὴν

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[48] Contemporary measurement theory works with such a dimensionless number series whatever our metaphysics of numbers may be.

[49] To put it very roughly, in *Metaphysics* 992a10 ff. and 1085a7 ff. Aristotle distinguishes bodies, surfaces, and lines as three different genera and 1092b30 ff. can be read as excluding the possibility that different genera could be measured by the same measure. See also 1016b17-31 and Aristotle’s criticism of the dimensional confusion he thinks Plato gets into with his assumption of basic triangles forming the elemental bodies in *De Caelo*. These passages suggest that Aristotle would reject calculating the volume of a cube as length times breadth times height as theoretically inadequate, even though he may allow it for practical purposes in calculations. Aristotle is unfortunately not very explicit, and he seems to allow different things for the purposes of calculation and constructions (see, for example, *De Anima* 413a11-20 and *Metaphysics* 996b18-22 for squaring a rectangle, that is, finding an equilateral rectangle equal to an oblong rectangle with the help of a mean proportional between two straight lines). But for Aristotle there does not seem to be an operation like length times length and he does not work with complex measurement units. By contrast, in the *Theaetetus*, 147eff. Plato seems to suggest that an area will be determined as a product, one unit times one unit. And in *Laws* 820a ff., in the context of introducing the idea of the incommensurable, the Athenian stranger claims that usually it is assumed that we can measure a length and a breath against each other.

[50] And while Aristotle seems to allow a comparison of length to surfaces in *Topics* 158b 30ff., what he does in fact compare there are the *ratios* of the areas to the *ratios* of the sides.

> κίνησιν γὰρ ὁμαλὴν ὑποτίθενται καὶ ταχίστην τὴν τοῦ οὐρανοῦ, πρὸς ἣν κρίνουσι τὰς ἄλλας)
>
> And indeed [they know] movement too by the simple movement and the fastest (for this takes least time). Thus also in astronomy, such a 'one' is the starting-point and measure (for they assume the movement of the heavens to be uniform and the fastest, and judge the others by reference to it) (1053a8-12).

Here we are told that in order to measure a movement we take another, simple motion and that is one that needs least time[51] – thus time is the only dimension taken into account for this measurement.[52] Hence, the dimension to be measured cannot be further analysed as the relation of two simpler dimensions (like the relation of time and distance),[53] but is treated as simple in itself. To this it may be objected that what we measure with is another motion, and so something that is in itself complex (covering some distance in a certain time). However, all that is taken into account of this motion with which we measure is the time it takes, as Aristotle states explicitly; and while we could use a motion also to measure a distance,[54] we normally cannot use the very same motion to measure time and distance of another motion simultaneously. The

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[51] By the motion that takes the least time Aristotle probably has in mind that part of the heavenly motion that produces the smallest temporal unit, namely a day (which does not take least time in the sense of coming to a halt then, as the motion of the heavens is eternal, but in the sense that it requires the least duration to get back to its starting point).

[52] Aristotle understands motion and speed here in a way that the fastest is what takes least time (no matter what distance is traversed). Given this understanding, the quickest movement is indeed the smallest quantum available and thus the one most appropriate as a unit. If we thought that Aristotle here understands speed more or less in the way we do as being determined by how much distance is covered in a certain time, the quickest movement would actually be not the adequately small quantum available in the sense specified by Aristotle for we would not notice whether it is a little bit faster or slower. For a discussion of the problems this understanding of the fastest movement raises cf. my *Motion*, chapter 8.

[53] Or the relation of time and angle, in case we deal with angular velocity.

[54] As Aristotle does in his *Physics*, cf. below.

distance the sun covers during a day, for example, plays no role when I use the sun's motion to measure the journey from Athens to Sparta as needing, say, three days.[55] Now we may understand the passage as tracing back one dimension (motion) to another one (time); however, we are tracing back motion to a dimension that is simple in itself, solely time. We will see this prominently taken up in Aristotle's *Physics* where time (and only time) is claimed to be "the measure of motion".[56] For the *Metaphysics* we can conclude that Aristotle's model of measurement for movement is a one-dimensional measure employing one-dimensional units.[57]

Aristotle rounds off his discussion of the homogeneity criterion with a conceptual clarification, warning the listener or reader of a conceptual mess she may get into if she transfers this criterion to the case of numbers without sufficient adjustment:

> οὕτω γὰρ δεῖ λαμβάνειν, ἀλλ' οὐχ ὅτι ἀριθμῶν ἀριθμός· καίτοι ἔδει, εἰ ὁμοίως· ἀλλ' οὐχ ὁμοίως ἀξιοῖ ἀλλ' ὥσπερ εἰ μονάδων μονάδας ἀξιώσειε μέτρον ἀλλὰ μὴ μονάδα· ὁ δ' ἀριθμὸς πλῆθος μονάδων.
>
> We must state the matter so, and not say that the measure of numbers is a number; we ought indeed to say this if we were to use the corresponding form of words, but the claim does not really correspond – it is as if one claimed that the measure of units is units and not a unit; number is a plurality of units" (1053a27-30).

[55] Otherwise we would get into troubles like how to measure a motion if the motion we measure with proceeds in another direction than the one that is measured, or if the motion used for measurement purposes is a different kind of *kinesis*, etc.

[56] Note that Aristotle claims that we measure motion by time and time by motion (for example, in *Physics* 220b31-32) as well as that time is the measure of motion (*Physics* 220b32-221a1, cf. below).

[57] The units have to be one-dimensional as well, for they have to be chosen in accordance with the dimension.

This passage has to be understood against the background of the ancient Greek idea that numbers start with two: the one is not a number itself, because all it does is to specify the unit for what we quantify, and as long as there is just one thing, no counting is going on.[58] Accordingly, Aristotle claims that if we just follow the wording so far, which tells us that we use a length to quantify a bigger length, it seems we need a number to quantify a bigger number. However, if we really understand the sense of what we are thus claiming, we see that a number within the ancient Greek context is necessarily already more than one; so we would quantify a number by a "plurality of units", rather than by one unit (singular) that we need in order to quantify something.

(3) According to the third feature of Aristotle's notion of measurement, a measure has to be in some sense indivisible:

> πανταχοῦ γὰρ τὸ μέτρον ἕν τι ζητοῦσι καὶ ἀδιαίρετον· τοῦτο δὲ τὸ ἁπλοῦν ἢ τῷ ποιῷ ἢ τῷ ποσῷ.
>
> for everywhere they seek the measure to be one and indivisible; and this is what is simple either in quality or quantity (1052b33-35).

In order to understand this passage we should take into account that each measure necessarily requires dealing with quantity *as well as* with quality in some sense, since it is always a quality (what we called "dimension") which is to be quantified by our measurement procedure. Therefore, it would be strange if Aristotle's "indivisibility in quality" referred to dimension in general as discussed above. For dimension is involved in every measurement process; hence, if Aristotle were to use the term "quali-

[58] Not understanding one as a number was common in Ancient Greece, cf. Euclid, *Elements*, book VII, def. 1-2, and T. Heath, *A History of Greek Mathematics* (Oxford 1921) for an overview of what was common in Aristotle's time. For reasons why Plato and Aristotle sometimes treat one as a number nevertheless, cf. my *Motion*, chapter 8.

ty" in order to refer to dimension in the quotation above, we would expect him to talk about indivisibility in quantity *and* quality. This should already warn us that Aristotle may employ a special understanding of quality and quantity in the passage quoted (our understanding of quality and quantity deriving from *Categories* 6 and 8 may not apply).[59]

But what then does Aristotle mean by claiming that the measure is simple "either in quality or quantity"?[60] It seems to me that this question can be answered only if we take the context of this quotation into account. Then we see that indivisibility in quality or quantity is introduced once the question of the appropriate "one" for measuring is discussed (cf. 1052b25-35 and 1088a2-3) – what we today may understand as a discussion of the appropriate scale.[61] Thus these different indivisibilities seem to indicate a further specification of the kinds of magnitude measured. Apparently, Aristotle wants to examine two different kinds of dimensions that need different kinds of units to measure them: Some magnitudes will need units simple in quality as a basis – he also calls them "indivisible in *eidos*" (1087b34-1088a3) or "indivisible for knowledge" (1052a32-33). Other magnitudes need units that are quantitatively simple, which Aristotle explains as indivisible with respect to perception (1088a2-3).[62] In both cases the measurement units have to be treated as indivisible because they are

[59] For example, we will see that certain quantities, like ἀκοή and φωνή, are "indivisible in quality".
[60] L. Castelli, *Problems and Paradigms of Unity, Aristotle's Accounts of the One* (Sankt Augustin 2010), 77 assumes that indivisibility in quality indicates a looser notion of indivisibility, without, however, giving any reasons for this assumption.
[61] We can follow Suppes, *Measurement* and understand a scale "as a class of measurement procedures having the same transformation properties". The modern distinction between ordinal, interval, and ratio scales will, however, not be useful here (for a discussion of different scale types in contemporary measurement theory cf. Suppes, *Measurement*).
[62] For the connection of indivisibility in quality with indivisibility in *eidos*, and of indivisibility in quantity with indivisibility for *aesthesis* cf. also Ross, *Metaphysics*, 472.

serving as the basis in such a way that the measurand can be expressed as the multiple of this basis.

Unfortunately, Aristotle does not explain this difference any further. But he gives a couple of examples for each of them. So let us look at some examples Aristotle gives in order to make this distinction clearer. As an example for indivisibility in quantity we are given a foot:

> τὸ δ’ εἰς ἀδιαίρετα πρὸς τὴν αἴσθησιν θετέον, ὥσπερ εἴρηται ἤδη· ἴσως γὰρ πᾶν συνεχὲς διαιρετόν.
>
> The one [the foot] must be placed among things which are undivided with respect to perception, as has been said already – for every continuum is equally divisible (1053a23-24).

Of course a foot is divisible in principle, since it is a continuum. However, for measurement purposes we can treat it as being indivisible, since for our perception the foot is given as a whole in nature – originally our own foot – and hence as something we perceive as one thing. It is something that cannot be further divided in so far as it is a given whole (cf. 1052a22-23). So the foot is not indivisible as such, but indivisible in so far as it is treated as a unit for measurement purposes.

We use such units that are indivisible with respect to perception in order to quantify continua. These units seem to be called “quantitatively indivisible” because in order to serve as a basis for measuring, these units have to be treated as undivided in their quantity for measurement purposes, for example, the foot is seen as undivided in its extension.

Measurement units that are not indivisible in quantity but rather in quality, Aristotle seems to illustrate, among other things, with the example of a human being, a letter in speech (φωνή), a foot in the sense of a metrical unit (βάσις) or a syllable in rhythm (1087b35-36), and a semitone/quartertone (δίεσις) in music.[63] I would suggest understanding these examples as follows: if we attempt to divide a letter like "ε", epsilon, we will not get any properly articulate sound any longer and thus we are leaving in some sense the realm of speech. Similarly, an attempt to divide a foot when we are scanning metres would lead us outside the business of scanning; to prevent leaving the conceptual basis, we have to take the letter or the foot as an undivided unit. Similarly, if we take the semitone in the diatonic scale as our basic unit,[64] we can of course divide the string on a monochord further, for example, between what we would call E and F, but this will not get us a tone that belongs to our musical scale

[63] Cf. 1052b33-1053a24 and 1087b34-88a11. It is semitones or quartertones depending on which scale is used. Cf. also LSJ for the understanding of βάσις in this very passage. Unfortunately, Aristotle does not specify which of the examples he takes to be units indivisible in quantity, and which ones indivisible in quality and his examples have been interpreted in different ways. Alexander (799, 21) claims a finger to be indivisible in *eidos*, because it is not divided into fingers, but into 'half-fingers' (in the context of measurement, a 'finger' (δακτυλιαῖον) for the Greeks refers to the length of a finger, sixteen of which make a foot). By contrast, Alexander understands the *diesis*, the semi-tone, as indivisible with respect to perception, because it is the smallest perceptible interval (cf. also 368.26). Against this understanding Ross, p. 472 points out that from other passages in the *Metaphysics* we know that 'indivisible in *eidos*' applies to infimae species (999a3) and to genera and species (1016a19), which have a conceptual unity. While I think Ross is right to understand genera and species as indivisible in *eidos*, because of their conceptual unity, Ross also mentions "that which cannot be divided into parts different in kind from the whole" (1014a27), i.e. elements. However, in 1014a27ff. Aristotle does not claim that elements are "indivisible in *eidos*", but only that they are not divisible into what is different in *eidos*, that is, "even if we divide them, they will be divided into homogenous parts (1014a30). In any case, I think it is clear that the examples of the letter, the semitone, of human beings, and horses do not fit what Aristotle has said about units indivisible in quantity in 1053a, namely that they are continua that are in principle further divisible; and also Alexander 369,1 and Ross p. 472 understand man and horse as indivisible in *eidos*.

[64] The example of the *diesis* is complicated by at least three factors: (a) the *diesis* could either refer to empirical harmonics or to mathematical ratio theory; (b) ancient musical theory recognized different harmonies with different *diesis* (see Barker, *Musical Writings*, volume I, 215-216); I have chosen one of them here, the diatonic one, to make the point clear; and (c) musical intervals are not additive measures in the way other measures are, since they are conceived within a system of intervals, which are akin to ratios. But even if this example in the way I spell it out here may make too many assumptions not shared by the reader, I think there are enough other examples in Aristotle that show what he understands by indivisibility in quality.

any longer. Rather, we will interpret such a tone as a badly played E or F.[65] Attempting to divide such a unit indivisible in quality would lead to something that is no longer a part of the respective field (of our musical scale, of scanning, or of speech). By contrast, units indivisible in quantity could in principle be divided and still be used as a unit for measuring the same magnitude, for example, in principle we could use a half foot (understood as a unit for length) as our basic unit to measure the length of our table.[66]

With respect to the example of human beings, I take Aristotle's idea to be the following: let us say we take rational animal as the basis to capture human beings and we want to find out how many human beings there are in a given group. If we now “divide” this basis, for example by leaving out the feature “rational”, we cannot be sure any longer to capture only human beings with our account, and not owls or eagles as well. Thus the concept must not be divided further in order to provide a basic unit that can be used to determine the number of a group. Accordingly, the units used to figure out how many human beings are in a group are units indivisible in *eidos* or to knowledge, because if one tries to divide such a unit further, it will not fit its defi-

[65] In ancient musical theory an octave consists of two tetrachords (each spanning a perfect fourth, in a diatonic scale these are divided into two tones and a *diesis*) and a tone in between the two tetrachords. A *diesis*, although it can be called a semitone, is a little bit less than a half tone in our modern diatonic scale (and a tone minus this *diesis* could also be called a *diesis*, but would be slightly more than a half tone – this would be one explanation why Aristotle refers to two half tones in 1053a 14-18). A *diesis* is the smallest division in the scale; it measures the octave (or more exactly speaking, the two kinds of *diesis* together measure the octave) and can as such not be divided. Mathematically we can divide the half tone further of course, but then we do not have a unit for our octave of sounds any longer, rather we would move from measuring the octave musically to measuring mathematical intervals.

[66] And this is in some sense what the Greek system of subunits does. But this system only makes sense in cases where dividing our one will still give us something of the same kind that we can use as a smaller unit, as when we divide our centimetre and then use a millimetre as out new unit; it does not work in cases where dividing the basic unit will lead us outside the realm of what we want to measure, as when we divide a φωνή, a βάσις or a δίεσις.

nition any longer. Units that are indivisible in quality or *eidos* or knowledge are conceptually indivisible. Unlike those units indivisible with respect to perception, these units indivisible in quality are not continuously extended (and thus further divisible in principle); rather, they are given as something discrete.

The important difference between the two magnitudes pointed out by Aristotle is that with magnitudes measured by units indivisible in quantity we are quantifying something continuous, while with magnitudes measured by units that are qualitatively, i.e., conceptually indivisible, we are quantifying something that is not continuous.[67] Many modern measurement theories are not really concerned with what Aristotle calls “indivisible in quality”, since what is “indivisible in quality” is connected with counting, rather than with measuring. And indeed also Aristotle does not always count it as part of the field of measurable quantities. For the difference between the two scales corresponds to the difference between counting and measuring given by Aristotle in Δ 13, 1020a8-11:

> πλῆθος μὲν οὖν ποσόν τι ἐὰν ἀριθμητὸν ᾖ, μέγεθος δὲ ἂν μετρητὸν ᾖ. λέγεται δὲ πλῆθος μὲν τὸ διαιρετὸν δυνάμει εἰς μὴ συνεχῆ, μέγεθος δὲ τὸ εἰς συνεχῆ·
>
> A quantum is a plurality if it is numerable, a magnitude if it is measurable. ‘Plurality’ means that which is divisible potentially into non-continuous parts, ‘magnitude’ that which is divisible into continuous parts.

This passage shows that Aristotle understands measuring in a strict sense as quantifying a continuous magnitude. Quite often, however, we find a wider notion of meas-

[67] We see here why the understanding of quantity and quality in these passages with the help of the *Categories* may be confusing. Thus it is probably better to think of the division between indivisibility in quantity and quality simply in terms of indivisibility with respect to perception and indivisibility in *eidos*, which we today understand as referring to the difference between continuous and discrete magnitudes.

urement in Aristotle which comprises what we would call measuring and counting; for example, when he calls measuring what we would call the counting of horses in 1088a8-11. And we should not be too surprised by Aristotle's employment of measuring here, since this usage of the term 'measuring' can also be found in Euclid, for example in *Elements* book 7, definition 13 and 14;[68] and it is helped by the fact that the Greek word '*μετρέω*' can mean counting as well as measuring.

(4) Since units indivisible with respect to perception are divisible in principle, we may wonder how we can find an appropriate unit as a basis at all. This question leads to the fourth feature of measurement in Aristotle, that a measure is most precise if nothing can be added or taken away without it being noticed:

> ὅπου μὲν οὖν δοκεῖ μὴ εἶναι ἀφελεῖν ἢ προσθεῖναι, τοῦτο ἀκριβὲς τὸ μέτρον (διὸ τὸ τοῦ ἀριθμοῦ ἀκριβέστατον· τὴν γὰρ μονάδα τιθέασι πάντῃ ἀδιαίρετον)· ἐν δὲ τοῖς ἄλλοις μιμοῦνται τὸ τοιοῦτον· ἀπὸ γὰρ σταδίου καὶ ταλάντου καὶ ἀεὶ τοῦ μείζονος λάθοι ἂν καὶ προστεθέν τι καὶ ἀφαιρεθὲν μᾶλλον ἢ ἀπὸ ἐλάττονος· ὥστε ἀφ' οὗ πρώτου κατὰ τὴν αἴσθησιν μὴ ἐνδέχεται, τοῦτο πάντες ποιοῦνται μέτρον καὶ ὑγρῶν καὶ ξηρῶν καὶ βάρους καὶ μεγέθους·
>
> Where it seems to be impossible to subtract or add something, there the measure is exact (therefore the measure of number is most exact, for one takes the unit (*monas*) to be indivisible in every respect);[69] but in the other cases we imitate this sort of measure. For in the case of a furlong or a talent or of anything

[68] 13. Σύνθετος ἀριθμός ἐστιν ὁ ἀριθμῷ τινι μετρούμενος, "A 'composite number' is that which is *measured* by some number" (my italics).
14. Σύνθετοι δὲ πρὸς ἀλλήλους ἀριθμοί εἰσιν οἱ ἀριθμῷ τινι μετρούμενοι κοινῷ μέτρῳ, "Numbers 'relatively composite' are those which are *measured* by some number as a common *measure*" (my italics).

[69] Elders, *One*, 75 thinks that this passage shows the close connection of "the art of measuring with the theory of ideas" – without, however, giving any reasons for this assumption and without clarifying the passage in any way.

> comparatively large, any addition or subtraction might more easily escape our notice than in the case of something smaller; so that the first thing from which, as far as our perception goes, nothing can be subtracted, all men make the measure, whether of liquids or of solids, whether of weight or of size (1052b35-1053a7, Ross' translation with alterations).

If a unit is of the right size relative to what is to be measured in that nothing can be added or subtracted without it being detected by perception, then that unit is accurate. The difference in precision discussed here is not the same as the difference between indivisibility in quantity and quality; Aristotle contrasts continuous and mathematical units (*monas*) when distinguishing different grades of precision (1053a21ff.), while he distinguishes between continuous and discrete physical entities when talking about the difference between indivisibility in quality and quantity, as we saw above.

Aristotle's talk about more or less exactness of the measure also suggests that to some degree he recognizes the problem of error, that is, that there can be variability or uncertainty in measurement.[70] However, the problem of error is dealt with in terms of more and less accuracy, not in probabilistic terms, as we do in modern science, and it is not built into the calculation by ancient mathematicians.[71]

4. Aristotle's notion of measure as a predecessor of our modern conception

We saw that Aristotle is not thinking of measurement in terms of axiomatization, representation and uniqueness theorems. And we may even think that *Metaphysics* Iota,1

[70] Cf., for example, Suppes, *Measurement*, Section VII "Variability, Thresholds and Errors".

[71] In Hellenistic times, we get upper and lower bounds, e.g., with Aristarchus' account of measurement of distances, which may seem like a predecessor to our idea of a tolerance interval or a margin of error. I owe the point about the problem of error to an anonymous reader for OSAP.

simply gives a general list of measurands and makes some general observations about them. However, it seems to me that this would seriously underestimate this chapter, which does not just provide some remarks about measurement, but rather clarifies the central task of a measure and details its essential features, as we saw in the last section. If we now compare Aristotle's understanding of measurement with the basic structure of measurement presented above, we will see that Aristotle's account is a special case of the latter: Aristotle's account possesses all the basic features we find in a modern context – thus there is enough communality that it can be seen at least in part as a predecessor of modern conceptions of measurement – but these features are determined in Aristotle's *Metaphysics* in a very specific way.[72]

All three features of measurement we postulated as necessary for measurement – dimensionality, the assignment of the measurand to numbers, and measurement units – can be found in the passages of Aristotle's *Metaphysics* referred to above. As for dimensionality and measurement units, things seem to get complicated by the fact that the same term, "*metron*", can be used in Greek for both. That Aristotle is nevertheless aware of the dimensionality of a measure, and distinguishes it clearly from the units used for measuring can be seen from passages like 1052b25-31:

> […] τὸ μέτρον ἑκάστου ἕν, ἐν μήκει, ἐν πλάτει, ἐν βάθει, ἐν βάρει, ἐν τάχει (τὸ γὰρ βάρος καὶ τάχος κοινὸν ἐν τοῖς ἐναντίοις· διττὸν γὰρ ἑκάτερον αὐτῶν, οἷον βάρος τό τε ὁποσηνοῦν ἔχον ῥοπὴν καὶ τὸ ἔχον ὑπεροχὴν ῥοπῆς, καὶ τάχος τό τε ὁποσηνοῦν κίνησιν ἔχον καὶ τὸ ὑπεροχὴν κινήσεως· ἔστι γάρ τι τάχος καὶ τοῦ βραδέος καὶ βάρος τοῦ κουφοτέρου).

[72] Aristotle does not seem to deal with actual representation theorems. However, he deals with measurement in a way that we could formulate as homomorphisms of physical structures into mathematical structures (though under certain provisos as we will see below), even if this presupposes a rather different perspective and uses formal treatment foreign to ancient Greek thought.

> […] the measure of each is a one – in length, in breadth, in depth, in weight, in speed (for 'weight' and 'speed' are common to both contraries; for each of them has two meanings – 'weight' means both that which has any amount of heaviness and that which has an excess of heaviness, and 'speed' both that which has any amount of movement and that which has an excess of movement; for even the slow has a certain speed and the (comparatively) light a certain weight).

In this passage Aristotle expressly differentiates two meanings of terms like “weight” and “speed”: they signify on the one hand what we would call the dimension of certain magnitudes (weight and speed as *dimensions* are also attributed to light and slow things, and thus to “both contraries”, the light and the heavy, the slow and the fast) and on the other hand the different amount or grades of these magnitudes. The latter tells us the times a unit is contained in a measurand (heavy and speedy is called what has a considerable degree of weight and speed, that is, what contains the basic unit multiple times).[73]

Aristotle’s awareness of the need to determine the *dimension* to be measured is also evident in his claim that the measure has to be of the same kind as the thing measured. And Aristotle presents a couple of different *measurement units* throughout his measurement discussion, a foot or a semitone, for example,[74] and thus shows that he takes into account that different basic units of measurement have to be found for

[73] βάρος and τάχος are genuinely ambiguous in Greek (τάχος can mean swiftness as well as speed, βάρος heaviness as well as weight) in a way that is difficult to mirror exactly in English.
[74] Cf. 1053a22 and 1053a12.

measurands of different dimensions – the semitone quantifies the dimension of pitch; the foot quantifies the dimension of length.[75]

I should, however add, that the ancient understanding of measurement units is slightly different than our modern one: we saw that for Aristotle a unit has to be understood as indivisible (in some sense) so that the measurand can be seen as a multiple of this basic one. By contrast, there is no expectation of indivisibility in modern theories of measurement. Furthermore, the ancient Greeks have two words for units: μονάς and μέτρον: *monas* is primarily tied to numbers – it is a unit of numbers in the sense that, for example, the number 5 consists of 5 *monades*.[76] By contrast, *metron* can be a unit of all kinds of things, a centimetre, a kilogram, etc.

As for the last of the three features we postulated as necessary for measurement from a contemporary background, the assignment of the measurand to numbers, Aristotle's remark that number means a measured plurality (1088a5) shows that the *measurand is* indeed *assigned to numbers*. This assignment makes it possible for the question, "How much (many) of the measurand in question are we dealing with?" to be answered.[77]

However, it is with this feature that we also see clearly a peculiarity of Aristotle that we do not find in modern accounts: assigning the measurand to numbers is not

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[75] The fact that Aristotle discusses different basic units of length just shows that due to problems in dealing with incommensurables, etc., he has to assume different basic units for the treatment of length, one for the side of the square, another one for the diagonal of the square.

[76] In this sense of unit, counting obviously involves units, cf. Euclid book VII, definition 2: "A number is a multitude composed of units (*monades*)." *Monas* is also defined as a point without location.

[77] Aristotle's solution to deal with the application problem for mathematics lies in understanding perceptible objects qua mathematical objects, cf. Mendell, *Aristotle*.

an operation separate from determining the dimension and the measurement unit.[78] For Aristotle understands numbers as necessarily always dimensional, that is, a two is always the two of two cups, or two turtles, etc. Numbers just are the multiple of a basic one that has a certain dimension, for example, if I count the cups in my kitchen, "cup" is the one of which the number I end up with in my counting process is a multiple.[79] Within the realm of natural philosophy, the dimension of the unit will always be a perceptible one and numbers are the quantification of a plurality of physical objects. By contrast, the mathematician deals with the "dimension" "*monas*" so to say, she also deals with numbers *of* something, only the something in this case is a mathematical *monas*, not physical aspects or things, two *monades*; but this is not what the natural philosopher interested in measurement of the physical world does.[80] Given this account of numbers as the multiple of a basic perceptible one, things in the perceptible world, which we count or measure, and numbers can be understood to be directly connected, that is, they are not members of different realms, as they may be seen on a Platonic picture where numbers belong to the intelligible realm, while physical things belong to the clearly distinguished sensible realm. While on a Platonic picture the question thus arises how to connect the perceptible things with the intelli-

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[78] If we understand Aristotle's theory of measurement as providing a homomorphism to his theory of number, then another peculiarity is that the homomorphism is an isomorphism (concatentation just *is* addition).

[79] Cf. J. Annas, "Die Gegenstände der Mathematik bei Aristoteles", in A. Graeser, *Mathematics and Metaphysics in Aristotle* (Bern/Stuttgart 1987), 142-143, where she interprets *Metaphysics* M 1-3 as showing that mathematical objects are not *ousiai*, but merely *onta*, for *ousiai* exist separately on their own, and also J. Cleary, *Aristotle and Mathematics. Aporetic Method in Cosmology and Metaphysics* (Leiden 1995), 372. For the thought that mathematical objects and hence numbers must have some sort of matter with Aristotle cf. Hussey, *Physics*, 183, and Cleary, p.375. Understanding numbers as natural numbers and hence as a logical series like, e.g., Peano did, is not possible within Aristotle's conceptual framework. Cf. also Henry Mendell, "Plato by the Numbers", in *Logos and Language*, ed. D. Follesdal and J. Woods [*Plato*] (London 2009), 137.

[80] The mathematicians may also abstract a two from the "two cups", but again this is not how for Aristotle the natural philosopher should think of numbers.

gible numbers, with Aristotle, by contrast, we do not need an additional operation of assigning the measurand to numbers.

This understanding of numbers also makes dealing with what we call fractions more difficult than it is anyway in Greek mathematics. While Greeks tended to avoid fractions, they would have to deal with things like 3/5 of the land of Sparta for purposes of land surveying, etc. But Aristotle's account cannot really handle these fractions: if your basic unit is a cup, then 5/16 does not seem to be a proper number. There were ways to deal with this problem – for example, an adequate subunit can be used as the new unit.[81] But an adequate subunit may not always be available. In the context of baking, the subunit of the cup may be the tablespoon, but in other contexts there may be no suitable subunit for the cup. If fractions are not possible, then the comparison of some different lengths, say, with the help of a single basic unit, will also not be possible, as this method might lead to fractions. Rather we will need different "ones" to measure such different lengths.[82]

5. The measure of motion in Aristotle's *Physics*

Let us now turn to Aristotle's account of the measure of motion in his *Physics*. His account there will be sketched against the background of Zeno's paradoxes of motion for two reasons: first, Zeno's paradoxes challenged the intelligibility of motion fundamentally and one main way to prove the intelligibility of motion against Zeno is to show that a motion can be measured – if motion can be quantified it can be related to numbers and thus grasped as something intelligible.[83] So it is only fitting that Aristo-

[81] Cf. Mendell, *Plato*.

[82] Our modern treatment of numbers and quantities does not face these problems.

[83] Cf. 1053a7-8: καὶ τότ' οἴονται εἰδέναι τὸ ποσόν, ὅταν εἰδῶσι διὰ τούτου τοῦ μέτρου ("they think they *know* the quantity when they know it by means of this [a most accurate] measure", my italics).

tle employs a measure of motion as one way to counter Zeno's paradoxes, as we will see below. Secondly, and more importantly for our purposes, Aristotle's treatment of Zeno is one of the most prominent places (though by no means the only one) where Aristotle clearly uses a complex measure – and he needs to use a complex measure in order to solve one aspect of the paradoxes. The discussion of Zeno thus also makes obvious that Aristotle really has to lose something if he sticks with the measurement account from *Metaphysics* Iota and what he gains with a complex measure of motion.

Of a motion we can either measure the time it takes, its duration, or the distance it covers. Or we measure its speed, that is, how much distance it covers in a certain time. The latter gives us not only a measure of duration or distance, but a real measure of motion in its complexity, its relation of time and space.[84] Thus measuring the speed of a motion allows us to determine the quantity of motion in its full sense (not just either its temporal or its spatial aspect). And it is this measure of motion that we will investigate in Aristotle in the following.

5.1. Aristotle's usage of the measure of motion

How then does Aristotle conceptualise the measure of motion in his *Physics*? We will see that in order to answer this question, a distinction has to be made between Aristotle's *explicit concept* of the measure of motion and the measure of motion he *implicitly uses* when fighting Zeno's paradoxes, especially the dichotomy and the Achilles paradox. Let me give a brief sketch of Zeno's paradoxes so that we can see how Aris-

[84] I will talk about the relation of time and space, but some scholars may want to think of it as the relation between time and spatial distance. I argue in "Space in Ancient Times: From the Presocratics to Aristotle", in: "Space", *Oxford Philosophical Concepts*, ed. Andrew Janiak (Oxford, 2017) that contrary to common opinion in the literature, Aristotle does develop a notion of space, and not just of place, in *Physics* IV, 1-5. But for our purposes here it is enough if the reader who objects to talking about space in an Aristotelian context reads "distance" instead.

totle's understanding of measurement can react to Zeno. Since for Aristotle the Achilles paradox is basically a variation of the dichotomy paradox,[85] I will restrict myself to this first paradox for the sake of simplicity.[86]

5.1.1 Zeno's Dichotomy Paradox

Zeno's dichotomy paradox claims that if a runner wants to cover a certain finite distance ΓΔ in a finite time ZH, he first has to cover half of this distance. For the continuation of this paradox, there are two versions, sometimes called "progressive" and "regressive": In the progressive form, the problem arises that the runner again has to cover first half of the remaining distance and then again the first half of the still remaining distance etc. Thus it seems that he will have to pass an *infinite* number of spatial pieces before reaching the end which does not seem to be possible in a *finite* time. The regressive version seems to intensify this paradox as it shows that the runner must have already gone through an infinite number of spatial pieces to cover even the first half of the distance: [87]

Γ Ξ M   K   Δ distance

— 3—2———1—————

Z   H time

——————————————

[85] As he claims in 239b14-26.
[86] For Aristotle's account of the paradox see *Physics* 239b9-14, 233a21-26, and 263a4-11. Different reconstructions of this paradox have been given. I defend mine in *Motion,* chapter 3.
[87] In the following diagram, "1", "2", etc. indicates the first, the second, etc. division of the distance. To make comparison easier with a passage from *Physics* VI 2 that is discussed in section 5.1.2 and 5.1.3, I have used Greek letters here following the lettering Aristotle uses in that passage.

So before the runner can cover the distance ΓΔ in a finite time ZH, she must have covered already half of this distance, ΓK, and before that half of this half, ΓM, etc., *ad infinitum,* which seems to imply that she cannot even get started.

No matter which form of the paradox we choose, the progressive or the regressive one, this paradox raises two logical problems: (1) By covering a *finite* distance a runner has to get over an *infinite* number of spatial pieces which, it seems, cannot be done. (2) This *infinite number of spatial pieces* shall be covered in a *finite time,* which seems to be impossible. The first problem I want to call the continuum problem since it arises as a problem for all magnitudes which we would call "continua". The second one is the problem specific for motion and can thus be called the problem of motion. It is this second problem to which we will get an answer from the passage in Aristotle's *Physics* I will investigate in a minute.

The paradox of motion arises due to the fact that the infinitely many parts, which something moving seems to be forced to cover, have to be traversed in a finite time. No matter how the infinity of the parts is understood, the paradox of "not-moving", as Aristotle calls it in 239b11-12, is that an infinite distance has to be covered in a finite time. Thus the time available always seems to be too short to cover something infinite, whilst in an infinite time it may be possible to traverse something infinite. This difference in the characterisation of time and distance as presented in the paradox seems to prevent that they can be combined in order to give an account of motion.

5.1.2 The measure of motion used by Aristotle

The first time Aristotle reacts to this paradox is in *Physics* book VI, a book where he aims to show that motion, time, and space are continua, and what that means in *his* theoretical framework.[88] So let us have a look at the sixth book of Aristotle's *Physics*. In the first chapter Aristotle tries to establish that a continuum cannot be thought of as being made up of extensionless points which are per se not divisible. Rather, parts of continua have to be extended. Now, while being extended, these parts could still be indivisible. The second chapter of book VI is then meant to demonstrate that the continua time and space are as divisible as one likes, and thus not atomistic, with the help of a comparison between two motions of different speed. According to Aristotle, it is this very proof in chapter two comparing two different motions demonstrating time and space to be divisible as much as one likes that can be used to show Zeno to be wrong. Let us look at the most important parts of this second chapter (232a24-233a34):

> Ἐπεὶ δὲ πᾶν μέγεθος εἰς μεγέθη διαιρετόν […], ἀνάγκη τὸ θᾶττον ἐν τῷ ἴσῳ χρόνῳ μεῖζον καὶ ἐν τῷ ἐλάττονι ἴσον καὶ ἐν τῷ ἐλάττονι πλεῖον κινεῖθαι, καθάπερ ὁρίζονταί τινες τὸ θᾶττον. […]
>
> λέγω δὲ συνεχὲς τὸ διαιρετὸν εἰς αἰεὶ διαιρετά· τούτου γὰρ ὑποκειμένου τοῦ συνεχοῦς, ἀνάγκη συνεχῆ εἶναι τὸν χρόνον. ἐπεὶ γὰρ δέδεικται ὅτι τὸ θᾶττον ἐν ἐλάττονι χρόνῳ δίεισιν τὸ ἴσον, ἔστω τὸ μὲν ἐφ’ ᾧ Α θᾶττον, τὸ δ’ ἐφ’ ᾧ Β βραδύτερον, καὶ κεκινήσθω τὸ βραδύτερον τὸ ἐφ’ ᾧ ΓΔ μέγεθος ἐν τῷ ΖΗ χρόνῳ.[89] δῆλον τοίνυν ὅτι τὸ θᾶττον ἐν ἐλάττονι τούτου κινήσεται τὸ αὐτὸ μέγεθος· καὶ κεκινήσθω ἐν τῷ ΖΘ. πάλιν δ’ ἐπεὶ τὸ θᾶττον ἐν τῷ ΖΘ διελήλυθεν τὴν ὅλην τὴν ΓΔ, τὸ βραδύτερον ἐν τῷ αὐτῷ χρόνῳ τὴν

[88] Before Aristotle the term we translate as 'continuum', the Greek '*syneches*', was prominently used by Parmenides but with quite different implications, cf. my 'Parmenides on "being *suneches*"', *Philosophical Inquiry,* Festschrift for Alexander Mourelatos (forthcoming 2016).
[89] The following lines may be easier to grasp with the help of the diagram provided below.

ἐλάττω δίεισιν· ἔστω οὖν ἐφ’ ἧς ΓΚ. ἐπεὶ δὲ τὸ βραδύτερον τὸ Β ἐν τῷ ΖΘ χρόνῳ τὴν ΓΚ διελήλυθεν, τὸ θᾶττον ἐν ἐλάττονι δίεισιν, ὥστε πάλιν διαιρεθήσεται ὁ ΖΘ χρόνος. τούτου δὲ διαιρουμένου καὶ τὸ ΓΚ μέγεθος διαιρεθήσεται κατὰ τὸν αὐτὸν λόγον. εἰ δὲ τὸ μέγεθος, καὶ ὁ χρόνος. […] διαιρήσει γὰρ τὸ μὲν θᾶττον τὸν χρόνον, τὸ δὲ βραδύτερον τὸ μῆκος. εἰ οὖν αἰεὶ μὲν ἀντιστρέφειν ἀληθές, ἀντιστρεφομένου δὲ αἰεὶ γίγνεται διαίρεσις, φανερὸν ὅτι πᾶς χρόνος ἔσται συνεχής. ἅμα δὲ δῆλον καὶ ὅτι μέγεθος ἅπαν ἐστὶ συνεχές· τὰς αὐτὰς γὰρ καὶ τὰς ἴσας διαιρέσεις ὁ χρόνος διαιρεῖται καὶ τὸ μέγεθος. […] καὶ εἰ ὁποτερονοῦν ἄπειρον, καὶ θάτερον, καὶ ὡς θάτερον, καὶ θάτερον […]

διὸ καὶ ὁ Ζήνωνος λόγος ψεῦδος λαμβάνει τὸ μὴ ἐνδέχεσθαι τὰ ἄπειρα διελθεῖν ἢ ἅψασθαι τῶν ἀπείρων καθ’ ἕκαστον ἐν πεπερασμένῳ χρόνῳ. διχῶς γὰρ λέγεται καὶ τὸ μῆκος καὶ ὁ χρόνος ἄπειρον, καὶ ὅλως πᾶν τὸ συνεχές, ἤτοι κατὰ διαίρεσιν ἢ τοῖς ἐσχάτοις. τῶν μὲν οὖν κατὰ τὸ ποσὸν ἀπείρων οὐκ ἐνδέχεται ἅψασθαι ἐν πεπερασμένῳ χρόνῳ, τῶν δὲ κατὰ διαίρεσιν ἐνδέχεται· καὶ γὰρ αὐτὸς ὁ χρόνος οὕτως ἄπειρος. ὥστε ἐν τῷ ἀπείρῳ καὶ οὐκ ἐν τῷ πεπερασμένῳ συμβαίνει διιέναι τὸ ἄπειρον, καὶ ἅπτεσθαι τῶν ἀπείρων τοῖς ἀπείροις, οὐ τοῖς πεπερασμένοις. οὔτε δὴ τὸ ἄπειρον οἷόν τε ἐν πεπερασμένῳ χρόνῳ διελθεῖν, οὔτ’ ἐν ἀπείρῳ τὸ πεπερασμένον· ἀλλ’ ἐάν τε ὁ χρόνος ἄπειρος ᾖ, καὶ τὸ μέγεθος ἔσται ἄπειρον, ἐάν τε τὸ μέγεθος, καὶ ὁ χρόνος.

And since every magnitude is divisible into magnitudes […] it necessarily follows that the faster of two things traverses a greater magnitude in an equal time, an equal magnitude in less time, and even a greater magnitude in less time, in conformity with the definition sometimes given of the faster. […]

By continuous I mean that which is divisible into divisibles that are always further divisible: and if we take this as the definition of continuous, it follows necessarily that time is continuous. For since it has been shown that the faster will pass over an equal magnitude in less time than the slower, suppose that A is faster and B slower, and that the slower has traversed the magnitude ΓΔ in the time ZH. Now it is clear that the faster will traverse the same magnitude in less time than this: let us say in the time ZΘ. Again, since the faster has passed over the whole ΓΔ in the time ZΘ, the slower will in the same time pass over ΓK, say, which is less than ΓΔ. And since B, the slower, has passed over ΓK in the time ZΘ, the faster will pass over it in less time: so that the time ZΘ will again be divided. And if this is divided the magnitude ΓK will also be divided according to the same rule: and again, if the magnitude is divided, the time will also be divided. […] the faster will divide the time and the slower will divide the length. If, then, this alternation always holds good, and at every turn involves a division, it is evident that all time must be continuous. And at the same time it is clear that all magnitude is also continuous; for the divisions of which time and magnitude respectively are susceptible are the same and equal. […] And if either is infinite, so is the other, and the one is so in the same way as the other […]

Hence Zeno's argument makes a false assumption in asserting that it is impossible for a thing to pass over or severally to come in contact with infinite things in a finite time. For there are two senses in which length and time and generally anything continuous are called 'infinite': they are called so either in respect of divisibility or in respect of their extremities. So while a thing in a finite time cannot come in contact with things quantitatively infinite, it can

> come in contact with things infinite in respect of divisibility: for in this sense the time itself is also infinite; and so we find that the time occupied by the passage over the infinite is not a finite but an infinite time, and the contact with the infinites is made by means of moments not finite but infinite in number. The passage over the infinite, then, cannot occupy a finite time, and the passage over the finite cannot occupy an infinite time: if the time is infinite the magnitude must be infinite also, and if the magnitude is infinite, so also is the time" (translation by Hardie and Gaye with alterations).

The first paragraph of the second chapter states that (a) what is faster moves in the same time over more magnitude,[90] and (b) in less time over the same or (c) even more magnitude.[91] In the second paragraph a faster and a slower motion are compared. And it turns out that if we compare the two, the slower always divides the magnitude while the faster divides time: The slower has traversed the magnitude ΓΔ in the time ZH; the quicker – let us assume it moves twice as fast as the slower one – will then traverse the same magnitude in half the time:

Γ Ξ M K Δ distance

—–3—2——– 1——————

[90] Since Aristotle wants to give a general account of a *kinêsis* being faster than another, he literally only talks about the faster moving "in the same time over more", etc. Hardie and Gaye add that the faster is moving over more "magnitude"; for our purposes we can focus on locomotion and understand the more that is covered as more "distance" or "spatial extension".

[91] Cf. H. Mendell, "Two traces of a Two-Step Eudoxian Proportion Theory in Aristotle: a Tale of Definitions in Aristotle, with a Moral", *Archive for History of Exact Sciences* 61 [*Proportion Theory*] (2007), pp. 3-37 for a discussion of these three claims, their relations, their status, and the problems they raise as possible definitions of "being faster". Mendell concludes that what we get here are "three necessary conditions for showing that A is faster than B under different conditions" (p. 22).

Z N L Θ H time

——3—2——— 1——————

The slower body will divide the distance – in time ZH the faster has covered distance ΓΔ, but the slower has only covered ΓK. The faster body will divide the time – in order to cover ΓK the slower will need the time ZH, while the faster will only need time ZΘ.

This comparison of two bodies moving with different speed shows that time and distance are divisible as much as one likes. There is a constant rule of division which stays the same as long as the speed stays the same.[92] While the rule of division stays the same, the time and distance investigated vary, since both are always further divided according to the different speeds of the things in motion. The current size of the relata, that is, of the respective pieces of time and space, is not relevant (we start out looking at the magnitude of size ΓΔ, but then we turn to the magnitude of half the size, of size ΓK, etc.).

This example of two bodies moving with different speed leads Aristotle to one of his aims of argumentation, namely that time and distance can be divided as much as one likes, there are no indivisible parts. The way he proves this, however, proves even more: he shows that with respect to motion, time and distance are divided in the

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[92] Mendell, *Proportion Theory* 12 n.16 points out that throughout *Physics* Z Aristotle does not need constant speed; rather the weaker notion of uniform periodicity (that is, if "a movement over a distance divided into equal distances (periods), every equal distance traveled is traveled in an equal time") is sufficient. I think this is right for the passages Mendell explicitly names as examples (233b4–5 and 233b26–7). However, in order to solve Zeno's motion paradox, where we deal with infinite divisibility, it seems to me that mere uniform periodicity would not guarantee that the ratio or rule of division would always stay the same, no matter where we cut (moreover, the usage of "*aiei*" (always) in our passage suggests that Aristotle uses uniform motion here). Also T. Heath, *Mathematics in Aristotle* (Oxford 1949), 129 assumes constant speed. And in book IV, 222b30-223a4 Aristotle explicitly talks about uniform motion (ὁμαλὴν κίνησιν) when comparing two motions of different speed.

very same way, "the divisions of time and of magnitude will be the same"; they are divided *kata ton auton logon*, "according to the same ratio" or "according to the same rule".[93] When comparing different speeds, time and distance can vary in getting as small as one likes, we can always divide them further – this is what Aristotle wants to prove. In addition, he thereby shows that the only thing that stays constant is the specific relation of time and distance of each motion. For example, we may think that the slower body will cover one unit of space in two units of time, while the faster body will cover one unit of space in one unit of time; then the relationship of one spatial unit per two temporal ones for the slower motion and the relationship of one spatial unit per one temporal unit for the faster motion will stay the same, no matter how far we divide time and distance. If we want to measure the speed of the two motions, we have to measure these relations. It is this very relation that provides Aristotle's main argument against what we called the 'motion problem' of Zeno's paradox, as we will see in the next section.

5.1.3 Implications for solving Zeno's Dichotomy Paradox

In order to explain why the relation between time and distance that Aristotle employs helps to solve the motion problem we have to remember that Zeno's paradox showed motion to require infinitely many spatial parts to be passed in a finite time – an impossible undertaking, since a finite time seems to be too short to cover something infinite. Accordingly, the finite time cannot be correlated with the infinitely many spatial parts.

The idea that motion requires infinitely many spatial parts to be passed in a finite time rests, however, on an implicit assumption that Aristotle in this second chap-

[93] For *logos* qua "rule" cf. LSJ *logos* 2.d: "rule, principle, law, as embodying the result of λογισμός", which we find as early as Pindar, and also in Plato and Aristotle.

ter of book VI proves wrong, namely that each division of the distance covered, which leads to infinitely many parts, *does not entail* an equal division of time. Zeno does not divide the time of a motion whenever he divides its space – the distance to be covered in the paradox gets divided *ad infinitum* and so each part gets smaller and smaller, but the time is not divided and thus stays finite. Given this *ad infinitum* division of the distance, it seems that the infinitely many parts could at best be covered in an infinitely extended time. Such an inference can, however, only be drawn if we do not sufficiently distinguish between infinity of division and infinity of extension; the former is employed with respect to distance, but the latter seems to be required with respect to time. But even if the runner of the dichotomy paradox had indeed an infinitely extended time available (if, for example, we asked immortal Apollo to do the run for us), we would still be caught in a paradox, since the infinitely many spatial parts are the parts of a finitely extended distance, which could not be paired with an infinitely extended time.

So the problem really is that Zeno does not divide the time of a motion whenever he divides the distance covered. Otherwise, Zeno would have needed to put the division somewhat like this: first the runner has to cover half the racing course in half the time. But before he can do that, he first has to cover half of this half, that is, a quarter of the race course, in half of the half time, that is, in a quarter of the time, and so on *ad infinitum*:

Γ Ξ M K Δ distance

——3—2———1——————

Z N L Θ H time

——3—2——— 1——————

We see that this is exactly the relation Aristotle employed when comparing two motions of different speed – each division of the distance covered by a motion leads to an equal division of the time taken. The fact that distance and time have to be divided by the same rule frees us from the problem that the infinitely many spatial parts cannot be related to a finite time (or the finite extension of the distance cannot be related to an infinitely extended time, respectively). Time and spatial magnitude are infinite in the very same sense – they are infinitely divisible.

Showing that the same kind of infinity has to be ascribed to both time and distance can thus solve the motion problem – as Aristotle demonstrates in the last paragraph of the long passage just quoted:

> διὸ καὶ ὁ Ζήνωνος λόγος ψεῦδος λαμβάνει τὸ μὴ ἐνδέχεσθαι τὰ ἄπειρα διελθεῖν ἢ ἅψασθαι τῶν ἀπείρων καθ’ ἕκαστον ἐν πεπερασμένῳ χρόνῳ. διχῶς γὰρ λέγεται καὶ τὸ μῆκος καὶ ὁ χρόνος ἄπειρον, καὶ ὅλως πᾶν τὸ συνεχές, ἤτοι κατὰ διαίρεσιν ἢ τοῖς ἐσχάτοις. τῶν μὲν οὖν κατὰ τὸ ποσὸν ἀπείρων οὐκ ἐνδέχεται ἅψασθαι ἐν πεπερασμένῳ χρόνῳ, τῶν δὲ κατὰ διαίρεσιν ἐνδέχεται· καὶ γὰρ αὐτὸς ὁ χρόνος οὕτως ἄπειρος. ὥστε ἐν τῷ ἀπείρῳ καὶ οὐκ ἐν τῷ πεπερασμένῳ συμβαίνει διιέναι τὸ ἄπειρον
>
> Hence Zeno's argument makes a false assumption in asserting that it is impossible for a thing to pass over or severally to come in contact with infinite things in a finite time. For there are two senses in which length and time and generally anything continuous are called 'infinite': they are called so either in respect of divisibility or in respect of their extremities. So while a thing in a

> finite time cannot come in contact with things quantitatively infinite, it can come in contact with things infinite in respect of divisibility: for in this sense the time itself is also infinite: and so we find that the time occupied by the passage over the infinite is not a finite but an infinite time (233a21-30).

Both time and distance covered are infinite in the sense of being divisible *ad infinitum* – a feature of their continuous structure. According to Aristotle, this very structure allows us to connect time and distance, for example when conceptualising speed. If we look at Plato's *Timaeus* we see that such a connection of time and space is not just a matter of course – there it is unclear how two entities of such different characteristics and ontological status as time and space, Plato's *chôra*, can be consistently combined in such a way as to determine speed.[94]

And Aristotle goes even further in his comparison of different speeds: not only *can* time and distance be divided in the very same way, since they are both infinitely divisible, they also *have to be divided according to the same ratio or rule*, since the one will be divided according to the other with regard to a certain motion. Every division of the distance covered leads necessarily to a division of the time taken, and vice versa (232b26-233a5).[95] Accordingly, it cannot be the case that the one is infinite while the other is not (cf. also 233a34-b15).

[94] For the problems Plato's *Timaeus* raises for an account of speed see my *Motion*, chapter 6; for a discussion of why I think Plato's *chôra* is closer to a notion of space than of matter, see my "A Likely Account of Necessity, Plato's Receptacle as a Physical and Metaphysical Basis of Space", *Journal of the History of Philosophy* (2012), 159-195.

[95] However, time and distance are only dependent on each with respect to the specific motion in question. Different motions require different relationships between them, as we saw with the two different motions Aristotle is comparing in his *Physics* – in the way sketched above, the faster motion was characterized by the relationship of one unit of space per one unit of time, while the slower motion was characterized by the relationship of one unit of space per two units of time.

Although the dichotomy paradox does not explicitly deal with measurement, it deals with the quantitative aspect of motion and thus with an area falling under the domain of a measure. And the relationship of time and distance we saw Aristotle employ in his reaction to this paradox is also what is required for measuring motion, as when we measure motion as km/h.

5.2 Aristotle's explicit conception of the measure of motion

When dealing with Zeno's paradoxes, Aristotle obviously uses the relation of time and distance to determine and measure motion. And at the beginning of the long *Physics* passage quote, he understands faster in a way that also takes into account time and distance: "the faster of two things traverses a greater magnitude in an equal time, an equal magnitude in less time, and even a greater magnitude in less time, in conformity with the definition sometimes given of the faster". So he (and those people who gave this definition of faster)[96] clearly take distance and time into account when talking about being faster. Thus we may think that Aristotle in fact understands distance as a measure of motion in determining whether something is faster. And there are other passages where we read that motion is long if the distance is, and distance, if motion is.[97] If we look at Aristotle's explicit definition of the measure of motion, however, we see that he never explicitly calls distance a measure of motion and, more importantly, distance is not integrated into a complex measure that would allow

[96] See below.

[97] 220b28-31: "καὶ μετροῦμεν καὶ τὸ μέγεθος τῇ. κινήσει καὶ τὴν κίνησιν τῷ μεγέθει· πολλὴν γὰρ εἶναί
φαμεν τὴν ὁδόν, ἂν ἡ πορεία πολλή, καὶ ταύτην πολλήν, ἂν ἡ ὁδὸς [ᾖ] πολλή·" ("And we measure both the distance by the movement and the movement by the distance; for we say that the road is long, if the journey is long, and that this is long, if the road is long"). However, it is not speed that is measured here, how fast a motion is, but the length of motion.

us to measure how fast a motion is. For the measure of motion Aristotle explicitly only takes time into account:

> ἐπεὶ δ’ ἐστὶν ὁ χρόνος μέτρον κινήσεως καὶ τοῦ κινεῖσθαι
>
> Time is a measure of motion and of being moved (220b32-221a1).

And more precisely,

> τοῦτο γάρ ἐστιν ὁ χρόνος, ἀριθμὸς κινήσεως κατὰ τὸ πρότερον καὶ ὕστερον
>
> For this is time – number of motion in respect of before and after (219b1-2).[98]

Time as the number of motion has to be understood as the number resulting from measurement, that is, time serves for quantifying one aspect of motion – it tells us how long a motion has lasted. But is time really enough to measure motion? Does time on its own actually fulfil the four criteria of a measure named in the *Metaphysics*?

The first criterion, to quantify motion, seems to be fulfilled in some sense by time alone. As for the units of measurement and their indivisibility either in quantity or quality, which we took to be Aristotle’s third criterion, it seems to be clear that Aristotle uses units indivisible in quantity, that is, units indivisible for perception, like days.[99] As for precision, the fourth criterion, for our eyes temporal units were not ter-

[98] While most scholars do not think that Aristotle makes a distinction between time as number and time as measure of motion in the *Physics*, some have taken these two accounts of time, as number and as measure, to be significantly different, cf., for example, P. Conen, *Die Zeittheorie des Aristoteles* (München 1964) and R. Sorabji, *Time, Creation and the Continuum. Theories in antiquity and the early middle ages* (London 1983). Coope, *Time*, even argues that time is defined only as a number, not as a measure. For a full discussion of the position of these scholars and an account why I think we have to understand time qua number and time qua measure as expressing the same notion see my *Motion*, chapter 8.

[99] We may originally have assumed that just as the concept “rational animal” is indivisible, so is the concept of motion; hence we would work with units indivisible in

ribly precise in ancient Greece, given the ancient measurement tools (mainly sun and water clocks).

What is really problematic, however, is the homogeneity criterion, Aristotle's second criterion, since time seems to be a dimension different from motion. Thus, time would be a measure homogenous with the motion to be measured in case we wanted to measure only the duration of a motion. But we have seen above that Aristotle does not only try to determine the duration of a motion, but its being faster and slower. Aristotle's interest in the speed of motion is confirmed by a passage in the context of his explicit definition of time as the measure of motion:

> φανερὸν ὅτι πᾶσα μεταβολὴ καὶ ἅπαν τὸ κινούμενον ἐν χρόνῳ. τὸ γὰρ θᾶττον καὶ βραδύτερον κατὰ πᾶσάν ἐστιν μεταβολήν (ἐν πᾶσι γὰρ οὕτω φαίνεται)· λέγω δὲ θᾶττον κινεῖσθαι τὸ πρότερον μεταβάλλον εἰς τὸ ὑποκείμενον κατὰ τὸ αὐτὸ διάστημα καὶ ὁμαλὴν κίνησιν κινούμενον
> […] it is evident that every change and everything that moves is in time; for the faster and slower exist in reference to all change, since it is found in every instance. I say that moves faster that changes before another into the condition in question, when it moves over the same interval and with a regular movement (222b30-223a2).

Similarly, in the long passage from the *Physics* quoted above we saw that Aristotle determines being faster as moving in less time over the same distance or in the same time over more distance. Measuring speed, measuring that something is faster or

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eidos. However, with the help of such units we could only find out what belongs to the concept of motion, that is, what is a motion and what is not a motion, and thus count the number of motions in a given context and probably of moving things. But we could not quantify an individual motion.

slower, thus cannot be reduced to measuring the duration of a motion. And a simple example can confirm that: Let us assume we want to compare who is faster, Achilles or the tortoise, and we suppose that Achilles runs 100 metres in 10 seconds and the tortoise moves 2 metres in 2 seconds. If we thought we could measure how fast their respective motions are by measuring only the time each motion takes without taking into account the distance covered, it would mean that the tortoise is faster than Achilles: the tortoise finishes her run in 2 seconds and thus much earlier than Achilles, who takes 10 seconds for his course; if we disregard the distance we cannot account for the fact that Achilles covers a much bigger distance during this 10 seconds than the tortoise does in her 2 seconds, and that it would take her 100 seconds to cover the same distance.

This example only stresses a point obvious from the quotation of the *Physics*, namely that distance is an essential element in order to account for speed. Accordingly, time is only understood as "something *of* motion" ("ὁ χρόνος […] τί τῆς κινήσεώς ἐστιν", 219a2-3). This seems to show that Aristotle himself realises that the homogeneity criterion with respect to measuring the full quantity of motion cannot be met by time alone, as this criterion demands that a measure, which is homogeneous with the measurand, is (I repeat the quotation from above) "of magnitudes a magnitude, and in particular of length a length, of breadth a breadth, of sounds a sound, of weights a weight", and now of motion – a time.

We see that the measure of motion confronts Aristotle with a dilemma in which he has to choose between two requirements of his measurement account, (a) simplicity and (b) homogeneity: (a) either he endorses the implicit requirement of the *Metaphysics* that the measure has to be simple in the sense of one-dimensional – then the measure of motion is solely time. This measure, however, does not fit the homogeneity requirement if he wants to measure more than the duration of a motion. It makes it impossible to determine how fast a motion is, and will get him into trouble when he attempts to compare two motions of different speed. (b) Or Aristotle employs a complex measure of motion, which takes into account time as well as distance covered. Such a measure is homogenous with the measurand if we want to measure how fast it is. But it will go against the implicit simplicity requirement of the *Metaphysics*. And it is unclear how Aristotle could in fact conceptualise a complex measure – there is no explicit example for a complex measure in the Aristotelian corpus.

Contemporary measurement theory usually opts for (b) homogeneity.[100] As for Aristotle, we saw that he *uses* a complex measure when dealing with Zeno's paradoxes and when giving an account of a thing being faster than another; thus he seems to have a clear understanding of a complex measure. However, in his explicit account of what he calls the measure of motion, he seems to opt for the first horn of this dilemma, for (a), simplicity. As it is clear that he runs into severe problems when giving an account of the measure of motion only in terms of one simple dimension, while he has a way to avoid them, he must have some serious reasons for sticking to time as a simple, one-dimensional measure. And it is indeed certain assumptions from his metaphysics and philosophy of mathematics (some of the latter were briefly mentioned above) that let him go down this route, as I have tried to show elsewhere.[101]

But even if Aristotle ends up in a dilemma, his accounts of measurement in the *Physics* as well as in the *Metaphysics* provide us with the first systematic discussion of measurement relevant for natural philosophy we find in ancient Greek think-

[100] At least for the cases of interest for our purpose, and, in any case, there is no simplicity requirement.
[101] See *Motion*, chapter 9.

ing.[102] For natural philosophy a measure is one important way how the perceptible world can be connected with mathematical structures and how perceptible properties can be, as we would call it, “represented” faithfully by numerical properties. Aristotle’s discussion of measuring motion in the *Physics* may have been part of a larger debate on the conceptualisation of speed at his time, which Aristotle seems to refer to in the first paragraph of the long *Physics* passage quoted above.[103] Mendell, *Proportion Theory*, 21 suggests that it may refer to Eudoxus’ influential but unfortunately lost treatise *peri tachôn* or to Archytas. We may thus also get a glimpse of a discussion of speed at the time which otherwise seems to be lost. And we see the important use Aristotle puts this understanding of speed to – in his fight against Zeno. Finally, even though Aristotle’s explicit account of measurement in *Metaphysics* Iota cannot deal with complex measures, such as speed, we saw that in some way it already employed implicitly the most important features on which also a modern theories of measurement (for all its differences in formulation and perspective) rests.[104]

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[102] As mentioned above, Plato’s discussion of measure in the *Philebus* seems to be more metaphysical in kind.

[103] 232a24ff., when he says “in conformity with the definition sometimes given of the faster”

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